**Linear Algebra** **Unseen Question 1**

Recall that for a prime number $p$, the field of $p$ elements is $\mathbb{F}_p = \{0, 1, 2, \ldots, p-1\}$, with addition and multiplication modulo $p$.

Also the *general linear group* $GL(n, \mathbb{F}_p)$ is the group consisting of all invertible $n \times n$ matrices over $\mathbb{F}_p$ (the group peration is matrix multiplication).

In this question you will calculate the order of the group $GL(n, \mathbb{F}_p)$.

(a) Let $V = \mathbb{F}_p^n$, the $n$-dimensional vector space of column vectors over $\mathbb{F}_p$. What is the total number of vectors in $V$?

(b) Recall from last year's Linear Algebra that an $n \times n$ matrix is invertible iff its columns form a basis of $V$. Calculate the order of $GL(2, \mathbb{F}_p)$.

(Hint: count the number of possibilities for the first column, and then the number for the second column.)

(c) Following the same procedure (counting the number of possibilities for each successive column), calculate $|GL(n, \mathbb{F}_p)|$.

(d) What is the full power of $p$ that divides $|GL(n, \mathbb{F}_p)|$?

(e) What is $|GL(2, \mathbb{F}_2)|$? What well-known group is isomorphic to $GL(2, \mathbb{F}_2)$?

(f) Define the *special linear group* $SL(n, \mathbb{F}_p)$ to be the subgroup of $GL(n, \mathbb{F}_p)$ consisting of $n \times n$ matrices of determinant 1. Calculate the order of $SL(n, \mathbb{F}_p)$ .