

# MATH60005/70005: Optimization (Autumn 22-23)

## Week 10: Exercises

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1. Solve the primal and dual problem for

$$\begin{array}{ll}\min & x_1^2 + x_2^2 + 2x_1 \\ \text{s.t.} & x_1 + x_2 = 0.\end{array}$$

2. Study the duality gap (difference between  $f^*$  and  $q^*$ ) for the problem

$$\min \left\{ e^{-x_2} : \sqrt{x_1^2 + x_2^2} - x_1 \leq 0 \right\}.$$

3. Recompute the dual of the convex quadratic problem from the notes under that assumption that the matrix  $\mathbf{Q} \geq 0$  instead of  $\mathbf{Q} > 0$ .
4. Consider the Chebyshev center problem where we have a set of points  $\mathbf{a}_1, \dots, \mathbf{a}_m \in \mathbb{R}^n$  for which we seek a point  $\mathbf{x} \in \mathbb{R}^n$  that is the center of a ball of minimum radius  $r > 0$  containing the points

$$\begin{array}{ll}\min_{\mathbf{x}, r} & r \\ \text{s.t.} & \|\mathbf{x} - \mathbf{a}_i\| \leq r, \quad i = 1, 2, \dots, m.\end{array}$$

Compute the dual of this problem. (*Hint: use an equivalent formulation over the squared radius*)

