MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Solutions to Quiz 1

Question 1. Correct answer: (b).

Due to the chain rule we get $\dot{\mu}(t) = -\dot{\lambda}(-t) = -f(\lambda(-t)) = -f(\mu(t))$, so μ solves the differential equation $\dot{x} = -f(x)$.

Question 2. Correct answer: (c).

Due to the chain rule we get $\dot{\mu}(t) = -\dot{\lambda}(-t) = -f(-t, \lambda(-t)) = -f(-t, \mu(t))$, so μ solves the differential equation $\dot{x} = -f(-t, x)$.

Question 3. Correct answer: (a).

Note that this differential equation has two constant solutions, the values which are zeros of the right hand side (see Proposition 1.3) and given by 0 and 1, as indicated in all the phase portrait. Outside of the equilibria, solutions are monotone, and the direction is given by the sign of the right hand side: positive on $(-\infty, 0) \cup (1, \infty)$ and negative on $(0, 1)$. This shows that (a) is the only possibility.

Question 4. Correct answer: (b).

Consider the differential equation $\dot{x} = t^2$. Then for $(t_0, x_0) = (0, 0)$, the right hand side is zero, but all solutions are of the form $t \mapsto \frac{1}{3}t^3 + c$ for some constant $c \in \mathbb{R}$, which are clearly not constant. Note that this differential equation does not depend on x, so we can just integrate and find all solutions without the need of any specific theory on differential equations.

Question 5. Correct answer: (b).

Using *Repetition Material 1*, we transform this differential equation into a first-order two-dimensional differential equation

$$\begin{aligned} \dot{x} &= y\,, \\ \dot{y} &= -ax\,. \end{aligned}$$

In polar coordinates this differential equation reads as

$$\begin{aligned} \dot{r} &= r\sin(\phi)\cos(\phi) - ar\cos(\phi)\sin(\phi) = r\sin(\phi)\cos(\phi)(1-a)\,, \\ \dot{\phi} &= \frac{1}{r}\big(-ar\cos^2(\phi) - r\sin^2(\phi)\big) = -a\cos^2(\phi) - \sin^2(\phi)\,, \end{aligned}$$

so $\dot{\phi}$ is constant if and only if $a = 1$.