Effective Programming Practices for Economists

Numerical Optimization

Derivative-Free Trust Region Algorithms

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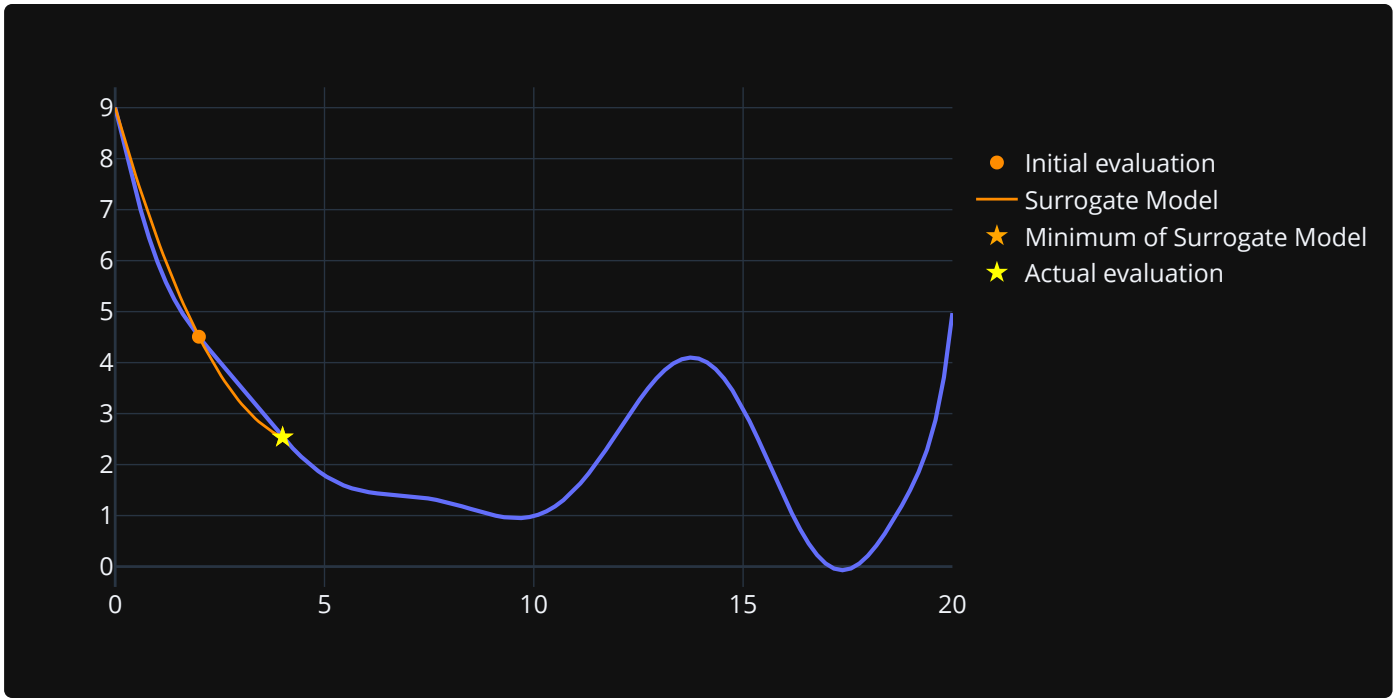
Basic Idea (optimagic docs)

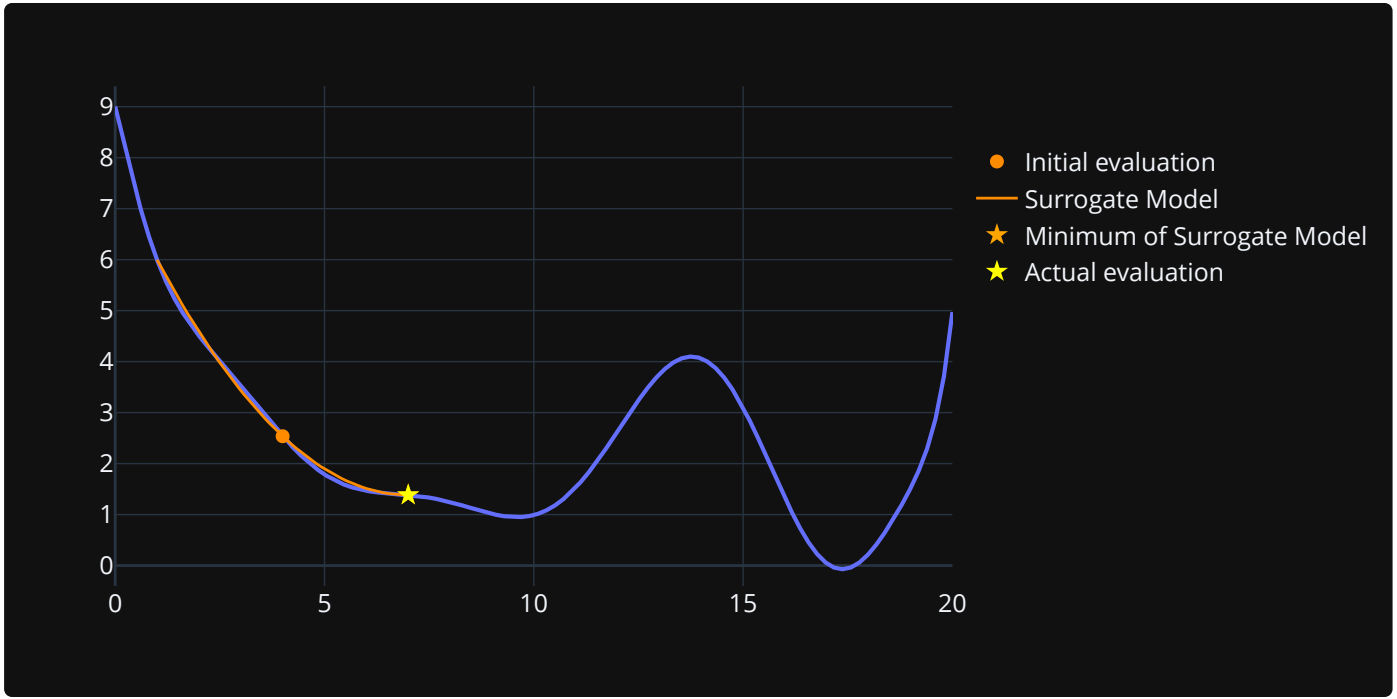
- Similar to derivative based trust region algorithm.
- Instead of Taylor approximation, use a surrogate model based on interpolation or regression.
  - Interpolation: Function is evaluated at exactly as many points as you need to fit the model.
  - Regression: Function is evaluated at more points than you strictly need. Better for noisy functions.
  - In general: Evaluation points are spread further out than for numerical derivatives.
- How the evaluation points are determined is complicated. It is also crucial for the efficiency of the algorithm.

Initial Evaluation



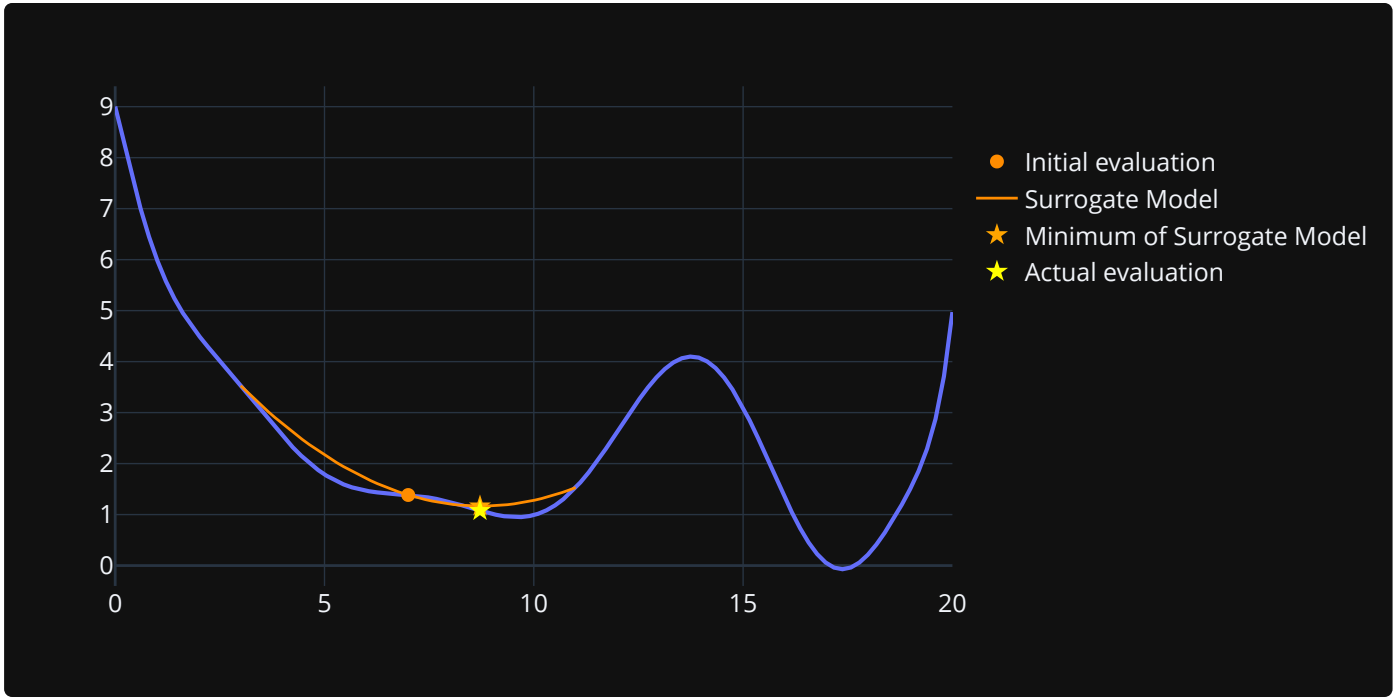
Actual $=$ expected $\Rightarrow$ accept, increase trust region radius

Iteration 1



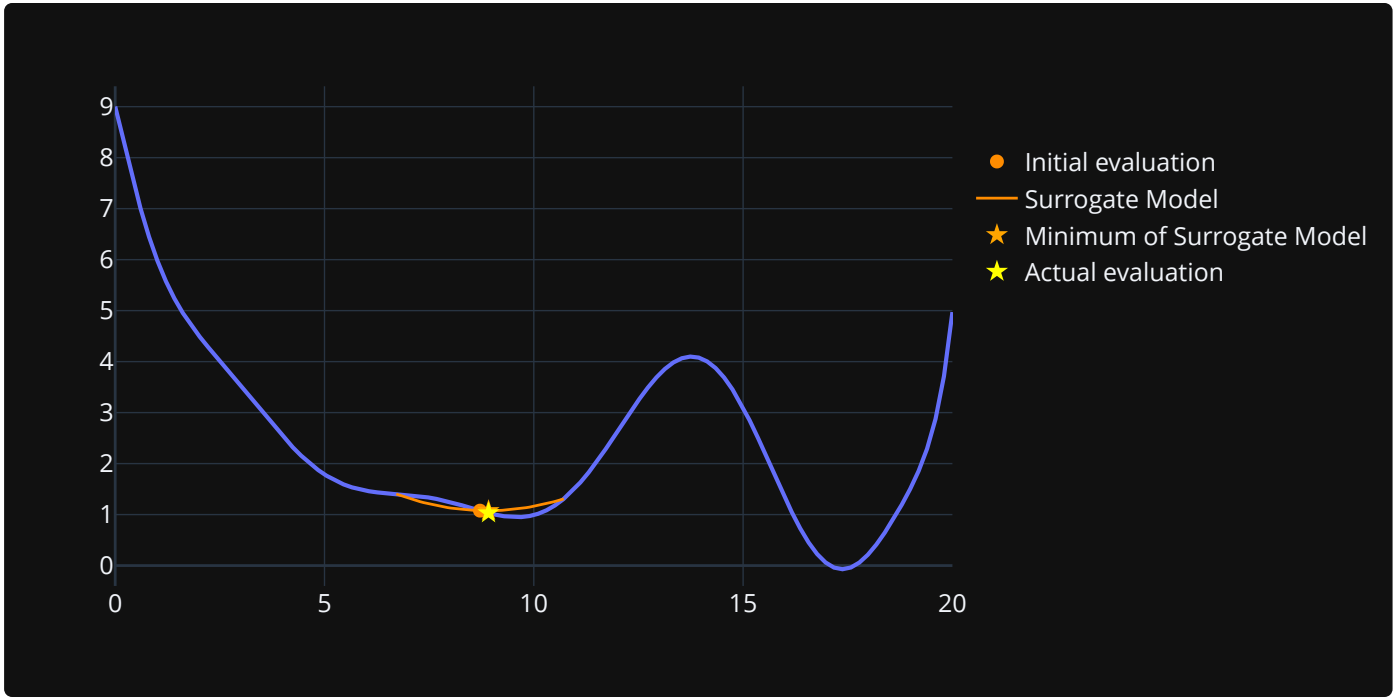
Actual $=$ expected $\Rightarrow$ accept, increase trust region radius

Iteration 2



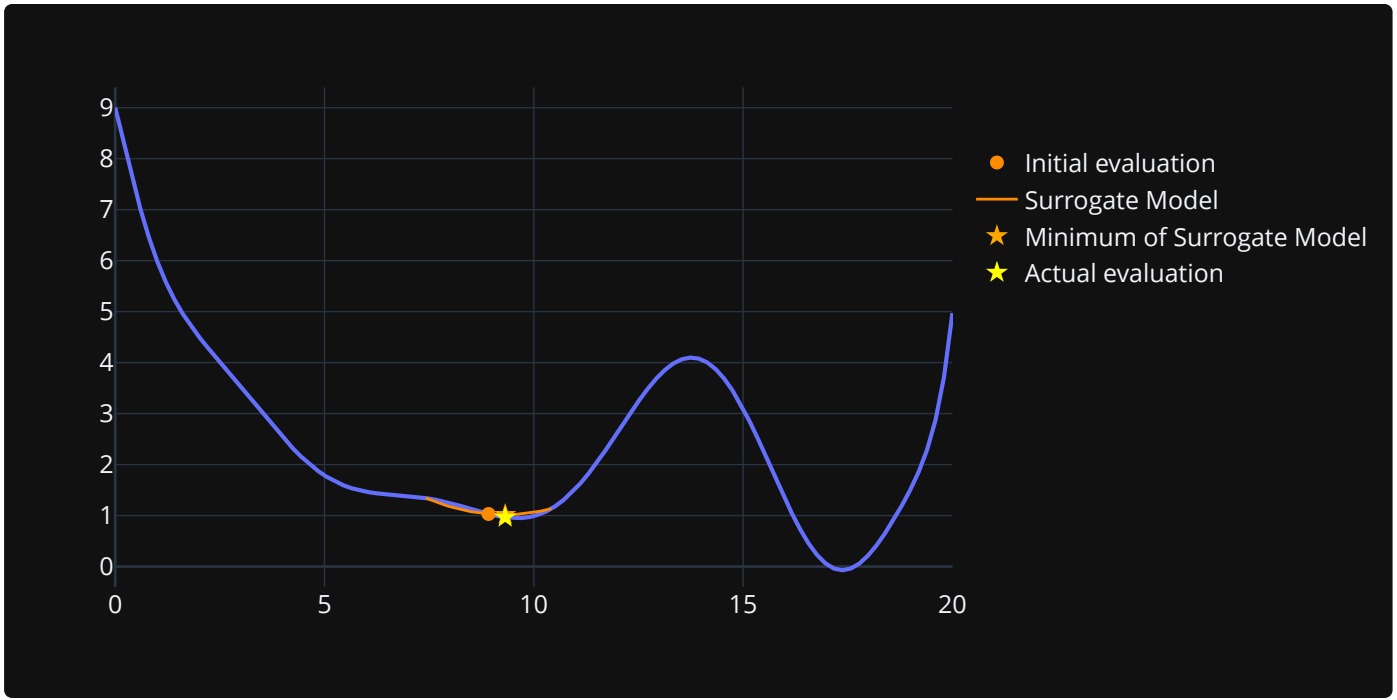
Actual $\approx$ expected, but step small $\Rightarrow$ accept, decrease trust region radius

Iteration 3



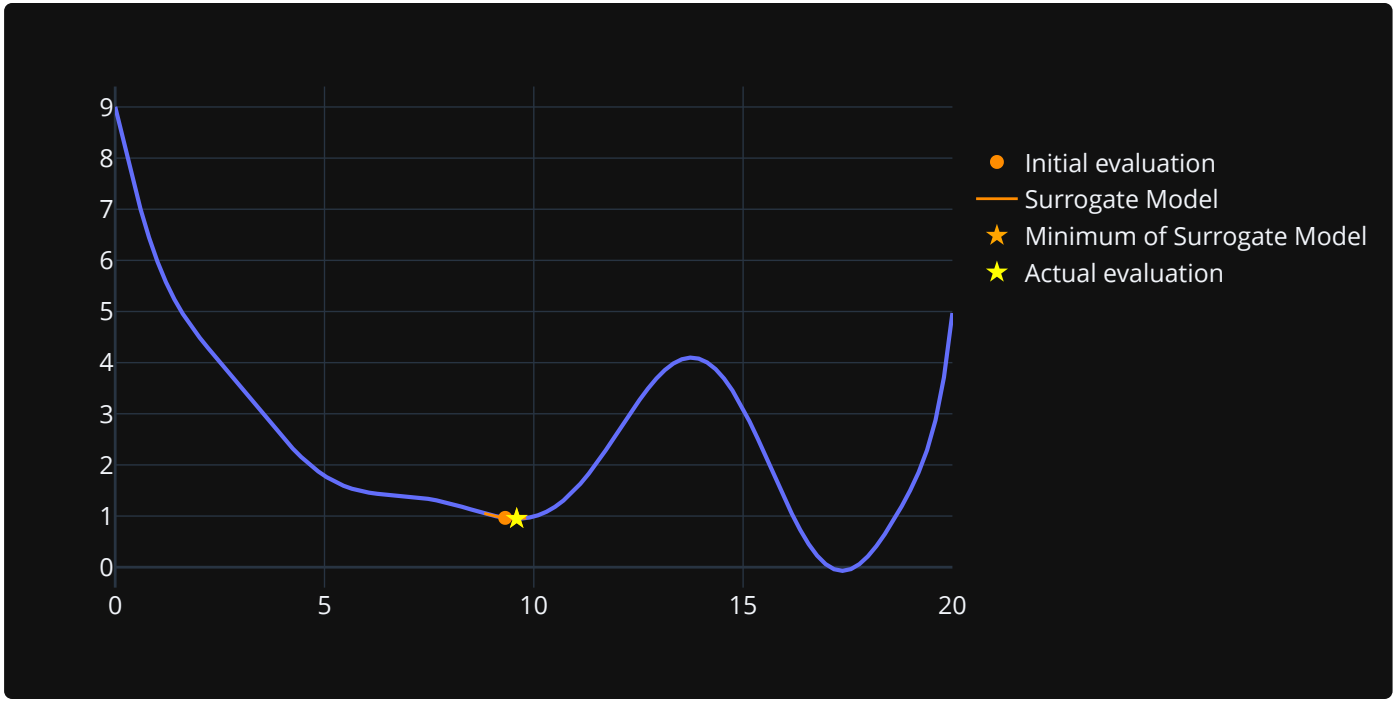
Actual $\approx$ expected, but step small $\Rightarrow$ accept, decrease trust region radius

Iteration 4



Actual $\approx$ expected, but step small $\Rightarrow$ accept, decrease trust region radius

Iteration 5



Converge because trust region radius shrinks to zero.

Some Remarks

- Within the trust region, the fit is generally better than the gradient based trust region algorithm
- By construction at the boundaries of the trust region for interpolation
- Choose between the two based on computation speed
  - If you have fast closed form derivatives, use the derivative based algorithm
  - If you only have numerical derivatives, use this instead
- It is intuitively very clear how this can work for noisy functions if enough evaluations are used for each surrogate model

A real algorithm: COBYLA