MIP Formulation for Human-Centered Dispatch

Sets:

- R : Set of resources (indexed by r)
- S : Set of shifts (indexed by s)
- B : Set of demand types/roles (indexed by b)
- P : Set of construction sites (baustellen, indexed by p)
- T : Set of timeslots (indexed by t)

Parameters:

- $d_{s,b}$: Number of resources required for demand b in shift s
- z_s : Timeslot of shift s
- p_s : Baustelle of shift s
- c_r : Cost of assigning resource r (higher for externals)
- $a_{r,b}$: 1 if resource r can cover demand b, 0 otherwise

Decision Variables:

$$x_{r,s,b} = \begin{cases} 1 & \text{if resource } r \text{ is assigned to shift } s \text{ for demand } b \\ 0 & \text{otherwise} \end{cases}$$

$$y_p \in Z_{\geq 0} \quad \text{(penalty for instability at baustelle } p\text{)}$$

$$\text{change}_{p,s_1,s_2,r,b} \in \{0,1\} \quad \text{(penalty if assignment changes between consecutive shifts } s_1, s_2 \text{ at } p\text{)}$$

$$\text{total\_shifts}_r \in Z_{\geq 0} \quad \text{(total shifts assigned to } r\text{)}$$

$$\text{min\_shifts}, \text{max\_shifts} \in Z_{\geq 0}$$

Objective:

$$\min \quad \sum_{r\in R}\sum_{s\in S}\sum_{b\in B} c_r x_{r,s,b} + \sum_{p\in P} y_p + \sum_{p\in P}\sum_{(s_1,s_2)\in \text{consec}(p)}\sum_{r\in R}\sum_{b\in B} 10\cdot \text{change}_{p,s_1,s_2,r,b}$$
$$+ 5\cdot(\text{max\_shifts} - \text{min\_shifts})$$

Constraints:

1. **Demand coverage:**

$$\sum_{r\in R} x_{r,s,b}\cdot a_{r,b} \geq d_{s,b} \qquad \forall s\in S,\, b\in B$$

2. **Resource assignment per timeslot:**

$$\sum_{s\in S: z_s = t}\sum_{b\in B} x_{r,s,b} \leq 1 \qquad \forall r\in R,\, t\in T$$

3. **Max 7 shifts in any 9-day window:**

$$\sum_{s \in S : z_s \in [t, t+17]} \sum_{b \in B} x_{r,s,b} \leq 7 \qquad \forall r \in R,\, t \in T,\, t \text{ odd}$$

4. **Max 14 night shifts in any 30-day window:**

$$\sum_{s \in S : z_s \in [t, t+59],\, z_s \text{ even}} \sum_{b \in B} x_{r,s,b} \leq 14 \qquad \forall r \in R,\, t \in T,\, t \text{ odd}$$

5. **No consecutive timeslot assignments:**

$$x_{r,s_1,b_1} + x_{r,s_2,b_2} \leq 1$$

for all $r \in R$, $t \in T$, $s_1 \in S : z_{s_1} = t$, $s_2 \in S : z_{s_2} = t+1$, $b_1, b_2 \in B$

6. **Penalty for assignment changes (stability):**

$$|x_{r,s_1,b} - x_{r,s_2,b}| \leq \text{change}_{p,s_1,s_2,r,b}$$

for all $p \in P$, consecutive shifts s_1, s_2 at p, $r \in R$, $b \in B$

7. **Fairness in shift allocation:**

$$\text{total\_shifts}_r = \sum_{s \in S} \sum_{b \in B} x_{r,s,b} \qquad \forall r \in R$$

$$\text{min\_shifts} \leq \text{total\_shifts}_r \leq \text{max\_shifts} \qquad \forall r \in R$$