NATIONAL UNIVERSITY OF SINGAPORE
SCHOOL OF COMPUTING

1st Midterm Assessment for CS4261/5461

September 18, 2025 Time Allowed: 45 minutes (6:30–7:15pm)

INSTRUCTIONS:

- This paper consists of **four** parts for a total of 40 points.
- This is a **closed book/notes** examination. No calculators or other electronic devices are allowed.
- Write your answers **clearly** in the given space. Justification is required only for question 4(d).
- For questions with a "Don't know" option, the answer "Don't know" guarantees 1 point.

Name: ______________________________

Student Number: ______________________________

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Grader's use only

Part	**Points**
1	
2	
3	
4	
Total	

1. (12 points)

   For question (a), consider the following game, where t is a positive real number:

	R	S
P	$4, 1$	$3, 2$
Q	$5, 3$	$0, t$

   (a) (4 points) Determine all integers k for which there exists a (positive real) value of t such that the number of **pure** Nash equilibria of this game is exactly k.
   Answer:

   For question (b), consider the following scenario: Alice and Bob are deciding whether to work overtime or go home.

   - If both work overtime, each person gets a payoff of 0.
   - If both go home, each person gets a payoff of 5.
   - If one works overtime and the other goes home, the person who works overtime gets a payoff of 7 and the person who goes home gets a payoff of 0.

   (b) (4 points) Find all Nash equilibria of this game. (Answer in the simplest form.)
   Answer:

   For question (c), consider the following game:

	L	M	R
T	$5, 6$	$2, 4$	$3, 9$
C	$7, 5$	$8, 3$	$4, 2$
B	$4, 4$	$1, 5$	$5, 7$

   (c) (4 points) Find all Nash equilibria of this game. (Answer in the simplest form.)
   Answer:

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2. (8 points) For questions (a) and (b), suppose there are three bidders, and one item to be sold. For each of the following auction format, is truthful bidding a dominant strategy? (Write "Yes", "No", or "Don't know".)

   (a) (2 points) Give the item to the highest bidder but do not charge anything.
   Answer:

   (b) (2 points) Give the item to the second-highest bidder and charge the third-highest bid.
   Answer:

   For questions (c) and (d), consider two bidders who are bidding for two items, A and B. Both bidders have value 0 for an empty set of items. Otherwise, their values for the items are as follows:

   Bidder 1: $v_1(A) = 3, v_1(B) = 4, v_1(AB) = 5$

   Bidder 2: $v_2(A) = 2, v_2(B) = 8, v_2(AB) = 10$

   Suppose that both bidders submit their true valuations to the VCG mechanism.

   (c) (2 points) What is the **payment** charged by the VCG mechanism to each of the two bidders?
   Answer:

   Suppose that a third bidder joins with the following valuation (again, value 0 for an empty set):

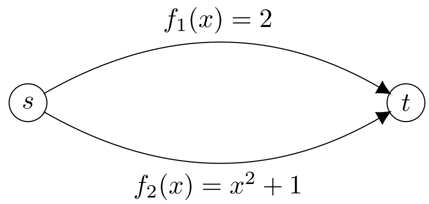
   Bidder 3: $v_3(A) = 6, v_3(B) = 7, v_3(AB) = 11$

   This bidder also submits her true valuation to the VCG mechanism.

   (d) (2 points) What is the **allocation** of the items to the three bidders that the VCG mechanism makes?
   Answer:

Scratch paper area

3. (8 points) For questions (a) and (b), consider the (non-atomic) routing game shown in the following figure, where we want to route one unit of traffic from s to t, and the cost functions of the edges are as shown.



(a) (2 points) Determine the amount of traffic routed on the **bottom** edge in the **optimal** flow.
Answer:

(b) (2 points) Determine the total cost of the **equilibrium** flow.
Answer:

For questions (c) and (d), consider the **atomic** routing game where there are only two nodes, s and t, and we want to route **three units** of traffic from s to t. There are three edges from s to t: the first edge with cost function $d_1(x) = x$, and the second edge with cost function $d_2(x) = 2$, and the third edge with cost function $d_3(x) = 3$.

(c) (2 points) Determine all (pure) equilibrium flows of this game.
Answer:

(d) (2 points) Determine the **largest** total cost among all (pure) equilibrium flows.
Answer:

Scratch paper area

4. (12 points) For questions (a), (b), and (c), suppose there are **three players** on a street represented by the interval $[0, 1]$, and one facility to be located. The cost of a player is his/her distance to the facility. Is each of the following mechanisms truthful? (Write “Yes”, “No”, or “Don’t know”.)

   (a) (2 points) Locate the facility at the point $(a+b+c)/3$, where a, b, c are the three reported locations (i.e., the arithmetic mean of the three reported locations).
   Answer:

   (b) (2 points) If the leftmost reported location is to the **left** of the point 0.5, locate the facility at the leftmost reported location. Else, locate the facility at 0.5.
   Answer:

   (c) (2 points) If the leftmost reported location is to the **right** of the point 0.5, locate the facility at the leftmost reported location. Else, locate the facility at 0.5.
   Answer:

   For question (d), suppose there are **three players** on a street represented by a **unit-length circle** (which can be viewed as the interval $[0, 1]$ with the two endpoints joined), and one facility to be located. The cost of a player is his/her **shortest** distance to the facility along the circle. (For example, if a player is at 0.1 and the facility is located at 0.9, then the player’s cost is $(0.1 - 0) + (1 - 0.9) = 0.2$.)

   (d) (6 points) Is there a **deterministic** mechanism f that is truthful and minimizes the total cost (i.e., the approximation ratio of f with respect to the total cost is 1)?
   Either give an answer **with justification** or write “Don’t know”. (No point will be awarded for the “Yes”/“No” answer alone.)
   Answer:

Scratch paper area. You may continue your answer for question (d) here (or on the back side if needed).

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