ANALYSIS I - SPRING 2023

LECTURER: MARCO A. M. GUARACO

updated: January 30, 2023 [1]

Lectures: Our first lecture will be on Friday, 13 January at Clore Lecture Theater. Afterwards, lectures will take place on Monday, 10am and Thursday, 12pm at Clore Lecture Theater for the rest of the term (these hours might vary on some weeks, please check your calendar for updates).

Office hours, problem sessions and contact: Office hours will take Thursday 5:30pm at HXLY 139. Problem sessions will take place Tuesday, 2pm at HXLY 340, 341 and 342. There hours might vary on some weeks, please check your calendar for updates. For questions related to the content of the course or problem sheets, please use the edSTEM forum, where you can ask questions anonymously. The more questions you ask the quicker you will be in control of the material!

Problem sheets: Please note that one problem sheet will cover material for several weeks and we will have several problem sessions and office hours to discuss them. You can also get feedback through the edSTEM Forum.

Coursework and Unseen problem sheets: There will be three (3) marked courseworks which you have to submit only via Turnitin by the end of the day on Friday 3 Feb, 3 Mar and 17 Mar by 1PM, corresponding to 1%, 2% and 2% of the total mark, respectively.

For each coursework you have to submit a number of indicated problems from the UNSEEN problem sheet, which will be circulating several weeks in advance.

You are allowed to discuss your progress and show us your writing for feedback during the office hours and problem sessions. **Please DO NOT upload (or type) your coursework to the edSTEM forum.**

Midterm: We will have a midterm on Friday, 17 Feb at 1pm, covering the material of Lectures 1 to 10. This is for 5% of the total mark.

Book: I will be following M. Spivak - *Calculus*. Specifically, Chapters 5 to Chapter 14. There are a few copies in the library, which you can photocopy, and I believe you can find some copies online as well. I will also post a scanned copy of the chapter we are discussing each week.

The tentative schedule at the end of this syllabus is arranged so that each lecture corresponds roughly to 8 pages of Spivak's book! Please, do a superficial reading of the corresponding pages before each lecture.

[1] If you spot any mistakes or inconsistencies in this syllabus please let me know as soon as possible via the edSTEM forum.

Lecture Notes: I will use an iPad during each lecture to write and comment over my own set of handwritten lecture notes, which will be updated on Blackboard regularly. This set of lecture notes should be self-contained.

TENTATIVE SCHEDULE

Last update: January 30, 2023[2]

PART I - CONTINUITY

Lecture 1 (13 Jan): Introduction and the definition of the limit of a function. *(Spivak: Ch. 5 - Limits)*

Lecture 2 (16 Jan): Properties of the notion of limit of a function; uniqueness of limits, limits and arithmetic operations, squeeze theorem, etc. *(Spivak: Ch. 5 - Limits)*

Lecture 3 (19 Jan): Definition of continuity. Properties of the notion of continuity: arithmetic operations and continuity, continuity and composition, etc. Statements of the intermediate value theorem and the extreme value theorem. *(Spivak: Ch. 6 - Continuous functions and Ch. 7 - Three hard theorems)*

Lecture 4 (23 Jan): Proofs and applications of the intermediate and extreme value theorems. *(Spivak: Ch. 7 - Three hard theorems and Ch. 8 - Least upper bounds)*

Lecture 5 (26 Jan): More on the intermediate and extreme value theorems. A global concept: Uniform Continuity. *(Spivak: Ch. 8 - Least Upper bounds and Appendix to Ch. 8 - Uniform Continuity)*

PART II - THE DERIVATIVE

Lecture 6 (30 Jan): Definition of the derivative. Existence of the derivative implies continuity. Higher order derivatives. *(Spivak: Ch. 9 - Derivatives)*

Lecture 7 (2 Feb): Properties of the notion of derivative. The derivative of elementary functions: polynomials, trigonometric functions, exponential and logarithm *(Spivak: Ch. 10 - Differentiation)*

CW 1 (3 Feb): Please ensure that your CID is written on all pages of your work. Submit your work only in Turnitin/Blackboard by the end of the day.

Lecture 8 (6 Feb): More on the derivative. Proof of the chain rule (or the derivative of a composition) *(Spivak: Ch. 10 - Differentiation)*

Lecture 9 (9 Feb): The derivative at maxima and minima. Critical points and critical values. Rolle's theorem. *(Spivak: Ch. 11 - Significance of the derivative)*

[2]Schedule continues on the next page. Please, check regularly for updates.

Lecture 10 (13 Feb): The Mean Value Theorem. Monotonicity and the sign of the derivative. *(Spivak: Ch. 11 - Differentiation)*

Midterm (17 Feb): This midterm will cover material from Lectures 1 to 10.

Lecture 11 (20 Feb): Second derivatives at minima and maxima. *(Spivak: Ch. 11 - Differentiation)*

Lecture 12 (23 Feb): Convexity, concavity and derivatives. *(Spivak: Appendix to Ch. 11 - Convexity and Concavity)*

Lecture 13 (27 Feb): The Inverse Function Theorem. *(Spivak: Ch. 12 - Inverse Functions)*

Lecture 14 (2 Mar): Planar curves. *(Spivak: Ch. 12 - Appendix. Parametric representation of curves)*

CW 2 (3 Mar): Please ensure that your CID is written on all pages of your work. Submit your work both only in Turnitin/Blackboard by the end of the day.

PART III - THE INTEGRAL

Lecture 15 (6 Mar): Partitions. Lower and Upper sums and the definition of Integrable functions. *(Spivak: Ch. 13 - Integrals)*

Lecture 16 (9 Mar): Continuous functions are integrable. Sumability of the integral with respect to the domain. Linearity and monotonicity of the integral. *(Spivak: Ch. 13 - Integrals)*

Lecture 17 (13 Mar): The Fundamental Theorem of Calculus *(Spivak: Ch. 14 - The Fundamental Theorem of Calculus)*

Lecture 18 (16 Mar): The Fundamental Theorem of Calculus *(Spivak: Ch. 14 - The Fundamental Theorem of Calculus)*

CW 3 (17 Mar): Please ensure that your CID is written on all pages of your work. Submit your work both only in Turnitin/Blackboard by the end of the day.

PART IV - TAYLOR'S THEOREM AND SERIES

Lecture 19 (20 Mar): Taylor's theorem *(Spivak: Ch. 20 - Approximation by polynomial functions)*

Lecture 20 (23 Mar): Uniform convergence. Integrals and derivatives of infinite sums. *(Spivak: Ch. 24 - Uniform convergence and power series)*