CS4261/5461: Assignment for Week 7

Due: Sunday, 12th Oct 2025, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 12th Oct, 11:59 pm to `Assignments/Assignment7/Submissions`. Name the file **Assignment7_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

Note: For this assignment, justification is required for all questions.

1. (7 points, graded for correctness) Consider the following matching instance with four students (A, B, C, D) and four hospitals (W, X, Y, Z).

   The students' preferences are:

   $$A : Z \succ W \succ Y \succ X \qquad B : Y \succ X \succ W \succ Z$$

   $$C : X \succ W \succ Z \succ Y \qquad D : Z \succ W \succ Y \succ X$$

   The hospitals' preferences are:

   $$W : A \succ B \succ C \succ D \qquad X : D \succ A \succ B \succ C$$

   $$Y : A \succ B \succ D \succ C \qquad Z : B \succ D \succ A \succ C$$

   (a) (2 points) Is the matching $\{(A, Z), (B, Y), (C, X), (D, W)\}$ stable? If not, name all blocking pairs.

   (b) (1 point) What is the output of Gale–Shapley with the students proposing?

   (c) (1 point) What is the output of Gale–Shapley with the hospitals proposing?

   (d) (3 points) How many stable matchings are there in this instance?

2. (1 point) Show that if all hospitals have the same preference list over students, then there exists a unique stable matching.

3. (1 point)

   (a) Find a Nash bargaining solution of the following problem:

   $$S = \{(x, y) \in \mathbb{R}^2_{\geq 0} \mid 4x + \sqrt{y} = 1\},$$

   where $u_1(x) = x$ and $u_2(y) = y$. Answer with the values (x, y).

   (b) Find a utilitarian optimal solution of the problem in part (a). Answer with the values (x, y).