Multi-Country OLG model

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Introduction

- Primary research question:
  - What the effects of different tax policy in various countries on the rest of the world?
- What we are doing:
  - Reconstructing OLG model of a large open economy in Python developed by various authors
  - Model economies of 7 different regions (U.S.A, E.U., Japan, China, India, Russia, and Korea) and how they interact
- Motivation
  - Inform policy makers of macroeconomic impact of various tax reforms
  - Incorporate theory into general OSPC model

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  - Book that developed multi-generational model to examine effects of tax policy
- Fehr, Jokisch, and Kotlikoff (2008), ”Dynamic Globalization and Its Potentially Alarming Prospects for Low-Wage Workers”
  - 6-good, 5-region general equilibrium, life-cycle model that focuses on wage difference between low-skill and high-skill workers

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Answer to research question

- ....We don't have an answer to our research question yet since we are still building the model in Python
- How we are building the model:
    - Stage 1. Simple OLG Model
    - Stage 2. Demographics and Growth

The Data

- Time t=0 is year 2008
- Initial population of each country by age
- Net migration by age
- Fertility rates by age
- Mortality rates by age
- Data for taxes, government, etc. not yet in model
- All the data (except for Korea) comes from Kotlikoff and coauthors and was pulled from a variety of data sources.

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Korean Data

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  - Fix a sixth degree polymonial of weighted average of 2005 and 2010 census data
- Fertility rates:
  - 2005-2010 data grouped into 5 age brackets
  - Fit to sixth degree polynomial for continuous function
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- β (Time preference ratio) and δ (depreciation rate) are functions of S to help with convergence
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Model Beginnings

- Began by taking Larry Kotlikoff's original Fortran Code and making a direct translation to Python.
- Posed problems because of the organization and differences between indexing arrays in Python and Fortran.
- Instead we decided to not do the translation and instead went with coding from the ground up in Python.

Model Summary

- Dynamic Life-Cycle Model
  - 80-period lived agents
  - 7 Countries: U.S.A, E.U., Japan, China, India, Russia, Korea
  - We are building up to have similiar models as Kotlikoff papers
- What the Python code does:
  - Demographic/population dynamics steady state prior to rest of model
  - Get equilibrium for fixed steady-state year
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Demographics

- The population of age s people in country i in period t is represented by N_{ist}
- Births occur as fractions of people born each period. This fraction born to people of age s in country i in period t is f_{ist}
  - People stop having kids at age 45 and begin at age 23.
- Mortality rates, as stated before, are represented as ρ_{ist}
  - People don't begin to die until age 68, this way, parents never outlive their children.
- Labor is mobile, migration occurs every year up until people are age 68 and is idential every year
- Bequests
  - Those the die leave their assets and they are spread equally across all households.

Stationarization of Demographics

- Why we must stationarize the population growth:
  - In order to calculate a steady state, population growth rates must be constant
  - Population projections are very hard beyond 50-60 years
- How we do it:
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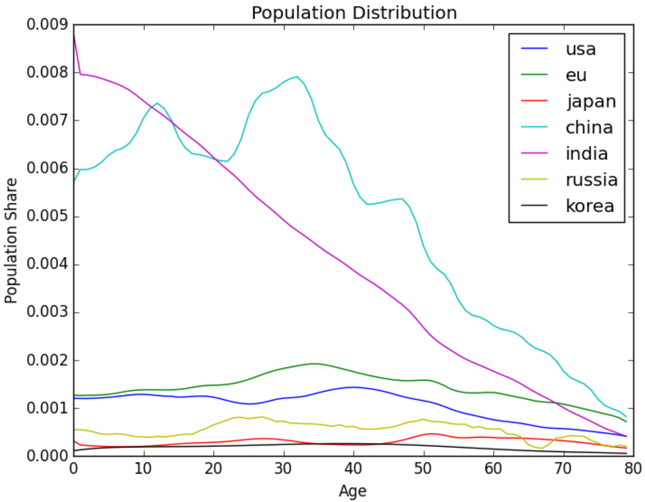
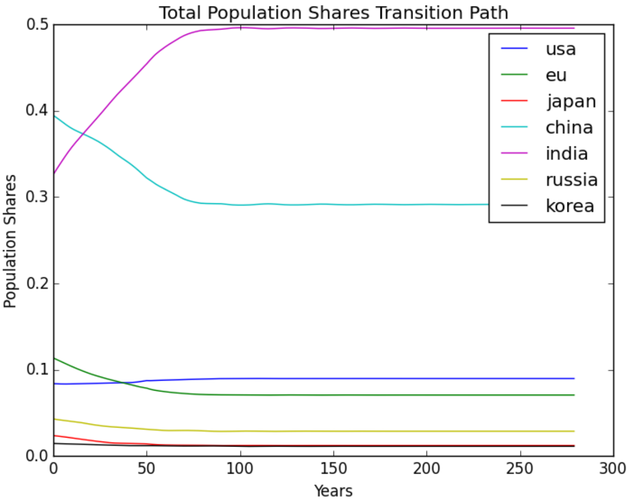
Initial Year Population Share Distribution



Population Share Equilibrium Transition



Households

- Households Maximize Utility according to:

$$\max_{\{c_{i,s+j,t+j}\}_{j=0}^{S-s}} U_{ist} = \sum_{j=0}^{S-s} \beta^j \rho_{ist}^c \frac{1}{1-\sigma} c_{i,s+j,t+j}^{1-\sigma}$$

subject to the budget constraint in each period

$$\hat{c}_{ist} = w_{it} e_{st} + (1 + r_{1t} - \delta)\hat{a}_{ist} + \hat{bq}_{ist} - \hat{a}_{i,s+1,t+1} e^{g^A}; \forall i, s \quad (1)$$

- Mortality Function:

$$\rho_{i,s+J,t+J}^c = \prod_{j=1}^{J} (1 - \rho_{i,s+j,t+j})$$

Firms

- The Representative Firm Maximizes According to:

$$\max_{n_{i,t},k_{i,t}} \Pi_{ist} = k + it^{\alpha}(A_i n_{it})^{1-\alpha} - w_{it} n_{it} - r_{it} k_{it}$$

Trade

- Mechanism of Trade:
  - Trade occurs through capital
  - Total amount of foreign-owned domestic capital sums to zero. (In other words, our model represents the entire world)
- There is one Global Interest Rate

$$r_{it} = r_t \tag{2}$$

Dynamic equations

$$g_t^N = \sum_{i=1}^{I}\sum_{s=1}^{S} \hat{N}_{ist}(f_{ist} + m_{ist} - \rho_{ist}); \forall i \tag{3}$$

$$\hat{N}_{i,1,t+1} = e^{-g_t^N} \sum_{s=23}^{45} \hat{N}_{ist} f_{ist}; \forall i \tag{4}$$

$$\hat{N}_{i,s+1,t+1} = e^{-g_t^N} \hat{N}_{ist}(1 + m_{ist} - \rho_{ist}); \forall i, 1 < s \leq S \tag{5}$$

$$\hat{k}_{it} = \sum_{s=1}^{S} \hat{a}_{ist}\hat{N}_{ist} - \hat{k}_{it}^{f}; \forall i \tag{6}$$

$$\hat{n}_{it} = \sum_{s=1}^{S} e_{is}\hat{N}_{ist}; \forall i \tag{7}$$

Dynamic equations 2

$$\hat{y}_{it} = \hat{k}_{it}^{\alpha} \left(A_i \hat{n}_{it} \right)^{1-\alpha} ; \forall i \qquad (8)$$

$$r_{it} = \alpha \frac{\hat{y}_{it}}{\hat{k}_{it}} ; \forall i \qquad (9)$$

$$w_{it} = (1-\alpha) \frac{\hat{y}_{it}}{\hat{n}_{it}} ; \forall i \qquad (10)$$

$$bq_{ist} = \frac{BQ_{it}}{\sum_{s=23}^{67} (1-\rho_{ist}) N_{ist}}$$

$$\hat{B}Q_{it} = \sum_{s=67}^{S} \hat{a}_{ist} \rho_{ist} \hat{N}_{ist} ; \forall i \qquad (11)$$

$$\hat{B}Q_{it} = \sum_{s=23}^{67} \hat{b}q_{ist} (1-\rho_{ist}) \hat{N}_{ist} ; \forall i \qquad (12)$$

Euler Equations

$$\hat{c}_{ist}^{-\sigma} - \beta(1 - \rho_{i,s+1,t+1}) \left(\hat{c}_{i,s+1,t+1} e^{g^A}\right)^{-\sigma} (1 + r_{1,t+1} - \delta) = 0; \forall i, s \tag{14}$$

$$r_{it} - r_{1t} = 0; \forall i > 1 \tag{15}$$

$$\sum_{i=1}^{I} \hat{k}_{it}^{f} \left(\sum_{s=1}^{S} \hat{N}_{ist}\right) = 0 \tag{16}$$

Solving for Model at Equilibrium

Finding the steady state

1. Make an initial guess for $\{r_t^0\}$ and $\{w_{it}^0\}$.
2. Impose $a_{ist} = \bar{a}_{is}$ and $k_{it}^f = \bar{k}_i^f$ for all t
3. Use the previous dynamic equations and search over the values to find the values of $\bar{a}_{is}$ and $\bar{k}_i^f$ that satisfy the euler equations. In Python we use an fsolve.
4. Use our values for $\bar{a}_{is}$ and $\bar{k}_i^f$ to get the equilibrium values for the rest of the system

Solving for Model Transition Path

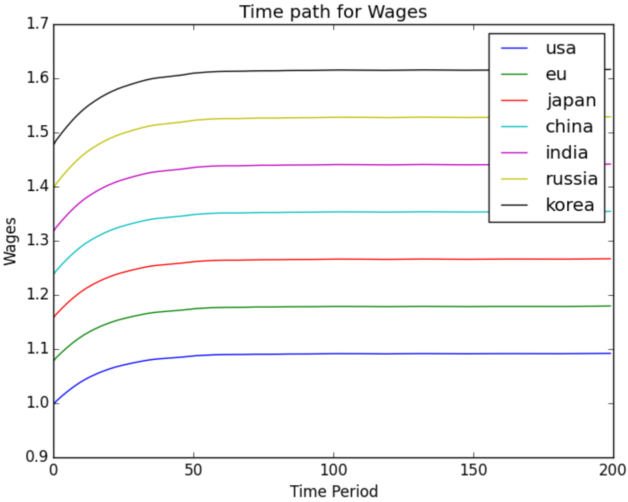
Time path iteration calculation

1. Make an initial guess for $\{r_t^0\}$ and $\{w_{it}^0\}$.
2. Solve (1) and (14) to consumption and assets decision paths for each agents' lifetime in each year.
3. Sum assets in each year to get aggregate capital K_{it} and use (6), (8), and (9) to get $\hat{k}_{it}^f \forall i > 0$
4. Take $k_{0t}^f = -\sum_{i=1}^{I} k_{it}^f$ to satisfy (16).
5. Get new paths $\{r_t^{new}\}$ based on $k_{0,t}^f$ and $\{w_{it}^{new}\}$.
6. If difference between old and new paths not within a given tolerance, take a convex combination for new guesses and iterate until a solution is found.

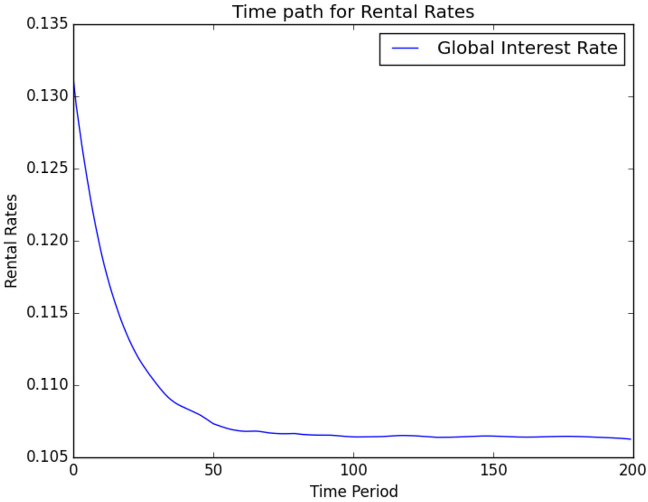
Robustness Checks

- We have employed a special script using parallel that checks multiple combinations of numbers of countries, numbers of cohorts and σ (intertemporal elasticity of substitution).
- Then, the root processor creates a .csv file that track where the TPI converges and where it fails.
- It can also save graphs of each of the attempts of running the code.

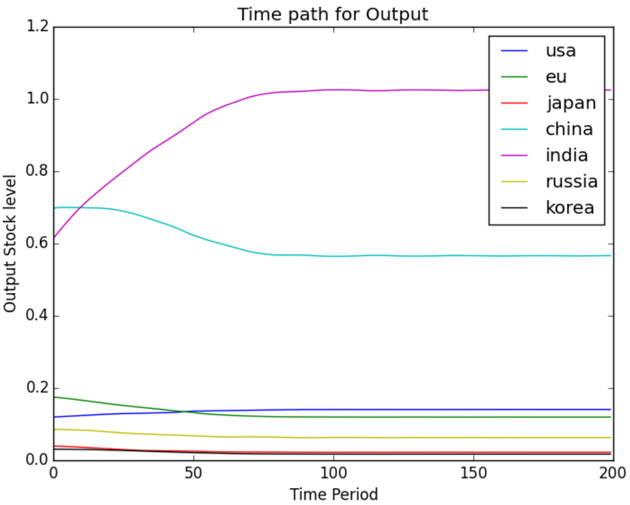
Calculated Wages



Global Interest Rate



Aggregate Output



Work To Be Done

- This code is being built in stages, what we've shown is the first two stages.
- Stage 3 (Current): Adding a Labor/Leisure Decision
- Stage 4: Add children
    - Essentially, changing the utility function to reflect children's consumption, since people tend to value their children's consumption.
- Stage 5: Adding Labor Classes
- Stage 6: Adding Corporate Taxes