MATH60005/70005: Optimization (Autumn 24-25)

Chapter 1: solutions

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E.1 Note that for the weighted dot product we need that $\mathbf{w} \in \mathbb{R}^n_{++}$ to ensure positive definiteness of the product: $\langle \mathbf{x}, \mathbf{x} \rangle \geq 0 \ \forall \mathbf{x} \in \mathbb{R}^n$ and $\langle \mathbf{x}, \mathbf{x} \rangle = 0$ if and only if $\mathbf{x} = 0$.

E.2 The $\ell_{1/2}$ "norm" is not a norm because it violates the triangular inequality: pick $\mathbf{x} = (1, 0)$ and $\mathbf{y} = (0, 1)$!

E.3 Try to work a proof of Cauchy-Schwarz inequality by taking any $\mathbf{x}, \mathbf{y}$ (the case when one of them is $\mathbf{0}$ follows directly), and working with

$$\mathbf{z} = \mathbf{x} - \frac{\langle \mathbf{x}, \mathbf{y} \rangle}{\langle \mathbf{y}, \mathbf{y} \rangle} \mathbf{y},$$

noting that $\mathbf{z}$ and $\mathbf{y}$ are perpendicular, and using Pythagoras theorem.

E.4 Here you simply need to check the properties of a matrix norm, one by one.

E.5 Here it depends where are we working. In $\mathbb{R}$, $int([\mathbf{x}, \mathbf{y}]) = (\mathbf{x}, \mathbf{y})$, but in $\mathbb{R}^2$, with two-dimensional balls, $int([\mathbf{x}, \mathbf{y}]) = \{\emptyset\}$

E.6
$$bd(B(c, r)) = bd(B([c, r]) = \{\|\mathbf{x} - c\| = r\}.$$
$$bd(\mathbb{R}^n_{++}) = bd(\mathbb{R}^n_{+}) = \{\mathbf{x} \in \mathbb{R}^n_{+} \text{ such that there exists } i \ : \ x_i = 0\}$$
$$bd(\mathbb{R}^n) = \emptyset$$

E.7
$$cl(\mathbb{R}^n_{++}) = \mathbb{R}^n_{+}$$
$$cl((\mathbf{x}, \mathbf{y})) = [\mathbf{x}, \mathbf{y}]$$