

CS4261/5461: Assignment for Week 6

Due: Wednesday, 1st Oct 2025, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 1st Oct, 11:59 pm to **Assignments/Assignment6/Submissions**. Name the file **Assignment6_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

Note 1: Because of the recess week, this assignment is due on Wednesday, 1st Oct.

Note 2: For this assignment, justification is required only for Question 1.

1. (7 points, graded for correctness) For each of the following games, compute the Shapley value of all players.
 - (a) (2 points) The weighted voting game with four players who have weights 1, 2, 2, 4, and the threshold is 5.
 - (b) (2 points) The game with three boys and three girls such that if a coalition contains x boys and y girls, then the value of the coalition is $x + 2y$.
 - (c) (3 points) The game with 7 players (numbered 1, 2, ..., 7) such that

$$v(S) = \begin{cases} 1 & \text{if } S \text{ contains player 1, player 2, and at least one other player;} \\ 0 & \text{otherwise,} \end{cases}$$

for each set S of players.

2. (1 point) For each of the three games in Question 1, determine whether the vector of Shapley values belongs to the core. (Answer “Yes” or “No” for each game.)

3. (1 point) True or false?

- (a) If the core is nonempty, then the vector of Shapley values is always a core payoff vector.
- (b) If the payoff vectors \vec{x} and \vec{y} belong to the core of a game \mathcal{G} , then for every $\alpha \in [0, 1]$, the payoff vector $\alpha\vec{x} + (1 - \alpha)\vec{y}$ belongs to the core of \mathcal{G} as well.
- (c) The core of a cooperative game can be of size exactly two.