Analysis II, Complex Analysis
Assessed Coursework 1

Deadline is 1pm, 12th of February, 2024.

Q1. [5]

Let $f(z) = \operatorname{Log} z$. Describe $f(\Omega)$ where:

(a. [1]) $\Omega = \{z \in \mathbb{C} : |z| < 1 \text{ and } -\pi/2 < \operatorname{Arg} z < \pi/2\}$.

(b. [1]) $\Omega = \{z \in \mathbb{C} : |z| > 1 \text{ and } 0 \leq \operatorname{Arg} z \leq \pi\}$.

(c. [3]) Find the location of the branch cut of

$$\operatorname{Log}\left(\frac{z-i}{z+i}\right),$$

where with Log we associate the principle value of logarithm.

Q2. [5]

(a. [2]) If $|\sin z| \leq 1$, then what you can say about $z = x + iy$ in terms of its real and imaginary parts? Justify your answer.

(b. [3]) Find Taylor series for

$$f(z) = \frac{1}{(z+i)(z-2)}$$

at $z_0 = 1$. What is its radius of convergence?

Q3. [5] Evaluate the integral

$$\oint_\gamma \left(\frac{3}{z+2} - \frac{1}{z-2i}\right) dz$$

(a. [1]) if $\gamma = \{z \in \mathbb{C} : |z| = 5\}$,

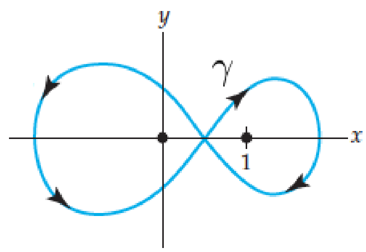
(b. [1]) if $\gamma = \{z \in \mathbb{C} : |z - 2i| = 1/2\}$.

(see page 2)

(c. [3]) Evaluate the integral

$$\oint_\gamma \frac{8z-3}{z^2-z}\,dz,$$

where γ is the "figure-eight" curve



Q 4. [5] Let f be entire and assume that $\lim_{|z|\to\infty} \frac{|f(z)|}{|z|^2} = 0$. Show that $f(z) = a + bz$, where $a, b \in \mathbb{C}$.