

MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Quiz 5

Question 1 (Periodic orbits of two-dimensional linear systems).

Consider a matrix $A \in \mathbb{R}^{2 \times 2}$ and the linear differential equation $\dot{x} = Ax$, and assume that there exists a non-constant periodic orbit. Which of the following two statements is true?

- (a) All non-constant orbits of $\dot{x} = Ax$ are periodic.
- (b) Non-constant orbits of $\dot{x} = Ax$ are not necessarily periodic.

Question 2 (Bounded solutions of linear systems).

Is the following statement true? Any maximal solution $\mu : \mathbb{R} \rightarrow \mathbb{R}^2$ of the linear differential equation

$$\dot{x} = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} x$$

is bounded.

- (a) The statement is true.
- (b) The statement is false.

Question 3 (Multiplicative property of matrix exponential function).

Is the following statement true? For matrices $A, B \in \mathbb{R}^{2 \times 2}$, we have $e^{(A+B)t} = e^{At}e^{Bt}$ for all $t \in \mathbb{R}$.

- (a) The statement is true.
- (b) The statement is false.

Question 4 (Inhomogeneous linear differential equations).

Consider $A \in \mathbb{R}^{d \times d}$ and two continuous functions $g, h : \mathbb{R} \rightarrow \mathbb{R}^d$, and let $\mu : \mathbb{R} \rightarrow \mathbb{R}^d$ be a solution to $\dot{x} = Ax + g(t)$ and $\nu : \mathbb{R} \rightarrow \mathbb{R}^d$ be a solution to $\dot{x} = Ax + h(t)$. Is the following statement true? The function $\lambda : \mathbb{R} \rightarrow \mathbb{R}^d$, $\lambda(t) := \mu(t) - \nu(t)$, is a solution to $\dot{x} = Ax + g(t) - h(t)$.

- (a) The statement is true.
- (b) The statement is false.

Question 5 (Matrix exponential function of skew symmetric matrices).

Let $A \in \mathbb{R}^{d \times d}$ be skew symmetric (i.e. $A + A^\top = 0$), and let $t \in \mathbb{R}$. Which of the following two statements is true?

- (a) e^{At} is orthogonal.
- (b) e^{At} is not necessarily orthogonal.