Table 1 Conditions of numerical experiments

1) Orbit	near synchronous orbit
2) Perturbation forces considered	gravitational force of the Moon gravitational force of the Sun perturbation forces due to the Earth's nonsphericity up to orders 15 solar radiation pressure
3) Epoch of used orbital elements	77.01.12.
4) Latitude of the satellite	60°E
5) Calculation of Eq. (7)	double precision
6) Calculation of Eq. (8)	single precision except for the calculation of $F_0(X_0+X_1)-F_0(X_0)$

Table 2 Deviations of satellite's position from DODS' values after one day integration from epoch

Δt	
2 h	about 20 m
1 h	about 3 m
30 min	less than 1 m
10 min	less than 1 m

from

$$V_{1,i+1}=V_{1,i}+\frac{\Delta t}{6}\{(f_{1,i}+f_{2,i})+4(f_{1,i+\frac{1}{2}}+f_{2,i+\frac{1}{2}})+(f_{1,i+1}+f_{2,i+1})\} \tag{13}$$

Thus, X_1 and V_1 at the next time step are obtained.

From the foregoing equations we notice that the predictor-corrector formulas chosen are very simple, requiring no starter at the beginning of integration. These facts enable the orbit generation programs to be compact. Next we introduce some approximations. As the values of $X^c_{1,i+\frac{1}{2}}-X^p_{1,i+\frac{1}{2}}$ and $X^c_{1,i+1}-X^p_{1,i+1}$ are small, we approximate $f^c_{2,i+\frac{1}{2}}$ by $f^p_{2,i+\frac{1}{2}}$ and $f^c_{2,i+1}$ by $f^p_{2,i+1}$. By introducing these approximations, we can reduce the frequency of calculating the perturbation forces to as little as twice per step, which contributes to the reduction of computation time. Successive corrections of the solution are continued by the correctors, Eqs. (11) and (12) applied only to the f_1 part. The error in the solution occurring from this approximation proves to be small as seen from the numerical experiment in the next section.

III. Results and Conclusions

According to the method described in the previous section, the results of numerical experiments are presented for ephemeris generation of satellites. Conditions for these experiments are shown in Table 1. The results are compared with those generated by the Adams-Cowell method of orders 12 using NASA's DODS (Table 2). In Table 2, deviations of the satellite's position from DODS's values are shown after one day of integration from the epoch. From these results, we can see that the choice of $\Delta t = 1$ h can be sufficient for the ephemeris generation, and the errors become negligibly small even if approximations noted in the last section are used. In our method, these approximations are very effective to reduce computational time. The fact that numerical integrations are all performed by a single type of algorithm in the computer is also effective in reducing computation time.

When a long-range ephemeris is generated, some additional calculations are required. At the time when X_1 and V_1 become large, we calculate X and V from Eqs. (3) and convert them to osculating orbital elements. Calculations are continued using these new elements setting X_1 and V_1 to zero.

When we apply the integration method described in this Note to orbit determination, Eqs. (8) can also be applicable to the calculation of ephemeris partials. In this case, Eqs. (8) are integrated with appropriate initial conditions. Thus, partials of finite-difference formulas are calculated easily.

The integration method described in this Note has proved to be useful. Using this method, orbit determination programs can be very compact and high-speed computations can be realized without reduction of accuracy or neglecting perturbation forces. According to our experience, it is possible to realize accurate orbit determination using minicomputers.

References

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Quaternion from Rotation Matrix

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Introduction

FOR the purposes of this Note, a quaternion will be thought of as a four-parameter representation of a coordinate transformation matrix. The quaternion (q) may be written in many different forms, three of which are

$$q \equiv \begin{bmatrix} s \\ v \end{bmatrix} \equiv \begin{bmatrix} \cos\dfrac{\alpha}{2} \\ -u\sin\dfrac{\alpha}{2} \end{bmatrix} \equiv \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{1}$$

The first representation, with a scalar (s) component and a vector (v) component, is perhaps the most traditional. The second representation is in the same format, but is expressed in terms of the parameters α and u which lend themselves to a geometrical interpretation of the quaternion. The unit vector u is the eigenvector of the rotation matrix, and α is the angle about u that one coordinate frame is rotated with respect to the other. This interpretation is included only to emphasize the relevance of the quaternion extraction algorithm to the important problem of finding the eigenvector of a rotation matrix.

The third representation of the quaternion shown in Eq. (1) will be the one used throughout the remainder of this Note. This choice best reflects the unifying philosophy of the new approach, which is that each of the components should be treated the same way. Other authors perhaps have been misled by the special nature of the scalar component. Grubin[1] presents a very simple algorithm; however, it degrades for large rotation angles and, indeed, has a zero-over-zero singularity at exactly 180 deg. Klumpp,[2] on the other hand,

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