

CS5275 The Algorithm Designer's Toolkit

S2 AY2025/26

Tutorial problem set 1

(There is no need to submit your solution for tutorial problem sets)

Problem 1 (Min s - t cut as a special case of generalized conductance). Let $G = (V, E)$ be a graph, and let $s, t \in V$. Show that there exists a choice of an unweighted graph H such that $\Phi(G, H)$ equals the size of a minimum s - t cut in G . Note that H is not required to be connected.

Problem 2 (Eigenvalue calculation). Prove the following bounds.

1. $\lambda_2(\mathbf{N}) \in O(1/n^2)$ for the n -vertex cycle.
2. $\lambda_2(\mathbf{N}) \in O(1/n^2)$ for the graph resulting from the following construction.
 - (a) Let A and B be two $(n/2)$ -cliques.
 - (b) Let $e_A = \{u, v\}$ be an edge in A , and let $e_B = \{w, x\}$ be an edge in B .
 - (c) Replace e_A and e_B with the two edges $\{u, w\}$ and $\{v, x\}$.
3. $\lambda_2(\mathbf{N}) \in \Omega(1/(ndD))$ for any n -vertex d -regular graph with diameter at most D .

Problem 3 (Random graphs). Prove that there exists a function $p \in O\left(\frac{\log n}{n}\right)$ such that an Erdős-Rényi random graph $G = (V, E) \sim \mathcal{G}(n, p)$ satisfies the following properties with probability at least $1 - 1/n$.

1. $\deg(v) \in \Theta(\log n)$ for every $v \in V$.
2. $\Phi(G) \in \Omega(1)$.