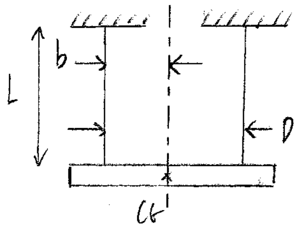
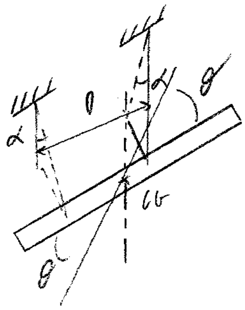
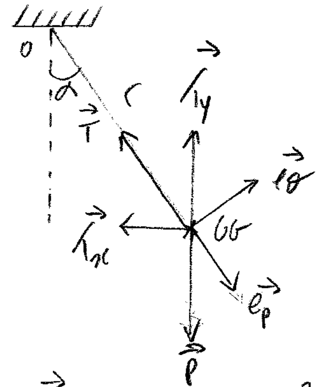
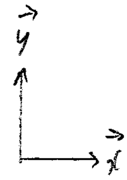
BORENSZTEJN
Titouan

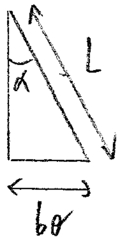
Mesure du moment d'inertie d'une fusée expérimentale





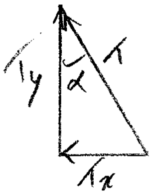






$\frac{b\theta}{L} = \sin\alpha$

$\frac{b\theta}{L} \simeq \alpha$



$T_x = T\sin\alpha$

$= \frac{mg}{2}\sin\alpha$ (2 fils)

$\boxed{T_x \simeq \frac{mgb\theta}{2L}}$

avec $\sin\alpha \simeq \alpha$
et $\cos\alpha \simeq 1$

$\vec a = -r\frac{d\alpha}{dt}^2 \vec e_p + r\frac{d^2\alpha}{dt^2}\vec e_\alpha$

PFD: $m\vec a = \sum \vec F_{ext}$

$m\vec a = \vec P + \vec T$

Sur $\vec e_p$: $-T + mg\cos\alpha = -mr\frac{d\alpha}{dt}^2$

$T = mr\frac{d\alpha}{dt}^2 + mg\cos\alpha$ $\alpha \simeq 0$

$\boxed{T = mg}$

$M_{CG} = -bT_x$

$\boxed{M_{CG} = -\frac{mgb^2}{2L}\theta}$

$\sum M_{CG} = I\ddot\theta$

$-\frac{mgb^2}{L}\theta = I\ddot\theta$

Le système peut être assimiller à un "oscillateur" oscillant à sa "fréquence propre" (sans excitateur)

Méthode des complexes:

$\theta = \theta_m \cos(\omega t)$

$\underline{\theta} = \theta_m e^{i(\omega t)}$

$-\frac{mgb^2}{L}\theta = I\ddot\theta$

$I\ddot\theta + \frac{mgb^2}{L}\theta = 0$

$I\ddot{\underline{\theta}} + \frac{mgb^2}{L}\underline{\theta} = 0$ avec $\ddot{\underline{\theta}} = (\theta_m e^{i(\omega t)})''$
$= -\omega^2\theta_m e^{i(\omega t)}$

$-I\omega^2\underline{\theta} + \frac{mgb^2}{L}\underline{\theta} = 0$

$\frac{mgb^2}{L}\theta_m e^{i(\omega t)} - I\omega^2\theta_m e^{i(\omega t)} = 0$

$I = \frac{mgb^2}{L\omega^2}$

$\omega = 2\pi f = \frac{2\pi}{T}$ avec $f = \frac{1}{T}$

$\omega^2 = 4\pi^2 f^2 = \frac{4\pi^2}{T^2}$

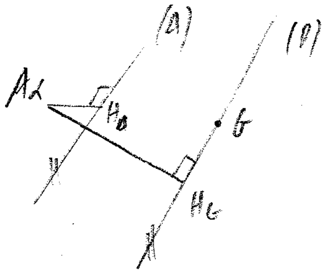
$\frac{mgb^2}{L} = I\left(\frac{4\pi^2}{T^2}\right)$

$\frac{mgb^2T^2}{4\pi^2 L} = I$ avec $D = 2b \Rightarrow b = \frac{D}{2}$

$\boxed{I = \frac{mgD^2T^2}{16\pi^2 L}}$

Mesurer le moment d'inertie

Théorème des axes parallèles (Huygens):



Solide de masse m

$(\Delta) \parallel (D)$

$J_{/\Delta} = J_{/D} + md^2$
$= J_G + md^2$

$J_{/A} = J_{/B} + md^2$
$= J_{/B} + m(x_{BC}^2 + y_{BC}^2)$

$$J_{/A} = \frac{mgD^2\tau^2}{16\pi^2 L}$$

$$J_{/G} = J_{/A} - md^2 = \frac{mgD^2\tau^2}{16\pi^2 L} - md^2$$

$$\boxed{J_{/G} = m\left(\frac{gD^2\tau^2}{16\pi^2 L} - d^2\right)}$$

Résolution de l'éq harmonique:

$J\ddot{\theta} + \frac{mgb^2}{L}\theta = 0$

$\frac{d^2\theta}{dt^2} + \frac{mgb^2}{LJ}\theta = 0$

$\omega_0^2 = \frac{mgb^2}{LJ}$

$\omega_0 = \sqrt{\frac{mgb^2}{LJ}}$

$\theta(t) = \theta_m \cos(\omega_0 t + \cancel{\varphi_0})$ $=0$

Forme canonique:

$\frac{d^2f}{dt^2} + \omega_0^2 f = 0$

$f(t) = f_m \cos(\omega_0 t + \varphi_0)$ (Oscillateur harmonique)

$f(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$

$J = \frac{mgb^2}{L\omega_0^2} \dashrightarrow \boxed{J = \frac{mgD^2\tau^2}{16\pi^2 L}}$