This report summarises the types of questions the past papers from 2012, 2013, 2014, 2015, 2017, 2018 (the in-person tests devised by Professor Almut). Note, mastery questions are not included. Each number in the bracket indicates the number of marks this type of question appeared.

Chapter	Questions	Total Marks

Discrete Markov Chains	- Definition (23)
	  - discrete-time Markov-chain, time-homogeneity, simple random walk, stationary distribution, recurrence & transience, accessible, communicate, Time reversibility
	- Draw Transition diagram (4)
	- Prove some process is a Markov chain (2)
	- Dynamics of Markov chain
	  - Write how the dynamics are determined (2)
	  - write initial distribution (1)
	  - Find transition probabilities/write transition matrix (19)
	- Calculate probability, e.g. of a realisation (1)
	- Find and classify communicating classes (21)
	- Judge irreducibility of Markov chain (3)
	- Find stationary distributions (may involve infinite state space) (38)
	  - Uniqueness of stationary distribution (6)
	- Use existence of stationary distribution and irreducibility to deduce positive recurrence (3)
	- Prove some given vector is stationary distribution (10)
	- Prove some the detailed balance equation (5)
	- Prove a proposition from the book/Problem Sheets (56)
	  - Theorem 3.1.4, Theorem 3.3.4, Lemma 3.4.7, Corollary 3.5.6, Theorem 3.7.2, Theorem 3.7.4, Theorem 3.7.11, Theorem 3.9.2, Theorem 3.10.1, Theorem 3.10.3, Exercise 1-2, Exercise 2-20
	- Use an example from the book (6)
	  - Example 3.3.7, Example 3.6.3
		200
		(41.7%)

Exponential Distribution	- Definition (3) - Lack of memory, Lack of memory iff exponential - Practical question: find probabilities associated with one or multiple exponential distributions (5) - Calculate probability of an event involving one or several exponentially distributed variables (10)	18 **(3.75%)**

Poisson Processes	- Definition (11)
	  - homogeneous Poisson process, non-homogeneous Poisson process, Compound Poisson process
	- Prove a proposition from the book/Problem Sheets (21)
	  - Theorem 5.4.1, Theorem 5.5.7, Theorem 5.5.13
	- find a probability associated with Poisson process, e.g. marginal probability (50)
	  - may involve solving an integral using pdf of a continuous distribution, or using Gamma function.
	  - may comes from a practical question, you have to convert the words into mathematical language
	- Practical question: give a class of stochastic Process better than the one described in the question (2)
	  - consider non-homogeneous Poisson process or
	- Derive a probability distributions associated with Poisson process (13)
	- Argue about stationary increment property(2)
		100
		(20.8%)

Chapter	Questions	Total Marks

Continuous-time Markov Chains	- Definition (15)
	  - Markov property for CTMC, Birth-death process, Generator
	- Find generator (7)
	- Convert between generator, transition probabilities of jump chain (9)
	- Find stationary distribution (5)
	- Explosion of Markov chain (6)
	- Find and/or solve forward equations (51)
	  - You may need to recognise the equation given in the question is a forward equation, and then you use the techniques for deriving forward equation.
	- Find expectation associated with CTMC (3)
	- Calculate marginal probability associated with CTMC (8)
	- Prove a proposition from the book/Problem Sheets (7)
	  - Exercise 6.1.5
	- Practical question: identify a birth-death process from the context (6)
		117
		(24.4%)

Brownian motion	- definition
	  - standard Brownian motion (3)
	  - reflection principle (2)
	- Prove a proposition from the book/Problem Sheets (5)
	  - Proposition 7.6.1
		10
		(2.1%)

Others	- Evaluate a complicated summation (16)
	  - may use PMF of a discrete distribution
	  - there can be double or triple summation
	- use Laplace transform to draw conclusions on distributions (7)
	- manipulation of Covariance (4)
	- Find the distribution (PDF or CDF or PMF) (8)
		35
		(7.3%)

Total marks		480

Distribution of Marks by Question Type

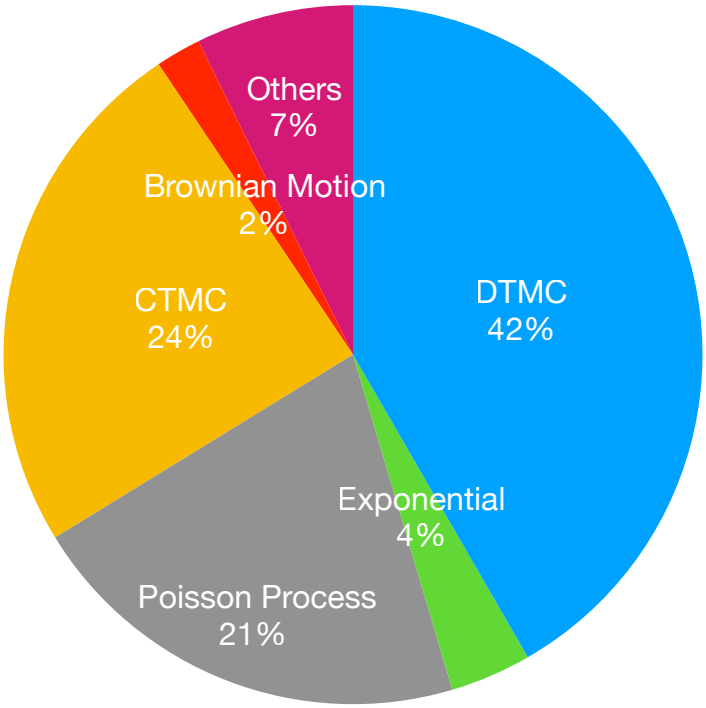
Bookwork(writing down or using definitions from books):	71 (14.8%)
Draw Diagrams	4 (0.8%)
Calculation:	224 (46.7%)
Example (find example of a Markov chain satisfying certain conditions)	6 (1.3%)
Logical Deductions/proofs:	167 (34.8%)
— of which: Proving theorems/propositions from lectures	98 (20.4%)
Explanation:	8 (1.7%)

Claim: Many parts of logical deductions involves simply using some results from the book to argue e.g. “stationary distribution is not unique because there are many positive recurrent communicating classes”. But some questions may ask you to prove a small theorem/proposition/lemma from the book.

Top Questions

- Proving a theorem/proposition from the lectures (20.4%)
- Write down a definition (11.5%)
- Find and/or solve forward equation for continuous-time Markov chain (10.6%)
- Find probabilities associated with Poisson process (10.4%)
- Find stationary distribution for discrete-time Markov chain (7.9%)
- Find and classify communicating classes (4.4%)
- Find transition probabilities (4.0%)

Distribution of Marks by Topic



Theorems already assessed (in 2012, 2013, 2014, 2015, 2017, 2018, 2020):

Theorem 3.1.4, Theorem 3.3.4, Lemma 3.4.7, Corollary 3.5.6, Theorem 3.7.2, Theorem 3.7.4 (2), Theorem 3.7.4(3), Theorem 3.7.11, Theorem 3.9.2, Lemma 3.9.3, Theorem 3.9.14, Theorem 3.10.1, Theorem 3.10.3, Exercise 1-2, Exercise 2-20, Exercise 2-17
Theorem 5.4.1, Theorem 5.5.7, Theorem 5.5.13, Exercise 6.1.5, Proposition 7.6.1 (rescaling)

Important theorems not yet assessed

Theorem 3.2.3, Proposition 3.4.5, Theorem 3.5.3, Theorem 3.5.5, Theorem 3.5.8, Theorem 3.5.9, Theorem 3.7.4 (1), Theorem 3.7.4(4), Theorem 3.7.8, Lemma 3.9.5, Lemma 3.9.6
Theorem 5.3.6, Theorem 5.4.5(Exercise 3-25), Theorem 5.4.6, Exercise 5.4.8, Theorem 5.5.2, Theorem 5.5.5,
Proposition 7.6.1 (reflection, translation, inversion)

Recommended challenging questions in past papers

2013 Q3 (unseen 17/20), 2014 Q3 b(ii), 2017 Q3 b(iii), 2018 Q4 d, e

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