Answers to Test 1

1. (i) (a)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0.$$

[**2 marks**]

(b) A conservative force is a position dependent force derivable from a potential energy function V so that $\mathbf{F} = -\nabla V$. [**3 marks**]

(ii) (a) The E-L equations give

$$\ddot{x} - b\dot{y} = 0, \quad \ddot{y} + b\dot{x} + g = 0,$$

so that $F_x = \ddot{x} = b\dot{y}$, $F_y = \ddot{y} = -b\dot{x} - g$ which is velocity dependent and hence not conservative. The force is conservative if $b = 0$. If $g = 0$ but $b \neq 0$ the force is not conservative. [**5 marks**]

(b) Integrating the x E-L equation gives $\dot{x} - by = C$. Substituting into the y equation gives

$$\ddot{y} + b(C + by) + g = 0$$

or

$$\ddot{y} + b^2 y = -g - bC$$

with general solution

$$y = A\cos(bt + \alpha) - \frac{g + bC}{b^2}.$$

Therefore $\dot{x} = C + by = bA\cos(bt + \alpha) - g/b$ which integrates to

$$x = A\sin(bt + \alpha) - gt/b + D.$$

[**8 marks**]

(c) There are no periodic solutions (although y is periodic, x isn't).

[**2 marks**]

(d) $p_x = \dot{x} - by$, $p_y = \dot{y}$ so that

$$H = (\dot{x} - by)\dot{x} + \dot{y}\dot{y} - \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + by\dot{x} + gy = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + gy.$$

[3 marks]

(e) As x is cyclic p_x is constant (this is what is obtained by integrating the E-L equation for x as suggested in part (b)).

[2 marks]

[Total: 25 marks]