Network Science
Spring 2024
Problem sheet 5

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1. Consider the set of graphs generated by the configuration model with degree sequence, $\mathrm{d} = (3, 1, 1, 1)$).

   (a) How many distinct matchings are there? A "matching" is a graph corresponding to a particular set of pairings of the 6 stubs.

   (b) How many distinct graphs can be generated? What is the probability of generating each of these distinct graphs? For this exercise, two graphs should be considered equivalent if 1) their node degrees match the given degree sequence and 2) the number of edges between each distinct pair of nodes is the same in both graphs.

2. Now consider the configuration model applied to a degree sequence with $k_1 = 3$, $k_2 = 2$, $k_3 = 1$ and with total degree $K >= 6$ (and even).

   (a) What is the expected number of self-loops on node 2?

   (b) What is the probability that nodes 3 and 1 are linked?

   (c) What is the probability that nodes 1 and 2 are linked?

3. Consider the set of graphs with $N$ nodes generated by the configuration model with a specified degree sequence. If node $i$ has degree 2, what is $\langle m_i \rangle$ the expected number of multiedges node $i$ will form in a graph? A multiedge here is 2 links between node $i$ and another node. Provide an approximate result in terms of $N$ and the the moments of the degree sequence ($\overline{k}$, $\overline{k^2}$, ...) when the total degree $K \gg 1$.

4. Consider graphs generated by the configuration model with a specified degree sequence, $\mathrm{d} = (k_1, k_2, ..., k_N)$. Choose a random stub on a node and follow it to the node it connects to. What is $\tilde{\pi}_k$, the probability that the connecting node has degree $k + 1$?