Scientific Computation
Autumn 2024
Problem sheet 2

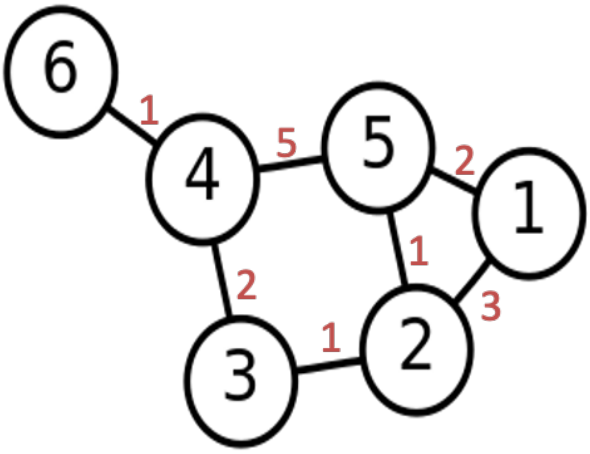

Figure 1: Graph for question 1

1. Apply Dijkstra's algorithm to find the distance between nodes 4 and 5 in the graph above. Set node 4 as the source and terminate the search once the distance to node 5 has been found. What is the sequence in which nodes are finalized?

2. Consider the code shown below:

```
import networkx as nx
def graph_code(G,x,L = []):
    """
    G: networkx graph with nodes numbered from 0 to N-1
    x: node number
    """
    if L==[]:
        N = G.number_of_nodes()
        L = [-1]*N
        L[x]=1
    for i in G.adj[x]:
        if L[i]==-1:
            L[i]=1
            L=graph_code(G,i,L)
    return L
```

What is the problem that this code is attempting to solve? Discuss the correctness of the code and its big-O time complexity.

3. You would like to find all connected components in an undirected graph with N nodes, L links, and N_c components. The graph is provided as an adjacency matrix. What is the cost of applying BFS to this problem?

4. Explain how to sort the following list in non-decreasing order using a binary heap: `L = [3,5,8,9,4,2,1]`. Can your approach be used generally? What is the big-O time complexity of your approach applied to a list with n elements? When analyzing the cost, you may assume that the list has $n = 2^m - 1$ elements where m is a positive integer.

5. Consider the following system of 2 ODEs:

$$\frac{d\theta}{dt} = \phi \tag{1}$$

$$\frac{d\phi}{dt} = -\sin\theta. \tag{2}$$

Find an approximate solution for the initial evolution of the system given the initial condition, $\theta(0) = \pi + 0.009$, $\phi(0) = -0.01$. Provide a qualitative description of the system's behavior. Explain why the approximation loses validity at sufficiently large times. (It's fine to use Python to help you with the calculations, but in principle, you should be able to work through this on paper.)