CS4261/5461 Algorithmic Mechanism Design

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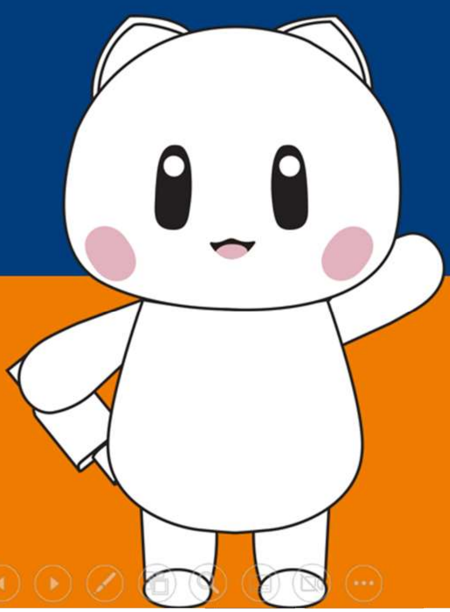
2025

SOC STUDENT LIFE SURVEY

**(plus, a potential $10 Grab Voucher)*



WE WANT TO HEAR FROM YOU

****Complete the survey by Sunday, 7 September 2025 for a chance to win Grab vouchers!***

	W	X	Y	Z
A	8, 6	1, 6	2, 7	6, 4
B	2, 4	5, 5	4, 5	7, 4
C	0, 2	5, 2	3, 6	6, 3
D	3, 5	4, 6	3, 5	0, 4

- Which of the eight actions are strictly dominated (in the original game)?
- D is strictly dominated by (1/5)A + (4/5)B
- A is not strictly dominated in the original game

	X	Y
B	5, 5	4, 5
C	5, 2	3, 6

- Make sure you cover all possible cases
  - Case 1: At least one player plays a pure strategy
  - Case 2: Both players strictly mix
- Case 1 includes the possibility that one player plays a pure strategy and the other player strictly mixes
- (B, q*X + (1-q)*Y) for any q in [0,1]

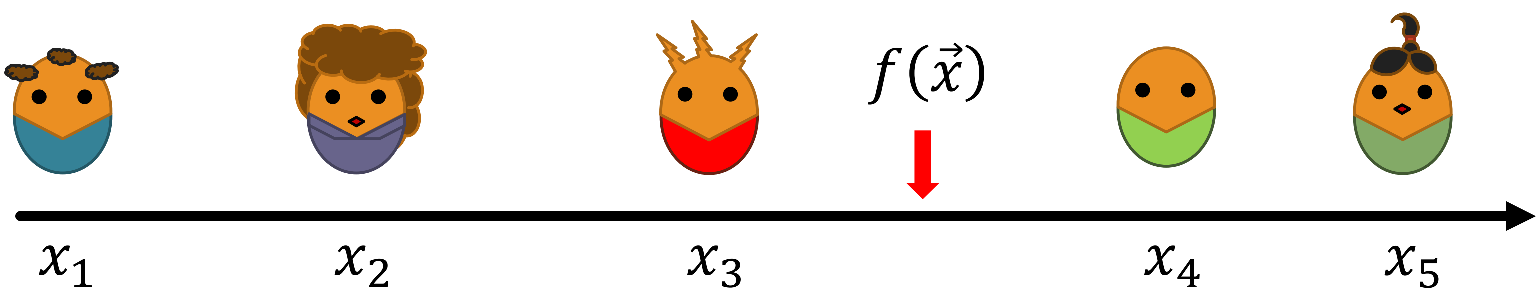
Facility Location

Model:

- Players: $N = \{1, \dots, n\}$
- Each with a location $x_i \in \mathbb{R}$
- Mechanism: $f: \mathbb{R}^n \to \mathbb{R}$
- For notational convenience, assume $x_1 \le x_2 \le \cdots \le x_n$ (the actual order may be different)

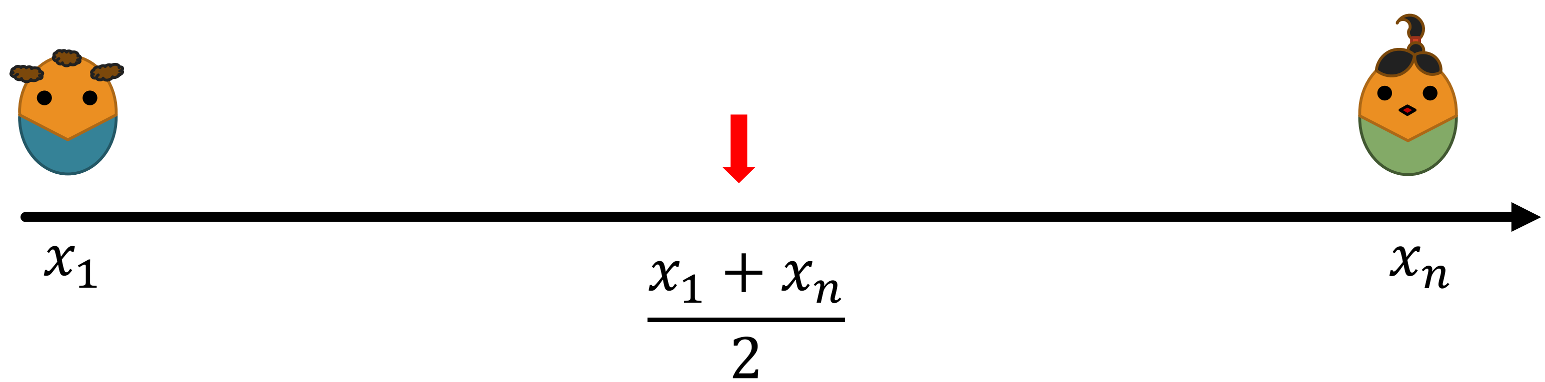
Social Cost

- Cost of player i = $|f(\vec{x}) - x_i|$
- Total Cost: $\sum_{i \in N} |f(\vec{x}) - x_i|$
- Max Cost: $\max_i |f(\vec{x}) - x_i|$



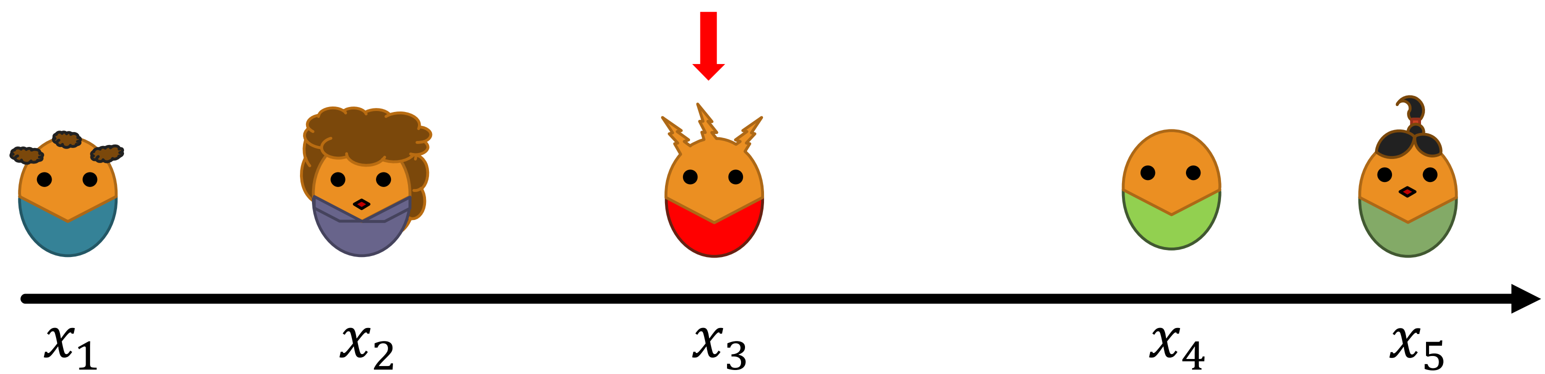
Minimize the maximum cost?

- Optimal solution: $\frac{x_1+x_n}{2}$
- Not truthful



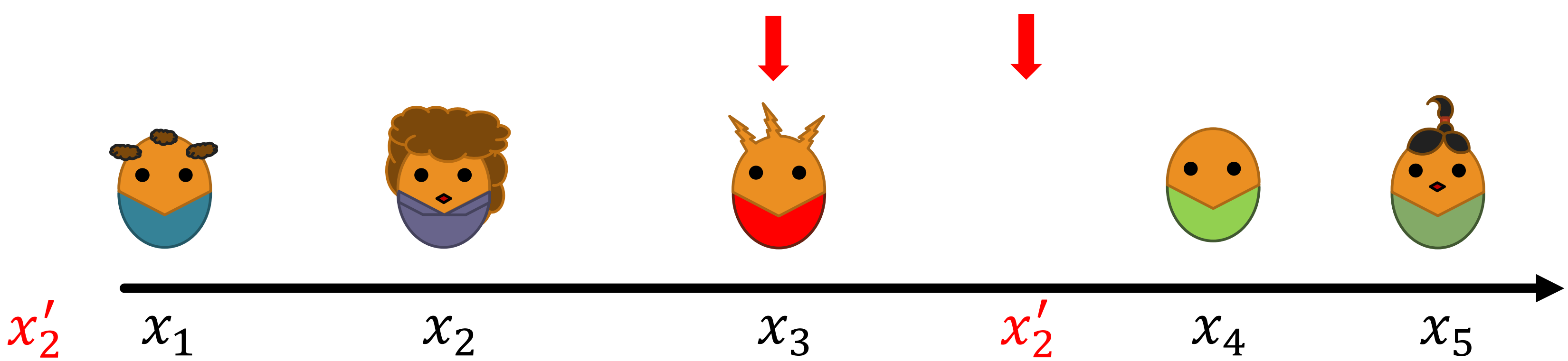
Median Mechanism:
Choose the median player's location (rounded down if there are two median players)

$$m(\vec{x}) = x_3$$



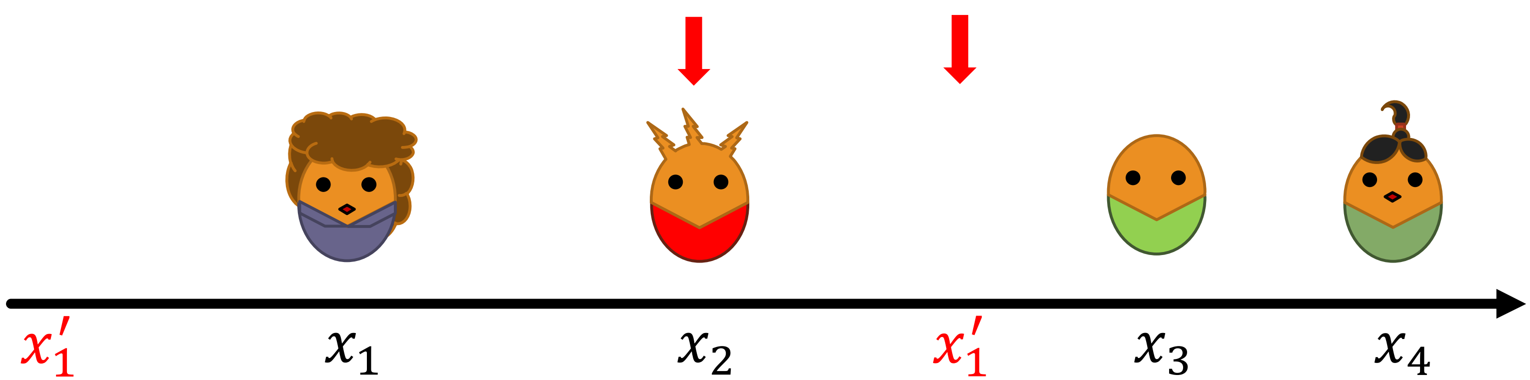
The median is:

- Strategyproof (= truthful)
- Socially optimal for the total cost objective



(Even n case) The median is:

- Strategyproof
- Socially optimal for the total cost objective

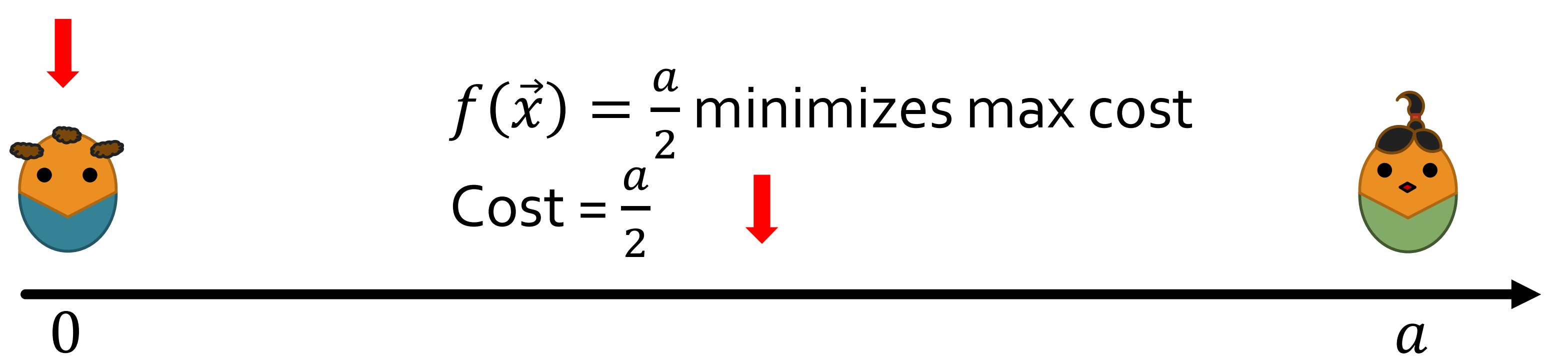


Choose the location of the leftmost agent

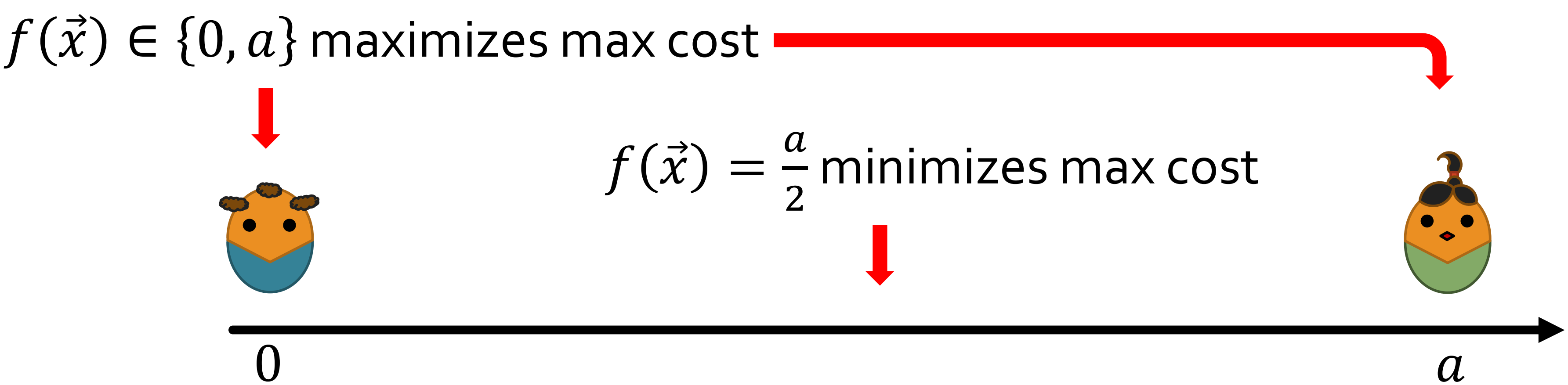
- Strategyproof
- What's the approximation ratio for max cost?

$f(\vec{x}) = 0$

Cost = a



Theorem: any deterministic truthful mechanism has a worst-case approximation ratio of at least 2 to the maximum cost.

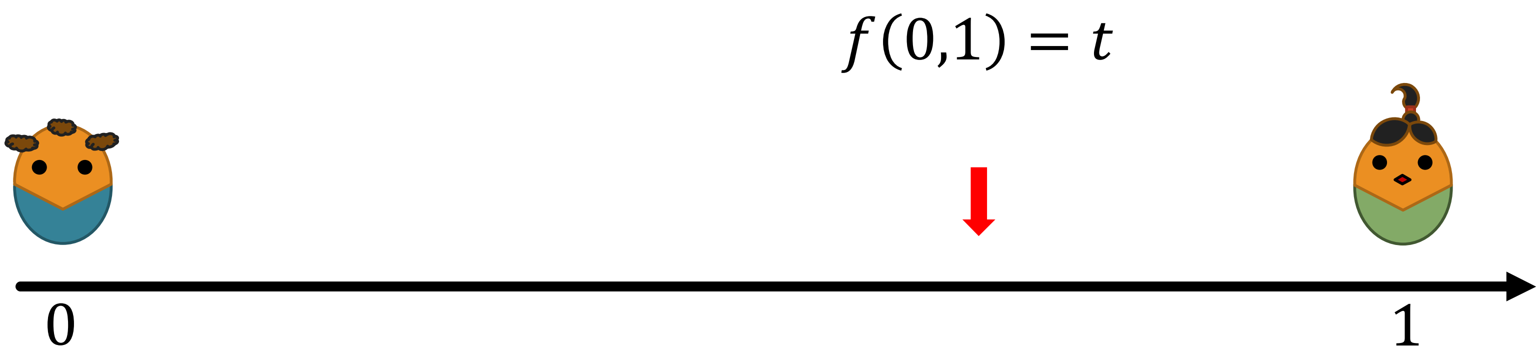


Proof:

Assume for contradiction that f is a deterministic truthful mechanism with ratio < 2 for max cost.

Consider two agents located at 0 and 1.

Suppose that $f(\vec{x}) = t$ for some $0 < t < 1$.



Suppose next that player 2's **true location** is t.
To maintain max-cost ratio better than 2, output of mechanism must be strictly between the players.

But then player 2 can benefit by reporting...

$$f(0, t) \in (0, t)$$

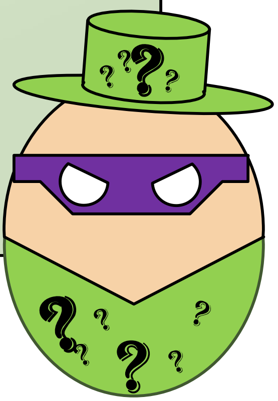
What about randomized mechanisms?

Randomized Mechanism:

- Choose x_1 with probability $\frac{1}{4}$
- Choose x_n with probability $\frac{1}{4}$
- Choose $\frac{x_1+x_n}{2}$ with probability $\frac{1}{2}$

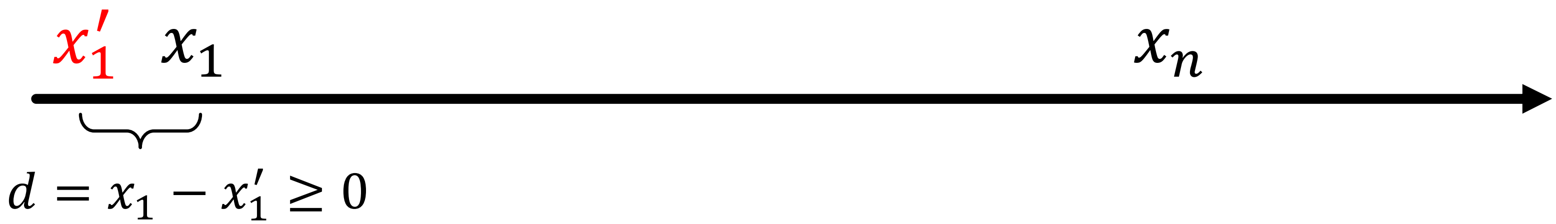
This mechanism offers a max-cost approximation ratio of…

1. $\frac{3}{2}$
2. 2
3. $\frac{5}{4}$
4. $\sqrt{2}$



The mechanism is strategyproof!

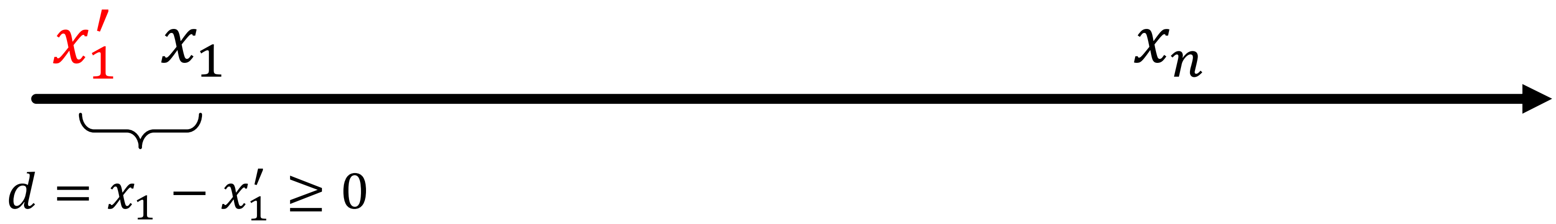
Proof: In order for the mechanism to change anything, either the leftmost point (x_1) or the rightmost point (x_n) must be changed.



Does the leftmost player have an incentive to misreport?
Certainly not to the right…
If the player misreports to the left by distance d:

- Cost from x_1' moving to the left = $\frac{1}{4} \cdot d$
- Benefit from $\frac{x_1'+x_n}{2}$ moving to the left = $\frac{1}{2} \cdot \frac{d}{2} = \frac{d}{4}$

Hence, the leftmost player has no incentive to misreport!



Similarly, the rightmost player has no incentive to misreport.

Any other player would have to move to the left of x_1 or to the right of x_n to change the outcome.

But by similar calculations, this cannot be beneficial.

Theorem: Any randomized strategyproof mechanism has a max-cost approximation ratio of at least $\frac{3}{2}$

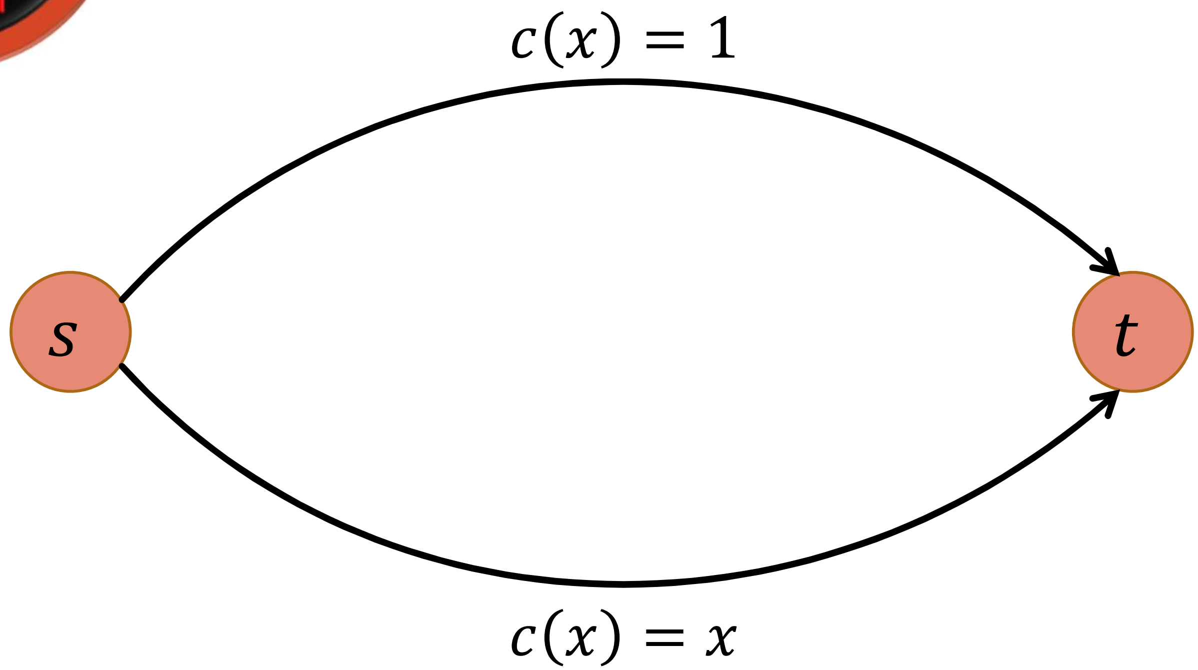
Objective Function	Deterministic	Randomized
Total cost	1	1
Max cost	2	3/2

Further reading: Procaccia and Tennenholtz, "Approximate Mechanism Design without Money", ACM TEAC 2013

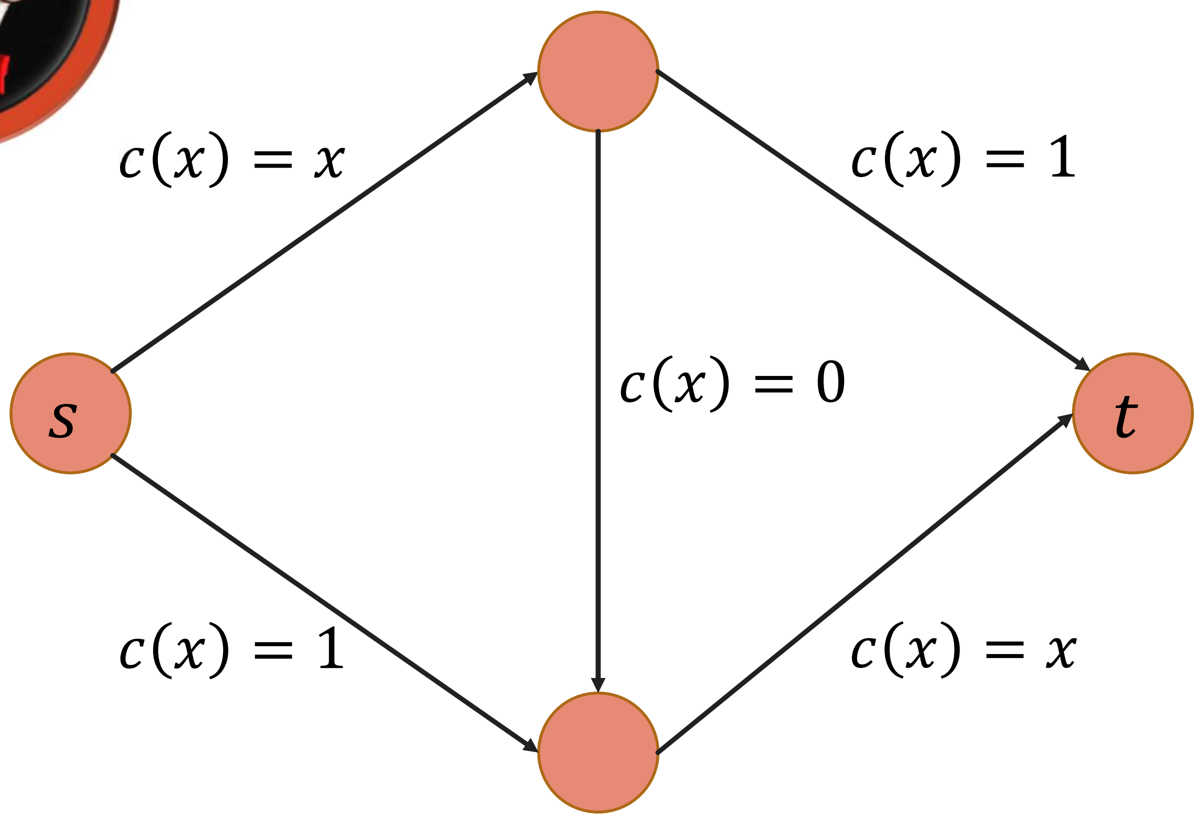
Routing Games

Pigou’s Example

One unit of traffic needs to be routed from s to t



Braess's Paradox



Key Insight: Selfish Behavior Hurts Social Welfare

- **Price of Anarchy**: ratio of the social cost under the *worst Nash Equilibrium* and the socially optimal solution

$$PoA(G) = \frac{WorstNash(G)}{OPT(G)}$$

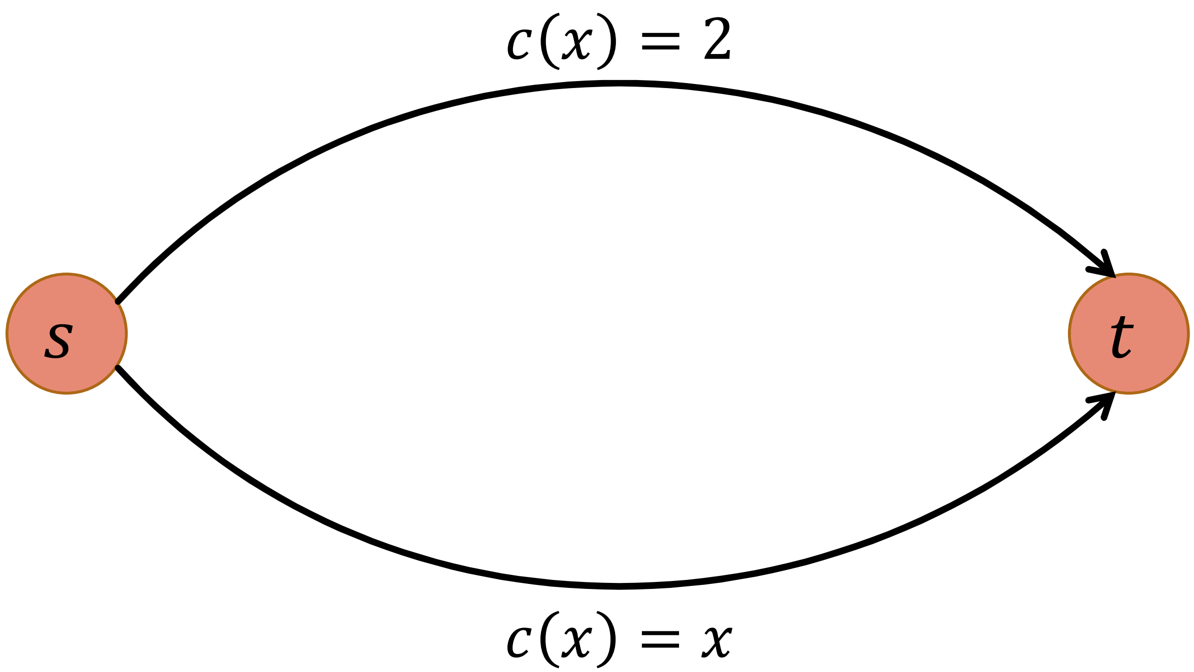
In **non-atomic** routing games, all equilibrium flows have the same cost, so we can take any equilibrium in the numerator above.

Routing Games: atomic version

- k units of traffic, where k is a positive integer
- Each unit must be routed as a whole (we can think of each unit as a player)
- Each edge $e \in E$ has a cost function $c_e\colon \mathbb{N} \to \mathbb{R}_{\geq 0}$

Atomic routing Game

Two units of traffic needs to be routed from s to t



Routing Games: equilibrium

Theorem: In an atomic routing game, a pure Nash equilibrium flow always exists.

- **High-level idea:** Show that every atomic routing game is a potential game.
- All players are inadvertently and collectively striving to optimize a potential function.
- Potential function: $\Phi(f) = \sum_{e \in E} \sum_{i=1}^{f_e} c_e(i)$, where f_e is the number of players that choose a path that includes the edge e.

Crucial Property: If a player deviates from path P to $\hat{P}$, the change in the potential function is

$$\Phi(\hat{f}) - \Phi(f) = \sum_{e \in \hat{P}} c_e(\hat{f}_e) - \sum_{e \in P} c_e(f_e)$$

- In other words, when a player deviates, the change in the potential function is the same as the change in the deviator's individual cost!
- So, a flow that minimizes the potential function is an equilibrium.

- A similar proof works for non-atomic routing games.
- Use integral instead of sum over the cost function.