MATH60005/700055: Optimisation (Autumn 22-23)

Module Guide

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Administrative details

- **Instructor**: Dr Dante Kalise (dkaliseb@imperial.ac.uk)
- **Lectures**: Wednesdays 10:00-11:50 am weeks 2-11, Thursdays 15:00-15:50 weeks 1-10. All lectures will be held in Huxley 130. These live sessions will consist of:
    1. Brief overview of the material of the week,
    2. Several illustrative examples.
    3. Discussion of selected problems from a weekly problem sheet.

  Live sessions will be recorded and available via Blackboard
- **Content structure:** the module material consists of:
    1. Lecture notes,
    2. Pre-recorded videos,
    3. Exercise list,
    4. Additional material such as code, external resources, and the reading list (at the end of this document),

5. Mock exams, previous year exam and coursework.

   Contents are usually releasead a couple of weeks in advance starting from week 2.

- **Module assessment:** 90% exam based on contents from weeks 2 to 9 (or until week 11 for those following the Year 4/MSc version with mastery topic), and a 10% coursework evaluating exercises and computational aspects up to week 5 (due date: 24/11/22).
- **Office hours:** Right after our lectures. Otherwise, feel free to contact me by email to find a suitable alternative.

Module overview

This module is an introduction to the theory and practice of mathematical optimization and its many applications in mathematics, data science, and engineering. The module aims at endowing students with the necessary mathematical background and a thorough methodological toolbox to formulate optimization problems and developing an algorithmic approach to its solution.

The module is structured around the following topics:

1. Formulation and classification of optimization problems;
2. Unconstrained optimization, including stochastic optimization;
3. Convex and constrained optimization;
4. Introduction to optimal control and dynamic optimization.

Intended learning outcomes

On successful completion of this module you will be able to:

1. formulate a mathematical optimization problem by identifying a suitable objective and constraints;
2. identify the mathematical structure of an optimization problem and, based on this classification, choose an appropriate methodological approach;
3. develop a mathematical and computational appreciation of convexity as a fundamental feature in optimization;
4. implement different computational optimization algorithms such as gradient descent and related variants;
5. analyse the results of a computational optimization method in terms of optimality guarantees, sensitivities, and performance.

6. interpret the role played by optimization in its application to computational data science;
7. design optimal control approaches relevant to tackling large-scale nonlinear problems.

Module Content

- Mathematical preliminaries. Relevant subspaces in optimization, linear algebra, and multivariable calculus.
- Week 2: unconstrained optimization. Classification of maxima and minima, saddle points, necessary and sufficient optimality conditions, quadratic forms.
- Week 3: linear and non-linear least square problems. Problem statement, optimality conditions, solution algorithms, and applications.
- Week 4: gradient descent method. Algorithmic formulation, convergence, stepsize choice, quasi-Newton methods.
- Week 5: stochastic gradient descent. Motivation, the Kaczmarz method, stochastic gradient descent, batch methods.
- Week 6: convexity. Convex sets and functions, implications in optimization.
- Week 7: convex optimization. Stationarity conditions, projection operators, and projected gradient descent.
- Week 8: optimality conditions. Necessary and sufficient optimality conditions for linearly constrained problems, KKT conditions, nonlinear constraints.
- Week 9: duality. Weak and strong duality, and applications.
- Weeks 10-11 (mastery topic): optimal control. Introduction to dynamic optimization, optimality conditions, the linear-quadratic case.

Prerequisites

Basic knowledge of linear algebra, real analysis, and multivariable calculus is required. Please ask for advice if you struggle when studying the mathematical preliminaries notes.

A part of the coursework involves a light computational component. For this part, coding proficiency will not be assessed. You can work in your preferred language as the evaluation will be focused on your results and analysis. Adequate support will be provided in Matlab and Python. Computational aspects are excluded from the final exam.

Feedback

Works bi-directionally. We will receive feedback from your coursework within a few days via Blackboard, and we can discuss your attempts at the exercises during live sessions, office hours, or forums. For any other matters please feel free to drop me an e-mail.

Reading list

Sometimes our understanding of a mathematics topic is guided by a certain book which we find particularly well-written. If your thirst for knowledge does not stop with the lecture notes, I propose the following reading list. These textbooks have been chosen so that you can access them online, some of them are free, others can be accessed through Imperial's library online subscriptions.

- "Introduction to Nonlinear Optimization", by Amir Beck. This book covers most of the material of the lecture notes very thoroughly, and is avaiable through Imperial's SIAM subscription.
- "Convex Optimization" by Stephen Boyd and Lieven Vandenberghe. This is a classic reference on convex optimization, with lots of resources freely available at: `https://web.stanford.edu/~boyd/cvxbook/`
- "Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares", by Stephen Boyd and Lieven Vandenbergh. This is a companion book to "Convex Optimization" and discussed relevant linear algebra knowledge to approach an optimization module. Freely available at:

  `https://web.stanford.edu/~boyd/vmls/`