IMPERIAL

MATH70142

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May 2024

This paper is also taken for the relevant examination for the
Associateship of the Royal College of Science

The Mathematics of Business and Economics

Date: Friday, May 24, 2024

Time: 14:00 – 16:30 (BST)

Time Allowed: 2.5 hours

This paper has 5 Questions.

Please Answer All Questions in 1 Answer Booklet

Candidates should start their solutions to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

Any required additional material(s) will be provided.

Allow margins for marking.

Credit will be given for all questions attempted.

Each question carries equal weight.

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO

1. (a) Decide if the following statements are true or false. Justify each answer in around two to three sentences.

   (i) The law of demand holds for all goods. (2 marks)

   (ii) For the same level of output, the long-run costs never exceed the short-run costs. (2 marks)

   (iii) Profit-maximizing monopolists will never go out of business, since they can control the output prices. (2 marks)

   (iv) Two different isoquant curves cannot intersect one another. (2 marks)

   (v) As a nominal value, the Gross Domestic Product (GDP) of a country is inflation-adjusted. An increase in prices does not directly affect the GDP. (1 mark)

(b) Suppose a firm's production is based on two input goods x_1 and x_2, with respective prices $w_1 > 0$ and $w_2 > 0$, and for $\alpha, \beta \in (0, 2)$ its production function is

$$f(x_1, x_2) = \sqrt{x_1^{\alpha} x_2^{\beta}}. \tag{1}$$

(i) Compute the marginal product with respect to good x_2. What is the limit of the marginal product for $x_2 \to \infty$ (x_2 tends to infinity)? Provide an *economic interpretation* of your result. (3 marks)

(ii) Compute the conditional factor demand functions $x_1^*(w_1, w_2, y)$ and $x_2^*(w_1, w_2, y)$, with output $y > 0$. [*You may assume that the second-order condition is satisfied.*] (5 marks)

(iii) You can now assume that the values for α and β are chosen such that $x_1^*(w_1, w_2, y) = y\sqrt{\frac{w_2}{w_1}}$, and $x_2^*(w_1, w_2, y) = y\sqrt{\frac{w_1}{w_2}}$. Let $p > 0$ be the given output price, show that the profit function $\pi^*(y, p, w_1, w_2)$ is positively homogeneous of degree 1 in prices p, w_1, w_2. (3 marks)

(Total: 20 marks)

2. (a) Prove the following statement or contradict via a counterexample:
"Any real valued function $f : \mathbb{R} \to \mathbb{R}$ cannot be both quasi-concave and quasi-convex."

(3 marks)

(b) Suppose at time point t you observe two price-taking firms *A* and *B*, each producing a single output good with price p, using two input goods x_1, x_2 with prices $w_1 > 0, w_2 > 0$. You have

Dataset of firm A:

t	p	w_1	w_2	x_1	x_2	y
1	5	2	1	1	1	1
2	11	1	2	3	3	2

Dataset of firm B:

t	p	w_1	w_2	x_1	x_2	y
1	5	2	1	12	3	1
2	11	1	2	4	7	3

(i) Show that firm *A* satisfies the WAPM. Does it also satisfy the WACM? (4 marks)

(ii) Demonstrate that firm *B* is not operating rationally. (3 marks)

(iii) Assume you would be able to adjust the input prices w_1 and w_2 at time point $t = 1$, denoted by w_1^1 and w_2^1 (all other quantities staying fixed). Determine the set of all w_1^1 and w_2^1, such that datasets A and B satisfy the WACM. (4 marks)

(c) (i) Briefly discuss differences between Marshallian demand and Hicksian demand.

(2 marks)

(ii) Suppose $u : \mathbb{R}^2_{\geq 0} \to \mathbb{R}$ is a strictly increasing, continuous, and quasi-convcave utility function of some consumer with budget $m > 0$, and for two goods with strictly positive prices $p_1, p_2 > 0$. Let $\underline{x}^* = (x_1^*, x_2^*)$ be the corresponding Marshallian demand, e be the expenditure function, and v be the indirect utility function. Assuming good 1 is an inferior good, use the Slutsky equation to show that for $q_1 > p_1$ (with $q_2 = p_2$), it holds that

$$\int_{p_1}^{q_1} \frac{\mathrm{d}\, e\Big\{(t, p_2), v(\underline{p}, m)\Big\}}{\mathrm{d}t}\, \mathrm{d}t < \int_{p_1}^{q_1} x_1^*\Big\{(t, p_2), m\Big\}\, \mathrm{d}t. \tag{2}$$

(You may assume that the usual regularity conditions for the Slutsky equation hold.)

(4 marks)

(Total: 20 marks)

3. (a) Anne likes two goods, and she is too attached to each of them to let go of one. Her strongly monotonic preferences can be represented by the following utility function

$$u(x_1, x_2) = \sqrt{x_1 x_2}.$$

The good x_i has price $p_i > 0, i = 1, 2$, and Anne has a fixed budget $m > 0$.

(i) Write down Anne's budget set, and briefly describe the Walras' law. (2 marks)

(ii) Compute the bundle that maximizes Anne's utility *[You may assume that the second-order condition is satisfied.]*.

(5 marks)

(iii) Using your solution in (ii), answer the following questions and justify your answer.

(I) Is good x_2 a normal or inferior good for Anne? (2 marks)

(II) Are goods x_1 and x_2 complements, substitutes, or unrelated? (2 marks)

(iv) Anna's friends Elisa and George both claim that they have the same preferences as Anna.

(I) George's preferences can be represented by the utility function

$$u_{\mathsf{log}}(x_1, x_2) = \frac{1}{2}\log(x_1) + \frac{1}{2}\log(x_2).$$

(II) Elisa's preferences can be represented by the utility function

$$u_2(x_1, x_2) = \{u_{\mathsf{log}}(x_1, x_2)\}^2.$$

Determine if their claims are correct (justify your reasoning). (5 marks)

(b) Suppose $\succeq$ is a preference relation over $\mathbb{R}^n_{\geq 0}$ that satisfies the strong monotonicity property. Show that the strong monotonicity property implies local nonsatiation of the preference relation $\succeq$ on $\mathbb{R}^n_{\geq 0}$. Briefly describe the local nonsatiation property in your own words. *[Hint: For the proof, you may use $||e|| = \sqrt{n}$, where $e = \{1, \ldots, 1\} \in \mathbb{R}^n_{\geq 0}$ is a vector of n ones.]* (4 marks)

(Total: 20 marks)

4. (a) Tom, Anne and Oliver are late for their breakfast buffet. Everything has been consumed already except for exactly one cup of coffee and one bagel. Tom does not like bagels, but cannot start a day without a cup of coffee. Anne does not drink coffee, but likes bagels, and Oliver enjoys both.

   (i) Write down all Pareto efficient allocations of the two goods. (2 marks)

   (ii) Reason why Pareto efficiency might not lead to fair allocations (you may use your answer from part (i) for an example). (1 mark)

(b) Consider the market for tobacco, and suppose the market demand for tobacco is given by $X^*(p) = 20 - 2p$, for $0 \leq p \leq 10$, with p being the price of one unit tobacco. The long-run cost function of a typical tobacco firm is given by $c^*(y) = 2y^2 + 2$.

   (i) Show that the individual long-run supply curve for some firm j in the market is given via $y_j^*(p) = p/4$. [*You may assume that the second-order condition is satisfied.*] (1 mark)

   (ii) Verify that in the long-run a total of exactly $J = 12$ firms will operate in the market. Determine the long-run equilibrium price and equilibrium quantity. (3 marks)

   (iii) Compute the long-run consumers' surplus, producers' surplus, and community surplus. (3 marks)

   (iv) Sketch the situation graphically (quantity on the horizontal axis, price on the vertical axis). Depict the market supply, market demand, equilibrium price and quantity, and the consumers' and producers' surplus. (2 marks)

   (v) To prevent health issues, the government wants to limit consumption of tobacco in the market to not more than $1/3$ of the current long-run equilibrium quantity. To this end, it wants to distribute licenses to firms, and only licensed firms would then be allowed to supply the market. What is the maximum amount of licenses that the government could issue to reach its goal? (2 marks)

   (vi) Instead of distributing licenses to firms, the political opposition suggests to introduce an ad valorem tax on tobacco. What is the tax percentage on tobacco that needs to be introduced to reduce the long-run market supply of tobacco to the maximum level requested by the government in part (v)? (3 marks)

   (vii) In consideration of community surplus, should the government follow the suggestions of the political opposition, or should it implement its own plan? Justify your reasoning by means of graphical illustrations. (You may add the graphical justifications to your plot from part (iv)). (3 marks)

(Total: 20 marks)

5. (a) You own an asset with current price p_0, and you know that in two years the asset will be worth $p_2 = £36$. Furthermore, as a separate investment, your local bank offers an interest rate of 20% per year on each £ invested in the bank. What would be the lowest price p_0 that you would still accept to sell your asset for today? (Justify your answer.) (3 marks)

(b) Briefly explain (in around three to four sentences) why expected utility is a reasonable objective for choice problems in the face of uncertainty. (3 marks)

(c) A consumer considers a lottery with two possible outcomes, and the consumer's preferences can be represented by an expected utility function. Show that any positive affine transformation of the expected utility function is again an expected utility function that describes the consumer's behaviour. (4 marks)

(d) Peter has some wealth $\tilde{w} \in \mathbb{R}_{>0}$, and he is considering investing some amount x in a risky asset, where $0 \leq x \leq \tilde{w}$. With a good future outcome, which happens with a probability of $\pi \in [0, 1]$, the asset earns a return of $r_g \in (0, 1)$, in which case Peter's wealth increases to $\tilde{w} + xr_g$. With a bad future outcome, the asset earns a return of $r_b \in (-1, 0)$, such that his wealth decreases to $\tilde{w} + xr_b$. How much of his wealth would Peter invest in the risky asset if:

(i) His preferences over wealth w can be represented by the utility function $u(w) = \log(w)$ (here $\log(\cdot)$ denotes the natural logarithm), and $\pi = 0.4$, $r_g = 0.2$, and $r_b = -0.1$. (3 marks)

(ii) His preferences over wealth w can be represented by the utility function $u(w) = \sqrt{w}$, and $\pi = 0.2$, $r_g = 0.1$, and $r_b = -0.05$. (2 marks)

(e) A risk-averse consumer is contemplating a gamble that might increase or reduce his current wealth. The consumer's preferences of the gamble can be represented by an expected utility function. Let $u(w)$ be the consumer's utility of wealth $w \in \mathbb{R}$. ($u : \mathbb{R} \to \mathbb{R}$ being continuous and strictly monotonic).

(i) Reason why $A(w) = -u''(w)/u'(w)$ provides a reasonable measure to assess the consumer's attitude towards risk. (4 marks)

(ii) Let $A(w)$ be decreasing for increasing w, what can you say about the consumer's attitude towards risk? (Briefly describe in one sentence.) (1 mark)

(Total: 20 marks)

BSc and MSci EXAMINATIONS (MATHEMATICS)

May 2024

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MATH60142/70142

Mathematics of Business and Economics (Solutions)

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1. (a) (i) This is *false*. The law of demand states that (ceteris paribus) an increase in price of a good will usually lead to a drop in its demand. However, we have seen in the lecture that for Giffen goods (e.g., staple food) an increase in price will lead to an increase in demand. seen ⇓ 2, A

(ii) This is *true*. In the long-run all input goods are variable, including the ones that are fixed in the short-run. In the short-run, we thus have fixed costs, which in the long-run can be optimized as variable costs. (This still holds in the presence of quasi-fixed costs, which might be present in the long-run and short-run scenario.) 2, A

(iii) This is *false*. We have seen in the lecture that monopolists must respect the market demand for their product, i.e., they set their prices according to the inverse of the market's demand function. Analogously to competitive firms, if the monopolist's (short-run) average (variable) costs are greater than the output price they can achieve, then the shutdown-condition is fulfilled and their profit maximising position is to go out of business. 2, A

(iv) This is *true*. Each isoquant represents a unique level of output. If two isoquants intersect, they would have the same level of production at the point of intersection, which is not possible. (They only intersect if they are identical.) unseen ⇓ 2, C

(v) This is *false*. We have discussed in the lecture that the nominal GDP refers to a current monetary value, and it does not adjust for effects of inflation (other than the real GDP). 1, A

(b) (i) The marginal product with respect to good x_2 is computed via seen ⇓ 1, A

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = \frac{\beta}{2} x_1^{\alpha/2} x_2^{\beta/2-1} = \frac{\beta}{2} \frac{x_1^{\alpha/2}}{x_2^{1-\beta/2}}. \tag{1}$$

unseen ⇓

Since $1-\frac{\beta}{2} > 0$, for $x_2 \to \infty$, we have $\lim_{x_2\to\infty} \frac{\partial f(x_1,x_2)}{\partial x_2} = 0$. This means that as the level of x_2 tends to infinity (in the limit) there is no additional increase in productivity when adding an additional unit of x_2, when keeping all other input factors fixed. Since the marginal product in (1) is non-negative, this can be interpreted as a result of the *law of diminishing marginal productivity*. (Ceteris paribus, for higher levels of input, there is a smaller increase in production when adding another unit.) 2, B

(ii) We seek to find meth seen ⇓

$$\underset{x_1,x_2\in\mathbb{R}_{\geq 0}}{\operatorname{argmin}}\ w_1x_1 + w_2x_2, \quad \text{s.t., } f(\underline{x}) = x_1^{\alpha/2} x_2^{\beta/2} = y.$$

The Lagrangian of the cost minimisation problem is then given via

$$\mathcal{L}(x_1, x_2, \lambda) = w_1x_1 + w_2x_2 - \lambda(x_1^{\alpha/2} x_2^{\beta/2} - y)$$

The first-order conditions are given by

$$\frac{\partial}{\partial \lambda}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow x_1^{\alpha/2} x_2^{\beta/2} = y \tag{1}$$

$$\frac{\partial}{\partial x_1}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow \lambda \frac{\alpha}{2} x_1^{\alpha/2-1} x_2^{\beta/2} = w_1 \tag{2}$$

$$\frac{\partial}{\partial x_2}\mathcal{L}(x_1, x_2, \lambda) = 0 \Rightarrow \lambda \frac{\beta}{2} x_1^{\alpha/2} x_2^{\beta/2-1} = w_2 \tag{3}$$

$\boxed{3, \text{A}}$

To obtain the cost minimising input bundle as a function of y, w_1, w_2, we simply solve the first-order conditions. By dividing (2) by (3), we obtain

$$\begin{aligned} \frac{\alpha}{\beta}\frac{x_2}{x_1} &= \frac{w_1}{w_2} \\ \Rightarrow x_2 &= \frac{w_1}{w_2}\frac{\beta}{\alpha} x_1 \end{aligned} \tag{4}$$

Substituting (4) into (1), we obtain

$$x_1^{\frac{\alpha+\beta}{2}} \left(\frac{w_1}{w_2}\frac{\beta}{\alpha}\right)^{\beta/2} = y \tag{5}$$

From that, we obtain the solution for the conditional factor demand for good 1 via

$$x_1^*(w_1, w_2, y) = y^{\frac{2}{\alpha+\beta}} \left(\frac{w_1}{w_2}\frac{\beta}{\alpha}\right)^{-\frac{\beta}{\alpha+\beta}} \tag{6}$$

and thus, by substituting (6) back into (4), we obtain

$$x_2^*(w_1, w_2, y) = y^{\frac{2}{\alpha+\beta}} \left(\frac{w_1}{w_2}\frac{\beta}{\alpha}\right)^{1-\frac{\beta}{\alpha+\beta}} \tag{7}$$

$\boxed{2, \text{B}}$

(iii) The profit of the firm is computed via revenue minus costs. As a function of output level y and prices p, w_1, w_2, the profit function can thus be written via $\boxed{\text{unseen} \Downarrow}$

$$\pi^*(y, p, w_1, w_2) = py - c^*(w_1, w_2, y), \tag{8}$$

where $c^*(w_1, w_2, y)$ is the cost function of the firm, which can be computed via

$$\begin{aligned} c^*(w_1, w_2, y) &= w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y) \\ &= 2y\sqrt{w_1 w_2} \end{aligned} \tag{9}$$

Then, via (8) and (9), we obtain

$$\begin{aligned} \pi^*(y, tp, tw_1, tw_2) &= tpy - 2y\sqrt{tw_1 tw_2} \\ &= t(py - 2y\sqrt{w_1 w_2}) \\ &= t\pi^*(y, p, w_1, w_2), \end{aligned}$$

which shows the required positive homogeneity of degree 1 (in prices p, w_1, and w_2). $\boxed{3, \text{C}}$

2. (a) The statement is false, and many examples of real valued functions $f : \mathbb{R} \to \mathbb{R}$ can be chosen as a counterexample for contradiction. For instance, we can choose a linear function $f(x) = x$, which is monotonically increasing and thus quasi-concave (as shown on problem sheet 1). Further, we have seen that a function is quasi-convex if its negative is quasi-concave. Since $g(x) = -f(x) = -x$ is monotonically decreasing, g is quasi-concave. f is thus both quasi-convex and quasi-concave, which contradicts the statement. (An alternative solution is to follow the definitions of quasi-concavity and quasi-convexity to demonstrate a counterexample.) $\boxed{\text{meth seen} \Downarrow}$ $\boxed{3, \text{B}}$

(b) (i) We first check the WAPM. We compare the profit of firm A obtained at time point $t = 1$, with the profit that they could have obtained by choosing the input and output from time point $s = 2$, with the prices at $t = 1$, i.e., $\boxed{\text{meth seen} \Downarrow}$

$$p^1y^1 - w_1^1x_1^1 - w_2^1x_2^1 = 5 - 2 - 1 = 2$$

(recall that here superscripts indicate the respective time points), which is greater than

$$p^1y^2 - w_1^1x_1^2 - w_2^1x_2^2 = 10 - 6 - 3 = 1$$

Analogously, for time point $t = 2$ and $s = 1$, we obtain

$$p^2y^2 - w_1^2x_1^2 - w_2^2x_2^2 = 22 - 3 - 6 = 13$$

and

$$p^2y^1 - w_1^2x_1^1 - w_2^1x_2^1 = 11 - 1 - 2 = 8 < 13.$$

Thus, for both time points $t = 1, 2$, the firm A chose the profit maximizing bundle. It thus satisfies the WAPM, which directly implies the WACM as shown in the lecture (alternatively, one can use the definition of the WACM to show that it is indeed satisfied). $\boxed{4, \text{A}}$

(ii) To demonstrate that firm B does not operate rationally, we can show that the WAPM and/or the WACM is violated for the observed dataset of firm B. We start with the WACM, and actually only have to consider the constellation $t = 1$ and $s = 2$, since $y^2 = 3 > 1 = y^1$. Then,

$$w_1^1x_1^1 + w_2^1x_2^1 = 24 + 3 = 27 \quad > \quad 15 = 8 + 7 = w_1^1x_1^2 + w_2^1x_2^2,$$

which shows that the WACM is violated at time point $t = 1$, since choosing the input bundle from time point $s = 2$ would lead to lower cost, while achieving a higher level of output. $\boxed{3, \text{A}}$

(iii) To find the required set of w_1^1 and w_2^1, we first note that we only need to consider the WACM at time point $t = 1$, since for both firms, we have $y^2 > y^1$. We first derive conditions to satisfy the WACM for firm B, i.e. $\boxed{\text{unseen} \Downarrow}$

$$\begin{gathered} w_1^1x_1^1 + w_2^1x_2^1 < w_1^1x_1^2 + w_2^1x_2^2 \Leftrightarrow \\ 12w_1^1 + 3w_2^1 < 4w_1^1 + 7w_2^1 \Leftrightarrow \\ w_1^1 < \frac{1}{2}w_2^1 \end{gathered}$$

$\boxed{2, \text{C}}$

Now, for firm A, we have

$$\begin{aligned} w_1^1 x_1^1 + w_2^1 x_2^1 < w_1^1 x_1^2 + w_2^1 x_2^2 &\Leftrightarrow \\ w_1^1 + w_2^1 < 3w_1^1 + 3w_2^1 &\Leftrightarrow \\ 2w_1^1 + 2w_2^1 > 0, \end{aligned}$$

where the last inequality always holds, since prices are strictly positive. The required set is thus given via $\{(w_1^1, w_2^1) \in R_{>0}^2 : w_1^1 < \frac{1}{2} w_2^1\}$. $\boxed{2, \text{D}}$

(c) (i) The Marshallian demand (also called 'uncompensated' demand) is a demand function that maximizes the utilities of a consumer given the prices of a good and the consumers budget/income/wealth. The Marshallian demand is observable/measurable, since we can observe the consumer's optimal consumption of a good for different prices and the same budget/income. The Hicksian demand is a function of prices and utility, it minimizes the expenditure needed to (at least) obtain the given level of utility for given prices. It is also called 'compensated' demand since the level of utility is fixed, which e.g., for increasing prices would require some compensation of the budget. Since utilities are not directly observable, the Hicksian demand is not observable. $\boxed{\text{seen} \Downarrow}$ $\boxed{2, \text{A}}$

(ii) We want to show that (under the given conditions) the following holds $\boxed{\text{meth seen} \Downarrow}$

$$\int_{p_1}^{q_1} \frac{\mathrm{d}\, e\big\{(t, p_2), v(\underline{p}, m)\big\}}{\mathrm{d}t}\, \mathrm{d}t < \int_{p_1}^{q_1} x_1^*\big\{(t, p_2), m\big\}\, \mathrm{d}t. \tag{10}$$

We start by noting that via Shephard's lemma, we have

$$\int_{p_1}^{q_1} \frac{\mathrm{d}\, e\big\{(t, p_2), v(\underline{p}, m)\big\}}{\mathrm{d}t}\, \mathrm{d}t = \int_{p_1}^{q_1} x_{H,1}^*\{(t, p_2), v(\underline{p}, m)\}\, \mathrm{d}t,$$

where $x_{H,1}^*\{(t, p_2), v(\underline{p}, m)\}$ denotes the Hicksian demand function. Further, in the lecture we have seen that

$$x_{H,1}^*\{\underline{p}, v(\underline{p}, m)\} = x_1^*(\underline{p}, m), \tag{11}$$

which means that the both integrands in (10) coincide at their left endpoint. We can then use the Slutsky equation to compare the two integrands for $p_1 \le t \le q_1$, obtaining

$$\frac{\mathrm{d}\, x_{H,1}^*\big\{(t, p_2), v(\underline{p}, m)\big\}}{\mathrm{d}t} - \frac{\mathrm{d}\, x_1^*\big\{(t, p_2), m\big\}}{\mathrm{d}t} = \frac{\mathrm{d}x_1^*\big\{(t, p_2), m\big\}}{\mathrm{d}m} x_1^*\big\{(t, p_2), m\big\} < 0,$$

where we use the fact that good 1 is an inferior good (i.e., an increase in income will result in a decrease in demand). This means for an inferior good the Marshallian demand grows faster than the Hicksian demand, and using (11), we obtain for $q_1 \ge t > p_1$,

$$x_{H,1}^*\big\{(t, p_2), v(\underline{p}, m)\big\} < x_1^*\big\{(t, p_2), m\big\},$$

which leads to (10) via the monotonicity of the integral.

$\square$ $\boxed{4, \text{D}}$

3. (a) (i) Anne's budget set $B_{\underline{p},m}$ contains the consumption bundles that Anne can afford, i.e., $\boxed{\text{seen} \Downarrow}$

$$B_{\underline{p},m} = \{\underline{x} \in R^2_{\geq 0} : p_1 x_1 + p_2 x_2 \leq m\}. \tag{12}$$

Walras' law states that utilities are maximised only if people spend all their money, i.e., the utility maximizing consumption bundle is on the budget line, i.e., $\underline{x} \in \partial B_{\underline{p},m}$. $\boxed{2, \text{A}}$

(ii) We seek to maximise the utility function $u(\underline{x})$ subject to the budget constraint $p_1 x_1 + p_2 x_2 = m$. $\boxed{\text{meth seen} \Downarrow}$

The Lagrangian is given via

$$\mathcal{L}(x_1, x_2, \lambda) = u(\underline{x}) - \lambda(\underline{p}\underline{x} - m) = \sqrt{x_1 x_2} - \lambda(p_1 x_1 + p_2 x_2 - m).$$

Then, the first-order conditions are given by:

$$\frac{\partial}{\partial \lambda}\mathcal{L}(x_1, x_2, \lambda) = 0 \Leftrightarrow p_1 x_1 + p_2 x_2 = m \tag{1}$$

$$\frac{\partial}{\partial x_1}\mathcal{L}(x_1, x_2, \lambda) = 0 \Leftrightarrow \frac{1}{2} x_1^{-1/2} x_2^{1/2} = \lambda p_1 \tag{2}$$

$$\frac{\partial}{\partial x_2}\mathcal{L}(x_1, x_2, \lambda) = 0 \Leftrightarrow \frac{1}{2} x_1^{1/2} x_2^{-1/2} = \lambda p_2 \tag{3}$$

Dividing (2) by (3) gives:

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

and rearranging allows us to express, e.g. x_2 in terms of x_1:

$$x_2 = \frac{p_1}{p_2} x_1. \tag{4}$$

Substituting (4) into the constraint in (1) gives:

$$\begin{aligned} p_1 x_1 + p_2 \left(\frac{p_1}{p_2} x_1\right) &= m \\ \Leftrightarrow 2 p_1 x_1 &= m. \end{aligned}$$

We thus obtain the utility maximizing bundle

$$x_1^*(\underline{p}, m) = \frac{m}{2p_1} \tag{5}$$

and by symmetry,

$$x_2^*(\underline{p}, m) = \frac{m}{2p_2} \tag{6}$$

$\boxed{5, \text{A}}$

(iii) (I) We can compute

$$\frac{\partial x_2^*(\underline{p}, m)}{\partial m} = \frac{1}{2p_2},$$

which is strictly positive, since prices p_2 are strictly positive. That means an increase in income will lead to an increase in demand of good x_2. It is thus a normal good.

seen ⇓

2, B

(II) The relation of the two goods can be determined via the cross-price elasticity of demand, which is given via

$$\frac{\partial x_1^*(\underline{p}, m)}{\partial p_2} \frac{p_2}{x_1^*(\underline{p}, m)} = 0,$$
$$\frac{\partial x_2^*(\underline{p}, m)}{\partial p_1} \frac{p_1}{x_2^*(\underline{p}, m)} = 0,$$

and the goods x_1 and x_2 are thus unrelated (neither substitutes, nor complements).

2, B

(iv) (I) George's claim is correct. We can directly see that $u_{\log}(x_1, x_2) = \log(u(x_1, x_2))$, which is a strictly increasing transformation of u, and it thus represents the same preferences.

2, B

(II) Elisa's claim is incorrect. $u_2(x_1, x_2) = \{u_{\log}(x_1, x_2)\}^2$ is not a strictly increasing transformation of u, and we can find an example to contradict her claim. For instance, let $x_1', x_2' \in (0, 1)$, s.t. $u(x_1', x_2') \in (0, 1)$ (such x_1', x_2' exist since $m > 0$). Then, $u(\frac{1}{2}\underline{x}') < u(\underline{x}') < 1$, and $\frac{1}{2}\underline{x}' \prec \underline{x}'$ (strong monotonicity). However,

unseen ⇓

$$u_2(\underline{x}') = \left[\log\left\{u(x_1', x_2')\right\}\right]^2 < \left[\log\left\{u\left(\frac{1}{2}x_1', \frac{1}{2}x_2'\right)\right\}\right]^2 = u_2(\frac{1}{2}\underline{x}'), \quad (7)$$

which would lead to $\underline{x}' \prec \frac{1}{2}\underline{x}'$.

3, C

(b) An interpretation of the local nonsatiation property is, that there is always some room for improvement, i.e., for any bundle of goods there is always another bundle of goods (arbitrary close) that is strictly preferred to it (leading to higher utility).

unseen ⇓

To prove the assertion, let $e = \{1, \ldots, 1\} \in \mathbb{R}^n_{\geq 0}$ be a vector of n ones, and consider some fixed $\epsilon > 0$. Then, for any $\underline{x} \in \mathbb{R}^n_{\geq 0}$, it holds that $(\underline{x} + \lambda e) \in \mathbb{R}^n_{\geq 0}$ for any $\lambda > 0$. Further, due to strong monotonicity, since clearly $(\underline{x} + \lambda e) > \underline{x}$, it holds that $(\underline{x} + \lambda e) \succ \underline{x}$. We further have

$$||(\underline{x} + \lambda e) - \underline{x}|| = \lambda ||e|| = \lambda\sqrt{n}. \quad (8)$$

Thus, for $\lambda < \frac{\epsilon}{\sqrt{n}}$, it follows that $||(\underline{x} + \lambda e) - \underline{x}|| < \epsilon$, and $(\underline{x} + \lambda e) \succ \underline{x}$ as required. □

4, D

4. (a) (i) There are exactly 4 Pareto efficient allocations. (i) Oliver gets both, the coffee and the bagel. (ii) Anne gets the bagel, and Tom the coffee. (iii) Anne gets the bagel, and Oliver the coffee, and (iv) Tom gets the coffee, and Oliver gets the bagel. seen/sim.seen ⇓ 2, B

(ii) A Pareto efficient allocation includes the option of Oliver having both goods, while Anne and Tom do not get anything, even though they might have higher utilities in consuming the respective goods. 1, A

(b) Consider the market for tobacco, and suppose the market demand for tobacco is given by $X^*(p) = 20 - 2p$, for $0 \leq p \leq 10$, with p being the price of one unit tobacco. The long-run cost function of a typical tobacco firm is given by $c^*(y) = 2y^2 + 2$.

meth seen ⇓

(i) To obtain the supply function $y_j^*(p)$, we need to solve the profit maximization of firm j. The FOC of the profit maximization yields $p = c^{*\prime}(y) = 4y$, which (provided the SOC) directly yields $y_j^*(p) = p/4$. Further, note that the converse of the shutdown condition is that $2y \leq p$. Inserting $y_j^*(p)$ yields $p/2 \leq p$, which indeed holds for all $p \geq 0$. 1, A

(ii) To verify that $J = 12$ firms operate in the long-run, we first compute the market supply for $J = 12$, firms via $Y(p^*) = Jp/4 = 3p$. The equilibrium price is then obtained via meth seen ⇓

$$X^*(p^*) = Y^*(p^*) \Leftrightarrow 20 - 2p^* = 3p^* \Leftrightarrow p^* = 4.$$

For a price of $p^* = 4$, the firm would have a profit of

$$\begin{aligned}\pi_j^*(4) &= 4y_j^*(4) - c^*(y_j^*(4)) \\ &= \frac{p^{*2}}{4} - \frac{2p^{*2}}{16} - 2 = 4 - 2 - 2 = 0\end{aligned}$$

Firms will enter the market as long as their individual profit is non-negative, i.e., $\pi^*(y) \geq 0$. Since, $p^* = 4$ leads to a profit of exactly zero, no other firms will enter the market, and thus $J^* = 12$. Further, $p^* = 4$ is indeed the market equilibrium price and the market equilibrium quantity is $q^* = Y^*(p^*) = X^*(p^*) = 12$.

3, B

(iii) The consumers' surplus (CS) at $p^* = 4$ is given by seen ⇓

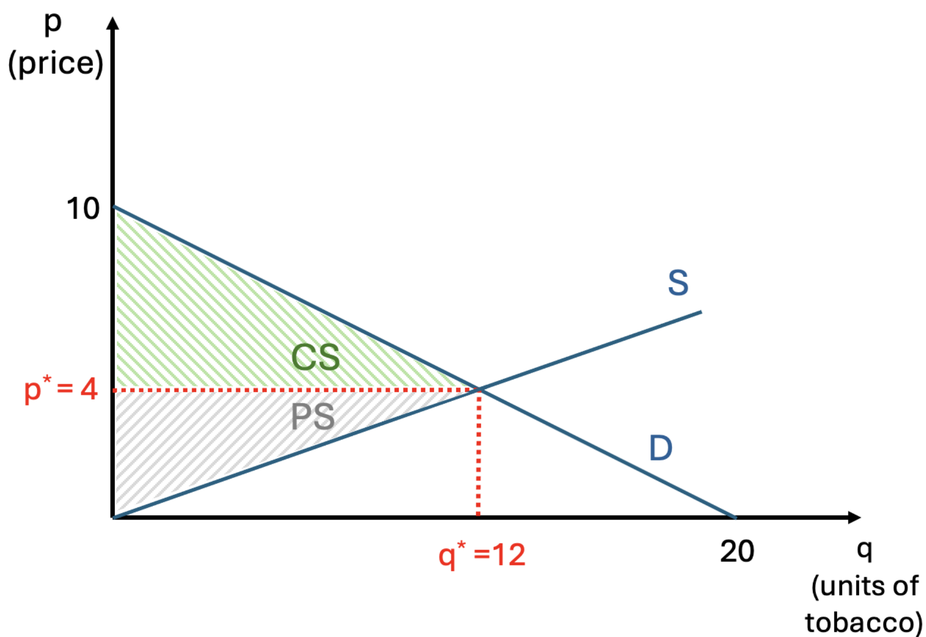
$$\text{CS}(p^*) = \int_{p^*}^{\infty} X^*(p)dp = \int_4^{10} (20 - 2p)dp = 36.$$

The producers' surplus (PS) is

$$\text{PS}(p^*) = \int_0^{p^*} Y^*(p)dp = \int_0^4 (3p)dp = 24.$$

The community surplus is then given by $\text{PS}(p^*) + \text{CS}(p^*) = 60$. 3, A

(iv) In the sketch below, CS and PS denote consumers' and producers' surplus, S and D denote the market supply and demand curves. (Note that, since quantity is shown on the x-axis and price on the y-axis, the inverse demand and inverse supply curves have to be drawn.) seen/sim.seen ⇓



2, B

(v) Since the market demand does not change, we can calculate the price p' that would lead to a demand of exactly $q' = 1/3q^* = 4$, via unseen ⇓

$$X^*(p') = q' \Leftrightarrow 20 - 2p' = 4 \Leftrightarrow p' = 8.$$

For the market to be in the equilibrium at price $p' = 8$ and quantity $q' = 4$, we need

$$Y^*(p') = X^*(p') \Leftrightarrow J\frac{p'}{4} = q' \Leftrightarrow J = 2.$$

The government could thus issue at most $J = 2$ licenses to reach its goal. 2, C

(vi) In order to quantify the effect of the tax on the equilibrium price and equilibrium quantity, we first compute the inverse market supply via $p_Y(q) = 1/3q$, and inverse market demand via $p_X(q) = 10 - q/2$. While market supply and market demand do not change per-se, the price 'facing the consumers' will change. unseen ⇓

Now, let $R\%$ be the percentage of tax added as an ad valorem tax. Then, (with tax) the consumers face the inverse supply function

$$p_{Y,\text{tax}}(q) = \frac{1}{3}q\left(1 + \frac{R}{100}\right). \tag{9}$$

We can quickly verify (also known from part (v)) that the requested market demand of $q' = 4$ is achieved via a price of $p' = 8$.

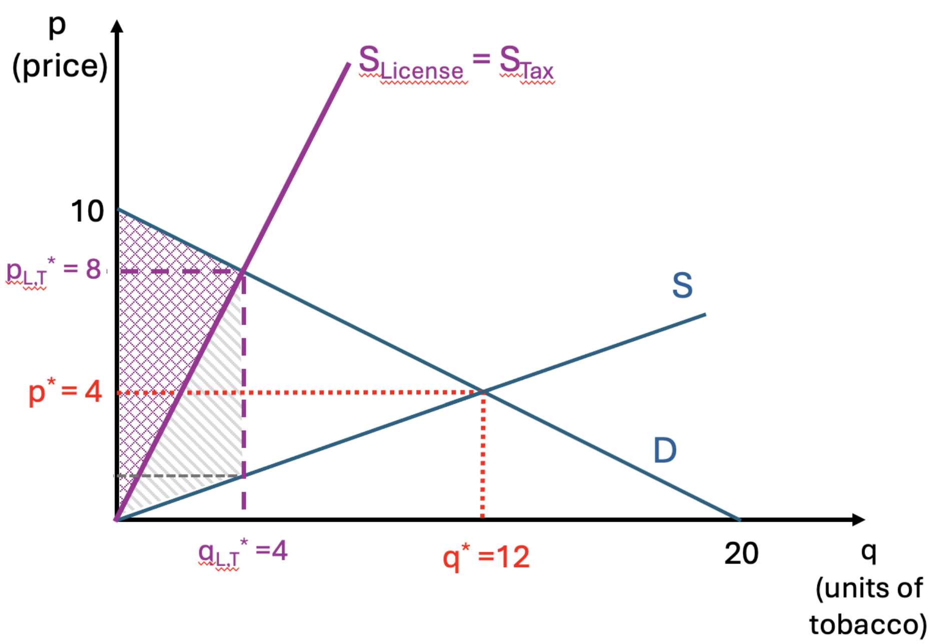
We can thus obtain the tax percentage $R\%$ by solving

$$p_{Y,\text{tax}}(4) = 8 \Leftrightarrow R = 500.$$

Thus, an ad valorem tax of $R = 500\%$ would have to be introduced to reduce the consumption of tobacco to the required level. 3, D

unseen ⇓

(vii) In order to increase the community surplus, the government should follow the suggestions of the political opposition. A graphical justification is provided below. More precisely, in the scenario with licenses, our newly restricted market supply function is given by the purple curve S_{License}, and thus the community surplus can be computed by the marked purple area (as the sum of the new CS and PS). It is worth noting that most of the surplus is on the producer side.
When introducing taxes instead of licenses, the market supply curve itself does not change, since all $J = 12$ firms operate in the market as specified in (ii). However, the price 'facing the consumer' changes due to the tax, corresponding to S_{tax} (in turn corresponding with S_{License}). In the tax scenario, the community surplus is now calculated as the sum of PS, CS and tax. CS does not change compared to the license scenario (upper purple dashed triangle). However, the PS substantially changes, since it is now given by the lower triangle (up to the price obtained by the producers, illustrated as a dashed grey horizontal line). Finally, the tax surplus is given by the rectangle between consumer and producer surpluses. One can directly see that the tax scenario includes a much larger area of community surplus (purple and grey) than the license scenario (only the purple area). In consideration of community surplus, the tax scenario should thus be implemented.



3, D

5. (a) Using the no arbitrage condition, the current price of the asset should be its present value, that is

meth seen ⇓

$$p_0 = \frac{p_2}{(1+r)^2} = \frac{36}{(1+r)^2} = \frac{36}{(\frac{6}{5})^2} = 25.$$

The lowest price that you should still accept to sell your asset for today is thus $p_0 = 25$. You could make arbitrage if you would receive an offer higher than p_0, but you would lose money, if you accepted offers below p_0.

3, M

(b) A reason to use expected utilities in choice under uncertainty is the *independence assumption*. Only one of the contingent consumption plans will actually be realized. Choices that people plan to make in one state of nature should not depend on choices planned in different states of nature. E.g., if your house happens to burn down, then the value of extra consumption/wealth should not depend on the amount of wealth/consumption that you would have had if it did not burn down. This independence leads to the form of the expected utility function.

seen ⇓

3, M

(c) Let the consumer's consumption in state/outcome one be denoted as c_1, and c_2 in state/outcome two. Further, let $\pi \in [0,1]$ be the probability that state one occurs. The consumer's preferences can then represented via some expected utility function

unseen ⇓

$$U(c_1, c_2, \pi) = \pi u(c_1) + (1-\pi)u(c_2),$$

where u denotes the consumer's utility function of consumption in one state. Let's now consider an affine transformation $v(U) = aU + b$, with $a > 0, b \in \mathbb{R}$, then

$$\begin{aligned} v(U) &= a\pi u(c_1) + a(1-\pi)u(c_2) + b \\ &= \pi(au(c_1) + b) + (1-\pi)(au(c_2) + b) \\ &= \pi u_a(c_1) + (1-\pi)u_a(c_2), \end{aligned}$$

which clearly has the form of an expected utility function, and $u_a = au + b$ is a strictly monotonic transformation of u and thus represents the same preferences.

4, M

(d) (i) We first note that u is concave, since $u''(w) = -\frac{1}{w^2} < 0, \forall w \in \mathbb{R}_{>0}$. Peter is thus a risk inverse consumer. Next, we calculate the expected return via

meth seen ⇓

$$\begin{aligned} \pi r_g + (1-\pi)r_b &= \frac{2}{5}\frac{1}{5} - \frac{3}{5}\frac{1}{10} \\ &= \frac{1}{50}, \end{aligned}$$

which is clearly positive, which means Peter will invest some positive amount $x > 0$ in the risky asset.

The optimal amount can be found by optimizing the expected utility via setting

$$
\begin{aligned}
EU'(x) = \pi u'(\tilde{w} + x r_g) r_g + (1-\pi) u'(\tilde{w} + x r_b) r_b &\overset{!}{=} 0 \\
\Leftrightarrow \frac{2}{5} \frac{1}{\tilde{w} + \frac{1}{5}x} \frac{1}{5} - \frac{3}{5} \frac{1}{\tilde{w} - \frac{1}{10}x} \frac{1}{10} &\overset{!}{=} 0 \\
\Leftrightarrow 4\tilde{w} - \frac{4}{10}x = 3\tilde{w} + \frac{6}{10}x \\
\Leftrightarrow x = \tilde{w}.
\end{aligned}
$$

Thus, $x^* = \tilde{w}$ is the optimal amount of investment, i.e., Peter will invest his entire wealth in the risky asset. Since u is concave, $x^* = \tilde{w}$ is indeed a global maximum. $\boxed{3, \mathrm{M}}$

(ii) Since $u''(w) = -\frac{1}{4} w^{-3/2} < 0, \forall w \in \mathbb{R}_{>0}$, Peter's utility function is again concave, and Peter is thus risk averse. We again calculate the expected return

$$
\begin{aligned}
\pi r_g + (1-\pi) r_b &= \frac{1}{5}\frac{1}{10} - \frac{4}{5}\frac{1}{20} \\
&= -\frac{1}{50},
\end{aligned}
$$

which is clearly negative. Since Peter is risk averse and the expected return is negative, we can conclude that he will have the highest expected utility at $x^* = 0$, which means he will not at all invest in the risky asset. $\boxed{2, \mathrm{M}}$

(e) (i) The curvature of the utility function measures the consumer's attitude towards risk. We know that a risk-averse consumer has a concave utility function, and the more concave the utility function, the more risk-averse the consumer will be. The concavity of the utility function can be assessed via the second derivative $u''(w)$ (being negative), which explains the numerator. $\boxed{\text{unseen} \Downarrow}$
However, expected utilities are only unique up to a positive affine transformation. We thus need some normalization term that keeps the risk measure invariant under an affine transformation, which explains the first derivative $u'(w)$ in the denominator. One can clearly see that for any positive affine transformation $u_a(u) = au + b$, with $a > 0$, we have $-\frac{u_a''(w)}{u_a'(w)} = \frac{au''(w)}{au'(w)} = \frac{u''(w)}{u'(w)} = A(w)$. Finally, since $u'' < 0$ (concavity) and $u' > 0$ (strict monotonicity), multiplying by minus one projects the $A(w)$ to positive values, s.t., higher $A(w)$ implies higher risk-aversion.[1] $\boxed{4, \mathrm{M}}$

(ii) Decreasing $A(w)$ for increasing w, means that the consumer is becoming less averse to risk as wealth w of the consumer increases. $\boxed{1, \mathrm{M}}$

[1] $A(W)$ is known in the economic literature as the Arrow–Pratt measure of absolute risk aversion.

Review of mark distribution:
Total A marks: 32 of 32 marks
Total B marks: 20 of 20 marks
Total C marks: 12 of 12 marks
Total D marks: 16 of 16 marks
Total marks: 100 of 80 marks
Total Mastery marks: 20 of 20 marks

MATH60142 Mathematics of Business and Economics

Question	Marker's comment
1	Question 1 was overall answered very well.nbsp;Part (a) (i) Almost all students identified cases/goods which do not follow the law of demand. Some students missed to explain the principle of the law of demand, and/or why it might not hold in the case they have stated.(a) (ii) Mostly answered correctly, although some students did not mention the additional variability in the long-run, which allows improved cost optimization compared to the short-run case, where at least one input factor is fixed.(a) (iii) Mostly answered correctly, although some students did not identify that monopolists still have to respect the market demand when setting their product prices. Some students missed to mention the shutdown condition for monopolists.(a) (iv) Mostly answered correctly. Some students chose examples of isoquants from different technologies, which can then of course contradict the statement by having intersections. Following the definitions and conventions in the lecture, it should have been clear that isoquants are compared under the same technology. Students still received a point if their answer correctly explains the concept of isoquants.(a) (v) Mostly answered correctly. Some students did not justify their answer, which was requested as part of the question. (b) (i) Most students correctly calculated the marginal product wrt. good x2. Some students made a mistake when applying the chain rule to obtain the partial derivative. Some students could not provide a sound economic interpretation of the limit of the marginal product being zero.(b) (ii) The question was very well answered. Some students made mistakes in the derivation of the first-order-conditions, which consequently led to incorrect results. Some students missed to indicate that their derived values for x1 and x2 are the conditional factor demand functions, denoted as x*1(w1,w2,y)nbsp;and x*2(w1,w2,y).b) (iii) This question was extremely well answered. Most students could correctly identify the profit function nbsp;\pi*, and correctly showed the positive homogeneity property. When showing positive homogeneity, some students missed that the conditional factor demand functions also depend onnbsp;w1,w2,nbsp;and they thus missed to scale these parts by factornbsp;t.

MATH60142 Mathematics of Business and Economics

Question	Marker's comment
2	Question 2 was generally answered well.Part (a) Most students correctly stated a function as a counterexample, but some missed the justification that the provided function actually is quasi-concave and quasi-convex. Some students used the definitions of concavity/convexity, without mentioning that this implies quasi-concavity/quasi-convexity. Few students did not attempt the question.Part (b)(i) Most students correctly demonstrated that the WAPM is satisfied for firm A, and noticed that the WACM is implied by the WAPM. Some students only showed that the WAPM holds at time t=1 and s=2, but missed to show t=2, s=1, which is needed to satisfy the WAPM. Some students incorrectly contradict the WACM, by comparing costs at t=2 with input at s=1, which cannot be considered for the WACM, since clearly y2nbsp;gt; y1.Part (b)(ii) Almost all students correctly demonstrated that firm B is not operating rationally, either by contradicting the WACM, WAPM, or demonstrating negative profit at t=1, which could be zero by shutting down.nbsp;Part (b)(iii) Only few students received full marks. Most students missed that here the inequality in the WACM must be strict, since y2nbsp;gt; y1, so the firm has to have lower cost in order to justify lower output. Many students missed to indicate the set of prices that satisfies the WACM for both firms. Some students used a formula of the WACM that only holds whennbsp;y2 nbsp;=nbsp;y1, which is clearly not the case here. Part (c)(i) Mostly answered correctly by all students. Some students missed to state that the both demands refer to consumption bundles (and their respective constraints).nbsp;Part (c)(ii) Many students did not approach this more challenging question. Some students correctly identified that here the Marshallian demand grows faster than the Hicksian demand, using Slutsky equation and the fact that good 1 is an inferior good. Some students correctly used Shephard's lemma to obtain the Hicksian demand as the derivative of the expenditure function. Only few students spotted that the Marshallian demand and the Hicksian demand coincide at the left endpoint of the integral, which leads to the desired result (given the monotonicity of the integral).
3	Question 3 was overall answered very well.Part a i: Mostly answered correctly, although some students gave only the boundary of the budget set ("=") rather than the full set ("lt;=") and some students omitted to state Walras's law.Part a ii: Answered extremely will, with the only (rare) errors coming through algebra slip-ups.Part a iii: Almost all students correctly identified x_2 as a normal good for Anne; most similarly identified x_1 and x_2 as independent, but a common error was to state these are substitutes.Part a iv: Almost all students correctly identified George's preferences as equivalent to Anne's (or Anna, as she's become at this point); most also demonstrated that Elisa's are different, but some failed to correctly spot that her preference is not a monotonic transform of Anne's over the whole range.Part b: A significant number of students did not attempt this challenging question; of those who did many either did not fully demonstrate the required result or did not explain it in their own words.

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MATH70142 Mathematics of Business and Economics

Question	Marker's comment
5	Question 5 was generally answered well.Part (a) Most students correctly calculated the current price as its present value. Some students missed to justify that this would be the lowest price to sell the asset for.Part (b) Only few students correctly identified the independence assumption as main reason to use expected utilities in choice under uncertainty.nbsp;Part (c) Most students correctly identified the structure of an expected utility function and applied an affine transformation to it. Only few students showed that the affine transformation preserves the expected utility structure, and that the positive affine transformation is strictly monotonic and thus represents the same preferences.nbsp;Part (d) (i) and (ii) Some students did not approach this part of the question. Only few students correctly identified the optimisation problem and obtained correct solutions for parts (i) and (ii).Part (e) (i) None of the students obtained full marks for this question. Some students did not approach this question. Some students correctly noticed that the risk aversion of the consumer can be identified via concavity of the utility function, which in turn can be assessed via the second derivative (being negative) of the utility function (and similarly a positive attitude towards risk via a convex utility function). Only few students noticed that the more concave the utility function, the more risk averse the consumer will be. None of the students correctly noticed that A(w) (the Arrow-Pratt measure) is particularly helpful since it is invariant under affine transformations, which explains the first derivative in the denominator.nbsp;Part (ii) (i) This part was mostly well answered. Some students did not approach this part.nbsp;