CS4261/5461: Assignment for Week 2 Solutions

Due: Sunday, 31st Aug 2025, 11:59 pm SGT.

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1. (a) W and Z are both strictly dominated by $\frac{1}{2}X + \frac{1}{2}Y$, while D is strictly dominated by $\frac{1}{5}A + \frac{4}{5}B$.

   A, B, C are not strictly dominated, because they yield the highest payoff when the row player plays W, X, X, respectively.

   X and Y are not strictly dominated, because they both yield the highest payoff when the column player plays B.

   Hence, $\boxed{D, W, Z}$ are the strictly dominated actions.

   (b) From part (a), we can immediately eliminate D, W, Z. Moreover, once these are eliminated, A is strictly dominated by B, so we can eliminate A. The game reduces to:

	X	Y
B	$5, 5$	$4, 5$
C	$5, 2$	$3, 6$

   First, we consider equilibria in which at least one player plays a pure strategy.

   - If the row player plays B, the column player's best response is any mixture between X and Y. Since B is a best response to any such mixture, we get the equilibria $\boxed{(B, qX + (1-q)Y) \text{ for any } 0 \le q \le 1}$.
   - If the row player plays C, the column player's best response is Y, but C is *not* a best response to Y.
   - If the column player plays X, the row player's best response is any mixture $pB + (1-p)C$ between B and C. The column player playing X is a best response to this if and only if $5p + 2(1-p) \ge 5p + 6(1-p)$, which is equivalent to $p \ge 1$, i.e., $p = 1$. Hence, we get the equilibrium (B, X). However, this is already covered by $(B, qX + (1-q)Y)$ for $0 \le q \le 1$.
   - If the column player plays Y, the row player's best response is B. Playing Y is a best response to this, so we get the equilibrium (B, Y). Again, this is already covered by $(B, qX + (1-q)Y)$ for $0 \le q \le 1$.

   To summarize, so far we have found the equilibria $(B, qX + (1-q)Y)$ for any $0 \le q \le 1$.

Now, suppose that both players put positive probability on both actions, say $(p, 1-p)$ and $(q, 1-q)$, respectively, where $0 < p, q < 1$. Since the row player is indifferent between the two actions,

$$5q + 4(1-q) = 5q + 3(1-q) \quad \Longrightarrow \quad q = 1.$$

But this contradicts the assumption that $0 < p, q < 1$.

Hence, the Nash equilibria are $\boxed{(B, qX + (1-q)Y) \text{ for any } 0 \le q \le 1}$.

2. (a) We observe the following:
   - (T, L) is never a Nash equilibrium.
   - (B, L) is always a Nash equilibrium.
   - (T, R) is a Nash equilibrium if and only if $t \le 2$.
   - (B, R) is a Nash equilibrium if and only if $t \ge 2$.

   Hence, the game has exactly two pure Nash equilibria if and only if $\boxed{t \ne 2}$.

   (b) If $t \ge 2$, then $(B, qL + (1-q)R)$ is a Nash equilibrium for any $0 \le q \le 1$, so there are more than two Nash equilibria overall.

   Suppose now that $t < 2$. We will find all Nash equilibria. First, we consider equilibria in which at least one player plays a pure strategy.
   - If the row player plays T, the column player's best response is R. Since T is a best response to R, we get the equilibrium (T, R).
   - If the row player plays B, the column player's best response is any mixture $qL + (1-q)R$. The row player playing B is a best response to such a mixture if and only if $q = 1$, which yields the equilibrium (B, L).
   - If the column player plays L, the row player's best response is any mixture $pT + (1-p)B$. The column player playing L is a best response to such a mixture if and only if $p = 0$, which yields the equilibrium (B, L).
   - If the column player plays R, the row player's best response is T. The column player playing R is a best response to T, so we get the equilibrium (T, R).

   Now, suppose that both players put positive probability on both actions, say $(p, 1-p)$ and $(q, 1-q)$, respectively, where $0 < p, q < 1$. Since the column player is indifferent between the two actions,

   $$2p + 0(1-p) = 3p + 0(1-p) \quad \Longrightarrow \quad p = 0.$$

   But this contradicts the assumption that $0 < p, q < 1$.

   Hence, when $t < 2$, there are two Nash equilibria overall: (T, R) and (B, L). The answer is therefore $\boxed{t < 2}$.

3. (a) False. For example:

	L	R
T	1, 1	0, 0
B	0, 0	1, 1

(T, L) and (B, R) are Nash equilibria, but neither (T, R) nor (B, L) are.

(b) False. For example:

	L	R
T	$0, 0$	$1, 1$
B	$0, 0$	$0, 0$

The conditions on the payoffs are satisfied, but (B, L) is a Nash equilibrium.

(c) True. If playing T yields payoff x to the row player when the column player plays $\frac{1}{3}L + \frac{2}{3}R$, and playing B also yields payoff x, then by linearity of payoffs, playing any mixture of T and B also yields payoff x.