CS4261/5461: Assignment for Week 4 Solutions

Due: Sunday, 14th Sep 2025, 11:59 pm SGT.

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1. (a) Yes. Consider any player. If the player's location is already rightmost, the player incurs cost 0, so has no incentive to lie. Else, the only way the player can change the outcome is to report a location further to the right than the rightmost location. But then the player's cost would increase rather than decrease.

   (b) No. Suppose the locations of the four players are $1, 3, 5, 7$, respectively. If all players report truthfully, the mechanism places the facility at point 4, and player 2 incurs cost 1. However, if player 2 instead reports 1, the mechanism places the facility at point 3, and player 2 incurs cost $0 < 1$.

   (c) Yes. From part (a), we know that if a player lies, the distance between her true location and the rightmost reported location cannot decrease (compared to when she reports truthfully). Similarly, the distance between her true location and the leftmost reported location cannot decrease. Hence, the average between the two distances also cannot decrease.

   (d) No. Suppose the locations of the four players are $1, 7, 9, 10$, respectively. If all players report truthfully, the mechanism places the facility at point 10, and player 1 incurs cost 9. However, if player 1 instead reports 8, the mechanism places the facility at point 7, and player 1 incurs cost $6 < 9$.

2. (a) Since the top and bottom paths are symmetric with respect to cost, in the equilibrium flow, half of the traffic goes on each path. Hence, the amount of traffic on the edge from s to a is $\boxed{0.5}$.

   (b) The cost of each path is $(0.5 + 1) + 2 = 3.5$. Since there is one unit of traffic, the total cost is $\boxed{3.5}$.

   (c) With the edge from a to b added, all of the traffic takes the path $s \to a \to b \to t$ (because if not all of the traffic does so, then this path will have strictly lower cost than any other path). The amount of traffic on the edge from s to a is therefore $\boxed{1}$.

   (d) The cost of the path $s \to a \to b \to t$ is $(1 + 1) + (1 + 1) = 4$. Since there is one unit of traffic, the total cost is $\boxed{4}$.

3. The costs of the edges for all possible amounts of traffic are shown in the following table:

x	0	1	2	3
$g_1(x)$	0	1	4	9
$g_2(x)$	2	3	4	5

- If all three units of traffic go on the top edge, each unit has an incentive to switch to the bottom edge, as that would reduce the cost from 9 to 3.
- If all three units of traffic go on the bottom edge, each unit has an incentive to switch to the top edge, as that would reduce the cost from 5 to 1.
- Suppose two units of traffic go on the top edge and one unit on the bottom edge. A unit switching from the top edge to the bottom edge would not change its cost of 4, while a unit switching from the bottom edge to the top edge would increase its cost from 3 to 9. Hence, this is an equilibrium.
- Suppose one unit of traffic goes on the top edge and two units on the bottom edge. A unit switching from the top edge to the bottom edge would increase its cost from 1 to 5, while a unit switching from the bottom edge to the top edge would not change its cost of 4. Hence, this is an equilibrium.

In summary, there are two equilibrium flows: two units on the top edge and one on the bottom edge, or two units on the bottom edge and one on the top edge.