MATH50001/50017/50018 - Analysis II Complex Analysis

MTExam

Tuesday 20th February 2024, 09:00-09:50

1. [5p]

Find $\Omega \subset \mathbb{C}$, where

$$\operatorname{Log}\left(\frac{z}{2z+i}\right)$$

is holomorphic. Here $\operatorname{Log}(w) = \ln|w| + i \operatorname{Arg} w$ and $\operatorname{Arg} w$ is the principal value of the argument.

2. [5p]

Compute the integral

$$\oint_{|z-i|=2} \frac{1}{(z+2)(z-2i)^2}\, dz.$$

3. [5p]

Find Taylor series for

$$f(z) = \frac{1}{z^2+1}$$

about $z_0 = 1$. What is its radius of convergence?

4. [5p]

Let f and g be entire. Assume that $\operatorname{Im} f \leq \operatorname{Im} g$. Show that $f(z) = g(z) + \mathtt{const}$.