Course Content

- Vector Calculus. Preliminary ideas and some revision of vectors: tensor notation and the summation convention. Gradient: components of grad in different coordinate systems, tangent planes, divergence and curl, operations with the gradient, the Laplacian, scalar and vector fields. Path integrals: conservative forces, circulation. Surface integrals: definition, projection theorem. Volume integrals: definition. Results relating line, surface and volume integrals: Green's theorem, flux, the divergence theorem, Gauss' theorem, Stokes' theorem. Curvilinear coordinates: implicit/inverse function theorems, line and volume elements, gradient, divergence, curl, Laplacian. Changes of variable in surface integration: Jacobian.

- Calculus of Variations. Preliminary motivational examples: shortest distance, curve of quickest descent, minimal surface of revolution. The Vanishing Lemma. The Euler-Lagrange equations and its short forms. Examples revisited. Extension to more variables. Problems involving constraints. The isoperimetric inequality. Euler-Lagrange theory for higher-dimensional integrals.

Recommended reading

1. My notes! These will be put on Blackboard with gaps in them and we will fill these in during the lectures. Problem sheets will be made available online. Hints will be provided after one week and full solutions usually after two weeks. There will also be some (non-examinable) material on applications of the methods learned, usually in the form of powerpoints.

2. A useful reference book with lots of solved problems is *Vector Analysis* by Spiegel and is part of the Schaum's Outline Series. This covers the whole of the Vector Calculus part of the course.

3. *Differential and Integral Calculus*, R. Courant. There are two volumes; Volume II is more relevant to this course. (Volume I is good for revision of first year material). Volume II covers topics on the integral theorems, changes of variables, curvilinear coordinates and multi-dimensional integrals.

4. *Advanced Engineering Mathematics* by Erwin Kreyszig. There is a good discussion here of vector analysis and the integral theorems.

5. For the calculus of variations the textbook *Advanced Engineering Mathematics* by Wylie and Barrett (McGraw-Hill) has a relevant chapter on both unconstrained and constrained problems.

6. For the latter part of the module I consulted several online sources. Here are a few that I found useful:

   http://www-users.math.umn.edu/~jwcalder/CalculusOfVariations.pdf
   http://www.maths.ed.ac.uk/~jmf/Teaching/Lectures/CoV.pdf
   http://math.uchicago.edu/~may/REU2019/REUPapers/Zheng,SiqiClover.pdf

7. Other online sources

   http://tutorial.math.lamar.edu/Classes/CalcIII/CalcIII.aspx A good source of elementary examples.
   http://mathinsight.org/ Very good visualization tools.

8. Online sources mentioned in the Powerpoint presentations

   http://www.pitt.edu/~jdnorton/Goodies/Zurich_Notebook/
   Einstein's notebook.
   http://phet.colorado.edu/en/simulation/faraday
   Faraday's electromagnetic experiments.
   http://en.wikipedia.org/wiki/Orthogonal_coordinates#Table_of_orthogonal_coordinates
   Table of orthogonal coordinate systems.
   http://www.youtube.com/watch?v=1BdO8J0iynY
   Experimental demonstration of brachistochrone.
   http://www.youtube.com/watch?v=HnIJD9vxyXk
   Soap films and minimal surfaces.