Imperial College London
MATH 50004/50015 Multivariable Calculus 2023
Quiz 2

This quiz consists of 10 questions worth 2 marks each. Please record all your responses on the Blackboard Quiz and don't forget to press 'submit' at the end. The quiz is open for 24 hours from 9am on Monday 11th December.

1. If we represent the vector field

$$\mathbf{A} = \alpha x\mathbf{i} + \beta y\mathbf{j} + \gamma z\mathbf{k}$$

in cylindrical polar coordinates as

$$A_r\widehat{\mathbf{r}} + A_\theta\widehat{\theta} + A_z\widehat{\mathbf{k}},$$

then we have

$$\begin{array}{ll} (a) & A_\theta = r(\alpha+\beta)\cos\theta\sin\theta \\ (b) & A_r = r(\alpha\cos^2\theta + \beta\sin^2\theta) \\ (c) & A_\theta = r(\beta-\alpha)\sin 2\theta \\ (d) & A_r = (\alpha+\beta)r \end{array}$$

2. The vector field $\mathbf{F}$ is given in terms of cylindrical polar coordinates (r, θ, z) by

$$\mathbf{F} = (r^2z^3\sin\theta)\widehat{\mathbf{r}} + (r^2z\sin\theta)\widehat{\theta} + (r^3z^2\cos\theta)\widehat{\mathbf{k}}.$$

The quantity $\operatorname{div}\mathbf{F}$ is given by

$$\begin{array}{ll} (a) & 3z^3\sin\theta + r(1+2r^2)z\cos\theta \\ (b) & 3rz^3\sin\theta + r(1+2r^2)z\cos\theta \\ (c) & 3rz^3\sin\theta + rz\cos\theta \\ (d) & 2rz^3\sin\theta + r(1+2r^2)z\cos\theta \end{array}$$

3. For the same vector field $\mathbf{F}$ as in Question 2, the $\widehat{\mathbf{r}}$ component of $\operatorname{curl}\mathbf{F}$ is

$$\begin{array}{ll} (a) & -r^2(1+z^2)\sin\theta \\ (b) & 3r^2z^2(\sin\theta-\cos\theta) \\ (c) & -r(1+z^2)\sin\theta \\ (d) & r^2(z^2-1)\sin\theta \end{array}$$

4. The function Φ is given in spherical polar coordinates (r, θ, ϕ) by

$$\Phi = (r^2 - r^{-4})\cos^2\theta.$$

If $\nabla\Phi = A_1\widehat{\mathbf{r}} + A_2\widehat{\theta}$, then

$$\begin{array}{ll} (a) & A_2 = (r^{-4}-r^2)\sin 2\theta \\ (b) & A_1 = (2r+4r^{-5})\cos 2\theta \\ (c) & A_2 = (r^{-5}-r)\sin 2\theta \\ (d) & A_1 = (2r-4r^{-5})\cos^2\theta \end{array}$$

5. For the same function Φ given in Q4, the Laplacian of Φ is given by:

$$\begin{array}{ll} (a) & 2-5r^{-6}-3r^{-6}\cos 2\theta \\ (b) & -6r^{-6}\cos^2\theta \\ (c) & 2-2r^{-6}+4(1-4r^{-6})\cos^2\theta \\ (d) & 0 \end{array}$$

6. Evaluate

$$\int_V (z^4)\ dV$$

where V is the volume contained between concentric spheres centred at the origin with radii a and b ($b > a$). The answer is

$(a)\ \ 4\pi(b^5 - a^5)/5$
$(b)\ \ 4\pi(b - a)^5/5$
$(c)\ \ 4\pi(b - a)^7/35$
$(d)\ \ 4\pi(b^7 - a^7)/35$

7. Evaluate the integral

$$\int_R (x + y)\, dx\, dy$$

where R is the region in the first quadrant bounded by $x^2 + y^2 = 16$, $x^2 + y^2 = 25$ and the x and y axes. The answer is

$(a)\ \ 68\pi$
$(b)\ \ 122\pi/12$
$(c)\ \ 122/3$
$(d)\ \ 0$

8. Evaluate

$$\int_R (\alpha x + \beta y)\ dx\ dy \quad (\alpha, \beta \text{ constants})$$

where R is the region in the $x-y$ plane inside the four-sided figure with vertices at $(0, 0)$, $(1, -1)$, $(2, 0)$ and $(1, 1)$, by using the transformation $u = x + y$, $v = x - y$. The answer is

$(a)\ \ 4\alpha$
$(b)\ \ 2\alpha$
$(c)\ \ 2\alpha + \beta$
$(d)\ \ 2\beta$

Questions 9 and 10 involve an open surface S whose side is the cylinder $x^2 + y^2 = 4$, $z \geq 0$. The top is capped with the plane $z = 3 + x$. Let S_1 be the curved surface and S_2 be the planar top.

9. Evaluate

$$\int_{S_1} (x + y + z)\ dS$$

The answer is

$(a)\ \ 30\pi$
$(b)\ \ 7\pi$
$(c)\ \ 2\pi$
$(d)\ \ 5\pi$

10. Evaluate

$$\int_{S_2} (x + y + z)\ dS$$

The answer is

$(a)\ \ 3\pi\sqrt{2}/2$
$(b)\ \ 12\pi\sqrt{2}$
$(c)\ \ 4\pi$
$(d)\ \ 0$

Remember to press SUBMIT once you have answered all the questions. The test will not auto-submit.