Beliefs under Ambiguity

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Motivation

- Beliefs are important determinants of real world decisions.
- If people are not ambiguity neutral, eliciting subjective probabilites becomes very hard.

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- Experiments: Under ambiguity (probabilites are unknown) people behave differently than under risk (probabilites are known)
- Recently it was shown that ambiguity attitudes can also be estimated for real world events (Baillon et al. 2018). (-> subject-specific)

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- Contributions:
  - Elicitation of ambiguity attitudes over the stock-market in representative data set.
  - Methodological: Elicitation of both subjective beliefs and ambiguity attitudes

Overview

Data sets

- Representative Sample of the Dutch population:
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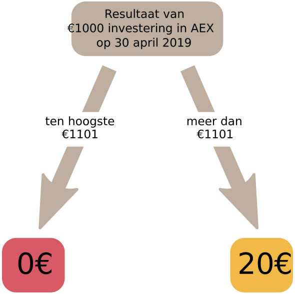
  Longitudinal Internet Studies for the Social sciences (LISS)
- 2169 participants
- First wave in Mai 2018, second wave in November 2018 (planned: 8 waves)
- 21–28 binary choices
- One choice is payed out.

Two Options

Option A

- Bet on AEX
- Win 20 EUR depending on AEX performance



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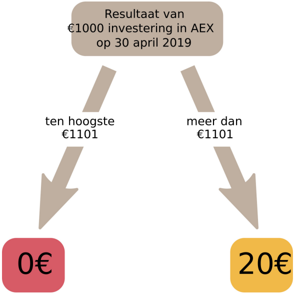
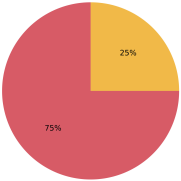


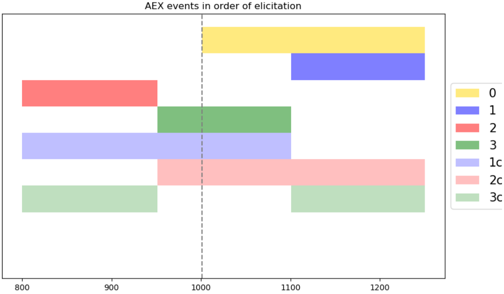
Option B

- Lottery with known probability
- Win 20 EUR with probability p



Two Options

- seven AEX-events:



- (values adjusted by interest on savings account)
- For each event: 3–4 choices with varying p in Option B

Distribution of Matching Probabilities

- Matching probability for each event calculated by switch point:

$$\begin{gathered} W(AEX)U(20 \text{ EUR}) = W(LOT)U(20 \text{ EUR}) \\ W(AEX) = p \end{gathered} \tag{1}$$

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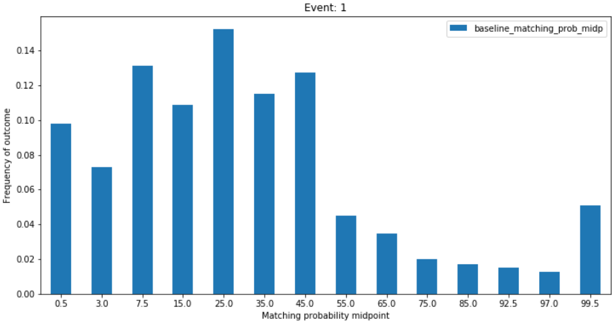
- One example:



Overview

Introduction

Data

Choice Model

First Results

Three components

1. Subjective probabilities/beliefs
2. Ambiguity attitudes
3. Error parameter

Subjective Probabilities

- Basic idea: Subjects have subjective probabilities in mind (that satisfies the requirements of probability theory)
- $Pr(\emptyset) = 0$, $Pr(\Omega) = 1$, additivity

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- But subjects make decisions based on probability weights that only satisfy a weaker condition.
- $W(\emptyset) = 0$, $W(\Omega) = 1$, subset condition

$$\forall \text{ disjoint } A, B : W(A \cup B) \geq W(A)$$

Source function

- Relation between subjective probability and probability weights via source function w_{so}:

$$W(E) = w_{so}(\Pr(E))$$

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  - Probability weighting, but no ambiguity: $w_{so}(x) = w(x)$
- We will assume: $w_{Risk}(x) = x$
- If this assumption does not hold, we measure the change in the source function introduced by ambiguity.

Ambiguity Indices (Baillon et al. 2018)

- Goal: Measure ambiguity attitudes (i.e. the source function) based on two indices
- Measuring the source function is not trivial because subjective probabilites are unobserved.

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- $b = 1 - (\bar{m}(E) + \bar{m}(E^C))$

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- Common finding: Ambiguity seeking for low prob. events. Ambiguity aversion for medium and high prob. events.
- $\implies$ Second index is needed for full picture
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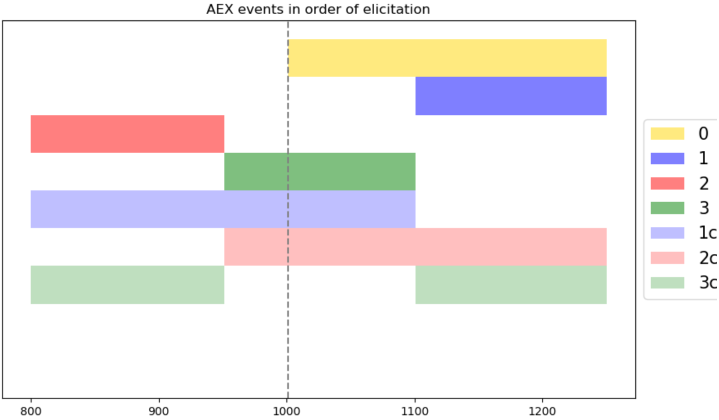
- Basic idea: Composite events on average twice as large as single events.
- $a = 3 \cdot \left(\frac{1}{3} - \left(\bar{m}(E^C) - \bar{m}(E) \right) \right)$

a-insensitivity: a



Linear source function

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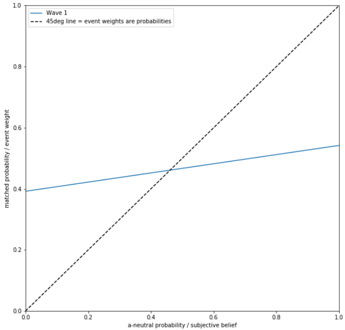
- Ambiguity indices easily interpretable functions of σ, τ:
  - $b = 1 - 2\tau - \sigma$
  - $a = 1 - \sigma$



Error Parameter

- Deterministic model as described so far is rejected by data:
  - Implication of basic model: Subset condition
  - 55 % of participants violate one subset condition (6 possibilites to do so)

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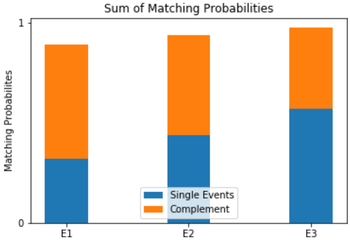
  - Implication of basic model: Subset condition
  - 55 % of participants violate one subset condition (6 possibilites to do so)
  - Implication of linear source function: $P(E_1) + P(E_{1^c}) = 2\tau + \sigma = P(E_2) + P(E_{2^c})$



- $\implies$ add stochastic component.

Parameters to be estimated

1. Subjective probabilities for 2 events: π_1, π_2
   (probability for the third event is inferred)
2. Ambiguity attitudes σ, τ / (a, b)
3. Trembling hand error parameter ω

Likelihood

$$L(c^{\mathsf{AEX}}_{E_1,p}|\pi_1,\pi_2,\sigma,\tau,\omega) = \frac{\omega}{2} + (1-\omega)\mathbb{1}_{\{\tau+\sigma\pi_1>p\}}$$

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$$L(c^{\text{AEX}}_{E_1,p}|\pi_1, \pi_2, \sigma, \tau, \omega) = \frac{\omega}{2} + (1-\omega)\mathbb{1}_{\{\tau+\sigma\pi_1 > p\}}$$

- Open question: Which parameters can be identified on individual level?

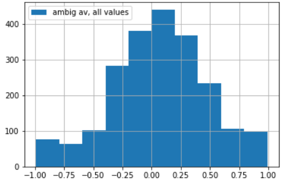
Intuition for identification

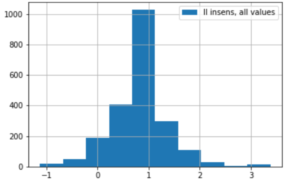
- π_1: Subject chooses AEX-options which include E_1 more often.
- τ: Subject chooses AEX-options more often in general.
- σ: Choice behavior for complementary and single events is more different.
- ω: Subject makes more choices not rationalizable with deterministic model (especially subset conditions)

Overview

Ambiguity attitudes

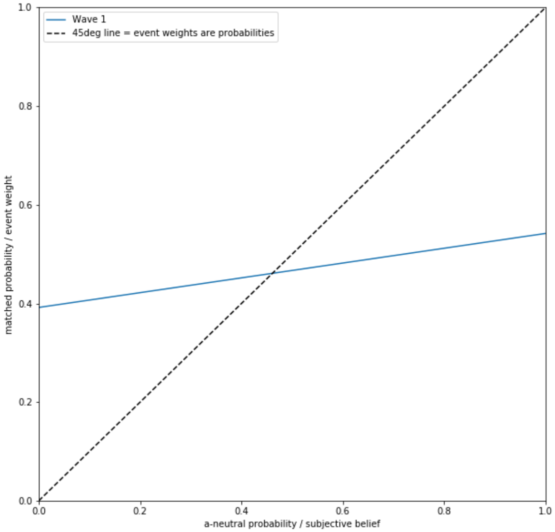
- We can identify ambiguity attitudes





- More subjects are ambiguity averse (not all)
- Some subjects show a-insensitivity outside the unit interval.

Source function of representative agent



Beliefs

- We can calculate subjective beliefs (considering ambiguity attitudes, but without error model)
- Subjective probabilities often outside of unit interval

Relation to general ambiguity (Elsberg urns)

- Compare individual ambiguity indices with Elsberg urn elicitation from an earlier study
- a-insensitivity: corr 0.012

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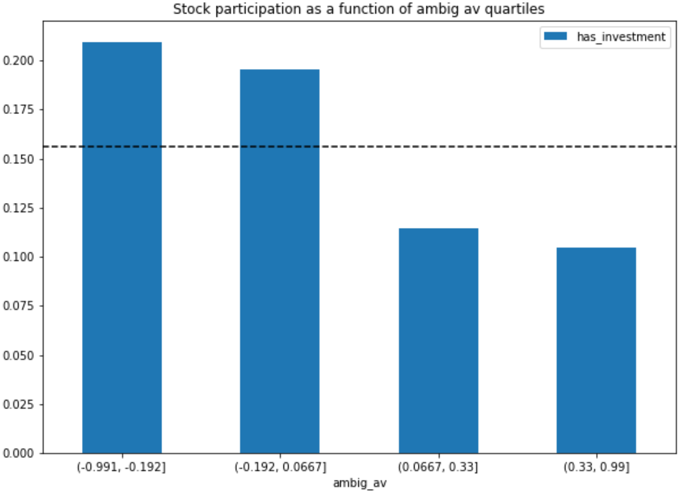
- Compare individual ambiguity indices with Elsberg urn elicitation from an earlier study
- a-insensitivity: corr 0.012
- ambiguity aversion: corr 0.058

Relation to portfolio choice



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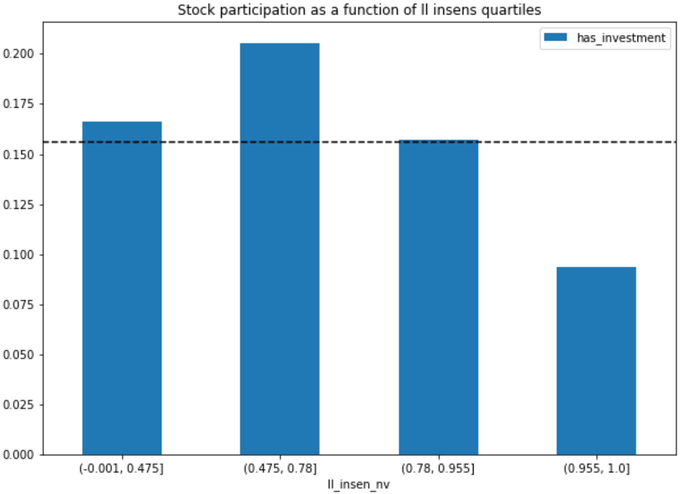

Relation to portfolio choice

	I	II	III	IV	V	VI
ll_insen	0.017				0.031	0.026
	(0.015)				(0.021)	(0.020)
ambig_av		-0.046***			-0.048***	-0.040***
		(0.010)			(0.010)	(0.009)
event_e1_adj			0.014		0.017	0.017
			(0.011)		(0.011)	(0.011)
event_e2_adj			-0.004		-0.004	-0.003
			(0.007)		(0.008)	(0.008)
subset_errors				0.004	-0.004	-0.001
				(0.006)	(0.008)	(0.008)
wealth						0.004***
						(0.001)
I(wealth ** 2)						0.000**
						(0.000)
R^2	0.001	0.015	0.002	0.000	0.019	0.097
N	1337	1337	1337	1337	1337	1337

The End