

Variable Scope

Output?

```
x = 10  
print(x)
```

- it will print “10”, easy!

- How about this?

```
print(x)  
x = 10  
print(x)
```

- Why?

Traceback (most recent call last):

File "G:\My Drive\Courses\CS1010E\Lectures
)\W04 Variable Scope.py", line 3, in <module>
 print(x)

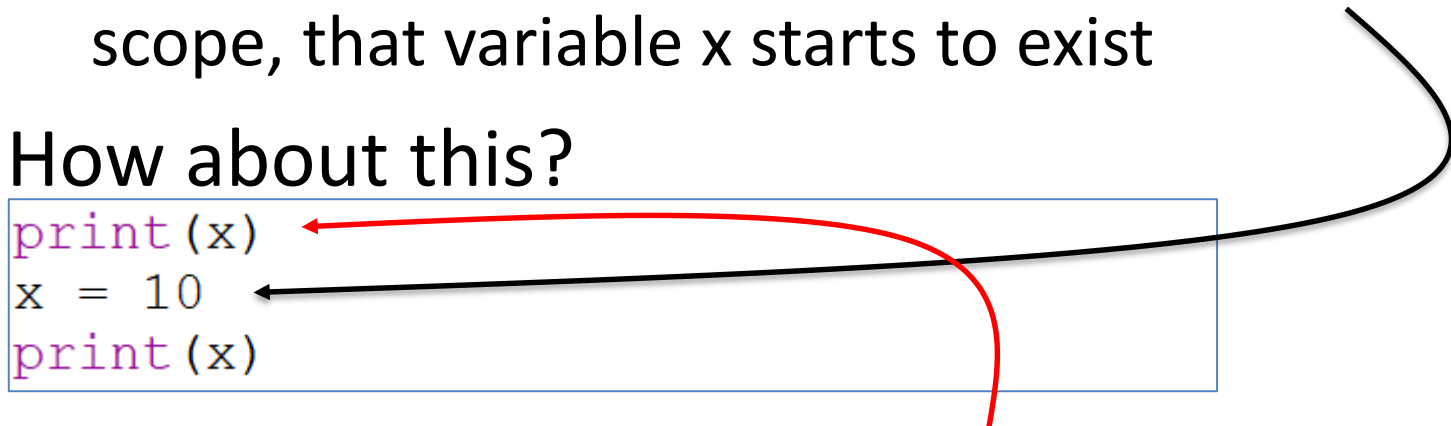
NameError: name 'x' is not defined

Output?

- When is x “defined”?
- “Golden rule”
 - Whenever we define a variable x by “x = ...” in a scope, that variable x starts to exist

- How about this?

```
print(x)  
x = 10  
print(x)
```



- Why?

```
Traceback (most recent call last):  
  File "G:\My Documents\Python\Scripts\1010E\Lectures\W04 Variable Scope.py", line 3, in <module>  
    print(x)  
NameError: name 'x' is not defined
```

Before the line “x = ...”, x did not exist

How about this?

```
x = 10

def foo():
    x = 20
    print("Inside foo(), x =", x)

foo()
print("At the end, x =", x)
```

- Which output is correct?

```
Inside foo(), x = 10
At the end, x = 10
```



```
Inside foo(), x = 20
At the end, x = 20
```



```
Inside foo(), x = 20
At the end, x = 10
```

Why?

- This created an `x`

```
x = 10

def foo():
    x = 20
    print("Inside foo(), x =", x)

foo()
print("At the end, x =", x)
```

- However,
 - When we define a function, the function will create a new “scope”
 - Meaning, if you start with “`x = ...`”, it will create a new `x`
 - However, if you created a new `x` in a function, it will disappear also in that function

```
Inside foo(), x = 20
At the end, x = 10
```

Further

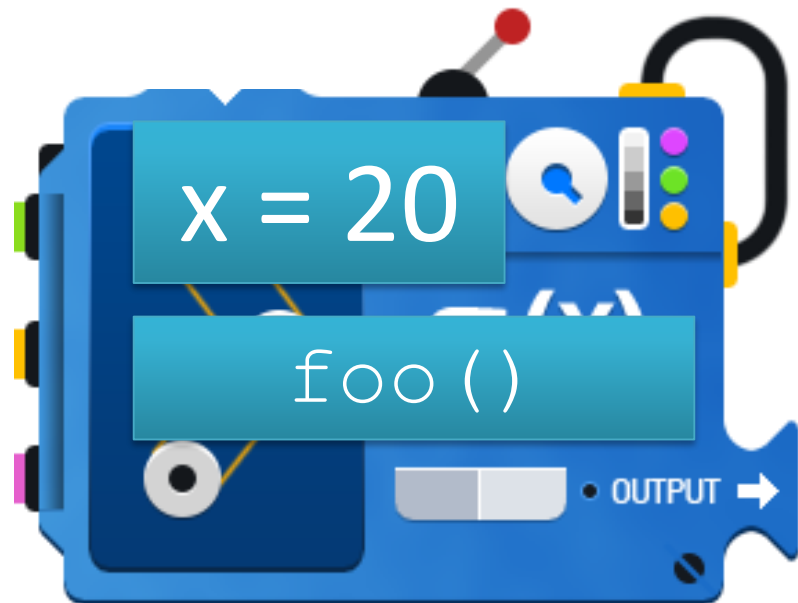
- Two copies of x
 - One outside
 - One inside the function

```
x = 10

def foo():
    x = 20
    print("Inside foo(), x =", x)

foo()
print("At the end, x =", x)
```

x = 10



Tracing the Code

- Create `x`, `x = 10`
- Define `foo()`
 - But didn't execute
- Call `foo()`
 - So jump into `foo()`
 - Create another `x` in function `foo()`
 - Print the inside `x`, in which is 20
 - When `foo()` finished, the `x` inside `foo()` disappears
- Finally, print the “outside” `x`, in which is 10

```
x = 10

def foo():
    x = 20
    print("Inside foo(), x =", x)

foo()
print("At the end, x =", x)
```

```
Inside foo(), x = 20
At the end, x = 10
```

Local Variables

- A variable created inside a function belongs to the *local scope* of that function, and can only be used inside that function.
- It will obscure another “outer” variable

```
x = 10
```

```
def foo():  
    x = 20  
    print("Inside foo(), x =", x)
```

```
foo()  
print("At the end, x =", x)
```


Global Variables

- Variables that are created outside of a function are called *global variables*.

```
x = 10
```

```
foo()  
print("At the end, x =", x)
```

Can we do this?

```
x = 10

def foo():
    print("Inside foo(), x =", x)

foo()
print("Global scope, x =", x)
```

- Error?
 - No error!
- Inside `foo()`, if there is no `x` created, it will try to look for any “existing” one from the global scope

Which one(s) will create errors?

- A

```
x = 10
def foo():
    print("Inside foo(), x =", x)
foo()
print("Global scope, x =", x)
```

- B

```
def foo():
    print("Inside foo(), x =", x)
x = 10
foo()
print("Global scope, x =", x)
```

- C

```
def foo():
    print("Inside foo(), x =", x)
foo()
x = 10
print("Global scope, x =", x)
```

Global vs Local Variables

- A variable which is defined in the main body of a file is called a **global** variable. It will be **visible throughout the file**, and also inside any file which imports that file. EXCEPT...
- A variable which is defined inside a function is **local** to that function. It is accessible **from the point at which it is defined until the end of the function**, and exists for as long as the function is executing.
- The parameter names in the function definition behave like local variables, but they contain the values that we pass into the function when we call it.

Crossing Boundary

- What if we really want to modify a global variable from inside a function?
- Use the “global” keyword
- (No local variable x is created)

```
x = 0
```

```
def foo_printx():  
    global x  
    x = 999  
    print(x)
```

```
foo_printx()  
print(x)
```

Output:
999
999

How about... this?

```
x = 0
```

```
def foo_printx():
```

```
    print(x)
```

```
    x = 999
```

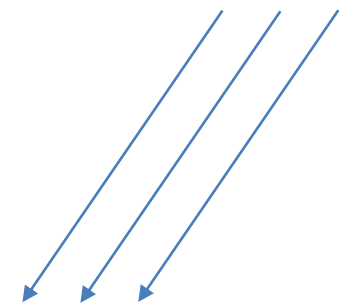
```
    print(x)
```

```
foo_printx()
```

- Local or global?
- Error!
- Because the line “x=999” creates a local version of ‘x’
- Then the first print(x) will reference a **local** x that is not assigned with a value
- The line that causes an error

Pass by Value

Function Arguments are Local Variables



```
def solve_qe(a,b,c):  
    delta = b**2 - 4*a*c  
    if delta >= 0:  
        ans1 = (-b + sqrt(delta))/(2*a)  
        ans2 = (-b - sqrt(delta))/(2*a)  
        print("The two solutions are " + str(ans1)  
              + " and " + str(ans2))  
    else:  
        print("The equation has no real root")
```


Can I Create an Increment Function?

- Will it print 2, 3, 4 or ...?

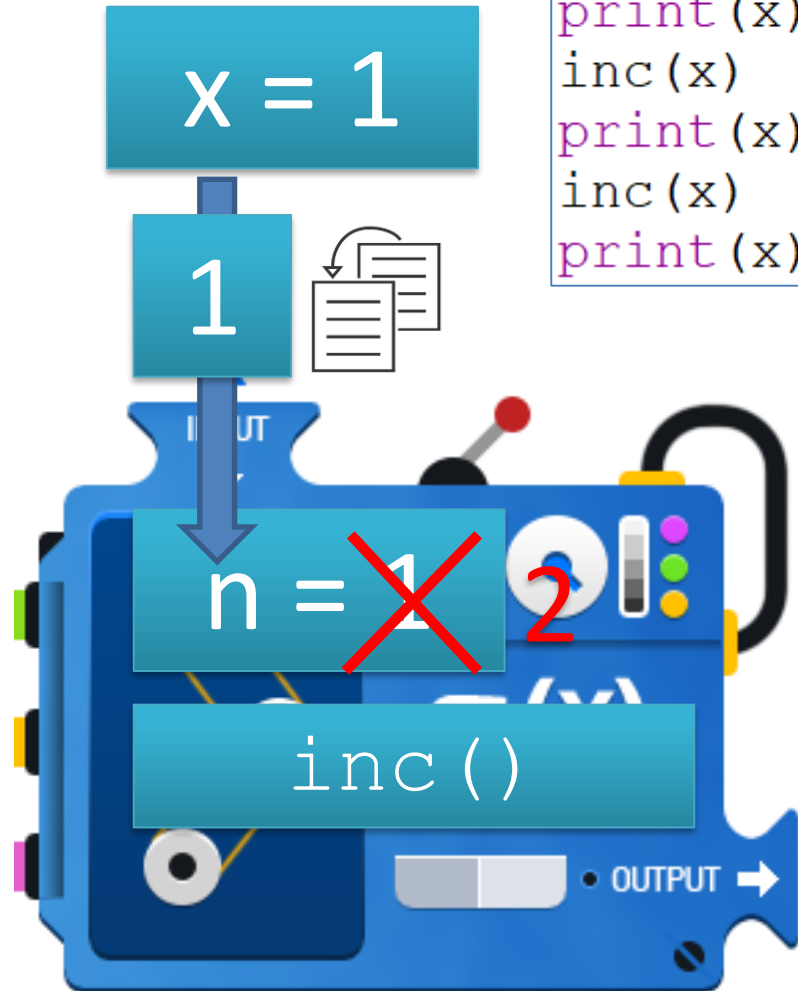
```
def inc(n):  
    n = n + 1  
  
x = 1  
inc(x)  
print(x)  
inc(x)  
print(x)  
inc(x)  
print(x)
```

Function

- “`inc()`” is a function
 - ~~`x = 1` (global)~~
 - ~~Input `x` into the function~~
 - **DUPLICATE** the value of `x` (global) into `n` (local)
 - And you changed that `n` (a local variable), it doesn't affect `x` (the global variable)

```
def inc(n):  
    n = n + 1
```

```
x = 1  
inc(x)  
print(x)  
inc(x)  
print(x)  
inc(x)  
print(x)
```



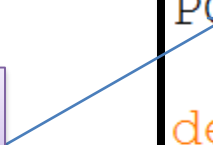
For Parameters that are Primitives

- Primitive data:
 - `int`, `float`, `bool`, etc.
- Parameters are passed by values
- But NOT for *some* parameters
 - E.g. sequences
 - Will discuss about this in later lectures

Practices (Convention)

- Global variables are VERY **bad** practices, especially if modification is allowed
- 99% of time, global variables are used as CONSTANTS
 - Variables that every function could access
 - But not expected to be modified

Convention:
Usually in all CAPs



```
POUNDS_IN_ONE_KG = 2.20462

def kg2pound(w) :
    return w * POUNDS_IN_ONE_KG

def pound2kg(w) :
    return w / POUNDS_IN_ONE_KG
```

Calling Other Functions

Year Book...



What scares you
the most?

"Werewolves!"

-Paul



What scares you
the most?

"Sharks"

-Nina



What scares you
the most?

"The unstoppable marching
of time that is slowly guiding
us all towards an inevitable
death."

-Dylan



What scares you
the most?

-Dylan

-Catherine

Local Variables

- Each function has its own scope

```
x = 10
```

```
def foo():  
    x = 20 #This x belongs to foo()  
    boo()  
    print("Inside foo(), x =", x)
```

```
def boo():  
    x = 30 #This x belongs to boo()  
    print("Inside boo(), x =", x)
```

```
foo()  
print("Global scope, x =", x)
```

- Output:

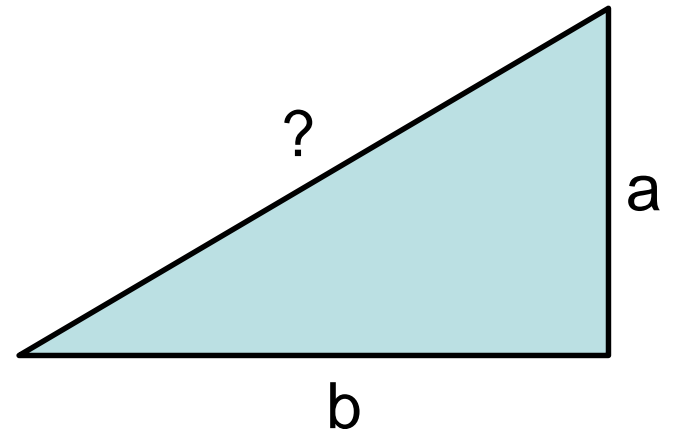
```
Inside boo(), x = 30  
Inside foo(), x = 20  
Global scope, x = 10
```

Compare:

```
def hypotenuse(a, b):  
    return sqrt(sum_of_squares(a, b))
```

```
def sum_of_squares(x, y):  
    return square(x) + square(y)
```

```
def square(x):  
    return x * x
```



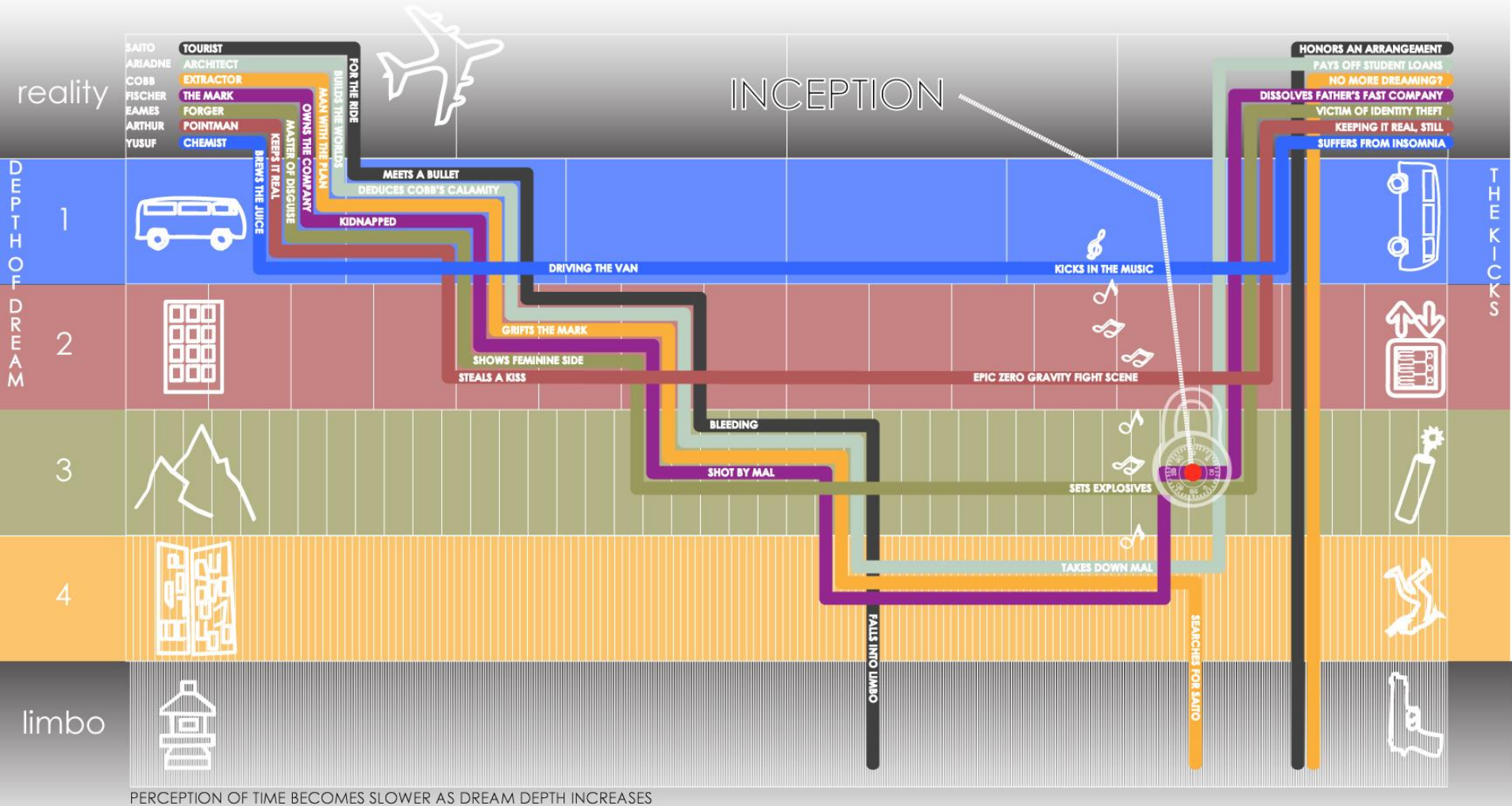
Versus:

```
def hypotenuse(a, b):  
    return sqrt((a*a) + (b*b))
```




The Call Stack



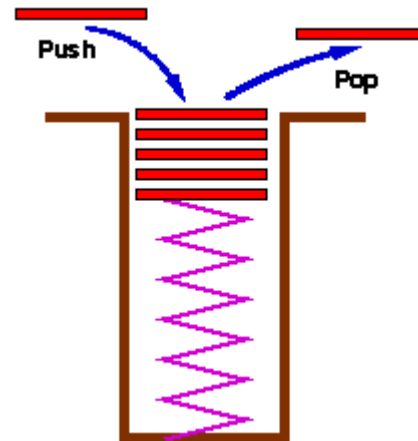


2010 INFOGRAPHIC
BY DANIEL WANG

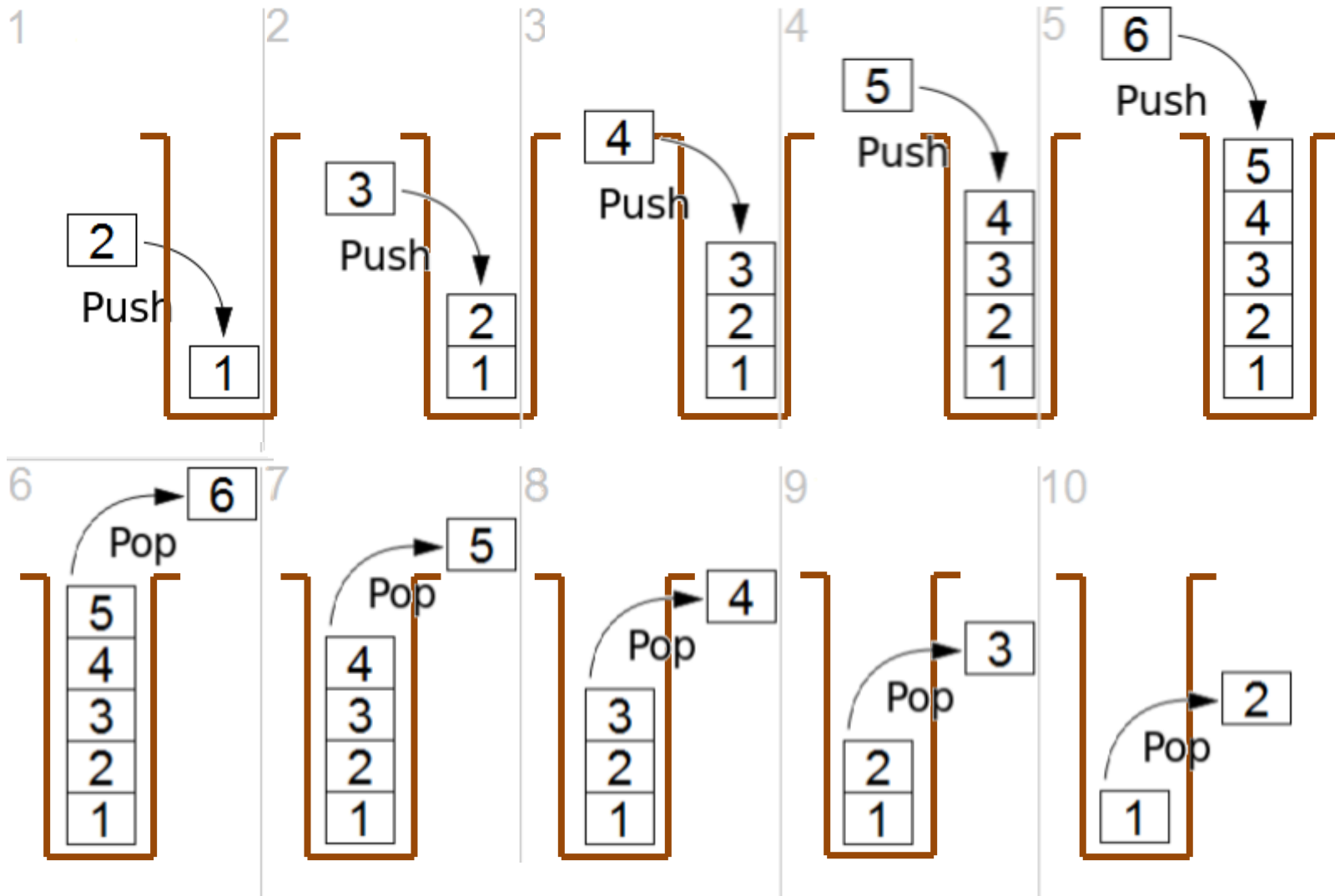
[Michael Caine Just Ended An Eight Year Long Debate Over The Ending Of "Inception"](#)

Stack

- First in last out order



First in Last Out



The Stack (or the Call Stack)

```
def p1(x):  
    print('Entering function p1')  
    output = p2(x)  
    print('Line before return in p1')  
    return output
```

```
def p2(x):  
    print('Entering function p2')  
    output = p3(x)  
    print('Line before return in p2')  
    return output
```

```
def p3(x):  
    print('Entering function p3')  
    output = x * x  
    print('Line before return in p3')  
    return output
```

```
print(p1(3))
```

The Stack (or the Call Stack)

```
>>> p1(3)
```

```
Entering function p1
```

```
Entering function p2
```

```
Entering function p3
```

```
Line before return in p3
```

```
Line before return in p2
```

```
Line before return in p1
```

```
9
```



FILO!

```
print(p1(3))
```

```
def p1(x):
```

```
    print('Entering function p1')
```

```
    output = p2(x)
```

```
    print('Line before return in p1')
```

```
    return output
```

```
def p2(x):
```

```
    print('Entering function p2')
```

```
    output = p3(x)
```

```
    print('Line before return in p2')
```

```
    return output
```

```
def p3(x):
```

```
    print('Entering function p3')
```

```
    output = x * x
```

```
    print('Line before return in p3')
```

```
    return output
```

→ Going in

→ Exiting a function



Debug Control



Go Step Over Out Quit

☒ Stack ☐ Source

☒ Locals ☐ Globals

W03a Call Stack.py:16: p3()

'bdb'.run(), line 431: exec(cmd, globals, locals)

'__main__'.<module>(), line 1: p1(3)

'__main__'.p1(), line 3: output = p2(x)

'__main__'.p2(), line 10: output = p3(x)

> '__main__'.p3(), line 16: output = x * x

Locals

x 3

p3()

p2()

p1()

Parameters are LOCAL variables

Scope of x in
p1

```
def p1(x):  
    print('Entering function p1')  
    output = p2(x)  
    print('Line before return in p1')  
    return output
```

Scope of x in
p2

```
def p2(x):  
    print('Entering function p2')  
    output = p3(x)  
    print('Line before return in p2')  
    return output
```

Scope of x in
p3

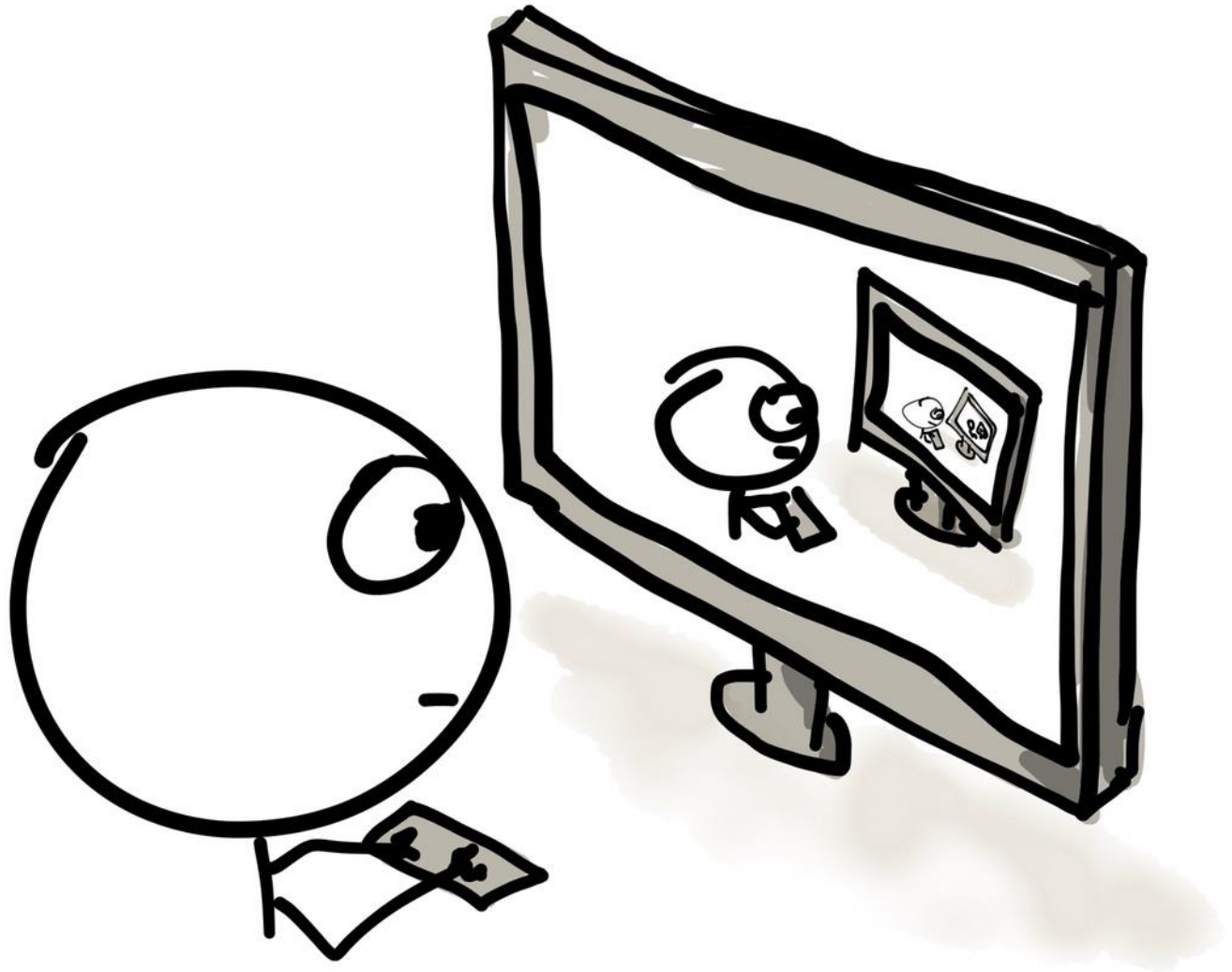
```
def p3(x):  
    print('Entering function p3')  
    output = x * x  
    print('Line before return in p3')  
    return output
```

Does not refer to

```
print(p1(3))
```

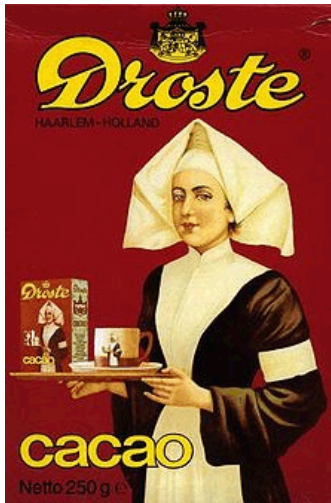
Calling ~~Other Functions~~ Yourself?

Recursion

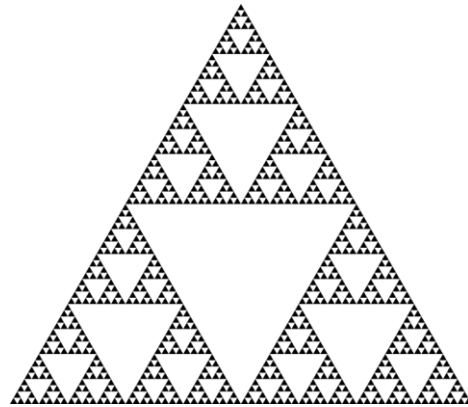


A Central Idea of CS

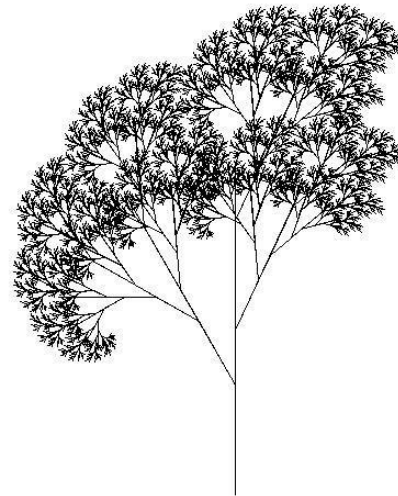
Some examples of recursion (inside and outside CS):



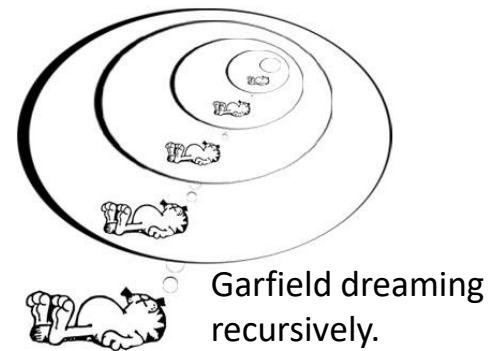
Droste effect



Sierpinski triangle



Recursive tree



Recursion

- A function that calls itself
- And extremely powerful technique
- Solve a big problem by solving a smaller version of itself
 - Mini-me



Factorial

- The factorial $n!$ is defined by

$$n! = 1 \times 2 \times 3 \times \cdots \times n$$

- Write a function for factorial?

```
def factorial(n):  
    ans = 1  
    i = 1  
    while i <= n:  
        ans = ans * i  
        i = i + 1  
    print(ans)
```

```
>>> factorial(3)  
6  
>>> factorial(6)  
720  
>>>
```

Factorial

$$n! = 1 \times 2 \times 3 \times \cdots \times n$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ (n - 1)! \times n & \text{otherwise} \end{cases}$$



Factorial



```
def factorial(n):  
    ans = 1  
    i = 1  
    while i <= n:  
        ans = ans * i  
        i = i + 1  
    print(ans)
```



```
def factorialR(n):  
    if n == 1:  
        return 1  
    else:  
        return n * factorialR(n-1)
```


Recursion

- Rules of recursion

Must have a **terminal** condition

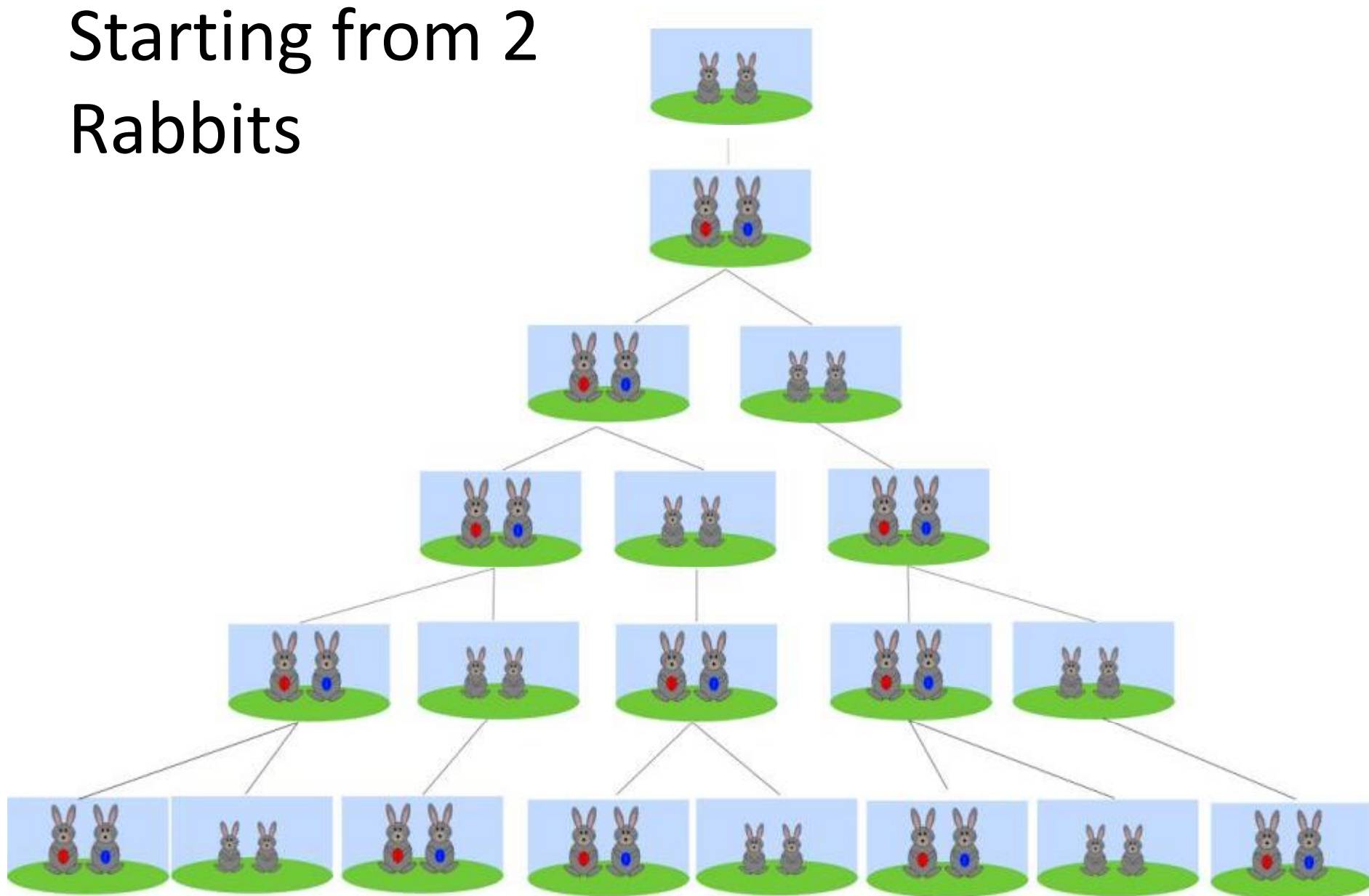
```
def factorialR(n):  
    if n == 1:  
        return 1  
    else:  
        return n * factorialR(n-1)
```

Must **reduce** the **size** of the problem for every layer

Fibonacci Number

(Recursion)

Starting from 2 Rabbits



How many ways to arrange cars?

- Let's say we have two types of vehicles, cars and buses



- And each car can park into one parking space, but a bus needs two consecutive ones
- If we have 1 parking space, I can only park a car



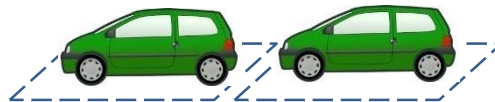
1 way

- But if there are 2 parking spaces, we can either park a bus or two cars



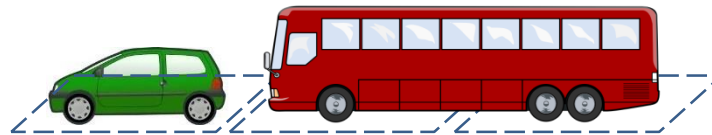
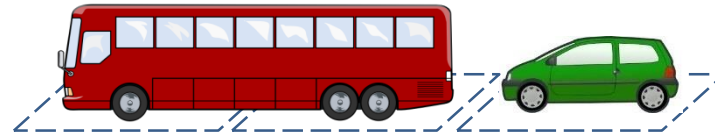
2 ways

-



How many ways to arrange cars?

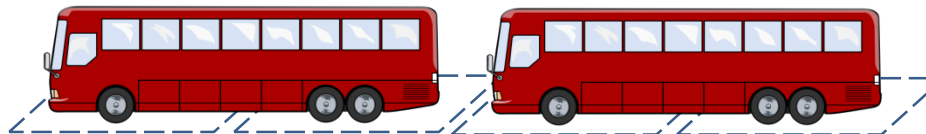
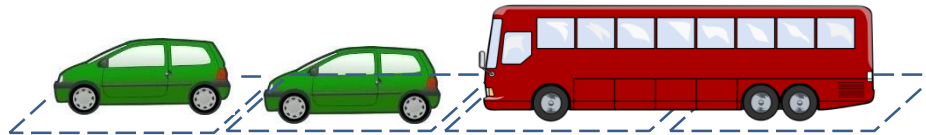
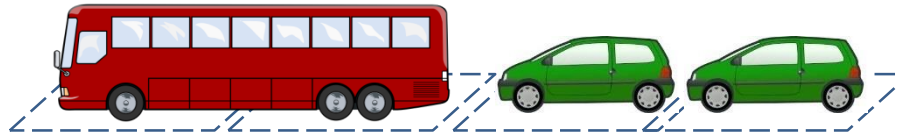
- So if we have 3 parking spaces, how many different ways can we park cars and buses?



3 ways

How many ways to arrange cars?

- So if we have 4 parking spaces, how many different ways can we park cars and buses?



5 ways

How many ways to arrange cars?

- 5 parking spaces?
- 6 parking spaces?

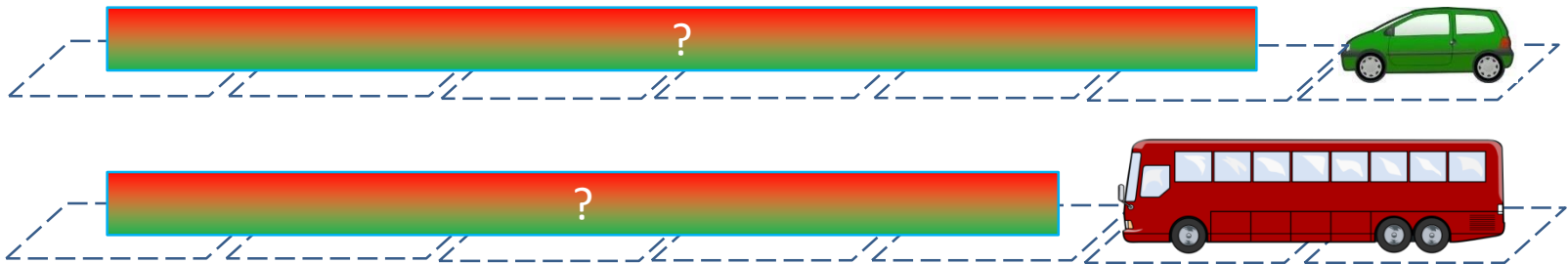


#parking spaces	#ways
0	1
1	1
2	2
3	3
4	5
5	8
6	13

- Can you figured out THE pattern?
 - 1, 1, 2, 3, 5, 8, 13, ...
 - What is the next number?

How many ways to arrange cars?

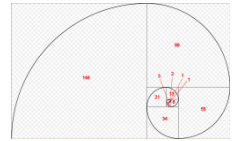
- In general, if we have n parking spaces, how many ways can we park the vehicles?
- You can think backward, the last parking space can be either a car or a bus



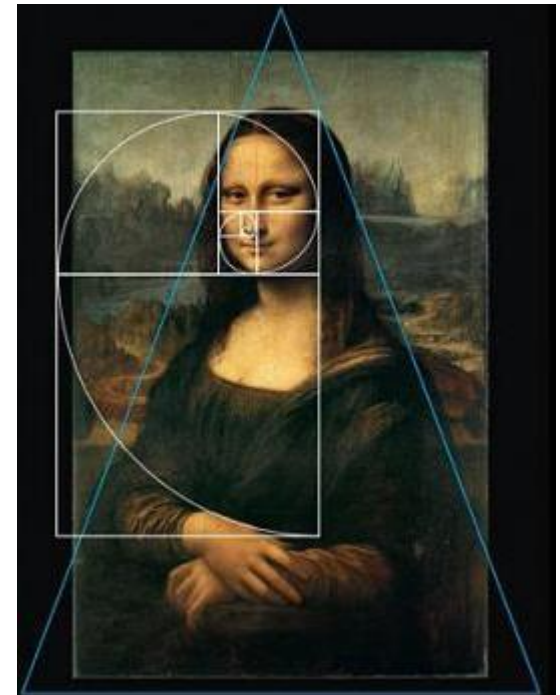
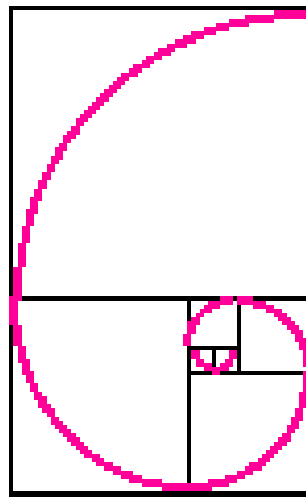
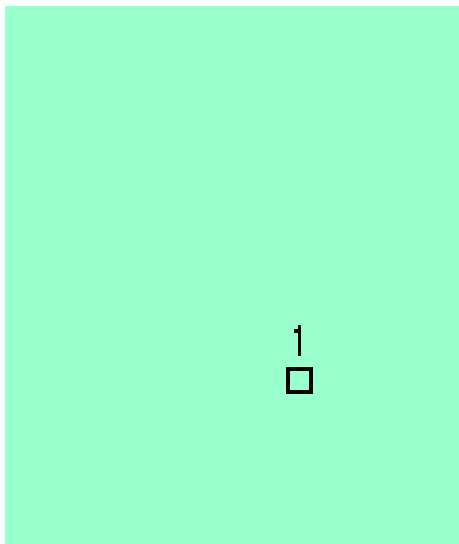
- If it's a car, there are $n - 1$ spaces left, you can have the number of way for $n - 1$ spaces
 - Otherwise, it's the number of way for $n - 2$ spaces
- So

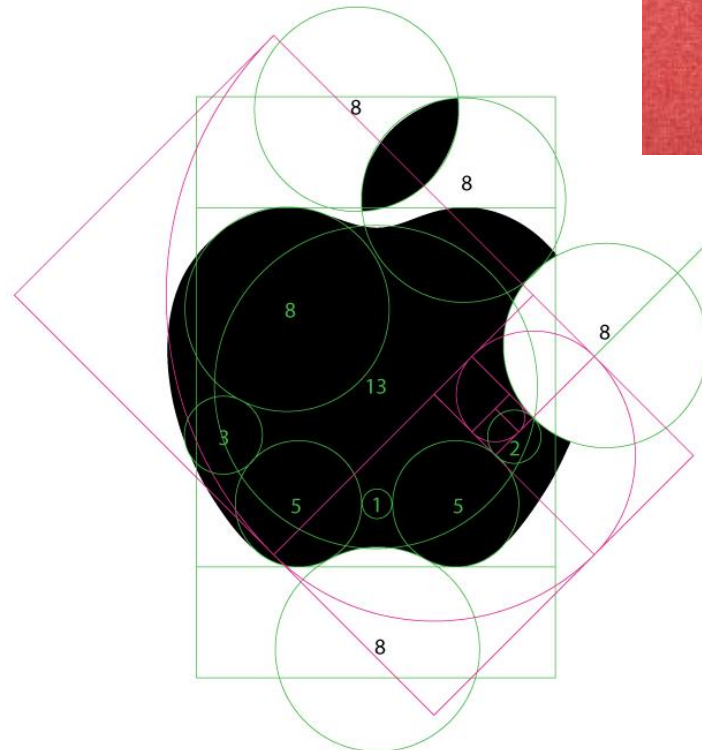
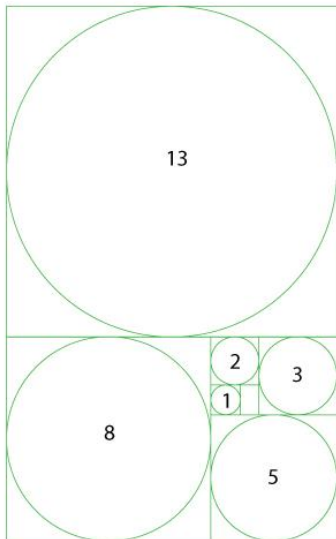
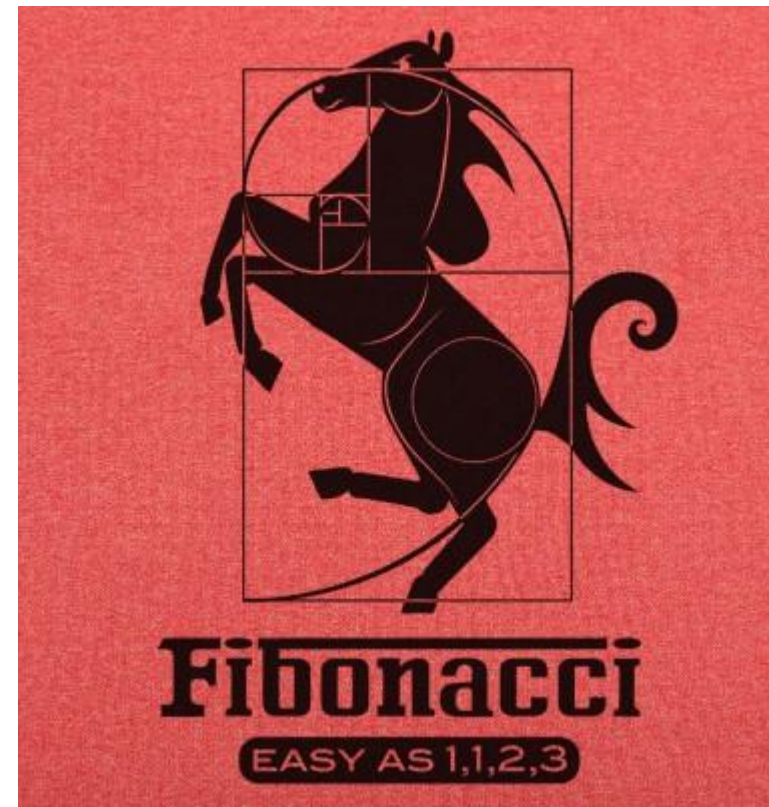
$$f(n) = f(n - 1) + f(n - 2) \quad \text{for } f(0) = f(1) = 1$$

Fibonacci Numbers



- Fibonacci numbers are found in nature (sea-shells, sunflowers, etc)
- <http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html>





```
def fibonacci(n):  
    if n == 1 or n == 0:  
        return 1  
    else:  
        return fibonacci(n-1) + fibonacci(n-2)
```

```
>>> fibonacci(10)
```

```
89
```

```
>>> fibonacci(20)
```

```
10946
```

```
>>> fibonacci(994)
```

```
Traceback (most recent call last):
```

```
File "<pyshell#16>", line 1, in <module>  
    fibonacci(994)
```

```
File "<pyshell#5>", line 5, in fibonacci  
    return fibonacci(n-1) + fibonacci(n-2)
```

```
File "<pyshell#5>", line 5, in fibonacci  
    return fibonacci(n-1) + fibonacci(n-2)
```

```
File "<pyshell#5>", line 5, in fibonacci  
    return fibonacci(n-1) + fibonacci(n-2)
```

```
[Previous line repeated 989 more times]
```

```
File "<pyshell#5>", line 2, in fibonacci  
    if n == 1 or n == 0:
```

```
RecursionError: maximum recursion depth exceeded in comparison
```

```
>>> fibonacci(50)
```



Challenge

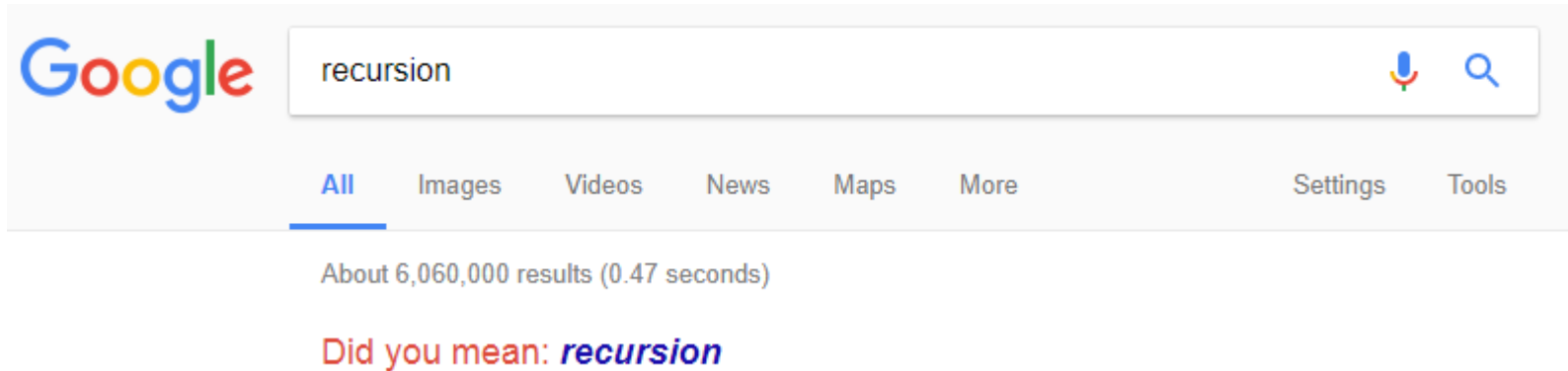
- Write a Fibonacci function that can compute $f(n)$ for $n > 1000$

```
>>> fibonacci(1000)
70330367711422815821835254877183549770181269836358732742604905087154537118196933
57974224949456261173348775044924176599108818636326545022364710601205337412127386
7339111198139373125598767690091902245245323403501
>>> fibonacci(2000)
68357022595758066470453965491705801070554080293655245654075533677980824544080540
14954534318953113802726603726769523447478238192192714526677939943338306101405105
41481970566409090181363729645376709552810486826470491443352935557914873104468563
41354877358979546298425169471014942535758696998934009765395457402148198191519520
85089538422954565146720383752121972115725761141759114990448978941370030912401573
418221496592822626
```



**TAKE THE
CHALLENGE**

Google about Recursion



- Try to search these in Google:
 - Do a barrel roll
 - Askew
 - Anagram
 - Google in 1998
 - Zerg rush
- More in [Google Easter Eggs](#)