Eckstein-Keane-Wolpin models

An invitation for transdisciplinary collaboration

The OSE team

February 19, 2021

Open Source Economics

Computational modeling in economics

Motivation

- Facilitate academic rigor
- Study mechanisms
- Predict public policies

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- Predict public policies

Transdisciplinary in nature

- Economic model
- Mathematical framework
- Computational implementation

Understanding individual decisions

- Human capital investment
- Consumption–savings decision

Eckstein–Keane–Wolpin models

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Predicting effects of policies

- Welfare programs
- Tax schedules

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Mathematical framework and implementation

- Finite-horizon discrete Markov decision problem
- Backward induction algorithm

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$\Longrightarrow$ Transdisciplinary research on their **economics**, data, and computation

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Partners

Institute for
Numerical Simulation

DIW BERLIN

Statistisk sentralbyrå
Statistics Norway

UNIL | Université de Lausanne

Roadmap

- Economic model
- Mathematical formulation
- Calibration

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- Economic model
- Mathematical formulation
- Calibration
- Example
- Pipeline
- Projects

Economic model

Decision Problem

$t = 1, \ldots, T$	decision period
$s_t \in S$	state
$a_t \in A$	action
$a_t(s_t)$	decision rule
$u_t(s_t, a_t)$	immediate utility

Timing of events

Choose

Learn a_t Receive

$\{u(s_t, a_t)\}_{a_t \in \mathcal{A}}$ $u(s_t, a_t)$

t

Choose

Learn a_{t+1} Receive

$\{u(s_{t+1}, a_{t+1})\}_{a_{t+1} \in \mathcal{A}}$ $u(s_{t+1}, a_{t+1})$

$t+1$

$\pi = (a_1^\pi(s_1), \dots, a_T^\pi(s_T))$	policy
δ	discount factor
$p_t(s_t, a_t)$	conditional distribution

Individual's objective

$$\max_{\pi \in \Pi} \mathrm{E}_{s_1}^{\pi} \left[\sum_{t=1}^{T} \delta^{t-1} u_t(s_t, a_t^{\pi}(s_t)) \,\middle|\, \mathcal{I}_1 \right]$$

Core economics

- Rational expectations
- Exponential discounting
- Time-separability

Mathematical formulation

Dynamic programming

Policy evaluation

$$v_t^{\pi}(s_t) = \mathrm{E}_{s_t}^{\pi}\left[\sum_{j=0}^{T-t} \delta^j \, u_{t+j}(s_{t+j}, a_{t+j}^{\pi}(s_{t+j})) \;\middle|\; \mathcal{I}_t\right]$$

Optimality equations

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A}\left\{u_t(s_t, a_t) + \delta \, \mathrm{E}_{s_t}^{\pi^*}\left[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t\right]\right\}$$

Backward induction algorithm

for $t = T, \ldots, 1$ **do**

if t = T **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in A} \Big\{ u_T(s_T, a_T) \Big\} \qquad \forall\, s_T \in S$$

else

Compute $v_t^{\pi^*}(s_t)$ for each $s_t \in S$ by

$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \Big\{ u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi}\Big[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \Big] \Big\}$$

and set

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in A} \Big\{ u_t(s_t, a_t) + \delta\, \mathrm{E}_{s_t}^{\pi}\Big[v_{t+1}^{\pi^*}(s_{t+1}) \mid \mathcal{I}_t \Big] \Big\}$$

end if

end for

Calibration procedure

Data

Dataset

$$\mathcal{D} = \{a_{it}, \bar{s}_{it}, \bar{u}_{it} : i = 1, \ldots, N; t = 1, \ldots, T_i\}$$

State variables

- $s_t = (\bar{s}_t, \varepsilon_t)$
  - $\bar{s}_t$ observed
  - ε_t unobserved

Procedures

Likelihood-based

$$\hat{\vartheta} = \arg\max_{\vartheta \in \Theta} \prod_{i=1}^{N} \prod_{t=1}^{T_i} p_{it}(a_{it}, \bar{u}_{it} \mid \bar{s}_{it}, \vartheta)$$

Simulation-based

$$\hat{\vartheta} = \arg\min_{\vartheta \in \Theta} \left(M_D - M_S(\vartheta)\right)' W \left(M_D - M_S(\vartheta)\right)$$

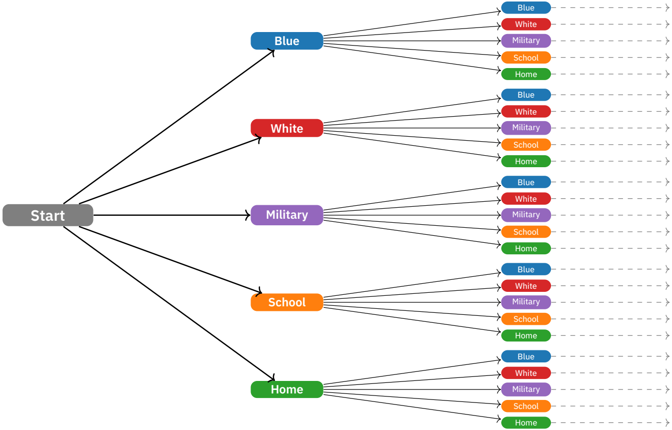
Example

Seminal paper

Michael P Keane and Kenneth I Wolpin. 1997. "The career decisions of young men." *Journal of Political Economy* 105 (3): 473–522 .

- The study follows individuals over their working life from young adulthood at age 16 to retirement at age 65 where the decision period $t = 16, \dots, 65$ is a school year.
- Individuals decide $a \in \mathcal{A}$ whether to work in a blue-collar or white-collar occupation ($a = 1, 2$), to serve in the military ($a = 3$), to attend school ($a = 4$), or to stay at home ($a = 5$).

Decision tree



Immediate utility

$$u_t(s_t) = \begin{cases} \zeta_a(s_t) + w_a(s_t) & \text{if } a \in \{1, 2, 3\} \\ \zeta_a(s_t) & \text{if } a \in \{4, 5\} \end{cases}$$

Informed by reduced-form evidence

- Mincer equation
- Sheepskin effects
- Skill depreciation
- Mobility and search costs
- Monetary and psychic cost of schooling

Transitions

- Work experience $\boldsymbol{k}_t$ and years of completed schooling h_t evolve deterministically.

$$
\begin{aligned}
k_{a,t+1} &= k_{a,t} + 1[a_t = a] \qquad \text{if } a \in \{1, 2, 3\} \\
h_{t+1} &= h_t \quad + 1[a_t = 4]
\end{aligned}
$$

- Productivity shocks $\boldsymbol{\epsilon}_t$ are uncorrelated across time and follow a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}$.

- Given the structure of the utility functions and the distribution of the shocks, the state at time t is $s_t = \{\boldsymbol{k}_t, h_t, t, a_{t-1}, \boldsymbol{e}, \boldsymbol{\epsilon}_t\}$.

Utility of blue-collar occupation

- Non-pecuniary

$$\zeta_1(\cdot) = \alpha_1 + c_{1,1} \cdot 1[a_{t-1} \neq 1] + c_{1,2} \cdot 1[k_{1,t} = 0] \\ + \vartheta_1 \cdot 1[h_t \geq 12] + \vartheta_2 \cdot 1[h_t \geq 16] + \vartheta_3 \cdot 1[k_{3,t} = 1]$$

- Wage component

$$w_1(\cdot) = r_1 \, x_1(\cdot),$$

where $x_1(\cdot)$ is the occupation-specific skill level.

Skill production for blue-collar occupation

$$x_1(\cdot) = \exp\left(\Gamma_1(\boldsymbol{k}_t, h_t, t, a_{t-1}, e_{j,1}) \cdot \varepsilon_{1,t}\right)$$

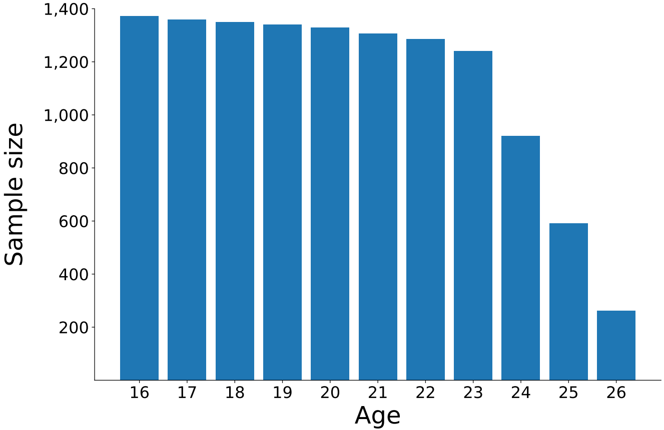
- Parameterization of the deterministic component of the skill production function:

$$\begin{aligned}\Gamma_1(\cdot) = e_{j,1} &+ \beta_{1,1} \cdot h_t + \beta_{1,2} \cdot 1[h_t \geq 12] + \beta_{1,3} \cdot 1[h_t \geq 16] \\ &+ \gamma_{1,1} \cdot k_{1,t} + \gamma_{1,2} \cdot (k_{1,t})^2 + \gamma_{1,3} \cdot 1[k_{1,t} > 0] \\ &+ \gamma_{1,4} \cdot t + \gamma_{1,5} \cdot 1[t < 18] \\ &+ \gamma_{1,6} \cdot 1[a_{t-1} = 1] + \gamma_{1,7} \cdot k_{2,t} + \gamma_{1,8} \cdot k_{3,t}\end{aligned}$$

National Longitudinal Survey of Youth 1979

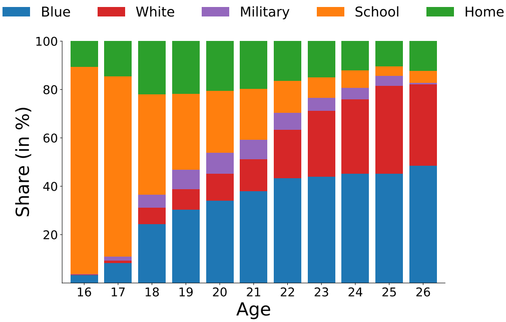
- 1,373 individuals starting at age 16
- Life cycle histories
  - School attendance
  - Occupation-specific work status
  - Wages

Sample size

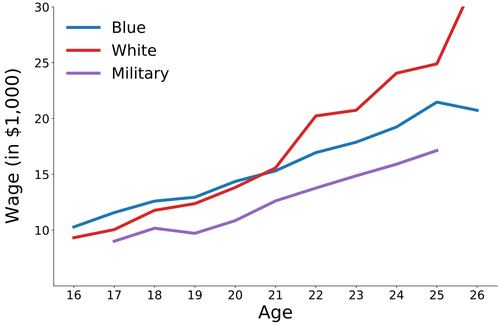


1,400
1,200
1,000
800
600
400
200
Sample size
16
17
18
19
20
21
22
23
24
25
26
Age

Data descriptives

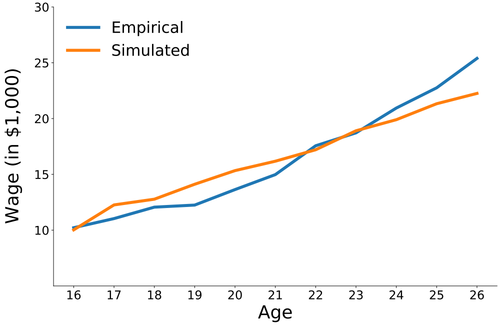


(a) Choices

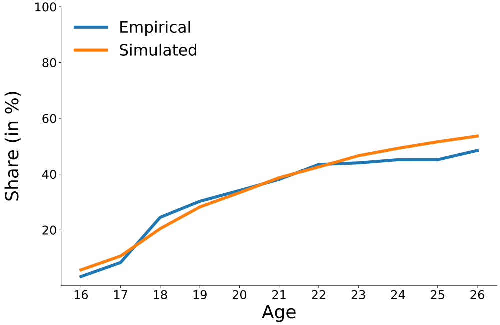


(b) Wages

Calibration results

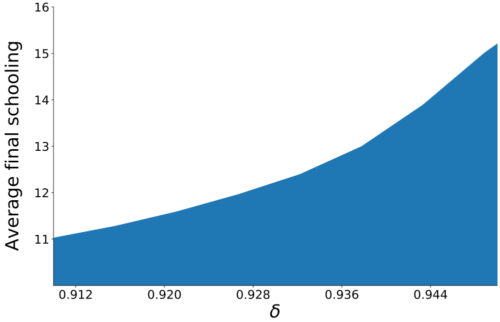


(c) Average wage

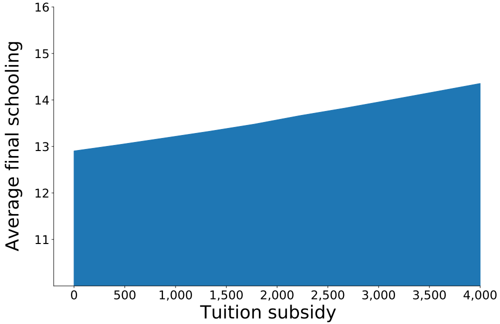


(d) Blue-collar

Economic insights



(e) Time preference



(f) Tuition subsidy

Pipeline

Tooling

respy

GitHub	OpenSourceEconomics/respy
Docs	respy.readthedocs.io

estimagic

GitHub	OpenSourceEconomics/estimagic
Docs	estimagic.readthedocs.io

Workflow

```python
import respy as rp
from estimagic import maximize

# obtain model input
params, options, df = rp.get_example_model("kw_97_extended_respy")

# process model specification
log_like = rp.get_log_like_func(params, options, df)
simulate = rp.get_simulate_func(params, options)

# perform calibration
results, params_rslt = maximize(log_like, params, "nlopt_bobyqa")

# conduct analysis
df_rslt = simulate(params_rslt)
```

Model parameterization

category	name	value	name
delta	**delta**	9.370735e-01	delta_delta
wage_white_collar	**constant**	8.741888e+00	wage_white_collar_constant
	exp_school	6.548940e-02	wage_white_collar_exp_school
	exp_white_collar	1.763655e-02	wage_white_collar_exp_white_collar
	exp_white_collar_square	-4.215936e-02	wage_white_collar_exp_white_collar_square
	exp_blue_collar	3.431936e-02	wage_white_collar_exp_blue_collar
	exp_military	1.406945e-02	wage_white_collar_exp_military
	hs_graduate	-3.599855e-03	wage_white_collar_hs_graduate
	co_graduate	2.301313e-03	wage_white_collar_co_graduate
	period	9.577717e-03	wage_white_collar_period
	is_minor	-1.509984e-01	wage_white_collar_is_minor

Model options

	value
estimation_draws	200
estimation_seed	500
estimation_tau	500
interpolation_points	-1
n_periods	50
simulation_agents	5000
simulation_seed	132
solution_draws	500
solution_seed	456
monte_carlo_sequence	random
covariates	{'hs_graduate': 'exp_school >= 12', 'co_gradua...

Projects

Research projects

Economics and data

- **Biased expectations**
- Robust decisions
- Option value

Incorporate subjective expectations

Collaboration with DIW for SOEP-IS data collection

Research projects

Economics and data

- Biased expectations
- **Robust decisions**
- Option value

Account for ubiquitous uncertainties

Robust decision in light of model misspecification

Research projects

Economics and data

- Biased expectations
- Robust decisions
- **Option value** Schooling reform for identification and validation
  Collaboration with Statistics Norway

Research projects

Computation

- **Uncertainty quantification**
- Global optimization
- HPC implementation

Capture parametric uncertainty

Assess competing policy implications

Research projects

Computation

- Uncertainty quantification
- **Global optimization**
- HPC implementation

Explore estimation uncertainty

Acknowledge multiplicity of local minima

Research projects

Computation

- Uncertainty quantification
- Global optimization
- **HPC implementation**

  Enable increased realism and auditing of economic models

  Exploit large-scale parallelism on supercomputers

Conclusion

Join us!

http://bit.ly/ose-github

http://bit.ly/ose-zulip

https://twitter.com/open_econ

https://open-econ.org

Open Source Economics

respy

econsa

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