CS4261/5461: Assignment for Week 11 Solutions

Due: Sunday, 9th Nov 2025, 11:59 pm SGT.

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1. (a) Note that a receives 3 votes, b receives 4 votes, c receives 3 votes, and d receives 2 votes. Hence, the only AV committee is $\boxed{\{a, b, c\}}$.

   (b) The committees $\boxed{\{a, c, d\}}$ and $\boxed{\{b, c, d\}}$ cover all voters. They are also the only committees of size 3 to do so, since choosing c and d is necessary due to voters 4 and 6.

   (c) The utilities of the voters for $\{a, b, c\}$ are $3, 3, 2, 1, 1, 0$, respectively. Hence, the PAV score of this committee is $2\left(1 + \frac{1}{2} + \frac{1}{3}\right) + \left(1 + \frac{1}{2}\right) + 2 \cdot 1 = \boxed{\frac{43}{6}}$.

   (d) Each voter begins with a budget of $k/n = 1/2$. First, b has the highest number of approvals and is chosen. Voters $1, 2, 3, 5$ pay $1/4$ each, and has a budget of $1/2 - 1/4 = 1/4$ left. Next, a and d are not affordable while c is affordable, so c is chosen. Voters $1, 2, 4$ pay their remaining budget $(1/4, 1/4, 1/2$, respectively). No more candidate is affordable, so a is chosen ahead of d by the approval score tie-breaking. Hence, the only MES committee is $\boxed{\{a, b, c\}}$.

2. Assume for contradiction that a committee W returned by GreedyCC fails JR. This means there exists a cohesive group of voters S such that no member of S has an approved candidate in W. Let x be a candidate approved by all voters in S. Since $|S| \geq n/k$, adding x would have added additional coverage of at least n/k at any stage throughout the execution of GreedyCC. Since GreedyCC did not select x, in each step it must have selected a candidate with additional coverage at least n/k, so the total coverage of W is at least $(n/k) \cdot k = n$. Hence, every voter has an approved candidate in W, contradicting the earlier assumption that no member of S has an approved candidate in W.

3. From the condition that S is a t-cohesive group of voters, we have that $|S| \geq t \cdot \frac{n}{k}$ and the voters in S commonly approve t candidates. Since W satisfies EJR, there exists a voter $i_1 \in S$ such that $u_{i_1}(W) \geq t$. Remove i_1 from S, and repeat the argument. At any stage, if $|S| \geq \ell \cdot \frac{n}{k}$ for some nonnegative integer $\ell \leq t$, then S is ℓ-cohesive, and EJR implies that there exists a

voter $i \in S$ such that $u_i(W) \geq \ell$. Hence, we get the following utility guarantees for the voters in S:

$$\overbrace{t, t, \dots, t}^{\geq 1 \text{ time}}, \overbrace{t-1, t-1, \dots, t-1}^{n/k \text{ times}}, \dots, \overbrace{1, 1, \dots, 1}^{n/k \text{ times}}, \overbrace{0, 0, \dots, 0}^{n/k-1 \text{ times}}.$$

Observe that if we decrease one of the t's to 0 and remove the remaining t's (if any), the average of this sequence would be exactly $\frac{t-1}{2}$. Thus, the actual average of the sequence is greater than $\frac{t-1}{2}$.