IT5005 Artificial Intelligence

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Propositional Logic Tutorial

Grammar for CNF

$$
\begin{aligned}
CNFSentence &\rightarrow Clause_1 \wedge \cdots \wedge Clause_n \\
Clause &\rightarrow Literal_1 \vee \cdots \vee Literal_m \\
Fact &\rightarrow Symbol \\
Literal &\rightarrow Symbol \mid \neg Symbol \\
Symbol &\rightarrow P \mid Q \mid R \mid \ldots \\
HornClauseForm &\rightarrow DefiniteClauseForm \mid GoalClauseForm \\
DefiniteClauseForm &\rightarrow Fact \mid (Symbol_1 \wedge \cdots \wedge Symbol_l) \Rightarrow Symbol \\
GoalClauseForm &\rightarrow (Symbol_1 \wedge \cdots \wedge Symbol_l) \Rightarrow False
\end{aligned}
$$

Figure 7.12 A grammar for conjunctive normal form, Horn clauses, and definite clauses. A CNF clause such as $\neg A \vee \neg B \vee C$ can be written in definite clause form as $A \wedge B \Rightarrow C$.

1. Use resolution to show that the statement

$$(P \vee Q) \wedge (\neg P \vee R) \wedge (\neg P \vee \neg R) \wedge (P \vee \neg Q)$$

is UNSAT.

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Solution:

- $L_1 : P \vee Q$ (given)
- $L_2 : \neg P \vee R$ (given)
- $L_3 : \neg P \vee \neg R$ (given)
- $L_4 : P \vee \neg Q$ (given)
- $L_5 : P$ (L_1, L_4, resolution)
- $L_6 : R$ (L_2, L_5, resolution)
- $L_7 : \neg R$ (L_3, L_5, resolution)
- $L_8 :$ false (L_6, L_7, resolution)

2. Suppose we are given the following premises:

- $P_1 : P \Rightarrow Q$
- $P_2 : R \Rightarrow P$
- $P_3 : \neg Q$
- $P_4 : R \vee P \vee S$

Use resolution to prove that S is always True under the premises.

2. Suppose we are given the following premises:

- $P_1 : P \Rightarrow Q$
- $P_2 : R \Rightarrow P$
- $P_3 : \neg Q$
- $P_4 : R \vee P \vee S$

Use resolution to prove that S is always True under the premises.

Solution:

- $L_1 : \neg P \vee Q$ (given)
- $L_2 : \neg R \vee P$ (given)
- $L_3 : \neg Q$ (premise)
- $L_4 : R \vee P \vee S$ (given)
- $L_5 : \neg S$ (negation of conclusion)
- $L_6 : R \vee P$ (L_4, L_5, resolution)
- $L_7 : P$ (L_2, L_6, resolution)
- $L_8 : Q$ (L_1, L_7, resolution)
- $L_9 :$ false (L_3, L_8, resolution)

3. Consider the following Horn clauses:

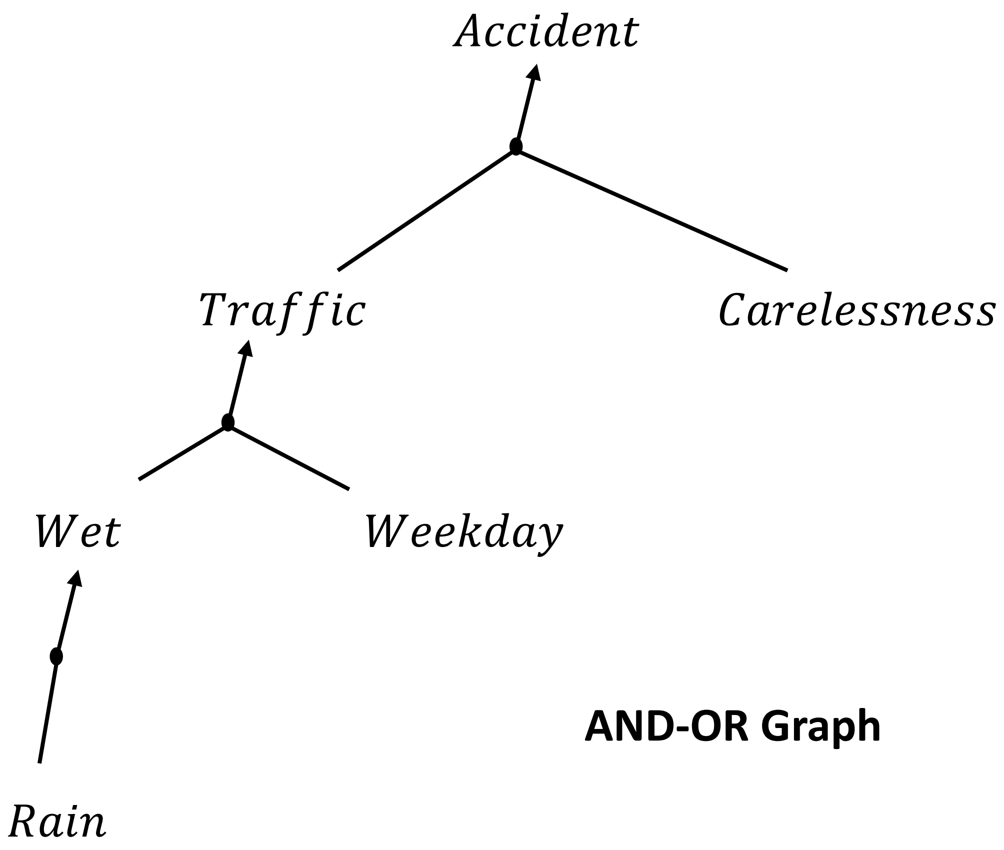
- $P_1 : Rain$
- $P_2 : Weekday$
- $P_3 : Rain \Rightarrow Wet$
- $P_4 : Wet \wedge Weekday \Rightarrow Traffic$
- $P_5 : Traffic \wedge Careless \Rightarrow Accident$

Prove $Traffic$ with both forward and backward chaining algorithms

Could you prove $Accident$ for the given KB?

KB

- $P_1 : Rain$
- $P_2 : Weekday$
- $P_3 : Rain \Rightarrow Wet$
- $P_4 : Wet \wedge Weekday \Rightarrow Traffic$
- $P_5 : Traffic \wedge Careless \Rightarrow Accident$



AND-OR Graph

Forward Chaining

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional definite clauses
          q, the query, a proposition symbol
  count ← a table, where count[c] is initially the number of symbols in clause c's premise
  inferred ← a table, where inferred[s] is initially false for all symbols
  queue ← a queue of symbols, initially symbols known to be true in KB

  while queue is not empty do
      p ← POP(queue)
      if p = q then return true
      if inferred[p] = false then
          inferred[p] ← true
          for each clause c in KB where p is in c.PREMISE do
              decrement count[c]
              if count[c] = 0 then add c.CONCLUSION to queue
  return false
```

Clause	c	count(c) = # of antecedents of c
$Rain$	1	0
$Weekday$	2	0
$Rain \Rightarrow Wet$	3	1
$Wet\ \wedge Weekday \Rightarrow Traffic$	4	2
$Traffic\ \wedge Careless \Rightarrow Accident$	5	2

Query: $Traffic$

Queue	p=Pop(Queue)	p=Query	Remark
$[Rain, Weekday]$	$Rain$	No	$Rain$ is already in KB; inferred($Rain$) = True; count(3) = 1-1 = 0; so we can derive $Wet\ = \ True$ Add Wet to queue
$[Weekday, Wet]$	$Weekday$	No	$Weekday$ is already in KB; inferred($Weekday$) = True; count(4) = 2-1=1
$[Wet]$	Wet	No	Inferred(Wet) = True count(4) = 0; so we can derive $Traffic\ = \ True$ add $Traffic$ to queue
$[Traffic]$	$Traffic$	Yes	Return $Traffic\ = \ True$

Clause	c	count(c) = # of antecedents of c
$Rain$	1	0
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$Wet \wedge Weekday \Rightarrow Traffic$	4	2
$Traffic \wedge Careless \Rightarrow Accident$	5	2

Query: $Accident$

Queue	p=Pop(Queue)	p=Query	Remark
$[Rain, Weekday]$	$Rain$	No	$Rain$ is already in KB; inferred($Rain$) = True; count(3) = 1-1 = 0; so we can derive $Wet = True$ Add Wet to queue
$[Weekday, Wet]$	$Weekday$	No	$Weekday$ is already in KB; inferred($Weekday$) = True; count(4) = 2-1=1
$[Wet]$	Wet	No	Inferred(Wet) = True count(4) = 0; so we can derive $Traffic = True$ add $Traffic$ to queue
$[Traffic]$	$Traffic$	Yes	Inferred$[Traffic] = True$ count(5) = 1
[]			Queue is empty and return $\text{Accident} = False$

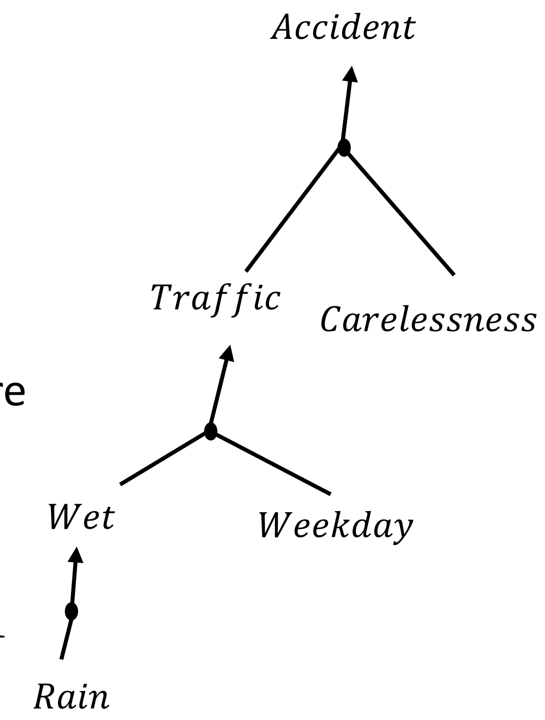
Backward Chaining (Pseudocode)

function $PL - BC - ENTAILS?\,(KB, q)$ returns true or false
inputs: KB, the knowledge base, a set of propositional definite clauses
q: the query, a propositional symbol

If q matches a fact, **then return** true
If there is no premise with a consequent that matches q, **then return** false

for each clause c in KB where q is in $c.CONCLUSION$ **do**
  $count$ = Number of symbols in $c.PREMISE$
  for all premises p in $c.PREMISE$ do
    if $PL - BC - ENTAILS?\,(KB, p)$
      if p not in KB **then** add p to KB
      $count$ = $count$ - 1
    else break
  if $count$ == 0, **then return** true
return false

Backward Chaining



Query: $Traffic$

The query $Traffic$ is not a fact
The query $Traffic$ is a consequent in P_4. To prove Traffic, both Wet and $Weekday$ needs to be proved, i.e., $count(Traffic) = 2$; make recursive calls to check whether Wet and $Weekday$ are true

Recursive call to check $\boldsymbol{Wet}$:

Wet is not a fact

Wet is a consequent in P_3. To prove Wet we need to prove $Rain$, i.e., $count(Wet) = 1$

Recursive call to check $\boldsymbol{Rain}$:

$Rain$ is a fact and the function returns $Rain = True$

$count(Wet) = 0$ and returns $Wet = True$

Add Wet to KB; $and\ count(Traffic) = 2 - 1 = 1$

Recursive call to check $\boldsymbol{Weekday}$:

$Weekday$ is a fact and the call returns $Weekday = True$

$count(Traffic) = 0$;
returns $Traffic = True$

- $P_1 : Rain$
- $P_2 : Weekday$
- $P_3 : Rain \Rightarrow Wet$
- $P_4 : Wet \wedge Weekday \Rightarrow Traffic$
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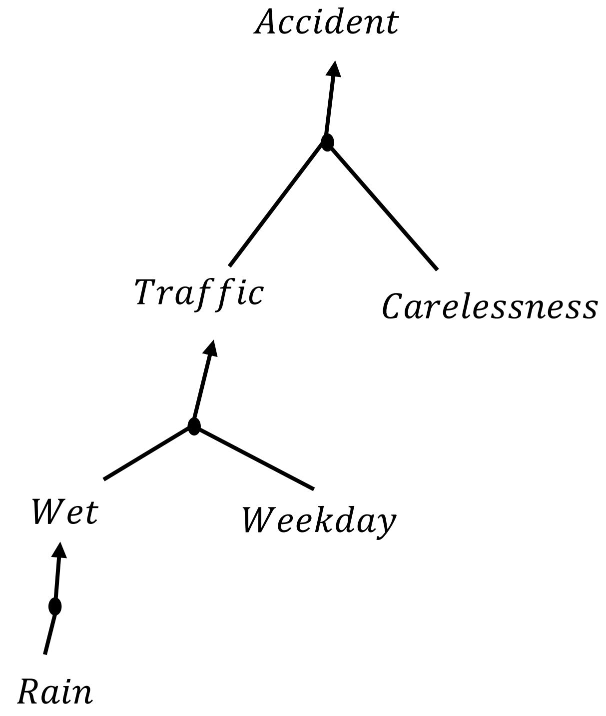
Backward Chaining



Query: $Accident$

The query $Accident$ is not a fact

The query$Accident$ is a consequent in P_5. To prove $Accident$, both $Traffic$ and $Carelessness$ needs to be proved, i.e., $count(Accident) = 2$; make recursive calls to check whether $Traffic$ and $Carelessness$ are true

Recursive call to check $\boldsymbol{Traffic}$:

returns $Traffic = True$

(Trace is shown in the previous question)

Add Traffic to KB

$count(Accident) = 2 - 1 = 1$

Recursive call to check $Carelessness$:

The query $Carelessness$is not a fact

There is no premise with consequent that matches the query $Carelessness$;

return $Carelessness = False$

Returns $Accident$ = False

- $P_1 : Rain$
- $P_2 : Weekday$
- $P_3 : Rain \Rightarrow Wet$
- $P_4 : Wet \land Weekday \Rightarrow Traffic$
- $P_5 : Traffic \land Careless \Rightarrow Accident$

4. Consider the following knowledge base (KB) with propositional symbols A, B, C, and D. The KB contains two sentences $\mathbf{P1}$ and $\mathbf{P2}$:

$$\mathbf{P1} \; : \; C \lor D$$
$$\mathbf{P2} \; : \; B \Rightarrow ((A \land B) \Rightarrow C)$$

Identify the models of the sentence $\mathbf{P2}$.

4. Consider the following knowledge base (KB) with propositional symbols A, B, C, and D.
The KB contains two sentences **P1** and **P2**:

$$\mathbf{P1} \; : \; C \vee D$$
$$\mathbf{P2} \; : \; B \Rightarrow ((A \wedge B) \Rightarrow C)$$

Identify the models of the sentence **P2**.

A	B	C	$(A \wedge B) \Rightarrow C$	$B \Rightarrow ((A \wedge B) \Rightarrow C)$
true	true	true	true	true
true	true	false	false	false
true	false	true	true	true
true	false	false	true	true
false	true	true	true	true
false	true	false	true	true
false	false	true	true	true
false	false	false	true	true

Except
{A = true, B = true, C = false, D = false}
And
{A = true, B = true, C = false, D = true},

all other assignments to the variables A, B, C, D are models of the KB.

Total number of models: 14

5. Using logic, you need to determine whether the following statement is true: "The vase was broken". The following clues are given:

- **R1**: Charlie was outside.
- **R2**: The vase was broken if and only if the cat was in the house or Bob was playing indoors.
- **R3**: If Bob was playing indoors, then Charlie was outside.
- **R4**: If Charlie was outside, then cat was in the house.

Use the propositional symbols shown in Table 1 to represent the sentences in the puzzle. Answer the following questions:

Table 1: Propositional Symbols

O: Charlie was outside.	**B**: The vase was broken.
H: The cat was in the house.	**I**: Bob was playing indoors.

(a) Translate the clues **R1** to **R4** into propositional form.

(b) Convert the above sentences in propositional form to CNF.

(c) Check whether the statement "The vase was broken" is true or not using resolution-refutation algorithm.

(a) Propositional Form

- **R1**: O
- **R2**: $B \iff H \vee I$
- **R3**: $I \Rightarrow O$
- **R4**: $O \Rightarrow H$

(b) Conjunctive Normal Form (CNF)

- **R1**: O
- **R2**: $(\neg B \vee H \vee I) \wedge (\neg H \vee B) \wedge (\neg I \vee B)$
- **R3**: $\neg I \vee O$
- **R4**: $\neg O \vee H$

(c) Resolution-Refutation

Query: The vase is broken: B

- **R1**: O (given)
- **R2a**: $(\neg B \vee H \vee I)$. (given)
- **R2b**: $(\neg H \vee B)$. (given)
- **R2c**: $(\neg I \vee B)$ (given)
- **R3**: $\neg I \vee O$ (given)
- **R4**: $\neg O \vee H$ (given)
- **R5**: $\neg B$. (Negation of Query)
- **R6**: H. (Resolution of **R1** and **R4**)
- **R7**: $\neg H$. (Resolution of **R5** and **R2b**)
- **R8**: Contradiction (from **R6** and **R7**)