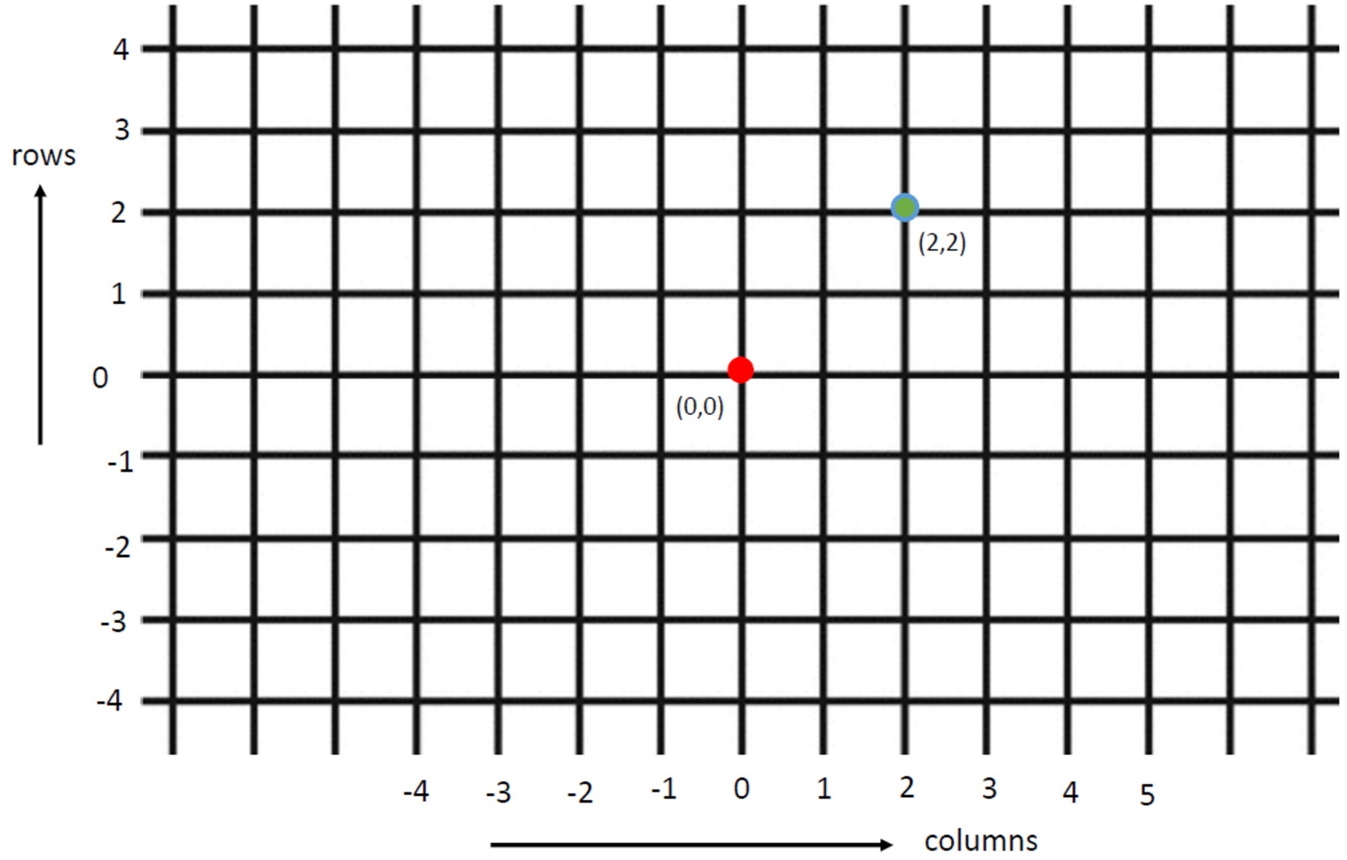
IT5005 Artificial Intelligence

Sirigina Rajendra Prasad
AY2025/2026: Semester 1

Tutorial 1: Uninformed and Informed Search

1. Consider the grid problem shown in Fig. 1. The grid extends to infinity in all directions. The agent can only move along the grid lines. Action *East*, *West*, *North*, and *South* takes the agent from state (r, c) to $(r, c+1)$, $(r, c-1)$, $(r+1, c)$, $(r-1, c)$, respectively. Each action costs one unit. The agent is initially assumed to be at state $(0, 0)$, and objective is to reach state $(2, 2)$. All action costs are same and equal to 1.



1. tree search/graph search?

Graph Search is preferred.
Tree search results in infinite loops

Comparison of Algorithms

Algorithm	**Complete**	**Optimal**
Breadth-First Search	Yes	Yes Optimal because of the unit step costs
Depth-limited Search	Yes	Yes (if the depth-limit is 3, cannot generalize it; counterexample in lecture slide 38)
Depth-first Search	No Because grid extends to infinity, the agent keep exploring towards east and never returns	No Incomplete, so not optimal
Iterative Deepening Search	Yes Complete as it searches each state in the increasing order of depth and goal can be reached at a depth of 4	Yes Optimal because of the unit step costs

2. An explicit state space graph is shown in Fig. 2. Several heuristics (i.e., estimated costs to reach the goal state from a node) are presented in Table 1. As an example, the column corresponding to h_5 can be interpreted as $h_5(A) = 5$, $h_5(B) = 7$, $h_5(C) = 2$, and $h_5(D) = 0$.

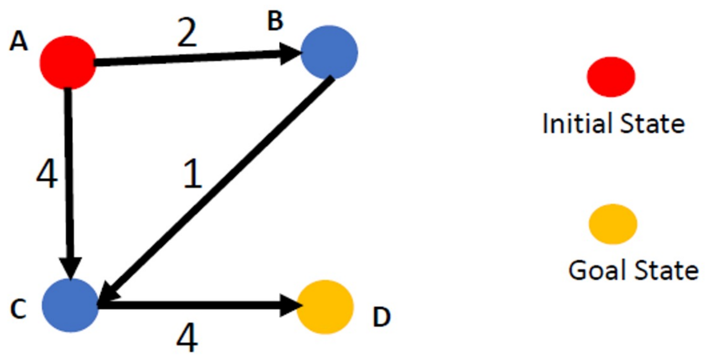


Figure 2: Explicit state space graph

Table 1: Heuristics for the state space graph shown in Fig. 2

Node	h_1	h_2	h_3	h_4	h_5
A	0	7	7	5	5
B	0	5	4	3	7
C	0	4	1	2	2
D	0	0	0	0	0

(a) Comment on admissibility and consistency of the heuristic functions h_i, $i = \{1, \ldots, 5\}$.

Checking for Admissibility: A heuristic is admissible if estimated cost at each node to reach the goal state is less than true cost to reach the goal state.

Let us denote the actual cost of reaching the goal by h^*. Then the actual costs for each state in the given graph are:

$$\begin{aligned} h^*(A) &= 7 \\ h^*(B) &= 5 \\ h^*(C) &= 4 \\ h^*(D) &= 0 \end{aligned}$$

It can be verified from Table 1 that h_1, h_2, h_3, and h_4 are admissible. h_5 is not admissible, because $h_5(B) > h^*(B)$.

Checking for Consistency: A heuristic is consistent if at each node n, the following triangle inequality is satisfied.

$$\begin{aligned} h(n) &\leq h(n') + c(n, a, n') \\ \implies h(n) - h(n') &\leq c(n, a, n') \end{aligned}$$

where $c(n, a, n')$ indicates the cost of reaching node n' from node n with action a.

$$h_1(A) - h_1(B) = 0 \leq C(A, B)$$
$$h_1(B) - h_1(C) = 0 \leq C(B, C)$$
$$h_1(A) - h_1(C) = 0 \leq C(A, C)$$
$$h_1(C) - h_1(D) = 0 \leq C(C, D)$$

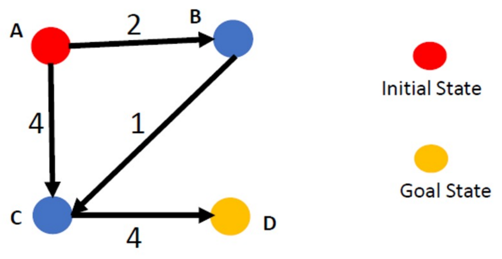


Figure 2: Explicit state space graph

Therefore, h_1 is consistent. Similarly, it can be verified that h_2 and h_4 are consistent.
On the other hand, the heuristics h_3 and h_5 are not consistent. They violate the triangle inequality as shown below:

$$h_3(A) - h_3(B) = 3 > C(A, B)$$
$$h_5(B) - h_5(C) = 5 > C(B, C)$$

Uniform Cost Search

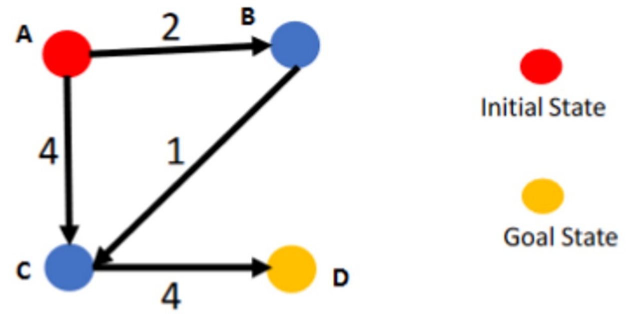
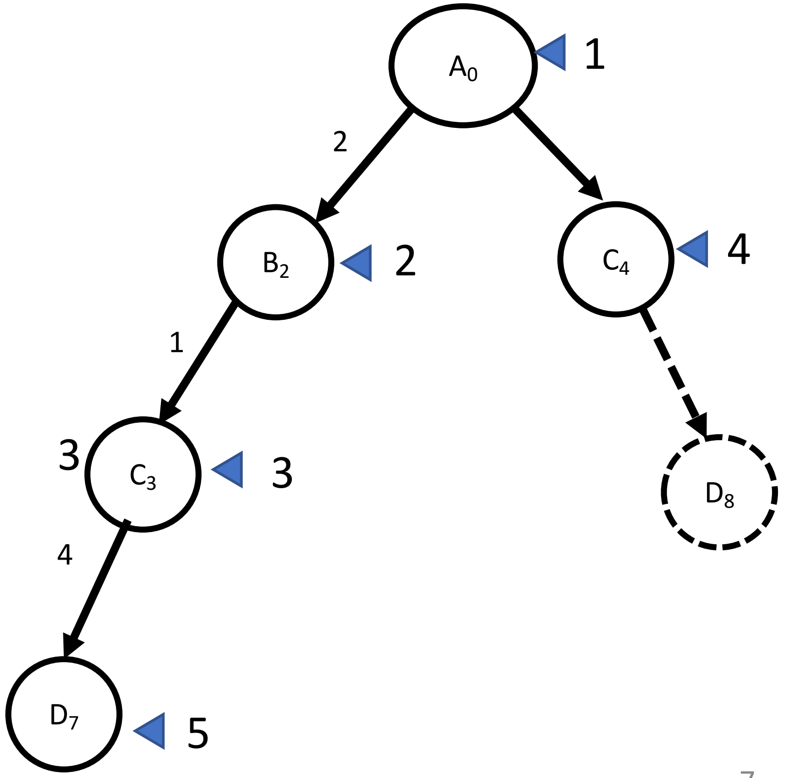


Table 2: Uniform-Cost Search Algorithm

#	Node	Is-Goal(Node)	s	Frontier	Reached
0	A_0			$[A_0]$	$\{A_0\}$
1	A_0	No	B_2	$[B_2]$	$\{A_0, B_2\}$
2			C_4	$[B_2, C_4]$	$\{A_0, B_2, C_4\}$
3	B_2	No	C_3	$[C_4, C_3]$	$\{A_0, B_2, \mathbf{C_3}\}$
4	C_3	No	D_7	$[C_4, D_7]$	$\{A_0, B_2, C_3, D_7\}$
5	C_4	No	D_8	$[D_7]$	$\{A_0, B_2, C_3, D_7\}$
6	D_7	Yes			



Solution: $A - B - C - D$

A* Search with Heuristic h_1

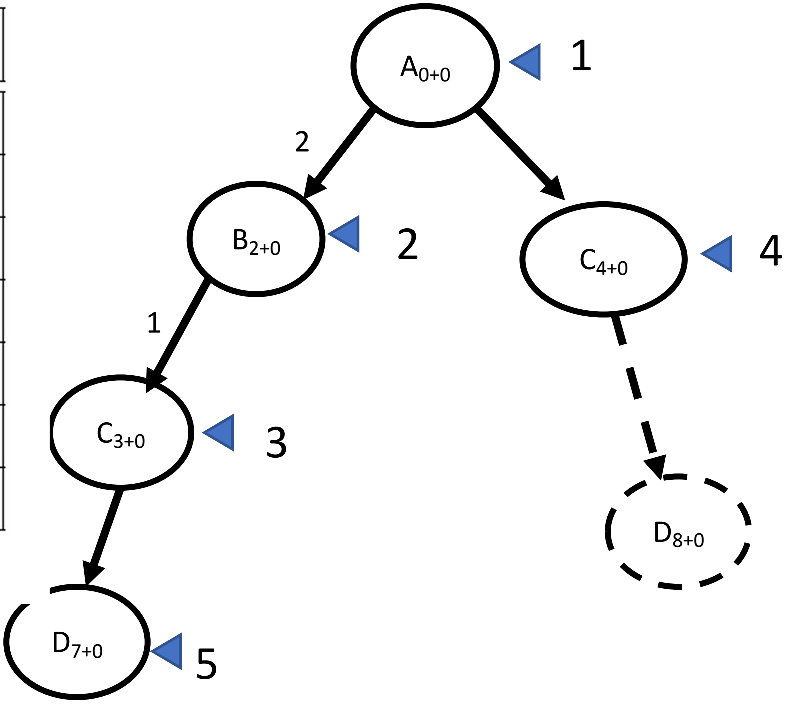
Node	h_1	h_2	h_3	h_4	h_5
A	0	7	7	5	5
B	0	5	4	3	7
C	0	4	1	2	2
D	0	0	0	0	0

h_1 is consistent and admissible

Table 3: A* Search for heuristic h_1; $h_1(A) = h_1(B) = h_1(C) = h_1(D) = 0$

#	Node	Is-Goal(Node)	s	Frontier	Reached
0	A_{0+0}			$[A_{0+0}]$	$\{A_{0+0}\}$
1	A_{0+0}	No	B_{2+0}	$[B_{2+0}]$	$\{A_{0+0}, B_{2+0}\}$
2			C_{4+0}	$[B_{2+0}, C_{4+0}]$	$\{A_{0+0}, B_{2+0}, C_{4+0}\}$
3	B_{2+0}	No	C_{3+0}	$[C_{4+0}, C_{3+0}]$	$\{A_{0+0}, B_{2+0}, \mathbf{C_{3+0}}\}$
4	C_{3+0}	No	D_{7+0}	$[C_{4+0}, D_{7+0}]$	$\{A_{0+0}, B_{2+0}, C_{3+0}, D_{7+0}\}$
5	C_{4+0}	No	D_{8+0}	$[D_{7+0}]$	$\{A_{0+0}, B_{2+0}, C_{3+0}, D_{7+0}\}$
6	D_{7+0}	Yes		$[]$	$\{A_{0+0}, B_{2+0}, C_{3+0}, D_{7+0}\}$

A* Search with h_1



Solution: $A - B - C - D$

f-costs are monotonic

A* Search with Heuristic h_2

Node	h_1	h_2	h_3	h_4	h_5
A	0	7	7	5	5
B	0	5	4	3	7
C	0	4	1	2	2
D	0	0	0	0	0

h_2 is consistent and admissible

A* Search with h_2

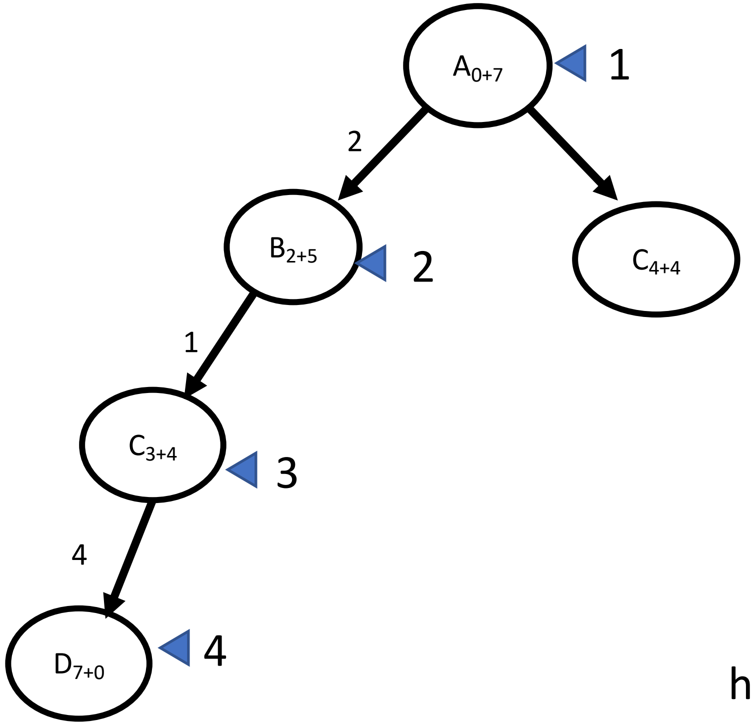


Table 4: A* Search for heuristic h_2; $h_2(A) = 7, h_2(B) = 5, h_2(C) = 4, h_2(D) = 0$

#	Node	Is-Goal(Node)	s	Frontier	Reached
0	A_{0+7}			$[A_{0+7}]$	$\{A_{0+7}\}$
1	A_{0+7}	No	B_{2+5}	$[B_{2+5}]$	$\{A_{0+7}, B_{2+5}\}$
2			C_{4+4}	$[B_{2+5}, C_{4+4}]$	$\{A_{0+7}, B_{2+5}, C_{4+4}\}$
3	B_{2+5}	No	C_{3+4}	$[C_{4+4}, C_{3+4}]$	$\{A_{0+7}, B_{2+5}, \mathbf{C_{3+4}}\}$
4	C_{3+4}	No	D_{7+0}	$[C_{4+4}, D_{7+0}]$	$\{A_{0+7}, B_{2+5}, C_{3+4}, D_{7+0}\}$
5	D_{7+0}	Yes		-	-

f-costs are monotonic

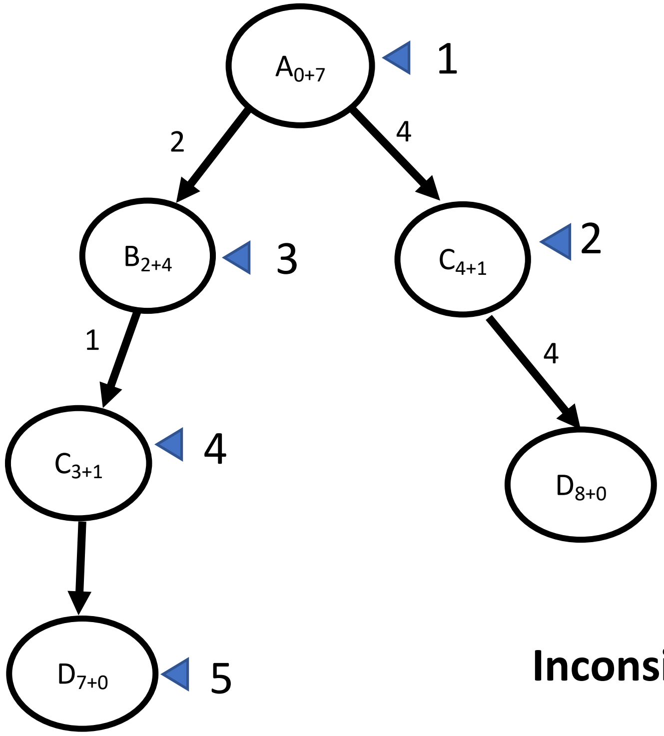
h_2 is an accurate estimate of path costs, so only nodes along optimal path are expanded

Solution: $A - B - C - D$

A* Search with Heuristic h_3

Node	h_1	h_2	h_3	h_4	h_5
A	0	7	7	5	5
B	0	5	4	3	7
C	0	4	1	2	2
D	0	0	0	0	0

A* Search with h_3



$$h_3(A) - h_3(B) = 3 > C(A,B)$$

h_3 is inconsistent but admissible

Table 5: A* Search for heuristic h_3; $h_3(A) = 7, h_3(B) = 4, h_3(C) = 1, h_3(D) = 0$

#	Node	Is-Goal(Node)	s	Frontier	Reached
0	A_{0+7}			$[A_{0+7}]$	$\{A_{0+7}\}$
1	A_{0+7}	No	B_{2+4}	$[B_{2+4}]$	$\{A_{0+7}, B_{2+4}\}$
2			C_{4+1}	$[B_{2+4}, C_{4+1}]$	$\{A_{0+7}, B_{2+4}, C_{4+1}\}$
3	C_{4+1}	No	D_{8+0}	$[B_{2+4}, D_{8+0}]$	$\{A_{0+7}, B_{2+4}, C_{4+1}, D_{8+0}\}$
4	B_{2+4}	No	C_{3+1}	$[D_{8+0}, C_{3+1}]$	$\{A_{0+7}, B_{2+4}, \mathbf{C_{3+1}}, D_{8+0}\}$
5	$\boxed{C_{3+1}}$	No	D_{7+0}	$[D_{8+0}, D_{7+0}]$	$\{A_{0+7}, B_{2+4}, C_{3+1}, \mathbf{D_{7+0}}\}$
6	D_{7+0}	Yes		-	-

Inconsistent: f-costs are nonmonotonic

State C is expanded twice

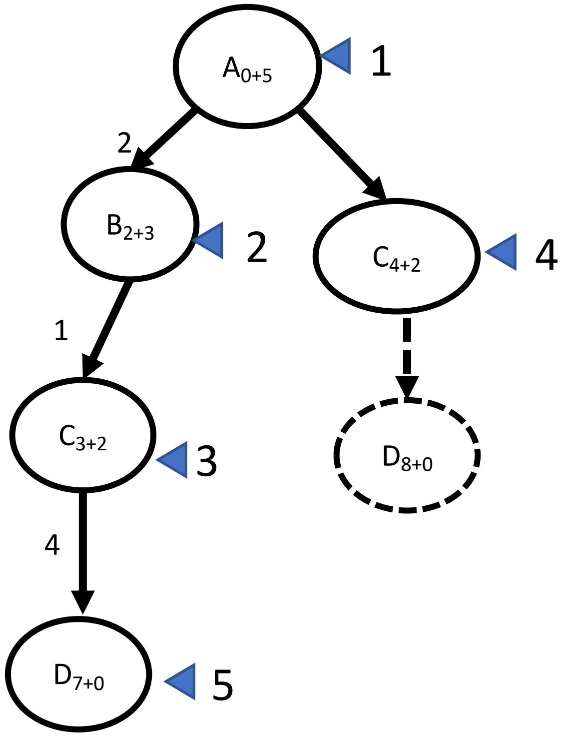
Need to replace state D in *reached*

Solution: $A - B - C - D$

A* Search with Heuristic h_4

Node	h_1	h_2	h_3	h_4	h_5
A	0	7	7	5	5
B	0	5	4	3	7
C	0	4	1	2	2
D	0	0	0	0	0

A* Search with h_4



Solution: $A - B - C - D$

h_4 is consistent and admissible

Table 6: A* Search for heuristic h_4; $h_4(A) = 5, h_4(B) = 3, h_4(C) = 2, h_4(D) = 0$

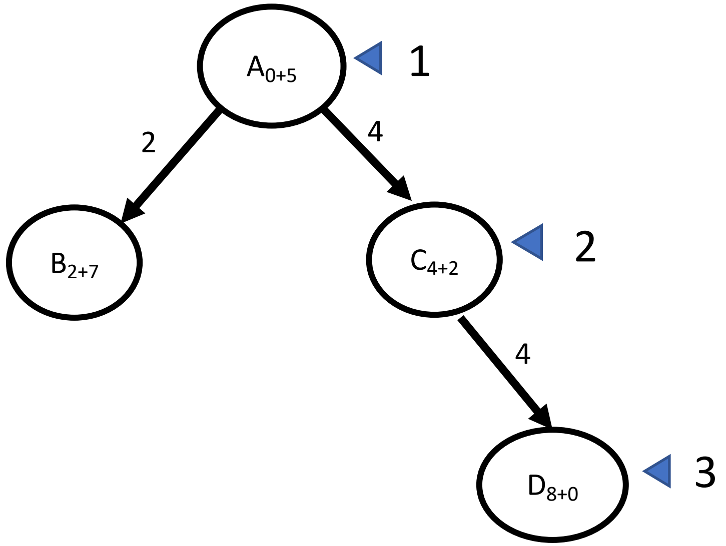
#	Node	Is-Goal(Node)	s	Frontier	Reached
0	A_{0+5}			$[A_{0+5}]$	$\{A_{0+5}\}$
1	A_{0+5}	No	B_{2+3}	$[B_{2+3}]$	$\{A_{0+5}, B_{2+3}\}$
2			C_{4+2}	$[B_{2+3}, C_{4+2}]$	$\{A_{0+5}, B_{2+3}, C_{4+2}\}$
3	B_{2+3}	No	C_{3+2}	$[C_{4+2}, C_{3+2}]$	$\{A_{0+5}, B_{2+3}, \mathbf{C_{3+2}}\}$
4	C_{3+2}	No	D_{7+0}	$[C_{4+2}, D_{7+0}]$	$\{A_{0+5}, B_{2+3}, C_{3+2}, D_{7+0}\}$
5	C_{4+2}	No	D_{8+0}	$[D_{7+0}]$	$\{A_{0+5}, B_{2+3}, C_{3+2}, D_{7+0}\}$
6	D_{7+0}	Yes		-	-

f-costs are monotonic

A* Search with Heuristic h_5

Node	h_1	h_2	h_3	h_4	h_5
A	0	7	7	5	5
B	0	5	4	3	7
C	0	4	1	2	2
D	0	0	0	0	0

A* Search with h_5



Solution: $A - C - D$

$h_5(B) > 5$ So, h_5 is inadmissible

$h_5(B) - h_5(C) = 5 > C(B,C)$ So, h_5 is inconsistent

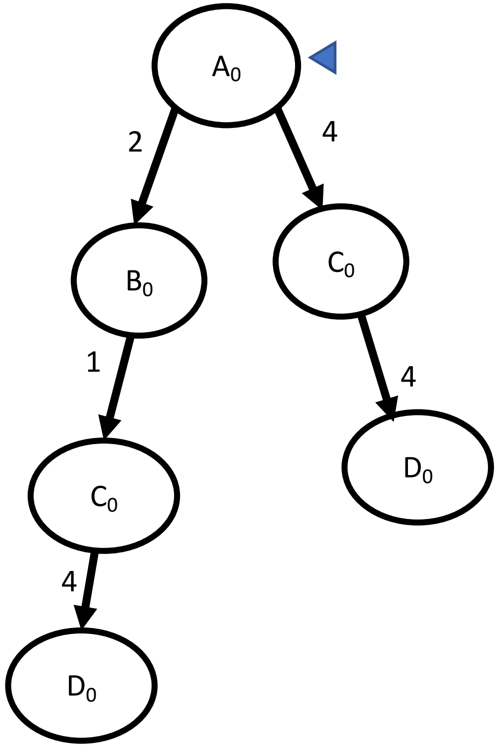
Table 7: A* Search for heuristic h_5; $h_5(A) = 5, h_5(B) = 7, h_5(C) = 2, h_5(D) = 0$

#	Node	Is-Goal(Node)	s	Frontier	Reached
0	A_{0+5}			$[A_{0+5}]$	$\{A_{0+5}\}$
1	A_{0+5}	No	B_{2+7}	$[B_{2+7}]$	$\{A_{0+5}, B_{2+7}\}$
2			C_{4+2}	$[B_{2+7}, C_{4+2}]$	$\{A_{0+5}, B_{2+7}, C_{4+2}\}$
3	C_{4+2}	No	D_{8+0}	$[B_{2+7}, D_{8+0}]$	$\{A_{0+5}, B_{2+7}, C_{4+2}, D_{8+0}\}$
4	D_{8+0}	Yes	-	-	-

Greedy Best First Search

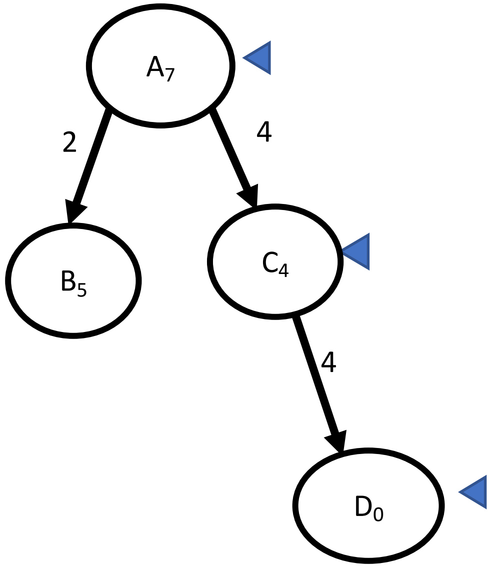
Node	h_1	h_2	h_3	h_4	h_5
A	0	7	7	5	5
B	0	5	4	3	7
C	0	4	1	2	2
D	0	0	0	0	0

Greedy Best-First Search with h_1



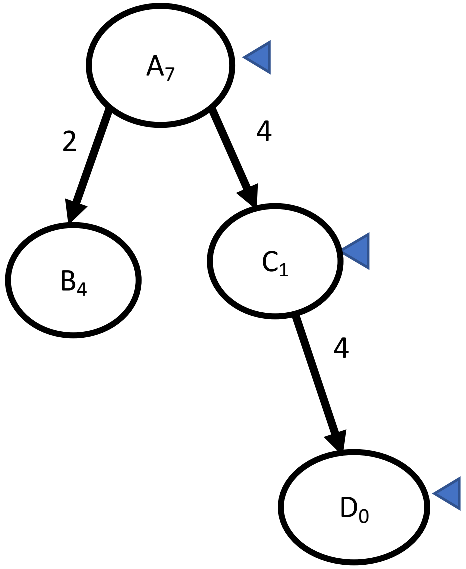
Solution: $A - C - D$
Or $A - B - C - D$

Greedy Best-First Search with h_2



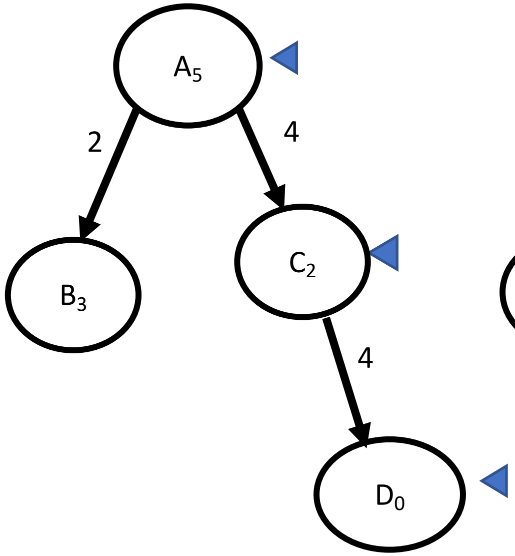
Solution: $A - C - D$

Greedy Best-First Search with h_3



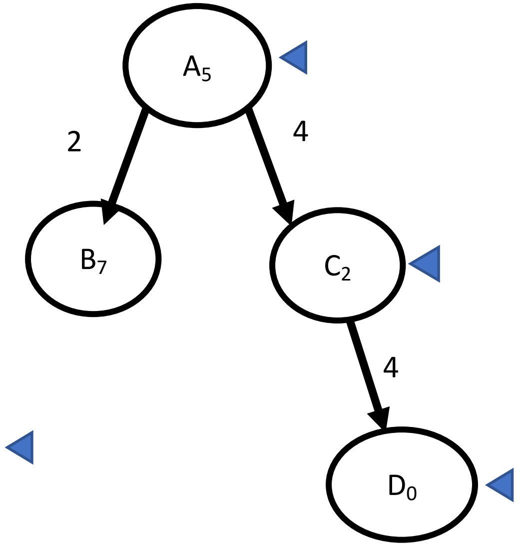
Solution: $A - C - D$

Greedy Best-First Search with h_4



Solution: $A - C - D$

Greedy Best-First Search with h_5



Solution: $A - C - D$

Key Messages

- Inadmissible heuristic -> suboptimal solution
- Admissible -> optimal solution

- Consistent -> monotonic f-costs -> efficient
- Inconsistent -> nonmonotonic f-costs -> Inefficient

8-Puzzle and Heuristics

3. In informed search, we relax problem settings to enable the estimation of path-cost from a given state to a goal state. In an n-puzzle, assume that square B is blank; if square A is horizontally or vertically adjacent to square B, then a tile can move from square A to square B horizontally or vertically, respectively. Consider the following heuristic costs:

   - Heuristic h_1: number of misplaced tiles compared to goal state
   - Heuristic h_2: Manhattan distance of each tile to its position in goal state. It is defined as

$$h_2 = \sum_{i=1}^{n} |x_{i,a} - x_{i,t}| + |y_{i,a} - y_{i,t}| \tag{1}$$

     where $(x_{i,a}, y_{i,a})$ is the actual position of a tile and $(x_{i,t}, y_{i,t})$ is its target position. The top left position is (1,1) and the bottom right position is (3,3).

(a) Show that both heuristics are admissible. Which heuristic can be a good estimate of the actual path-cost?

Solution:

(a) We can show that they're both admissible by considering two "relaxed problems", which are simplified versions of the game. The resulting optimal heuristic for such games will necessarily be admissible, since the relaxed problems allow for shortcuts.

Actual game:

A tile can move from A to B if A is adjacent to B and B is blank.

Relaxed game for heuristic h_1:

A misplaced tile can be placed directly into its correct position. In this case the optimal heuristic is h_1, since, if we have n misplaced tiles, we place these n tiles into their respective positions.

Relaxed game for heuristic h_2:

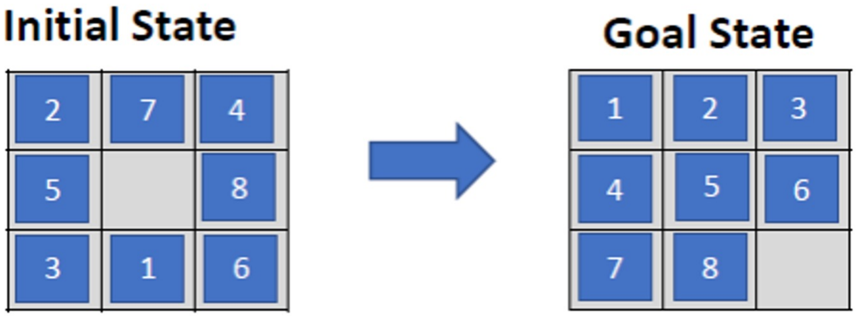
A misplaced tile can be moved to any horizontally or vertically adjacent tile regardless of whether it is empty.

The resulting optimal heuristic is h_2 Since both h_1 and h_2 are optimal for the relaxed problem, they are admissible for the actual problem.

As an aside, this is the main way to derive admissible heuristics.

Note that $h_2 \geq h_1$, i.e. h_2 dominates h_1 since version 2 does not allow for a direct placement.

(b) Calculate the heuristic costs for the initial state of 8-puzzle



Misplaced Tiles:

$$h_1 = 8$$

Manhattan Distance:

Table 9: Calculation of Manhattan Distance for Initial State

Tile (i)	Target Position $(x_{i,t}, y_{i,t})$	Actual Position $(x_{i,a}, y_{i,a})$
1	(1,1)	(3,2)
2	(1,2)	(1,1)
3	(1,3)	(3,1)
4	(2,1)	(1,3)
5	(2,2)	(2,1)
6	(2,3)	(3,3)
7	(3,1)	(1,2)
8	(3,2)	(2,3)

$$\begin{aligned} h_2 &= \sum_{i=1}^{8} |x_{i,a} - x_{i,t}| + |y_{i,a} - y_{i,t}| \\ &= 3 + 1 + 4 + 3 + 1 + 1 + 3 + 2 = 18 \end{aligned}$$

Manhattan Distance Vs Misplaced Tiles

- Manhattan distance is always greater than or equal to misplaced tiles
- Manhattan dominates misplaced tiles
- Therefore, Manhattan distance is better than misplaced tiles