MATH60005/70005: Optimization (Autumn 24-25)

Chapter 8: exercises

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1. Solve the primal and dual problem for

$$\begin{array}{ll} \min & x_1^2 + x_2^2 + 2x_1 \\ \text{s.t.} & x_1 + x_2 = 0\,. \end{array}$$

2. Study the duality gap (difference between f^* and q^*) for the problem

$$\min \left\{ e^{-x_2} : \sqrt{x_1^2 + x_2^2} - x_1 \leq 0 \right\}\,.$$

3. Recompute the dual of the convex quadratic problem from the notes under that assumption that the matrix $\mathbf{Q} \succeq 0$ instead of $\mathbf{Q} \succ 0$.

4. Consider the Chebyshev center problem where we have a set of points $\mathbf{a}_1 \ldots, \mathbf{a}_m \in \mathbb{R}^n$ for which we seek a point $\mathbf{x} \in \mathbb{R}^n$ that is the center of a ball of minimum radius $r > 0$ containing the points

$$\begin{array}{ll} \min_{\mathbf{x},r} & r \\ \text{s.t.} & \|\mathbf{x} - \mathbf{a}_i\| \leq r\,, \quad i = 1, 2, \ldots, m\,. \end{array}$$

Compute the dual of this problem. *(Hint: use an equivalent formulation over the squared radius)*