

# Mathematics Year 1, Calculus and Applications I

## Challenge Problems 2

These are a little more physically inspired problems but with a rich mathematical history behind them as I will point out.

### 1. The Catenary - *linea catenaria funicularis*<sup>1</sup>

Find the shape assumed by a flexible chain under the action of gravity, when it is hung from its two ends. A schematic of the problem is given in the figure - the chain is in blue and hangs from A and B.

Hint: As shown in the figure, take an arbitrary piece PQ of the chain. The forces at P and Q are shown schematically (the force  $\mathbf{T}(x)$  is tangent to the curve and the angle  $\theta$  to the horizontal has been defined as shown). The force  $\mathbf{T}_c$  represents the weight of the chain piece PQ.

- (a) Balance forces and come up with a differential equation that you need to solve.
- (b) Assume that the density of the chain,  $\rho$  say, is constant. Then you should be able to solve the equation and get the shape in closed form (define your preferred geometry - I picked everything to me 1 but that of course is a schematic - the only things you need to prescribe are (i) the chain length  $L$ , (ii) the distance AB; clearly  $AB < L$ ).
- (c) What happens if  $\rho$  is not constant, e.g. take the density to be constant and  $\rho_0$  in the middle third of the chain and another constant  $\rho_1$  in the two outer thirds of the chain. If  $\rho_1 = \rho_2$  you get the answer in part (b).

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<sup>1</sup>Historical comments: Galileo wrote in 1638 that the curve is a parabola to within tiny errors (*ad unguem*). He is of course wrong, as was pointed out about 20 years later by Huygens. A solution was finally given independently by Leibniz and Johann Bernoulli in 1691. If your scientific Latin is reasonable, let me know and I can give you the references!

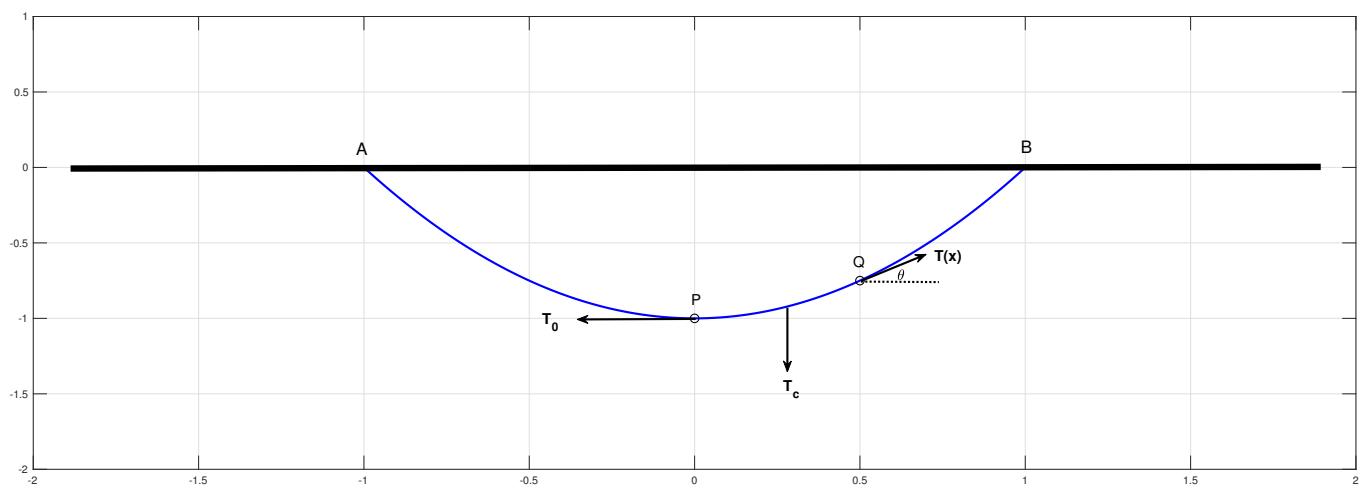


Figure 1: Schematic of the hanging chain.