Robust investments under risk and ambiguity.

Harold Zurcher's robust replacement policy

Maximilian Blesch & Philipp Eisenhauer

February 17, 2020

Table of Content

Introduction

Introduction

- Policy makers vote for climate change policies facing uncertainty about future costs and benefits (Barnett, Brock, & Hansen, 2019).

Introduction

- Policy makers vote for climate change policies facing uncertainty about future costs and benefits (Barnett et al., 2019).
- Young adults decide on their careers in light of uncertainty about future job outcomes (Keane & Wolpin, 1997).

Introduction

- Policy makers vote for climate change policies facing uncertainty about future costs and benefits (Barnett et al., 2019).
- Young adults decide on their careers in light of uncertainty about future job outcomes (Keane & Wolpin, 1997).
- Doctors decide on the timing of an organ transplant in light of uncertainty about future patient health (Kaufman et al., 2017).

Introduction

- Economic models formalize objectives.

Introduction

- Economic models formalize objectives.
- Agents use past data to inform their decisions.

Introduction

- Economic models formalize objectives.
- Agents use past data to inform their decisions.
- Models often induce a unique distribution over sequences of future events.

Introduction

- Economic models formalize objectives.
- Agents use past data to inform their decisions.
- Models often induce a unique distribution over sequences of future events.
- Thus unique model describing decisions. (optimal model)

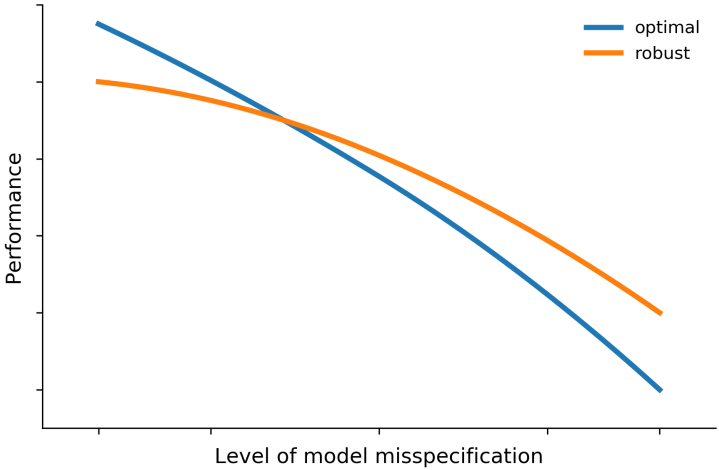
Introduction

- Economic models formalize objectives.
- Agents use past data to inform their decisions.
- Models often induce a unique distribution over sequences of future events.
- Thus unique model describing decisions. (optimal model)
- No role of ambiguity about model.

Introduction

- Economic models formalize objectives.
- Agents use past data to inform their decisions.
- Models often induce a unique distribution over sequences of future events.
- Thus unique model describing decisions. (optimal model)
- No role of ambiguity about model.
- Need of robust decision rule.

Notion of Robust decision rule



Questions addressed in our research

- Do or/and can agents account for ambiguity in the past data?

Questions addressed in our research

- Do or/and can agents account for ambiguity in the past data?
- How sensitive is the performance of the optimal decision rule?

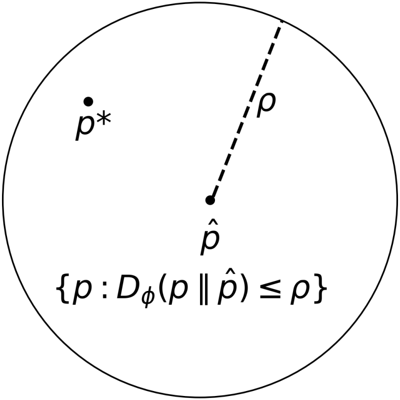
Questions addressed in our research

- Do or/and can agents account for ambiguity in the past data?
- How sensitive is the performance of the optimal decision rule?
- When does a robust decision rule perform better?

Assessing ambiguity

Confidence sets

Following the ideas of Ben-Tal, den Hertog, De Waegenaere, Melenberg, and Rennen (2013):



Size of confidence set

Calibrate the confidence set size ρ, by the number of observations per state N_s, $\forall s \in S$:

$$\omega = P\{p_s^* \in P_s\}$$

Size of confidence set

Calibrate the confidence set size ρ, by the number of observations per state N_s, $\forall s \in S$:

$$\begin{aligned}\omega &= P\{p_s^* \in P_s\} \\ &= P\{p_s^* \in \{p : D(p||\hat{p}_s) \leq \rho\}\}\end{aligned}$$

Size of confidence set

Calibrate the confidence set size ρ, by the number of observations per state N_s, $\forall s \in S$:

$$\begin{aligned} \omega &= P\{p_s^* \in P_s\} \\ &= P\{p_s^* \in \{p : D(p||\hat{p}_s) \leq \rho\}\} \\ &\approx F_{|S|-1}(2N_s\rho_s) \end{aligned}$$

Size of confidence set

Calibrate the confidence set size ρ, by the number of observations per state N_s, $\forall s \in S$:

$$
\begin{aligned}
\omega &= P\{p_s^* \in P_s\} \\
&= P\{p_s^* \in \{p : D(p||\hat{p}_s) \leq \rho\}\} \\
&\approx F_{|S|-1}(2N_s\rho_s)
\end{aligned}
$$

Therefore

$$\rho_s = F_{|S|-1}^{-1}(\omega)/2N_s$$

Harold Zurcher's dynamic investments: Rust (1987)

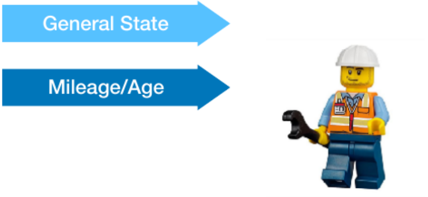
A brief summary

A brief summary

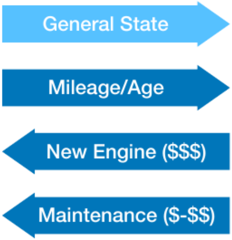
A brief summary

General State

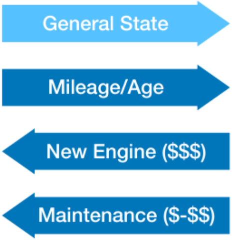
A brief summary



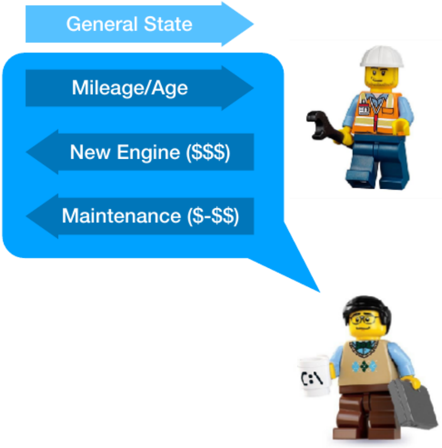
A brief summary



A brief summary



A brief summary



The framework

The optimal decision is given by the Bellman equation (Bellman (1954)):

$$V_\theta(x_t) = \max_{i_t \in \{0,1\}} \left[u(x_t, i_t, \theta_1) + \epsilon_t(i_t) + \beta EV_\theta(x_t, i_t) \right]$$

The framework

The optimal decision is given by the Bellman equation (Bellman (1954)):

$$V_\theta(x_t) = \max_{i_t \in \{0,1\}} \Big[u(x_t, i_t, \theta_1) + \epsilon_t(i_t) + \beta E V_\theta(x_t, i_t) \Big]$$

with

$$u(x_t, i_t, \theta_1) = \begin{cases} -c(x_t, \theta_1) & \textbf{if} \quad i_t = 0 \\ -RC & \textbf{if} \quad i_t = 1 \end{cases}$$

and utilitiy shock $\epsilon_t(i_t)$

The framework

Rust imposes the conditional independence (CI):

$$p(x_{t+1}, \epsilon_{t+1} | x_t, \epsilon_t, i_t, \theta_2, \theta_3) = q(\epsilon_{t+1} | \theta_2) p(x_{t+1} | x_t, i_t, \theta_3)$$

The framework

Rust imposes the conditional independence (CI):

$$p(x_{t+1}, \epsilon_{t+1} | x_t, \epsilon_t, i_t, \theta_2, \theta_3) = q(\epsilon_{t+1} | \theta_2) p(x_{t+1} | x_t, i_t, \theta_3)$$

which allows to estimate the model in two steps:

$$\max_{\theta_3} \prod_{t=1}^{T} p(x_t | x_{t-1}, i_{t-1}, \theta_3)$$

The framework

Rust imposes the conditional independence (CI):

$$p(x_{t+1}, \epsilon_{t+1} | x_t, \epsilon_t, i_t, \theta_2, \theta_3) = q(\epsilon_{t+1} | \theta_2) p(x_{t+1} | x_t, i_t, \theta_3)$$

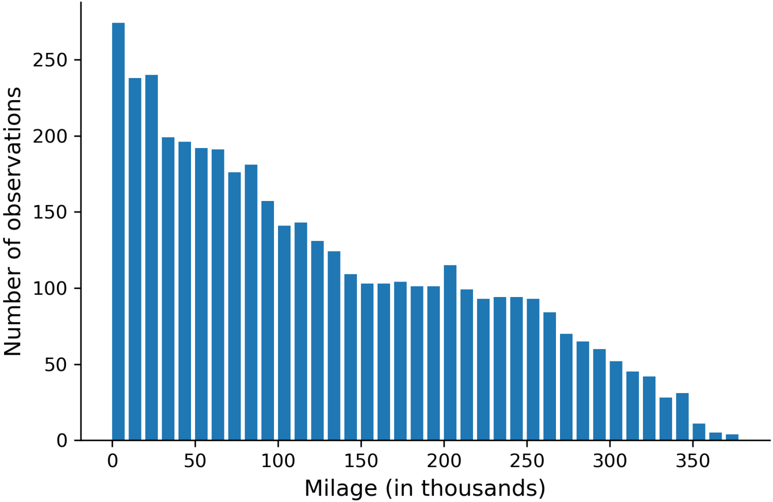
which allows to estimate the model in two steps:

$$\max_{\theta_3} \prod_{t=1}^{T} p(x_t | x_{t-1}, i_{t-1}, \theta_3)$$

Second:

$$\max_{RC, \theta_1} \prod_{t=1}^{T} P(i_t | x_t, \theta)$$

Differences in estimation quality



Robust decision making

Theory

Ben-Tal, El Ghaoui, and Nemirowski (2009) develop the following idea of robust decision making:

Theory

Ben-Tal, El Ghaoui, and Nemirowski (2009) develop the following idea of robust decision making:

- Robust decision making is a game nature vs. agent.

Theory

Ben-Tal, El Ghaoui, and Nemirowski (2009) develop the following idea of robust decision making:

- Robust decision making is a game nature vs. agent.
- First the agent chooses his action to maximize his present value.

Theory

Ben-Tal, El Ghaoui, and Nemirowski (2009) develop the following idea of robust decision making:

- Robust decision making is a game nature vs. agent.
- First the agent chooses his action to maximize his present value.
- Then nature chooses the transition probabilities accordingly to minimize the agent's value.

Theory

Ben-Tal, El Ghaoui, and Nemirowski (2009) develop the following idea of robust decision making:

- Robust decision making is a game nature vs. agent.
- First the agent chooses his action to maximize his present value.
- Then nature chooses the transition probabilities accordingly to minimize the agent's value.
- As the agent and nature have common information, the agent chooses in the first step the alternative with the highest (max) minimal (min) value.

Theory

This leads to a robust Bellman equation, which in the framework of Rust (1987) is:

$$V_\theta(x_t) = \max_{i_t \in \{0,1\}} \left[u(x_t, i_t, \theta_1) + \epsilon_t(i_t) + \beta \min_{p_s \in P_s^{i_t}} EV_\theta(x_t, i_t) \right]$$

Theory

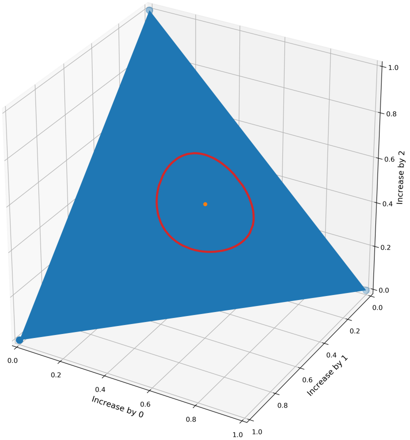
This leads to a robust Bellman equation, which in the framework of Rust (1987) is:

$$V_\theta(x_t) = \max_{i_t \in \{0,1\}} \left[u(x_t, i_t, \theta_1) + \epsilon_t(i_t) + \beta \min_{p_s \in P_s^{i_t}} EV_\theta(x_t, i_t) \right]$$

compared to the standard Bellman equation:

$$V_\theta(x_t) = \max_{i_t \in \{0,1\}} \left[u(x_t, i_t, \theta_1) + \epsilon_t(i_t) + \beta EV_\theta(x_t, i_t) \right]$$

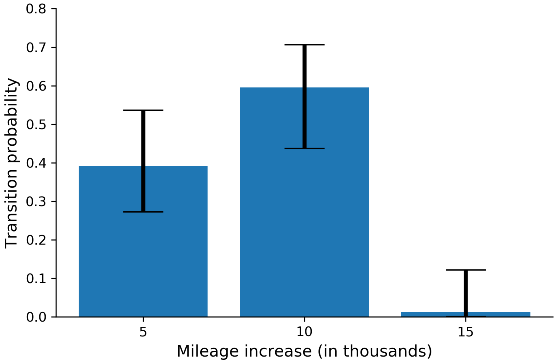
Probability simplex



Robust investments

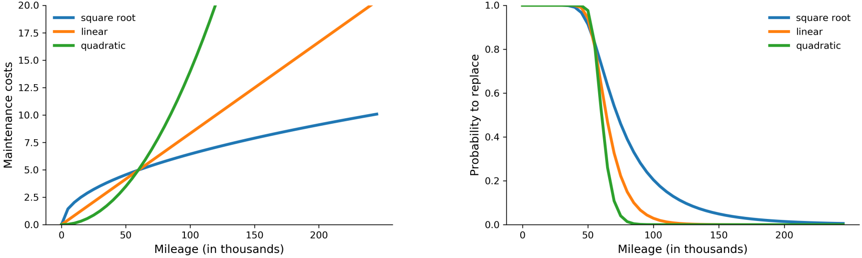
Estimated transition probabilities

Estimated transition probabilities



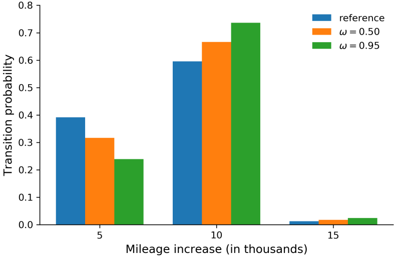
Setup cost parameters

Setup cost parameters

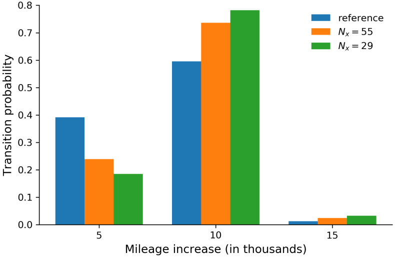


Worst case distribution

Worst case distribution

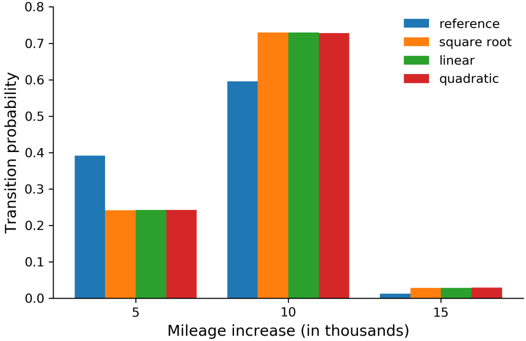


Variation in ω, ($N_x = 55$)



Variation in N_x, ($\omega = 0.95$)

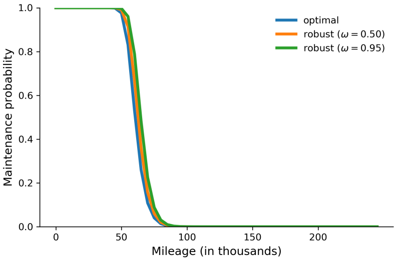
Worst case distribution



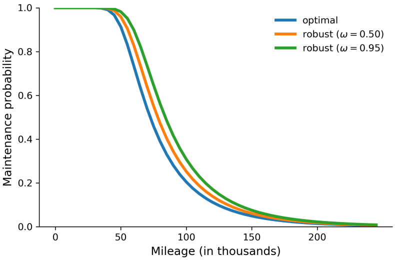
Variation cost functions,
$(N_x = 55,\, \omega = 0.95)$

Shift in maintenance probabilities

Shift in maintenance probabilities



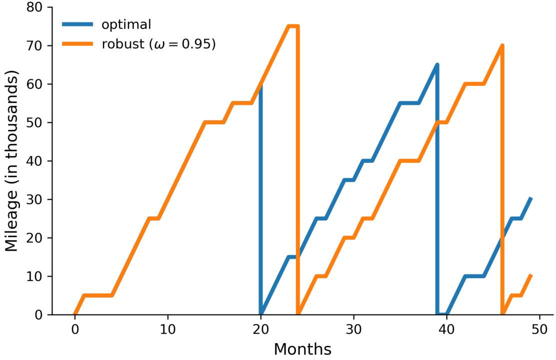
Quadratic



Square root

Simulation setup

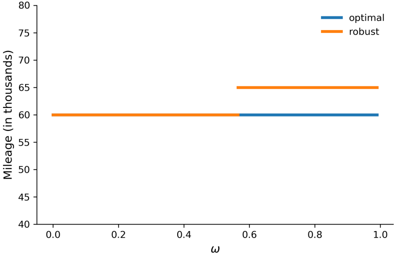
Simulation setup



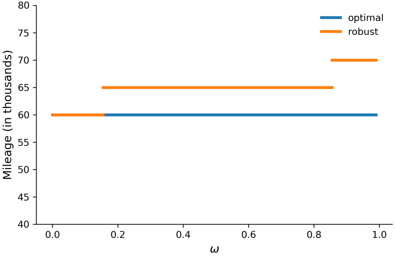
Mean mileage at replacement

Mean mileage at replacement

Actual transitions are determined by maximum likelihood.



Quadratic

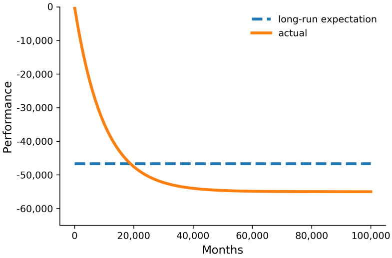


Square root

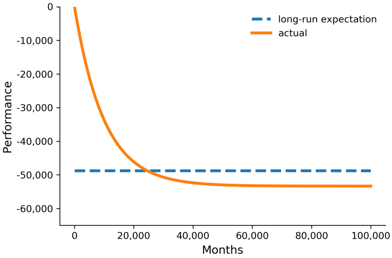
Performance of optimal policy

Performance of optimal policy

Actual transitions are determined by worst-case distribution of $\omega = 0.95$.



Quadratic

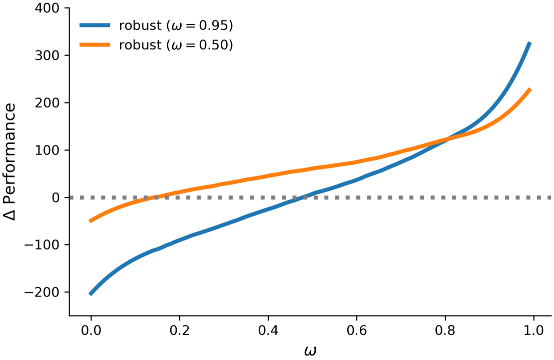


Square root

Performance on the boundary

Performance on the boundary

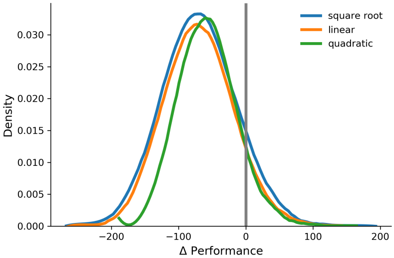
Actual transitions are determined by the worst-case distribution of varying ω.

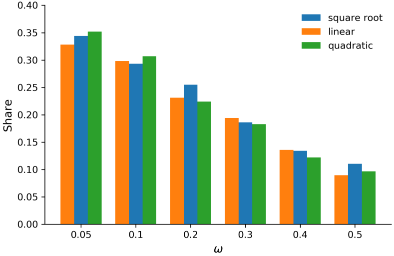


Validation performance

Validation performance

5,000 draws from the asymptotic distribution.





$\omega = 0.50$

Thank you for your attention

Our project online

Code, documentation, examples, and much more available online at

`https://github.com/OpenSourceEconomics`

Visit us!

References I

Barnett, M., Brock, W., & Hansen, L. P. (2019). Pricing uncertainty induced by climate change. *Working Paper*.

Bellman, R. E. (1954). The theory of dynamic programming. *Bulletin of the American Mathematical Society*, *60*(6), 503–515.

Ben-Tal, A., den Hertog, D., De Waegenaere, A., Melenberg, B., & Rennen, G. (2013). Robust solutions of optimization problems affected by uncertain probabilities. *Management Science*, *59*(2), 341–357.

Ben-Tal, A., El Ghaoui, L., & Nemirowski, A. (2009). *Robust optimization*. Princeton, NJ: Princeton University Press.

Kaufman, D. L., Schaefer, A. J., & Roberts, M. S. (2017). Living-donor liver transplantation timing under ambiguous health state transition probabilities. *Working Paper*.

References II

Keane, M. P., & Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, *105*(3), 473–522.

Rust, J. (1987). Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher. *Econometrica*, *55*(5), 999–1033.