**NATIONAL UNIVERSITY OF SINGAPORE**
**SCHOOL OF COMPUTING**

3rd Midterm Assessment for CS4261/5461

November 13, 2025 Time Allowed: 80 minutes (6:30–7:50pm)

**INSTRUCTIONS:**

- This paper consists of **six** parts for a total of 60 points.
- This is a **closed book/notes** examination. No calculators or other electronic devices are allowed.
- Write your answers **clearly** in the given space. Justification is required only for questions 3(d) and 5(d).
- For questions with a "Don't know" option, the answer "Don't know" guarantees 1 point.

Name: ____________________

Student Number: ____________________

**DO NOT TURN THIS PAGE OVER UNTIL INSTRUCTED**

Grader's use only

| **Part** | **Points** | **Part** | **Points** |
|---|---|---|---|
| 1 | | 4 | |
| 2 | | 5 | |
| 3 | | 6 | |
| – | – | Total | |

1. (10 points) For questions (a) and (b), write "Yes", "No", or "Don't know".

   (a) (2 points) Does there exist a normal-form game with two players, each having three actions, such that the total number of Nash equilibria (whether pure or not) is 0?
   **Answer:**

   (b) (2 points) Does there exist a normal-form game with two players, each having three actions, such that the total number of Nash equilibria (whether pure or not) is 2?
   **Answer:**

For questions (c) and (d), consider the following game:

| | W | X | Y | Z |
|---|---|---|---|---|
| A | $6,3$ | $1,4$ | $5,5$ | $4,6$ |
| B | $9,6$ | $2,4$ | $8,5$ | $5,7$ |
| C | $8,5$ | $3,8$ | $9,6$ | $6,5$ |
| D | $1,8$ | $3,7$ | $4,7$ | $4,9$ |

   (c) (2 points) Which of the eight actions A, B, C, D, W, X, Y, Z are strictly dominated **in the original game**? (If there is more than one such action, you must specify all of them.)
   **Answer:**

   (d) (4 points) Find all Nash equilibria of this game. (Answer in the simplest form.)
   **Answer:**

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Scratch paper area

2. (10 points) For questions (a) and (b), consider the weighted voting game with four players who have weights $1, 2, 3, 4$, and the threshold is 9.

   (a) (3 points) Compute the Shapley value of all players.
   **Answer:**

   (b) (2 points) Determine all payoff vectors in the core.
   **Answer:**

   For questions (c) and (d), consider the game with 10 players such that if a coalition contains $x$ players, then the value of the coalition is $\lfloor x/3 \rfloor$ (that is, the largest integer not exceeding $x/3$).

   (c) (3 points) Compute the Shapley value of all players.
   **Answer:**

   (d) (2 points) Determine all payoff vectors in the core.
   **Answer:**

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3. (12 points) For questions (a), (b), and (c), consider a cake-cutting instance with four agents and the following density functions:

$$f_1(x) = 4x^3 \text{ for all } x \in [0,1]; \qquad f_2(x) = 3x^2 \text{ for all } x \in [0,1];$$
$$f_3(x) = 2x \text{ for all } x \in [0,1]; \qquad f_4(x) = 1 \text{ for all } x \in [0,1].$$

(a) (2 points) What is agent 1's value for the interval $[1/2, 1]$?
**Answer:**

(b) (2 points) Which agent receives the second leftmost piece from the Dubins–Spanier protocol?
**Answer:**

(c) (2 points) Which agent receives the leftmost piece from the Even–Paz protocol?
**Answer:**

For question (d), assume that there are four agents. Each agent has a nonnegative, additive, nonatomic, and normalized valuation function over the cake. For an allocation of the cake to the four agents, its *total number of intervals* is the sum of the number of intervals allocated to each agent. (For example, an allocation that gives $[0, 0.2] \cup [0.8, 1]$ to agent 1, $[0.2, 0.4]$ to agent 2, $[0.4, 0.6]$ to agent 3, and $[0.6, 0.8]$ to agent 4 has a total of $2 + 1 + 1 + 1 = 5$ intervals.)

(d) (6 points) Determine a large integer $k$ with the following property:

There exist valuation functions of the four agents and positive real numbers $r_1, r_2, r_3, r_4$ with $r_1 + r_2 + r_3 + r_4 = 1$ such that no allocation that has a total of at most $k$ intervals gives agent $i$ a value of at least $r_i$ for every $i \in \{1, 2, 3, 4\}$. (You should try to make $k$ as large as possible, but do not need to show that your $k$ is optimal.)

Either give an answer **with justification** or write "Don't know". (The answer "Don't know" or a correct answer with $k \leq 3$ will get 1 point.)

**Answer:**

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Scratch paper area. You may continue your answer for question (d) here (or on the back side if needed).

4. (8 points) For questions (a) and (b), consider a rent division setting with two players and two rooms. The total rent is 1000.

   - Player 1 has value 1000 for the first room and 0 for the second room.
   - Player 2 has value 500 for the first room and 500 for the second room.

   Suppose we use the mechanism which chooses an allocation of the rooms to the players that maximizes the sum of players' values for the rooms, and then chooses an envy-free price vector that maximizes the minimum utility of the players.

   (a) (2 points) Can Player 1 misreport her valuations so as to obtain a higher utility? (Write "Yes", "No", or "Don't know".)
   **Answer:**

   (b) (2 points) Can Player 2 misreport her valuations so as to obtain a higher utility? (Write "Yes", "No", or "Don't know".)
   **Answer:**

   For questions (c) and (d), suppose that Alice, Bob, and Charlie are moving into their new apartment with three bedrooms and a total rent of 1000.

   - Alice has value 300 for Room 1, 300 for Room 2, and 400 for Room 3.
   - Bob has value 300 for Room 1, 400 for Room 2, and 300 for Room 3.
   - Charlie has value 400 for Room 1, 300 for Room 2, and 300 for Room 3.

   (c) (2 points) What is the allocation of the rooms to the players that maximizes the sum of players' values for the rooms?
   **Answer:**

   (d) (2 points) For the welfare-maximizing room allocation, determine **all** values of $t$ such that there exists an envy-free price vector that charges Room 1 at $t$.
   **Answer:**

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5. (12 points) For questions (a), (b), and (c), consider an instance of approval committee voting with $n = 12$ voters, $m = 12$ candidates, and a target committee size of $k = 4$. The approval sets are:

   - $A_1 = A_2 = A_3 = A_4 = \{c_1, c_2, c_3, c_4, c_5, c_6\}$
   - $A_5 = A_6 = A_7 = \{c_5, c_6, c_7, c_8\}$
   - $A_8 = A_9 = A_{10} = \{c_5, c_9, c_{10}\}$
   - $A_{11} = \{c_1, c_{11}\}$
   - $A_{12} = \{c_{12}\}$

   (a) (2 points) If we run GreedyCC, determine **all** candidates that can be chosen in the first step.
   **Answer:**

   (b) (2 points) Determine the PAV score of the committee $\{c_9, c_{10}, c_{11}, c_{12}\}$.
   **Answer:**

   (c) (2 points) Does the committee $\{c_1, c_2, c_9, c_{10}\}$ satisfy JR? (Answer "Yes", "No", or "Don't know".)
   **Answer:**

   For question (d), consider **all** possible instances of approval committee voting with $n = 12$ voters, $m = 12$ candidates, and a target committee of size $k = 4$, such that each candidate is approved by at least one voter. (The instance shown above is one such instance.) Assume that in the instance, there are at least two JR committees.

   (d) (6 points) Is it true that for every such instance and every JR committee, it is possible to replace some candidate in the committee by some candidate outside the committee in such a way that the resulting committee also satisfies JR?
   Either give an answer **with justification** or write "Don't know". (No point will be awarded for the "Yes"/"No" answer alone.)
   **Answer:**

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Scratch paper area. You may continue your answer for question (d) here (or on the back side if needed).

6. (8 points) Consider a tournament $T$ with 2025 alternatives $x_1, \ldots, x_{2025}$ such that $x_i$ dominates $x_j$ whenever $i < j$, except that $x_{2025}$ dominates $x_1$ and $x_{2024}$.

   (a) (2 points) Is there a Condorcet loser in $T$? (Answer "Yes", "No", or "Don't know".)
   **Answer:**

   (b) (2 points) Determine the outdegree of $x_1$.
   **Answer:**

   (c) (2 points) Determine the top cycle of $T$.
   **Answer:**

   (d) (2 points) Determine the uncovered set of $T$.
   **Answer:**

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