Classical Dynamics **Autumn Term 2021**

Test 1

Instructions

The deadline is 1pm on Wednesday 27 October.

Upload your answers in a single PDF file.

Your answers should be hand-written.

Include your name and CID on your script.

If you are unable to submit via Blackboard/Turnitin email your script to
`maths-student-office@imperial.ac.uk`

1. (a) A particle moving in two dimensions is subject to the conservative force $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$. Given that $F_x = xe^{y^2}$, what can you say about F_y?

(4 marks)

(b) A particle of unit mass moving in one dimension obeys the equation of motion

$$\ddot{x} = \frac{1}{2}e^x.$$

(i) Show that $\dot{x}^2 - e^x$ is a constant of the motion.

(ii) At $t = 0$ the particle is at $x = 0$ and at rest, i.e. $x(0) = \dot{x}(0) = 0$. Show that the particle reaches $x = +\infty$ at *finite* $t > 0$.

Hint: use the result of part (i) to write the time taken to reach $x = +\infty$ as an integral.

(iii) What is a suitable Lagrangian for this system?

(9 marks)

(c) The motion of a particle in a plane is governed by the Lagrangian

$$L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2}(y\dot{x} - x\dot{y}).$$

(i) Obtain the equations of motion (simplify if possible). Is the force acting on the particle conservative?

(ii) Verify that

$$x = R\cos t, \qquad y = R\sin t,$$

is a solution of the equations of motion (here R is a constant).

(iii) What is the general form of the solutions?

Hint: The solution from part (ii) has one arbitrary constant R. The general solution includes four arbitrary constants. (12 marks)

(Total: 25 marks)