CS4261/5461: Assignment for Week 3 Solutions

Due: Sunday, 7th Sep 2025, 11:59 pm SGT.

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1. (a) The allocation that maximizes social welfare gives two durians to Alice (for a value of 6) and one durian to Bob (for a value of 3).

   (b) If Alice is not present, Bob gets all three durians and receives value 7. So Alice's payment is $7 - 3 = \boxed{4}$.

   If Bob is not present, Alice gets all three durians and receives value 8. So Bob's payment is $8 - 6 = \boxed{2}$.

   (c) The allocation that maximizes social welfare gives all three durians to Cindy (for a value of 10).

   (d) If Alice is not present, the welfare-maximizing allocation remains the same, so Alice's payment is $\boxed{0}$.

   If Bob is not present, the welfare-maximizing allocation remains the same, so Bob's payment is $\boxed{0}$

   If Cindy is not present, the welfare-maximizing allocation gives two durians to Alice and one to Bob, for a welfare of 9. So Cindy's payment is $9 - 0 = \boxed{9}$.

   (e) No. Truthful bidding is a dominant strategy in VCG.

2. No. If there are only two bidders and both bidders simply say that they value the item at 0, then the bidders obtain a higher joint utility than if they were to bid truthfully. (If there are more than two bidders, a similar situation occurs when all but two bidders have value 0—the two colluding bidders can bid some small ε each.) Collusion can be highly beneficial in VCG!

3. No. Suppose the bidders' private values are $70, 50, 30$. Assuming that the first and third bidders bid truthfully, the second bidder is better off bidding 80 (which results in a utility of $50 - 30 = 20$) than truthfully bidding 50 (which results in a utility of 0).