

Probability for Statistics

Unseen Problem 4

1. Suppose that X and Y are absolutely continuous random variables with joint pdf given by

$$f_{X,Y}(x,y) = cx(1-y), \text{ for } 0 < x < 1 \text{ and } 0 < y < 1,$$

and zero otherwise, for some constant c .

- (a) Find the value of c .
 - (b) Are X and Y independent random variables?
 - (c) Find $\Pr(X < Y)$.
2. Let X be a 2×2 symmetric matrix with random entries. Suppose $X_{11}, X_{22} \sim N(0, 1)$, $X_{12} \sim N(0, \frac{1}{2})$, with all mutually independent, and $X_{2,1} = X_{1,2}$. Let the eigenvalues of X be λ_1 and λ_2 . Find the distribution of the eigenvalue spacing $|\lambda_1 - \lambda_2|$.