

# Lecture 13: Review, Summary, and Potential Directions

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# Admin

Due end next week

- Quiz
- Homework
- Project

Let me know if you need an extension

No office hours next week

- Must be conducted virtually in the evening
- I should be around on Friday
- Or simply email me

Teaching feedback (please submit!)

# Today's lecture

This class only touched the tip of the surface

- Modeling vs theory/algorithms

Hopefully, people outside of Game Theory have learnt something

- Originally meant to be a pure research seminar

We covered a broad range of topics (we'll review them in a moment)

An overview of what research topics I'm interested in

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An overview of what research topics I'm interested in

I'm **actively** looking for students (more later)

# First: Any Questions For Quiz/Homework?

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(Do on whiteboard)

# Classic Material

## Nash equilibrium in matrix games

- Definition, Best responses, existence via Brouwer/Kakutani
  - Refinements: Strong Nash equilibrium, Stable Nash equilibrium, Coalition Proof NE, Evolutionary Stable Strategies
  - Weakening: correlated equilibria, coarse CE, rationalizable strategy (players are BR-ing to some *beliefs* as to what opponents are doing)
- Limitations
  - PPAD-hard, inapproximable, hard to find → weak predictive power?
  - Need complete specification of game, payoffs, actions
  - Bounded rationality: Quantal response equilibria
- Algorithms
  - General-sum games
    - Vertex/Support enumeration, Nash as LCP, Lemke Howson (2 players) & path following, Integer programming (2 players) [Qn: what are some advantages/disadvantages?] Multiplayer general-sum game: homotopy methods: Govindan-Wilson, social welfare optimum equilibria, index of equilibria
  - Zero-sum games (next few lectures)
- Public libraries for game solving
  - gambit, NashPy
  - Most aren't super performant, but gets the job done

# 2-player zero-sum games

## Minimax Theorem (important!)

- Conditions, Uses, Proof (many ways, e.g., Farkas Lemma, existence of no-regret methods)
- Extensions: Sion's minimax theorem, Glicksberg theorem for continuous games (mathematically, not all have a value!), useful for market equilibria
- Equivalence to finding saddle-point

## Importance of Exploitability (holds for general-sum games)

- Even if you are not into game solving, at least **evaluate** things properly
  - [Point out recent work in evaluation]
- Should evaluate based on worst case performance, BR of opponent
  - What makes a strong player? Beating noobs all the time or beating everyone by a small margin?
  - Non-transitivity in performance, A beats B, B beats C, C beats A
  - Example: Go players beat AlphaGo by exploiting oversights in AI (executed manually)
- In practice, difficult to evaluate exploitability for large games, but still an objective
- Alternatives to ELO scores that are based on equilibria (many others exist)
  - AlphaRank, Nash

Strategy spaces beyond simplexes: graphs, other constraints

# Algorithms for solving 2p0s games

## Linear programming + dualization

- Change min-max problem to min-min problem
- Works on domains beyond the simplex too!

## Decoupled methods

- Approximate equilibria, saddle point residual
- Fictitious play
- No-regret learning (outside of games)
  - Putting 2 no-regret agents against each other  $\rightarrow$  converges in average iterate to Nash
    - Average iterate convergence vs Last iterate convergence. Best iterate convergence
    - Usually much faster (and space efficient) than LPs
  - Regret minimizers: RM, RM+, Hedge (+others, even projected gradient descent works)
    - Bandits (not full feedback, e.g., Exp3)
    - Relationship to optimization, in particular online mirror descent, e.g., Hedge = OMD with entropy DGF
    - Relationship to Blackwell approachability
    - Can be used to prove minimax theorem



# Extensive Form Games

## Definition, Game Trees

- Perfect information games, backward induction, subgame perfect nash equilibria, pure strategy equilibria, credible vs non-credible threats
- Imperfect information games
  - Information sets
    - Generalizes matrix games, Lying/bluffing/playing randomly
    - More “nice” structures: perfect recall, no-absent mindedness, timeability
  - Conversion to normal form strategies
  - Behavioral strategies
  - Sequence form (perfect recall), treeplexes

## Algorithms

- General-sum: 2 players, Lemke’s algorithm, LCPs, integer programming
- Zero-sum games
  - Online learning, CFR, Scaled Extensions
- Meta-game algorithms: double oracle, PSRO

# Other equilibria

## Mediated equilibria

- Correlated equilibria, Coarse correlated equilibria
- Strong connections to no-regret learning, external regret vs. internal regret
- Poly-time in matrix games (LP or self-play), only very recently poly-time for EFGs (was open for a long time!)
- EFCE/EFCCE, Phi-regret, hindsight rationality

## Stackelberg leadership

- Bi-level optimization: Usually super general and hard, but in certain cases can be solved efficiently using multiple LP method, poly-time algorithms, in EFG is NP-hard (Conitzer & Sandholm) apart from 1-2 special cases (perfect info without chance)

## Some applications of both

- Security, Battleship, Social welfare optimal strategies

Some other “rare” equilibria, e.g., Nash Stackelberg equilibria

# Markov Games

## Generalization of MDPs

- POMDPs would be POSGs (very hard to scale in practice)

## Difference with EFGs

- Trajectories with cycles exist

## Centralized method

- Minimax-Q learning (very, very old)

## Decentralized methods

- V-learning (CE, CCE, NE for 2p0s)

## Deep learning-based methods

- MADDPG + many, many others
- Check out Stefano Albrecht's textbook for a reasonable coalition

# Other topics

## Learning Dynamics and Games

- Hedge, "energy" conservation, Euler discretization, difference between continuous dynamics/EGT and discrete variants
- Last iterate convergence, RVU bounds

## Subgame solving

- Value function at a node level does **not** exist in IIEFG
- Need value "functional" sometimes, value depends on what opponent strategy chooses. Key difference between multiplayer and single player, e.g., POMDP has belief state space and belief-values
- Extend method in chess (roughly) to IIEFG
  - What makes a good value in practice? E.g., darkchess → use perfect information value?
- Safe subgame solving, Max-margin solving (other methods: ReBel, reach subgame solving, DeepStack), compatibility with deep learning based methods
- Extension to general-sum games, set of achievable payoffs

# My interests and thoughts

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# What I'm interested in

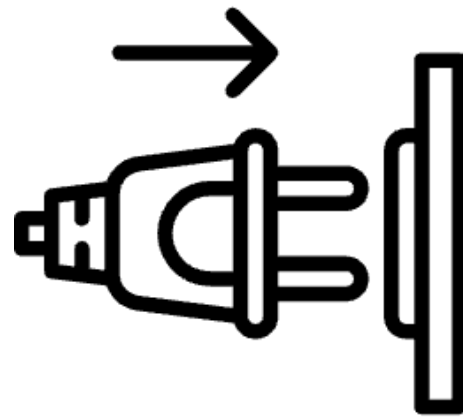
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... because everyone has their own agenda

“So far the field has struggled to take techniques like this *out of the laboratory and game environments and into the real world...*”

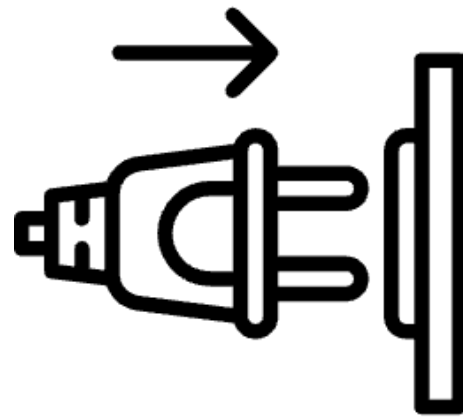
-Gary Marcus on Alphastar, 2019





Game Theory in the real-world



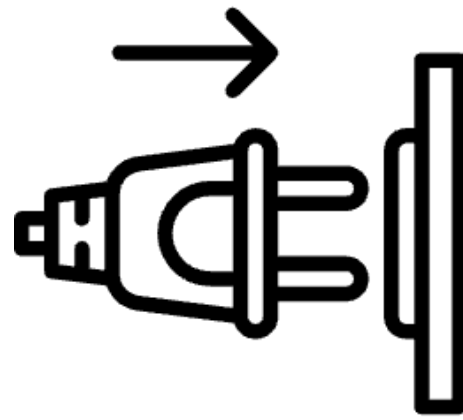


Game Theory in the real-world

Traditional

More modern





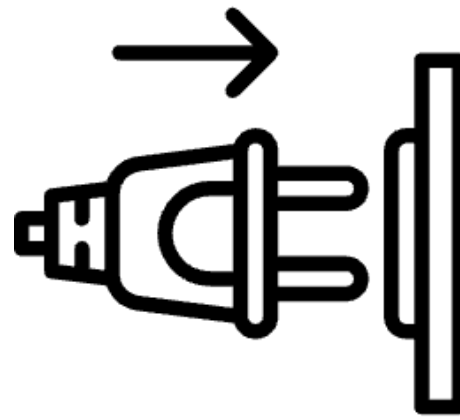
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1. Algorithms &  
Structure



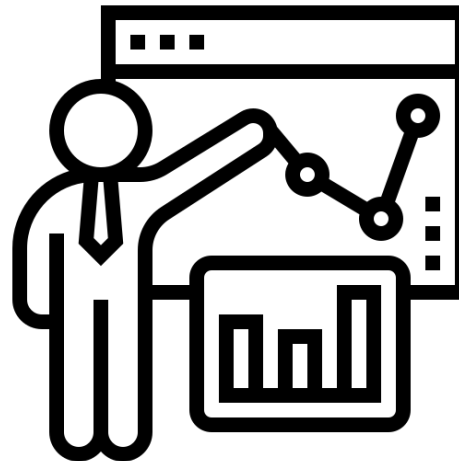
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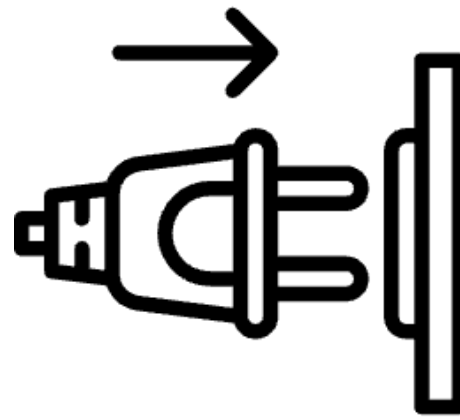
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1. Algorithms &  
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2. Interpretability &  
sensemaking



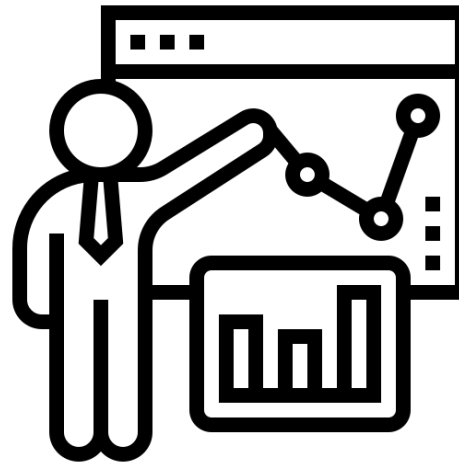
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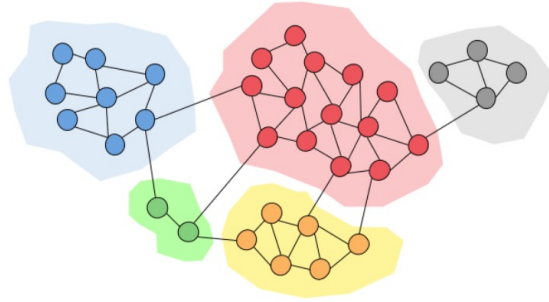
3. Challenging assumptions

# Direction 1: Algorithms & Structure

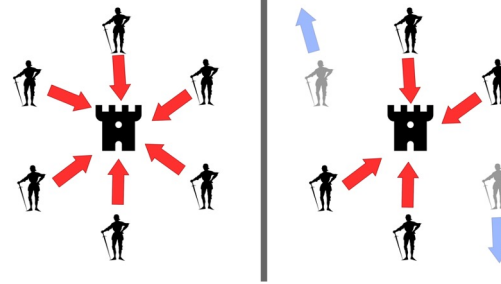
Real world problems are tough (large) but have lots of structure!



Ridesharing



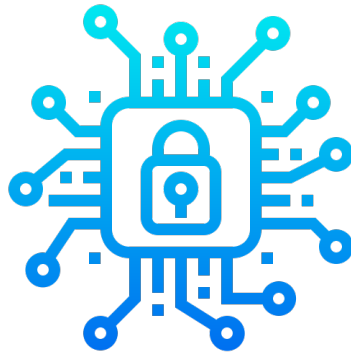
Social media analysis



Distributed systems



Logistics



Cybersecurity



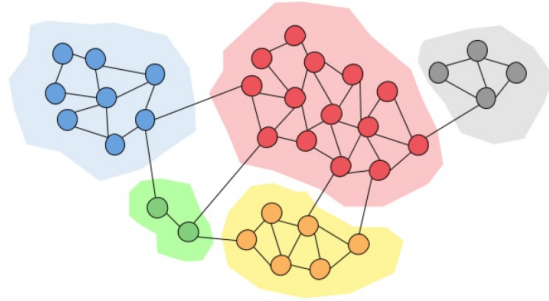
Politics

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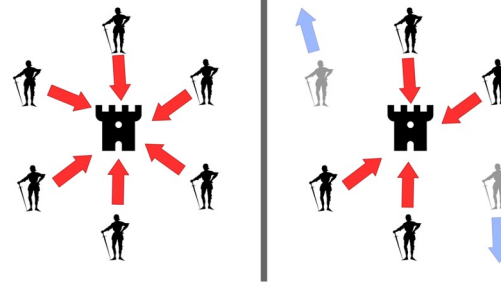
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Social media analysis



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Politics

- Multiplayer, team games
- General-sum
- Imperfect information
- Jointly continuous & discrete
- Huge, combinatorial structure

## How do we **model** real-world scenarios?

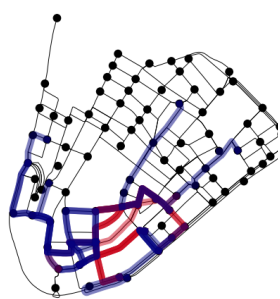
- Applications
  - Logistics and Patrolling
  - Cyber and network security
- Equilibrium concepts
  - Multi-defender security games
  - Extensive-form correlated equilibria
  - Sparse equilibrium

## How do we **solve** these games we have defined?

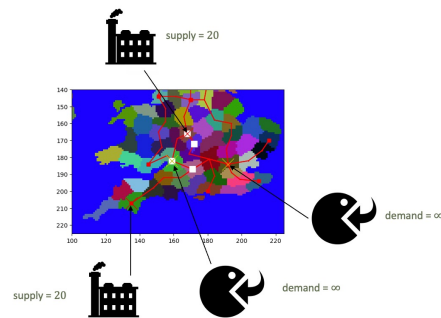
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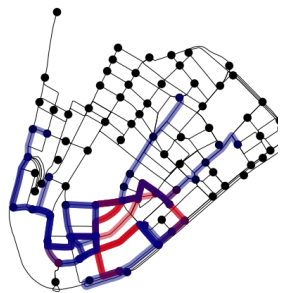
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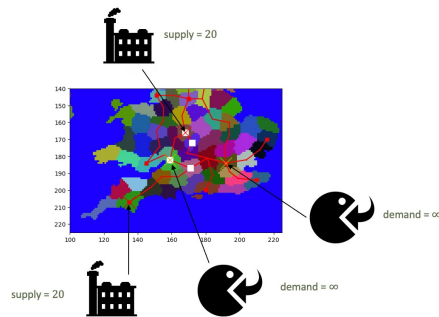


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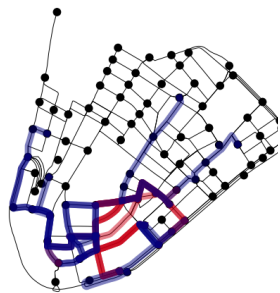
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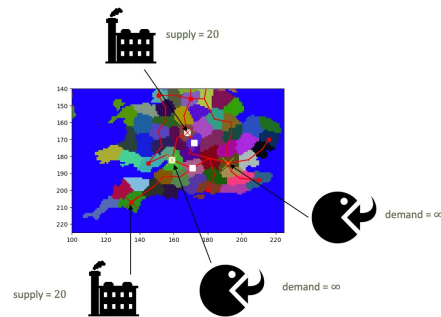
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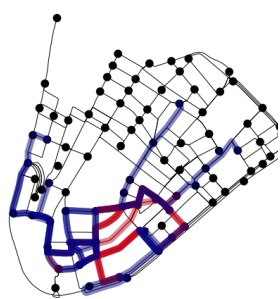
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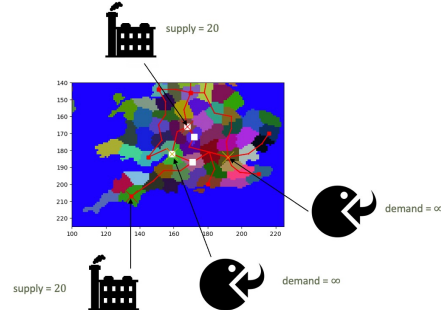
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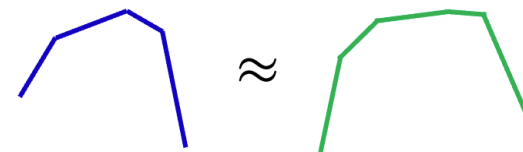
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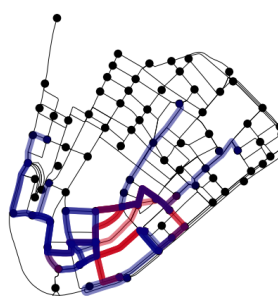
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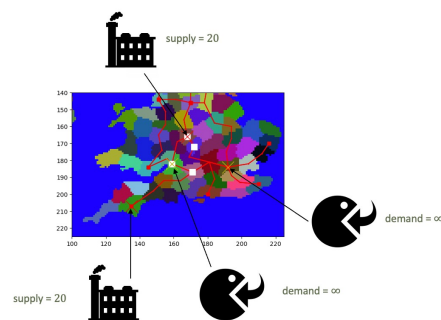
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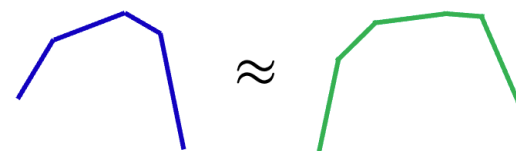
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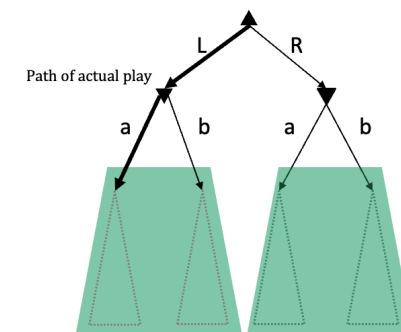
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# Inverse Game Theory

Learning player utilities from their past actions played in games

			
	0	$-b_1(x)$	$b_2(x)$
	$b_1(x)$	0	$-b_3(x)$
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Learn  
←

i.i.d samples from  
equilibrium strategies

$$a^{(1)} = ( \text{rock}, \text{paper} )$$

$$a^{(2)} = ( \text{rock}, \text{scissors} )$$

$$a^{(3)} = ( \text{paper}, \text{scissors} )$$

...

Side Information

$$x^{(1)} = [0.1, 0.5]$$







$$x^{(2)} = [0.3, 0.7]$$

$$x^{(3)} = [0.9, 0.2]$$

...

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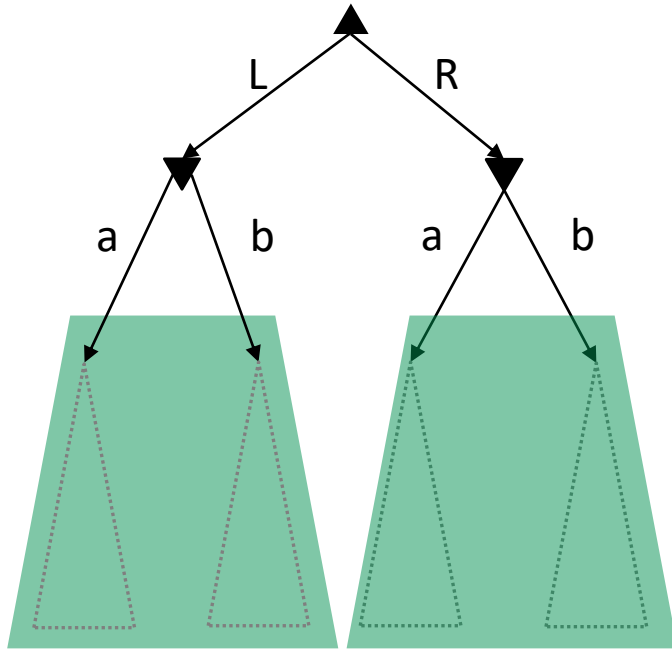
$$x^{(2)} = [0.3, 0.7]$$

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- Utilities are *functions* of side information  $x$ 
  - E.g., utilities depend on age, demographic of players

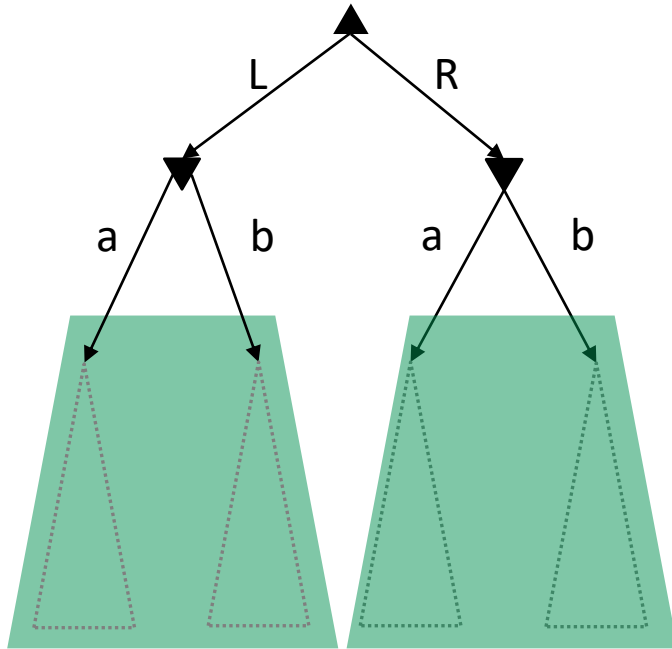
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# Subgame Resolving





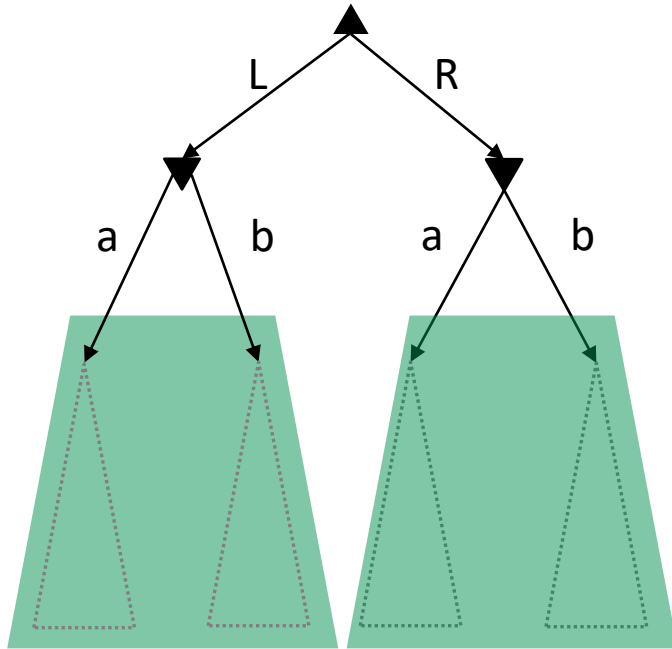
# Subgame Resolving



## Consider Chess

- Resolving is only initiated from states faced in **actual play**
- Avoid search from states which we've never reached

# Subgame Resolving



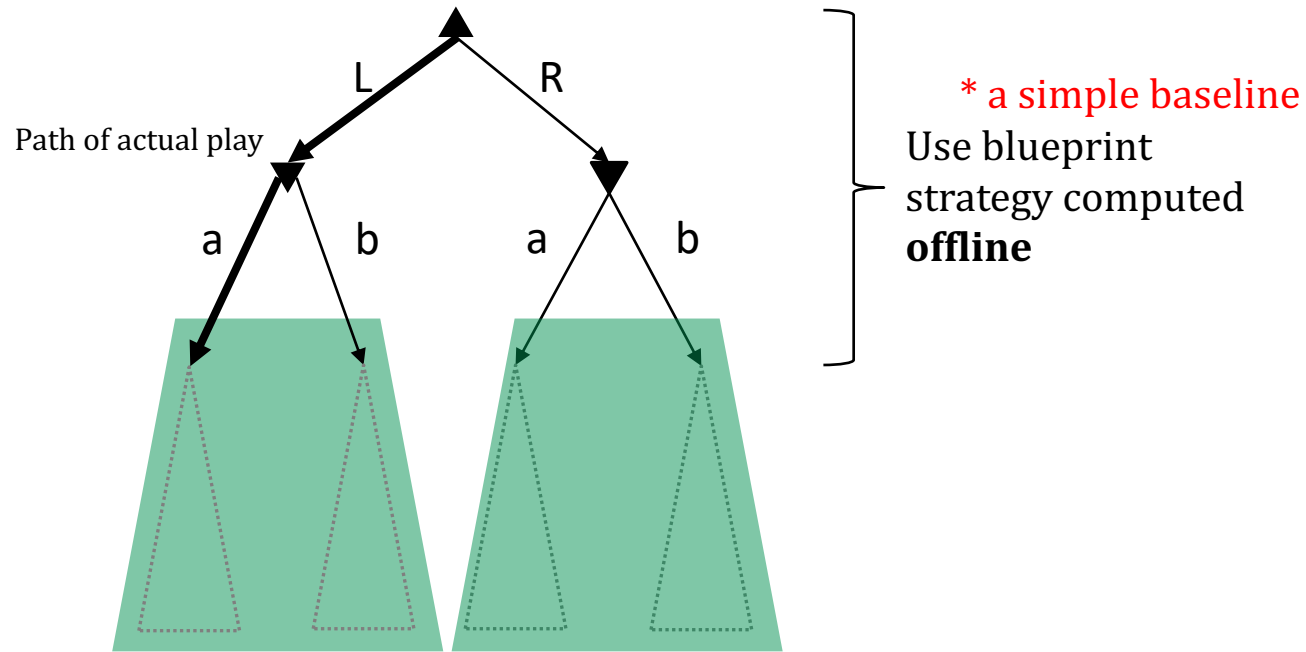
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## Extensions to zero-sum imperfect information games

- Crucial component of top poker bots (Deepstack, Libratus)
- Us: Can be extended to various general-sum games safely!

# Subgame Resolving



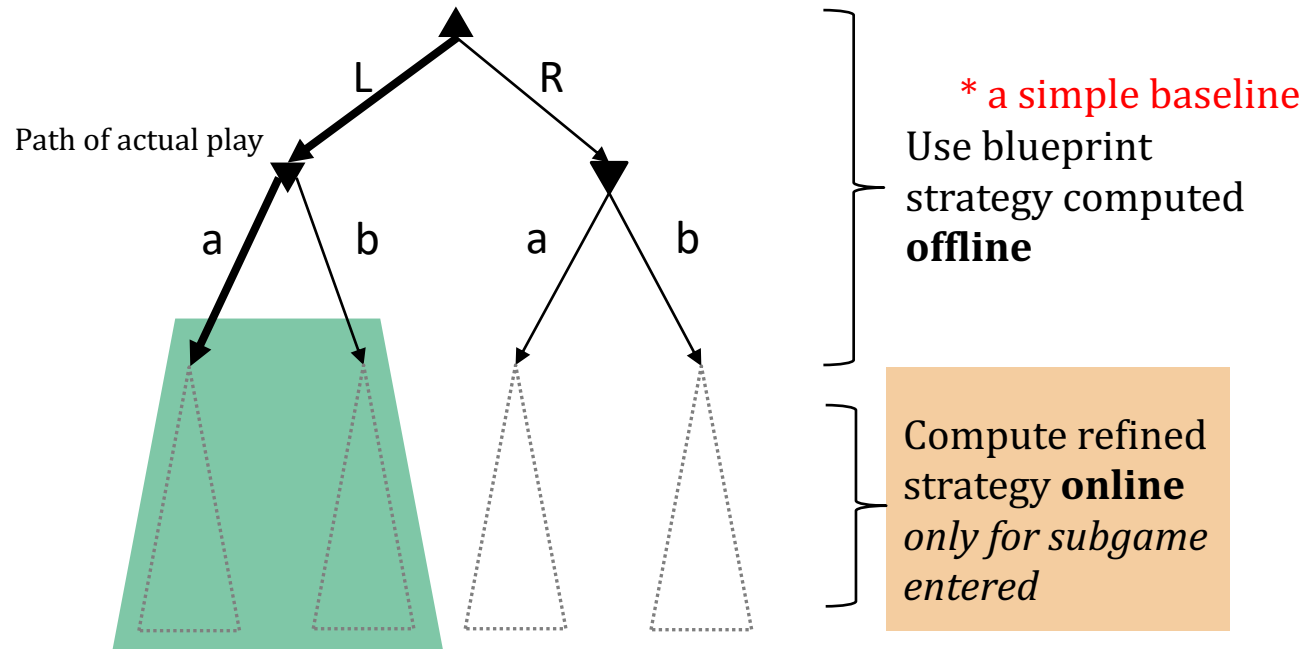
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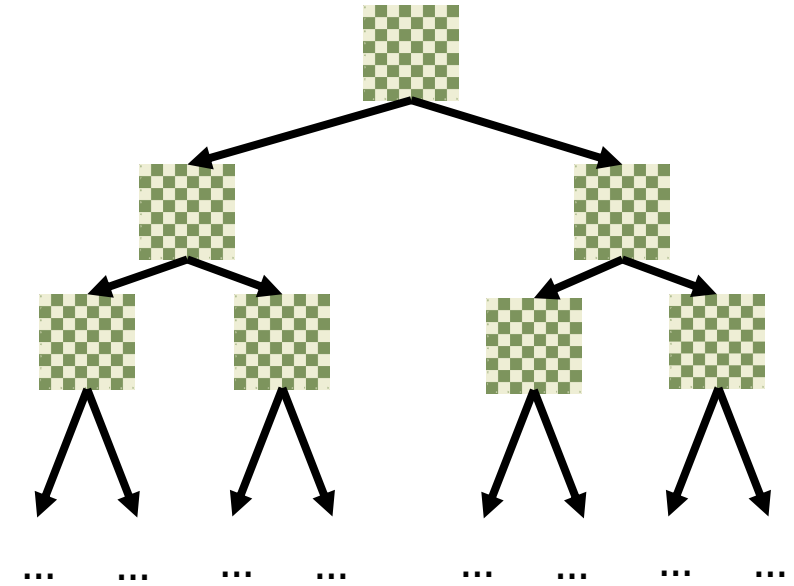
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Resolving will do at least as well as the blueprint!

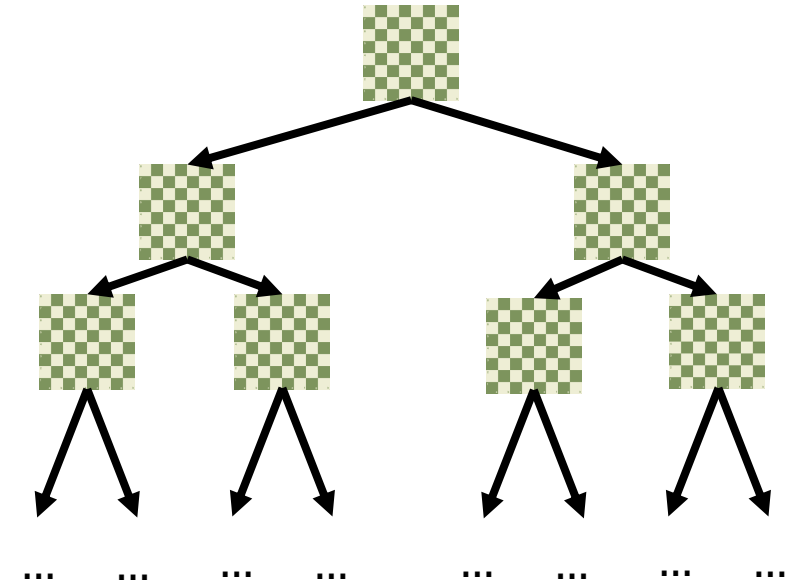
# Game Solving with Function Approximation



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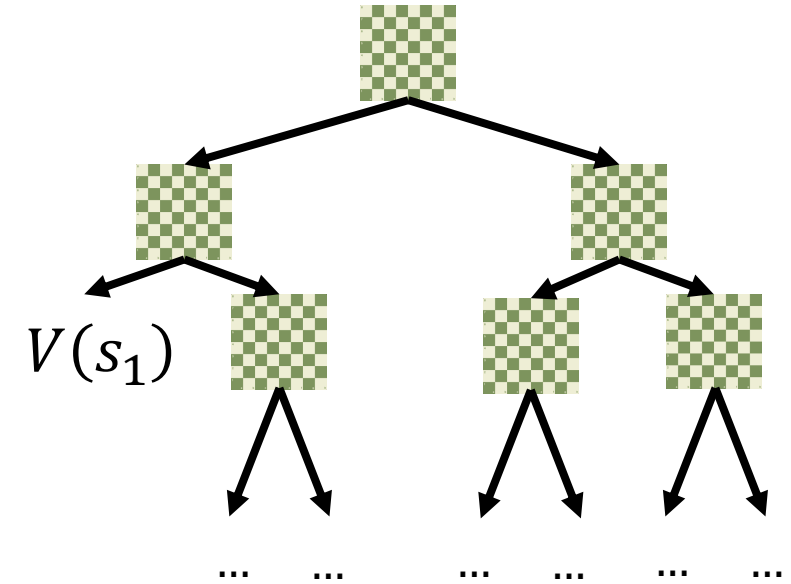
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- **Value Function**  $V(s)$  approximates how “good or bad” each state is



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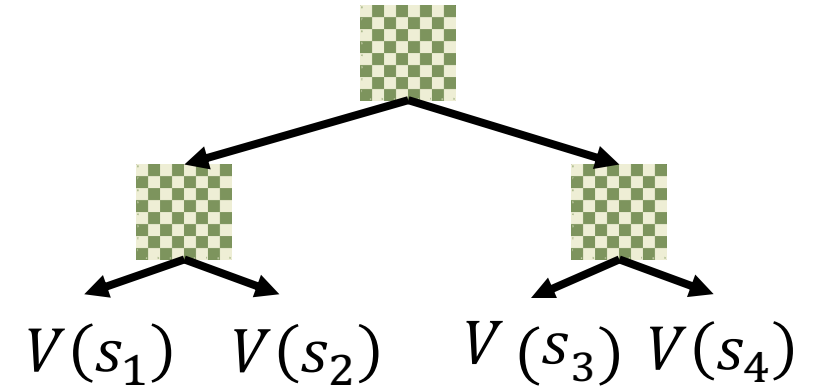
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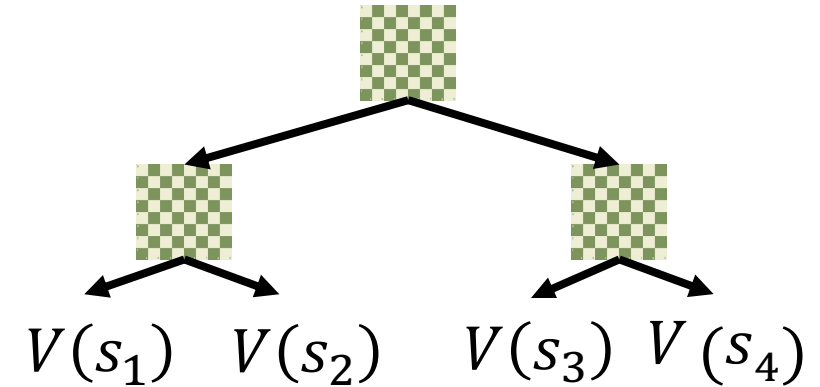
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Consider zero-sum games

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Previously (e.g., Deep Blue)

- Handcrafted evaluation function  $V(s)$  based on heuristics from experts



# Game Solving with Function Approximation

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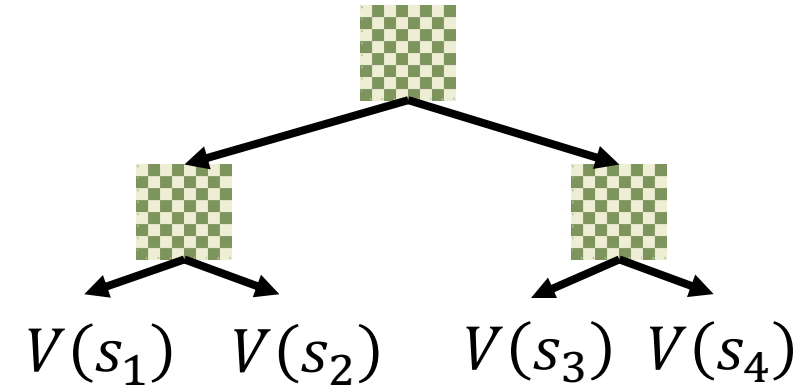
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Today (e.g., AlphaGo/Zero, DeepStack)

- **Learn** how good each state  $s$  is, generalize to states not seen before
- $V_\phi(s)$  is a network parameterized by  $\phi$  approximating the value of  $s$



# Game Solving with Function Approximation

Consider zero-sum games

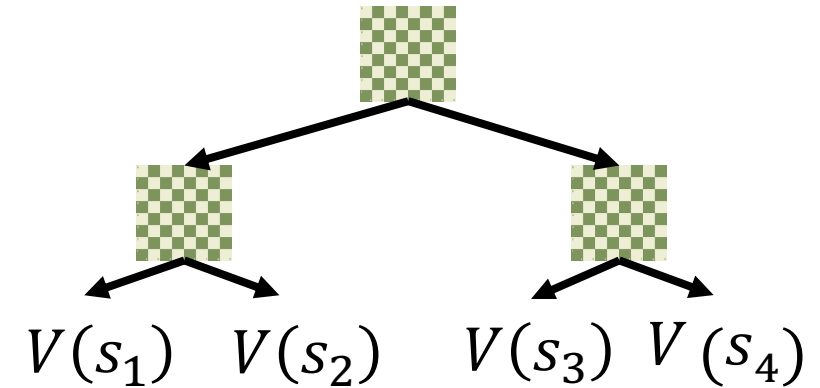
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Also applies to general sum (Stackelberg) games in a principled manner!

Need to predicting **achievable sets** of payoffs, “value” of vertex is not just a scalar or fixed size vector

# Multidefender Security Games



# Multidefender Security Games



attacker

v.s.



defenders



# Multidefender Security Games



v.s.



attacker

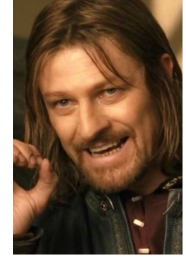
defenders

Defenders *each* choose how to distribute their armies. Orcs choose a target to attack.

# Multidefender Security Games



v.s.



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Not always but yes, under some assumptions.



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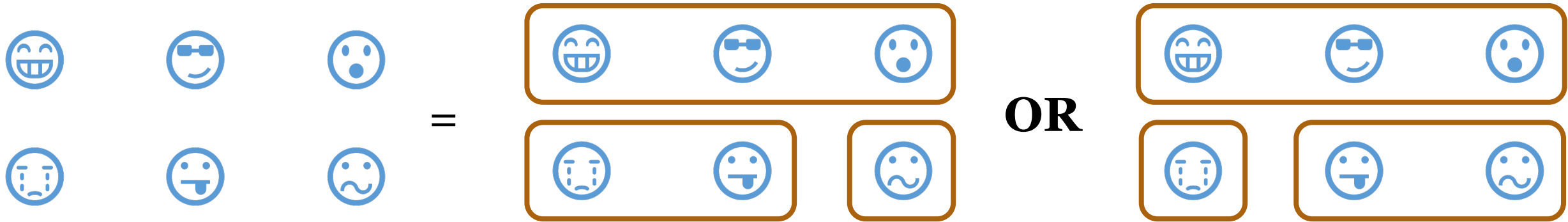
~~Democracy is two wolves and a sheep voting on whats for dinner.~~

Democracy is two or more sheet deciding who to throw to the wolves.

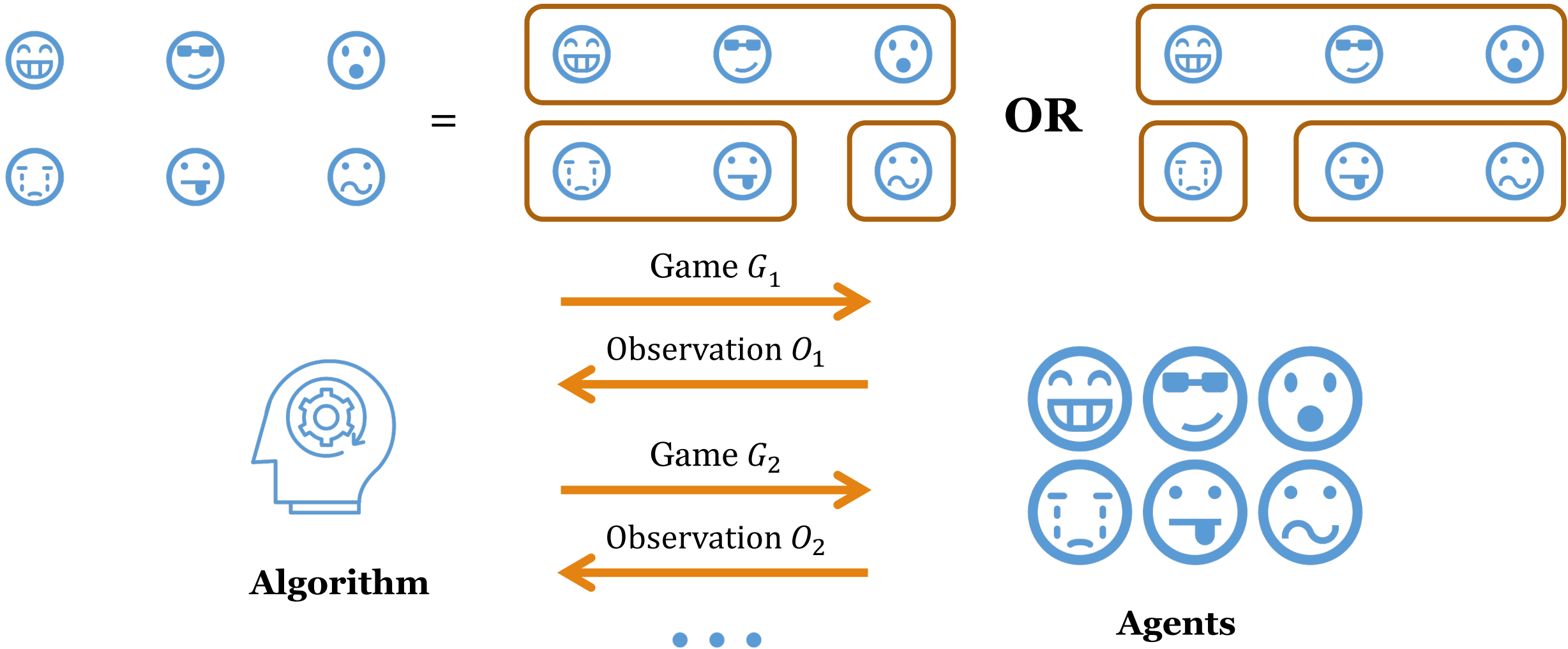
# Coalition Structure Learning



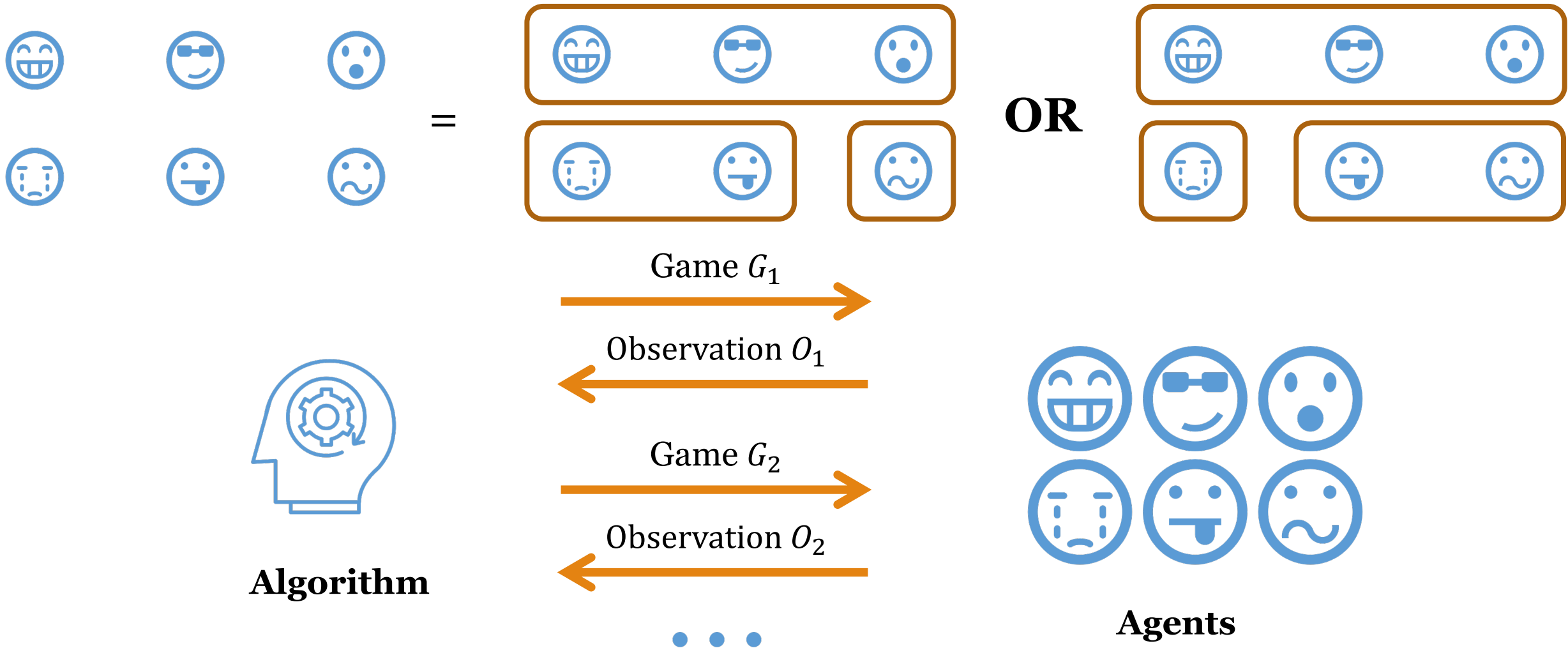
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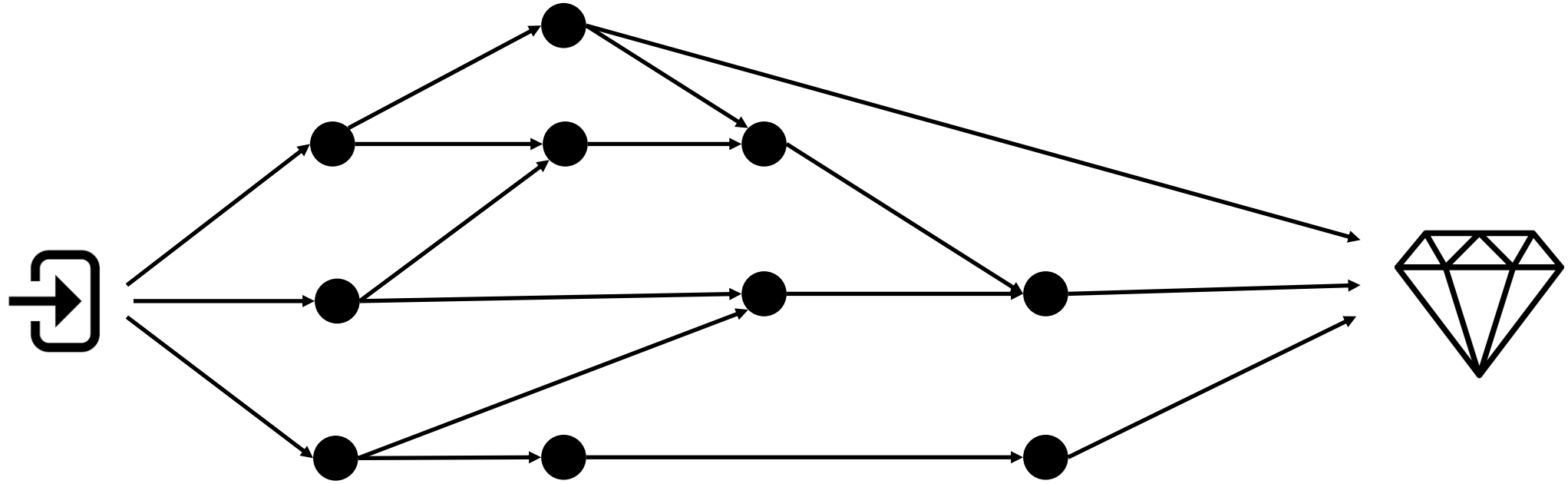
# Coalition Structure Learning



Discovers true optimal in smallest number of tests!

# Example: Network Protection Game

\*Abstracted and super simplified



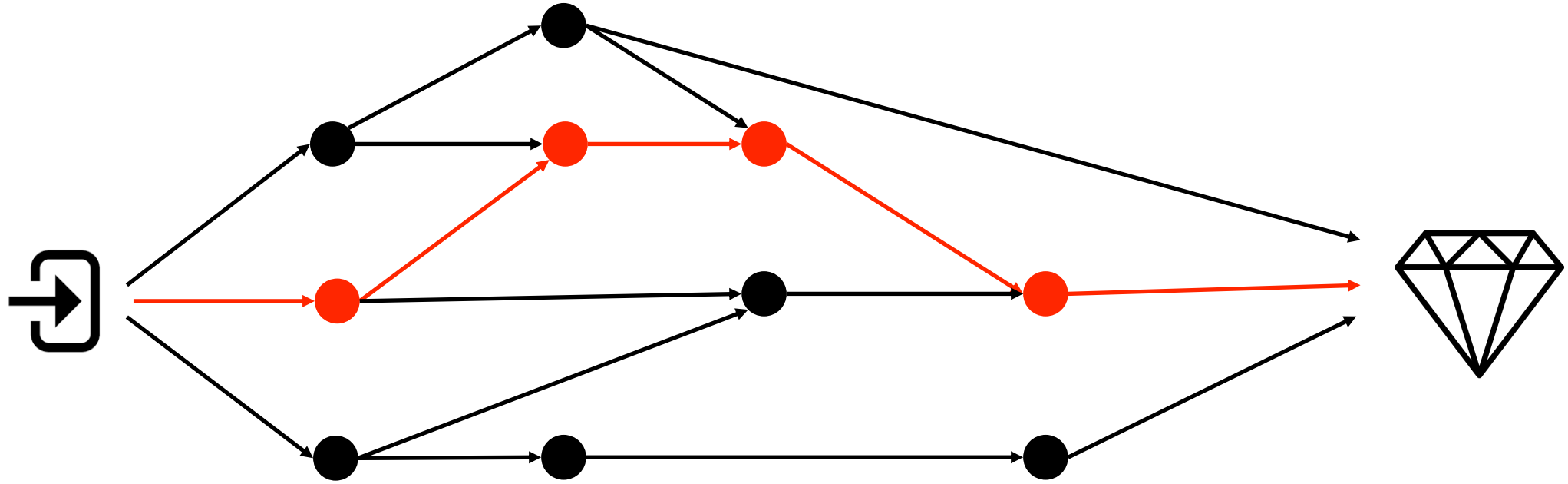
● Nodes: Machines, servers, routers

➔ Edges: Potential vulnerabilities



# Example: Network Protection Game

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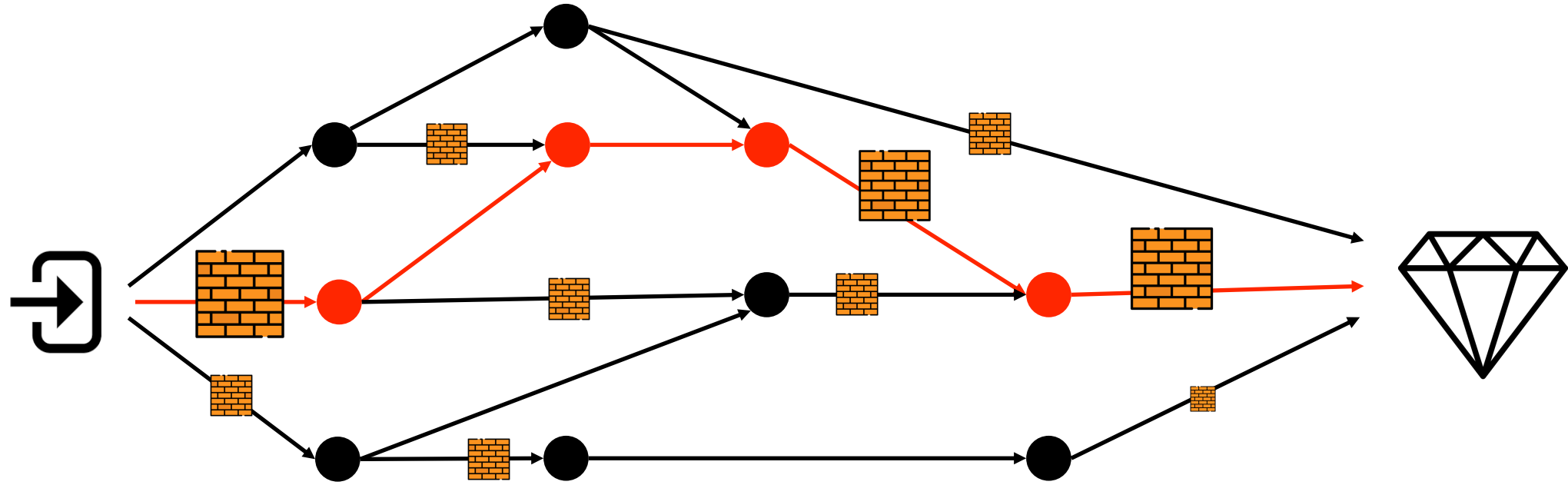


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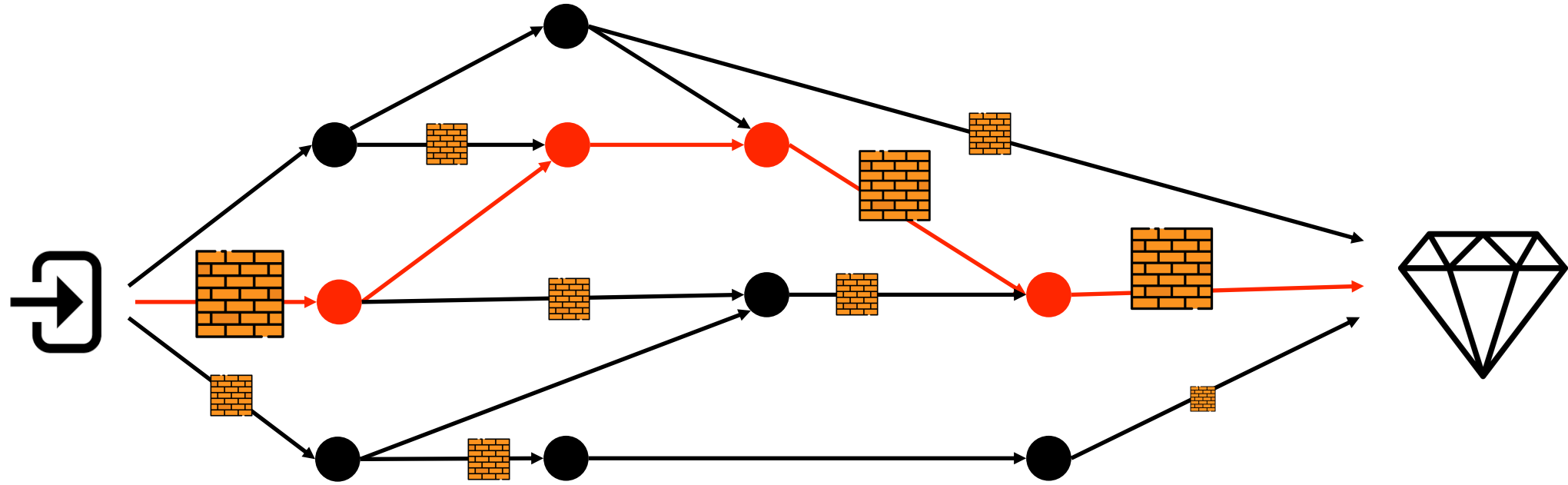
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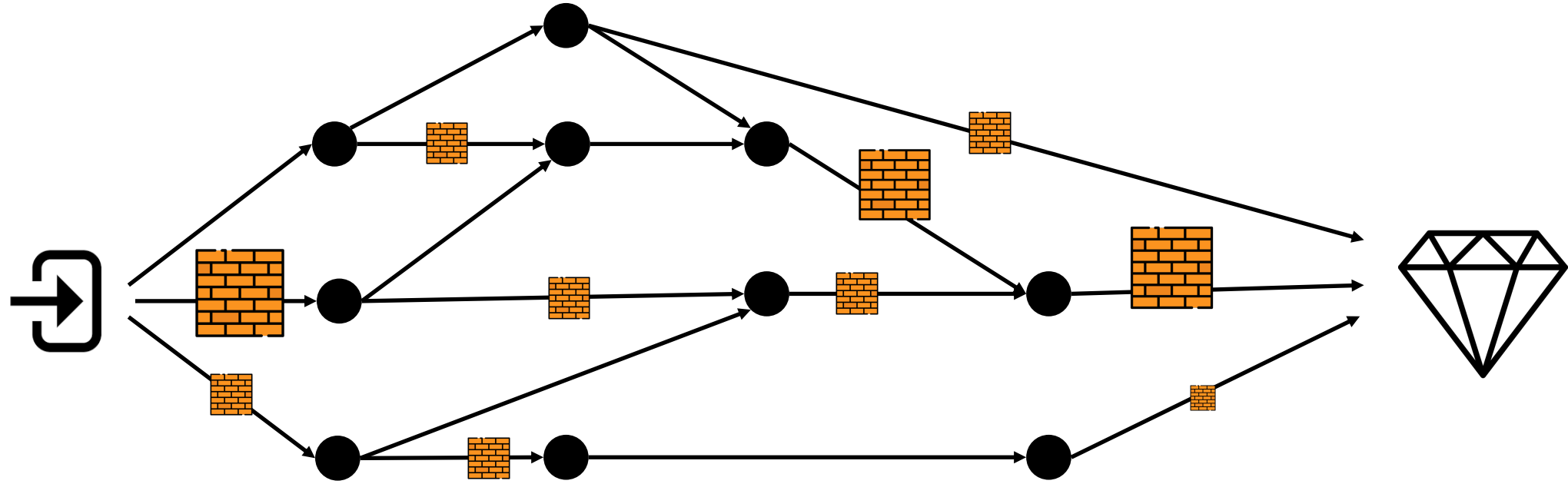


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Reduces to a longest shortest path problem  
(super easy for simple constraints, easily  
scale to around tens of millions of nodes)

# Pivoting: Intruder is not stupid!

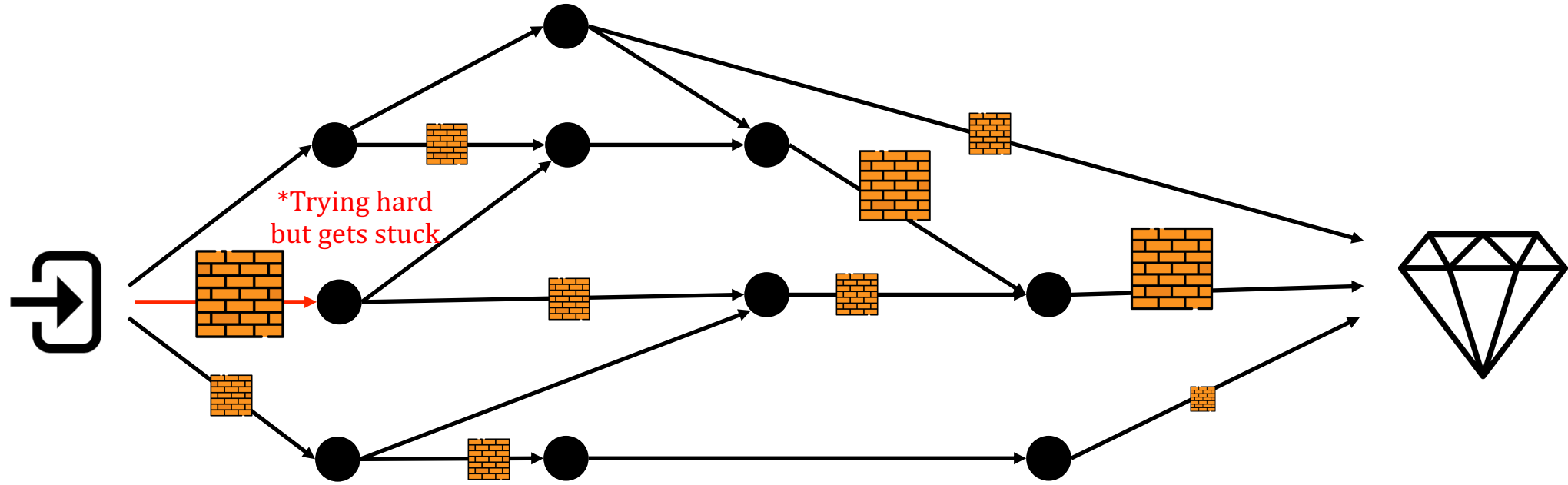
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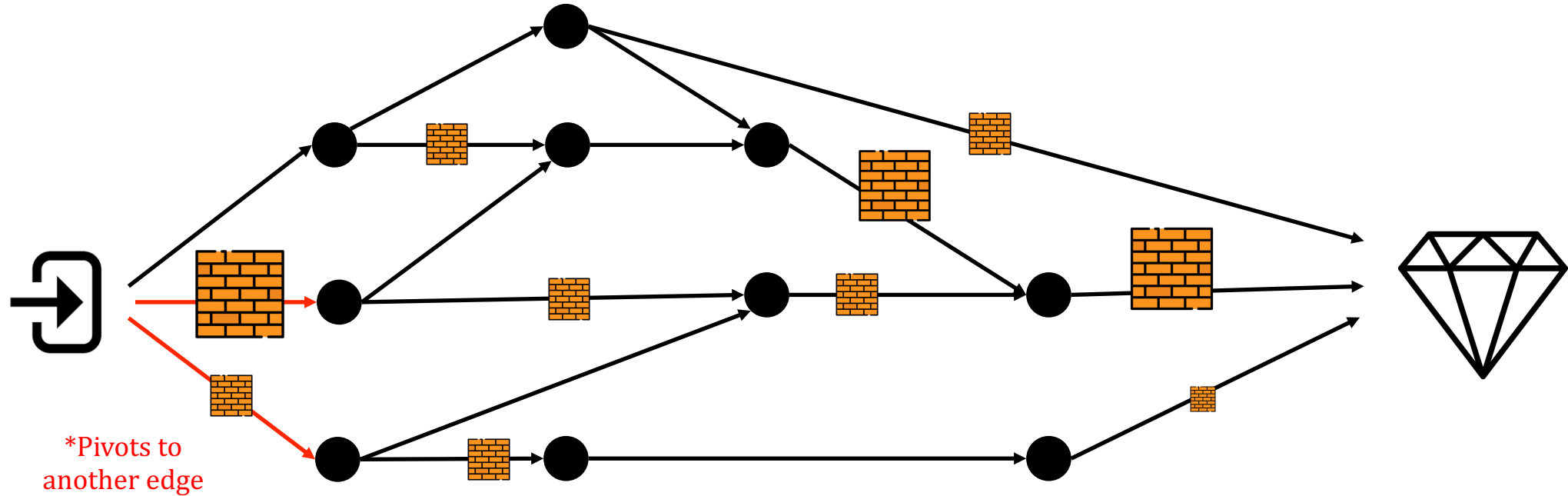
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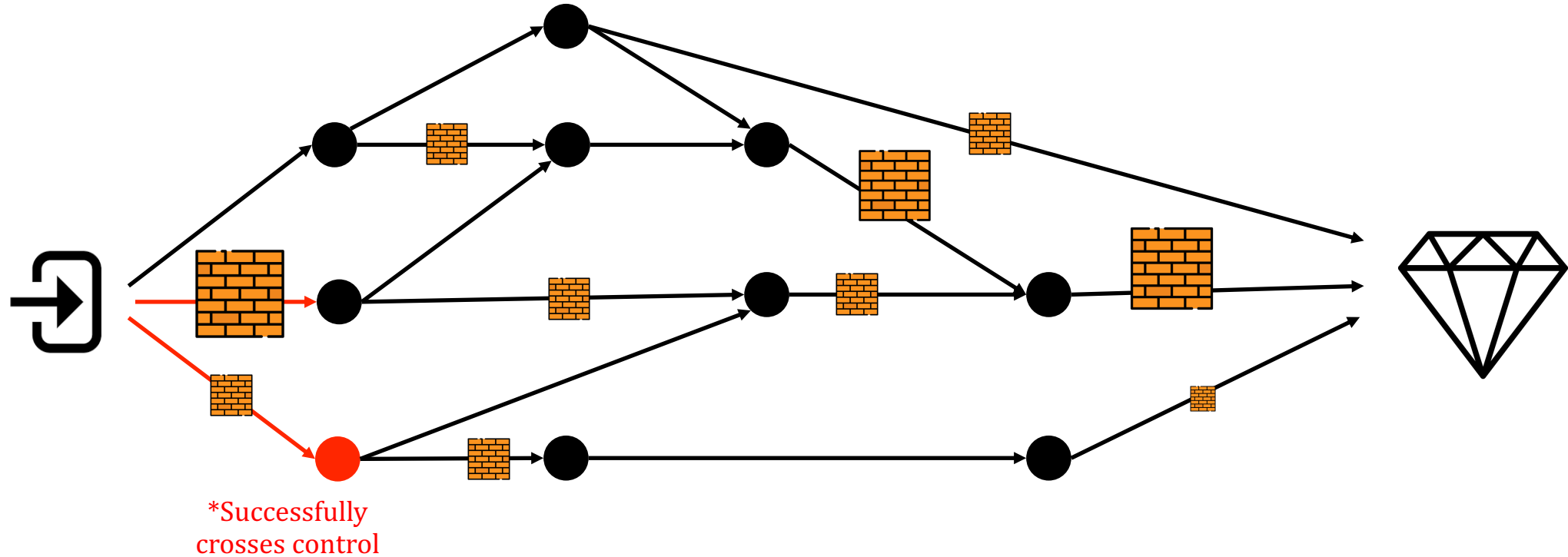
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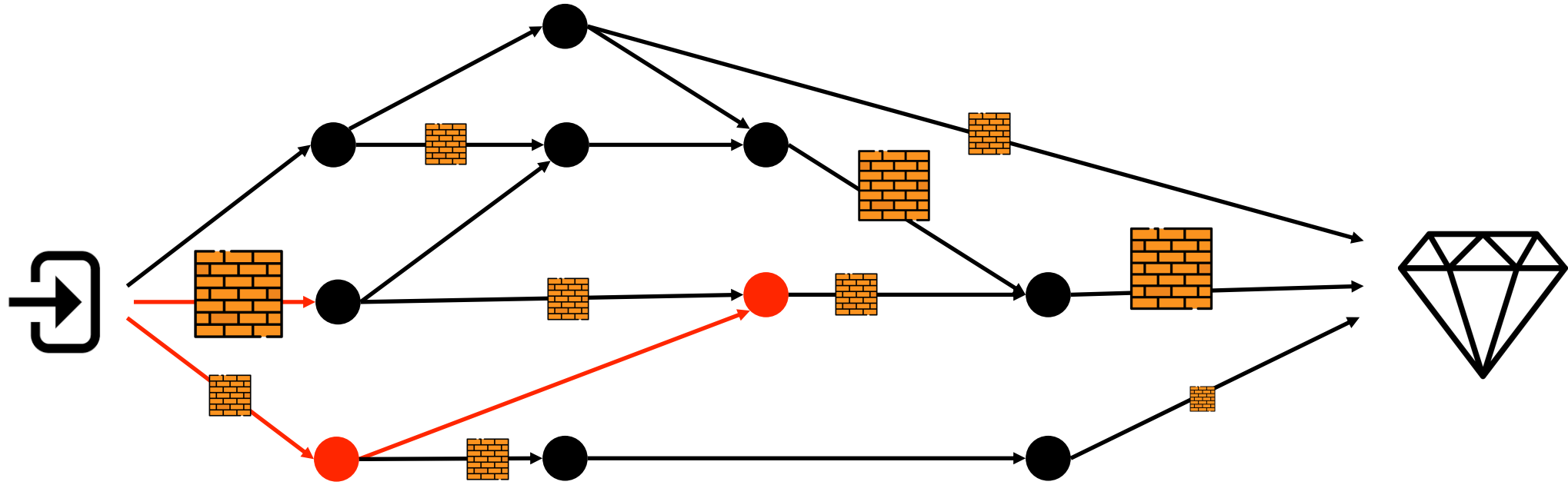
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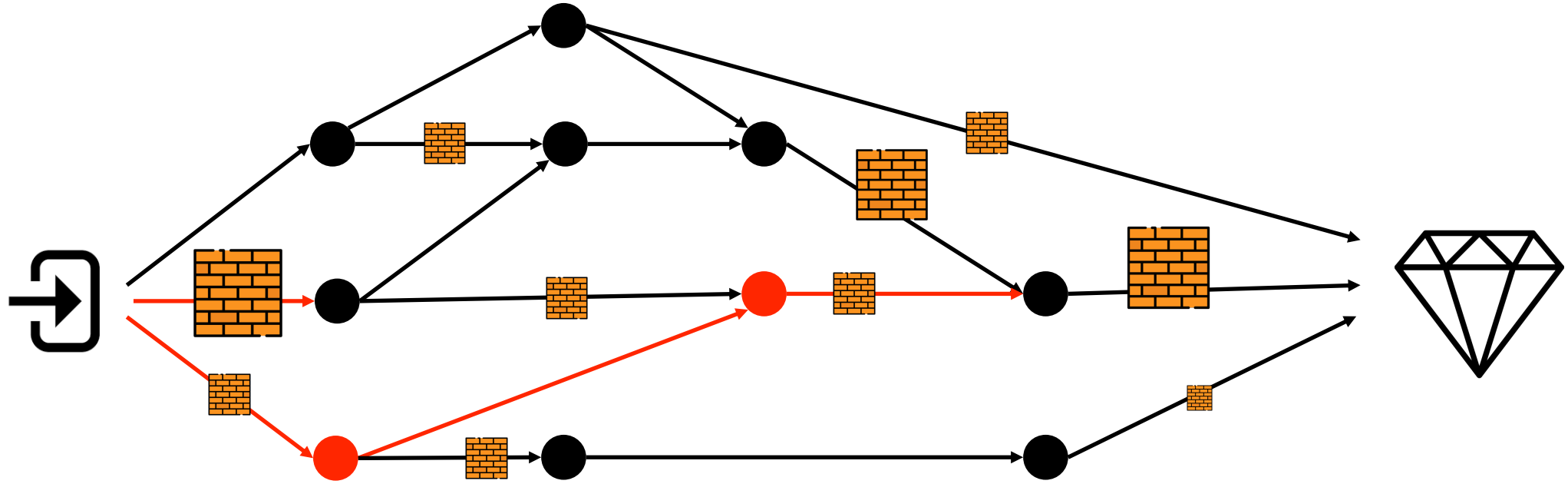


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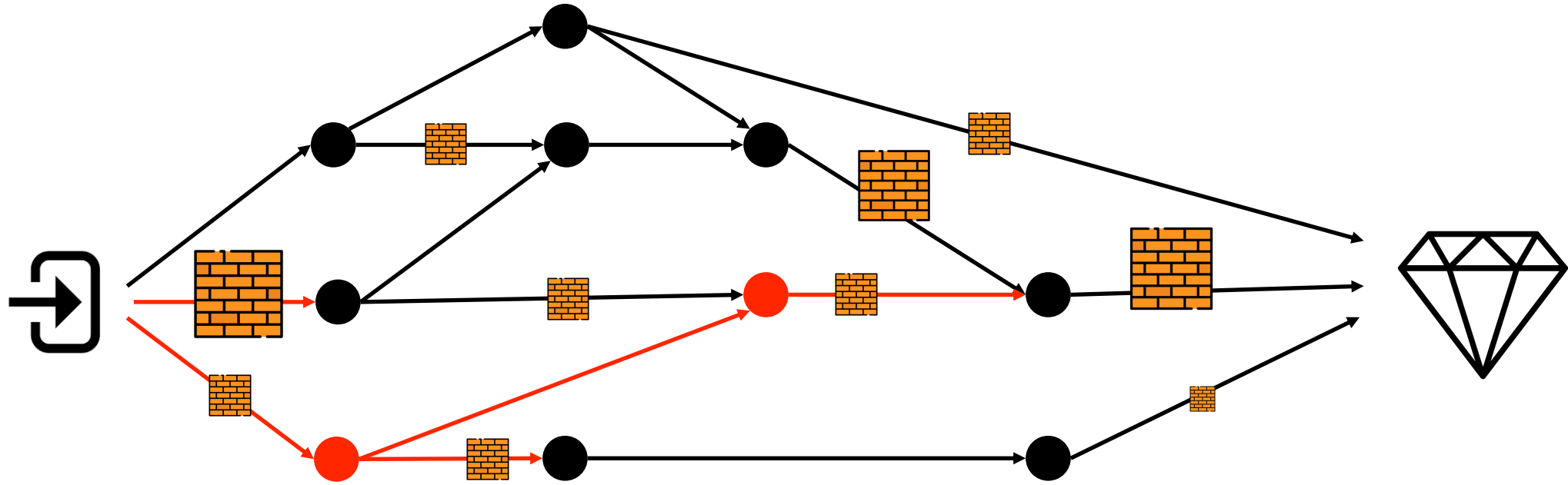
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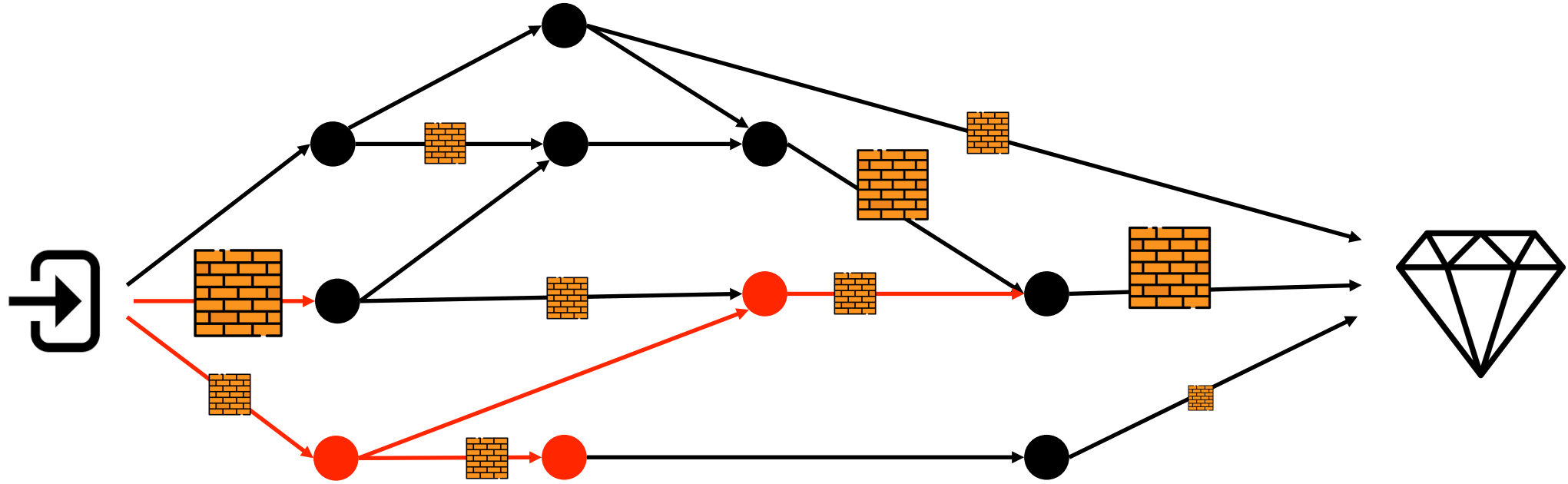
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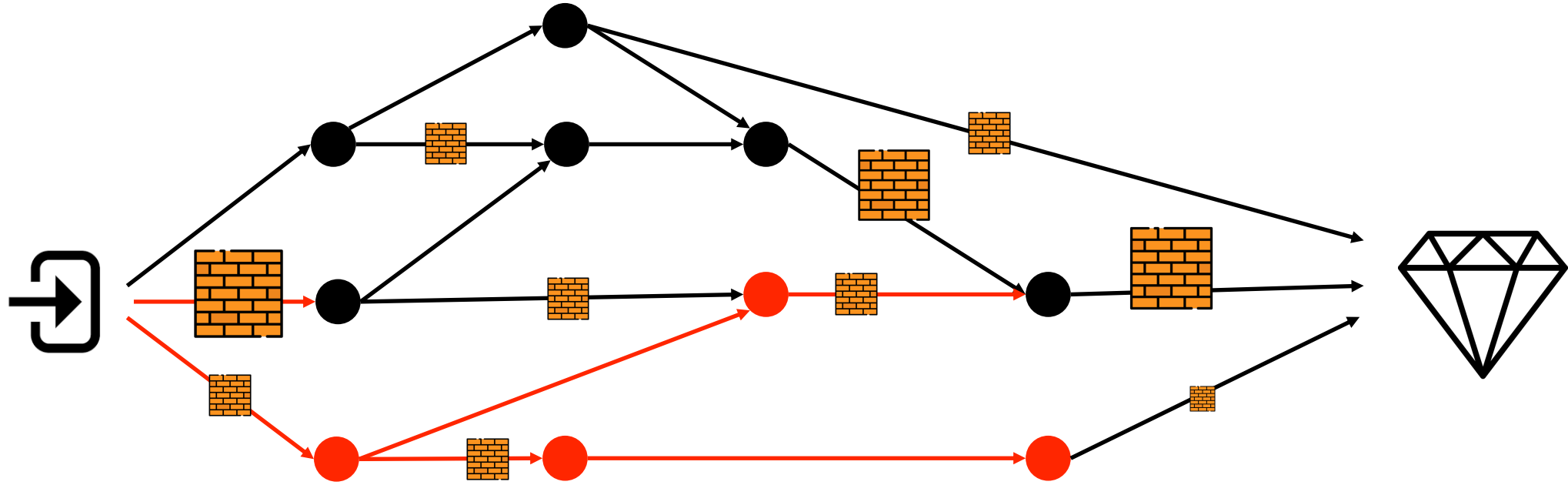
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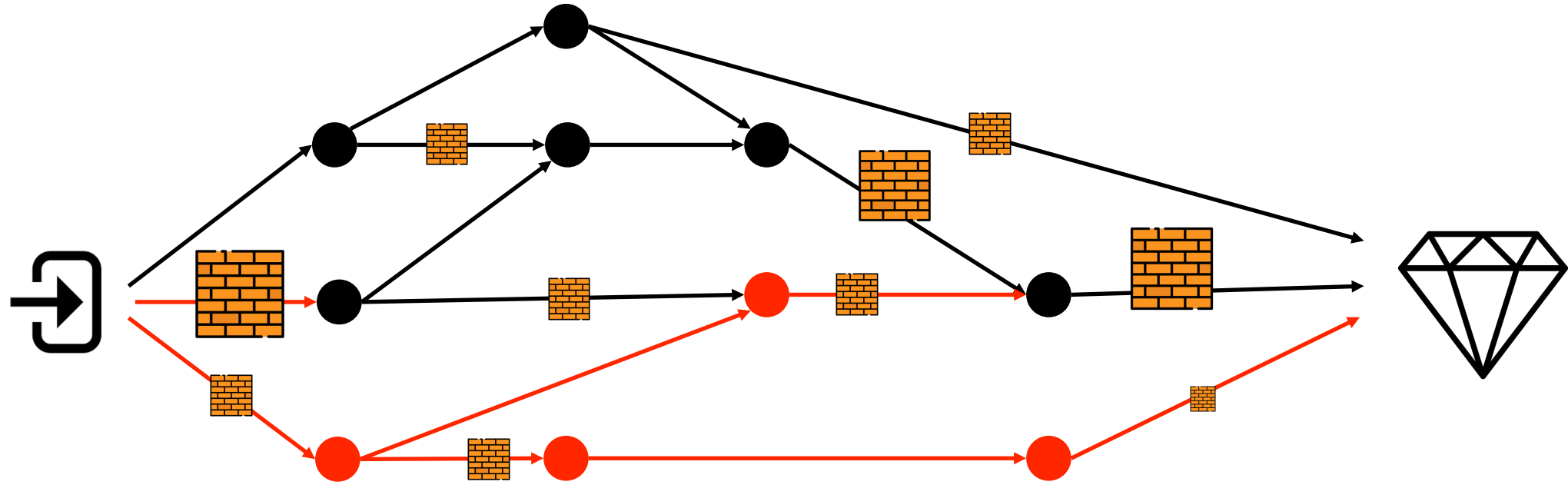
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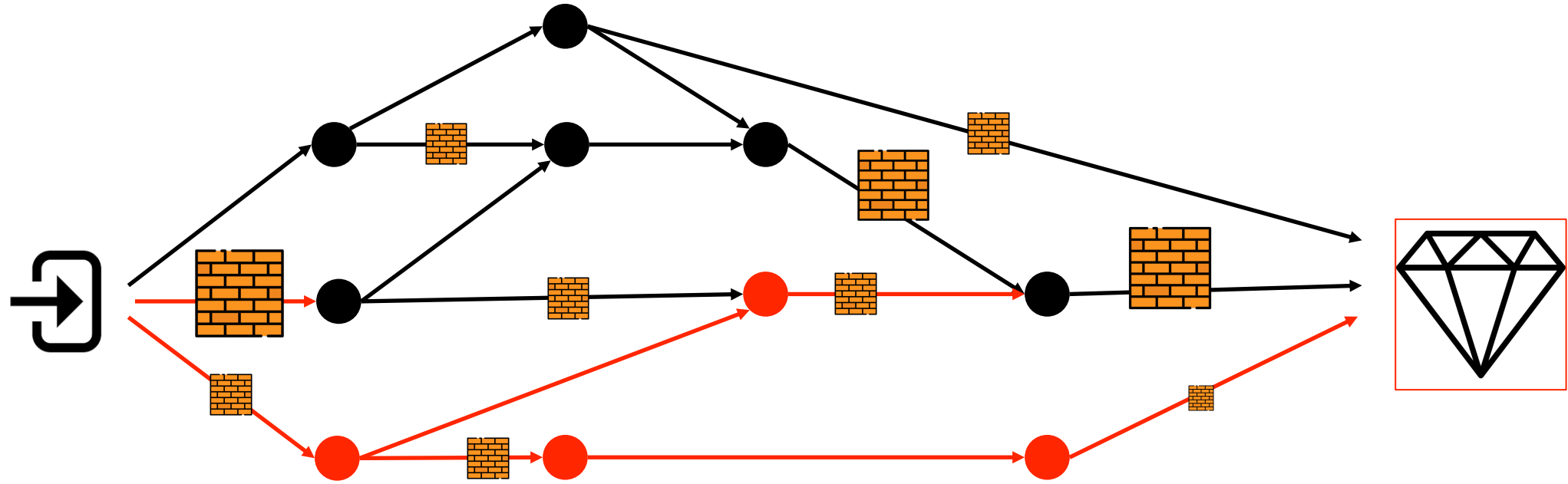
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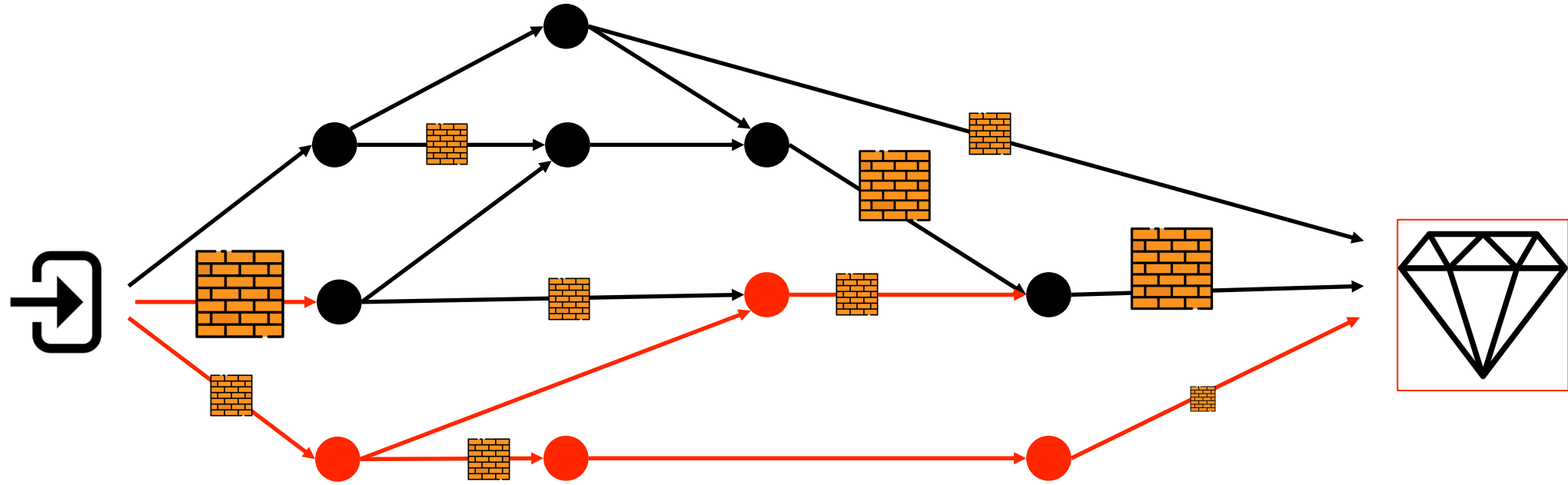
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Given intruder's action **changes with time as they explore the network**, how should defenses be placed?

\*Interesting connection with the Gittins index, ongoing work

# Ongoing research

Dealing with stochastic actions sets (looking for students!)

- Not all actions available, can we still solve this **efficiently**?

Security games with discovery (looking for students)

- Attacker doesn't actually know the full network, need to account for that or else defender is at too big a disadvantage

Structured reward functions that are defined formally by commonly used semantics (e.g., PCTL, LTL)

- Can we still get fast algorithms?

Structured eqm are very interesting, but assumptions not necessarily true

- E.g., Gittins policy can be quite brittle, make several assumptions, but necessary for tractability
- How can we handle slight deviations from these assumptions?
- Do we swing all the way to deep learning? Or is there a better balance between the 2?



# Direction 2: Sensemaking in Gams

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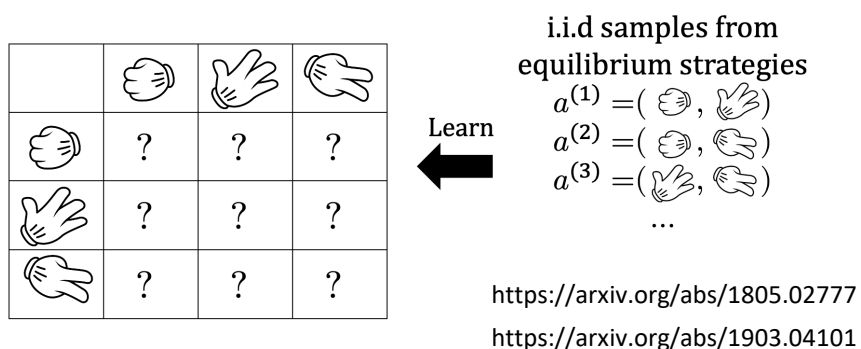
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  - Some quantitative guarantees
    - Saying XYZ “converged” isn't enough
- Exact solutions: very rarely (outside of recreational games)
  - Does the 6<sup>th</sup> significant digit matter if the model itself is inaccurate?

*\*Yes, I know the audience here...*

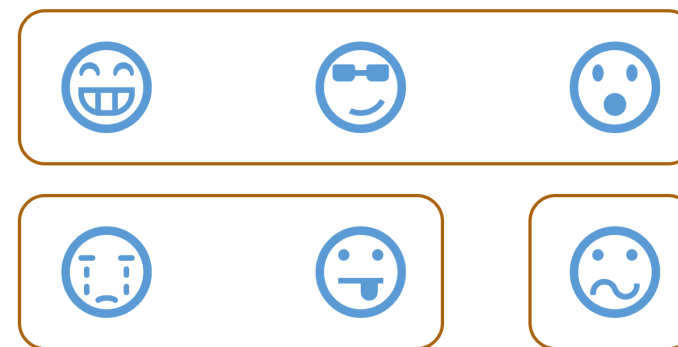


# Inverse Game Theory/Computational Rationalization

- Explain **why** players are behaving the way they do



IGT via differentiable optimization

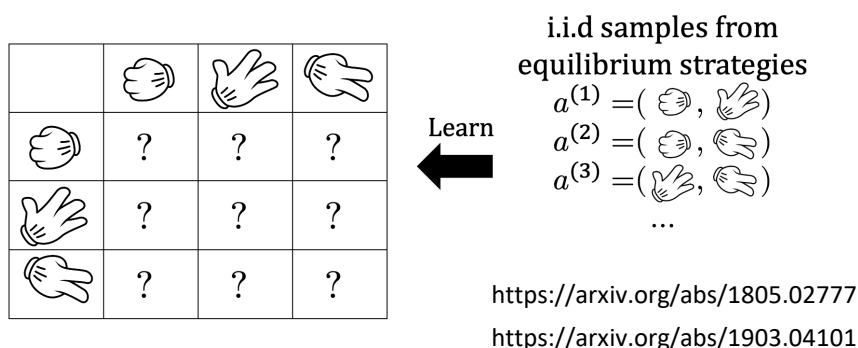


<https://arxiv.org/abs/2312.09058>

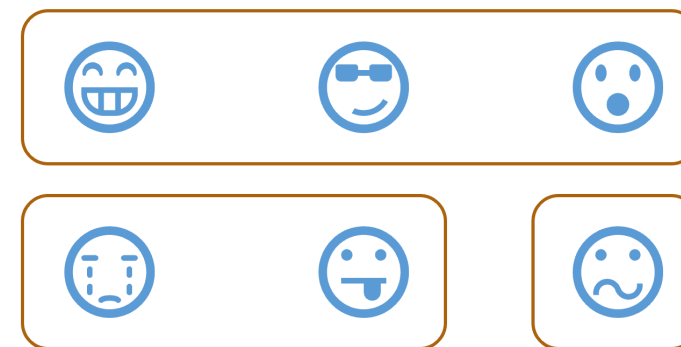
Coalition Structure Learning

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Coalition Structure Learning

## Learning strategies that **are** interpretable/operationalizable

- Restriction to strategies that are compact e.g., product distributions, sparse
- Regularization to “human-like” strategies

Explaining equilibrium selection *\*ongoing work*

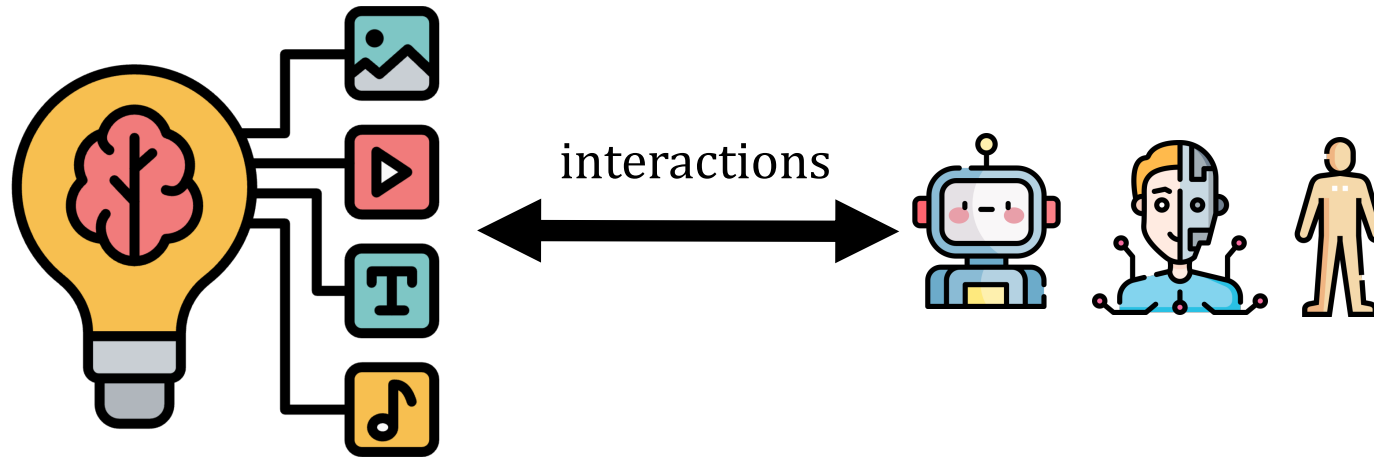


# Direction 3: Challenging Assumptions

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Tackle new *problems* by allowing interactions in human modalities

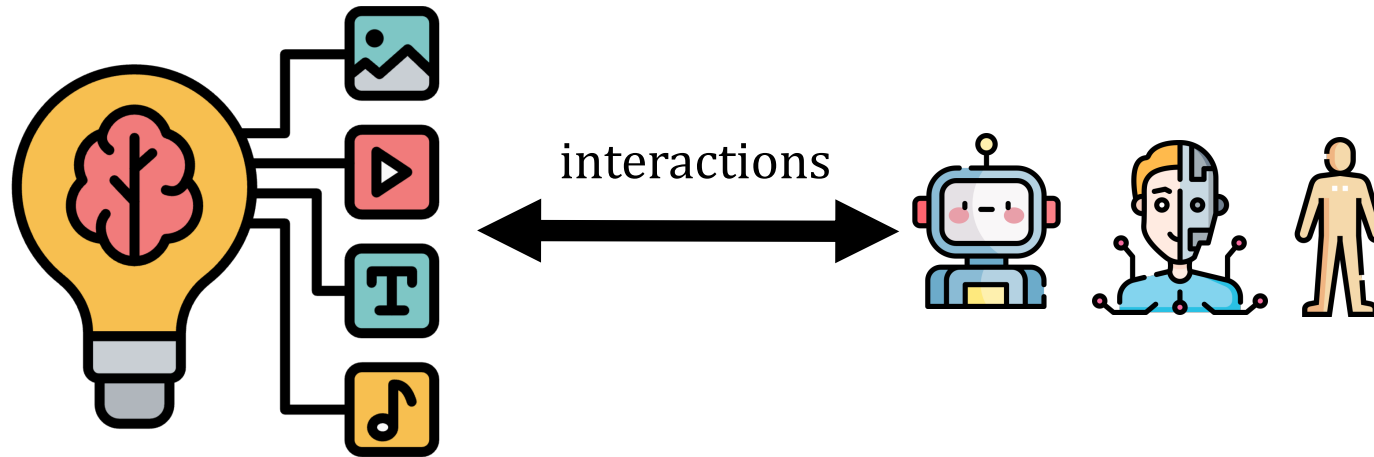
- Example: language models used in bargaining, negotiations
- How to incorporate strategy into such high-modality, fluid action spaces?
  - Strategically eliciting information from other language agents *\*ongoing work*



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What about games without common prior?

Are equilibrium even the right concept?

# Ongoing Projects

Learning **robust** strategies in a interactive setting

- Game of twenty questions

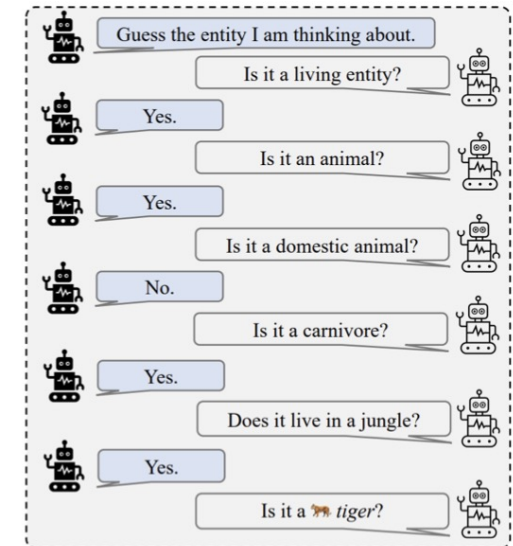
Explaining **strategies** to humans in a **meaningful** way

- Useful for persuasion, or just learning in general
- Much more tricky, need to care about **counterfactuals**

In general, for many of these problems...

- Behavior off the equilibrium path matters **a lot**
- Big difference between single agent and multiagent

Game of 20 Questions as an Extensive Form Game (EFG)



# Any Questions?

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lingchunkai.github.io

Contact me if you'd like to collaborate!