CS5461 Assignment 6

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1. (a) We are given the game $[5; 1, 2, 2, 4]$, so $4! = 24$ permutations in total. In all permutations:
   - Player 1 is pivotal only in the cases $(4, 1, 2, 2)$ and $(2, 2, 1, 4)$, each of which having 2 permutations. Therefore $\text{Sh}_1 = 4/24 = 1/6$.
   - Player 4 is pivotal unless they are first or last, so $\text{Sh}_4 = 1/2$.
   - Players 2 and 3 are symmetric, so by efficiency $\text{Sh}_2 = \text{Sh}_3 = (1 - 1/6 - 1/2)/2 = 1/6$.

   Therefore the Shapley values are $\boxed{(1/6, 1/6, 1/6, 1/2)}$.

   (b) Here each boy always adds 1 and each girl always adds 2 regardless of the coalition. Hence each player's marginal contribution is constant. By definition, the Shapley values are $\boxed{(1, 1, 1, 2, 2, 2)}$ for the three boys and the three girls.

   (c) There are $7! = 5040$ permutations in total. In all permutations:
   - Players 3 to 7 are symmetric and pivotal only when they are third and the first two are players 1 and 2 in any order, giving $2 \times 4! = 48$ permutations. Therefore $\text{Sh}_i = 48/5040 = 1/105$ for $i \in \{3, 4, 5, 6, 7\}$.
   - Players 1 and 2 are symmetric, so by efficiency $\text{Sh}_1 = \text{Sh}_2 = (1 - 6 \times 1/105)/2 = 10/21$.

   Therefore the Shapley values are $\boxed{(10/21, 10/21, 1/105, 1/105, 1/105, 1/105, 1/105)}$.

2. (a) No.

   (b) Yes.

   (c) No.

3. (a) False.

   (b) True.

   (c) False.