MATH60005/70005: Optimization (Autumn 22-23)

Week 8: Problem Session

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1. Derive the orthogonal projection formula for a closed ball centered at $\mathbf{x}_0 \in \mathbb{R}^n$, $B[\mathbf{x}_0, r]$.

2. Show that the stationarity condition over the unit ball in $\mathbb{R}^n$, that is,

$$\min\{f(\mathbf{x}) : \|\mathbf{x}\| \leq 1\}$$

is given by $\nabla f(\mathbf{x}^*) = 0$, or $\|\mathbf{x}^*\| = 1$ and there exists $\lambda \leq 0$ such that $\nabla f(\mathbf{x}^*) = \lambda \mathbf{x}^*$.

3. Consider the minimization problem

$$\begin{aligned} \min \quad & 2x_1^2 + 3x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 - 8x_1 - 4x_2 - 2x_3 \\ \text{subject to} \quad & x_1, x_2, x_3 \geq 0\,. \end{aligned}$$

   - Show that the vector $(\frac{17}{7}, 0, \frac{6}{7})^\top$ is an optimal solution.
   - Implement a projected gradient method with constant stepsize $\frac{1}{L}$, where L is the Lipschitz constant of the gradient of the function.