IMPERIAL

MATH70034

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
Summer 2025

This paper is also taken for the relevant examination for the
Associateship of the Royal College of Science

Algebraic Topology

Date: Thursday, May 22, 2025

Time: Start time 10:00 – End time 12:30 (BST)

Time Allowed: 2.5 hours

This paper has 5 Questions.

Please Answer All Questions in 1 Answer Booklet

This is a closed book examination.

Candidates should start their solutions to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Allow margins for marking.

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO DO SO

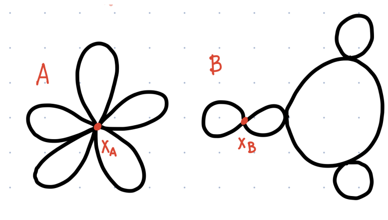
1. (a) State the definition of covering space. (2 marks)

(b) Let $p : \tilde{X} \to X$ be a covering space. Let $x_0 \in X$ and $\tilde{x}_0 \in p^{-1}(x_0)$,

(i) Define $p_* : \pi(\tilde{X}, \tilde{x}_0) \to \pi_1(X, x_0)$. (2 marks)

(ii) Show that p_* is injective. (3 marks)

(c) Consider the following two pointed spaces $(A, x_A), (B, x_B)$.



(i) Are A and B homeomorphic? Are they homotopy equivalent? Briefly justify.

(3 marks)

(ii) One of A and B is a covering of $\mathbb{S}^1 \vee \mathbb{S}^1$. Which one? Justify your answer. Give the degree for that covering. (4 marks)

(d) For which $n \in \mathbb{N}$ does there exist an injective group homomorphism from the free group on n generators to the free group on two generators? Briefly justify your answer. (6 marks)

(Total: 20 marks)

2. (a) State the Seifert van-Kampen theorem.

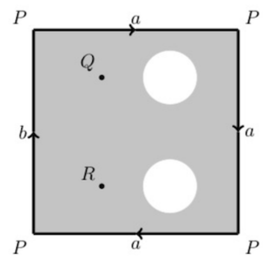
(3 marks)

(b) Let $X = U \cup V$. Provide an example with a brief justification to show that the Seifert-van Kampen theorem cannot be applied in the following situations.

(i) U and V are not both open, but that all other assumptions of the Seifert-van Kampen theorem are satisfied. (4 marks)

(ii) $U \cap V$ is non-empty and not path-connected, but that all other assumptions of the Seifert-van Kampen theorem are satisfied. (4 marks)

(c) Consider the following space, a square with two circles removed and identifications on the edges (for now ignore Q and R).



(i) What is its fundamental group? Justify your answer. (5 marks)

(ii) Consider the same space after identifying the points Q and R together. What is its fundamental group now? Justify your answer. (4 marks)

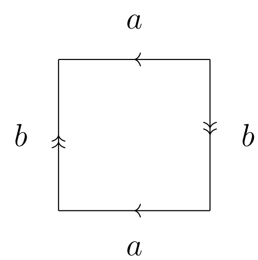
(Total: 20 marks)

3. (a) Write down the long exact sequence on homology for a pair of topological spaces (X, A). Define all terms involved but you do not have to define the connecting homomorphism.

(3 marks)

(b) Show that if $x_0 \in X$, $H_n(X, \{x_0\}) \cong \tilde{H}_n(X)$ for all n. (5 marks)

(c) Consider the Klein bottle K.



(i) What is the relative homology $H_n(K, b)$? Justify your answer. (4 marks)

(ii) Does K deformation retract to b? Justify your answer. (2 marks)

(d) Describe all possible exact sequences

$$0 \to \mathbb{Z}/3\mathbb{Z} \to G \to \mathbb{Z}/2\mathbb{Z} \to \mathbb{Z} \xrightarrow{\alpha} \mathbb{Z} \to 0$$

where G is an abelian group and $\alpha : \mathbb{Z} \to \mathbb{Z}$ is a group homomorphism. (6 marks)

(Total: 20 marks)

4. (a) Let $C_\bullet$ and $D_\bullet$ be chain complexes and let $p, q : C_\bullet \to D_\bullet$ be chain morphisms. Define what it means for p and q to be chain-homotopic. (2 marks)

(b) Show that two chain-homotopic maps induce the same map on homology. (4 marks)

(c) If two chain morphisms induce the same map on homology, are they chain homotopic? Justify your answer. (5 marks)

(d) Provide an example for the following statements (give a brief justification).

(i) A path-connected space X and a point $x \in X$ such that $\pi_1(X)$ is abelian, but $\pi_1(X \backslash \{x\})$ is not abelian. (3 marks)

(ii) A continuous injective map $i : X \to Y$ such that the map $i_* : H_1(X) \to H_1(Y)$ is not injective. (3 marks)

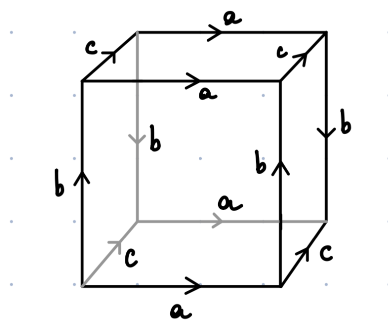
(iii) A continuous bijection which is not a homeomorphism. (3 marks)

(Total: 20 marks)

5. (a) State the Mayer–Vietories Theorem (no need to write down the connecting homomorphism). (4 marks)

(c) Use the Mayer–Vietories Theorem to compute the homology groups of M_g, the surface of genus g. (8 marks)

(d) Use cellular homology to compute the homology groups of the space CW-complex in the picture (this is $K \times \mathbb{S}^1$ for the Klein bottle K). (8 marks)



(Total: 20 marks)

BSc and MSci EXAMINATIONS (MATHEMATICS)

May 2025

This paper is also taken for the relevant examination for the Associateship.

MATH60034/MATH70034

Algebraic Topology (Solutions)

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1. (a) A covering space (or covering) is a map $p : \tilde{X} \to X$ such that there exists an open cover $\{U_\alpha\}$ of X such that for every α

seen ⇓

$$p^{-1}(U_\alpha) = \bigcup_\beta V_\alpha^\beta$$

and such that the restrictions $p\ |_{V_\alpha^\beta}: V_\alpha^\beta \to U_\alpha$ are homeomorphisms and $V_\alpha^\beta \cap V_\alpha^\gamma = \emptyset$ for all $\beta \neq \gamma$.

2, A

(b) (i) Given a covering space $p : \tilde{X} \to X$ with $x_0 \in X$ and $\tilde{x}_0 \in p^{-1}(x_0)$, we define $p_* : \pi_1(\tilde{X}, \tilde{x}_0) \to \pi_1(X, x_0)$ as

$$p_*([\gamma]) = [p \circ \gamma]$$

2, A

(ii) Given a covering space $p : \tilde{X} \to X$, with $x_0 \in X$ and $\tilde{x}_0 \in p^{-1}(x_0)$, we We want to show that if $p_*([\tilde{\alpha}]) = [e_{x_0}]$ then $\tilde{\alpha}$ is homotopic to the constant path. By path lifting property we have $\alpha = p \circ \tilde{\alpha}$ and $\tilde{\alpha}$ is the unique lift such that $\tilde{\alpha}(0) = \tilde{x}_0$. Therefore we have that $\alpha = p \circ \tilde{\alpha} \simeq e_{x_0}$. Let $F : I \times I \to X$ be the homotopy such that $f_0 = \alpha, f_1 = e_{x_0}$. By homotopy lifting property we have $F = p \circ \tilde{F}$, which is a homotopy from $\tilde{\alpha}$ to $e_{\tilde{x}_0}$ (since constant paths lift to constant paths).

3, A

(c)

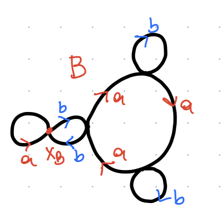
meth seen ⇓

(i) Yes homotopic, no homeomorphic. They are homotopic equivalent (since they are both deformation retracts of $\mathbb{R}^2$ minus five points), but not homeomorphic. They are not homeomorphic because I can remove x_A from A to create five connected components. But there is no point of B that removed gives five connected components.

1, B

2, C

(ii) A cannot be a covering, because there is no point in A that has a neighbourhood that is locally homeomorphic to the point where the two circles are joined up. B is a covering (a drawing of where a and b go is enough here).



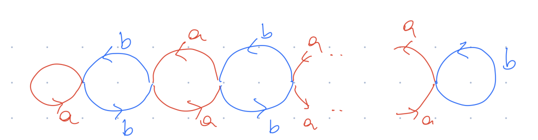
It has degree four.

4, B

(d) For all n.

2, C

We can always construct a n covering of $\mathbb{S}^1 \vee \mathbb{S}^1$ that looks like the following.



If we have a wedge of n circles this gives us a $n-1$ covering of $\mathbb{S}^1 \vee \mathbb{S}^1$, and its fundamental group is the free group on n generators. Since the map induced by the covering is injective, this will give an injective homomorphism from the free group on n generators into the free group on 2 generators. $\boxed{4, \text{D}}$

2. (a) Let $X = \cup_\alpha X_\alpha$ be an open cover and assume $x_0 \in \cap_\alpha X_\alpha$. Then,

1. if for all α, β $X_\alpha \cap X_\beta$ is path-connected, then the map

$$\Phi = *_\alpha (\iota_\alpha)_* : *_\alpha \pi_1(X_\alpha, x_0) \to \pi_1(X, x_0)$$

is surjective,

2. if for all α, β, γ $X_\alpha \cap X_\beta \cap X_\gamma$ is path-connected then $\ker(\Phi) = N$ and

$$\pi_1(X, x_0) \cong *_\alpha \pi_\alpha(X_\alpha, x_0)/N$$

where N is defined as the normal closure of

$$U = \{(\iota_{\alpha\beta})_*(w)(\iota_{\beta\alpha})_*(w)^{-1} \mid w \in \pi_1(X_{\alpha\beta}, x_0)\} \subset *_\alpha \pi_1(X_\alpha, x_0)$$

$\boxed{3, \mathrm{A}}$

(b) (i) Let us take $X = \mathbb{S}^1$ and consider it as the quotient of $[0,1]$ by identifying $0 \sim 1$. Let U be the image under the quotient map of $(0,1)$, which is open, and V the image under the quotient map of $[0, 1/2]$, which is closed. $U \cap V$ is path connected, $\mathbb{S}^1 = U \cup V$, but U and V are both simply-connected, so we do not a surjection as in the first part of the theorem. $\boxed{\text{unseen} \Downarrow}$ $\boxed{4, \mathrm{B}}$

(ii) For example consider $\mathbb{S}^1$ with the cover $U = \mathbb{S}^1 \setminus \{1\}$ and $V = \mathbb{S}^1 \setminus \{-1\}$. U and V are both open, but $U \cap V$ is not path connected. We note that U and V are contractible, so they have trivial fundamental groups, however $\mathbb{S}^1$ does not. So Seifert-van Kampen does not work here. $\boxed{4, \mathrm{B}}$

(c) (i) This exercise can be solved with Seifert-van Kampen (correct solutions using this will be given full marks), but a much faster way considers the space after adding an extra edge c connecting opposite vertices and separating the two holes. Then the space deformation retracts to the two triangles, which (with the same identifications as in X) is the wedge of three circles. Hence (by results from the course) its fundamental group is $\mathbb{Z} * \mathbb{Z} * \mathbb{Z}$. $\boxed{5, \mathrm{C}}$

(ii) Finally, after we have identified P and Q, that is the same as connecting them with an interval. So we are doing the same as adding another circle to our wedge, so the fundamental group is $\mathbb{Z} * \mathbb{Z} * \mathbb{Z} * \mathbb{Z}$. $\boxed{4, \mathrm{D}}$

3. (a) If $A \subset X$, then there exists a long exact sequence of relative homology $\boxed{\text{seen} \Downarrow}$

$$\begin{tikzcd}
{} \arrow[r] & H_n(A) \arrow[r, "\iota_*"] & H_n(X) \arrow[r, "q_*"] & H_n(X,A) \arrow[dll, "\partial"] \\
& H_{n-1}(A) \arrow[r, "\iota_*"] & H_{n-1}(X) \arrow[r, "q_*"] & H_{n-1}(X,A) \arrow[dll, dashed, "\partial"] \\
& H_0(A) \arrow[r, "\iota_*"] & H_0(X) \arrow[r, "q_*"] & H_0(X,A) \arrow[r] & 0
\end{tikzcd}$$

where ι_* is the map induced by the inclusion $\iota : A \to X$, and q_* is the map induced by the quotient map on chains $C_\bullet(X) \to C_\bullet(X,A) = C_\bullet(X)/C_\bullet(A)$. The map ∂ is called the connecting homomorphism $\boxed{3, \text{A}}$

(b) Considering the pair (X, x_0), we have the long exact sequence on homology

$$\begin{tikzcd}
{} \arrow[r] & \cancelto{0}{\tilde{H}_n(\{x_0\})} \arrow[r, "\iota_*"] & \tilde{H}_n(X) \arrow[r, "q_*"] & H_n(X,\{x_0\}) \arrow[dll, "\partial"] \\
& \cancelto{0}{\tilde{H}_{n-1}(\{x_0\})} \arrow[r, "\iota_*"] & \tilde{H}_{n-1}(X) \arrow[r, "q_*"] & H_{n-1}(X,\{x_0\}) \arrow[dll, dashed, "\partial"] \\
& \cancelto{0}{\tilde{H}_0(\{x_0\})} \arrow[r, "\iota_*"] & \tilde{H}_0(X) \arrow[r, "q_*"] & H_0(X,\{x_0\}) \arrow[r] & 0
\end{tikzcd}$$

$\boxed{5, \text{A}}$

So q_* must be and isomorphism. $\boxed{\text{meth seen} \Downarrow}$

(c) (i) We need to look at the long exact sequence of the pair (K, b).

$$\begin{tikzcd}
{} \arrow[r] & \cancelto{0}{\tilde{H}_2(b)} \arrow[r, "\iota_*"] & \cancelto{0}{\tilde{H}_2(K)} \arrow[r, "q_*"] & H_2(K,b) \arrow[dll, "\partial"] \\
& \cancelto{\mathbb{Z}}{\tilde{H}_1(b)} \arrow[r, "\iota_*"] & \cancelto{\mathbb{Z}\oplus\mathbb{Z}/2\mathbb{Z}}{\tilde{H}_1(K)} \arrow[r, "q_*"] & H_1(K,b) \arrow[dll, "\partial"] \\
& \cancelto{0}{\tilde{H}_0(b)} \arrow[r, "\iota_*"] & \cancelto{0}{\tilde{H}_0(K)} \arrow[r, "q_*"] & H_0(K,b) \arrow[r] & 0
\end{tikzcd}$$

Again, $H_0(K,a) = 0$ and $H_n(K,a) = 0$ for $n > 1$ by exactness. In this case the map $\tilde{H}_1(b) \to H_1(K,b)$ maps the generator to the part with torsion, $\mathbb{Z}/2\mathbb{Z}$, so by exactness $H_1(K,b) \cong \mathbb{Z}$. $\boxed{4, \text{B}}$

(ii) K does not deformation retract to b, since if it did the relative homology groups would have to be trivial, and they are not. $\boxed{2, \text{A}}$

(d) α is an isomorphism since $\mathbb{Z}/2\mathbb{Z} \to \mathbb{Z}$ is the zero map, and it is also followed by another zero map. $\boxed{2, \text{C}}$

$G = \mathbb{Z}/6\mathbb{Z}$ since $\mathbb{Z}/3\mathbb{Z} \to G$ must be injective and $G \to \mathbb{Z}/2\mathbb{Z}$ must be surjective. And we also have that the image of the first is equal to the second, so the first homomorphism sends $1 \mapsto 2$ and the second homomorphism sends $1 \mapsto 1$. $\boxed{4, \text{D}}$

4. (a) p, q are chain homotopic if there exists and operator $P : C_n \to D_{n+1}$ such that $p - q = \partial P + P\partial$. seen $\Downarrow$

2, A

(b) Assume p, q are chain homotopic, i.e. there exists an operator P such that $p - q = \partial P + P\partial$. Take $[c] \in H_n(C_\bullet)$, then for a representative c (where $\partial c = 0$) 4, A

$$p(c) - q(c) = \partial(P(c)) + P(\partial(c)) = \partial(P(c))$$

so $p(c)$ and $q(c)$ differ by a boundary, meaning that

$$p_*([c]) = q_*([c])$$

So they induce the same map on homology. unseen $\Downarrow$

(c) No.

1, C

For example consider the following short exact sequence.

$$0 \to \mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z} \to \mathbb{Z}/2\mathbb{Z} \to 0$$

Since it is exact, all its homology groups are zero, therefore the induced map on homology given by the identity and the zero map will be the same. However, the identity on chain and the zero map on chain are not chain homotopic. In fact, if they were there would exist an operator P such that

$$\text{id} = P\partial + \partial P$$

Specifically we would need for $c \in \mathbb{Z}/2\mathbb{Z}$ non-zero

$$c = P\partial(c) + \partial P(c)$$

however, $P : \mathbb{Z}/2\mathbb{Z} \to \mathbb{Z}$ must be the zero map and $\partial(c) = 0$, so the right hand side is zero, giving a contradiction. 4, D

(d) (i) $\pi_1(\mathbb{R}^2) \cong 0$, $\pi_1(\mathbb{R}^2 \setminus \{0\}) \cong \mathbb{Z}$. 3, A

(ii) The inclusion $i : \mathbb{S}^1 \to \mathbb{R}^2$ does not induce an injective map on the first homology group, since $H_1(\mathbb{S}^1) \cong \mathbb{Z}$ but $H_1(\mathbb{R}^2) \cong 0$. 3, A

(iii) An example is the map $f : [0, 2\pi) \to \mathbb{S}^1, t \mapsto (\cos(t), \sin(t))$. It is bijective and continuous, but its inverse is discontinuous at 1 (and it cannot be a homeomorphism, since $\mathbb{S}^1$ is compact while $[0, 2\pi)$ is not). 3, B

5. (a) Let X_1, X_2 be open in X, and $X = X_1 \cup X_2$. Then there exists a long exact sequence

$$\begin{tikzcd}
{} \arrow[r] & H_n(X_1 \cap X_2) \arrow[r] & H_n(X_1) \oplus H_n(X_2) \arrow[r] & H_n(X_1 \cup X_2) \arrow[dll, "\partial"] \\
& H_{n-1}(X_1 \cap X_2) \arrow[r] & H_{n-1}(X_1) \oplus H_{n-1}(X_2) \arrow[r] & H_{n-1}(X_1 \cup X_2) \arrow[dll, dashed, "\partial"] \\
& H_0(X_1 \cap X_2) \arrow[r] & H_0(X_1) \oplus H_0(X_2) \arrow[r] & H_0(X_1 \cup X_2) \arrow[r] & 0
\end{tikzcd}$$

$\boxed{4, \text{M}}$

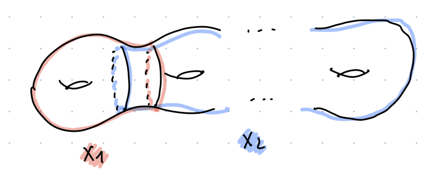
(b) Let M_g be the surface of genus g. We prove this by induction. We know the homology of the torus $\boxed{\text{meth seen} \Downarrow}$

$$H_n(M_1) = \begin{cases} \mathbb{Z} & n = 0, 2 \\ \mathbb{Z}^2 & n = 1 \\ 0 & \text{otherwise} \end{cases}$$

Assume that we know the homology of M_{g-1} is

$$H_n(M_1) = \begin{cases} \mathbb{Z} & n = 0, 2 \\ \mathbb{Z}^{2g-2} & n = 1 \\ 0 & \text{otherwise} \end{cases}$$

Then we can apply Mayer-Vietoris with X_1 and X_2 as in the picture.



Note that X_1 is homotopic to the punctured torus, and X_2 is homotopic to the punctured surface of genus $g - 1$. The intersection $X_1 \cap X_2$ is just a cylinder. So we have the following long exact sequence from Mayer-Vietoris

$$\begin{tikzcd}
{} \arrow[r] & \cancelto{0}{H_2(X_1 \cap X_2)} \arrow[r, dashed, "\iota_*"] & H_2(X_1) \oplus \cancelto{0}{H_2(X_2)} \arrow[r, dashed, "q_*"] & H_2(M_g) \arrow[dll, "\partial"] \\
& H_1(X_1 \cap X_2) \arrow[r, dashed, "\iota_*"] & H_1(X_1) \oplus H_1(X_2) \arrow[r, dashed, "q_*"] & H_1(M_g) \arrow[dll, "\partial"] \\
& H_0(X_1 \cap X_2) \arrow[r, dashed, "\iota_*"] & H_0(X_1) \oplus H_0(X_2) \arrow[r, dashed, "q_*"] & H_0(M_g) \arrow[r] & 0
\end{tikzcd}$$

where

$$H_n(X_1 \cap X_2) = \begin{cases} \mathbb{Z} & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

since it is homotopy equivalent to a circle, and

$$H_n(X_1) = \begin{cases} \mathbb{Z} & n = 0 \\ \mathbb{Z}^2 & n = 1 \\ 0 & \text{otherwise} \end{cases}$$

since the punctured torus deformation retracts to $\mathbb{S}^1 \vee \mathbb{S}^1$. By a similar reasoning

$$H_n(X_2) = \begin{cases} \mathbb{Z} & n = 0 \\ \mathbb{Z}^{2g-2} & n = 1 \\ 0 & \text{otherwise} \end{cases}$$

since the punctured genus $g-1$ surface deformation retracts to a wedge of $2g-2$ circles. We have that the map $H_1(X_1 \cap X_2) \to H_1(X_1) \oplus H_1(X_2)$ is the zero map, since the generator of the first homology group of the cylinder is trivial when seen as a loop in the punctured torus or the punctured genus $g-1$ surface. Then by exactness $H_2(M_g) \cong \mathbb{Z}$. Finally the map $H_0(X_1 \cap X_2) \to H_0(X_1) \oplus H_0(X_2)$ is $[1,1]^T$ (induced by the inclusion of one path-connected space into two path-connected spaces). Therefore, by exactness $H_1(M_g) \cong H_1(X_1) \oplus H_1(X_2) \cong \mathbb{Z}^{2g}$.

$\boxed{8, \text{M}}$

(c) The cellular complex is

$$0 \to \mathbb{Z} \to \mathbb{Z}^3 \to \mathbb{Z}^3 \to \mathbb{Z} \to 0$$

$d_1 = 0$ since it is just the connecting homomorphism. We compute d_2, d_3 using the cellular boundary formula. If we call A, B, C the three faces perpendicular to a, b, c (respectively) we obtain

$$\begin{aligned} d_2(A) &= 2b \\ d_2(B) &= 0 \\ d_2(C) &= 2b \end{aligned}$$

and

$$d_3(e^3) = d_A A + d_B B + d_C C$$

We can take $x \in A$ and compute the local degrees: the two faces differ by the antipodal map, so the local degrees are ± 1 and the cancel out. The same for B, but for C, they two faces differ by a rotation of 180 degrees so they have the same sign, and $d_C = 2$. Therefore,

$$H_n(X) = \begin{cases} \mathbb{Z} & n = 0 \\ \mathbb{Z}^2 \oplus \mathbb{Z}_2 & n = 1 \\ \mathbb{Z} \oplus \mathbb{Z}_2 & n = 2 \\ 0 & \text{otherwise} \end{cases}$$

$\boxed{8, \text{M}}$

Review of mark distribution:
Total A marks: 32 of 32 marks
Total B marks: 20 of 20 marks
Total C marks: 12 of 12 marks
Total D marks: 16 of 16 marks
Total marks: 100 of 80 marks
Total Mastery marks: 20 of 20 marks

MATH70034 Algebraic Topology Markers Comments

Question 1	This question was overall answered well. Parts (a) and (b) were completely seen content, and were answered well with the exception of b(ii) which often lacked some details. Question (c) was unseen but very similar questions were included in the example sheet and the revision session. A part from imprecisions (and sometimes lack of a full answer -- e.g. specifying why B is a covering space, but not specifying why A is not a covering space), this question was answered fine. Question (d) was completely unseen and the hardest part of the question, but most student succeeded to see the connection between subgroups of the free group and covering spaces. Not everyone was able to build a cover for each n in N.
Question 2	This question was answered okay. Question (a) was bookwork, but many students did not state the complete version of Seifert-van Kampen theorem, but just the statement for two open sets. Question (b) was unseen, and was attempted by most students, who came up with a wide range of examples. The most common mistake was that the example given did not satisfy the other assumptions of the theorem (as it was requested in both cases) or that the example did not satisfy one of the assumption but either was not giving a contradiction or the fact that it was giving a contradiction was not explained. Question (c) was unseen and had a wide range of answers. (c)(i) could be solved with either Seifert-van Kampen (more time consuming and potentially leading to mistakes) or by realising that the space deformation retracts to the wedge of three circles. Students that realised the second fact usually got to the correct answer. (c)(ii) boiled down to realising that identifying two points is the same as adding a loop: students that realised that got full marks regardless of whether they answered the previous part correctly.

Question 3

This question was answered okay.
Questions (a) and (b) were both seen. Using reduced homology solve (b) makes the argument much quicker than using ordinary singular homology. Some students tried to use the fact that (X,x_0) is a good pair, but that is not necessarily true.
Question (c) could be solved in two ways. Some students noted that (K,b) is a good pair and tried to compute the homology of the quotient K/b (which is a sphere with two points identified). This argument works well, but many students failed to identify exactly what K/b was. The other way is to look at the long exact sequence of the pair (K,b) and computing the maps. This was more time consuming, but generally more successful, even if there are a lot of places where one can make some small mistakes. Question (c)(ii) could be answered correctly without having answered correctly the previous part (and students that did so were given full marks here).
Question (d) was completely unseen and was not answered fully correctly by many students. The main mistakes were to not identify specifically what the group G can be (just saying that it has a certain subgroup), and forgetting to specify what alpha can be (and just focusing on G).

Question 4

This question was not answered well overall (potentially because of time constraints).
Questions (a) and (b) were seen, but they had not been the main focus of any of the example sheets, so unfortunately they were not answered well by many students. For similar reasons question (c) (which was unseen) was not answered well by the majority of students.
Question (d) was attempted by many students, who provided a wide range of examples to the statements. The main problem here was when an example was provided by no justification of why this was satisfying the conditions required was given.

Question 5

This question had mixed outcomes, and I think overall it was mostly due to time constraints.
Part (a) was seen material and was answered well by basically everyone.
Part (c) was unseen, but we had covered the case g=2 in an example sheet. Some students tried to solve it by induction, but an induction argument was not really needed.
Part (d) was unseen, but it is a worked out example in Hatcher in the cellular homology section (and the argument is similar to the three-torus, which was in an example sheet). Most students that attempted this question got the set-up correct, but could not compute the map d_3 correctly (which required a local degree computation).