MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Quiz 7

Question 1 (Invariant sets I).

Consider a differential equation $\dot{x} = f(x)$, where $f : \mathbb{R}^d \to \mathbb{R}^d$ is continuously differentiable. Is the following statement true? For every $x \in \mathbb{R}^d$, $O^+(x)$ is positively invariant.

(a) The statement is true.

(b) The statement is false.

Question 2 (Invariant sets II).

Is the following statement true? There exists a differential equation $\dot{x} = f(x)$, where $f : \mathbb{R}^d \to \mathbb{R}^d$ is continuously differentiable such that $O^+(x)$ is negatively invariant for all $x \in \mathbb{R}^d$.

(a) The statement is true.

(b) The statement is false.

Question 3 (Invariant sets III).

Consider a differential equation $\dot{x} = f(x)$, where $f : \mathbb{R}^d \to \mathbb{R}^d$ is locally Lipschitz continuous. Is the following statement true? If M_1 and M_2 are invariant, then $M_1 \setminus M_2$ is invariant.

(a) The statement is true.

(b) The statement is false.

Question 4 (Stability).

Consider the two-dimensional differential equation

$$\dot{x} = x + by + x^2\,,$$
$$\dot{y} = -bx + y + yx\,.$$

Which of the following two statements is true?

(a) There exists a $b \in \mathbb{R}$ such that the equilibrium $(0,0)$ is stable.

(b) For all $b \in \mathbb{R}$, the equilibrium $(0,0)$ is unstable.

Question 5 (Stable and unstable sets).

Is the following statement true? There exists a differential equation $\dot{x} = f(x)$, where $f : \mathbb{R}^2 \to \mathbb{R}^2$ and x^* is an equilibrium, such that $W^u(x^*) \cap W^s(x^*) \supsetneq \{x^*\}$.

(a) The statement is true.

(b) The statement is false.