Imperial College London

MATH70036

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May 2023

This paper is also taken for the relevant examination for the
Associateship of the Royal College of Science

Group Theory

Date: 9 May 2023

Time: 10:00 – 12:30 (BST)

Time Allowed: 2.5hrs

This paper has 5 Questions.

Please Answer All Questions in 1 Answer Booklet

Candidates should start their answers to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

Any required additional material(s) will be provided.

Allow margins for marking.

Credit will be given for all questions attempted.

Each question carries equal weight.

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO

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Setter:	Liebeck
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BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)

May – June 2023

MATH60036/MATH70036 Group Theory

The following information must be completed:

Is the paper suitable for resitting students from previous years: No

Category A marks: available for basic, routine material (excluding any mastery question) (40 percent = 32/80 for 4 questions):
25/80, as indicated in Solutions

Category B marks: Further 25 percent of marks (20/ 80 for 4 questions) for demonstration of a sound knowledge of a good part of the material and the solution of straightforward problems and examples with reasonable accuracy (excluding mastery question):
23/80, as indicated in Solutions

Category C marks: the next 15 percent of the marks (= 12/80 for 4 questions) for parts of questions at the high 2:1 or 1st class level (excluding mastery question):
14/80, as indicated in Solutions

Category D marks: Most challenging 20 percent (16/80 marks for 4 questions) of the paper (excluding mastery question):
18/80, as indicated in Solutions

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BSc, MSc and MSci EXAMINATIONS (MATHEMATICS)

May – June 2023

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science.

Group Theory

Date: ??
Time: ??
Time Allowed: 2 Hours
This paper has *4 Questions (MATH96 version); 5 Questions (MATH97 versions)*.
Statistical tables will not be provided.

- Credit will be given for all questions attempted.
- Each question carries equal weight.

Throughout the paper, you may use any results from the course that you require provided you state them clearly.

1. Let G be a group, and for $g \in G$ define $\iota_g : G \mapsto G$ by

$$\iota_g(x) = gxg^{-1} \quad \text{for all } x \in G.$$

Let $\mathrm{Inn}(G) = \{\iota_g : g \in G\}$.

(a) Show that $\iota_g \in \mathrm{Aut}(G)$, and that $\mathrm{Inn}(G)$ is a subgroup of $\mathrm{Aut}(G)$. (3 marks)

(b) Prove that $\mathrm{Inn}(G) \cong G/Z(G)$, where $Z(G)$ is the centre of G. (3 marks)

(c) Prove that $\mathrm{Inn}(G)$ cannot be a nontrivial cyclic group. (4 marks)

(d) Recall that Q_8 denotes the quaternion group of order 8:

$$Q_8 = \langle a, b : a^4 = b^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1} \rangle.$$

Suppose G is a finite group such that $\mathrm{Inn}(G) \cong G/Z(G) \cong Q_8$.

(i) Prove that G has two abelian subgroups A, B of index 2.

(ii) Show that $G = AB$ and find the value of $|G : A \cap B|$.

(iii) Prove that $A \cap B \leq Z(G)$.

(iv) Deduce that G does not exist. (Hence there is no finite group G such that $\mathrm{Inn}(G) \cong Q_8$.)

(10 marks)

(Total: 20 marks)

2. (a) State the four Sylow theorems. (4 marks)

(b) Using the Sylow theorems, and any other standard results you need, prove the following:

(i) There is no simple group of order 84.

(ii) There is no simple group of order 132.

(8 marks)

(c) Let p be a prime, and suppose that G is a simple group of order $p^m r$, where $r > 1$ and p does not divide r (and m is a positive integer). Prove that p^m divides $(r-1)!$ (5 marks)

(d) Prove that there is no simple group of order $12p^m$, where $p \geq 7$ is prime and $m \in \mathbb{N}$.

(3 marks)

(Total: 20 marks)

3. (a) Let G be a finite group. Define what is meant by the statement that G is *nilpotent*. Define also the *Frattini subgroup* $\Phi(G)$. (4 marks)

   (b) (i) Let p be an odd prime. Prove that the dihedral group D_{2p} is not nilpotent.

   (ii) Deduce that for any $n \in \mathbb{N}$, the dihedral group D_{2n} is nilpotent if and only if n is equal to a power of 2.

   (6 marks)

   (c) Now let G be a finite nilpotent group.

   (i) Let M be a maximal subgroup of G. Assuming the standard result that $M \lhd G$, prove that G/M is cyclic of prime order.

   (ii) Prove that $G' \leq \Phi(G)$.

   (iii) Hence prove that if G/G' is cyclic, then G must be cyclic.

   (10 marks)

(Total: 20 marks)

4. (a) Let G be a finite group and let H be a subgroup of G. Define the *normalizer* $N_G(H)$ and the *centralizer* $C_G(H)$. Prove that $N_G(H)/C_G(H)$ is isomorphic to a subgroup of $\mathrm{Aut}(H)$. (5 marks)

   (b) Show that $|\mathrm{Aut}(C_4)| = 2$ and $|\mathrm{Aut}(C_2 \times C_2)| = 6$. (Here C_n denotes a cyclic group of order n.) (3 marks)

   (c) Let G be a group of order $4k$, where k is an odd integer. Let P be a Sylow 2-subgroup of G, and suppose that $N_G(P) \neq C_G(P)$. Prove the following:

   (i) $N_G(P)/C_G(P)$ is isomorphic to a nontrivial subgroup of $\mathrm{Aut}(P)$ of odd order.

   (ii) $P \cong C_2 \times C_2$.

   (iii) k is divisible by 3.

   (5 marks)

   (d) State Burnside's Transfer Theorem. (2 marks)

   (e) Let G be a finite simple group such that $|G|$ is an even number and $|G| > 2$. Prove that $|G|$ is divisible by either 8 or 12. (Hint: suppose 8 does not divide $|G|$ and consider the possibilities for a Sylow 2-subgroup of G using (d).) (5 marks)

(Total: 20 marks)

5. **(Mastery Question)**

(a) Let F be a field and let $n \geq 2$ be an integer. Define the *special linear group* $SL_n(F)$. (1 mark)

(b) For $i, j \leq n$ with $i \neq j$, and $\lambda \in F$, recall the *elementary* matrix

$$E_{ij}(\lambda) = I_n + \lambda e_{ij},$$

where e_{ij} is the matrix with 1 in the ij-entry and 0 elsewhere.

Prove that $SL_n(F)$ is generated by the set of all elementary matrices. (4 marks)

(c) Let $n \geq 3$ and let $G = SL_n(F)$. Prove that $G' = G$. (5 marks)

(d) Now let $n \geq 2$ and $G = SL_n(F)$. Let V be the vector space F^n, and let $PG(V)$ be the set of 1-dimensional subspaces of V. Let G act on $PG(V)$ in the natural way (i.e. $A \in G$ sends $\langle v \rangle \mapsto \langle Av \rangle$ for any $\langle v \rangle \in PG(V)$).

(i) Prove that this action of G on $PG(V)$ is 2-transitive. (4 marks)
(Recall that an action of a group H on a set X is defined to be 2-*transitive* if for any two ordered pairs (x_1, x_2) and (y_1, y_2) in $X \times X$, with $x_1 \neq x_2$, $y_1 \neq y_2$, there exists $h \in H$ sending $x_1 \mapsto x_2$, $y_1 \mapsto y_2$.)

(ii) Let $\alpha \in PG(V)$, and let G_α be the stabilizer of α in G. Prove that G_α has an abelian normal subgroup that is isomorphic to $(F^{n-1}, +)$. (6 marks)

(Total: 20 marks)

Group Theory MATH 60036/70036 May 2023 Solutions

1. (a) $\iota_g(x_1x_2) = gx_1x_2g^{-1} = gx_1g^{-1}gx_2g^{-1} = \iota_g(x_1)\iota_g(x_2)$, os ι_g is a homomorphism; it is clearly a bijection (by cancellation rules in groups). So $\iota_g \in \mathrm{Aut}(G)$.

Let $H = \mathrm{Inn}(G)$. Since $\iota_{g_1}\iota_{g_2} = \iota_{g_1g_2}$ and $\iota_g^{-1} = \iota_{g^{-1}}$, H is closed under product and inversion, and also contains the identity, so H is a subgroup of $\mathrm{Aut}(G)$. (**Bookwork, 3 marks, Category A**)

(b) The map $\pi : G \mapsto \mathrm{Inn}(G)$ sending $g \mapsto \iota_g$ is a homomorphism, and

$$g \in \mathrm{Ker}(\pi) \Leftrightarrow \iota_g = \mathrm{id}_G \Leftrightarrow gxg^{-1} = x\ \forall x \in G \Leftrightarrow g \in Z(G).$$

So $\mathrm{Ker}(\pi) = Z(G)$, and by the 1st Isomorphism Thm, $G/Z(G) \cong \mathrm{Im}(\pi) = \mathrm{Inn}(G)$. (**Bookwork, 3 marks, A**)

(c) Suppose $G/Z(G)$ is cyclic; say $G/Z(G) = \langle gZ(G)\rangle$. Then every element of G is of the form g^iz for some integer i and some $z \in Z(G)$. Let $x = g^iz$, $y = g^jz'$ be two such elements. Then $xy = g^izg^jz' = g^ig^jzz' = g^{i+j}zz'$, and $yx = g^jz'g^iz = g^jg^iz'z = g^{i+j}zz'$. Hence $xy = yx$, showing that G is abelian. But this means that $G = Z(G)$, so $G/Z(G)$ is the trivial group. (**Set as exercise, 4 marks, B**)

(d) (i) Let $\pi : G \mapsto Q_8 \cong G/Z(G)$ be the canonical map, and define $A = \pi^{-1}(\langle a\rangle)$, $B = \pi^{-1}(\ ab\rangle)$, the preimages of the cyclic subgroups $\langle a\rangle$ and $\langle b\rangle$ of Q_8. Then $|G : A| = |Q_8 : \langle a\rangle| = 2$, and similarly $|G : B| = 2$. Also $Z(G) \leq Z(A)$ and $A/Z(G)$ is cyclic, so $A/Z(A)$ is also cyclic, and so by (c), A is abelian. Similarly B is abelian. (**3 marks, C**)

(ii) As they have index 2, a standard result implies that A and B are normal subgroups of G. Hence AB is also a subgroup, and $A < AB \leq G$. As $|G : A| = 2$, it follows that $G = AB$.

By a standard result, $|G| = |AB| = \frac{|A|\,|B|}{|A\cap B|}$. As $|A| = |B| = \frac{1}{2}|G|$, it follows that $|G| = \frac{1}{4}\frac{|G|^2}{|A\cap B|}$. Hence $|G : A\cap B| = 4$. (**2 marks, B**)

(iii) The centralizer in G of $A \cap B$ contains A, as A is abelian; similarly it contains B. Since $G = AB$, it follows that G centralizes $A\cap B$, i.e. $A\cap B \leq Z(G)$. (**3 marks, D**)

(iv) Part (iii) implies that $|G : Z(G)| \leq |G : A\cap B| = 4$. However $|G : Z(G)| = |Q_8| = 8$. This is a contradiction, showing that G does not exist. (**2 marks, B**)

Part (d) is unseen

2. (a) The Sylow theorems:

Sylow I: Let $|G| = p^a m$, where p is prime and p does not divide m. Then G has a subgroup of order p^a.

Sylow II: If $n_p(G)$ denotes the number of Sylow p-subgroups of G, then $n_p(G) \equiv 1 \bmod p$.

Sylow III: Let Q be a p-subgroup of G. Then there exists $P \in Syl_p(G)$ such that $Q \leq P$.

Sylow IV: $Syl_p(G)$ is a single conjugacy class of subgroups of G; that is, for any $P, Q \in Syl_p(G)$, there exists $g \in G$ such that $Q = {}^gP$.

(**Bookwork, 4 marks, A**)

(b) (i) Suppose $|G| = 84 = 2^2 \cdot 3 \cdot 7$. By Sylow II, $n_7(G) \equiv 1 \bmod 7$, and by a standard result it divides $|G|$. Hence $n_7(G) = 1$. But this means that G has a normal Sylow 7-subgroup, so G is not simple. (**Similar seen, 3 marks, A**)

(ii) Suppose $|G| = 132 = 2^2 \cdot 3 \cdot 11$, and suppose also that G is simple. Then $n_{11}(G) \neq 1$, so by the reasoning in (i), $n_{11}(G) = 12$. Any two Sylow 11-subgroups intersect only in 1, so the total number of elements in G of order 11 is $N_{11} = 12 \times 10 = 120$. Also $n_3(G) \geq 4$, so if N_3 is the number of elements of order 3 then $N_3 \geq 8$. Hence the number of non-identity elements that can lie in Sylow 2-subgroups of G is at most $132 - 120 - 8 - 1 = 3$ (the -1 is for the identity). This means there can only be 1 Sylow 2-sybgroup, which is a contradiction as G is simple. (**Similar seen, 5 marks, C**)

(c) Suppose $|G| = p^m r$ and G is simple. Let $k = n_p(G)$. As G is simple, $k > 1$, and also $k \equiv 1 \bmod p$ and divides $|G|$. Hence $k|r$. The action of G by conjugation on $Syl_p(G)$ gives a homomorphism $\pi : G \mapsto S_k$ with nontrivial image. As G is simple, the normal subgroup $\mathrm{Ker}(\pi) = 1$, so $G \cong \mathrm{Im}(\pi) \leq S_k$. Hence $|G = p^m r$ divides $k!$, which divides $r!$, and so $p^m|(r-1)!$ (**Unseen, 5 marks, D**)

(d) Suppose $|G| = 12p^m$, where $p \geq 7$ is prime. If G is simple, then p^m divides $11!$ by part (c), and hence $p^m = 7$ or 11. So $|G| = 84$ or 132. But there are no simple groups of these orders, by (b). Hence G is not simple. (**3 marks, B**)

3. (a) There are quite a few correct definitions - here is the original one in lectures: G is nilpotent if there is a series of normal subgroups $1 = G_0 < G_1 < \cdots < G_r = G$ such that $G_{i+1}/G_i \leq Z(G/G_i)$ for all i. (**Bookwork, 2 marks, A**)

The Frattini subgroup $\Phi(G)$ is the intersection of all the maximal subgroups of G. (**Bookwork, 2 marks, A**)

(b) (i) Let $G = D_{2p} = \langle x, y : x^p = y^2 = 1, y^{-1}xy = x^{-1}\rangle$. If G is nilpotent, then by definition $Z(G) \neq 1$. However $Z(D_{2p}) = 1$ (the non-identity powers x^i do not commute with y, the other elements $x^i y$ do not commute with x). So D_{2p} is not nilpotent. (**Similar seen, 3 marks, B**)

(ii) Consider $G = D_{2n}$. If $n = 2^a$, then G is a 2-group, which is nilpotent (standard result). If not, then n has an odd prime factor p, and then G has a subgroup $\langle x^{n/p}, y\rangle \cong D_{2p}$. This is non-nilpotent by (i), so G has a non-nilpotent subgroup, hence is itself not nilpotent (standard result). (**Similar seen, 3 marks, B**)

(c) (i) Let G be nilpotent, and M a maximal subgroup.As stated in the question, $M \triangleleft G$. Since M is maximal, G/M can have no nontrivial subgroups, hence $G/M \cong C_p$ for some prime p. (**Set as exercise, 2 marks, B**)

(ii) By (i), G/M is cyclic, hence abelian, for every maximal subgroup M. Hence $G' \leq M$ for every max subgroup, and so $G' \leq \Phi(G)$. (**Unseen, 3 marks, C**)

(iii) Suppose G/G' is cyclic. Then by (ii), $G/\Phi(G)$ is cyclic, so $G/\Phi(G) = \langle g\Phi(G)\rangle$ for some $g \in G$. Then $G = \langle g, \phi(G)\rangle$. Hence by the 'non-generation' property of $\Phi(G)$ (standard result), $G - \langle g\rangle$, cyclic. (**Unseen, 5 marks, D**)

4. (a) $N_G(H) = \{g \in G : gHg^{-1} = H\}$, $C_G(H) = \{g \in G : gh = hg \ \forall h \in H\}$. (**Bookwork, 2 marks, A**)

For $n \in N_G(H)$, let $\iota_n : H \mapsto H$ be the conjugation map sending $h \mapsto nhn^{-1}$. Then $\iota_n \in \mathrm{Aut}(H)$, and the map $\iota : N_G(H) \mapsto \mathrm{Aut}(H)$ sending $n \mapsto \iota_n$ is a homomorphism with kernel $C_G(H)$. Hence $N_G(H)/C_G(H) \cong \mathrm{Im}(\iota) \leq \mathrm{Aut}(H)$. (**Bookwork, 3 marks, A**)

(b) Let $C_4 = \langle x \rangle$. If $\alpha \in \mathrm{Aut}(C_4)$ then α sends x to x or x^{-1}, and both possibilities gives automorphisms. So $|\mathrm{Aut}(C_4)| = 2$. (**Seen as exercise, 1 mark, A**)

Since $C_2 \times C_2 \cong (\mathbb{F}_2^2, +)$, a vector space of dim 2 over $\mathbb{F}_2$, we have $\mathrm{Aut}(C_2 \times C_2) \cong GL_2(\mathbb{F}_2)$. From lectures (or from direct calculation), this group has order 6. (**Seen as exercise, 2 marks, B**)

(c) (i) Let $P \in Syl_2(G)$, so $|P| = 4$. Then $P \cong C_4$ or $C_2 \times C_2$. Since we are supposing $N_G(P) \neq C_G(P)$, part (a) gives that $N_G(P)/C_G(P)$ is isomorphic to a nontrivial subgroup of $\mathrm{Aut}(P)$. Also P is abelian, so $P \leq C_G(P)$ and so $N_G(P)/C_G(P)$ has odd order. (**3 marks, C**)

(ii) If $P \cong C_4$ then $|\mathrm{Aut}(P)| = 2$ by (b). This is not possible by (i). Hence $P \cong C_2 \times C_2$. (**1 mark, B**)

(iii) By (b) we have $|\mathrm{Aut}(P)| = 6$. Hence by (i), $N_G(P)/C_G(P)$ must have order 3. So 3 divides $|G|$, hence divides k. (**1 mark, B**)

Part (c) is unseen

(d) **Burnside's transfer theorem**: Let p be prime, $P \in Syl_p(G)$, and suppose that $P \leq Z(N_G(P))$. Then G has a normal p-complement (ie. a normal subgroup N such that $G = PN$ and $P \cap N = 1$). (**Bookwork, 2 marks, A**)

(e) Let G be simple with $|G| > 2$ even, and suppose $|G|$ is not a multiple of 8. Let $P \in Syl_2(G)$. Then $|P| = 2$ or 4. If $|P| = 2$ then $N_G(P)/C_G(P) \leq \mathrm{Aut}(P) = 1$, so $P \leq C_G(P) = N_G(P)$; now Burnside's transfer theorem shows that G is not simple, a contradiction. If $|P| = 4$ and $N_G(P) = C_G(P)$, we obtain the same contradiction. Finally, if $|P| = 4$ and $N_G(P) \neq C_G(P)$, then part (c)(iii) shows that 3 divides $|G|$, and hence 12 divides $|G|$. (**Unseen, 5 marks, D**)

Mastery Q5. (a) $SL_n(F) = \{A \in M_n(F) : \det(A) = 1\}$. (**Bookwork, 1 mark, A**)

(b) Let $A \in SL_n(F)$. Recall that if $r_1, \ldots, r_n$ are the rows of A, then $E_{ij}(\lambda)A$ has the same rows, except that r_i is replaced by $r_i + \lambda r_j$.

Let $A = (a_{ij})$. Adding some row to row 2, we can assume that $a_{21} \neq 0$. Then add $a_{21}^{-1}(1-a_{11})r_2$ to r_1 to get $a_{11} = 1$. Now subtract multiples of r_1 from the other rows to get a matrix $U_1 A$ with first column $(1, 0, \ldots, 0)^T$, where U_1 is a product of elementary matrices. Repeat with columns $2, \ldots, n-1$ to get

$$U_{n-1}A = \begin{pmatrix} I_{n-1} & * \\ 0 & \mu \end{pmatrix},$$

where again U_{n-1} is a product of elementary matrices. Since $\det(A) = 1$, we have $\mu = 1$. Finally, clearing the last column gives $U_n A = I_n$. Then $A = U_n^{-1}$. Since the inverse of an elementary matrix is also elementary, this shows that A is a product of elementary matrices. Hence $SL_n(F)$ is generated by the elementary matrices. (**Bookwork, 4 marks, A**)

(c) From the equation

$$\begin{pmatrix} 1 & -x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

applied to all 3×3 sub-blocks of $n \times n$ matrices, we see that every elementary matrix is a commutator in $G = SL_n(F)$. Hence $G' = G$ by (b). (**Argument in the mastery notes, 5 marks, B**)

(d) Let $\langle v_1 \rangle$, $\langle v_2 \rangle$ and $\langle w_1 \rangle$, $\langle w_2 \rangle$ be two pairs of distinct 1-spaces in $PG(V)$. Then v_1, v_2 are linearly independent, so can be extended to a basis $\mathrm{B} = v_1, v_2, \ldots, v_n$ of V. Similarly there is a basis $C = w_1, w_2, \ldots, w_n$. There is a linear map g sending $v_i \mapsto w_i$ for all i, and $g \in GL_n(F)$. If $\det(g) = \lambda$, then the map sending $v_1 \mapsto \lambda^{-1}w_1$ and $v_i \mapsto w_i$ for $i \geq 2$ has determinant 1 and sends $\langle v_1 \rangle \mapsto \langle w_1 \rangle$ and $\langle v_2 \rangle \mapsto \langle w_2 \rangle$. Hence the action of $G = SL_n(F)$ is 2-transitive. (**Refinement of argument in mastery notes, 4 marks, C**)

(e) Since G is transitive on $PG(V)$, all point-stabilizers are conjugate, hence isomorphic, so we can take $\alpha = \langle e_1 \rangle$, where $e_1, \ldots, e_n$ is the standard basis. Then

$$G_\alpha = \left\{ \begin{pmatrix} \lambda & x \\ 0 & A \end{pmatrix} : x \in F^{n-1}, \lambda \in F^*, A \in GL_{n-1}, \det(A) = \lambda^{-1} \right\}.$$

Let

$$N = \left\{ \begin{pmatrix} 1 & x \\ 0 & I_{n-1} \end{pmatrix} : x \in F^{n-1} \right\}.$$

Then N is a subgroup of G_α and $N \cong F^{n-1}$, and also $N \triangleleft G_\alpha$ since

$$\begin{pmatrix} \lambda^{-1} & * \\ 0 & A^{-1} \end{pmatrix} \begin{pmatrix} 1 & x \\ 0 & I_{n-1} \end{pmatrix} \begin{pmatrix} \lambda & * \\ 0 & A \end{pmatrix} = \begin{pmatrix} 1 & * \\ 0 & I_{n-1} \end{pmatrix} \in N.$$

(**Refinement of argument in mastery notes, 6 marks, C**)

If your module is taught across multiple year levels, you might have received this form for each level of the module. You are only required to fill this out once for each question.		
ExamModuleCode	**QuestionNumber**	**Comments for Students**
MATH60036/70036	1	The bookwork parts were well done. The non-bookwork parts were also quite good, although a number of solutions showed confusion when working in quotient groups.
MATH60036/70036	2	This question was really well done. I thought the last two parts might prove tricky, but most candidates did them very well.
MATH60036/70036	3	Some candidates struggled with the first part, deciding which dihedral groups are nilpotent. But the second (unseen) part was really well done.
MATH60036/70036	4	This question was not quite so well done as the others, particularly the first two parts on automorphism groups.
MATH70036	5	There were no really substantial attempts as this question. The question was mainly material taken fairly directly from the mastery notes, so I don't know what the issue was.