

MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Quiz 8

This is an assessed quiz. The quiz will open on Friday, 15 March at 10am
and close on Saturday, 16 March at 10.30am.

Question 1 (Linear decomposition).

Consider a linear differential equation $\dot{x} = Ax$, where $A \in \mathbb{R}^{d \times d}$. Is the following statement true? The stable set $W^s(0, 0)$ and unstable set $W^u(0, 0)$ form a linear decomposition of \mathbb{R}^d , i.e.

$$W^s(0, 0) \oplus W^u(0, 0) = \mathbb{R}^d.$$

- (a) The statement is true.
- (b) The statement is false.

Question 2 (Intersection of stable and unstable set).

Let $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$ be continuously differentiable, and consider the differential equation $\dot{x} = f(x)$ with an equilibrium $x^* \in \mathbb{R}^d$. Is it true that $W^s(x^*) \cap W^u(x^*) = \{x^*\}$?

- (a) The statement is true.
- (b) The statement is false.

Question 3 (Invariant sets I).

Let $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$ be continuously differentiable, and consider the differential equation $\dot{x} = f(x)$ with a positively invariant set A and a negatively invariant set B . Is it true that the set $A \setminus B$ is positively invariant?

- (a) The statement is true.
- (b) The statement is false.

Question 4 (Invariant sets II).

Let $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$ be continuously differentiable, and consider the differential equation $\dot{x} = f(x)$ with a negatively invariant set A and a positively invariant set B . Is it true that the set $A \setminus B$ is negatively invariant?

- (a) The statement is true.
- (b) The statement is false.

Question 5 (Omega limit sets).

Is the following statement true or false? There exists a differential equation $\dot{x} = f(x)$, where $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$ is continuously differentiable, and an $x \in \mathbb{R}^d$ such that $\omega(x) = \alpha(x)$ and $x \notin \omega(x)$.

- (a) The statement is true.
- (b) The statement is false.