

Database Programming and Management

Tutorial 8: Normalization

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$$R = \{A, B, C, D, E, F, G, H\}$$
$$\Sigma = \{ \{A\} \rightarrow \{C, E\}, \quad \{A, B\} \rightarrow \{D\}, \quad \{F\} \rightarrow \{H\}, \quad \{C, E\} \rightarrow \{A\}, \quad \{B, C, E\} \rightarrow \{D\}, \\ \{A, B, F\} \rightarrow \{D, G\}, \quad \{B, C, E, F\} \rightarrow \{G\} \}$$

1.(a) Is R with Σ in 3NF?

A relation R is in **3NF** if, for every functional dependency $X \rightarrow Y$ in Σ :

1. The dependency is **trivial** (i.e, $Y \subseteq X$),

OR

2. X is a **superkey** for R,

OR

3. Every attribute in Y is **prime** (i.e., part of some candidate key).



NOTE: Satisfying any one of the conditions will suffice and hence OR.

1.(a) Is R with Σ in 3NF?

Consider $\{A, B\} \rightarrow \{D\}$.

1. The dependency is **trivial** (i.e, $Y \subseteq X$),

OR

2. X is a **superkey** for R ,

OR

3. Every attribute in Y is **prime**



NOTE: To prove something is NOT in 3NF, all 3 conditions must be False.

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1.(a) Is R with Σ in 3NF?

Consider $\{A, B\} \rightarrow \{D\}$.

- It is non-trivial (i.e., $\{D\} \not\subseteq \{A, B\}$).
- $\{A, B\}$ is not a key (i.e., $\{A, B\}^+ = \{A, B, C, D, E\} \subset R$).
 $\{A, B\}$ is *also* not a superset of a key (keys are $\{A, B, F\}$ and $\{B, C, E, F\}$).
 \Rightarrow This is a simpler way to check superkey if we have computed keys.

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- D is not a prime attribute.

Prime attributes are $\{A, B, C, E, F\}$.

1. The dependency is **trivial** (i.e, $Y \subseteq X$),

OR

2. X is a **superkey** for R ,

OR

3. Every attribute in Y is **prime**

NO! NOT in 3NF!

1.(b) Is R with Σ in BCNF?

A relation R is in **BCNF** if, for every functional dependency $X \rightarrow Y$ in Σ :

1. The dependency is **trivial** (i.e, $Y \subseteq X$),

OR

2. X is a **superkey** for R,



NOTE:

- Every BCNF relation is in 3NF, but not every 3NF relation is in BCNF
- BCNF is stricter!

1.(b) Is R with Σ in BCNF?

From Question 1a, we know that R with Σ is not in 3NF. Therefore, it cannot be in BCNF. However, let us verify this from the definition of BCNF. Obviously, we can consider $\{A, B\} \rightarrow \{D\}$, but let us consider a different functional dependency. Consider $\{A\} \rightarrow \{C\}$.

NO! NOT in BCNF!

1.(b) Is R with Σ in BCNF?

From Question 1a, we know that R with Σ is not in 3NF. Therefore, it cannot be in BCNF. However, let us verify this from the definition of BCNF. Obviously, we can consider $\{A, B\} \rightarrow \{D\}$, but let us consider a different functional dependency. Consider $\{A\} \rightarrow \{C\}$.

- It is non-trivial (i.e., $\{C\} \not\subseteq \{A\}$).
- $\{A\}$ is not a superkey (i.e., $\{A\}^+ = \{A, C, E\} \subset R$).
 $\{A\}$ is *also* not a superset of a key (keys are $\{A, B, F\}$ and $\{B, C, E, F\}$).
⇒ This is a simpler way to check superkey if we have computed keys.

NO! NOT in BCNF!

2.(a) Decompose R with Σ into a lossless-join 3NF.

Algorithm : 3NF Synthesis (Bernstein Algorithm)

When a relation is not in 3NF, we can *synthesize* a schema in 3NF from a *canonical cover* of the set of functional dependencies.

- ▶ For each functional dependency $X \rightarrow Y$ in the minimal cover, create a relation

$$R_i = X \cup Y$$

Unless it already exists or is *subsumed* by another relation

- ▶ If none of the created relations contain one of the keys, pick any candidate key and create a relation with that candidate key.

2.(a) Decompose R with Σ into a lossless-join 3NF.

We can start from a canonical cover directly.

$$\{ \{A\} \rightarrow \{C, E\}, \quad \{F\} \rightarrow \{H\}, \quad \{C, E\} \rightarrow \{A\}, \quad \{B, C, E\} \rightarrow \{D\}, \quad \{B, C, E, F\} \rightarrow \{G\} \}$$

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For each functional dependency, we synthesize a fragment.

$$\{ \{A, C, E\}, \quad \{F, H\}, \quad \{A, C, E\}, \quad \{B, C, D, E\}, \quad \{B, C, E, F, G\} \}$$

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$$\{ \{A, C, E\}, \quad \{F, H\}, \quad \{A, C, E\}, \quad \{B, C, D, E\}, \quad \{B, C, E, F, G\} \}$$

If there is any fragments that can be *subsumed*, we remove them from the result.

$$\{ \{A, C, E\}, \quad \{F, H\}, \quad \cancel{\{A, C, E\}}, \quad \{B, C, D, E\}, \quad \{B, C, E, F, G\} \}$$

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We can start from a canonical cover directly.

$$\{ \{A\} \rightarrow \{C, E\}, \quad \{F\} \rightarrow \{H\}, \quad \{C, E\} \rightarrow \{A\}, \quad \{B, C, E\} \rightarrow \{D\}, \quad \{B, C, E, F\} \rightarrow \{G\} \}$$

For each functional dependency, we synthesize a fragment.

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If there is any fragments that can be *subsumed*, we remove them from the result.

$$\{ \{A, C, E\}, \quad \{F, H\}, \quad \cancel{\{A, C, E\}}, \quad \{B, C, D, E\}, \quad \{B, C, E, F, G\} \}$$

If no candidate keys is present as a subset of any fragment, we add. Luckily, the key $\{B, C, E, F\}$ is a subset of $\{B, C, E, F, G\}$. So, we do not have to add another relation.

$$\{ \{A, C, E\}, \quad \{F, H\}, \quad \{B, C, D, E\}, \quad \{B, C, E, F, G\} \}$$

If no key is present

We know the keys of R are $\{A, B, F\}$ and $\{B, C, E, F\}$

Suppose after subsumption we ended up with this:

$$\{ \{A, C, E\}, \{F, H\}, \{B, C, D, E\}, \boxed{\{B, C, E, F\}} \}$$

Add any of the keys! That's it

NOTE:

- If the questions asks you to just make fragments - Make fragments using the **3NF synthesis algorithm** and you can stop there.
- Skip **FD projections** and **BCNF checks** unless the question **explicitly asks** or you need to verify extras.

Projection steps

5 steps to compute projection of R with Σ onto X .

1. Find all subset X' of attributes of X .
2. For each subset X' , compute the closure (i.e., $\varphi_1 := \text{AttrClose}(X', \Sigma)$).
3. Keep only the relevant attributes (i.e., $\varphi_2 := \varphi_1 \cap X$).
4. Remove attributes that does not contribute new information (i.e., $\varphi_3 := \varphi_2 - X'$).
5. If φ_3 is not empty, form a functional dependency $X' \rightarrow \varphi_3$.



Question:

Calculate the projection R with Σ onto $X = \{A, C, E\}$

Calculating Projections

Do this for
all possible
subsets

- 1 = choose subset $X' \subseteq X$
- 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
- 3 = $\varphi_2 := \varphi_1 \cap X$
- 4 = $\varphi_3 := \varphi_2 - X'$
- 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Onto $X = \{A, C, E\}$ (call this Σ_1)

| Step 1 X' | Step 2 φ_1 (closure) | Step 3 φ_2 | Step 4 φ_3 | Step 5 (FD) |
|-------------|----------------------------------|--------------------|--------------------|-------------------------------------|
| {A} | {A,C,E} (by $A \rightarrow CE$) | {A,C,E} | {C,E} | A → CE |
| {C,E} | {A,C,E} (by $CE \rightarrow A$) | {A,C,E} | {A} | CE → A |
| {A,C} | {A,C,E} | {A,C,E} | {E} | AC → E (<i>implied by A → CE</i>) |
| {A,E} | {A,C,E} | {A,C,E} | {C} | AE → C (<i>implied by A → CE</i>) |

All other X' give $\varphi_3 = \emptyset$. Keep the bold FDs for a minimal projection: $\Sigma_1 = \{\mathbf{A \rightarrow CE}, \mathbf{CE \rightarrow A}\}$.

Calculating Projections

Exhaustive check (all 8 subsets of X)

| Step 1 X' | Step 2 $\varphi_1 = (X')^+$ | Step 3 $\varphi_2 = \varphi_1 \cap X$ | Step 4 $\varphi_3 = \varphi_2 \setminus X'$ | Step 5 (FD if any) | |
|-------------|--------------------------------------|---------------------------------------|---|---|---|
| \emptyset | \emptyset | \emptyset | \emptyset | — | 1 = choose subset $X' \subseteq X$ |
| $\{E\}$ | $\{E\}$ | $\{E\}$ | \emptyset | — | 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$ |
| $\{C\}$ | $\{C\}$ | $\{C\}$ | \emptyset | — | 3 = $\varphi_2 := \varphi_1 \cap X$ |
| $\{A\}$ | $\{A,C,E\}$ (by $A \rightarrow CE$) | $\{A,C,E\}$ | $\{C,E\}$ | $A \rightarrow CE$ | 4 = $\varphi_3 := \varphi_2 - X'$ |
| $\{C,E\}$ | $\{A,C,E\}$ (by $CE \rightarrow A$) | $\{A,C,E\}$ | $\{A\}$ | $CE \rightarrow A$ | 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$ |
| $\{A,E\}$ | $\{A,C,E\}$ | $\{A,C,E\}$ | $\{C\}$ | $AE \rightarrow C$ (implied by $A \rightarrow CE$) | |
| $\{A,C\}$ | $\{A,C,E\}$ | $\{A,C,E\}$ | $\{E\}$ | $AC \rightarrow E$ (implied by $A \rightarrow CE$) | |
| $\{A,C,E\}$ | $\{A,C,E\}$ | $\{A,C,E\}$ | \emptyset | — | |

Calculating Projections

Do this for
all possible
subsets

- 1 = choose subset $X' \subseteq X$
- 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
- 3 = $\varphi_2 := \varphi_1 \cap X$
- 4 = $\varphi_3 := \varphi_2 - X'$
- 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Onto $X = \{F, H\} (\Sigma_2)$

| Step 1 X' | Step 2 φ_1 | Step 3 φ_2 | Step 4 φ_3 | Step 5 |
|-------------|-------------------------------|--------------------|--------------------|-------------------|
| {F} | {F,H} (by $F \rightarrow H$) | {F,H} | {H} | $F \rightarrow H$ |

Other subsets yield $\varphi_3 = \emptyset$. So $\Sigma_2 = \{F \rightarrow H\}$.

📌 Early Exit Rule:

Let $S = \text{AttrClose}(X', \Sigma)$. If $\varphi_2 = X'$, stop for this X' (no FD).

Calculating Projections

Do this for
all possible
subsets

- 1 = choose subset $X' \subseteq X$
- 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
- 3 = $\varphi_2 := \varphi_1 \cap X$
- 4 = $\varphi_3 := \varphi_2 - X'$
- 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Onto $X = \{B, C, D, E\}$ (Σ_3)

| Step 1 X' | Step 2 φ_1 | Step 3 φ_2 | Step 4 φ_3 | Step 5 |
|-------------|------------------------------|--------------------|--------------------|--------------|
| {B,C,E} | {B,C,E,D} (by BCE→D) | {B,C,E,D} | {D} | BCE→D |

All other subsets inside $\{B, C, D, E\}$ do not gain D (or add nothing beyond themselves), so $\varphi_3 = \emptyset$.

Thus $\Sigma_3 = \{\text{BCE} \rightarrow \text{D}\}$.

Calculating Projections

Do this for
all possible
subsets

- 1 = choose subset $X' \subseteq X$
- 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
- 3 = $\varphi_2 := \varphi_1 \cap X$
- 4 = $\varphi_3 := \varphi_2 - X'$
- 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Onto $X = \{B, C, E, F, G\}$ (Σ_4)

| Step 1 X' | Step 2 φ_1 | Step 3 φ_2 | Step 4 φ_3 | Step 5 |
|-------------|--|--------------------|--------------------|------------------------|
| {B,C,E,F} | {B,C,E,F,G} (by BCEF → G) | {B,C,E,F,G} | {G} | BCEF → G |

Others give $\varphi_3 = \emptyset$. Hence $\Sigma_4 = \{\text{BCEF} \rightarrow \text{G}\}$.

Final Projections

Do this for all
possible subsets

- 1 = choose subset $X' \subseteq X$
- 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
- 3 = $\varphi_2 := \varphi_1 \cap X$
- 4 = $\varphi_3 := \varphi_2 - X'$
- 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$
- $\{B, C, D, E\}$ with $\Sigma_3 = \{ \{B, C, E\} \rightarrow \{D\} \}$
- $\{B, C, E, F, G\}$ with $\Sigma_4 = \{ \{B, C, E, F\} \rightarrow \{G\} \}$

Mistake : “Keep only FDs that use attributes from X”

$R(A, B, C)$, $\Sigma = \{ A \rightarrow B, B \rightarrow C \}$, project onto $X = \{A, C\}$.

Naive keeps none (since both FDs mention B).

Correct (by the algorithm):

- For $X' = \{A\}$: $A^+ = \{A, B, C\} \Rightarrow (A^+ \cap X) - \{A\} = \{C\} \Rightarrow A \rightarrow C$
Projection is $\{ A \rightarrow C \}$.
The naive method **misses** $A \rightarrow C$ (transitive via B).



NOTE:

Using shortcuts can lead to error in **MANY** different ways!
This is not the only one!

Projection – Caution Points

- Projection is **error-prone** → don't skip steps!
- Shortcuts can omit valid **derived FDs**.
- Always follow all **5 steps** for correctness .
- Tedious  , but ensures **accurate results**.
-  Shortcuts **don't guarantee** correctness.

(b) Is the result dependency preserving?

Definition

- Let R be a relation schema and Σ a set of functional dependencies (FDs).
- Decompose R into $\Delta = \{R_1, \dots, R_n\}$.
- For each piece R_i , let $\Sigma|_{R_i}$ be the projection of Σ onto R_i .
- The decomposition is *dependency-preserving* iff

$$\Sigma^+ = (\Sigma|_{R_1} \cup \dots \cup \Sigma|_{R_n})^+.$$

Note (how to check)

- For every FD $X \rightarrow Y$ in Σ , compute X^+ using $\Sigma|_{R_1} \cup \dots \cup \Sigma|_{R_n}$.
- If $Y \subseteq X^+$ for all FDs in Σ , the decomposition is dependency-preserving.

(b) Is the result dependency preserving?

Yes. This is guaranteed by the algorithm. 3NF synthesis algorithm produces lossless-join dependency-preserving decomposition in 3NF.

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The 3NF synthesis only guarantees that the result is in 3NF, it may not be in BCNF.

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- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$



NOTE:

All non-trivial FDs here ($A \rightarrow C$, $A \rightarrow E$, $CE \rightarrow A$, and $A \rightarrow CE$) have $LHS \in \{A, CE\}$ (keys) \Rightarrow BCNF

(c) Is the result in BCNF?

The 3NF synthesis only guarantees that the result is in 3NF, it may not be in BCNF.

- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$

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- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$
- $\{B, C, D, E\}$ with $\Sigma_3 = \{ \{B, C, E\} \rightarrow \{D\} \}$

(c) Is the result in BCNF?

The 3NF synthesis only guarantees that the result is in 3NF, it may not be in BCNF.

- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$
- $\{B, C, D, E\}$ with $\Sigma_3 = \{ \{B, C, E\} \rightarrow \{D\} \}$
- $\{B, C, E, F, G\}$ with $\Sigma_4 = \{ \{B, C, E, F\} \rightarrow \{G\} \}$

YES! It is in BCNF!

(c) Is the result in BCNF?

The 3NF synthesis only guarantees that the result is in 3NF, it may not be in BCNF.

- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$
- $\{B, C, D, E\}$ with $\Sigma_3 = \{ \{B, C, E\} \rightarrow \{D\} \}$
- $\{B, C, E, F, G\}$ with $\Sigma_4 = \{ \{B, C, E, F\} \rightarrow \{G\} \}$

Note that it is not always the case that we are lucky to obtain a BCNF decomposition using 3NF synthesis, but it may happen.

 One Last Lap

❤️ **One last class to go!** (BCNF + Extra problems + doubt clearing) — hope to all of you there!  

💬 If you enjoyed the module, please take a moment to leave your **feedback/rating** after the course — it really helps me grow as a teacher!



👋 I'll see you all around campus — don't hesitate to say hi or wave!  

Thank you for joining!

Got questions? Post them on the forum or email me:

biswadeep@u.nus.edu

(I reply within 2 working days — faster if coffee is strong ☕)

Because your learning matters to me! 😊



You can generate all implied FDs by running the above closure for **every** subset $X \subseteq UX \setminus \text{subseteq } UX \subseteq U$ ($U = \text{all attributes}$):

```
 $\Sigma_{\text{plus}} := \emptyset$ 
for each  $X \subseteq U$ :
   $S := \text{AttrClosure}(X, \Sigma)$ 
  for each attribute  $A$  in  $(S - X)$ :
    add FD  $X \rightarrow A$  to  $\Sigma_{\text{plus}}$  // (often we ignore trivial  $A \in X$ )
```

Subsumption

Keep

In some cases like $R = \{A, B, C\}$ with $\Sigma = \{ \{A,B\} \rightarrow \{C\}, \{C\} \rightarrow \{B\} \}$, we cannot remove $R_2 = \{B, C\}$ even when it is subsumed by $R_1 = \{A, B, C\}$.

$R_2(B, C)$

```
CREATE TABLE R2 (
    B  INT,
    C  INT
        UNIQUE,
    PRIMARY KEY (B, C)
);
;
```

$R_1(A, B, C)$

```
CREATE TABLE R1 (
    A  INT,
    B  INT,
    C  INT,
    PRIMARY KEY (A, B),
    FOREIGN KEY (B, C) REFERENCES R2(B, C)
);
;
```

Lemma #1: Lossless-Join Binary Decomposition

A **binary decomposition** of R into R_1 and R_2 is lossless-join if $R = R_1 \cup R_2$ and $(R_1 \cap R_2) \rightarrow R_1$ or $(R_1 \cap R_2) \rightarrow R_2$

Lemma #2: Lossless-Join Decomposition

A **decomposition** of R into R_1, R_2, \dots, R_n is lossless-join if there exists **at least one sequence** of binary lossless-join decomposition that generates that decomposition.

Note

If $(R_1 \cap R_2)$ is the primary key of one of the two tables, then it can be a foreign key in the other table referencing the primary key.

0) Pick an anchor that contains a key

Compute $(BCEF)^*$ under Σ :

- $BCEF \rightarrow G$ ($BCEF \rightarrow G$) \Rightarrow add G
- $BCE \rightarrow D$ ($BCE \rightarrow D$) \Rightarrow add D
- $CE \rightarrow A$ ($CE \rightarrow A$) \Rightarrow add A
- $F \rightarrow H$ ($F \rightarrow H$) \Rightarrow add H

So $(BCEF)^* = A B C D E F G H \Rightarrow BCEF$ is a key.

Fragment $R_4(BCEFG)$ contains BCEF \Rightarrow use R_4 as the anchor.

1) Join $R_4(BCEFG)$ with $R_3(BCDE)$

- Intersection: $R_4 \cap R_3 = \{B, C, E\}$.
- Check binary test: does $\{B, C, E\} \rightarrow R_3$?
Yes: $\{B, C, E\} \rightarrow D$ (given) and trivially $\rightarrow \{B, C, E\}$, so $\{B, C, E\} \rightarrow BCDE$.

✓ Step 1 is lossless.

2) Join the result with $R_1(ACE)$

(Current attributes after step 1: $\{B, C, E, F, G, D\}$)

- Intersection: $\{C, E\}$.
- Check: $\{C, E\} \rightarrow R_1$?
Yes: $\{C, E\} \rightarrow A$ (given) and trivially $\rightarrow \{C, E\} \Rightarrow \{C, E\} \rightarrow ACE$.

✓ Step 2 is lossless.

3) Join the result with $R_2(FH)$

(Current attributes after step 2: $\{A, B, C, D, E, F, G\}$)

- Intersection: $\{F\}$ (H isn't in the current set yet).
- Check: $\{F\} \rightarrow R_2$?
Yes: $\{F\} \rightarrow H$ (given) and trivially $\rightarrow F \Rightarrow \{F\} \rightarrow FH$.

Problem Overview

Design a relational schema for the management of coffee bean, drinks and cafes

The Coffee Bean Entity



- Identified by *BrandName OR (Cultivar, Region)*
- One bean → many drinks
- Attributes: (*BrandName, Cultivar, Region*)
- PK: (*BrandName*) or (*Cultivar, Region*)

Drink Entity



- Made from **one coffee bean**
- Name unique **per bean**
- Attributes: (*BeanID, DrinkName, Price*)
- PK: (*BeanID, DrinkName*)

Branch



- Represents a **physical coffee shop branch**
- Each branch has a **unique name**
- Attributes:** (*BranchName, Address*)
- PK:** (*BranchName*)