

$$\begin{aligned}
\int \frac{dx}{a^2+x^2} &= \frac{1}{a} \arctg \frac{x}{a} \quad (a > 0) & \int \frac{dx}{a^2-x^2} &= \frac{1}{2a} \ln \left| \frac{a+1}{a-x} \right| & \int \frac{dx}{\sqrt{a^2-x^2}} &= \arcsin \frac{x}{a} \\
\int \frac{dx}{\sqrt{x^2 \pm a^2}} &= \ln |x + \sqrt{x^2 \pm a^2}| & \int \frac{dx}{sh^2 x} &= -\operatorname{cth} x & \int \frac{dx}{ch^2 x} &= \operatorname{th} x \\
\int \frac{P_n(x)}{Q_m(x)} dx &= \frac{P_{m-k-1}}{Q_{m-k}} + \int \frac{R_{k-1}(x)}{Q_k(x)} dx \\
\int \frac{P_n(x)}{y} dx &= Q_{n-1}(x)y + \lambda \int \frac{dx}{y}, \quad y = \sqrt{ax^2 + by + c} \\
t = \frac{x+a}{x+b} &\implies dt = \frac{(b-a)dx}{(x+b)^2} \quad \text{ИЛИ} \quad x = \frac{bt-a}{t-1} \implies x+a = \frac{(a-b)t}{t-1} \quad \text{и} \quad x+b = \frac{a-b}{t-1} \\
t = \operatorname{tg} \frac{x}{2}, \quad \sin x &= \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2} \\
\int \frac{a \sin x + b \cos x + c}{a \sin x + b \cos y + c} dx &= Ax + B \ln |a \sin x + b \cos y + c| + C \int \frac{dx}{a \sin x + b \cos y + c} \\
\int \sin^n x dx &= -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x dx \\
\int \cos^n x dx &= \frac{1}{n} \sin x \cos^{n-1} x + \frac{n-1}{n} \int \cos^{n-2} x dx
\end{aligned}$$

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\int_0^{\frac{\pi}{2}} f(\sin x) dx &= \int_0^{\frac{\pi}{2}} f(\cos x) dx & \int_0^{\pi} x f(\sin x) dx &= \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx \\
F(x) &= \int_{\varphi_1(x)}^{\varphi_2(x)} f(t) dt \implies F'(x) = f(\varphi_2(x)) \varphi_2'(x) - f(\varphi_1(x)) \varphi_1'(x) \\
S &= \int_{T_0}^{T_1} x(t) y'(t) dt = - \int_{T_0}^{T_1} x'(t) y(t) dt = \frac{1}{2} \int_{\alpha}^{\beta} r^2(\varphi) d\varphi \\
V_{ox} &= \pi \int_a^b f^2(x) dx = \pi \int_a^b y^2(x) x' dt \\
V_{oy} &= \pi \int_a^b f(x) dx^2 = 2\pi \int_{t_1}^{t_2} x(t) y(t) dx(t) \\
|\Gamma| &= \int_{T_0}^{T_1} \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt \\
|\Gamma| &= \int_a^b \sqrt{1 + (y')^2} dx \\
|\Gamma| &= \int_{\alpha}^{\beta} \sqrt{(r(\varphi))^2 + (r'(\varphi))^2} d\varphi = \int_{\alpha}^{\beta} \sqrt{r^2 \cdot (\varphi')^2 + (r')^2} \cdot \operatorname{sgn}(\varphi') d\varphi
\end{aligned}$$