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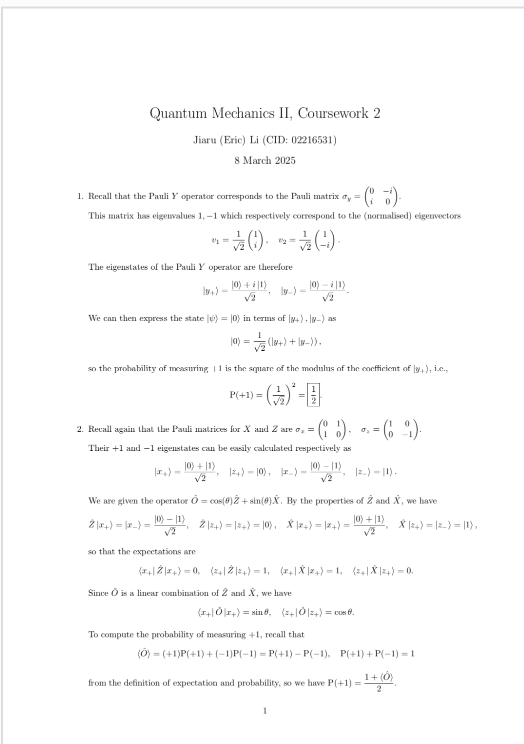

Quantum Mechanics II, Coursework 2

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1. Recall that the Pauli Y operator corresponds to the Pauli matrix $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

   This matrix has eigenvalues $1, -1$ which respectively correspond to the (normalised) eigenvectors
   $$v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}, \quad v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}.$$
   The eigenstates of the Pauli Y operator are therefore
   $$|y_+\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}}, \quad |y_-\rangle = \frac{|0\rangle - i|1\rangle}{\sqrt{2}}.$$
   We can then express the state $|\psi\rangle = |0\rangle$ in terms of $|y_+\rangle, |y_-\rangle$ as
   $$|0\rangle = \frac{1}{\sqrt{2}}(|y_+\rangle + |y_-\rangle),$$
   so the probability of measuring $+1$ is the square of the modulus of the coefficient of $|y_+\rangle$, i.e.,
   $$\mathrm{P}(+1) = \left(\frac{1}{\sqrt{2}}\right)^2 = \boxed{\frac{1}{2}}.$$

2. Recall again that the Pauli matrices for X and Z are $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

   Their $+1$ and -1 eigenstates can be easily calculated respectively as
   $$|x_+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |z_+\rangle = |0\rangle, \quad |x_-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}, \quad |z_-\rangle = |1\rangle.$$
   We are given the operator $\hat{O} = \cos(\theta)\hat{Z} + \sin(\theta)\hat{X}$. By the properties of $\hat{Z}$ and $\hat{X}$, we have
   $$\hat{Z}|x_+\rangle = |x_-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}, \quad \hat{Z}|z_+\rangle = |z_+\rangle = |0\rangle, \quad \hat{X}|x_+\rangle = |x_+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad \hat{X}|z_+\rangle = |z_-\rangle = |1\rangle,$$
   so that the expectations are
   $$\langle x_+|\hat{Z}|x_+\rangle = 0, \quad \langle z_+|\hat{Z}|z_+\rangle = 1, \quad \langle x_+|\hat{X}|x_+\rangle = 1, \quad \langle z_+|\hat{X}|z_+\rangle = 0.$$
   Since $\hat{O}$ is a linear combination of $\hat{Z}$ and $\hat{X}$, we have
   $$\langle x_+|\hat{O}|x_+\rangle = \sin\theta, \quad \langle z_+|\hat{O}|z_+\rangle = \cos\theta.$$
   To compute the probability of measuring $+1$, recall that
   $$\langle\hat{O}\rangle = (+1)\mathrm{P}(+1) + (-1)\mathrm{P}(-1) = \mathrm{P}(+1) - \mathrm{P}(-1), \quad \mathrm{P}(+1) + \mathrm{P}(-1) = 1$$
   from the definition of expectation and probability, so we have $\mathrm{P}(+1) = \frac{1 + \langle\hat{O}\rangle}{2}$.

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