MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Hints for Problem Sheet 4

Exercise 16.

To prove (i), assume there exists a non-monotone solution, and look at the right hand side in the local extremum. To prove (ii), assume that $f(c) \neq 0$, and use arguments you have seen before (such as in Exercise 12 (iii)).

Exercise 17.

There are some hints on how to solve this exercise in the comments section on the problem sheet. From Exercise 16 (i), one gets that solutions are monotone, and Exercise 16 (ii) implies that thus every solution converges to either an equilibrium or to ∞ or $-\infty$. So the half-orbits are given by an interval, one boundary point is given by where the orbit starts, and the other boundary point is given by the limit (forward or backward in time, depending on which half orbit is considered).

Exercise 18.

To find out in (i) which functions are flows, you may want to check the properties of a flow given in Proposition 2.24 (or listed in the question itself). If these properties are not fulfilled, then the function is not a flow. Using (ii) and under the differentiability property required in (ii), the converse can be established as well. For solving (ii), differentiate the identity given in the hint with respect to s and set $s = 0$.

Exercise 19.

Look at suitable partial differentiation of the identities $\frac{\partial \lambda}{\partial t}(t, t_0, x_0) = f(t, \lambda(t, t_0, x_0))$ and $\lambda(t_0, t_0, x_0) = x_0$, which are defined for all $(t, t_0, x_0) \in \Omega$.

Exercise 20.

The main objective is to prove (iii), and (i) and (ii) provide a good starting point for that, and are relatively straightforward compared to (iii). To establish (iii), show first that the sequence $\{\nu_k\}_{k \in \mathbb{N}_0}$ converges uniformly on $[0, T]$ and that the limit function is a solution of the differential equation.