NTCIR-10 Math Pilot Task Overview

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Introduction & Motivation for a Math Pilot Task



Introduction/Background

- ► Mathematics plays a fundamental role in Science, Technology, and Engineering (learn from Math, apply for STEM)
- ► Mathematical knowledge is rich in content, sophisticated in structure, and technical in presentation!



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- ► There is a lot of documents with maths
 - ▶ there are 120.000 journal articles per year in pure/applied math, 3.5 Million overall
 - ▶ 50 million science articles in 2010 [Jin10] with a doubling time of 8-15 years [Lvl10] And this excludes gray literature, engineering, and school textbooks.
 - ▶ Even in the Renaissance, polymaths like Leonardo de Vinci were a rare exception.



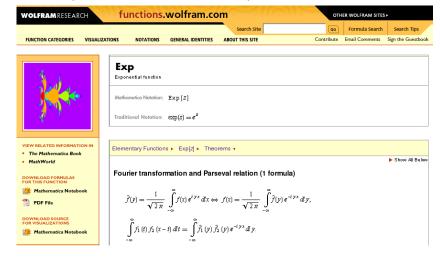
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 - ▶ Even in the Renaissance, polymaths like Leonardo de Vinci were a rare exception.
- ► We need IR support to deal with this! (~> NTCIR-10 Math Pilot Task)



Mathematics Resources on the Web

► Example 1 (The Wolfram Functions Site) contains ≥ 307k Formuae







More Mathematics on the Web

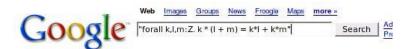
... Engineering Company Intranets, ...

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    ► The Connexions project (http://cnx.org)
    ► Wolfram Inc. (http://functions.wolfram.com)
    ► Eric Weisstein's MathWorld (http://mathworld.wolfram.com)
    ► Digital Library of Mathematical Functions (http://dlmf.nist.gov)
    ► Cornell ePrint arXiv (http://www.arxiv.org)
    ► Zentralblatt Math (http://www.zentralblatt-math.org)
```

- ▶ Question: How will we find content that is relevant to our needs
- ▶ Idea: try Google (like we always do)
- Scenario: Try finding the distributivity property for \mathbb{Z} $(\forall k, l, m \in \mathbb{Z}.k \cdot (l+m) = (k \cdot l) + (k+m))$



Searching for Distributivity



Web

Tip: Try removing quotes from your search to get more results.

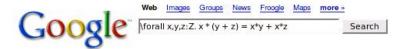
Your search - "forall k,l,m:Z. k * (I + m) = k*I + k*m" - did not match any documents.

Suggestions:

- Make sure all words are spelled correctly.
- · Try different keywords.
- Try more general keywords.



Searching for Distributivity



Web

Untitled Document

... theorem distributive Ztimes Zplus: distributive Z Ztimes Zplus. change with (\forall x,y,z:Z. x * (y +

z) = x*y + x*z). intros.elim x. ...

matita.cs.unibo.it/library/Z/times.ma - 21k - Cached - Similar pages





Searching for Distributivity



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Web Images Groups News Froogle Maps more
```

\forall a,b,c:Z. a * (b + c) = a*b + a*c

Search

Web

Mathematica - Setting up equations

Try "Reduce" rather than "Solve" and use "ForAll" to put a condition on x, y, and z. In[1]:=

Reduce[ForAll[$\{x, y, z\}, 5^*x + 6^*y + 7^*z == a^*x + b^*y + c^*z], ...$

www.codecomments.com/archive382-2006-4-904844.html - 18k - Supplemental Result -

Cached - Similar pages

[PDF] arXiv:nlin.SI/0309017 v1 4 Sep 2003

File Format: PDF/Adobe Acrobat - View as HTML

7.2 Appendix B. Elliptic constants related to gl(N,C). ... 1 for all s ≤]. (4.14). The first condition means that the traces (4.13) of the Lax operator ...

www.citebase.org/cgi-bin/fulltext?format=application/pdf&identifier=oai:arXiv.org:nlin/0309017 -

Supplemental Result - Similar pages

\documentclass{article} \usepackage{axiom} \usepackage{amssymb ...

i+1) bz:= (bz - 2**i)::NNI else bz:= bz + 2**i z.bz := z.bz + c z x * y == z ... b,i-1)] be := reduce(***, ml)

c = 1 => be c::Ex * be coerce(x): Ex == tl ...

wiki, axiom-developer.org/axiom-test--1/src/algebra/CliffordSpad/src - 20k - Supplemental Result -Cached - Similar pages



Of course Google cannot work out of the box

- ► Formulae are not words:
 - \triangleright a, b, c, k, l, m, x, y, and z are (bound) variables.

(do not behave like words/symbols)

where are the word boundaries for "bag-of-words" methods?



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- where are the word boundaries for "bag-of-words" methods?
- ► Idea: Need a special treatment for formulae Indeed this is done

(translate into "special words") ([MY03, MM06, LM06, MG11])

...and works surprisingly well

(using Lucene as an indexing engine)

► Idea: Use database techniques Indeed this is done for the Cog/HELM corpus (extract metadata and index it) ([AGC⁺06])

▶ Idea: Use Automated Reasoning Techniques

(Term Indexing [Nor06, K\$06, KMP12])

▶ Idea: Use standard IR techniques

(Learn from the NTCIR crowd?)

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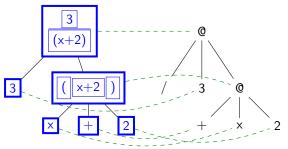
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(Term Indexing [Nor06, K\$06, KMP12])

- ► Idea: Use standard IR techniques (Learn from the NTCIR crowd?)
- ▶ Which one is best?: We do not really know, evaluation is very difficult
- ▶ Future: maybe even mix/integrate the respective best features (once we know)

Markup Markup e.g. in MathML and LATEX

- ▶ MathML3 is a W3C Recommendation for representing Formulae [ABC+10]
- ▶ Idea: Combine the presentation and content markup and cross-reference



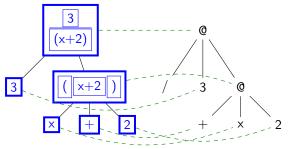
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(click on presentation, follow link and copy content)



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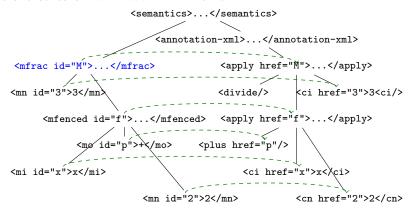


- use e.g. for semantic copy and paste.
 (click on presentation, follow link and copy content)
- ▶ But: Formulae are mostly written in LATEX, e.g. \frac{3}{(x+2)}
- ► Solution: Write LATEX, convert to HTML5 HTML+MathML+SVG



Parallel Markup Markup in MathML

► Concrete Realization in MathML: semantics element with presentation as first child and content in annotation-xml child





Task Description





Task Overview

- ▶ Math Retrieval Subtask: Given a document collection, retrieve relevant mathematical formulae or documents for a given query.
- ▶ Math Understanding Subtask: Extract natural language definitions of mathematical expressions in a document for their semantic interpretation.



- ► NTCIR-10 Math Dataset: 100.000 Documents transformed to HTML5 from http://arxiv.org (10.000 for dry run)
 - ▶ 63 GiB overall size, 35 MFormulae, 297 MSubformulae (size challenge for systems)
 - every formula given in content MathML, presentation MathML, and LATEX (23 GiB)





The uncertainty principle requires that sequential measurements of non-commuting spin components cannot achieve a resolution of $\varepsilon = 1$ at zero back-action. For orthogonal spin components, the quantitative limit can be expressed in terms of the uncertainty relation [18, 19]

$$\varepsilon^2 + (1 - \eta)^2 \le 1. \tag{5}$$

It is therefore impossible to measure the intrinsic joint probabilities $P_{\psi}(s_2, s_3)$ directly. However, the spin flip model allows us to reconstruct this joint probability from the experimentally observed distribution of sequential outcomes, $P_{\exp}(s_2, s_3)$. Due to the spin flip errors, each measurement outcome (s_2, s_3) can also originate from different spin values, with probabilities determined by the spin flip probabilities of (1 - e)/2 and $\pi/2$. The relation between the experimental probabilities and the intrinsic probabilities is then given by

$$P_{\exp(s_2, s_3)} = \left(\frac{1+\epsilon}{2}\right) \left(1 - \frac{\eta}{2}\right) P_{\psi}(s_2, s_3) + \left(\frac{1-\epsilon}{2}\right) \left(1 - \frac{\eta}{2}\right) P_{\psi}(-s_2, s_3) + \left(\frac{1+\epsilon}{2}\right) \left(\frac{\eta}{2}\right) P_{\psi}(s_2, -s_3) + \left(\frac{1-\epsilon}{2}\right) \left(\frac{\eta}{2}\right) P_{\psi}(-s_2, -s_3).$$
(6)

This linear map can be inverted to reconstruct the intrinsic joint probabilities $P_{\psi}(s_2, s_3)$ from the experimentally observed joint probabilities $P_{\exp}(s_2, s_3)$. If the measurement resolution and the back-action are known, the same joint probabilities $P_{\psi}(s_2, s_3)$ should be obtained at any measurement strength. The relations that describe the reconstruction of intrinsic joint probabilities from the measurement data are given by

$$P_{\psi}(s_2, s_3) = \frac{(1 + \epsilon)(2 - \eta)}{4\epsilon(1 - \eta)} P_{\exp}(s_2, s_3) - \frac{(1 - \epsilon)(2 - \eta)}{4\epsilon(1 - \eta)} P_{\exp}(-s_2, s_3)$$

$$- \frac{(1 + \epsilon)\eta}{4\epsilon(1 - \eta)} P_{\exp}(s_2, -s_3) + \frac{(1 - \epsilon)\eta}{4\epsilon(1 - \eta)} P_{\exp}(-s_2, -s_3).$$
(7)

Note that the spin flip model used to reconstruct the intrinsic joint probabilities of the quantum state does not require any assumptions from quantum theory and is based entirely on the experimentally observable spin flip rates $(1 - \epsilon)/2$ and n/2.



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- ► NTCIR-10 Topics:

Three Math Retrieval Subtasks: (write LATEX+?, transform to MathML Query)

- ► Formula Search (FS; automated): Formulae with named wildcards e.g. $\int_{?!}^{?h} ?f(x)^2 dx$
- ► Full Text Search (FT; automated): (Formulae and keywords) e.g. Bell curve in the form of $\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- Open Math IR (OMIR; semi-automated): human-readable questions e.g. For which n and k is PSL(n, k) not commutative?

Math Understanding Task: Manually created content MathML as gold standard



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Math Understanding Task: Manually created content MathML as gold standard

► State of Play: Establishing community (16/6 Teams), ran successful Task (made mistakes, learnt a lot)



Participation, Evaluation & Results



NTCIR-10 Math Pilot Task Participants

Group ID	Organization
BRKLY	University of California, USA
FSE	Technische Universität Berlin, Germany
KWARC	Jacobs University, Germany
MCAT	National Institute of Informatics, Japan
MIRMU	Masaryk University, Czech Republic
NAK	Keio University, Japan



Number of runs for each subtask category.

Group ID	Subtasks						
	MIR/FS	MIR/FT	MIR/OIR	MU			
BRKLY	4	1*	_				
FSE	1	1	_	_			
KWARC	1	_	_	_			
MCAT	1	2	_	4			
MIRMU	4	1*	1*	_			
NAK	1	_	_	_			
Total	12	3(2*)	0 (1*)	4			

^{*} Reported only document URIs without formula IDs and were not included in the relevance judgment pool.

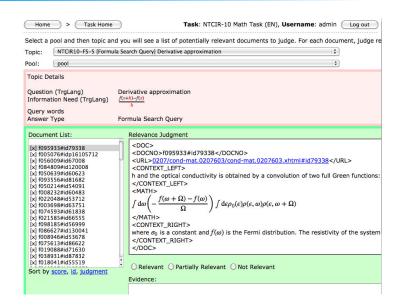


Total number of topics.

Query type	Distributed	Evaluated	
Formula Search	22	21	
Full Text Search	15	15	
Open Search	19	0	



Assessment: Math Extension for SEPIA

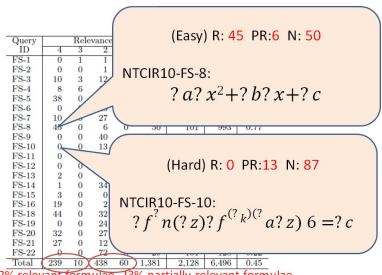




Math Retrieval Subtask: Pooling

- ▶ Select formulae as evenly as possible from all the runs
- ▶ The current top ranked formulae were taken from all the ranking lists, and added to the pool if they were not found.
- ► This process was repeated until the total size of the pool becomes equal or greater than 100.

Pooling Results



12% relevant formulae, 23% partially-relevant formulae



Relevance judgment statistics (Formula Search).

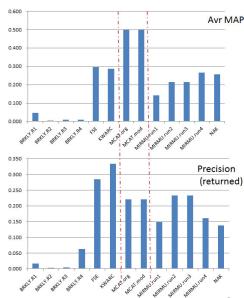
Query	Relevance score				Total	Total	Uniq	
ID	4	3	2	1	0	judged	hit	ratio
FS-1	0	1	1	30	69	101	155	0.30
FS-2	0	0	1	1	102	104	453	0.25
FS-3	10	3	12	10	66	101	284	0.33
FS-4	8	6	17	19	52	102	278	0.56
FS-5	38	0	25	0	38	101	274	0.34
FS-6	0	0	25	0	77	102	261	0.53
FS-7	10	0	27	0	68	105	382	0.46
FS-8	45	0	6	0	50	101	993	0.77
FS-9	0	0	40	0	63	103	361	0.58
FS-10	0	0	13	0	87	100	281	0.49
FS-11	0	0	42	0	58	100	161	0.29
FS-12	0	0	26	0	74	100	135	0.26
FS-13	2	0	0	0	98	100	245	0.49
FS-14	1	0	34	0	65	100	231	0.40
FS-15	3	0	0	0	98	101	304	0.23
FS-16	19	0	2	0	81	102	357	0.38
FS-18	44	0	32	0	28	104	610	0.58
FS-19	0	0	24	0	76	100	195	0.29
FS-20	32	0	27	0	41	100	100	0.00
FS-21	27	0	12	0	61	100	178	0.31
FS-22	0	0	72	0	29	101	128	0.22
Total	239	10	438	60_	1,381	2,128	6,496	0.45



Evaluation Measure & Results

- Formula-based evaluation: It turned out document-based evaluation cost too much for human assessors!
- ► Evaluation measures:
 - ► Trec-eval (MAP, P-5, P-10) (similarity-based systems)
 - P-hit: The ratio of the relevant and the submitted hits for all the queries.

(matching-based systems)



Math Understanding Task: Dataset & Participants

- Development Data: 35 papers selected from ArXiv.org dataset which were also used in Math Retrieval Task.
- ▶ Data for Formal Run: 10 papers selected from ArXiv.org dataset which were also used in Math Retrieval Task.



- ▶ There was only one participant
- ▶ The achieved performance was
 - ▶ 0.45-0.55 (F1-measure) for strict matching
 - ▶ 0.55-0.65 (F1-measure) for soft matching
- ▶ The best precision for soft matching was 0.87



Lessons Learnt & Conclusions



Four Types of Approaches

► Math-agnostic IR systems:

(BRKLY)

- Keyword-based search
- ▶ Did not perform well
- ► Batch Math processors:

(FSE)

- ▶ Distributed system, but does not use a search index
- Not suitable for interactive IR
- ► Matching/Unification-based Math IR systems:

(KWARC, NAK)

- ▶ Return exact instances of the query
- ► Similarity-search Math IR systems:

(MIRMU, MCAT)

- Return large sets of similar formulae scored by closeness.
- Query variables are similar to any sub-formula

Analysis of Retrieval Performances

- Without explicitly dealing with mathematical formulae, it is very difficult (impossible?) to achieve high effectiveness
 - ► Math-agnostic systems did not perform well
- ► Similarity-search Math IR systems did better overall
 - Especially in terms of MAP
 - Investing into partial matches can be rewarding
- Matching /Unification-based Math IR systems perform better if unanswered topics are discarded



Relating the Subtasks

- ▶ Math Search has normally two components:
 - ► Math Understanding (semantic extraction)
 - ► Semantic Search
- Math Understanding can therefore be seen as a step towards more intelligent Math Search
- ▶ In the Formulae Search Subtask, all groups used a possible baseline (LATEXML) to approximate Math understanding
- ▶ In the future, the efforts in creating more sophisticated and effective Math understanding systems can directly feed into Formulae Search enriching the indexing of documents



Conclusions: Achievements

- First time a task dedicated to Math IR was run as part of an evaluation forum
- ► The NTCIR-9 Pilot Math Task has been successful in creating an experimental platform for conducting Math Retrieval experiments:
 - The development of a new collection of 100K of documents and over 35M of formulae
 - ▶ The definition of 2 natural Math Search Subtasks
 - ▶ The development of a reusable relevance assessment system for Math Tasks



Conclusions: Assessment of the state

- ▶ A great deal of work has been done in the first NTCIR-10 Math Pilot Task:
 - Identification of reasonable baselines
 - Shaping the details of the tackled Math Subtasks
 - ► Facilitating the formation of a pluri-disciplinary community of researchers



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 - Identification of reasonable baselines
 - ▶ Shaping the details of the tackled Math Subtasks
- ► Facilitating the formation of a pluri-disciplinary community of researchers
- ▶ A great more deal of work is still needed:
 - ► Refinement of the topic development process (e.g. easy vs hard topics)
 - Conducting an inter-assessor agreement study
 - Developing common/standard baselines
 - ... Converging perspectives from the two main types of participating groups: IR scientists and Mathematicians/Logicians



Looking Forward

- ▶ It is our intention to run a new iteration of Math IR Task in NTCIR-11
 - ▶ Using the same created collection in NTCIR-10
 - Focussing more on the Formulae Search Subtask
 - Achieving a reusable test collection for the Formulae Search Subtask
 - Developing the Math Understanding Subtask
- ▶ ... Growing and supporting the Math IR community

Visit the new community portal: https://trac.mathweb.org/NTCIR-Math/



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