$$f(\underline{x}) = \sqrt{\underline{x}^T Q \underline{x} + 1} = \underset{\underset{\sqrt{1+\underline{y}^2}}{\parallel}}{h} \circ \underset{\underset{\|L x\|}{\parallel}}{g} (\underline{x}) \quad \text{convex}$$

because of composing $h \circ g$ w/ g convex and h convex and non-decreasing.

$$\min \; - x_1 x_2 x_3$$

$$\text{s.t.} \quad x_1 + 3x_2 + 6x_3 \leq 48$$

$$-x_1 \leq 0$$

$$-x_2 \leq 0$$

$$-x_3 \leq 0$$

i) KKT System

$$\mathcal{L} = -x_1 x_2 x_3 + \lambda_1 (x_1 + 3x_2 + 6x_3 - 48) - \lambda_2 x_1 - \lambda_3 x_2 - \lambda_4 x_3$$

$$\nabla_{\underline{x}} \mathcal{L} = 0 \iff \begin{aligned} -x_2 x_3 + \lambda_1 - \lambda_2 &= 0 \\ -x_1 x_3 + 3\lambda_1 - \lambda_3 &= 0 \\ -x_1 x_2 + 6\lambda_1 - \lambda_4 &= 0 \end{aligned}$$

$$\lambda_1 (x_1 + 3x_2 + 6x_3 - 48) = 0$$

$$\lambda_2 x_1 = 0 \qquad \lambda_i\text{'s} \geq 0$$

$$\lambda_3 x_2 = 0 \qquad i = 1, \dots, 4$$

$$\lambda_4 x_3 = 0$$

1) $\lambda_1 = 0$, $\lambda_2 = \lambda_3 = \lambda_4 = 0$

$$\left.\begin{array}{l} -x_2 x_3 = 0 \\ -x_1 x_3 = 0 \\ -x_1 x_2 = 0 \end{array}\right\} \begin{array}{l} \to x_2 = 0 \quad \text{or} \quad x_3 = 0 \\ \to x_1 = 0 \quad \text{or} \quad x_3 = 0 \\ \to x_1 = 0 \quad \text{or} \quad x_2 = 0 \end{array}$$

We need at least 2 out of 3 x_i's to be 0

$\Longrightarrow$ That all the KKT points that follow from this system have an optimal value equal to 0.

2) $\lambda_1 > 0, \ \lambda_2 = \lambda_3 = \lambda_4 = 0$

$\lambda_1 = x_2 x_3$ but $\lambda_1 > 0$, so $\Rightarrow x_1, x_2, x_3 \neq 0$

$3\lambda_1 = x_1 \cdot x_3$

$\Rightarrow 3 = x_1 / x_2$

$6\lambda_1 = x_1 \cdot x_2$

$\Rightarrow x_1 = 3x_2$ &

$x_2 = 2x_3$

and plug into $x_1 + 3x_2 + 6x_3 - 48 = 0$

$\Rightarrow x_3 = 8/3, \ x_2 = 16/3,$

$x_1 = 16$

$\Rightarrow$ the value is $-x_1 x_2 x_3$

$-16 \cdot 16/3 \cdot 8/3$

Now, for the remaining cases, if <u>any</u> of $\lambda_2, \lambda_3, \lambda_4$ is $> 0 \Rightarrow$ there exists at least one coordinate $= 0$ in the solution $\Rightarrow$ its cost is 0

$(-x_1 \cdot x_2 \cdot x_3)$

$\Rightarrow$ That the optimal solution is the only KKT point with 3 coordinates $\neq$ from 0.

(case 2))