

IT5002 Computer Systems and Applications
Tutorial 1
Number Systems

1. In 2's complement representation, "sign extension" is used when we want to represent an n -bit signed integer as an m -bit signed integer, where $m > n$. We do this by copying the MSB (most significant bit) of the n -bit number $m - n$ times to the left of the n -bit number to create the m -bit number.

For example, we want to sign-extend 0b0110 to an 8-bit number. Here $n = 4$, $m = 8$, and thus we copy the MSB bit 0 four ($8 - 4$) times, giving 0b00000110.

Similarly, if we want to sign-extend 0b1010 to an 8-bit number, we would get 0b11111010.

Show that IN GENERAL, sign extension is value-preserving. For example, 0b00000110 = 0b0110 and 0b11111010 = 0b1010.

2. We generalize $(r - 1)$'s-complement (also called *radix diminished complement*) to include fraction as follows:

$$(r - 1)\text{'s complement of } N = r^n - r^m - N$$

where n is the number of integer digits and m the number of fractional digits. (If there are no fractional digits, then $m = 0$ and the formula becomes $r^n - 1 - N$ as given in class.)

For example, the 1's complement of 011.01 is $(2^3 - 2^{-2}) - 011.01 = (1000 - 0.01) - 011.01 = 111.11 - 011.01 = 100.10$. (Since 011.01 represents the decimal value 3.25 in 1's complement, this means that -3.25 is represented as 100.10 in 1's complement.)

Perform the following binary subtractions of values represented in 1's complement representation by using addition instead. (Note: Recall that when dealing with complement representations, the two operands must have the same number of digits.)

(a) $0101.11 - 010.0101$

(b) $010111.101 - 0111010.11$

Is sign extension used in your working? If so, highlight it.

Check your answers by converting the operands and answers to their actual decimal values.

3. Convert the following numbers to fixed-point binary in 2's complement, with 4 bits for the integer portion and 3 bits for the fraction portion.

(a) 1.75

(b) -2.5

(c) 3.876

(d) 2.1

Using the binary representations you have derived, convert them back into decimal. Comment on the compromise between range and accuracy of the fixed-point binary system.

4. [AY2010/2011 Semester 2 Term Test #1]

How would you represent the decimal value -0.078125 in the IEEE 754 single-precision representation? Express your answer in hexadecimal. Show your working.

5. Write the following in MIPS Assembly, using as few instructions as possible. You may rewrite the equations if necessary to minimize instructions.

In all parts you can assume that integer variables **a**, **b**, **c** and **d** are mapped to registers \$s0, \$s1, \$s2 and \$s3 respectively. Each part is independent of the others.

- a. $c = a + b$
- b. $d = a + b - c$
- c. $c = 2b + (a - 2)$
- d. $d = 6a + 3(b - 2c)$