Unseen Sheet

U.1. Let p and q be distinct primes. Prove that $\mathbb{F}_{pq}$ is not a field.

U.2. Show that the following are vector spaces. If the field or operations are unspecified, then find ones that make the set a vector space.

(a) $X = \{r \in \mathbb{R} : r > 0\}$ with $x \oplus y = xy$ and $r \odot x = x^r$.

(b) The game "Lights Out" consists of a 5×5 grid of lights, which be be either on or off. If you press one of the lights, it and its 4 adjacent neighbours switch state (i.e. if on, they become off, and vice versa). Let V be the set of all possible states of the grid.

(c) Let P be a set of propositions, and let $\delta : P \to \{T, \perp\}$ be a function that assigns each proposition a true/false value. Let $\mathcal{S}(P)$ be the set of all sentences we can build from P using $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$, and $\neg$. Using truth tables, we can uniquely extend δ to $\mathcal{S}(P)$. Let Δ be the set of extensions of all the possible δ. This Δ is our candidate for a vector space. For the operations, consider how you can apply $\neg$ and $\wedge$ to these functions.

U.3. Fill in the following table, putting a Y if the row title is a vector space over the column title.

	$\mathbb{Q}$	$\mathbb{R}$	$\mathbb{C}$
$\mathbb{Q}$			
$\mathbb{R}$			
$\mathbb{C}$			