IMPERIAL

MATH70038

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
Summer 2025

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Graph Theory

Date: Thursday, May 29, 2025

Time: Start time 14:00 – End time 16:30 (BST)

Time Allowed: 2.5 hours

This paper has 5 Questions.

Please Answer All Questions in 1 Answer Booklet

This is a closed book examination.

Candidates should start their solutions to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Allow margins for marking.

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO DO SO

Throughout this paper, all graphs are assumed to be finite and simple unless otherwise stated. The set $\mathbb{N}$ consists of the positive integers.

1. (a) Provide the examples requested below. For each example, give a brief justification of why it has the required properties.

    (i) Find two graphs G and H such that H is a minor of G, $G \neq H$, and $\mathrm{Aut}(G) \cong \mathrm{Aut}(H)$. [*You may use graphs with known automorphism groups without justifying why they have that group of automorphisms.*] (2 marks)

    (ii) Find a connected graph G such that $\delta(G)$ is minimal among all connected non-trivial graphs. (Recall that $\delta(G) = \min\{\deg(v) \mid v \in V_G\}$ is the minimum degree of G.) (2 marks)

    (iii) Provide an example of a graph that is tripartite but not bipartite. (2 marks)

    (iv) Provide an example of a graph that is triangle-free but not bipartite. (2 marks)

   (b) Let G and H be graphs. Prove that $G \cong H$ implies that $\mathrm{Aut}(G) \cong \mathrm{Aut}(H)$. (5 marks)

   (c) Let G be a graph and let s, t be two vertices. We define $\lambda(s, t)$ to be the minimal size of a set of edges $A \subseteq E_G$ such that:

    (i) $G \setminus A$ is disconnected, and

    (ii) s and t lie in different connected components of $G \setminus A$.

   Further we define $\sigma(s, t)$ to be the maximal size of a set of pair-wise edge-disjoint paths between s and t (two paths are *edge-disjoint* if they never traverse the same edge). Use the max-flow min-cut theorem to prove that $\lambda(s, t) = \sigma(s, t)$. (7 marks)

(Total: 20 marks)

2. (a) Let G_1 and G_2 be two bipartite graphs and let $H = G_1 \oplus G_2$ be the disjoint union of G_1 and G_2.

   (i) Prove that H is bipartite. (2 marks)

   (ii) Let X_1 be a minimal cover of G_1 and X_2 be a minimal cover of G_2. Prove that $X_1 \cup X_2$ is a minimal cover of H. (2 marks)

   (iii) Let M_1 be a maximal matching of G_1 and M_2 be a maximal matching of G_2. Prove that $M_1 \cup M_2$ is a maximal matching of H. (2 marks)

   (iv) Suppose that both G_1 and G_2 have a perfect matching. Prove that H has a perfect matching. (2 marks)

(b) Let G be a graph, $M \subseteq E_G$ be a matching in G, and $X \subseteq V_G$ be a cover of G. Prove that $|M| \leq |X|$. (2 marks)

(c) Let Γ be a finite group and let Δ be a subgroup of Γ. Use Hall's marriage theorem to prove that there exist elements $g_1, g_2, ..., g_k \in \Gamma$ such that $g_1\Delta, \dots, g_k\Delta$ are the left cosets of Δ and $\Delta g_1, \dots, \Delta g_k$ are the right cosets of Δ.

(Recall that if $g \in \Gamma$ the left coset of Δ containing g is $g\Delta = \{gd \mid d \in \Delta\}$ and similarly the right coset of Δ containing g is $\Delta g = \{dg \mid d \in \Delta\}$.)

[*You may use without proof the following facts about the cosets of Δ:*

- *two left cosets (respectivelly right cosets) of Δ are either equal or have empty intersection,*
- *if $h \in g\Delta$, then $h\Delta = g\Delta$ (and the analogue holds for right cosets),*
- *the number of left cosets is the same as the number of right cosets.*

Hint: let $X_1, \dots, X_k$ be the left cosets of Δ and $Y_1, \dots, Y_k$ be the right cosets of Δ. Define a bipartite graph with parts $\{x_1, \dots, x_k\}$ and $\{y_1, \dots, y_k\}$, and such that, for all $i, j \in \{1, \dots, k\}$, there is an edge between x_i and y_j if and only if $X_i \cap Y_j \neq \emptyset$.]

(10 marks)

(Total: 20 marks)

3. (a) For each of the following statements say if it is true or false. Justify your answer with a short proof or a counterexample. Credit will only be given for the part of your answer containing the justification. For this part of the question you may use without proof that certain graphs are known to be non-planar.

(i) Every 2-colourable graph is planar. (2 marks)

(ii) For all graphs G, the chromatic polynomial $P_G(X)$ has a zero at 1. (2 marks)

(iii) For all graphs G with $|G| = n$, the chromatic polynomial $P_G(X)$ has a zero at n. (2 marks)

(iv) The chromatic polynomial of $\overline{K_n}$ (that is, the null graph on n vertices) is X^n. (2 marks)

(b) Let G and H be two graphs with $|G| = |H| = n$ and let $m \leq n$. Suppose that both G and H have the following properties

- G and H are complete m-partite (that is, they have m parts and all possible edges between the parts).
- the number of vertices in each part is as uniform as possible. That is, if V_1 and V_2 are two of the m parts of G (or two of the m parts of H), then $\big||V_1| - |V_2|\big| \leq 1$.

Prove that $G \cong H$. (5 marks)

(c) Let $k \in \mathbb{N}$. Use a version of Ramsey's theorem to prove the following statement: there is an $R_k \in \mathbb{N}$ such that, for all $n \geq R_k$, if the edges of K_n are coloured with k colours, then there is always a monochromatic triangle (a triangle with all edges of the same colour). Prove further that, it is possible to take $R_k \leq k(R_{k-1} - 1) + 2$ for all $k \geq 2$.

[*Note that in this problem we allow incident edges to have the same colour.*]

(7 marks)

(Total: 20 marks)

4. (a) Let $n \in \mathbb{N}$ and H be a graph on $m \leq n$ vertices. Recall that $\mathrm{ex}(n, H)$ is defined as the maximum number of edges that an H-free graph on n vertices may have. A graph G is called extremal H-free if it is H-free and has $\mathrm{ex}(n, H)$ edges.

For each of the following statements say if it is true or false. Justify your answer with a short proof or a counterexample. Credit will only be given for the part of your answer containing the justification. You may assume any standard result from the notes for this part.

(i) If $H = K_2$ (the complete graph on two vertices), then the only H-free graph is the null graph (the graph with no edges). (2 marks)

(ii) Two triangle-free extremal graphs on n vertices are isomorphic. (2 marks)

(iii) Two H-free extremal graphs on n vertices are isomorphic. (Note that H is arbitrary again here.) (4 marks)

For the rest of this question $p \in [0, 1]$, $n \in \mathbb{N}$, and $\mathcal{G}(n, p)$ is the Erdős-Rényi model for random graphs, where the graphs are simple and there is an edge between two vertices with probability p for every pair of vertices.

(b) Let $G \in \mathcal{G}(n, p)$ be a random graph on n. Fix $m \in \mathbb{N}$ and let K_m be the complete graph on m vertices. Prove that, if $p \neq 0$, then $\mathbb{P}[K_m \subseteq G] \to 1$ as $n \to \infty$. (5 marks)

(c) Let $f(n)$ be the probability that a random graph in $\mathcal{G}(n, p)$ is connected. Prove that

$$f(n) = 1 - \sum_{i=1}^{n-1} f(i) \binom{n-1}{i-1} (1-p)^{i(n-i)}.$$

[*Hint: compute the probability a random graph has more than one connected component.*]

(7 marks)

(Total: 20 marks)

5. (a) (i) Let G be a graph, let $\{V_0, \dots, V_k\}$ be a partition of V_G, and let ε be a positive real number. Define what it is meant for $\{V_0, \dots, V_k\}$ to be ε-*regular*. (3 marks)

(ii) Let G be a graph and let $X, Y \subseteq V_G$ be non-empty and disjoint. Prove that the density $d(X, Y)$ is a rational number between 0 and 1. (3 marks)

(iii) Find an example of a graph G and $X, Y \subseteq V_G$ such that the density $d(X, Y) = 1$. Justify your answer. (2 marks)

(iv) Let G be a complete graph on $n > 1$ vertices, and $A, B \subseteq V_G$ be non-empty and disjoint. Prove that the pair (A, B) is ε-regular for all $\varepsilon > 0$. (2 marks)

(b) Let $p \in [0, 1]$, $n \in \mathbb{N}$, and $\mathcal{G}(n, p)$ be the Erdős-Rényi model for random graphs, where the graphs are simple and there is an edge between two vertices with probability p for every pair of vertices. Let $d \in [0, 1]$, $G \in \mathcal{G}(n, p)$, $X, Y \subseteq V_G$ be non-empty and disjoint. Compute the probability that $d(X, Y) = d$. (4 marks)

(c) Let $\varepsilon > 0$. Let G be a graph with n vertices and suppose that $V_0, V_1, \dots, V_k$ is an $(\varepsilon/4)$-regular partition. Suppose further that for $i, j \in \{1, \dots, k\}$ and $i < j$ we remove all edges between V_i and V_j if at least one of the following holds:

(i) (V_i, V_j) is not $\varepsilon/4$-regular,

(ii) the density $d(V_i, V_j) < \varepsilon/2$,

(iii) V_i or V_j has at most $(\varepsilon/4k)n$ vertices.

Prove that we removed at most εn^2 edges at the end of this process. (6 marks)

(Total: 20 marks)

Solutions

1. (a) i. The null graph and the complete graph on $n > 1$ vertices are examples, as they both have S_n as group of automorphisms. Clearly the null graph $\overline{K_n}$ is a minor of K_n, obtained by deleting all edges. Another example could be $H = K_5$ and G the Petersen graph as they both have S_5 as automorphism group. (2 marks) **Cat A** seen

   ii. A non-trivial connected graph must have $\delta(v) \geq 1$ otherwise it has some isolated vertex and it is cannot be connected. Therefore an example is K_2 . (2 marks) **Cat A** seen

   iii. K_3 is tripartite as every graph of order n is n-partite but cannot be bipartite as for every choice of two vertices there is an edge between them. (2 marks) **Cat A** seen

   iv. The cycle graph with 5 vertices is an example as it cannot be coloured with two colours: if we use red and blue and start with red, then we must alternate red and blue and the last vertex is also red. Clearly C_5 is triangle-free. (2 marks) **Cat A** seen

   (b) Let $\varphi : G \to H$ be an isomorphism. Then $\varphi\sigma\varphi^{-1}$ is a composition of graph isomorphisms and hence it is an isomorphism $H \to H$. The rule $\sigma \mapsto \varphi\sigma\varphi^{-1}$ defines a map

$$C_\varphi : \mathrm{Aut}(G) \longrightarrow \mathrm{Aut}(H).$$

   Its inverse is defined as $\tau \mapsto \varphi^{-1}\tau\varphi$, thus C_φ is bijective. It now remains to show that C_φ is a group homomorphism and this is immediate: for all $\sigma_1, \sigma_2 \in \mathrm{Aut}(G)$, we see that

$$C_\varphi(\sigma_1\sigma_2) = \varphi^{-1}\sigma_1\sigma_2\varphi = \varphi^{-1}\sigma_1\varphi\varphi^{-1}\sigma_2\varphi = C_\varphi(\sigma_1)C_\varphi(\sigma_2).$$

   (5 marks) **Cat B** seen

   (c) We construe G as a network N by replacing every edge with two directed edges of capacity 1 and setting s as the source and t as the sink (so we need to delete some edges: all the ingoing ones for s and all the outgoing ones for t). We use the max-flow min-cut theorem. If we restrict to flows that only have integer values on each edge, then the existence of a flow f with $v(f) = k$ corresponds to having k edge-distinct paths s–t, where f has value 1 on each edge. Thus the value of a max-flow in N is exactly $\sigma(s, t)$.

   (3 marks) **Cat C** unseen

   If C is a cut in N, the set of outgoing edges $E(C)$ corresponds to a set of edges $X \subseteq E_G$ of the same cardinality and such that

   - $G \setminus X$ is disconnected
   - s and t are in different connected components.

   Vice-versa if X is such a set of edges then we may choose one endpoint other than t for each of them. These endpoints together with s form a cut of N of capacity $|X|$. It follows that $\lambda(s, t)$ is the value of a min-cut in N. The result follows from the (integer) max-flow min-cut theorem (Theorem 4.1.14, but referencing this is not necessary for full marks)

   (4 marks) **Cat D** unseen

   (Total: 20 marks)

2. (a) i. If we colour one part with red and one with blue in both graphs then their disjoint union is still 2-colourable. (2 marks) **Cat A** seen

ii. The union is a cover because it still contains one vertex for every edge. It is minimal because if there is a cover C of H with less vertices than either $C \cap V_{G_1}$ or $C \cap V_{G_2}$ violates the minimality of C_1 and C_2. (2 marks) **Cat A** seen

iii. The same reasoning applies for matchings or one could use Kőnig's theorem as the cardinality of $M_1 \cup M_2$ is the same as $C_1 \cup C_2$. One need only check that the union of two matchings is a matching. (2 marks) **Cat A** seen

iv. If every vertex is matched in both G_1 and G_2 by two matchings M_1 and M_2, then the union $M_1 \cup M_2$ matches every vertex in H. (It is not enough to just use the previous point because that only says that the union is maximal, not that it is perfect.)

(2 marks) **Cat A** seen

(b) A cover contains at least a vertex for every edge of G. Hence it contains a vertex for every edge in M, so the cardinality of C must be larger than or equal to the cardinality of M.

(2 marks) **Cat B** seen

(c) Let G be the graph in the hint. If we find a perfect matching in this graph we will have found the required set. Indeed it suffices to choose one element in each intersection for each edge in the matching (by the second property of cosets listed before the hint).

(3 marks) **Cat B** unseen

We know that the number of left cosets is the same as the number of right cosets and is the index of Δ in Γ. It follows that a finding a perfect matching of L gives the set we are looking for. Indeed, if L has a perfect matching $M \subseteq E_G$, then it is possible to select the desired k elements as follows: let $M = \{e_1, \dots, e_k\}$, for each $i \in \{1, \dots, k\}$ choose $g_i \in l_i \cap r_i$ where l_i is the left coset at one end of e_i and r_i is the right coset at the other end of e_i.

(3 marks) **Cat C** unseen

Clearly $g_i\Delta = \Delta g_i$ and we get a complete list of cosets this way because M is a matching.

(1 mark) **Cat D** unseen

It remains to show that the graph G has Hall's condition. Consider ℓ left cosets, their union has cardinality $\ell|\Delta|$ so at least ℓ right cosets are needed to cover that. This means that every selection of ℓ vertices in L has at least ℓ neighbours in R and that Hall's condition is satisfied. Therefore G has a matching of L by Hall's marriage theorem. (3 marks) **Cat D** unseen

(Total: 20 marks)

3. (a) i. False the complete bipartite graph $K_{3,3}$ is a counterexample. (2 marks) $\mathbf{Cat\ A}$ seen

ii. False some graphs, like the null graphs, are 1-colourable. (2 marks) $\mathbf{Cat\ A}$ seen

iii. False, a graph of order n is always n colourable.

(2 marks) $\mathbf{Cat\ A}$ seen

iv. True, the chromatic polynomial is X^n. This is because every vertex of the null graph may be coloured independently of the colour already given to the other vertices. (2 marks) $\mathbf{Cat\ A}$ seen

(b) Let $q, r \in \mathbb{N}_0$ be such that $n = qm+r$ with $r < m$ (these exist by Euclid's division algorithm). Then both G and H have $m - r$ parts of size q and r parts of size $q + 1$.

Let $V_1, \ldots, V_m$ be the parts of G and $U_1, \ldots, U_m$ be the parts of H, with both partitions ordered by increasing cardinality of the sets. Then any map $V_G \to V_H$ that sends bijectively $V_i \to U_i$ (for all $i = 1, \ldots, m$) is an automorphism because both graphs are complete m-partite.

(5 marks) $\mathbf{Cat\ B}$ seen, set as an exercise

(c) The existence of R_k is a consequence of the finite combinatorial version of Ramsey's theorem. Let us identify the vertices of K_n with $[n] = \{1, \ldots, n\}$ and its edges with the set of 2-subsets of $[n]$, that is, $\mathcal{P}_2([2])$. With this conventions, a k-colouring of the edges of K_n is just a k-colouring $c : \mathcal{P}_2([2]) \to [k]$. By the finite version of the combinatorial Ramsey's theorem there is an R_k such that for $n \geq R_k$, the set $[n]$ has a 3-subset M such that the restriction of c to $\mathcal{P}_2(M)$ is constant. This set M forms a monochromatic triangle.

(3 marks) $\mathbf{Cat\ C}$ unseen

To show the recursive bound we assume $k \geq 2$ and $n \geq k(R_{k-1} - 1) + 2$ and show that there is a monochromatic triangle. Pick a vertex x of K_n. There are $n - 1$ edges incident with x, which is more than $k(R_{k-1} - 1)$. It follows that there is an $i \in [k]$ such that more than $(R_{k-1} - 1)$ of these edges are coloured with i. In other words, let X be the set of the corresponding vertices (those $v \in N(x)$ such that $\{v, x\}$ has colour i). Then $|X| \geq R_{k-1}$. Note that (setting $G = K_n$) $G[X]$ is a complete graph. We have two cases:

- there is an edge with colour i in $G[X]$. Then this forms a monochromatic triangle with the edges connecting its endpoints to x.
- there is no edge of colour i in $G[X]$. Then the edges of $G[X]$ have been coloured with $k-1$ colours and, since $|X| \geq R_{k-1}$, there is a monochromatic triangle already in $G[X]$.

(4 marks) $\mathbf{Cat\ D}$ unseen but similar to other years

(Total: 20 marks)

4. (a) i. True, the only way that a graph can be K_2-free is when it has no edges.

(2 marks) **Cat A** seen

ii. True, this is a consequence of Turán's theorem as all extremal triangle-free graphs are isomorphic to $T(n, 2)$. (2 marks) **Cat A** seen

iii. False. Let $n = 4$ and H be the disjoint union of two copies of K_2. Then the graph G whose connected components are a triangle and a vertex is H-free, so $\mathrm{ex}(n, H) \geq 3$. However, every graph with 4 edges is not H-free (one sees this by listing all possible graphs with 3 edges and then adding and extra edge to each of them). Hence $\mathrm{ex}(n, H) = 3$. It follows that G is extremal H-free but it is not the only extremal H-free graph up to isomorphism: the graph with only three coincident edges is also extremal H-free but is not isomorphic to G.

(4 marks) **Cat B** unseen

(b) Since we are interested in what happens when $n \to \infty$, we may assume that $n > m$. Let $n = am + b$. Then there are a pairwise disjoint m-element subsets of $V = V_G$, called $A_1, \ldots, A_a$. The probability that $A_1 \cong K_m$ is $p^{\binom{m}{2}}$. Let $E = \{K_m \cong G[U] \mid \exists U \subseteq V\}$ (this is the event that K_m is isomorphic to a (induced) subgraph of G). We have

$$\mathbb{P}[K_m \lesssim G] > \mathbb{P}[A_i \cong K_m \text{ for some i}] = 1 - \mathbb{P}[A_i \ncong K_m \text{ for all } i] = 1 - (1 - p^{\binom{m}{2}})^a \to 1$$

Fix graph H, and let $|H| = m$. We know that $H \subseteq K_m$, so if $K_m \subseteq G$ then $H \subseteq G$. Thus $\mathbb{P}[K_m \subseteq G) \leq \mathbb{P}[H \subseteq G]$, and we just saw that $\mathbb{P}[K_m \subseteq G] \to 1$.

(5 marks) **Cat A** seen in Problem Sheet 5

(c) Let $G \in \mathcal{G}(n, p)$ the probability that G is connected is

$$f(n) = 1 - P[G \text{ disconnected}].$$

Let $v \in V_G$. A graph of order n is disconnected if and only the connected component of v, say C, is of size $i < n$ (and $n - i$ vertices non adjacent to any of the vertices in C).

(3 marks) **Cat C** unseen

Choose $i - 1$ other vertices, the probability that these i vertices will be a connected induced subgraph is $f(i)$. There are $\binom{n-1}{i-1}$ choices for the other $i - 1$ vertices and the condition that C is not connected to any of the other vertices has probability $(1 - p)^{i(n-i)}$ as $i(n - i)$ is the number of edges to avoid. This gives the recursive formula.

(Note that by fixing a vertex v and considering only its connected component we avoid overcounting when we add the probabilities that there is a connected component of size less than n.) (4 marks) **Cat D** unseen

(Total: 20 marks)

5. (a) i. Definition from Diestel's book. The partition is regular if it satisfies the following 3 conditions

A. $|V_0| \leq \varepsilon |V|$;

B. $|V_1| = \cdots = |V_k|$;

C. all but at most εk^2 of the pairs (V_i, V_j) with $1 \leq i < j \leq k$ are ε-regular.

(3 marks) seen

ii. The maximum amount of possible edges between X and Y is $|X||Y|$, in which case $d(X,Y) = 1$. Similarly the minimum amount of possible edges is 0, in which case $d(X,Y) = 0$. The density is a non-negative rational number as it is defined as a quotient of two non-negative integers, moreover it is monotone increasing with respect to $\|X, Y\|$ once the size of these two sets is fixed. We conclude that $d(X,Y)$ is rational between 0 and 1.

(3 marks) seen

iii. An example is the complete graph with any two non-empty subsets of vertices.

(2 marks) seen

iv. A complete graph with A and B any two non-empty subsets. In this case (A, B) is ε-regular because any pair of non-empty subsets $X \subseteq A, Y \subseteq B$ has density $d(X,Y) = 1 = d(A,B)$.

(2 marks) seen

(b) Let $m_x = |X|$ and $m_y = |Y|$. If d is not a rational number, then the probability of $d(X,Y) = d$ is 0 as, by definition d is a rational number. The same is true, more in general, if $\ell = m_x m_y \cdot d$ is not an integer.

Assume d is a rational number and that $\ell = m_x m_y \cdot d$ is an integer. The probability that there are exactly ℓ (chosen) edges between X and Y is $P = p^\ell (1-p)^{(m_x m_y - \ell)}$. The choices of edges are

$$f = \binom{m_x m_y}{\ell}$$

Since the probability that $d(X,Y) = d$ is the same as the probability that there are exactly ℓ edges between X and Y, the answer is $f \cdot P$.

(4 marks) unseen

(c) By definition of regular partition the sum

$$\sum_{\substack{i,j \\ (V_i,V_j) \text{ irregular}}} |V_i||V_j| \leq \frac{\varepsilon}{4} k^2 \cdot \frac{n^2}{k^2} = \frac{\varepsilon}{4} n^2.$$

Hence at most $\frac{\varepsilon}{4} n^2$ edges have been removed in the first of the three cases.

The number of edges between each V_i and V_j is, by definition of density

$$d(V_i, V_j)|V_i||V_j|.$$

This is at most $d(V_i, V_j)n^2$. Hence, in the second case, we removed at most $\frac{\varepsilon}{2} n^2$ edges.

Finally suppose that V_i has size at most $(\varepsilon/4k)n$. Then the third case will happen when it is paired with any of the other parts. Hence, summing over all the times a pair containing V_i is considered, we are removing at most

$$n\frac{\varepsilon}{4k}n = n^2\frac{\varepsilon}{4k}$$

edges. There are at most k such parts (in case V_0 is large enough all parts may have such "small size"), so the removal just described happens at most k times. In total we have removed at most

$$n^2\frac{\varepsilon}{4k}$$

edges. The total of the three cases is therefore εn^2.

(6 marks) unseen

(Total: 20 marks)

MATH70038 Graph Theory Markers Comments

Question 1

Part (a) was solved well (b) some did not find an explicit isomorphism or found the wrong one (c) there were several issues in the solutions to this question, mainly only few people proved that a min cut in the network they defined was giving a set A and (and vice versa).

Question 2

Part (a)(b) solved well generally (c) Only few complete solutions, most solutions were missing the justification of why a perfect matching gives the result to prove, other mistakes included saying that every left coset corresponded to a unique right coset in the bipartite graph (this is not the case in general: for instance take \Delta = (12) in S_3 and list its right and left cosets), and missing a full justification of why the graph had Hall's property.

Question 3

(a) Solved well generally (b) It does not make sense to prove this by invoking Turán's theorem. This is about showing that Turán's graph are well defined up to isomorphism.
(c) only a handful of full solutions for this. The most common confusion was to use 3-sets instead of 2-sets (and a monochromatic 3-set).

Question 4

(a) solved well generally (b) a few mistakes using equality instead of inequalities for estimating the probability (c) only a few solutions explaining why (to avoid overcounting) there was a (n-i choose i - 1) as a factor.

Question 5

The question was in general solved well, although only few people attempted the last part.