Introduction to the Large-scale Overlapping Generations Model of India
Day 1

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Tax Policy Research Unit and World Bank

Our goals

By the end of this week, TPRU staff should:

- calibrate some important parts of `OG-India`
- be familiar with theory, code, and solution methods for `OG-India`
- be able to customize and run experiments using `OG-India`

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Medium-term to long-term goals:

- Fully calibrate `OG-India` to Indian economy
- Integrate microsimulation models with `OG-India`
- Have TPRU staff have mastery of software, collaboration, and modeling best practices

Before we begin

- Remove any old (last year) `OG-India` repositories from your GitHub accounts accounts
- We will continue to place supplementary materials in https://github.com/OpenRG/WB-India-2019
- Fork and clone `OG-India` repo https://github.com/TPRU-India/OG-India

Today's outline

1. OG-India brief tour
2. OG-India documentation
3. OG-India theory and computation

Lunch break: 1-2pm

4. Map and tour of files and modules
5. Running OG-India, setting a policy
6. Experiments and results (TCJA)

Why overlapping generations?

	Microsim	OG	Microsim + OG
revenue estimates	X		
marginal tax rates	X		
distributional analysis	X		
macroeconomic effects			
macroeconomic feedback			

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- How does a change in the GST affect wages?

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- How does tax change affect wealthy and old if $w \uparrow$ and $r \downarrow$?

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- How does a change in the GST affect wages?
- How does tax change affect wealthy and old if $w \uparrow$ and $r \downarrow$?
- Are feedback effect bigger in India with significant hand-to-mouth population?

Brief tour

- Give brief tour of `OG-India` repo
  https://github.com/TPRU-India/OG-India
- Jason will give more detailed tour of file structure and modules

OG-India Documentation

- Show the PDF documentation at https://github.com/TPRU-India/OG-India/blob/master/docs/OGINDIAdoc.pdf
- Discuss source LaTeX files and where to put them

OG-India model summary overview

- 100-period-lived agents
- heterogeneous age, lifetime income
- realistic demographics (fertility rates, mortality rates, immigration rates, population growth)
- intentional and unintentional bequests
- representative perfectly competitive firms
- productivity growth
- closed, large open, small open economy specifications

OG-India model summary overview

- Fiscal policy
  - Household ETR, MTR from microsimulation model
  - Corporate taxation (Corp inc tax rate, depreciation expensing rate)
  - Household transfers
  - Non-transfer government spending
  - Non-balanced government budget constraint
  - Exogenous interest rate wedge

Some shining components

- Incorporation of microsimulation tax data
- Realistic demographics
- Calibration of lifetime income process
- Flexible closure rule (technical)
- Flexible openness assumptions

Household demographics

- Fertility rates f_s, mortality rates ρ_s, immigration rates i_s

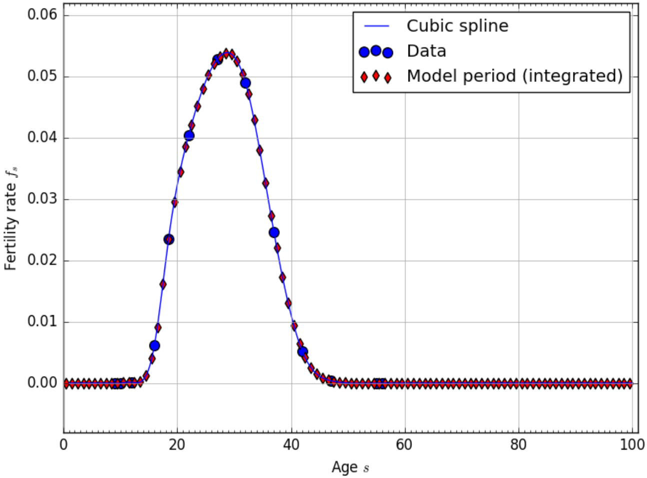
$$\omega_{1,t+1} = (1-\rho_0)\sum_{s=1}^{E+S} f_s\omega_{s,t} + i_1\omega_{1,t} \quad \forall t$$

$$\omega_{s+1,t+1} = (1-\rho_s)\omega_{s,t} + i_{s+1}\omega_{s+1,t} \quad \forall t \quad \text{and} \quad 1 \leq s \leq E+S-1$$

$$N_t \equiv \sum_{s=1}^{E+S}\omega_{s,t} \quad \forall t$$

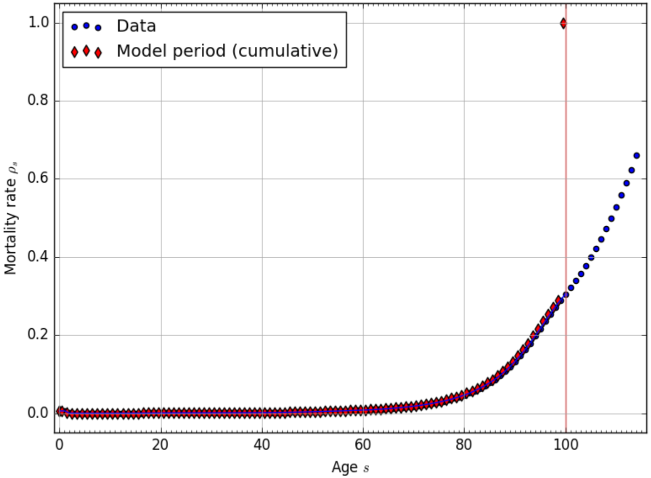
$$g_{n,t+1} \equiv \frac{N_{t+1}}{N_t} - 1 \quad \forall t$$

Demographics: fertility rates



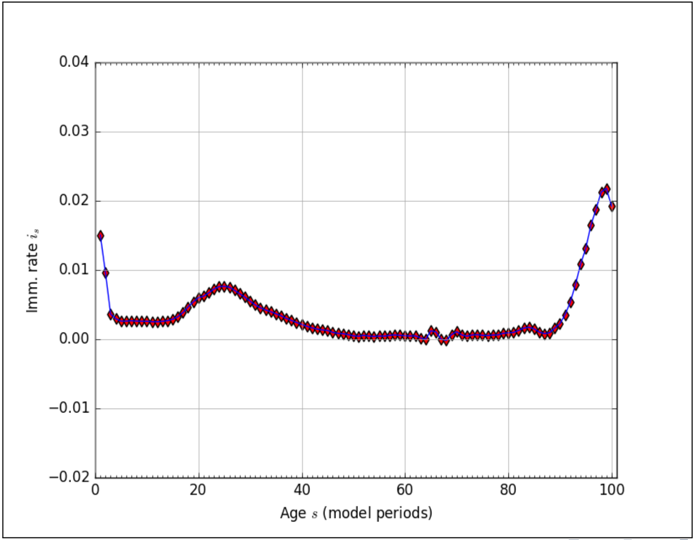
Source: National Vital Statistics Reports, Volume 64, Number 1, January 15, 2015.

Demographics: mortality rates

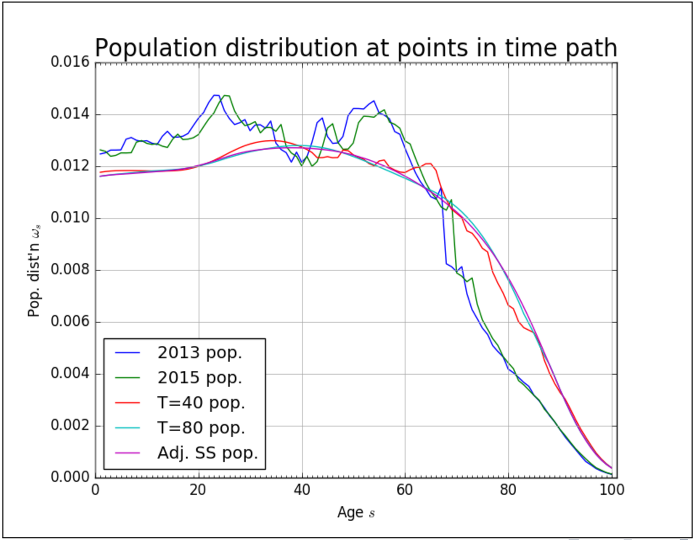


Source: Actuarial Life table, 2011 Social Security Administration.

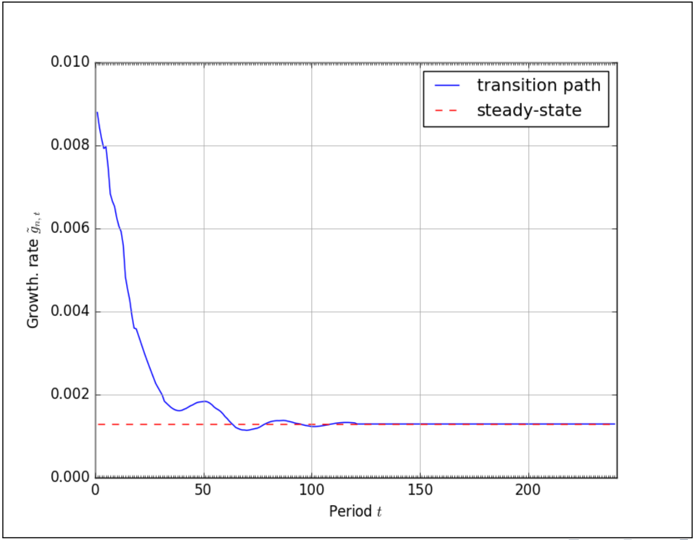
Demographics: immigration rates



Demographics: population distribution



Demographics: population growth



Household maximization problem

- Maximize lifetime utility

$$\max_{\{(c_{j,s,t}),(n_{j,s,t}),(b_{j,s+1,t+1})\}_{s=E+1}^{E+S}} \sum_{s=1}^{S} \beta^{s-1} \left[\Pi_{u=E+1}^{E+s}(1-\rho_u)\right] u(c_{j,s,t+s-1}, n_{j,s,t+s-1}, b_{j,s+1,t+s})$$

- Subject to a budget constraint

$$c_{j,s,t} + b_{j,s+1,t+1} = (1+r_t)b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \zeta_{j,s}\frac{BQ_t}{\lambda_j \omega_{s,t}} + \eta_{j,s,t}\frac{TR_t}{\lambda_j \omega_{s,t}} - T_{s,t}$$

- and occasionally binding constraints

$$c_{j,s,t} \geq 0,\ n_{j,s,t} \in [0,\tilde{l}],\ \text{and } b_{j,1,t} = 0 \quad \forall j,t,\ \text{and } E+1 \leq s \leq E+S$$

Household optimality conditions

- S static labor supply Euler equations

$$w_t e_{j,s}\left(1-\tau^{mtrx}_{s,t}\right)\left(c_{j,s,t}\right)^{-\sigma} = e^{g_y(1-\sigma)}\chi^n_s\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}$$
$$\forall j,t, \quad \text{and} \quad E+1\leq s\leq E+S$$

- $S-1$ dynamic savings Euler equations

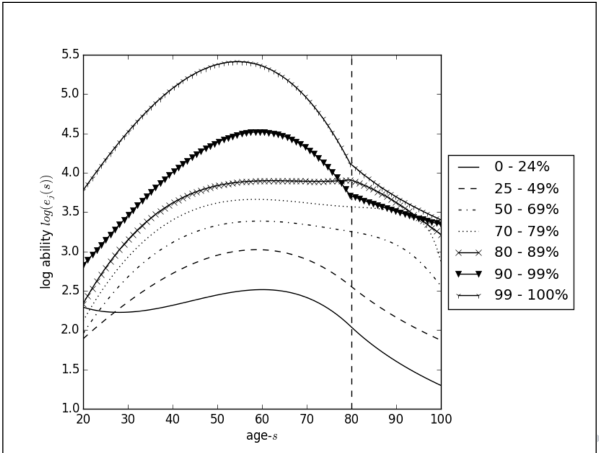
$$(c_{j,s,t})^{-\sigma} = \chi^b_j\rho_s(b_{j,s+1,t+1})^{-\sigma} + \beta(1-\rho_s)\Big(1+r_{t+1}\left[1-\tau^{mtry}_{s+1,t+1}\right]\Big)(c_{j,s+1,t+1})^{-\sigma}$$
$$\forall j,t, \quad \text{and} \quad E+1\leq s\leq E+S-1$$

- One end-of-life savings (bequests) equation

$$(c_{j,E+S,t})^{-\sigma} = \chi^b_j(b_{j,E+S+1,t+1})^{-\sigma} \quad \forall j,t \quad \text{and} \quad s=E+S$$

Lifetime earnings profiles in USA

$$c_{j,s,t} + b_{j,s+1,t+1} = (1 + r_t)b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \zeta_{j,s}\frac{BQ_t}{\lambda_j \omega_{s,t}} + \eta_{j,s,t}\frac{TR_t}{\lambda_j \omega_{s,t}} - T_{s,t}$$



OG-India Tax integration

DeBacker, Jason, Richard W. Evans, and Kerk L. Phillips, “Integrating Microsimulation Models of Tax Policy into a DGE Macroeconomic Model,” *Public Finance Review*, 47:2, pp. 207-275 (Mar. 2019).

$$c_{j,s,t} + b_{j,s+1,t+1} = (1 + r_t)b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \zeta_{j,s}\frac{BQ_t}{\lambda_j \omega_{s,t}} + \eta_{j,s,t}\frac{TR_t}{\lambda_j \omega_{s,t}} - T_{s,t}$$

$$\forall j,t \quad \text{and} \quad s \geq E+1 \quad \text{where} \quad b_{j,E+1,t} = 0 \quad \forall j,t$$

$$w_t e_{j,s}\left(1 - \tau^{mtrx}_{s,t}\right)(c_{j,s,t})^{-\sigma} = e^{g_y(1-\sigma)}\chi^n_s\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1 - \left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}$$

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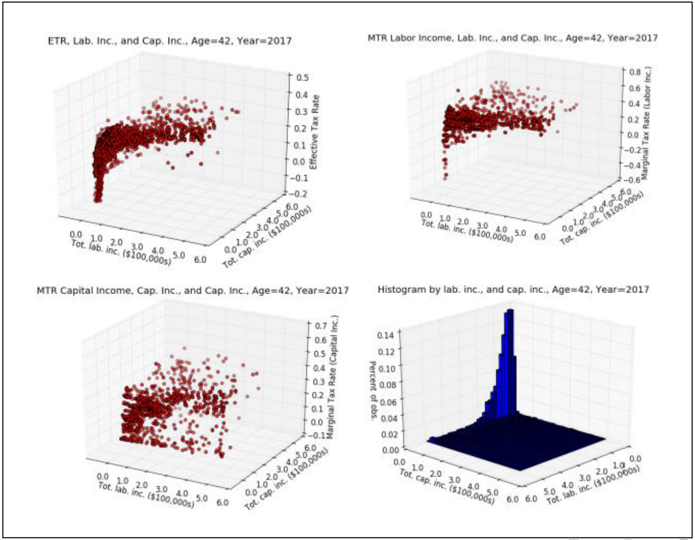
OG-India Tax integration, USA 42-yr-old, 2017



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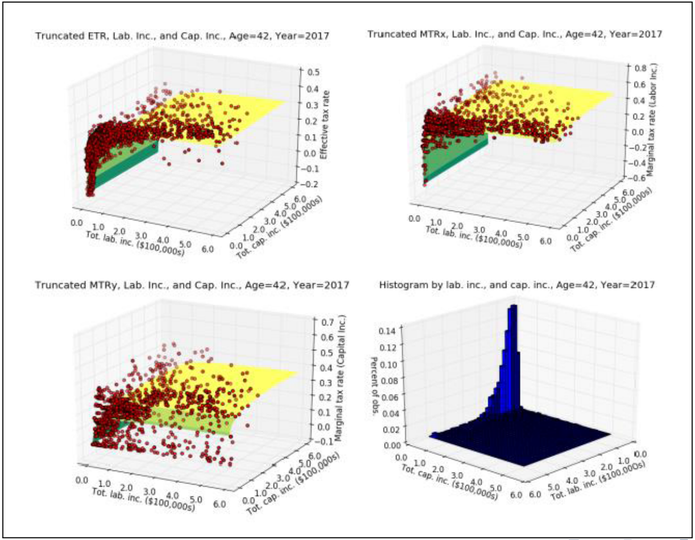
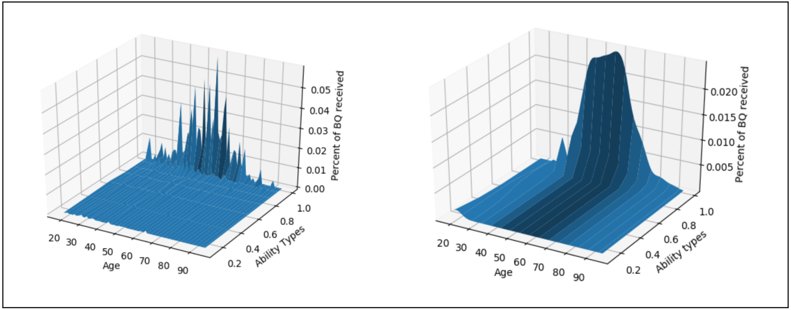


Bequests and Transfers

$$c_{j,s,t} + b_{j,s+1,t+1} = (1 + r_t)b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \zeta_{j,s}\frac{BQ_t}{\lambda_j \omega_{s,t}} + \eta_{j,s,t}\frac{TR_t}{\lambda_j \omega_{s,t}} - T_{s,t}$$

- It matters how bequests are distributed $\zeta_{j,s}$
- We take data and incorporate it into the model



Firms optimality conditions

- Representative perfectly competitive CES production

$$Y_t = F(K_t, L_t) \equiv Z_t \left[(\gamma)^{\frac{1}{\varepsilon}} (K_t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{\frac{1}{\varepsilon}} (e^{g_y t} L_t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t$$

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- Firms choose K_t and L_t to maximize profit

$$PR_t = (1-\tau^{corp})\Big[F(K_t, L_t) - w_t L_t\Big] - \big(r_t + \delta\big)K_t + \tau^{corp}\delta^{\tau}K_t \quad \forall t$$

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- FOC for labor demand L_t and capital demand K_t

$$w_t = e^{g_y t} (Z_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[(1-\gamma) \frac{Y_t}{e^{g_y t} L_t} \right]^{\frac{1}{\varepsilon}} \quad \forall t$$

$$r_t = (1-\tau^{corp})(Z_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\gamma \frac{Y_t}{K_t} \right]^{\frac{1}{\varepsilon}} - \delta + \tau^{corp} \delta^{\tau} \quad \forall t$$

Government budget conditions

- Government revenue

$$Rev_t = \underbrace{\tau^{corp}\left[Y_t - w_t L_t\right] - \tau^{corp}\delta^{\tau} K_t}_{\text{corporate tax revenue}} + \underbrace{\sum_{s=E+1}^{E+S}\sum_{j=1}^{J}\lambda_j \omega_{s,t}\tau^{etr}_{s,t}\left(x_{j,s,t}, y_{j,s,t}\right)\left(x_{j,s,t} + y_{j,s,t}\right)}_{\text{household tax revenue}}$$

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- Government spending

$$Spend_t = \underbrace{G_t}_{\text{public goods}} + \underbrace{Tr_t}_{\text{Transfers to HH}}$$

$$\text{where} \quad G_t = g_{g,t}\alpha_g Y_t \quad \text{and} \quad Tr_t = g_{tr,t}\alpha_{tr} Y_t$$

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- Law of motion for government debt

$$D_{t+1} + Rev_t = (1 + r_t)D_t + G_t + TR_t \quad \forall t$$

Government interest rate wedge

- Government might borrow at different rate than households

$$r_{gov,t} = (1 - \tau_{d,t})r_t - \mu_d$$

Government interest rate wedge

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$$r_{gov,t} = (1 - \tau_{d,t})r_t - \mu_d$$

- Households hold government and firm assets in proportion from data (average interest rate, assume perfect substitutes)

$$r_{hh,t} = \frac{r_{gov,t} D_t + r_t K_t}{D_t + K_t}$$

Government budget and closure

- Model must be stationary and stable (growth and balance)
- (Closure rule): Most models change policy in future to stabilize debt-to-GDP.
- OG-USA allows for multiple stabilization options
  - Gov't discretionary spending
  - Gov't transfers to households
  - Taxes
  - Combinations of the three
- These options can be implemented over time

Market clearing: 3 markets, one LOM

$$L_t = \sum_{s=E+1}^{E+S} \sum_{j=1}^{J} \omega_{s,t} \lambda_j e_{j,s} n_{j,s,t} \quad \forall t$$

$$K_t + D_t = \sum_{s=E+2}^{E+S+1} \sum_{j=1}^{J} \Big(\omega_{s-1,t-1} \lambda_j b_{j,s,t} + i_s \omega_{s,t-1} \lambda_j b_{j,s,t} \Big) \quad \forall t$$

$$Y_t = C_t + K_{t+1} - \left(\sum_{s=E+2}^{E+S+1} \sum_{j=1}^{J} i_s \omega_{s,t} \lambda_j b_{j,s,t+1} \right) - (1-\delta) K_t + G_t \quad \forall t$$

and

$$BQ_t = (1 + r_t) \left(\sum_{s=E+2}^{E+S+1} \sum_{j=1}^{J} \rho_{s-1} \lambda_j \omega_{s-1,t-1} b_{j,s,t} \right) \quad \forall t$$

Open Economy Options: Small and Large

Small Open Economy

- Capital flows freely to keep interest rate at an exogenous world interest rate
- Capital market clearing condition satisfied by foreign capital flows:

$$K_t^f = K_t^{demand} - B_t - D_t \tag{1}$$

Large Open Economy

- Lies between closed and small open economy cases
- *This should be the default option for* `OG-India`
- Foreign investors purchase some fraction of new debt issues:

$$D_{t+1}^f = D_t^f + \zeta_D(D_{t+1} - D_t) \tag{2}$$

- and own some fraction of capital:

$$K_t^f = \zeta_K K_t^{open} \tag{3}$$

Steady-state (long-run) equilibrium

Definition (**Stationary steady-state equilibrium**)

A non-autarkic stationary steady-state equilibrium in the OG-India model is defined as constant allocations of stationary household labor supply $n_{j,s,t} = \bar{n}_{j,s}$ and savings $\hat{b}_{j,s+1,t+1} = \bar{b}_{j,s+1}$ for all j, t, and $E + 1 \leq s \leq E + S$, and constant prices $\hat{w}_t = \bar{w}$ and $r_t = \bar{r}$ for all t such that the following conditions hold:

1. the population has reached its stationary steady-state distribution $\hat{\omega}_{s,t} = \bar{\omega}_s$ for all s and t,
2. households optimize according to *JS* labor supply Euler equations and *JS* savings Euler equations,
3. firms optimize according to capital and labor first order conditions,
4. Government activity behaves according to government budget constraint and closure rule, and
5. markets clear according to labor and capital market clearing (and bequests law of motion.

Non-steady-state (transition path) equilibrium

Definition (**Non-stationary (transition path) equilibrium**)

A non autarkic nonsteady-state functional equilibrium in the OG-India model is defined as stationary allocation functions of the state $\left\{n_{j,s,t} = \phi_s\big(\hat{\boldsymbol{\Gamma}}_t\big)\right\}_{s=E+1}^{E+S}$ and $\left\{\hat{b}_{j,s+1,t+1} = \psi_s\big(\hat{\boldsymbol{\Gamma}}_t\big)\right\}_{s=E+1}^{E+S}$ for all j and t and stationary price functions $\hat{w}(\hat{\boldsymbol{\Gamma}}_t)$ and $r(\hat{\boldsymbol{\Gamma}}_t)$ for all t such that:

1. households have symmetric beliefs $\Omega(\cdot)$ about the evolution of the distribution of savings, and those beliefs about the future distribution of savings equal the realized outcome (rational expectations),
2. households optimize according to S labor supply Euler equations and S savings Euler equations in all periods,
3. firms optimize according to capital and labor first order conditions in all periods,
4. Government activity behaves according to government budget constraint and closure rule in all periods, and
5. markets clear according to labor and capital market clearing (and bequests law of motion in all periods.

Solution method

- See documentation

1. Guess outer loop aggregates
   - Choose the aggregates needed to solve the household problem
   - r_t, w_t, BQ_t, TR_t

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Solution method

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1. Guess outer loop aggregates
    - Choose the aggregates needed to solve the household problem
    - r_t, w_t, BQ_t, TR_t
2. Given outer loop aggregates, solve household decisions
3. Update outer loop aggregates based on household decisions

Things to add

1. Link with TPRU household microsim
2. Dynamic firms (own capital, adj. costs) WIP PR
3. Multiple industries, multiple goods
4. Link with TPRU corporate microsim
5. Calibration, revenue estimation
6. Stochastic income