

Mathematics Pre-arrival course

Solutions to Weekly Quiz 2 – Calculus: Integration and Differentiation

Note in all problems that $\log(x)$ represents the natural logarithm of x (sometimes written $\ln(x)$).

1. The derivative of the function:

$$y = \sin(x^3)$$

is

- (a) $y = 3x^2 \cos(x^3)$
- (b) $y = 3x^2 \sin(x^3)$
- (c) $y = -3x^2 \cos(x^3)$
- (d) $y = -3x^2 \sin(x^3)$

2. The derivative of the function:

$$y = 2x^2 \cos(3x^2)$$

is

- (a) $y = 4x(\cos(3x^2) + 3x^2 \sin(3x^2))$
- (b) $y = 2x(2 \cos(3x^2) - x \sin(3x^2))$
- (c) $y = 2x(2 - x^2) \cos(3x^2)$
- (d) $y = 4x(\cos(3x^2) - 3x^2 \sin(3x^2))$

3. The derivative of the function:

$$y = \frac{e^{\sin(x)}}{1 + 2x}$$

is

- (a) $y = \frac{-e^{\cos(x)}((1+2x)\cos(x)+2)}{(1+2x)^2}$
- (b) $y = \frac{e^{\sin(x)}((1+2x)\cos(x)-2)}{(1+2x)^2}$
- (c) $y = \frac{e^{\sin(x)}(2-(1+2x)\cos(x))}{(1+2x)^2}$
- (d) $y = \frac{e^{\sin(x)}(1+2x)\cos(x)}{(1+2x)^2}$

4. The value of $\frac{dy}{dx}$ when:

$$\sqrt{x^2y} + x = y, \quad x, y > 0$$

is

- (a) $\frac{dy}{dx} = \frac{\sqrt{y}(1+\sqrt{y})}{\sqrt{y}-x}$
- (b) $\frac{dy}{dx} = \frac{2\sqrt{y}(1-y)}{2\sqrt{y}+x^2}$
- (c) $\frac{dy}{dx} = \frac{2\sqrt{y}(1+\sqrt{y})}{2\sqrt{y}-x}$
- (d) $\frac{dy}{dx} = \frac{\sqrt{y}(1-\sqrt{y})}{\sqrt{y}+x}$

5. Find the value of

$$\int_0^1 (x^3 - 6x + 1)(x^2 - 2)dx$$

- (a) 1
- (b) -1
- (c) $\frac{1}{12}$
- (d) $\frac{5}{2}$

6. Find the value of

$$\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{2x^2 + 1}} dx$$

- (a) $\log |\sqrt{2}|$
- (b) 1
- (c) $\frac{1}{\sqrt{2}} \log |\sqrt{2} + 1|$
- (d) $\frac{1}{\sqrt{2}} \log |\sqrt{2}|$

7. Find the value of

$$\int_0^1 \frac{e^{2x}}{e^{2x} + 4} dx$$

- (a) $\frac{1}{2} \log \left| \frac{e^2 + 4}{5} \right|$
- (b) $1 - \log |5|$
- (c) $\log \left| \frac{e^2 + 4}{5} \right|$
- (d) $\frac{1}{2} \log \left| \frac{e^2 - 4}{5} \right|$

8. Find the value of

$$\int_0^1 \frac{1}{2x^2 + 4x + 4} dx$$

- (a) $\arctan(3) - \frac{\pi}{4}$
- (b) $\frac{1}{2} \arctan(2) - \frac{\pi}{8}$
- (c) $\frac{1}{2} \arctan(2) - \frac{\pi}{4}$
- (d) $\frac{1}{2} \arctan(3) - \frac{\pi}{8}$

9. Find the value of

$$\int_0^1 \frac{1}{x^2 + 3x + 2} dx$$

- (a) $\frac{1}{2} \log \left| \frac{2}{3} \right|$
- (b) $\frac{1}{2} \log \left| \frac{4}{3} \right|$
- (c) $\log \left| \frac{2}{3} \right|$
- (d) $\log \left| \frac{4}{3} \right|$

10. Find the value of

$$\int_0^1 \arcsin(x) dx$$

- (a) π
- (b) $\frac{\pi}{2} - 1$
- (c) $\arcsin(2) - 1$
- (d) $\arcsin(2)$

11. The solution to the equation

$$\cosh(3x) = 6 \cosh(x)$$

for $x \in \mathbb{R}$ is

- (a) $x = 1$
- (b) $x = \log(2 + \sqrt{5})$
- (c) $x = \log(3 + \sqrt{8})$
- (d) $x = \log\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)$

12. The value of the integral

$$\int_{\frac{1}{2} \log(3)}^{\log(3)} \frac{1}{5 \cosh(x) - 4 \sinh(x)} dx$$

is

- (a) $\frac{\pi}{18}$
- (b) $\frac{\pi}{9}$
- (c) $\frac{1}{2} \log(3)$
- (d) $\log(3)$