

1. (i) (a) The Lagrangian $L(q, \dot{q}, t)$ describes the time evolution of a physical system with one coordinate q . Write down the Euler-Lagrange equation.

(b) What is meant by a conservative force?

- (ii) Consider the Lagrangian

$$L = \frac{1}{2} (\dot{x}^2 + \dot{y}^2) - by\dot{x} - gy,$$

where b and g are non-zero constants.

- (a) Use the 2 Euler-Lagrange equation to show that this Lagrangian describes a particle of unit mass acted on by a non-conservative force. Is the force conservative if g or b is set to zero?
- (b) Find the general solution of the Euler-Lagrange (E-L) equations obtained in part (a).
Hint: integrate the E-L equation for x and use the result to eliminate \dot{x} from the E-L equation for y .
- (c) Are any solutions periodic?
- (d) Determine the conserved quantity

$$H = p_x \dot{x} + p_y \dot{y} - L.$$

Express your answer in terms of x , y , \dot{x} , \dot{y} (you do not need the solution to part (b) to answer this).

- (e) Are there any other conserved quantities?