

MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Solutions to Quiz 8

Question 1. Correct answer: (b).

This is false. Consider the zero matrix $A = (0) \in \mathbb{R}^{1 \times 1}$. Then $W^s(0) = W^u(0) = \{0\} \neq \mathbb{R}$.

Question 2. Correct answer: (b).

This is false. Consider an equilibrium that is limit of a homoclinic orbit, e.g. as given in Example 4.3 (ii).

Question 3. Correct answer: (a).

This is true. Let $x \in A \setminus B$ and $t \geq 0$. Then $\varphi(t, x) \in A$ and assume that $\varphi(t, x) \in B$, which implies that there exists a $y \in B$ with $\varphi(t, x) = y$, and thus, $\varphi(-t, y) = x$. This implies that B is not negatively invariant, which is a contradiction.

Question 4. Correct answer: (a).

This is true. Let $x \in A \setminus B$ and $t \leq 0$. Then $\varphi(t, x) \in A$ and assume that $\varphi(t, x) \in B$, which implies that there exists a $y \in B$ with $\varphi(t, x) = y$, and thus, $\varphi(-t, y) = x$. This implies that B is not positively invariant, which is a contradiction.

Question 5. Correct answer: (a).

This is true. For any homoclinic orbit $O(x)$, there exists an equilibrium $y \in \mathbb{R}^d$ such that $\omega(x) = \alpha(x) = \{y\}$. x is not an equilibrium, so $x \notin \omega(x)$.