CS4261/5461: Assignment for Week 10

Due: Sunday, 2nd Nov 2025, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 2nd Nov, 11:59 pm to `Assignments/Assignment10/Submissions`. Name the file **Assignment10_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

Note: For this assignment, justification is required only for Questions 2 and 3.

1. (1 point) Consider a rent division setting with two players and two rooms. The total rent is 1000. Player 1 has value 900 for the first room and 100 for the second room. Player 2 has value 700 for the first room and 300 for the second room.

   (a) What is the allocation of the rooms to the players that maximizes the sum of players' values for the rooms?

   (b) Given the allocation in part (a), what is the envy-free price vector that maximizes the minimum utility of the players? (Recall that the utility of a player is the player's value for the assigned room minus the price of the room.)

2. (7 points, graded for correctness) Consider the rent division mechanism described in lecture, which chooses an allocation of the rooms to the players that maximizes the sum of players' values for the rooms, and then chooses an envy-free price vector that maximizes the minimum utility of the players. (In each step, if there is a tie, the mechanism may break ties arbitrarily.)

   (a) (2 points) Is this mechanism strategyproof?

   (b) (2 points) Assuming that all players report truthfully, is the outcome of this mechanism Pareto optimal?

   (c) (3 points) If there are two players (who report truthfully), is it true that both players necessarily obtain the same utility from the outcome of this mechanism?

3. (1 point) Suppose that all players have the same valuation for each room (i.e., for each room j, there exists v_j such that $v_{ij} = v_j$ for all players i). Explain whether each statement below is true or false.

    (a) There is always a unique envy-free price vector.

    (b) There is always a unique envy-free room allocation.