

Problem Sheet 5

We will discuss the solutions of this problem sheet in the problem class on Thursday, 14 March 2024.

1. Suppose Anne has a utility function $u: \mathbb{R}_{\geq 0}^2 \rightarrow \mathbb{R}$ which is strictly increasing, continuous and strictly quasi-concave.

Assume Anne has an initial budget of $m > 0$ and that the prices for the two goods are $p_1, p_2 > 0$. Now assume that there is an increase in price for good 1 such that the new price for good 1 is $q_1 > p_1$. The price for good 2 stays fixed, so $q_2 = p_2$. Of course, Anne is not happy with this increase in price.

- a) Explain why it would be reasonable to pay Anne an amount of

$$e(\underline{q}, v(\underline{p}, m)) - m$$

to compensate for this increase in price.

- b) Show that:

$$\begin{aligned} e(\underline{q}, v(\underline{p}, m)) - m &= e(\underline{q}, v(\underline{p}, m)) - e(\underline{p}, v(\underline{p}, m)) \\ &= \int_{p_1}^{q_1} x_{H,1}^*((z, p_2), v(\underline{p}, m)) dz \end{aligned}$$

- c) Assume that good 1 is a normal good. Use Slutsky's equation to show that:

$$\int_{p_1}^{q_1} x_1^*((z, p_2), m) dz \leq \int_{p_1}^{q_1} x_{H,1}^*((z, p_2), v(\underline{p}, m)) dz. \quad (1)$$

2. Consider the market for one good. Suppose that the market demand is given by $X(p) = 10 - p$, where $p \geq 0$ is the price for the good. Suppose there are two firms with the same technology. Each of the firms has a short-run cost function of $c_S^*(y) = 2 + y^2$ (reflecting the actual economic costs), where $y \geq 0$ is the firm's output. Suppose that we have a perfect competition, that is, consumers and firms are price takers.

- a) Compute the short-run equilibrium price as well as the short-run equilibrium quantity produced.
- b) Compute the consumers' surplus, the producers' surplus and the community surplus at the short-run equilibrium.
- c) Suppose there are no entry barriers for firms to the market and any new firm has the same production technology as the 2 firms that are already in the market. How many firms will operate in the long-run equilibrium?
Also determine the long-run equilibrium price and the long-run equilibrium quantity.
- d) Compute the consumers' surplus, the producers' surplus and the community surplus at the long-run equilibrium.
- e) Sketch the situation graphically and demonstrate that there is a gain in the community surplus.

3. Consider a market demand that is described by the following demand function:

$$X(p) = \frac{16}{p^2}.$$

On the other hand, the industry supply is given by:

$$Y(p) = 2p.$$

- a) Compute the equilibrium price and quantity.
- b) Compute the producers' surplus, the consumers' surplus and the community surplus.
- c) Now suppose the government introduces a tax of 10% on the good. Calculate the new equilibrium price and the new equilibrium quantity.
- d) Calculate the producers' surplus, the consumers' surplus and the revenue of the tax. How would you define the community surplus in this situation? What is the deadweight loss?
- e) Now suppose that the government introduces a subsidy of 10%. Again, calculate the new equilibrium price and quantity, as well as consumers' surplus, producers' surplus and community surplus. What is the absolute size of the subsidy? What is the deadweight loss?