BT5110: Tutorial 8

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Example

Example Relation,
$$R = \{A, B, C\}$$
 and $\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}\}$

The column in red decides the color of the row: The rows having same values in the red column, have same color.

Task: Check which other columns are following "same value if same color" for each deciding (red) column. The columns for which this holds, are implied by the red column.

Α	В	С
a1	b1	c1
a1	b1	c1
a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2

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{ <mark>A} −</mark>	→ { <i>A</i> ,	B, C

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a5	b2	c2
{ <mark>A} −</mark>	→ { <i>A</i> ,	<i>B</i> , <i>C</i> }

Α	В	С
a1	b1	c1
a1	b1	c1
a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2
{ B }	$\rightarrow \{F$	3 C}

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a4	b1	c1
a5	b2	c2
(A) -	→ { <i>A</i> ,	<i>B</i> , <i>C</i> }

Α	В	С
a1	b1	c1
a1	b1	c1
a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2
{ B }	$\rightarrow \{E$	3, <i>C</i> }

Α	В	С
a1	b1	c1
a1	b1	c1
a2	b2	c2
a2	b2	c2
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(A) -	→ { <i>A</i> ,	<i>B</i> , <i>C</i> }

Α	В	С
a1	b1	c1
a1	b1	c1
a2	2 b2	c2
a2	2 b2	c2
a3	B b3	c2
a3	3 b3	c2
a4	l b1	c1
a4	₽ b1	c1
a5	5 b2	c2
{ B	$\{ \} \rightarrow \{ \}$	<i>B</i> , <i>C</i> }

Α	В	С
a1	b1	c1
a1	b1	c1
a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2
{ C	$\rightarrow \{$	<i>C</i> }

Question

Your company, Apasaja Private Limited, is commissioned by an online company offering several services to design the relational schema the management of their users' profiles. A service is fully described and identified by its name. Each user can register to one or more services. A user is uniquely identified by her email as well as by her mobile number. Each user has both a postal address and a country of residence. The postal address, however, unambiguously identifies the country in which it is located. There can be several users with the same address. However, we are only given an abstract schema for this application. Consider the relations $R = \{A, B, C, D, E\}$ with the set of functional dependencies $\Sigma = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}, \{B, C\}, \{B, C\} \rightarrow \{A\}, \{B, C\}, \{B, C\} \rightarrow \{A\}, \{B, C\}, \{$ $\{A, D\}, \{B\} \rightarrow \{A, B\}, \{C\} \rightarrow \{D\}\}.$



Question

- 1. Functional Dependencies
 - (a) From the functional dependencies in Σ and the text description of the application, can you figure out a mapping of the attributes and the letters?
- (b) Compute the attribute closures of the attributes in R with Σ in order to find the candidate keys of R with Σ .
- (c) Find the candidate keys of R with Σ .

Question

- (d) Find the prime attributes.
- 2. Minimal Cover
 - (a) Compute a minimal cover of R with Σ .
- (b) Compute a compact minimal cover of R with Σ .



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Solutions •000000

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 $\{A\} \to \{A,B,C\} \implies \{A\} \to \{B\}$ and $\{B\} \to \{A\}$: A and B are Email and Mobile number or vice versa.

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 $\{C\} \rightarrow \{D\}$: C is the postal address and D is the country.

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 $\{C\} \rightarrow \{D\}$: C is the postal address and D is the country.

Service does not depend on anything, neither implies anything. Thus E is the Service attribute.

```
\{B\}^+ = \{B, C, A, D\}
\{C\}^+ = \{C, D\}
\{\mathsf{D}\}^+ = \{D\}
\{E\}^+ = \{E\}
\{B, A\}^+ = \{B, C, A, D\}
\{C, A\}^+ = \{B, C, A, D\}
\{D, A\}^+ = \{A, B, C, D\}
\{E, A\}^+ = \{B, C, A, E, D\}
\{B, C\}^+ = \{A, B, C, D\}
```

 ${A}^+ = {B, C, A, D}$

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$$\{A\}^+ = \{B, C, A, D\}
 \{B\}^+ = \{B, C, A, D\}
 \{C\}^+ = \{C, D\}
 \{D\}^+ = \{D\}
 \{E\}^+ = \{E\}
 \{B, A\}^+ = \{B, C, A, D\}
 \{C, A\}^+ = \{B, C, A, D\}
 \{D, A\}^+ = \{A, B, C, D\}
 \{E, A\}^+ = \{B, C, A, E, D\}
 \{B, C\}^+ = \{A, B, C, D\}$$

$$\{B, D\}^{+} = \{B, C, A, D\}$$

$$\{E, B\}^{+} = \{B, C, A, E, D\}$$

$$\{C, D\}^{+} = \{C, D\}$$

$$\{E, C\}^{+} = \{E, C, D\}$$

$$\{E, D\}^{+} = \{E, D\}$$

$$\{B, C, A\}^{+} = \{B, C, A, D\}$$

$$\{B, D, A\}^{+} = \{A, B, C, D\}$$

$$\{C, D, A\}^{+} = \{A, B, C, D\}$$

$$\{C, D, A\}^{+} = \{A, B, C, D\}$$

$$\{C, D, A\}^{+} = \{B, C, A, E, D\}$$

$$\{B, D\}^+ = \{B, C, A, D\}$$

$$\{E, B\}^+ = \{B, C, A, E, D\}$$

$$\{C, D\}^+ = \{C, D\}$$

$$\{E, C\}^+ = \{E, C, D\}$$

$$\{B, C, A\}^+ = \{B, C, A, D\}$$

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$$\{E, C, D, A\}^{+} = \{A, B, C, D, E\}$$

$$\{E, B, C, D\}^{+} = \{A, B, C, D, E\}$$

$$\{C, D, E, B, A\}^{+} = \{A, B, C, D, E\}$$

$$\{A\}^{+} = \{B, C, A, D\}$$

$$\{B\}^{+} = \{B, C, A, D\}$$

$$\{B\}^{+} = \{B, C, A, D\}$$

$$\{C\}^{+} = \{C, D\}$$

$$\{C, D\}^{+} = \{C, D\}$$

$$\{E, C\}^{+} = \{E, C, D\}$$

$$\{E, C, C\}^{+} = \{E, C, C\}$$

$$\{E, C\}$$

It's tedious to compute them all on paper!



Solutions

Question 1.b.

$$\{A\}^{+} = \{B, C, A, D\} \\ \{B\}^{+} = \{B, C, A, D\} \\ \{E, B\}^{+} = \{B, C, A, D\} \\ \{C\}^{+} = \{C, D\} \\ \{D\}^{+} = \{D\} \\ \{E, C\}^{+} = \{E, C, D\} \\ \{E, C, C\}^{+} = \{E, C, D, A\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C, C\} \\ \{E, C, C\}^{+} = \{E, C, C$$

It's tedious to compute them all on paper!

Here's some help: Check it out! (also find mistakes if any)



The ones in red are the superkeys.



The ones in red are the superkeys.

A close look reveals that either {A,E} or {B,E} is the smallest subset of all the superkeys and are superkeys themselves, and thus are the candidate keys.



We have $\{A,E\}$ and $\{B,E\}$ as the candidate keys.

The attributes participating in the candidate keys are called the prime attributes.

Here the prime attributes are A, B and E.



```
We start with \Sigma=
\{\{A\} \rightarrow \{A, B, C\}
\{A, B\} \rightarrow \{A\}
\{B, C\} \rightarrow \{A, D\}
\{B\} \rightarrow \{A, B\}
\{C\} \rightarrow \{D\}\}
```

We start with $\Sigma =$ $\{\{A\} \rightarrow \{A, B, C\}$ $\{A, B\} \rightarrow \{A\}$ $\{B, C\} \rightarrow \{A, D\}$ $\{B\} \rightarrow \{A, B\}$ $\{C\} \rightarrow \{D\}\}$

```
Step 1: Simplify the
right hand sides
\{A\} \rightarrow \{A\}
\{A\} \rightarrow \{B\}
\{A\} \rightarrow \{C\}
\{A, B\} \rightarrow \{A\}
\{B, C\} \rightarrow \{A\}
\{B, C\} \rightarrow \{D\}
\{B\} \rightarrow \{A\}
\{B\} \rightarrow \{B\}
\{C\} \rightarrow \{D\}
```

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 $\{C\} \rightarrow \{D\}$

Question 2.a.

We start with $\Sigma = \{\{A\} \rightarrow \{A, B, C\} \\ \{A, B\} \rightarrow \{A\} \\ \{B, C\} \rightarrow \{A, D\} \\ \{B\} \rightarrow \{A, B\} \\ \{C\} \rightarrow \{D\} \}$

```
Step 1: Simplify the right hand sides \{A\} \rightarrow \{A\} \{A\} \rightarrow \{B\} \{A\} \rightarrow \{C\} \{A, B\} \rightarrow \{A\} \{B, C\} \rightarrow \{A\} \{B, C\} \rightarrow \{A\} \{B, C\} \rightarrow \{D\} \{B\} \rightarrow \{B\} \rightarrow \{B\} \{C\} \rightarrow \{D\}
```

Question 2.a. contd.

Step 3: Remove redundancies

 ${A} \rightarrow {A}$ Trivial

 $\{A\} \rightarrow \{B\}$

 $\{A\} \rightarrow \{C\}$

 $\{B\} \rightarrow \{A\}$

 $\{B\} \rightarrow \{D\}$

Can be derived from the others.

 $\{B\} \rightarrow \{B\}$

Trivial

 $\{A\} \rightarrow \{A\}$

Trivial

 $\{C\} \rightarrow \{D\}$

Question 2.a. contd.

Step 3: Remove redundancies

$$A \rightarrow A$$
 Trivial $A \rightarrow B$ $A \rightarrow C$

$$\{B\} \rightarrow \{A\}$$

 $\{B\} \rightarrow \{D\}$

 $\{B\} \rightarrow \{B\}$

Trivial

$$\{A\} \rightarrow \{A\}$$

Trivial

$$\{C\} \rightarrow \{D\}$$

Minimal cover

$$\{\{A\} \rightarrow \{B\},\$$

 $\{A\} \rightarrow \{C\},\$

$$\{A\} \rightarrow \{C\},\$$

$$\{\mathsf{B}\} \to \{A\},\\ \{\mathsf{C}\} \to \{D\}\}$$

Compact minimal cover

Just merge the dependencies according to the left hand sides:

We get:

$$\{\{A\} \to \{B,C\}, \{B\} \to \{A\}, \{C\} \to \{D\}\}$$

Tutorial recording



Scan to see the recording of the tutorial Link to recording

