Mock Exam 2: Question 1a

March 25, 2022

Consider the function

$$f(\mathbf{x}) = f_1(\mathbf{x})^2 + f_2(\mathbf{x})^2\,, \quad \mathbf{x} \in \mathbb{R}^2\,,$$

with

$$\begin{aligned} f_1(\mathbf{x}) &= -13 + x_1 + ((5 - x_2)x_2 - 2)x_2\,, \\ f_2(\mathbf{x}) &= -29 + x_1 + ((x_2 + 1)x_2 - 14)x_2\,. \end{aligned}$$

Knowing that there exists an $\mathbf{x}^*$ such that $f(\mathbf{x}^*) = 0$, find a minimizer for this function, and discuss whether it is local or global.

Answer. Since $f(\mathbf{x})$ is the sum of two positive functions, the information about an $\mathbf{x}^*$ such that $f(\mathbf{x}^*) = 0$ requires that both $f1_(\mathbf{x}^*) = 0$ and $f_2(\mathbf{x}^*) = 0$, which is found by solving the system of equations, obtaining $x_1 = 5$ and $x_2 = 4$. It is clear that for $f(\mathbf{x}) = 0 \leq f(\mathbf{y})$ for any $\mathbf{y} \in \mathbb{R}^2$, which is the definition of a global minimizer. $\mathbf{x}^* = (5, 4)^\top$ is thus a global minimizer of $f(\mathbf{x})$.