



Small-angle approximation

$$\sin \theta \cong \theta$$

$$\cos \theta \cong 1 - \frac{\theta^2}{2} \cong 1 - 0 \cong 1$$

$$\tan \theta \cong \theta$$

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \text{ 繞X軸旋轉, 即Roll}$$

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \text{ 繞Y軸旋轉, 即Pitch}$$

$$R_z(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ 繞Z軸旋轉, 即YAW}$$

Apply Small-angle approximation



$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\phi \\ 0 & \phi & 1 \end{bmatrix} \text{ 繞X軸旋轉, 即Roll}$$

$$R_y(\theta) = \begin{bmatrix} 1 & 0 & \theta \\ 0 & 1 & 0 \\ -\theta & 0 & 1 \end{bmatrix} \text{ 繞Y軸旋轉, 即Pitch}$$

$$R_z(\psi) = \begin{bmatrix} 1 & -\psi & 0 \\ \psi & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ 繞Z軸旋轉, 即YAW}$$

($\psi = \text{YAW}$, $\phi = \text{ROLL}$, $\theta = \text{PITCH}$)

$$V_G = \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix}$$

Small-angle approximation

$$\sin \theta \cong \theta$$

$$\cos \theta \cong 1 - \frac{\theta^2}{2} \cong 1 - 0 \cong 1$$

$$\tan \theta \cong \theta$$

$$\phi\theta\psi \cong 0,$$

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$$\theta\psi \cong 0$$

$$V_{new_G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \cdot \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix}$$

$$V_{new_G} \cong \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\phi \\ 0 & \phi & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & \theta \\ 0 & 1 & 0 \\ -\theta & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -\psi & 0 \\ \psi & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix}$$

$$V_{new_G} \cong \begin{bmatrix} 1 & 0 & \theta \\ \phi\theta & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -\psi & 0 \\ \psi & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix}$$

$$V_{new_G} \cong \begin{bmatrix} 1 & -\psi & \theta \\ \phi\theta + \psi & -\phi\theta\psi + 1 & -\phi \\ -\theta + \phi\psi & \theta\psi + \phi & 1 \end{bmatrix} \cdot \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix}$$

$$V_{new_G} \cong \begin{bmatrix} 1 & -\psi & \theta \\ \cancel{\phi\theta} + \psi & \cancel{-\phi\theta\psi} + 1 & -\phi \\ -\theta + \cancel{\phi\psi} & \cancel{\theta\psi} + \phi & 1 \end{bmatrix} \cdot \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix}$$

$$V_{new_G} \cong \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \cdot \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} \cong \begin{bmatrix} X_G - \psi Y_G + \theta Z_G \\ Y_G + \psi X_G - \phi Z_G \\ Z_G - \theta X_G + \phi Y_G \end{bmatrix}$$

```
int vector[3] = {0, 0, 1};
int x_g = vector[0];
int y_g = vector[1];
int z_g = vector[2];
int yaw = 1, pitch = 1, roll = 1;
```

```
/* Small-angle approximation matrix rotation */
vector[0] = vector[0] - yaw * y_g + pitch * z_g; /* x_g_new */
vector[1] = vector[1] + yaw * x_g - roll * z_g; /* y_g_new */
vector[2] = vector[2] - pitch * x_g + roll * y_g; /* z_g_new */
```