## Quiz

Illustrate some trajectories in the phase plane near an unstable, aymptotically stable or Lyapanov stable fixed point.

# Intro to ODEs - Qualitative analysis of linear ODEs

## Phase plane analysis for the linear systems of first order ODEs

For linear systems, the vector field has some very nice properties.

$$\frac{d\vec{y}}{dt} = A\vec{y}$$

We have eigenvectors defining very special directions in the phase plane $A\vec{v}_1 = \lambda_1\vec{v}_1$. The line defined by $\vec{v}_1$ in the phase plane is an *invariant*, meaning that if we start on $\vec{v}_1$, we will remain on it.

# Intro to ODEs - Qualitative analysis of linear ODEs

To check this explicitly, we go back to the general solution of systems of ODE (2 dimensional case):

$$\vec{y}(t) = c_1 e^{\lambda_1 t} \vec{v_1} + c_2 e^{\lambda_2 t} \vec{v_2}$$

# Intro to ODEs - Qualitative analysis of linear ODEs

## First example of qualitative and phase plane analysis of linear systems of ODE

$$\frac{d\vec{y}}{dt} = A\vec{y}; \quad A = \begin{bmatrix} -4 & -3 \\ 2 & 3 \end{bmatrix}$$

# Intro to ODEs - Qualitative analysis of linear ODEs

- Compute the vector field:

- Asymptotically, solutions blow up parallel to $\vec{v}_1$.

- Asymtotically, if we start on $\vec{v}_2$, we approach $\vec{y}^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

# Intro to ODEs - Qualitative analysis of linear ODEs

## Phase portrait

Aim of phase plane analysis is to obtain the *phase portrait* of the system, which is a summary of all distinct solutions, with qualitatively different trajectories in the phase plane.

# Intro to ODEs - Qualitative analysis of linear ODEs

## Obtaining the family of solutions explicitly

$$\begin{aligned} \frac{dx}{dt} &= -4x - 3y \\ \frac{dy}{dt} &= 2x + 3y \end{aligned} \implies \frac{dy}{dx} = \frac{2x + 3y}{-4x - 3y}$$

# Intro to ODEs - Qualitative analysis of linear ODEs

Quiz: Alternatively, obtain the trajectories in the phase portrait by using the general soltuion

$$\vec{y}(t) = \begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{\lambda_1 t} \vec{v}_1 + c_2 e^{\lambda_2 t} \vec{v}_2$$