MATH40005 Probability and Statistics

Midterm [20 points] **23 November 2021**

Please remember to define your notation and justify all your answers.

Question 1: Three fair six-sided dice are rolled together once.

(a) (1 point) Find the sample space for this experiment.

(b) (2 points) What is the probability that one die shows 4, another 3, and another 1?

Question 2: (4 points) Suppose there are three identical urns. Each urn contains 6 identical balls labelled with the numbers $\{1, 2, 3, 4, 5, 6\}$. Suppose you take one ball from each urn. What is the probability that one of the numbers on the three labels equals the sum of the other two numbers?

Question 3: If the letters H, I, P, P, O, P, O, T, A, M, U, S are arranged at random, what is the probability that

(a) (2 points) the arrangements spell the word HIPPOPOTAMUS?

(b) (2 points) the arrangements have three adjacent P's?

Question 4: (2 points) Consider a probability space $(\Omega, \mathcal{F}, \mathrm{P})$. Let $A, B \in \mathcal{F}$. Suppose that $\mathrm{P}(A) = 0$. Can we conclude that $\mathrm{P}(A \cap B) = 0$? If so, prove the result, otherwise give a counterexample.

Question 5: Suppose there are only two types of books on your bookshelf, they either cover probability or analysis (but never both topics). 70% of your books are analysis books. 65% of the probability books contain graphics, whereas only 30% of the analysis books contain graphics, the other books only contain text and formulas.

(a) (3 points) Suppose you randomly take one book from your bookshelf. What is the probability that it does not contain any graphics?

(b) (1 point) Suppose that one of your books does not contain any graphics. What is the probability that this book is a probability book?

Question 6: Suppose that $(\Omega, \mathcal{F}, \mathrm{P})$ is a probability space and consider events $A, B \in \mathcal{F}$. Suppose that $\mathrm{P}(A) = 1/3, \mathrm{P}(B) = 2 \cdot \mathrm{P}(A), \mathrm{P}(A \cap B) = \mathrm{P}(A)/2$.

(a) (2 points) Find $\mathrm{P}(A) + \mathrm{P}(B)$. Can you conclude that $A \cup B = \Omega$?

(b) (1 point) Find $\mathrm{P}(B^c|A)$.