Solve

$$\max \quad 2x_1^2 + x_2^2 + x_3^2 + 2x_1 - 3x_2 + 4x_3$$

$$\text{s.t.} \quad x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0 \quad .$$

A] The cost is a quadratic function

$$\underline{x}^T \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{x} + [2 \quad -3 \quad 4]\, \underline{x}$$

$\hookrightarrow Q \succ 0 \Rightarrow$ Cost is convex.

The constraint is the unit simplex in 3D

Δ_3, convex. $\Rightarrow$ Convex cost + convex constraint

$\Rightarrow$ Maximizer is one of the extreme points!

Extreme points of $\Delta_3 = \begin{cases} (1,0,0) \\ (0,1,0) \\ (0,0,1) \end{cases}$

$$\Rightarrow f(1,0,0) = 2 \cdot 1^2 + 2 \cdot 1 = 4$$
$$f(0,1,0) = 1 \cdot 1^2 - 3 \cdot 1 = -2$$
$$f(0,0,1) = 1 \cdot 1^2 + 4 \cdot 1 = 5$$

Hence, the maximizer is $(0,0,1)$ //.