

Exercise 8.1. Consider a discrete metric space (X, d_{disc}) , that is d_{disc} is a discrete metric on X . Show that d_{disc} induces the discrete topology on X .

Exercise 8.2. Let (X, τ) be a topological space, $Y \subset X$, and

$$\tau_Y = \{U \cap Y \mid U \in \tau_X\}.$$

Show that τ_Y is a topology on Y .

Exercise 8.3. Let τ_{Eucl} be the Euclidean topology on \mathbb{R} , that is τ_{Eucl} is the collection of all open sets in (\mathbb{R}, d_1) . Show that the collection

$$\{U \times V \mid U \in \tau_{\text{Eucl}}, V \in \tau_{\text{Eucl}}\}.$$

is not a topology on $\mathbb{R} \times \mathbb{R}$. Is condition T2 satisfied? How about condition T3?

Exercise 8.4. Let (A, τ) be a topological space, and let S and T be subsets of A . The following properties hold:

- (i) if $S \subset T$ then $S^\circ \subset T^\circ$,
- (ii) S is open in A if and only if $S = S^\circ$,
- (iii)* S° is the largest open set contained in S .

Exercise 8.5. Let (X, d) be a metric space, and let τ be the topology on X induced from the metric d . Show that (X, τ) is a Hausdorff topological space.

Exercise 8.6. Assume that the topological spaces (X, τ_X) and (Y, τ_Y) are topologically equivalent. Then, (X, τ_X) is Hausdorff if and only if (Y, τ_Y) is Hausdorff.