IMPERIAL

MATH70062

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May 2024

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Lie Algebras

Date: Tuesday, May 14, 2024

Time: 14:00 – 16:30 (BST)

Time Allowed: 2.5 hours

This paper has 5 Questions.

Please Answer All Questions in 1 Answer Booklet

Candidates should start their solutions to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

Any required additional material(s) will be provided.

Allow margins for marking.

Credit will be given for all questions attempted.

Each question carries equal weight.

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO

Every Lie algebra and every module over a Lie algebra is finite dimensional over $\mathbb{C}$. You can use, without proof, any results from the course provided you state them correctly and clearly.

1. Let $L \subseteq K$ be a Lie subalgebra of a Lie algebra K. The normaliser of L in K is

$$N_K(L) = \{x \in K \mid [x, y] \in L \ (\forall y \in L)\}.$$

(a) Show that $N_K(L)$ is a subalgebra of K and that it contains the centraliser $C_K(L)$ of L in K.

(3 marks)

(b) Give the dimension of $N_K(L)$ when $K = \mathfrak{gl}(n, \mathbb{C})$ and $L = \mathfrak{u}(n, \mathbb{C})$. (Justify your answer.)

(4 marks)

(c) Assume that K is solvable and $L \neq K$. Show that L is strictly contained in $N_K(L)$.

(7 marks)

(d) Assume that L is semisimple. Show that $N_K(L) = C_K(L) \oplus L$. (4 marks)

(e) Give an example of a pair $L \subsetneq K$ such that $N_K(L)$ is not the direct sum $C_K(L) \oplus L$. Justify your answer. (2 marks)

(Total: 20 marks)

2. For every Lie algebra L let $\mathrm{Der}(L)$ and $Z(L)$ denote the algebra of derivations and the centre of L, respectively.

(a) Show that for every $\delta \in \mathrm{Der}(L)$ we have $\delta(Z(L)) \subseteq Z(L)$. (1 mark)

(b) Let L be a solvable Lie algebra of dimension n. Prove that L is abelian if and only if the dimension of $\mathrm{Der}(L)$ is n^2. (4 marks)

(c) Let $L = A \oplus B$ be a direct sum of two Lie algebras A, B.

(i) Show that $\mathrm{Der}(L) \cong \mathrm{Der}(A) \oplus \mathrm{Der}(B)$, when A and B are semisimple. (2 marks)

(ii) Give an example when $\mathrm{Der}(L)$ is not isomorphic to $\mathrm{Der}(A) \oplus \mathrm{Der}(B)$. Justify your answer. (2 marks)

We say that L is a reductive Lie algebra if it is the direct sum of a semisimple Lie algebra and an abelian Lie algebra.

(d) Is $\mathfrak{gl}(n, \mathbb{C})$ reductive? Justify your answer. (2 marks)

(e) Let L be a reductive Lie algebra and let $I \triangleleft L$ be an ideal. Prove that the quotient algebra $\overline{L} = L/I$ is also reductive. (4 marks)

(f) Give an example of a Lie algebra L such that $\mathrm{Der}(L)$ is not reductive. Justify your answer.

(5 marks)

(Total: 20 marks)

3. Let L be a Lie algebra and let V, W be L-modules. Let $\cdot : L \times \mathrm{Hom}(V, W) \to \mathrm{Hom}(V, W)$ be the map given by the rule:

$$(x \cdot f)(v) = xf(v) - f(xv) \quad (\forall x \in L, f \in \mathrm{Hom}(V, W), v \in V).$$

(a) Show that $\cdot$ above defines a structure of an L-module on $\mathrm{Hom}(V, W)$. (5 marks)

(b) Assume that V, W are irreducible L-modules. Show that

$$\mathrm{Hom}(V, W)^L = \{f \in \mathrm{Hom}(V, W) \mid x \cdot f = 0 \ (\forall x \in L)\}$$

is either a zero- or one-dimensional $\mathbb{C}$-linear subspace of $\mathrm{Hom}(V, W)$. (3 marks)

(c) Assume now that $L = \mathfrak{sl}(2, \mathbb{C})$ and for $d \geq 0$ let V_d denote the $d+1$-dimensional irreducible L-module, unique up to isomorphism. Determine all pairs $m, n \in \mathbb{N}$ such that the L-module $\mathrm{Hom}(V_m, V_n)$ is irreducible. (7 marks)

(d) Give the definition of a faithful L-module V. (1 mark)

(e) Assume that V is a faithful irreducible L-module and let $N \triangleleft L$ be a nilpotent ideal in L. Prove that N is zero. (4 marks)

(Total: 20 marks)

4. Let L be a Lie algebra and let $K(\cdot, \cdot)$ denote its Killing form.

(a) State both Cartan's first and second criteria. (2 marks)

Assume that L is a semi-simple Lie algebra.

(b) Prove that there is a semi-simple element $x \in L$ such that $K(x, x) \neq 0$. (3 marks)

(c) Is there a nilpotent element $x \in L$ such that $K(x, x) \neq 0$? Justify your answer. (3 marks)

(d) Show that for every invertible $\phi \in \mathfrak{gl}(n, \mathbb{C})$ the conjugation $X \mapsto \phi^{-1} X \phi$ furnishes an automorphism of $\mathfrak{sl}(n, \mathbb{C})$. (2 marks)

(e) Let H be a commutative subalgebra of $\mathfrak{sl}(n, \mathbb{C})$ such that every element of H is semisimple as an element of the simple algebra $\mathfrak{sl}(n, \mathbb{C})$. Show that the dimension of H is at most $n - 1$, and it is a Cartan subalgebra if and only if it has dimension $n - 1$. (6 marks)

(f) Let H, H' be two Cartan subalgebras of $\mathfrak{sl}(n, \mathbb{C})$. Show that there is an automorphism of $\mathfrak{sl}(n, \mathbb{C})$ mapping H to H'. (4 marks)

(Total: 20 marks)

5. For every set S let $|S|$ denote its cardinality. For every root system (V, R) let $W(V, R)$ denote its Weyl group.

(a) Show that for every root system (V, R) we have $|R| \leq 2 \cdot |W(V, R)|$. (2 marks)

(b) Let (V, R) be a reducible root system, that is, R is a disjoint union of root systems R_1 and R_2 such that $(\alpha, \beta) = 0$ for every $\alpha \in R_1$ and $\beta \in R_2$. Let V_1, V_2 be the span of R_1, R_2, respectively. Show that $W(V, R)$ is the direct product of the Weyl groups of (V_1, R_1) and (V_2, R_2). (5 marks)

(c) Let (V, R) be a root system such that $W(V, R) \cong (\mathbb{Z}/2\mathbb{Z})^n$ for some positive integer n. Compute $|R|$. (Justify your answer.) (6 marks)

Let $V = \mathbb{R}^3$ be equipped with the usual dot product, and let R be the set:

$$R = \Big\{(a_1, a_2, a_3) \in \mathbb{Z}^3 \mid 2a_1^2 + a_2^2 + a_3^2 \leq 2\Big\} - \{(0, 0, 0)\}.$$

(d) Show that (V, R) is a root system. (3 marks)

(e) Find the cardinality $|R|$ of R. (1 mark)

(f) Find the Weyl group of (V, R) up to isomorphism. (You are required to give details of your calculation.) (3 marks)

(Total: 20 marks)

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BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)

May – June 2024

MATH70062 Lie Algebras Exam

The following information must be completed:

Is the paper suitable for resitting students from previous years: Yes

Category A marks: available for basic, routine material (excluding any mastery question) (40 percent = 32/80 for 4 questions):
1(a) 3 marks; 1(b) 4 marks; 2(a) 1 mark; 2(b) 4 marks; 2(c) 4 marks; 2(d) 2 marks; 2(f) 5 marks; 3(a) 5 marks; 3(d) 1 mark; 4(a) 2 marks.

Category B marks: Further 25 percent of marks (20/ 80 for 4 questions) for demonstration of a sound knowledge of a good part of the material and the solution of straightforward problems and examples with reasonable accuracy (excluding mastery question):
1(e) 2 marks; 2(e) 4 marks; 3(b) 3 marks; 4(c) 3 marks; 4(d) 2 marks; 4(e) 6 marks.

Category C marks: the next 15 percent of the marks (= 12/80 for 4 questions) for parts of questions at the high 2:1 or 1st class level (excluding mastery question):
1(d) 4 marks; 3(e) 4 marks; 4(b) 3 marks.

Category D marks: Most challenging 20 percent (16/80 marks for 4 questions) of the paper (excluding mastery question):
1(c) 7 marks; 3(c) 7 marks; 4(f) 4 marks.

Signatures are required for the final version:

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BSc, MSc and MSci EXAMINATIONS (MATHEMATICS)

May – June 2024

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science.

Lie Algebras Exam

Date: ??
Time: ??
Time Allowed: 2 Hours for MATH96 paper; 2.5 Hours for MATH97 papers
This paper has *4 Questions (MATH96 version); 5 Questions (MATH97 versions)*.
Candidates should start their solutions to each question in a new main answer book.
Supplementary books may only be used after the relevant main book(s) are full.
Statistical tables will not be provided.

- DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO.
- Affix one of the labels provided to each answer book that you use, but DO NOT USE THE LABEL WITH YOUR NAME ON IT.
- Credit will be given for all questions attempted.
- Each question carries equal weight.
- Calculators may not be used.

1. (a) Since the centraliser is

$$C_K(L) = \{x \in K \mid [x, y] = 0 \ (\forall y \in L)\},$$

the second claim is trivially true. The normaliser is defined by a linear equation, and for every $x, y \in N_K(L), z \in L$ we have:

$$[[x, y], z] = [x, [y, z]] - [y, [x, z]] \subset [x, L] + [y, L] \subset L$$

using the Jacobi identity. So $N_K(L)$ is a subalgebra. **A**, similar seen (3 marks)

(b) It will be sufficient to show that $N_K(L) = \mathfrak{t}(n, \mathbb{C})$ since $\dim(\mathfrak{t}(n, \mathbb{C})) = \frac{(n+1)n}{2}$. Note that if $A = (a_{ij}) \in \mathfrak{gl}(n, \mathbb{C})$ then for every $i < j$ we have:

$$[A, E_{k,k+1}] = \sum_{i=1}^{n} a_{ik} E_{i,k+1} - \sum_{j=1}^{n} a_{k+1,j} E_{kj},$$

so $A \in N_K(L)$ if and only if $a_{ij} = 0$ for $j < i$. **A**, similar seen (4 marks)

(c) We are going to show the claim by induction on $\dim(K)$. Since K is solvable, its centre $Z(K) \neq 0$, and the quotient $K/Z(K)$ is also solvable. There are two cases: if L does not contain $Z(K)$, then every $x \in Z(K) \backslash L$ is in $C_K(L) \subseteq N_K(L)$, and hence $L \neq N_K(L)$. If L contains $Z(K)$, then $N_{K/Z(K)}(L/Z(K))$ strictly contains $L/Z(K)$ by the induction hypothesis. Let N be the pre-image of $N_{K/Z(K)}(L/Z(K))$ under the quotient map $K \to K/Z(K)$. If $x \in N$ then $[x, L] \subseteq L + Z(K) \subseteq L$ since the quotient map is a homomorphism, and hence $N \subseteq N_K(L)$. So the latter is strictly larger than L. **D**, not seen (7 marks)

(d) For every $x \in N_K(L)$ the restriction $\mathrm{ad}(x)|_L$ is a derivation. Since L is semisimple there is a $y \in L$ such that $\mathrm{ad}(x)|_L = \mathrm{ad}(y)|_L$, so $x - y \in C_K(L)$ by the linearity of the adjoint representation. Therefore $N_K(L) = C_K(L) + L$. But the centre of L is trivial, since L is semisimple, therefore $C_K(L) \cap L = 0$, and hence $N_K(L) = C_K(L) \oplus L$. **C**, similar seen (4 marks)

(e) If K is abelian then $C_K(L) = K$, and hence $K = N_K(L)$ is not the direct sum $C_K(L) \oplus L$ unless $L = 0$. **B**, not seen (2 marks)

(Total: 20 marks)

2. (a) If $z \in Z(L)$ and $x \in L$ then $[\delta(z), x] = \delta([z, x]) - [z, \delta(x)] = 0 - 0$, so $\delta(z) \in Z(L)$.

A, similar seen (1 mark)

(b) If L is abelian then $\mathrm{Der}(L) = \mathfrak{gl}(L)$, so it has dimension n^2. If L is not abelian, we have $Z(L) \neq L$. Since L is solvable, we have $Z(L) \neq 0$. By part (a) we have $\mathrm{Der}(L)(Z(L)) \subseteq Z(L)$, so

$$\dim(\mathrm{Der}(L)) \leq \dim(Z(L))^2 + (\dim(L) - \dim(Z(L))) \cdot \dim(L) < \dim(L)^2 = n^2.$$

A, not seen (4 marks)

(c) (i) As A, B are semisimple, the algebra L is also semisimple. For every semisimple Lie algebra S we have $\mathrm{Der}(L) = S$. The claim is now clear. **A**, not seen (2 marks)

(ii) Set $A, B = \mathbb{C}$. Then $\mathrm{Der}(L) = \mathfrak{g}l(2, \mathbb{C})$, but $\mathrm{Der}(A) \oplus \mathrm{Der}(B) = \mathbb{C}^2$. These have different dimensions. **A**, not seen (2 marks)

(d) We have $\mathfrak{gl}(n, \mathbb{C}) = \mathfrak{c}(n, \mathbb{C}) \oplus \mathfrak{sl}(n, \mathbb{C})$, where $\mathfrak{c}(n, \mathbb{C})$ is the subalgebra of scalar matrices. Since $\mathfrak{c}(n, \mathbb{C})$ is abelian and $\mathfrak{sl}(n, \mathbb{C})$ is semisimple, we get that $\mathfrak{gl}(n, \mathbb{C})$ is reductive. **A**, similar seen (2 marks)

(e) Let $L = A \oplus B$, where A is abelian and B is semisimple. Let $\overline{A}, \overline{B}$ be the image of A, B in $\overline{L}$, respectively. Since the quotient map is surjective, we have $\overline{L} = \overline{A} + \overline{B}$, and $\overline{A}, \overline{B}$ is an abelian, respectively a semisimple ideal. Their intersection $\overline{A} \cap \overline{B}$ is both abelian and semisimple, so trivial. So $\overline{L} = \overline{A} \oplus \overline{B}$. **B**, not seen (4 marks)

(f) Let $L = \langle x, y \rangle$ with $[x, y] = y$. It is solvable, but not abelian, so it is not reductive. If $\delta = (a_{ij}) \in \mathfrak{gl}(L)$ then

$$\delta([x, y]) = a_{21}x + a_{22}y, \quad [\delta(x), y] + [x, \delta(y)] = (a_{11} + a_{22})y,$$

so $\delta \in \mathrm{Der}(L)$ if and only if $a_{21} = 0 = a_{11}$. So $\mathrm{Der}(L) = \langle -E_{22}, E_{12} \rangle$ with $[-E_{22}, E_{12}] = E_{12}$. In particular it is isomorphic to L, and hence not reductive. **A**, similar seen (5 marks)

(Total: 20 marks)

3. (a) Clearly $\cdot$ is bilinear, so we only need to check its compatibility with the Lie bracket. For every $x, y \in L$, $f \in \mathrm{Hom}(V, W)$, $v \in V$ we have:

$$xy \cdot f(v) = x(yf(v) - f(yv)) = (xy)f(v) - yf(xv) - xf(yv) + f(yxv),$$

so by substracting the same equations with the role of x, y reversed, we get that

$$(xy - yx) \cdot f(v) = (xy - yx)f(v) - f((xy - yx)v) = [x, y]f(v) - f([x, y]v),$$

using that V, W are L-modules. So $\cdot$ does define an L-module structure on $\mathrm{Hom}(V, W)$. **A**, not seen (5 marks)

(b) Note that $\mathrm{Hom}(V, W)^L$ is just the $\mathbb{C}$-linear subspace of L-module homomorphisms from V to W. So it is zero, when V and W are not isomorphic, and it is one dimensional by the Schur lemma, otherwise. **B**, not seen (3 marks)

(c) Let $e_0, \dots, e_m$ and $f_0, \dots, f_n$ be the basis of V_m, V_n, respectively, such that $h(e_i) = (m-2i)e_i$ and $h(f_j) = (n - 2j)f_j$ for every i, j. Let $E_{ij} \in \mathrm{Hom}(V_m, V_n)$ be the unique linear map such that $E_{ij}(e_k) = \delta_{ik} f_j$, where δ_{ik} is the Kronecker delta. A simple computation shows that $h(E_{ij}) = (n - m) + 2(i - j)$. Arguing similarly to the proof of part (1) of Proposition 11.5, i.e. using the existence of Jordan-Hölder decomposition for representations, we get that an L-module is irreducible if and only if each eigenvalue of the action of h on L has the same parity and has multiplicity one. Clearly this is the case if and only if either m or n is zero. **D**, not seen

(7 marks)

(d) The L-module V is faithful if for every $x \in L$ such that $xv = 0$ for all $v \in V$ we have $x = 0$. **A**, seen (1 mark)

(e) By Engel's theorem there exists a basis of V such that all the elements of N are given by strictly upper triangular matrices. This implies that $NV \neq V$. Note that $W = NV$ is an L-submodule. Indeed for every $x \in L$, $n \in N$, $v \in V$ we have $xnv = [x, n]v + nxv \in W$ since $[xn] \in N$. By the irreducibility of V we must have $NV = 0$ which implies that $N = 0$. **C**, not seen (4 marks)

(Total: 20 marks)

4. (a) A Lie algebra L is solvable if and only if $K(L, L') = 0$. (Cartan's first criterion)

A Lie algebra L is semi-simple if and only if its Killing form is non-degenerate. (Cartan's second criterion) **A**, seen (2 marks)

(b) Let H be a Cartan subalgebra of L. Then the restriction of $K(\cdot, \cdot)$ onto H is non-degenerate, so there is an $x \in H$ such that $K(x, x) \neq 0$. But all elements of H are semisimple by definition. **C**, similar seen (3 marks)

(c) By definition $\mathrm{ad}(x)$ is nilpotent, so $\mathrm{ad}(x)^2$ is also nilpotent. Therefore the trace $\mathrm{Tr}(\mathrm{ad}(x)^2) = K(x, x)$ is zero. So the answer is no. **B**, similar seen (3 marks)

(d) Since the conjugation by ϕ is an automorphism of the matrix algebra, it is also an automorphism of the underlying Lie algebra $\mathfrak{gl}(n, \mathbb{C})$. Since conjugation does not change the trace, it leaves $\mathfrak{sl}(n, \mathbb{C})$ invariant, so it induces an automorphism of the latter. **B**, not seen (2 marks)

(e) Let $x \in \mathfrak{sl}(n, \mathbb{C})$ be arbitrary. Then the following are equivalent:

(i) the map $\mathrm{ad}(x) : \mathfrak{sl}(n, \mathbb{C}) \to \mathfrak{sl}(n, \mathbb{C})$ is semisimple,

(ii) the map $\mathrm{ad}(x) : \mathfrak{gl}(n, \mathbb{C}) \to \mathfrak{gl}(n, \mathbb{C})$ is semisimple (where we consider x as an element of $\mathfrak{gl}(n, \mathbb{C})$),

(iii) the map $x : \mathbb{C}^n \to \mathbb{C}^n$ is semisimple.

The equivalence of (ii) and (iii) was seen in the lectures. Since the action of $\mathrm{ad}(x) : \mathfrak{gl}(n, \mathbb{C}) \to \mathfrak{gl}(n, \mathbb{C})$ on scalar matrices is trivial, we get the equivalence of (i) and (ii). Therefore H consists of commuting semisimple linear maps $\mathbb{C}^n \to \mathbb{C}^n$, and hence the elements of H have a simultaneous diagonalisation in some basis $B \subset \mathbb{C}^n$. So H is a subalgebra of the algebra of diagonal matrices of trace 0 in the basis B, and hence its dimension is at most $n-1$. Moreover it is Cartan, i.e. maximal with respect to these properties, if and only if it has dimension $n - 1$. **B**, not seen (6 marks)

(f) By the above there are two bases B, B' of $\mathbb{C}^n$ such that $H, H' \subset \mathfrak{sl}(n, \mathbb{C})$ is the subalgebra of diagonal matrices of trace 0 in the basis B, B', respectively. Let ϕ be the invertible linear map taking B to B'. Then conjugation by ϕ takes H to H', and this is an automorphism of $\mathfrak{sl}(n, \mathbb{C})$ by part (b). **D**, not seen (4 marks)

(Total: 20 marks)

5. (a) Since for every $\alpha \in R$ the -1 eigenspace of s_α is spanned by α, for every $\alpha, \beta \in R$ we have $s_\alpha = s_\beta$ if and only if $\alpha = \pm\beta$. Therefore $|R| \leq 2 \cdot |W(V,R)|$. **M**, not seen (2 marks)

(b) Every reflexion s_α, where $\alpha \in R_1$, leaves V_2 fixed, since these vectors are orthogonal to α. So these reflections generate a subgroup W_1 of the Weyl group of the orthogonal sum isomorphic to $W(V_1, R_1)$. Similarly the reflections s_α, where $\alpha \in R_2$, generate a subgroup W_2 of the Weyl group of the orthogonal sum isomorphic to $W(V_2, R_2)$, leaving V_1 fixed. So W_1 and W_2 commute, and since they generate $W(R,V)$, the latter is isomorphic to $W(V_1,R_1) \times W(V_2,R_2)$. **M**, not seen

(5 marks)

(c) The group $W(V,R)$ is commutative, therefore $s_\beta \circ s_\alpha = s_\alpha \circ s_\beta$ for every $\alpha, \beta \in R$. So for every $v \in V$ we have:

$$v - \frac{2(v,\alpha)}{(\alpha,\alpha)}\alpha - \frac{2(v,\beta)}{(\beta,\beta)}\beta + \frac{4(v,\alpha)(\alpha,\beta)}{(\alpha,\alpha)(\beta,\beta)}\beta = v - \frac{2(v,\beta)}{(\beta,\beta)}\beta - \frac{2(v,\alpha)}{(\alpha,\alpha)}\alpha + \frac{4(v,\beta)(\beta,\alpha)}{(\beta,\beta)(\alpha,\alpha)}\alpha.$$

This is only possible when $\alpha = \pm\beta$ or $(\alpha,\beta) = 0$. Therefore R is reducible such that (V_1, R_1) has rank 1 and the Weyl group of (V_2, R_2) is $(\mathbb{Z}/2\mathbb{Z})^{n-1}$ by part (b), using the notation of the latter. So by induction on n we have $|R_2| = 2n-2$, so $|R| = |R_1| + |R_2| = 2n$. **M**, not seen (6 marks)

(d) We have $R = R_1 \cup R_2$ with $R_1 = \{(\pm1,0,0)\}$ and $R_2 = \{(0,a_2,a_3) \neq (0,0,0) \mid a_2, a_3 = 0, \pm1\}$. If V_1, V_2 is the span of R_1, R_2, respectively, then (V_1,R_1), (V_2,R_2) are root systems of type A_1, B_2, respectively, and all vectors in R_1 are orthogonal to vectors in R_2. Therefore (V,R) is a reducible root system. **M**, similar seen (3 marks)

(e) We have $|R| = 2 + 8 = 10$. **M**, similar seen (1 mark)

(e) The Weyl group of (V_1,R_1) is $\mathbb{Z}/2\mathbb{Z}$. The Weyl group of (V_2,R_2) is the group generated by the reflections around the axes $x = 0$, $y = 0$, $x = y$ and $x = -y$ on the (x,y)-plane, so it is the dihedral group D_4. So by part (b) the Weyl group of (V,R) is $\mathbb{Z}/2\mathbb{Z} \times D_4$. **M**, similar seen

(3 marks)

(Total: 20 marks)

MATH70062 Lie Algebras

Question	Marker's comment
1	The exam was more difficult than I anticipated, although there were several solutions by the students of problems which were different, and simpler than the solutions I gave. In many cases the problem was not so much with the Lie algebras material, but will more basic algebra. Almost all students attempted almost all problems, so the difficulty do not seem to be with time constraints, but more with some of the questions requiring ideas which were did not come so naturally. One example is the question on the invariant dimension of Homs between irreducible modules, which is immediate from Schur's lemma, but many students did not recognise that invariant homomorphisms are Lie module homomorphisms, which is just the definition, and did not do this problem. This is of course a very typical situation in algebra, so I expected that students will notice this easily, even if it is in the Lie Algebra context. Question 5 was very elementary, but many students did not even remember the definition of the Weyl group right; it is not the set of reflections, but the group generated by them. Doing one of the problems actually gives a quick counterexample.