

CS4261/5461: Assignment for Week 3

Due: Sunday, 7th Sep 2025, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 7th Sep, 11:59 pm to **Assignments/Assignment3/Submissions**. Name the file **Assignment3_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

Note: For this assignment, justification is required for all questions **except** 1(a) and 1(c).

1. (7 points, graded for correctness) Consider the following scenario: Two bidders, Alice and Bob, are bidding for three durians.
 - Alice receives value 0 if she receives no durian, 2 if she receives one durian, 6 if she receives two durians, and 8 if she receives all three durians.
 - Bob receives value 0 if he receives no durian, 3 if he receives one durian, 5 if he receives two durians, and 7 if he receives all three durians.

Suppose both bidders submit their true valuations to the VCG mechanism.

- (a) (1 point) What is the allocation of durians to the bidders that the VCG mechanism makes?
- (b) (2 points) What is the payment charged by the VCG mechanism to each of the two bidders?

Suppose now that a third bidder, Cindy, joins the previous two bidders and also bids truthfully.

- Cindy receives value 0 if she receives no durian, 1 if she receives one durian, 4 if she receives two durians, and 10 if she receives all three durians.
- (c) (1 point) What is the allocation of durians to the bidders that the VCG mechanism makes?
 - (d) (2 points) What is the payment charged by the VCG mechanism to each of the three bidders?
 - (e) (1 point) Given Alice's and Bob's bids (which may or may not be their truthful valuations), is it possible for Cindy to submit false valuations so as to strictly improve her own utility under the VCG mechanism?

2. (1 point) Consider a case where two bidders in a single-item auction *cooperate*; that is, they both submit bids to the mechanism, but they want to maximize their joint utility. Can VCG (which is simply the Vickrey—i.e., second-price—auction in this case) still extract truthful valuations from the bidders?

3. (1 point) Assume that there are at least three bidders. Suppose we run a single-item auction where we award the item to the highest bidder but charge her only the third-highest bid. Is this auction truthful?