**Classical Dynamics** **Autumn Term 2021**

**Test 2**

## Instructions

The deadline is 1pm on Wednesday 24 November.

Upload your answers in a single PDF file.

Your answers should be hand-written (electronic tablets may be used).

Include your name and CID on your script.

If you are unable to submit via Blackboard/Turnitin email your script to `maths-student-office@imperial.ac.uk`

1. The time-evolution of a physical system with one coordinate $q$ is described by the Lagrangian

$$L = \frac{1}{2}\dot{q}^2 + a\dot{q}\sin q \sin t + b\cos q,$$

where $a$ and $b$ are constants.

(a) Show that the corresponding Hamiltonian is

$$H = \frac{1}{2}\left(p - a\sin q \sin t\right)^2 - b\cos q.$$

Is $H$ a constant of the motion? (7 marks)

(b) Obtain a type 2 generating function, $F_2(q, P, t)$, for the canonical transformation

$$Q = q, \qquad P = p - a\sin q \sin t.$$

$$\left[\quad \textit{Definition of a type 2 generating function:} \qquad p = \frac{\partial F_2}{\partial q}, \qquad Q = \frac{\partial F_2}{\partial P}. \quad\right]$$

(5 marks)

(c) Use $K = H + \partial F_2/\partial t$ to find the new Hamiltonian, $K(Q, P, t)$, obtained by applying the transformation from part (b) to the Hamiltonian given in part (a). (4 marks)

(d) Using your result from part (c), or otherwise, derive the equation of motion

$$\ddot{Q} = -(a\cos t + b)\sin Q.$$

(4 marks)

(e) Give the (four) Hamilton's equations for

$$H = \frac{1}{2}(p_1 - a\sin q_1 \sin q_2)^2 + p_2 - b\cos q_1.$$

Explain briefly how solving the equations of motion solves Hamilton's equations deriving from the Hamiltonian quoted in part (a).

(5 marks)

(Total: 25 marks)