

CS5461 Assignment 3

Li Jiaru (SID: A0332008U)

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1.
 - (a) The VCG mechanism allocates 2 durians to Alice and 1 durian to Bob.
 - (b) With Alice present, Bob only gets 1 durian so his utility is 3. If Alice is absent, Bob gets all 3 durians, so his utility becomes 7. Therefore Alice should be charged with $7 - 3 = 4$.
With Bob present, Alice only gets 2 durians so her utility is 6. If Bob is absent, Alice gets all 3 durians, so her utility becomes 8. Therefore Bob should be charged with $8 - 6 = 2$.
 - (c) The VCG mechanism allocates all 3 durians to Cindy.
 - (d) With Cindy present, both Alice and Bob get no durians so their utility is 0. If Cindy is absent, from previous parts we know that the optimal allocation guarantees a utility of $3 + 6 = 9$. Therefore Cindy should be charged with $9 - 0 = 9$.
Since both Alice and Bob get nothing, they should pay nothing either.
 - (e) No, this is impossible. As given in the lecture, in the VCG mechanism, truthful reporting is a dominant strategy. Submitting false valuations will only make at most as much as truthful reporting does, so strict improvement is not possible.
2. No, not necessarily. We consider a simple counterexample. Let us denote v_1 and v_2 to be the value for bidder 1 and 2, respectively, with $v_1 > v_2 > 0$.
Under truthful valuations, bidder 1 submits v_1 and bidder 2 submits v_2 , so bidder 1 will win and pay v_2 , giving a total utility of $v_1 - v_2$.
However, if they could cooperate, bidder 2 could submit a bid of 0 and bidder 1 could then submit any positive bid. In this case, bidder 1 will still win but they pay nothing, giving a total utility of $v_1 - 0 = v_1$.
Since $v_2 > 0$, $v_1 > v_1 - v_2$ so truthful reporting is no longer dominant.
3. No, not necessarily. We consider a simple counterexample. Suppose that there are three bidders. Your value for the item is 2, and the other two bids are 3 and 1.
Under truthful bidding, you lose and get a utility of 0.
However, you can bid anything greater than 3 and get the item, so that you only pay the third-highest bid, which is 1 in this case, and your utility becomes $2 - 1 = 1$. Therefore, the auction is no longer truthful.