Preface.

While the profound and sublime Philosophy, which the most benevolent Divinity has finally granted to mortals in these times, must be attributed to the advancement of Geometry; and the remarkable progress that many other Arts and Disciplines, most useful to the human race, have made in our age have sprung from the same source; no sensible person would deny that any new developments in this Science should not be disregarded, and that it should be cultivated above all by those who seek truth. Nor will anyone disparage the various efforts of Geometers to advance pure Mathematics (even if the utility of such inventions may not be apparent at first glance) who remembers how many things, which at first seemed of little use, have later, through the industry of others, been adapted to excellent purposes.

Nor does Geometry seem to be cultivated by its students only because it brings great benefits and is an ornament; but also because it greatly attracts with its inherent beauty; indeed, it generates in our minds a pleasure not unlike that which we derive from the humanities. For to anyone considering the various pleasures of life, it will become clear that they arise chiefly from the perception of proportion, in which all beauty resides; and that our minds are so constituted by nature, *But of whom are these things; if not that we may understand in some measure whence so much charm resides in those Theories, which all sciences, especially Mathematics, present to the intellect; so that the praise and commendation of pure Geometry may be established? Indeed, the preeminence of Mathematical Philosophy is today universally acknowledged: For it directs the mind to the grandiloquent eloquence of the heavens and presents the most beautiful form of the whole work to the eyes, as it were. By its aid, whether we investigate the Phenomena of the motion of any separate body of the world; or whether we contemplate the disposition and structure of the entire System, as far as it is yet known; everywhere discovering the highest order and most consummate harmony of things, we cannot but be filled with the sweetest admiration of so great a work and of its great Author.*

But pure Mathematics, which claims for itself by right the greatest part of this glory, however great it is, although it does not strike us with such obvious pleasure; yet it supplies the mind with the most beautiful concepts; which everywhere desire Harmony and Proportion. To these contemplations whoever is devoted, whether to the various agreeing properties of some one accurate Line or Figure; or to some complete

If one considers the appearance of Figures, or the System of appearances, or compares those various universal systems; one examines in the mind the idea of all proportion, and thus of beauty, varied in infinite ways; since every possible law or rule of harmony can have a curve as its observer. Meanwhile, the powers of the mind are wonderfully increased and strengthened by such exercises. But as, with the help of pure Geometry, we investigate both the present structure of the World and the phenomena that would have been formed according to other laws, indications everywhere emerge most clearly of a most wise Deity, the supreme Lord of the universe; whose work we perceive on every side to be worthy of its Author, insofar as we, spectators of so small a part of its extent and duration, are able to attain. Let these things be said to this end, that it may be evident that those very theories of pure Geometry, which do not immediately appear to pertain to any part of life, are not to be called useless or unpleasant; and whose deeper investigation has been perpetually approved by the example of the greatest men.

The geometry of the ancients extended only to Figures which are circumscribed by straight lines or curves of the first kind. More recent geometers, however, have admitted infinite orders of lines into Geometry; and they define them by equations involving the ordinates and abscissas of curves. No one, however, before the illustrious Newton, attempted a universal organic description of lines of the second or higher order. His method provides a most convenient way of mechanically describing third-order lines endowed with a double point; it also includes some lines of higher orders. Following in his footsteps, we present a most universal Method in the following Treatise; by which a curve of any higher order is generated by the continuous motion of given angles along straight lines, or even along curves of any lower order.

In the first part, we show how second-order lines can be described by means of a single straight line. Then, from two straight lines, we deduced the generation of third-order lines endowed with a double point. Next, we described lines of this order, devoid of a double point, and of the fourth, by means of three straight lines. Finally, we have most universally demonstrated the method of describing any curves by means of several straight lines; and some.

In the second part, we have demonstrated that curves of all higher orders can be described equally well by means of any lower ones. We have treated universally epicycloids of any kind, which are generated by the revolution of curves upon themselves as bases. We have opened an expeditious way to measure well nigh many infinite series of these epicycloids. Then we have presented a method of investigating, with very little labor, the resistance and density of the media in which given curves describe the motion of bodies. Lastly, we have shown, in all cases which we have hitherto been able to consider accurately, how a geometric curve of a given order can be drawn through given points.

In most of the propositions of the first part, we have not only demonstrated to which order the described curves belong, but also shown how their algebraic equations can be obtained. However, I would like to warn the readers that very different signs of quantities arise in those equations, depending on the various positions of the given straight lines and the various kinds of given angles; which in only some cases it was worth the effort to designate.

In the last section of the first part, we postulated that the maximum number of points in which any two lines intersect is always equal to the product of the numbers expressing both orders. Since the matter seemed to require demonstration, we have added it, as far as possible accommodated to all cases, at the end, perhaps to be dealt with by its own numbers hereafter. However, the reason why we are compelled to bring this, rather crudely handled by whatever unskilled hand, into the light is that leisure and excellent opportunity have ceased.