CS4261/5461: Assignment for Week 8

Due: Sunday, 19th Oct 2025, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 19th Oct, 11:59 pm to `Assignments/Assignment8/Submissions`. Name the file **Assignment8_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

Note: For this assignment, justification is required for all questions.

1. (7 points, graded for correctness) Consider an instance with three players and four goods, and the following (additive) valuations:

Player	g_1	g_2	g_3	g_4
1	40	30	20	10
2	0	40	30	30
3	30	0	20	50

   (a) (1 point) What is the maximum utilitarian welfare among all allocations?

   (b) (2 points) What is the maximum egalitarian welfare among all allocations?

   Consider the allocation A that gives g_1 to player 1, g_4 to player 2, and g_2 and g_3 to player 3.

   (c) (2 points) Is the allocation A Pareto optimal?

   (d) (2 points) In the allocation A, do all players obtain at least their maximin share?

2. (1 point) Recall that an MMS allocation does not always exist when there are at least three agents. Show that if there exists at least one MMS allocation in a certain instance, then there also exists a Pareto-optimal MMS allocation in that instance.

3. (1 point) An allocation $A = (A_1, \ldots, A_n)$ is said to be *strong EF1 (s-EF1)* if for every player j with $A_j \neq \varnothing$, there exists a good $g_j \in A_j$ such that for all $i \in N \setminus \{j\}$, it holds that $v_i(A_i) \geq v_i(A_j \setminus \{g_j\})$. (Think about why this definition is stronger than EF1.)

   Is the output of the round-robin algorithm always s-EF1?