

Section A

A.1. Exercise 2.7.8: Let R_θ be the anticlockwise rotation of \mathbb{R}^2 around the origin through θ radians. Find the matrix representing R_θ .

A.2. Prove that clockwise rotation of \mathbb{R}^2 around $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ by 1 radian can not be represented by a matrix.

A.3. Exercise 3.1.4: Which of the following are examples of vector spaces over \mathbb{R} :

(a) $V = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ with the usual vector addition and scalar multiplication.

(b) $V = \left\{ \begin{pmatrix} a+1 \\ 2 \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ with the usual vector addition and scalar multiplication.

(c) $V = \mathbb{R}^2$ with standard addition and scalar multiplication defined to be:

$$r \odot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ rb \end{pmatrix}$$

A.4. Exercise 3.2.5: Prove that all subspaces of a vector space over F are vector spaces over F in their own right.

A.5. Find U and V , two subspaces of \mathbb{R}^2 , such that $U \cup V$ is *not* a subspace of \mathbb{R}^2 .

Section B

- B.1. (a) Let M_θ be the reflection in the line $L_\theta = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 = x_1 \tan \theta\}$. Using any school geometry or trigonometry you like, show that the matrix representing M_θ is

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}.$$

- (b) Let R_α be a rotation about the origin, and let M_β be the reflection in a line through the origin. Prove that $M_\beta R_\alpha$ is a reflection.
- (c) Let M_α and M_β be reflections in straight lines through the origin. Prove that $M_\alpha M_\beta$ is a rotation.
- B.2. Let $\mathbb{R}[x]$ be the set of all polynomials with variable x and real coefficients, with standard addition and scalar multiplication. Show that this is a vector space over \mathbb{R} .
- B.3. For each of the following sets and operations, decide whether it is a vector space over \mathbb{R} or not.
- (a) The set \mathbb{R}^2 with standard addition, but with scalar multiplication defined by

$$r \odot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ry \\ rx \end{pmatrix}$$

- (b) The set \mathbb{R}^2 with standard scalar multiplication, but with addition defined by

$$\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b + y \\ a + x \end{pmatrix}$$

- (c) The set \mathbb{R}^2 with addition and scalar multiplication defined by

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} a + x + 1 \\ b + y \end{pmatrix} \\ r \odot \begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} r(a + 1) - 1 \\ rb \end{pmatrix} \end{aligned}$$

- B.4. Let F be a field. Show every F -vector space V with additive identity 0_V has the following properties:

- (a) The vector 0_V is the unique vector satisfying the equation $0_V \oplus v = v$ for all vectors v in V .
- (b) Let 0 be the additive identity in F . Then $0 \odot v = 0_V$ for all vectors v in V .

- B.5. Describe all subspaces of \mathbb{R}^3 .

- B.6. Let U and W be subspaces of vector space V over F . Prove that $U \cup W$ is a subspace of V if and only if $U \subseteq W$ or $W \subseteq U$.