	EXAMINATION SOLUTIONS 2015-16	Course M3B
Question 1		Marks & seen/unseen
Part (a)	The given preference relation is complete, transitive and continuous Therefore, by Debreu's Theorem, there must exist a continuous utility function that represents this preference relation.	1, Seen 1
Part (b)	We solve the constrained maximisation problem: find $\max_{\underline{x}\in X} u(\underline{x})$ such that $\underline{p}\underline{x} = m$.	Seen
	Define the Lagrangian: $\mathcal{L} = u_A(\underline{x}) - \lambda(p_1x_1 + p_2x_2 - m_A)$ and determine the first order conditions (FOCs) for maximisation: $\frac{\partial \mathcal{L}}{\partial x_i} = 0 \Rightarrow \frac{\partial u_A}{\partial x_i} = \lambda p_i \qquad i = 1, 2$ $\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow p_1x_1 + p_2x_2 = m_A$	1 2
	Solving the FOCs yields the Marshallian demand for each good: $$x^*_{A,1}(\underline{p}, m_A) = \frac{m_Ap_1}{p_1^2 + p_2^2}, \qquad x^*_{A,2}(\underline{p}, m_A) = \frac{m_Ap_2}{p_1^2 + p_2^2},$$	2
	[Note: The above approach adopts the usual constraint $\underline{p}\underline{x} = m$. It is also justifiable to assume that the consumer's expenditure can be less than their budget. This leads to a utility-maximising demand of $\underline{x} = (0, 0)$, corresponding to an infinite indirect utility in Part (c).]	
Part (c)	Since $u_B(\underline{x})$ is a monotonic transformation of $u_A(\underline{x})$ for $\underline{x} \in X$, Barbara's preferences can also be represented by u_A, i.e. Barbara and Alfred have the same preferences. $\Rightarrow$ Barbara and Alfred have the same Marshallian demand. So we can write Barbara's indirect utility function as $$v_B(\underline{p}, m_B) = u_B(\underline{x}^*_A(\underline{p}, m_B)) = \left(\frac{m_B^2}{p_1^2 + p_2^2}\right)^{-3/2}$$	Seen 1 1
Part (d)	We replace $(\underline{p}, m_B)$ with $(t\underline{p}, tm_B)$ in $v_B(\underline{p}, m_B)$: $$v_B(t\underline{p}, tm_B) = \left(\frac{t^2m_B^2}{t^2p_1^2 + t^2p_2^2}\right)^{-3/2} = \left(\frac{m_B^2}{p_1^2 + p_2^2}\right)^{-3/2}$$ $$\Rightarrow v_B(t\underline{p}, tm_B) = v_B(\underline{p}, m_B)$$ and so v_B is homogeneous of degree 0 in $(\underline{p}, m_B)$, as required.	Seen 2
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<tr><td>Question 1</td><td></td><td>Marks & seen/unseen</td></tr>
<tr><td>Part (e)(i)</td><td>We know that Barbara's Marshallian demand is linked to her Hicksian demand through the identity

$$x^*_{B,1}(\underline{p}, m_B) \equiv x^*_{H,B,1}(\underline{p}, v_B(\underline{p}, m_B))$$

$$\Rightarrow \frac{\partial x^*_{B,1}(\underline{p}, m_B)}{\partial m_B} = \frac{\partial x^*_{H,B,1}(\underline{p}, u)}{\partial u} \frac{\partial v_B(\underline{p}, m_B)}{\partial m_B}$$

We also know that the Hicksian demand function for good 1 is simply the derivative of the expenditure function with respect to p_1:

$$x^*_{H,B,1}(\underline{p}, u) = \frac{\partial e_B(\underline{p}, u)}{\partial p_1}$$

$$\Rightarrow \frac{\partial x^*_{B,1}(\underline{p}, m_B)}{\partial m_B} = \frac{\partial^2 e_B(\underline{p}, u)}{\partial p_1 \partial u} \frac{\partial v_B(\underline{p}, m_B)}{\partial m_B} = -3\frac{\left(p_1^2 + p_2^2\right)^{3/2}}{m_B^4} \frac{\partial^2 e_B(\underline{p}, u)}{\partial p_1 \partial u}.$$</td><td>Seen similar

2

2</td></tr>
<tr><td>Part (e)(ii)</td><td>Good 1 will be a luxury good for Barbara if and only if her income elasticity of demand is greater than 1, i.e.

$$\frac{\partial x^*_{B,1}}{\partial m_B} \frac{m_B}{x^*_{B,1}} > 1$$

And we also have that Barbara and Alfred have the same Marshallian demand:

$$\Rightarrow x^*_{B,1}(\underline{p}, m_B) = \frac{m_B p_1}{p_1^2 + p_2^2}$$

We can combine these results with the result in part (e)(i) to express the luxury good condition in terms of the expenditure function:

$$\frac{\partial x^*_{B,1}}{\partial m_B} > \frac{x^*_{B,1}}{m_B} \Leftrightarrow \frac{\partial^2 e_B(\underline{p}, u)}{\partial p_1 \partial u} < \frac{p_1}{p_1^2 + p_2^2}\left(-\frac{1}{3}\frac{m_B^4}{\left(p_1^2 + p_2^2\right)^{3/2}}\right)$$

$$\Leftrightarrow \frac{\partial^2 e_B(\underline{p}, u)}{\partial p_1 \partial u} < -\frac{m_B^4 p_1}{3\left(p_1^2 + p_2^2\right)^{5/2}}$$</td><td>Seen similar
2

1

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Question 2 — Marks & seen/unseen

Part (a)

Factors 1 and 2 are perfect substitutes for one another — 1, Seen
Factors 3 and 4 are perfect substitutes for one another
The factor pairs (1,2) and (3,4) are perfect complements, and should be used in fixed proportions — 1

Part (b)

Define — Seen

$$MRTS(x_1, x_2) = -\frac{\partial f_2(\underline{x})}{\partial x_1} \bigg/ \frac{\partial f_2(\underline{x})}{\partial x_2}$$

$$= -\frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}} = -\frac{a x_2}{b x_1}$$ — 2

Part (c)

Define the profit function: — Seen

$$\pi(\underline{x}, p, \underline{w}) = p x_1^a x_2^b - w_1 x_1 - w_2 x_2$$ — 1

The first-order conditions for maximising $\pi(\underline{x}, p, \underline{w})$ are

$$\frac{\partial f_2(\underline{x})}{\partial x_i} = \frac{w_i}{p}, \qquad i = 1, 2$$

$$\Rightarrow \qquad a x_1^{a-1} x_2^b = \frac{w_1}{p}, \qquad b x_1^a x_2^{b-1} = \frac{w_2}{p}$$ — 2

which can be solved to yield the factor demand functions:

$$x_1^*(p, \underline{w}) = \left[p w_1^{b-1} w_2^{-b} a^{1-b} b^b\right]^{\frac{1}{1-a-b}}$$
$$x_2^*(p, \underline{w}) = \left[p w_1^{-a} w_2^{a-1} a^a b^{1-a}\right]^{\frac{1}{1-a-b}}$$ — 2

The firm's supply function $y^*(p, \underline{w})$ is then given by

$$y^*(p, \underline{w}) = \{x_1^*(p, \underline{w})\}^a \{x_2^*(p, \underline{w})\}^b$$

$$= \left[p^{a+b} w_1^{-a} w_2^{-b} a^a b^b\right]^{\frac{1}{1-a-b}}$$ — 1

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	# EXAMINATION SOLUTIONS 2015-16	Course M3B
Question 2		Marks & seen/unseen
Part (d)	The maximised profit function $\pi^*(p, \underline{w})$ is: (any 4 of 5)	Seen
	...nondecreasing in p	
	...nonincreasing in $\underline{w}$	
	...homogeneous of degree 1 in $(p, \underline{w})$	
	...convex in $(p, \underline{w})$	
	...continuous in $(p, \underline{w})$	4
Part (e)	Define	Seen similar
	$h(p) = \pi_L^*(p, \underline{w}) - \pi_S^*(p, \underline{w}, \underline{x}_F(p', \underline{w}))$	
	The factors that a firm may adjust in the long-run include all those that may also be adjusted in the short-run. The long-run profit-maximising position will therefore always yield profits that are at least as large as those in the short-run profit-maximising position.	
	So, $h(p) \geq 0$	1
	We can relate the maximum profits in the long-run and the short-run, in particular for output price p':	
	$\pi_L^*(p', \underline{w}) = \pi_S^*(p', \underline{w}, \underline{x}_F^*(p', \underline{w}))$	
	So $h(p') = 0$	1
	Since p' therefore minimises $h(p)$, we have that $\frac{d^2h}{dp^2} \geq 0$ at $p = p'$	1
	$\Rightarrow \frac{\partial^2 \pi_L^*(p, \underline{w})}{\partial p^2} - \frac{\partial^2 \pi_S^*(p, \underline{w}, \underline{x}_F(p', \underline{w}))}{\partial p^2} \geq 0$ at $p = p'$	
	By Hotelling's Lemma, we also have that the output supply can be written as the price-derivative of the maximised profit function:	Seen
	$y^*(p, \underline{w}) = \frac{\partial \pi^*(p, \underline{w})}{\partial p}$,	1
	which we can combine with the above to yield the Le Chatelier principle:	Unseen
	$\Rightarrow \frac{\partial^2 y_L^*(p, \underline{w})}{\partial p} - \frac{\partial y_S^*(p, \underline{w}, \underline{x}_F(p', \underline{w}))}{\partial p} \geq 0$ at $p = p'$	2
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	EXAMINATION SOLUTIONS 2015-16	Course M3B
Question 3		Marks & seen/unseen

Part (a) — Seen

The long-run supply function for a firm is found by setting price equal to (the increasing part of) the marginal cost function, over and above the average costs.

So for firm $i \in \{1,2,3\}$, denoting the price of coffee by p_C, we have

$$p_C = MC_i(y) \text{ when } MC_i(y) \geq AC_i(y). \qquad 1$$

Now, for $i \in \{1,2,3\}$,

$$MC_i(y) = \frac{\partial c_i^*(\underline{w}, y)}{\partial y} = \frac{5}{i}(w_1 + w_2)y$$

$$AC_i(y) = \frac{c_i^*(\underline{w}, y)}{y} = \frac{2.5}{i}(w_1 + w_2)y + \frac{w_1 w_2}{y} \qquad 1$$

so we can deduce that $p = MC_i(y) \geq AVC_i(y)$ for

$$p_C \geq \left[\frac{10}{i} w_1 w_2 (w_1 + w_2)\right]^{1/2}, \qquad 1$$

and we can obtain the long-run supply functions for the three firms:

$$y_1^*(p_C, \underline{w}) = \frac{p_C}{5(w_1 + w_2)}, \qquad p_C \geq \sqrt{10 w_1 w_2 (w_1 + w_2)}$$

$$y_2^*(p_C, \underline{w}) = \frac{2p_C}{5(w_1 + w_2)}, \qquad p_C \geq \sqrt{5 w_1 w_2 (w_1 + w_2)}$$

$$y_3^*(p_C, \underline{w}) = \frac{3p_C}{5(w_1 + w_2)}, \qquad p_C \geq \sqrt{10 w_1 w_2 (w_1 + w_2)/3} \qquad 1$$

Part (b) — Seen

The market demand function $X^*(p_C, p_T, m_1, \ldots, m_{n_C})$ is found by summing the individual consumers' demand functions:

$$X^*(p_C, p_T, m_1, \ldots, m_{n_C}) = \sum_{i=1}^{n_C} x_j^*(p_C, p_T, m_j)$$

$$\Rightarrow X^*(p_C, p_T, M) = \frac{M p_C}{p_C^2 + p_T}, \qquad 1$$

where M is the sum of the consumers' budgets.

Similarly, the market supply function $Y^*(p_C, \underline{w})$ is found by summing the individual firms' supply functions:

$$Y^*(p_C, \underline{w}) = \frac{1.2 p_C}{w_1 + w_2} \qquad 1$$

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	EXAMINATION SOLUTIONS 2015-16	Course M3B
Question 3		Marks & seen/unseen
Part (b) (cont.)	To find the equilibrium price of coffee, $p^{(1)}_{C,0}$, we set market supply equal to market demand and solve:	1
	$\dfrac{Mp_C}{p_C^2 + p_T} = \dfrac{1.2p_C}{w_1 + w_2}$	
	yields a cubic in p_C, which we can solve to give	
	$p^{(1)}_{C,0} = 0,$ or $\pm\sqrt{\dfrac{M(w_1 + w_2) - 1.2p_T}{1.2}}$	1
	since $p_C \geq 0$, and $p^{(0)}_{C,0} = 0$ corresponds to all firms going out of business, we take	1
	$p^{(1)}_{C,0} = \sqrt{\dfrac{M(w_1 + w_2) - 1.2p_T}{1.2}}.$	1
Part (c)	The subsidy will increase market supply for fixed prices and decrease prices for fixed market supply; the market supply curve will shift down and to the right on a standard supply and demand plot.	Seen 1
	The subsidy will not effect the market demand function (or its curve).	1
Part (d)(i)	The post-subsidy market supply function is given by scaling the price by 0.8; to keep the level of supply constant, the firm can reduce its prices to 80% of their previous value.	Seen similar
	$Y^{(2)*}(p_C, \underline{w}) = \dfrac{1.2}{(w_1 + w_2)}\dfrac{p_C}{0.8} = \dfrac{1.5p_C}{w_1 + w_2}$	2
	and following the same approach as in part (b) will yield a new equilibrium price of	
	$p^{(2)}_{C,0} = \sqrt{\dfrac{M(w_1 + w_2) - 1.5p_T}{1.5}}.$	2
Part (d)(ii)	Evaluating the equilibrium prices $p^{(1)}_{C,0}$, $p^{(2)}_{C,0}$ at the given parameter values,	
	$p^{(1)}_{C,0} = 2$ and $p^{(2)}_{C,0} = \sqrt{3}.$	2
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	EXAMINATION SOLUTIONS 2015-16	Course M3B
Question 3		Marks & seen/unseen
Part (d)(ii)	The quantity sold at the post-subsidy equilibrium point can be found by substituting $p_{C,0}^{(2)}$ into the post-subsidy market supply $Y^{*(2)}$: $$Y^{(2)^*}(p_{C,0}^{(2)}, \underline{w}) = \frac{1.5 \times p_{C,0}^{(2)}}{0.06} = 25\sqrt{3}$$	1
	Now, the benefit to the consumer is calculated as the product of the change in prices and the post-subsidy quantity traded: $$\text{Consumer benefit} = (p_{C,0}^{(1)} - p_{C,0}^{(2)}) \times Y^{(2)^*}(p_{C,0}^{(2)}, \underline{w}).$$ $$= 50\sqrt{3} - 75.$$	1
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	EXAMINATION SOLUTIONS 2015-16	Course M3B
Question 4		Marks & seen/unseen
Part (a)	Define the K-firm concentration ratio: $$CR_K = \frac{\sum_{i=1}^{K} y^{(i)}}{\sum_{i=1}^{n_F} y^{(i)}}$$ for a market comprising n_F firms, where $y^{(i)}$ is the output of the i^{th}-most-productive firm.	Seen 1
	We should use CR_4 for industry A: $$CR_4^{(A)} = \frac{17}{20} = 0.85.$$	1
	This level of CR_4 indicates that the market A is an oligopoly.	1
Part (b)	The 4-firm concentration ratios for markets A and B are: $CR_4^{(A)} = 0.85$ (from part (a)) and $CR_4^{(B)} = 0.85$	Unseen 1
	The Herfindahl indices for markets A and B are: $H^{(A)} = 0.18875$ (given), and $$H^{(B)} = \frac{49 + 0.25 + 0.25 + 0.25 + 0.25 + 0.25 + 0.25}{100} = 0.505$$	1
	The 4-firm concentration ratios indicate that the same level of competition exists in industries A and B.	1
	But industry B clearly has a much lower level of competition than industry A.	1
	The Herfindahl index is able to pick up this discrepancy	1
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	EXAMINATION SOLUTIONS 2015-16	Course M3B
Question 4		Marks & seen/unseen
Part (c)	The inverse demand function facing the firm is found by setting output equal to the market demand function and inverting:	Seen
	$$y_B = a_1 - a_2 p_B \Rightarrow p_B = \frac{a_1 - y_B}{a_2}$$	1
	For a monopolist with minimised costs, a profit maximising position will only exist in the short-run if they face elastic demand, i.e. for price elasticity of demand ϵ_D,	
	$$\lvert \epsilon_D \rvert = \left\lvert \frac{\partial y_B}{\partial p_B} \frac{p_B}{y_B} \right\rvert \geq 1 \qquad \text{or} \qquad \lvert \epsilon_D^{-1} \rvert = \left\lvert \frac{\partial p_B}{\partial y_B} \frac{y_B}{p_B} \right\rvert \leq 1$$	2
	where the second expression gives the elasticity of the inverse demand function with respect to output. Candidates may use either expression; I use the first expression here.	
	Now, $$\frac{\partial y_B}{\partial p_B} \frac{p_B}{y_B} = -\frac{a_2 p_B}{a_1 - a_2 p_B} \leq 0, \text{ as } \epsilon_D \leq 0 \text{ always}$$ $$\Rightarrow \left\lvert \frac{\partial y_B}{\partial p_B} \frac{p_B}{y_B} \right\rvert = \frac{a_2 p_B}{a_1 - a_2 p_B}$$ $p_B \geq 0$ as it is a price, so the numerator, and therefore also the denominator, are both ≥ 0	
	So $$\left\lvert \frac{\partial y_B}{\partial p_B} \frac{p_B}{y_B} \right\rvert \geq 1 \Rightarrow p_B \geq \frac{a_1}{2a_2}$$	
	and using the inverse demand facing the firm,	
	$$\frac{a_1 - y_B}{a_2} \geq \frac{a_1}{2a_2} \Rightarrow y_B \leq \frac{a_1}{2} \text{ as required.}$$	3
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	EXAMINATION SOLUTIONS 2015-16	Course M3B
Question 4		Marks & seen/unseen
Part (d)	Unconstrained maximisation problem: $$\text{find } \max_{y_B \geq 0} \{p_B(y_B)y_B - c^*(\underline{w}, y_B)\}.$$	Seen
	FOCs for maximisation: $$p_B(y_B)\left[1 + \frac{1}{\epsilon_D(y_B)}\right] = SMC(y_B)$$	2
	Now, the short-run marginal costs $SMC(y_B)$ are $$\frac{\partial c^*(\underline{w}, y_B)}{\partial y_B} = 4y_B\sqrt{w_1 w_2}$$	1
	and the second term in the bracket is the inverse of the price elasticity of demand, which we require in terms of y_B: $$\epsilon_D^{-1}(y_B) = \frac{\partial p_B}{\partial y_B}\frac{y_B}{p_B} = -\frac{y_B}{a_1 - y_B}$$	1
	and substituting these, along with the inverse demand function facing the firm, into our FOCs yields the profit-maximising output: $$y_B^* = \frac{a_1}{2 + 4a_2\sqrt{w_1 w_2}}$$	2
	(and this is $\leq \frac{a_1}{2}$, as required).	
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