DIW GRADUATE CENTER

The Discrete-Continuous Endogenous Grid Method (DC-EGM)

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Endogenous Grid Method (EGM)

Introduced to the economic literature by Carroll (2006)

- computational method for solving dynamic structural models with a continuous choice variable
- value functions typically concave, solution via Euler Equation
- controlling post-decision state:
  - set up exogenous grid over endogenous variable (savings)
  - solve for optimal policy rule (consumption)
  - endogenously determine pre-decision state, assets.

Dynamic models with discrete and continuous choices

- discrete choice introduce non-concave regions in the value function $\rightarrow$ discontinuities in the policy function for continuous choice var.
- multiple solutions to the Euler Equation, EGM not able to select the global optimum

Iskhakov et al. (2017) extend EGM to discrete-continuous models:

- EGM-nested steps to resolve above problems: upper and secondary envelope
- choice-specific taste shocks, income shocks acting to smooth out value function and simplify solution

The retirement model

Agents' problem:

$$\max_{\{c_t, d_t\}_{t=1}^{T}} \sum_{t=1}^{T} \left(\frac{u(c_t)^{1-\theta} - 1}{1-\theta} - \delta d_t \right) \tag{1}$$

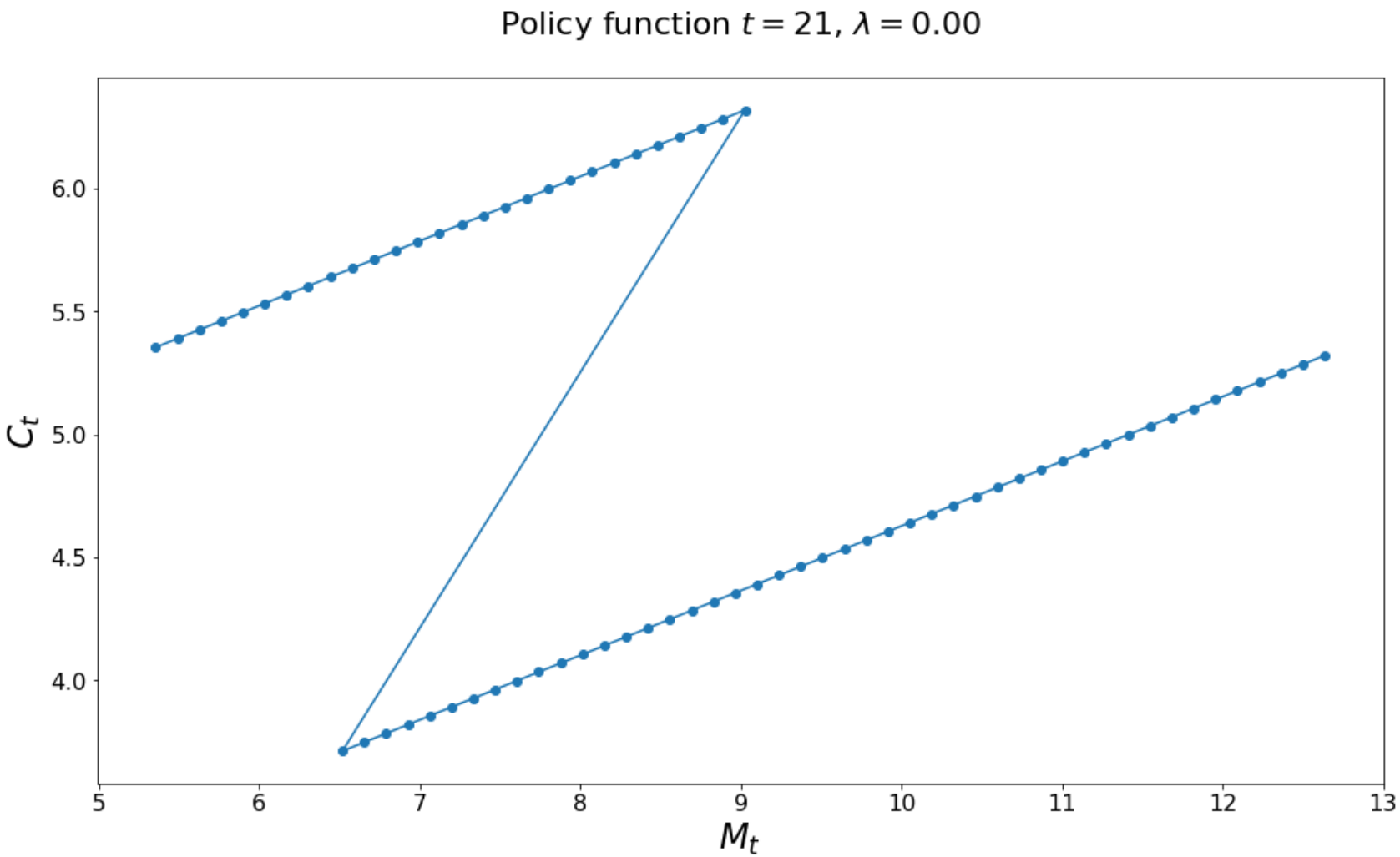
- $d_t = 0$ retirement
- $d_t = 1$ continue work

Budget constraint sequence:

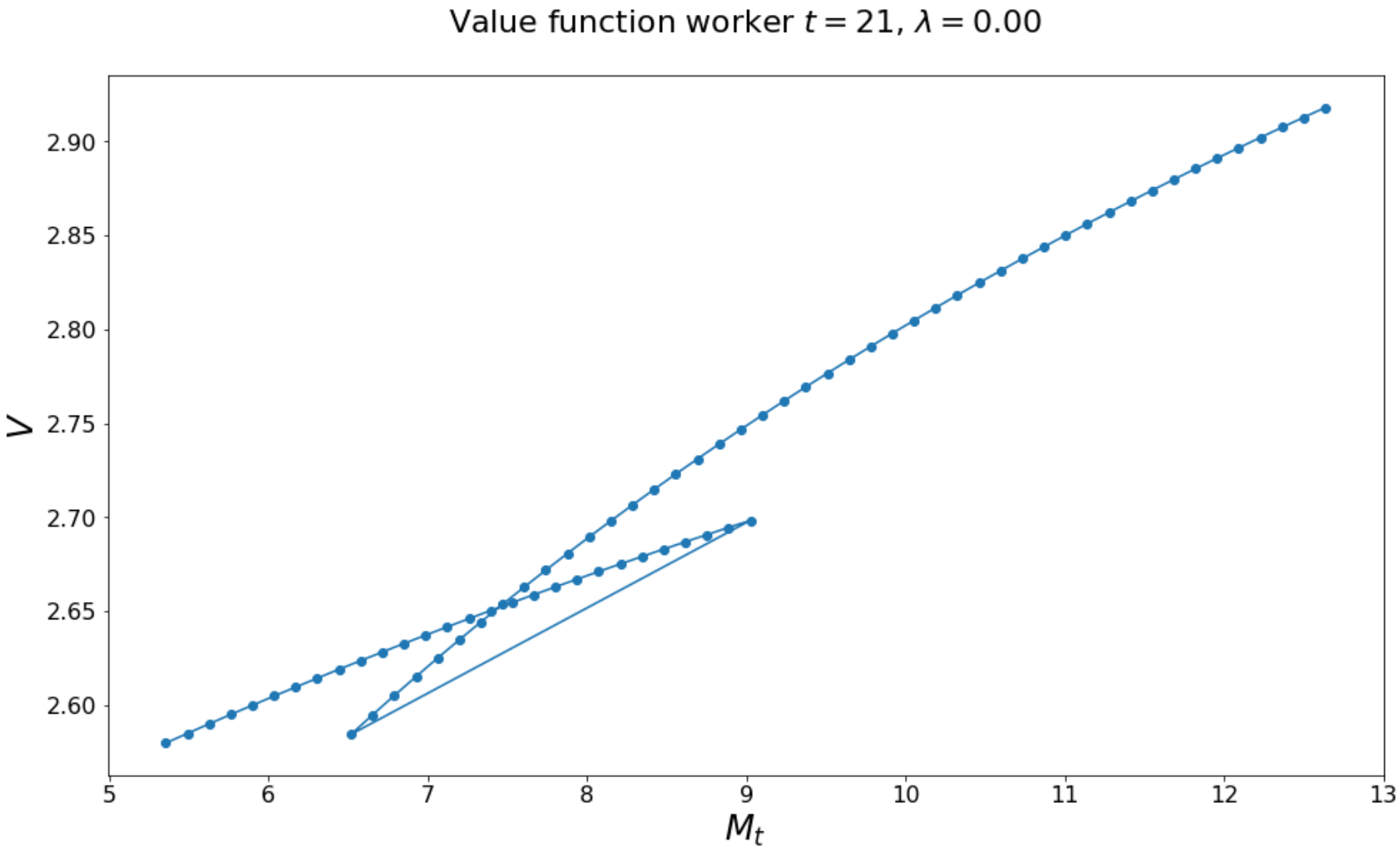
$$c_t \leq M_t$$

$$M_t = R(M_{t-1} - c_{t-1}) + y d_{t-1}$$

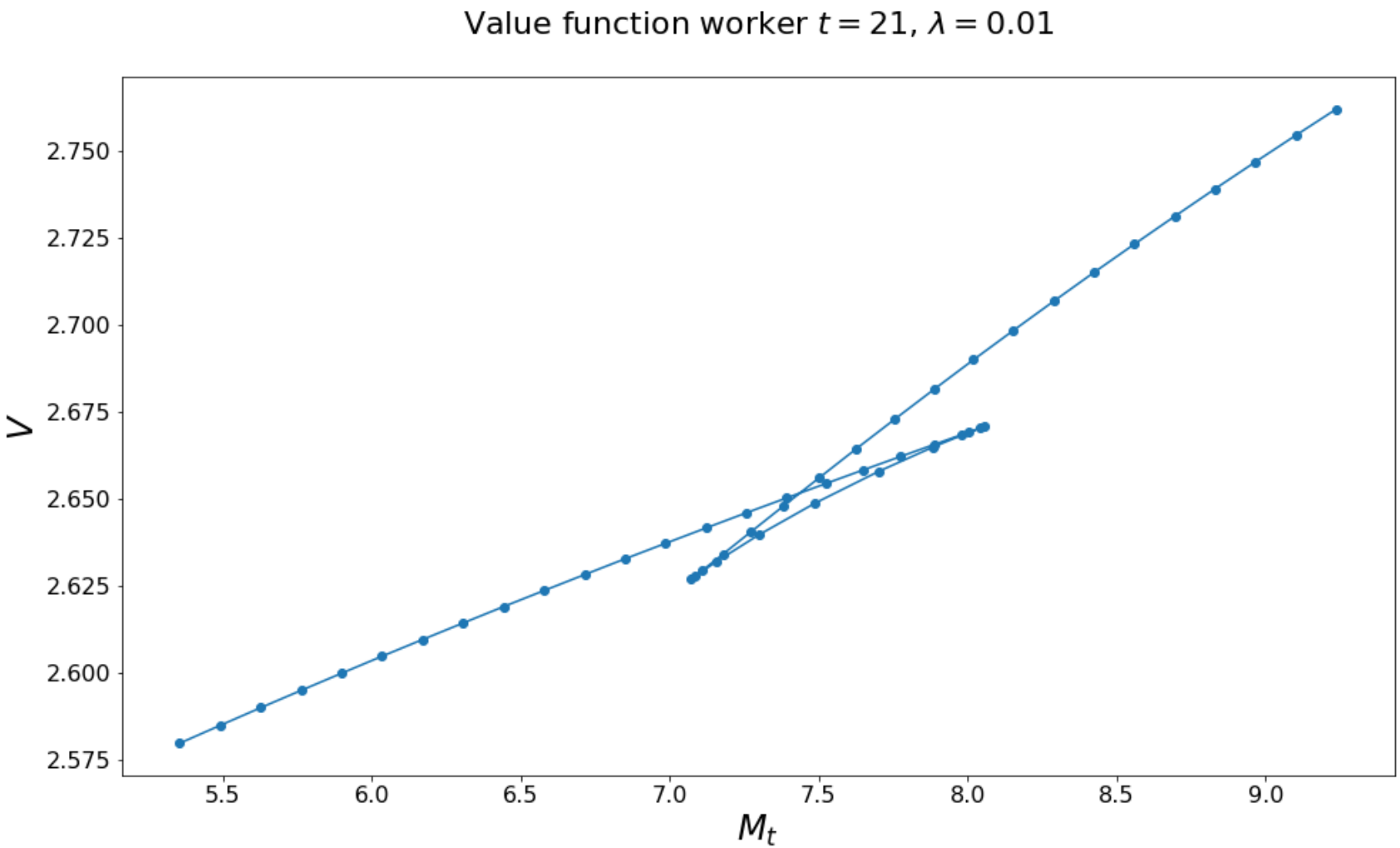
Non-monotonic endogenous grid



Non-concave region in value function (1)



Nonconcave region in value function (2)



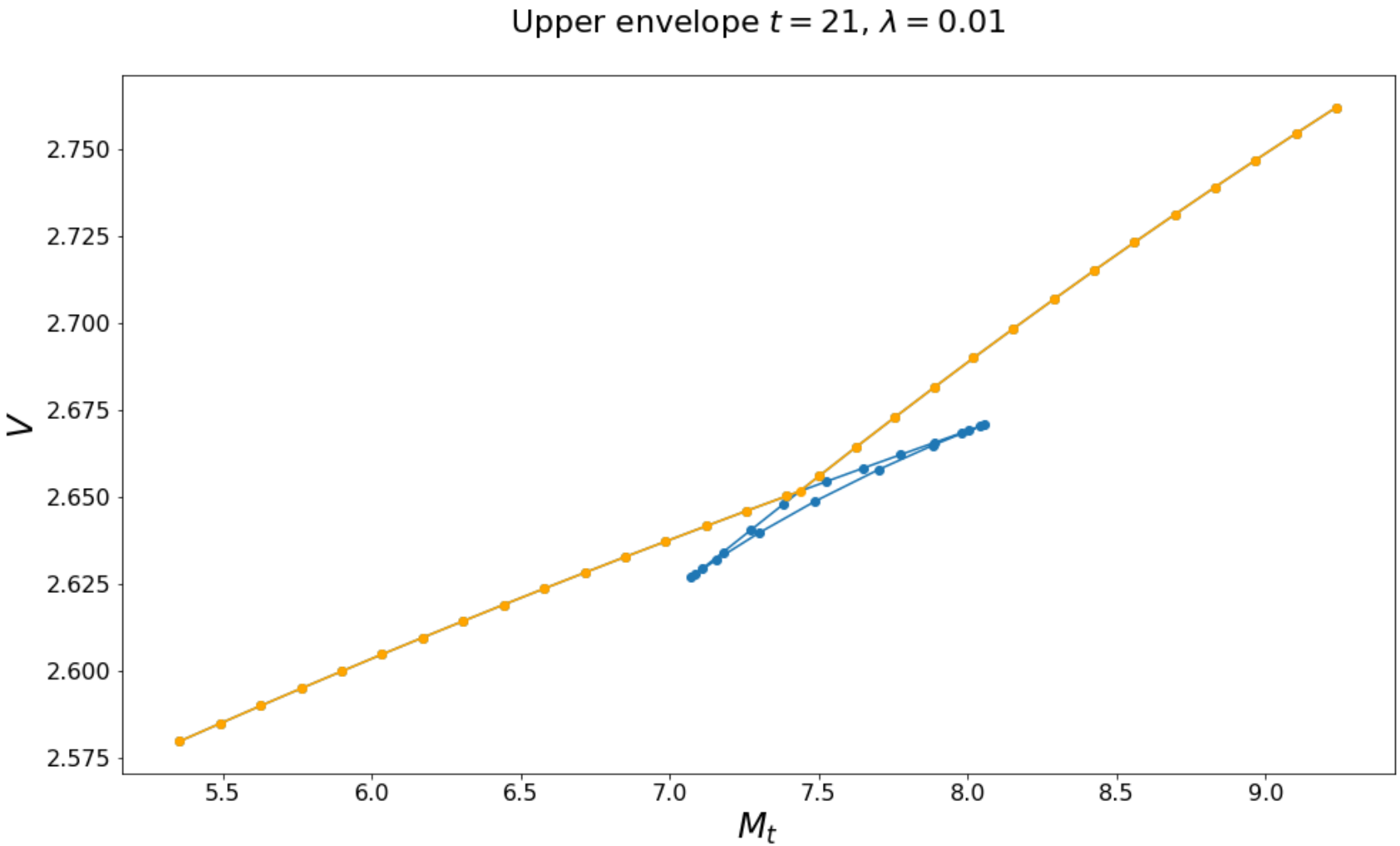
Handling the discrete choice

- regions of M_t where multiple local optima for consumption exist
- Euler equation is only necessary, but not sufficient, nonconcave region in the value function
- EGM produces a consumption correspondence and not an optimal consumption function

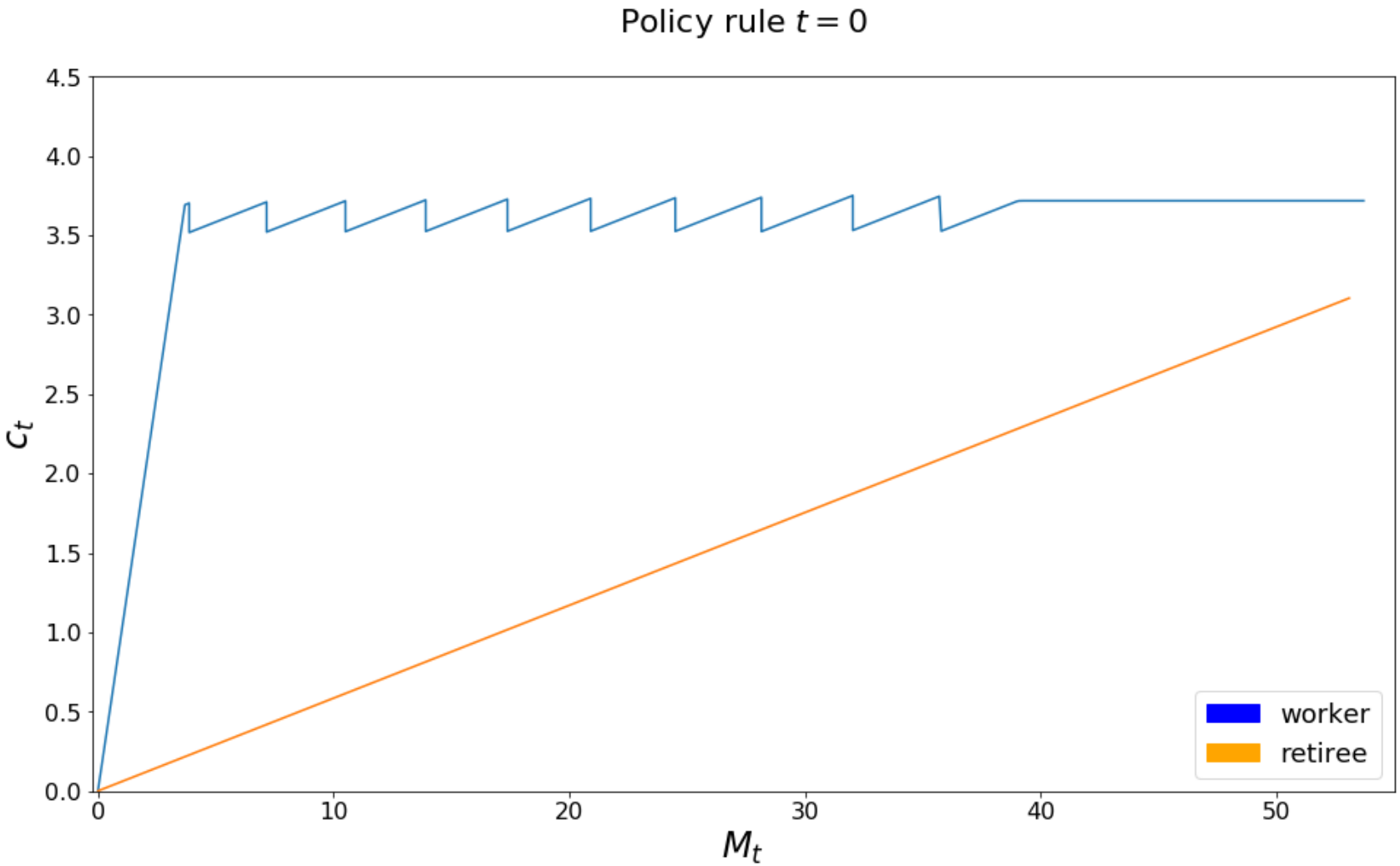
The "dc step":

- calculate upper envelope over overlapping segments of decision specific value functions
- eliminate dominated grid points
- select the "correct" solution of the Euler equation, i.e. obtain optimal consumption

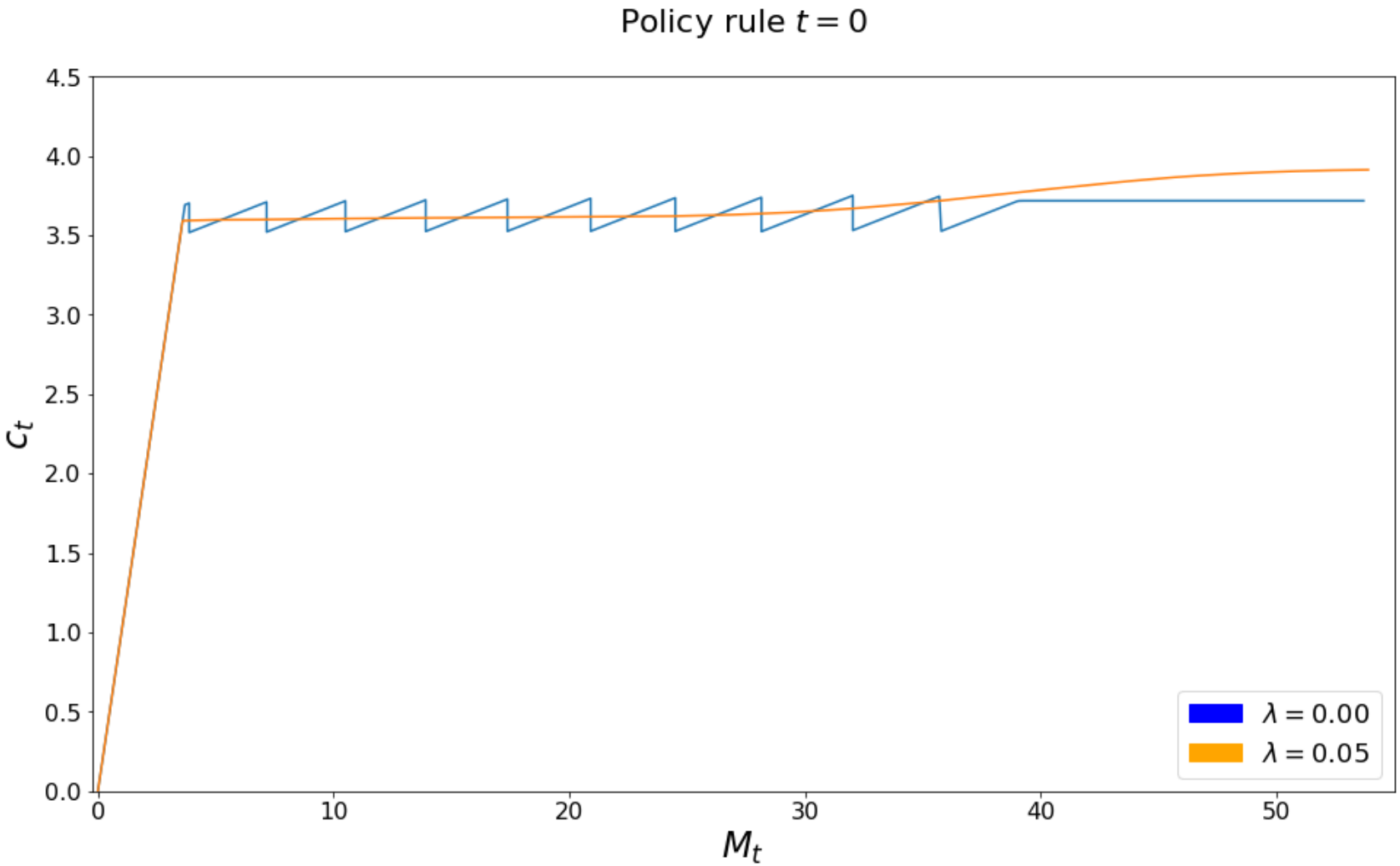
Upper envelope



Policy rule


Policy rule $t = 0$
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
c_t
0
10
20
30
40
50
M_t
worker
retiree

Smoothing: Choice specific taste shocks



Have a great day!

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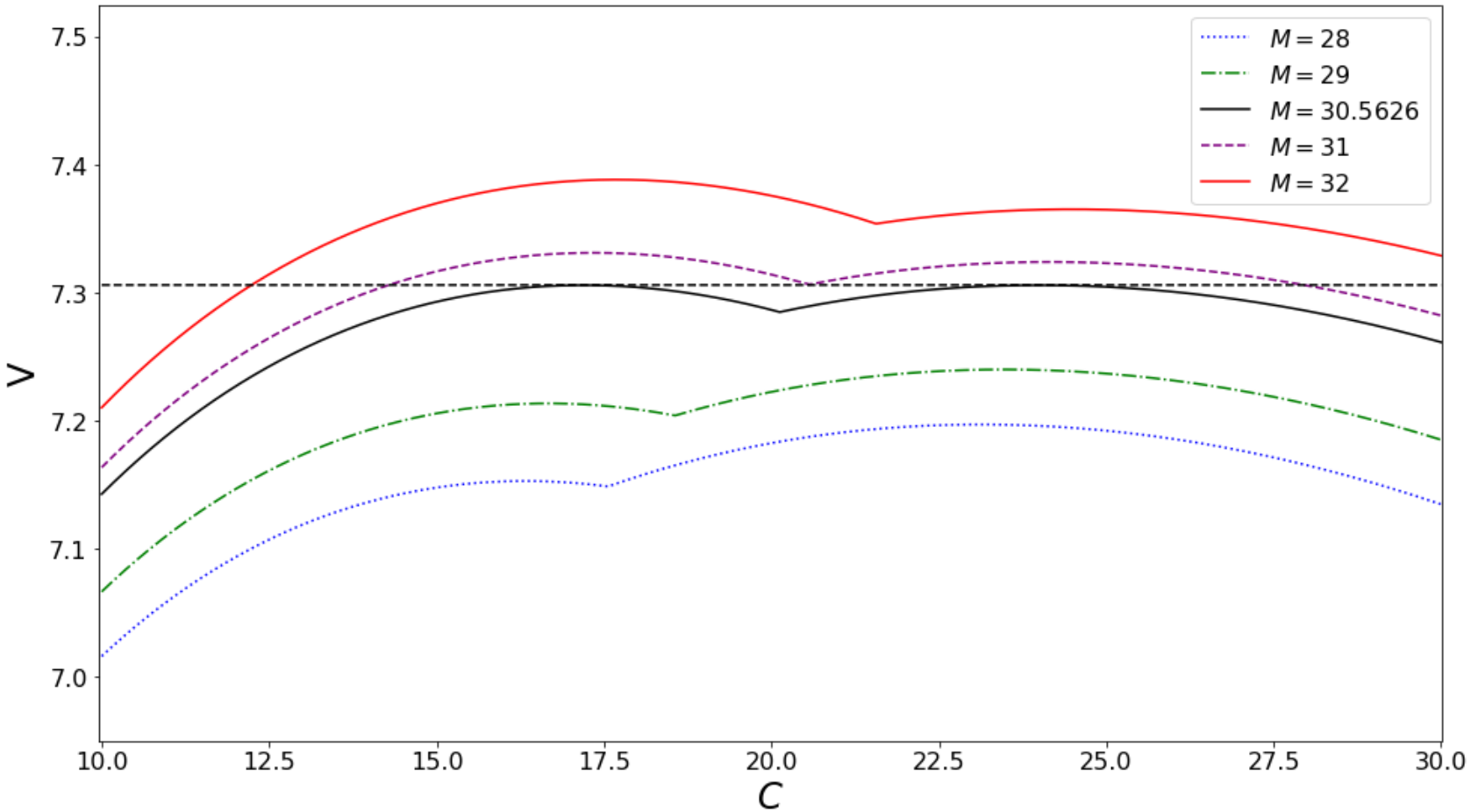
Appendix (1)


7.5
7.4
7.3
7.2
7.1
7.0
V
10.0
12.5
15.0
17.5
20.0
22.5
25.0
27.5
30.0
C
M = 28
M = 29
M = 30.5626
M = 31
M = 32

Appendix (2)