Imperial College London

MATH60013 MATH96011

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May-June 2022

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Mathematics of Business and Economics

Date: 23 May 2022

Time: 09:00 – 11:00 (BST)

Time Allowed: 2:00 hours

Upload Time Allowed: 30 minutes

This paper has 4 Questions.

Candidates should start their solutions to each question on a new sheet of paper.

Each sheet of paper should have your CID, Question Number and Page Number on the top.

Only use 1 side of the paper.

Allow margins for marking.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Each question carries equal weight.

SUBMIT YOUR ANSWERS AS SEPARATE PDFs TO THE RELEVANT DROPBOXES ON BLACKBOARD (ONE FOR EACH QUESTION) WITH COMPLETED COVERSHEETS WITH YOUR CID NUMBER, QUESTION NUMBERS ANSWERED AND PAGE NUMBERS PER QUESTION.

1. (a) Consider a firm that produces a single output using two input factors. Briefly explain the following notions, assuming that the reader is a non-expert. Do not use any mathematical expressions or graphs.

   (i) Law of Demand
   (ii) Law of Supply
   (iii) Marginal Rate of Technical Substitution
   (iv) Returns to Scale

(4 marks)

(b) (i) The Law of Diminishing Marginal Productivity is also known as the *Low-Fruit Hanging Principle*. Can you briefly justify this and give a real-world example where this principle can be used? (2 marks)

   (ii) The Leontief production function is also know as *fixed proportions production function*. Can you briefly justify this and give a real-world example where this function can be used? [An example with two inputs is sufficient.] (2 marks)

(c) You observe a firm's behaviour at two different time points $t = 1, 2$. Assume that the firm produces a single output, y, using two input factors, x_1 and x_2.

t	x_1	x_2	y
1	10	20	100
2	14	10	110

We've seen in the lectures three examples of production functions that are often used in microeconomic analysis:

- Linear technology: $f \colon \mathbb{R}^2_{\geq 0} \to \mathbb{R}$, $f(x_1, x_2) = ax_1 + bx_2$,
- Leontief technology: $f \colon \mathbb{R}^2_{\geq 0} \to \mathbb{R}$, $f(x_1, x_2) = \min\{ax_1, bx_2\}$,
- Cobb-Douglas technology: $f \colon \mathbb{R}^2_{\geq 0} \to \mathbb{R}$, $f(x_1, x_2) = x_1^a x_2^b$.

   (i) Determine the parameters $a, b \geq 0$ in each case. [You may use that $\log 2 \approx 0.3$, $\log 7 \approx 0.85$ and $\log 11 \approx 1.04$.] (9 marks)

   (ii) Now, you have access to the input prices, w_1 and w_2, and the complete information is of the following form:

t	w_1	w_2	x_1	x_2	y
1	2	1	10	20	100
2	1	2	14	10	110

   Comment on the suitability of the three production functions in the light of this additional piece of information.

(3 marks)

(Total: 20 marks)

2. The UK automotive industry is a vital part of the UK economy. Consider an automotive firm in the UK that produces cars, using three input factors, raw materials (in quantity x_R), labour (x_L) and machinery (x_M), and for the purposes of this question we assume that its production function is given by:

$$f(x_R, x_L, x_M) = x_R x_L x_M.$$

Assume that the output price p and the input prices w_R, w_L, w_M are all positive.

(a) Compute the conditional factor demand function $\underline{x}^*(w_R, w_L, w_M, y)$. Verify that it satisfies the required homogeneity property and briefly interpret it. (6 marks)

(b) Briefly describe the variable and fixed costs that the firm faces. (3 marks)

(c) Compute the average cost function $AC(y)$ and briefly justify the shape of the average cost curve. (4 marks)

(d) Consider a second automotive firm in the UK with the same production function. Do you anticipate that the government will approve a merger between the two firms? Justify your answer. [*Hint: Use the returns to scale behaviour.*] (3 marks)

(e) The automotive firm in the UK wishes to analyse the actions of a rival firm in Japan. In the first week of the financial year, the Japanese firm purchases 500 units of raw materials, and produces and sells 4,000 cars at £14,000 each; in the following week, it purchases 700 units of raw materials, and produces and sells 5,000 cars at £13,500 each. What can you deduce about the unit costs at which the Japanese firm was able to obtain raw materials in each of these weeks? (4 marks)

(Total: 20 marks)

3. (a) Consider a consumer whose preferences for a pair of goods can be represented by the following utility function:

$$u(x_1, x_2) = x_1^2 x_2^4.$$

(i) Show that the underlying preferences are convex. (2 marks)

(ii) Compute the bundle that maximises the consumer's utility. [*You may assume that the second-order condition is satisfied.*] (5 marks)

(iii) Briefly describe the firm-side analogue problem. (2 marks)

(iv) Using your answer in (ii), asnwer the folowing questions and justify your answers.

(i) Are the goods x_1 normal or inferior goods for the consumer? (2 marks)

(ii) Are the goods x_2 ordinary or Giffen goods for the consumer? (2 marks)

(iii) Are the goods x_1 and x_2 complements, substitutes or neither? (2 marks)

(v) Give two additional utility functions that represent the same preferences. (1 mark)

(b) Consider a consumer that consumes two goods. Show that if two utility functions represent the same preferences, then the marginal utility must always have the same sign at each bundle, i.e. if u and v represent the same preferences, then $\frac{\partial u(x)}{\partial x_i} > 0$ if and only if $\frac{\partial v(x)}{\partial x_i} > 0, i = 1, 2.$ (4 marks)

(Total: 20 marks)

4. ~~(a) (i) Sketch the five-sector circuit flow of income of a *national* economy. (2 marks)~~

~~(ii) What changes would you make to your answer in (i) if you were to describe the *global* economy? (2 marks)~~

(b) Consider the oil market and suppose that we have a perfect competition, that is, consumers and firms are price takers. Let the inverse market demand be of the linear form $39 - 0.009q$, where $q \geq 0$ is the quantity demanded measured in millions of barrels, and let the long-run profit function for an individual firm be of the quadratic form $p^2 - 2p - 399$, where $p \geq 0$ is the oil price in pounds per barrel. [You may assume that the output of an individual firm, y is non-negative.]

(i) State clearly the conditions that need to be satisfied such that J identical firms operate in the long-run equilibrium and no new firms wish to enter or exit the market.

(1 mark)

(ii) Using the conditions in (i), compute the long-run equilibrium price, p_l^*, the long-run equilibrium quantity, q_l^*, and the number of firms operating in the long-run equilibrium, J^*. (5 marks)

(iii) Sketch a graph of the market demand curve and the market supply curve. (2 marks)

(iv) Sketch a graph of the marginal cost curve and the average cost curve of an individual firm. (2 marks)

(v) In the short run, there are $n < J$ identical firms in the market and each firm has fixed costs $F > 0$. Describe, how, if at all, an increase in F for all firms simultaneously would affect:

(i) the output supply function of a typical firm,

(ii) the market equilibrium price, and

(iii) the profit of a typical firm. (6 marks)

(Total: 20 marks)

BSc and MSci EXAMINATIONS (MATHEMATICS)

May 2022

This paper is also taken for the relevant examination for the Associateship.

MATH60013/MATH96011

Mathematics of Business and Economics (Solutions)

Setter's signature	Checker's signature	Editor's signature
....	....	....

1. (a) (i) The law of demand tells us that at a higher price consumers will demand a lower quantity of a certain good. $\boxed{\text{seen} \Downarrow}$

(ii) The law of supply tells us that at higher prices, sellers will supply more of a certain good.

(iii) The marginal rate of technical substitution tells us the rate of change of one input factor with respect to the other, in order to keep the level of output constant.

(iv) Returns to scale tells us how the output changes as both inputs change by the same factor. $\boxed{4, \text{A}}$

(b) (i) Low-hanging fruit is a metaphor for "doing the simplest or easiest work first". Getting the easy-to-reach fruit at the lower end of a tree's branches will increase the productivity, but it will gradually decrease, when it is not possible to reach the higher branches. Multiple examples possible, e.g. retargeting former customers to increase sales, as this could be considered an easy-to-reach target group to start with. $\boxed{\text{seen} \Downarrow}$ $\boxed{2, \text{B}}$

(ii) The factors of production are used in fixed proportions, as there is no substitutability between factors. Multiple examples possible, where two goods are perfect complements, e.g. the Leontief production can be used to describe the production of bikes, using wheels and frames as the input factors. $\boxed{\text{unseen} \Downarrow}$ $\boxed{2, \text{B}}$

(c) (i) - Linear technology: We have two linear equations and two unknowns:

$$10a + 20b = 100$$

$$14a + 10b = 110$$

With a standard procedure, we can determine the unique solution $a = 20/3$ and $b = 5/3$.

- Leontief technology: The first time point gives the condition: $\boxed{3, \text{A}}$

$$(a = 10 \quad \text{and} \quad b \geq 5) \quad \text{or} \quad (a \geq 10 \quad \text{and} \quad b = 5).$$

The second time point gives the condition:

$$(a = 55/7 \quad \text{and} \quad b \geq 11) \quad \text{or} \quad (a \geq 55/7 \quad \text{and} \quad b = 11).$$

These conditions hold simultaneously for $a = 10$ and $b = 11$.

- Cobb-Douglas technology: $\boxed{3, \text{C}}$

$$\left.\begin{aligned} 10^a 20^b &= 100 \\ 14^a 10^b &= 110 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} a + b\log 20 &= \log 100 \\ a\log 14 + b &= \log 110 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} a + b(1+\log 2) &= 2 \\ a(\log 7 + \log 2) + b &= 1 + \log 11 \end{aligned}\right\}$$

Using the given algorithms, we get the following equations:

$$a + 1.3b = 2$$

$$1.15a + b = 2.04$$

and we can determine $a \approx 1.3$ and $b \approx 0.54$.

[*Students will still get full marks if they get sligthly different values for a and b that satisfy the equations.*] $\boxed{3, \text{B}}$

(ii) At time point $t = 1$ we have costs:

$$w_1^1 x_1^1 + w_2^1 x_2^1 = 40,$$

with an output of 100. However, if they had used the input combination from time point $t = 2$, they would have had a cost of only:

$$w_1^1 x_1^2 + w_2^1 x_2^2 = 38$$

and they could have produced more, since $y^2 = 110 > 100 = y^1$, meaning that the firm has not minimised its costs and thus violates the Weak Axiom of Cost Minimisation. Hence, the firm does not act rationally and any inference of the firm's technology must be deemed highly doubtful, i.e. none of the production functions is suitable. $\boxed{3, \text{D}}$

2. (a) We determine the minimiser of $w_R x_R + w_L x_L + w_M x_M$ subject to $f(x_R, x_L, x_M) = y$. meth seen ⇓

First, we define the Lagrangian:

$$\mathcal{L}(x_R, x_L, x_M, \lambda) = w_R x_R + w_L x_L + w_M x_M - \lambda x_R x_L x_M + \lambda y$$

The first-order conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial x_i} = 0, \quad i = R, L, M \Leftrightarrow \frac{\partial f(\underline{x})}{\partial x_i} = \frac{w_i}{\lambda}, \quad i = R, L, M \tag{1}$$

$$\frac{\partial \mathcal{L}}{\lambda} = 0 \Leftrightarrow f(\underline{x}) = y \tag{2}$$

and we can note that (1) can be more usefully written as:

$$\frac{\partial f(\underline{x})}{\partial x_i} \Big/ \frac{\partial f(\underline{x})}{\partial x_j} = \frac{w_i}{w_j}.$$

i.e.

$$\frac{x_L}{x_R} = \frac{w_R}{w_L} \tag{3}$$

$$\frac{x_M}{x_R} = \frac{w_R}{w_M} \tag{4}$$

$$\frac{x_M}{x_L} = \frac{w_L}{w_M} \tag{5}$$

Rearranging (3) and (4) allows us to express, e.g. x_L and x_M in terms of x_R:

$$x_L = \frac{w_R}{w_L} x_R, \quad x_M = \frac{w_R}{w_M} x_R$$

Substituting these into the constraint (2) gives:

$$x_R^*(w_R, w_L, w_M, y) = \left(\frac{y w_L w_M}{w_R^2}\right)^{1/3}$$

By symmetry:

$$x_L^*(w_R, w_L, w_M, y) = \left(\frac{y w_R w_M}{w_L^2}\right)^{1/3}$$

$$x_M^*(w_R, w_L, w_M, y) = \left(\frac{y w_R w_L}{w_M^2}\right)^{1/3}$$

Indeed, the conditional factor demand function is homogeneous of degree 0:

$$x_R^*(t w_R, t w_L, t w_M, y) = x_R^*(w_R, w_L, w_M, y)$$

$$x_L^*(t w_R, t w_L, t w_M, y) = x_L^*(w_R, w_L, w_M, y)$$

$$x_M^*(t w_R, t w_L, t w_M, y) = x_M^*(w_R, w_L, w_M, y)$$

meaning that it is invariant under change of currency. 6, A

(b) The firm is facing variable costs (costs associated with raw materials, labour, and machinery) and quasi-fixed costs (fixed costs paid unrelated to the number of cars produced, e.g. business licence). $\boxed{3, \text{A}}$

(c) The cost function is given by:

$$\begin{aligned} c^*(w_R, w_L, w_M, y) &= w_R x_R^*(w_R, w_L, w_M, y) + w_L x_L^*(w_R, w_L, w_M, y) + w_M x_M^*(w_R, w_L, w_M, y) \\ &= 3(y w_R w_L w_M)^{1/3}, \end{aligned}$$

and the average costs are:

$$AC(y) = \frac{c^*(w_R, w_L, w_M, y)}{y} = 3y^{-2/3}(w_R w_L w_M)^{1/3}.$$

Hence, the average cost curve is decreasing and converges to 0 as $y \to \infty$. This is not surprising, since the production function we are dealing with is a Cobb-Douglas production function with increasing returns to scale ($\alpha + \beta + \gamma > 1$).

[*Students will still get full marks if they do not notice that this is a Cobb-Douglas production function and show the increasing returns to scale using the definition.*] $\boxed{4, \text{B}}$

(d) This is a Cobb-Douglas production function with increasing returns to scale ($\alpha + \beta + \gamma > 1$), therefore the government will likely approve the merger between the two firms, as they could save, e.g. administrative costs.

[*Again, students will still get full marks if they do not notice that this is a Cobb-Douglas production function and show the increasing returns to scale using the definition.*]

$\boxed{3, \text{C}}$

$\boxed{\text{unseen} \Downarrow}$

(e) We use the Weak Axiom of Profit Maximisation, which states that, for net output vectors $z = (y, -\underline{x})$ and corresponding price vectors $\underline{r} = (\underline{p}, \underline{w})$, observed at two times t_1 and t_2,

$$\underline{r}^{t_2}\underline{z}^{t_2} \geq \underline{r}^{t_2}\underline{z}^{t_1}$$

and implies (through switching the indices), using the same notation as before,

$$(\underline{r}^{t_2} - \underline{r}^{t_1})(\underline{z}^{t_2} - \underline{z}^{t_1})^T \geq 0 \Rightarrow (p^{t_2} - p^{t_1})(y^{t_2} - y^{t_1}) - \sum_{i \in R, L, M} (w_i^{t_2} - w_i^{t_1})(x_i^{t_2} - x_i^{t_1}) \geq 0.$$

We take t_1 and t_2 to denote the observed behaviour of the automotive firm in Japan in weeks 1 and 2, respectively. This is clearly a short-run scenario, and so we adopt the reasonable assumption that the quantities of labour and machinery used by the Japanese firm are kept constant, reducing the above relation to:

$$(p^{t_2} - p^{t_1})(y^{t_2} - y^{t_1}) - (w_R^{t_2} - w_R^{t_1})(x_R^{t_2} - x_R^{t_1}) \geq 0.$$

The question gives us:

$$\begin{aligned} p^{t_1} &= 14,000 \quad y^{t_1} = 4,000, \quad x_R^{t_1} = 500 \\ p^{t_2} &= 13,500 \quad y^{t_2} = 5,000, \quad x_R^{t_2} = 700 \end{aligned}$$

which we plug in to obtain:

$$(-500)(1,000) - 200(w_R^{t_2} - w_R^{t_1}) \geq 0 \Rightarrow w_R^{t_2} \leq w_R^{t_1} - 2,500.$$

$\boxed{4, \text{D}}$

3. (a) (i) One can plot the Cobb-Douglas utility function (which is a convex function), or can show that the marginal rate of technical substitution is decreasing in x_1.

2, A

(ii) We look to maximise $u(\underline{x})$ subject to the budget constraint $p_1x_1 + p_2x_2 = m$. First, we define the Lagrangian:

meth seen ⇓

$$\mathcal{L}(x_1, x_2, \lambda) = u(\underline{x}) - \lambda(\underline{p}\underline{x}, m) = x_1^2x_2^4 - \lambda(p_1x_1 + p_2x_2 - m)$$

The first-order conditions for maximisation are given by:

$$\frac{\partial}{\partial\lambda}\mathcal{L}(x_p, x_e, \lambda) = 0 \Rightarrow p_1x_2 + p_2x_2 = m \tag{1}$$

$$\frac{\partial}{\partial x_1}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow 2x_1x_2^4 - p_1\lambda = 0 \tag{2}$$

$$\frac{\partial}{\partial x_2}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow 4x_1^2x_2^3 - p_2\lambda = 0 \tag{3}$$

Dividing (2) by (3) and rearranging allows us to express, e.g. x_2 in terms of x_1:

$$x_2 = \frac{2p_1}{p_2}x_1$$

Substituting into the constraint (1) gives $x_1^*(\underline{p}, m) = \frac{m}{3p_1}$ and $x_2^*(\underline{p}, m) = \frac{2m}{3p_2}$.

5, A

(iii) The firm-side analogue problem is as follows: given a Cobb-Douglas production function, we look to maximise the firm's profit.

seen ⇓

2, A

(iv) (i) The income elasticity of demand is given by:

$$\frac{\partial x_1}{\partial m}\frac{m}{x_1} = 1 > 0$$

and thus, x_1 are normal goods.

2, B

(ii) The price elasticity of demand is given by:

$$\frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1} = -1 < 0$$

and thus, x_2 are ordinary goods.
Alternatively, one may show that x_2 are normal goods:

$$\frac{\partial x_2}{\partial m}\frac{m}{x_2} = 1 > 0$$

and we've seen/proved in the lectures that normal goods are always ordinary goods.

2, B

(iii) The cross-price elasticity of demand is given by:

unseen ⇓

$$\frac{\partial x_1}{\partial p_2}\frac{p_2}{x_1} = 0$$

$$\frac{\partial x_2}{\partial p_1}\frac{p_1}{x_2} = 0$$

and thus, the goods are neither complements, nor substitutes.

2, C

(v) Any strictly increasing transformation of u represents the same preferences, e.g. $u_1(x_1, x_2) = 2\log x_1 + 4\log x_2$ and $u_2(x_1, x_2) = 50x_1^2 x_2^4$.

seen ⇓

1, A

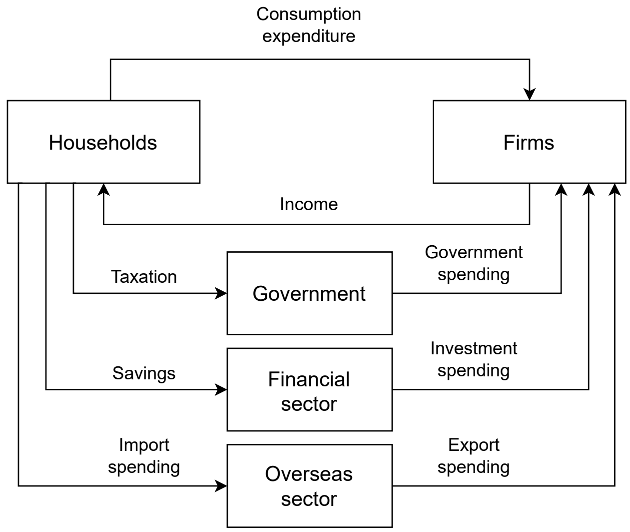
(b) If u and v represent the same preferences, then we can write $u(x_1, x_2) = f(v(x_1, x_2))$ for some strictly increasing function f. We can show that the marginal utilities hold the same signs by an application of the chain rule:

unseen ⇓

$$\frac{\partial u(x_1, x_2)}{\partial x_i} = \frac{\partial f(v(x_1, x_2))}{\partial x_i} = f'(u(x_1, x_2))\frac{\partial v(x_1, x_2)}{\partial x_i}, i = 1, 2.$$

4, D

4. (a) (i) The five-sector model of a national economy is illustrated below: seen ⇓



2, A

(ii) When considering the global economy, one should exclude the overseas sector and the govenrment sector and include international organisations, e.g. the United Nations. unseen ⇓

2, D

meth seen ⇓

(b) (i) Let J^* denote the number of firms acting in the market in the long-run equilibrium. Then J^* is the integer J such that market supply meets market demand and the individual profits are zero (so that no new firms wish to enter or exit the market), i.e.

$$X(p_l^*) = Y(p_l^*) \quad \text{and} \tag{1}$$

$$\pi_j^*(p_l^*) = 0 \quad \forall j = 1, \ldots, J \tag{2}$$

1, A

(ii) Solving (2) gives $p_l^* = 21$ or $p_l^* = -19$; however, the price is non-negative, therefore the long-run equilibrium price is $p_l^* = £21$ per barrel. Before considering (1), we have to solve the profit maximisation problem for each firm to calculate their supply function, $y_j^*(p), j = 1, \ldots, J$. The profit function of each firm is $p^2 - 2p - 399$, which is a strictly convex function, and thus, the solution of the optimisation problem is unique. The first-order condition yields $2p - 2 = 0$, so the optimal supply is given by:

$$y^*(p) = 2p - 2, \quad p \geq 1.$$

Hence the market supply is given by:

$$Y^*(p_l) = \sum_{j=1}^{J} y_j^*(p_l) = J(2p_l - 2).$$

The inverse market demand is $D(q) = 39 - 0.009q$, therefore the market demand is given by:

$$X^*(p_l) = \frac{1000}{9}(39 - p_l).$$

Now, we consider (1). This yields:

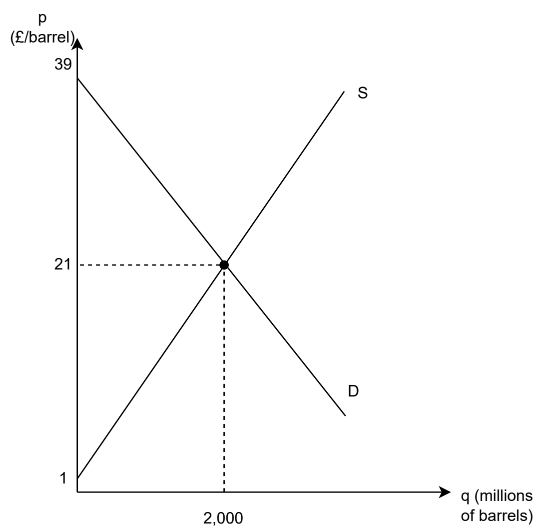
$$J^*(2p_l^* - 2) = \frac{1000}{9}(39 - p_l^*)$$

and substituting $p_l^* = 21$, gives $J^* = 50$ firms.
Hence, the long-run equilibrium quantity is:

$$q_l^* = X(p_l^*) = 50(2 \cdot 41 - 2) = 2,000 \quad \text{millions of barrels.}$$

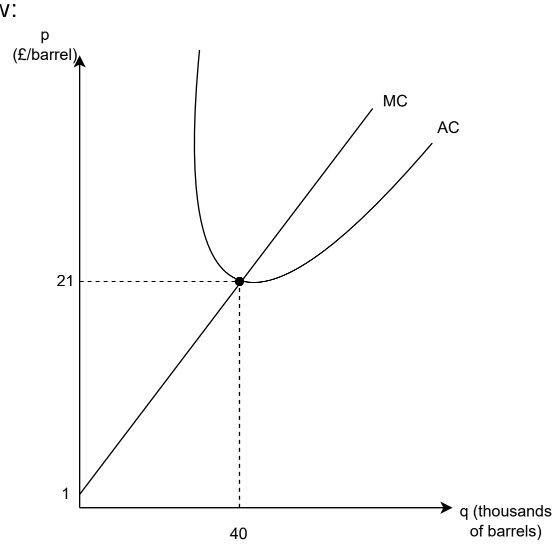
[Alternatively, one can also compute the long-run equilibrium quantity using the market supply, i.e. $q_l^* = Y(p_l^*) = \frac{1000}{9}(39 - 21) = 2,000$ millions of barrels.] $\boxed{5, \text{B}}$

(iii) The market demand curve and the market supply curve are illustrated below:



[The p-axis intercepts are the prices when market demand is zero ($p = 39$) and when market supply is zero ($p = 1$). Students will not be penalised if they use different notation, i.e. X^* and Y^*.] $\boxed{2, \text{C}}$

(iv) The marginal cost curve and the average cost curve of an individual firm are illustrated below:



[The quantity for an individual firm is given by $2 \cdot 21 - 2 = 40$.] $\boxed{2, \text{C}}$

(v) (i) No change in the output supply function of a typical firm. The output supply curve for a firm in the short run is the part of the marginal cost curve above the average variable cost curve, and zero output for prices below the mininum average cost curve (if there are such prices). Neither the marginal costs nor the average variable costs are affected by F, so there is no effect on the output supply curve. unseen ⇓ 1, A 1, D

(ii) No effect on the market equilibrium price. Since the output supply functions for individual firms do not change, the market supply function $S(p)$ does not change. F has no effect on market demand $D(p)$, so the equilibrium priced where $S(p) = D(p)$ also does not change. 1, A 1, D

(iii) All firms are expected to have lower profit. Suppose firm i operates where $p = SMC$ with $p \geq SAVC(y_i^*)$. The profit is $py_i^* - VC(y_i^*) - F$. The increase in F does not change y^*, but does reduce profit. Alternatively, suppose firm i shuts down, so $y_i = 0$. Then its profit is just $-F$, and it falls if the fixed costs increase. 1, A 1, D

Review of mark distribution:

Total A marks: 32 of 32 marks
Total B marks: 20 of 20 marks
Total C marks: 12 of 12 marks
Total D marks: 16 of 16 marks
Total marks: 80 of 80 marks
Total Mastery marks: 0 of 20 marks

If your module is taught across multiple year levels, you might have received this form for each level of the module. You are only required to fill this out once for each question.

Please record below, some brief but non-trivial comments for students about how well (or otherwise) the questions were answered. For example, you may wish to comment on common errors and misconceptions, or areas where students have done well. These comments should note any errors in and corrections to the paper. These comments will be made available to students via the MathsCentral Blackboard site and should not contain any information which identifies individual candidates. Any comments which should be kept confidential should be included as confidential comments for the Exam Board and Externals. If you would like to add formulas, please include a separate pdf file with your email.

ExamModuleCode	QuestionNumber	Comments for Students
Mathematics of Business and Economics_MATH60013 MATH96011	1	Question 1 was mostly well done, except parts (b) (i) and (c) (ii). In part (b) (i) many students explained the law of diminishing marginal productivity, instead of justifying why it is also known as 'low-hanging fruit principle'. Part (c) (ii) was a challenging task, where students were expected to check whether the weak axiom of cost minimisation is satisfied/violated. In this case, the axiom is violated, meaning that the firm does not act rationally, and therefore none of the production functions is suitable. Students also lost marks for not showing their work in part (c) (i).
Mathematics of Business and Economics_MATH60013 MATH96011	2	Many students either did not attempt Q2, or did not approach it correctly. One approach was to use the Weak Axiom of Profit Maximisation.
Mathematics of Business and Economics_MATH60013 MATH96011	3	In general this question was answered well. Some specific comments: - For 3(a)(i) several students attempted to show the result directly, but it was sufficient to state that from lecture notes the utility is Cobb-Douglas and so the preferences convex - For 3(a)(iii) most students stated the analogue was profit maximization but failed to state that this only applied because the function is Cobb-Douglas - 3(b) was well answered when attempted. Several different methods were used, most of these were correct but some answers were missing steps in the derivation.
Mathematics of Business and Economics_MATH60013 MATH96011	4	- 4(a)(i) most students got this correct, although some incompletely labelled the edges with letters without explaining what the letter represent. - 4(a)(ii) generally answered well, although some students missed that we would add organisations such as the UN - 4(b)(i) very few students stated the equations with equality - 4(b)(ii) answered OK, several students calculated p^* and q^* but not J^* - 4(b)(iii)&(iv) answered OK, some axis ticks were missing (particularly y intercept) - 4(b)(v) mixed responses, several students gave answers without explanations.