

**Mathematics Pre-arrival course**

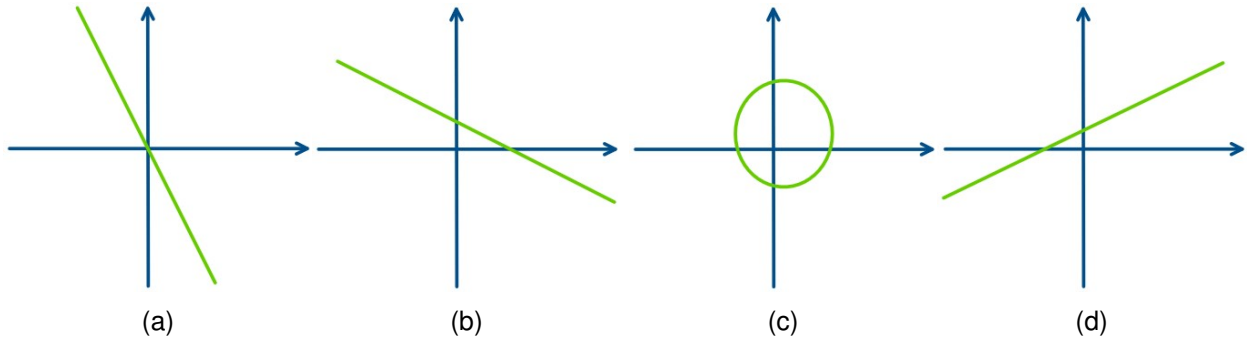
**Weekly Quiz 4 – Complex Numbers**

1. Which of the following is equal to  $i^{103}$ :
  - (a)  $i$
  - (b)  $-1$
  - (c)  $-i$
  - (d)  $1$
2. Given that the quadratic equation  $z^2 + az + b$ , has a complex root equal to  $2 - 3i$  and  $a$  and  $b$  are real numbers. Find  $a$ :
  - (a)  $4$
  - (b)  $25$
  - (c)  $-4$
  - (d)  $6$
3. What is the argument of  $-3 + i\sqrt{3}$ :
  - (a)  $\frac{11\pi}{6}$
  - (b)  $\frac{2\pi}{3}$
  - (c)  $\frac{\pi}{6}$
  - (d)  $\frac{5\pi}{6}$
4. Which of the following are solutions to  $z^3 = 2$ :
  - (a)  $\sqrt[3]{2}$
  - (b)  $-\frac{\sqrt[3]{2}}{2} + i\frac{\sqrt[3]{2}\sqrt{3}}{2}$
  - (c)  $\frac{\sqrt[3]{2}}{2} + i\frac{\sqrt[3]{2}\sqrt{3}}{2}$
  - (d)  $-\frac{\sqrt[3]{2}}{2} - i\frac{\sqrt[3]{2}}{2}$
5. Find all complex solutions to the following cubic:  $3z^3 - 5z^2 - 10z - 6$ :
  - (a)  $z = 3, \quad z = \pm 2\sqrt{2}i$
  - (b)  $z = -3, \quad z = \pm 2\sqrt{2}i$
  - (c)  $z = 3, \quad z = -\frac{2}{3} \pm \frac{\sqrt{2}}{3}i$
  - (d)  $z = 3, \quad z = \frac{2}{3} \pm \frac{\sqrt{2}}{3}i$
6. Find all complex solutions to  $z^2 + z^* = 2z$ :
  - (a)  $z = 0$  and  $\frac{\sqrt{3}}{\sqrt{2}} \pm i\frac{\sqrt{3}}{\sqrt{2}}$
  - (b)  $z = 0, z = 1$  and  $\frac{3}{2} \pm i\frac{\sqrt{3}}{2}$
  - (c)  $z = 0, z = 1$  and  $\frac{\sqrt{3}}{2} \pm i\frac{\sqrt{3}}{\sqrt{2}}$
  - (d)  $z = 0$

7. Which of the following are true:

- (a)  $\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1$
- (b)  $\cos 4\theta = 8 \cos^4 \theta + 8 \cos^2 \theta + 1$
- (c)  $\cos 4\theta = 4 \cos^3 \theta \sin \theta - 4 \cos \theta \sin^3 \theta$
- (d)  $\cos 4\theta = \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$

8. Which of the following (in green) is a sketch of the loci  $|z + 1| = |z - 2i|$ :



9. Let  $\arg\left(\frac{z+1}{z-1}\right) = \frac{\pi}{2}$  and  $z^* = -z$ . Select all possibilities for  $z$ :

- (a)  $z = \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}$
- (b)  $z = i$
- (c)  $z = i \frac{1}{\sqrt{2}}$
- (d)  $z = -i$