National University of Singapore
School of Computing
IT5005 Artificial Intelligence

Probabilistic Reasoning

1. Consider the joint distribution table for three variables $ToothAche$, $Cavity$, and $Catch$.

$ToothAche$	$Cavity$	$Catch$	$P(ToothAche, Cavity, Catch)$
t	t	t	0.108
t	t	f	0.012
t	f	t	0.016
t	f	f	0.064
f	t	t	0.072
f	t	f	0.008
f	f	t	0.144
f	f	f	0.576

   (a) Show that $Toothache$ and $Catch$ are independent given $Cavity$.

   (b) Show that

$$\mathbf{P}(ToothAche, Cavity, Catch) = \mathbf{P}(Cavity)\mathbf{P}(ToothAche|Cavity)\mathbf{P}(Catch|Cavity)$$

2. Consider the Bayesian network shown in Fig. 1. Calculate $\mathbf{P}(J|a)$ and $\mathbf{P}(J|b)$.

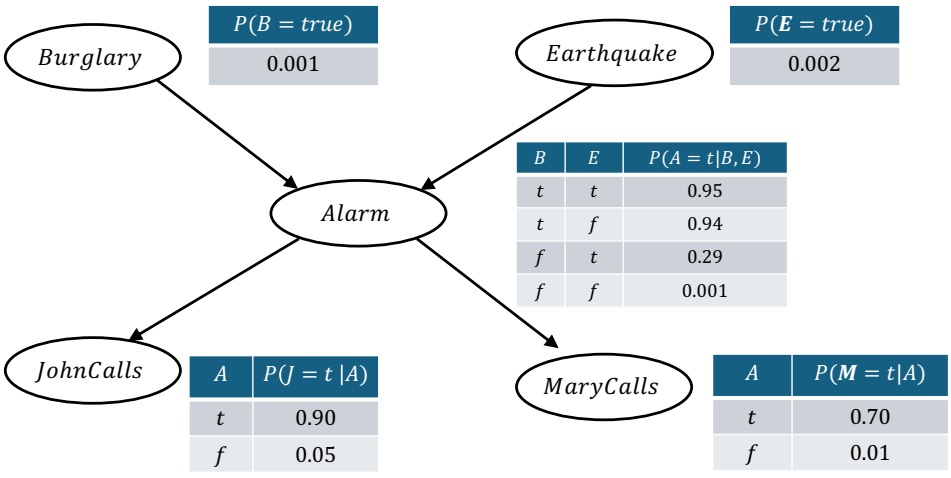


Figure 1: Bayes Net (Fig. 13.2 of AIMA4e)

3. Consider the Bayesian network shown in Fig. 2. Identify whether the following Independence's are guaranteed to be true or not.

   (a) $X_1 \perp X_2$

   (b) $X_1 \perp X_2 \mid X_4$

   (c) $X_3 \perp X_7 \mid X_5$

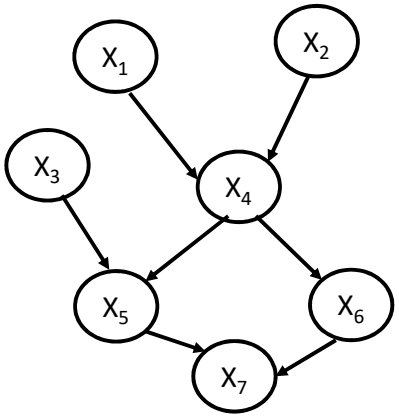
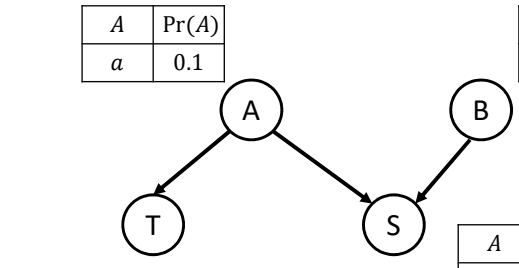


Figure 2: Bayes Net

4. Consider the Bayesian network shown in Fig. 3.



A	$\Pr(A)$
a	0.1

B	$\Pr(B)$
b	0.5

A	$P(T = t\|A)$
a	1
$\neg a$	0.2

A	B	$P(S =$s$\|A, B)$
a	b	1
a	$\neg b$	0.8
$\neg a$	b	1
$\neg a$	$\neg b$	0

Figure 3: Bayesian Network for Disease

(a) Compute the probability $P(\neg a, \neg t, b, s)$

(b) What is the probability that the patient has disease A given that they have symptom S and test T returns positive?

(c) Suppose that both diseases become more likely as person ages. Add a new variable and the related arcs to the Bayesian network to reflect this new knowledge. Note that you only need to draw the revised Bayesin network and the CPTs need not be defined for the new variable.