Lecture 3: Role of Financial Heterogeneity in Monetary Transmission and (if time) Details of Winberry (2018) Method

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Financial Heterogeneity and the Investment Channel of Monetary Policy (paper with Pablo Ottonello)

Motivation

- Want to understand the role of financial frictions in shaping the investment channel of monetary policy

- Which firms respond the most to monetary policy?

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- Want to understand the role of financial frictions in shaping the investment channel of monetary policy
- Which firms respond the most to monetary policy?
- Firms more affected by financial frictions:
  - Have steeper marginal cost of investment $\implies$ dampen
  - More sensitive to cash flows + collateral values $\implies$ amplify (financial accelerator across firms)
- We revisit this question with
  1. New **cross-sectional evidence**
  2. **Heterogeneous firm New Keynesian** model

Our Contributions

Descriptive evidence on heterogeneous responses
using high-frequency shocks and quarterly Compustat

1. Firms with low leverage, good ratings, and large "distance to default" are more responsive

$\Longrightarrow$ Heterogeneity in default risk is key driver of micro response

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   $\Longrightarrow$ Heterogeneity in default risk is key driver of micro response

Heterogeneous firm New Keynesian model
with financial frictions arising from default risk

1. Model consistent with heterogeneous responses
   - Firms with low risk have flatter marginal cost curve
2. Aggregate response depends on distribution of default risk
   - Driven by low-risk firms, which is time-varying

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$\implies$ **Default risk dampens response to monetary policy**

Related Literature

1. **Household Heterogeneity and Monetary Policy**
   Doepke and Schneider (2006); Auclert (2015); Werning (2015); Wong (2016); Gornermann, Kuester, Nakajima (2016); Kaplan, Moll, and Violante (2018)

2. **Financial Heterogeneity and Investment**
   Khan and Thomas (2013); Gilchrist, Sim and Zakrajsek (2014); Khan, Senga and Thomas (2016)

3. **Financial Frictions and Monetary Transmission**
   - Gertler, and Gilchrist (1994); Kashyap, Lamont, and Stein (1994); Kashyap and Stein (1995); Jeenas (2018); Cloyne et al. (2018)
   - Bernanke, Gertler, and Gilchrist (1999)

Descriptive Empirical Evidence

Data Sources

1. **Monetary policy shocks** $\varepsilon_t^{\mathrm{m}}$: high-frequency identification
   - Compare FFR future before vs. after FOMC announcement
     - Assume nothing else affects FFR in window
   - Time aggregate to quarterly frequency ▸ Summary Statistics

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2. **Firm-level outcomes**: quarterly Compustat
   - Investment $\Delta \log k_{it+1}$: capital stock from net investment
   - Leverage ℓ_{it}: debt divided by total assets
   - Credit rating cr_{jt}: S&P rating of firm's long-term debt
   - Distance to default dd_{jt}: constructed following Gilchrist and Zakrasjek (2012) ▸ Sample Construction ▸ Compustat vs. NIPA ▸ DD details

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Merge 1990q1 - 2007q2

Summary Statistics of Firm-Level Variables

(a) Marginal Distributions

Statistic	$\Delta \log k_{jt+1}$	ℓ_{jt}	$\mathbb{1}\{\mathrm{cr}_{jt} \geq A\}$	dd_{jt}
Mean	0.005	0.267	0.024	5.744
Median	-0.004	0.204	0.000	4.704
S.D.	0.093	0.361	0.154	5.032
95th Percentile	0.132	0.725	0.000	14.952

(b) Correlation Matrix (raw variables)

	ℓ_{jt}	$\mathbb{1}\{\mathrm{cr}_{jt} \geq A\}$	dd_{jt}
ℓ_{jt}	1.00		
(p-value)			
$\mathbb{1}\{\mathrm{cr}_{jt} \geq A\}$	-0.02	1.00	
	(0.00)		
dd_{jt}	-0.46	0.21	1.00
	(0.00)	(0.00)	

(c) Correlation matrix (residualized)

	ℓ_{jt}	$\mathbb{1}\{\mathrm{cr}_{jt} \geq A\}$	dd_{jt}
ℓ_{jt}	1.00		
(p-value)			
$\mathbb{1}\{\mathrm{cr}_{jt} \geq A\}$	-0.02	1.00	
	(0.00)		
dd_{jt}	-0.38	0.05	1.00
	(0.00)	(0.00)	

Baseline Empirical Specification

Firm fixed effect (always include)

$$\Delta \log k_{it+1} = \beta y_{it-1}\varepsilon_t^{\mathrm{m}} + \alpha_i + \alpha_{st} + \Gamma' Z_{it-1} + \varepsilon_{it}$$

- Coefficient of interest β: how semi-elasticity of investment w.r.t. monetary policy depends on financial position y_{it-1}
- Want to isolate differences due to financial position
  - α_{st}: compare within a sector-quarter
  - Z_{it-1}: conditional on financial position y_{it-1}, sales growth, log total assets, current assets share, fiscal quarter dummy
- Standard errors clustered two-way by firm and quarter

Low-Risk Firms More Responsive

	(1)	(2)	(3)	(4)	(5)
leverage $\times$ shock	-0.66**	-0.52**			
	(0.27)	(0.25)			
$\mathbb{1}\{\text{cr}_{jt} \geq A\}$			2.69**		
			(1.16)		
dd $\times$ shock				1.06**	
				(0.45)	
ffr shock					
Observations	239259	239259	239259	151433	
R^2	0.108	0.119	0.116	0.137	
Firm controls	no	yes	yes	yes	
Time sector FE	yes	yes	yes	yes	
Time clustering	yes	yes	yes	yes	

$$\Delta \log k_{it+1} = \beta y_{it-1}\varepsilon_t^{\mathrm{m}} + \alpha_i + \alpha_{st} + \Gamma' Z_{it-1} + \varepsilon_{it}$$

- Monetary expansion has positive sign ($-\varepsilon_t^{\mathrm{m}}$)
- Standardize leverage and distance to default over all firms and quarters

Low-Risk Firms More Responsive

	(1)	(2)	(3)	(4)	(5)
leverage × shock	-0.66**	-0.52**			-0.24
	(0.27)	(0.25)			(0.38)
$\mathbb{1}\{\text{cr}_{jt} \geq A\}$			2.69**		
			(1.16)		
dd × shock				1.06**	1.07**
				(0.45)	(0.52)
ffr shock					1.63**
					(0.72)
Observations	239259	239259	239259	151433	151433
R^2	0.108	0.119	0.116	0.137	0.126
Firm controls	no	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes	no
Time clustering	yes	yes	yes	yes	yes

$$\Delta \log k_{it+1} = \gamma \varepsilon_t^m + \beta y_{it-1} \varepsilon_t^m + \alpha_i + \Gamma_1' Z_{it-1} + \Gamma_2' Y_{t-1} + \varepsilon_{it}$$

- Monetary expansion has positive sign ($-\varepsilon_t^m$)
- Standardize leverage and distance to default over all firms and quarters

Results Hold Using Only Within-Firm Variation

	(1)	(2)	(3)	(4)	(5)
lev_wins_dem_std_wide	-0.80**	-0.67**		-0.33	-0.21
	(0.31)	(0.28)		(0.37)	(0.38)
d2d_wins_dem_std_wide			1.08***	0.87**	1.11**
			(0.39)	(0.38)	(0.47)
ffr shock					1.64**
					(0.77)
Observations	219674	219674	151422	151422	151422
R^2	0.113	0.124	0.137	0.139	0.126
Firm controls	no	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes	no
Time clustering	yes	yes	yes	yes	yes

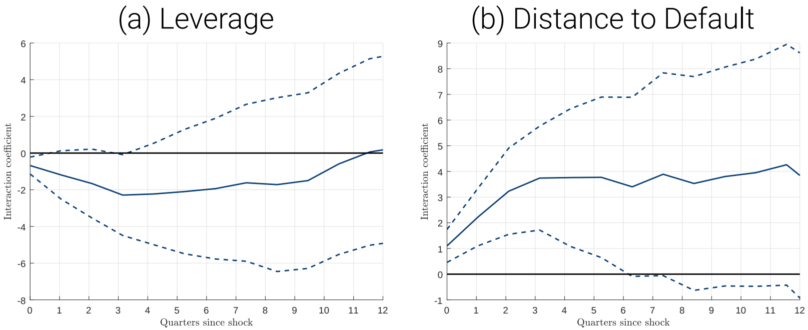
$$\Delta \log k_{it+1} = \beta(y_{it-1} - \mathbb{E}_i[y_{it}])\varepsilon_t^{\mathrm{m}} + \alpha_i + \alpha_{st} + \Gamma_1' Z_{it-1} + \Gamma_2(y_{it-1} - \mathbb{E}_i[y_{it}])Y_{t-1} + \varepsilon_{it}$$

▸ Positive vs. Negative ▸ Information channel ▸ Relation to Gertler-Gilchrist ▸ Relation to Cloyne et al.

- Monetary expansion has positive sign ($-\varepsilon_t^{\mathrm{m}}$)
- Standardize demeaned leverage and distance to default over all firms and quarters

Dynamics of Differences Across Firms

Comparison to Jeenas (2018)



$$\log k_{it+h+1} - \log k_{it} = \beta_h (y_{it-1} - \mathbb{E}_i[y_{it}])\varepsilon_t^m + \alpha_{ih} + \alpha_{sth} + \\ + \Gamma'_{1h} Z_{it-1} + \Gamma_{2h}(y_{it-1} - \mathbb{E}_i[y_{it}]) Y_{t-1} + \varepsilon_{ith}$$

Robustness of Empirical Results

1. **Sorting variables**
    - Control for interaction w/ other covariates ▸ Details
    - Control for lagged investment ▸ Details
    - Decomposition of leverage ▸ Details
    - Instrument w/ lagged financial position ▸ Details
2. **Monetary policy variable**
    - Interaction with other cyclical variables ▸ Details
    - Use raw changes in FFR ▸ Details
    - Results post 1994 ▸ Details
3. **Outcome variable**
    - Financing flows and interest rates ▸ Details

Heterogeneous Firm New Keynesian Model

Model Overview

"There are lots of moving blocks"

1. **Investment block**
   - Heterogeneous firms invest s.t. default risk
   - Intermediary lends resources from household to firms

2. **New Keynesian block** From Gali book, 3 equations
   - Retailers differentiate output s.t. sticky prices
   - Final good producer combines goods into final output
   - Monetary authority follows Taylor rule (monetary shock)
   - Capital good producer with adjustment costs

3. **Representative household** GE effects
   - Owns firms + labor-leisure choice

Heterogeneous Firms

Enter period with state variables z_{jt}, ω_{jt}, k_{jt}, and b_{jt}

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2. **Default decision**
   - If default, value = 0
   - If continue, repay debt b_{jt} and pay operating cost ξ

Heterogeneous Firms

Enter period with state variables z_{jt}, ω_{jt}, k_{jt}, and b_{jt}

1. **Exogenous exit**: w/ i.i.d. prob π_d, forced to exit at end of period
2. **Default decision**
   - If default, value = 0
   - If continue, repay debt b_{jt} and pay operating cost ξ
3. **Production**: $y_{jt} = z_{jt}(\omega_{jt}k_{jt})^{\theta}n_{jt}^{\nu}$, $\theta + \nu < 1$ at price p_t
   - $\log z_{jt+1} = \rho \log z_{jt} + \varepsilon^{z}_{jt+1}$, $\varepsilon^{z}_{jt+1} \sim N(0, \sigma^2)$
   - $\log \omega_{jt} \sim N(-\sigma^2_{\omega}/2, \sigma^2_{\omega})$ i.i.d. truncated above at 0
     - Undepreciated captial $(1-\delta)\omega_{jt}k_{jt}$

Heterogeneous Firms

Enter period with state variables z_{jt}, ω_{jt}, k_{jt}, and b_{jt}

1. **Exogenous exit**: w/ i.i.d. prob π_d, forced to exit at end of period
2. **Default decision** market value
   - If default, value = 0
   - If continue, repay debt b_{jt} and pay operating cost ξ
3. **Production**: $y_{jt} = z_{jt}(\omega_{jt}k_{jt})^{\theta}n_{jt}^{\nu}$, $\theta + \nu < 1$ at price p_t
   - $\log z_{jt+1} = \rho \log z_{jt} + \varepsilon^z_{jt+1}$, $\varepsilon^z_{jt+1} \sim N(0, \sigma^2)$
   - $\log \omega_{jt} \sim N(-\sigma^2_\omega/2, \sigma^2_\omega)$ i.i.d. truncated above at 0
     - Undepreciated captial $(1-\delta)\omega_{jt}k_{jt}$
4. **Investment**: choose $q_t k_{jt+1}$ and financing b_{jt+1}, d_{jt}
   - External finance b_{jt+1} at price $\mathcal{Q}_t(z_{jt}, k_{jt+1}, b_{jt+1})$
   - Internal finance subject to $d_{jt} \geq 0$

Heterogeneous Firms' Bellman Equation

- Default if and only if no feasible choice s.t. $d \geq 0$

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- If receive exit shock ($\zeta = 1$):

$$v_t^{\text{exit}}(z, \omega, k, b) = \max_n p_t z(\omega k)^\theta n^\nu - w_t n - b - \xi + q_t(1-\delta)\omega k$$

Heterogeneous Firms' Bellman Equation

- Default if and only if no feasible choice s.t. $d \geq 0$
- If receive exit shock ($\zeta = 1$):

$$v_t^{\text{exit}}(z, \omega, k, b) = \max_n p_t z(\omega k)^\theta n^\nu - w_t n - b - \xi + q_t(1-\delta)\omega k$$

- If do not receive exit shock ($\zeta = 0$):

$$\begin{aligned} v_t^{\text{cont}}(z, \omega, k, b) = & \max_{n,k',b'} p_t z(\omega k)^\theta n^\nu - w_t n - b - \xi + q_t(1-\delta)\omega k \\ & - q_t k' + \mathcal{Q}_t(z, k', b')b' \\ & + \mathbb{E}_t\left[\Lambda_{t+1} v_{t+1}^0(z', \omega', \zeta', k', b'/\Pi_{t+1})\right] \\ & \text{such that } d \geq 0, \text{ where} \end{aligned}$$

where $v_t^0(z, \omega, \zeta, k, b) = \mathbb{1}\{\zeta = 1\}\chi_t^1(z, \omega, k, b) v_t^{\text{exit}}(z, \omega, k, b)$
$+\mathbb{1}\{\zeta = 0\}\chi_t^2(z, \omega, k, b) v_t^{\text{cont}}(z, \omega, k, b)$

Financial Intermediary

- **Financial intermediary** lends from households to firms
  - No default: get $1/\Pi_{t+1}$ (nominal debt)
  - Default: get up to $\alpha q_{t+1}\omega_{jt+1}k_{jt+1}$ per unit of debt

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calibrate to match data recovery rate

$$\mathcal{Q}_t(z,k',b') = \mathbb{E}_t[\Lambda_{t+1}((1-\mathbb{1}\{\text{default}_{t+1}(z',\omega',\zeta',k',b')\}) \times \frac{1}{\Pi_{t+1}})$$

$$+\mathbb{1}\{\text{default}_{t+1}(z',\omega',\zeta',k',b')\} \times \min\{1,\alpha\frac{q_{t+1}\omega'k'}{b'/\Pi_{t+1}}\})]$$

Firm Entry

- Firms exit due to exit shocks and default

- One **new entrant** for each exiting firm
  1. Draw productivity z_{jt} from shifted distribution

interpretate as revenue shock

calibrate to match firm lifecycle

$$\log z_{jt} \sim N\left(-m\frac{\sigma}{\sqrt{1-\rho^2}}, \frac{\sigma^2}{1-\rho^2}\right)$$

  2. Draw capital quality ω_{jt}
  3. Endowed with k_0 units of capital and $b_0 = 0$ units of debt

$\Longrightarrow$ incumbent w/ initial state $(z_{jt}, \omega_{jt}, k_0, 0)$

Retailers and Final Good Producer

- Monopolistically competitive **retailers**
  - Technology: $\tilde{y}_{it} = y_{it} \implies$ real marginal cost $= p_t$
  - Set price $\tilde{p}_{it}$ s.t. quadratic cost $-\frac{\varphi}{2}\left(\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1\right)^2 Y_t$

- Perfectly competitive **final good producer**
  - Technology: $Y_t = \left(\int \tilde{y}_{it}^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} \implies P_t = \left(\int \tilde{p}_{it}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$

how monetary policy affects real economy

- Implies New Keynesian Phillips Curve linking inflation π_t to marginal cost p_t

The Rest of the Model

- **Monetary authority** follows Taylor rule

$$\log R_t^{\text{nom}} = \log \frac{1}{\beta} + \varphi_\pi \Pi_t + \varepsilon_t^{\text{m}}$$

- **Capital good producer** with technology

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t \implies q_t = 1/\Phi'\left(\frac{I_t}{K_t}\right) = \left(\frac{I_t/K_t}{\delta}\right)^{\frac{1}{\phi}}$$

- **Representative household** with preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \Psi N_t\right)$$

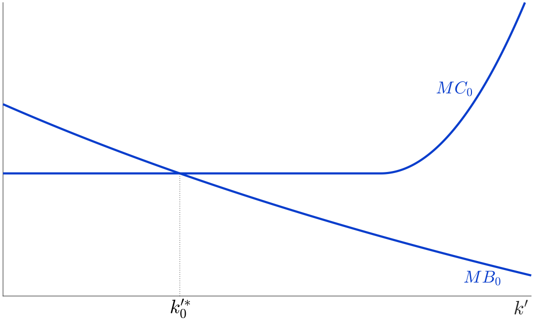
  - Owns firms $\implies \Lambda_{t+1} = \beta \frac{C_t}{C_{t+1}}$
  - Labor-leisure choice $\implies w_t C_t^{-1} = \Psi$
  - Euler equation for bonds $\implies 1 = \beta R_t^{\text{nom}} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Pi_{t+1}}\right]$

An Equilibrium of this Model Satisfies

1. **Heterogeneous firms** choose investment $k'_t(z, \omega, k, b)$, financing $b'_t(z, \omega, k, b)$, and default decision
2. **Financial intermediaries** price default risk $\mathcal{Q}_t(z, k', b')$
3. **Firm entry** with shifted initial distribution
4. **Retailers and final good producer** generate Phillips Curve
5. **Monetary authority** follows Taylor rule
6. **Capital good producer** generates capital price q_t
7. **Household** supplies labor N_t and generates SDF w/ Λ_{t+1}

Channels of Investment Response to Monetary Policy

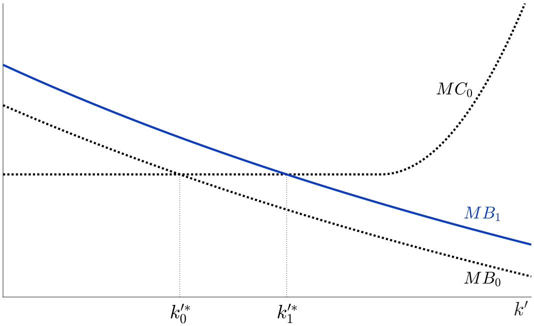
Risk-Free Firms' Response



$$q_t = \frac{1}{R_t}\left(\mathbb{E}_t\left[\text{MRPK}_{t+1}(z',k')\right] + \frac{\mathbb{C}ov_t(\text{MRPK}_{t+1}(z',k'), 1+\lambda_{t+1}(z',k',b'))}{\mathbb{E}_t[1+\lambda_{t+1}(z',k',b')]}\right)$$

$$\text{MRPK}_{t+1}(z',k') = \frac{\partial}{\partial k'}\left(\max_{n'} p_{t+1} z'(\omega' k')^{\theta}(n')^{\nu} - w_{t+1}n' + q_{t+1}(1-\delta)\omega' k'\right)$$

Risk-Free Firms' Response: Discount Rate Falls



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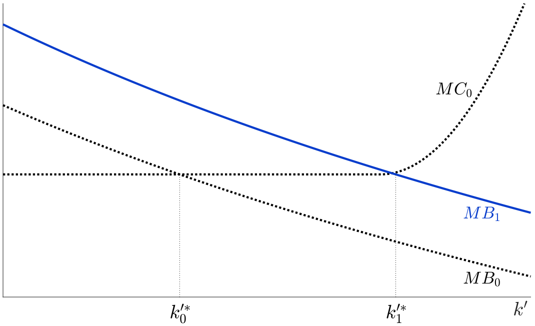
Risk-Free Firms' Response: Future Revenue Rises



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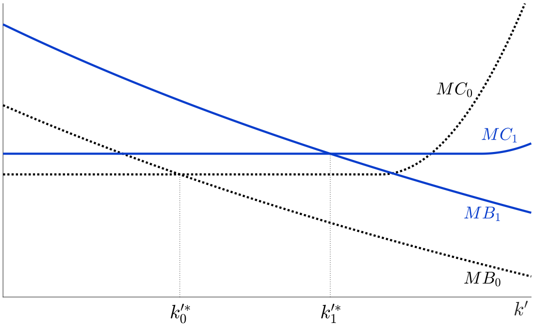
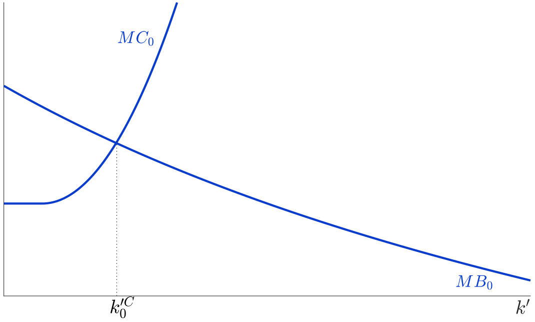
Risk-Free Firms' Response: Price of Capital Rises



$$q_t = \frac{1}{R_t}\left(\mathbb{E}_t\left[\mathrm{MRPK}_{t+1}(z', k')\right] + \frac{\mathbb{C}ov_t(\mathrm{MRPK}_{t+1}(z', k'), 1 + \lambda_{t+1}(z', k', b'))}{\mathbb{E}_t[1 + \lambda_{t+1}(z', k', b')]}\right)$$

$$\mathrm{MRPK}_{t+1}(z', k') = \frac{\partial}{\partial k'}\left(\max_{n'} p_{t+1} z' (\omega' k')^{\theta} (n')^{\nu} - w_{t+1} n' + q_{t+1}(1 - \delta)\omega' k'\right)$$

Risky Firms' Response

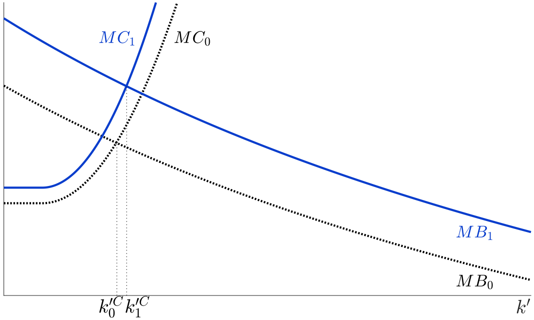


$$\left(q_t - \varepsilon_{R,k'}\frac{b'}{k'}\right)\frac{R_t^{\text{sp}}(z,k',b')}{1-\varepsilon_{R,b'}} = \frac{1}{R_t}\left(\mathbb{E}_t\left[\text{MRPK}_{t+1}(z',k')\right] + \frac{\mathbb{C}ov_t(\text{MRPK}_{t+1}(z',k'), 1+\lambda_{t+1}(z',k',b'))}{\mathbb{E}_t[1+\lambda_{t+1}(z',k',b')]}\right)$$

$$d = 0 \implies \quad q_t k' = \max_n p_t z(\omega k)^\theta n^\nu - w_t n - b - \xi + q_t(1-\delta)\omega k + \frac{1}{R_t(z,k',b')}b'$$

$$\text{MRPK}_{t+1}(z',k') = \frac{\partial}{\partial k'}\left(\max_{n'} p_{t+1} z'(\omega' k')^\theta (n')^\nu - w_{t+1}n' + q_{t+1}(1-\delta)\omega' k'\right)$$

Risky Firms' Response: Previous Channels



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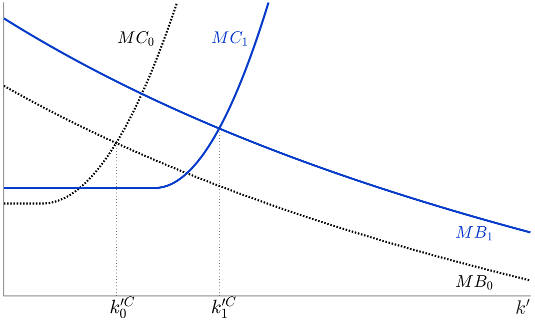
Risky Firms' Response: Cash Flow Rises

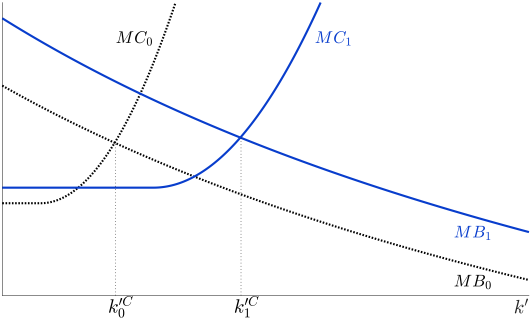


$$\left(q_t - \varepsilon_{R,k'}\frac{b'}{k'}\right)\frac{R_t^{sp}(z,k',b')}{1-\varepsilon_{R,b'}} = \frac{1}{R_t}\left(\mathbb{E}_t\left[\text{MRPK}_{t+1}(z',k')\right] + \frac{\mathbb{C}ov_t(\text{MRPK}_{t+1}(z',k'),1+\lambda_{t+1}(z',k',b'))}{\mathbb{E}_t[1+\lambda_{t+1}(z',k',b')]}\right)$$

$$d=0 \implies \quad q_t k' = \max_n p_t z(\omega k)^\theta n^\nu - w_t n - b - \xi + q_t(1-\delta)\omega k + \frac{1}{R_t(z,k',b')}b'$$

$$\text{MRPK}_{t+1}(z',k') = \frac{\partial}{\partial k'}\left(\max_{n'} p_{t+1}z'(\omega' k')^\theta (n')^\nu - w_{t+1}n' + q_{t+1}(1-\delta)\omega' k'\right)$$

Risky Firms' Response: Recovery Value Rises

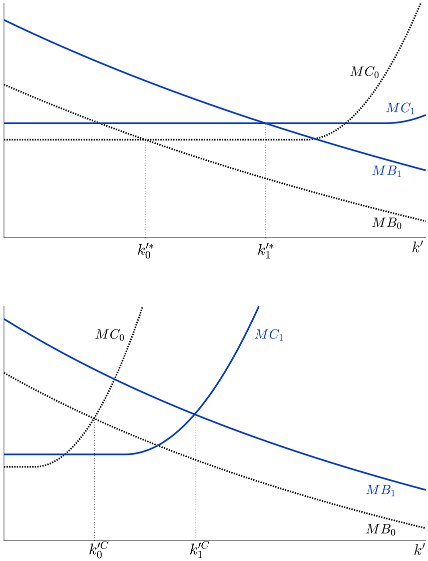


$$\left(q_t - \varepsilon_{R,k'}\frac{b'}{k'}\right)\frac{R_t^{sp}(z,k',b')}{1-\varepsilon_{R,b'}} = \frac{1}{R_t}\left(\mathbb{E}_t\left[\mathrm{MRPK}_{t+1}(z',k')\right] + \frac{\mathbb{C}ov_t(\mathrm{MRPK}_{t+1}(z',k'),1+\lambda_{t+1}(z',k',b'))}{\mathbb{E}_t[1+\lambda_{t+1}(z',k',b')]}\right)$$

$$d=0 \implies \quad q_t k' = \max_n p_t z(\omega k)^\theta n^\nu - w_t n - b - \xi + q_t(1-\delta)\omega k + \frac{1}{R_t(z,k',b')}b'$$

$$R_t^{sp}(z,k',b') = \mathrm{Prob}\left(\mathrm{default}_{t+1}(z',k',b')\right)\left(1-\min\{1,\alpha\frac{q_{t+1}\omega' k'}{b'/\Pi_{t+1}}\}\right)$$

Which Is More Responsive? Quantitative Question



Calibration

Overview of Calibration

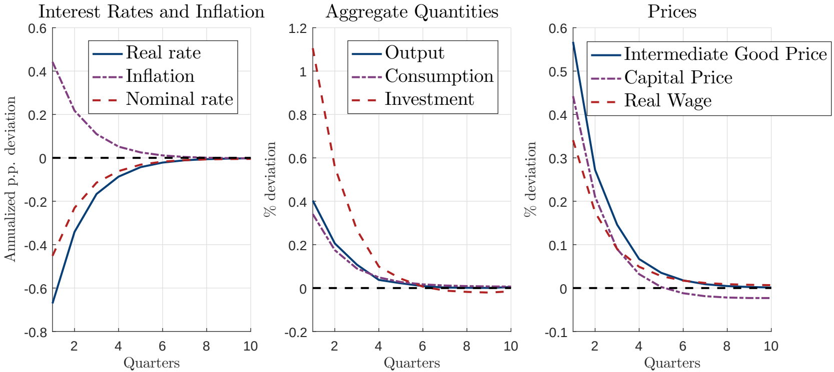
- **Fix** subset of parameters to standard values ▸ Details
- **Choose** parameters governing idiosyncratic shocks, financial frictions, and lifecycle to match empirical targets ▸ Details
    1. Cross-sectional dispersion of investment rates
    2. Mean default rate, credit spread, and leverage ratio
    3. Employment shares + establishment shares by age group

Overview of Calibration

- **Fix** subset of parameters to standard values ▸ Details
- **Choose** parameters governing idiosyncratic shocks, financial frictions, and lifecycle to match empirical targets ▸ Details
- **Analyze** sources of financial heterogeneity ▸ Details
  1. Lifecycle dynamics
  2. Productivity shocks
- **Verify** model (roughly) matches untargetted statistics
  1. Lifecycle dynamics ▸ Details
  2. Distribution of investment and leverage ▸ Details
  3. Investment-cash flow sensitivity ▸ Details

Quantitative Analysis of Monetary Transmission Mechanism

Aggregate Monetary Transmission Mechanism



- Peak responses in line with VARs (CEE 2005)
- Not designed to generate hump-shaped responses

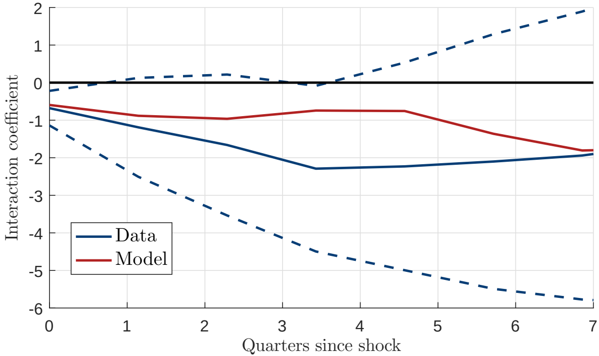
Heterogeneous Responses Consistent with Data

LHS:	$\Delta \log k_{jt}$	
	Data	Model
	(1)	(2)
leverage × ffr shock	-0.68^{**}	-0.59
	(0.28)	
Firm controls	yes	yes
Time FE	yes	yes
R^2	0.12	0.58

R^2 always higer than data

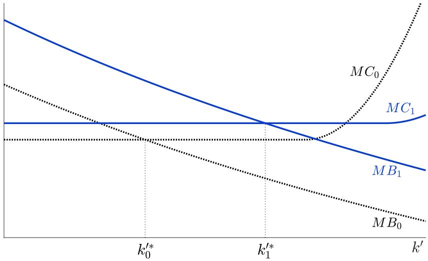
$$\Delta \log k_{jt+1} = \beta(\ell_{jt-1} - \mathbb{E}_j[\ell_{jt}])\varepsilon_t^{m} + \alpha_i + \alpha_{st} + \Gamma' Z_{jt-1} + \varepsilon_{jt}$$

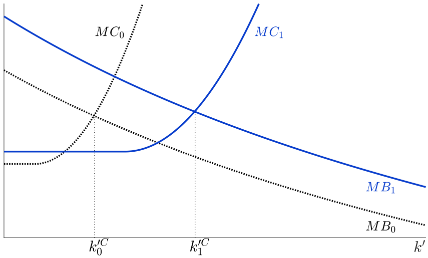
Heterogeneous Responses Consistent with Data



$$\log k_{it+h+1} - \log k_{it} = \beta_h (y_{it-1} - \mathbb{E}_i[y_{it}])\varepsilon_t^{m} + \alpha_{ih} + \alpha_{sth} + \\ + \Gamma_{1h}' Z_{it-1} + \Gamma_{2h}(y_{it-1} - \mathbb{E}_i[y_{it}]) Y_{t-1} + \varepsilon_{ith}$$

Heterogeneous Responses Consistent with Data



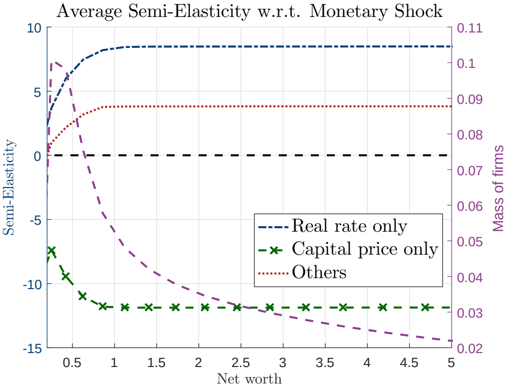


Heterogeneous Responses Consistent with Data

	LHS:	$\Delta \log k_{jt}$	**LHS:**	Δr_{jt}
	Data	Model	Data	Model
	(1)	(2)	(3)	(4)
leverage × ffr shock	-0.68^{**}	-0.59	0.17^{**}	0.26
	(0.28)		(0.06)	
Firm controls	yes	yes	yes	yes
Time FE	yes	yes	yes	yes
R^2	0.12	0.58	0.55	0.99

$$\Delta r_{jt} = \beta(\ell_{jt-1} - \mathbb{E}_j[\ell_{jt}])\varepsilon_t^{\mathrm{m}} + \alpha_i + \alpha_{st} + \Gamma' Z_{jt-1} + \varepsilon_{jt}$$

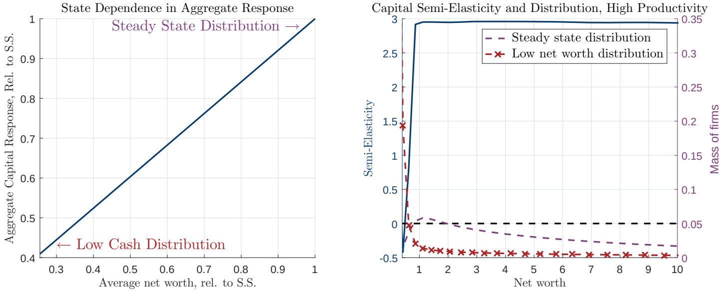
Risky Firms Less Responsive to All Channels



1. **Real interest rate** shifts out MB
2. **Capital price** shifts up MC + shifts out MB
3. **Other prices** shift out MB + move along x-axis

Both direct and indirect effects quantitatively important

Aggregate Effect Depends on Distribution of Risk



Back of the envelope calculation:

- Fix investment response across state space
- Vary initial distribution of net worth:

$$\mu(z,n) = \omega \underbrace{\mu_{\text{normal}}(z,n)}_{\text{s.s.}} + (1-\omega) \underbrace{\mu_{\text{bad}}(z,n)}_{\text{s.s., low prod.}}$$

Conclusion

Financial Heterogeneity and Investment Channel

Default risk dampens response of investment to monetary policy

Financial Heterogeneity and Investment Channel

Default risk dampens response of investment to monetary policy

1. **Which firms respond the most?**
    - Firms with low leverage, good credit ratings, and large distance to default
    - Indicates default risk is key to micro response

2. **Implications for aggregate transmission?**
    - Low-risk firms drive aggregate response
    - Suggests that aggregate effect depends on distribution of default risk

Details of Winberry (2018)

Recursive Competitive Equilibrium

A set of $\widehat{v}(\varepsilon, k; z, g)$, $C(z, g)$, $w(z, g)$, $\Lambda(z'; z, g)$, and $g'(z, g)$ such that

1. **Firm optimization**: Taking $\Lambda(z'; z, g)$ and $w(z, g)$ as given, $\widehat{v}(\varepsilon, k; z, g)$ solves Bellman equation
2. **Household optimization**: $w(z, g)C(z, g)^{-\sigma} = \chi N(z, g)^{\alpha}$
3. **Market clearing**:

$$N(z, g) = \int n(\varepsilon, k; z, g)g(\varepsilon, k)d\varepsilon dk$$

$$\Lambda(z'; z, g) = \beta \left(\frac{C(z', g'(z, g))}{C(z, g)} \right)^{-\sigma}$$

4. **Consistency**:

$$C(z, g) = \int (y(\varepsilon, k, \xi; z, g) - i(\varepsilon, k, \xi; z, g))dG(\xi)g(\varepsilon, k)d\varepsilon dk$$

$g'(\varepsilon, k)$ satisfies law of motion for distribution

Overview of the Method

1. Solve the steady state without aggregate shocks using global approximation

2. Solve for dynamics using local approximation

Overview of the Method

1. Solve the steady state without aggregate shocks using global approximation
    - **Discretize model in clever way**

2. Solve for dynamics using local approximation

Steady State Recursive Competitive Equilibrium

A set of $v^*(\varepsilon, k)$, C^*, w^*, and $g^*(\varepsilon, k)$ such that

1. **Firm optimization**: Taking w^* as given: $v^*(\varepsilon, k)$ solves Bellman equation
2. **Household optimization**: Taking w^* as given: $w^*(C^*)^{-\sigma} = \chi(N^*)^{\alpha}$
3. **Market clearing**:

$$N^* = \int n(\varepsilon, k) g(\varepsilon, k) d\varepsilon dk$$

4. **Consistency**:

$$C^* = \int (y(\varepsilon, k, \xi) - i(\varepsilon, k, \xi)) dG(\xi) g^*(\varepsilon, k) d\varepsilon dk$$

$g^*(\varepsilon, k)$ satisfies law of motion for distribution given g^*

Discretizing the Distribution

- Approximate p.d.f. $g(\varepsilon, k)$ with exponential polynomial from Algan, Allais, and Den Haan (2008)

$$g(\varepsilon, k) \approx g_0 \exp\{g_1^1 \left(\varepsilon - m_1^1\right) + g_1^2 \left(\log k - m_1^2\right) + \sum_{i=2}^{n_g} \sum_{j=0}^{i} g_i^j \left[\left(\varepsilon - m_1^1\right)^{i-j} \left(\log k - m_1^2\right)^j - m_i^j \right] \}$$

- Moments $\mathbf{m}$ pin down parameters $\mathbf{g}$ through

$$m_1^1 = \int \int \varepsilon g(\varepsilon, k)\, d\varepsilon dk,$$

$$m_1^2 = \int \int \log k g(\varepsilon, k)\, d\varepsilon dk, \text{ and}$$

$$m_i^j = \int \int \left(\varepsilon - m_1^1\right)^{i-j} \left(\log k - m_1^2\right)^j g(\varepsilon, k)\, d\varepsilon dk$$

Discretizing the Distribution

- Law of motion for the distribution = law of motion for moments

$$m_1^{1\prime} = \int (\rho_\varepsilon \varepsilon + \omega_\varepsilon')p\left(\omega_\varepsilon'\right) g\left(\varepsilon, k; \mathbf{m}\right) d\omega_\varepsilon' d\varepsilon dk$$

$$m_1^{2\prime} = \int \left[\begin{array}{c} \frac{\widehat{\xi}(\varepsilon,k)}{\bar{\xi}} \log k^a\left(\varepsilon, k\right) \\ + \left(1 - \frac{\widehat{\xi}(\varepsilon,k)}{\bar{\xi}}\right) \log k^n\left(\varepsilon, k\right) \end{array} \right]$$
$$\times\, p\left(\omega_\varepsilon'\right) g\left(\varepsilon, k; \mathbf{m}\right) d\omega_\varepsilon' d\varepsilon dk$$

$$m_i^{j\prime} = \int \left[\begin{array}{c} (\rho_\varepsilon \varepsilon + \omega_\varepsilon' - m_1^{1\prime})^{i-j} \{ \frac{\widehat{\xi}(\varepsilon,k)}{\bar{\xi}} \left(\log k^a\left(\varepsilon, k\right) - m_1^{2\prime}\right)^j \\ + \left(1 - \frac{\widehat{\xi}(\varepsilon,k)}{\bar{\xi}}\right) \left(\log k^n\left(\varepsilon, k\right) - m_1^{2\prime}\right)^j \} \end{array} \right]$$
$$\times\, p\left(\omega_\varepsilon'\right) g\left(\varepsilon, k; \mathbf{m}\right) d\omega_\varepsilon' d\varepsilon dk$$

$\Longrightarrow$ distribution: $\mathbf{m}$

Discretizing the Value Function

- Approximate value function with Chebyshev polynomials (Judd 1998 textbook)

$$\widehat{v}(\varepsilon, k) \approx \sum_{i=1}^{n_\varepsilon} \sum_{j=1}^{n_k} \theta_{ij} T_i(\varepsilon) T_j(k)$$

- Coefficients θ_{ij} solve Bellman at collocation points ε_i, k_j

$$\begin{aligned}
\widehat{v}(\varepsilon_i, k_j) = \max_n & \left\{ e^z e^{\varepsilon_i} k_j^\theta n^\nu - w^* n \right\} + (1-\delta) k \\
& + \left(\frac{\widehat{\xi}(\varepsilon_i, k_j)}{\overline{\xi}} \right) \left(\begin{array}{c} -\left(k^a(\varepsilon_i, k_j) + w^* \frac{\widehat{\xi}(\varepsilon_i, k_j)}{2} \right) \\ +\beta \int \widehat{v}\left(\rho_\varepsilon \varepsilon_i + \sigma_\varepsilon \omega'_\varepsilon, k^a(\varepsilon_i, k_j) \, p(\omega'_\varepsilon) \right) d\omega'_\varepsilon \end{array} \right) \\
& + \left(1 - \frac{\widehat{\xi}(\varepsilon_i, k_j)}{\overline{\xi}} \right) \left(\begin{array}{c} -k^n(\varepsilon_i, k_j; z, \mathbf{m}) \\ +\beta \int \widehat{v}\left(\rho_\varepsilon \varepsilon_i + \sigma_\varepsilon \omega'_\varepsilon, k^n(\varepsilon_i, k_j) \right) p(\omega'_\varepsilon) d\omega'_\varepsilon \end{array} \right)
\end{aligned}$$

$\Longrightarrow$ value function: θ

Hopenhayn-Rogerson (1993) Algorithm

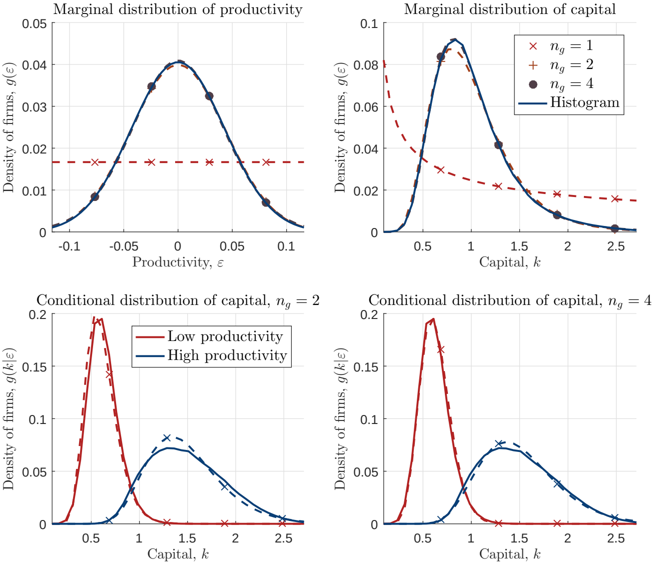
Start with guess of w^*

- Solve firm optimization problem by iterating on Bellman equation $\implies \theta$
- Use k' to compute stationary distribution by iterating on law of motion $\implies \mathbf{m}$
- Compute implied labor demand $N^d = \int n^*(\varepsilon, k) g^*(\varepsilon, k) d\varepsilon dk$
- Compute labor supply $N^s = \left(\frac{w^*(C^*)^{-\sigma}}{w^*}\right)^{\frac{1}{\alpha}}$

Update guess of w^* based on $N^d - N^s$

Iterate to convergence

Accuracy of Distribution Approximation



Overview of the Method

1. Solve the steady state without aggregate shocks using global approximation
    - Discretize model in clever way
2. **Solve for dynamics using local approximation**

Discretizing the Distribution Outside Steady State

- Law of motion for the distribution

$$m_1^{1\prime}(z, \mathbf{m}) = \int (\rho_\varepsilon \varepsilon + \omega_\varepsilon')p\left(\omega_\varepsilon'\right) g\left(\varepsilon, k; \mathbf{m}\right) d\omega_\varepsilon' d\varepsilon dk$$

$$m_1^{2\prime}(z, \mathbf{m}) = \int \left[\begin{array}{c} \frac{\widehat{\xi}(\varepsilon,k;z,\mathbf{m})}{\overline{\xi}} \log k^a\left(\varepsilon, k; z, \mathbf{m}\right) \\ + \left(1 - \frac{\widehat{\xi}(\varepsilon,k;z,\mathbf{m})}{\overline{\xi}}\right) \log k^n\left(\varepsilon, k; z, \mathbf{m}\right) \end{array} \right]$$
$$\times p\left(\omega_\varepsilon'\right) g\left(\varepsilon, k; \mathbf{m}\right) d\omega_\varepsilon' d\varepsilon dk$$

$$m_i^{j\prime}(z, \mathbf{m}) = \int \left[\begin{array}{c} (\rho_\varepsilon \varepsilon + \omega_\varepsilon' - m_1^{1\prime})^{i-j} \{ \frac{\widehat{\xi}(\varepsilon,k;z,\mathbf{m})}{\overline{\xi}} \left(\log k^a\left(\varepsilon, k; z, \mathbf{m}\right) - m_1^{2\prime}\right)^j \\ + \left(1 - \frac{\widehat{\xi}(\varepsilon,k;z,\mathbf{m})}{\overline{\xi}}\right) \left(\log k^n\left(\varepsilon, k; z, \mathbf{m}\right) - m_1^{2\prime}\right)^j \} \end{array} \right]$$
$$\times p\left(\omega_\varepsilon'\right) g\left(\varepsilon, k; \mathbf{m}\right) d\omega_\varepsilon' d\varepsilon dk$$

$\Longrightarrow$ distribution: $\mathbf{m}$

Discretizing the Value Function Outside Steady State

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- Coefficients θ_{ij} chosen to solve Bellman at collocation points ε_{ij}

$$\begin{aligned}
\widehat{v}\left(\varepsilon_i, k_j; z, \mathbf{m}\right) &= \max_n \left\{e^z e^{\varepsilon_i} k_j^\theta n^\nu - w\left(z, \mathbf{m}\right) n\right\} + \left(1-\delta\right) k \\
&+ \left(\frac{\widehat{\xi}\left(\varepsilon_i, k_j; z, \mathbf{m}\right)}{\bar{\xi}}\right) \left(\begin{array}{c} -\left(k^a\left(\varepsilon_i, k_j; z, \mathbf{m}\right) + w\left(z, \mathbf{m}\right) \frac{\widehat{\xi}\left(\varepsilon_i, k_j; z, \mathbf{m}\right)}{2}\right) \\ +\beta \mathbb{E}_{z'|z}\left[\int \widehat{v}\left(\rho_\varepsilon \varepsilon_i + \sigma_\varepsilon \omega_\varepsilon', k^a\left(\varepsilon_i, k_j; z, \mathbf{m}\right); z', \mathbf{m}'\left(z, \mathbf{m}\right)\right) p\left(\omega_\varepsilon'\right) d\omega_\varepsilon'\right] \end{array}\right) \\
&+ \left(1 - \frac{\widehat{\xi}\left(\varepsilon_i, k_j; z, \mathbf{m}\right)}{\bar{\xi}}\right) \left(\begin{array}{c} -k^n\left(\varepsilon_i, k_j; z, \mathbf{m}\right) \\ +\beta \mathbb{E}_{z'|z}\left[\int \widehat{v}\left(\rho_\varepsilon \varepsilon_i + \sigma_\varepsilon \omega_\varepsilon', k^n\left(\varepsilon_i, k_j; z, \mathbf{m}\right); z', \mathbf{m}'\left(z, \mathbf{m}\right)\right) p(\omega_\varepsilon') d\omega_\varepsilon'\right] \end{array}\right)
\end{aligned}$$

$\Longrightarrow$ value function: θ

Overview of the Method

1. Solve the steady state without aggregate shocks using global approximation
   - Discretize model in clever way

2. **Solve for dynamics using local approximation**

$$\mathbf{x} = (\mathbf{m}, z)' \text{ and } \mathbf{y} = (\theta, C)'$$

$$f(\mathbf{y}', \mathbf{y}, \mathbf{x}', \mathbf{x}; \psi) = \begin{bmatrix} \text{Bellman} \\ \text{Evolution of } \mathbf{m} \\ \text{Consistency of } C \\ z' = \rho_z z + \psi \omega_z' \end{bmatrix}$$

Overview of the Method

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$$\mathbf{x} = (\mathbf{m}, z)' \text{ and } \mathbf{y} = (\theta, C)'$$

$$f(\mathbf{y}', \mathbf{y}, \mathbf{x}', \mathbf{x}; \psi) = \begin{bmatrix} \text{Bellman} \\ \text{Evolution of } \mathbf{m} \\ \text{Consistency of } C \\ z' = \rho_z z + \psi \omega_z' \end{bmatrix}$$

$$\text{Equilibrium} : \mathbb{E}_{\omega_z'}\left[f(\mathbf{y}', \mathbf{y}, \mathbf{x}', \mathbf{x}; \psi) = 0\right]$$

Overview of the Method

1. Solve the steady state without aggregate shocks using global approximation
   - Discretize model in clever way

2. **Solve for dynamics using local approximation**

$$\mathbb{E}_{\omega_z'}\left[f\left(\mathbf{y}', \mathbf{y}, \mathbf{x}', \mathbf{x}; \psi\right) = 0\right]$$

Overview of the Method

1. Solve the steady state without aggregate shocks using global approximation
   - Discretize model in clever way

2. **Solve for dynamics using local approximation**

$$\Longrightarrow \mathbb{E}_{\omega_z'}\left[f\left(\mathbf{y}', \mathbf{y}, \mathbf{x}', \mathbf{x}; \psi\right) = 0\right]$$

$$\mathbf{y} = g(\mathbf{x}; \psi = 1)$$

$$\mathbf{x}' = h(\mathbf{x}; \psi = 1) + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \omega_z'$$

Perturbation Methods

- Approximate solution using Taylor expansion around steady state

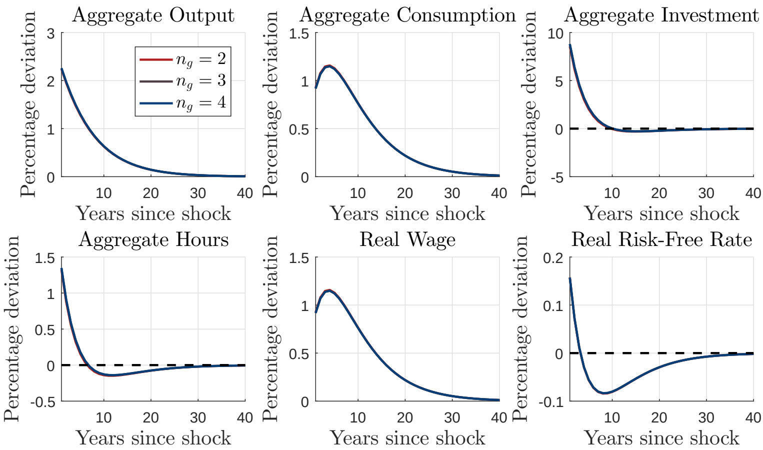
$$g(\mathbf{x};\psi=1) \approx \mathbf{y}^* + g_{\mathbf{x}}(\mathbf{x}^*;\psi=0)(\mathbf{x}-\mathbf{x}^*) + \text{(higher order terms)}$$
$$h(\mathbf{x};\psi=1) \approx \mathbf{x}^* + h_{\mathbf{x}}(\mathbf{x}^*;\psi=0)(\mathbf{x}-\mathbf{x}^*) + \text{(higher order terms)}$$

- Unknowns in this approximation are $g_{\mathbf{x}}(\mathbf{x}^*;\psi=0)$ and $h_{\mathbf{x}}(\mathbf{x}^*;\psi=0)$

- Perturbation methods: how to solve for unknowns using derivatives of the equilibrium conditions $f(\mathbf{x},\mathbf{x}',\mathbf{y},\mathbf{y}';\psi)$

- Largely automated by `Dynare`

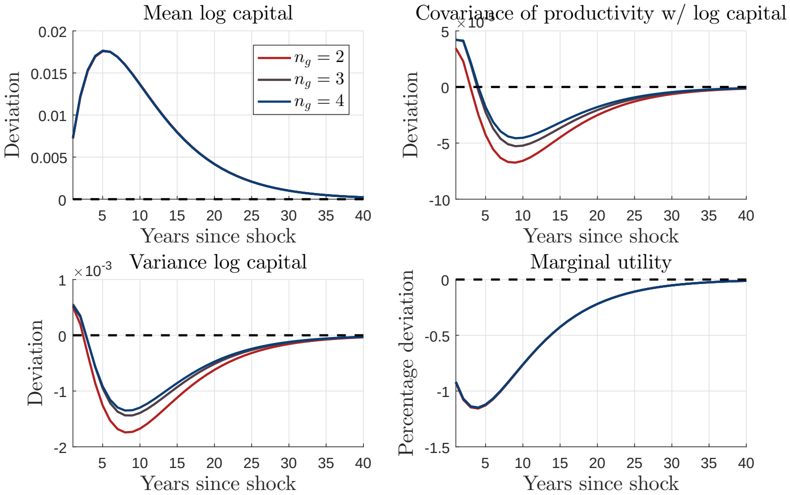
Impulse Responses of Aggregate Variables



Business Cycle Statistics of Aggregate Variables

SD (rel. to output)	$\mathbf{n}_g = 2$	$\mathbf{n}_g = 4$	**Corr. with Output**	$\mathbf{n}_g = 2$	$\mathbf{n}_g = 4$
Output	(2.14%)	(2.16%)	$\times$	$\times$	$\times$
Consumption	0.48	0.47	Consumption	0.90	0.90
Investment	3.86	3.93	Investment	0.97	0.97
Hours	0.61	0.61	Hours	0.95	0.94
Real wage	0.48	0.47	Real wage	0.90	0.90
Real interest rate	0.08	0.08	Real interest rate	0.80	0.79

Impulse Responses of Distributional Variables



Business Cycle Statistics of Distributional Variables

$\mathbb{E}[\log k]$	$\mathbf{n}_g = \mathbf{2}$	$\mathbf{n}_g = \mathbf{3}$	$\mathbf{n}_g = \mathbf{4}$	$\mathbb{C}ov\,(\varepsilon, \log k)$	$\mathbf{n}_g = \mathbf{2}$	$\mathbf{n}_g = \mathbf{3}$	$\mathbf{n}_g = \mathbf{4}$
Mean	-0.0899	-0.0822	-0.0824	Mean	0.0123	0.0121	0.0122
SD	0.0125	0.0126	0.0127	SD	6.7e-5	7.2e-5	6.9e-5
Corr w/ Y	0.6017	0.6095	0.6117	Corr w/ Y	0.7157	0.8276	0.8432
Autocorr	0.8280	0.8268	0.8264	Autocorr	0.7472	0.7339	0.7240
$\mathbb{V}ar\,(\log k)$	$\mathbf{n}_g = \mathbf{2}$	$\mathbf{n}_g = \mathbf{3}$	$\mathbf{n}_g = \mathbf{4}$	**Marginal Utility**	$\mathbf{n}_g = \mathbf{2}$	$\mathbf{n}_g = \mathbf{3}$	$\mathbf{n}_g = \mathbf{4}$
Mean	0.1529	0.1476	0.1485	Mean	0.8995	0.8934	0.8931
SD	0.0014	0.0013	0.0012	SD	0.0103	0.0102	0.0101
Corr w/ Y	0.5752	0.6608	0.6539	Corr w/ Y	-0.8999	-0.9001	-0.8999
Autocorr	0.7980	0.7823	0.7782	Autocorr	0.6704	0.6712	0.6715

Wrapping Up Discussion of the Method

- Relative to Krusell-Smith:
  - Advantages: fast, complicated distribution
  - Disadvantages: local approximation, parametric form for distribution

Wrapping Up Discussion of the Method

- Relative to Krusell-Smith:
  - Advantages: fast, complicated distribution
  - Disadvantages: local approximation, parametric form for distribution

- Other analysis (in the paper)
  1. Nonlinear approximation of dynamics
     - Set `order = 2` in `Dynare`
  2. Occasionally binding constraints and mass points (e.g., Krusell-Smith)
     - Separately approximate (i) mass at borrowing constraint and (ii) distribution away from borrowing constraint

Dynare Implementation

- Automate perturbation step in `Dynare`
  - Takes derivatives of equilibrium conditions f
  - Solve for approximate solution g and h

- Online code template provides basic structure:
  1. **Inputs:** `.m` file to compute steady state + `.mod` file to define equilibrium conditions
  2. **Outputs:** impulse responses, business cycle statistics, variance decompositions, option to estimate model

- Two worked-out examples: Krusell-Smith (1998) and Khan-Thomas (2008)

Appendix for Ottonello-Winberry (2019)

Constructing Investment ▸ Back

1. Start with firms' reported level of plant, property, and equipment (`ppegtq`) as firms' initial value of capital
2. Compute differences of net plant, property, and equipment (`ppentq`) to get net investment
3. Interpolate missing values when missing a single quarter in the data
4. Compute gross investment using depreciation rates of Fixed Asset tables from NIPA at the industry level
5. Trim the data: extreme values and short spells

Sectoral Controls ▸ Back

Sectors considered:

1. Agriculture, Forestry, And Fishing: $\texttt{sic} < 10$
2. Mining: $\texttt{sic} \in [10, 14]$
3. Construction: $\texttt{sic} \in [15, 17]$
4. Manufacturing: $\texttt{sic} \in [20, 39]$
5. Transportation, Communications, Electric, Gas, And Sanitary Services: $\texttt{sic} \in [40, 49]$
6. Wholesale Trade: $\texttt{sic} \in [50, 51]$
7. Retail Trade: $\texttt{sic} \in [52, 59]$
8. Services: $\texttt{sic} \in [70, 89]$

Sectors not considered:

1. Finance, Insurance, and Real Estate: $\texttt{sic} \in [60, 67]$
2. Public Administration: $\texttt{sic} \in [91, 97]$

Sample Selection ▸ Back

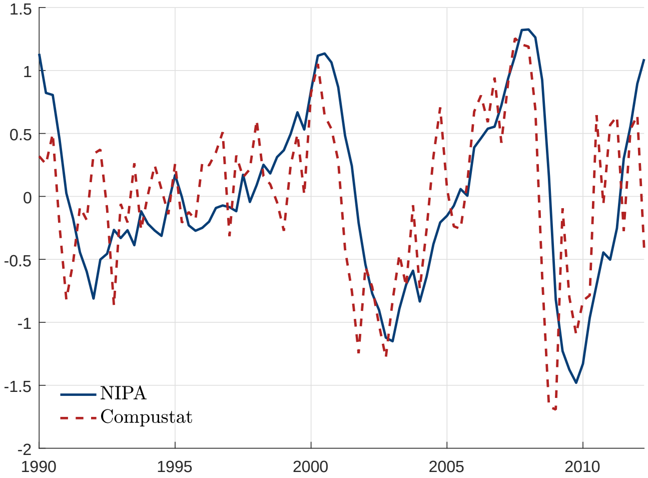
1. Drop observations with investment rate in the top and bottom 0.5% of the distribution
2. Drop observations with leverage ratios higher than 10
3. Drop observations with net current assets higher than 10 or lower than -10
4. Drop observations with quarterly sales growth higher than 1 or lower than -1
5. Winsorize the top and bottom 0.5% of investment and financial positions

Monetary Shocks ▸ Back

	high frequency	smoothed	sum
mean	-0.0185	-0.0429	-0.0421
median	0	-0.0127	-0.00509
std	0.0855	0.108	0.124
min	-0.463	-0.480	-0.479
max	0.152	0.233	0.261
num	164	71	72

Investment: Compustat and NIPA

Back



Distance to Default: Theory ▸ Back

- A1: Total value of firm follows

$$dV = \mu_V V dt + \sigma_V V dW$$

  μ_V: expected continuously compounded return on V
  σ_V: volatility of firm value
  dW: increment of standard Weiner process

- A2: Firm has just issued single discount bond that will mature in T periods

- A3: Firm's default occurs when $V < D$

$\Rightarrow$ Merton (1974): Equity of firm can be seen as a call option on firm's value with a strike price equal to the face value of the firm's debt

Distance to Default: Definition ▸ Back

- Follows Merton (1974) and Gilchrist and Zakrajsek (2012):

$$dd \equiv \frac{\log(V/D) + (\mu_V - 0.5\sigma_V^2)}{\sigma_V}$$

where

  - V: total value of the firm
  - μ_V: expected return on V
  - σ_V: volatility of the firm's value
  - D: firm's debt

- Interpretation:
  - Number of standard deviations that $\log V$ must deviate from its mean for $V < D$ (default)

Distance to Default: Measurement ▸ Back

Iterative procedure:

1. Initialize procedure with $\sigma_V = \sigma_E[D/(E+D)]$,
   where E: market value of equity,
   σ_E: estimated volatility from daily returns (250-day moving window)

2. Infer market value of firm's asset for every day of the 250-day moving window from the Black-Scholes-Merton option-pricing framework

$$E = V\Phi(\delta_1) - e^{-rT} D\Phi(\delta_2)$$

   where $\delta_1 = \frac{\log(V/D)+(r+0.5\sigma_V^2)T}{\sigma_V^2\sqrt{T}}$, $\delta_2 = \delta_1 - \sigma_V\sqrt{T}$

3. Calculate implied daily log-return on assets ($\Delta \log V$) and use resulting series to generate new estimates of σ_V and μ_V

Extensive Margin Measure of Investment ▸ Back

Dependent variable: $\mathbb{1}\{\frac{i_{it}}{k_{it}} \geq 1\%\}$

	(1)	(2)	(3)	(4)
leverage $\times$ ffr shock	-2.81**		-4.12**	-3.69*
	(1.40)		(1.93)	(1.91)
dd $\times$ ffr shock		5.30***	3.44*	4.09*
		(1.70)	(1.74)	(2.32)
ffr shock				7.47
				(4.59)
Observations	219702	151433	151433	151433
R^2	0.223	0.234	0.235	0.222
Firm controls	yes	yes	yes	yes
Time sector FE	yes	yes	yes	no
Time clustering	yes	yes	yes	yes

Expansionary vs. Contractionary Shocks Back

Dependent variable: $\Delta \log k_{it+1}$

	(1)	(2)	(3)	(4)
leverage × ffr shock	-0.68**			
	(0.28)			
leverage × pos ffr shock		-0.71**		
		(0.30)		
leverage × neg ffr shock		-0.56		
		(0.96)		
dd × ffr shock			1.10***	
			(0.39)	
dd × pos ffr shock				1.38***
				(0.50)
leverage × neg ffr shock				0.12
				(0.77)
Observations	219702	219702	151433	151433
R^2	0.124	0.124	0.137	0.137
Firm controls	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes

Information: Controlling for Fed Forecasts ▸ Back

Greenbook Forecast Revisions

	(1)	(2)	(3)	(4)	(5)	(6)
leverage × ffr shock	-0.80***		-0.96***		-1.10***	
	(0.29)		(0.35)		(0.34)	
dd × ffr shock		1.11***		0.78*		0.74*
		(0.40)		(0.44)		(0.43)
Forecast controls	GDP	GDP	GDP, Infl.	GDP, Infl.	GDP, Un.	GDP, Un.
Observations	219702	151433	219702	151433	219702	151433
R^2	0.124	0.137	0.124	0.137	0.124	0.137
Firm controls	yes	yes	yes	yes	yes	yes

Information: Controlling for Fed Forecasts ▸ Back

Greenbook Forecasts

	(1)	(2)	(3)	(4)	(5)	(6)
leverage × ffr shock	-1.08***		-0.73**		-0.75*	
	(0.29)		(0.32)		(0.44)	
dd × ffr shock		1.14***		0.92**		0.90*
		(0.41)		(0.37)		(0.53)
Forecast controls	GDP	GDP	GDP, Infl.	GDP, Infl.	GDP, Un.	GDP, Un.
Observations	219702	151433	219702	151433	219702	151433
R^2	0.124	0.137	0.124	0.137	0.124	0.137
Firm controls	yes	yes	yes	yes	yes	yes

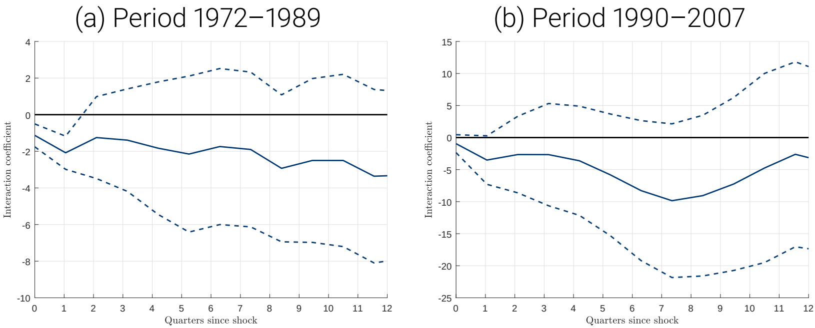
Information: Target vs. Path Decomposition

▸ Back

Dependent variable: $\Delta \log k_{it+1}$	(1)	(2)	(3)	(4)
leverage × ffr shock	-0.68**			
	(0.28)			
leverage × target shock		-0.98**		
		(0.45)		
leverage × path shock		-0.70		
		(1.30)		
dd × shock			1.10***	
			(0.39)	
dd × target shock				1.47**
				(0.67)
dd × path shock				-0.41
				(1.65)
Observations	219702	214301	151433	147986
R^2	0.124	0.125	0.137	0.138
Firm controls	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes

Relation to Gertler and Gilchrist (1994) ▸ Back

Replicate spirit of Gertler-Gilchrist in our sample



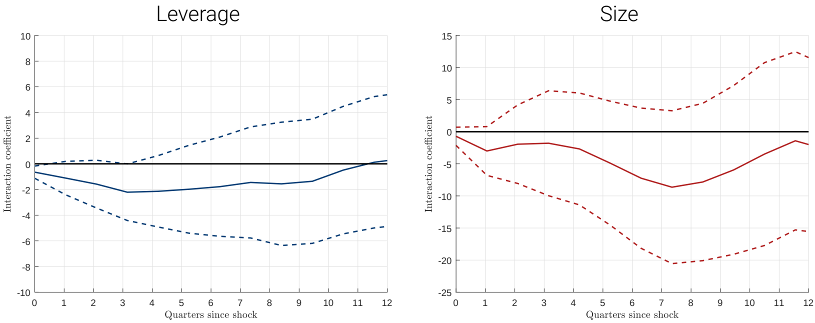
$$\log k_{jt+h} - \log k_{jt} = \alpha_{jh} + \alpha_{sth} + \beta_h size^s_{jt-1}\varepsilon^m_t + \Gamma'_{1h} Z_{jt-1} + \Gamma'_{2h} size^s_{jt-1} Y_{t-1} + \varepsilon_{jth}$$

where $size^s_{jt-1}$ = 1 if average sales over last ten years above p30

Relation to Gertler and Gilchrist (1994)

▸ Back

Our results robust to controlling for Gertler-Gilchrist size



Relation to Gertler and Gilchrist (1994) Back

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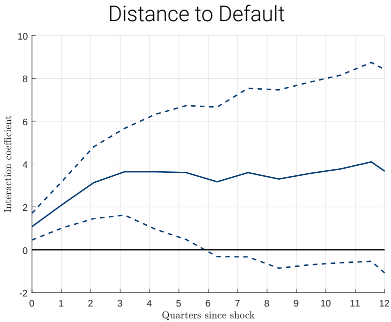
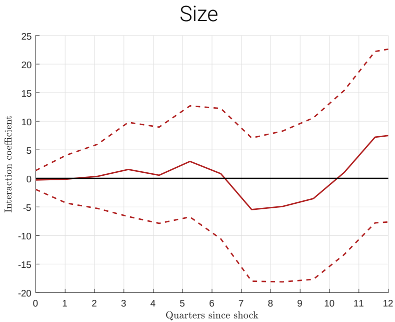
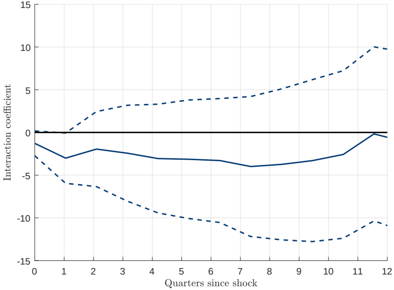




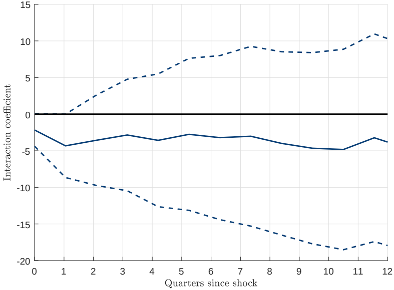
Relation to Cloyne et al. (2018) ▸ Back

Replicate spirit of Cloyne et al. (2018) in our sample

(a) Middle-age Firms



(b) Older Firms

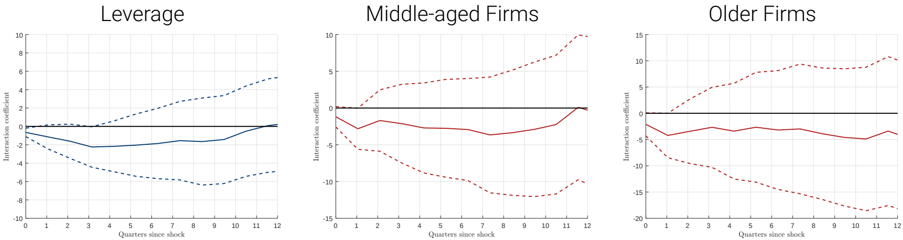


$$\log k_{jt+h} - \log k_{jt} = \alpha_{jh} + \alpha_{sth} + \boldsymbol{\beta}'_h age_{jt} \varepsilon^m_t + \Gamma'_{1h} Z_{jt-1} + \Gamma'_{2h} age_{jt} Y_{t-1} + \varepsilon_{jth}$$

where age = young (< 15 years), middle-aged (15-50 years), or old (> 50 years)

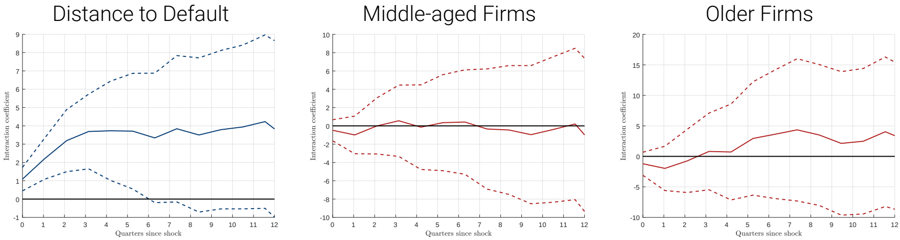
Relation to Cloyne et al. (2018) ▸ Back

Our results robust to controlling for age



Relation to Cloyne et al. (2018) ▸ Back

Our results robust to controlling for age



Comparison to Jeenas (2018): Dynamics ▸ Back

Two key differences between our specification and Jeenas (2018)'s:

1. Trimming top 1% rather than winsorizing top 0.5%
2. Sorting firms based on past year's average leverage $\widehat{\ell}_{jt}$

Comparison to Jeenas (2018): Dynamics ▸ Back

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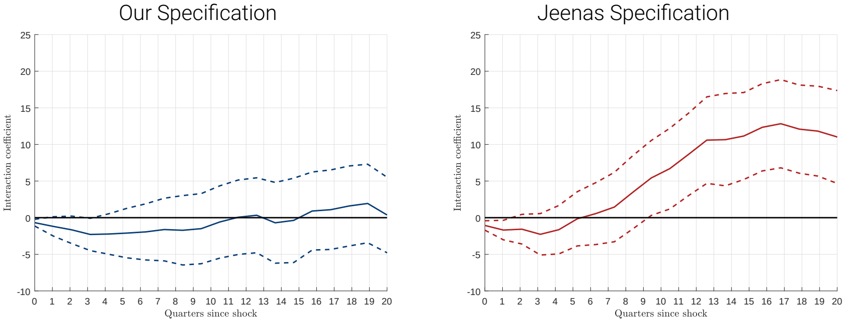

Comparison to Jeenas (2018): Dynamics

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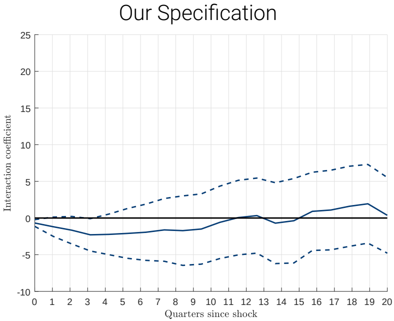

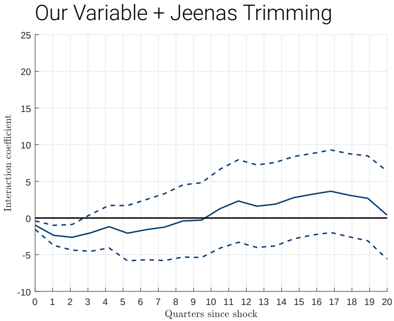

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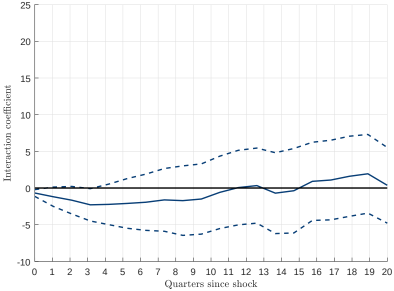
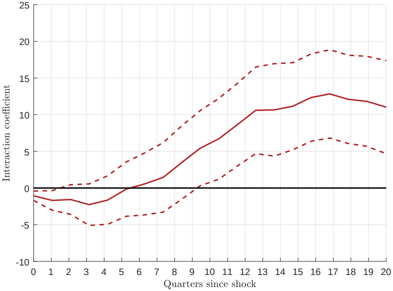


Jeenas Trimming + Jeenas Variable



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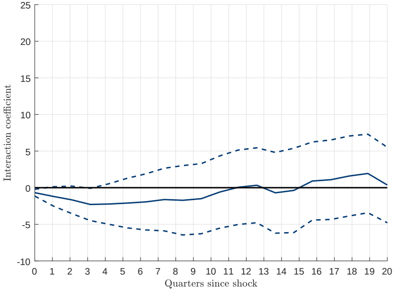
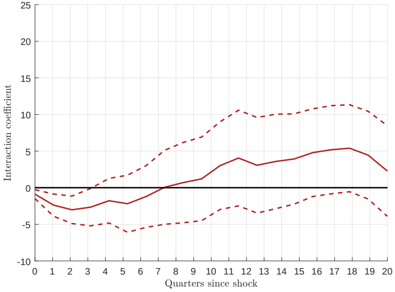


Jeenas Trimming + Demeaned Jeenas Variable



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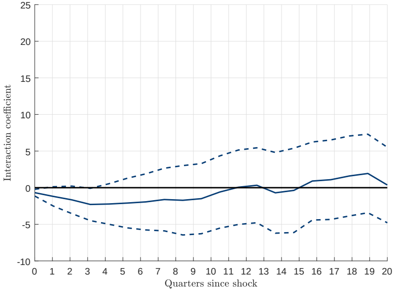

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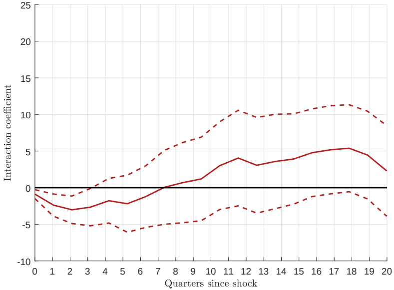
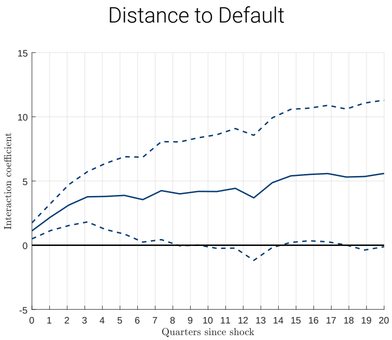


$\Longrightarrow$ **Long-run dynamics driven by permanent heterogeneity**

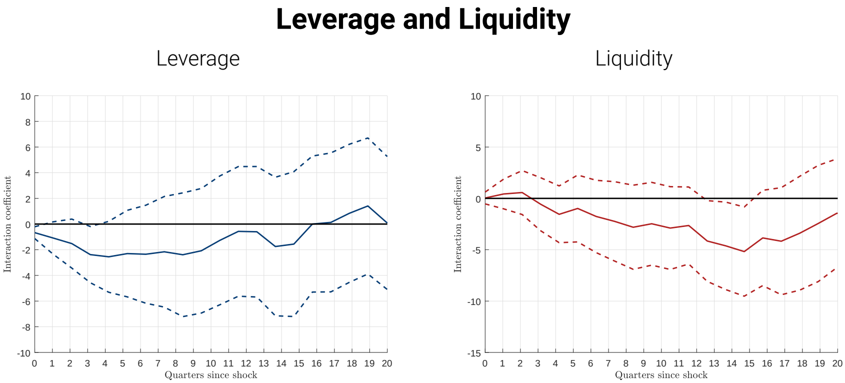
Focus on impact effects because robust + significant

Comparison to Jeenas (2018): Results Not Driven by Liquidity ▸ Back

Distance to Default and Liquidity



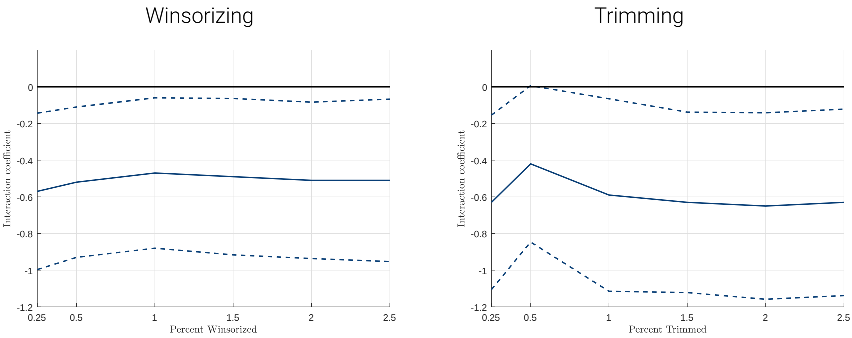
Comparison to Jeenas (2018): Results Not Driven by Liquidity ▸ Back



Comparison to Jeenas (2018): Results Not Driven by Outliers

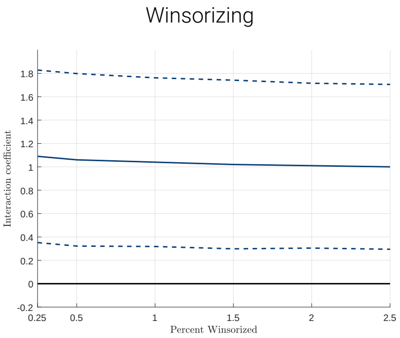
Back

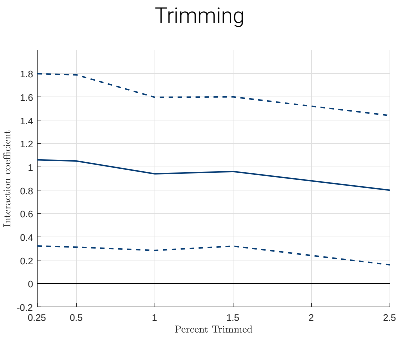
Leverage



Comparison to Jeenas (2018): Results Not Driven by Outliers
Back

Distance to Default

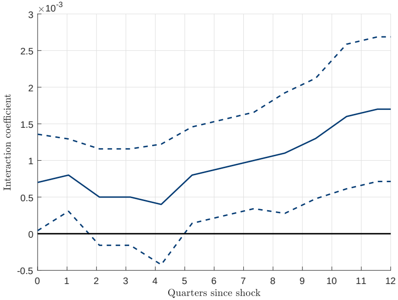




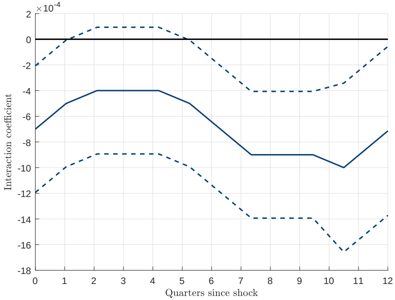
Response of average interest payments ▸ Back

Response of average interest payments

by Leverage

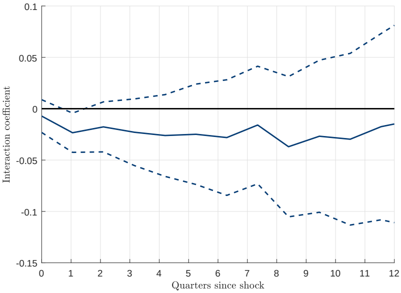


by Distance to Default

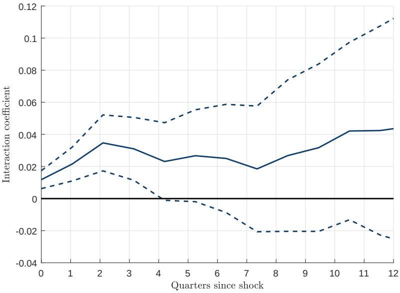


Response of financing flows

by Leverage



by Distance to Default



Instrumenting Financial Position with Lags ‣ Back

Dependent variable: $\Delta \log k_{it+1}$

	(1)	(2)	(3)	(4)	(5)	(6)
leverage × ffr shock	-0.72**	-0.84**	-1.35***			
	(0.36)	(0.39)	(0.47)			
dd × ffr shock				1.17***	1.23**	1.24*
				(0.44)	(0.53)	(0.70)
Observations	219674	217179	213207	138989	128745	122547
R^2	0.020	0.019	0.018	0.021	0.021	0.019
Firm controls, Time-Sector FE	yes	yes	yes	yes	yes	yes
Instrument	1q lag	2q lag	4q lag	1q lag	2q lag	4q lag

Decomposition of Leverage ▸ Back

Dependent variable: $\Delta \log k_{it+1}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
leverage × ffr shock	-0.68**						
	(0.28)						
net leverage × ffr shock		-0.71**					
		(0.30)					
ST debt × ffr shock			-0.37		-0.44		
			(0.31)		(0.31)		
LT debt × ffr shock				-0.20	-0.35		
				(0.25)	(0.24)		
other liabilities × ffr shock						-0.23	
						(0.28)	
liabilities × ffr shock							-0.69**
							(0.31)
Observations	219702	219702	219702	219702	219702	219682	219682
R^2	0.124	0.125	0.124	0.121	0.125	0.124	0.126
Firm controls	yes	yes	yes	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes	yes	yes	yes

Using Raw Changes in FFR ▸ Back

Dependent variable: $\Delta \log k_{it+1}$				
	(1)	(2)	(3)	(4)
leverage × ffr shock	-0.67**			
	(0.28)			
leverage × Δffr		-0.12**		
		(0.06)		
dd × ffr shock			1.08***	
			(0.39)	
dd × Δffr				0.16*
				(0.08)
Observations	219674	278800	151422	195672
R^2	0.124	0.114	0.137	0.122
Firm controls	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes

Results Post-1994 ▸ Back

Dependent variable: $\Delta \log k_{it+1}$

	(1)	(2)	(3)	(4)
leverage $\times$ ffr shock	-0.80**		-0.54	-0.55
	(0.37)		(0.49)	(0.52)
dd $\times$ ffr shock		0.80*	0.54	0.75
		(0.43)	(0.40)	(0.56)
ffr shock				0.25
				(1.19)
Observations	174546	118782	118782	118782
R^2	0.138	0.150	0.152	0.137
Firm controls	yes	yes	yes	yes
Time sector FE	yes	yes	yes	no
Time clustering	yes	yes	yes	yes

Robustness: Interaction with Cyclical Variables

Back

Dependent variable: $\Delta \log k_{it+1}$

	(1)	(2)	(3)	(4)	(5)	(6)
leverage $\times$ ffr shock	-0.68**		-0.64**		-0.36	
	(0.28)		(0.29)		(0.26)	
dd $\times$ ffr shock		1.10***		1.12***		0.88**
		(0.39)		(0.39)		(0.35)
leverage $\times$ dlog gdp	-0.14**		-0.15***			
	(0.06)		(0.06)			
dd $\times$ dlog gdp		0.11		0.09		
		(0.11)		(0.11)		
leverage $\times$ dlog cpi			-0.12			
			(0.09)			
dd $\times$ dlog gdp				-0.09		
				(0.12)		
leverage $\times$ ur					0.00	
					(0.00)	
dd $\times$ ur						0.00
						(0.00)
Observations	219702	151433	219702	151433	219702	151433
R^2	0.124	0.137	0.124	0.137	0.124	0.137
Firm controls	yes	yes	yes	yes	yes	yes

Robustness: Interaction with Firm Characteristics

Back

Dependent variable: $\Delta \log k_{it+1}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
leverage $\times$ ffr shock	-0.68**		-0.70**		-0.68**		-0.73**	
	(0.28)		(0.30)		(0.28)		(0.28)	
dd $\times$ ffr shock		1.10***		1.13***		1.12***		1.13***
		(0.39)		(0.39)		(0.39)		(0.39)
sales growth $\times$ ffr shock	-0.18	0.07						
	(0.25)	(0.27)						
future sales growth $\times$ ffr shock			-0.37	-0.69				
			(0.44)	(0.57)				
size $\times$ ffr shock					0.37	0.56		
					(0.29)	(0.40)		
liquidity $\times$ ffr shock							-0.24	-0.31
							(0.31)	(0.35)
Observations	219702	151433	208917	145073	219702	151433	219578	151353
R^2	0.124	0.137	0.128	0.140	0.124	0.137	0.126	0.138
Firm controls	yes	yes	yes	yes	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes	yes	yes	yes	yes

Robustness: Interaction with Other Financial Characteristics ▸ Back

Dependent variable: $\Delta \log k_{it+1}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
leverage × ffr shock	-0.68**		-0.67**		-0.68**		-0.73**	
	(0.28)		(0.28)		(0.28)		(0.28)	
dd × ffr shock		1.12***		1.09***		1.09***		1.13***
		(0.39)		(0.39)		(0.39)		(0.39)
size × ffr shock	0.37	0.56						
	(0.29)	(0.40)						
cash flows × ffr shock			-0.02	-0.35				
			(0.46)	(0.63)				
$\mathbb{I}\{\text{dividends} > 0\} \times$ ffr shock					0.39	0.24		
					(0.60)	(0.64)		
liquidity × ffr shock							-0.24	-0.31
							(0.31)	(0.35)
Observations	219702	151433	218185	150350	219482	151311	219578	151353
R^2	0.124	0.137	0.130	0.142	0.125	0.137	0.126	0.138
Firm controls	yes	yes	yes	yes	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes	yes	yes	yes	yes

Robustness: Alternative Time Aggregation

Back

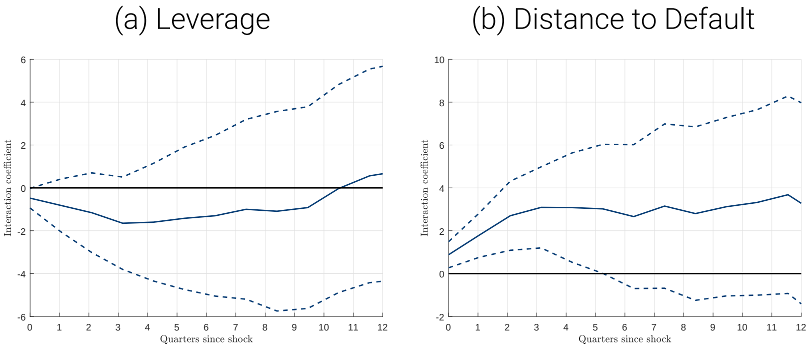
	(1)	(2)	(3)	(4)
leverage × ffr shock (sum)	-0.68***		-0.61**	-0.54**
	(0.19)		(0.25)	(0.27)
dd × ffr shock (sum)		0.81***	0.54**	0.69**
		(0.26)	(0.25)	(0.32)
ffr shock (sum)				0.47
				(0.53)
Observations	222475	153520	153520	151433
R^2	0.123	0.135	0.138	0.126
Firm controls	yes	yes	yes	yes
Time sector FE	yes	yes	yes	no
Time clustering	yes	yes	yes	yes

Robustness: Controlling for Lagged Investment ▸ Back

	(1)	(2)	(3)	(4)
lev_wins_dem_std_wide	-0.47		-0.20	-0.10
	(0.28)		(0.37)	(0.39)
Ldl_capital	0.20***	0.15***	0.15***	0.16***
	(0.01)	(0.01)	(0.01)	(0.01)
d2d_wins_dem_std_wide		0.87**	0.72**	0.93**
		(0.37)	(0.35)	(0.41)
ffr shock				1.14*
				(0.65)
Observations	219674	151422	151422	151422
R^2	0.159	0.156	0.158	0.148
Firm controls	yes	yes	yes	yes
Time sector FE	yes	yes	yes	no
Time clustering	yes	yes	yes	yes

Robustness: Dynamics Controlling for Lagged Investment

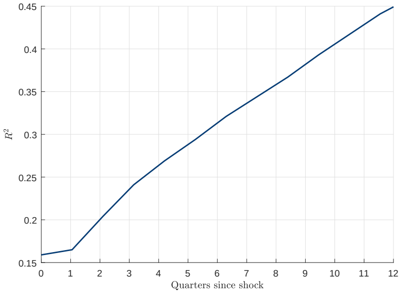
Back



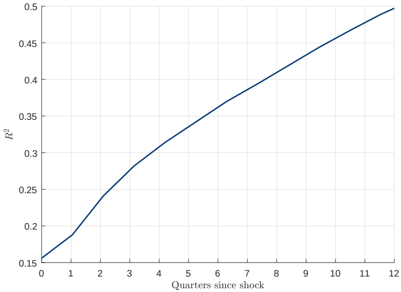
Robustness: R^2 Controlling for lagged Investment

▸ Back

(a) Leverage



(b) Distance to Default



Retailers and Final Good Producer ▸ Back

- Monopolistically competitive **retailers**
  - Technology: $\tilde{y}_{it} = y_{it} \implies$ real marginal cost $= p_t$
  - Set price $\tilde{p}_{it}$ s.t. quadratic cost $-\frac{\varphi}{2}\left(\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1\right)^2 Y_t$

- Perfectly competitive **final good producer**
  - Technology: $Y_t = \left(\int \tilde{y}_{it}^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} \implies P_t = \left(\int \tilde{p}_{it}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$

- Implies New Keynesian Phillips Curve linking inflation π_t to marginal cost p_t

The Rest of the Model ▸ Back

- **Monetary authority** follows Taylor rule

$$\log R_t^{\text{nom}} = \log \frac{1}{\beta} + \varphi_\pi \Pi_t + \varepsilon_t^{\text{m}}$$

- **Capital good producer** with technology

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t \implies q_t = 1/\Phi'\left(\frac{I_t}{K_t}\right) = \left(\frac{I_t/K_t}{\delta}\right)^{\frac{1}{\phi}}$$

- **Representative household** with preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \Psi N_t\right)$$

  - Owns firms $\implies \Lambda_{t+1} = \beta \frac{C_t}{C_{t+1}}$
  - Labor-leisure choice $\implies w_t C_t^{-1} = \Psi$
  - Euler equation for bonds $\implies 1 = \beta R_t^{\text{nom}} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Pi_{t+1}}\right]$

Fixed Parameters ▸ Back

Parameter	Description	Value
Household		
β	Discount factor	0.99
Firms		
ν	Labor coefficient	0.64
θ	Capital coefficient	0.21
δ	Depreciation	0.025
New Keynesian Block		
ϕ	Aggregate capital AC	4
γ	Demand elasticity	10
φ_π	Taylor rule coefficient	1.25
φ	Price adjustment cost	90

Parameters to be Computed ▸ Back

Parameter	Description	Value
Idiosyncratic shock processes		
ρ	Persistence of TFP (fixed)	0.90
σ	SD of innovations to TFP	
σ_ω	Dispersion of capital quality	
Financial frictions		
ξ	Operating cost	
α	Loan recovery rate	
Firm lifecycle		
m	Mean shift of entrants' prod.	
k_0	Initial capital	
π_d	Exogeneous exit rate	

Choose labor disutility Ψ to ensure steady state employment = 0.6

Empirical Targets

Back

Moment	Description	Data	Model
Investment behavior (annual)			
$\sigma\left(\frac{i}{k}\right)$	SD investment rate	33.7%	
Financial behavior (annual)			
$\mathbb{E}\left[\text{default rate}\right]$	Mean default rate	3.00%	
$\mathbb{E}\left[\text{credit spread}\right]$	Mean credit spread	2.35%	
$\mathbb{E}\left[b/k\right]$	Mean gross leverage ratio	34.4%	
Firm Growth (annual)			
N_1/N	Emp. share in age ≤ 1	2.6%	
N_{1-10}/N	Emp. share in age $\in (1, 10)$	21%	
N_{11+}/N	Emp. share in age ≥ 10	76%	
Firm Exit (annual)			
$\mathbb{E}\left[\text{exit rate}\right]$	Mean exit rate	8.7%	
$\mathbb{E}\left[M_1\right]/\mathbb{E}\left[M\right]$	Share of firms at age 1	10.5%	
$\mathbb{E}\left[M_2\right]/\mathbb{E}\left[M\right]$	Share of firms at age 2	8.1%	

Empirical Targets ▸ Back

Moment	Description	Data	Model
Investment behavior (annual)			
$\sigma\left(\frac{i}{k}\right)$	SD investment rate	33.7%	35.2%
Financial behavior (annual)			
$\mathbb{E}$ [default rate]	Mean default rate	3.00%	3.05%
$\mathbb{E}$ [credit spread]	Mean credit spread	2.35%	0.70%
$\mathbb{E}[b/k]$	Mean gross leverage ratio	34.4%	41.3%
Firm Growth (annual)			
N_1/N	Emp. share in age ≤ 1	2.6%	2.8%
N_{1-10}/N	Emp. share in age $\in (1, 10)$	21%	36%
N_{11+}/N	Emp. share in age ≥ 10	76%	61%
Firm Exit (annual)			
$\mathbb{E}$ [exit rate]	Mean exit rate	8.7%	8.92%
$\mathbb{E}[M_1]/\mathbb{E}[M]$	Share of firms at age 1	10.5%	7.8%
$\mathbb{E}[M_2]/\mathbb{E}[M]$	Share of firms at age 2	8.1%	6.0%

Parameters to be Computed Back

Parameter	Description	Value
Idiosyncratic shock processes		
ρ	Persistence of TFP (fixed)	0.90
σ	SD of innovations to TFP	0.03
σ_ω	Dispersion of capital quality	0.035
Financial frictions		
ξ	Operating cost	0.03
α	Loan recovery rate	0.45
Firm lifecycle		
m	Mean shift of entrants' prod.	3.00
k_0	Initial capital	0.22
π_d	Exogeneous exit rate	0.02

Choose labor disutility Ψ to ensure steady state employment = 0.6

Identification of Fitted Parameters

Back

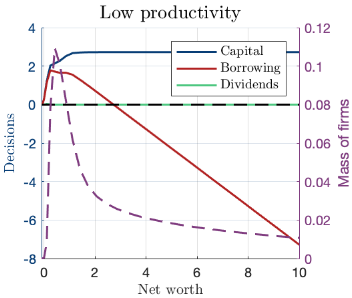
Elasticity of moments w.r.t. parameters

	σ	ξ	k_0	m	α	σ_ω
$\sigma(\frac{i}{k})$	1.05	0.13	0.13	-2.57	2.10	0.11
E[default rate]	-2.55	0.63	-3.30	0.41	-0.63	15.30
E[credit spread]	2.42	0.86	0.02	0.10	3.48	1.47
E[gross leverage]	-1.26	-0.17	-0.90	0.10	-0.14	3.73
N_1/N	-0.44	-1.04	0.10	-0.17	-0.16	1.23
N_{1-10}/N	0.50	-0.12	0.10	-0.06	0.00	0.15
M_1/M	-0.06	-0.34	-0.12	0.07	-0.10	1.67
M_2/M	0.39	-0.26	0.12	0.03	-0.03	0.56

Elasticity of parameters w.r.t. moments

	σ	ξ	k_0	m	α	σ_ω
$\sigma(\frac{i}{k})$	-0.14	0.27	0.60	-0.40	-0.01	0.11
E[default rate]	0.03	0.22	0.02	-0.05	-0.10	0.06
E[credit spread]	0.06	-0.17	-0.31	0.24	0.30	-0.05
E[gross leverage]	0.35	-1.30	-2.60	0.15	0.27	-0.44
N_1/N	0.21	-1.32	-2.06	0.17	0.34	-0.35
N_{1-10}/N	3.91	-5.30	-13.14	0.16	-0.48	-1.99
M_1/M	-1.24	1.41	6.13	-0.06	0.02	1.08
M_2/M	-1.04	3.00	7.29	-0.32	-0.58	1.25

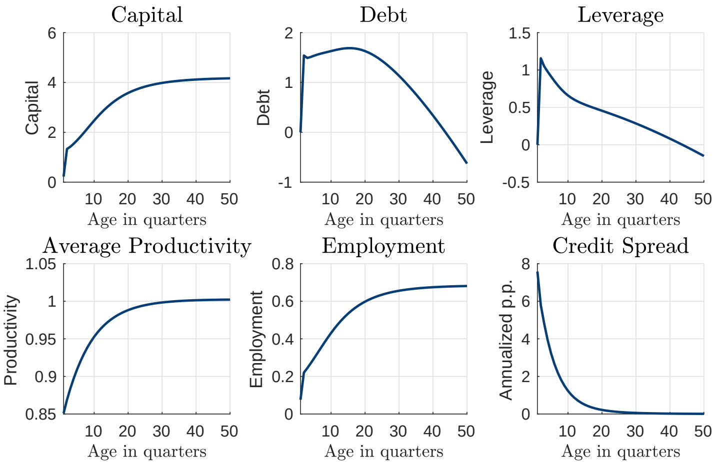
Steady State Decision Rules ▸ Back



Two key sources of **financial heterogeneity**

1. Lifecycle dynamics
2. Productivity shocks

Firm Lifecycle Dynamics ▸ Back

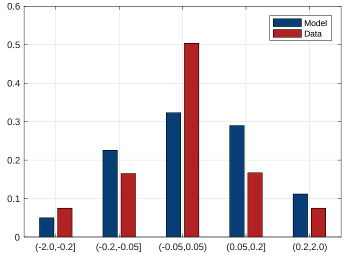


- Young firms riskier than average
- But default risk spread out over large set of firms

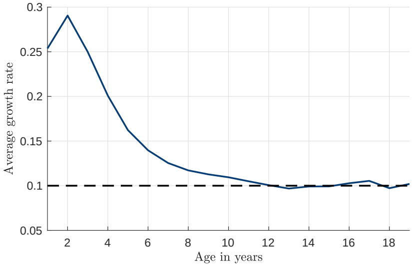
Firm Lifecycle Dynamics in the Model and Data

Back

(a) Distribution of growth rates



(b) Age-growth profile



Financial Heterogeneity in the Model and Data

Back

Investment and leverage heterogeneity

Moment	Description	Data	Model (selected)	Model (full)
Investment heterogeneity (annual LRD)				
$\mathbb{E}\left[\frac{i}{k}\right]$	Mean investment rate	12.2%	9.59%	22.3%
$\sigma\left(\frac{i}{k}\right)$	SD investment rate (calibrated)	33.7%	31.8%	44.8%
$\rho\left(\frac{i}{k}, \frac{i}{k}_{-1}\right)$	Autocorr investment rate	0.058	-0.16	-0.16
Joint investment and leverage heterogeneity (quarterly Compustat)				
$\rho\left(\frac{b}{k}, \frac{b}{k}_{-1}\right)$	Autocorr leverage ratio	0.94	0.95	0.09
$\rho\left(\frac{i}{k}, \frac{b}{k}\right)$	Corr. of leverage and investment	-0.08	-0.10	-0.20

Measured investment-cash flow sensitivity

	Without cash flow		With cash flow	
	Data	*Model*	*Data*	*Model*
Tobin's q	0.01***	0.01	0.01***	0.01
cash flow			0.02***	0.07