# Instructions

This quiz comprises 5 short problems. Attempt all questions and submit your solutions on Canvas on the quizzes section. Please use Canvas, do not turn in hard (nor soft) copies.

This homework is to be completed **individually** and is due in $\sim$ 2.5 weeks. You are encouraged to complete the assignment without AI usage, though I cannot and will not police or enforce this.

**Important erratas are given in red.**

- Q1: fixed typos. Options B and D are changed from $I'$ to $\tilde{I}$.
- Q5: clarify that $A$ is the sequence form payoff matrix.

# 1 CFR

You are Player 1 in an two-player zero-sum extensive form game with perfect recall and your treeplex contains exactly two infosets $I$ and $\tilde{I}$, where $\tilde{I}$ is a descendent of $I$ after taking action 1. $I$ contains $n > 1$ actions and $\tilde{I}$ contains $m > 1$ actions.

Suppose you have run self-play via the CFR algorithm for $T$ timesteps using the regret matching algorithm. The iterates obtained by the *local regret minimizers* are $\beta^{(t)}$ and $\tilde{\beta}^{(t)}$ for $t = \{1, \ldots, T\}$. Note that $\beta^{(t)} \in \Delta_n$ and $\tilde{\beta}^{(t)} \in \Delta_m$. The algorithm is deemed to have converged (the saddle-point gap is close to 0).

Which of the following are true of the (approximate) equilibrium of Player 1 in terms of $\beta$ and $\tilde{\beta}$?

(A) The approximate behavioral strategy at equilibrium for infoset $I$ is $\frac{1}{T}\sum_{t=1}^{T} \beta^{(t)}$.

(B) The approximate behavioral strategy at equilibrium for infoset $\tilde{I}$ is $\frac{1}{T}\sum_{t=1}^{T} \tilde{\beta}^{(t)}$.

(C) The approximate behavioral strategy at equilibrium for infoset $I$ is $\beta^{(T)}$.

(D) The approximate behavioral strategy at equilibrium for infoset $\tilde{I}$ is $\tilde{\beta}^{(T)}$.

## 2 Compare payoffs between eqm

Let $G = (A, B)$ be a general-sum bimatrix game where $A, B \in \mathbb{R}^{n \times m}$. Consider the following quantities:

- $q_{\text{NE}}$ is the highest expected payoff that Player 1 obtains out of all the mixed strategy Nash equilibria in $G$.
- $q_{\text{CE}}$ is the highest expected payoff that Player 1 obtains out of all the Correlated equilibria in $G$.
- $q_{\text{CCE}}$ is the highest expected payoff that Player 1 obtains out of all the Coarse corelated equilibria in $G$.
- $q_{\text{SSE}}$ is the (highest) expected payoff that Player 1 obtains out of all the Strong Stackelberg equilibria in $G$. You can assume Player 1 plays the role of the Stackelberg leader.

Which of the following is **always** true?

(A) $q_{\text{NE}} \leq q_{\text{CCE}}$.

(B) $q_{\text{CE}} \leq q_{\text{CCE}}$.

(C) $q_{\text{CCE}} \leq q_{\text{SSE}}$.

(D) $q_{\text{SSE}} \leq q_{\text{CE}}$.

# 3 Poker game and variable payoffs

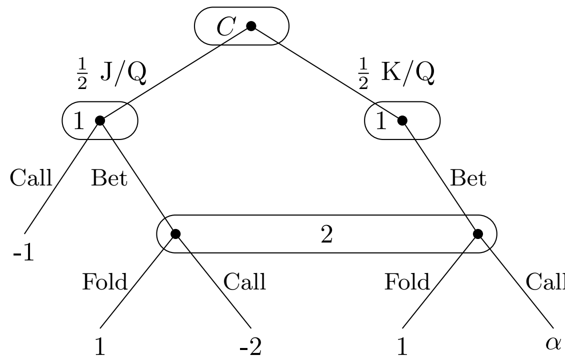


Figure 1: Modified poker game.

Consider the poker game in Figure 1 that we looked at many times in class with the slight modification that if the opponent calls when we (P1) holds a King, then instead of us getting 2, he pays us a value of $\alpha \in [1, \infty)$ instead. Note that the game continues to be zero-sum. The infoset belonging to P1 after being dealt a King is dummy infoset with only one action.

Which of the following is true?

(A) The set of NE for player 1 is independent of the value of $\alpha$.

(B) The set of NE for player 2 is independent of the value of $\alpha$.

(C) Regardless of how high $\alpha$ is, there exists a NE in which Player 2 always Calls with some strictly positive probability.

(D) The value of the game (in terms of player 1's utility) is monotonically non-decreasing as $\alpha$ increases.

# 4 Perfect recall, merge infosets

Let $G$ be a 2-player imperfect information game, possibly with imperfect recall. Let $G'$ be another imperfect information game (still possibly with imperfect recall) sharing the same game tree and rewards (in the leaves) as $G$, but with more fine-grained infosets for Player 1, i.e., the infosets in $G$ can be obtained by merging infosets of $G'$. Note that this merging can of course, only be done between infosets that have the same actions.

Which of the following statements is true?

NOTE: For the definition of Nash equilibrium, we are referring to the version where we convert the game to normal (matrix) form and solve it by standard methods.

(A) If $G$ (and $G'$) are zero-sum, then the value of $G$ and $G'$ is such that $\text{Value}(G) \leq \text{Value}(G')$, where $\text{Value}(\cdot)$ is Player 1's payoff in the Nash equilibrium. In other words, Player 1 performs no worse with more information.

(B) Same as (A), but with the added restriction that $G'$ is *perfect recall*. Note that (A) implies (B).

(C) If $G$ (and $G'$) are *cooperative* between both players, *and* $G'$ is perfect recall, then the optimal NE (one that maximizes either player's payoff) has a payoff no lower in $G'$ than $G$.

(D) If $G$ (and $G'$) are general-sum games with a unique payoff (to both players) for all their NE and both $G, G'$ have perfect recall, then the payoff to P1 under $G'$ is no lower than that of $G$, i.e., extra information will not hurt Player 1.

## 5 Treeplexes and the Sequence Form

Let $G$ is an 2-player zero-sum perfect recall imperfect information extensive form game with $|V|$ number of vertices. Let $\Sigma_1$ and $\Sigma_2$ be the treeplexes such that $|\Sigma_i|$ is the size of the treeplex for Player $i$ (i.e., the number of sequences for player $i$). Furthermore, $A$ is the **sequence form payoff matrix**. Which of the following are **always** true?

(A) $|\Sigma_1| + |\Sigma_2| = \mathcal{O}(|V|)$.

(B) $|V| = \mathcal{O}(|\Sigma_1| + |\Sigma_2|)$

(C) $\text{nnz}(A) = \mathcal{O}(|V|)$, where $\text{nnz}(A)$ is the number of non-zero entries in $A$.

(D) $\text{nnz}(A) = \Omega(|\mathcal{L}|)$, where $\text{nnz}(A)$ is the number of non-zero entries in $A$ and $|\mathcal{L}|$ is the number of leaves in the game tree. Assume that all leaves have nonzero payoffs.