Imperial College
London

MATH96011

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May-June 2020

This paper is also taken for the relevant examination for the
Associateship of the Royal College of Science

Mathematics of Business and Economics

Date: 29th May 2020

Time: 13.00pm - 15.00pm (BST)

Time Allowed: 2 Hours

Upload Time Allowed: 30 Minutes

This paper has 4 Questions.

Candidates should start their solutions to each question on a new sheet of paper.

Each sheet of paper should have your CID, Question Number and Page Number on the top.

Only use 1 side of the paper.

Allow margins for marking.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Each question carries equal weight.

SUBMIT YOUR ANSWERS AS SEPARATE PDFs TO THE RELEVANT DROPBOXES ON BLACKBOARD (ONE FOR EACH QUESTION) WITH COMPLETED COVERSHEETS WITH YOUR CID NUMBER, QUESTION NUMBERS ANSWERED AND PAGE NUMBERS PER QUESTION.

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1. (a) A bicycle is assembled out of a bicycle frame and two wheels.

(i) Write down a production function of a firm that produces bicycles out of frames and wheels. The firm requires no assembly, so labour is not an input in this case. Draw the isoquant that shows all combinations of frames and wheels that result in producing 100 bicycles. (4 marks)

(ii) Suppose that initially the price of a frame is £100 and the price of a wheel is £50. Compute the cost level and draw the corresponding isocost line on the graph you drew in part (i). (3 marks)

(iii) Repeat the part (ii) if the price of a frame rises to £200, while the price of a wheel remains £50. (3 marks)

(b) Consider a consumer whose preferences for a pair of goods can be represented by the following utility function:

$$u_1(x_1, x_2) = \frac{3}{2}\log(x_1) + \log(x_2)$$

(i) If the goods are priced at $p = (p_1, p_2) = (3, 4)$ and the consumer's total budget for both goods is $m = 100$, compute the bundle that maximises the consumer's utility. (*The second-order condition doesn't need to be checked.*)

(5 marks)

(ii) Explain briefly the difference between Marshallian demand and Hicksian demand and thus decide whether the bundle computed in part (i) is the consumer's Marshallian demand or Hicksian demand. (3 marks)

(iii) Suppose a second consumer has the same budget for the same two goods and that their utility function is given by:

$$u_2(x_1, x_2) = x_1^{\frac{3}{2}} x_2$$

Assuming the same prices for the goods as in part (i), write down the bundle that maximises the second consumer's utility and justify your answer. (2 marks)

2. Consider a firm that produces a single output using two input factors. The production function is given by:

$$f(x_1, x_2) = 4\sqrt{x_1 x_2}$$

(a) Show that f is a homothetic function. (3 marks)

(b) Compute the elasticity of scale of f. (4 marks)

(c) Does f exhibit increasing, decreasing or constant returns to scale? Justify your answer. (2 marks)

(d) Compute the marginal rate of technical substitution (MRTS) of f and show that it is positively homogeneous of degree 0. (4 marks)

(e) Compute the conditional factor demand function $\underline{x}^*(w_1, w_2, y)$ and the cost function $c^*(w_1, w_2, y)$, where $w_1, w_2 > 0$ are the input prices. (4 marks)

(f) Compute the average cost function and the marginal cost function. (2 marks)

(g) Using the scale behaviour, explain whether the results in part (f) seem reasonable. (1 mark)

3. (a) Consider a firm that produces a single output using two input factors.

(i) Prove that the Weak Axiom of Profit Maximisation (WAPM) implies the Weak Axiom of Cost Minimisation (WACM). (5 marks)

(ii) Which of the following datasets:
(i) satisfies the WACM, but not the WAPM?
(ii) satisfies both WAPM and WACM?
(iii) violates both WAPM and WACM?
Show your work and justify your answers.

Dataset A:

t	p	w_1	w_2	x_1	x_2	y
1	4	1	1	1	1	1
2	12	1	2	4	4	2

Dataset B:

t	p	w_1	w_2	x_1	x_2	y
1	4	1	1	1	99	1
2	4	1	2	9	9	3

Dataset C:

t	p	w_1	w_2	x_1	x_2	y
1	4	1	1	1	1	1
2	4	1	2	9	9	3

where t is the time point, p is the output price, $w_i, i = 1, 2$ are the prices for the input factors, $x_i, i = 1, 2$ are the levels of input factors and y is the level of output.

(9 marks)

(b) Let $f\colon \mathbb{R}^n_{\geq 0} \to \mathbb{R}^m_{\geq 0}$ be a differentiable, non-decreasing and quasi-concave production function.

(i) Prove that the profit function π^* is convex. (4 marks)

(ii) Using (i) prove that:

$$\frac{\partial y_i^*(\underline{p}, \underline{w})}{\partial p_i} \geq 0, i = 1, \ldots, n$$

where $\underline{p}$ is the vector of output prices and $\underline{w}$ is the vector of input prices.

(2 marks)

4. (a) Consider the market for electricity. Suppose market demand for electricity is given by $X^*(p) = 2000 - 200p$, where $p \geq 0$ is the price in pounds per MWh (megawatt hour) and the quantity demanded is in thousands of MWh.

We have the following information about the market supply:

- Market research shows that for an increase in price by £1/MWh, quantity supplied increases by 100,000 MWh.
- Additionally, when the price is £1/MWh, 900,000 MWh are supplied.

(i) Use the above information to determine the market supply $Y^*(p)$ as a linear function of the price p. (2 marks)

(ii) Determine the equilibrium price and the equilibrium quantity. (3 marks)

(iii) Draw a graph (quantity on the horizontal axis, price on the vertical axis) and depict the market supply, market demand, equilibrium price and equilibrium quantity as well as producers' and consumers' surplus. (3 marks)

(iv) Compute the producers' surplus, the consumers' surplus and the community surplus. (3 marks)

~~(b) (i) Define the Gross Domestic Product (GDP). (2 marks)~~

~~(ii) State the three approaches used to compute GDP. (3 marks)~~

~~(iii) Define the Gross National Product (GNP). (2 marks)~~

~~(iv) A British citizen is working for the European Commission in Belgium. Describe the contribution of his income in the above products. (2 marks)~~

Module:	MATH96011
Setter:	Papatsouma
Checker:	Pakkanen
Editor:	Hallsworth
External:	Jennison
Date:	April 17, 2020
Version:	Final version

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)

May – June 2020

MATH96011 Mathematics of Business and Economics

The following information must be completed:

Is the paper suitable for resitting students from previous years: Yes

Category A marks: available for basic, routine material (excluding any mastery question) (40 percent = 32/80 for 4 questions):
1(b) (ii) 3 marks; 2(a) 3 marks; 2(b) 4 marks; 2(d) 4 marks; 2(f) 2 marks; 3(a) (ii) (i) 3 marks; 4(a) (ii) 3 marks; 4(a) (iv) 3 marks; 4(b) (i) 2 marks; 4(b) (ii) 3 marks; 4(b) (iii) 2 marks

Category B marks: Further 25 percent of marks (20/ 80 for 4 questions) for demonstration of a sound knowledge of a good part of the material and the solution of straightforward problems and examples with reasonable accuracy (excluding mastery question):
1(b) (i) 5 marks; 2(c) 2 marks; 2(e) 4 marks; 3(a) (ii) (ii) 3 marks; 3(a) (ii) (iii) 3 marks; 4(a) (iii) 3 marks

Category C marks: the next 15 percent of the marks (= 12/80 for 4 questions) for parts of questions at the high 2:1 or 1st class level:
1(a) (ii) 3 marks; 1(a) (iii) 3 marks; 3(b) (i) 4 marks; 4(a) (i) 2 marks

Category D marks: Most challenging 20 percent (16/80 marks for 4 questions) of the paper (excluding mastery question):
1(a) (i) 4 marks; 1(b) (iii) 2 marks; 2(g) 1 mark; 3(a) (i) 5 marks; 3(b) (ii) 2 marks; 4(b) (iv) 2 marks

Signatures are required for the final version:

Setter's signature	Checker's signature	Editor's signature
....	Mikko Pakkanen	....

BSc, MSc and MSci EXAMINATIONS (MATHEMATICS)

May – June 2020

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science.

Mathematics of Business and Economics

Date: Friday, 29th May 2020
Time: 09:00 - 11:00
Time Allowed: 2 Hours
This paper has *4 Questions*.
Candidates should start their solutions to each question in a new main answer book.
Supplementary books may only be used after the relevant main book(s) are full.
Statistical tables will not be provided.

- DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO.
- Affix one of the labels provided to each answer book that you use, but DO NOT USE THE LABEL WITH YOUR NAME ON IT.
- Credit will be given for all questions attempted.
- Each question carries equal weight.
- Calculators may not be used.

1. (a) A bicycle is assembled out of a bicycle frame and two wheels.

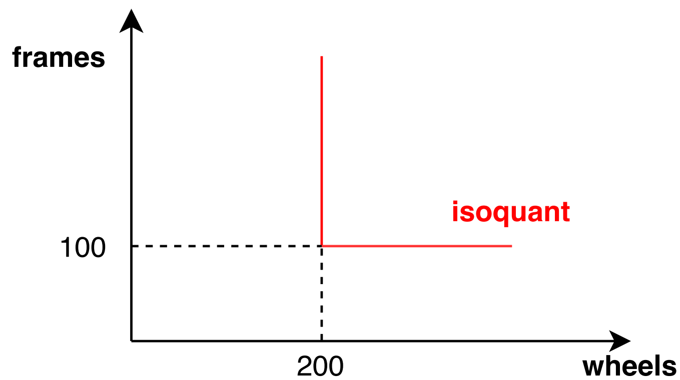
(i) Write down a production function of a firm that produces bicycles out of frames and wheels. The firm requires no assembly, so labour is not an input in this case. Draw the isoquant that shows all combinations of frames and wheels that result in producing 100 bicycles.

Solution: (UNSEEN)

The production function is a Leontief production function given by:

$$f(x_1, x_2) = \min\left\{x_1, \frac{1}{2}x_2\right\}$$

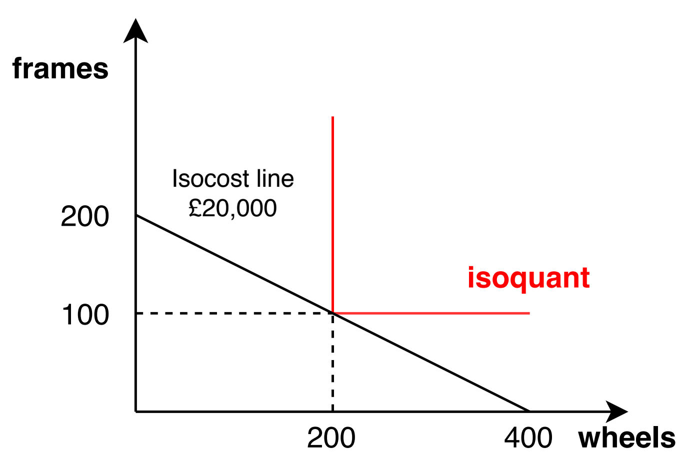
where x_1 denotes the number of frames and x_2 denotes the number of wheels.
The isoquant will be L-shaped:



(4 marks)

(ii) Suppose that initially the price of a frame is £100 and the price of a wheel is £50. Compute the cost level and draw the corresponding isocost line on the graph you drew in part (i).

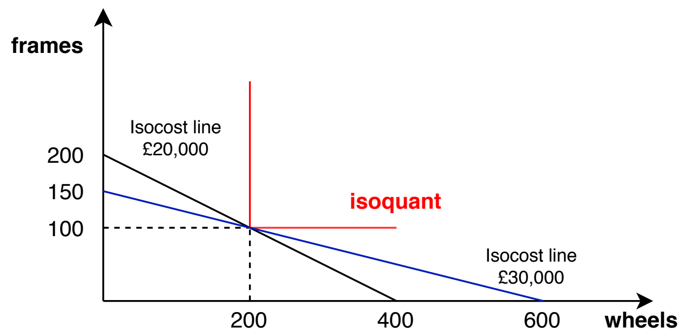
Solution: (UNSEEN)



(3 marks)

(iii) Repeat the part (ii) if the price of a frame rises to £200, while the price of a wheel remains £50.

Solution: (UNSEEN)



(3 marks)

(b) Consider a consumer whose preferences for a pair of goods can be represented by the following utility function:

$$u_1(x_1, x_2) = \frac{3}{2}\log(x_1) + \log(x_2)$$

(i) If the goods are priced at $p = (p_1, p_2) = (3, 4)$ and the consumer's total budget for both goods is $m = 100$, compute the bundle that maximises the consumer's utility. (*The second-order condition doesn't need to be checked.*)

Solution: (SEEN SIMILAR)

We look to maximise $u_1(\underline{x})$ subject to the constraint $3x_1 + 4x_2 = 100$.

First, we define the Lagrangian:

$$\mathcal{L}(x_1, x_2, \lambda) = u_1(\underline{x}) - \lambda(\underline{p}\underline{x}, m) = \frac{3}{2}\log(x_1) + \log(x_2) - \lambda(3x_1 + 4x_2 - 100)$$

First-order conditions for maximisation are therefore:

$$\frac{\partial}{\partial \lambda}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow \underline{p}\underline{x} = m \Rightarrow 3x_1 + 4x_2 = 100$$

$$\frac{\partial}{\partial x_1}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow \frac{3}{2x_1} = 3\lambda$$

$$\frac{\partial}{\partial x_2}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow \frac{1}{x_2} = 4\lambda$$

which we can solve to get $x_1^*(\underline{p}, m) = 20, x_2^*(\underline{p}, m) = 10.$

(5 marks)

(ii) Explain briefly the difference between Marshallian demand and Hicksian demand and thus decide whether the bundle computed in part (i) is the consumer's Marshallian demand or Hicksian demand.

Solution: (SEEN)

The Marshallian demand is the quantity of each good that a consumer requires in order to maximise their utility for a fixed budget. In contrast, the Hicksian demand is the quantity of each good that the consumer requires in order to minimise their expenditure for a fixed level of utility.

The bundle computed in part (i) is the consumer's Marshallian demand for the two goods.

(3 marks)

(iii) Suppose a second consumer has the same budget for the same two goods and that their utility function is given by:

$$u_2(x_1, x_2) = x_1^{\frac{3}{2}} x_2$$

Assuming the same prices for the goods as in part (i), write down the bundle that maximises the second consumer's utility and justify your answer.

Solution: (UNSEEN)

Since u_2 is a positive monotonic transformation of u_1, both consumers have the same preferences, and thus the same Marshallian demand:

$$x_1^*(\underline{p}, m) = 20, x_2^*(\underline{p}, m) = 10.$$

(2 marks)

2. Consider a firm that produces a single output using two input factors. The production function is given by:

$$f(x_1, x_2) = 4\sqrt{x_1 x_2}$$

(a) Show that f is a homothetic function.

Solution: (SEEN SIMILAR)

Let $g\colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, $g(z) = 4z^{\frac{1}{2}}$ and $h\colon \mathbb{R}^2_{\geq 0} \to \mathbb{R}_{\geq 0}$, $h(x_1, x_2) = x_1 x_2$.
Then g is (strictly) increasing, h is positively homogeneous (of degree 2) and $f = g \circ h$.

(3 marks)

(b) Compute the elasticity of scale of f.

Solution: (SEEN SIMILAR)

Let $(x_1, x_2) \in \mathbb{R}^2_{\geq 0}$. Applying the chain rule, we obtain the partial derivatives:

$$\partial_1 f(x_1, x_2) = \frac{2\sqrt{x_2}}{\sqrt{x_1}}, \qquad \partial_2 f(x_1, x_2) = \frac{2\sqrt{x_1}}{\sqrt{x_2}}.$$

Hence, the elasticity of scale of f at (x_1, x_2) is given by:

$$\begin{aligned} e(x_1, x_2) &= \frac{\langle \nabla f(x_1, x_2), (x_1, x_2) \rangle}{f(x_1, x_2)} \\ &= \frac{\partial_1 f(x_1, x_2) x_1 + \partial_2 f(x_1, x_2) x_2}{f(x_1, x_2)} \\ &= 1\,. \end{aligned}$$

(4 marks)

(c) Does f exhibit increasing, decreasing or constant returns to scale? Justify your answer.

Solution: (SEEN SIMILAR)

f exhibits constant returns to scale because the elasticity of scale of f is 1.
Alternative answer: f exhibits constant returns to scale because the sum of the exponents of x_1 and x_2 is 1.

(2 marks)

(d) Compute the marginal rate of technical substitution (MRTS) of f and show that it is positively homogeneous of degree 0.

Solution: (SEEN SIMILAR)

Let $(x_1, x_2) \in \mathbb{R}^2_{\geq 0}$, $x_1 > 0$. Then the MRTS of f at (x_1, x_2) is given by:

$$MRTS(x_1, x_2) = -\frac{\partial_1 f(x_1, x_2)}{\partial_2 f(x_1, x_2)} = -\frac{x_2}{x_1}\,.$$

For any $t > 0$ $MRTS(tx_1, tx_2) = MRTS(x_1, x_2)$.

(4 marks)

(e) Compute the conditional factor demand function $\underline{x}^*(w_1, w_2, y)$ and the cost function $c^*(w_1, w_2, y)$, where $w_1, w_2 > 0$ are the input prices.

Solution: (SEEN SIMILAR)

We determine the minimiser of $w_1x_1 + w_2x_2$ subject to $4\sqrt{x_1x_2} = y$. For $y = 0$, we clearly have $x_1^*(w_1, w_2, 0) = x_1^*(w_1, w_2, 0) = 0$. For $y > 0$, we must have $x_1, x_2 > 0$. We can either consider the first-order conditions or use the marginal rate of technical substitution (part d) that coincides with the economic rate of substitution and get:

$$-\frac{x_2}{x_1} = -\frac{w_1}{w_2}$$

Solving the above equation for x_2 and substituting into the constraint yields:

$$x_1^*(w_1, w_2, y) = \frac{y}{4}\left(\frac{w_2}{w_1}\right)^{1/2} .$$

By symmetry:

$$x_2^*(w_1, w_2, y) = \frac{y}{4}\left(\frac{w_1}{w_2}\right)^{1/2} .$$

Finally, the cost function is given by:

$$c^*(w_1, w_2, y) = w_1x_1^*(w_1, w_2, y) + w_2x_2^*(w_1, w_2, y) = \frac{y}{2}\Big(w_1w_2\Big)^{1/2} .$$

(4 marks)

(f) Compute the average cost function and the marginal cost function.

Solution: (SEEN SIMILAR)

The average cost function is given by:

$$AC(y) = \frac{c^*(w_1, w_2, y)}{y} = \frac{1}{2}\left(w_1w_2\right)^{1/2}.$$

The marginal cost function is given by:

$$MC(y) = \frac{\partial}{\partial y}c^*(w_1, w_2, y) = \frac{1}{2}\left(w_1w_2\right)^{1/2}.$$

(2 marks)

(g) Using the scale behaviour, explain whether the results in part (f) seem reasonable.

Solution: (UNSEEN)

The results seem reasonable, because f exhibits constant returns to scale and that scale behaviour implies constant marginal cost and constant average cost.

(1 mark)

3. (a) Consider a firm that produces a single output using two input factors.

(i) Prove that the Weak Axiom of Profit Maximisation (WAPM) implies the Weak Axiom of Cost Minimisation (WACM).

Solution: (UNSEEN)

Indeed, WAPM asserts that for all s, t:

$$p^t y^t - w_1^t x_1^t - w_2^t x_2^t \geq p^t y^s - w_1^t x_1^s - w_2^t x_2^s \,. \tag{1}$$

Now consider a pair s, t such that $y^s \geq y^t$. Then (1) implies that:

$$-w_1^t x_1^t - w_2^t x_2^t \geq \underbrace{p^t(y^s - y^t)}_{\geq 0} - w_1^t x_1^s - w_2^t x_2^s \geq -w_1^t x_1^s - w_2^t x_2^s \,.$$

Therefore:

$$w_1^t x_1^t + w_2^t x_2^t \leq w_1^t x_1^s + w_2^t x_2^s \,.$$

(5 marks)

(ii) Which of the following datasets:
(i) satisfies the WACM, but not the WAPM?
(ii) satisfies both WAPM and WACM?
(iii) violates both WAPM and WACM?
Show your work and justify your answers.

Dataset A:

t	p	w_1	w_2	x_1	x_2	y
1	4	1	1	1	1	1
2	12	1	2	4	4	2

Dataset B:

t	p	w_1	w_2	x_1	x_2	y
1	4	1	1	1	99	1
2	4	1	2	9	9	3

Dataset C:

t	p	w_1	w_2	x_1	x_2	y
1	4	1	1	1	1	1
2	4	1	2	9	9	3

where t is the time point, p is the output price, $w_i, i = 1, 2$ are the prices for the input factors, $x_i, i = 1, 2$ are the levels of input factors and y is the level of output.

Solution:

(i) **(UNSEEN)** Dataset C satisfies the WACM, but not the WAPM. Indeed, we have that $y^2 > y^1$. Hence we only need to check that:

$$2 = w_1^1 x_1^1 + w_2^1 x_2^1 \leq w_1^1 x_1^2 + w_2^1 x_2^2 = 18 \,.$$

On the other hand, the WAPM is violated since at time point 2 one has a profit of $12 - 27 = -15$. However, if they had produced with the inputs and outputs at time 1, they could have had a profit of $4 - 3 = 1$.

(3 marks)

(ii) **(UNSEEN)** Dataset A satisfies both WACM and WAPM. We have that $y^2 > y^1$ and

$$2 = w_1^1 x_1^1 + w_2^1 x_2^1 \leq w_1^1 x_1^2 + w_2^1 x_2^2 = 8\,.$$

Moreover, at time point $t = 1$ the firm makes a profit of:

$$p^1 y^1 - w_1^1 x_1^1 - w_2^1 x_2^1 = 4 - 2 = 2\,.$$

If they had used the output and input at time $s = 2$ with the prices at time $t = 1$, they would have obtained:

$$p^1 y^2 - w_1^1 x_1^2 - w_2^1 x_2^2 = 8 - 8 = 0 < 2\,.$$

(3 marks)

(iii) **(SEEN SIMILAR)** Dataset B violates both WACM and WAPM. Clearly, the costs at time point 1 are 100 with an output of $y^1 = 1$. At time point 2, there is a higher output of $y^2 = 3$. Using prices at time 1, that would have caused costs of 18. Since we have already established the implication WAPM implies WACM, WAPM cannot be satisfied.

(3 marks)

(b) Let $f\colon \mathbb{R}^n_{\geq 0} \to \mathbb{R}^m_{\geq 0}$ be a differentiable, non-decreasing and quasi-concave production function.

(i) Prove that the profit function π^* is convex.

Solution: (SEEN)

The profit function at prices $(\underline{p}, \underline{w}) \in \mathbb{R}^m_{\geq 0} \times \mathbb{R}^n_{\geq 0}$ is defined as:

$$\pi^*(\underline{p}, \underline{w}) = \max_{\underline{x} \in \mathbb{R}^n_{\geq 0}} \pi(\underline{x}, \underline{p}, \underline{w})\,.$$

Let $(\underline{p}, \underline{w}), (\underline{p}', \underline{w}') \in \mathbb{R}^m_{\geq 0} \times \mathbb{R}^n_{\geq 0}$, $\lambda \in [0,1]$. Define $(\underline{p}'', \underline{w}'') = (1-\lambda)(\underline{p}, \underline{w}) + \lambda(\underline{p}', \underline{w}')$. Then:

$$\begin{aligned}
\pi^*(\underline{p}'', \underline{w}'') &= \underline{p}'' f\left(\underline{x}^*(\underline{p}'', \underline{w}'')\right)^\top - \underline{w}'' \underline{x}^*(\underline{p}'', \underline{w}'')^\top \\
&= (1-\lambda)\Big[\underline{p} f\left(\underline{x}^*(\underline{p}'', \underline{w}'')\right)^\top - \underline{w}\underline{x}^*(\underline{p}'', \underline{w}'')^\top\Big] \\
&\quad + \lambda\Big[\underline{p}' f\left(\underline{x}^*(\underline{p}'', \underline{w}'')\right)^\top - \underline{w}'\underline{x}^*(\underline{p}'', \underline{w}'')^\top\Big] \\
&\leq (1-\lambda)\Big[\underline{p} f\left(\underline{x}^*(\underline{p}, \underline{w})\right)^\top - \underline{w}\underline{x}^*(\underline{p}, \underline{w})^\top\Big] \\
&\quad + \lambda\Big[\underline{p}' f\left(\underline{x}^*(\underline{p}', \underline{w}')\right)^\top - \underline{w}'\underline{x}^*(\underline{p}', \underline{w}')^\top\Big] \\
&= (1-\lambda)\pi^*(\underline{p}, \underline{w}) + \lambda\pi^*(\underline{p}', \underline{w}')\,.
\end{aligned}$$

(4 marks)

(ii) Using (i) prove that:

$$\frac{\partial y_i^*(\underline{p}, \underline{w})}{\partial p_i} \geq 0, i = 1, \ldots, n$$

where $\underline{p}$ is the vector of output prices and $\underline{w}$ is the vector of input prices.

Solution: (UNSEEN)

The convexity of the profit function π^* implies that:

$$\frac{\partial^2 \pi^*(\underline{p}, \underline{w})}{\partial p_i^2} \geq 0$$

and from Hotelling's Lemma, we have that:

$$\frac{\partial \pi^*(\underline{p}, \underline{w})}{\partial p_i} = f_i(x^*(\underline{p}, w)) = y_i^*(\underline{p}, \underline{w}).$$

(2 marks)

4. (a) Consider the market for electricity. Suppose market demand for electricity is given by $X^*(p) = 2000 - 200p$, where $p \geq 0$ is the price in pounds per MWh (megawatt hour) and the quantity demanded is in thousands of MWh.

We have the following information about the market supply:

- Market research shows that for an increase in price by £1/MWh, quantity supplied increases by 100,000 MWh.
- Additionally, when the price is £1/MWh, 900,000 MWh are supplied.

(i) Use the above information to determine the market supply $Y^*(p)$ as a linear function of the price p.

Solution: (UNSEEN)

We consider the linear function $Y^*(p) = a + bp$, where $a = 800$, $b = 100$, $p \geq 0$ is the price in pounds per MWh and the quantity supplied is in thousands of MWh.

(2 marks)

(ii) Determine the equilibrium price and the equilibrium quantity.

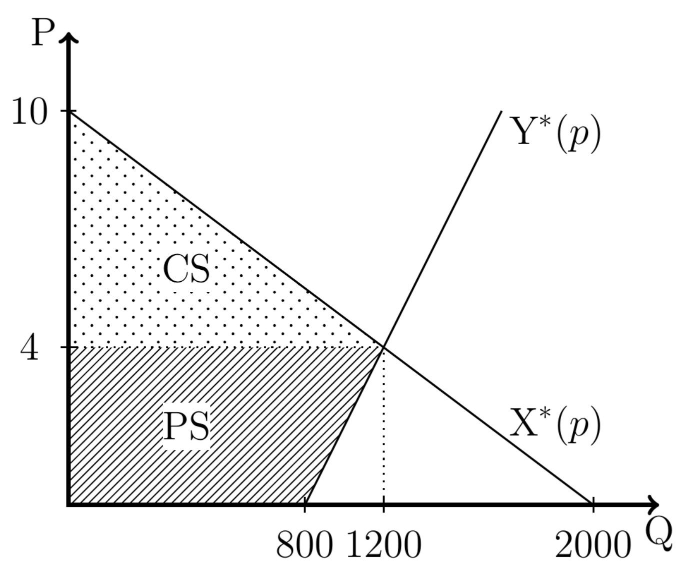
Solution: (SEEN SIMILAR)

The equilibrium price is obtained by equating $X^*(p) = Y^*(p)$. This yields an equilibrium price of $p^* = £4$/MWh and an equilibrium quantity of $X^*(p^*) = Y^*(p^*) = 1200$ thousands of MWh.

(3 marks)

(iii) Draw a graph (quantity on the horizontal axis, price on the vertical axis) and depict the market supply, market demand, equilibrium price and equilibrium quantity as well as producers' and consumers' surplus.

Solution: (SEEN SIMILAR)



(3 marks)

(iv) Compute the producers' surplus, the consumers' surplus and the community surplus.

Solution: (SEEN SIMILAR)
The producers' surplus at $p^* = 4$ is given by:

$$PS(p^*) = \int_0^{p^*} Y^*(p)\mathrm{d}p = \int_0^4 (100p + 800)\mathrm{d}p = 4000.$$

Alternatively, the producer's surplus can be found from the area of the trapezoid:

$$PS(p^*) = \frac{1}{2}(1200 + 800)4 = 4000$$

The consumers' surplus at $p^* = 2$ is given by:

$$CS(p^*) = \int_{p^*}^{10} X^*(p)\mathrm{d}p = \int_2^{10} (2000 - 200p)\mathrm{d}p = 3600$$

Alternatively, it can be found from the area of the triangle:

$$CS(p^*) = \frac{1}{2}(1200)(10 - 4) = 3600$$

The community surplus is the sum of the two, that is, 7600.

(3 marks)

(b) (i) Define the Gross Domestic Product (GDP).
Solution: (SEEN)
The Gross Domestic Product (GDP) measures the nominal gross value of all goods and services produced in a certain country in a certain period of interest.

(2 marks)

(ii) State the three approaches used to compute GDP.
Solution: (SEEN)
Production approach
Expenditure approach
Income approach

(3 marks)

(iii) Define the Gross National Product (GNP).
Solution: (SEEN)
The Gross Domestic Product (GNP) measures the nominal gross value of all goods and services produced by all citizens of a certain nationality in a certain period of interest.

(2 marks)

(iv) A British citizen is working for the European Commission in Belgium. Describe the contribution of his income in the above products.
Solution: (UNSEEN)
Contribution to the GNP of United Kingdom and the GDP of Belgium.

(2 marks)

If your module is taught across multiple year levels, you might have received this form for each level of the module. You are only required to fill this out once for each question.

Please record below, some brief but non-trivial comments for students about how well (or otherwise) the questions were answered. For example, you may wish to comment on common errors and misconceptions, or areas where students have done well. These comments should note any errors in and corrections to the paper. These comments will be made available to students via the MathsCentral Blackboard site and should not contain any information which identifies individual candidates. Any comments which should be kept confidential should be included as confidential comments for the Exam Board and Externals. If you would like to add formulas, please include a sperate pdf file with your email.

ExamModuleCode	QuestionN	Comments for Students	
MATH96011	1	Question 1 was generally answered well. Part (a) (i) required to spot that wheels and frames must be used in fixed proportions (Leontief production function); Parts (a) (ii) and (iii) a notable number of students did not compute the cost level as required and did not mark the intercepts in their drawings; Part (b) (i) mostly well done, although some students did not state the optimisation problem; Part (b) (ii) the question was not always answered with the appropriate level of detail; Part (b) (iii) some students did not spot that u2 is a positive monotonic transformation of u1 and thus did a lot of extra work.	
MATH96011	2	Question 2 was generally answered well. Students have seen questions similar to parts (a)-(f), most of the errors were algebraic and some students did not state the optimisation problem in part (e); Part (g) was intended to be challenging and students were required to spot that the average cost and the marginal cost are constant. Marks were awarded for a different sensible interpretation, i.e. link to zero maximised profit.	
MATH96011	3	Question 3: part (a)(i) required a proof that the WAPM implies the WACM from the definitions. There were a few logical fallacies, but this was mostly answered well. Part (a)(ii) required some calculations on three simple datasets to find out if the WAPM and WACM are satisfied. The main errors here were thoroughly checking the conditions of each axiom; in some cases not all conditions were fully checked. Part (b) required a proof that the profit function is convex. This was mostly answered well, the main mistake being overlooking that the profit function is a function of prices p and w (some students just proved convexity in p, for example). In part (ii), this was usually answered correctly or not at all, but always make sure to reference results (e.g. Hotelling's Lemma) when you use them.	
MATH96011	4	Question 4: part (a)(i) was straightforward and generally answered correctly, likewise for part (a)(ii). Part (a)(ii) was mostly answered well, aside from some graphs being too imprecise. The market supply was occasionally drawn incorrectly. Part (iv) required the computation of two integrals (or calculation of areas in the graph), and this is a part where many students lost marks, either because of incorrectly evaluating the integrals, or omitting the community surplus from the answer. All parts of part (b) were generally answered well.	