A Network-Cognizant Bid-Based Transactive Energy System Design

I. Appendix A: Proof of Proposition 1

[Proof of Proposition 1]: The market is cleared when the aggregate network-cognizant bid function $\sum_{\psi\in\Psi} p_{\psi}^{n*}(\lambda_H)$ is equal to the offer function $H^*(\lambda_H)$. It can be observed (17) that $H^*(\lambda_H) - \sum_{\psi\in\Psi} p_{\psi}^{n*}(\lambda_H)$ is the subgradient of the dual problem (17). Therefore, this clearing signal λ_H^*, satisfying $H^*(\lambda_H^*) - \sum_{\psi\in\Psi} p_{\psi}^{n*}(\lambda_H^*) = 0$, is the optimal dual variable for the dual problem (17). As the strong duality holds, then the optimal dual solution d^* for (16) is equal to the optimal primal solution p^* for (14). Thus, we have:

$$\begin{aligned}
p^* = d^* &= \min_{\lambda_H} \max_{\boldsymbol{p}_\Psi\in\mathcal{P}_\Psi, \underline{H}\le H\le\overline{H}} L(\boldsymbol{p}_\Psi, H, \lambda_H) \\
&= \max_{\boldsymbol{p}_\Psi\in\mathcal{P}_\Psi, \underline{H}\le H\le\overline{H}} L(\boldsymbol{p}_\Psi, H, \lambda_H^*) \\
&= \max_{\boldsymbol{p}_\Psi\in\mathcal{P}_\Psi} \sum_{\psi\in\Psi} [U_\psi(p_\psi) - \lambda_H^* p_\psi S_{base}\Delta t] \\
&+ \max_{\underline{H}\le H\le\overline{H}} \lambda_H^* H S_{base}\Delta t - \mu_g C(H) \\
&= \sum_{\psi\in\Psi} [U(p_\psi^{n*}(\lambda_H^*)) - \lambda_H^* p_\psi^{n*}(\lambda_H^*) S_{base}\Delta t] \\
&+ \lambda_H^* H(\lambda_H^*) S_{base}\Delta t - \mu_g C(H(\lambda_H^*))
\end{aligned} \tag{1}$$

Plus $H^*(\lambda_H^*) - \sum_{\psi\in\Psi} p_{\psi}^{n*}(\lambda_H^*) = 0$, it follows that:

$$p^* = \sum_{\psi\in\Psi} U(p_\psi^{n*}(\lambda_H^*)) - \mu_g C(H(\lambda_H^*)) \tag{2}$$

Thus, $\boldsymbol{p}_\Psi^{n*}(\lambda_H^*) = \{p_\psi^{n*}(\lambda_H^*)|\psi\in\Psi\}, H(\lambda_H^*)$ are the optimal primal variables for the problem (14) to achieve the social welfare maximization. Q.E.D.

II. Appendix B: Proof of Propositions 2 and 3

[Proof of Proposition 2]: From $\underline{\lambda}_H = \max\{\lambda_{v_{min}}, \lambda_{\overline{p}}\}$, it follows $\underline{\lambda}_H$ is the minimum discretized λ_H, ensuring there are no violations of both the minimum voltage limits and peak demand limit due to $\boldsymbol{p}_\Psi^{d*}(\lambda_H)$. We also know that $\overline{\lambda}_H$ is the maximum discretized λ_H, ensuring there are no violations of maximum voltage limits due to $\boldsymbol{p}_\Psi^{d*}(\lambda_H)$.

As $\lambda_H \in [\underline{\lambda}_H, \overline{\lambda}_H]$, there are no violations of the minimum and maximum voltage limits, and peak demand limit due to $\boldsymbol{p}_\Psi^{d*}(\lambda_H)$. Thus, we have $p_\psi^{n*}(\lambda_H) = p_\psi^{d*}(\lambda_H)$, for $\lambda_H \in [\underline{\lambda}_H, \overline{\lambda}_H]$. In addition, (19b) indicates $p_\psi^{d*}(\lambda_H)$ is non-increasing as λ_H increases. followed by $p_\psi^{d*}(\overline{\lambda}_H) \le p_\psi^{d*}(\underline{\lambda}_H)$. We can say the feasible region for p_ψ, satisfying the network constraints, is $[p_\psi^{d*}(\overline{\lambda}_H), p_\psi^{d*}(\underline{\lambda}_H)]$.

The value of objective function $U_\psi(p_\psi) - \lambda_H p_\psi$ is non-decreasing as $p_\psi \in (-\infty, p_\psi^{d*}(\lambda_H)]$, and it is non-increasing as $p_\psi \in [p_\psi^{d*}(\lambda_H), +\infty)$. As $\lambda_H \in [\lambda_0, \underline{\lambda}_H)$, there must be either violations of the minimum voltage limits or peak demand limit due to $\boldsymbol{p}_\Psi^{d*}(\lambda_H)$. When $\lambda_H < \underline{\lambda}_H$, it leads to $\boldsymbol{p}_\Psi^{d*}(\lambda_H) \ge \boldsymbol{p}_\Psi^{d*}(\underline{\lambda}_H)$, indicating this feasible region $[p_\psi^{d*}(\overline{\lambda}_H), p_\psi^{d*}(\underline{\lambda}_H)] \subseteq (-\infty, p_\psi^{d*}(\lambda_H)]$. Thus, we can know that the value of objective function $U_\psi(p_\psi) - \lambda_H p_\psi$ is non-decreasing as $p_\psi \in [p_\psi^{d*}(\overline{\lambda}_H), p_\psi^{d*}(\underline{\lambda}_H)]$. In this case, we can conclude that $\boldsymbol{p}_\Psi = \{p_\psi = p_\psi^{d*}(\underline{\lambda}_H)|\forall\psi\in\Psi\}$ maximizes the objective function in (18b) while satisfying the network constraints as $\lambda_H \in [\lambda_0, \underline{\lambda}_H)$. That is, $\boldsymbol{p}_\Psi^{n*}(\lambda_H) = \boldsymbol{p}_\Psi^{d*}(\underline{\lambda}_H)$ as $\lambda_H \in [\lambda_0, \underline{\lambda}_H)$. As $\lambda_H \in (\overline{\lambda}_H, \lambda_K]$, there must be violations of the maximum voltage limits due to $\boldsymbol{p}_\Psi^{d*}(\lambda_H)$. When $\overline{\lambda}_H < \lambda_H$, it leads to $\boldsymbol{p}_\Psi^{d*}(\lambda_H) \le \boldsymbol{p}_\Psi^{d*}(\overline{\lambda}_H)$, indicating this feasible region $[p_\psi^{d*}(\overline{\lambda}_H), p_\psi^{d*}(\underline{\lambda}_H)] \subseteq [p_\psi^{d*}(\lambda_H), +\infty)$. Thus, we can know that the value of objective function $U_\psi(p_\psi) - \lambda_H p_\psi$ is non-increasing as $p_\psi \in [p_\psi^{d*}(\overline{\lambda}_H), p_\psi^{d*}(\underline{\lambda}_H)]$. In this case, we can conclude that $\boldsymbol{p}_\Psi = \{p_\psi = p_\psi^{d*}(\overline{\lambda}_H)|\forall\psi\in\Psi\}$ maximizes the objective function in (18b) while satisfying the network constraints as $\lambda_H \in (\overline{\lambda}_H, \lambda_K]$. That is, $\boldsymbol{p}_\Psi^{n*}(\lambda_H) = \boldsymbol{p}_\Psi^{d*}(\overline{\lambda}_H)$ as $\lambda_H \in (\overline{\lambda}_H, \lambda_K]$. Q.E.D.

[Proof of Proposition 3]: Proposition 3 can be easily proved by replacing $\underline{\lambda}_H, \overline{\lambda}_H$, $p_\psi^{n*}(\lambda_H)$, $p_\psi^{d*}(\lambda_H)$, $p_\psi^{d*}(\underline{\lambda}_H)$, $p_\psi^{d*}(\overline{\lambda}_H)$, $U_\psi(p_\psi) - \lambda_H p_\psi S_{base}\Delta t$, λ_0, λ_K in the proof of Proposition 2 with $\underline{\pi}, \overline{\pi}$, $p_\psi^{n*}(\pi)$, $p_\psi^{d*}(\pi)$, $p_\psi^{d*}(\underline{\pi})$, $p_\psi^{d*}(\overline{\pi})$, $U_\psi(p_\psi) - \mu_\psi \pi p_\psi S_{base}\Delta t$, π_0, π_K. Q.E.D.