U.1. Are both bullet points of Definition 2.3.5 necessary? In other words, given any two inconsistent systems of linear equations, can the augmented matrix of the first be transformed into the augmented matrix of the second using row operations?

If your answer is 'yes', please give a proof. The ideal proof would describe how to find the row operations from the equations, but if you can show these operations exist without being able to name them explicitly that would still work.

If your answer is 'no', can you find when two inconsistent systems have these row operations, and when they don't?

U.2. Prove that for any $A \in M_{m\times n}(\mathbb{R})$, if there are $B_1, B_2 \in M_{n\times m}$ such that $B_1A = I_n$ and $AB_2 = I_m$ then $B_1 = B_2$.

U.3. Find a system of linear equations $Ax = b$ in variables $x_1, \ldots, x_k$ (over an arbitrary field), such that the following are equivalent:

i. $(\alpha_1, \ldots, \alpha_k)$ is a solution to $Ax = b$.

ii. For every system of linear equations (in n variables) $A'x = b'$ over the same field of definition as A, if $v_1, \ldots, v_k$ are all solutions to $A'x = b'$, then $\alpha_1 v_1 + \cdots + \alpha_k v_k$ is also a solution to $A'x = b'$.

iii. There is some system of linear equations (in n variables) $A'x = b'$ over the same field of definition as A, **such that** $b' \neq 0$ and $v_1, \ldots, v_k$ such that $v_1, \ldots, v_k, \alpha_1 v_1 + \cdots + \alpha_k v_k$ are all solutions to $A'x = b'$.