

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May-June 2022

This paper is also taken for the relevant examination for the
Associateship of the Royal College of Science

Special Relativity and Electromagnetism

Date: 16 May 2022

Time: 09:00 – 11:30 (BST)

Time Allowed: 2:30 hours

Upload Time Allowed: 30 minutes

This paper has 5 Questions.

Candidates should start their solutions to each question on a new sheet of paper.

Each sheet of paper should have your CID, Question Number and Page Number on the top.

Only use 1 side of the paper.

Allow margins for marking.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Each question carries equal weight.

**SUBMIT YOUR ANSWERS AS ONE PDF TO THE RELEVANT DROPBOX ON BLACKBOARD
WITH COMPLETED COVERSHEETS WITH YOUR CID NUMBER, QUESTION NUMBERS
ANSWERED AND PAGE NUMBERS PER QUESTION.**

1. In the following the spatial origins of frames K and K' are chosen so that $(0, 0, 0, 0)$ refers to the same event in both frames. All axes are parallel at all time. The origin of frame K' moves with velocity V relative to frame K along its x -axis. Dashed variables refer to measurements in the dashed frame K' , undashed variables to measurements in the undashed frame K .

To ease notation, use $\gamma = 1/\sqrt{1 - V^2/c^2}$ and $\beta = V/c$.

- (a) Lorentz transform the following four-vectors and tensors from frame K to frame K' .

(i) $x^i = (ct, Vt, 0, 0)$. (2 marks)

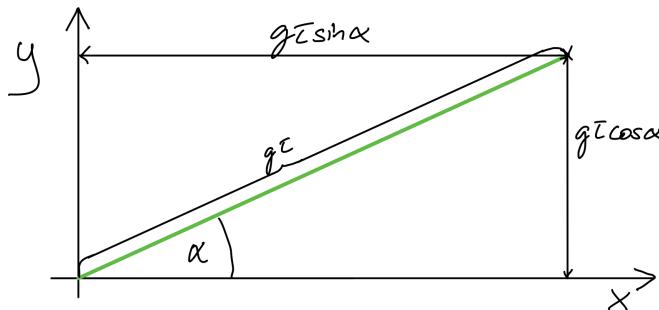
(ii) (2 marks)

$$A^{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

(iii) (2 marks)

$$B^{ij} = \begin{cases} -1 & \text{if } i = j = 0 \\ 1 & \text{if } i = j \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$

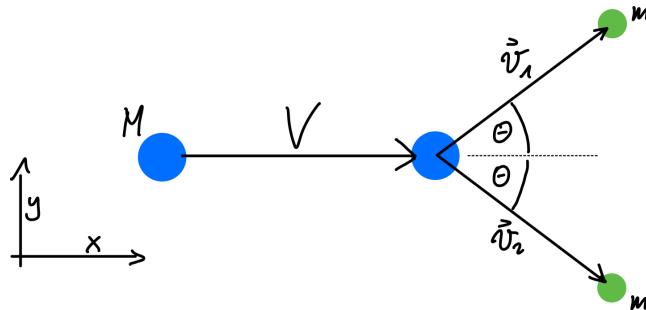
- (b) A signal departs from the origin of K at time $t = 0$, arriving at time $t = \tau$ at position $\mathbf{r} = (g\tau \cos(\alpha), g\tau \sin(\alpha), 0)$. The speed of the signal in frame K is therefore g and the distance travelled by it is $\ell = g\tau$.



- (i) State the four-vector d^i of the event of departure in K . (2 marks)
- (ii) Transform the four-vector d^i to frame K' . (2 marks)
- (iii) State the four-vector a^i of the event of arrival in K . (2 marks)
- (iv) Transform the four-vector a^i to frame K' . (2 marks)
- (v) State the distance ℓ' travelled by the signal in K' . (2 marks)
- (vi) State the time τ' travelled by the signal in K' . (2 marks)
- (vii) Show that $\ell'/\tau' = c$ if $g = c$. (2 marks)

(Total: 20 marks)

2. We consider the spontaneous decay of a particle with rest mass M that moves with velocity V relative to an observer along the x -axis. This particle decays into two daughter particles with identical rest masses m , each moving with velocity $\mathbf{v}_1 = (v_{1x}, v_{1y}, 0)$ and $\mathbf{v}_2 = (v_{2x}, v_{2y}, 0)$ respectively. The velocities are mirror symmetric, such that $v_{1x} = v_{2x} = v_x$ and $v_{1y} = -v_{2y}$ and therefore $|\mathbf{v}_1| = |\mathbf{v}_2| = v$. The angle θ characterises the trajectory by $v_x = \cos(\theta)v$.



To simplify notation use $\Gamma = 1/\sqrt{1 - V^2/c^2}$ and $\gamma = 1/\sqrt{1 - v^2/c^2}$.

Observables in the observer frame are undashed, observables in the inertial frame where M is at rest are dashed.

- (a) State the equations of the two conservation laws governing the decay. (4 marks)
- (b) Express $\cos(\theta)$ in terms of v as well as the masses M and m . (4 marks)
- (c) Which value of m results in $\theta = 0$? Express your answer in terms of M . (3 marks)
- (d) Which value of m results in $\theta = \pi/2$? Express your answer in terms of M and v .
Hint: The answer is independent of V . (3 marks)
- (e) In the (dashed) frame of reference of the initial particle with mass M the two daughter particle travel with velocities $\mathbf{v}'_1 = (0, v')$ and $\mathbf{v}'_2 = (0, -v')$ respectively. Find v' in terms M and m . (3 marks)
- (f) Determine v_{1x} and v_{1y} by transforming the velocity of the daughter particle from M 's frame to the observer's frame. (3 marks)

(Total: 20 marks)

3. (a) In the following the spatial origins of frames K and K' are chosen so that $(0, 0, 0, 0)$ refers to the same event in both frames. All axes are parallel at all time. The origin of frame K' moves with velocity V relative to frame K along its x -axis. Dashed variables refer to measurements in the dashed frame K' , undashed variables to measurements in the undashed frame K .

To ease notation, use $\gamma = 1/\sqrt{1 - V^2/c^2}$ and $\beta = V/c$.

- (i) A stick of length ℓ is at rest in K along its x -axis. What is the length measured in K' ? (3 marks)
- (ii) A stick of length ℓ is at rest in K along its y -axis. What is the length measured in K' ? (3 marks)
- (iii) A constant electric field $\mathbf{E} = (E, 0, 0)$ and no magnetic field, $\mathbf{H} = (0, 0, 0)$, are observed in K . What are the electric and magnetic fields measured in K' ? (3 marks)
- (iv) A constant electric field $\mathbf{E} = (0, E, 0)$ and no magnetic field, $\mathbf{H} = (0, 0, 0)$, are observed in K . What are the electric and magnetic fields measured in K' ? (3 marks)

- (b) In four-vector form, Maxwell's equations can be succinctly summarised as

$$\partial^\ell F^{ik} + \partial^i F^{k\ell} + \partial^k F^{\ell i} = 0 \quad \text{and} \quad \partial_k F^{ik} = -\frac{4\pi}{c} j^i ,$$

where F^{ik} is the electromagnetic field tensor and $\partial_i = \partial/\partial x^i$.

- (i) Express

$$\partial_\ell \partial^\ell F^{ik}$$

in terms of the four-current j^i . (4 marks)

- (ii) On the basis of Maxwell's equations in four-vector form, express the charge density in terms of electric and magnetic fields, which are related to the electromagnetic field tensor via

$$F^{ij} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z & H_y \\ E_y & H_z & 0 & -H_x \\ E_z & -H_y & H_x & 0 \end{pmatrix} .$$

(4 marks)

(Total: 20 marks)

4. (a) The wave equation in four-vector form is

$$\partial^i \partial_i A^j = 0$$

using the notation $\partial_i = \partial/\partial x^i$. State the wave equation of the scalar and the vector potentials.

(4 marks)

- (b) The Lorenz gauge

$$\partial_k A^k = 0 \quad (\text{Lorenz gauge})$$

is a convenient choice to arrive at a simple form of the wave equation. Another choice is the Coulomb gauge in vacuum

$$\phi \equiv 0 \quad \text{and} \quad \nabla \cdot \mathbf{A} = 0 \quad (\text{Coulomb gauge}).$$

Which gauge implies which other gauge? Provide an explanation. (4 marks)

- (c) A (complex) solution of the wave-equation is the four-potential

$$A^i = B^i \exp(i k^j x_j)$$

at space-time point x^j with constant polarisation four-vector B^i , wave four-vector k^j and imaginary unit i .

NB: Reality of the four-potential can be restored by adding its complex conjugate here and below, as it enters only linearly.

- (i) Use

$$F^{ij} = \partial^i A^j - \partial^j A^i$$

to determine $F^{ij} F_{ij}$ in terms of the polarisation four-vector B^i and the wave four-vector k^j . (5 marks)

- (ii) Use $\partial_i F^{ij} = 0$ (in the vacuum) to show that $F^{ij} F_{ij} = 0$. (5 marks)

- (iii) Indicate which of the following are Lorentz scalars: $F^{ij} F_{ij}$, $F^{ij} F_{ji}$ and $F^{ii} F_{jj}$. (2 marks)

(Total: 20 marks)

5. In the following, we consider a point particle with charge $q \neq 0$ and a (time averaged) magnetic moment $\bar{\mathbf{m}} \neq \mathbf{0}$ located and at rest at the origin. In the following, overhead bars denote time averages as used by Landau and Lifshitz.
- (a) State the resulting electric field $\mathbf{E}(\mathbf{R})$ as a function of position \mathbf{R} in the frame of reference of the particle. (4 marks)
 - (b) State the resulting (time averaged) magnetic field $\bar{\mathbf{H}}(\mathbf{R})$ as a function of position \mathbf{R} in the frame of reference of the particle. (4 marks)
 - (c) Show that
- $$\mathbf{E}(\mathbf{R}) \cdot \bar{\mathbf{H}}(\mathbf{R}) = \frac{2q\bar{\mathbf{m}} \cdot \mathbf{n}}{R^5}$$
- where $\mathbf{R} = R\mathbf{n}$ is the position at distance $R = |\mathbf{R}|$ and \mathbf{n} is its normal vector. (4 marks)
- (d) State the conditions on $\mathbf{E}(\mathbf{R})$ and $\bar{\mathbf{H}}(\mathbf{R})$ in the inertial frame K of the particle that need to be met at \mathbf{R} , so that there exists another inertial frame of reference K' which has vanishing electric field $\mathbf{E}'(\mathbf{R}')$ at this same location, denoted by \mathbf{R}' in K' . (4 marks)
 - (e) Determine the region in space of the resting particle K , where $\mathbf{E}'(\mathbf{R}') = \mathbf{0}$ can possibly be observed in a suitably moving inertial frame K' . (4 marks)

(Total: 20 marks)

Solutions of M345A6 2021/2022

Some of the questions are motivated by or rooted in the book M Blennow and T Ohlsson, *300 Problems in special and general relativity*, Cambridge University Press, Cambridge, UK, 2022.

1.a.i (2 marks, easy, A, seen)

Only hurdle is the sign of V and the marker should be generous here:

$$x'^i = (\gamma(ct - \beta Vt), \gamma(Vt - \beta ct), 0, 0) = (ct/\gamma, 0, 0, 0).$$

1.a.ii (2 marks, easy, B, seen similar)

Best written in matrix form

$$\begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \mathbf{1} \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} (1+\beta^2)\gamma^2 & -2\beta\gamma^2 & 0 & 0 \\ -2\beta\gamma^2 & (1+\beta^2)\gamma^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.a.iii (2 marks, easy, A, seen)

B^{ij} is the negative of the metric tensor and thus known to be invariant. Doing it explicitly,

$$\begin{aligned} & \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} (1-\beta^2)\gamma^2 & 0 & 0 & 0 \\ 0 & (1-\beta^2)\gamma^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

1.b.i (2 marks, trivial, A)

$$d^i = (0, 0, 0, 0)$$

as stated in the question.

1.b.ii (2 marks, easy, A)

$$d'^i = (0, 0, 0, 0)$$

as stated in the question.

1.b.iii (2 marks, easy, A)

$$a^i = (c\tau, g\tau \cos(\alpha), g\tau \sin(\alpha), 0)$$

as stated in the question.

1.b.iv (2 marks, easy, A, seen similar)

This is a matter of a simple transform, keeping in mind that we are transforming from K to

$$K': a'^i = (\gamma(c\tau - \beta g\tau \cos(\alpha)), \gamma(g\tau \cos(\alpha) - \beta c\tau), g\tau \sin(\alpha), 0).$$

1.b.v (2 marks, medium, C, seen similar)

$$\text{Extracting the components of } x' \text{ and } y' \text{ from } a'^i: \boxed{\ell' = \sqrt{(\gamma(g\tau \cos(\alpha) - \beta c\tau))^2 + (g\tau \sin(\alpha))^2}}$$

1.b.vi (2 marks, easy, B, seen similar)

$$\text{Extracting the time } \tau' \text{ from } a'^i: \boxed{\tau' = \gamma\tau(1 - \beta(g/c) \cos(\alpha))}.$$

1.b.vii (2 marks, medium, D, seen similar)

For $g = c$ the expression for ℓ'^2 can be rewritten as follows: $\ell'^2 = c^2\tau^2(\gamma^2(\cos(\alpha) - \beta)^2 + \sin^2(\alpha)) = c^2\tau^2\gamma^2(1 - \beta \cos(\alpha))^2$. The central identity needed for this is $1 - \gamma^2 + \beta^2\gamma^2 = 0$, which cancels the sin. The square root can now be taken, so that $\ell' = c\tau\gamma(1 - \beta \cos(\alpha))$.

$$\text{For } g = c, \text{ the time becomes } \tau' = \gamma\tau(1 - \beta \cos(\alpha)) \text{ and therefore } \boxed{\ell'/\tau' = c}.$$

2.a (4 marks, easy, A, seen)

Conservation of energy, $Mc^2\Gamma = 2mc^2\gamma$, and conservation of momentum $MV\Gamma = 2mv_x\gamma$, as the momentum in the y -direction vanishes.

2.b (4 marks, medium, C, unseen)

From conservation energy we have that $M\Gamma = 2m\gamma$ and therefore from conservation of momentum $V = v_x = v \cos(\theta)$. To determine $\cos(\theta)$, we thus need to determine the ratio $V/v = \cos(\theta)$. Squaring kinetic energies gives $M^2(1 - v^2/c^2) = 4m^2(1 - V^2/c^2) = 4m^2(1 - \cos^2(\theta)v^2/c^2)$ which can be rearranged for

$$\cos^2(\theta) = \frac{M^2}{4m^2} + \frac{c^2}{v^2} - \frac{M^2c^2}{4m^2v^2}$$

2.c (3 marks, easy, A, unseen)

Even without the expression for $\cos^2(\theta)$ above, the solution is easily found by considering $\theta = 0$ as no decay at all, in which case $m = M/2$. Of course, this is also a solution of $\cos^2(\theta) = 1$ above.

2.d (3 marks, hard, D, unseen)

This is hard without b above, but also doable, given the hint that the solution is independent of V . Choosing $V = 0$ produces $\theta = \pi/2$ because v_x must vanish by momentum conservation and any $v > 0$ will therefore result in a trajectory with $\theta = \pi/2$. Since $\Gamma = 1$ for $V = 0$, energy conservation gives the desired answer $m = \sqrt{1 - v^2/c^2}M/2$.

Setting $\cos^2(\theta) = 0$ in b equivalently gives $(M^2/(4m^2))(c^2/v^2 - 1) = c^2/v^2$ and after some algebra the expression for m above.

2.e (3 marks, medium, B, unseen)

This is very much the situation considered in 2.d. Energy conservation gives $Mc^2\sqrt{1 - v'^2/c^2} = 2mc^2$ and therefore

$$v' = c\sqrt{1 - \frac{4m^2}{M^2}}$$

2.f (3 marks, hard, C, seen similar)

The velocities observed in M' 's frame are $v'_{1x} = 0$ and $v'_{1y} = v'$. To calculate the velocities in the observer frame we need to use the addition formula of velocities, which does not need to be rederived. The expressions simplify significantly as $v'_{1x} = 0$:

$$v_{1x} = V \quad \text{and} \quad v_{1y} = v'\sqrt{1 - \frac{V^2}{c^2}}$$

3.a.i (3 marks, easy, A, seen)

This is a matter of conventional Lorentz contraction, $\ell' = \ell/\gamma$.

3.a.ii (3 marks, easy, A, seen similar)

This is a matter of knowing that there is no effect on the perceived length of the stick, so $\ell' = \ell$.

3.a.iii (3 marks, medium, B, seen similar)

This can be done either on the basis of the transformation of the electromagnetic field tensor, or by knowing the transformation law, $\mathbf{E}' = (E, 0, 0)$ and $\mathbf{H}' = (0, 0, 0)$.

3.a.iv (3 marks, medium, B, seen similar)

This can be done either on the basis of the transformation of the electromagnetic field tensor, or by knowing the transformation law, $\boxed{\mathbf{E}' = (0, E\gamma, 0)}$ and $\boxed{\mathbf{H}' = (0, 0, -\beta E\gamma)}$.

3.b.i (4 marks, hard, D, unseen)

Applying ∂_ℓ on the left of the homogeneous equation gives

$$\partial_\ell \partial^\ell F^{ik} = -\partial_\ell \partial^i F^{k\ell} - \partial_\ell \partial^k F^{\ell i}$$

The first term on the right is essentially the inhomogeneous equation, the second term requires an exchange of the indices of $F^{\ell i}$, which is antisymmetric, so that finally

$$\boxed{\partial_\ell \partial^\ell F^{ik} = \frac{4\pi}{c} \partial^i j^k - \frac{4\pi}{c} \partial^k j^i}$$

3.b.ii (4 marks, medium, C, unseen)

The charge density can be extracted from the current four-vector as $\rho = j^0/c$, so that

$$\boxed{\rho = \frac{-1}{4\pi} \partial_k F^{0k} = \frac{1}{4\pi} \nabla \cdot \mathbf{E}}$$

by inspection of the electromagnetic field tensor as given in the question. A sign mistake is avoided by noting that

$$\partial_1 = \frac{\partial}{\partial^1} = \frac{\partial}{\partial x}$$

4.a (4 marks, medium, B, unseen)

This is a matter of writing the four potential in the form $A^j = (\phi, \mathbf{A})$ and the d'Alembert operator as $\partial_t^2 - \Delta$, so that

$$\boxed{\partial_t^2 \phi = \Delta \phi \quad \text{and} \quad \partial_t^2 \mathbf{A} = \Delta \mathbf{A}}$$

4.b (4 marks, easy, A, unseen)

The Lorenz gauge spelt out is $\partial_t \phi + \nabla \cdot \mathbf{A} = 0$, which is implied by the Coulomb gauge.

The Coulomb gauge implies the Lorenz gauge.

4.c.i (5 marks, difficult, C, unseen)

The key is to observe that

$$\partial^j A^i = \frac{\partial A^i}{\partial x_j} = ik^j A^i$$

so that $F^{ij} = ik^i A^j - ik^j A^i$. The contraction of F^{ij} with itself therefore gives

$$\boxed{F^{ij} F_{ij} = -2k^i k_i A^j A_j + 2k^i A_i k^j A_j}$$

4.c.ii (5 marks, difficult, D, unseen)

To take the second derivative of A^i , we rewrite the argument of the potential as $k_j x^j$, so that

$$\partial_\ell \partial^j A^i = -k^j k_\ell A^i$$

In the vacuum $\partial_i F^{ij} = 0 = \partial_i \partial^i A^j - \partial_i \partial^j A^i$, which implies

$$0 = -k^i k_i A^j + k^j k_i A^i$$

Contracting with A_j gives

$$\boxed{0 = -k^i k_i A^j A_j + k^j A_j k_i A^i}$$

which implies that $F^{ij} F_{ij} = 0$, as it is twice the right hand side according to c.i.

4.c.iii (2 marks, easy, A, seen)

All of them.

5.a (4 marks, easy, A)

This is Eq. 36.6 in LL2,

$$\mathbf{E}(\mathbf{R}) = \frac{q\mathbf{R}}{R^3}$$

5.b (4 marks, easy, A)

This is Eq. 44.4 in LL2,

$$\overline{\mathbf{H}}(\mathbf{R}) = \frac{3\mathbf{n}(\overline{\mathbf{m}} \cdot \mathbf{n}) - \overline{\mathbf{m}}}{R^3}$$

5.c (4 marks, easy, B)

This is a matter of projecting a vector to another. Knowing the quoted result helps not only in c, but also in b. Using $\mathbf{n} \cdot \mathbf{R} = R$,

$$\mathbf{E}(\mathbf{R}) \cdot \overline{\mathbf{H}}(\mathbf{R}) = R^{-6} (3q(\mathbf{n} \cdot \mathbf{R})(\overline{\mathbf{m}} \cdot \mathbf{n}) - q\overline{\mathbf{m}} \cdot \mathbf{R}) = 2q \frac{\overline{\mathbf{m}} \cdot \mathbf{n}}{R^5}$$

5.d (4 marks, medium, C)

There are two invariants of the fields: $\mathbf{E} \cdot \overline{\mathbf{H}} = \mathbf{E}' \cdot \overline{\mathbf{H}}'$ and $\mathbf{E}^2 - \overline{\mathbf{H}}^2 = \mathbf{E}'^2 - \overline{\mathbf{H}}'^2$. If the electric field \mathbf{E}' is to vanish somewhere in K' , the first invariant is 0 at that location. It must therefore also vanish somewhere in K , i.e. $\mathbf{E}(\mathbf{R}) \cdot \overline{\mathbf{H}}(\mathbf{R}) = 0$ for those \mathbf{R} where $\mathbf{E}' = \mathbf{0}$ in K' . Further, if $\mathbf{E}'^2 = 0$, then $\mathbf{E}^2 - \overline{\mathbf{H}}^2 = \mathbf{E}'^2 - \overline{\mathbf{H}}'^2 \leq 0$ because of the second invariant, given that $\overline{\mathbf{H}}'^2 \geq 0$.

5.e (4 marks, difficult, D)

Firstly, $\mathbf{E}(\mathbf{R}) \cdot \overline{\mathbf{H}}(\mathbf{R}) = 0$ implies that $\overline{\mathbf{m}} \cdot \mathbf{n} = 0$ using the result in c, given that all other parameters are non-zero and the limit $R \rightarrow 0$ does not exist. The relevant region is thus in the plane orthogonal to the magnetic moment. Using $\overline{\mathbf{m}} \cdot \mathbf{n} = 0$ in b gives $\overline{\mathbf{H}} = -\overline{\mathbf{m}}/R^3$ there. To meet the second invariant, we need $\mathbf{E}^2 - \overline{\mathbf{H}}^2 \leq 0$, which gives

$$\frac{q^2}{R^4} - \frac{\overline{\mathbf{m}}^2}{R^6} \leq 0$$

in the relevant region. Rearranging this expression and taking the square root gives the second condition $R \leq |\overline{\mathbf{m}}|/|q|$.

If your module is taught across multiple year levels, you might have received this form for each level of the module. You are only required to fill this out once for each question.

Please record below, some brief but non-trivial comments for students about how well (or otherwise) the questions were answered. For example, you may wish to comment on common errors and misconceptions, or areas where students have done well. These comments should note any errors in and corrections to the paper. These comments will be made available to students via the MathsCentral Blackboard site and should not contain any information which identifies individual candidates. Any comments which should be kept confidential should be included as confidential comments for the Exam Board and Externals. If you would like to add formulas, please include a separate pdf file with your email.

ExamModuleCode	QuestionNumber	Comments for Students
Special Relativity and Electromagnetism_MATH60016 MATH97022 MATH70016	1	Mostly done very well. A lot of this material was learnt in the first coursework and I realise that most students were able to make use of that learning. Some got tripped up when transforming tensors of rank 2, some brutally multiplied everything by γ in the hope that will do.
Special Relativity and Electromagnetism_MATH60016 MATH97022 MATH70016	2	This was mostly done well. The first question, when I ask for the equations, some students didn't state them and I had to fish for them in the rest of the answers. Most succeeded in deriving subsequently a useful expression for the angle and then had no trouble determining the special cases. Most of those who didn't have a good expression for the angle succeeded nevertheless, with a bit more work. The last two parts were generally done well.
Special Relativity and Electromagnetism_MATH60016 MATH97022 MATH70016	3	Generally well answered. There was some confusion about $1/\gamma$ versus γ in the very first question. This is something I have repeated a couple of times in the lectures, the classes and the coursework. Transformation of fields generally well done, too, and even 3.b.i, which I thought was pretty difficult.
Special Relativity and Electromagnetism_MATH60016 MATH97022 MATH70016	4	This was the most difficult of the four regular questions and it was not answered as well as the other three. One of the issues seemed to be an apparent lack of time. However, some of it is down to an apparent lack of understanding, in particular in regards to the tensor notation. Many students got (a) and (b) right, but many struggled with c.i and c.ii. There were many correct answers of c.iii, but surprisingly many mistakes were made there as well.
Special Relativity and Electromagnetism_MATH60016 MATH97022 MATH70016	5	The first three parts of the question were easy and generally well answered. Part d and e were harder, with only a few students writing down both invariants in d and very few using the right reasoning in e. A very small numbers ended up with the correct answer in e.