# MATH50004/MATH50015/MATH50019 Differential Equations
## Spring Term 2023/24
## Solutions to Quiz 4

**Question 1**. Correct answer: (a).

Translation invariance and the cocycle property implies that

$$\lambda(t,s,\lambda(u,s,x)) = \lambda(t-s+u,u,\lambda(u,s,x)) = \lambda(t-s+u,s,x) \quad \text{for all } u,s\in\mathbb{R} \text{ and } x\in\mathbb{R}^d\,.$$

**Question 2**. Correct answer: (a).

We have

$$\begin{aligned} O(x) &= \{\varphi(\tau,x) : \tau \in J_{max}(x)\} = \{\varphi(\tau+t,x) = \varphi(\tau,\varphi(t,x)) : \tau \in J_{max}(x)-t\} \\ &= \{\varphi(\tau,\varphi(t,x)) : \tau \in J_{max}(\varphi(t,x))\} = O(\varphi(t,x))\,, \end{aligned}$$

where (2.29) was used.

**Question 3**. Correct answer: (a).

Consider $\dot{x} = t$, the general solution of which is given by $\lambda(t,t_0,x_0) = \frac{1}{2}(t^2-t_0^2)+x_0$. For any $(t_0,x_0)\in\mathbb{R}^2$, this function is non-monotone in $t$.

**Question 4**. Correct answer: (b).

Due to Proposition 3.4 (ii), $e^{A5}$ is invertible for any $A\in\mathbb{R}^{5\times 5}$, so such an $x\in\mathbb{R}^5\setminus\{0\}$ does not exist.

**Question 5**. Correct answer: (a).

For any $n\in\mathbb{N}$, we have

$$\sum_{k=0}^{n}\frac{t^k(A^\top)^k}{k!} = \sum_{k=0}^{n}\frac{t^k(A^k)^\top}{k!} = \left(\sum_{k=0}^{n}\frac{t^kA^k}{k!}\right)^\top.$$

Since in the limit $n\to\infty$, the left hand side converges to $e^{A^\top t}$, and the right hand side converges to $(e^{At})^\top$, the statement is true.