Simulation of Choice Probabilities

Tobias Raabe

November 4, 2019

The probability for choosing an alternative is computed with the softmax function.

$$p_i = \text{softmax}(x) \tag{0.1}$$
$$= \frac{e^{x_i}}{\sum_j e^{x_j}} \tag{0.2}$$

A numerically robust solutions is to subtract the maximum, x^*, and use logsumexp.

$$p_i = \frac{e^{x_i - x^*}}{\sum_j e^{x_j - x^*}} \tag{0.3}$$
$$= \exp\{x_i - x^* - \log(\sum_j e^{x_j - x^*})\} \tag{0.4}$$
$$= \exp\{x_i - x^* - \text{logsumexp}(x)\} \tag{0.5}$$

As choices are affected by uncertainty, this probability is simulated.

$$\hat{p}_i = \frac{1}{n} \sum^D p_i \tag{0.6}$$

Furthermore, we want the log probability and replace p_i with the aforementioned calculation.

$$\begin{aligned}
\log \hat{p}_i &= \log(\sum_d^D p_{id}) - \log(n) \\
&= \log(\sum_d^D \exp\{x_{id} - x_d^* - \text{logsumexp}(x_d)\}) - \log(n) \\
&= \text{logsumexp}(x_{id} - x_d^* - \text{logsumexp}(x_d)) - \log(n)
\end{aligned}$$