MATH60005/70005: Optimization (Autumn 22-23)

Week 2: Exercises

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1. Find the global minimum and maximum points of $f(x_1, x_2) = x_1 + x_2$ over the unit ball in $\mathbb{R}^2$, $S = B[0, 1] = \{(x_1, x_2)^T : x_1^2 + x_2^2 \leq 1\}$. Repeat with $f(x_1, x_2) = 2x_1 - 3x_2$ over the set $S = \{(x_1, x_2) : 2x_1^2 + 5x_2^2 \leq 1\}$.

2. Classify the matrices

$$\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0.1 \end{pmatrix}.$$

3. Use a computational tool of your preference to classify

$$\begin{pmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{pmatrix}, \quad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} -5 & 1 & 1 \\ 1 & -7 & 1 \\ 1 & 1 & -5 \end{pmatrix}$$

4. Classify the stationary points of

   a) $f(\mathbf{x}) = x_1^2 + x_2^2 + x_3^2 + x_1 x_2 + x_1 x_3 + x_2 x_3 + \left(x_1^2 + x_2^2 + x_3^2 \right)^2$.

   b) $f(x_1, x_2) = x_1^4 + 2x_1^2 x_2 + x_2^2 - 4x_1^2 - 8x_1 - 8x_2$.

   c) $f(x_1, x_2) = x_1^2 + 4x_1 x_2 + x_2^2 + x_1 - x_2$.