

MATH60005/70005/97405: Optimisation (Autumn 24-25)

Chapter 1: exercises

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- E.1 Show that the “usual” and the weighted dot products satisfy all the properties of an inner product in \mathbb{R}^n . For the weighted dot product, why do we require that $\mathbf{w} \in \mathbb{R}_{++}^n$?
- E.2 Why $\ell_{1/2}$ is not a norm? Can you spot a difficulty for ℓ_p norms with $p < 1$?
- E.3 Prove the Cauchy-Schwarz inequality in \mathbb{R}^n .
- E.4 Prove the Frobenius norm is indeed a norm.
- E.5 What is $\text{int}([\mathbf{x}, \mathbf{y}])$?
- E.6 Find the boundary of the following sets:

$$\begin{aligned}(\mathbf{c} \in \mathbb{R}^n, r \in \mathbb{R}_{++}), \text{bd}(B(\mathbf{c}, r)) &= \\(\mathbf{c} \in \mathbb{R}^n, r \in \mathbb{R}_{++}), \text{bd}(B[\mathbf{c}, r]) &= \\ \text{bd}(\mathbb{R}_{++}^n) &= \\ \text{bd}(\mathbb{R}_+^n) &= \\ \text{bd}(\mathbb{R}^n) &= \\ \text{bd}(\Delta_n) &= \end{aligned}$$

- E.7 Find the closure of the following sets:

$$\begin{aligned}\text{cl}(\mathbb{R}_{++}^n) &= \\(\mathbf{x} \neq \mathbf{y}), \text{cl}((\mathbf{x}, \mathbf{y})) &= \end{aligned}$$

- E.8 Redo the calculation of the gradient and the Hessian for a quadratic function at the end of the notes.

