Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher.

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Table of Content

Introduction

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- Rust discretises the odometer data into 90 states of 5000 miles length.
- If a bus engine is replaced, the odometer state is set to 0.

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$$u(x_t, i_t, \theta) = \begin{cases} -c(x_t, \theta_1) & \textbf{if} \quad i_t = 0 \\ -[RC + c(0, \theta_1)] & \textbf{if} \quad i_t = 1 \end{cases}$$

where θ_1 is the parameter determining the cost function.

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In each period Harold Zurcher chooses his optimal action according to:

$$V_\theta(x_t) = \max_{i_t \in \{0,1\}} [u(x_t, i_t, \theta) + \epsilon_t(i_t) + \beta EV_\theta(x_t, i_t)]$$

where β is the discount factor, θ contains all parameters to be estimated, ϵ_t is unobserved information and $EV_\theta(x_t, i_t)$ is the expected value.

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As by replacement the state is set to 0:

$$EV_\theta(x_t, 1) = EV_\theta(0, 0) =: EV_\theta(0)$$

$$EV_\theta(x_t, 0) =: EV_\theta(x_t)$$

for all x_t.

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$$EV_\theta(x_t) = \sum_{j \in \{1,2,3\}} p_j * \ln\{ \sum_{i_t \in \{0,1\}} \exp[u(x_t, i_t, \theta_1, RC) + \beta EV_\theta(i_t * (x_t + j))]\}$$

With p_j being the transition probabilities for a state increase by $j \in \{1, 2, 3\}$ and the choice probabilities therefore by:

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$$P(i_t | x_t, \theta) = \frac{\exp[u(x_t, i_t, \theta_1, RC) + \beta EV_\theta(i_t * x_t)]}{\sum_{j \in \{0,1\}} \exp[u(x_t, j, \theta_1, RC) + \beta EV_\theta(j * x_t)]}$$

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The likelihood for the estimation of θ can be split up into two separate functions:

$$l^1(x_1, ..., x_T, i_1, ..., i_T | x_0, i_0, \theta) = \prod_{t=1}^{T} p(x_t | x_{t-1}, i_{t-1}, \theta_3)$$

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for the transition probabilities θ_3 and

$$l^2(x_1, ..., x_T, i_1, ..., i_T | \theta) = \prod_{t=1}^{T} P(i_t | x_t, \theta_1, RC, \theta_3)$$

for the cost parameters RC and θ_1.

Rust's achievement

- With this result he implements the Nested Fixed Point Algorithm (NFXP).

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- The first time a single agent decision problem could be estimated by a bottom-up approach.