Classical Dynamics **Autumn Term 2020**

Test 2

Instructions

The deadline is 12 noon on Friday 27 November.

Upload your answers in a single PDF file.

Your answers should be hand-written.

Include your name and CID on your script.

If you are unable to submit via Blackboard/Turnitin email your script to
`maths-student-office@imperial.ac.uk`

1. (i) The motion of a raindrop is described by the Lagrangian

$$L(y, \dot{y}, t) = e^{\gamma t}\left(\frac{1}{2}\dot{y}^2 - gy\right),$$

where γ and g are positive constants. Obtain the corresponding Hamiltonian.

(5 marks)

(ii) A bead of unit mass moves on a catenary-shaped wire. The motion of the bead is governed by the Lagrangian

$$L(q, \dot{q}) = \frac{1}{2}\dot{q}^2 \cosh^2 q - g \cosh q,$$

where g is a positive constant.

(a) Show that the corresponding Hamiltonian is

$$H(q, p) = \frac{p^2}{2\cosh^2 q} + g \cosh q,$$

and write down Hamilton's equations for the bead. (7 marks)

(b) Consider the time-independent canonical transformation

$$Q = \sinh q, \qquad P = \frac{p}{\cosh q}.$$

What is the transformed Hamiltonian, $K(Q, P)$, for the motion of the bead? (4 marks)

(c) Obtain type 2 and type 3 generating functions for the canonical transformation defined in (b) and explain briefly why a type 1 generating function does not exist.

$$\left[\ \textit{Definition of a type 3 generating function:} \qquad q = -\frac{\partial F_3}{\partial p}, \qquad P = -\frac{\partial F_3}{\partial Q}.\ \right]$$

(9 marks)

(Total: 25 marks)