Introduction to Quantum Mechanics – Problem sheet 2

1. **A quantum wave function**

   Consider a quantum particle in one dimension described by the wave function

   $$\psi(x, t_0) = N x \,\mathrm{e}^{-\frac{(x-x_0)^2}{2}},$$

   with a real constants $N > 0$ and $x_0 > 0$.

   (a) Sketch the probability distribution, for the numerical value $x_0 = 1/2$. You may use a computer if you wish.

   (b) At which location is the particle most likely to be found?

2. **A quantum wave function, probabilities, and energies**

   A particle is described by the wave function $\psi(x) = b(a^2 - x^2)$ for $|x| \leq a$ and $\psi(x) = 0$ for $|x| > a$, where a and b are real positive constants.

   (a) Sketch the wave function.

   (b) Use the normalisation condition to find the normalisation constant b in terms of a.

   (c) Calculate the probability of finding the particle between $-a/2$ and $-a/4$.

   (d) Calculate the energy expectation value assuming there is no external potential, i.e. $V(x) = 0$.

3. **Stationary states and complex energies**

   Consider the two stationary states

   $$\psi_A(x,t) = e^{-iE_A t}\psi_A(x,0)$$
   $$\psi_B(x,t) = e^{-iE_B t}\psi_B(x,0),$$

   where $E_A \neq E_B$.

   (a) Calculate the probability densities of ψ_A, ψ_B, $\psi_A + \psi_B$ and $\psi_A - \psi_B$.

   (b) In standard quantum mechanics the energies are strictly real. But let us suppose for a moment that E_A is not. Expressing E_A as $E_A = E_R + iE_I$, describe what impact this has on the overall norm $|\psi_A(t)|^2$.