Find stationary points of

$$f(x_1, x_2) = \frac{x_1^3}{3} + \frac{x_1^2}{2} + 2x_1x_2 + \frac{x_2^2}{2} - x_2 + 1$$

$\nabla f = 0$

$$\frac{\partial f}{\partial x_1} = x_1^2 + x_1 + 2x_2 = 0$$

$$\frac{\partial f}{\partial x_2} = 2x_1 + x_2 - 1 = 0$$

$$\hookrightarrow x_2 = 1 - 2x_1$$

$$\Rightarrow x_1^2 + x_1 + 2(1 - 2x_1) = 0$$

$$(x_1 - 1)(x_1 - 2) = 0$$

$$x_1 = 1 \qquad x_1 = 2$$

$$\downarrow \qquad\qquad \downarrow$$

$$x_2 = -1 \qquad x_2 = -3$$

$$\nabla^2 f = \quad \frac{\partial f}{\partial x_1^2} = 2x_1 + 1 \qquad \frac{\partial^2 f}{\partial x_2^2} = 1$$

$$\frac{\partial f}{\partial x_1 \partial x_2} = \frac{\partial f}{\partial x_1 \partial x_1} = 2 \qquad \nabla^2 f = \begin{bmatrix} 2x_1 + 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\nabla^2 f(1,-1) = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{matrix} \nearrow \text{det} < 0 \\ \searrow \text{trace} > 0 \end{matrix} \Rightarrow \text{Saddle point}$$

$$\nabla^2 f(2,-3) = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{matrix} \nearrow \text{det} > 0 \\ \searrow \text{trace} > 0 \end{matrix} \Rightarrow \underline{\underline{\text{minimum.}}}$$

local or global? check convexity

$$\frac{x_1^3}{3} + \frac{x_1^2}{2} + \ldots \text{ lower-order terms}$$

Also $x_1 > 0 \longrightarrow \infty \Longrightarrow f(x_1, x_2) \longrightarrow +\infty$

$x_1 < 0$, take $x_1 \longrightarrow -\infty$ $f(x_1, x_2) \longrightarrow -\infty$

$\Longrightarrow$ We can only obtain local optimality.