MATH40004 - Calculus and Applications - Term 2

Assessed course work - due date Tuesday March 14, 1pm

Please show reasonable steps of your calculations. Each part has 10 marks for a total of 50 marks.

1. Consider the following system of differential equations:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}. \tag{1}$$

   (a) Find the solution for this system in terms of real functions.

   (b) Sketch the phase portrait for this system in the phase plane. Describe the asymptotic behavior explaining your reasoning.

   (c) Find γ and ω such that the equation

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega^2 x = 0$$

   has the same dynamics as the system (1).

   (d) Consider now the system:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 3 & -3+\epsilon \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

   Using a graph of τ and $Delta$ plane, find the value of $\epsilon \in \mathbb{R}$ at which the system undergoes a bifurcation (where the system changes stability).

   (e) Finally consider the non-homogenous system:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}.$$

   Find the general solution and particularize it when $x(0) = 1$ and $y(0) = 0$.