CS4261/5461: Assignment for Week 5

Due: Sunday, 21st Sep 2025, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 21st Sep, 11:59 pm to `Assignments/Assignment5/Submissions`. Name the file **Assignment5_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

Note: For this assignment, justification is required only for Questions 2 and 3.

1. (1 point) Consider the following cooperative game with three players, $N = \{1, 2, 3\}$:

$$v(\{1\}) = v(\{2\}) = 0$$

and

$$v(\{3\}) = v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = v(\{1, 2, 3\}) = 1.$$

   (a) Is this game monotone?

   (b) Is this game simple?

   (c) Is this game superadditive?

   (d) Is this game convex?

2. (7 points, graded for correctness) For the following games, determine all payoff vectors in the core.

   (a) (1 point) The weighted voting game with three players who have weights $1, 2, 3$, and the threshold is 4.

(b) (2 points) The game with three players with weights $10, 20, 30$, respectively, with the following property:

For any coalition, if the sum of the weights of its members is at most 45, then its value is 0, while if the sum of the weights of its members is more than 45, then its value is 6.

(Note: This game is **not** a weighted voting game.)

(c) (2 points) The game with 10 players (numbered $1, 2, \ldots, 10$) such that

$$v(S) = \begin{cases} 1 & \text{if } S \text{ contains player 1, player 2, and at least one other player;} \\ 0 & \text{otherwise,} \end{cases}$$

for each set S of players.

(d) (2 points) The game with 10 players (numbered $1, 2, \ldots, 10$) such that for any coalition $S \subseteq N$,

$$v(S) = \begin{cases} 0 & \text{if } S \text{ is empty;} \\ |S| + 1 & \text{if } S \text{ is non-empty.} \end{cases}$$

(Recall that $|S|$ denotes the size of the set S.)

3. (1 point)

(a) Give an example of a four-player monotone, superadditive cooperative game such that the core is empty. Explain why the core of your game is empty (but no need to explain why the game is monotone and superadditive).

(b) Give an example of a three-player monotone, superadditive game that is not convex. Explain why your game is not convex (but no need to explain why it is monotone and superadditive).