It is clear from the Slutsky equation that the income effect plays a major part in determining how the demand for a set of goods will react to changes in their prices. For firms, consumers and economists alike, then, it is important to ascertain how the demand of certain goods will react to changes in consumer budget.

Indeed, economists class goods according to the manner in which they react to changes in consumer income:

- For ***normal goods***, an increase in income will result in an increase in demand;

$$\frac{\partial}{\partial m} x_j^*(\underline{p}, m) \geq 0 .$$

- For ***inferior goods***, an increase in income will result in a decrease in demand.

$$\frac{\partial}{\partial m} x_j^*(\underline{p}, m) < 0 .$$

It is also worth noting the different subclasses of normal goods: suppose we have an increase in consumer income m...

- ...for ***luxury goods***, demand will increase more than proportionally to income;

i.e., $$\frac{\partial x_j^*(\underline{p}, m)}{\partial m} \cdot \frac{m}{x_j^*(\underline{p}, m)} > 1$$

$\parallel\!\!\mid$

Income elasticity of demand (IED)

(roughly speaking, $\dfrac{\delta x_j^*}{x_j^*} > \dfrac{\delta m}{m}$)

Consumers are likely to already be buying non-luxury goods, so any increase in their income is likely to result in a greater (relative) increase in their demand for luxury goods than for non-luxury goods.

- ...for ***necessary goods***, demand will increase less than proportionally $(0 \leq IED < 1)$
- ...and if demand increases proportionally to income, the consumer is said to have ***homothetic preferences*** for the set of goods under consideration. $(IED = 1)$

Finally, we note that goods can also be classified according to how changes in price impact their consumer demand:

- For ***ordinary goods***, a decrease in price will lead to an increase in their demand;

  i.e., $\frac{\partial x_j^*(\underline{p}, m)}{\partial p_j} \leq 0$

- For ***Giffen goods***, a decrease in price will lead to a decrease in demand

  i.e., $\frac{\partial x_j^*(\underline{p}, m)}{\partial p_j} > 0$

That means our previously stated law of demand only holds for ordinary goods, but not for Giffen goods.

What is an example for a Giffen good? Some theoretical considerations can help us finding necessary conditions for Giffen goods. In particular, the Slutsky equation helps to establish a relation between ordinary and normal goods on the one hand side, as well as Giffen and inferior goods on the other side.

Recall that from Slutsky's Equation with $i = j$:

$$\partial_j x_j^*(\underline{p}, m) =$$

$$= \partial_j x_{H,j}^*(\underline{p}, V(\underline{p}, m)) - (\partial_{n+1} x_j^*(\underline{p}, m)) x_j^*(\underline{p}, m)$$

But $$x^*_{H,j}(\underline{p}, V(\underline{p}, m)) = \partial_j e(\underline{p}, V(\underline{p}, m))$$

$$\Rightarrow \quad \partial_j x^*_{H,j}(\underline{p}, V(\underline{p}, m)) = \partial_j^2 e(\underline{p}, V(\underline{p}, m))$$

Recall now that $e(\underline{p}, u)$ is concave in $\underline{p}$.

Aside: Let $f(\underline{x})$ be a function defined on a convex subset U of $\mathbb{R}^n$. If f is concave, then its restriction to every line segment in U is a concave function of a single variable.

Proof: For $\underline{x}$ and $\underline{x}'$ in U, define

$$g(t) = f(t\underline{x} + (1-t)\underline{x}') \quad \forall t \in [0,1]$$

i.e., $g(t)$ is the restriction of f to the line segment that joins $\underline{x}$ and $\underline{x}'$ (note that this is wholly contained in U as U is convex).

Then, for t, t' and s in $[0,1]$,

$$g(st + (1-s)t')$$

$$= f((st + (1-s)t')\underline{x} + (1-(st + (1-s)t'))\underline{x}')$$

$$= f\left(st\underline{x} + (1-s)t'\underline{x} + \underline{x}' - st\underline{x}' - (1-s)t'\underline{x}'\right)$$

$$= f\left(st(\underline{x}-\underline{x}') + (1-s)t'(\underline{x}-\underline{x}') + \underline{x}'\right)$$

$$= f\Big(s\big(t\underline{x} + (1-t)\underline{x}'\big) - s\underline{x}' +$$

$$+ (1-s)t'(\underline{x}-\underline{x}') + \underline{x}'\Big)$$

$$= f\Big(s\big(t\underline{x} + (1-t)\underline{x}'\big) + (1-s)\big(t'\underline{x} + (1-t')\underline{x}'\big)\Big)$$

$\downarrow$ f is concave means

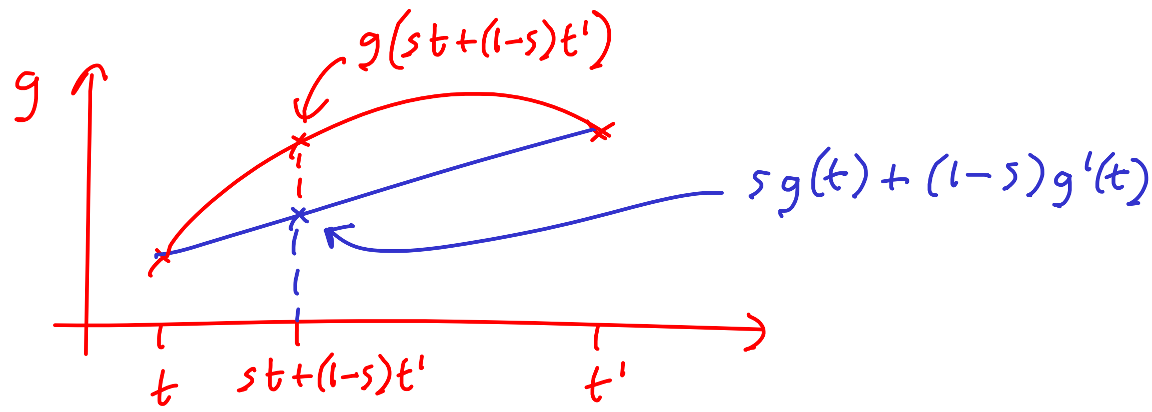
$$f(s\underline{w} + (1-s)\underline{w}') \geq sf(\underline{w}) + (1-s)f(\underline{w}') \quad \forall s \in [0,1]$$

$$\geq sf\big(t\underline{x} + (1-t)\underline{x}'\big) + (1-s)f\big(t'\underline{x} + (1-t')\underline{x}'\big)$$

$$\geq s\,g(t) + (1-s)\,g(t').$$

$\Rightarrow$ g is concave in t. //

Eg



So $g''(t) \leq 0 \quad \forall t \in [0,1]$.

$\Rightarrow$ $e(\underline{p}, u)$ is a concave function of p_j if all other p_i's and u are fixed

$\Rightarrow$ $\partial_j^2 e(\underline{p}, u) \leq 0$

$\Rightarrow$ $\partial_j x^*_{H,j}(\underline{p}, v(\underline{p}, m)) \leq 0$

So, recalling

$$\partial_j x_j^*(\underline{p}, m) =$$

$$= \partial_j x^*_{H,j}(\underline{p}, v(\underline{p}, m)) - \left(\partial_{n+1} x_j^*(\underline{p}, m)\right) x_j^*(\underline{p}, m)$$

and the fact that $x_j^*(\underline{p}, m) \geq 0$, one may deduce that

$$\frac{\partial x_j^*(\underline{p}, m)}{\partial m} \geq 0 \Rightarrow \frac{\partial x_j^*(\underline{p}, m)}{\partial p_j} \leq 0$$

So a normal good is always an ordinary good.
On the other hand

$$\frac{\partial x_j^*(\underline{p}, m)}{\partial p_j} > 0 \Rightarrow \frac{\partial x_j^*(\underline{p}, m)}{\partial m} < 0$$

So a Giffen good is always an inferior good.

Example of Giffen goods: inferior quality staple foods. If a consumer bases most – but not all of – their diet on these, supplemented by smaller quantities of some better quality foodstuffs, then if the prices of these staple foods rises, the consumer may rely on them so much that they must forgo the better quality foods and replace these with <u>more</u> of the (now more expensive) staple foods. (Note, this assumes that there are no cheaper substitute goods.)