National University of Singapore

School of Computing

CS2040S - Data Structures and Algorithms
Midterm Test

(AY2025/26)

Time Allowed: 70 minutes

INSTRUCTIONS TO CANDIDATES:

1. Do **NOT** open this assessment paper until you are told to do so.

2. This assessment paper contains just ONE section.
   It comprises SEVEN (7) printed pages, including this page.

3. This is an **Open Book Assessment**.
   You cannot use any electronic device except one non-programmable calculator.

4. You can use either pen or pencil. Just make sure that you write **legibly**!

5. Important tips: Pace yourself! Do **not** spend too much time on one (hard) question.
   Read all the questions first! Some (sub-)questions might be easier than they appear.

6. You can use **pseudo-code** in your answer but beware of penalty marks for **ambiguous answer**.
   You can use **standard, non-modified** classic algorithm in your answer by just mentioning its
   name, e.g. run Merge Sort on list L, use Stack S, use Binary Min Heap $MinPQ$, etc.

7. The total marks is 50. All the best :)

A Application Questions ($3 \times 16 + 2 = 50$ marks)

A.1 Special Largest Gap (16 marks)

You are given an integer array *arr* that contains n *distinct* positive integers $\in [1..10^7 - 1]$ (i.e., at most 7 digits). Your task is to output the largest gap between any two integers in *arr* that have the same sum of digits. If no two integers with the same sum of digits exists, output -1 instead.

Sum of digits (`sod`) of an integer is as the name implies, e.g., `sod(123) = 1+2+3 = 6`, `sod(77) = 7+7 = 14`, etc.

No	Input array *arr*	Output	Explanation
1	$[63, 123, 27, 77, 7, 239, 45]$	162	63, 27, and 45 have the same sum of digits: 9,
			$63 - 27 = 36$ is the largest gap among these 3 integers,
			but 77 and 239 have the same sum of digits: 14,
			and $239 - 77 = 162$ is the largest gap overall
2	$[7, 16, 25, 34, 43, 52, 61, 70]$	63	All integers have the same sum of digits: 7,
			$70 - 7 = 63$ is the largest gap among these 8 integers
3	$[10, 2, 77, 14]$	-1	There is no pair that satisfy the conditions

Table 1: A Few Examples for the Special Largest Gap Problem

Design algorithm that solves this task and analyze its worst-case time and space complexities. Note that the size of the input $O(n)$ of *arr* is not factored in the space complexity analysis. Your solution will be given marks according to Table 4 below.

Time complexity	Space complexity	Comments	Marks
$O(n^2)$	$O(\infty)$	you can use any amount of extra memory	8
$O(n \log n)$	$O(n)$	additional array of size n is allowed	12
(Expected) $O(n \log n)$	(Expected) $O(\log n)$	expected analysis is ok	13
$O(n)$	$O(1)$	*a few constant* extra variables are allowed	16

Table 2: Scoring Scheme

A.2 Linked List Splicing (16 marks)

You are given two Singly Linked Lists (SLLs): SLL1 and SLL2. The head (first element) of SLL1/SLL2 is `head1`/`head2`, respectively. The tail (last element) of SLL1/SLL2 is `tail1`/`tail2`, respectively. The length of SLL1/SLL2 is n/m integers, respectively ($4 \leq n, m$). All integers in either SLL are $\in [0..99]$. It is guaranteed that there exists exactly one special sublist of size 4 vertices in SLL1 that contains value 2, 0, 4, and then 0.

Your task is to remove those special sublist from SLL1 and 'splice' (insert) the entire content of list2 into the space vacated by that special sublist. Return the `head1` of the updated SLL1 after this action. ~~You may have to update `tail1` if necessary~~. For this problem, you do not have to worry about updating `tail1`/`head2`/`tail2`. Check Figure 1 for an example.
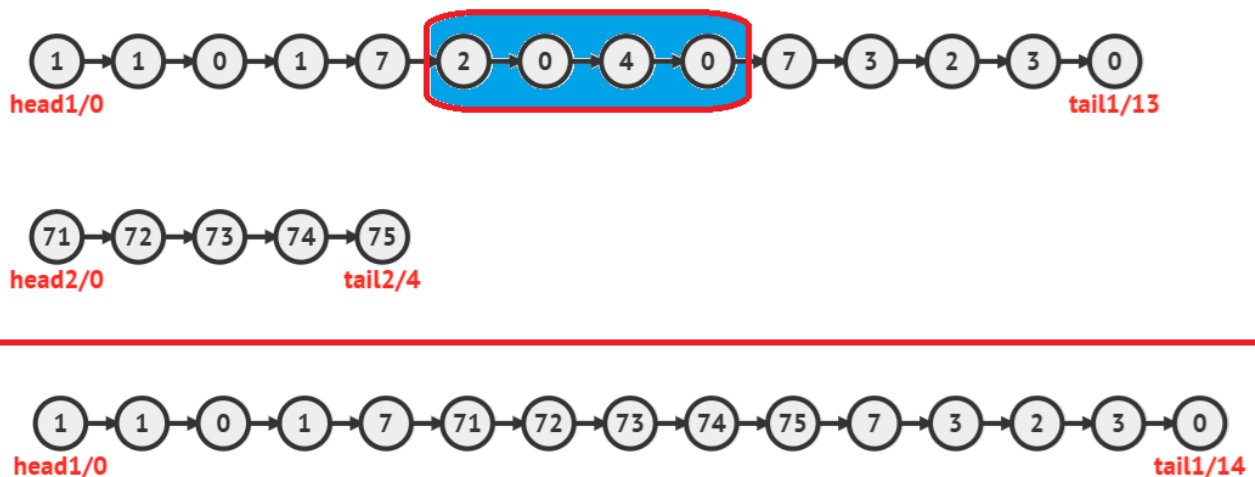
Figure 1: An example SLL1 (top), SLL2 (middle), and the updated SLL1 (bottom)

For full 16 marks, implement this SLL operation (use the given template at answer sheet).

Analyze the tightest worst-case time complexity of your solution.

Be-careful of corner cases, -3 marks per corner case bug.

A.3   Special Operations (16 marks)

You are given an integer array arr. There are up to n $(n > 1)$ integers in arr, and each integer $\in [7..7^7]$. Additionally, you are also given another integer m and you have to do m operations (described below). Your starting $total$ is 0. You are told that $m \in O(n^7)$.

In one operation, you can choose *any* integer x in arr, increase $total$ by x, and then replace (just one copy of) x with $max(\lfloor \frac{x}{7} \rfloor, 7)$.

Your task is to output the maximum possible $total$ that you can attain after applying exactly m operations. Check a few examples below to aid your understanding of this task:

No	Input arr A	m	Output	Explanation
1	$[49, 49, 49]$	4	154	For the first three operations,
				do the operation to *each* array element once,
				then do the fourth operation on one of the 7,
				The total is $49 + 49 + 49 + 7 = 154$
2	$[441, 7, 8, 34]$	4	547	Do the following operations:
				1). select 441, $total = 441$, now $arr = [63, 7, 8, 34]$,
				2). select 63, $total = 504$, now $arr = [9, 7, 8, 34]$,
				3). select 34, $total = 538$, now $arr = [9, 7, 8, 7]$,
				4). select 9, $total = 547$, now $arr = [7, 7, 8, 7]$
3	$[441, 7, 8, 34]$	77	1059	Think about it

Table 3: A Few Examples for The Special Operations Problem

Design algorithm that solves this task and analyze its worst-case time complexities.

Your solution will be given marks according to Table 4 below.

Time-complexity	Comments	Marks
$O(mn^2)$ or $O(n^9)$	Dependent on m	4
$O(mn \log n)$ or $O(n^8 \log n)$	Dependent on m	6
$O(mn)$ or $O(n^8)$	Dependent on m	8
$O(m \log n)$ or $O(n^7 \log n)$	Dependent on m	10
$O(n^2)$	Independent of m	12
$O(n \log n)$	Independent of m	16

Table 4: Scoring Scheme

A.4 Midterm Feedback ($2 \times 1 = 2$ marks)

Prof Halim had run S1 version of CS2040S in AY23/24 and AY24/25. For this S1 AY25/26, he is implementing these changes. So just focus on writing non-blank non-anonymous feedback on them:

- Does adding LeetCode – more concise problem description, closer to coding interview tasks, mostly for lecture demos (on top of Kattis – longer/more cryptic problem description from programming competition, mostly for programming assignments) helps you in understanding CS2040S contents so far?

- What is your opinion about the plan to drop the PE (15% in AY23/24 and 11% in AY24/25) to 0% (none) this AY25/26 (and moving its weightage to no-electronic device final assessment, now becomes a 60% paper)?

The Answer Sheet for Semester 1 AY2025/26

Write your Student Number in the box below using **(2B) pencil**. **Do NOT write your name**.

STUDENT NUMBER

Section	Maximum Marks	Your Marks	Grading Remarks
Total	50		

Box A.1. Special Largest Gap

I claim that my solution runs in $O(_____)$ and uses $O(_____)$ additional space.

Box A.2. Linked List Splicing

```
/* public class Vtx { // Definition for Singly-Linked List.
 *      int val;
 *      Vtx next;
 *      Vtx() {}
 *      Vtx(int val, Vtx next) { this.val = val; this.next = next; }
 * } */
class MidtermQ2 { // implement your answer here
    public Vtx splice(Vtx head1, Vtx tail1, Vtx head2, Vtx tail2) {




        return head1; // obviously, after you modify something above
    }
}
```

I claim that my solution runs in $O(_____)$.

Box A.3. Special Operations

I claim that my solution runs in $O(\underline{\hspace{3cm}})$.

Box A.4. Feedback about CS2040S S1 AY25/26 changes: Add LeetCode, Drop PE $\rightarrow$ 60% Final

– END OF PAPER; All the Best –