Database Programming and Management

Tutorial 7: Functional Dependencies

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Problem Overview

Design a relational schema for the management of coffee bean, drinks and cafes

The Coffee Bean Entity

- Identified by *BrandName* **OR** *(Cultivar, Region)*
- One bean → many drinks
- Attributes: *(BrandName, Cultivar, Region)*
- PK: *(BrandName)* or *(Cultivar, Region)*

Drink Entity

- Made from **one coffee bean**
- Name unique **per bean**
- Attributes: *(BeanID, DrinkName, Price)*
- PK: *(BeanID, DrinkName)*

Branch

- Represents a **physical coffee shop branch**
- Each branch has a **unique name**
- **Attributes:** (BranchName, Address)
- **PK:** (BranchName)

1(a). Given the schema find the mapping of vars and letters

$$R = \{A, B, C, D, E, F, G, H\}$$

$$\Sigma = \{ \{A\} \rightarrow \{C, E\},\ \ \{A, B\} \rightarrow \{D\},\ \ \{F\} \rightarrow \{H\},\ \ \{C, E\} \rightarrow \{A\},\ \ \{B, C, E\} \rightarrow \{D\},$$
$$\{A, B, F\} \rightarrow \{D, G\},\ \ \{B, C, E, F\} \rightarrow \{G\} \}$$

Finding Functional Dependencies

- You may find it easier to **first draw an ER diagram** based on the text description of the application but **it is optional**.

- From the ER diagram, **figure out the functional dependencies (FDs)**.

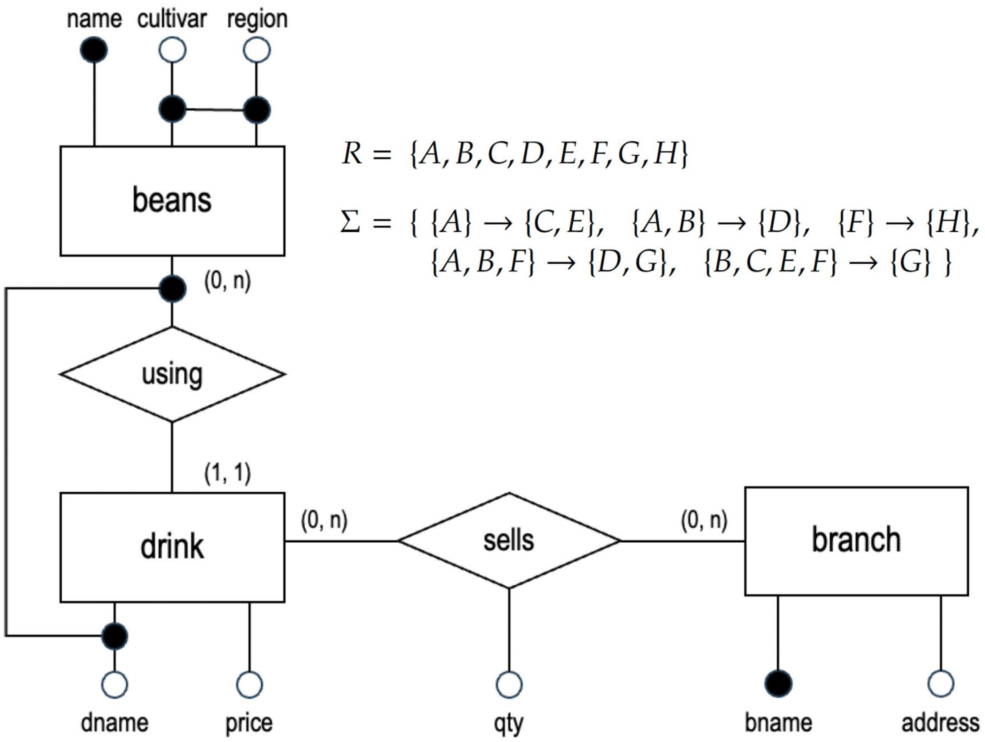
- Alternatively, once you have the ER diagram, you can **translate it into schema** and then derive the FDs.

Finding Functional Dependencies from ER

> 📌NOTE: FDs come from multiple sources—ER, schema, the problem text, and domain rules ("common sense"). The bullets below are examples, **not exhaustive**; use judgment to infer others.

- **Entity:** key → all other (non-key) attributes.
- **(0,1) or (1,1) participation:** key of the ≤1 side → relationship attributes **and** attributes of the other entity.
    - **Exception:** if the (1,1) involves a **weak entity**, the strong entity's key **alone does not** determine the weak entity; you need **(strong key + weak partial key)**.
- **All sides (0,n)/(1,n):** the **concatenation of keys of all participating entities** → relationship attributes (no single entity key determines the others).
- **Relationships with their own attrs: relationship key** → relationship non-key attributes.

Drawing the ER diagram

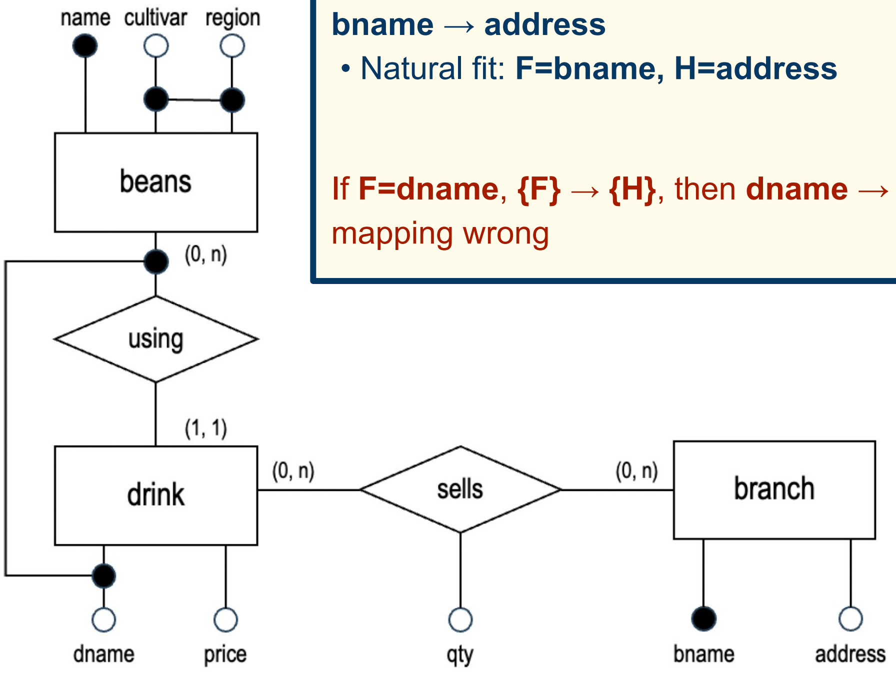


$$R = \{A, B, C, D, E, F, G, H\}$$

$$\Sigma = \{ \{A\} \rightarrow \{C, E\}, \quad \{A, B\} \rightarrow \{D\}, \quad \{F\} \rightarrow \{H\}, \quad \{C, E\} \rightarrow \{A\}, \quad \{B, C, E\} \rightarrow \{D\}, \quad \{A, B, F\} \rightarrow \{D, G\}, \quad \{B, C, E, F\} \rightarrow \{G\} \}$$

Drawing the ER diagram

A=name
C=cultivar
E=region
F=bname
H=address



name ↔ {cultivar, region}

- Natural fit: **A=name, C=cultivar, E=region**

bname → address

- Natural fit: **F=bname, H=address**

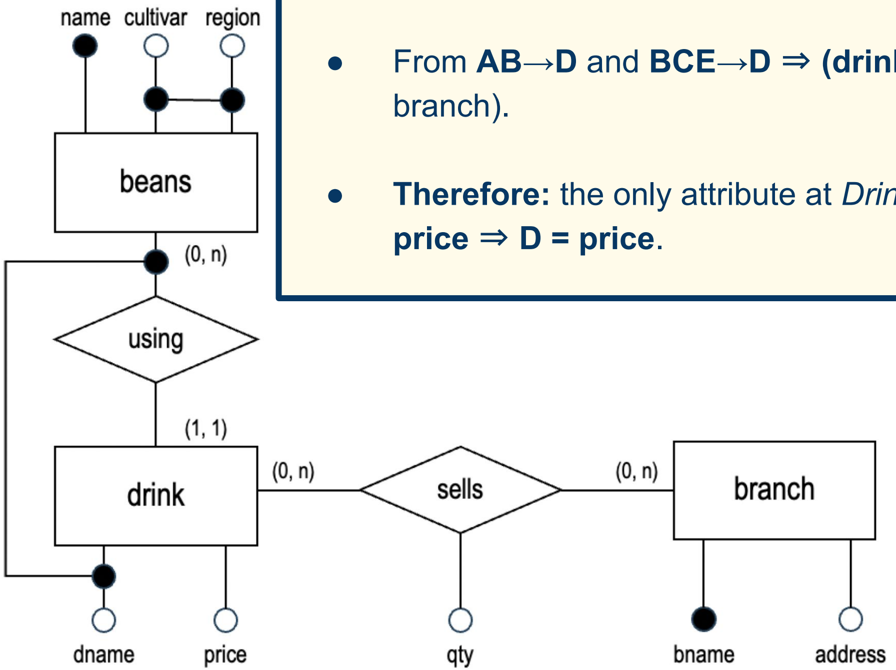
If **F=dname**, **{F}** → **{H}**, then **dname** → **address** ❌ → mapping wrong

Drawing the ER diagram

A=name
B=dname
C=cultivar
D=price
E=region
F=bname
H=address

Identify B (only in determinants with A/CE and F)

- B pairs with A (or CE) to determine **D**
- Natural fit: **B = dname** (drink identifier).

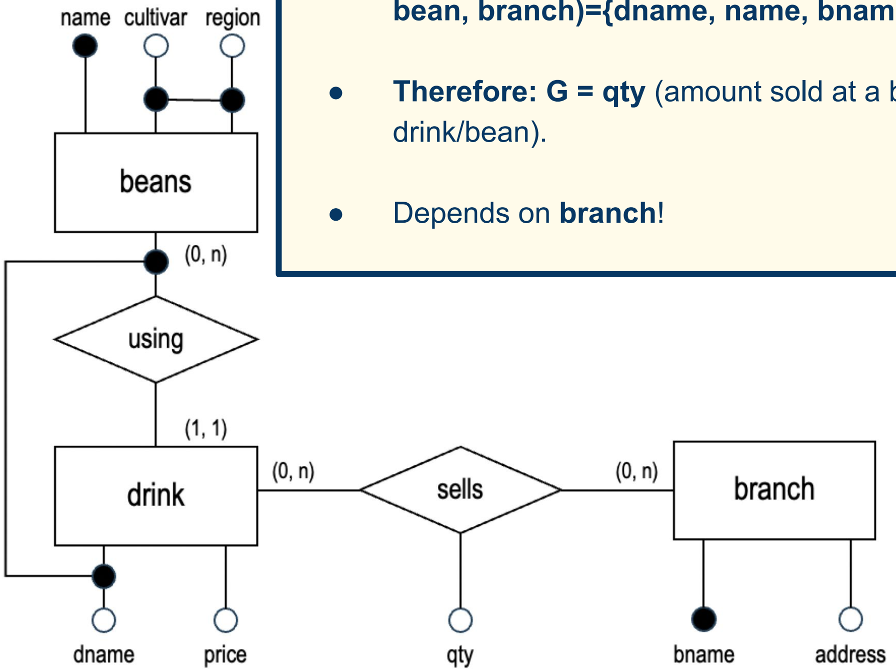


Determine D = price

- **A↔(C,E)** ⇒ “bean identity.”
- From **AB→D** and **BCE→D** ⇒ **(drink + bean)** → **D** (no branch).
- **Therefore:** the only attribute at *Drink×Bean* level is **price** ⇒ **D = price**.

Drawing the ER diagram

A=name
B=dname
C=cultivar
D=price
E=region
F=bname
G=qty
H=address



Identify G = qty

- From **ABF→DG** and **BCEF→G** with **A↔CE** ⇒ **(drink, bean, branch)={dname, name, bname}** → **G**.
- **Therefore: G = qty** (amount sold at a branch for that drink/bean).
- Depends on **branch**!

1(a). Given the schema find the mapping of vars and letters

Attribute	Character
`name`	A
`dname` (drink name)	B
`cultivar`	C
`price`	D

Attribute	Character
`region`	E
`bname` (branch name)	F
`qty` (quantity)	G
`address`	H

(b) Compute the attribute closures of the subset of attributes of R with Σ. Find the candidate keys.

$$R = \{A, B, C, D, E, F, G, H\}$$

$$\Sigma = \{\ \{A\} \rightarrow \{C, E\},\ \ \{A, B\} \rightarrow \{D\},\ \ \{F\} \rightarrow \{H\},\ \ \{C, E\} \rightarrow \{A\},\ \ \{B, C, E\} \rightarrow \{D\},\ \ \{A, B, F\} \rightarrow \{D, G\},\ \ \{B, C, E, F\} \rightarrow \{G\}\ \}$$

📌DEFINITION: A **candidate key** for a relation schema R with a set of functional dependencies Σ is a minimal set of attributes K ⊆ R such that:

1. Closure covers all attributes: K^+ (the closure of K under Σ) = R.
2. Minimality: No proper subset of K has a closure equal to R.

📌NOTE: Since B and F never appear on the right-hand side of any FD, they must be included in every candidate key!

(b) Compute the attribute closures of the subset of attributes of R with Σ. Find the candidate keys.

- Sets of 2 attributes.
  - $\{B, F\}^+ = \{B, F, H\}$ NOT a Key! 🔒

(b) Compute the attribute closures of the subset of attributes of R with Σ. Find the candidate keys.

- Sets of 3 attributes superset of $\{B, F\}$.
  - $\{\mathbf{A}, B, F\}^+$

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(b) Compute the attribute closures of the subset of attributes of R with Σ. Find the candidate keys.

- Sets of 5 attributes superset of $\{B, F\}$ but not superset of $\{A, B, F\}$ or $\{B, C, E, F\}$.

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  .....other sets, also not a key

📌NOTE:
Any 5/6-attribute superset of {B, F} that is not a superset of {A, B, F} or {B, C, E, F} necessarily omits A and at least one of C/E.

Such sets can't determine all attributes, so they can't be keys.

Therefore, there are no additional 6-attribute sets to check.

Conclusion: The only candidate keys 🔑 are {A, B, F} and {B, C, E, F}—**we've found them all!**

(c) Find the prime attributes of R with Σ.

Since the keys are *{A, B, F}* and *{B, C, E, F}*, the prime attributes are:

{A, B, F} U *{B, C, E, F}* = ***{A, B, C, E, F}***.

2 (a) Compute a minimal cover of R with Σ.

We start from Σ.

$$\{A\} \rightarrow \{C, E\}$$

$$\{A, B\} \rightarrow \{D\}$$

$$\{F\} \rightarrow \{H\}$$

$$\{C, E\} \rightarrow \{A\}$$

$$\{B, C, E\} \rightarrow \{D\}$$

$$\{A, B, F\} \rightarrow \{D, G\}$$

$$\{B, C, E, F\} \rightarrow \{G\}$$

2 (a) Compute a minimal cover of R with Σ.

DEFINITION:

A set of FDs G is a **minimal cover** for a set of FDs H if and only if:

- Every FD in G is of the form $X \rightarrow A$ where X is a set of attributes, A is a single attribute, and X has no redundant attributes
- There are no redundant FDs in G
- G and H are equivalent

2 (a) Compute a minimal cover of R with Σ.

We start from Σ.

$$\{A\} \rightarrow \{C, E\}$$
$$\{A, B\} \rightarrow \{D\}$$
$$\{F\} \rightarrow \{H\}$$
$$\{C, E\} \rightarrow \{A\}$$
$$\{B, C, E\} \rightarrow \{D\}$$
$$\{A, B, F\} \rightarrow \{D, G\}$$
$$\{B, C, E, F\} \rightarrow \{G\}$$

Step 1: Simplify the right-hand side.

$$\{A\} \rightarrow \{C\} \quad (split)$$
$$\{A\} \rightarrow \{E\} \quad (split)$$
$$\{A, B\} \rightarrow \{D\}$$
$$\{F\} \rightarrow \{H\}$$
$$\{C, E\} \rightarrow \{A\}$$
$$\{B, C, E\} \rightarrow \{D\}$$
$$\{A, B, F\} \rightarrow \{D\} \quad (split)$$
$$\{A, B, F\} \rightarrow \{G\} \quad (split)$$
$$\{B, C, E, F\} \rightarrow \{G\}$$

2 (a) Compute a minimal cover of R with Σ.

Step 2: Simplify the left-hand side.

$\{A\} \to \{C\}$

$\{A\} \to \{E\}$

$\{A, B\} \to \{D\}$

$\{F\} \to \{H\}$

$\{C, E\} \to \{A\}$

$\{B, C, E\} \to \{D\}$

$\{A, B, F\} \to \{D\}$

$\{A, B, F\} \to \{G\}$

$\{B, C, E, F\} \to \{G\}$

STEP 2: To check if an attribute A ∈ X is redundant in X -> B, compute (X − A)+ w.r.t. **the current FD set Σ.**

A is redundant in X -> B iff B ∈ (X − A)+. (If **B** is in that closure, then **A** is extraneous and can be removed from the left side.)

For ABF→D:

(BF)+ = {B,F,H} ⇒ D absent → A not redundant.

(AF)+ = {A,F,C,E,H} ⇒ D absent → B not redundant.

(AB)+ = {A,B,C,E,D} ⇒ D present → F is extraneous.

So ABF→D simplifies to AB→D (remove the duplicate with the existing AB→D).

2 (a) Compute a minimal cover of R with Σ.

Step 2: Simplify the left-hand side.

$$\{A\} \rightarrow \{C\}$$
$$\{A\} \rightarrow \{E\}$$
$$\{A, B\} \rightarrow \{D\}$$
$$\{F\} \rightarrow \{H\}$$
$$\{C, E\} \rightarrow \{A\}$$
$$\{B, C, E\} \rightarrow \{D\}$$
$$\{A, B, \cancel{F}\} \rightarrow \{D\} \qquad (\{A, B\} \rightarrow \{D\})$$
$$\{A, B, F\} \rightarrow \{G\}$$
$$\{B, C, E, F\} \rightarrow \{G\}$$

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Note: The rule of simplifying LHS is iterative.

- If an attribute on the left is removed, you must re-check the remaining attributes on the new left side for further possible drops.
- Here, after dropping F from ABF -> D, we re-test AB -> D: (B)+ = {B} ⇒ D absent, and (A)+ = {A, C, E} ⇒ D absent. So neither A nor B is extraneous, and no further simplification occurs.

2 (a) Compute a minimal cover of R with Σ.

Step 2: Simplify the left-hand side.

$$\{A\} \rightarrow \{C\}$$

$$\{A\} \rightarrow \{E\}$$

$$\{A, B\} \rightarrow \{D\}$$

$$\{F\} \rightarrow \{H\}$$

$$\{C, E\} \rightarrow \{A\}$$

$$\{B, C, E\} \rightarrow \{D\}$$

$$\{A, B, \cancel{E}\} \rightarrow \{D\} \qquad (\{A, B\} \rightarrow \{D\})$$

$$\{A, B, F\} \rightarrow \{G\}$$

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To check if an attribute A ∈ X is redundant in X -> B, compute (X − A)+ w.r.t. **the current FD set Σ.**

A is redundant in X -> B iff B ∈ (X − A)+. (If **B** is in that closure, then **A** is extraneous and can be removed from the left side.)

For CE→A:

(E)+ = {E} ⇒ A absent → C not redundant.

(C)+ = {C} ⇒ A absent → E not redundant.

(So CE→A keeps both C and E.)

2 (a) Compute a minimal cover of R with Σ.

Step 2: Simplify the left-hand side.

$\{A\} \rightarrow \{C\}$

$\{A\} \rightarrow \{E\}$

$\{A, B\} \rightarrow \{D\}$

$\{F\} \rightarrow \{H\}$

$\{C, E\} \rightarrow \{A\}$

$\{B, C, E\} \rightarrow \{D\}$

$\{A, B, \cancel{E}\} \rightarrow \{D\}$ $(\{A, B\} \rightarrow \{D\})$

$\{A, B, F\} \rightarrow \{G\}$

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A is redundant in X -> B iff B ∈ (X − A)+. (If **B** is in that closure, then **A** is extraneous and can be removed from the left side.)

For BCE⟶D:

(CE)+ = {C,E,A} ⇒ D absent ⟶ B not redundant.

(BE)+ = {B,E} ⇒ D absent ⟶ C not redundant.

(BC)+ = {B,C} ⇒ D absent ⟶ E not redundant.

(Result: keep B,C,E.)

2 (a) Compute a minimal cover of R with Σ.

Step 2: Simplify the left-hand side.

$\{A\} \rightarrow \{C\}$

$\{A\} \rightarrow \{E\}$

$\{A, B\} \rightarrow \{D\}$

$\{F\} \rightarrow \{H\}$

$\{C, E\} \rightarrow \{A\}$

$\{B, C, E\} \rightarrow \{D\}$

$\{A, B, \cancel{E}\} \rightarrow \{D\}$ $\quad (\{A, B\} \rightarrow \{D\})$

$\{A, B, F\} \rightarrow \{G\}$

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To check if an attribute A ∈ X is redundant in X -> B, compute (X − A)+ w.r.t. **the current FD set Σ.**

A is redundant in X -> B iff B ∈ (X − A)+. (If **B** is in that closure, then **A** is extraneous and can be removed from the left side.)

For AB⟶D:

(B)+ = {B} ⇒ D absent ⟶ A not redundant.

(A)+ = {A,C,E} ⇒ D absent ⟶ B not redundant.

(Result: keep A,B.)

2 (a) Compute a minimal cover of R with Σ.

Step 2: Simplify the left-hand side.

$\{A\} \rightarrow \{C\}$

$\{A\} \rightarrow \{E\}$

$\{A, B\} \rightarrow \{D\}$

$\{F\} \rightarrow \{H\}$

$\{C, E\} \rightarrow \{A\}$

$\{B, C, E\} \rightarrow \{D\}$

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For ABF→G:

(BF)+ = {B,F,H} ⇒ G absent.

(AF)+ = {A,F,C,E,H} ⇒ G absent.

(AB)+ = {A,B,C,E,D} ⇒ G absent.

(No attribute is extraneous; keep A,B,F.)

2 (a) Compute a minimal cover of R with Σ.

Step 2: Simplify the left-hand side.

$\{A\} \rightarrow \{C\}$

$\{A\} \rightarrow \{E\}$

$\{A, B\} \rightarrow \{D\}$

$\{F\} \rightarrow \{H\}$

$\{C, E\} \rightarrow \{A\}$

$\{B, C, E\} \rightarrow \{D\}$

$\{A, B, \cancel{E}\} \rightarrow \{D\}$ $(\{A, B\} \rightarrow \{D\})$

$\{A, B, F\} \rightarrow \{G\}$

$\{B, C, E, F\} \rightarrow \{G\}$

To check if an attribute A ∈ X is redundant in X -> B, compute (X - A)+ w.r.t. **the current FD set Σ.**

A is redundant in X -> B iff B ∈ (X - A)+. (If **B** is in that closure, then **A** is extraneous and can be removed from the left side.)

For BCEF⟶G:

(CEF)+ = {C,E,F,A,H} ⇒ G absent ⟶ B not redundant.

(BEF)+ = {B,E,F,H} ⇒ G absent ⟶ C not redundant.

(BCF)+ = {B,C,F,H} ⇒ G absent ⟶ E not redundant.

(BCE)+ = {B,C,E,A,D} ⇒ G absent ⟶ F not redundant.

(Result: keep B,C,E,F.)

2 (a) Compute a minimal cover of R with Σ.

Step 2: Simplify the left-hand side.

$\{A\} \rightarrow \{C\}$

$\{A\} \rightarrow \{E\}$

$\{A, B\} \rightarrow \{D\}$

$\{F\} \rightarrow \{H\}$

$\{C, E\} \rightarrow \{A\}$

$\{B, C, E\} \rightarrow \{D\}$

$\{A, B, \cancel{F}\} \rightarrow \{D\} \qquad (\{A, B\} \rightarrow \{D\})$

$\{A, B, F\} \rightarrow \{G\}$

$\{B, C, E, F\} \rightarrow \{G\}$

STEP 3: REDUNDANCY CHECK

📌 **RULE:** For each FD X → Y, temporarily remove it. Compute the **closure of X** using the remaining FDs. If Y is in that closure, then X → Y is **redundant**—delete it

📌 **ELIMINATIONS:**

- **Check 1: AB → D**
  - Remove AB → D. (AB)+ = {A, B} → add C,E (from A→C,E) → {A,B,C,E} → add D (from BCE→D).
  - ✅ D appears → **AB → D is redundant.**
- **Check 2: ABF → G**
  - Remove ABF → G. (ABF)+ = {A, B, F} → add C,E (from A→C,E) → add H (from F→H) → add G (from BCEF→G).
  - ✅ G appears → **ABF → G is redundant.**

2 (a) Compute a minimal cover of R with Σ.

Step 3: Simplify the set.

$$\{A\} \rightarrow \{C\}$$
$$\{A\} \rightarrow \{E\}$$
$$\cancel{\{A,B\} \rightarrow \{D\}}$$
$$\{F\} \rightarrow \{H\}$$
$$\{C,E\} \rightarrow \{A\}$$
$$\{B,C,E\} \rightarrow \{D\}$$
$$\cancel{\{A,B\} \rightarrow \{D\}}$$
$$\cancel{\{A,B,F\} \rightarrow \{G\}}$$
$$\{B,C,E,F\} \rightarrow \{G\}$$

STEP 3: REDUNDANCY CHECK (ALTERNATIVE)

📌 **RULE:** For each FD X → Y, temporarily remove it. Use **Armstrong's Axioms** (augmentation, transitivity, etc.) to see if Y can still be derived from X. If yes, X → Y is **redundant** — delete it.

📌 **ELIMINATIONS:**

- **{A,B} → {D}:** From **{A} → {C,E}**, by augmentation with **{B}** we get **{A,B} → {B,C,E}**; since **{B,C,E} → {D}**, by transitivity **{A,B} → {D}** → **redundant**.
- **{A,B,F} → {G}:** From **{A} → {C,E}**, by augmentation with **{B,F}** we get **{A,B,F} → {B,C,E,F}**; since **{B,C,E,F} → {G}**, by transitivity **{A,B,F} → {G}** → **redundant**.

2 (a) Compute a minimal cover of R with Σ.

Step 3: Simplify the set.

$\{A\} \rightarrow \{C\}$

$\{A\} \rightarrow \{E\}$

$\cancel{\{A, B\} \rightarrow \{D\}}$

$\{F\} \rightarrow \{H\}$

$\{C, E\} \rightarrow \{A\}$

$\{B, C, E\} \rightarrow \{D\}$

$\cancel{\{A, B\} \rightarrow \{D\}}$

$\cancel{\{A, B, F\} \rightarrow \{G\}}$

$\{B, C, E, F\} \rightarrow \{G\}$

One possible minimal cover is shown below.

$$\{\ \{A\} \rightarrow \{C\},\ \ \{A\} \rightarrow \{E\},\ \ \{F\} \rightarrow \{H\},\ \ \{C, E\} \rightarrow \{A\},\ \ \{B, C, E\} \rightarrow \{D\},\ \ \{B, C, E, F\} \rightarrow \{G\}\ \}$$

2 (a) Compute a minimal cover of R with Σ.

Step 3: Simplify the set.

$\{A\} \rightarrow \{C\}$

$\{A\} \rightarrow \{E\}$

$\cancel{\{A,B\} \rightarrow \{D\}}$

$\{F\} \rightarrow \{H\}$

$\{C,E\} \rightarrow \{A\}$

$\{B,C,E\} \rightarrow \{D\}$

$\cancel{\{A,B\} \rightarrow \{D\}}$

$\cancel{\{A,B,F\} \rightarrow \{G\}}$

$\{B,C,E,F\} \rightarrow \{G\}$

One possible minimal cover is shown below.

$$\{\ \{A\} \rightarrow \{C\},\ \ \{A\} \rightarrow \{E\},\ \ \{F\} \rightarrow \{H\},\ \ \{C,E\} \rightarrow \{A\},\ \ \{B,C,E\} \rightarrow \{D\},\ \ \{B,C,E,F\} \rightarrow \{G\}\ \}$$

There can be other minimal cover such as the following.

$$\{\ \{A\} \rightarrow \{C\},\ \ \{A\} \rightarrow \{E\},\ \ \{F\} \rightarrow \{H\},\ \ \{C,E\} \rightarrow \{A\},\ \ \{A,B\} \rightarrow \{D\},\ \ \{A,B,F\} \rightarrow \{G\}\ \}$$

📌NOTE:

Minimal covers are not unique—different valid answers can result.

(b) Compute a canonical cover of R with Σ.

$$\{\ \{A\} \rightarrow \{C, E\},\ \ \{F\} \rightarrow \{H\},\ \ \{C, E\} \rightarrow \{A\},\ \ \{B, C, E\} \rightarrow \{D\},\ \ \{B, C, E, F\} \rightarrow \{G\}\ \}$$

📌NOTE:

Canonical cover of R: Start from a **minimal cover** and merge all FDs that share the same left-hand side. You end up with at most one FD per left-hand side.

Example: {X} -> {A} and {X} -> {B} becomes {X} -> {A,B}.

In practice: compute minimal cover → then merge all FDs with the same left-hand side into one by unioning their right-hand sides!

Thank you for joining!

Got questions? Post them on the forum or email me:

biswadeep@u.nus.edu

(I reply **within 2 working days** — *faster if coffee is strong* ☕)

Because your learning matters to me! 😊

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