Imperial College London

Lecture 02: Point Estimation

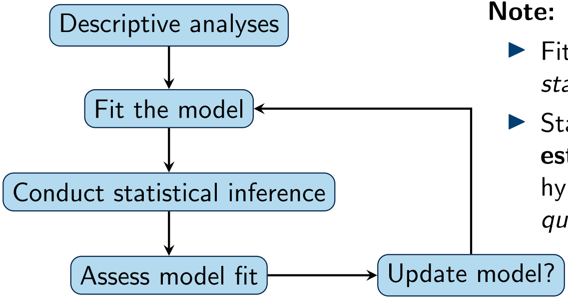
Statistical Modelling I

Dr. Riccardo Passeggeri

Outline

Point Estimation

Statistical Analysis



Note:

- Fit the model $\Leftrightarrow$ **estimate** θ in the *statistical model*
- Statistical inference $\Leftrightarrow$ **point estimate**, interval estimate, hypothesis test to address *scientific question*

Statistics, Estimates and Estimators

Data $y_1, \ldots, y_n$ is one realisation of $Y_1, \ldots, Y_n$.

Definition

- **Statistic**: a function t of observable random variables
- **Estimate** (of θ): $t(y_1, \ldots, y_n)$
- **Estimator** (of θ): $T = t(Y_1, \ldots, Y_n)$

Example: $Y_1, \ldots, Y_n$ iid $N(\theta, \sigma^2) \Rightarrow$ how to estimate θ?

Candidate Estimators

- Sample mean:

$$\frac{1}{n}\sum_{i=1}^{n} Y_i$$

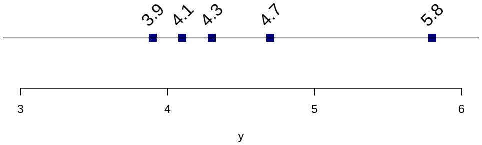
- Median (n odd):

$$Y_{(1)} < Y_{(2)} < \cdots < Y_{(n+1)/2} < \cdots < Y_{(n)}$$

- k-Trimmed mean:

$$\frac{1}{n-2k}\sum_{i=k+1}^{n-k} Y_{(i)}$$

Data

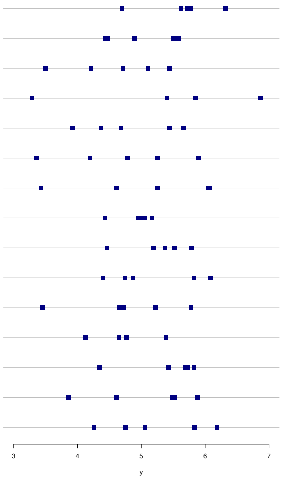


Candidate Estimates

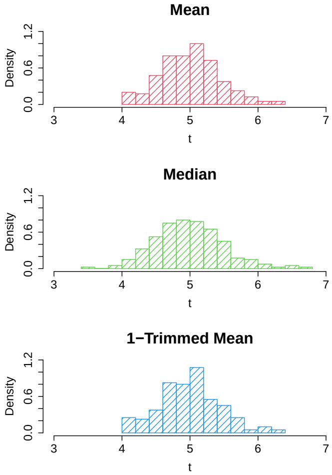
- Sample mean:
- Median:
- 1-Trimmed mean:

Example: $Y_1, \ldots, Y_n$ iid $N(\theta, \sigma^2) \Rightarrow$ repeat the experiment

New data sets ($n = 5$)



Sampling distributions of estimates



Properties of Estimators

Properties of estimators

Key idea: $T = t(Y_1, \ldots, Y_n)$ is a random variable and summaries of its sampling distribution

$$P_\theta(T \in \mathcal{A}) \qquad E_\theta(T) \qquad Var_\theta(T) \qquad etc\ldots$$

can be computed. Comparing different estimators means comparing the properties of their summaries.

Common properties of estimators:

- Bias
- Standard error
- Mean square error

Definition: Bias (general)

If $\Theta \subset \mathbb{R}^k$, $g(\theta)$ for $g : \Theta \to \mathbb{R}$ and T is an estimator of $g(\theta)$, then $bias_\theta(T) = E_\theta(T) - g(\theta)$

Example: $Y_1, \ldots, Y_n \sim N(\mu, \sigma^2)$ iid, $\theta = (\mu, \sigma^2) \in \Theta = \mathbb{R} \times (0, \infty)$ unknown

Definition: Unbiased Estimator

If $bias_\theta(T) = 0$ for all $\theta \in \Theta$, then T is unbiased for $g(\theta)$

Example: $X \sim \text{Binomial}(n, p)$, $p \in [0, 1]$ unknown

$$S = X/n \qquad\qquad T = \frac{X+1}{n+2}$$

Definition: Standard Error and MSE

Let T be an estimator for $\theta \in \Theta \subset \mathbb{R}$.

The standard error (SE) is the standard deviation of the sampling distribution of T: $SE_\theta(T) = \sqrt{Var_\theta(T)}$

The mean square error (MSE) of T is defined by $MSE_\theta(T) = E_\theta[(T - \theta)^2]$.

Worked Examples

Example: $Y_1, \ldots, Y_n$ iid with mean μ and variance $\sigma^2 \Rightarrow$ estimating $\theta = (\mu, \sigma^2)$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \qquad\qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

Example: $X \sim \text{Binomial}(n, p) \Rightarrow$ estimating p

$$S = X/n \qquad\qquad T = \frac{X+1}{n+2}$$