Classical Dynamics   **Autumn Term 2022**

Test 1

1. (a) A particle of unit mass moving in one dimension is subject to the conservative force
$$F = \frac{1}{x^2} - 1.$$

(i) Determine the potential energy.

(ii) Find the frequency of small oscillations about $x = 1$.

(iii) Write down a suitable Lagrangian for the particle.

[9 marks]

(b) The motion of a particle of unit mass moving in the plane is governed by the Lagrangian

$$L = \dot{x}\dot{y} - xy,$$

where x and y are cartesian coordinates.

(i) Obtain the equations of motion.

(ii) Give the general solution of the equations of motion.

(iii) Is the force acting on the particle conservative?

[8 marks]

(c) A bead of mass m moves without friction or gravity on an expanding hoop of radius $R = t$ for $t > 0$.

(i) Obtain a Lagrangian for the motion of the bead.

Hint: use the kinetic energy formula

$$T = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right),$$

where r and θ are polar coordinates, to obtain a Lagrangian depending on $\dot{\theta}$ and t.

(ii) Solve the equation of motion.

[8 marks]

[Total: 25 marks]