BORENSZTEJN
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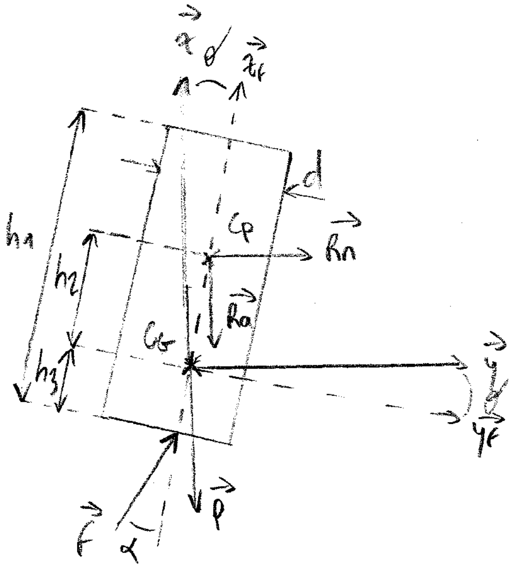
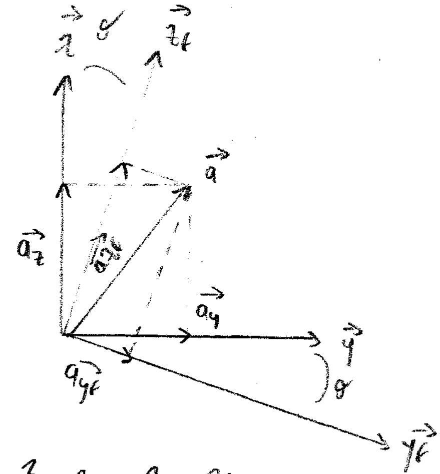
Modélisation du système de correction d'altitude d'une fusée expérimentale par poussée vectorielle





$$a_y^2 + a_z^2 = a_{yf}^2 + a_{zf}^2$$

$$\begin{pmatrix} a_y \\ a_z \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_{yf} \\ a_{zf} \end{pmatrix}$$

$$\begin{cases} a_y = a_{yf}\cos\theta - a_{zf}\sin\theta \\ a_z = a_{yf}\sin\theta + a_{zf}\cos\theta \end{cases}$$

$$\vec{R_n} = Q.S_{ref}.C_n.\vec{e_y}$$
$$= (P_t - P_s)\frac{\pi d^2}{4}C_n.\vec{e_y}$$

$$\vec{R_a} = -Q.S_{ref}.C_a.\vec{e_z}$$
$$= -(P_t - P_s)\frac{\pi d^2}{4}C_a.\vec{e_z}$$
$$= (P_s - P_t)\frac{\pi d^2}{4}C_a.\vec{e_z}$$

$$\vec{P} = -mg.\vec{e_z}$$
$$\vec{F} = F(\sin\alpha.\vec{e_{yf}} + \cos\alpha.\vec{e_{zf}})$$
$$= F(\sin(\alpha+\theta).\vec{e_y} + \cos(\alpha+\theta).\vec{e_z})$$

Principe fondamental de la Dynamique (PFD):

$$m\vec{a} = \sum \vec{F_{ext}}$$

$$m\vec{a} = \vec{R_n} + \vec{R_a} + \vec{P} + \vec{F}$$

$$m\begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix} = Q.S_{ref}.C_n\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + Q.S_{ref}.C_a\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + mg\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + F\begin{pmatrix} 0 \\ \sin(\alpha+\theta) \\ \cos(\alpha+\theta) \end{pmatrix}$$

$$F\begin{pmatrix} 0 \\ \sin(\alpha+\theta) \\ \cos(\alpha+\theta) \end{pmatrix} = m\begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix} - mg\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} - Q.S_{ref}\left[C_n\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + C_a\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\right]$$

$$F\sqrt{\sin^2(\alpha+\theta) + \cos^2(\alpha+\theta)} = \sqrt{(ma_{y_f} - Q.S_{ref}.C_n)^2 + (ma_{z_f} + mg + QS_{ref}C_a)^2}$$

$$F = \sqrt{(ma_{y_f} - Q.S_{ref}C_n)^2 + [m(a_{z_f}+g) + QS_{ref}C_a]^2}$$

$$F = \sqrt{\left(ma_{yf} - (P_t - P_s)\frac{\pi d^2}{4}C_n\right)^2 + \left[m(a_{zf}+g) + (P_t - P_s)\frac{\pi d^2}{4}C_a\right]^2}$$

$$F = \sqrt{\left(ma_{yf} + (P_s - P_t)\frac{\pi d^2}{4}C_n\right)^2 + \left[m(a_{zf}+g) + (P_t - P_s)\frac{\pi d^2}{4}C_a\right]^2}$$

Théorème du moment cinétique (TMC):

$$I\frac{d^2\theta}{dt^2} = \sum M_{CG}$$

$$I\frac{d^2\theta}{dt} = F\cos\left(\frac{\pi}{2} - \alpha\right)h_3 - (R_n\cos\theta - R_a\sin\theta)h_2$$

On considère l'angle θ "très petit"

$\Rightarrow \theta \simeq 0$ donc $\cos\theta \simeq 1$ et $\sin\theta \simeq 0$

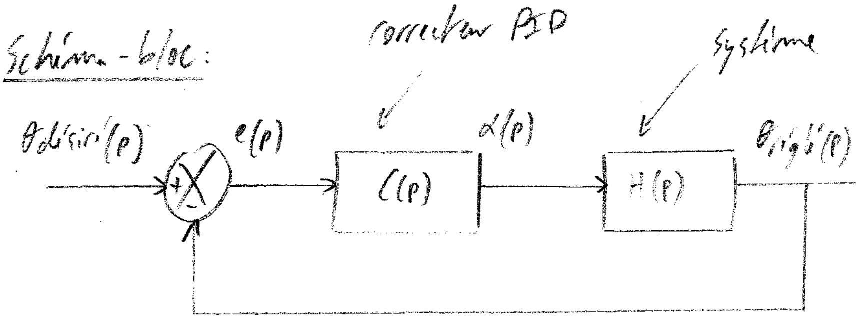
d'où $I\frac{d^2\theta}{dt^2} = F\sin\alpha\, h_3 - R_n h_2$

On obtient donc $\alpha = \arcsin\left(\frac{I\frac{d^2\theta}{dt^2} + R_n h_2}{F h_3}\right)$

$$\alpha = \arcsin\left(\frac{I\frac{d^2\theta}{dt^2} + R_n h_2}{h_3\left[\left(ma_{yf} + (P_s - P_t)\frac{\pi d^2}{4}C_n\right)^2 + \left[m(a_{zf}+g) + (P_t - P_s)\frac{\pi d^2}{4}C_a\right]^2\right]^{1/2}}\right)$$

Recherche de la fonction de transfert du modèle 2D :

Schéma-bloc :



$$H(p) = \frac{\theta(p)}{\alpha(p)}$$

$$J\frac{d^2\theta}{dt^2} = F\sin\alpha\, h_3 - F_n h_2$$

transformée de Laplace

$$J\theta p^2 = F\sin\alpha\, h_3 - F_n h_2$$

α "petit"

$$J\theta p^2 = F\alpha h_3 - F_n h_2$$

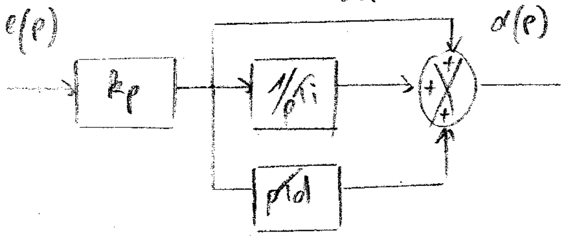
$$\theta(p) = \frac{F\alpha h_3}{Jp^2} - \frac{F_n h_2}{Jp^2}$$

$$\frac{\theta(p)}{\alpha(p)} = \frac{F h_3}{Jp^2} - \frac{F_n h_2}{Jp^2\alpha(p)}$$

$F_n \ll F$

$$\frac{\theta(p)}{\alpha(p)} = \frac{F h_3}{Jp^2} \longrightarrow \boxed{H(p) = \frac{F h_3}{Jp^2}}$$

$$K_p = K\left(1 + \frac{\tau_d}{\tau_i}\right)$$
$$T_i = \tau_i + \tau_d$$
$$T_d = \frac{\tau_d \tau_i}{\tau_d + \tau_i}$$



$$\alpha(p) = \varepsilon(p)\cdot K_p\left(1 + \frac{1}{pT_i} + pT_d\right)$$

$$\left[\frac{\theta(p)}{\alpha(p)}\right] = rad = \varnothing$$

$$\left[\frac{F h_3}{Jp^2}\right] = \frac{kg\cdot m^2\cdot s^{-2}}{kg\cdot m^2\cdot s^{-2}} = \varnothing$$

$$[p] = s^{-1} \qquad [\tau p] = \left[\frac{p}{\omega}\right] = \varnothing$$

$$s\cdot s^{-1} = \frac{s^{-1}}{s^{-1}} = \varnothing$$

Homogénéité :

$$F = \sqrt{\left(m a_{yf} + (\rho_s - \rho_f)\frac{\pi d^2}{4}(a)\right)^2 + \left[m(a_{zf} + g) + (\rho_f - \rho_s)\frac{\pi d^2}{4}(a)\right]^2}$$

$$[\rho] = \left[\frac{F}{S}\right] = \frac{kg\cdot m\cdot s^{-2}}{m^2} = kg\cdot m^{-1}\cdot s^{-2}$$

$$[F] = N = kg\cdot m\cdot s^{-2}$$

$$\left((kg\cdot m\cdot s^{-2} + kg\cdot m^{-1}\cdot s^{-2}\cdot m^2)^2 + (kg\cdot m\cdot s^{-2} + kg\cdot m^{-1}\cdot s^{-2}\cdot m^2)^2\right)^{1/2} = (kg^2\cdot m^2\cdot s^{-4})^{1/2} = kg\cdot m\cdot s^{-2} = [F]$$