CS4261/5461 Algorithmic Mechanism Design

Instructor: Warut Suksompong

2025

Auctions

Auctions Around Us

eBay

- Online Auction Website
- $22.3 billion in revenue in 2016

Spectrum auctions

- selling access to the wireless spectrum to the telecom companies
- US: $60 billion in revenue since 1994

Sponsored search auctions

- selling context-dependent ads on search pages
- Google: $116B in ad revenue in 2018

Single-item auction:

- the seller has one object for sale
- bidders compete to purchase the object

Multi-unit auction:

- seller has several identical items for sale
- each bidder wants to get one or more units
- license plates, airline tickets, US treasury bills

Combinatorial auction:

- seller has several distinct items for sale
- each bidder is interested in some combination of items
- airport departure and landing slots, spectrum auctions

Single Item Auctions

- We have a single item for sale.
- Each bidder $i \in N$ values the item at v_i.
- Seller can decide on a price.
- What are the outcomes?

Single Item Auction Formats

- English auction:
  - auctioneer sets a starting price
  - bidders take turns raising their bids
  - the person who makes the last bid wins and pays his bid

English Auction, Dynamics

- $v_1 = 50$, $v_2 = 30$, $v_3 = 70$, $\delta = 1$.
- The auction starts at $p = 0$.
- While $p < 30$, all bidders are submitting bids.
- At $p = 30$, player 2 stops bidding.
- At $p = 50$, player 1 stops bidding.
- If player 3 was the one to bid 50, she wins and pays 50.
- If player 1 was the one to bid 50, player 3 bids 51 and wins.
- Winning bid is either v_1 or $v_1 + \delta$.

English Auction

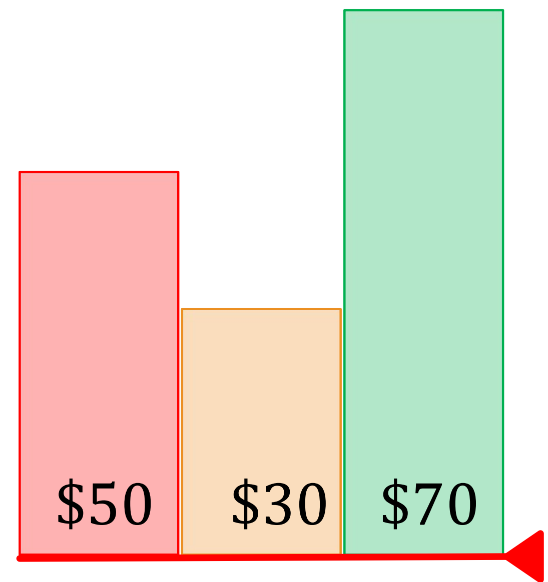
- Suppose your value for the object is v, the current price is p, and the minimum bid increment is δ
- It is rational to bid if and only if $p + \delta \leq v$, and your bid should be $p + \delta$
  - If $p + \delta > v$, and you end up winning, you will pay more than the object is worth to you.
  - If you bid more than $p + \delta$, but no one else was willing to pay more than p, you pay more than is necessary to win.

Single Item Auction Formats

- Japanese auction:
  - auctioneer sets a starting price and then starts raising it
  - a bidder can drop out, and cannot return once he dropped
  - the bidder who stays in last gets the object, pays the current price

Japanese Auction, Dynamics

- $v_1 = 50, v_2 = 30, v_3 = 70, \delta = 1$
- The auction starts at $p = 0$
- At $p = 30$, player 2 drops out
- At $p = 50$, player 1 drops out
- Player 3 wins and pays 50
- Communication:
  - English auction: 50 messages
  - Japanese auction: 2 messages



Single Item Auction Formats, Continued

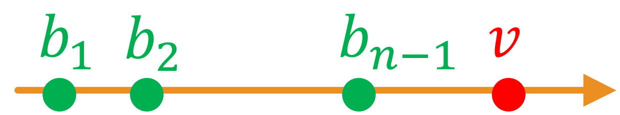
- Dutch auction:
  - auctioneer sets a (high) starting price and then starts lowering it
  - the auction ends when some bidder accepts the price
  - used in the Amsterdam flower market
- Sealed-bid auction:
  - all bidders simultaneously submit their bid
  - the highest bidder gets the item and pays….
    - his bid (first-price auction)
    - 2nd highest bid (second-price, or Vickrey, auction)

Vickrey (Second-Price) Auction

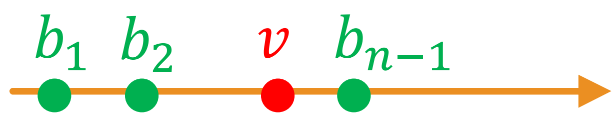
- All bidders submit bids simultaneously in sealed envelopes
- The highest bidder wins and pays the second highest price
- Strategic game
  - n players (bidders)
  - actions: bids (continuous action space)
  - payoff: if a player values the object at v and the 2nd highest bid is p, her payoff is
    - $v - p$ if she gets the object
    - 0 if she does not get the object

In a Vickrey auction, truthful bidding is a dominant strategy

- Proof:
  - suppose your value is v
  - suppose other players' bids are $b_1 \leq \cdots \leq b_{n-1}$
  - Case 1: $v \geq b_{n-1}$
    - if you bid $b \geq b_{n-1}$, you win and pay b_{n-1}
    - payoff is $v - b_{n-1}$, same as what you would get from truthfully reporting v
    - if you bid $b < b_{n-1}$, you lose; payoff is 0.



- Proof:
  - suppose your value is v
  - suppose other players' bids are $b_1 \leq \cdots \leq b_{n-1}$
  - Case 2: $v < b_{n-1}$
    - if you bid $b \geq b_{n-1}$, you win and pay b_{n-1}
    - payoff is $v - b_{n-1} < 0$
    - if you bid $b < b_{n-1}$, you lose; payoff is 0, same as what you get from truthfully bidding v.



English auction

- person with the highest value wins
- pays 2^{nd} highest value or 2^{nd} highest value+δ

Japanese auction

- person with the highest value wins
- pays 2^{nd} highest value

Vickrey auction

- person with the highest value wins
- pays 2^{nd} highest value
- + easy to implement, computationally efficient
- - bidders must trust the auctioneer

Nash Equilibria in Auction

- $v_1 = 50, v_2 = 30, v_3 = 70$
- $b_1 = 50, b_2 = 30, b_3 = 70$ is an equilibrium in dominant strategies.
- $b_1 = 0, b_2 = 0, b_3 = 70$ is also a NE
  - auctioneer makes no profit
- $b_1 = 70, b_2 = 0, b_3 = 0$ is also a NE
  - Player 1 gets the object and pays nothing
  - if player 3 increases her bid in order to beat player 1, she will end up paying at least 70, so she cannot increase her profit

Are 1st Price Auctions Truthful?

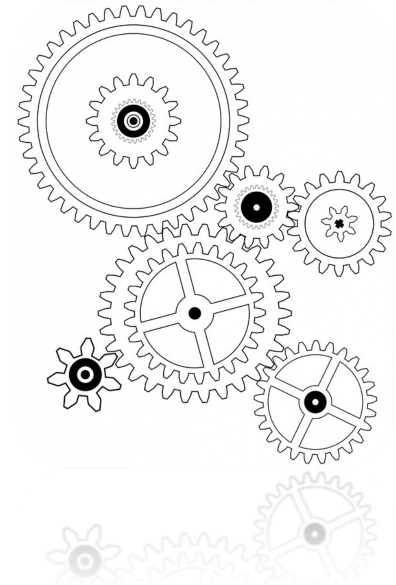
Let's play a game!

- Suppose you're participating in a first-price auction
- Your true value for the item is the last two digits of your NUS login ID
- For example, if your student number is e0123456, your value is $56.
- Your bid should be an integer amount of dollars.
- The prize for the winner is his/her utility divided by 10.

Multi Unit Auctions

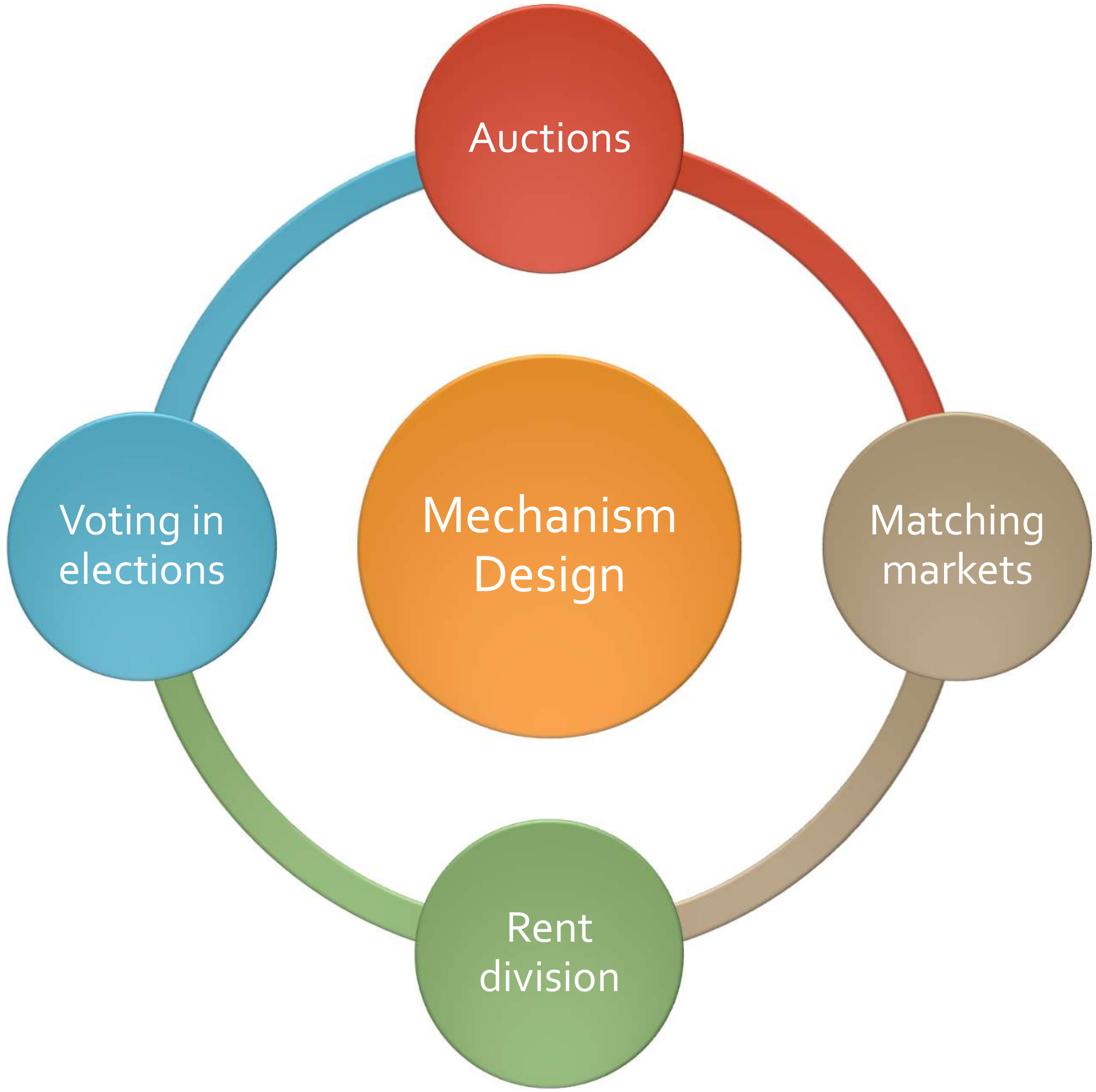
- We have $k \leq n$ **identical** items for sale.
- Each bidder $i \in N$ wants one item; values the item at v_i.
- What is the analog of Vickrey auction?

Multi Unit Auctions



Design a mechanism where:

- Truthful bidding is a dominant strategy
- Items are allocated to the k highest bidders.


Auctions
Voting in elections
Mechanism Design
Matching markets
Rent division

Mechanism Design

- Players: $N = \{1, \ldots, n\}$
- Outcomes $O = o_1, \ldots, o_m$.
- Each player i has a valuation function $v_i: O \to \mathbb{R}$.
- Can sometimes assign payments $\pi_1, \ldots, \pi_n$; utility is then $u_i(o) = v_i(o) - \pi_i$.
- Center chooses on outcome o^* to maximize some function (perhaps $\sum_i v_i(o^*)$)

What are the outcomes in:

- Single item auctions? Multi unit auctions?
- Rent division?
- Allocation of indivisible goods?
- Matching Markets?

Incentive Compatibility

- Reporting your true valuations is a NE

Dominant Strategy Incentive Compatibility

- Reporting your true valuations is a (weakly) dominant strategy.

Why do we want truthful reporting?

Vickrey Clarke Groves (VCG) Mechanism

- A general framework of truthful mechanisms
- Selects socially optimal outcome
- Ensures truthful reporting by careful payment design

Choose some outcome o^* that maximizes $\Sigma_i b_i(o^*)$

To determine the payment that agent j must make:

- Pretend j does not exist, and choose o^*_{-j} that maximizes $\sum_{i \neq j} b_i(o_{-j})$.
- j pays $\sum_{i \neq j} b_i(o^*_{-j}) - \sum_{i \neq j} b_i(o^*) = \sum_{i \neq j}[b_i(o^*_{-j}) - b_i(o^*)]$

Each agent pays the **externality** that she imposes on the other agents.

- Agent j's externality:
  (max. welfare of others if j were absent) - (max. welfare of others when j is present)

In VCG mechanisms, truthful reporting is a dominant strategy.

Utility of player j when she reports truthfully:

$$v_j(o^*) - \left(\sum_{i \neq j} b_i\left(o^*_{-j}\right) - \sum_{i \neq j} b_i(o^*) \right) =$$

$$\underbrace{v_j(o^*) + \sum_{i \neq j} b_i(o^*)}_{\text{Total social welfare under } o^* \text{ - optimal}} - \underbrace{\sum_{i \neq j} b_i\left(o^*_{-j}\right)}_{\text{Does not depend on } j\text{'s report}}$$

Utility of player j when she misreports:

$$v_j(o') - \left(\sum_{i\neq j} b_i\left(o^*_{-j}\right) - \sum_{i\neq j} b_i(o')\right) =$$

$$\underbrace{v_j(o') + \sum_{i\neq j} b_i(o')}_{\substack{\textbf{Total social}\\ \textbf{welfare}\\ \textbf{under } o' \textbf{ -}\\ \textbf{at most}\\ \textbf{under } o^*}} - \underbrace{\sum_{i\neq j} b_i\left(o^*_{-j}\right)}_{\substack{\textbf{Does not}\\ \textbf{depend on}\\ j\textbf{'s report}\\ \textbf{(same as}\\ \textbf{before)}}}$$

Combinatorial Auctions

- We have m (possibly distinct) items for sale.
- $(n+1)^m$ possible outcomes (allocation of the items to the bidders)
- Bidders can have valuations for different subsets of items
- VCG is truthful, but some challenges:
  - Preference elicitation: Each bidder has 2^m private parameters
  - Computational issues: Finding the optimal outcome o^* may be NP-hard
  - Revenue non-monotonicity (see next slide)

Revenue non-monotonicity of VCG

- Two bidders, two items A and B
- First bidder only wants both items together
  - $v_1(A, B) = 1, v_1(A) = v_1(B) = 0$
- Second bidder only wants item A
  - $v_2(A, B) = v_2(A) = 1, v_2(B) = 0$
- Revenue of VCG is 1
- Third bidder is added, who only wants item B
  - $v_3(A, B) = v_3(B) = 1, v_3(A) = 0$
- Revenue of VCG drops to 0!