

MATH60005/70005: Optimisation (Autumn 22-23)

Weeks 6 and 7: Exercises

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1. Show the convexity of the following functions:

- the quad-over-lin function

$$f(x_1, x_2) = \frac{x_1^2}{x_2}$$

defined over $\mathbb{R} \times \mathbb{R}_{++} = \{(x_1, x_2) : x_2 > 0\}$.

- the generalized quad-over-lin function

$$g(\mathbf{x}) = \frac{\|\mathbf{Ax} + \mathbf{b}\|^2}{\mathbf{c}^\top \mathbf{x} + d} \quad (\mathbf{A} \in \mathbb{R}^{m \times n}, \mathbf{b} \in \mathbb{R}^m, \mathbf{c} \in \mathbb{R}^n, d \in \mathbb{R})$$

is convex over $D = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{c}^\top \mathbf{x} + d > 0\}$.

- $f(x_1, x_2) = -\log(x_1 x_2)$, over \mathbb{R}_{++}^2 .
- $h(\mathbf{x}) = e^{\|\mathbf{x}\|^2}$.

2. Show that $\sqrt{1 + \mathbf{x}^\top Q \mathbf{x}}$ is convex for Q positive definite.

3. Find the optimal solution of

$$\max_{\mathbf{x} \in \mathbb{R}^3} 2x_1^2 + x_2^2 + x_3^2 + 2x_1 - 3x_2 + 4x_3$$

subject to $x_1 + x_2 + x_3 = 1$

$x_1, x_2, x_3 \geq 0$

