Classical Dynamics **Autumn Term 2020**

Test 1

Instructions

The deadline is 12 noon on Friday 30 October.

Upload your answers in a single PDF file.

Your answers should be hand-written.

Include your name and CID on your script.

If you are unable to submit via Blackboard/Turnitin email your script to
`maths-student-office@imperial.ac.uk`

1. (i) The motion of a particle of unit mass in the plane is governed by the Lagrangian

$$L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \alpha r,$$

where r, θ are polar coordinates and α is a positive constant.

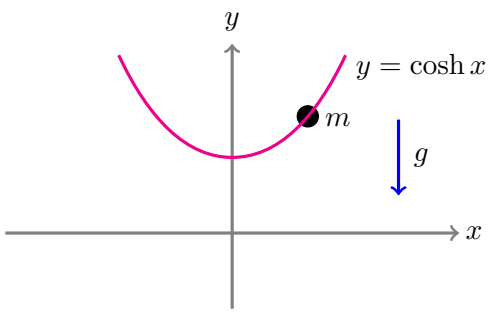
(a) Obtain the equations of motion.

(b) Show that there are circular orbits of the form $r =$ constant.

Determine the period of these orbits (this depends on r).

(9 marks)

(ii) A bead of mass m moves on a wire without friction. The (downwards) acceleration due to gravity has magnitude g. The shape of the wire is described by the equation $y = \cosh x$ (a catenary) as shown in the diagram below. Find a Lagrangian, $L(x, \dot{x})$, that describes the motion of the bead.



(6 marks)

(iii) The motion of a raindrop is described by the Lagrangian

$$L = e^{\gamma t}\left(\frac{1}{2}\dot{y}^2 - gy\right),$$

where γ and g are positive constants. Obtain and solve the equation of motion. At what speed does a raindrop hit the ground?

(10 marks)

(Total: 25 marks)