1 Demographics

1.1 Assumptions

Ages 0-20 don't work and are supported by parents
Age 21 start working and become individual households
Ages 23-45 give birth each year to fractions of children
Ages 43-66 children leave home each year
Ages 68-90 they die
Immigration exogeneously set to match population projections, has perfect assimilation

1.2 Variables

$N(a,t,k)$ = matrix of agents a = age
t = year
k = skill class

1.3 Functions/Formulas

$KID(a,t,k)$ = total number of kids of agent age a with skill k in year t

$$KID(a,t,k) = \sum_{l=u}^{m} \frac{N(l,t,a-l,k)}{\sum_{s=23}^{45} N(a,t,s,k)} \quad 23 \leq a \leq 65,\ k = 1,2,3$$

2 Households

2.1 Assumptions

- Given taxes, interest rates $r(i)$, and wages $w(k,i)$:
- Agents maximize utility subject to the intertemporal budget constraint and the constraint that leisure in each period doesn't exceed the time endowment, i.e $\ell(l,t,k \leq h(l,t)$.
- They do this by choosing leisure and consumption $\ell(l,t,k), \bar{c}(a,i,k),$ and $\bar{c}_K(a,i,k)$

Additionally:

- Given individual consumption and leisure, we can get agents' asset levels from the function a$(l+1,t+1,k)$
- Aggregate values of assets, private consumption goods, and labor supply obey the Functions/Formulas $A(t+1), C(t),$ and $L^S(k,t)$

2.2 Variables

$i = t + a - l$
δ: rate of time preference
ρ : intratemporal elasticity of substitution between consumption and leisure
ω : intratemporal elasticity of substitution between goods
ϵ : leisure preference
γ : intertemporal elasticity of substitution between consumption and leisure
$r(t)$: pre-tax return on intrest
$w(k, t)$: wage rate
$E(a, i)$: productivity per time-unit in year i and age a
$\bar{A}(t, k)$: aggretate assets of skill-class k agents who die in year t.
$\Gamma(l)$: share of bequests $\bar{A}$ dedicated to inheritants aged l of the same skill class
where $\sum\limits_{l=21}^{49} \Gamma(l) = 1$
λ : common growth rate of time endowment of successive generations
$c(j, a, i, k)$: private consumptions of goods j
$\kappa(j, a)$: consumption share of good j at age a, where $\sum_{j \in G_c} \kappa(j, a) = 1$
$\bar{c}(a, i, k)$: aggregate private good consumpiton
$\bar{c}_K(a, i, k)$: aggregate consumption of children
$\ell(a, i, k)$: leisure
$d(l, t)$: mortality probability of an agent age l in year t
a(l, t, k) asset endowment of the agent in year t
$T(l, t, k)$: Net taxes: consumption, capital income, wage taxes, and benefits received in the form of transfer payments
$\bar{A}$: Aggregation across all agents alive at the end of the prior period of dead households

2.3 Functions/Formulas

$U(l, t, k) = V(l, t, k) + H(l, t, k)$:
Remaining lifetime utility of a generation age l at time t of skill-class k

$V(l, t, k) =$ Utility from own goods/leisure:

$$\frac{1}{1-\frac{1}{\gamma}} \sum_{a=l}^{90} (\frac{1}{1+\delta})^{a-l} P(a, i) \bar{c}(a, i, k)^{1-\frac{1}{\rho}} + \epsilon \ell(a, i, k)^{1-\frac{1}{\rho}}]^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\rho}}}$$

$H(l, t, k) =$ Utility from children's consumption:

$$\frac{1}{1-\frac{1}{\gamma}} \sum_{a=l}^{90} (\frac{1}{1+\delta})^{a-l} P(a, i) KID * (a, i, k) \bar{c}_K(a, i, k)^{1-\frac{1}{\gamma}}$$

$P(a, i) = \prod\limits_{u=l}^{a} [1 - d(u, u - a + i)]$: survival probability of reaching age a in year i

$\bar{p}(a,i) = [\sum_{j\in G_c} \kappa(j,a)^{\omega} p(j,i)^{1-\omega}]^{\frac{1}{1-\omega}}$: price index of $\bar{c}(a,i,k)$

$$\mathrm{a}(l+1,t+1,k) = \mathrm{a}(l,t,k) + I(l,t,k)][1 + r(t) + w(t,k)E(l,t)][h(l,t) - \ell(l,t,k)]$$
$$-T(l,t,k) - \bar{p}(a,i)[\bar{c}(l,t,k) - KID(l,t,k)\bar{c}_K(l,t,k)]$$

$c(j,a,i,k) = \left(\frac{\kappa(j,a)}{p(j,i)}\right)\bar{p}(a,i)^{\omega}\bar{c}(a,i,k)$: Demand for specific goods $j \in G_c$

$I(l,t,k) = \Gamma(l)\frac{\bar{A}(t,k)}{N(l,t,k)}$: inheritances received in year t

$h(a,i) = (1+\lambda)h(a,i-1)$: Time endowment of an agent age a at time i

$A(t+1) = \sum_{k=1}^{3}\sum_{a=21}^{90} \mathrm{a}(a+1,t+1,k)N(a,t,k)$: Aggregate values of assets

$\bar{A}(t+1,k) = \sum_{a=21}^{90} d(a+1,t+1)\mathrm{a}(a+t,t+1,k)N(a,t,k)$: bequests

$C(j,t) = \sum_{k=1}^{3}\sum_{a=21}^{90}[c(j,a,t,k) + KID(a,t,k)c_K(j,a,t,k)]N(a,t,k)$: Private consumption of good j

$L^S(t,k) = \sum_{a=21}^{90} E(a,t)[h(a,t) - \ell(a,t,k)]N(a,t,k)$: Labor supply

3 Production Sector

3.1 Assumptions:

Profit maximization requires the following conditions:

$$[r(t) + \delta_K]K(j,t) = \alpha(j)q(j,t)Y(j,t)$$
$$w(t,k)L(j,t,k) = [1-\alpha(j)]\beta(j,k)q(j,t)Y(j,t)$$

3.2 Variables:

ϕ : total factor productivity
$\alpha(j)$: shares of capital inputs in production
$\beta(j,k)$: shares of skill-specific labor inputs in production, where $\sum_{k=1}^{3}\beta(j,k) = 1$
δ_K: depreciation rate
$q(j,t)$: producer price of good j in year t
$\tau^K(t)$: Corporate tax rate

3.3 Functions/Formulas:

$Y(j,t) = \phi K(j,t)^{\alpha(j)}\big[\prod_{k=1}^{3} L(j,t,k)^{\beta(j,k)}\big]^{1-\alpha(j)}$: Aggregate output of good j

$T^k = \tau^K(t)\big[Y(t) - \sum_{k=1}^{3} w(k,t)L(k,t) - \delta_K K(t)\big]$: Corporate taxes

$r(t) = \phi\alpha q(j,t)\big(\frac{\prod_{k=1}^{3} L(j,t,k)^{\beta(j,k)}}{K(j,t)}\big)^{1-\alpha}$: rental rate in year t

$w(t,k) = \phi(1-\alpha)\beta(j,k)q(j,t)\big(\frac{K(j,t)}{L(j,t,k)^{\beta(j,k)}}\big)^{\alpha}$: wage rate for skill level k in year t

4 Government

4.1 Assumptions:

Each government maintains its debt-to-output ratio over time
Pension lindearly depend on working-life average earnings: $Pen(a,t,k) = v_0 + v_1\bar{W}(i,k)$

4.2 Variables:

$B(t)$: Governemnt Debt $\Delta B(t)$: Government Deficit in year t
$C^g(t)$: Government Expenditures in year t
$SB(t)$: General-revenue financed social benefits, (e.g. pension, health care, etc)
ϱ: Share of transfer payments financed by general revenue
$PY(t)$: Aggregate payroll tax base with fixed ceiling at a percentage of average income in a particular country
$\hat{\tau}^p(t)$: Region-specific average employer and employee payroll tax rates
$Pen(a,t,k)$: Pension benefits
$\cap a$: Retirement age
$\bar{W}(i,k)$: Average earnings during working life

4.3 Functions/Formulas:

Main Government budget equation:

$$\Delta B(t) + \sum_{k=1}^{3}\sum_{a=21}^{90} T(a,t,k)N(a,t,k) + T^K(t) = C^g(t) + \varrho SB(t) + r(t)B(t)$$

$\hat{\tau}^p(t)PY(t) = (1-\varrho)SB(t)$

5 All variables

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