Imperial College London

DEPARTMENT OF MATHEMATICS
IMPERIAL COLLEGE LONDON
Academic Year 2022-2023

Mathematics Pre-arrival course

Solutions to Weekly Quiz 3 – Linear Algebra, Sequences and Series

1. The matrix A is given by
$$A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}.$$
The determinant of A is
   (a) 1
   (b) -1
   (c) 11
   (d) -11

2. Which of the following matrices represents a rotation of 30° anti-clockwise about the origin.
   (a) $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}.$
   (b) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}.$
   (c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$
   (d) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$

3. How many of the following statements are true?
   - ☐ If a line is invariant under a transformation matrix A and a transformation matrix B, then it's also invariant under BA.
   - ☐ Every rotation in 2D space about the origin through an angle of $\theta \neq 0$ anti-clockwise has at least one line of invariant points.
   - ☐ Every rotation in 3D space about a given axis through an angle of $\theta \neq 0$ anti-clockwise has at least one line of invariant points.

   (a) 0
   (b) 1
   (c) 2
   (d) 3

4. All of the invariant **lines** under the transformation

$$\begin{bmatrix} -3 & 2 \\ -8 & 5 \end{bmatrix}$$

are

(a) $y = 2x$
(b) $y = 2x + c$, c any constant
(c) $y = 3x$
(d) $y = 3x$ and $y = 2x$

5. The triangle with vertices $(-2, 4)$, $(1, 4)$ and $(0, -6)$ is transformed with a transformation represented by

$$\begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}.$$

The area of the image is:

(a) 15 square units
(b) 30 square units
(c) 60 square units
(d) 120 square units

6. What conclusion can we reach about the planes represented by the equations

$$\begin{aligned} 2x + 3y - z &= 9 \\ x - 2y - 3z &= 2 \\ x + 5y + 2z &= 7. \end{aligned}$$

(a) They form a sheaf
(b) They form a prism
(c) They are parallel
(d) They meet at a unique point

7. An expression for

$$\sum_{r=1}^{n} (2r^2 - 1) + \sum_{r=1}^{n} (3r + 1)$$

is

(a) $\frac{1}{6}n(n+1)(2n+11)$
(b) $\frac{1}{2}n(n+1)(2n+11)$
(c) $\frac{1}{6}n(n+1)(4n+11)$
(d) $\frac{1}{2}n(n+1)(4n+11)$

8. An expression for

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$$

is

(a) $\frac{n(n+2)}{(n+1)^2}$
(b) $\frac{n^2-1}{n^2}$
(c) $\frac{(n-2)n}{(n-1)^2}$
(d) $\frac{(n+1)(n+2)}{n^2}$