CS4261/5461: Assignment for Week 4

Due: Sunday, 14th Sep 2025, 11:59 pm SGT.

Please upload PDFs containing your solutions (hand-written & scanned, or typed) by 14th Sep, 11:59 pm to `Assignments/Assignment4/Submissions`. Name the file **Assignment4_SID.pdf**, where SID should be replaced by your student ID.

You may discuss the problems with your classmates or read material online, but you should write up your solutions on your own. Please note the names of your collaborators or online sources in your submission; failure to do so would be considered plagiarism.

Note: For this assignment, justification is required for all questions **except** Question 3.

1. (7 points, graded for correctness) There are four players on an infinite street, and one facility to be located. The cost of an agent is his/her distance to the facility. Is each of the following mechanisms truthful?

   (a) (1 point) Locate the facility at the rightmost reported location.

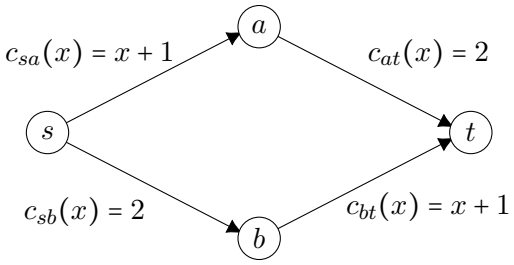
   (b) (2 points) Locate the facility at the midpoint between the second leftmost reported location and the third leftmost reported location.

   (c) (2 points) Locate the facility at the leftmost reported location with probability 1/2, and at the rightmost reported location with probability 1/2.

   (d) (2 points) Let a be the distance between the leftmost reported location and the second leftmost reported location, and let b be the distance between the second leftmost reported location and the rightmost reported location.

   If $a \leq b$, locate the facility at the leftmost reported location. Else, locate the facility at the rightmost reported location.

2. (1 point) Consider the (non-atomic) routing game shown in the following figure, where we want to route **one unit** of traffic from s to t, and the cost functions of the edges are as shown.



(a) Determine the amount of traffic routed on the edge from s to a in the equilibrium flow.

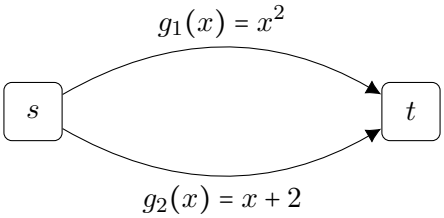
(b) Determine the total cost of the equilibrium flow.

For parts (c) and (d), consider the routing game that results from augmenting the game above by adding an edge from a to b with cost function $c_{ab}(x) = 0$.

(c) Determine the amount of traffic routed on the edge from s to a in the equilibrium flow of the augmented game.

(d) Determine the total cost of the equilibrium flow of the augmented game.

3. (1 point) Consider the **atomic** routing game shown in the following figure, where we want to route three units of traffic from s to t. There are two edges, the top edge with cost function $g_1(x) = x^2$ and the bottom edge with cost function $g_2(x) = x + 2$.



Determine all (pure) equilibrium flows of this game.