Fitting Hazard Rates with Polynomials

We have data on mortality hazard rates from age 1 to age 90.

We will fit these to the following univariate polynomial function.

$$\ln \rho_{is} = \sum_{j=1}^{J} \beta_{is} \left(\frac{s}{90}\right)^j \tag{1}$$

where i indexes the country and s is the age. We fit each country separately in different regressions.

The vector of beta coefficients, B_i, can be estimated using OLS

$$B_i = (X'X)^{-1}X'Y_i \tag{2}$$

where X is an $S \times (J+1)$ matrix of ages raised to various powers, and Y_i is a $(J+1) \times 1$ vector of the natural log of mortality hazard rates for country i.

We fit these polynomials and save only the regression coefficients, the B_is, to pass to the program.

In our Python program, to generate mortality hazard rates for agents that live for S periods we use the regression equation above replacing 90 with S. Note this gives us the one-year hazard rate at various age intervals. To adjust for changes in the length of the period we must do further adjustments as shown below.

$$\rho_{is} = 1 - \left(1 - e^{\left[\sum_{j=1}^{J} \beta_{is} \left(\frac{s}{S}\right)^j\right]}\right)^{\frac{70}{S}} \tag{3}$$

References