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Quantum Mechanics II, Coursework 1

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Question 1

(a) We are given that $\phi(x) = Axe^{-\alpha^2/a^2}$, which must satisfy the normalisation condition

$$\int_{-\infty}^{\infty} |\phi(x)|^2 \mathrm{d}x = 1.$$

We have $|\phi(x)|^2 = A^2x^2e^{-2\alpha^2/a^2}$, and by setting $\alpha = 2/a^2$, we get

$$\begin{aligned}\int_{-\infty}^{\infty} A^2x^2e^{-2\alpha^2/a^2}\mathrm{d}x &= A^2\int_{-\infty}^{\infty} x^2e^{-\alpha x^2}\mathrm{d}x \\ &= -A^2\int_{-\infty}^{\infty}\frac{\mathrm{d}}{\mathrm{d}\alpha}\left(e^{-\alpha x^2}\right)\mathrm{d}x \\ &= -A^2\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_{-\infty}^{\infty} e^{-\alpha x^2}\mathrm{d}x,\end{aligned}$$

where we recall the Gaussian integral

$$\int_{-\infty}^{\infty} e^{-\alpha x^2}\mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}.$$

This gives

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\int_{-\infty}^{\infty} e^{-\alpha x^2}\mathrm{d}x = -\frac{\sqrt{\pi}}{2}\alpha^{-3/2} = -\frac{a^3\sqrt{\pi}}{2^{5/2}}.$$

Substituting into the normalisation condition gives

$$\begin{aligned}(-A^2)\left(-\frac{a^3\sqrt{\pi}}{2^{5/2}}\right) &= 1 \\ A^2 &= \frac{2^{5/2}}{a^3\sqrt{\pi}},\end{aligned}$$

so we take A to be

$$\boxed{A = \frac{2^{5/4}}{a^{3/2}\pi^{1/4}}}.$$

(b) To compute $\tilde{\phi}(p)$ in the momentum basis, we use the Fourier transform relation, namely

$$\tilde{\phi}(p) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\phi(x)e^{-ixp/\hbar}\mathrm{d}x.$$

Recall the standard definition of Fourier transform in the form of

$$\mathcal{F}\{f(x)\}(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-ikx}\mathrm{d}x.$$

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