Linear Algebra **Unseen Question 7**

We've seen in Corollary 15.4 of the lecture notes (and also I think you saw it last year) that every real symmetric matrix is diagonalisable. The same is far from true for *complex* symmetric matrices, as we'll see in this project. In fact we'll prove

Theorem *Every $n \times n$ matrix over $\mathbb{C}$ is similar to a complex symmetric matrix.*

So complex symmetric matrices can have any JCF.

Here's a neat proof of the theorem. Let $K = K_n$ be the $n \times n$ matrix with 1's on the 'reverse diagonal', ie.

$$K = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & \cdot & & \\ \cdot & & & \\ 1 & & & \end{pmatrix},$$

and define $L = \frac{1}{\sqrt{2}}(I + iK)$. Also let $J = J_n(0)$, the $n \times n$ Jordan block with eigenvalue 0.

(a) Show that K is symmetric and $K^2 = I$.

(b) Show that L is a unitary matrix (ie. $L^T \bar{L} = I$).

(c) Show that KJ and JK are both symmetric, and that $KJK^{-1} = J^T$.

(d) Deduce that LJL^{-1} is symmetric.

(e) Hence show that any Jordan block $J_n(\lambda)$ is similar to a complex symmetric matrix.

(f) Complete the proof of the theorem!