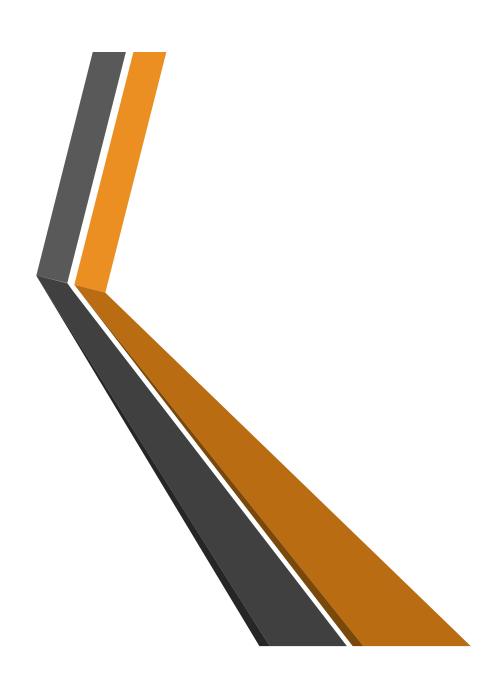


Instructor: Warut Suksompong

2025



# Rent Division







Assign Credit

Determine the contribution of

each individual to a school

project, academic paper, or

business endeavor.





#### Share Rent

Moving into a new apartment with roommates? Create harmony by fairly assigning rooms and sharing the rent.



#### Split Fare

Fairly split taxi fare, or the cost of an Uber or Lyft ride, when sharing a ride with friends.

#### STAR

#### Divide Goods

Fairly divide jewelry, artworks, electronics, toys, furniture, financial assets, or even an entire estate.

#### START >

#### Distribute Tasks

Divvy up household chores, work shifts, or tasks for a school project among two or more people.

START >



# **SHARE RENT**

splíddít



Spliddit's rent calculator helps roommates to fairly share rent when moving into a new house or apartment. This is especially useful when bedrooms differ in size, closet space, bathrooms, and more. You begin by providing a list of roommates, a list of bedrooms, and the monthly rent. We then send everyone a link where they specify how much they would pay to live in each bedroom. Our algorithm uses these evaluations to determine who should live in which room, and how much each housemate should contribute to the monthly rent.

LIVE DEMO

START >

### The Problem

Fairly distribute rooms and rent amongst roommates.

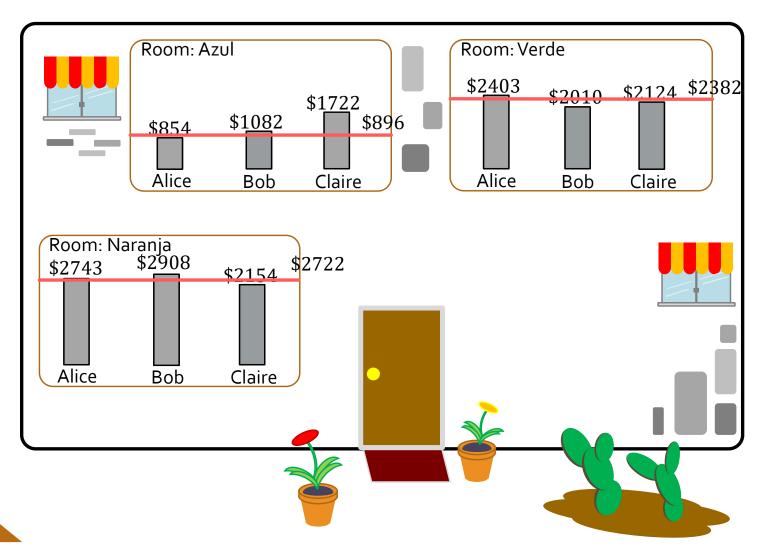
Input

- Roommates (players):  $N = \{1, ..., n\}$
- r: rent-price for the whole apartment
- $v_{ij}$ : player i's value of room j such that  $\sum_{i} v_{ij} = r$

• Output

- Room allocation:  $\sigma: N \to N$
- Rent division  $\vec{p}=p_1,\ldots,p_n$ , such that  $\sum_j p_j=r$
- $(p_i \text{ is the rent for room } j)$

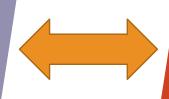
r = \$6000



### The Valuations

 $v_{ij}$ : player i's value of room j such that  $\sum_j v_{ij} = r$ 





### Not Fully Expressive

"I prefer room 1, but can't afford to pay more than S\$500"

# Equal Incomes All roommates have equal opportunities



#### **External Factors**

roommate has more disposable income?

### Envy-Free (EF) Outcome

Outcome  $\langle \sigma, \vec{p} \rangle$  such that

$$v_{i\sigma(i)} - p_{\sigma(i)} \ge v_{ij} - p_j$$
 for all  $i, j \in N$ 

"I do not prefer your room for the price you're being charged"

#### Fairness Properties

#### **Envy-freeness**

Our algorithm assigns rooms in such a way that each roommate feels that they got the best deal. In other words, no roommate would want to swap places with another.

This property is guaranteed, assuming that each roommate wishes to maximize the difference between what she thinks her room is worth and how much rent she has to pay.





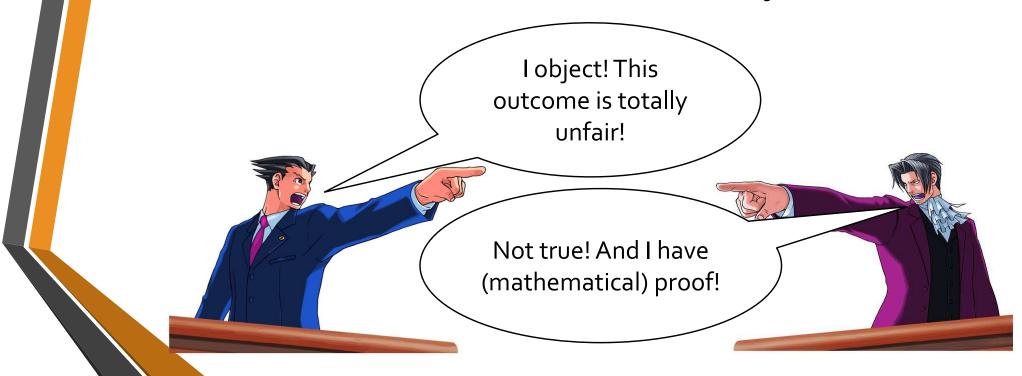
#### Efficiency

Our algorithm divides the rent and assigns rooms in such a way that it would be impossible to find another assignment that benefits a roommate without making another roommate worse off.

This property is guaranteed, assuming that each roommate wishes to maximize the difference between what she thinks her room is worth and how much rent she has to pay.



- Can be easily related to the users: "You wouldn't want room 3 for \$847 based on your valuations"
- Satisfies fundamental ideas of economic/social justice



### Properties of EF Allocations

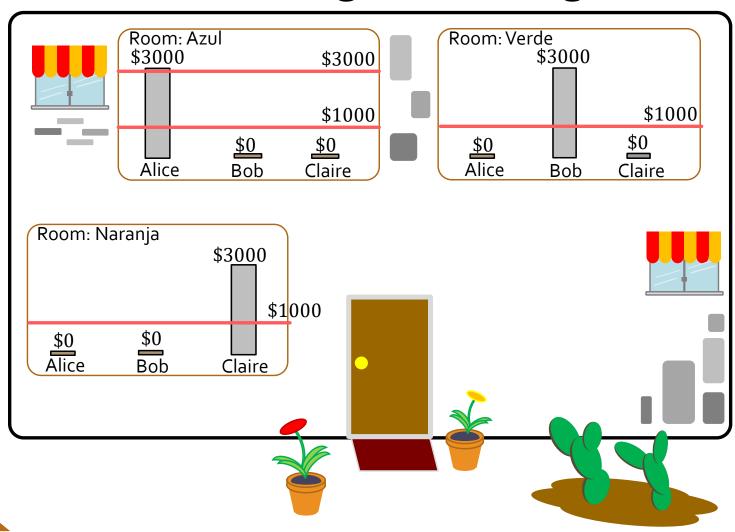
- Does an EF allocation always exist?
- Is it Unique?
- Can it be efficiently computed?



# General Algorithmic Framework

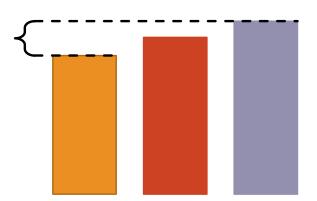
- Compute a socially optimal allocation (max. weighted matching)
- Find an EF price vector (using linear programming).

# Where EF goes wrong



### Equitability (under EF constraint):

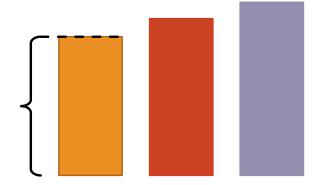
$$D(\vec{p}) = \max_{i,j \in N} (u_i(\vec{p}) - u_j(\vec{p}))$$
$$\min D(\vec{p})$$
$$s. t. \vec{p} \text{ is } EF$$



Here, 
$$u_i(\vec{p}) = v_{i\sigma(i)} - p_{\sigma(i)}$$

Maximin (under EF constraint):

$$U(\vec{p}) = \min u_i(\vec{p})$$
  
 $\max U(\vec{p})$   
 $s. t. \vec{p} \text{ is } EF$ 



#### Theorem:

There is a unique maximin price vector, and it is also equitable!

However, there exist equitable price vectors that are not maximin.

Maximin is good in theory.

Is it also good in practice?

### **Participants**

Spliddit users who participated in rent division instances with 2, 3 or 4 players

Invited over email

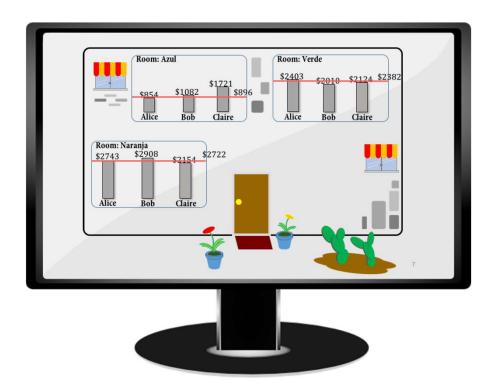
Offered \$10 fixed compensation

# **Experimental Design**

• The subjects were shown the two solutions — maximin and arbitrary EF for their own instance

• The two solution outcomes were shown in sequence, and in random

order



# Survey

#### Individual

This question relates to your own allocation.

In other words, we would like you to pay attention only to your own benefit.

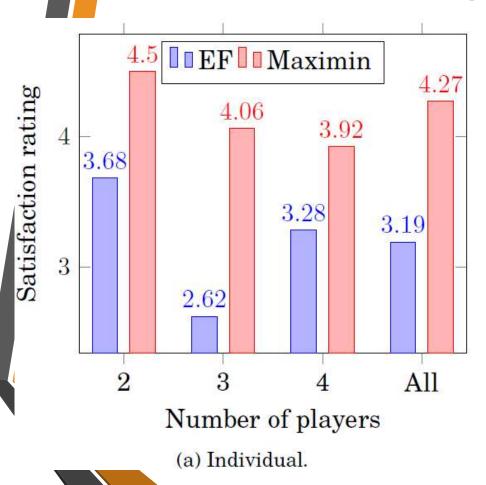
How happy are you with getting the room called Verde for \$2,382?

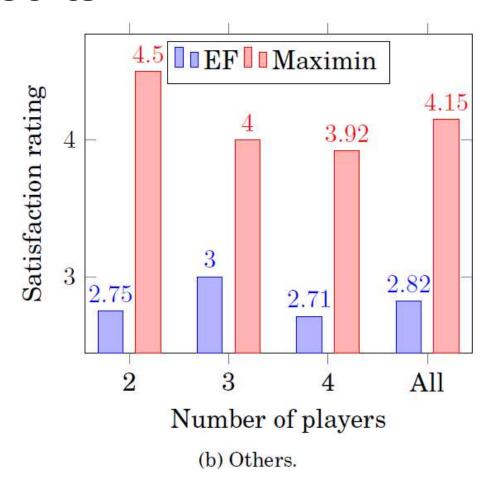
#### Other

This question relates to the allocation for everyone else.

How fair do you rate the allocation for Bob and Claire?

### Results





### References

 Gal, Mash, Procaccia and Zick "Which is the Fairest (Rent Division) of Them All?", JACM 2017