

# 现代密码学

3.1.  $SPN(\alpha, \bar{z}_s, \bar{z}_p, (k_1, k_2, \dots, k_{N+1}))$

$$w_0 \leftarrow \alpha$$

for  $r=1$  to  $N-1$

$$\begin{cases} u_r = w_{r-1} \oplus k_r \\ v_r = \bar{z}_s(u_r) \\ w_r = \bar{z}_p(v_r) \end{cases}$$

$$w_N = w_{N-1} \oplus k_N$$

$$v_N = \bar{z}_s(w_N)$$

$$y = v_N \oplus k_{N+1}$$

$SPN(y, \bar{z}_s^\alpha, \bar{z}_p^\alpha, (L_{N+1}, L_N, k, \dots, L_1))$

$$w_0 \leftarrow y$$

$$v_0 = y \oplus k_{N+1}$$

$$u_0 = \bar{z}_s^{-1}(v_0)$$

$$w_1 = u_0 \oplus k_N$$

for  $r=1$  to  $N-1$

$$\begin{cases} v_r = \bar{z}_p^{-1}(w_r) \\ u_r = \bar{z}_s^{-1}(v_r) \\ w_{r+1} = u_r \oplus k_{N-r} \end{cases}$$

$$\alpha \leftarrow w_N$$

~~$SPN(y, \bar{z}_s^\alpha, \bar{z}_p^\alpha, (L_{N+1}, \dots, L_1))$~~

~~$$w_0 \leftarrow y$$~~

~~for  $r=1$  to  $N-1$~~

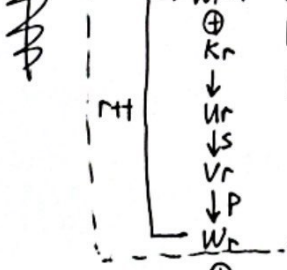
~~$$v_0 = y \oplus k_{N+1} = y \oplus L_{N+1}$$~~

~~$$u_0 = \bar{z}_s^{-1}(v_0)$$~~

~~$$w_1 = u_0 \oplus k_N = u_0 \oplus L_N$$~~

~~for  $r=1$  to  $N-1$~~

~~$$w_r = u_{r-1} \oplus k_r$$~~



$$\therefore \bar{z}_s^\alpha = \bar{z}_s^{-1}, \bar{z}_p^\alpha = \bar{z}_p^{-1}$$

$\bar{z}_s$	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
$\bar{z}_s^\alpha$	E	3	4	8	1	C	A	F	7	D	9	6	B	2	0	5

$\bar{z}_p$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\bar{z}_p^\alpha$	1	5	9	13	2	6	10	14	3	7	11	15	4	8	12	16

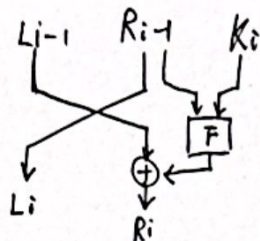
3.2.

Feistel 型密码.

对第  $i$  轮:

$$L_i = R_{i-1}$$

$$R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$$



$$\begin{cases} R_{i-1} = L_i \\ L_{i-1} = R_i \oplus f(R_{i-1}, K_i) = R_i \oplus f(L_i, K_i). \end{cases}$$

设 Feistel 型密码有  $N_r$  轮. 明文为  $x$ , 密文为  $y$ .

归纳 ①  $R_{N_r} L_{N_r} = IP^{-1}(x)$   $y = R_{N_r} L_{N_r} = L_0' R_0'$   
证明.

$$\begin{cases} L_1' = R_0' = L_{N_r} = R_{N_r-1} \\ R_1' = L_0' \oplus f(R_0', K_1) = R_{N_r} \oplus f(L_{N_r}, K_{N_r}) = L_{N_r-1} \end{cases}$$

$$\textcircled{2} i=k. \begin{cases} L_k' = R_{k-1}' \\ R_k' = L_{k-1}' \oplus f(R_{k-1}', K_k) \end{cases} \text{ (定义).}$$

$$\text{由于已证} \begin{cases} L_{k-1}' = R_{N_r-k+1} \\ R_{k-1}' = L_{N_r-k+1} \end{cases}$$

$$\therefore L_k' = L_{N_r-k+1} = R_{N_r-k} \text{ (定义).}$$

$$R_k' = L_{k-1}' \oplus f(R_{k-1}', K_k) = L_{N_r-k+1} \oplus f(L_{N_r-k+1}, K_{N_r-k+1}) = L_{N_r-k} \text{ (定义).}$$

$\therefore$  最终的第  $N_r$  轮输出  $L_{N_r}' R_{N_r}' = R_0 L_0$   
将明文交换  $x = L_0 R_0$ .

$\therefore$  得证.

3.3.

对于  $k: k \rightarrow (k_1, k_2, \dots, k_{16})$ . 由置换函数得出 若该置换不变, 则  $c(k) \rightarrow (c(k_1), c(k_2), \dots, c(k_{16}))$

对于轮函数:  $\begin{cases} L_i = R_{i-1} \\ R_i = L_{i-1} \oplus f(R_{i-1}, K_i) \end{cases} \rightarrow c(R_{i-1}) = c(L_i)$   
 $\rightarrow c(L_{i-1}) \oplus f(c(R_{i-1}), c(K_i))$   
函数中  $A$  对 48bit 扩展  $E(A)$ .

$$f(c(R_{i-1}), c(K_i)).$$

先将  $c(R_{i-1})$  扩展为 48bit.  $E(c(R_{i-1}))$ .

$$E(c(R_{i-1})) = c(E(R_{i-1})).$$

$$\text{再 } E(c(R_{i-1})) \oplus c(K_i) = c(E(R_{i-1}) \oplus c(K_i))$$

• 因为两者都取反, 所以异或结果不变.

$$\text{即 } f(c(R_{i-1}), c(K_i)) = f(R_{i-1}, K_i).$$

$$\therefore c(L_{i-1}) \oplus f(R_{i-1}, K_i) = c(R_i).$$

$\therefore$  每一轮函数的输出都为原  $L_i R_i$  取反

$\therefore$  最终的结果为  $c(y)$ .

$\therefore$  得证



3.7.  $x_1 x_2 \dots x_n \rightarrow y_1 y_2 \dots y_n$

ECB: 解密方式为分组密码的解密方式. 又, 分组之间相互不影响. 所以只影响出错的每一位所在的分组.

OFB: ~~同ECB~~. 但  $y_i$  出错与  $z_i$  无关. 除出错的  $y_i$  外其余都以正确通过  $x_i = y_i \oplus z_i$  得出.  $z_i$  由统一  $ek$  求出.

CBC: 设出错的为  $y_i$ :

$$x_i = dk(y_i) \oplus y_{i-1} \rightarrow \text{有错.}$$

$$x_{i+1} = dk(y_{i+1}) \oplus y_i \rightarrow \text{有错.}$$

其余: e.g.  $x_{i-1} = dk(y_{i-1}) \oplus y_{i-2} \rightarrow$  不涉及  $y_i$ . 正确

$$x_{i+2} = dk(y_{i+2}) \oplus y_{i+1} \rightarrow \text{不涉及 } y_i. \text{ 正确.}$$

又: 因为只是密文出错. 不会影响到后续所有分组.

加密时会用到上一分组的密文  $y_{i-1}$ . 但解密不会使用明文  $x_i$ .

CFB: 设出错的为  $y_i$ .

~~$$z_{i+1} = ek(y_i)$$~~ 
$$x_i = y_i \oplus z_i$$

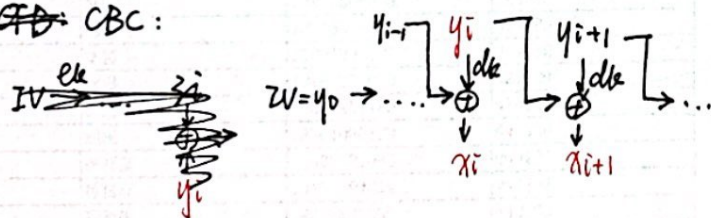
$$x_{i+1} = y_{i+1} \oplus z_{i+1} = y_{i+1} \oplus ek(y_i).$$

其余分组均正确. 理由同上.

}  $\Rightarrow$  使用  $z_i$ . 有错.

图解:

~~CFB~~ CBC:



CFB:

