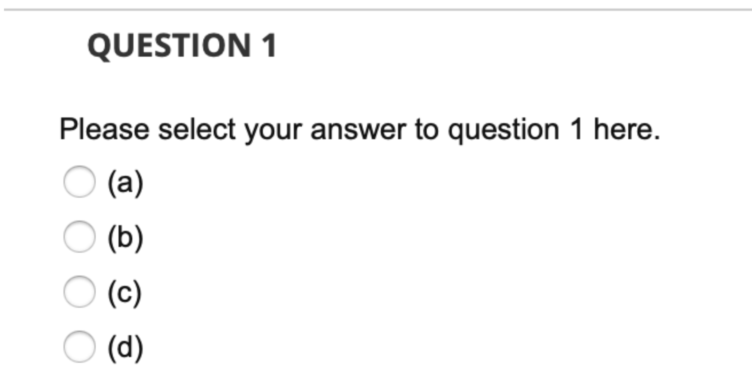
Math50004/50015 Multivariable Calculus Quiz 1

Instructions

This quiz contains ten multiple choice questions worth 2 marks each. Please record your answers on the Blackboard quiz titled ‘Multiple Choice Quiz 1.’

You may find it useful to note down the letter corresponding to your answer for each question as you go along. Below is a snippet of the Blackboard quiz:



You can save your progress and return to the quiz at any time within the 24 hour period. Once you are happy with all your answers please press ‘save and submit’ (only one submission is allowed). **The quiz will not auto-submit so please remember to press the submit button. The quiz is open for 24 hours from 9am on Monday 27th November.**

The questions will begin on the next page.

Let S be an open surface described by

$$z^2 = 25 - x^2 - y^2, \quad 3 \leq z \leq 5. \tag{1}$$

Let

$$\mathbf{A} = yz^3\,\hat{\mathbf{i}} + x^2y^2\,\hat{\mathbf{j}} + yz\,\hat{\mathbf{k}}. \tag{2}$$

During this quiz, we will evaluate the integral

$$I = \int_S (\operatorname{curl}\mathbf{A}) \cdot \hat{\mathbf{n}}\ \mathrm{d}S, \tag{3}$$

where $\hat{\mathbf{n}}$ is the unit normal to S with $\hat{\mathbf{n}} \cdot \hat{\mathbf{k}} > 0$.

We will evaluate the integral I in two different ways:

- By projecting S onto a plane and using the projection theorem
- By considering an appropriate path integral and using Stokes theorem

Each of the ten questions is worth 2 marks.

Questions 1-5: Projection theorem

1. Calculate $(\operatorname{curl}\mathbf{A}) \cdot \hat{\mathbf{n}}$.

   (a) $-\frac{1}{5}(xz + 3y^2z^2 + 2xy^2z - z^4)$

   (b) $\frac{1}{5}(xz - 3y^2z^2 + 2xy^2z - z^4)$

   (c) $\frac{1}{5}(xz + 3y^2z^2 + 2xy^2z - z^4)*$

   (d) $-\frac{1}{5}(xz - 3y^2z^2 + 2xy^2z - z^4)$

2. Let Σ_z denote the projection of S onto the plane $z = 0$. What is Σ_z?

   (a) A disc of radius 5, described by $x^2 + y^2 \leq 25$

   (b) A disc of radius 3, described by $x^2 + y^2 \leq 9$

   (c) The boundary of the circle $x^2 + y^2 = 16$

   (d) A disc of radius 4 described by $x^2 + y^2 \leq 16.*$

3. Using the projection theorem, we can write

(a) $I = \displaystyle\int_{\Sigma_z} \left(x + 2xy^2\right)\, \mathrm{d}\Sigma_z$

(b) $I = -\displaystyle\int_{\Sigma_z} \left(x + 2xy^2\right)\, \mathrm{d}\Sigma_z$

(c) $I = \displaystyle\int_{\Sigma_z} \left(x + 3y^2(25 - x^2 - y^2)^{1/2} + 2xy^2 - (25 - x^2 - y^2)^{3/2}\right)\, \mathrm{d}\Sigma_z*$

(d) $I = -\displaystyle\int_{\Sigma_z} \left(x + 3y^2(9 - x^2 - y^2)^{1/2} + 2xy^2 - (9 - x^2 - y^2)^{3/2}\right)\, \mathrm{d}\Sigma_z$

4. Using polar coordinates with the parameterisation $x = r\cos\theta$ and $y = r\sin\theta$, and an appropriate range for r and θ, the integral I can be written as which of the following?

(a) $\displaystyle\int_0^{2\pi}\int_0^3 (r^2\cos\theta + 3(25 - r^2)^{1/2} r^3 \sin^2\theta + 2r^4\sin^2\theta\cos\theta - r(25 - r^2)^{3/2})\, \mathrm{d}r\mathrm{d}\theta$

(b) $\displaystyle\int_0^{2\pi}\int_0^5 (r^2\cos\theta + 3(25 - r^2)^{1/2} r^3 \sin^2\theta + 2r^4\sin^2\theta\cos\theta - r(25 - r^2)^{3/2})\, \mathrm{d}r\mathrm{d}\theta$

(c) $\displaystyle\int_0^{2\pi}\int_0^4 (r\cos\theta + 3(25 - r^2)^{1/2} r^2 \sin^2\theta + 2r^3\sin^2\theta\cos\theta - (25 - r^2)^{3/2})\, \mathrm{d}r\mathrm{d}\theta$

(d) $\displaystyle\int_0^{2\pi}\int_0^4 (r^2\cos\theta + 3(25 - r^2)^{1/2} r^3 \sin^2\theta + 2r^4\sin^2\theta\cos\theta - r(25 - r^2)^{3/2})\, \mathrm{d}r\mathrm{d}\theta*$

5. Hence evaluate I. (You may use integration software if you wish).

(a) -576π

(b) 1024π

(c) 0

(d) $-432\pi*$

Questions continue on the next page.

Questions 6-10: Stokes theorem

6. According to Stokes theorem, the integral I defined by (1)-(3) is equivalent to

$$\oint_{\gamma} \mathbf{A} \cdot \mathrm{d}\mathbf{r} \tag{4}$$

where γ is traversed in which direction, as viewed from above?

(a) Anti-clockwise*

(b) Clockwise

(c) It could be either direction

(d) We need to be given more information to decide

7. γ can be described as the boundary

(a) $x^2 + y^2 = 16, z = 3*$

(b) $x^2 + y^2 = 25, z = 0$

(c) $x^2 + y^2 = 9, z = 4$

(d) $x^2 + y^2 = 9, z = 3$

8. Which of the following integrals **does not** describe I?

(a) $I = \oint_{\gamma} yz^3 \,\mathrm{d}x + x^2y^2 \,\mathrm{d}y + yz \,\mathrm{d}z$

(b) $I = \oint_{\gamma} yz^3 \,\mathrm{d}x + x^2y^2 \,\mathrm{d}y$

(c) $I = \oint_{\gamma} x^2y^2 \,\mathrm{d}y + 27y \,\mathrm{d}x$

(d) $I = \oint_{\gamma} x^2y^2 \,\mathrm{d}y + 64y \,\mathrm{d}x$

9. Consider the parameterisation $x = 4\cos\theta$ and $y = 4\sin\theta$ for $0 \le \theta \le 2\pi$. Which of the following integrals describes I?

(a) $I = -576 \int_0^{2\pi} \sin^2\theta \,\mathrm{d}\theta$

(b) $I = 1024 \int_0^{2\pi} \cos^3\theta \sin^2\theta \,\mathrm{d}\theta - 432 \int_0^{2\pi} \sin^2\theta \,\mathrm{d}\theta *$

(c) $I = 1024 \int_0^{2\pi} \cos^2\theta \sin^2\theta \,\mathrm{d}\theta - 432 \int_0^{2\pi} \sin^2\theta \,\mathrm{d}\theta$

(d) $I = 1024 \int_0^{2\pi} \cos^3\theta \sin^2\theta \,\mathrm{d}\theta + 432 \int_0^{2\pi} \sin^2\theta \,\mathrm{d}\theta$

10. Hence calculate I. (You may use integration software).

(a) 0

(b) -176π

(c) $-432\pi *$

(d) -576π

END OF QUIZ

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