CS5461 Assignment 1

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1. (a) The normal-form game is shown as below.

		Don	
		Tennis	Basketball
Carla	Tennis	$7, 3$	$1, 1$
	Basketball	$1, 1$	$3, 7$

   (b) (Tennis, Tennis) and (Basketball, Basketball) are the only two pure Nash equilibria.

   (c) We only need to compute mixed equilibria here.
   Therefore, we consider the case where Carla plays (Tennis, Basketball) with probability $(p, 1-p)$ and Don plays (Tennis, Basketball) with probability $(q, 1-q)$ where $0 < p, q < 1$.
   Then each player must be indifferent between these two actions, as stated in the lecture.
   For Carla, this gives
   $$7q + 1(1-q) = 1q + 3(1-q),$$
   so we have $q = 1/4$.
   For Don, this gives
   $$3p + 1(1-p) = 1p + 7(1-p),$$
   so we have $p = 3/4$.
   Therefore, Carlo playing (Tennis, Basketball) with probability $(3/4, 1/4)$ and Don playing (Tennis, Basketball) with probability $(1/4, 3/4)$ is a mixed Nash equilibrium.
   Together with the pure Nash equilibria (Tennis, Tennis) and (Basketball, Basketball) as found in part (b), these give all Nash equilibria of this game.

2. Consider the normal-form game as given.

	L	M	R
T	$1, 6$	$1, 0$	$2, 7$
C	$2, 4$	$2, 2$	$0, 3$
B	$2, 3$	$1, 4$	$8, 2$

To find all Nash equilibria, we use iterated removal of dominated strategies.

We first notice that T is strictly dominated by $1/2(\mathrm{C} + \mathrm{B})$ as $1 < 1/2(2+2)$ for L, $1 < 1/2(2+1)$ for M, $2 < 1/2(0+8)$ for R. Therefore we can remove T.

After that, notice that R is now strictly dominated by L as $3 < 4$ for C and $2 < 3$ for B. Therefore we can remove R.

We are now left with the game in normal form as

	L	M
C	$2, 4$	$2, 2$
B	$2, 3$	$1, 4$

We consider the case where the row player plays C. Then the column player will play L as a best response. In this case, the row player will be indifferent between C and B. However, when the row player plays B, the column player will play M as a best response.

Therefore we need to calculate their expected utility in each case.

For the row player, when the column player plays (L, M) with probability $(q, 1-q)$ where $0 \leq q \leq 1$, we have

$$u_{\text{row}}(\mathrm{C}) = 2q + 2(1-q) = 2, \quad u_{\text{row}}(\mathrm{B}) = 2q + 1(1-q) = 1 + q,$$

for which $2 = 1 + q \implies q = 1$ is the only possibility for indifference, i.e., the column player always plays L at Nash equilibrium.

For the column player, when the row player plays (C, B) with probability $(p, 1-p)$ where $0 \leq p \leq 1$, we have

$$u_{\text{column}}(\mathrm{L}) = 4p + 3(1-p) = 3 + p, \quad u_{\text{column}}(\mathrm{M}) = 2p + 4(1-p) = 4 - 2p,$$

but since the column player should always play L, we must have $3 + p \geq 4 - 2p \implies p \geq 1/3$ for L to be a best response.

Therefore, the Nash equilibria of the original game consist of the row player playing (T, C, B) with probability $(0, p, 1-p)$ for $1/3 \leq p \leq 1$ and the column player always playing L.

3. (a) Yes.
   (b) No.
   (c) Yes.