# Estimation Effects of Various Demographic Forecasting Techniques in Japan Using an Overlapping Generations Model

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# Acknowledgements

I would like to thank all those who have helped me (and continue to help me) along the way to finishing my thesis. This includes Dr. Rick Evans, Dr. Kotaro Yoshida, Dr. Victor Lima, and my many friends and family who have commented on my paper (especially Kei Irizawa and Ujaan Purakayastha), among others.

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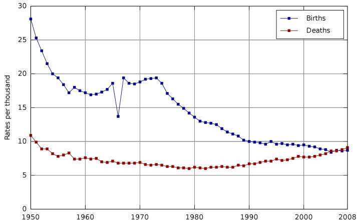
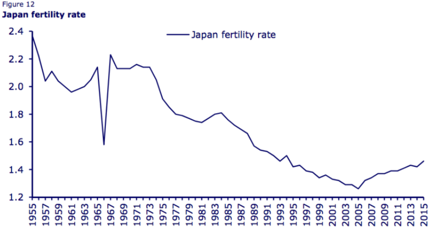
# Motivation




Source: Wikipedia




Source: Ministry of Health, Labour and Welfare

# Research Question

## Why Care?

What is the effect of COVID-19 mortality? How will public pensions change over time? How does predicted macroeconomic behavior respond?

## Research Question

How should demographics be forecast? I propose a new method for forecasting demographics.

## Economic Application

Compare macroeconomic forecasts from the most common demographic forecasting assumptions.

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# Presentation Overview

- Data
- Demographic Forecasting Methods
  - Static, PCA, partial dynamic, full dynamic
- Macroeconomic Model
- Macroeconomic Results

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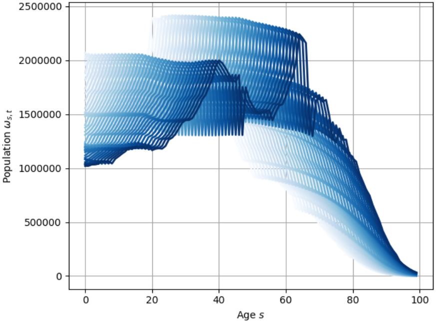
# Data

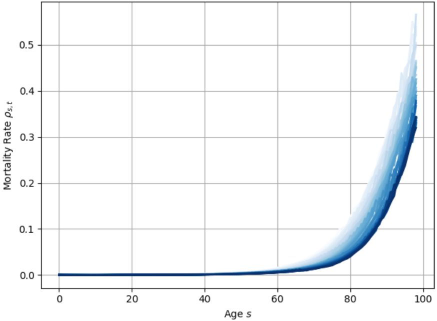
- Population and Mortality: Japanese Mortality Database (2018)
- Fertility: Human Fertility Collection

# Population



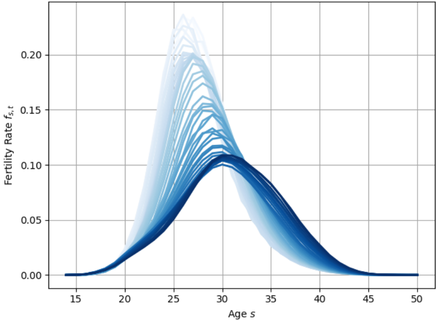

Population, 1970-2014

# Mortality



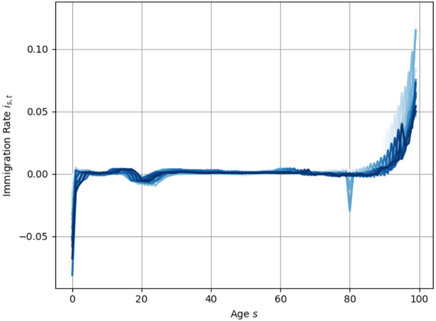

Mortality Rates, 1970-2014

# Fertility




Fertility Rates, 1970-2014

# Immigration




Immigration Rates, 1971-2014

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# Four Forecasting Methods

- Static
- Principal Components Analysis (PCA)
- Partial-Dynamic
- Full-Dynamic

# Static

- Constant fertility, mortality, immigration, and population (use 2014 data)
- Treat as baseline

# Dynamic Population Models

Population evolution given by the following:

$$\omega_{s+1,t+1} = (1-\rho_{s,t})\omega_{s,t} + i_{s+1,t}\omega_{s+1,t} \;\; \forall t \text{ and } 1 \leq s+1 \leq E+S-1$$

$$\omega_{0,t+1} = \sum_{s=1}^{E+S} f_{s,t}\omega_{s,t} + i_{0,t}\omega_{0,t} \;\; \forall t$$

$$\omega_{E+S+1,t} = 0 \;\; \forall t$$

- $\omega, f, \rho, i$: population, fertility, mortality, immigration
- $s, t, E, S$: age, year, years as child (outside economy), years as adult (contribute to economy)

# Principal Components Analysis (PCA)

- Based on Hyndman and Ullah (2007)
- Forecasts fertility, mortality, and immigration rates
- Start population forecast using true 2017 population

# Partial-Dynamic

- Based on DeBacker and Evans (2018)
- Fixed fertility, mortality, and immigration rates
- Start population forecast using true 2017 population

# Full-Dynamic

- Parametric forecasts of fertility, mortality, and immigration rates
- Start population forecast using true 2017 population

# Full-Dynamic - Fertility

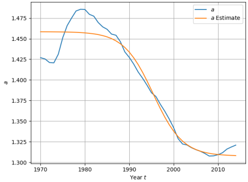
Fit to generalized beta 2 distribution:

$$f(x|a,b,p,q) = \frac{ax^{ap-1}}{b^{ap}B(p,q)\left(1+\left(\frac{x}{b}\right)^{a}\right)^{p+q}}$$

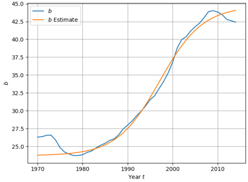
where $x \in [0,\infty); a,b,p,q > 0$

# Full-Dynamic - Fertility

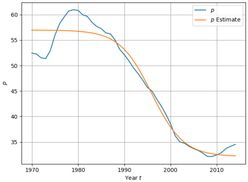
## Fertility Generalized Beta 2 Parameter Estimates



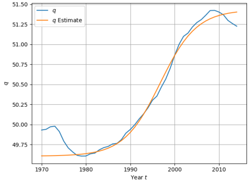

(a) $a$



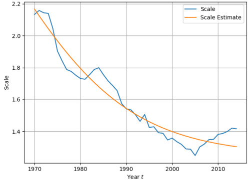

(b) $b$




(c) $p$



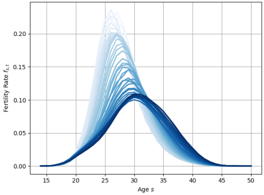

(d) $q$



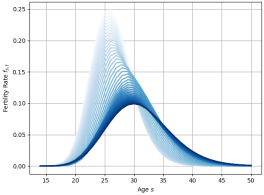

(e) Scale

# Full-Dynamic - Fertility

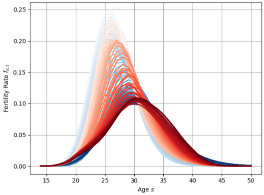
## Fertility Generalized Beta 2 Model Fit



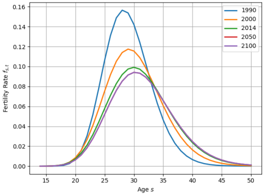

(a) Data




(b) Model




(c) Model Overlays Data

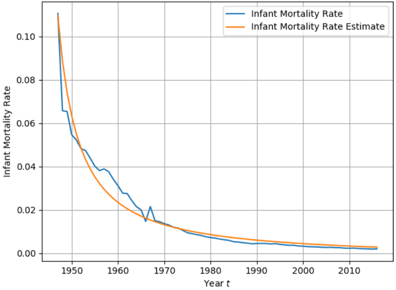



(d) Model Forecasts

# Full-Dynamic - Mortality

Infant mortality fit to a polynomial of the form:

$$f(x|a,b,c,d,e) = a(e \cdot x - b)^{\frac{1}{c}} + d$$



# Full-Dynamic - Mortality

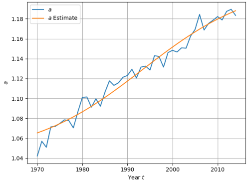
Mortality fit to generalized beta 2 distribution (same as fertility):

$$f(x|a,b,p,q) = \frac{ax^{ap-1}}{b^{ap}B(p,q)\left(1+\left(\frac{x}{b}\right)^{a}\right)^{p+q}}$$

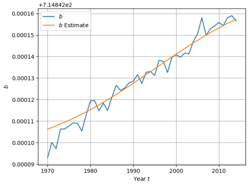
where $x \in [0,\infty); a,b,p,q > 0$

# Full-Dynamic - Mortality

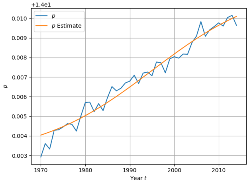
## Mortality Generalized Beta 2 Parameter Estimates



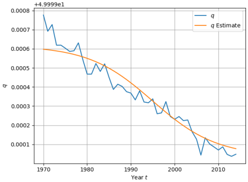

(a) $a$



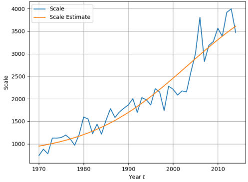

(b) $b$




(c) $p$



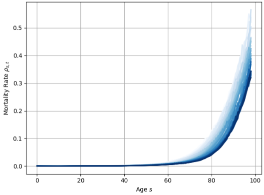

(d) $q$



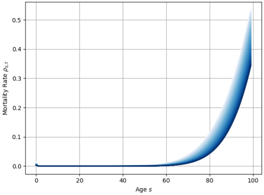

(e) Scale

# Full-Dynamic - Mortality

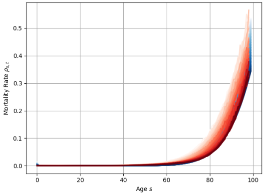
## Mortality Generalized Beta 2 Model Fit



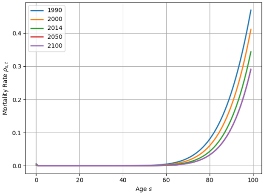
(a) Data



(b) Model



(c) Model Overlays Data



(d) Model Forecasts

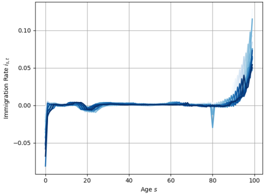
# Full-Dynamic - Immigration

Immigration fit to linear regression, then forecasted out using an exponential of the form:

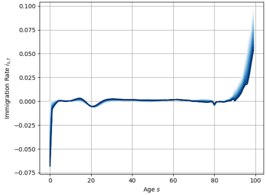
$$f(x|a, b, c, d, p, s, \beta_0, \beta_1) = e^{a(x-s)^2+b(x-s)+c} + p$$

# Full-Dynamic - Immigration

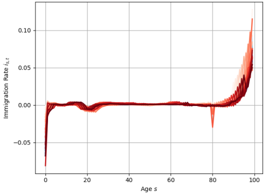
## Immigration Estimated by Linear Regression and Forecasted by Exponential



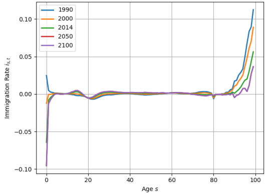

(a) Data




(b) Model



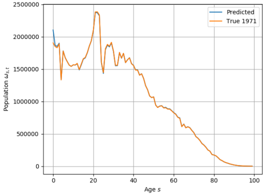

(c) Model Overlays Data



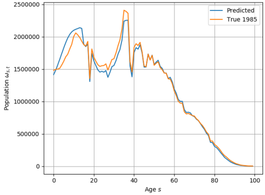

(d) Model Forecasts

# Full-Dynamic - Population

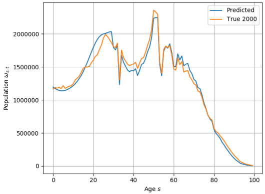
## Population Forecasts, Initial Population Set to 1970



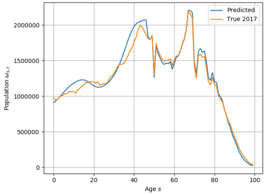

(a) 1971




(b) 1985



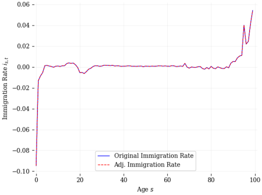

(c) 2000



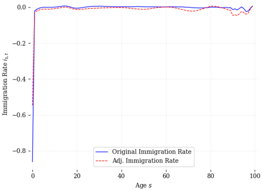

(d) 2017

# All Models - Steady State Immigration Rates

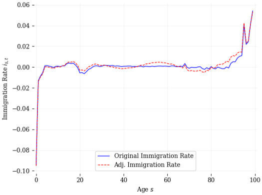
## Steady State Immigration Rates



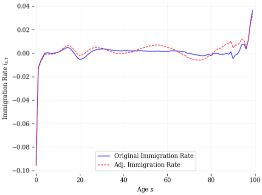

(a) Static




(b) PCA



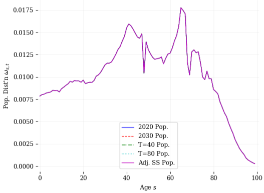

(c) Partial-Dynamic



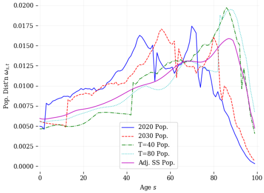

(d) Full-Dynamic

# All Models - Population Distribution Path

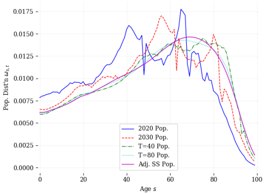
## Population Distribution Paths



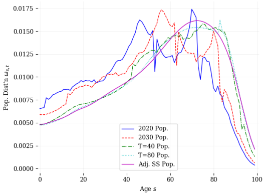

(a) Static




(b) PCA



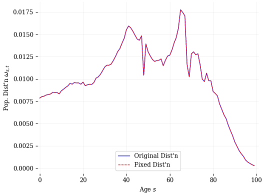

(c) Partial-Dynamic



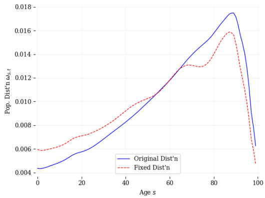

(d) Full-Dynamic

# All Models - Steady State Population Distribution

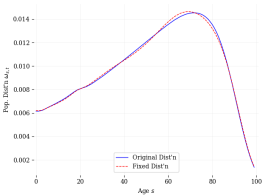
## Steady State Population Distributions



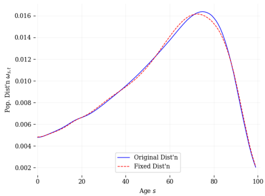

(a) Static




(b) PCA




(c) Partial-Dynamic




(d) Full-Dynamic

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# Macroeconomic Model: Short Description

- Overlapping generations model from Evans (2020)
- Households live for 100 periods: 20 periods of youth, outside the labor market; 80 periods of adulthood, contribute to economy
- Households choose consumption, labor, and savings to maximize lifetime utility
- Households subject to warm bequest motive
- Population demographics can evolve over time
- Firms choose capital and labor to maximize profits
- No government (no taxes or transfers)

# Macroeconomic Model: Households

Households choose consumption, labor, and savings to maximize

$$u(c_{s,t}, n_{s,t}, b_{s+1,t+1}) = \frac{(c_{s,t})^{1-\sigma} - 1}{1-\sigma} + e^{g_y t(1-\sigma)} \chi_{n,s} b \left[1 - \left(\frac{n_{s,t}}{\tilde{l}}\right)\right]^{\frac{1}{\upsilon}}$$
$$+\rho_{s,t}\chi_b \frac{(b_{s+1,t+1})^{1-\sigma} - 1}{1-\sigma} \quad \forall s, t$$

subject to

$$c_{s,t} + b_{s+1,t+1} = (1+r_t)b_{s,t} + w_t n_{s,t} + \frac{BQ_t}{\tilde{N}_t} \quad \forall t \quad \text{and} \quad s \geq E$$
$$b_{E,t} = b_{E+S,t} = 0 \quad \forall t$$
$$c_{s,t} \geq 0 \quad \forall s, t$$

# Macroeconomic Model: Firms

Firms have the following Cobb-Douglas production function:

$$Y_t = F(K_t, L_t) \equiv A(K_t)^{\alpha}(e^{g_y t} L_t)^{1-\alpha} \forall t \quad \alpha \in (0,1) \quad \text{and} \quad A > 0$$

Firms then choose capital and labor to maximize profits:

$$PR_t = A(K_t)^{\alpha}(e^{g_y t} L_t)^{1-\alpha} - (r_t + \delta)K_t - w_t L_t \quad \forall t$$

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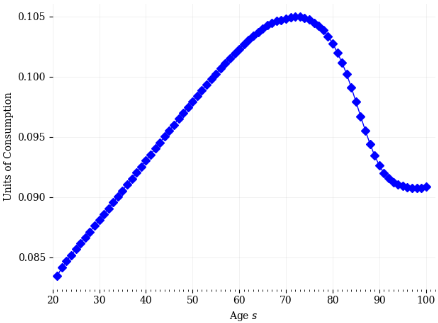
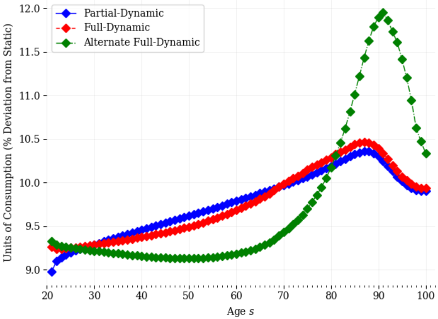
# Summary of Steady State Results

- Compared to baseline (static), consumption/savings everywhere higher in dynamic models
- PCA results: dramatically more consumption/savings with old population relative to partial- and full-dynamic
- Compared to baseline (static), labor everywhere lower in dynamic models
- PCA results: more labor for young population, less labor for old population relative to partial- and full-dynamic

# Steady State Consumption

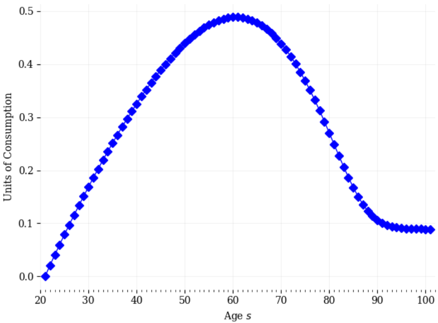



(a) Static Demographics

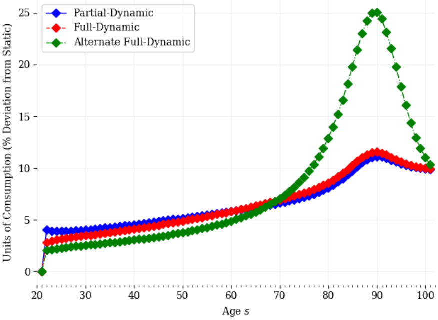



(b) All Other Demographics (% Deviation from Static)

# Steady State Savings

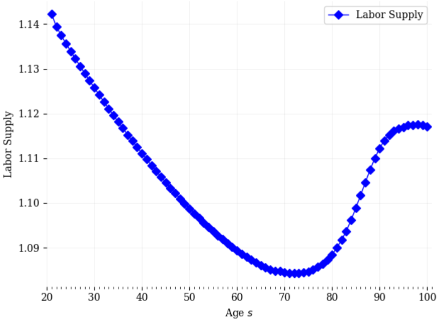



(a) Static Demographics

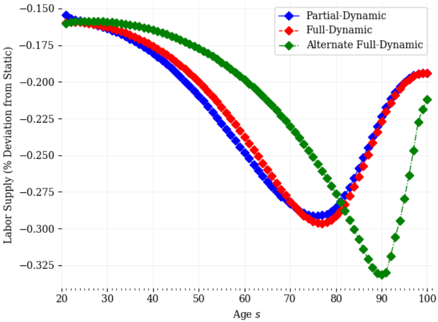



(b) All Other Demographics (% Deviation from Static)

# Steady State Labor




(a) Static Demographics

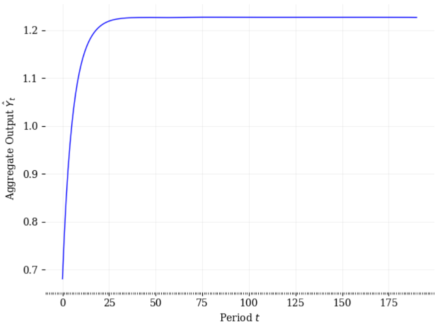



(b) All Other Demographics (% Deviation from Static)

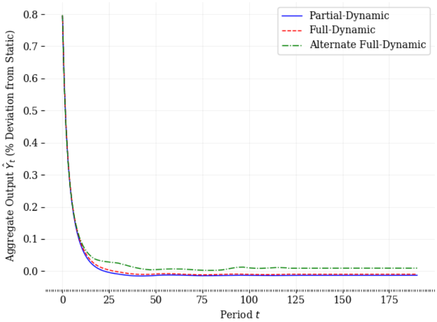
# Summary of Transition Path Results

- Transition path results difficult to interpret: difference between baseline (static) and dynamic models too large to explain

# Transition Path of Aggregate Output

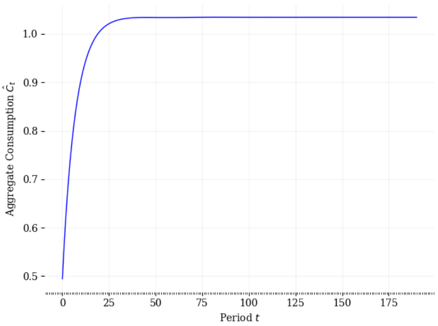



(a) Static Demographics

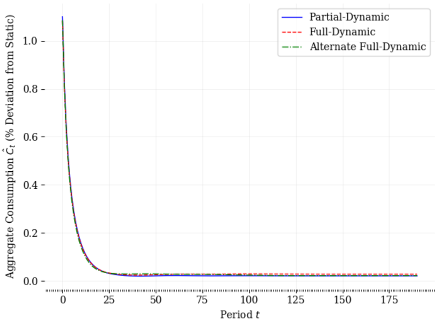



(b) All Other Demographics (% Deviation from Static)

# Transition Path of Aggregate Consumption



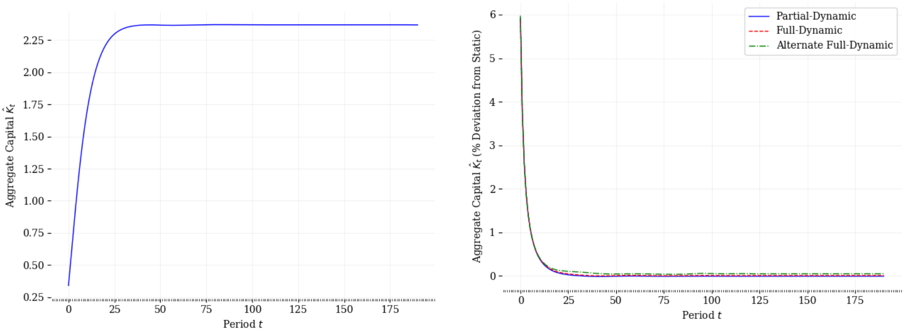

(a) Static Demographics




(b) All Other Demographics (% Deviation from Static)

# Transition Path of Aggregate Capital

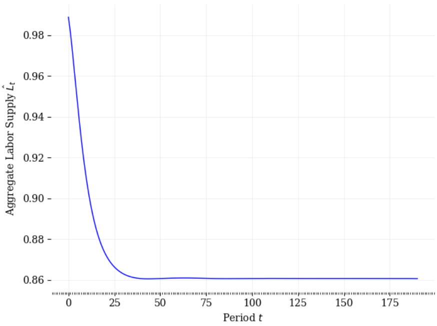
Figure: Time Path of Aggregate Capital $\hat{K}_t$



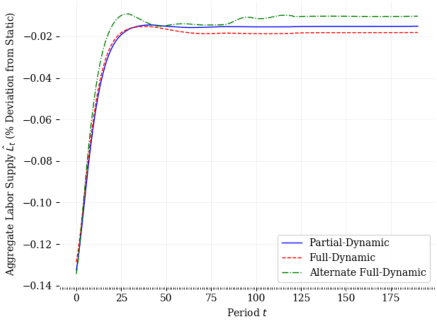

(a) Static Demographics

(b) All Other Demographics (% Deviation from Static)

# Transition Path of Aggregate Labor Supply




(a) Static Demographics




(b) All Other Demographics (% Deviation from Static)

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# Conclusion

- Full dynamic demographic forecasting seems realistic, but forecasts vary depending on model used
- Macroeconomic results differ by demographic assumptions
  - Distributional differences
  - Short-run and medium-run aggregate variable differences
- Results certainly apply to demographic assumptions used in other models
- Extensions: endogenous fertility, in the spirit of Barro and Becker (1989)

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