2 Step Derivation Concept -Time Speed Distance Triangle

Time (SI unit - seconds) and distance (SI unit - meters) are fundamental quantities of measurement. For a distance ‘d’ covered in a time duration ‘t', the average speed ‘s’ or simply the speed is defined as the rate of covering the given distance or distance covered per unit time.

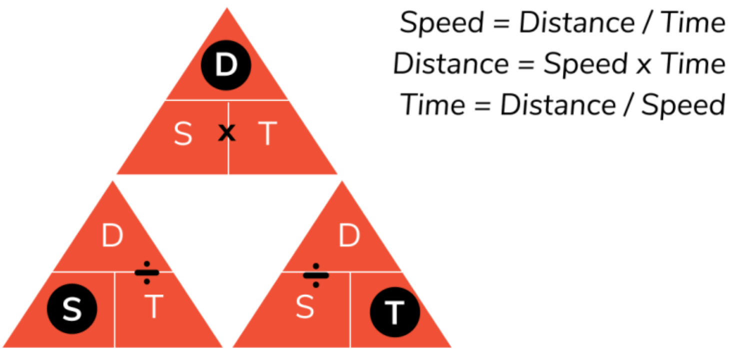
Therefore, **the formula for Time Speed Distance is:**

Speed = Distance / Time

This means,

Distance = Speed x Time

Time = Distance / Speed



Note: SI unit of speed is meters per second or m/sec. The other most commonly used unit of speed is kilometer per hour (kmph or km/h).

- To convert a **speed given in m/s into a speed in km/h**, multiply with 18/5.
- To convert a **speed given in km/h into a speed in m/s**, multiply with 5/18.

A simple way to remember the multiplication factor is to recall that particular speed, when expressed in km/h, is numerically larger than the same speed expressed in m/s. For other units like m/min or km/min, it is sufficient to remember that 1 km = 1000 m and 1 min = 60 sec.

Relation between Time Speed Distance

From the above, the following can be concluded about the relations between Time Speed Distance

- If two bodies move at the same speed, the distances covered by them are (directly) proportionality to the times of travel. i.e. when s is constant,
- If two bodies move for the same time period, the distances covered by them are (directly) proportional to the speeds of travel. i.e. when t is constant,
- If two bodies move for the same distance, their times of travel are inversely proportional to the speeds of travel. i.e. when d is constant,

Time Speed Distance Formula

Here are some important time speed distance formula you need to know to solve problems quickly.

a) Average Speed

- When a body travels different distances at different speeds, the average speed is the amount of time taken to travel the total distance in total time, Therefore, the formula for Time Speed Distance is:

$$Average\ speed = \frac{Total\ distance}{Total\ time}$$

- When the body travels at speeds of u and v units for equal distances, i.e. the distance segments are in the ratio 1:1, then

$$Average\ speed = \frac{2uv}{u+v}$$

b) Relative Speed

Consider two bodies moving at speeds u and v.

- When they are moving in the same direction, the **relative speed between the two bodies** is the difference of their speeds, i.e.u – v if u > v or v – u if v > u.
- When they are moving in the opposite directions, the relative speed between the two bodies is the sum of their speeds, i.e. u + v

Types of problems in Time Speed Distance

1) Key Rule for Ratio based Problems

- If two objects have their speeds in the ratio a:b, distance covered by the two objects in the same time will be in the ratio a:b and time taken by the two objects to cover same distance will be in the ratio b:a.
- The ratio concept can be applied only when one of the given distance, time or speed is constant.

2) Overtaking, passing bodies

Problems may often feature bodies that move while other bodies may move or remain stationary. In such cases, the following points are to be noted.

- When two bodies pass each other (one body may be stationary), the speed of passing is equal to the relative speed between the two bodies.
- Objects like man, car, cycle, telegraph pole and tree are to be taken as point objects, with negligible length. When a train of length ‘l’ passes such an object, the distance covered while passing is equal to the length of the train ‘l’.
- Objects like train, platform and bridge have length which needs to be taken into account when approaching the problem. When a train passes such an object of length ‘p’, the distance covered while passing is equal to the total length of the train and the object, i.e., ‘l + p’. This holds true even when the second object is another train.

3) Travel and meeting

When two persons start from two points at the same time and travel towards each other, the time taken by each of them to reach the meeting point is the same. Hence, the distances covered by them from their respective starting points to the meeting point will be proportional to their respective speeds.

$$\frac{d_1}{d_2} = \frac{s_1}{s_2}$$

For example, assume A and B start simultaneously with speeds SA and SB respectively from points P and Q towards each other. They meet on the way at some point of time. From then onwards, A takes a time TA and B takes a time TB to reach Q and P respectively by continuing their onward travel. Then the following relationship is present between the speeds and times.

$$\frac{s_A}{s_B} = \sqrt{\frac{t_B}{t_A}} \ or \ {s_A}^2 \times t_A = {s_B}^2 \times t_B$$

4) Boats and Streams based problems

The two quantities that come into the picture are the speed of the stream or river or current, and the speed of the boat or swimmer in still water.

- When the boat is moving against the current, it is said to move ‘upstream’. Then the effective speed at which it moves is the difference between its speed in still water and the speed of the current.
- When the boat is moving with the current, it is said to move ‘downstream’. Then the effective speed at which it moves is the sum of its speed in still water and the speed of the current.

If u and v are the upstream and downstream speeds of the boat, then

- Speed of the boat in still water = (u + v)/2
- Speed of the stream = (v - u)/2

5) Races

A race involves runners covering a certain distance and the person who reaches the finishing point first wins the race. A 'kilometer race' requires the participants to run for 1 km. Unless otherwise specified, a race is assumed to be run on a linear track. A race can also be run on a circular track.

Suppose two runners A and B run a race.

- If A reaches the finishing point first and B is still behind by 'x' meters, A is said to win the race by 'x' metes. In this case, both ran for equal amounts of time but only A ran the whole distance.
- If A reaches the finishing point first and B needs 't' more minutes to complete the race, A is said to win the race by 't' minutes. In this case, both ran for equal distances but B took more time than A.
- If B starts running only 't' minutes after A has started, A is said to have a start of 't' minutes.
- If B starts running only after A has covered 'x' meters, A is said to have a start of 'x' meters.

6) Circular tracks

Circular tracks may involve two or three participants. They usually start from the same point and run in the same or opposite directions. Let the length of the track by 'l' metres.

a) When two runners start from the same point and run in opposite directions with speeds 'a' and 'b' m/s, the following points apply:

- Their relative speed is equal to the sum of their individual speeds.
- Between any two meetings, the total distance covered by them together is equal to the length of the circular track.
- Time taken for the first meeting after starting = d/s = l/(a + b)
- Time taken for them to meet at the starting point for the first time = LCM of (l/a) and (l/b)

b) When two runners start from the same point and run in the same direction with speeds 'a' and 'b' m/s (assume a>b), the following points apply:

- Their relative speed is equal to the difference of their individual speeds.
- Between any two meetings, the faster participant covers an extra distance equal to the length of the circular track compared to the slower participant.
- Time taken for the first meeting after starting = d/s = l/(a - b)
- Time taken for them to meet at the starting point for the first time = LCM of (l/a) and (l/b) . This is the same even if they run in opposite directions.

c) When three runners start from the same point and run in the same direction with speeds 'a' and 'b' (assume a>b), the following points apply:

A and B running in the same direction	L/ a - b	LCM of L/a and L/b
A and B running in the opposite direction	L/ a + b	LCM of L/a and L/b

d) When three runners start from the same point and run in the same direction with speeds 'a', 'b' and 'c' m/s (assume a>b>c), the following points apply:

- Time taken for the first meeting after starting is LCM of l/(a - b) and l/(b - c)
- Time taken for them to meet at the starting point for the first time = LCM of (l/a), (l/b) and (l/c).

When the speed of B expressed in terms of the speed of A such as twice, thrice etc, then the following results will generate.

B's speed in terms of the speed of A and they are running on opposite directions	**Number of meeting points**
Equal to A	2
Twice of A	3
Thrice of A	4
N times of A	N+1

Sample problems

Question 1: A person crosses a 400 m long street in 6 minutes. What is his speed in km per hour?

Step 1: Speed = 400 / (6 x 60) m/sec = 1.11 m/sec

Step 2: Converting m/sec to km/hr

1.11 x (18/5) km/hr = 4 km/hr

Therefore the speed of the person is 4km/hr.

Question 2: A train covers a certain distance at a speed of 200 kmph in 4 hours. To cover the same distance in $2\frac{2}{3}$ hours, it must travel at a speed of?

Step 1: Distance = (200 x 4) = 800 km.

Step 2: Speed = Distance/Time = 1200/(8/3) km/hr = 3200kmph

Therefore the required speed is 3200 kmph.

Question 3: If a person walks at 12 km/hr instead of 9 km/hr, he would have walked 18 km more. What is the actual distance traveled by him?

Step 1: Let the actual distance travelled be x km.

$x/9 = (x+18) / 12$

Step 2: Solve for x. $x = 54$

Therefore the actual distance traveled by him is 54km.

Question 4: Excluding stoppages, the speed of a bus is 48 kmph and including stoppages, it is 34 kmph. For how many minutes does the bus stop per hour?

Step 1: Due to stoppages, it covers 14 km less.

Time taken to cover 14 km = $(14/48) \times 60 = 17.5$ min.

Question 5: The ratio between the speeds of two trains is 6 : 8. If the second train runs 300 km in 3 hours, then what is the speed of the first train?

Step 1: Let the speed of two trains be $6x$ and $8x$ km/hr.

Then, $8x = 300/3$

Step 2: Solving for x, $x=12.5$

Step 3: Therefore the required speed is $12.5 \times 6 = 75$kmph.

Question 6: In covering a distance of 20 km, Raj takes 3 hours more than Sam. If Raj doubles his speed, then he would take 2 hours less than Sam. What is Raj's speed?

Step 1: Let Raj's speed be x km/hr.

Step 2: Then, $(20/x) - (20/2x) = 5$

Step 3: Solve for x. $2x= 4 \Rightarrow x = 2$ km/hr

Question 7: A farmer traveled a distance of 53 km in 8 hours. He traveled partly on foot at 4 km/hr and partly on bicycle at 10 km/hr. What is the distance traveled on foot?

Step 1: Let the distance traveled on foot be x km.

Then, distance travelled on bicycle = $(53 - x)$ km.

Step 2: $(x/4) + ([53-x] / 10) = 8$

Step 3: Solve for x. $0.15x = 2.7 \Rightarrow 18$ km