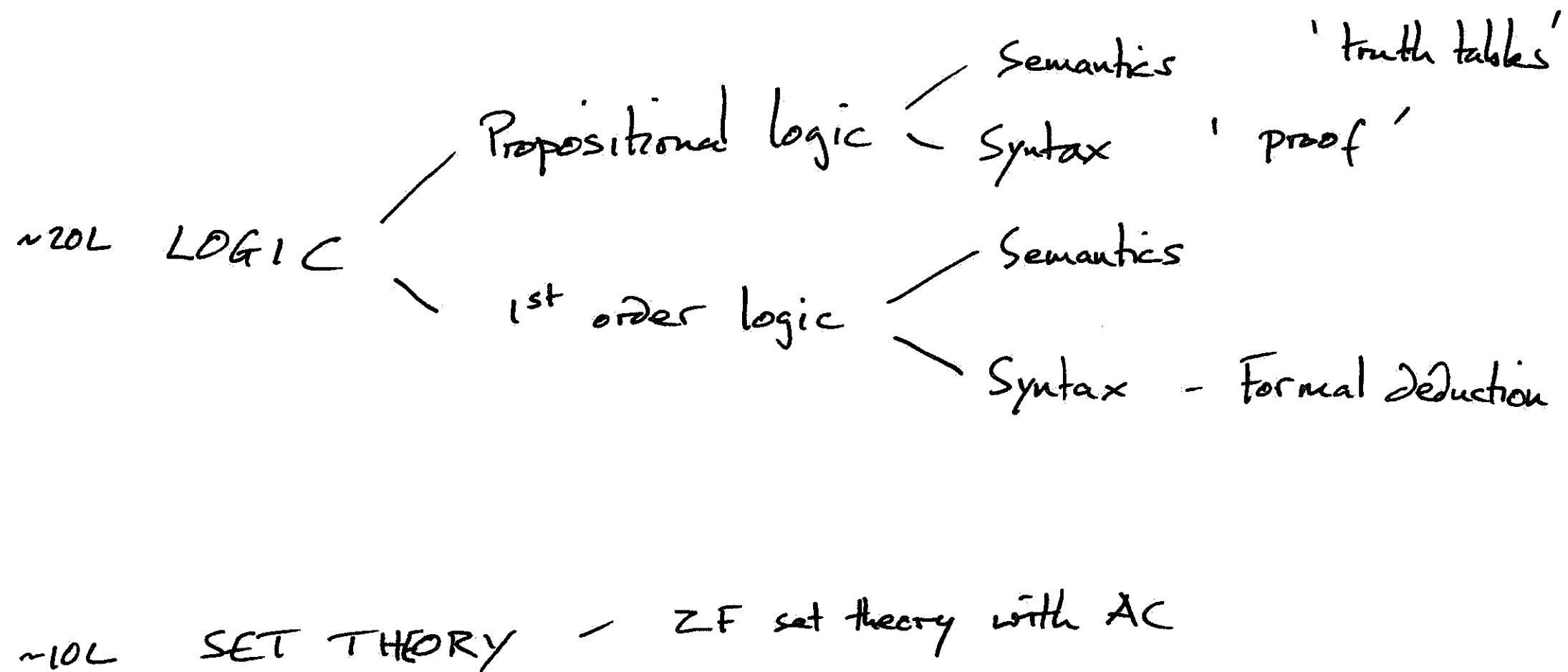


LI

(2)

$$\left( \left( (p \rightarrow q) \wedge (q \rightarrow (\neg p)) \right) \rightarrow (\neg p) \right)$$

$p$      $q$

---

" If Mr. Jones is happy, then Mrs. Jones is unhappy  
and if Mrs. J. is unhappy then Mr. J. is unhappy.  
So Mr. J. is unhappy. "

# 1. Propositional logic .

## 1.1 Propositional formulas .

'Proposition'                          'Statement'  
either                                  True ( $T$ )  
or                                        False ( $F$ )

Combine ~~less~~ basic props. using  
connectives :

### (1.1.1) Connectives + truth table rules .

| statements         |            |
|--------------------|------------|
| $P, q, \dots$      |            |
| <u>Connectives</u> |            |
| <u>Negation</u>    | $(\neg P)$ |

$\frac{P}{\begin{array}{c|c} & (\neg P) \\ \hline T & F \\ F & T \end{array}}$

- (3)
- Conjunction ('and')  
 $(P \wedge q)$  has value  $T$   
 $\Leftrightarrow p$  and  $q$  have value  $T$ .
- Disjunction ('or')  
 $(P \vee q)$  has value  $T$   
 $\Leftrightarrow$  at least one of  $p, q$  has value  $T$ .
- Implication  $(P \rightarrow q)$   
has value  $F$  only when  
 $p$  has value  $T$  and  $q$  has value  $F$ .
- Biconditional  $(P \leftrightarrow q)$   
has value  $T$  precisely when  
 $p, q$  have the same value.

## Summary .

(4)

| P | T | $(P \wedge q)$ | $(P \vee q)$ | $(P \rightarrow q)$ | $(P \leftrightarrow q)$ |
|---|---|----------------|--------------|---------------------|-------------------------|
| T | T | T              | T            | T                   | T                       |
| T | F | F              | T            | F                   | F                       |
| F | T | F              | T            | T                   | F                       |
| F | F | F              | F            | T                   | T                       |

(1.1.2) Def. A propositional formula is obtained from propositional variables  $p_1, p_2, \dots$  and connectives in

the following way :

- (i) Any propositional variable is a prop. formula ;
- (ii) if  $\phi, \psi$  are formulas then  
 $(\neg \phi)$   $(\phi \wedge \psi)$   $(\phi \rightarrow \psi)$   ~~$(\phi \wedge \psi)$~~   $(\phi \vee \psi)$   
 $(\phi \leftrightarrow \psi)$

are formulas .

- (iii) any formula arises in this way  
 (after a finite number of steps) .

Eg. Formulas

$$\begin{array}{c}
 p_1 \quad p_2 \quad (\neg p_2) \\
 (p_1 \rightarrow (\neg p_2)) \\
 ((p_1 \rightarrow (\neg p_2)) \rightarrow p_2) : \phi
 \end{array}$$

Not formulas :  $p_1 \wedge p_2$

$\neg (\neg p_2)$

Remarks. ① Every fmla. is either a prop. variable or is built from 'shorter' fmlas. in a unique way

$$\begin{array}{c}
 \phi \\
 / \quad \backslash \\
 (p_1 \rightarrow (\neg p_2)) \quad p_2 \\
 / \quad \backslash
 \end{array}$$

$$\begin{array}{c}
 p_1 \quad (\neg p_2) \\
 \backslash \quad \backslash \\
 p_2
 \end{array}$$

(2) Any assignment of truth values to the prop. variables in a formula  $\phi$  determines the truth value for  $\phi$  in a unique way, using 1.1.1.  
 Eg.  $\phi : ((p_1 \rightarrow (\neg p_2)) \rightarrow p_1)$

| $p_1$ | $p_2$ | $(\neg p_2)$ | $(p_1 \rightarrow (\neg p_2))$ | $\phi$ |
|-------|-------|--------------|--------------------------------|--------|
| T     | T     | F            | F                              | T      |
| T     | F     | T            | T                              | T      |
| F     | T     | F            | T                              | F      |
| F     | F     | T            | T                              | F      |

'truth table of  $\phi$ '.

(6)

(1.1.3) Def. Let  $n \in \mathbb{N}$ ① A truth function of  
 $n$  variables is afunction  $f: \{\text{T}, \text{F}\}^n \rightarrow \{\text{T}, \text{F}\}$ 

(where

$$\{\text{T}, \text{F}\} = \{(x_1, \dots, x_n) : \text{each } x_i \in \{\text{T}, \text{F}\}\}$$

② Suppose  $\phi$  is a formula  
whose variables are amongst  
 $p_1, \dots, p_n$ . We obtain afunction  $F_\phi: \{\text{T}, \text{F}\}^n \rightarrow \{\text{T}, \text{F}\}$ 

whose value at

$$(x_1, \dots, x_n) \in \{\text{T}, \text{F}\}^n$$

is the truth value of  $\phi$   
when  $p_i$  has value  $x_i$  (for $i \leq n$ ), according to  
(1.1.1). $F_\phi$  is the truth function  
of  $\phi$ .

Eg. in example

$$F_\phi(\text{F}, \text{T}) = \text{F}.$$

=

Read info, sheet.

Compute truth table for  
 $H_r + H_s \cdot J$ .

Qn1.