

Database Programming and Management

Tutorial 8: Normalization

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$$R = \{A, B, C, D, E, F, G, H\}$$

$$\Sigma = \{ \{A\} \rightarrow \{C, E\}, \{A, B\} \rightarrow \{D\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\}, \{B, C, E\} \rightarrow \{D\}, \\ \{A, B, F\} \rightarrow \{D, G\}, \{B, C, E, F\} \rightarrow \{G\} \}$$

1.(a) Is R with Σ in 3NF?

A relation R is in **3NF** if, for every functional dependency $X \rightarrow Y$ in Σ :

1. The dependency is **trivial** (i.e, $Y \subseteq X$),

OR

2. X is a **superkey** for R,

OR

3. Every attribute in Y is **prime** (i.e., part of some candidate key).

 **NOTE:** Satisfying any one of the conditions will suffice and hence **OR**.

1.(a) Is R with Σ in 3NF?

Consider $\{A, B\} \rightarrow \{D\}$.

1. The dependency is **trivial** (i.e, $Y \subseteq X$),

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 **NOTE:** To prove something is NOT in 3NF, all 3 conditions must be False.

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1.(a) Is R with Σ in 3NF?

Consider $\{A, B\} \rightarrow \{D\}$.

- It is non-trivial (i.e., $\{D\} \not\subseteq \{A, B\}$).
- $\{A, B\}$ is not a key (i.e., $\{A, B\}^+ = \{A, B, C, D, E\} \subset R$).
 $\{A, B\}$ is *also* not a superset of a key (keys are $\{A, B, F\}$ and $\{B, C, E, F\}$).
 \Rightarrow This is a simpler way to check superkey if we have computed keys.

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 $\{A, B\}$ is *also* not a superset of a key (keys are $\{A, B, F\}$ and $\{B, C, E, F\}$).
 \Rightarrow This is a simpler way to check superkey if we have computed keys.
- D is not a prime attribute.

Prime attributes are $\{A, B, C, E, F\}$.

1. The dependency is **trivial** (i.e., $Y \subseteq X$),

OR

2. X is a **superkey** for R ,

OR

3. Every attribute in Y is **prime**

NO! NOT in 3NF!

1.(b) Is R with Σ in BCNF?

A relation R is in **BCNF** if, for every functional dependency $X \rightarrow Y$ in Σ :

1. The dependency is **trivial** (i.e, $Y \subseteq X$),

OR

2. X is a **superkey** for R,

 **NOTE:**

- Every BCNF relation is in 3NF, but not every 3NF relation is in BCNF
- BCNF is stricter!

1.(b) Is R with Σ in BCNF?

From Question 1a, we know that R with Σ is not in 3NF. Therefore, it cannot be in BCNF. However, let us verify this from the definition of BCNF. Obviously, we can consider $\{A, B\} \rightarrow \{D\}$, but let us consider a different functional dependency. Consider $\{A\} \rightarrow \{C\}$.

NO! NOT in BCNF!

1.(b) Is R with Σ in BCNF?

From Question 1a, we know that R with Σ is not in 3NF. Therefore, it cannot be in BCNF. However, let us verify this from the definition of BCNF. Obviously, we can consider $\{A, B\} \rightarrow \{D\}$, but let us consider a different functional dependency.

Consider $\{A\} \rightarrow \{C\}$.

- It is non-trivial (i.e., $\{C\} \not\subseteq \{A\}$).
- $\{A\}$ is not a superkey (i.e., $\{A\}^+ = \{A, C, E\} \subset R$).

$\{A\}$ is *also* not a superset of a key (keys are $\{A, B, F\}$ and $\{B, C, E, F\}$).

\Rightarrow This is a simpler way to check superkey if we have computed keys.

NO! NOT in BCNF!

2.(a) Decompose R with Σ into a lossless-join 3NF.

Algorithm : 3NF Synthesis (Bernstein Algorithm)

When a relation is not in 3NF, we can *synthesize* a schema in 3NF from a *canonical cover* of the set of functional dependencies.

- ▶ For each functional dependency $X \rightarrow Y$ in the minimal cover, create a relation

$$R_i = X \cup Y$$

Unless it already exists or is *subsumed* by another relation

- ▶ If none of the created relations contain one of the keys, pick any candidate key and create a relation with that candidate key.

2.(a) Decompose R with Σ into a lossless-join 3NF.

We can start from a canonical cover directly.

$\{ \{A\} \rightarrow \{C,E\}, \{F\} \rightarrow \{H\}, \{C,E\} \rightarrow \{A\}, \{B,C,E\} \rightarrow \{D\}, \{B,C,E,F\} \rightarrow \{G\} \}$

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For each functional dependency, we synthesize a fragment.

$$\{ \{A, C, E\}, \{F, H\}, \{A, C, E\}, \{B, C, D, E\}, \{B, C, E, F, G\} \}$$

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$$\{ \{A,C,E\}, \{F,H\}, \{A,C,E\}, \{B,C,D,E\}, \{B,C,E,F,G\} \}$$

If there is any fragments that can be *subsumed*, we remove them from the result.

$$\{ \{A,C,E\}, \{F,H\}, \cancel{\{A,C,E\}}, \{B,C,D,E\}, \{B,C,E,F,G\} \}$$

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If there is any fragments that can be *subsumed*, we remove them from the result.

$$\{ \{A,C,E\}, \{F,H\}, \cancel{\{A,C,E\}}, \{B,C,D,E\}, \{B,C,E,F,G\} \}$$

If no candidate keys is present as a subset of any fragment, we add. Luckily, the key $\{B,C,E,F\}$ is a subset of $\{B,C,E,F,G\}$. So, we do not have to add another relation.

$$\{ \{A,C,E\}, \{F,H\}, \{B,C,D,E\}, \{B,C,E,F,G\} \}$$

If no key is present

We know the keys of R are $\{A,B,F\}$ and $\{B,C,E,F\}$

Suppose after subsumption we ended up with this:

$\{ \{A,C,E\}, \{F,H\}, \{B,C,D,E\}, \boxed{\{B,C,E,F\}} \}$

Add any of the keys! That's it

NOTE:

- If the questions asks you to just make fragments - Make fragments using the **3NF synthesis algorithm** and you can stop there.
- Skip **FD projections** and **BCNF checks** unless the question **explicitly asks** or you need to verify extras.

Projection steps

5 steps to compute projection of R with Σ onto X .

1. Find all subset X' of attributes of X .
2. For each subset X' , compute the closure (i.e., $\varphi_1 := \text{AttrClose}(X', \Sigma)$).
3. Keep only the relevant attributes (i.e., $\varphi_2 := \varphi_1 \cap X$).
4. Remove attributes that does not contribute new information (i.e., $\varphi_3 := \varphi_2 - X'$).
5. If φ_3 is not empty, form a functional dependency $X' \rightarrow \varphi_3$.

 **Question:**

Calculate the projection R with Σ onto $X = \{A, C, E\}$

Calculating Projections

Do this for
all possible
subsets

- 1 = choose subset $X' \subseteq X$
- 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
- 3 = $\varphi_2 := \varphi_1 \cap X$
- 4 = $\varphi_3 := \varphi_2 - X'$
- 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Onto $X = \{A, C, E\}$ (call this Σ_1)

Step 1 X'	Step 2 φ_1 (closure)	Step 3 φ_2	Step 4 φ_3	Step 5 (FD)
$\{A\}$	$\{A, C, E\}$ (by $A \rightarrow CE$)	$\{A, C, E\}$	$\{C, E\}$	$A \rightarrow CE$
$\{C, E\}$	$\{A, C, E\}$ (by $CE \rightarrow A$)	$\{A, C, E\}$	$\{A\}$	$CE \rightarrow A$
$\{A, C\}$	$\{A, C, E\}$	$\{A, C, E\}$	$\{E\}$	$AC \rightarrow E$ (implied by $A \rightarrow CE$)
$\{A, E\}$	$\{A, C, E\}$	$\{A, C, E\}$	$\{C\}$	$AE \rightarrow C$ (implied by $A \rightarrow CE$)

All other X' give $\varphi_3 = \emptyset$. Keep the bold FDs for a minimal projection: $\Sigma_1 = \{A \rightarrow CE, CE \rightarrow A\}$.

Calculating Projections

Exhaustive check (all 8 subsets of X)

Step 1 X'	Step 2 $\varphi_1 = (X')^+$	Step 3 $\varphi_2 = \varphi_1 \cap X$	Step 4 $\varphi_3 = \varphi_2 \setminus X'$	Step 5 (FD if any)
\emptyset	\emptyset	\emptyset	\emptyset	—
$\{E\}$	$\{E\}$	$\{E\}$	\emptyset	—
$\{C\}$	$\{C\}$	$\{C\}$	\emptyset	—
$\{A\}$	$\{A, C, E\}$ (by $A \rightarrow CE$)	$\{A, C, E\}$	$\{C, E\}$	$A \rightarrow CE$
$\{C, E\}$	$\{A, C, E\}$ (by $CE \rightarrow A$)	$\{A, C, E\}$	$\{A\}$	$CE \rightarrow A$
$\{A, E\}$	$\{A, C, E\}$	$\{A, C, E\}$	$\{C\}$	$AE \rightarrow C$ (implied by $A \rightarrow CE$)
$\{A, C\}$	$\{A, C, E\}$	$\{A, C, E\}$	$\{E\}$	$AC \rightarrow E$ (implied by $A \rightarrow CE$)
$\{A, C, E\}$	$\{A, C, E\}$	$\{A, C, E\}$	\emptyset	—

1 = choose subset $X' \subseteq X$
 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
 3 = $\varphi_2 := \varphi_1 \cap X$
 4 = $\varphi_3 := \varphi_2 - X'$
 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Calculating Projections

Do this for
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- 1 = choose subset $X' \subseteq X$
- 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
- 3 = $\varphi_2 := \varphi_1 \cap X$
- 4 = $\varphi_3 := \varphi_2 - X'$
- 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Onto $X = \{F, H\} (\Sigma_2)$

Step 1 X'	Step 2 φ_1	Step 3 φ_2	Step 4 φ_3	Step 5
$\{F\}$	$\{F, H\}$ (by $F \rightarrow H$)	$\{F, H\}$	$\{H\}$	$F \rightarrow H$

Other subsets yield $\varphi_3 = \emptyset$. So $\Sigma_2 = \{F \rightarrow H\}$.

Early Exit Rule:

Let $S = \text{AttrClose}(X', \Sigma)$. If $\varphi_2 = X'$, stop for this X' (no FD).

Calculating Projections

Do this for
all possible
subsets

1 = choose subset $X' \subseteq X$
 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
 3 = $\varphi_2 := \varphi_1 \cap X$
 4 = $\varphi_3 := \varphi_2 - X'$
 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Onto $X = \{B, C, D, E\} (\Sigma_3)$

Step 1 X'	Step 2 φ_1	Step 3 φ_2	Step 4 φ_3	Step 5
$\{B, C, E\}$	$\{B, C, E, D\}$ (by $BCE \rightarrow D$)	$\{B, C, E, D\}$	$\{D\}$	$BCE \rightarrow D$

All other subsets inside $\{B, C, D, E\}$ do not gain D (or add nothing beyond themselves), so $\varphi_3 = \emptyset$.

Thus $\Sigma_3 = \{BCE \rightarrow D\}$.

Calculating Projections

Do this for
all possible
subsets

- 1 = choose subset $X' \subseteq X$
- 2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
- 3 = $\varphi_2 := \varphi_1 \cap X$
- 4 = $\varphi_3 := \varphi_2 - X'$
- 5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

Onto $X = \{B, C, E, F, G\}$ (Σ_4)

Step 1 X'	Step 2 φ_1	Step 3 φ_2	Step 4 φ_3	Step 5
$\{B, C, E, F\}$	$\{B, C, E, F, G\}$ (by $\text{BCEF} \rightarrow G$)	$\{B, C, E, F, G\}$	$\{G\}$	$\text{BCEF} \rightarrow G$

Others give $\varphi_3 = \emptyset$. Hence $\Sigma_4 = \{\text{BCEF} \rightarrow G\}$.

Final Projections

Do this for all
possible subsets

1 = choose subset $X' \subseteq X$
2 = $\varphi_1 := \text{AttrClose}(X', \Sigma)$
3 = $\varphi_2 := \varphi_1 \cap X$
4 = $\varphi_3 := \varphi_2 - X'$
5 = if $\varphi_3 \neq \emptyset$, emit $X' \rightarrow \varphi_3$

- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$
- $\{B, C, D, E\}$ with $\Sigma_3 = \{ \{B, C, E\} \rightarrow \{D\} \}$
- $\{B, C, E, F, G\}$ with $\Sigma_4 = \{ \{B, C, E, F\} \rightarrow \{G\} \}$

Mistake : “Keep only FDs that use attributes from X”

$R(A, B, C)$, $\Sigma = \{ A \rightarrow B, B \rightarrow C \}$, project onto $X = \{A, C\}$.

Naive keeps none (since both FDs mention B).

Correct (by the algorithm):

- For $X' = \{A\}$: $A^+ = \{A, B, C\} \Rightarrow (A^+ \cap X) - \{A\} = \{C\} \Rightarrow A \rightarrow C$




Projection is $\{ A \rightarrow C \}$.

The naive method **misses** $A \rightarrow C$ (transitive via B).

 **NOTE:**

Using shortcuts can lead to error in **MANY** different ways!
This is not the only one!

Projection – Caution Points

- Projection is **error-prone** → don't skip steps!
- Shortcuts can omit valid **derived FDs**.
- Always follow all **5 steps** for correctness  .
- Tedious  , but ensures **accurate results**.
-  Shortcuts **don't guarantee** correctness.

(b) Is the result dependency preserving?

Definition

- Let R be a relation schema and Σ a set of functional dependencies (FDs).
- Decompose R into $\Delta = \{R_1, \dots, R_n\}$.
- For each piece R_i , let $\Sigma|_{R_i}$ be the projection of Σ onto R_i .
- The decomposition is *dependency-preserving* iff

$$\Sigma^+ = (\Sigma|_{R_1} \cup \dots \cup \Sigma|_{R_n})^+.$$

Note (how to check)

- For every FD $X \rightarrow Y$ in Σ , compute X^+ using $\Sigma|_{R_1} \cup \dots \cup \Sigma|_{R_n}$.
- If $Y \subseteq X^+$ for all FDs in Σ , the decomposition is dependency-preserving.

(b) Is the result dependency preserving?

Yes. This is guaranteed by the algorithm. 3NF synthesis algorithm produces lossless-join dependency-preserving decomposition in 3NF.

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The 3NF synthesis only guarantees that the result is in 3NF, it may not be in BCNF.

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- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$

 **NOTE:**

All non-trivial FDs here ($A \rightarrow C$, $A \rightarrow E$, $CE \rightarrow A$, and $A \rightarrow CE$) have $LHS \in \{A, CE\}$ (keys) \Rightarrow BCNF

(c) Is the result in BCNF?

The 3NF synthesis only guarantees that the result is in 3NF, it may not be in BCNF.

- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$

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- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$
- $\{B, C, D, E\}$ with $\Sigma_3 = \{ \{B, C, E\} \rightarrow \{D\} \}$

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- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
- $\{F, H\}$ with $\Sigma_2 = \{ \{F\} \rightarrow \{H\} \}$
- $\{B, C, D, E\}$ with $\Sigma_3 = \{ \{B, C, E\} \rightarrow \{D\} \}$
- $\{B, C, E, F, G\}$ with $\Sigma_4 = \{ \{B, C, E, F\} \rightarrow \{G\} \}$

YES! It is in BCNF!

(c) Is the result in BCNF?

The 3NF synthesis only guarantees that the result is in 3NF, it may not be in BCNF.

- $\{A, C, E\}$ with $\Sigma_1 = \{ \{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\} \}$
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- $\{B, C, E, F, G\}$ with $\Sigma_4 = \{ \{B, C, E, F\} \rightarrow \{G\} \}$

Note that it is not always the case that we are lucky to obtain a BCNF decomposition using 3NF synthesis, but it may happen.

One Last Lap

♥ **One last class to go!** (BCNF + Extra problems + doubt clearing) — hope to all of you there!  

💬 If you enjoyed the module, please take a moment to leave your **feedback/rating** after the course — it really helps me grow as a teacher!



👋 I'll see you all around campus — don't hesitate to say hi or wave! 😊 

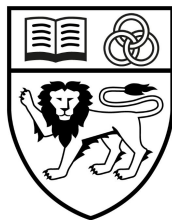
Thank you for joining!

Got questions? Post them on the forum or email me:

biswadeep@u.nus.edu

(I reply **within 2 working days** — *faster if coffee is strong* ☕)

Because your learning matters to me! 😊



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You can generate all implied FDs by running the above closure for **every** subset $X \subseteq U$ (U = all attributes):

```
 $\Sigma_{plus} := \emptyset$ 
```

```
for each  $X \subseteq U$ :
```

```
     $S := \text{AttrClosure}(X, \Sigma)$ 
```

```
    for each attribute  $A$  in  $(S - X)$ :
```

```
        add FD  $X \rightarrow A$  to  $\Sigma_{plus}$     // (often we ignore trivial  $A \in X$ )
```

Subsumption

Keep

In some cases like $R = \{A, B, C\}$ with $\Sigma = \{ \{A, B\} \rightarrow \{C\}, \{C\} \rightarrow \{B\} \}$, we cannot remove $R_2 = \{B, C\}$ even when it is subsumed by $R_1 = \{A, B, C\}$.

$R_2(B, C)$

```
CREATE TABLE R2 (  
  B INT,  
  C INT  
  UNIQUE,  
  PRIMARY KEY (B, C)  
  
);
```

$R_1(A, B, C)$

```
CREATE TABLE R1 (  
  A INT,  
  B INT,  
  C INT,  
  PRIMARY KEY (A, B),  
  FOREIGN KEY (B, C) REFERENCES R2(B, C)  
  
);
```


Lemma #1: Lossless-Join Binary Decomposition

A **binary decomposition** of R into R_1 and R_2 is lossless-join if $R = R_1 \cup R_2$ and $(R_1 \cap R_2) \rightarrow R_1$ or $(R_1 \cap R_2) \rightarrow R_2$

Lemma #2: Lossless-Join Decomposition

A **decomposition** of R into R_1, R_2, \dots, R_n is lossless-join if there exists **at least one sequence** of binary lossless-join decomposition that generates that decomposition.

Note

If $(R_1 \cap R_2)$ is the primary key of one of the two tables, then it can be a foreign key in the other table referencing the primary key.

0) Pick an anchor that contains a key

Compute $(BCEF)^+$ under Σ :

- $BCEF \rightarrow G$ ($BCEF \rightarrow G$) \Rightarrow add G
- $BCE \rightarrow D$ ($BCE \rightarrow D$) \Rightarrow add D
- $CE \rightarrow A$ ($CE \rightarrow A$) \Rightarrow add A
- $F \rightarrow H$ ($F \rightarrow H$) \Rightarrow add H

So $(BCEF)^+ = \mathbf{A B C D E F G H} \Rightarrow \mathbf{BCEF}$ is a **key**.
 Fragment $\mathbf{R_4(BCEFG)}$ contains BCEF \Rightarrow use $\mathbf{R_4}$ as the anchor.

1) Join $\mathbf{R_4(BCEFG)}$ with $\mathbf{R_3(BCDE)}$

- Intersection: $\mathbf{R_4 \cap R_3 = \{B,C,E\}}$.
- Check binary test: does $\{B,C,E\} \rightarrow R_3$?
 Yes: $\{B,C,E\} \rightarrow D$ (given) and trivially $\rightarrow \{B,C,E\}$, so $\{B,C,E\} \rightarrow \mathbf{BCDE}$.

✓ Step 1 is **lossless**.

2) Join the result with $\mathbf{R_1(ACE)}$

(Current attributes after step 1: $\{B,C,E,F,G,D\}$)

- Intersection: $\mathbf{\{C,E\}}$.
- Check: $\{C,E\} \rightarrow R_1$?
 Yes: $\{C,E\} \rightarrow A$ (given) and trivially $\rightarrow \{C,E\} \Rightarrow \{C,E\} \rightarrow \mathbf{ACE}$.

✓ Step 2 is **lossless**.

3) Join the result with $\mathbf{R_2(FH)}$

(Current attributes after step 2: $\{A,B,C,D,E,F,G\}$)

- Intersection: $\mathbf{\{F\}}$ (H isn't in the current set yet).
- Check: $\{F\} \rightarrow R_2$?
 Yes: $\{F\} \rightarrow H$ (given) and trivially $\rightarrow F \Rightarrow \{F\} \rightarrow \mathbf{FH}$.

✓ Step 3 is **lossless**.

Problem Overview

Design a relational schema for the management of coffee bean, drinks and cafes

The Coffee Bean Entity



- Identified by *BrandName* **OR** (*Cultivar*, *Region*)
- One bean → many drinks
- Attributes: (*BrandName*, *Cultivar*, *Region*)
- PK: (*BrandName*) or (*Cultivar*, *Region*)

Drink Entity



- Made from **one coffee bean**
- Name unique **per bean**
- Attributes: (*BeanID*, *DrinkName*, *Price*)
- PK: (*BeanID*, *DrinkName*)

Branch



- Represents a **physical coffee shop branch**
- Each branch has a **unique name**
- Attributes:** (*BranchName*, *Address*)
- PK:** (*BranchName*)