

MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Quiz 4

This is an assessed quiz. The quiz will open on Friday, 9 February at 10am
and close on Saturday, 10 February at 10.30am.

Question 1 (General solutions).

Let λ be the general solution to an autonomous differential equation $\dot{x} = f(x)$, where the right hand side $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$ is (globally) Lipschitz continuous. Is it true that then

$$\lambda(t, s, \lambda(u, s, x)) = \lambda(t - s + u, s, x) \quad \text{for all } t, s \in \mathbb{R} \text{ and } x \in \mathbb{R}^d?$$

- (a) The statement is true.
- (b) The statement is false.

Question 2 (Orbits).

Let φ be the flow of $\dot{x} = f(x)$, where $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$ is locally Lipschitz continuous. Is it true that then

$$O(x) = O(\varphi(t, x)) \quad \text{for all } x \in \mathbb{R}^d \text{ and } t \in J_{\max}(x).$$

- (a) The statement is true.
- (b) The statement is false.

Question 3 (Non-monotone solutions to one-dimensional differential equations).

Is the following statement true? There exists a one-dimensional differential equation $\dot{x} = f(t, x)$, where $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous and globally Lipschitz continuous (with respect to x) such that all maximal solutions (i.e. defined on \mathbb{R}) are non-monotone.

- (a) The statement is true.
- (b) The statement is false.

Question 4 (Matrix exponential function I).

Is the following statement true? There exists an $A \in \mathbb{R}^{5 \times 5}$ and $x \in \mathbb{R}^5 \setminus \{0\}$ such that $e^{A5}x = 0 \in \mathbb{R}^5$?

- (a) The statement is true.
- (b) The statement is false.

Question 5 (Matrix exponential function II).

Let $A \in \mathbb{R}^{d \times d}$ and $t \in \mathbb{R}$. Is it true that then $e^{A^\top t} = (e^{At})^\top$?

- (a) The statement is true.
- (b) The statement is false.