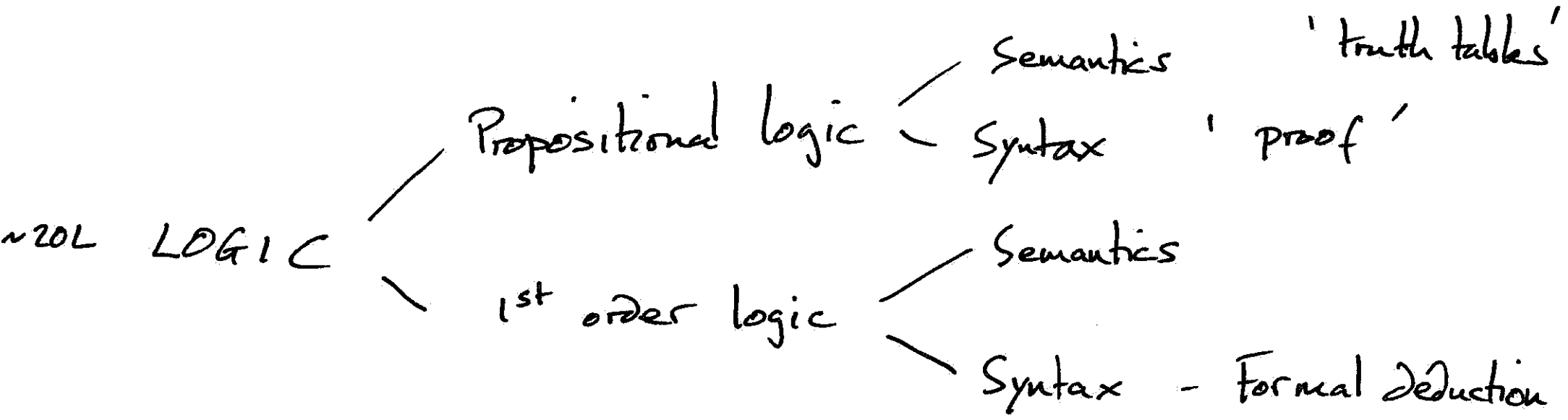


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①



~10L SET THEORY - ZF set theory with AC

$$\left(\left((p \rightarrow q) \wedge (q \rightarrow (\neg p)) \right) \right) \rightarrow (\neg p) \quad \textcircled{2}$$

p
└──────────┘

q
└──────────┘

" If Mr. Jones is happy, then Mr. Jones is unhappy
and if Mr. J. is unhappy then Mr. J. is unhappy.
So Mr. J. is unhappy. "

1. Propositional Logic

1.1 Propositional formulas

'Proposition'

'Statement'

either True (T)

or False (F)

Combine ~~logs~~ basic props. using
connectives :

(1.1.1) Connectives + truth table rules

P, q, \dots statements

Connectives

$(\neg P)$

Negation

P	$(\neg P)$
T	F
F	T

Conjunction ('and') (3)

$(P \wedge q)$ has value T

(\Rightarrow) p and q have value T.

Disjunction ('or')

$(P \vee q)$ has value T

(\Rightarrow) at least one of p, q has value T.

Implication $(P \rightarrow q)$

has value F only when
p has value T and q has value F.

Biconditional $(P \leftrightarrow q)$

has value T precisely when
p, q have the same value.

Summary.

(4)

P	q	$(P \wedge q)$	$(P \vee q)$	$(P \rightarrow q)$	$(P \leftrightarrow q)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

(1.1.2) Def. A propositional formula is obtained from propositional variables p_1, p_2, \dots and connectives in

the following way:

- (i) Any propositional variable is a prop. formula;
- (ii) if ϕ, ψ are formulas then

$(\neg \phi)$ $(\phi \wedge \psi)$ $(\phi \rightarrow \psi)$ ~~$(\phi \rightarrow \psi)$~~ $(\phi \vee \psi)$
 $(\phi \leftrightarrow \psi)$

are formulas.

- (iii) any formula arises in this way
(after a finite number of steps).

Eg. Formulas

$$p_1 \quad p_2 \quad (\neg p_2)$$

$$(p_1 \rightarrow (\neg p_2))$$

$$((p_1 \rightarrow (\neg p_2)) \rightarrow p_2) : \phi$$

Not formulas : $p_1 \wedge p_2$

$$)(\neg p_2$$

Remarks. ① Every formula is either a prop. variable or is built from 'shorter' formulas in a unique way

$$\begin{array}{c} \phi \\ / \quad \backslash \\ (p_1 \rightarrow (\neg p_2)) \quad p_2 \\ / \end{array}$$

$$\begin{array}{c} p_1 \quad (\neg p_2) \\ | \\ p_2 \end{array}$$

② Any assignment of truth values to the prop. variables in a formula ϕ determines the truth value for ϕ in a unique way, using I.I.I.

Eg. $\phi : ((p_1 \rightarrow (\neg p_2)) \rightarrow p_1)$

p_1	p_2	$(\neg p_2)$	$(p_1 \rightarrow (\neg p_2))$	ϕ
T	T	F	F	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

'truth table of ϕ '

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(1.1.3) Def. Let $n \in \mathbb{N}$

① A truth function of n variables is a function $f: \{T, F\}^n \rightarrow \{T, F\}$

(where

$\{T, F\} = \{(x_1, \dots, x_n) : \text{each } x_i \in \{T, F\}\}$)

② Suppose ϕ is a formula whose variables are amongst p_1, \dots, p_n . We obtain a function $F_\phi: \{T, F\}^n \rightarrow \{T, F\}$ whose value at

$(x_1, \dots, x_n) \in \{T, F\}^n$ is the truth value of ϕ when p_i has value x_i (for

$i \leq n$), according to (1.1.1). ⑥

F_ϕ is the truth function of ϕ .

Eg. in example

$$F_\phi(F, T) = F.$$

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Read info, sheet.

Compute truth table for $\neg p \rightarrow q$.

Qn 1.