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H_0 : No one in the class
can predict the future

H_1 : Someone can predict the future

6 tosses

Let A be the event of someone
guessing all 6 tosses correctly

$$P(A) = \frac{\text{card } A}{\text{card } \Omega} = \frac{1}{2^6} = \frac{1}{64}$$

Ω - space of 6 tosses $= 0.0156$

$$1/32 = 0.03$$

$$\alpha = 0.05$$

$$p = 0.03 \quad p < \alpha$$

Class today has 56 students

Suppose there were 100 students

there would be 100 p-values

$p \sim \text{Unif}(0,1)$ under H_0 is true

So for 100 independent p -values

How many would you expect are less than 0.05?

Let Z be the number of p -values
less than α ($= 0.05$)

$$Z = \sum_{i=1}^n \mathbb{I}(X_i < \alpha) \quad (n=100) \\ = A_i$$

each X_i is a p -value

$$\mathbb{I}(X_i < \alpha) = \begin{cases} 1 & \text{if } X_i < \alpha \\ 0 & \text{otherwise} \end{cases}$$

\uparrow
 $\sim \text{Unif}(0,1)$

each $\mathbb{I}(X_i < \alpha) \sim \text{Bern}(\alpha)$

$$Z \sim \text{Bin}(n, \alpha) \quad n=100 \quad \alpha=0.05$$

$$E[Z] = n\alpha = 100 \cdot (0.05) = 5$$

$$\hat{Z} \quad n=58 \\ \alpha=0.05$$

$$E[\xi] = 58 \cdot (0.05) = 2.9$$

$$\alpha = 0.03 \quad = 58(0.03) \approx \underline{1.74}$$

Let A_i be a hypothesis test

$$\text{Let } A = \bigcup_{i=1}^n A_i$$

$$P(A) = P\left(\bigcup_{i=1}^n A_i\right) \leq \sum_{i=1}^n P(A_i)$$

Suppose $P(A_i) = \alpha$ for $i=1, 2, \dots, n$

$$\begin{aligned} P(A) &= P\left(\bigcup_{i=1}^n A_i\right) \leq \sum_{i=1}^n P(A_i) \\ &= \sum_{i=1}^n \alpha = n\alpha \end{aligned}$$

Need to adjust threshold α when
there are multiple hypotheses

$$P(A) = n\alpha$$

let $\tilde{\alpha} = \frac{\alpha}{n}$ \leftarrow nominal threshold level
 \nwarrow adjusted significance level

Test H_0 at level $\tilde{\alpha} = \frac{\alpha}{n}$
for multiple hypotheses

then the probability that at least
one individual hypothesis is rejected
is α

BONFERRONI CORRECTION

H_0 : My colleague cannot see
the future

H_1 : My colleague can see
the future.

$\alpha = 0.02 \Rightarrow H_0$ rejected

The exam

Chapter 1: Central tendency + dispersion

Chapter 2: EDA

Chapter 3: Normal random variables

Chapter 4: Hypothesis testing

Excluding one-sided hypothesis testing (Section 4.4.2)

Chapter 5: Pitfalls in Statistics

Chapter 6: Covariance + Correlation

Chapter 7: Statistical models

Chapter 8: (Maximum) likelihood

Chapter 9: Simple linear regression

Excluding 10. Bayesian inference

11. Bootstrap

A. Additional details

Formula sheet?

— Term 1 3 Q

— Term 2 3 Q

Revision class(es) 3 May (Wed)