National University of Singapore
School of Computing
IT5005 Artificial Intelligence

Propositional Logic 2

1. Use resolution to show that the statement

$$(P \vee Q) \wedge (\neg P \vee R) \wedge (\neg P \vee \neg R) \wedge (P \vee \neg Q)$$

   is UNSAT.

2. Suppose we are given the following premises:

   - $P_1 : P \Rightarrow Q$

   - $P_2 : R \Rightarrow P$

   - $P_3 : \neg Q$

   - $P_4 : R \vee P \vee S$

   Use resolution to prove that S is always True under the premises.

3. Consider the following Horn clauses:

   - $P_1 : Rain$

   - $P_2 : Weekday$

   - $P_3 : Rain \Rightarrow Wet$

   - $P_4 : Wet \wedge Weekday \Rightarrow Traffic$

   - $P_5 : Traffic \wedge Careless \Rightarrow Accident$

   Prove $Traffic$ with both forward and backward chaining algorithms

   Could you prove $Accident$ for the given KB?

4. Consider the following knowledge base (KB) with propositional symbols A, B, C, and D. The KB contains two sentences **P1** and **P2**:

$$\begin{aligned} \mathbf{P1} &: \quad C \vee D \\ \mathbf{P2} &: \quad B \Rightarrow ((A \wedge B) \Rightarrow C) \end{aligned}$$

   Identify the models of the sentence **P2**.

5. Using logic, you need to determine whether the following statement is true: "The vase was broken". The following clues are given:

   - **R1**: Charlie was outside.
   - **R2**: The vase was broken if and only if the cat was in the house or Bob was playing indoors.
   - **R3**: If Bob was playing indoors, then Charlie was outside.
   - **R4**: If Charlie was outside, then cat was in the house.

   Use the propositional symbols shown in Table 1 to represent the sentences in the puzzle. Answer the following questions:

   Table 1: Propositional Symbols

O: Charlie was outside.	**B**: The vase was broken.
H: The cat was in the house.	**I**: Bob was playing indoors.

   (a) Translate the clues **R1** to **R4** into propositional form.

   (b) Convert the above sentences in propositional form to CNF.

   (c) Check whether the statement "The vase was broken" is true or not using resolution-refutation algorithm.