Imperial College
London

MATH96011

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May-June 2021

This paper is also taken for the relevant examination for the
Associateship of the Royal College of Science

Mathematics of Business and Economics

Date: Friday, 28 May 2021

Time: 09:00 to 11:00

Time Allowed: 2 hours

Upload Time Allowed: 30 minutes

This paper has 4 Questions.

Candidates should start their solutions to each question on a new sheet of paper.

Each sheet of paper should have your CID, Question Number and Page Number on the top.

Only use 1 side of the paper.

Allow margins for marking.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Each question carries equal weight.

SUBMIT YOUR ANSWERS AS SEPARATE PDFs TO THE RELEVANT DROPBOXES ON BLACKBOARD INCLUDING A COMPLETED COVERSHEET WITH YOUR CID NUMBER, QUESTION NUMBERS ANSWERED AND PAGE NUMBERS PER QUESTION.

1. (a) State the consumer-side analogue notions to the following:

(i) Profit maximisation problem

(ii) Production function

(iii) Cost function

(iv) Marginal rate of technical substitution

(v) Conditional factor demand function

(vi) Isoquant (6 marks)

(b) Decide if the following statements are true or false. Justify your answers.

(i) The market demand curve for heroin is extremely inelastic. The market is monopolised by the Mafia and the Mafia is only interested in maximising their profits. The two statements are consistent with one another. (2 marks)

(ii) A firm may be willing to accept losses in the short-run. (2 marks)

(iii) Ordinal utility is a much weaker notion than cardinal utility. (2 marks)

(c) (i) Briefly explain a U-shaped long-run average cost curve in terms of the scale behaviour. (3 marks)

(ii) Using your answer from part (c) (i), prove that there is no homogeneous production function (of any degree $k \in R$) that gives a U-shaped long-run average cost curve. (5 marks)

2. Consider a firm that produces a single output using two input factors. The production function is given by:

$$f(x_1, x_2) = x_1 x_2.$$

Assume that the output price p and the input prices w_1, w_2 are all positive.

(a) Compute the elasticity of scale of f and comment on it. (2 marks)

(b) Compute the conditional factor demand function $\underline{x}^*(w_1, w_2, y)$ and the cost function $c^*(w_1, w_2, y)$. (3 marks)

(c) Verify Shephard's Lemma. (1 mark)

(d) Verify that the conditional factor demand function and the cost function in (b) satisfy the required homogeneity properties. (2 marks)

(e) Compute the optimal output $y^*(p, w_1, w_2)$ and the profit function $\pi^*(p, w_1, w_2)$. (4 marks)

(f) Check if the conditions for the profit-maximising output are satisfied. (4 marks)

(g) Briefly explain what is the expected shape of the marginal cost curve and the average cost curve and why it does not agree with the results in the section "geometry of costs". (*You are not asked to sketch a graph.*) (4 marks)

3. (a) (i) Determine the relation between the Weak Axiom of Profit Maximisation and the Weak Axiom of Cost Minimisation. (2 marks)

(ii) Determine the relation between the utility maximisation and expenditure minimisation. (2 marks)

(b) Consider the following preference relation $\succeq$ on $\mathbb{R}^2$: for any $\underline{x} = (x_1, x_2) \in \mathbb{R}^2$ and $\underline{y} = (y_1, y_2) \in \mathbb{R}^2$ it holds that $\underline{x} \succeq \underline{y}$ if and only if

$$x_1 \geq y_1 \quad \text{and} \quad x_2 \geq y_2.$$

(i) Check whether completeness, transitivity, weak/strong monotonicity, local nonsatiation, and (strict) convexity are satisfied giving a counterexample or a proof. (8 marks)

(ii) Suppose there is a utility function $u \colon \mathbb{R}^2 \to \mathbb{R}$ representing $\succeq$. Show that u is an injection. (3 marks)

(c) Consider the following possible assumptions made to solve a cost-minimisation problem:

(i) the second order necessary condition holds at all points,

(ii) the production function is quasi-concave,

(iii) the production function is strictly quasi-concave,

(iv) the second order sufficient condition holds at all points.

Briefly explain the advantage of making assumption (ii) instead of (i); assumption (iii) instead of (ii); and assumption (iv) instead of (iii). (5 marks)

4. (a) ~~Give examples of economic activities that:~~

~~(i) are not included in the Gross Domestic Product.~~ ~~(1 mark)~~

~~(ii) are not included in the Gross National Product.~~ ~~(1 mark)~~

(b) ~~Suppose the economy is described by the five-sector model.~~

~~(i) Describe the equilibrium in the model and define the notation used.~~ ~~(2 marks)~~

~~(ii) Describe what happens if injections exceed leakages over a given time period.~~ ~~(4 marks)~~

(c) Consider the oil market. There are n consumers, each having the same utility function, which leads to the following market demand (measured in thousands of barrels):

$$X^*(p) = \begin{cases} n\left(\frac{a-p}{2}\right), & 0 \leq p < a, \\ 0, & p \geq a, \end{cases}$$

where $p \geq 0$ is the oil price in pounds per barrel and $a > 0$ constant.

There are also m firms producing oil in a perfect competition and each of the firms has a long-run cost function (reflecting the actual economic costs) $c^*(y) = y(b+y)$, where $y \geq 0$ is the firm's output in thousands of barrels and $b \in (0, a)$ constant.

(i) Compute the individual supply function and use it to compute the market supply, $Y^*(p)$. (*Note: You do not have to check the second order condition.*) (3 marks)

(ii) Can an individual firm ever have negative profit? Explain briefly. (2 marks)

(iii) Sketch a graph of the market demand curve and the market supply curve and compute the equilibrium price, p^* and the equilibrium quantity, q^*. (3 marks)

(iv) Using the graph from part (c) (iii), show graphically and justify what happens to the market equilibrium (p^*, q^*) when (i) n increases while m is held constant; (ii) m increases while n is held constant. (*It would be preferable to draw one graph for each case.*) (4 marks)

1. (a) State the consumer-side analogue notions to the following:

(i) Profit maximisation problem

(ii) Production function

(iii) Cost function

(iv) Marginal rate of technical substitution

(v) Conditional factor demand function

(vi) Isoquant

SOLUTION: (SEEN)

(i) Utility maximisation problem

(ii) Utility function

(iii) Expenditure function

(iv) Marginal rate of substitution

(v) Hicksian demand

(vi) Indifference curve (6 marks)

(b) Decide if the following statements are true or false. Justify your answers.

(i) The market demand curve for heroin is extremely inelastic. The market is monopolised by the Mafia and the Mafia is only interested in maximising their profits. The two statements are consistent with one another.

(ii) A firm may be willing to accept losses in the short-run.

(iii) Ordinal utility is a much weaker notion than cardinal utility.

SOLUTION: (UNSEEN/SEEN/SEEN)

(i) False. A monopolist can only maximise their profits when faced with an elastic market demand curve. (2 marks)

(ii) True. A firm may be willing to accept losses in the short-run, because there are fixed costs in the short-run, while in the long-run all inputs may vary. (2 marks)

(iii) True. Ordinal utility is a much weaker notion than cardinal utility because it only requires that the consumer be able to rank goods in the order of his/her preference. (2 marks)

(c) (i) Briefly explain a U-shaped long-run average cost curve in terms of the scale behaviour.

SOLUTION: (UNSEEN)

A downward-sloping long-run average cost curve shows increasing returns to scale, a flat long-run average cost curve shows constant returns to scale, and an upward-sloping long-run average cost curve shows decreasing returns to scale. (3 marks)

(ii) Using your answer from part (c) (i), prove that there is no homogeneous production function (of any degree $k \in R$) that gives a U-shaped long-run average cost curve.

SOLUTION: (UNSEEN)

A homogeneous function has $f(tx) = t^k f(x)$ for some $k \in R$. We differentiate wrt t:

$$\sum_{i=1}^{n} \frac{\partial f(tx)}{\partial x_i} x_i = kt^{k-1} f(x)$$

If we set $t = 1$, we get:

$$\sum_{i=1}^{n} \frac{\partial f(x)}{\partial x_i} x_i = kf(x) \Rightarrow e(x) = k$$

where $e(x)$ is the local elasticity wrt the scale at x. Since $e(x) = k$ is a constant, we either have $k > 1$, which implies increasing returns to scale everywhere, or $k = 1$, which implies constant returns to scale everywhere or $k < 1$, which implies decreasing returns to scale everywhere, so none of these gives a typical U-shaped long-run average cost curve.

(5 marks)

2. Consider a firm that produces a single output using two input factors. The production function is given by:

$$f(x_1, x_2) = x_1 x_2.$$

Assume that the output price p and the input prices w_1, w_2 are all positive.

(a) Compute the elasticity of scale of f and comment on it.

SOLUTION: (SEEN SIMILAR) Let $(x_1, x_2) \in \mathbb{R}^2_{\geq 0}$. The partial derivatives are given by:

$$\partial_1 f(x_1, x_2) = x_2 \,, \qquad \partial_2 f(x_1, x_2) = x_1.$$

Hence, the elasticity of scale of f at (x_1, x_2) is given by:

$$\begin{aligned} e(x_1, x_2) &= \frac{\langle \nabla f(x_1, x_2), (x_1, x_2) \rangle}{f(x_1, x_2)} \\ &= \frac{\partial_1 f(x_1, x_2) x_1 + \partial_2 f(x_1, x_2) x_2}{f(x_1, x_2)} \\ &= 2 \,. \end{aligned}$$

This coincides also with the degree of homogeneity and it shows us that f has increasing returns to scale.

(2 marks)

(b) Compute the conditional factor demand function $\underline{x}^*(w_1, w_2, y)$ and the cost function $c^*(w_1, w_2, y)$.

SOLUTION: (SEEN SIMILAR) We determine the minimiser of $w_1 x_1 + w_2 x_2$ subject to $x_1 x_2 = y$. For $y = 0$, we clearly have $x_1^*(w_1, w_2, 0) = x_1^*(w_1, w_2, 0) = 0$. For $y > 0$, we must have $x_1, x_2 > 0$. Hence, the constraint yields that $x_1 = y/x_2$. Substituting into the cost function yields

$$w_1 x_1 + w_2 \frac{y}{x_1} \,.$$

This is a convex function in x_1. So it suffices to consider the first order condition only. This yields

$$x_1^*(w_1, w_2, y) = \left(y \frac{w_2}{w_1} \right)^{1/2} \,.$$

Similarly, one obtains

$$x_2^*(w_1, w_2, y) = \left(y \frac{w_1}{w_2} \right)^{1/2} \,.$$

Finally, the cost function is given by

$$c^*(w_1, w_2, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y) = 2 \Big(y w_1 w_2 \Big)^{1/2} \,.$$

(3 marks)

(c) Verify Shephard's Lemma.

SOLUTION: (SEEN SIMILAR) We get indeed that:

$$x_1^*(w_1, w_2, y) = \frac{\partial}{\partial w_1} c^*(w_1, w_2, y), \qquad x_2^*(w_1, w_2, y) = \frac{\partial}{\partial w_2} c^*(w_1, w_2, y)\,.$$

(1 mark)

(d) Verify that the conditional factor demand function and the cost function in (b) satisfy the required homogeneity properties.

SOLUTION: (SEEN SIMILAR) We should verify that the conditional factor demand functions are homogeneous of degree 0 in $\underline{w}$ and the cost function is homogeneous of degree 1 in $\underline{w}$. We get indeed that:

$$x_1^*(tw_1, tw_2, y) = \left(y\frac{tw_2}{tw_1}\right)^{1/2} = x_1^*(w_1, w_2, y), \quad x_2^*(tw_1, tw_2, y) = \left(y\frac{tw_1}{tw_2}\right)^{1/2} = x_2^*(w_1, w_2, y)$$

$$c^*(tw_1, tw_2, y) = 2\Big(t^2\Big)^{1/2}\Big(yw_1w_2\Big)^{1/2} = tc^*(w_1, w_2, y)$$

(2 marks)

(e) Compute the optimal output $y^*(p, w_1, w_2)$ and the profit function $\pi^*(p, w_1, w_2)$.

SOLUTION: (UNSEEN) The profit at output y is given by:

$$\pi(p, w_1, w_2, y) = py - y^{1/2}\, 2(w_1w_2)^{1/2}\,.$$

We can see that – as the sum of two convex functions – it is a convex function in y. For $p > 0$ it has a global minimum at the critical point $y_0 = \frac{w_1 w_2}{p^2}$ and diverges to infinity as $y \to \infty$. Thus,

$$y^*(p, w_1, w_2) = \infty, \qquad \pi^*(p, w_1, w_2) = \infty$$

(4 marks)

(f) Check if the conditions for the profit-maximising output are satisfied.

SOLUTION: (UNSEEN)

First condition: The marginal costs are:

$$MC(y) = \frac{\partial}{\partial y} c^*(w_1, w_2, y) = y^{-1/2}(w_1w_2)^{1/2}.$$

For $y \to \infty$, they converge to 0. That means the first condition is not satisfied ($MC(y^*) \neq p$).

Second condition: Marginal costs are decreasing everywhere such that the second condition is also not satisfied.

Third condition: The average costs are

$$AC(y) = \frac{c^*(w_1, w_2, y)}{y} = y^{-1/2}\, 2(w_1 w_2)^{1/2}$$

and converge to 0 as $y \to \infty$. That means that $LAC(y^*) \leq p$ and the converse of the shutdown condition is satisfied.

(1 mark for justification/condition and
1 mark for computing marginal and average costs; total 4 marks)

(g) Briefly explain what is the expected shape of the marginal cost curve and the average cost curve and why it does not agree with the results in the section "geometry of costs". (*You are not asked to sketch a graph.*)

SOLUTION: (UNSEEN) We expect the two curves to have a different shape: both curves are actually <u>decreasing and converge to 0</u>. However, in the lectures we were always dealing with the situation that both the average and marginal costs are eventually increasing. The underlying argument in the lecture was the <u>law of diminishing marginal productivity</u>. This is equivalent to the fact that the elasticity of scale is <u>decreasing</u> (and eventually smaller than 1). However, the production function we are dealing with has a <u>constant elasticity of scale</u> (and has in particular increasing returns to scale). (This is also the reason why we can make infinite profit with this production function.)

(1 mark for mentioning each of the underlined points; total 4 marks)

3. (a) (i) Determine the relation between the Weak Axiom of Profit Maximisation and the Weak Axiom of Cost Minimisation.

Solution: (SEEN)

The Weak Axiom of Profit Maximisation (WAPM) implies the Weak Axiom of Cost Minimisation (WACM), meaning that if the firm is maximising profits, then it minimises costs. On the other hand side, it could minimise costs, but not maximise profits.

(2 marks)

(ii) Determine the relation between the utility maximisation and expenditure minimisation.

Solution: (SEEN)

The utility maximisation implies expenditure minimisation and vice versa.

(2 marks)

(b) Consider the following preference relation $\succeq$ on $\mathbb{R}^2$: for any $\underline{x} = (x_1, x_2) \in \mathbb{R}^2$ and $\underline{y} = (y_1, y_2) \in \mathbb{R}^2$ it holds that $\underline{x} \succeq \underline{y}$ if and only if

$$x_1 \geq y_1 \quad \text{and} \quad x_2 \geq y_2.$$

Check whether completeness, transitivity, weak/strong monotonicity, local nonsatiation, and (strict) convexity are satisfied giving a counterexample or a proof.

SOLUTION: (SEEN SIMILAR)

Completeness: Completeness is not satisfied. Consider the following counter example: if $x = (0, 1)$ and $y = (1, 0)$, clearly neither $x_i \geq y_i$ for all i nor $y_i \geq x_i$ for all i, so neither $\underline{x} \succeq \underline{y}$ or $\underline{y} \succeq \underline{x}$.

Transitivity: Let $x = (x_1, x_2), y = (y_1, y_2), z = (z_1, z_2) \in \mathbb{R}^2$. Assume that $\underline{x} \succeq \underline{y}$ and $\underline{y} \succeq \underline{z}$. We will show that $\underline{x} \succeq \underline{z}$. We get indeed that:

$$x_1 \geq y_1 \quad \text{and} \quad x_2 \geq y_2$$

and

$$y_1 \geq z_1 \quad \text{and} \quad y_2 \geq z_2.$$

Hence

$$x_1 \geq z_1 \quad \text{and} \quad x_2 \geq z_2.$$

Strong monotonicity: Let $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$. If $x_1 > y_1$ and $x_2 \geq y_2$, then $\underline{x} \succeq \underline{y}$, but not $\underline{y} \succeq \underline{x}$. Hence $\underline{x} \succ \underline{y}$.

Weak monotonicity: It follows from strong monotonicity.

Local nonsatiation: It follows from strong monotonicity.

Strict convexity: Let $x = (x_1, x_2), y = (y_1, y_2), z = (z_1, z_2) \in \mathbb{R}^2$. Suppose that $x \succ z$ and $y \succ z$. Let $a = (1 - \lambda)x + \lambda y$ for some $\lambda \in [0, 1]$. Then we have to show that $a \succ z$. If $\lambda \in \{0, 1\}$, the claim is true. So let $\lambda \in (0, 1)$.

$$\text{If} \quad x_1 > z_1 \quad \text{and} \quad y_1 > z_1 \quad \text{then} \quad a_1 > z_1$$

and

$$\text{if} \quad x_2 > z_2 \quad \text{and} \quad y_2 > z_2 \quad \text{then} \quad a_2 > z_2.$$

Hence in both cases we get that $a \succ z$.

Convexity: It follows from strict convexity.

(1 mark/property, except 2 marks for strict convexity; total 8 marks)

(ii) Suppose there is a utility function $u\colon \mathbb{R}^2 \to \mathbb{R}$ representing $\succeq$. Show that u is an injection.

SOLUTION: (UNSEEN) If $u\colon \mathbb{R}^2 \to \mathbb{R}$ represents $\succeq$ that means $u(x) \geq u(y)$ if and only if $x \succeq y$ for all $x, y \in \mathbb{R}^2$. Therefore, $u(x) = u(y)$ if and only if $x \sim y$. However, one can see that $x \sim y$ if and only if $x = y$. (3 marks)

(c) Consider the following possible assumptions made to solve a cost-minimisation problem:

(i) the second order necessary condition holds at all points,

(ii) the production function is quasi-concave,

(iii) the production function is strictly quasi-concave,

(iv) the second order sufficient condition holds at all points.

Briefly explain the advantage of making assumption (ii) instead of (i); assumption (iii) instead of (ii); and assumption (iv) instead of (iii).

SOLUTION: (UNSEEN) If we assume (ii) instead of (i), we know that any point satisfying FOC minimises cost (if all we have is (i), we can't be sure of this), if we assume (iii) instead of (ii), we know that the solution of the cost-minimisation problem is unique (we don't know this if all we have is (ii)) and if we assume (iv) instead of (iii), we know that we can use the implicit function theorem to differentiate FOC and get the derivative of the conditional factor demands wrt to prices ((iii) alone does not guarantee that we can do this). (5 marks)

4. (a) Give examples of economic activities that:

(i) are not included in the Gross Domestic Product.

(ii) are not included in the Gross National Product.

SOLUTION: (SEEN) Multiple solutions possible, for example:

(i) Intermediate goods purchased (that have been turned into final goods and services).

(ii) Goods and services produced within a nation's boundaries by foreign citizens and firms.

(1 mark/example; total 2 marks)

(b) Suppose the economy is described by the five-sector model.

(i) Describe the equilibrium in the model and define the notation used.

SOLUTION: (SEEN)

An equilibrium occurs when:

$$I + G + X = S + T + M$$

where I: Investment spending, G: Government spending on goods and services, X: Export spending, S: Savings, T: Taxes, M: Import spending.

(1 mark for the equation and 1 mark for the notation; total 2 marks)

(ii) Describe what happens if injections exceed leakages over a given time period.

SOLUTION: (SEEN)

- There will be an excess in aggregate demand, motivating an increase in aggregate supply to move towards equilibrium - this is economic growth.
- The resulting increase in aggregate supply may cause firms to increase their labour supply, leading to a fall in unemployment.
- An increase in demand will also increase prices - this leads to inflation.
- Excess in demand will increase imports as consumers buy elsewhere; exports will decrease due to rising prices.

(4 marks)

(c) Consider the oil market. There are n consumers, each having the same utility function, which leads to the following market demand (measured in thousands of barrels):

$$X^*(p) = \begin{cases} n\left(\frac{a-p}{2}\right), & 0 \leq p \leq a, \\ 0, & p \geq a, \end{cases}$$

where $p \geq 0$ is the oil price in pounds per barrel and $a > 0$ constant.

There are also m firms producing oil in a perfect competition and each of the firms has a long-run cost function (reflecting the actual economic costs) $c^*(y) = y(b + y)$, where $y \geq 0$ is the firm's output in thousands of barrels and $b \in (0, a)$ constant.

(i) Compute the individual supply function and use it to compute the market supply, $Y^*(p)$. (*Note: You do not have to check the second order condition.*) (3 marks)

SOLUTION: (SEEN SIMILAR)

First, we have to solve the profit maximisation problem for each firm to calculate their supply function $y_j^*(p), j = 1, \ldots, m$. Profit is given by $py - c^*(y) = py - by - y^2$. The first order condition yields $p - b - 2y = 0$ (for $y > 0$). If the derivative of the profit is non-positive at $y = 0$, then the optimal level of supply is 0; this is true when $p \leq b$.

So the optimal supply is given by:

$$y^*(p) = \frac{p-b}{2}, \quad \text{if} \quad p > b \quad \text{and} \quad y^*(p) = 0, \quad \text{if} \quad p \leq b.$$

Hence the market supply is given by:

$$Y^*(p) = \sum_{j=1}^{m} y_j^*(p) = \begin{cases} m\left(\frac{p-b}{2}\right), & p > b, \\ 0, & p \leq b. \end{cases}$$

(3 marks)

(ii) Can an individual firm ever have negative profit? Explain briefly.

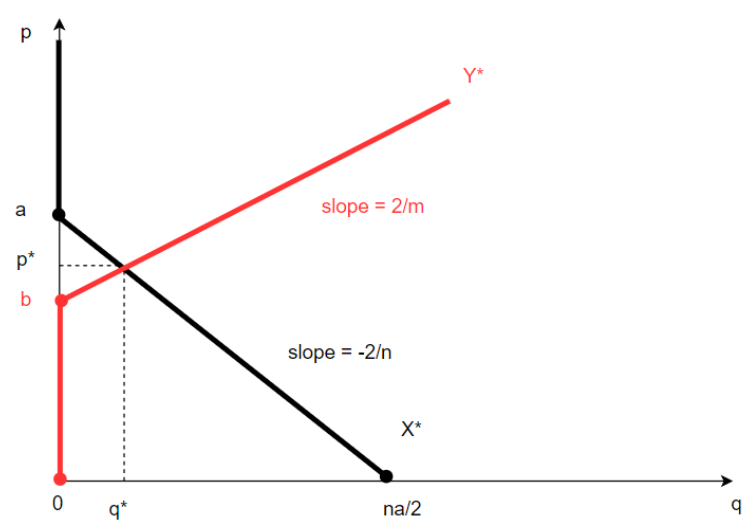
SOLUTION: (SEEN SIMILAR)

An individual firm cannot have negative profit. If $p \leq b$, the firm produces zero and has zero profit. If $p > b$, it has $p = MC(y) > AC(y)$, where $MC(y) = b + 2y$ and $AC(y) = b + y$. In this case, output is positive and profit is also positive.

Alternatively, one can say that an individual firm could never have a negative profit, as it can always produce nothing, giving zero cost and hence no loss. (2 marks)

(iii) Sketch a graph of the market demand curve and the market supply curve and compute the equilibrium price, p^* and the equilibrium quantity, q^*.

SOLUTION: (SEEN SIMILAR)



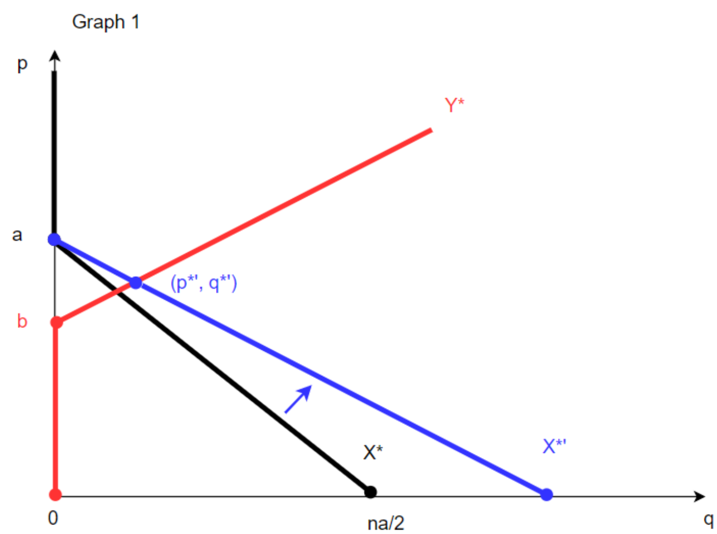
The equilibrium price p^* is the price where market demand and market supply meet:

$$X^*(p^*) = Y^*(p^*) \Leftrightarrow n\left(\frac{a-p^*}{2}\right) = m\left(\frac{p^*-b}{2}\right) \Leftrightarrow p^* = \frac{na+mb}{m+n} \quad \text{measured in pounds/barrel.}$$

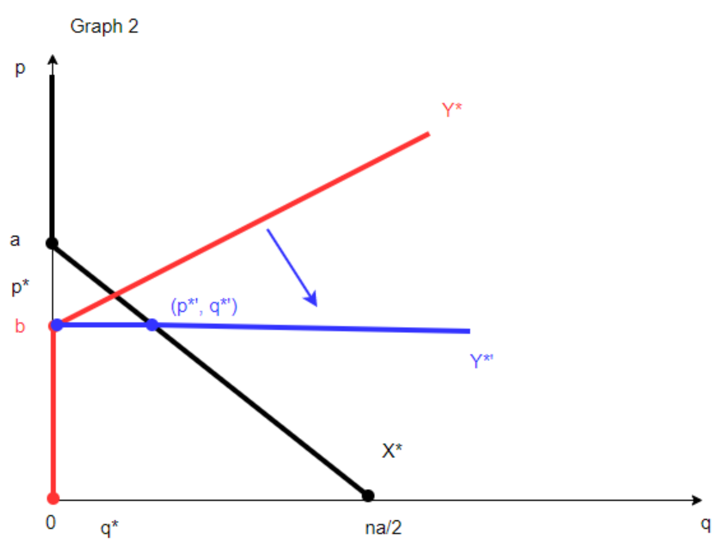
The equilibrium quantity will then be $q^* = X^*(p^*) = Y^*(p^*)$ measured in thousands of barrels. (3 marks)

(iv) Using the graph from part (c) (iii), show graphically and justify what happens to the market equilibrium (p^*, q^*) when (i) n increases while m is held constant; (ii) m increases while n is held constant. (*It would be preferable to draw one graph for each case.*)

SOLUTION: (UNSEEN)



If the number of consumers, n, increases, then quantity increases and thus, price increases, i.e. the new market equilibrium $(p^{*'}, q^{*'})$ is such that $p^{*'} > p^*$ and $q^{*'} > q^*$ (see Graph 1).



If the number of firms, m, increases, then quantity increases and price decreases, i.e. the new market equilibrium $(p^{*'}, q^{*'})$ is such that $p^{*'} < p^*$ and $q^{*'} > q^*$ (see Graph 2). (4 marks)

If your module is taught across multiple year levels, you might have received this form for each level of the module. You are only required to fill this out once for each question.
Please record below, some brief but non-trivial comments for students about how well (or otherwise) the questions were answered. For example, you may wish to comment on common errors and misconceptions, or areas where students have done well. These comments should note any errors in and corrections to the paper. These comments will be made available to students via the MathsCentral Blackboard site and should not contain any information which identifies individual candidates. Any comments which should be kept confidential should be included as confidential comments for the Exam Board and Externals. If you would like to add formulas, please include a sperate pdf file with your email.

ExamModuleCode	QuestionNumber	Comments for Students
MATH96011	1	Question 1 was generally answered well. Part (a) was straightforward; it required to state the consumer-side analogue notions to six firm-side notions. Part (b) was mostly well done, although some students did not justify their answers adequately. Part (c) was intented to be challenging and students lost marks, as it required to link the scale behaviour to the default U-shaped long-run average cost curve and then using this, prove that there is no homogeneous production function (of any degree k in R) that gives a U-shaped long-run average cost curve. In the first part, students were expected to mention the returns to scale (increasing returns to scale, descreasing returns to scale and constant returns to scale). However, some students mentioned economies and diseconomies of scale – this notion was never mentioned in the lectures, problems classes, etc., so it might be a result of an internet search. In the second part, the students were expected to link the homogeneity to the returns to scale through the elasticity of scale, which is a local measure of the scale behaviour (see solutions). However, full marks were given for an alternative reasonable approach, i.e. the direct link to the returns to scale.
MATH96011	2	Parts (a)-(d) were generally well answered. For part (e), many students did not recognise that finding a solution via differentiation lead to a minimum not a maximum in this case. Part (f) had varied responses, the most common errors were to use the wrong conditions or not state the conditions. It was common for studetns to loose marks in part (g). Here, it was expected that the elasticity of scale and law of diminishing marginal productivity would be discussed.

MATH96011	3	Part (a) was well answered. For part (b)(i), many students struggled with showing monotonicity, a common error was to assume a relation which was the same as the one we were trying to prove. Several students also lost marks by only discussing strict convexity or convexity (or strong or weak monotonicity) and not discussing how it related to the other. Part (b)(ii) was generally well answered. For part (c), some students got that (iii) meant that the minimum was unique, but very few other marks were awarded. Lots of students said that the assumptions were getting stronger, but we were looking for interpretation of what this meant we could say about the solution in each case.
MATH96011	4	Parts (a) and (b) were straightforward and were done well by the majority of students. Part (c) was where marks tended to be lost by some students. A complete description of y*(p) requires you to say y*(p) =(p-b)/2 when p > b, and = 0 otherwise. Answers to the question of whether a firm can have negative profit were varied. Most students evaluated the equilibrium values of p and q correctly. However the supply and