Imperial College London

**MATH60013**

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May 2023

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

# Mathematics of Business and Economics

Date: 26 May 2023

Time: 14:00 – 16:00 (BST)

Time Allowed: 2 hrs

**This paper has 4 Questions.**

**Please Answer Questions 1-2, and Questions 3-4 in Separate Answer Booklets**

Candidates should start their answers to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

Any required additional material(s) will be provided.

Allow margins for marking.

Credit will be given for all questions attempted.

Each question carries equal weight.

**DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO**

1. (a) We have seen in the lectures that a good's price is not the only determinant of its demand. Give three examples of other determinants. (3 marks)

(b) The newspaper The Guardian reported in an article entitled "'It's like a war': the fight for rice and toilet roll as coronavirus convulses Hong Kong" on 12th February 2020 the following:

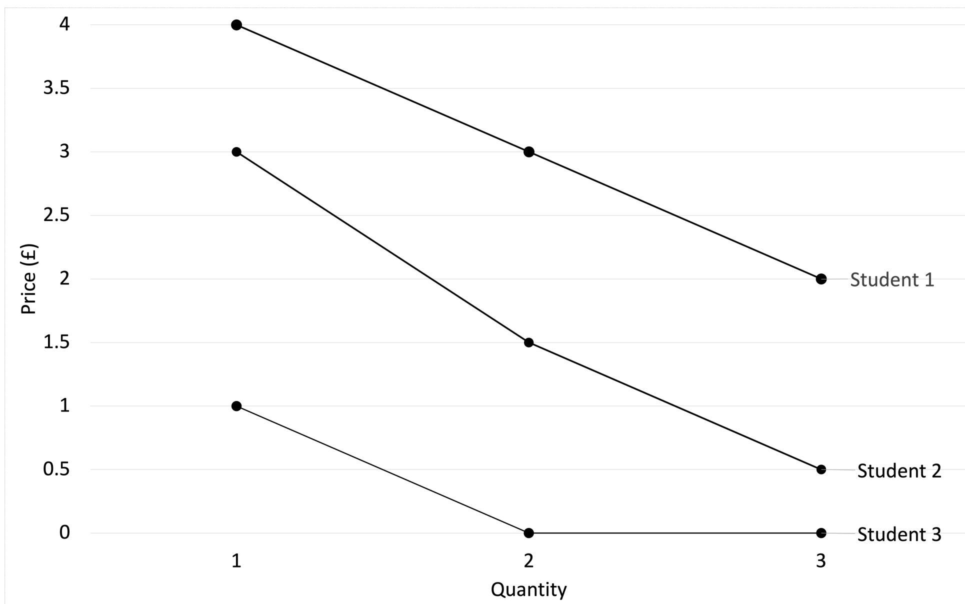
> "For the third week in a row, supermarkets shelves that usually heave with bags of rice – a staple food for Hong Kongers – are empty, and packs of noodles are running out too, although there is still plenty of meat and vegetables."

Does the law of demand hold for all goods stated above? Justify your answer. (2 marks)

(c) Imperial students are interviewed about their chocolate consumption. More specifically, they report their reservation prices (in £) for the quantity of bars they would be willing to purchase.

(i) Briefly explain the notion of reservation price both on the demand and the supply side and give a real-world example, other than the one described in this question. (3 marks)

The following line plot shows the reservation prices reported by three randomly chosen students.




(ii) Compute the individual demand functions $D_i : A \to B, i \in \{1,2,3\}$ and the inverse demand functions $P_i : C \to E, i \in \{1,2,3\}$. State clearly any assumptions made, and define the sets $A, B, C, E$. (5 marks)

(iii) Considering the three individuals as a group of students, compute the aggregate demand function and its inverse. (3 marks)

(d) Consider a firm that produces a single output using two input factors. If the production function of the firm exhibits diminishing marginal product with respect to each input, does it have to exhibit decreasing returns to scale? If this is true, prove it; if not, provide a counterexample. (4 marks)

(Total: 20 marks)

2. (a) (i) Briefly describe the difference between microeconomics and macroeconomics.

(2 marks)

(ii) Briefly explain the following financial terms used in macroeconomics, assuming that the reader is a non-expert. Do not use any mathematical expressions or graphs.

- Aggregate demand
- Aggregate supply
- Trade surplus
- Trade deficit

(4 marks)

(b) A restaurant chain has a limited menu, serving only burgers and lobsters. Their production process can be modelled as using beef ($x_1$) and lobster ($x_2$) to produce meals in a given time period, according to the production function:

$$f(x_1, x_2) = (x_1^\beta + x_2^\beta)^{\alpha/\beta},$$

where $\alpha \neq 0$ and $\beta \neq 0$ are constants.

(i) Show that, for this production function, decreasing returns to scale behaviour requires $\alpha < 1$ and constant returns to scale behaviour requires $\alpha = 1$. (3 marks)

(ii) Compute the conditional factor demand functions $x_1^*(w_1, w_2, y)$ and $x_2^*(w_1, w_2, y)$, where $w_1, w_2 > 0$ are the input prices, and $y > 0$ is the output. (5 marks)

(iii) Verify that the conditional factor demand functions satisfy the required homogeneity property and briefly interpret it. (2 marks)

(iv) Assume that the restaurant is profit-maximising, conditional on minimised costs. Show that when the production function exhibits constant returns to scale, we cannot determine the profit-maximising level of output. (4 marks)

(Total: 20 marks)

3. (a) Consider a consumer whose preferences for a pair of goods can be represented by the following strictly quasi-concave Cobb-Douglas utility function:

$$u(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2},$$

with $0 < \alpha_i < 1, i = 1, 2$. The good $x_i$ has price $p_i > 0, i = 1, 2$ and the individual has a fixed budget $m > 0$.

(i) Give an example of a utility function that represents the same preferences. Justify your answer. (2 marks)

(ii) Compute the marginal utility with respect to $x_1$. What is the limit of the marginal utility for $x_1 \to \infty$? Briefly interpret the *economic intuition* behind this feature. (3 marks)

(iii) Compute the bundle that maximises the consumer's utility. (5 marks)

(iv) Using your answer in (iii), show that Cobb-Douglas preferences have the feature that the share of money spent on each good does not depend on the budget or prices. (4 marks)

(b) Consider the consumer faced with a choice of two goods. Their preferences are described by:

$$x_1 + x_2 \leq y_1 + y_2 \Leftrightarrow \underline{x} \succeq \underline{y}$$

for any consumption bundles $\underline{x} = (x_1, x_2), \underline{y} = (y_1, y_2) \in \mathbb{R}^2$.

(i) Show that the preference relation $\succeq$ satisfies the three axioms of a complete weak order on $\mathbb{R}^2$, i.e. completeness, reflexivity and transitivity. (3 marks)

(ii) Prove that there exists some continuous utility function representing the consumer's preferences. (3 marks)

(Total: 20 marks)

4. For the purposes of this question, we assume that there are only two firms in the UK that produce fully electric vehicles, each having the same production function:

$$f(x_1, x_2) = (x_1 x_2)^{1/2}.$$

We also assume a short-run scenario, where input good $x_2 > 0$ is fixed for each firm, the input prices are $w_1 = w_2 = 1$, and the output price is $p > 0$. Quantities are reported in hundred thousand vehicles, and prices are reported in hundred thousand pounds.

(a) Show that the short-run cost function for each firm is given by $c_S^*(y) = \frac{y^2}{x_2} + x_2$, where $y > 0$ is the firm's output. (2 marks)

(b) Compute the individual supply function $y_i^*(p), i = 1, 2$. (3 marks)

(c) Plot the short-run marginal cost and the short-run average variable cost curves for an individual firm. Does an individual firm ever shut down? Use your plot to justify your answer. (4 marks)

(d) Let $x_{i2} = 2$ be the level of the fixed input for firm $i, i = 1, 2$, and $X^*(p) = \frac{1}{8p^3}$ be the market demand. Compute the market supply function $Y^*(p)$ and the short-run equilibrium price for electric vehicles $p_S^*$. (3 marks)

(e) The goverment decides to impose an indirect $x\%$ tax on electric vehicles. How do we call this type of tax? If the new-short run equilibrium quantity is $q_{S,tax}^* = 90,000$ vehicles, determine the tax rate. [You may use that $0.9^{3/4} \approx 0.87$.] (4 marks)

(f) Assume the market consists of $n > 2$ firms, each having a U-shaped long-run average cost curve. If the market has free entry and exit in the long run, and all firms are price takers, is there a price level $p$ where supply equals demand?
*Hint:* You may investigate the cases of $p > p_{min}, p < p_{min}$ and $p = p_{min}$, where $(y_{min}, p_{min})$ is the minimum point of the long-run average cost curve. (4 marks)

(Total: 20 marks)

BSc and MSci EXAMINATIONS (MATHEMATICS)

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# MATH60013

## Mathematics of Business and Economics (Solutions)

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| ................ | ................ | ................ |

1. (a) Multiple example possibles, e.g. number and price of substitute goods, number and price of complementary goods, level and distribution of income, consumer's tastes and habits, and consumer's expectations with regard to the future. seen ⇓

3, A

(b) The law of demand does not hold for rice in this case, as this is a staple food for Hong Kongers, and an increase in price would not lead to a drop in demand. 2, B

(c) (i) On the demand side, the reservation price is the highest price a buyer is willing to pay for an item, whereas on the supply side, the reservation price is the minimum price at which as seller is wiling to sell it.
Multiple examples possible, e.g. during auctions, a seller won't accept anything less than a specific bid amount, which is their reservation price.

3, A

(ii) We are working under the assumption that prices, although being reported discretely in pounds and pennies, are continuous quantities. seen/sim.seen ⇓
The demand functions are maps $D_i : \mathbb{Z}_{>0} \to \mathbb{R}_{\geq 0}, i \in \{1, 2, 3\}$, and take the form of step functions:

$D_1(p) = \mathbb{1}_{[4,0)}(p) + \mathbb{1}_{[3,0)}(p) + \mathbb{1}_{[2,0)}(p) \quad p > 0$
$D_2(p) = \mathbb{1}_{[3,0)}(p) + \mathbb{1}_{[1.5,0)}(p) + \mathbb{1}_{[0.5,0)}(p) \quad p > 0$
$D_3(p) = \mathbb{1}_{[1,0)}(p) \quad p > 0$

2, A

The inverse demand functions $P_i : \mathbb{R}_{>0} \to \mathbb{Z}_{\geq 0}, i \in \{1, 2, 3\}$ are as follows:

$P_1(1) = 4, \quad P_1(2) = 3, \quad P_1(3) = 2, \quad P_1(n) = 0 \quad \forall n \geq 4$
$P_2(1) = 3, \quad P_2(2) = 1.5, \quad P_2(3) = 0.5, \quad P_2(n) = 0 \quad \forall n \geq 4$
$P_3(1) = 1, \quad P_3(n) = 0 \quad \forall n \geq 2$

3, B

(iii) To compute the aggregate demand function, we add the individual demand functions: $D(p) = D_1(p)+D_2(p)+D_3(p) = \mathbb{1}_{[4,0)}(p)+2\mathbb{1}_{[3,0)}(p)+\mathbb{1}_{[2,0)}(p)+\mathbb{1}_{[1.5,0)}(p) + \mathbb{1}_{[1,0)}(p) + \mathbb{1}_{[0.5,0)}(p)$.
In stark contrast to the aggregate demand function, the inverse of the aggregate demand function cannot be computed as the sum of the inverses of the individual demand functions. Instead, we compute the (generalised) inverse of the aggregate demand function $P(q) = \max\{p \geq 0 \,|\, D(p) \geq q\}$, and thus we obtain $P(1) = 4, P(2) = P(3) = 3, P(4) = 2, P(5) = 1.5, P(6) = 1, P(7) = 0.5, P(n) = 0, \forall n \geq 8$.

3, C

(d) False. A counterexample is the Cobb-Douglas production function $f(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$ with $0 < \alpha < 1$. unseen ⇓
This function exhibits diminishing marginal product for each input:

$$\frac{\partial f(\underline{x})}{\partial x_1} = \alpha \left(\frac{x_2}{x_1}\right)^{1-\alpha}, \text{which is decreasing in } x_1$$

$$\frac{\partial f(\underline{x})}{\partial x_2} = (1-\alpha) \left(\frac{x_1}{x_2}\right)^{\alpha}, \text{which is decreasing in } x_2$$

and for any $t > 0$, $f(tx_1, tx_2) = (tx_1)^\alpha (tx_2)^{1-\alpha} = t^{\alpha+1-\alpha} x_1^\alpha x_2^{1-\alpha} = tf(x_1, x_2)$, which implies that it exhibits constant returns to scale instead of decreasing returns to scale.

4, D

2. (a) (i) In microeconomics, we study the behaviour of individual firms, individual consumers, and individual markets, whereas in macroeconomics, we study the economy as a whole, i.e. how the markets, businesses, consumers, and governments behave/interact. $\boxed{\text{seen} \Downarrow}$ $\boxed{2, \text{A}}$

(ii)
- Aggregate demand is the total demand of all final goods and services produced within an economy.
- Aggregate supply is the total supply of all final goods and services provided by firms to the economy.
- A trade surplus is a financial term used when an economy exports more goods than imports.
- A trade deficit is a financial term used when an economy imports more goods than exports. $\boxed{4, \text{A}}$

(b) (i) Decreasing returns to scale requires: $\boxed{\text{meth seen} \Downarrow}$

$$
\begin{aligned}
f(tx_1, tx_2) < tf(x_1, x_2) \quad \forall \quad t > 1 \Leftrightarrow \\
\left((tx_1)^\beta + (tx_2)^\beta\right)^{\alpha/\beta} < t(x_1^\beta + x_2^\beta)^{\alpha/\beta} \quad \forall \quad t > 1 \Leftrightarrow \\
t^\alpha \left(x_1^\beta + x_2^\beta\right)^{\alpha/\beta} < t(x_1^\beta + x_2^\beta)^{\alpha/\beta} \quad \forall \quad t > 1 \Leftrightarrow \\
\alpha < 1
\end{aligned}
$$

and constant returns to scale requires:

$$
\begin{aligned}
f(tx_1, tx_2) = tf(x_1, x_2) \quad \forall \quad t > 0 \Leftrightarrow \\
\left((tx_1)^\beta + (tx_2)^\beta\right)^{\alpha/\beta} = t(x_1^\beta + x_2^\beta)^{\alpha/\beta} \quad \forall \quad t > 0 \Leftrightarrow \\
t^\alpha \left(x_1^\beta + x_2^\beta\right)^{\alpha/\beta} = t(x_1^\beta + x_2^\beta)^{\alpha/\beta} \quad \forall \quad t > 0 \Leftrightarrow \\
\alpha = 1
\end{aligned}
$$

$\boxed{3, \text{C}}$

(ii) We determine the minimiser of $w_1x_1 + w_2x_2$ subject to $f(x_1, x_2) = y$. First, we define the Lagrangian:

$$\mathcal{L}(x_1, x_2, \lambda) = w_1x_1 + w_2x_2 - \lambda(x_1^\beta + x_2^\beta)^{\alpha/\beta} + \lambda y$$

The first-order conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial x_i} = 0, \quad i = 1, 2 \Leftrightarrow \frac{\partial f(\underline{x})}{\partial x_i} = \frac{w_i}{\lambda}, \quad i = 1, 2 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow f(\underline{x}) = y \text{ (constraint)} \tag{2}$$

We compute the partial derivatives:

$$\frac{\partial f(\underline{x})}{\partial x_1} = \alpha(x_1^\beta + x_2^\beta)^{\frac{\alpha}{\beta}-1}x_1^{\beta-1}, \tag{3}$$

$$\frac{\partial f(\underline{x})}{\partial x_2} = \alpha(x_1^\beta + x_2^\beta)^{\frac{\alpha}{\beta}-1}x_2^{\beta-1}. \tag{4}$$

Dividing $(3)$ by $(4)$ and rearranging allows us to express, e.g. $x_1$ in terms of $x_2$:

$$x_1 = x_2 \left(\frac{w_1}{w_2}\right)^{\frac{1}{\beta-1}}.$$

Substituting into the constraint $(2)$ gives:

$$x_2^*(w_1, w_2, y) = \frac{y^{\frac{1}{\alpha}}}{\left[\left(\frac{w_1}{w_2}\right)^{\frac{\beta}{\beta-1}} + 1\right]^{\frac{1}{\beta}}}.$$

By symmetry:

$$x_1^*(w_1, w_2, y) = \frac{y^{\frac{1}{\alpha}}}{\left[\left(\frac{w_2}{w_1}\right)^{\frac{\beta}{\beta-1}} + 1\right]^{\frac{1}{\beta}}}.$$

$\boxed{5, \text{B}}$

(iii) Indeed, the conditional factor demand function is positively homogeneous of degree 0:

$$x_1^*(tw_1, tw_2, y) = x_1^*(w_1, w_2, y)$$

$$x_2^*(tw_1, tw_2, y) = x_2^*(w_1, w_2, y)$$

meaning that it is invariant under change of currency.

$\boxed{2, \text{A}}$

(iv) The profit is given by revenue minus costs:

$\boxed{\text{unseen} \Downarrow}$

$$\pi = py - w_1 x_1 - w_2 x_2.$$

The first-order condition is given by:

$$\frac{\partial \pi}{\partial y} = p - \frac{w_1}{\alpha}\frac{y^{\frac{1}{\alpha}-1}}{A} - \frac{w_2}{\alpha}\frac{y^{\frac{1}{\alpha}-1}}{B} = 0$$

where $A = \left[\left(\frac{w_2}{w_1}\right)^{\frac{\beta}{\beta-1}} + 1\right]^{\frac{1}{\beta}}$ and $B = \left[\left(\frac{w_1}{w_2}\right)^{\frac{\beta}{\beta-1}} + 1\right]^{\frac{1}{\beta}}$.

If the profuction function exhibits constant returns to scale ($\alpha = 1$), then $y$ disappears from the first-order condition, and therefore we cannot determine the profit maximising level of output.

$\boxed{4, \text{D}}$

3. (a) (i) Any strictly increasing transformation of $u$ represents the same preferences. Multiple examples possible, e.g. $u_1(x_1, x_2) = \log u(\underline{x}) = \alpha_1 \log x_1 + \alpha_2 \log x_2$. $\boxed{\text{seen} \Downarrow}$

$\boxed{2, \text{A}}$

(ii) The marginal utility with respect to good $x_1$ is:

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = \alpha_1 x_1^{\alpha_1 - 1} x_2^{\alpha_2}.$$

For $x_1 \to \infty$ the marginal utility converges to 0. This means that for very high consumption of $x_1$, an additional unit of consumption has almost no added utility. $\boxed{1, \text{A}}$ $\boxed{\text{unseen} \Downarrow}$

(iii) We look to maximise $u(\underline{x})$ subject to the budget constraint $p_1 x_1 + p_2 x_2 = m$. The Lagrangian is: $\boxed{2, \text{B}}$ $\boxed{\text{meth seen} \Downarrow}$

$$\mathcal{L}(x_1, x_2, \lambda) = u(\underline{x}) - \lambda(\underline{p}\underline{x} - m) = x_1^{\alpha_1} x_2^{\alpha_2} - \lambda(p_1 x_1 + p_2 x_2 - m).$$

The Marshallian demand exists and is unique. That means we only need to check for first-order conditions and can leave out second-order conditions. The first-order conditions are given by:

$$\frac{\partial}{\partial \lambda}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow p_1 x_1 + p_2 x_2 = m \tag{1}$$

$$\frac{\partial}{\partial x_1}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow \alpha_1 x_1^{\alpha_1 - 1} x_2^{\alpha_2} - \lambda p_1 = 0 \tag{2}$$

$$\frac{\partial}{\partial x_2}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow \alpha_2 x_1^{\alpha_1} x_2^{\alpha_2 - 1} - \lambda p_2 = 0 \tag{3}$$

Dividing (2) by (3) gives:

$$\frac{\alpha_1}{\alpha_2}\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

and rearranging allows us to express, e.g. $x_2$ in terms of $x_1$:

$$x_2 = \frac{p_1}{p_2}\frac{\alpha_2}{\alpha_1} x_1$$

Substituting the solutions for $x_2$ and $x_3$ into the constraint (1) gives:

$$p_1 x_1 + p_2 \left(\frac{p_1}{p_2}\frac{\alpha_2}{\alpha_1} x_1\right) = m \Rightarrow$$

$$p_1 x_1 + \frac{\alpha_2}{\alpha_1} p_1 x_1 = m \Rightarrow$$

$$\frac{\alpha_1}{\alpha_1} x_1 + \frac{\alpha_2}{\alpha_1} x_1 = \frac{m}{p_1} \Rightarrow$$

$$\frac{\alpha_1 + \alpha_2}{\alpha_1} x_1 = \frac{m}{p_1} \Rightarrow$$

$$x_1^*(\underline{p}, m) = \frac{\alpha_1}{\alpha_1 + \alpha_2}\frac{m}{p_1}$$

By symmetry,

$$x_2^*(\underline{p}, m) = \frac{\alpha_2}{\alpha_1 + \alpha_2} \frac{m}{p_2}$$

$\boxed{5, \text{A}}$

(iv) We can rewrite the above expressions as follows:

$\boxed{\text{unseen} \Downarrow}$

$$\frac{x_i^* p_i}{m} = \frac{\alpha_i}{\alpha_1 + \alpha_2}, \quad i = 1, 2.$$

The left-hand side is the share of budget $m$ spent on good $i$, and the right hand side of the equation shows that this is a constant, it does not depend on prices or budget. Hence, the statement is true.

$\boxed{4, \text{D}}$

(b) (i) [Completeness] Since, for any two bundles $\underline{x}, \underline{y} \in \mathbb{R}^2$ we must have $x_1 + x_2 \leq y_1 + y_2$, or $x_1 + x_2 \geq y_1 + y_2$, or both, this allows us to deduce that: $\boxed{\text{meth seen} \Downarrow}$

$$\underline{x} \succeq \underline{y} \quad \text{or} \quad \underline{y} \succeq x \quad \text{or} \quad x \succeq \underline{y}, \underline{y} \succeq x$$

Hence, the preferences satisfy completeness.

[Reflexivity] Clearly, for all bundles $\underline{x} \in \mathbb{R}^2$, $x_1 + x_2 \leq x_1 + x_2$, and so we can deduce $\underline{x} \succeq \underline{x}$. Hence, the preferences satisfy reflexivity. Alternatively, one can say that reflexivity is implied by completeness.

[Transitivity] Let $x = (x_1, x_2), y = (y_1, y_2), z = (z_1, z_2) \in \mathbb{R}^2$. Assume that $\underline{x} \succeq \underline{y}$ and $\underline{y} \succeq \underline{z}$. We will show that $\underline{x} \succeq \underline{z}$. We get indeed that:

$$x_1 + x_2 \leq y_1 + y_2 \quad \text{and} \quad y_1 + y_2 \leq z_1 + z_2$$

Hence

$$x_1 + x_2 \leq z_1 + z_2.$$

$\boxed{3, \text{B}}$

(ii) The required conditions are completeness, transitivity and continuity. Completeness and transitivity are satisfied (part (i)), and we now verify the continuity. $\boxed{\text{unseen} \Downarrow}$

For any given $\underline{x} \in \mathbb{R}^2$,

$$\{\underline{y} \in \mathbb{R}^2 : \underline{y} \leq \underline{x}\} \quad \text{and} \quad \{\underline{y} \in \mathbb{R}^2 : \underline{x} \leq \underline{y}\}$$

are both closed sets. Hence

$$\{\underline{y} \in \mathbb{R}^2 : \underline{y} \succeq \underline{x}\} \quad \text{and} \quad \{\underline{y} \in \mathbb{R}^2 : \underline{x} \succeq \underline{y}\}$$

are also closed sets, and so the preferences are continuous.

A continuous utility function therefore exists for the preference relation. $\boxed{3, \text{C}}$

[Note: The proof we've seen in the lectures also assumed strong monotonicity, but this is not necessary for Debreu's Theorem to hold.]

4. (a) We have $y^2 = x_1 x_2$, or $x_1 = y^2/x_2$, where $x_2$ is fixed. The short-run cost function is given by: $\boxed{\text{seen} \Downarrow}$

$$c_S^*(y) = w_1 x_1 + w_2 x_2 = x_1 + x_2 = \frac{y^2}{x_2} + x_2.$$

$\boxed{2, \text{A}}$

(b) We solve the profit maximisation problem for each firm to compute their supply function $y_i^*(p), i = 1, 2$. $\boxed{\text{meth seen} \Downarrow}$

The profit is given by revenue minus costs:

$$py - c_S^*(y) = py - \frac{y^2}{x_2} - x_2.$$

The first-order condition yields:

$$p - \frac{2y}{x_2} = 0 \Rightarrow y(p) = \frac{px_2}{2}$$

and a sufficient second-order condition is satisfied, since the marginal cost is increasing.

Hence the individual supply function is given by:

$$y_i^*(p) = \frac{px_{i2}}{2}, i = 1, 2.$$

$\boxed{3, \text{A}}$

(c) The short-run marginal costs are:

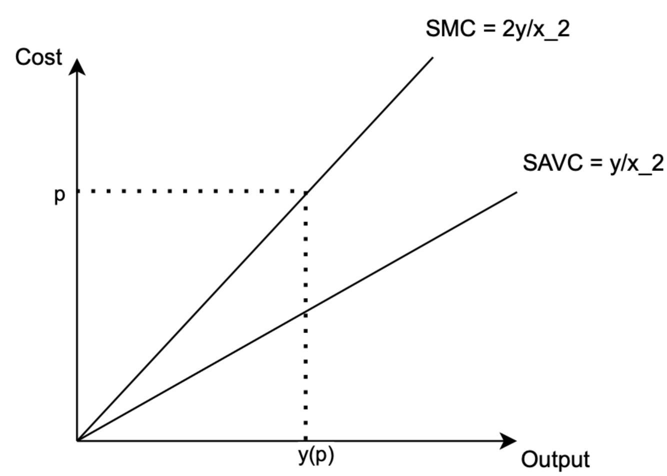
$$SMC(y) = \frac{\partial c_S^*(w_1, w_2, y)}{\partial y} = \frac{2y}{x_2}.$$

The short-run average variable costs are:

$$SAVC(y) = \frac{SVC}{y} = \frac{y^2/x_2}{y} = \frac{y}{x_2}.$$

$\boxed{2, \text{A}}$

The cost curves are:




Hence, the firm never shuts down because we always have $p = SMC > SAVC$. $\boxed{2, \text{B}}$

(d) The market supply is given by:

$$Y^*(p) = \sum_{i=1}^{2} y_i^*(p) = 2p.$$

Hence, the short-run equilibrium price is given by:

$$Y^*(p_S^*) = S^*(p_S^*) \Leftrightarrow 2p_S^* = \frac{1}{8p_S^{*3}} \Leftrightarrow p_S^* = 0.5 \text{ hundred thousand pounds [or £50,000].}$$

$\boxed{3, \text{B}}$

(e) This is an ad valorem tax. $\boxed{1, \text{A}}$

Without taxes, the consumers face the inverse supply function, which is $p(q) = q/2$, and with taxes, the inverse supply is $p_{tax}(q) = (1+x)p(q)$, where $x$ is the tax rate. $\boxed{\text{unseen} \Downarrow}$

The inverse demand function is $D(q) = 0.5q^{-1/3}$.

The market is in equilibrium:

$$p_{tax}(q^*_{S,tax}) = D(q^*_{S,tax}) \Rightarrow$$

$$(1+x)\frac{q^*_{S,tax}}{2} = 0.5(q^*_{S,tax})^{-1/3} \Rightarrow$$

$$x = \left(\frac{1}{q^*_{S,tax}}\right)^{4/3} - 1 \Rightarrow$$

$$x \approx \frac{100}{87} - 1 \approx 0.15 \text{ [or 15\%].}$$

$\boxed{3, \text{C}}$

(f) There is no price level $p$ where supply equals demand.

* If $p > p_{min}$, then the firms have positive profit, and since firms are price-takers, new firms enter the market, and supply exceeds demand.
* If $p < p_{min}$, then any firm with positive output has negative profit, so all firms exit and demand exceeds supply.
* If $p = p_{min}$, then no firm is willing to produce a fractional output, $0 < y < y_{min}$, because this would give negative profit, so supply cannot equal demand at this price either.

$\boxed{4, \text{D}}$

**Review of mark distribution:**
Total A marks: 32 of 32 marks
Total B marks: 20 of 20 marks
Total C marks: 12 of 12 marks
Total D marks: 16 of 16 marks
Total marks: 80 of 80 marks
Total Mastery marks: 0 of 20 marks

| If your module is taught across multiple year levels, you might have received this form for each level of the module. You are only required to fill this out once for each question. | | |
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| **ExamModuleCode** | **QuestionNumber** | **Comments for Students** |
| MATH60013 | 1 | Part (a): Most students addressed the question well. However, there were a few who mentioned determinants of the supply instead of the demand, or provided unclear answers without justifying why there are determinants of the demand.Part (b) was addressed well by most students. Just a few mistakes understanding the law of demand.Part(c)(ii): Most students mentioned that the reservation prices are "the prices at which  buyers/sellers are willing to buy/sell an item", but they didn't mention that these are the "highest"/"lowest" prices, in which case the answer is incomplete. Many examples were also ambiguous/unclear.Part(c)(ii) Only a few students stated the assumptions needed to get the functions (which was asked in the question). There were also many mistakes when computing the individual demand functions and the inverse demand function. Most common mistakes were the result of not considering prices as continuous and/pr not to work with quantity demanded as a discreet variable.Part(c)(iii) There were some difficulties to compute the aggregate demand. Same considerations as in part (c)(ii).Part(d) This was a true/false question and students were expected to provide a counterexample to show that a statement was false. Only a few students answered it correctly by providing a valid counterexample. |
| MATH60013 | 2 | No Comment Received |
| MATH60013 | 3 | Overall, the question was answered well.Some comments below:Part(a)(i) Most students answered this question well and provided a valid example.Part(a)(ii): Most students computed the marginal utility correctly, as well as the limit. However, in many cases, the economic intuition was unclear. Note that the marginal utility gives information about the utility provided by an additional unit of the product, not the utility of the product when the consumer has much of it.Part(a)(iii): Nearly all students computed the bundle that maximises the consumer's utility well.Part(a)(iv): Many students answered this question well. A common mistake was to compare the optimal quantities for each product instead of the amount of money spent on each product, given the optimal quantities. Some students left this question unanswered.Part(b)(i) Most students explained the 3 axioms well, though some confused the axioms, or explained one or two axioms.Part(b)(ii) Only a few students proved what it was asked. Many students didn't attempt to answer this question, or provided an example of a continuous function (which was not what was being asked), or gave a wrong justification. |
| MATH60013 | 4 | Q4 – Overall, answered well. A few points to note: in part (b) some students did not check that the second-order condition is satisfied, in part (c) some students included the fixed costs in the average variable costs, in part (e), full marks were given no matter what the numerical answer was, and part (f) was not attempted by many students, which could be due to lack of time. |