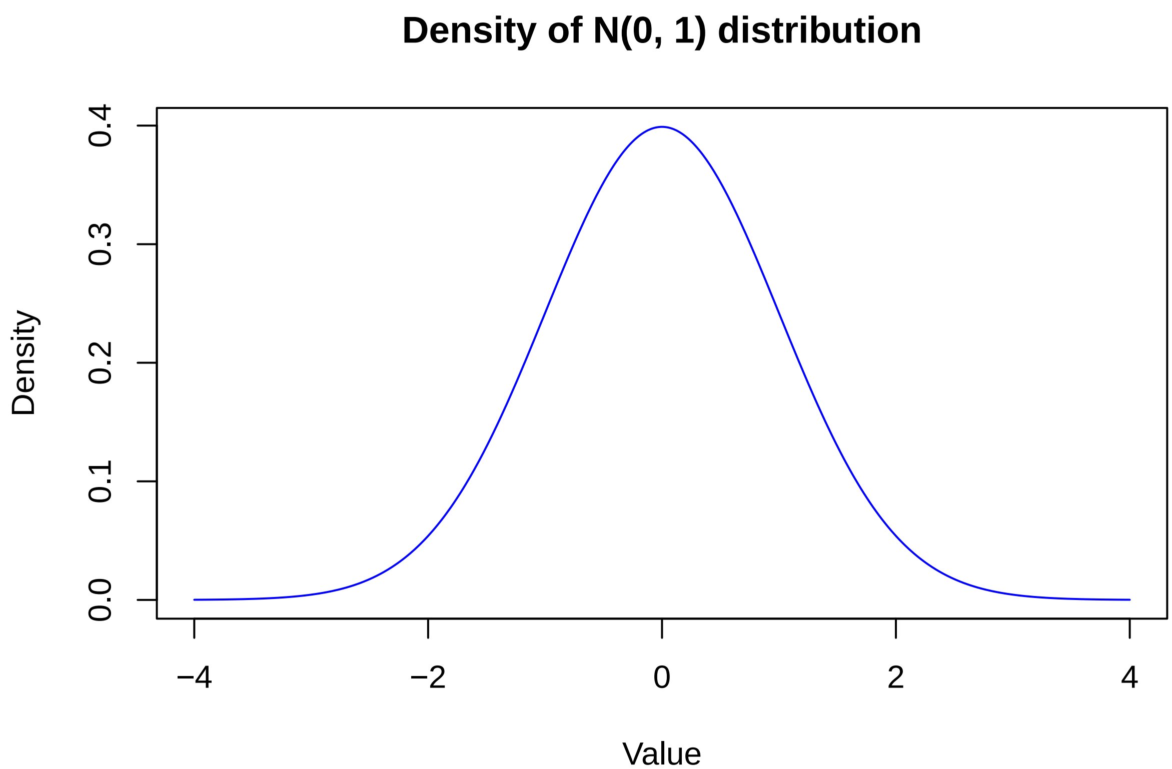
Problem sheet 11 Question 3

Part (a)

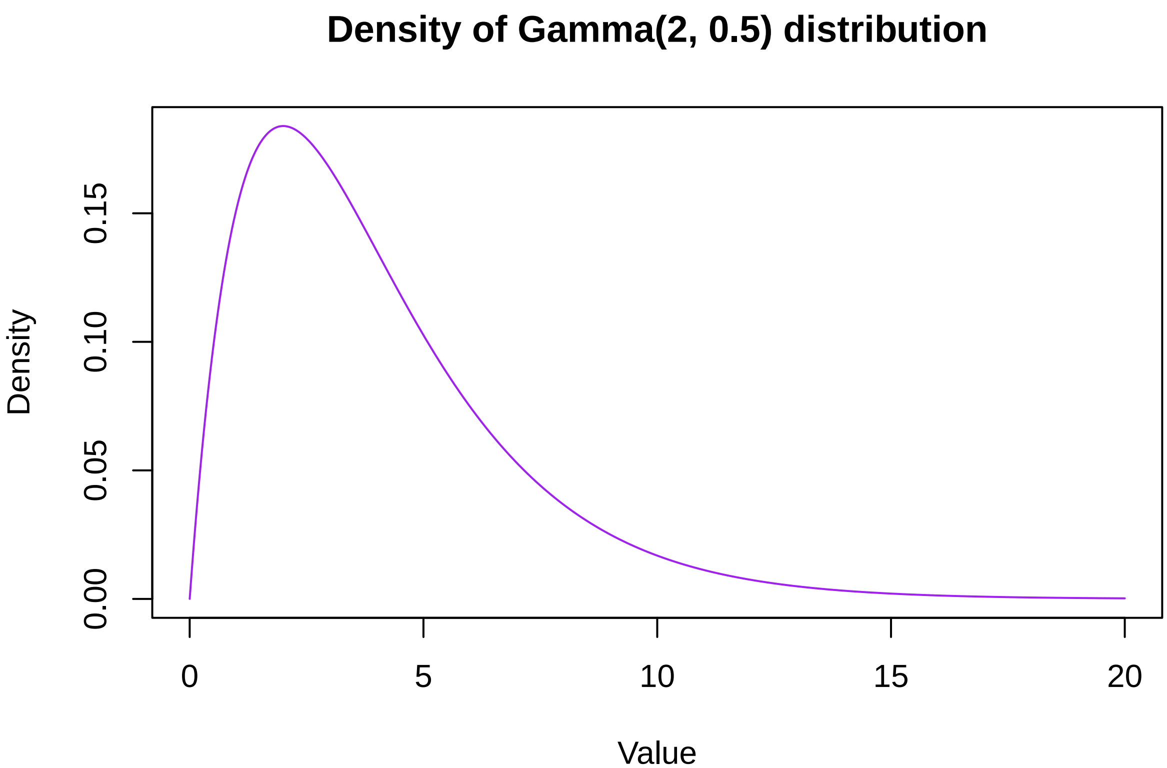
Plotting a standard normal density, i.e. $f_Z(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$ for $Z \sim N(0, 1)$.

```
mu <- 0
sigma <- 1
x <- seq(-4, 4, length=1000)
y <- dnorm(x, mean=mu, sd=sigma)
xlab <- "Value"
ylab <- "Density"
main <- paste0("Density of N(", mu, ", ", sigma, ") distribution")
plot(x, y, type='l', xlab=xlab, ylab=ylab, main=main, col="blue")
```



Part (b)

```
alpha <- 2
beta <- 0.5
x <- seq(0, 20, length=1000)
y <- dgamma(x, shape=alpha, rate=beta)
xlab <- "Value"
ylab <- "Density"
main <- paste0("Density of Gamma(", alpha, ", ", beta, ") distribution")
plot(x, y, type='l', xlab=xlab, ylab=ylab, main=main, col="purple")
```



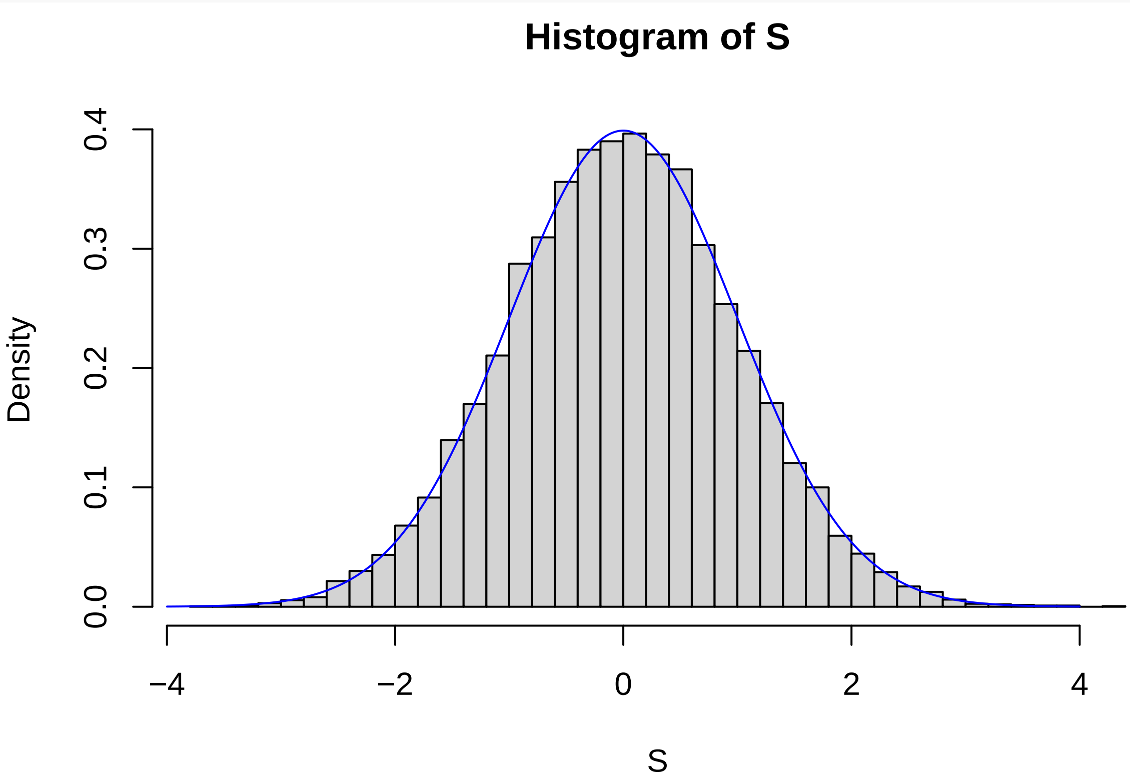
Part (c)

```r
n <- 1e3
numtrials <- 1e4
set.seed(1)
#shape
alpha <- 2
#rate
beta <- 0.5

#computing mean and variance
mu <- alpha/beta
sigma_sq <- alpha/(beta^2)
sigma <- sqrt(sigma_sq)

# initialise the sums, and run the trials
S <- rep(0, numtrials)
for (i in seq_len(numtrials)){
    x <- rgamma(n, shape=alpha, rate=beta)
    #standardise
    x <- (x - mu)/sigma
    S[i] <- sum(x) / sqrt(n)
}

# plot the histogram
hist(S, freq=F, breaks=30)
z <- seq(-4, 4, length=1000)
# add the normal density plot
d <- dnorm(z, mean=0, sd=1)
lines(z, d, col="blue")
```



Note how this histogram appears to show a standard normal distribution. This is not an accident; for interest look up the **central limit theorem** in a standard textbook. You will learn more about this in Year 2.