Writing Full-Mark Definitions

Check that you have defined EVERY new symbol
- especially m, n as natural numbers

check you have written the DOMAIN, CODOMAIN of every function in the definition

Integral domain must be **nonzero, commutative** ring

PID, UFD, ED: all require the ring to be ID
- Noetherian ring: no requirement for being ID

Euclidean domain: careful with 0 case
- Euclidean function ϕ has domain $R \setminus \{0\}$
- Euclidean division: **$b \neq 0$**, $a = bq + r$, with $\phi(r) < \phi(b)$ or **$r = 0$**

localisation on integral domain: the submonoid S cannot contain 0
localisation on general commutative ring: the submonoid S can contain 0
- don't forget to check the relations defined are equivalence relation
- don't forget to define operations and prove they are well-defined.

Well-defined
When giving definitions with operations, functions on cosets of subgroup, ideals, submodules, must prove they are WELL-DEFINED
- e.g. operations on R/I
- in general, has to be proven for definitions on any set of equivalence classes
  - e.g. maps defined on local rings

G/H, R/I are only well-defined if H, I are normal subgroup, ideal respectively
- no restriction for M/N (modules)

something can be well-defined up to minor change
- e.g. gcd well-defined up to multiplication by unit

Equivalence of two definitions
definitions with explicit constructions: establish one-to-one correspondence

- if you have definition (a), you can construct the object required in definition (b), vice versa
- show these constructions are inverses to each other

explicit vs property(defined by some object that satisfies given properties):

- prove the explicit construction in definition (a) satisfies properties in definition (b)
- prove that (a) is the unique object satisfying properties in definition (b)