

Additional Definitions from PS

A ring R is simple if it is non-trivial and only two-sided ideals are $\{0\}$, R
– when R is commutative, simple is equivalent to being a field

centre of R : $Z(R) := \{x : xy = yx \text{ for all } y\}$

idempotent element: e in R is idempotent if $e^2 = e$, $er = re$ for all r (central idempotent)

primitive idempotent element: non-zero idempotent element e is primitive if for any e' , $e'e = 0$ or $e'e = e$.

indecomposable ring: if there is no idempotent element other than $0, 1$

Endomorphism ring of M : $\text{End}_R(M) := \{f : M \rightarrow M : f \text{ is } R\text{-module homomorphism}\}$

- addition: pointwise
- multiplication: composition
- 0 : 0 function, $f(r) \equiv 0$
- 1 : id function.

automorphism group $\text{Aut}_R(M)$: group of units of $\text{End}_R(M)$

cyclic R -module: R -module generated by one element.

simple R -module: no R -submodule other than $0, M$