Math40002 Analysis 1   Problem Sheet 0

1. What is the biggest element of the set $\{x \in \mathbb{R} : x < 1\}$? Give a careful proof.

2. Prove that for every positive integer $n \neq 3$, the number $\sqrt{n} - \sqrt{3}$ is irrational.

3. * Show that any positive *eventually periodic* decimal expansion is rational, and in fact can be written as the fraction

$$p \Big/ 99\ldots9900\ldots00 \quad (m \text{ 9s and } n \text{ 0s})$$

for some integers $p, m, n \geq 0$.
Deduce that any integer divides some number of the form $99\ldots9900\ldots00$.

4. Kevin tries to show $\sqrt{12} - \sqrt{3}$ is rational, by the following argument.

$$\sqrt{12} - \sqrt{3} = p/q, \quad p, q \in \mathbb{N},$$
$$\Rightarrow \quad 12 - 2\sqrt{12}\sqrt{3} + 3 = p^2/q^2,$$
$$\Rightarrow \quad 15 - 2\sqrt{36} = p^2/q^2.$$

Since $\sqrt{36} = 6$ is indeed rational, this looks good to him. Can you help him by pointing out three ways in which he's gone wrong? Be kind to him!

*Starred questions * are good to prepare to discuss with your Personal Tutor.*