Imperial College London

MATH97050 MATH70057

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May-June 2022

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Riemannian Geometry

Date: 19 May 2022

Time: 09:00 – 11:30 (BST)

Time Allowed: 2:30 hours

Upload Time Allowed: 30 minutes

This paper has 5 Questions.

Candidates should start their solutions to each question on a new sheet of paper.

Each sheet of paper should have your CID, Question Number and Page Number on the top.

Only use 1 side of the paper.

Allow margins for marking.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Each question carries equal weight.

SUBMIT YOUR ANSWERS AS ONE PDF TO THE RELEVANT DROPBOX ON BLACKBOARD WITH COMPLETED COVERSHEETS WITH YOUR CID NUMBER, QUESTION NUMBERS ANSWERED AND PAGE NUMBERS PER QUESTION.

1. (a) Which of the following X define vector fields on $\mathbb{R}$? Here $x \in \mathbb{R}$ and $f \in C^\infty(\mathbb{R})$. Justify your answers.

(i)
$$X|_x f = \frac{\partial f}{\partial x}(x)$$
(2 marks)

(ii)
$$X|_x f = \frac{\partial^2 f}{\partial x^2}(x)$$
(2 marks)

(iii)
$$X|_x f = x^2 \frac{\partial f}{\partial x}(x)$$
(2 marks)

(iv)
$$X|_x f = \frac{\partial f}{\partial x}(0)$$
(2 marks)

(v)
$$X|_x f = f(x)$$
(2 marks)

(vi)
$$X|_x f = f(x) \frac{\partial f}{\partial x}(x)$$
(2 marks)

(b) Consider smooth manifolds $\mathcal{M}$, $\mathcal{N}$, $\mathcal{P}$, and smooth functions $F \in C^\infty(\mathcal{M}, \mathcal{N})$, $G \in C^\infty(\mathcal{N}, \mathcal{P})$.

(i) Show that if F is constant, i.e. $F(p) = q \in \mathcal{N}$ for all $p \in \mathcal{M}$, then $F_{*p} \equiv 0$ for all $p \in \mathcal{M}$. (4 marks)

(ii) Show that $(G \circ F)_{*p} = G_{*F(p)} \circ F_{*p}$ for all $p \in \mathcal{M}$. (4 marks)

(Total: 20 marks)

2. (a) Consider the metric

$$g = (1+4x^2)dx \otimes dx + (1+4y^2)dy \otimes dy + 4xy(dx \otimes dy + dy \otimes dx),$$

on $\mathbb{R}^2$, where (x, y) denote Cartesian coordinates.

(i) Show that the Christoffel symbols of g take the form

$$\Gamma^x_{xx} = \Gamma^x_{yy} = \frac{4x}{1+4x^2+4y^2}, \qquad \Gamma^y_{yy} = \Gamma^y_{xx} = \frac{4y}{1+4x^2+4y^2},$$

and

$$\Gamma^x_{xy} = \Gamma^x_{yx} = \Gamma^y_{xy} = \Gamma^y_{yx} = 0.$$

(5 marks)

(ii) Compute the length of the curve $\gamma\colon (0, 2\pi) \to \mathbb{R}^2$, defined by $\gamma(t) = (\cos t, \sin t)$, with respect to g.

(3 marks)

(iii) Is γ a geodesic of $(\mathbb{R}^2, g)$? Justify your answer.

(4 marks)

(iv) Find a nontrivial isometry $F\colon (\mathbb{R}^2, g) \to (\mathbb{R}^2, g)$, i.e. an isometry F which is not the identity map. Justify your answer.

(4 marks)

(b) Let $(\mathcal{M}, g)$ be a one dimensional Riemannian manifold. Show that, for all $p \in \mathcal{M}$, there exists an open neighbourhood $U \subset \mathcal{M}$ of p, an open set $V \subset \mathbb{R}$, and an isometry $\Phi\colon (U, g) \to (V, g_{\mathrm{Eucl}})$. (4 marks)

(Total: 20 marks)

3. (a) Consider Cartesian coordinates $x^1, \dots, x^n$ on $\mathbb{R}^n$. Justify your answers to the following questions.

(i) Does

$$\nabla_V X|_p = \Big(V(X^i)\Big)^2 \partial_{x^i}|_p,$$

define a connection on $\mathbb{R}^n$? (1 mark)

(ii) Does

$$\nabla_V X|_p = V(X^i)\partial_{x^i}|_p,$$

define a connection on $\mathbb{R}^n$? (1 mark)

(iii) Does

$$\nabla_V X|_p = X(V^i)\partial_{x^i}|_p,$$

define a connection on $\mathbb{R}^n$? (2 marks)

(iv) Does

$$\nabla_V X|_p = x^1 V(X^i)\partial_{x^i}|_p,$$

define a connection on $\mathbb{R}^n$? (2 marks)

(b) Justify your answers to the following questions.

(i) Is ∂_{x^1} a Killing vector of $(\mathbb{R}^n, g_{\mathrm{Eucl}})$? (2 marks)

(ii) Is $x^k \partial_{x^k}$ a Killing vector of $(\mathbb{R}^n, g_{\mathrm{Eucl}})$? (2 marks)

(iii) Is $x^2\partial_{x^1} - x^1\partial_{x^2}$ a Killing vector of $(\mathbb{R}^n, g_{\mathrm{Eucl}})$? (2 marks)

(c) Show that X is a Killing vector of $(\mathcal{M}, g)$ if and only if

$$g(\nabla_Y X, Z) + g(\nabla_Z X, Y) = 0,$$

for all $Y, Z \in \mathfrak{X}(\mathcal{M})$, where ∇ is the Levi-Civita connection of $(\mathcal{M}, g)$. (8 marks)

(Total: 20 marks)

4. (a) Justify your answers to the following questions.

(i) Does there exist a complete metric g on $\mathbb{R}^n$ such that

$$Ric(g) = 0?$$

(3 marks)

(ii) Does there exist a complete metric g on $\mathbb{R}^n$ such that

$$Ric(g) = \eta \otimes \xi - \xi \otimes \eta,$$

for some nowhere-zero one forms $\eta, \xi \in \Gamma(T^*\mathbb{R}^n)$? (3 marks)

(iii) Does there exist a complete metric g on $\mathbb{R}^n$ such that

$$Ric(g) = g?$$

(3 marks)

(iv) Does there exist a complete metric on $\mathbb{R}^n$ such that

$$Ric(g) = -g?$$

(3 marks)

(b) Let $(\mathcal{M}, g)$ be a Riemannian manifold with $n = \dim\mathcal{M} \geq 2$ and let $\gamma\colon [0,\pi] \to \mathcal{M}$ be a unit speed geodesic such that

$$Ric(\dot\gamma(t), \dot\gamma(t)) \geq 2(n-1),$$

for all $t \in [0,\pi]$. Show that γ does not minimise the length between $\gamma(0)$ and $\gamma(\pi)$. You may use any results from the lectures, provided you state them clearly. (8 marks)

(Total: 20 marks)

5. (a) Consider S^3 in polar coordinates

$$S^3 = \{(\cos\psi, \sin\psi\cos\theta, \sin\psi\sin\theta\cos\phi, \sin\psi\sin\theta\sin\phi) \in \mathbb{R}^4 \mid \psi, \theta \in (0,\pi), \phi \in (0, 2\pi)\},$$

with the round metric

$$g_{\mathrm{Round}} = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2).$$

Compute the second fundamental form

$$k(X,Y) = -g_{\mathrm{Round}}(\nabla_X n, Y)$$

of the submanifold

$$N = \left\{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\cos\theta, \frac{\sqrt{2}}{2}\sin\theta\cos\phi, \frac{\sqrt{2}}{2}\sin\theta\sin\phi\right) \in \mathbb{R}^4 \mid \theta \in (0,\pi), \phi \in (0,2\pi)\right\} \subset S^3,$$

in the induced (θ, ϕ) coordinate system, for an appropriate unit normal n. You may use, without proof, the fact that $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

(10 marks)

(b) Let (M, g) be a Riemannian manifold and let $F\colon M \to \mathbb{R}$ be a smooth immersion, so that $\overline{M} := F^{-1}(0)$ is a smooth submanifold of M of dimension $\dim M - 1$. Show that the second fundamental form

$$k(X,Y) = -g(\nabla_X n, Y),$$

of $\overline{M}$, for the appropriate unit normal n, takes the form

$$k(X,Y) = -\frac{1}{|dF|_g}\nabla^2 F(X,Y),$$

for all $X, Y \in \mathfrak{X}(\overline{M})$.

(10 marks)

(Total: 20 marks)

Riemannian geometry exam 2022 – solutions

1. (a) (i) This X does define a vector field. For each $x \in \mathbb{R}$, X_x is a map $X_x : C^\infty(x) \to \mathbb{R}$, which one easily checks is linear and satisfies the Leibniz rule.

Seen [2 mark].

(ii) This X does not define a vector field as it does not satisfy the Leibniz rule.

Unseen [2 mark].

(iii) This X does define a vector field. For each $x \in \mathbb{R}$, X_x is a map $X_x : C^\infty(x) \to \mathbb{R}$, which one easily checks is linear and satisfies the Leibniz rule.

Unseen [2 marks].

(iv) This X does not define a vector field as, for $x \neq 0$, X_x is not a map from $C^\infty(x) \to \mathbb{R}$, i.e. X_x is not an element of $T_x\mathbb{R}$.

Unseen [2 marks].

(v) This X does not define a vector field as it does not satisfy the Leibniz rule.

Unseen [2 marks].

(vi) This X does not define a vector field as X_x is not a linear map over $\mathbb{R}$.

Unseen [2 marks].

(b) (i) Consider $X \in T_p\mathcal{M}$ and $h \in C^\infty(\mathcal{N})$. One computes

$$(F_{*p}X)(h) = X|_p(h \circ F) = X|_p(h(q)) = 0,$$

for all $p \in \mathcal{M}$, since $h(q)$ is constant. Indeed, by the Leibniz rule,

$$X|_p(1) = X|_p(1 \cdot 1) = 2X|_p(1),$$

which implies $X|_p(1) = 0$ and thus $X|_p(c) = 0$ for any constant c by linearity. Hence $F_{*p} = 0$ for all $p \in \mathcal{M}$.

Seen as exercise [4 marks].

(ii) Consider some $p \in \mathcal{M}$, $X \in T_p\mathcal{M}$ and some $h \in C^\infty(G \circ F(p))$. By the definition of pushforward,

$$(G_{*F(p)} \circ F_{*p}X)(h) = (G_{*F(p)}(F_{*p}X))(h) = (F_{*p}X)(h \circ G) = X|_p(h \circ G \circ F) = ((G \circ F)_{*p}X)(h),$$

as desired.

Seen as exercise [4 marks].

2. (a) (i) Using the standard formula

$$\Gamma^k_{ij} = \frac{g^{kl}}{2}\Big(\frac{\partial g_{jl}}{\partial x^i} + \frac{\partial g_{il}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^l}\Big),$$

one computes

$$\Gamma^x_{xx} = \Gamma^x_{yy} = \frac{4x}{1 + 4x^2 + 4y^2}, \qquad \Gamma^x_{xy} = \Gamma^x_{yx} = 0,$$

and

$$\Gamma^y_{yy} = \Gamma^y_{xx} = \frac{4y}{1 + 4x^2 + 4y^2}, \qquad \Gamma^y_{xy} = \Gamma^y_{yx} = 0.$$

Unseen [5 marks].

(ii) Note that

$$\dot{\gamma}(t) = -\sin t \partial_x + \cos t \partial_y,$$

and so

$$|\dot{\gamma}(t)|_g^2 = (1 + 4\cos^2 t)\sin^2 t + (1 + 4\sin^2 t)\cos^2 t - 8\cos^2 t \sin^2 t = 1.$$

Hence

$$L(\gamma) = \int_0^{2\pi} |\dot{\gamma}(t)| dt = 2\pi.$$

Unseen [3 marks].

(iii) Using the above expressions for the Christoffel symbols, one sees that

$$\ddot{\gamma}^x + \dot{\gamma}^i \dot{\gamma}^j \Gamma^x_{ij}(\gamma(t)) = -\frac{1}{5}\cos t, \qquad \ddot{\gamma}^y + \dot{\gamma}^i \dot{\gamma}^j \Gamma^y_{ij}(\gamma(t)) = -\frac{1}{5}\sin t,$$

and so the geodesic equations are not satisfied. So γ is not a geodesic.

Unseen [4 marks].

(iv) Any rotation around the origin is an isometry. For example, take

$$F(x, y) = (y, -x).$$

Clearly $F : \mathbb{R}^2 \to \mathbb{R}^2$ is a diffeomorphsim. Moreover, a short computation gives

$$F_{*(x,y)}\partial_x = -\partial_y, \qquad F_{*(x,y)}\partial_y = \partial_x,$$

and so

$$F^* g|_{(x,y)}(\partial_x, \partial_x) = g|_{(y,-x)}(-\partial_y, -\partial_y) = 1 + 4x^2,$$
$$F^* g|_{(x,y)}(\partial_y, \partial_y) = g|_{(y,-x)}(\partial_x, \partial_x) = 1 + 4y^2,$$
$$F^* g|_{(x,y)}(\partial_x, \partial_y) = g|_{(y,-x)}(-\partial_y, \partial_x) = 4xy.$$

Hence

$$F^* g|_{(x,y)} = (1 + 4x^2) dx \otimes dx + (1 + 4y^2) dy \otimes +4xy(dx \otimes dy + dy \otimes dx) = g|_{(x,y)},$$

and so F is an isometry.

Unseen [4 marks].

(b) Consider some $p \in M$. Since M is a one dimensional smooth manifold, there exists $U \subset M$ open with $p \in U$, $W \subset \mathbb{R}$ open, and a homeomorphism $\phi : U \to W$. In the associated x coordinate

$$g = f(x) dx^2,$$

for some smooth function $f > 0$. Define $\psi : (x(p) - \varepsilon, x(p) + \varepsilon) \to \mathbb{R}$ by

$$\psi(x) = \int_{x(p)-\varepsilon}^{x} \sqrt{f(z)} dz.$$

Note that ψ is a smooth function, well defined if ε is sufficiently small, such that $\psi' > 0$. Hence ψ is a diffeomorphism onto its image V. Define $\Phi : U \to V$ by

$$\Phi = \psi \circ \phi.$$

Clearly (U, Φ) is a chart. In the associated $y = \psi(x)$ coordinate,

$$dy^2 = \psi'(x)^2 dx^2 = f(x) dx^2 = g,$$

i.e. $g = \Phi^* g_{\mathrm{Eucl}}$ and so Φ is an isometry.

Seen as exercise [4 marks].

3. (a) (i) No: the map $X \mapsto \nabla_V X$ is not linear over $\mathbb{R}$.

Unseen [1 mark].

(ii) Yes, one can check that $X \mapsto \nabla_V X$ is linear over $\mathbb{R}$, $V \mapsto \nabla_V X$ is linear over $C^\infty(\mathbb{R}^2)$, and $\nabla_V(fX) = V(f)X + f\nabla_V X$. In fact, ∇ is the Euclidean connection.

Seen [1 mark].

(iii) No: the map $V \mapsto \nabla_V X$ is not linear over $C^\infty(\mathbb{R}^2)$.

Unseen [2 marks].

(iv) No: $\nabla_V(fX) = x^1 V(f)X + f\nabla_V X$, so the product rule is not satisfied away from $\{x^1 = 1\}$.

Unseen [2 marks].

(b) Recall the formula for the Lie derivative of a $(0,2)$ tensor field:

$$(\mathcal{L}_V g)_{ij} = V(g_{ij}) + \partial_i V^k g_{kj} + \partial_j V^k g_{ik}.$$

(i) Yes: by the above

$$\mathcal{L}_{\partial_{x^1}} g = 0.$$

Unseen [2 mark].

(ii) No: if $V = x^k \partial_{x^k}$ then $\partial_i V^k = \delta_i^k$ and so, by the above,

$$(\mathcal{L}_{x^k \partial_{x^k}} g)_{ij} = \partial_i V^k g_{kj} + \partial_j V^k g_{ik} = 2g_{ij}.$$

Unseen [2 mark].

(iii) Yes: If $V = x^2\partial_{x^1} - x^1\partial_{x^2}$ then

$$\partial_{x^i} V^1 = -\delta_i^2, \qquad \partial_{x^i} V^2 = \delta_i^1, \qquad \partial_{x^i} V^k = 0, \quad k = 3, \dots, n.$$

Hence

$$(\mathcal{L}_{x^2\partial_{x^1} - x^1\partial_{x^2}} g)_{ij} = \delta_i^1 g_{2j} - \delta_i^2 g_{1j} + \delta_j^1 g_{2i} - \delta_j^2 g_{1i},$$

from which one easily checks that $\mathcal{L}_{x^2\partial_{x^1} - x^1\partial_{x^2}} g = 0$.

Unseen [2 marks].

(c) Recall that the Lie derivative satisfies

$$X(g(Y,Z)) = (\mathcal{L}_X g)(Y,Z) + g(\mathcal{L}_X Y, Z) + g(Y, \mathcal{L}_X Z).$$

Now for vector fields, the Lie derivative is equal to the Lie bracket, $\mathcal{L}_X Y = [X,Y]$, and so

$$(\mathcal{L}_X g)(Y,Z) = X(g(Y,Z)) - g([X,Y],Z) - g(Y,[X,Z]).$$

The covariant derivative similarly satisfies

$$X(g(Y,Z)) = (\nabla_X g)(Y,Z) + g(\nabla_X Y, Z) + g(Y, \nabla_X Z) = g(\nabla_X Y, Z) + g(Y, \nabla_X Z),$$

where the latter holds by the compatibility of ∇ with g. Hence

$$(\mathcal{L}_X g)(Y,Z) = g(\nabla_X Y, Z) + g(Y, \nabla_X Z) - g([X,Y],Z) - g(Y,[X,Z]) = g(\nabla_Y X, Z) + g(Y, \nabla_Z X),$$

by the torsion free property of ∇.

Unseen [8 marks].

4. (a) (i) Yes: the Euclidean metric $g = dx^2 + dy^2 + dz^2$.

Seen [3 marks].

(ii) No: by the symmetries of R it must be the case that $Ric(X,Y) = Ric(Y,X)$ for all X, Y, but $\eta \otimes \xi - \xi \otimes \eta$ is anti-symmetric and so such a metric cannot exist if η and ξ are nowhere vanishing.

Unseen [2 marks].

(iii) No: such a metric would satisfy the assumptions of the Bonnet–Myers theorem, which would then imply that $\mathbb{R}^n$ is compact.

Seen similar [3 marks].

(iv) Yes: the hyperbolic metric

$$g = \left(\frac{2}{1 - \left((x^1)^2 + \ldots + (x^n)^2\right)} \right)^2 \left((dx^1)^2 + \ldots + (dx^n)^2\right),$$

on the open unit ball $B^n(0,1) \subset \mathbb{R}^n$ satisfies $Ric(g) = -g$. Consider the diffeomorphism $\Phi \colon B^n(0,1) \to \mathbb{R}^n$ defined by $\Phi(x) = \frac{x}{\sqrt{1-|x|^2}}$. The pullback $(\Phi^{-1})^* g$ defines such a metric on $\mathbb{R}^n$.

Seen similar [3 marks].

(b) Recall the second variation formula: If A is a proper variation of γ with variational vector field $V = \partial_s A|_{s=0}$ then

$$\left.\frac{d^2}{ds^2}\right|_{s=0} L(A_s) = I(V,V),$$

where

$$I(V,V) = -\int_0^\pi g(D_t^2 V - R(\dot\gamma, V)\dot\gamma, V) dt,$$

is the index form of γ. Note that any proper vector field $V \in \mathfrak{X}(\gamma)$ arises from some variation variation (indeed, take $A(s,t) = \exp(\gamma(t), sV(t))$) and so it suffices to find a proper vector field V such that

$$I(V,V) < 0.$$

Extend $e_1 = \dot\gamma(0)$ to an orthonormal frame $\{e_i\}_{i=1}^n$ for $T_{\gamma(0)}\mathcal{M}$ and extend to a frame for $T_{\gamma(t)}\mathcal{M}$ by parallel transport along γ:

$$D_t e_i = 0, \qquad i = 2, \ldots, n.$$

Define vector fields $V_i \in \mathfrak{X}(\gamma)$ by

$$V_i(t) = \sin t e_i, \qquad i = 2, \ldots, n.$$

Then

$$D_t^2 V_i(t) = -V_i(t),$$

and so

$$\begin{aligned}\sum_{i=2}^n I(V_i, V_i) &= \sum_{i=2}^n \int_0^\pi |V_i(t)|_g^2 - \sin^2 t Rm(e_i, \dot\gamma, \dot\gamma, e_i) dt \\ &= \int_0^\pi (n-1)\sin^2 t - \sin^2 t Ric(\dot\gamma, \dot\gamma) dt \le -\int_0^\pi (n-1)\sin^2 t dt < 0,\end{aligned}$$

where the assumption is used in the penultimate step. It follows that there exists $i \in \{2, \ldots, n\}$ such that

$$I(V_i, V_i) < 0,$$

as desired.

Unseen [8 marks].

5. (a) Let $\overline{\partial}$ denote derivatives with respect to the induced (θ, ϕ) coordinate system for N, and let ∂ denote derivatives with respect to the ambient (ψ, θ, ϕ) coordinate system for S^3. First note that

$$\overline{\partial}_\theta = \partial_\theta, \qquad \overline{\partial}_\phi = \partial_\phi.$$

It follows that

$$g_{\text{Round}}(\partial_\psi, \overline{\partial}_\theta) = g_{\text{Round}}(\partial_\psi, \overline{\partial}_\phi) = 0,$$

and so $n = \partial_\psi$ is a unit normal to N.
Using the standard formula for the Christoffel symbols of the Levi-Civita connection, one computes

$$\nabla_{\partial_\theta}\partial_\psi = \frac{\cos\psi}{\sin\psi}\partial_\theta, \qquad \nabla_{\partial_\phi}\partial_\psi = \frac{\cos\psi}{\sin\psi}\partial_\phi.$$

Hence, on N,

$$g(\nabla_{\partial_\theta} n, \partial_\theta) = g_{\theta\theta} = \sin^2\frac{\pi}{4} = \frac{1}{2}, \qquad g(\nabla_{\partial_\phi} n, \partial_\phi) = g_{\phi\phi} = \frac{1}{2}\sin^2\theta,$$

$$g(\nabla_{\partial_\phi} n, \partial_\theta) = g(\nabla_{\partial_\theta} n, \partial_\phi) = 0,$$

and so

$$k = -\frac{1}{2}d\theta^2 - \frac{1}{2}\sin^2\theta d\phi^2.$$

Unseen [10 marks].

(b) Note first that the unit normal n is given by

$$n = \frac{dF^\flat}{|dF|_g}.$$

Indeed, if $X \in T_p\overline{M}$ then $g(dF^\flat, X) = X(F) = 0$, since $\overline{M}$ is a level hypersurface of F, and clearly $|n|_g = 1$. Now

$$\begin{aligned} k(X,Y) = -g(\nabla_X \frac{dF^\flat}{|dF|_g}, Y) &= -X\left(g\left(\frac{dF^\flat}{|dF|_g}, Y\right)\right) + g\left(\frac{dF^\flat}{|dF|_g}, \nabla_X Y\right) \\ &= -\frac{\nabla_X\nabla_Y F}{|dF|_g} - X(|dF|_g^{-1})Y(F) + \frac{\nabla_{\nabla_X Y}F}{|dF|_g} \\ &= -\frac{1}{|dF|_g}\nabla^2 F(X,Y), \end{aligned}$$

where the final equality follows from the fact that $Y(F) = 0$, since $Y \in \mathfrak{X}(\overline{M})$ and $\overline{M}$ is a level hypersurface of F.

Unseen [10 marks].

If your module is taught across multiple year levels, you might have received this form for each level of the module. You are only required to fill this out once for each question.
Please record below, some brief but non-trivial comments for students about how well (or otherwise) the questions were answered. For example, you may wish to comment on common errors and misconceptions, or areas where students have done well. These comments should note any errors in and corrections to the paper. These comments will be made available to students via the MathsCentral Blackboard site and should not contain any information which identifies individual candidates. Any comments which should be kept confidential should be included as confidential comments for the Exam Board and Externals. If you would like to add formulas, please include a separate pdf file with your email.

ExamModuleCode	QuestionNumber	Comments for Students
Riemannian Geometry_MATH97050 MATH70057	1	Question 1 was generally well answered. Some students incorrectly thought that (a) (iv) is a vector field.
Riemannian Geometry_MATH97050 MATH70057	2	Question 2(a) was well answered. Many students struggled with part (b), which involves showing that every one dimensional Riemannian manifold is flat.
Riemannian Geometry_MATH97050 MATH70057	3	Students answered part (a) of question 3 well. There were some minor errors in computing the Lie derivatives of the metric in part (b). Some students found part (c) difficult.
Riemannian Geometry_MATH97050 MATH70057	4	Parts (a) (i) and (ii) of question 4 were well answered. Many students recognised that the Bonnet--Myers Theorem is relevant for (iii). Few students successfully answered part (iv). Some students inncorrectly thought that the Bonnet--Myers Theorem should be used for (b), rather than adapting its proof.
Riemannian Geometry_MATH97050 MATH70057	5	Many students struggled with Question 5. Perhaps because they had run out of time.