Imperial College
London

M4/5P8

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)

May-June 2017

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Algebra III

Date: Wednesday 31 May 2017

Time: 10:00 - 12:30

Time Allowed: 2.5 Hours

This paper has 5 Questions.

Candidates should use ONE main answer book.

Supplementary books may only be used after the relevant main book(s) are full.

All required additional material will be provided.

- DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO.
- Affix one of the labels provided to each answer book that you use, but DO NOT USE THE LABEL WITH YOUR NAME ON IT.
- Credit will be given for all questions attempted, but extra credit will be given for complete or nearly complete answers to each question as per the table below.

Raw Mark	Up to 12	13	14	15	16	17	18	19	20
Extra Credit	0	½	1	1 ½	2	2 ½	3	3 ½	4

- Each question carries equal weight.
- Calculators may not be used.

1. For each of the following statements, state whether the assertion is true or false. Then, give either a proof (if true) or an explicit counterexample (if false).

   (i) Every PID is Noetherian.

   (ii) Every UFD is Noetherian.

   (iii) If the only two-sided ideals of R are $\{0\}$ and R itself, then R is a division ring.

   (iv) If V is an irreducible (otherwise known as a simple) module over a ring R, then $\mathrm{End}_R(V)$ is a division ring (note: the zero module is not considered irreducible).

   (v) Every finitely-generated module over $\mathbf{C}[x, y]$ is isomorphic to a product of cyclic modules.

2. For each commutative ring R and ideal I below, complete the following steps:

   (a) Determine whether I is prime or not (give full justification);

   (b) Determine whether I is maximal or not (again with full justification).

      (i) $R = \mathbf{Z}, I = (15, 20)$.

      (ii) $R = \mathbf{R}[x], I = (x^2 + x - 1)$.

      (iii) $R = \mathbf{Z}[i], I = (5 + 12i, 13)$.

      (iv) $R = \mathbf{Q}[x], I = (2x^4 + 6x^3 + 18x^2 - 12x + 30)$. Hint: Use Eisenstein's criterion.

3. (i) Find, with full justification, the factorisations of the following polynomials in $\mathbf{F}_3[x]$ into irreducibles. Hint: don't forget the Freshman's Dream formula.

   (a) $x^{27} - 1$

   (b) $x^{13} - x$

   (ii) Prove that $\mathbf{F}_3[x]/(x^4 - 1) \cong \mathbf{F}_9 \times \mathbf{F}_3 \times \mathbf{F}_3$ as rings, where $\mathbf{F}_9$ is a field of nine elements. Hint: use the Chinese Remainder Theorem.

4. (i) If V and W are finite-dimensional vector spaces over a field F, show that the natural map $\varphi : V^* \otimes_F W \to \mathrm{Hom}_F(V, W)$, given by $\varphi(\sum_i f_i \otimes_F w_i)(v) = \sum_i f_i(v) w_i$, is an isomorphism.

   (ii) Now suppose that V and W are not necessarily finite-dimensional vector spaces over F. Give an example where the natural map $V^* \otimes_F W \to \mathrm{Hom}_F(V, W)$ is not surjective (and show it is not surjective).

   (iii) For $R = \mathbf{Z}$, compute $(\mathbf{Z}/n)^*$ and $\mathrm{Hom}_{\mathbf{Z}}(\mathbf{Z}/n, \mathbf{Z}/m)$. Prove that the natural map $(\mathbf{Z}/n)^* \otimes_{\mathbf{Z}} (\mathbf{Z}/n) \to \mathrm{Hom}_{\mathbf{Z}}(\mathbf{Z}/n, \mathbf{Z}/n)$ is not surjective in general.

5. Prove that all finitely-generated modules over $\mathbf{Z}[\sqrt{2}]$ are isomorphic to finite products of the following basic modules:

(a) $\mathbf{Z}[\sqrt{2}]$;

(b) $\mathbf{Z}[\sqrt{2}]/(p^k)$ for p a prime number not of the form $|a^2 - 2b^2|$ and $k \geq 1$;

(c) $\mathbf{Z}[\sqrt{2}]/((a + b\sqrt{2})^k)$ where $|a^2 - 2b^2|$ is a prime number and $k \geq 1$.

You may use, without proof, the assertion from lecture and coursework that $\mathbf{Z}[\sqrt{2}]$ is a Euclidean domain.

As part of your answer, you will need to classify the irreducible elements of $\mathbf{Z}[\sqrt{2}]$.

Final Exam: Solutions
M3P8: Algebra 3, Fall 2016, Travis Schedler

1. (i) [Seen, 4 pts] True: if R is a PID, then every ideal is principal. In particular, every ideal is finitely-generated (as it is generated by a single element), and since R is commutative (so ideals are the same as left ideals and the same as right ideals), we conclude that R is Noetherian.

   (ii) [Seen, 4 pts] False: let $R = \mathbf{C}[x_1, x_2, \ldots]$, a polynomial algebra in infinitely many variables. This is not Noetherian since the ideal $I = (x_1, x_2, \ldots)$ cannot be generated by finitely many elements (indeed, already to span I/I^2, which is infinite-dimensional, would require infinitely many elements).

   (iii) [Seen, 4 pts] False: let $R = \mathrm{Mat}_2(\mathbf{C})$. Then as we saw in coursework, the only two-sided ideals of R are $\{0\}$ and R (since multiplying any nonzero matrix simultaneously on the right and left by elementary matrices yields a spanning set for all matrices, in fact we can get a nonzero multiple of every elementary matrix in this way). In the previous sentence, "elementary matrix" refers to a matrix all of whose entries are zero except for one entry which is one.

   (iv) [Seen, 4 pts] True: as seen on the coursework: Suppose that $a \in \mathrm{End}_R(V)$ is nonzero. Then the image of a is nonzero. Since V is simple, the image is all of V, so a is surjective. Similarly the kernel of a is not all of V, and simplicity implies it is zero. Thus a is injective. Therefore a is an isomorphism. Finally a^{-1} is then also an R-module endomorphism, so $\mathrm{End}_R(V)$ is a division ring.

   (v) [Seen but without full details, 4 pts] False: let $R = \mathbf{C}[x, y]$ and consider the ideal $I = (x, y)$, which is an R-module generated by two elements. We claim that this is not a product of cyclic modules. Indeed, if there were an isomorphism $V_1 \times \cdots \times V_k \to I$ with V_i cyclic modules, then let $W_i \subseteq I$ be the image of each factor $\{0\}^{i-1} \times V_i \times \{0\}^{k-i}$ in I. Since V_i is cyclic, so is W_i, so there are elements $w_1, \ldots, w_k \in I$ such that $W_i = Rw_i$ for all i. Now it would also follow that $W_i \cap W_j = \{0\}$ for all $i \neq j$, since this is true for the preimage in $V_1 \times \cdots \times V_k$. But if $W_i \cap W_j$ contains $w_i w_j \neq 0$ for all i and j. Thus we must have $k = 1$, i.e., I itself must be cyclic, which means it is a principal ideal. But (x, y) is not generated by a single element, since the only common factors of x and y are units, and I is not the unit ideal.

2. (i) [Similar problem seen, 5 points]: $I = (5)$, which is prime and maximal, since $\mathbf{Z}/I = \mathbf{Z}/5$ is a field.

   (ii) [Similar problem seen, 5 points]: Since the discriminant of $x^2 + x - 1$ is $5 > 0$, there are two real roots. So the polynomial is reducible, and hence $(x^2 + x - 1)$ is not prime (and hence not maximal).

   (iii) [Unseen, 5 points]: Note that $(5 + 12i)(5 - 12i) = 13^2$ and we know that $\mathbf{Z}[i]$ is a UFD, so there has to be a common factor of $5 + 12i$ and 13. It is easy to factor

$13 = 3^2 + 4^2 = (2+3i)(2-3i)$, and we can then check that $5+12i = -(2-3i)^2$. Thus the gcd of $5+12i$ and 13 is $2-3i$. Since $\mathbf{Z}[i]$ is a PID (as we proved in lecture that it is Euclidean and that Euclidean implies PID), this implies that $(5+12i, 13) = (2-3i)$. Next, $2-3i$ is irreducible, since $2-3i = ab$ would imply $|a|^2 \cdot |b|^2 = 13$, so that either $|a|^2$ or $|b|^2$ would be one, but that would imply that a or b is in the set $\{\pm 1, \pm i\}$, which consists only of units. Again applying the fact that $\mathbf{Z}[i]$ is a PID, we obtain that $(2-3i)$ is a maximal ideal, and hence prime.

(iv) [Unseen, 5 points] Up to associate we can replace the polynomial $2x^4 + 6x^3 + 18x^2 - 12x + 30$ with half of it, $x^4 + 3x^3 + 9x^2 - 6x + 15$, since we are working in $\mathbf{Q}[x]$. Now this is a primitive polynomial in $\mathbf{Z}[x]$. Eisenstein's criterion applied to the prime 3 shows it is irreducible: 3 divides all coefficients other than the leading coefficient, and $3^2 = 9$ does not divide the constant term. Therefore the polynomial is irreducible. Since $\mathbf{Q}[x]$ is a PID (since we proved in lecture that $F[x]$ is Euclidean for every field F and that Euclidean implies PID), we conclude that the ideal generated by it is maximal (hence prime).

3. (i) [10 points: 5 points per part.]

(a) By the Freshman's Dream formula (which says that $(a+b)^p = a^p + b^p$ modulo p), $(x-1)^3 = x^3 + (-1)^3 = x^3 - 1$. Iterating, $(x-1)^9 = ((x-1)^3)^3 = x^9 - 1$ and similarly $(x-1)^{27} = x^{27} - 1$. Since $x-1$ is evidently irreducible (as it is linear), this is the factorisation into irreducibles.

(b) We have $x^{13} - x = x(x^{12} - 1) = x(x^4 - 1)^3$, applying the Freshman's Dream formula as in (a). Now $x^4 - 1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x^2+1)$. Note that x^2+1 is irreducible in $\mathbf{F}_3[x]$ since it has not roots, and hence no linear factors: $1^2 + 1 = (-1)^2 + 1 = 2$ and $0^2 + 1 = 1 \neq 0$. Thus the desired factorisation is $x(x-1)^3(x+1)^3(x^2+1)^3$.

(ii) [Unseen, 10 points] As we worked out as part of part (b) above, $x^4 - 1 = (x-1)(x+1)(x^2+1)$ is a factorisation into irreducibles. Therefore, $(x^4-1) = (x-1) \cap (x+1) \cap (x^2+1)$, where here (f) denotes the ideal generated by f. The Chinese Remainder theorem then states that $\mathbf{F}_3[x]/(x^4-1) \cong \mathbf{F}_3[x]/(x-1) \times \mathbf{F}_3[x]/(x+1) \times \mathbf{F}_3[x]/(x^2+1)$. Now the evaluation map $\mathrm{ev}_1 : \mathbf{F}_3[x] \to \mathbf{F}_3$ has kernel $(x-1)$, so the first isomorphism theorem provides an isomorphism $\mathbf{F}_3[x]/(x-1) \xrightarrow{\sim} \mathbf{F}_3$. Similarlt using ev_{-1} we get an isomorphism $\mathbf{F}_3[x]/(x+1) \xrightarrow{\sim} \mathbf{F}_3$. Finally, since x^2+1 is an irreducible element of the PID $\mathbf{F}_3[x]$ (it is a PID since we proved in lecture that $F[x]$ is Euclidean for every field F and that Euclidean implies PID), a result from lecture shows that (x^2+1) is maximal and hence $\mathbf{F}_3[x]/(x^2+1)$ is a field. Finally this field has size nine, since every element can uniquely be written as $a + bx + (x^2+1)$ by long division. Put together we obtain the stated result.

4. (i) [Seen, 5 points] The natural map is obviously additive and F-linear. To see that the map is injective, first note that every element of $V^* \otimes_F W$ can be written in the form $\sum_{i=1}^n f_i \otimes_R w_i$ where the w_i are linearly independent (in fact, we can

take the w_i to consist of a basis of W, by rewriting general elements of W in terms of this basis and applying the tensor product identity $v \otimes_F (aw + bw') = av \otimes_F w + bv \otimes_F w'$.) Then in this case, if $\sum_{i=1}^n f_i(v)w_i = 0$, then $f_i(v) = 0$ for all i. If this is true for all v then $f_i = 0$ for all i. Thus the map is injective. Finally to see that the map is surjective: pick a basis $v_1, \ldots, v_n$ of V with dual basis $v_1^\vee, \ldots, v_n^\vee$. Then for all $f \in \mathrm{Hom}_R(V, W)$, we have $f = \sum_{i=1}^n v_i^\vee \otimes_F \phi(v_i)$, which follows since $f(v) = \sum_{i=1}^n f(v_i^\vee(v)v_i) = \sum_{i=1}^n v_i^\vee(v)f(v_i)$ for all v.

(ii) [Seen, 5 points] We have to use the natural map recalled in the solution to (i), namely $\varphi : V^* \otimes_F W \to \mathrm{Hom}_F(V, W)$ given by $\varphi(\sum_{i=1}^n f_i \otimes_F w_i)(v) = \sum_i f_i(v)w_i$. Now if V is infinite-dimensional and $W = V$, then we claim that $\mathrm{Id} \in \mathrm{Hom}(V, W)$ is not in the image of the map $V^* \otimes_F W \to \mathrm{Hom}(V, W)$. Indeed, every element f in the image of this map is a linear transformation such that $\mathrm{im}(f)$ is finite-dimensional: for $f = \varphi(\sum_{i=1}^n f_i \otimes_F w_i)$, then $\mathrm{im}(f)$ is contained in the span of the w_i, which has dimension at most n and hence is finite. But, $\mathrm{im}(\mathrm{Id}) = V$ is not finite-dimensional.

(iii) [Unseen, 10 points] Note first that $(\mathbf{Z}/n)^* = \{0\}$ since any homomorphism $\mathbf{Z}/n \to \mathbf{Z}$ has to send every element of $\mathbf{Z}/n$ to an element of $\mathbf{Z}$ of finite additive order, but the only such element is $\{0\}$. Thus every homomorphism is the zero homomorphism and hence $(\mathbf{Z}/n)^* = \{0\}$.

Next note that $\mathrm{Hom}_{\mathbf{Z}}(\mathbf{Z}/n, \mathbf{Z}/m)$ similarly has to send the generator $\overline{1}$ to an element of $\mathbf{Z}/m$ of order dividing n, and that is the only condition on the image of this element, which determines uniquely the homomorphism. The subset of $\mathbf{Z}/m$ of elements of order dividing n is isomorphic to $\mathbf{Z}/\gcd(m, n)$, since this subgroup is $\frac{m}{\gcd(m,n)}\mathbf{Z}/m$. Therefore $\mathrm{Hom}_{\mathbf{Z}}(\mathbf{Z}/n, \mathbf{Z}/m) \cong \mathbf{Z}/\gcd(m, n)$. Thus, if $\gcd(m, n) \neq 1$, then $(\mathbf{Z}/n)^* \otimes \mathbf{Z}/m = \{0\}$ and $\mathrm{Hom}_{\mathbf{Z}}(\mathbf{Z}/n, \mathbf{Z}/m) \neq \{0\}$, so the natural map cannot be a surjection.

5. [Unseen, 10 points] Let $R := \mathbf{Z}[\sqrt{2}]$. Since Euclidean domains are PIDs, we conclude that R is a PID. By the theorem on finitely-generated modules over PIDs, we conclude that every finitely-generated module is a product of cyclic modules, $R/(a)$ for elements $a \in R$. If $a = 0$ we obtain the factor (a). If $a \neq 0$, we apply the Chinese Remainder Theorem. Recall that PIDs are UFDs, and suppose the factorisation of a into irreducibles is $a = up_1^{r_1} \cdots p_k^{r_k}$, with $u \in R^\times$ and $p_1, \ldots, p_k$ irreducibles such that p_i is not associate to p_j for $i \neq j$. Now let $I = (p_1^{r_1} \cdots p_{k-1}^{r_{k-1}})$ and $J = (p_k^{r_k})$. Then $I \cap J = (a)$ (since R is a UFD) and $I + J = (\gcd(p_1^{r_1} \cdots o_{k-1}^{r_{k-1}}, p_k^{r_k})) = (1)$ (where for the first equality we use that R is a PID). By the Chinese Remainder Theorem, we obtain that $R/(a) \cong R/(p_1^{r_1} \cdots p_{k-1}^{r_{k-1}}) \times R/(p_k^{r_k})$. Iterating this by induction, we obtain $R/(a) \cong R/(p_1^{r_1}) \times \cdots \times R/(p_k^{r_k})$ (this induction was also all done on a coursework, so I won't mind much if the student simply refers to their work there, but strictly I am not sure they are allowed to use coursework exercises without including solutions.)

[Unseen, 10 pts] Now the problem will follow if we can prove that the irreducible elements of $\mathbf{Z}[\sqrt{2}]$ are of the form either $\pm p$ for p a prime integer not of the form

$|a^2 - 2b^2|$ for $a, b \in \mathbf{Z}$, or elements $a + b\sqrt{2}$ such that $|a^2 - 2b^2|$ is itself a prime integer. To see this, recall that the norm function $\nu : \mathbf{Z}[\sqrt{2}] \to \mathbf{Z}_{\geq 0}$ is defined as $\nu(a + b\sqrt{2}) := |a^2 - 2b^2| = |(a + b\sqrt{2})(a - b\sqrt{2})|$, and by definition it is multiplicative ($\nu(xy) = \nu(x)\nu(y)$). Therefore, all units have norm one. Conversely, if $a + b\sqrt{2}$ has norm one, it is obviously ± 1, so is a unit. Thus $\mathbf{Z}[\sqrt{2}]^\times = \{\pm 1\}$. Next, if p is a prime integer, then $\nu(p) = p^2$. Thus if $a+b\sqrt{2} \mid p$ and $a+b\sqrt{2}$ is not associate to p (i.e., is not $\pm p$), then $\nu(a+b\sqrt{2}) = p$. In this case, $p = |a^2-2b^2|$. Conversely if $p = |a^2-2b^2|$, then $p = \pm(a + b\sqrt{2})(a - b\sqrt{2})$, so it follows that p is not irreducible. Finally, it is obvious that any irreducible integer is a prime integer or negative a prime integer. Hence the irreducible integers are precisely $\pm p$ where p is a prime integer which is not of the form $|a^2 - 2b^2|$.

Next, suppose that $a + b\sqrt{2}$ is irreducible for $b \neq 0$. In particular this means that $\gcd(a, b) = 1$. Then we claim that $(a + b\sqrt{2})(a - b\sqrt{2}) = a^2 - 2b^2$, which is an integer, must be prime. Indeed, suppose that p is a prime integer such that $p \mid a^2 - 2b^2$. Then $p \mid (a + b\sqrt{2})(a - b\sqrt{2})$. This shows that p is not prime in $\mathbf{Z}[\sqrt{2}]$, since otherwise $p \mid a + b\sqrt{2}$ or $p \mid a - b\sqrt{2}$, impossible as $\gcd(a, b) = 1$. Since $\mathbf{Z}[\sqrt{2}]$ is a PID, and hence a UFD (by a theorem from lecture), every irreducible is prime (also by a theorem from lecture). Thus p is also not irreducible. By the preceding paragraph, $p = (c + d\sqrt{2})(c - d\sqrt{2})$ for some $c, d \in \mathbf{Z}$, with $d \neq 0$, and $c + d\sqrt{2}$ and $c - d\sqrt{2}$ are both irreducible. Since every irreducible is prime, $c+d\sqrt{2}$ and $c-d\sqrt{2}$ are both prime. Therefore one of these primes is a factor of $a + b\sqrt{2}$. By assumption, this means that one of these equals $\pm(a + b\sqrt{2})$. We conclude that $|a^2 - 2b^2| = p$, as desired.

Imperial College London
Department of Mathematics

Examiner's Comments

Exam: M345P8 Session: 2016-2107

Question 1

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

This exam was not easy, but given the depth of the course and strength of the students, it was fair.

This question (true/false) was among the easiest, partly because it required recalling important relationships between concepts which were stressed in lectures. Perhaps the most difficult aspect was in recalling how to think about noncommutative rings: especially in (c), but also in (d). In part (a) almost all students neglected to appreciate that Noetherian rings were originally defined using left ideals and right ideals separately.

Marker: Travis Scheller

Signature: Date: 5/6/17

Please return with exam marks (one report per marker)

Imperial College London
Department of Mathematics

Examiner's Comments

Exam: M345P8 **Session: 2016-2107**

Question 2

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

This question was relatively easy for most students. The main difficulties were: • remembering for a PID, irreducibles generate maximal ideals (which equal nonzero prime ideals);

• Finding the real roots (or their existence) to the quadratic in (ii)

• ~~Correctly recalling the freshman's Dream formula~~ in $\mathbb{Z}[i]$

• Correctly computing gcd in $\mathbb{Z}[i]$: note almost all students tried the Euclidean algorithm and not all succeeded. Using "prime factorisation" is easier

• Recalling Eisenstein's criterion in (iv). Most students did this fine.

Marker: Travis Scholler

Signature: [signature] **Date:** 5/6/17

Please return with exam marks (one report per marker)

Imperial College London
Department of Mathematics

Examiner's Comments

Exam: M345 P8 Session: 2016-2107

Question 3

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

This question was not too difficult. Part (i) merely required careful application (and recollection) of the Freshman's Dream formula.

Part (ii) was more difficult. Many students could not recall the precise statement of the Chinese Remainder Theorem, nor why $F_3[x]/(x^2+1)$ is a field (though the argument is used repeatedly in (ii)).

Many students incorrectly stated that $I+J+K=R \Rightarrow R/I\cap J\cap K \cong R/I \times R/J \times R/K$. It only works for two ideals (otherwise the sum of any two must be R). ~~AFS~~

Many referred to "$F_3[i]$". This does not exist as $i \in \mathbb{C}$.

Marker: Travis Scholler

Signature: [signature] Date: 5/6/17

Please return with exam marks (one report per marker)

Imperial College London
Department of Mathematics

Examiner's Comments

Exam: M345P8 Session: 2016-2107

Question 4

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

This was the hardest question for most students, probably due to the unit on tensor product being the hardest to understand.

For (i) many forgot to show φ was linear (or explain why) and many assumed w_i (or f_i) formed a basis which was not assumed. In principle this is just (multi)linear algebra but students got confused.

For (ii) few could explain why the map is not surjective and some tried not requiring both V, W to be infinite-dimensional (which is necessary).

(iii) was the hardest part of the exam. Many thought $(\mathbb{Z}/n)^* (= \mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}/n, \mathbb{Z}))$ was nonzero and failed to compute $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}/n, \mathbb{Z}/m)$.

Marker: Travis Scholler

Signature: [signature] Date: 5/6/17

Please return with exam marks (one report per marker)

Imperial College London
Department of Mathematics

Examiner's Comments

Exam: Alg M345 P8 **Session: 2016-2107**

Question 5

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

This question was fair, given a relatively similar one was asked as an assessed coursework: but it was still quite challenging.

Many students failed to recall the statement of the fundamental theorem of modules over a PID correctly (or at all), and more did not recall (or use) the Chinese Remainder Theorem which was needed.

Few classified the units in $\mathbb{Z}[\sqrt{2}]$, and I did not take much off for this given the difficulty. Few could explain why if $a+b\sqrt{2}$ is irreducible and $b \neq 0$, then $|a^2-2b^2|$ is a prime integer.

Marker: Travis Scheeler

Signature: [signature] **Date:** 5/6/17

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