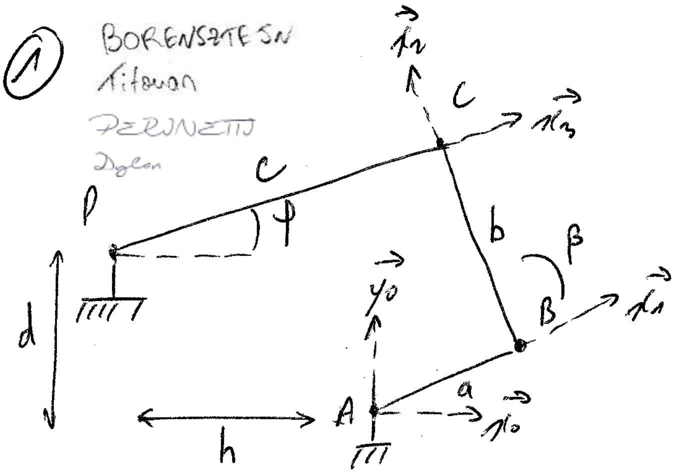
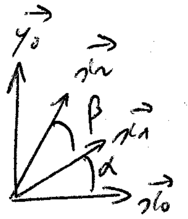
(1)

BORENSZTEJN
Tifouan
PERINETTI
Dylan




$$\vec{AB} = a\vec{x_1} = a\left(\cos\alpha\,\vec{x_0} + \sin\alpha\,\vec{y_0}\right)$$
$$\vec{BC} = b\vec{x_2} = b\left(\cos(\alpha+\beta)\,\vec{x_0} + \sin(\alpha+\beta)\,\vec{y_0}\right)$$
$$\vec{CD} = -c\vec{x_3} = -c\left(\cos\varphi\,\vec{x_0} + \sin\varphi\,\vec{y_0}\right)$$
$$\vec{DA} = h\vec{x_0} - d\vec{y_0}$$




Fermeture géométrique ABCD:

$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA} = 0$$

$$\begin{cases} \vec{x_0}: & a\cos\alpha + b\cos(\alpha+\beta) - c\cos\varphi + h = 0 \\ \vec{y_0}: & a\sin\alpha + b\sin(\alpha+\beta) - c\sin\varphi - d = 0 \end{cases}$$

$$\begin{cases} \vec{x_0}: & \left(b\cos(\alpha+\beta)\right)^2 = \left(c\cos\varphi + h - a\cos\alpha\right)^2 \quad (1) \\ \vec{y_0}: & \left(b\sin(\alpha+\beta)\right)^2 = \left(c\sin\varphi + d - a\sin\alpha\right)^2 \quad (2) \end{cases}$$

$$(1)+(2):\ b^2 = c^2 + a^2 + h^2 + d^2 - 2ac\cos(\alpha+\varphi) + 2c\left(d\sin\varphi - h\cos\varphi\right) - 2a\left(d\sin\alpha - h\cos\alpha\right)$$

"$\varphi$ petit"
$\cos\varphi \simeq 1$
$\sin\varphi \simeq \varphi$

$$b^2 = c^2 + a^2 + h^2 + d^2 + 2a\left[\cos\alpha(h-c) - d\sin\alpha\right] + 2c\left(d\varphi - h\right)$$

$$\underbrace{\cos\alpha(h-c)}_{L} - \underbrace{d}_{K}\sin\alpha = \underbrace{\frac{2c(d\varphi - h) + c^2 + a^2 + h^2 + d^2 - b^2}{2a}}_{M} \qquad K = d \text{ et } L = (h-c)$$

$$L\cos\alpha - K\sin\alpha = M$$

$$\frac{L}{\sqrt{K^2+L^2}}\cos\alpha - \frac{K}{\sqrt{K^2+L^2}}\sin\alpha = \frac{M}{\sqrt{K^2+L^2}}$$

$$\left(\frac{K}{\sqrt{K^2+L^2}}\right)^2 + \left(\frac{L}{\sqrt{K^2+L^2}}\right)^2 = \cos^2 x + \sin^2 x = 1$$

$$\sin x\cos\alpha - \cos x\sin\alpha = \frac{M}{\sqrt{K^2+L^2}}$$

$$\cos x = \frac{K}{\sqrt{K^2+L^2}} = \frac{(h-c)}{\sqrt{(h-c)^2+d^2}}$$

$$\sin(x+y) = \sin x\cos y + \sin y\cos x$$
$$\sin(x+y) = \sin x\cos y - \sin y\cos x$$

$$\sin x = \frac{L}{\sqrt{K^2+L^2}} = \frac{d}{\sqrt{(h-c)^2+d^2}}$$

(2) $\sin x \cos\alpha - \cos x \sin\alpha = \sin(x-\alpha)$

$$\sin(x-\alpha) = \frac{M}{\sqrt{K^2+L^2}} \qquad x-\alpha = \arcsin\left(\frac{M}{\sqrt{K^2+L^2}}\right)$$

$$\alpha = x - \arcsin\left(\frac{M}{\sqrt{K^2+L^2}}\right)$$

$$\alpha = \arcsin\left(\frac{d}{\sqrt{(h-c)^2+d^2}}\right) - \arcsin\left(\frac{\frac{2c(df-h)+c^2+a^2+h^2+d^2-b^2}{2a}}{\sqrt{(h-c)^2+d^2}}\right)$$

$$\boxed{\alpha = \arcsin\left(\frac{d}{\sqrt{(h-c)^2+d^2}}\right) - \arcsin\left(\frac{\sqrt{(h-c)^2+d^2}\,(2c(df-h)+c^2+a^2+h^2+d^2-b^2)}{2a}\right)}$$