

MATH50010 - Probability for Statistics

Unseen Problem 5

1. Consider a rod of unit length. The rod is broken at two points, whose locations can be modelled as independent, uniformly distributed random variables.
 - (a) What is the density function of the *ordered* breakpoints $(x_{(1)}, x_{(2)})$, where $x_{(1)} < x_{(2)}$?
 - (b) What is the probability that the three segments of the rod fit together to form a triangle?
2. Assume that the interval $[0, 1]$ is deterministically partitioned into n disjoint sub-intervals with lengths p_1, p_2, \dots, p_n , the *entropy* of this partition is defined to be

$$h = - \sum_{i=1}^n p_i \log p_i.$$

Let X_1, X_2, \dots be independent $\text{UNIFORM}[0, 1]$ random variables and let $Z_m(i)$ be the number of the X_1, \dots, X_m which lie in the i th interval of the partition. Show that

$$R_m = \prod_{i=1}^n p_i^{Z_m(i)}$$

satisfies $\frac{1}{m} \log R_m \xrightarrow{P} -h$ as $m \rightarrow \infty$.