Derivation FOC angle

What follows is a derivation of the FOC angle δ based on a AC analysis on the single phase equivalent circuit of a DC motor.

$\underline{u}_q$	inverter voltage
$\underline{u}_s$	phase voltage
$\underline{u}_e$	back emf voltage
$\underline{i}_s$	phase currents
δ	FOC angle
s	duty cycle
L	motor phase inductance
R	motor phase resistance
I_B	battery current
U_B	battery voltage

Table 1: nomenclature

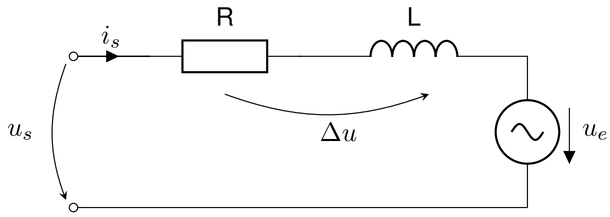


Figure 1: single phase equivalent circuit

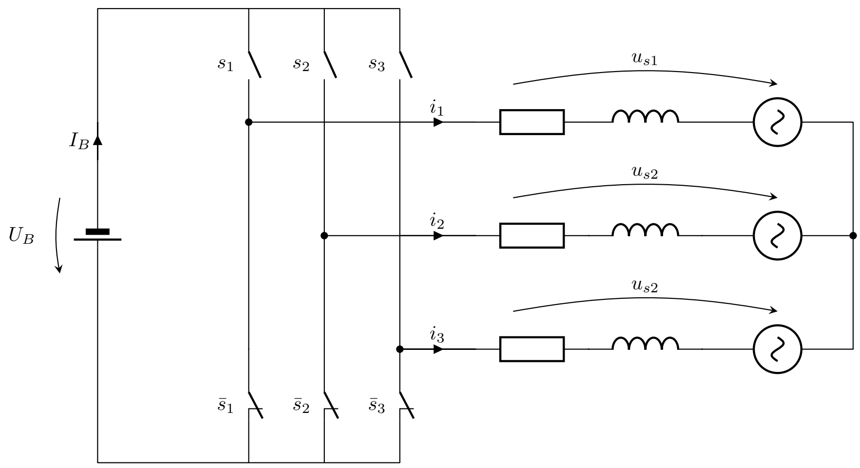


Figure 2: 3-phase equivalent circuit with inverter

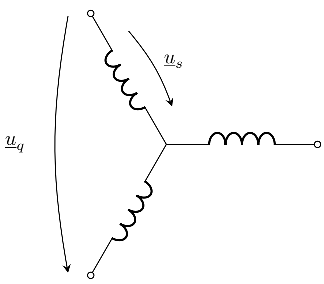
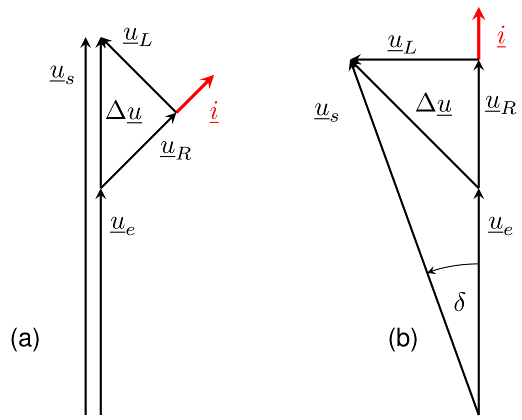


Figure 3: star connection

star connection relation: $\underline{u}_s = \frac{\underline{u}_q}{\sqrt{3}}$



FOC angle δ:

$$\delta = \arcsin\left(\frac{\hat{u}_L}{\hat{u}_s}\right) = \arcsin\left(\frac{\hat{\imath} \cdot \omega_{el} L}{\hat{u}_s}\right) \tag{1}$$

The task is to express $\hat{\imath}$ and $\hat{u}_s$ in terms of the measured battery current I_B and voltage U_B.
General relation for three phase current where m is the modulation index of the space vector pwm.

$$\hat{u}_q = s \cdot m \cdot U_B \qquad m = \frac{\sqrt{3}}{2} \tag{2}$$

$$\hat{u}_s = \frac{\hat{u}_q}{\sqrt{3}} = \frac{U_B \cdot s}{2} \tag{3}$$

The power delivered by the battery has to be equal to the power converted in the three phases. I use $\cos(\delta) \approx 1$ for small δ

$$\begin{aligned} U_B \cdot I_B &= \frac{3}{2}\hat{u}_s \cdot \hat{\imath} \cdot \cos(\delta) \approx \frac{3}{2}\hat{u}_s \cdot \hat{\imath} \\ \hat{\imath} &= \frac{4}{3}\frac{I_B}{s} \end{aligned} \tag{4}$$

Combine 2 and 4 to compute the FOC angle δ

$$\delta = \arcsin\left(\frac{\hat{\imath} \cdot \omega_{el} L}{\hat{u}_s}\right) = \frac{8}{3}\arcsin\left(\frac{\frac{I_B}{s} \cdot \omega_{el} L}{U_B \cdot s}\right) \approx \frac{8}{3}\frac{\frac{I_B}{s} \cdot \omega_{el} L}{U_B \cdot s} \tag{5}$$

Note: $\arcsin(x) \approx x$ for small x.