Imperial College
London

**M3E**

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)

May-June 2017

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Econometric Theory and Methods

Date: Wednesday 17 May 2017

Time: 14:00 - 16:00

Time Allowed: 2 Hours

**This paper has 4 Questions.**

Candidates should use ONE main answer book.

Supplementary books may only be used after the relevant main book(s) are full.

All required additional material will be provided.

- DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO.
- Affix one of the labels provided to each answer book that you use, but DO NOT USE THE LABEL WITH YOUR NAME ON IT.
- Credit will be given for all questions attempted, but extra credit will be given for complete or nearly complete answers to each question as per the table below.

| Raw Mark | Up to 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|
| Extra Credit | 0 | ½ | 1 | 1 ½ | 2 | 2 ½ | 3 | 3 ½ | 4 |

- Each question carries equal weight.
- Calculators may not be used.

1. Is the underlined statement in each of (i)-(v) true or false? Please explain your answers.

   (i) Let $(Y_i, \mathbf{X}_i)_{i=1}^n$ be i.i.d. copies of the random couple $(Y, \mathbf{X})$, whose relationship is described by the linear regression model $Y = \mathbf{X}^T\beta_0 + \epsilon$, where $\epsilon$ satisfies $\mathbb{E}[\epsilon|\mathbf{X} = \mathbf{x}] = 0$ for all $\mathbf{x}$. $Y$ is a scalar valued random variable and $\mathbf{X}$ is a random vector. The OLS estimator fails to exist if the dimension of $\mathbf{X}$ is larger than the sample size $n$.

   (ii) Consider the simple linear regression model, $Y = \alpha_0 + \beta_0 X + \epsilon$, where $\epsilon$ is normally distributed with mean zero and variance $\sigma^2$, independently of $X$. Both $Y$ and $X$ are scalar valued random variables.
   The square of the OLS estimator, $\widehat{\beta}$, based on $n$ i.i.d. copies of $(Y, X)$ is an unbiased estimator of $\beta_0^2$.

   (iii) Let $(Y_i, \mathbf{X}_i, \mathbf{Z}_i)_{i=1}^n$ be i.i.d. copies of the random triple $(Y, \mathbf{X}, \mathbf{Z})$, whose relationship is described by the linear regression model $Y = \mathbf{X}^T\beta_0 + \mathbf{Z}^T\gamma_0 + \epsilon$, where $\epsilon$ satisfies $\mathbb{E}[\epsilon|\mathbf{X} = \mathbf{x}] = 0$ for all $\mathbf{x}$. $Y$ is a scalar random variable and $\mathbf{X}$ and $\mathbf{Z}$ are random vectors of arbitrary length. Suppose the variables $\mathbf{Z}$ are ignored. The OLS estimator $\widehat{\beta} = (\sum_i \mathbf{X}_i\mathbf{X}_i)^{-1}\sum_i \mathbf{X}_i Y_i$ is biased for $\beta_0$.

   (iv) Let $(Y_i, \mathbf{X}_i)_{i=1}^n$ be i.i.d. copies of the random couple $(Y, \mathbf{X})$, whose relationship is described by the linear regression model $Y = \mathbf{X}^T\beta_0 + \epsilon$. $Y$ is a scalar valued random variable and $\mathbf{X}$ is a random vector. Let $\epsilon = (\epsilon_1, \ldots, \epsilon_n)^T$, let $\mathbf{Y} = (Y_1, \ldots, Y_n)^T$ and let $X$ be the $n \times p$ matrix whose rows are $\mathbf{X}_i^T$. When $\epsilon$ satisfies $\mathbb{E}[\epsilon|X = x] = 0$ for all $x$ but there exists an $x$ such that $\mathbb{E}[\epsilon\epsilon^T|X = x] = \Omega(x) \neq \mathsf{I}_n$, the OLS estimator $\widehat{\beta} = (X^TX)^{-1}X^T\mathbf{Y}$ is a biased estimator of $\beta_0$.

2. Consider the linear model $Y = \mathbf{X}^T\boldsymbol{\beta}_0 + \epsilon$ where $\mathbf{X}$ is a $p \times 1$ random vector, $\beta_0$ is a $p \times 1$ vector and $Y$ and $\epsilon$ are scalar random variables. Given i.i.d. random couples $\{Y_i, \mathbf{X}_i^T\}_{i=1}^n$, we write the linear regression model as $\mathbf{Y} = X\beta_0 + \epsilon$, where $X$ is an $n \times p$ random matrix with $i^{\text{th}}$ row $\mathbf{X}_i^T$ and $\mathbf{Y}$ and $\epsilon$ are $n \times 1$ random vectors with $i^{\text{th}}$ entries $Y_i$ and $\epsilon_i$ respectively.

   (a) Let $Z = XA$, where $A$ is a $p \times p$ nonsingular matrix of constants.

      (i) Let $\widehat{\beta}_Z$ denote the OLS coefficient estimator from the regression of $\mathbf{Y}$ on $Z$ and let $\widehat{\beta}_X$ denote the OLS coefficient estimator from the regression of $\mathbf{Y}$ on $X$. Show that $\widehat{\beta}_Z = A^{-1}\widehat{\beta}_X$.

      (ii) Show that the vector of residuals from an OLS regression of $\mathbf{Y}$ on $Z$ are the same as those from an OLS regression of $\mathbf{Y}$ on $X$.

   (b) Let $W$ be an $n \times 3$ matrix whose $i^{\text{th}}$ row is given by $\mathbf{W}_i = (1, X_{i,1}, X_{i,2})$ for $i = 1, \ldots, n$, and let $V$ be an $n \times 3$ matrix whose $i^{\text{th}}$ row is given by $\mathbf{V}_i = (1, X_{i,1} + X_{i,2}, X_{i,2})$ for $i = 1, \ldots, n$. Compare the OLS coefficient estimates and residuals from the OLS regression of $\mathbf{Y}$ on $W$ to those of the OLS regression of $\mathbf{Y}$ on $V$.

3. (a) Let $f_n$ be a positive nonstochastic sequence and let $T_n$ be a sequence of random variables. We write $T_n = O_{\mathbb{P}}(f_n)$ if for all $\delta > 0$, there exists a $C < \infty$ and a $0 < n_0 < \infty$ such that $\mathbb{P}(|T_n| > Cf_n) < \delta$ for all $n > n_0$.

   (i) Suppose that $V_n = O_{\mathbb{P}}(v_n)$ and $W_n = O_{\mathbb{P}}(w_n)$. Prove that $V_nW_n = O_{\mathbb{P}}(v_nw_n)$.

   (ii) Define what is meant by $W_n = o_{\mathbb{P}}(w_n)$.

   (iii) If $V_n = O_{\mathbb{P}}(v_n)$ and $W_n = o_{\mathbb{P}}(w_n)$, what can we say about $V_nW_n$? A proof is not required.

   (b) Let $Y_i = \phi_0 Y_{i-1} + \epsilon_i$, where $|\phi| < 1$ and $\{\epsilon_i\}$ is an independent and identically distributed sequence with mean zero and variance $\sigma^2$. What is the limit distribution of $\frac{1}{\sqrt{n}}\sum_{i=1}^{n} Y_i$? **Hint:** You may assume without proof that $Y_0 = O_{\mathbb{P}}(1)$ and $Y_n = O_{\mathbb{P}}(1)$

4. (a) (i) What is the relationship between convergence in probability and convergence in distribution?

   (ii) Let $g$ be a real valued continuous function. Does $T_n \to_p T$ imply $g(T_n) \to_d g(T)$? Justify your answer.

   (b) Let $\beta_0$ be a scalar parameter of interest and let $\widehat{\beta}$ be a consistent estimator of $\beta_0$ satisfying $\sqrt{n}(\widehat{\beta} - \beta_0) \to_d N(0, s_0)$. Use the mean value theorem and the properties of normal distributions to explain why $\sqrt{n}(e^{\widehat{\beta}} - e^{\beta_0}) \to_d N(0, (g(\beta_0))^2 s_0)$. Be explicit about the form of $g(\beta_0)$.

   (c) Suppose now that $Y_i = \alpha_0 + \beta_{0,1}X_{i,1} + \beta_{0,2}X_{i,2} + \epsilon_i$. Let $\beta_0 = (\alpha_0, \beta_{0,1}, \beta_{0,2})^T$ and let $\widehat{\boldsymbol{\beta}} = (\widehat{\alpha}, \widehat{\beta}_1, \widehat{\beta}_2)^T$ be an estimator of $\boldsymbol{\beta}_0$ such that $\sqrt{n}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0) \to N(\mathbf{0}, \Sigma)$, where

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{pmatrix}.$$

   What is the limit distribution of $\sqrt{n}\left(\frac{\widehat{\beta}_1}{\widehat{\beta}_2} - \frac{\beta_{0,1}}{\beta_{0,2}}\right)$?

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|---|---|
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| Checker: | Pakkanen |
| Editor: | Walden |
| External: | Jennison |
| Date: | March 8, 2017 |

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)

May – June 2017

# M3E (Solutions)

# Econometric Theory and Methods (Solutions)

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Imperial College London



BSc, MSc and MSci EXAMINATIONS (MATHEMATICS)

May – June 2017

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science.

# Econometric Theory and Methods (Solutions)

Date: ??

Time: ??

Time Allowed: 2 Hours

This paper has *4* Questions.

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1. [**Mostly seen**, 1 mark each for correct answer and 4 marks each for justification]

(i) True. The OLS estimator is $\widehat{\beta} = (X^T X)^{-1} X^T \mathbf{Y}$ where $\mathbf{Y} = (Y_1, \ldots, Y_n)^T$ and $X$ is the matrix with $i^{\text{th}}$ row $\mathbf{X}_i^T$. If the number of observations is smaller than the dimension of $\mathbf{X}_i$, i.e. if the number of columns of $X$ is larger than the number of rows, $X^T X$ is not invertible and therefore $\widehat{\beta}$ does not exist.

(ii) False. For any $x$

$$\begin{aligned}
\mathbb{E}[\widehat{\beta}^2 | X = x] &= \mathbb{E}\Big[\Big(\widehat{\beta} - \mathbb{E}[\widehat{\beta}|X = x] + \mathbb{E}[\widehat{\beta}|X = x]\Big)^2 \Big| X = x\Big] \\
&= \mathbb{E}\Big[\Big(\widehat{\beta} - \mathbb{E}[\widehat{\beta}|X = x]\Big)^2 |X = x\Big] + \mathbb{E}\Big[(\mathbb{E}[\widehat{\beta}|X = x])^2 \Big| X = x\Big] \\
&\quad + 2\mathbb{E}\Big[\Big(\widehat{\beta} - \mathbb{E}[\widehat{\beta}|X = x]\Big)\mathbb{E}[\widehat{\beta}|X = x]\Big| X = x\Big] \\
&= \text{Var}(\widehat{\beta}|X = x) + \beta_0^2 > \beta_0^2.
\end{aligned}$$

(iii) True. In matrix notation, $\widehat{\beta} = \beta_0 + (X^T X)^{-1} X^T Z \gamma_0 + (X^T X)^{-1} X^T \epsilon$. This is not equal to $\beta_0$ in expectation unless $X^T Z = 0$.

(iv) False. The OLS estimator is unbiased but inefficient in this case. Unbiasedness should be demonstrated.

2. (a) (i) [**seen**, 6 marks] In a multiple regression model the estimated parameter vector is $\widehat{\beta}_X = (X^T X)^{-1} X^T \mathbf{Y}$ and the residuals are $\widehat{\epsilon} = \mathbf{Y} - X\widehat{\beta}_X$. Regressing $\mathbf{Y}$ on $\mathbf{Z}$, the corresponding OLS estimator is

$$\widehat{\beta}_Z = (\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{Y} = ((XA)^T(XA))^{-1}(XA)^T\mathbf{Y} = (A^T X^T XA)^{-1}A^T X^T\mathbf{Y}.$$

Both $A$ and $X^T X$ are square $(k \times k)$ and nonsingular. If $\mathbf{M}, \mathbf{N}, \mathbf{P}$ are square and nonsingular, then $(\mathbf{MNP})^{-1} = \mathbf{P}^{-1}\mathbf{N}^{-1}\mathbf{M}^{-1}$ therefore $\widehat{\beta}_Z$ becomes

$$\widehat{\beta}_Z = A^{-1}(X^T X)^{-1}(A^T)^{-1}A^T X^T\mathbf{Y} = A^{-1}(X^T X)^{-1}X^T\mathbf{Y} = A^{-1}\widehat{\beta}_X\,.$$

There is a linear relation between the coefficient estimated in the two regressions.

(ii) [**seen**, 4 marks] The residuals are

$$\widehat{\epsilon}_Z = \mathbf{Y} - \mathbf{Z}\widehat{\beta}_Z = \mathbf{Y} - (XA)(A^{-1}\widehat{\beta}_X) = \mathbf{Y} - X\widehat{\beta}_X = \widehat{\epsilon}\,.$$

(b) [**seen similar**, 10 marks]

$$\begin{bmatrix} 1 & X_{1,1} & X_{2,1} \\ 1 & X_{1,2} & X_{2,2} \\ \vdots & \vdots & \vdots \\ 1 & X_{1,n} & X_{2,n} \end{bmatrix} A = \begin{bmatrix} 1 & X_{1,1} + X_{2,1} & X_{2,1} \\ 1 & X_{1,2} + X_{2,2} & X_{2,2} \\ \vdots & \vdots & \vdots \\ 1 & X_{1,n} + X_{2,n} & X_{2,n} \end{bmatrix}$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

so

$$\widehat{\beta}_V = A^{-1}\widehat{\beta}_W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \widehat{\beta}_{W,1} \\ \widehat{\beta}_{W,2} \\ \widehat{\beta}_{W,3} \end{bmatrix} = \begin{bmatrix} \widehat{\beta}_{W,1} \\ \widehat{\beta}_{W,2} \\ \widehat{\beta}_{W,3} - \widehat{\beta}_{W,2} \end{bmatrix}.$$

3. (a) (i) [**seen**, 9 marks] By the fact that $T_n = O_p(f_n)$ and $S_n = O_p(g_n)$ we know that, for any $\delta > 0$, there exist a $C, D < \infty$ and $n_1, n_2 > 0$ such that

$$\begin{aligned}\mathbb{P}(|T_n| > Cf_n) &< \frac{1}{2}\delta \quad \forall n \geq n_1 \\ \mathbb{P}(|S_n| > Dg_n) &< \frac{1}{2}\delta \quad \forall n \geq n_2.\end{aligned}$$

For all $n \geq \max\{n_1, n_2\} > 0$, we have

$$\begin{aligned} & \mathbb{P}(|T_n S_n| > CDf_n g_n) \\ \leq\ & \mathbb{P}(|T_n||S_n| > Cf_n Dg_n) \\ =\ & \mathbb{P}\Big(\Big\{\{|T_n||S_n| > Cf_n Dg_n\} \cap \{|S_n|/Dg_n > 1\}\Big\} \\ & \qquad \cup \Big\{\{|T_n||S_n| > Cf_n Dg_n\} \cap \{|S_n|/Dg_n \leq 1\}\Big\}\Big) \\ =\ & \mathbb{P}\Big(\Big\{|T_n||S_n| > Cf_n Dg_n\Big\} \cap \Big\{|S_n|/Dg_n > 1\Big\}\Big) \\ & \qquad + \mathbb{P}\Big(\Big\{|T_n||S_n| > Cf_n Dg_n\Big\} \cap \Big\{|S_n|/Dg_n \leq 1\Big\}\Big) \\ \leq\ & \mathbb{P}(|S_n| > Dg_n) + \mathbb{P}(|T_n| > Cf_n) < \delta. \end{aligned}$$

The final line follows because the probability of the joint event $\Big\{|T_n||S_n| > Cf_n Dg_n\Big\} \cap \Big\{|S_n|/Dg_n > 1\Big\}$ must be smaller than the probability of the single event $\{|S_n| > Dg_n\}$, whilst if events $A := \Big\{|T_n||S_n| > Cf_n Dg_n\Big\}$ and $B := \Big\{|S_n|/Dg_n \leq 1\Big\}$ both occur, a fortiori (replacing $|S_n|/Dg_n$ by 1), event $E := \Big\{|T_n| > Cf_n\Big\}$ occurs, i.e. $(A \cap B) \subseteq E$, thus $\mathbb{P}(A \cap B) \leq \mathbb{P}(E)$.

(ii) [**seen**, 1 mark] $W_n = o_{\mathbb{P}}(w_n)$ means that $W_n/w_n \to_p 0$.

(iii) [**seen**, 1 mark] $V_n W_n = o_{\mathbb{P}}(v_n w_n)$.

(b) [**seen proof strategy in a different context**, 9 marks] By the Lindeberg-Levy CLT for i.i.d. sequences, $\frac{1}{\sqrt{n}}\sum_{i=1}^n \frac{\epsilon_i}{\sigma} \to_d N(0,1)$. So re-writing, $\epsilon_i = Y_i - \phi Y_{i-1}$, $\frac{1}{\sqrt{n}}\sum_{i=1}^n (Y_i - \phi Y_{i-1}) \to_d N(0, \sigma^2)$. But

$$\begin{aligned}\frac{1}{\sqrt{n}}\sum_{i=1}^n (Y_i - \phi Y_{i-1}) &= \frac{1}{\sqrt{n}}\sum_{i=1}^n Y_i - \phi\frac{1}{\sqrt{n}}\sum_{i=1}^n Y_{i-1}) = \frac{1}{\sqrt{n}}\sum_{i=1}^n Y_i - \phi\frac{1}{\sqrt{n}}\sum_{i=0}^{n-1} Y_i \\ &= \frac{1-\phi}{\sqrt{n}}\sum_{i=1}^n Y_i - \phi\frac{1}{\sqrt{n}}Y_0 + \phi\frac{1}{\sqrt{n}}Y_n.\end{aligned}$$

Rearranging and noticing that $-\phi\frac{1}{\sqrt{n}}Y_0 = O_{\mathbb{P}}(n^{-1/2})$ and $\phi\frac{1}{\sqrt{n}}Y_n = O_{\mathbb{P}}(n^{-1/2})$, we obtain

$$\frac{1}{\sqrt{n}}\sum_{i=1}^n Y_i \to_d N\Big(0, \frac{\sigma^2}{(1-\phi)^2}\Big).$$

4. (a) (i) [**seen**, 2 marks] Convergence in probability implies convergence in distribution. The converse is not true.

   (ii) [**unseen**, 2 marks] Yes. By the continuous mapping theorem $T_n \to_p T$ implies $g(T_n) \to_p g(T)$, which in turn implies $g(T_n) \to_d g(T)$ because convergence in probability implies convergence in distribution.

(b) [**Proof strategy seen, context not seen**, 8 marks] For any continuously differentiable function $f$, the mean value theorem gives $f(\widehat{\beta}) = f(\beta_0) + (\widehat{\beta} - \beta_0)f'(\beta^*)$ where $\beta^*$ lies on a line segment between $\widehat{\beta}$ and $\beta_0$. This gives $\sqrt{n}(f(\widehat{\beta}) - f(\beta_0)) = f'(\beta^*)\sqrt{n}(\widehat{\beta} - \beta_0)$. Since $\widehat{\beta}$ is consistent and $\beta^*$ lies in a line segment between $\widehat{\beta}$ and $\beta_0$, $\beta^* \to_p \beta_0$ and by the continuous mapping theorem, provided $f$ has continuous derivatives, $f'(\widehat{\beta}) \to_p f'(\beta_0)$. By Slutsky's theorem, we obtain $f'(\beta^*)\sqrt{n}(\widehat{\beta} - \beta_0) \to_d N(0, f^2(\beta_0)s_0)$. Applying this in the context of the question, $\sqrt{n}(e^{\widehat{\beta}} - e^{\beta_0}) \to_d N(0, (g(\beta_0))^2 s_0)$, where $g(\beta_0) = e^{\beta_0}$.

(c) [**unseen**, 8 marks] Let

$$\Sigma_S = \begin{pmatrix} \Sigma_{22} & \Sigma_{23} \\ \Sigma_{32} & \Sigma_{33} \end{pmatrix},$$

$\beta_{S,0} = (\beta_{0,1}, \beta_{0,2})^T$ and $f(\beta_S) = \frac{\beta_1}{\beta_2}$. By the delta method,

$$\sqrt{n}\left(\frac{\widehat{\beta}_1}{\widehat{\beta}_2} - \frac{\beta_{0,1}}{\beta_{0,2}}\right) \to_d N\Big(0, \Big(\frac{\partial f}{\partial \beta_S}(\beta_{S,0})\Big)^T \Sigma_S \frac{\partial f}{\partial \beta_S}(\beta_{S,0})\Big)$$

where

$$\frac{\partial f}{\partial \beta_S}(\beta_{S,0}) = (1/\beta_{0,2}, -\beta_{0,1}/\beta_{0,2}^2)^T.$$

Imperial College London
Department of Mathematics

## Examiner's Comments

**Exam:** m3E **Session: 2016-2107**

**Question 1**

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

No errors consistently observed across candidates.
Mostly well done.

**Marker:** HEATHER BATTEY

**Signature:** HBattey **Date:** 23/05/2017

**Please return with exam marks (one report per marker)**

Imperial College London
Department of Mathematics

## Examiner's Comments

**Exam:** M3E **Session: 2016-2107**

### Question 2

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

~~[illegible]~~
Usually well done. Conclusion of part (b) was often ~~omitted~~ left out.

**Marker:** HEATHER BATTEY

**Signature:** HBattey **Date:** 23/05/2017.

**Please return with exam marks (one report per marker)**

Imperial College London
Department of Mathematics

## Examiner's Comments

**Exam:** M3E **Session: 2016-2107**

**Question 3**

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

When attempted, part (b) was well done.

Proof strategy in (a) (i) some seen in lectures but not always reproduced or adapted in a convincing way.

**Marker:** HEATHER BATTEY

**Signature:** HBattey **Date:** 23/05/2017 .

**Please return with exam marks (one report per marker)**

Imperial College London
Department of Mathematics

## Examiner's Comments

**Exam:** M3E **Session: 2016-2107**

### Question 4

Please use the space below to comment on the candidates' overall performance in the exam. A brief paragraph highlighting common mistakes and parts of questions done badly (or well) is sufficient. Do not refer to individual candidates. The purpose of this exercise is to provide guidance to the external examiners, and to the candidates themselves, on how you feel the cohort faired. Your comments will be available to students online.

Everyone correctly stated that convergence in probability implies convergence in distribution but the converse relationship ~~at~~ was either mistated or not mentioned.

The final calculation in part (c) was not always done correctly.

**Marker:** HEATHER BATTEY

**Signature:** [signature] **Date:** 23/05/2017

**Please return with exam marks (one report per marker)**