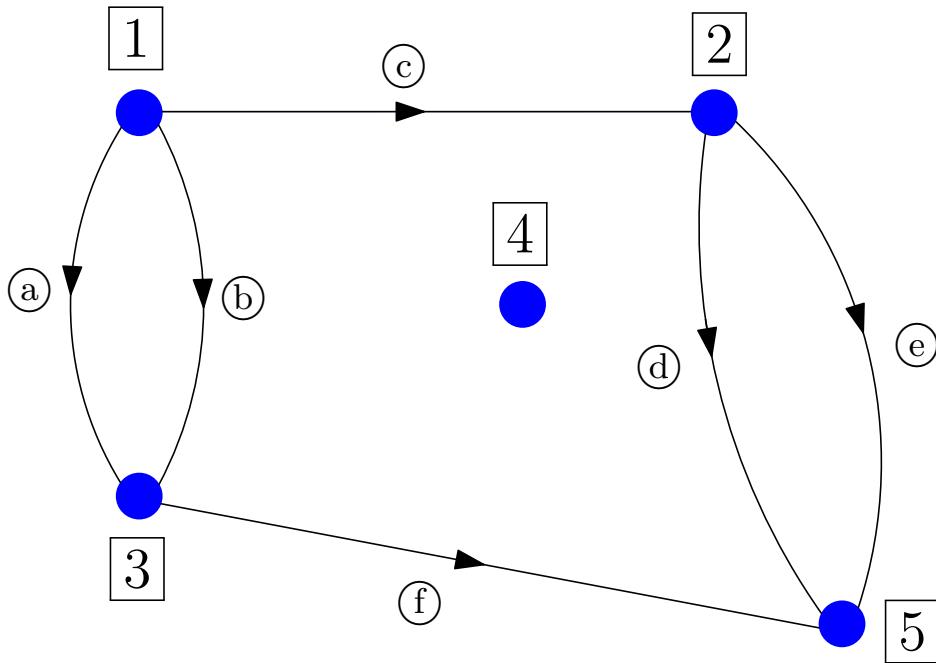


This examination consists of **two questions**. You should attempt both of them.  
For maximum credit, **you must explain your answers clearly.**

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1. Consider the graph shown in the figure where large blue dots indicate nodes and all lines with arrows are edges:

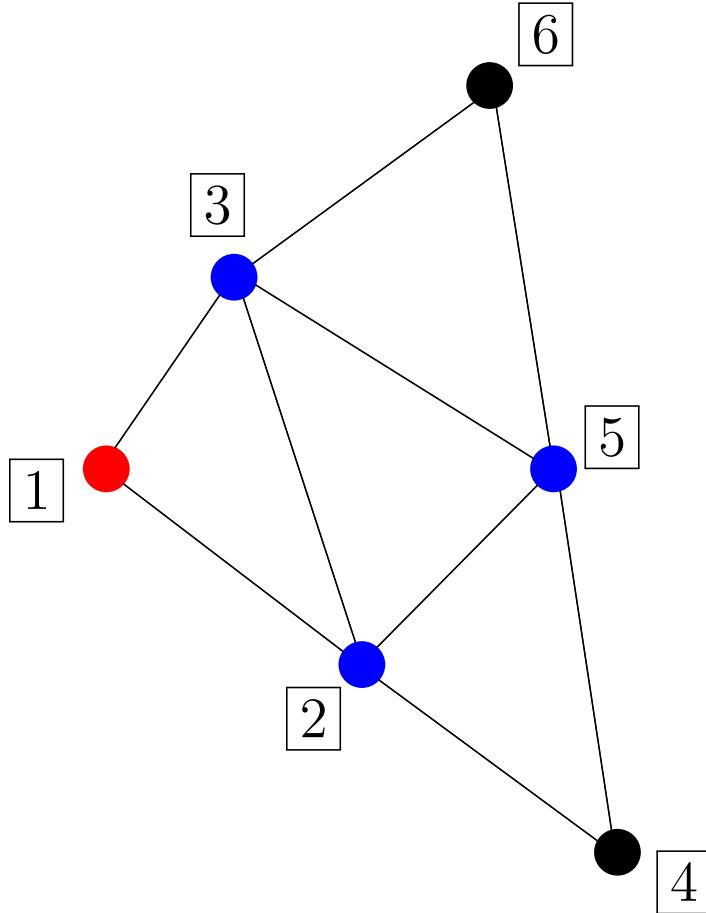


- (a) Write down the incidence matrix  $\mathbf{A}$  for this graph (you must use the numerical and alphabetical ordering of nodes and edges on the figure, and the edge directions indicated).
- (b) Find a basis for the right null space of  $\mathbf{A}$ .
- (c) Find a basis for the row space of  $\mathbf{A}$ .
- (d) Find a basis for the column space of  $\mathbf{A}$ .
- (e) Write down as many linearly independent vectors as you can that are orthogonal to all elements of the column space of  $\mathbf{A}$  (note: two vectors  $\mathbf{x}_1$  and  $\mathbf{x}_2$  are said to be orthogonal if  $\mathbf{x}_1^T \mathbf{x}_2 = 0$ ).

**(12 marks)**

2. The figure below shows an electric circuit in which node **[1]** is set to unit voltage and nodes **[4]** and **[6]** are set to zero voltage (“grounded”). Kirchhoff’s current law holds at all other nodes.

All edges have **unit conductance**.



Find the divergence of the currents at node **[6]**.

**(8 marks)**

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**THE END**

(If you have extra time, use it to ensure you have explained all your answers clearly and fully).