

Problem Sheet 6

1. Use the Hamilton-Jacobi method to solve the equations of motion for the Hamiltonian $H = xp$.
2. Consider the Hamiltonian

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + V(r, \theta),$$

where the potential has the form

$$V(r, \theta) = U(r) + f(r)g(\theta).$$

For what f and g is the Hamilton-Jacobi equation completely separable in polar coordinates? For the separable cases obtain integral expressions for Hamilton's characteristic function W .

3. Consider the Hamiltonian

$$H = \frac{p^2}{2} - cxt,$$

where c is a constant

- (i) Find a complete solution of the Hamilton-Jacobi equation for this Hamiltonian.

Hint: write Hamilton's Principal Function in the form $S = xf(t) + g(t)$.

- (ii) Use the result of part (i) to solve the equations of motion.

Remark: the Hamilton-Jacobi method is a clumsy way to solve this rather simple mechanics problem! However, this is an example of a simple Hamilton-Jacobi equation which is not separable.

4. What is the Hamilton-Jacobi equation if S is expressed as a function of p and t ?