

Aptitude Algebraic Expressions Concepts and Formulas

Points to Remember:

1. Quadratic Equations:

- (i) An equation of the type $ax^2 + bx + c = 0$ is called the quadratic equation.
- (ii) The highest power of the variable is called the degree of an equation.
- (iii) An equation will have as many solutions as its degree. If an equation is of n degree, it will have ' n ' solutions.
- (iv) The solution of an equation is the value by which equation is satisfied. The values of the solution of an equation are also called the roots of the equation. This quadratic equation has two solutions.

2. Solving Quadratic Equations:

Any quadratic equation can be either solved by the factor method or by formula.

- (i) **By the factor method:** First find the factors of the given equation making the right-hand side equal to zero and then by equating the factors to zero, we get the values of the variable.
- (ii) **By Formula:** Consider a quadratic equation $ax^2 + bx + c = 0$, for finding the roots of the equation, we use the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here + and - in the above formula is used to get the two values of x . Here the quantity $b^2 - 4ac$ is called the discriminant.

3. Roots of the Quadratic Equation:

The value of the x that we obtain from a quadratic equation is called the root of the equation; α and β are used to denote the roots of the equation.

- (i) Sum of the roots of a quadratic equation:

$$ax^2 + bx + c = 0 \text{ is equal to } -\frac{b}{a} \text{ i.e., } \alpha + \beta = -\frac{b}{a}$$

- (ii) The product of the roots is equal to $\frac{c}{a}$, i.e., $\alpha * \beta = \frac{c}{a}$

(iii) Consider a quadratic equation: $ax^2 + bx + c = 0$.

For this equation, the roots will be equal if $b^2 = 4ac$.

The roots will be unequal and real if $b^2 > 4ac$.

The roots will be unequal and unreal if $b^2 < 4ac$.

4. Whenever we are given the roots of a quadratic equation, then the equation will be

$$x^2 - (\text{Sum of the roots})x + \text{product of roots} = 0.$$

5.

(i) When a quadratic equation, $ax^2 + bx + c = 0$, has one root equal to zero, then $c = 0$.

(ii) A quadratic equation, $ax^2 + bx + c = 0$, will have reciprocal roots, if $a = c$.

(iii) When the roots of a quadratic equation, $ax^2 + bx + c$, are negative reciprocals of each other, then $c = -a$.

(iv) When both the roots are equal to zero, $b = 0$ and $c = 0$.

(v) When one root is infinite, then $a = 0$ and when both the roots are infinite, then $a = 0$ and $b = 0$.

(vi) When the roots have equal magnitude but are opposite in sign, then $b = 0$.

(vii) When two quadratic equations, $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$, and have a common root, then $(bc_1 - b_1c)(ab_1 - a_1b) = (c_1a - ca_1)^2$.

(viii) When they have both the roots common, then

$$\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$$

6. Linear Equations:

A statement of equality that contains an unknown quantity or variable is called an equation. In a linear equation the pattern of numbers increases or decreases by the same amount every step of the way, so the graph of a linear equation is always a straight line.

Aptitude Algebraic Expressions Problems

1) If $x^2 + \frac{1}{x^2} = 34$, $x + \frac{1}{x}$ is equal to

- A. 3
- B. 4
- C. 5
- D. None of these

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Answer: D

Explanation:

Adding 2 to the L.H.S and R.H.S of the equation:

$$x^2 + 2 + \frac{1}{x^2} = 34 + 2 \quad [\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$\left(x + \frac{1}{x}\right)^2 = 36$$

$$\left(x + \frac{1}{x}\right) = +6, -6$$

2) The value of x in the equation: $16x + \frac{1}{x} = 8$ is

- A. $\frac{1}{4}, \frac{1}{4}$
- B. $\frac{1}{4}, 2$
- C. $\frac{1}{4}, \frac{1}{2}$
- D. $\frac{1}{2}, \frac{1}{2}$

Answer: A

Explanation:

$$16x + \frac{1}{x} = 8$$

$$\frac{16x^2 + 1}{x} = 8$$

$$16x^2 + 1 = 8x$$

$$\Rightarrow 16x^2 + 1 - 8x = 0$$

$$\Rightarrow (4x - 1)^2 = 0$$

$$\Rightarrow x = \frac{1}{4}, \frac{1}{4}$$

3) The equation whose roots are 4 and 5, is

A. $x^2 + 9x - 20 = 0$

B. $x^2 + 9x + 20 = 0$

C. $x^2 - 9x + 20 = 0$

D. $x^2 - 9x - 20 = 0$

Answer: C

Explanation:

The required equation is:

$$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

$$\Rightarrow x^2 - (4 + 5)x + 4 \times 5 = 0$$

$$\Rightarrow x^2 - 9x + 20 = 0$$

4) The perimeter of a rectangle is 48 meters, and its area is 135 m². The sides of the rectangle are

A. 15 m, 9m.

B. 19m, 5m.

C. 45m, 3m.

D. 27m, 5m.

Answer: A

Explanation:

Let the sides of the rectangle be x and y meters respectively. Then

$$2x + 2y = 48$$

$$2(x+y) = 48$$

$$x+y = 24$$

$$y = 24 - x$$

We have, $xy = 135$

$$\text{So, } x(24 - x) = 135$$

$$24x - x^2 = 135$$

$$x^2 - 24x + 135 = 0$$

$$x^2 - 9x - 15x + 135 = 0$$

$$(x-9)(x-15) = 0$$

So, when $x = 9\text{m}$, $y = 24 - 9 = 15\text{m}$

And when $x = 15$, $y = 24 - 15 = 9\text{ m}$

So, the sides of rectangle are 15m and 9m.

5) The roots of the equation $(x + 3)(x - 3) = 160$ are

A. ± 13

B. 13, 13

C. ± 12

D. 12, 12

Answer: A

Explanation:

$$(x + 3)(x - 3) = 160$$

$$x^2 - 9 = 160$$

$$x^2 = 169$$

$$x = +13, -13$$