<u>Coursework 2</u>

Due Date: Wednesday, 18 Mar 2025 to be submitted electronically on Blackboard.

Please make the solutions you hand in as neat and concise as possible. Show your work. As some problems are shorter than others, the mark scheme will not allocate equal marks to each problem. You are encouraged to perform any necessary integrals, but looking them up or using software to compute them is also acceptable. As always, cite any sources used.

1. A qubit is in the state $|\psi\rangle = |0\rangle$. You measure the Pauli Y operator. What is the probability that you measure $+1$?

2. You have a bag of qubits. Half are in the $+1$ eigenstate of Pauli X and half are in the $+1$ Pauli Z eigenstate. You pick out one of the qubits randomly and measure the operator $\cos(\theta)\hat{Z}+\sin(\theta)\hat{X}$. What is the probability you measure $+1$?

3. Same setup as previous problem. Your aim is to correctly guess after measurement which of the two states you pulled out of the qubit bag. What is the best θ to choose for this purpose?

4. Consider the two-qubit state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. The first qubit is measured in the $\cos(\theta)\hat{Z} + \sin(\theta)\hat{X}$ eigenbasis. Suppose that the $+1$ eigenstate is measured. Then the second qubit is measured in the $\hat{Z}$ eigenbasis. What is the probability that the second qubit is measured to be in the $|1\rangle$ state?

5. Consider the driven harmonic oscillator $\hat{\mathcal{H}} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2 + \lambda\gamma\cos(\Omega t)\hat{x}$. At $t = 0$, the system is in the ground state of the Harmonic oscillator. Using time-dependent perturbation theory, what is the approximate probability that at later time t, the system is in the n^{th} excited state of the Harmonic oscillator.

6. Consider the Hamiltonian $\hat{\mathcal{H}} = \varepsilon\hat{S}_z + \lambda\gamma\hat{S}_x^2$ where $\hat{S}_a$ are spin-s operators. Using first-order perturbation energy, find the approximate eigenenergies of this Hamiltonian.