Mathematics Pre-arrival course

Solutions to Weekly Quiz 4 – Complex Numbers

1. Which of the following is equal to i^{103}:

    (a) i

    (b) -1

    (c) $-i$

    (d) 1

2. Given that the quadratic equation $z^2 + az + b$, has a complex root equal to $2 - 3i$ and a and b are real numbers. Find a:

    (a) 4

    (b) 25

    (c) -4

    (d) 6

3. What is the argument of $-3 + i\sqrt{3}$:

    (a) $\frac{11\pi}{6}$

    (b) $\frac{2\pi}{3}$

    (c) $\frac{\pi}{6}$

    (d) $\frac{5\pi}{6}$

4. Which of the following are solutions to $z^3 = 2$:

    (a) $\sqrt[3]{2}$

    (b) $-\frac{\sqrt[3]{2}}{2} + i\frac{\sqrt[3]{2}\sqrt{3}}{2}$

    (c) $\frac{\sqrt[3]{2}}{2} + i\frac{\sqrt[3]{2}\sqrt{3}}{2}$

    (d) $-\frac{\sqrt[3]{2}}{2} - i\frac{\sqrt[3]{2}}{2}$

5. Find all complex solutions to the following cubic: $3z^3 - 5z^2 - 10z - 6$:

    (a) $z = 3, \quad z = \pm 2\sqrt{2}i$

    (b) $z = -3, \quad z = \pm 2\sqrt{2}i$

    (c) $z = 3, \quad z = -\frac{2}{3} \pm \frac{\sqrt{2}}{3}i$

    (d) $z = 3, \quad z = \frac{2}{3} \pm \frac{\sqrt{2}}{3}i$

6. Find all complex solutions to $z^2 + z^* = 2z$:

    (a) $z = 0$ and $\frac{\sqrt{3}}{\sqrt{2}} \pm i\frac{\sqrt{3}}{\sqrt{2}}$

    (b) $z = 0$, $z = 1$ and $\frac{3}{2} \pm i\frac{\sqrt{3}}{2}$

    (c) $z = 0$, $z = 1$ and $\frac{\sqrt{3}}{2} \pm i\frac{\sqrt{3}}{\sqrt{2}}$
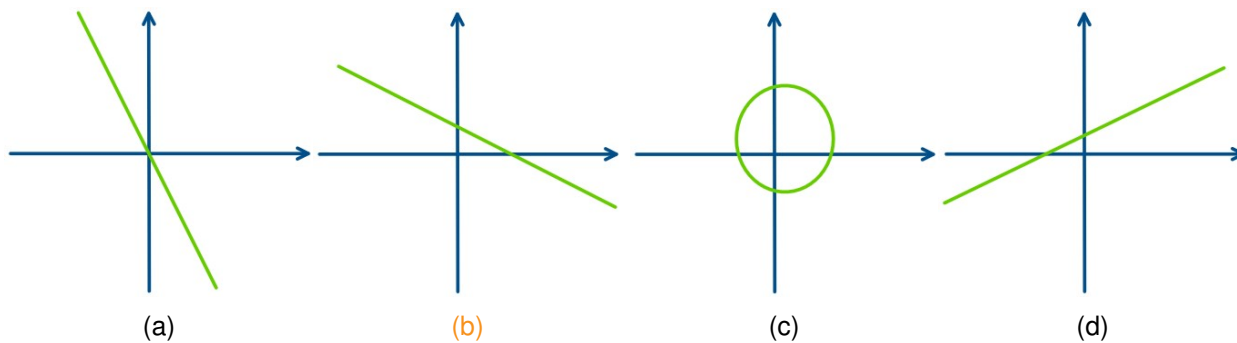
    (d) $z = 0$

7. Which of the following are true:

   (a) $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$

   (b) $\cos 4\theta = 8\cos^4\theta + 8\cos^2\theta + 1$

   (c) $\cos 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$

   (d) $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

8. Which of the following (in green) is a sketch of the loci $|z+1| = |z-2i|$:

(a) (b) (c) (d)

9. Let $\arg\left(\frac{z+1}{z-1}\right) = \frac{\pi}{2}$ and $z^* = -z$. Select all possibilities for z:

   (a) $z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

   (b) $z = i$

   (c) $z = i\frac{1}{\sqrt{2}}$

   (d) $z = -i$