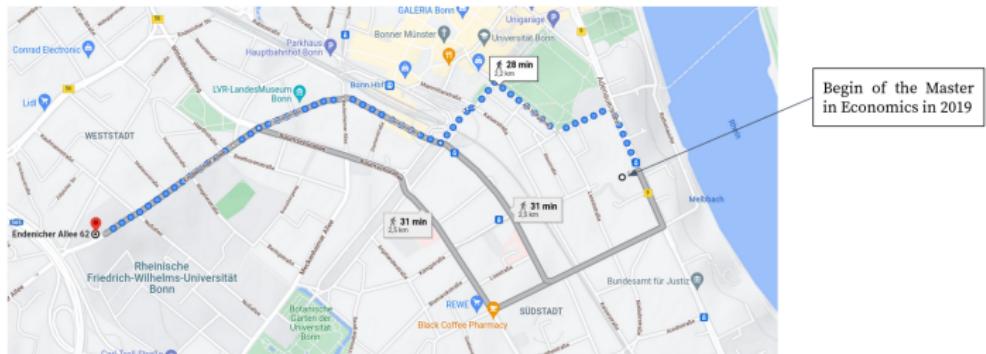




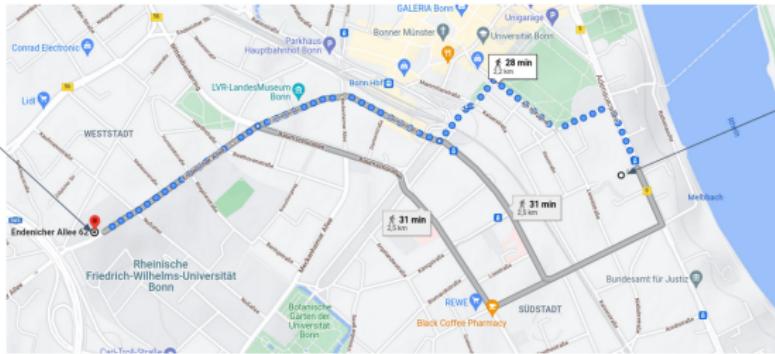
# About me



# About me

Begin of the PhD in the group  
of Jan Hasenauer in 2021  
(Computational Life Sciences)

Begin of the Master  
in Economics in 2019



# Hasenauer group - Computational Life Sciences

Hasenauer group



# Hasenauer group - Computational Life Sciences

Hasenauer group



Our work covers:

- Multi-cellular modelling
- Metabolic modelling
- Bioinformatics and machine learning
- Statistical inference for dynamical systems
- Infectious disease modelling

# Hasenauer group - Computational Life Sciences

Hasenauer group



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- Bioinformatics and machine learning
- Statistical inference for dynamical systems
- **Infectious disease modelling**

Topic of today

# SIRD models for the optimal allocation of vaccines across countries

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Manuel Huth

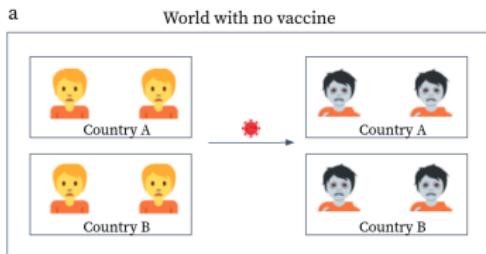
Supervision: Lorenzo Contento, Jan Hasenauer, Lena Janys

November 17, 2021

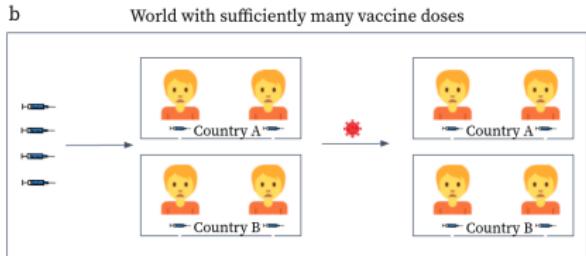
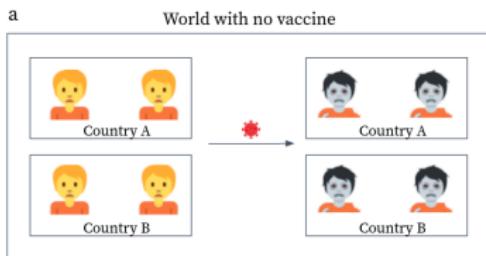


HelmholtzZentrum münchen  
German Research Center for Environmental Health

# Why do we need an optimized vaccine allocation scheme?



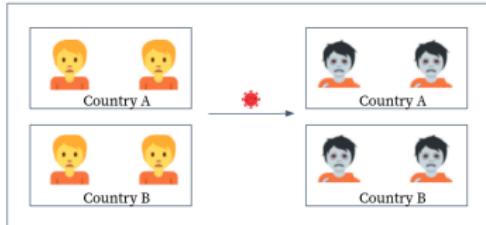
# Why do we need an optimized vaccine allocation scheme?



# Why do we need an optimized vaccine allocation scheme?

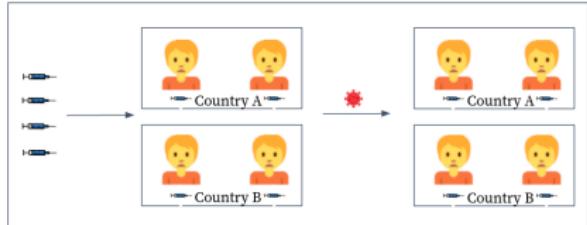
a

World with no vaccine



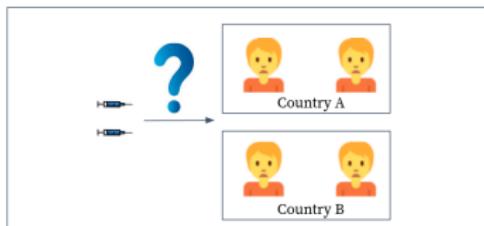
b

World with sufficiently many vaccine doses

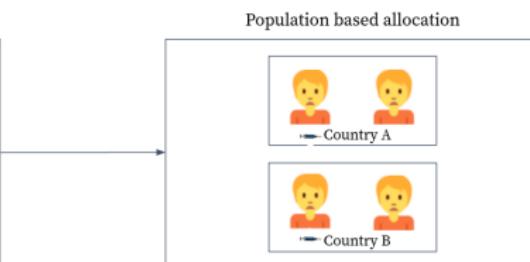
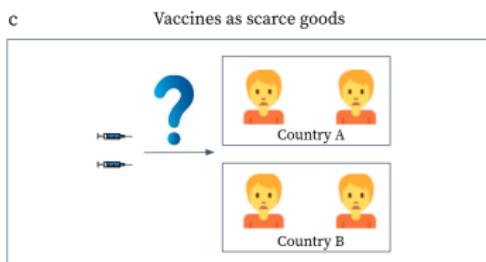
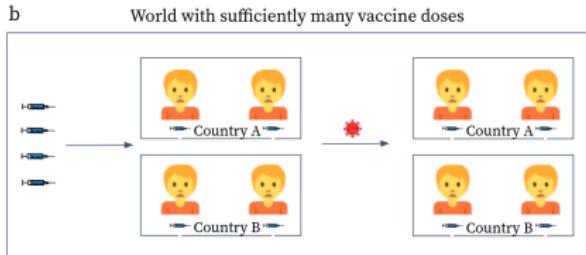
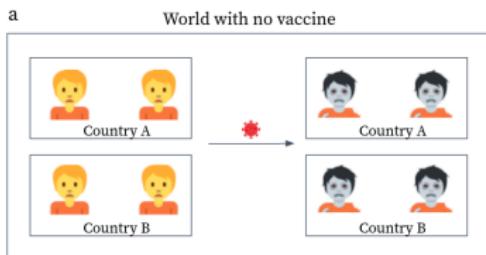


c

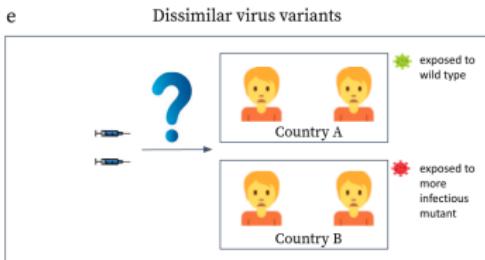
Vaccines as scarce goods



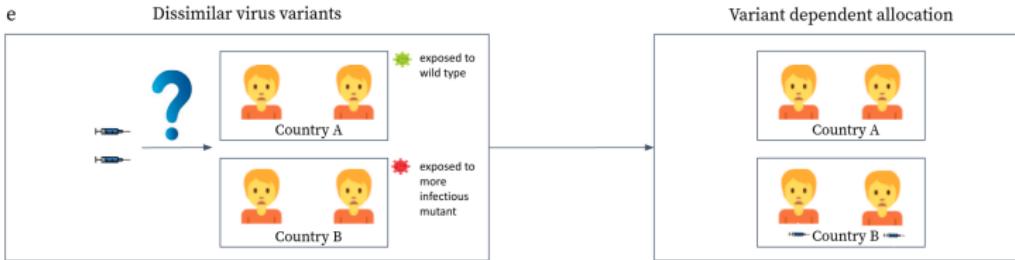
# Why do we need an optimized vaccine allocation scheme?



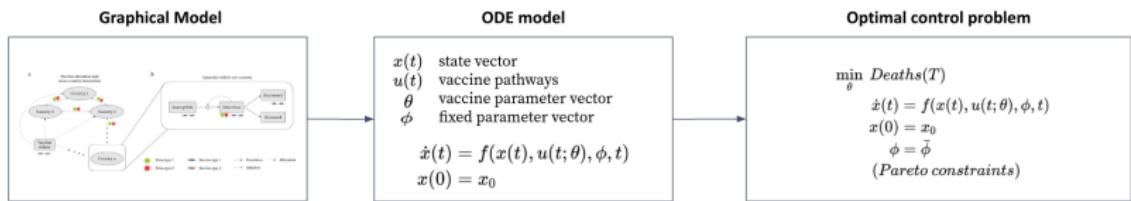
# Why do we need an optimized vaccine allocation scheme?



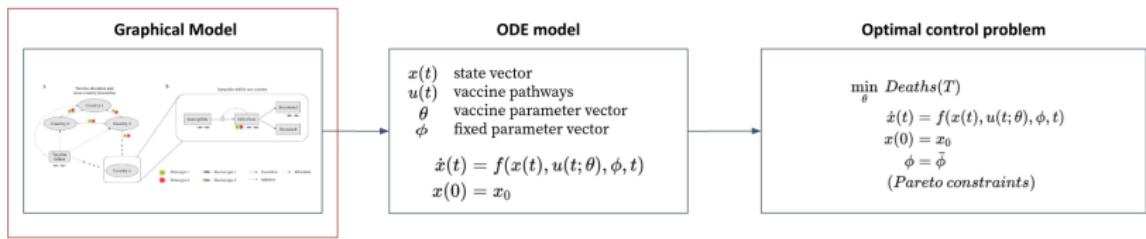
# Why do we need an optimized vaccine allocation scheme?



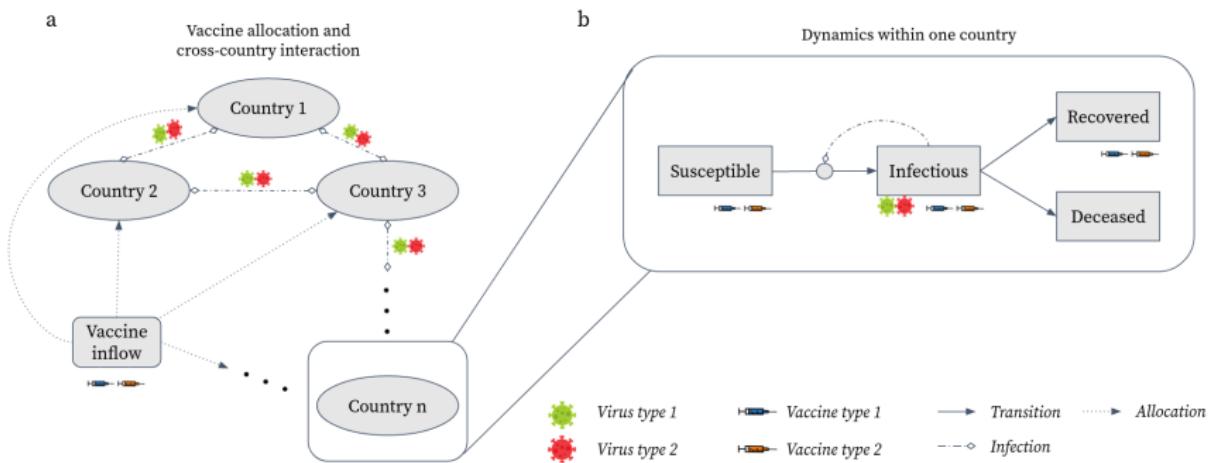
# General model building



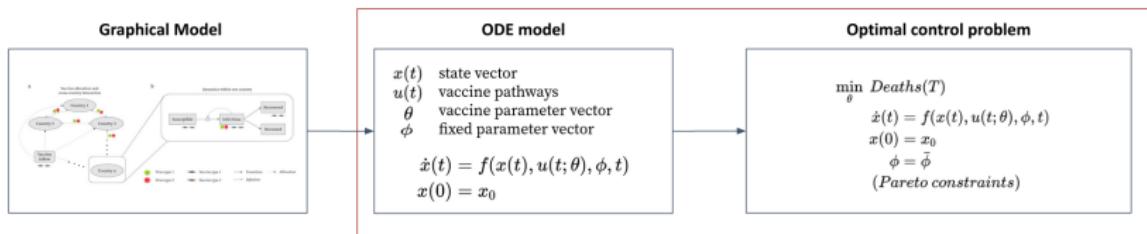
# General model building



# Graphical model



# General model building



# ODE model

$x(t)$  state vector  
 $u(t)$  vaccine pathways  
 $\theta$  vaccine parameter vector  
 $\phi$  fixed parameter vector

## Optimal control problem

$$\begin{aligned} & \min_{\theta} Deaths(T) \\ & \dot{x}(t) = f(x(t), u(t; \theta), \phi, t) \\ & x(0) = x_0 \\ & \phi = \bar{\phi} \\ & (Pareto constraints) \end{aligned}$$

# ODE model

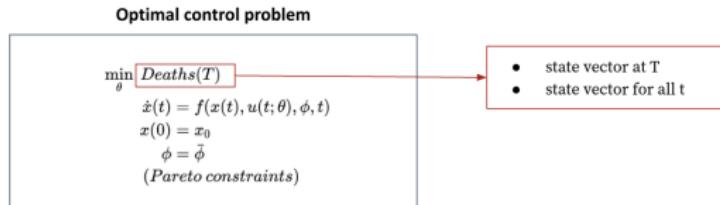
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 $u(t)$  vaccine pathways  
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$$\begin{aligned} \min_{\theta} & Deaths(T) \\ & \dot{x}(t) = f(x(t), u(t; \theta), \phi, t) \\ & x(0) = x_0 \\ & \phi = \bar{\phi} \\ & (Pareto constraints) \end{aligned}$$

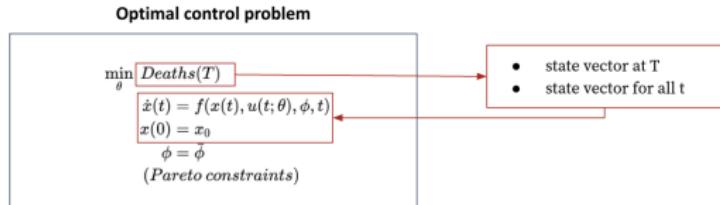
# ODE model

$x(t)$  state vector  
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# ODE model

$x(t)$  state vector  
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# ODE model

- $x(t)$  state vector
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(Pareto constraints)

- state vector at T
- state vector for all t

How to solve for the state vector?

- > 100 states
- non-linear ODE system
- no closed-form solution

# ODE model

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(Pareto constraints)

- state vector at T
- state vector for all t

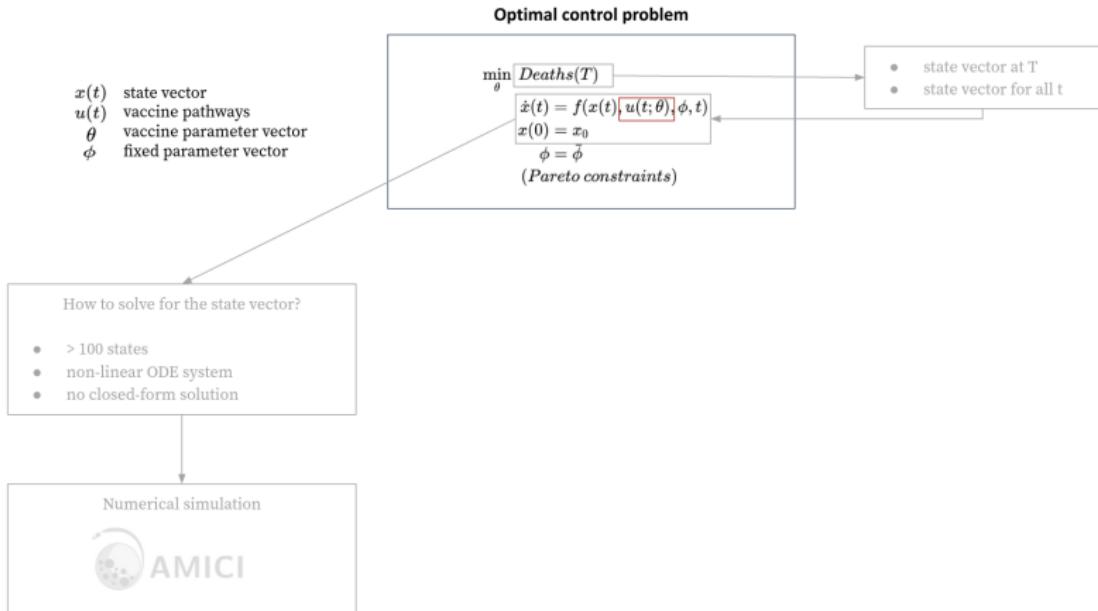
How to solve for the state vector?

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- non-linear ODE system
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Numerical simulation



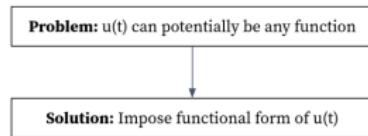
# ODE model



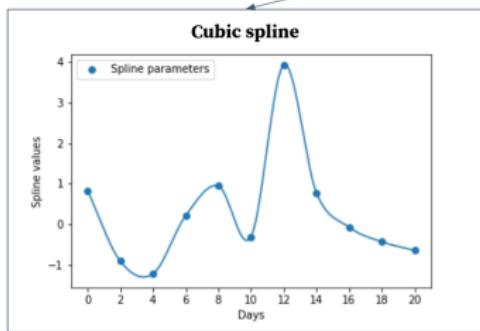
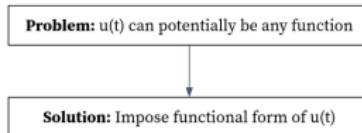
# Vaccination pathways

**Problem:**  $u(t)$  can potentially be any function

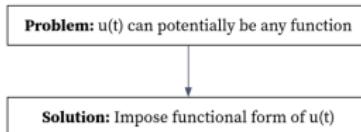
# Vaccination pathways



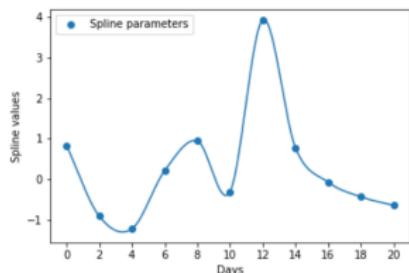
# Vaccination pathways



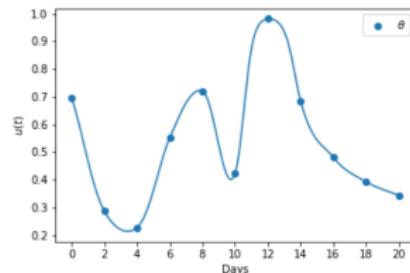
# Vaccination pathways



Cubic spline

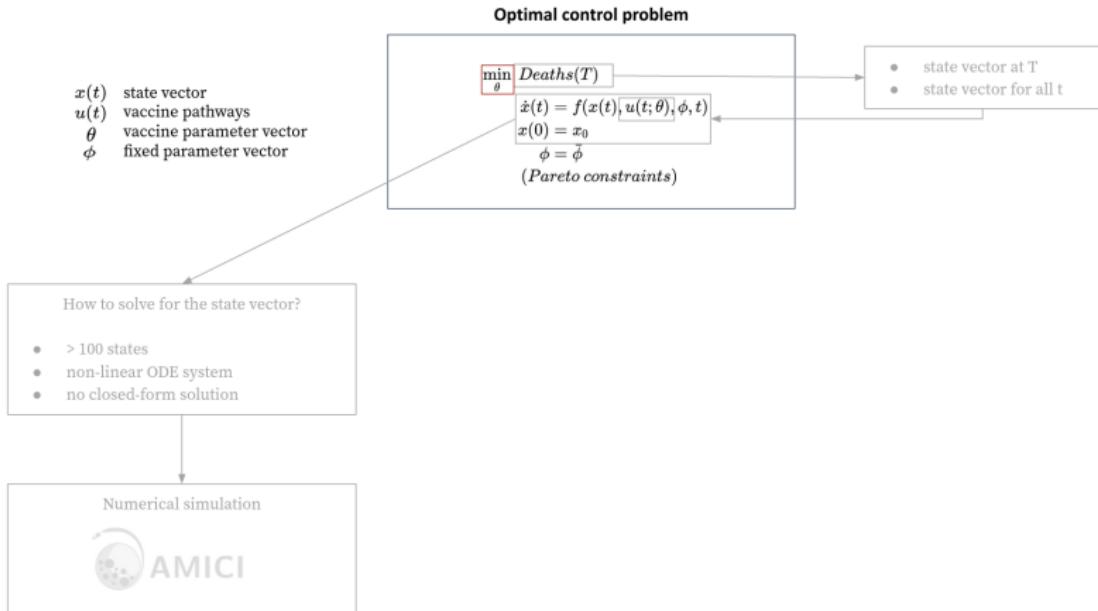


Vaccination pathway



logistic  
transformation

# ODE model



# Optimization

Optimal strategy  
(global minimum)

$$\min_{\theta} Deaths(T)$$

$$\dot{x}(t) = f(x(t), u(t; \theta), \phi, t)$$

$$x(0) = x_0$$

$$\phi = \bar{\phi}$$

# Optimization

Optimal strategy  
(global minimum)

$$\begin{aligned} \min_{\theta} & \text{Deaths}(T) \\ & \dot{x}(t) = f(x(t), u(t; \theta), \phi, t) \\ & x(0) = x_0 \\ & \phi = \bar{\phi} \end{aligned}$$

Pareto optimal strategy

$$\begin{aligned} \min_{\theta} & \text{Deaths}(T) \\ & \dot{x}(t) = f(x(t), u(t; \theta), \phi, t) \\ & x(0) = x_0 \\ & \phi = \bar{\phi} \\ & \boxed{(\text{Pareto constraints})} \end{aligned}$$

Every country needs to be at least as good as within a population based vaccine allocation.

# Optimization

Optimal strategy  
(global minimum)

$$\min_{\theta} \text{Deaths}(T)$$

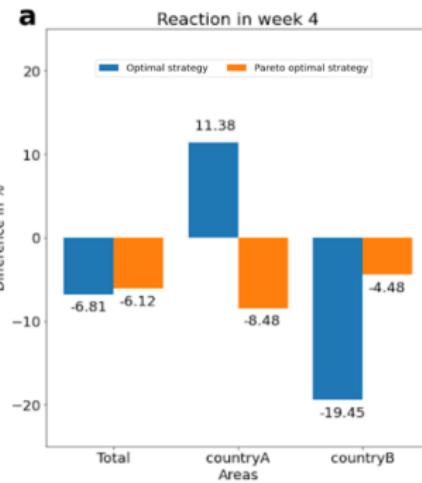
$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t; \theta), \phi, t) \\ x(0) &= x_0 \\ \phi &= \bar{\phi}\end{aligned}$$

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(Pareto constraints)



Every country needs to be at least as good as within a population based vaccine allocation.

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$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t; \theta), \phi, t) \\ x(0) &= x_0 \\ \phi &= \bar{\phi}\end{aligned}$$

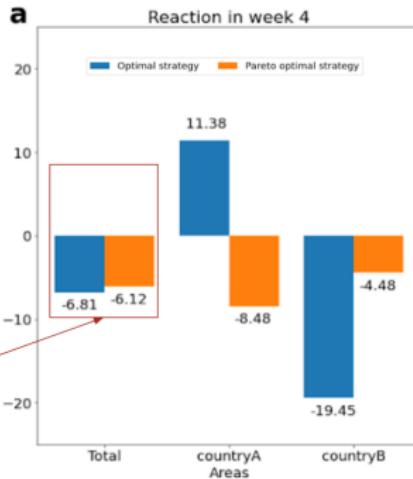
Pareto optimal strategy

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(Pareto constraints)

a



Every country needs to be at least as good as within a population based vaccine allocation.

How can we interpret these numbers?

# Optimization

Optimal strategy  
(global minimum)

$$\min_{\theta} \text{Deaths}(T)$$
$$\dot{x}(t) = f(x(t), u(t; \theta), \phi, t)$$
$$x(0) = x_0$$
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Pareto optimal strategy

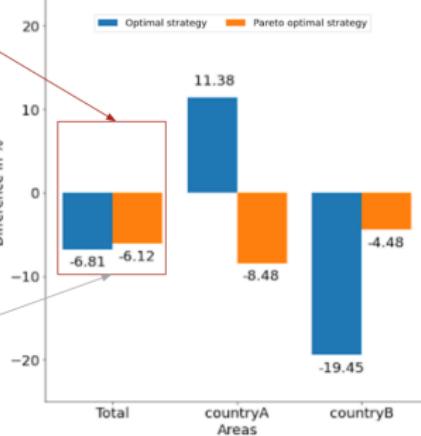
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$$\dot{x}(t) = f(x(t), u(t; \theta), \phi, t)$$
$$x(0) = x_0$$
$$\phi = \bar{\phi}$$

(Pareto constraints)



a

Reaction in week 4



Overall the globally optimal strategy performs better

Every country needs to be at least as good as within a population based vaccine allocation.

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Optimal strategy  
(global minimum)

$$\min_{\theta} \text{Deaths}(T)$$

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t; \theta), \phi, t) \\ x(0) &= x_0 \\ \phi &= \bar{\phi}\end{aligned}$$

Pareto optimal strategy

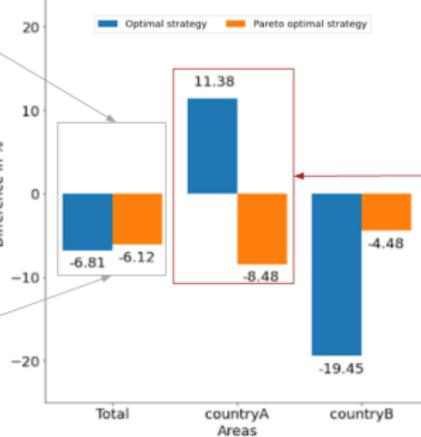
$$\min_{\theta} \text{Deaths}(T)$$

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t; \theta), \phi, t) \\ x(0) &= x_0 \\ \phi &= \bar{\phi}\end{aligned}$$

(Pareto constraints)

a

Reaction in week 4



Overall the globally optimal strategy performs better

Every country needs to be at least as good as within a population based vaccine allocation.

The globally optimal strategy yields no improvement for country A but the Pareto optimal strategy does so.

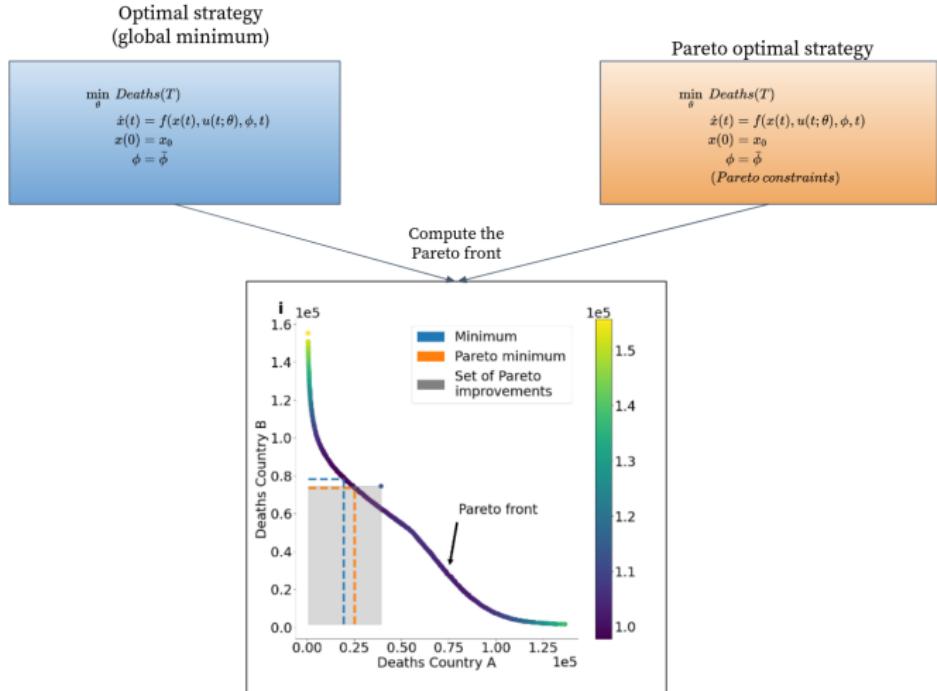
How can we interpret these numbers?

# Optimization - Pareto front

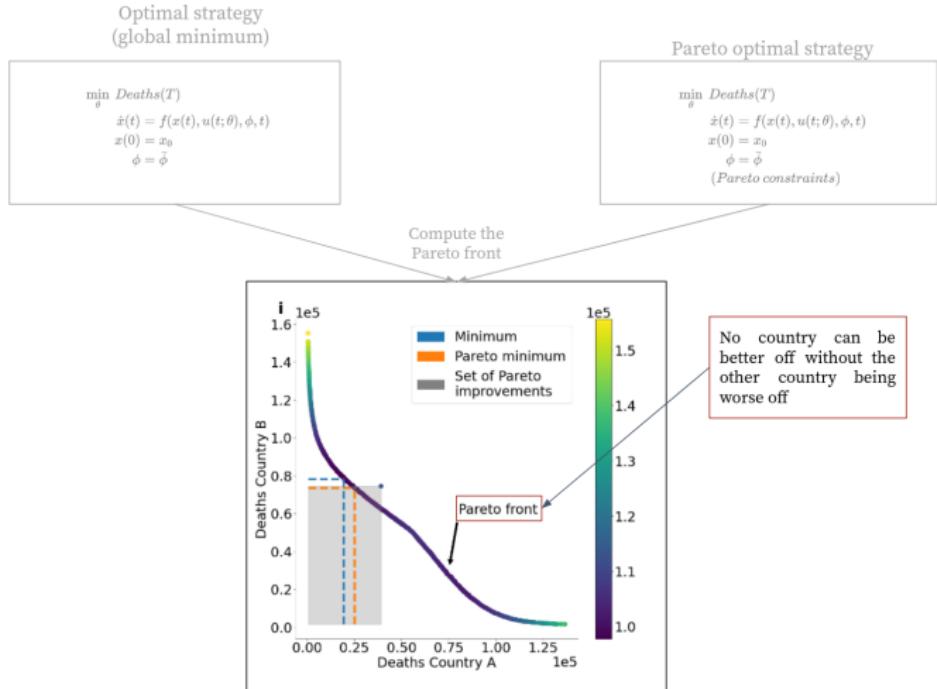
**Aim:** Find the minimal number of deaths in country A, given the number of deaths in country B.



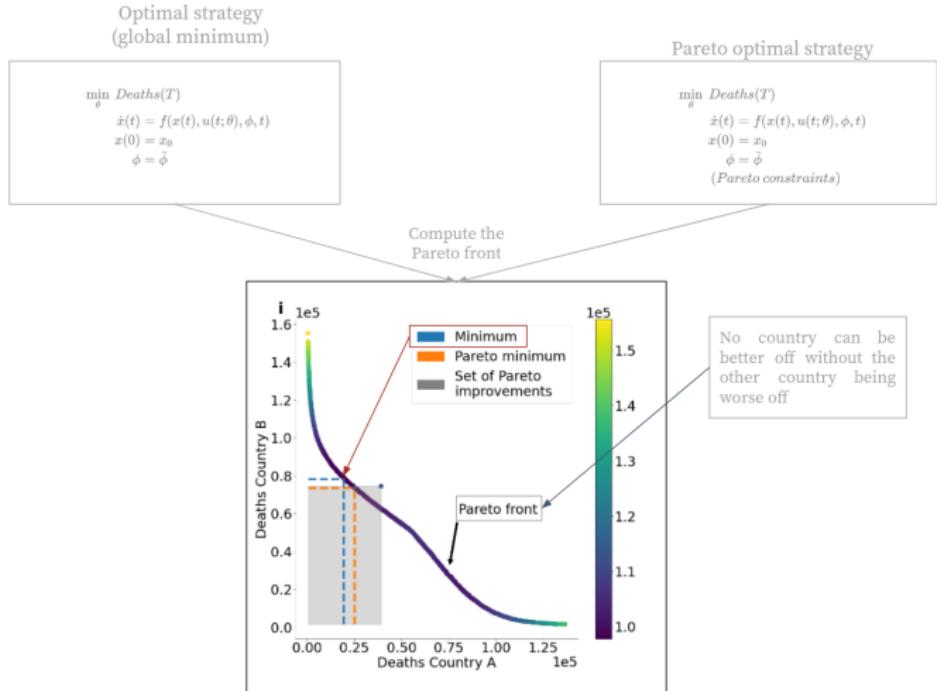
# Optimization - Pareto front



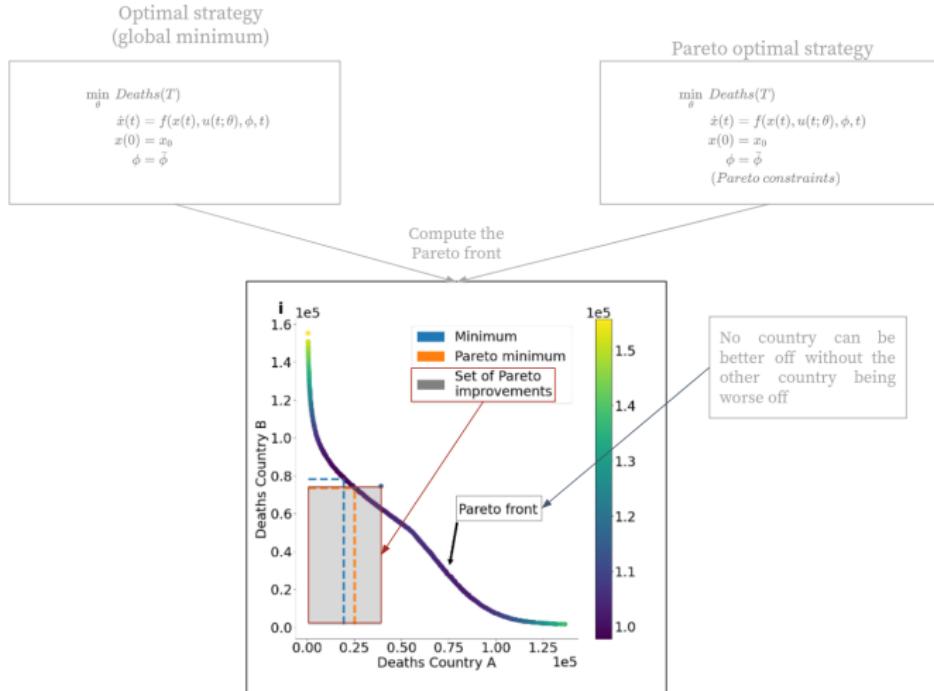
# Optimization - Pareto front



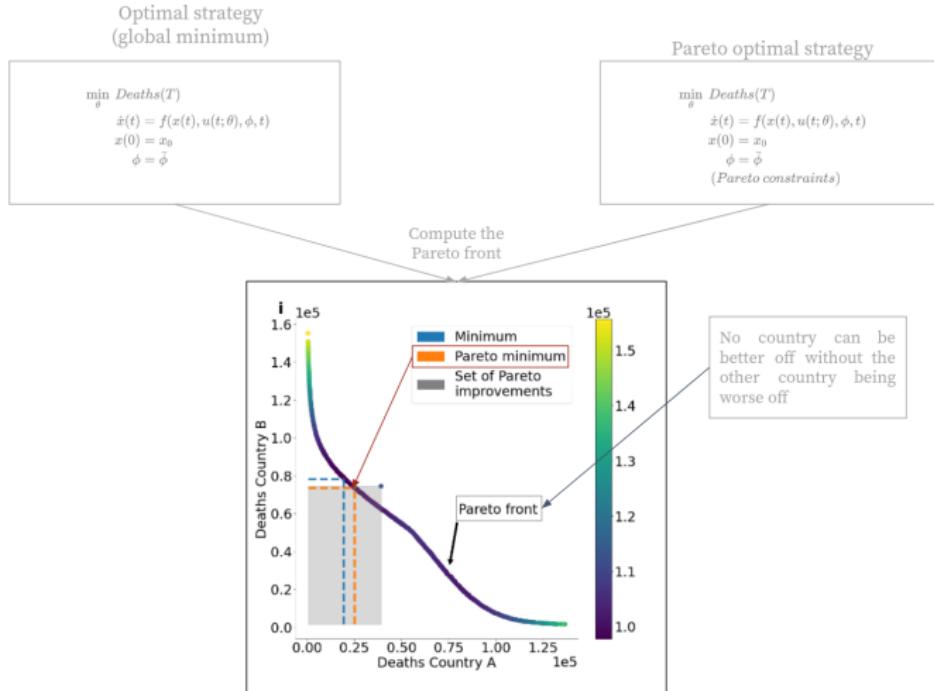
# Optimization - Pareto front



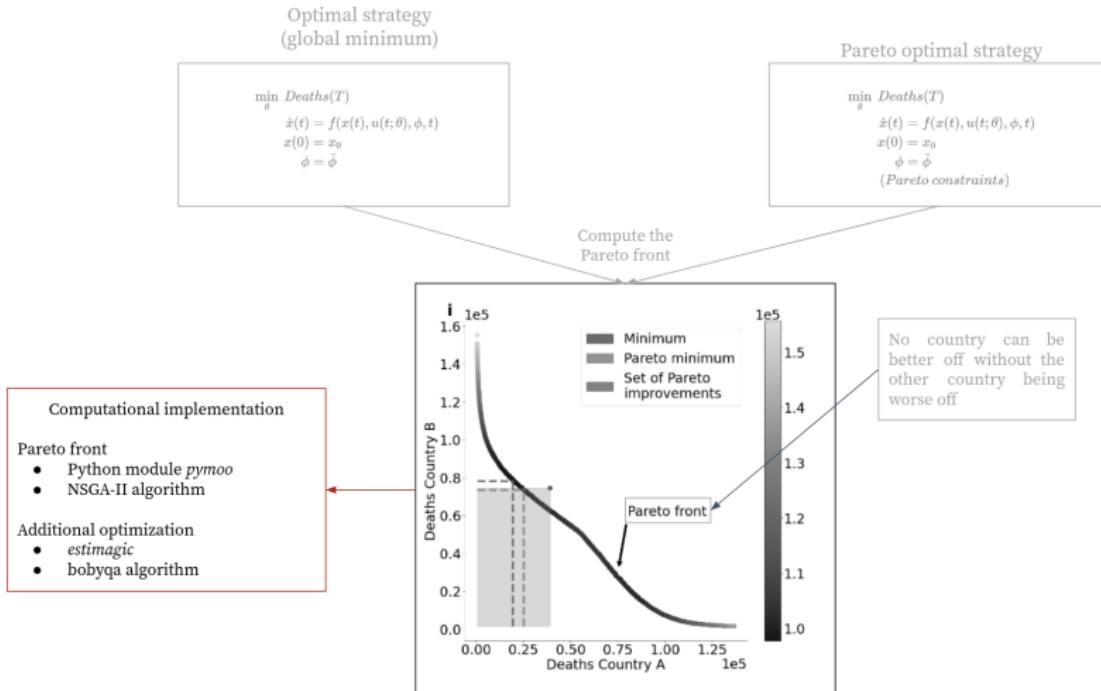
# Optimization - Pareto front



# Optimization - Pareto front



# Optimization - Pareto front



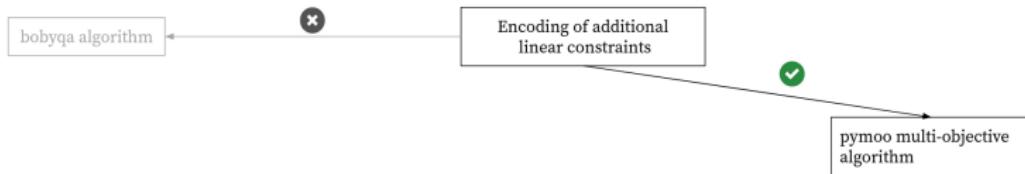
# Optimization - Many countries

Encoding of additional  
linear constraints

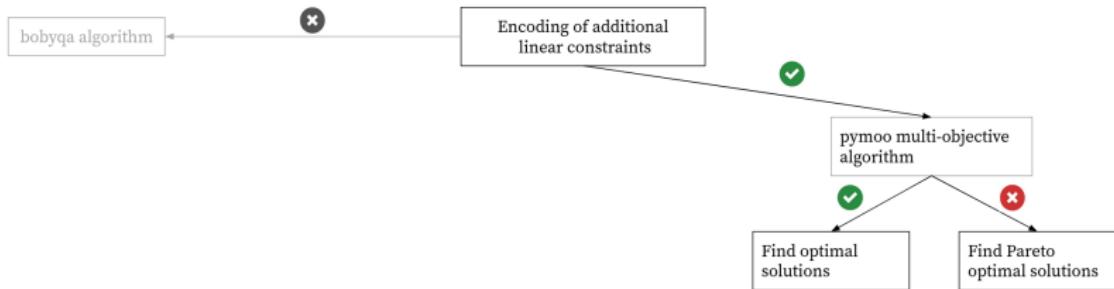
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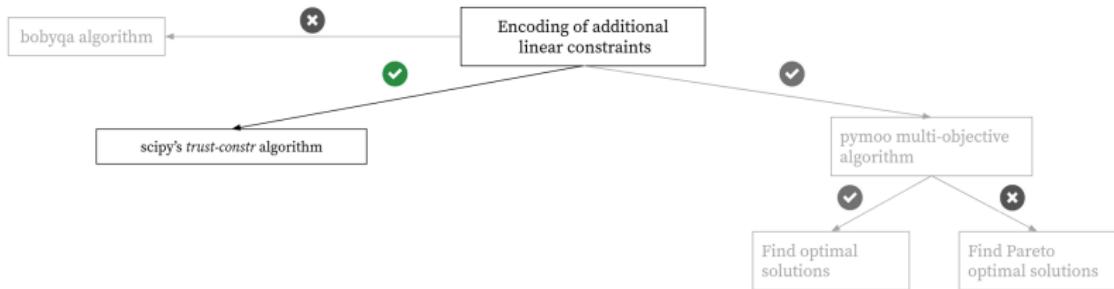
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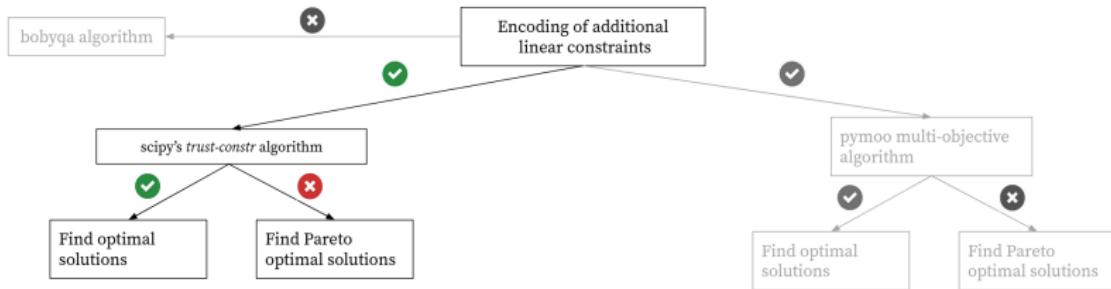
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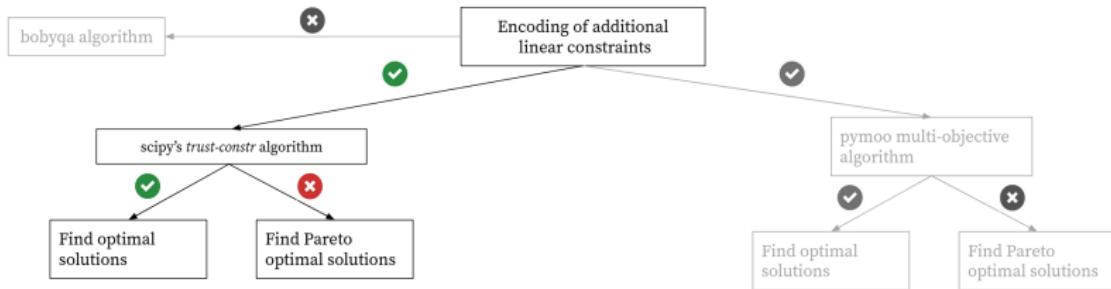
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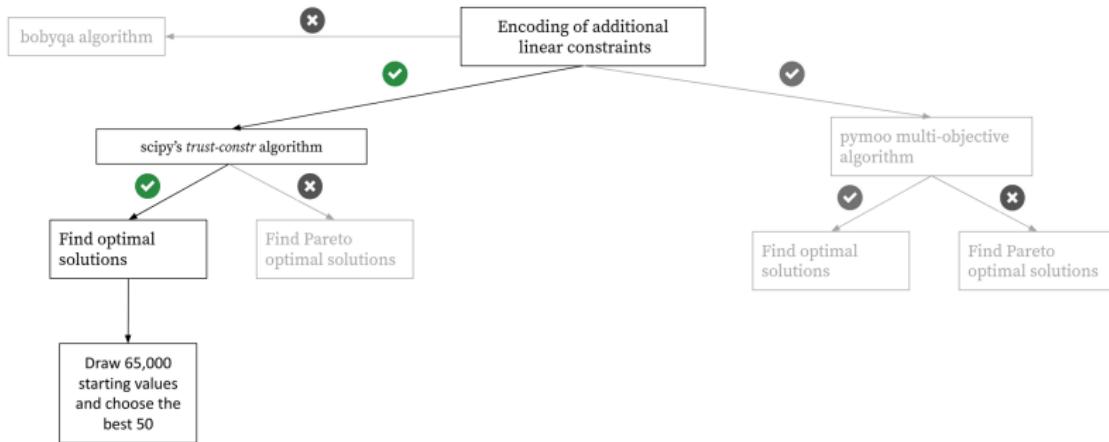
# Optimization - Many countries



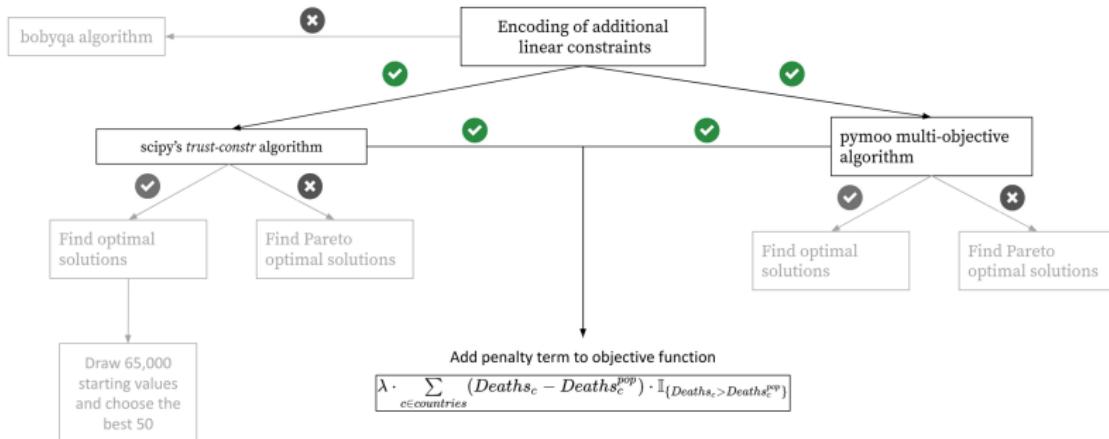
# Optimization - Many countries



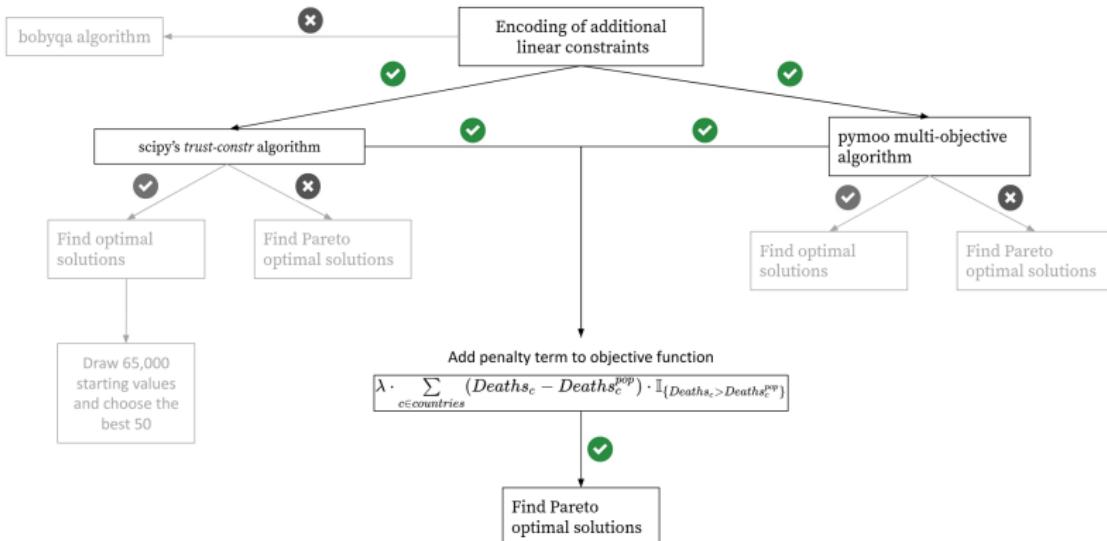
# Optimization - Many countries



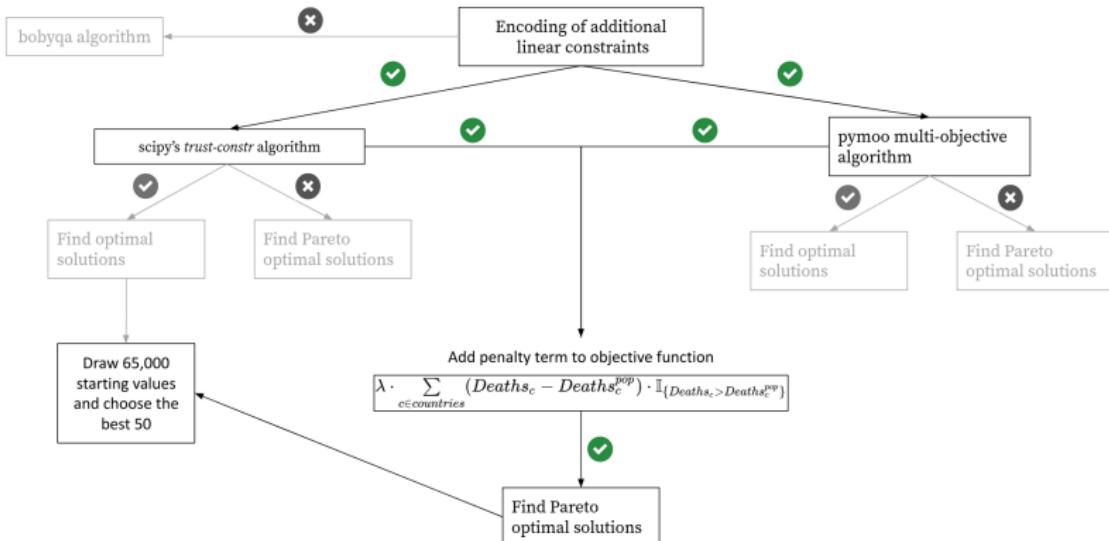
# Optimization - Many countries



# Optimization - Many countries



# Optimization - Many countries



## **Additional material**

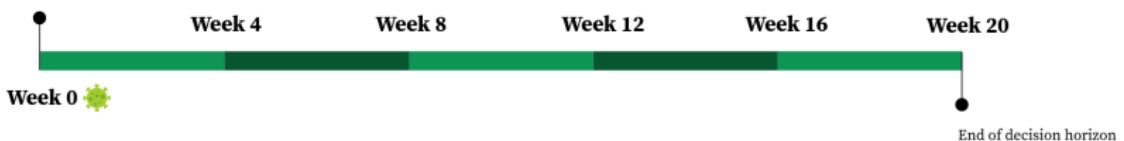
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# Examining policy decision delays

## General Scenario

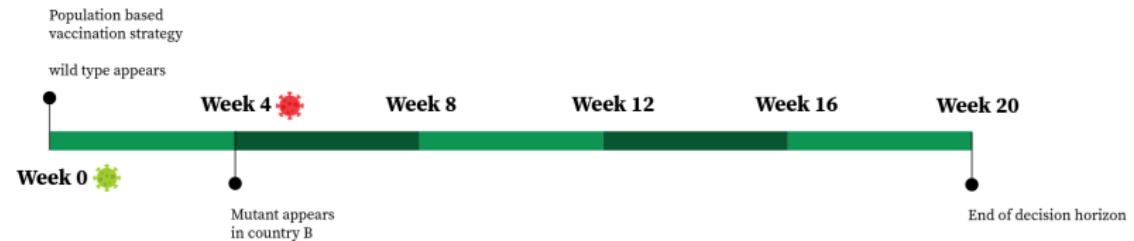
Population based  
vaccination strategy

wild type appears



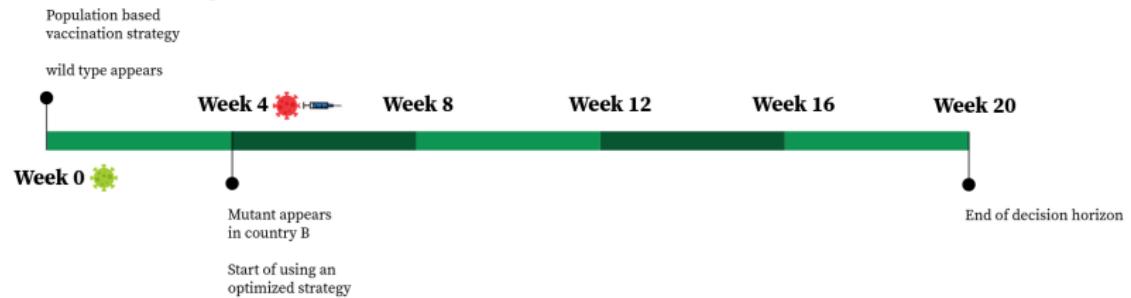
# Examining policy decision delays

## General Scenario



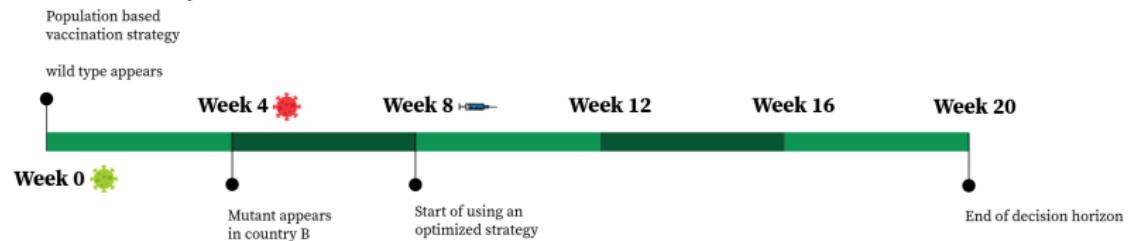
# Examining policy decision delays

## Scenario 1: Immediate Response



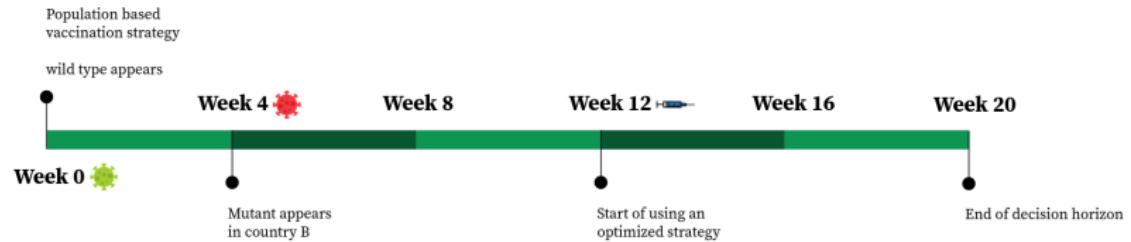
# Examining policy decision delays

## Scenario 2: 4 weeks delay



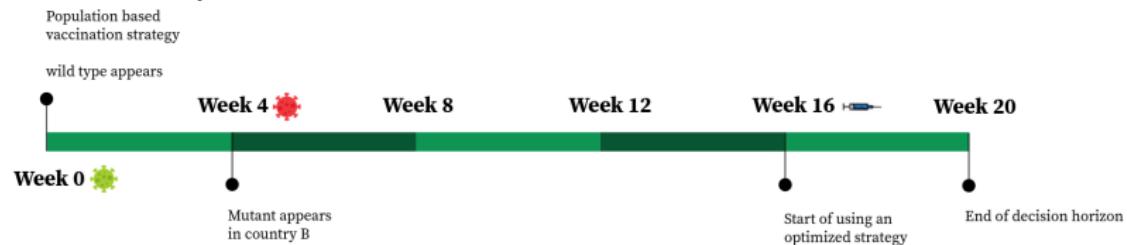
# Examining policy decision delays

## Scenario 3: 8 weeks delay

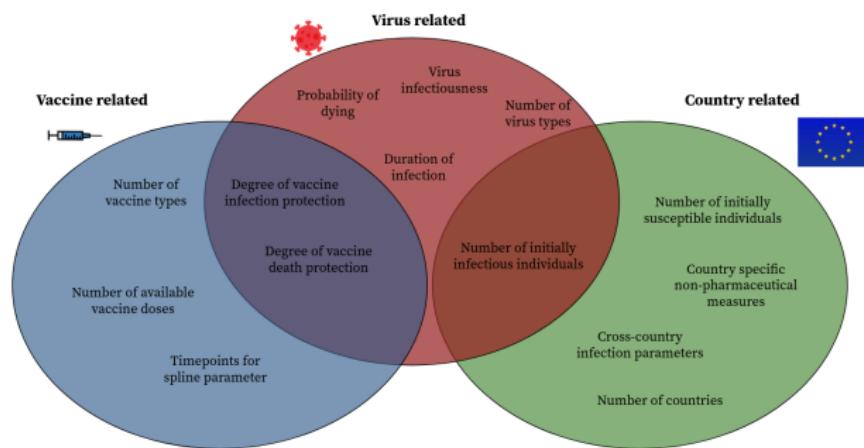


# Examining policy decision delays

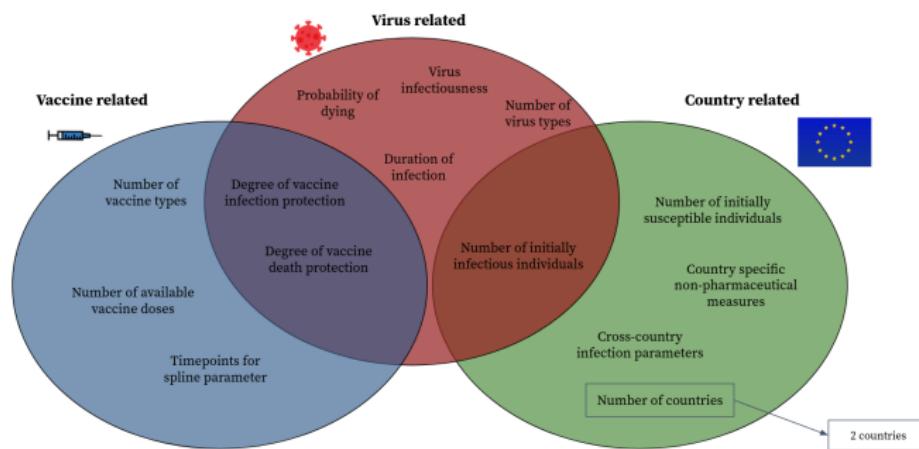
## Scenario 4: 12 weeks delay



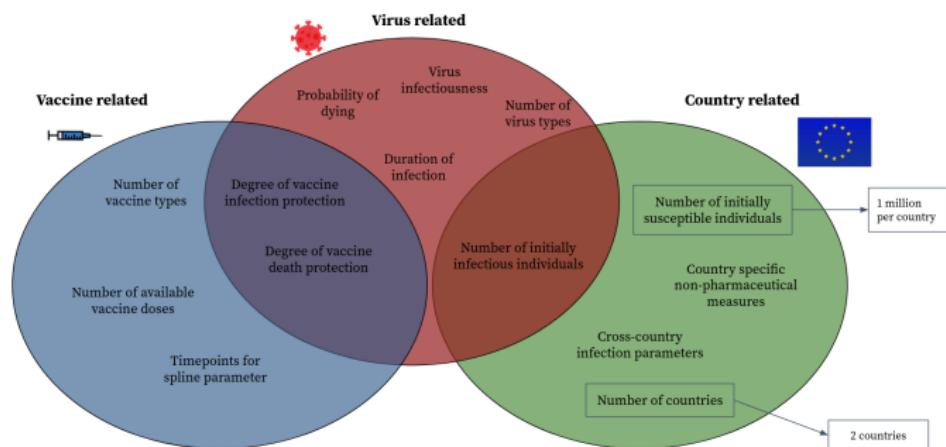
# Fixed Parameters



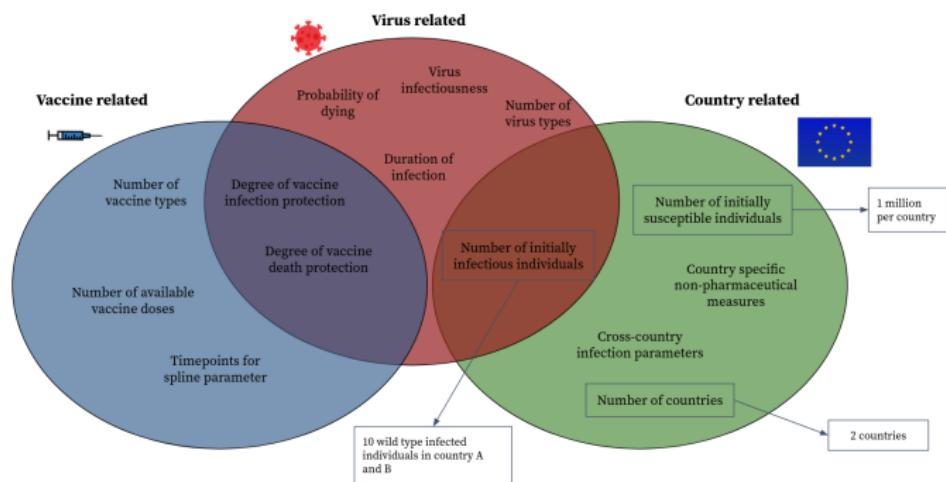
# Fixed Parameters



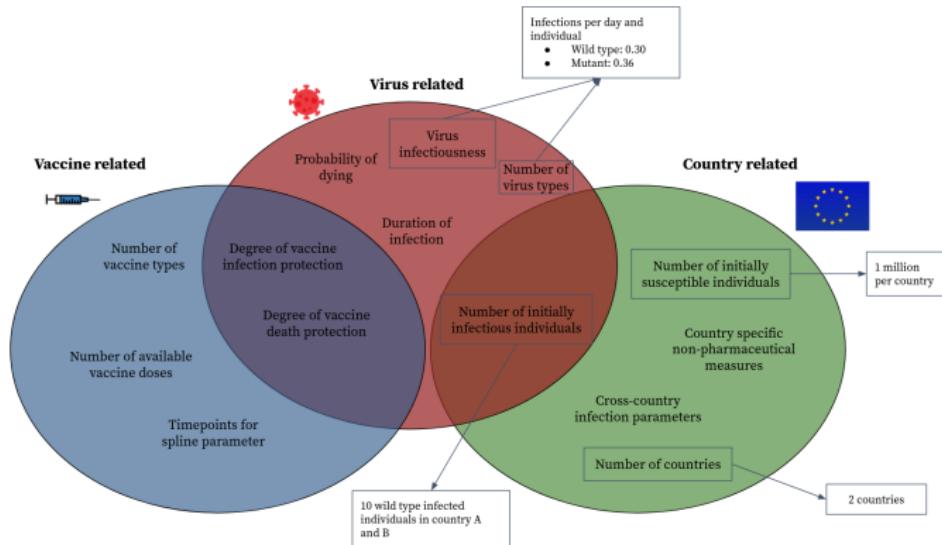
# Fixed Parameters



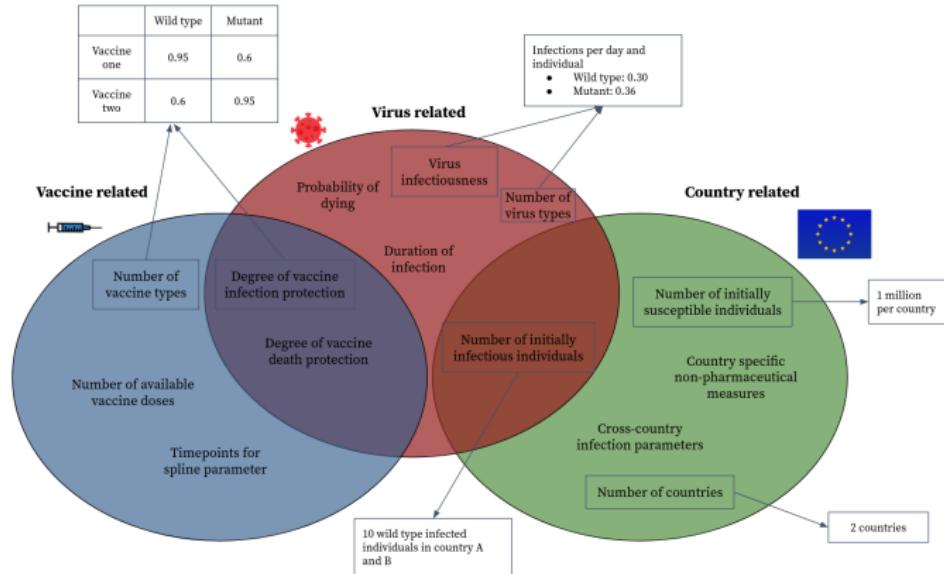
# Fixed Parameters



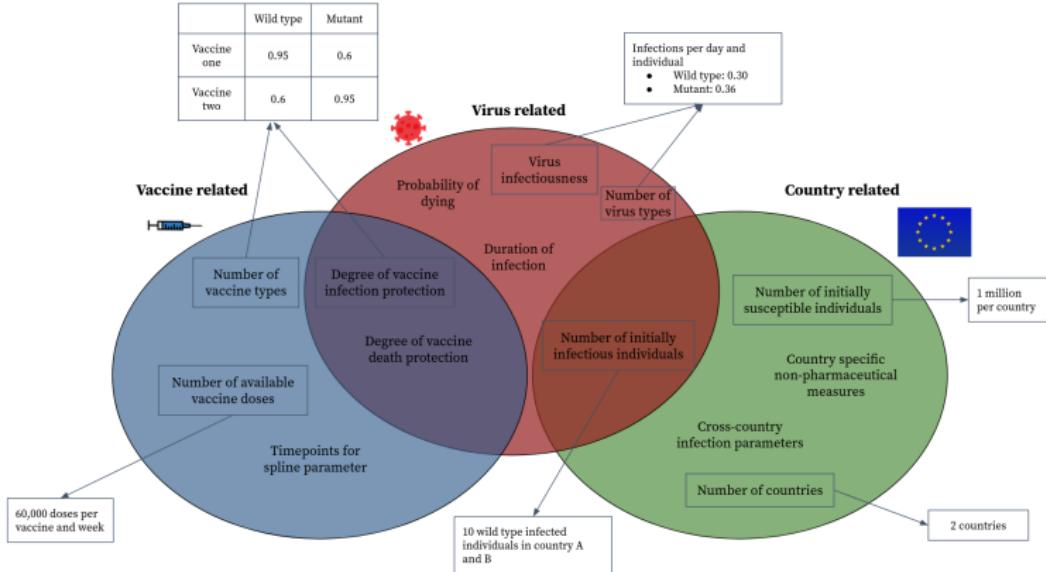
# Fixed Parameters



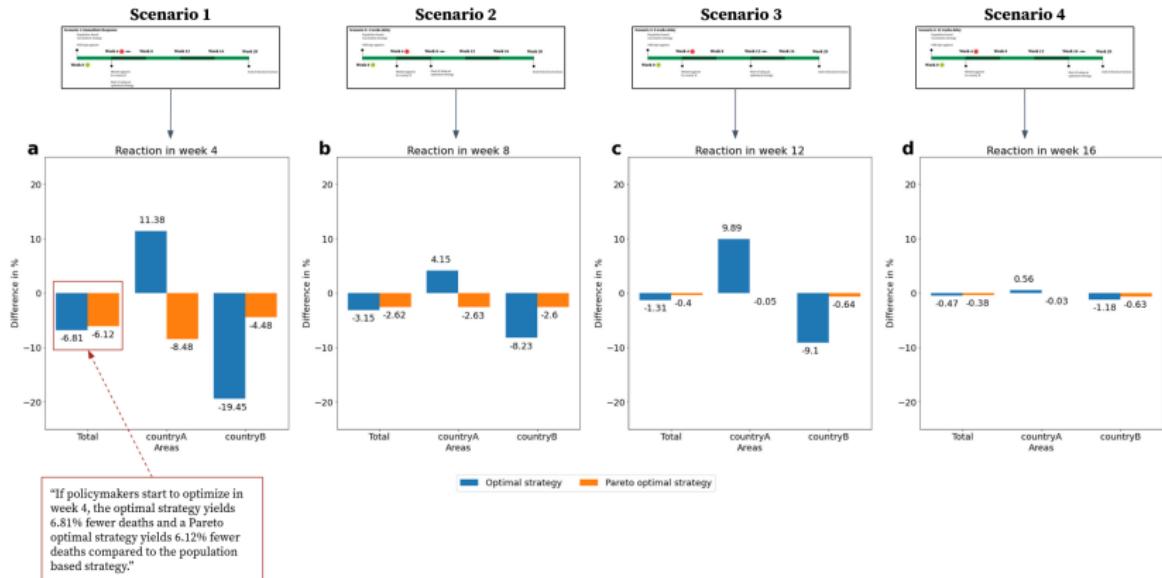
# Fixed Parameters



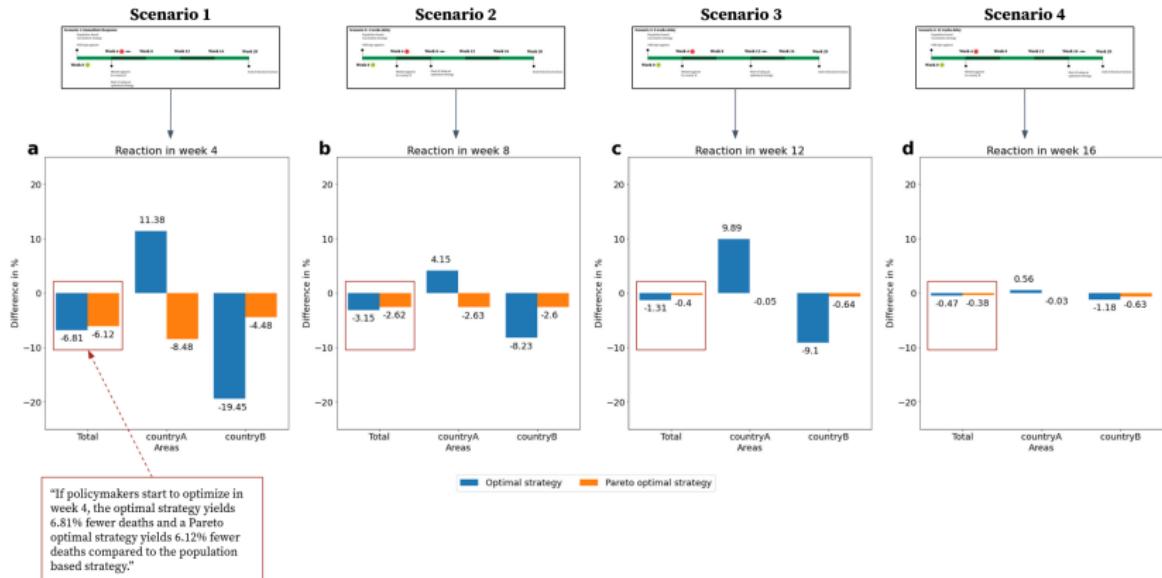
# Fixed Parameters



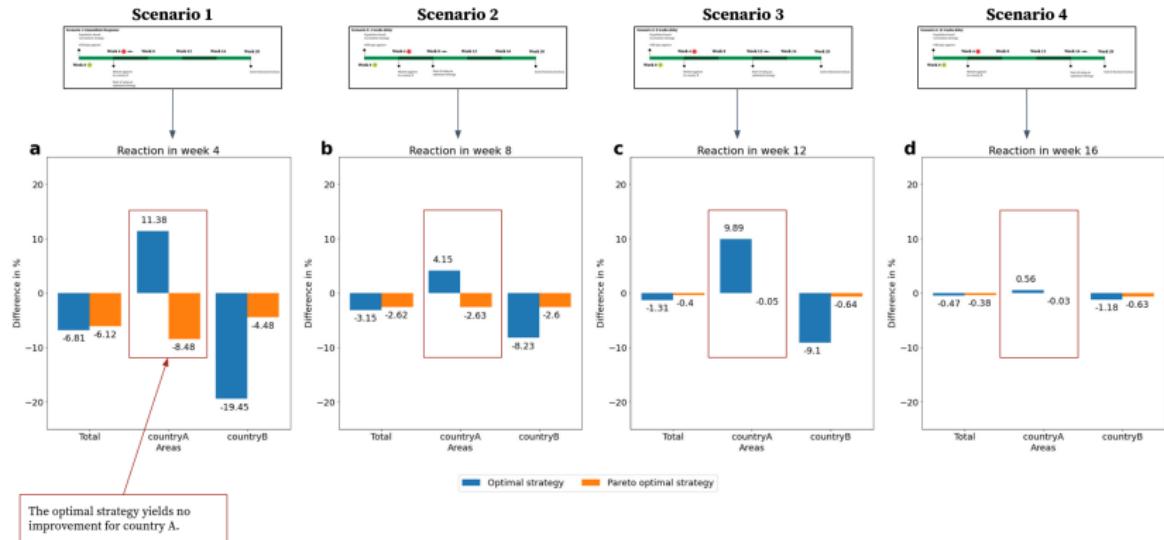
# Results relative to population based strategy



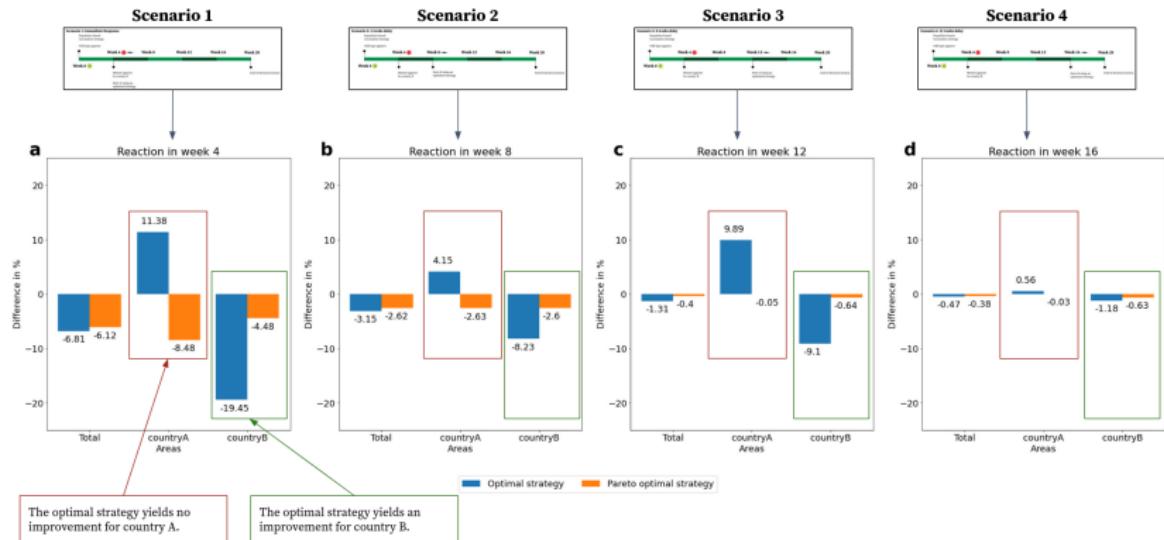
# Results relative to population based strategy



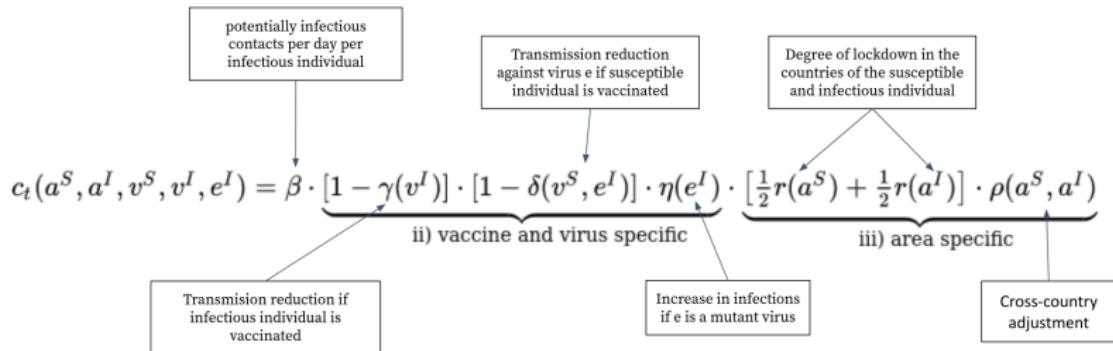
# Results relative to population based strategy



# Results relative to population based strategy



# Infection constant



## Extensions - Countries decide on lockdowns

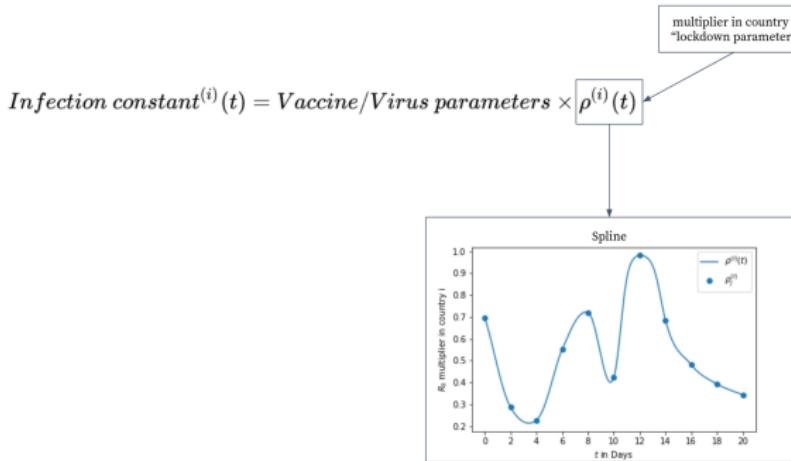
$$\text{Infection constant}^{(i)}(t) = \text{Vaccine/Virus parameters} \times \rho^{(i)}(t)$$

## Extensions - Countries decide on lockdowns

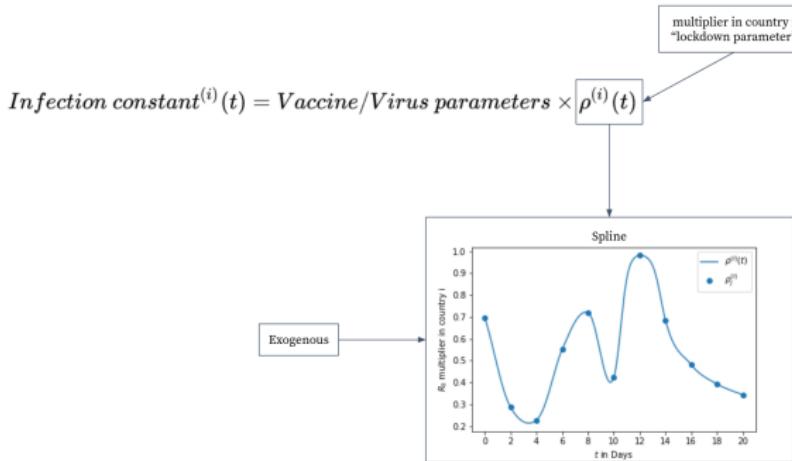
$$\text{Infection constant}^{(i)}(t) = \text{Vaccine/Virus parameters} \times \rho^{(i)}(t)$$

multiplier in country i  
"lockdown parameter"

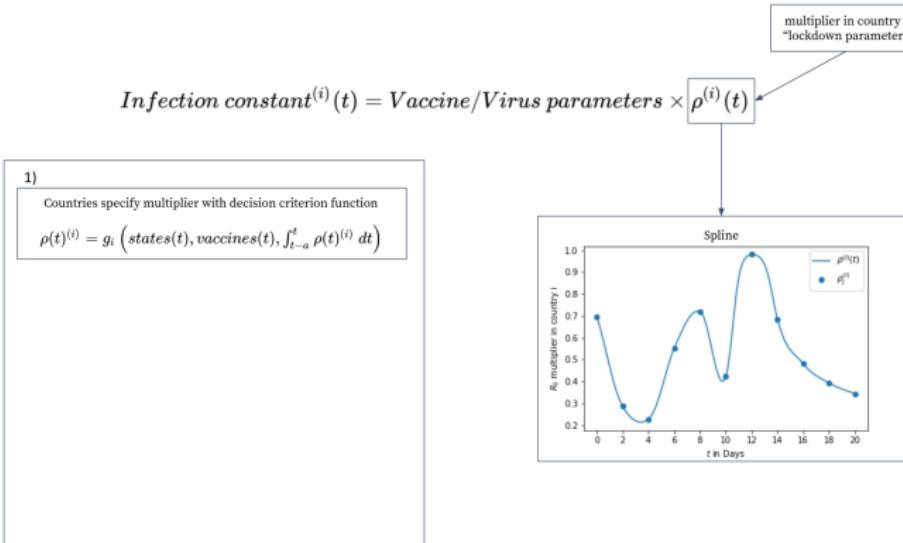
## Extensions - Countries decide on lockdowns



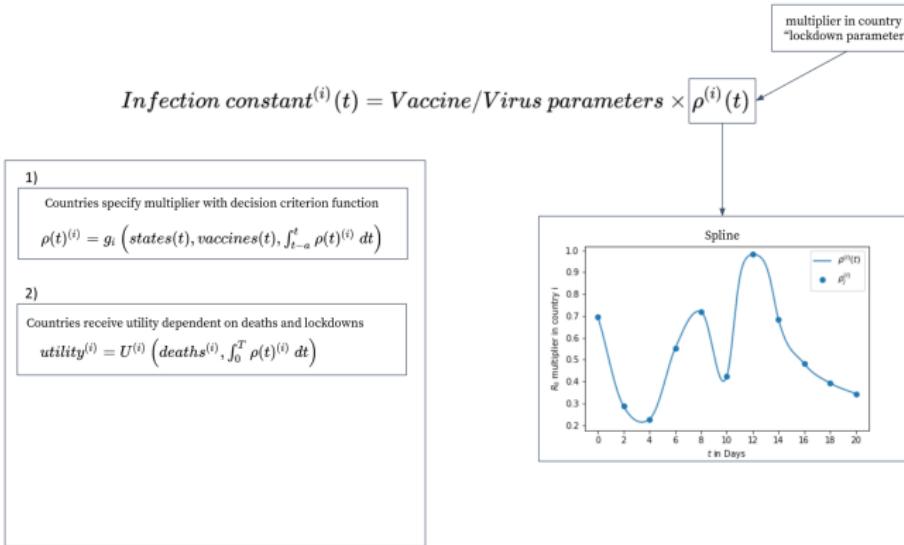
# Extensions - Countries decide on lockdowns



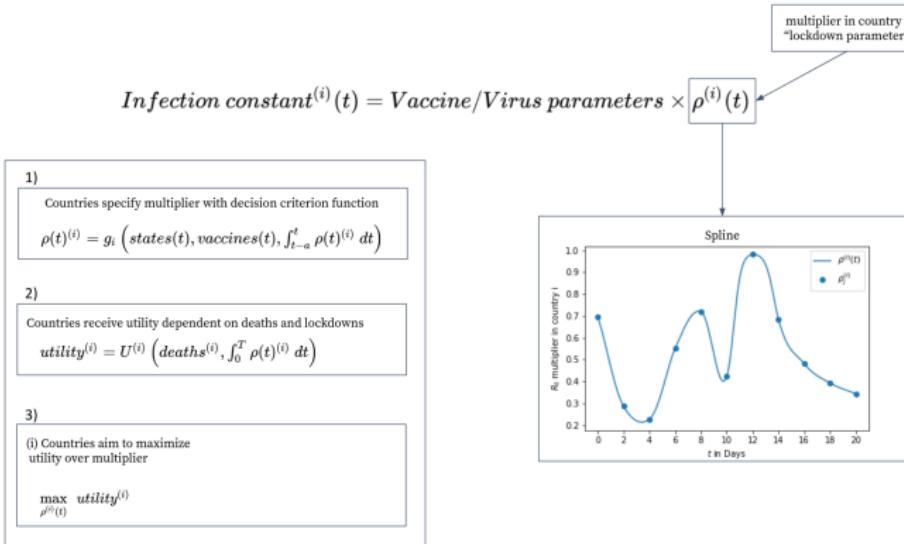
# Extensions - Countries decide on lockdowns



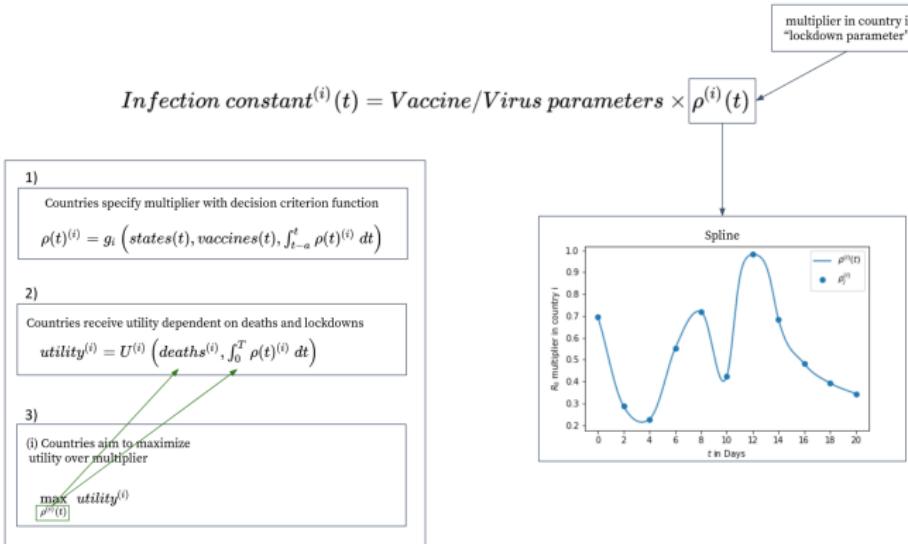
# Extensions - Countries decide on lockdowns



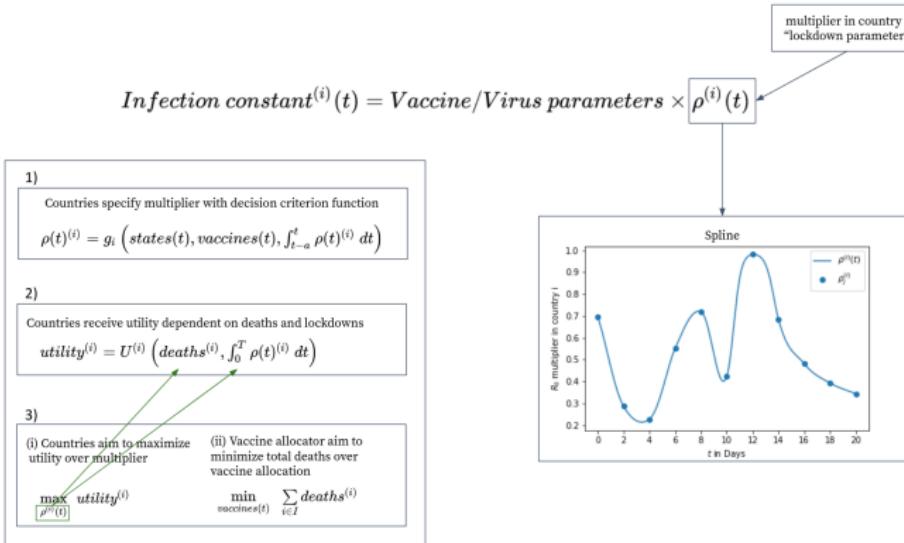
# Extensions - Countries decide on lockdowns



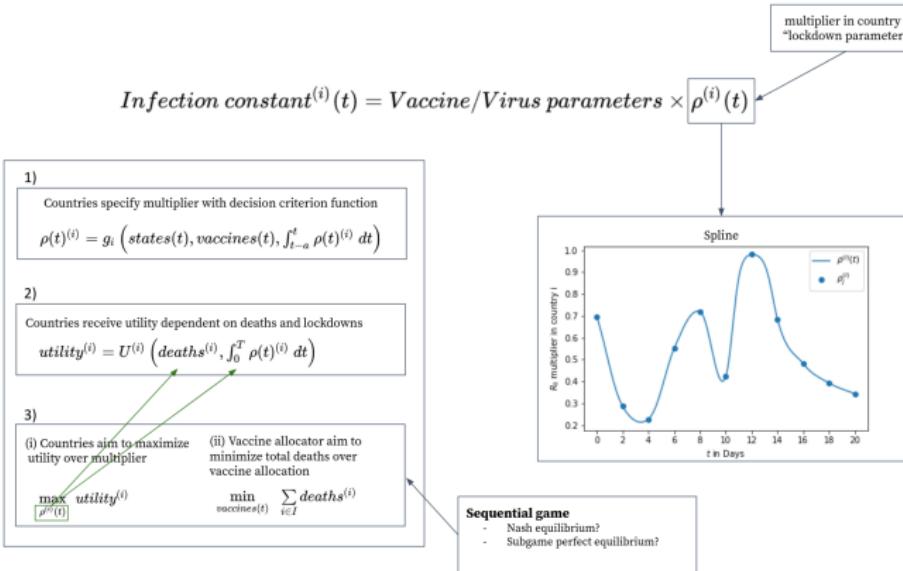
# Extensions - Countries decide on lockdowns



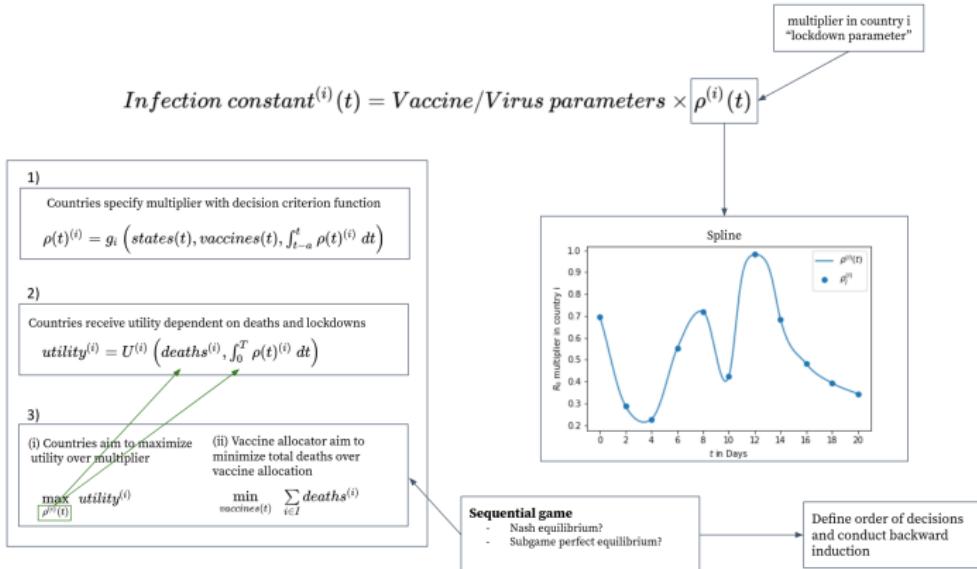
# Extensions - Countries decide on lockdowns



# Extensions - Countries decide on lockdowns



# Extensions - Countries decide on lockdowns



# SIRD models for the optimal allocation of vaccines across countries

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Manuel Huth

Supervision: Lorenzo Contento, Jan Hasenauer, Lena Janys

November 17, 2021



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German Research Center for Environmental Health