DEPARTMENT OF MATHEMATICS
IMPERIAL COLLEGE LONDON
Academic Year 2022-2023

Mathematics Pre-arrival course

Problem Sheet 2 – Calculus: Integration and Differentiation

Note in all problems that $\log(x)$ represents the natural logarithm of x (sometimes written $\ln(x)$).

1. Calculate the derivative of the following functions:

   (a) $y = x^{-4}$
   (b) $y = 4x^{-\frac{3}{2}}$
   (c) $y = \sin(3x+1)$
   (d) $y = \sqrt{x^2+4}$
   (e) $y = (x^2+2x-3)\log(x)$
   (f) $y = \sin^2(x)e^x$
   (g) $y = \frac{3x+2}{x-1}$ with $(x \neq 1)$
   (h) $y = \frac{e^{\cos(x)}}{x+2}$ with $(x \neq -2)$
   (i) $y = \cot(\sqrt{x})$
   (j) $y = \log\left(\cot\left(\frac{x}{2}\right)\right)$

2. Calculate an expression for $\frac{dy}{dx}$ when:

   (a) $xy = y^2$
   (b) $\sin(xy) = y$
   (c) $x^2y^2 = 1$
   (d) $\log(y^3) = \frac{x}{2}\log(x-2)$, $x > 2$, $y > 0$
   (e) $\sqrt{xy} + x + y^2 = 0$

3. Find:

   (a) $\int_1^2 (x+3)^8 \mathrm{d}x$
   (b) $\int (x^3+x+1)(3x^2+1)\mathrm{d}x$
   (c) $\int \sec^6(x)\tan(x)\mathrm{d}x$
   (d) $\int_1^2 \frac{1}{\sqrt{4x+2}}\mathrm{d}x$
   (e) $\int_1^2 \frac{1}{\sqrt{25-4x^2}}\mathrm{d}x$
   (f) $\int_0^\pi x^2\sin(x)\mathrm{d}x$
   (g) $\int_1^2 \frac{1}{(x-3)(x-4)}\mathrm{d}x$
   (h) $\int_1^2 (\log(x))^2\mathrm{d}x$
   (i) $\int \frac{x^2}{x^3+5}\mathrm{d}x$
   (j) $\int_4^5 \frac{9x^2-7x+10}{(x-3)(x+1)}\mathrm{d}x$
   (k) $\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\mathrm{d}x$
   (l) $\int_0^{\pi/2} 2^{\sin(x)}\cos(x)\mathrm{d}x$
   (m) $\int_{-1}^1 |xe^x|\mathrm{d}x$

4. Find constants a, b, c and d such that

$$\frac{ax+b}{x^2+2x+2}+\frac{cx+d}{x^2-2x+2}=\frac{1}{x^4+4}.$$

Find

$$\int_0^1 \frac{1}{x^4+4}\mathrm{d}x$$

5. Not until your second year will you learn how to evaluate the integral

$$\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2}\mathrm{d}x,$$

but one of the following answers is correct. Which one?

   (a) $-\pi$
   (b) 0
   (c) $\frac{\pi}{e}$
   (d) $\sqrt{2}\pi$
   (e) Infinity (the integral doesn't converge)

6. If $y=\sinh^{-1} x$ show that $y=\log(x+\sqrt{x^2+1})$.
Given that:

$$\sinh^{-1} x-\cosh^{-1} x=\log(2),$$

show that

$$\sqrt{x^2+1}-2\sqrt{x^2-1}=x.$$

Hence, find the value of x.

7. Solve the equation

$$7\,\mathrm{sech}(x)-\tanh(x)=5.$$

8. Show that

$$\cosh^{-1} x=\log(x+\sqrt{x^2-1}).$$

Show that the area of the region defined by the inequalities $y^2 \geq x^2-8$ and $x^2 \geq 25y^2-16$ is $(72/5)\log(2)$.