# The McCall Job Search Model

*A partial equilibrium search model*

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# Roadmap

Background

The McCall Model

The Fixed Point Algorithm

# Job Search

- Textbook model of the labor market has no involuntary unemployment; wages adjust to clear the market
- So how to we explain unemployment?
- Answer: labor market frictions
- One such friction: search frictions

# Job Search

- Basic structure, workers search for jobs
- Workers receive job offers with wages draw from some distribution
- Workers can accept offer and work at the posted wage or reject offer and continue to search

# What can the simple model tell us?

- Effects of unemployment insurance on duration of unemployment?
- How does the distribution of wages affect the duration of unemployment?
- What is the optimal unemployment insurance?

# Roadmap

Background

The McCall Model

The Fixed Point Algorithm

# McCall (1970)

- Simplest job search model
- Job offers arrive sequentially, with wages drawn from a known distribution
- Agents decide: which jobs to accept and why
- Partial equilibrium model
- Potential issues: Structure may break down in general equilibrium
  - → The Rothschild Critique - why would firms offer more than reservation wage?
  - → Diamond's Paradox - reservation wage goes to zero and workers accept first offer

# Environment

- Population: workers and firms
- Preferences of workers:
  - → Discount factor: $\beta$
  - → Risk neutral
  - → Utility: $U = \sum_{t=0}^{\infty} \beta^t c_t$
  - → Decide to accept or reject job offers
- Technology of Firms:
  - → Post wage offers
  - → Wages drawn from distribution $F(w)$ with finite support $W$
  - → Make no decisions, just post wage offers

# Environment (cont'd)

- Information technology:
  - $\rightarrow$ Workers know the wage distribution
- Enforcement technology:
  - $\rightarrow$ Once workers accept a job, they keep it and earn wage $w$ forever
- Matching technology:
  - $\rightarrow$ Undirected search
  - $\rightarrow$ No recall – workers cannot return to previous job offers
  - $\rightarrow$ Workers receive job offers sequentially, one per model period
  - $\rightarrow$ Wage offers are iid draws from $F(w)$

# Equilibrium Concept

- A partial equilibrium model
- Workers choose optimal strategy to accept or reject offers given offer process and unemployment benefits

# Worker Decisions

- If worker accepts offer at wage $w \in W$:

$$U = \sum_{t=0}^{\infty} \beta^t w = \frac{w}{1-\beta}$$

- Decision rule is accept ($a_t$=1) or reject ($a_t$=0):

$$a_t : W \rightarrow [0, 1]$$

# The dynamic programming problem

- The value of receiving offer $w$ is:

$$v(w) = \max \left\{ \frac{w}{1-\beta}, b + \beta \int_W v(w') dF(w') \right\}$$

- The first term in the brackets is the payoff from accepting the offer
- The term second term in the brackets is the continuation value of rejecting the offer
- We'll want to solve for $v(w)$ and $a(w)$, the decision rule

# DPP: existence and uniqueness

- Given $\beta < 1$ and a bounded distribution $F(w)$, the value function $v(w)$ is bounded, continuous, and quasi-concave
- Thus, all the properties for a contraction mapping are satisfied and the value function has a unique fixed point
- The value function will have a flat portion (where continue search) and an increasing portion (where accept offer and get payoff $\frac{w}{1-\beta}$)
- This implies that the decision rule $a(w)$ will be a cutoff rule: accept if $w \geq \bar{w}$ and reject if $w < \bar{w}$
- $\bar{w} \equiv R$ is the **reservation wage**

# The Reservation Wage

The reservation wage, $R$ solves:

$$R - b = \frac{\beta}{1-\beta}\left[\int_{W \geq R}(w - R)dF(w)\right]$$

- The left-hand side is the payoff from accepting the offer
- The right-hand side is the expected payoff from rejecting the offer
- At $R$, the worker is indifferent between accepting and rejecting the offer

# Roadmap

# The Contraction Mapping

- Since the value function is a contraction mapping, we can solve for it using the contraction mapping theorem
- Start with some initial guess for $v(w)$ and apply the Bellman operator, $T(v)(w)$ to it:
  → $T(v)(w) = \max\left\{\frac{w}{1-\beta}, b + \beta \int_W v(w')dF(w')\right\}$
- Applying the operator repeatedly will converge to the fixed point

# Another Contraction Mapping

- Let $h$ denote the continuation value:

$$h = b + \beta \int_W v(w) dF(w)$$

- The Bellman equation can be written as:

$$v(w) = \max \left\{ \frac{w}{1-\beta}, h \right\}$$

- Which means we can rewrite $h$ as:

$$h = b + \beta \int_W \max \left\{ \frac{w}{1-\beta}, h \right\} dF(w)$$

# Another Contraction Mapping (cont'd)

- This is a nonlinear equation in $h$
- We can solve for $h$ using the contraction mapping theorem
- Start with some initial guess for $h$ and apply the Bellman operator, $T(h)$ to it:
  - $\rightarrow$ $T(h) = b + \beta \int_W \max\left\{\frac{w}{1-\beta}, h\right\} dF(w)$
- Applying the operator repeatedly will converge to the fixed point
- Note that in the Bellman operator on the value function, we had vector of $v(w)$ (for each possible wage), here we just have a scalar $h$