CS4261/5461: Assignment for Week 1 Solutions

Due: Sunday, 24th Aug 2025, 11:59 pm SGT.

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1. (a) The game can be represented as a normal-form game as follows, with Carla being the row player and Don the column player:

	Tennis	Basketball
Tennis	$7, 3$	$1, 1$
Basketball	$1, 1$	$3, 7$

(b) The pure Nash equilibria are $\boxed{\text{(Tennis, Tennis)}}$ and $\boxed{\text{(Basketball, Basketball)}}$.

(c) First, we consider equilibria in which at least one player plays a pure strategy.

- If Carla chooses Tennis, Don's best response is Tennis, and we have the equilibrium $\boxed{\text{(Tennis, Tennis)}}$.
- If Carla chooses Basketball, Don's best response is Basketball, and we have the equilibrium $\boxed{\text{(Basketball, Basketball)}}$.

A similar reasoning can be made if Don plays a pure strategy, leading to the same two pure Nash equilibria.

Now, suppose that both players put positive probability on both actions, say $(p, 1-p)$ and $(q, 1-q)$, respectively, with $0 < p, q < 1$. Since Carla is indifferent between the two actions,

$$7q + 1(1-q) = 1q + 3(1-q) \implies q = 1/4.$$

Similarly, since Don is indifferent between the two actions,

$$3p + 1(1-p) = 1p + 7(1-p) \implies p = 3/4.$$

So we get the mixed Nash equilibrium $\boxed{(\frac{3}{4}\text{Tennis} + \frac{1}{4}\text{Basketball}, \frac{1}{4}\text{Tennis} + \frac{3}{4}\text{Basketball})}$.

In summary, the Nash equilibria are (Tennis, Tennis), (Basketball, Basketball), and $(\frac{5}{8}\text{Tennis} + \frac{3}{8}\text{Basketball}, \frac{3}{8}\text{Tennis} + \frac{5}{8}\text{Basketball})$.

2. We begin by iteratively removing dominated strategies. Note that T is strictly dominated by $\frac{1}{2}$C + $\frac{1}{2}$B, so we may eliminate T. With T gone, R is strictly dominated by L, so we may remove R. We are left with the following 2-by-2 game:

	L	M
C	$2,4$	$2,2$
B	$2,3$	$1,4$

First, we consider equilibria in which at least one player plays a pure strategy.

- If the row player chooses C, the column player's best response is L. The row player choosing C is a best response to L, so we have the equilibrium $\boxed{\text{(C, L)}}$.
- If the row player chooses B, the column player's best response is M. The row player choosing C is *not* a best response to M.
- If the column player chooses L, the row player's best response is any mixture between C and B. Suppose the row player chooses pC + $(1-p)$B for some $p \in [0,1]$. The column player choosing L is a best response to this if and only if $4p+3(1-p) \geq 2p+4(1-p)$, which is equivalent to $p \geq \frac{1}{3}$. Hence, we have the equilibrium $\boxed{(p\text{C} + (1-p)\text{B, L}) \text{ for } \frac{1}{3} \leq p \leq 1}$. Note that this subsumes the equilibrium (C, L) found earlier.
- If the column player chooses M, the row player's best response is C. The column player choosing M is *not* a best response to C.

Now, suppose that both players put positive probability on both actions, say $(p, 1-p)$ and $(q, 1-q)$, respectively, with $0 < p, q < 1$. Since the row player is indifferent between the two actions,

$$2q + 2(1-q) = 2q + 1(1-q) \quad \Longrightarrow \quad q = 1.$$

But this contradicts the assumption that $p < 1$.

In summary, the Nash equilibria are (pC + $(1-p)$B, L) for $\frac{1}{3} \leq p \leq 1$.

3. (a) True. For example:

	L	R
T	$1,0$	$0,1$
B	$0,1$	$1,0$

(b) False. By Nash's theorem, there always exists at least one Nash equilibrium.

(c) True. For example:

	L	R
T	$1,1$	$0,0$
B	$0,0$	$0,0$

(Try checking this as an exercise!)