IMPERIAL

Game Theory Behind Kuhn Poker
MATH60141 Coursework

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Introduction

Poker is a card game popular around the world. In most poker games, players start by placing a forced bet known as ante. Then each player could either check, place a bet, call or raise the existing bet or fold their hand. After some rounds of betting, the game ends with either (1) all but one player has folded, in which case the remaining player takes the pot, or (2) more than one player remains, and a showdown takes place and the player with the largest hand collects the pot. [1]

One of the most popular variants of poker is Texas hold 'em, in which there are multiple stages of betting, and comparison between strengths of hands is rather complex. To begin studying poker from a game theoretic perspective, we shall focus on a simple model known as Kuhn poker, proposed by Harold W. Kuhn. [2] We now introduce the rules.

Rules

(1) There are 2 players, P1 and P2. Each player is dealt one card from Ace, King and Queen. Each player also places an ante of £1.

(2) P1 can check or bet £1. If P1 checks, P2 can check or bet £1.

(3a) If P2 checks, the game goes to showdown and the player with higher card (A > K > Q) wins the pot.

(3b) If P2 bets, P1 can fold or call. If P1 folds, P2 takes the pot. If P1 calls, the game goes to showdown.

(4) If P1 bets in (2), P2 can fold or call. The game then proceeds as in (3b).

Analysis

When P1 bets in (2), P2 would always call with A and fold with Q. Therefore, P1 should never bet with K. When P1 checks in (2), P2 would not bet with K either due to the same reason.

However, when P1 has Q, they still have the incentive to bet with the hope that P2 will fold. This is known as bluffing in poker terms. And when P1 has A, they could still check to induce P2's bluff, known as underbidding.

P1 wants to make sure that P2 is indifferent to calling and folding, i.e., that P2's payoff from one option is no better than another, because otherwise P2 can exploit P1's strategy. Let's say that P1 bets with Q and A with probability p_Q and p_A respectively. The corresponding payoff of P2 calling (with K) is 3 and -1, and the payoff of P2 folding is 0. We then have $3p_Q + (-1)p_A = 0$, or $p_Q = 1/3p_A$. The result indicates that if P1 does want to underbid with A, they must also bluff with Q one third as often.

We might continue using this kind of argument on other cases to get all optimal strategies and draw a game tree. Due to the space constraint however, we will simply present the relevant results in the following table. The first column indicate the 6 possible cases. The second column gives the optimal opening strategy for P1. The third column is the corresponding strategy for P2. The last column is action by P1 when P2 bets.

From the table, we could see the following points:

Results

(P1,P2)	P1 Action	P2 Action	P1 Action (if P2 bets)
(A, K)	Bet (x) Check (1-x)	Call (1/3), Fold (2/3) Check	-
(A, Q)		Fold Bet (1/3), Check (2/3)	- Call
(K, A)	Check	Bet	Call ((x+1)/3), Fold ((2-x)/3)
(K, Q)		Bet (1/3), Check (2/3)	Call ((x+1)/3), Fold ((2-x)/3)
(Q, A)	Bet (x/3) Check (1-x/3)	Call Bet	- Fold
(Q, K)		Call (1/3), Fold (2/3) Check	-

Figure 1: Optimal strategies for P1 and P2 in all possible cases. Parentheses indicate the corresponding probabilities (frequencies).

- A Nash equilibrium exists for Kuhn poker, but only in the form of mixed strategies (not pure strategies).
- P1 could freely choose between $x = p_A \in [0, 1/3]$ and the results are still the same. If P1 does choose to underbid, however, they must match it with bluffing and further actions.
- P2, on the other hand, only has fixed actions.

We could also calculate the expected payoff for both P1 and P2 from here. By simple expectation argument, we have that the expected payoff is £$-1/18$ for P1 and £$1/18$ for P2. [3]

This means that P1 and P2 are not equivalent! From an intuitive perspective, P1 has to act first and has less information available for them, which leads to worse results.

Another takeaway from here is that bluffing, while appearing to be a bad decision, is actually part of the strategy. In more complex games like Texas hold 'em, bluffing and underbidding are essential.

Conclusion

In this poster, we mainly talked about Kuhn poker, which, while appearing to be extremely simple, is already rich enough for us to get a glimpse into the poker world. By using principles from game theory, we are able to deduce the optimal strategies and get some insight behind how poker works.

Figure 2: Texas Hold 'em. [1]

Nowadays, the best poker players rely on GTO (Game Theory Optimal) a lot. Depending on opponents, they might also deviate from GTO and use exploitation to maximise their expected value per hand. This poster is only a starting point; by utilising more advanced game theory, we could derive even more interesting results.

References

[1] https://en.wikipedia.org/wiki/Poker

[2] Kuhn, H. W. (1950). "Simplified Two-Person Poker". In Kuhn, H. W.; Tucker, A. W. (eds.). Contributions to the Theory of Games. Vol. 1. Princeton University Press. pp. 97–103.

[3] https://aipokertutorial.com/toy-poker-games/#finding-the-game-value