L7 Problem Class.

① 1. F
2. F
3. T
4. T
5. T
6. F
7. T
8. F

1, 2) P1

3, 4) by induction on length of fmla.

5. by MP

6. take ϕ to be an axiom

8. Not consistent (α an axiom, then $(\neg\alpha) \in \Delta$ and $\Delta \vdash \alpha$ $\Delta \vdash (\neg\alpha)$)

(It is complete.)

② (i) $\alpha \vee \beta$: $((\neg\alpha) \to \beta)$; $\vdash_L (\phi \vee (\neg\phi))$: "$((\neg\phi) \to (\neg\phi))$" "Theorem".

(ii) $\Gamma \cup \{(\neg\phi)\} \vdash \psi$ and $\Gamma \cup \{\neg\phi\} \vdash (\neg\psi)$ ...

(3) $\{\neg, \leftrightarrow\}$ not adequate :

(2)

Look at truth fns. of 2 vars. p, q obtained using fmlas. involving $\neg, \leftrightarrow$

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$(\neg p) \leftrightarrow q$	$p \leftrightarrow p$	$(p \leftrightarrow (\neg p))$	
T	T	F	F	T	F	T	F	
T	F	F	T	F	T	T	F	
F	T	T	F	F	T	T	F	$\phi_1 - \phi_8$
F	F	T	T	T	F	T	F	

Show $(\neg \phi_i)$

$(\phi_j \leftrightarrow \phi_k)$

gives no new truth fns.

- only 8 truth fns.

$2^{2^2} = 16$ truth fns. of 2 vars.