



1. Consider the “triangle graph” shown above. Solve the linearized naive network SI model for this graph with  $\beta = 1$  and  $\langle x_1(t=0) \rangle = 1$  and  $\langle x_2(t=0) \rangle = \langle x_3(t=0) \rangle = 0$
2. Consider the naive network-SI model on an  $N$ -node complete graph. Derive an approximate nonlinear ODE for  $\bar{x} = \frac{1}{N} \sum_{i=1}^N \langle x_i \rangle$  when  $N$  is large (the accuracy of the approximation should increase as  $N$  increases)
3. Consider a “cloud” of particles undergoing random walks on a simple  $N$ -node graph. Let  $n_i(t)$  and  $n_j(t)$  be the number of particles on nodes  $i$  and  $j$  at time  $t$ . Given  $n_i(t)$  and  $n_j(t)$ , what is the expected net flux of particles from node  $j$  to  $i$  between  $t$  and  $t + \Delta t$ . In other words, how many more particles are expected to move from node  $j$  to node  $i$  than from  $i$  to  $j$  during a time step? Give your answer in terms of  $n_i(t)$ ,  $n_j(t)$ ,  $k_i$ , and  $k_j$ . How does your result compare to the net flux of particles associated with diffusion on a graph? (It may be helpful to work with an indicator random variable,  $X_{ija}$ , which is 1 if particle  $a$  on node  $j$  moves to  $i$  and is zero otherwise.)
4. Consider the following definition of a step of a random walk on a connected directed  $N$ -node graph with adjacency matrix  $\mathbf{A}$ . Each step begins with a Bernoulli trial with probability of success,  $\pi$ . If successful, the walker chooses an outward link uniformly at random and follows it to a neighboring node. If unsuccessful, the walker finds itself picked up and “dropped” back onto the graph which we model as selecting a node uniformly at random.
  - (a) What is the master equation for this process? Let  $p_i^{(l+1)}$  be the probability a walker is at node  $i$  after  $l + 1$  steps. You should derive an equation relating  $p_i^{(l+1)}$  to the elements in the probability vector at iteration  $l$ , as well as  $N$ ,  $\mathbf{A}$ ,  $\pi$  and the node degrees (e.g.  $k_i^{out}$ ).
  - (b) Interpret the equation for the stationary state as a node centrality. What is the role of the probability,  $\pi$ ?

5. Consider the following modification to the network-SI model: the probability that an infectious node becomes susceptible in time  $\Delta t$  is  $\gamma\Delta t$ . Taking this modification into account, and applying the limit  $\Delta t \rightarrow 0$ , derive the resulting system of ODEs known as the *network-SIS model*