# General Information for Analysis 2 (MATH 50001)

- The lectures during term 1 are delivered by Dr Davoud Cheraghi.
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- The lectures during term 2 are delivered by Prof Ari Laptev.
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- **Syllabus:** We aim to cover the following topics during term 1:
  - **Differentiation in higher dimensions:** Euclidean spaces, continuity and derivative of a map from $\mathbb{R}^n$ to $\mathbb{R}^m$, chain rule, directional derivatives, derivative in terms of partial derivatives, higher derivatives, higher dimensional Taylor's theorem, symmetry of mixed partials, inverse function theorem, implicit function theorem.
  - **Metric spaces:** axiomatic definition, examples, open sets, closed sets, interior of a set, closure of a set, limit points, continuity of maps, topologically equivalent metrics, Lipschitz maps, separable metric spaces.
  - **Topological spaces:** axiomatic definition, examples, open sets, closed sets, convergence of sequences, continuity of maps, basis for a topological space.
  - **Connectedness:** definition, continuity and connectedness, connected components.
  - **Compactness:** definition in terms of open covers, convergent sub-sequences, continuous maps and compact sets, sequential compactness.
  - **Completeness:** motivation, examples, completeness of function spaces, incomplete function spaces, continuity of maps on function spaces, Arzela-Ascoli, fixed-point theorem.

- **Learning activities:** the activities designed to help you learn this module:
  (a) **Lectures**: there will be 2 hours of lectures each week, except for 1 hour lecture during the midterm week.
  (b) **Typed lecture notes**: there will be computer typed detailed contents of the material for each week. These will be released on BlackBoard, weekly.
  (c) **Problem sheets, hints, and solutions**: there will be weekly problem sheets, followed by hints, and solutions. All of these will be released on BlackBoard. The hints appear few days after the problems sheets, and the solutions appear about a week later.
  (d) **Problem classes**: there will be 1 hour of in-person classes where you can work on the problem sheets, and receive help from GTAs and the lecturer.
  (e) **Office hours**: One hour per week I will be available in my office to answer your questions about the module. These will be held in my office, Huxley 624, on Thursdays, 11:00 – 12:00.
  (f) **Online Forum**: You may anonymously ask questions, make comments, or open up discussions about any topic in the module. These will be monitored by a PhD student in mathematics, but the lecturer will also regularly check to make sure that everything receives proper feedback. To access the forum, follow the link on BB pages for the module.

• **Assessment:** There will be two quizzes, one midterm test, and a final exam for the module. For more information about these, including the dates and contents, you may refer to the item Assessments and Mark Schemes page for the module on BlackBoard.

• **Relevant books for the module:** We shall not follow any particular book for this module. There are many books on real analysis, metric spaces, and topological spaces, almost all of them covering similar material. Each book presents a point of view, with its advantages and disadvantages, so it is fun to explore different books on the topic. Below are some examples, with similar, and not so similar, style to ours. Imperial College library has access to online versions of many books in mathematics.

- Elements of Real Analysis, Robert Bartle.
- Mathematical Analysis I and II, Vladimir Zorich.
- Principles of Mathematical Analysis, Walter Rudin.
- The Way of Analysis, Strichartz.
- Introduction to Metric and Topological Spaces, by Thuserland.
- Topology, by James Munkres.

• **Few remarks about the module:**

The lecture material for this module will become available on BlackBoard on a weekly basis. This includes: typed lecture notes, problem sheets, solutions to problem sheets, etc. At the end of the term I will also release the hand-written notes delivered during the lectures on BlackBoard. I do this because some of you may wish to only listen during the lectures instead of making notes.

For each problem sheet, some hints will become available after two days, and the complete solutions will become available after about a week. These are all set up to streamline your studies, and help you to smoothly progress through the module. It is good to have a strategy and keep up with it. For example, during each week, attend the lectures, answer the quizzes for each week on BB, work through the typed lecture notes, and work on the exercises in the typed lecture notes as you come across them. The exercises of the problem sheets are also placed in carefully chosen places along the typed lecture notes, so that they help you progress. They are part of the learning process. These problems are set up to help you decided if you have learned each material and can move onto the next topic.

If you follow my suggested stream outlined above, the material for each week grows from easy to difficult. In the lectures I introduce the new concepts for that week, explain the motivation for those, and sketch proofs of the key results. I may skip long and tedious proofs, or small technical steps, but never skip conceptual bits. The quizzes for each week are not difficult. Then comes the typed lecture notes, where all the arguments are presented with absolute care, and the proofs are written as it is normally expected in mathematics. Most problems help you understand the concepts, but occasionally, there are challenging problems. Those represent the height of the difficultly in this module. It is natural that your intuition will grow over time as you progress through these. There is a strange phenomena that every mathematician knows about, but no one really understands it (as far as I know); It takes time to digest mathematical concepts. If you leave things to the last moment, the understanding will come too late!

Please note that the typed notes and the lectures are not identical – they serve different purposes. The lectures usually introduce new topics, explain the motivation, or sketch the key ideas/proofs. The typed notes present more details, which will not be possible to discuss in the allocated hours for the lectures. Typed notes are more rigorous and help you learn proper style of writing proofs in analysis. So, do not complain that the typed notes are not identical to the lectures, or you prefer the same level of details in the lectures. It is just impossible.

The quizzes and midterm test each carry little weight for the final mark in the module. Please don't think of them as a way of building incremental marks for the module. They are designed to confirm your understanding and help you progress through the material. Specially the portfolio element, which consists of two quizzes, is only meant to make sure that you are following the module. What you see in the tests will be new material, and you will need to apply your understanding of the topics. The purpose of this module is not to make you a library of facts in analysis and test your memory. It aims to teach you how to think logically in the framework of analysis, and how to apply the concepts you have learned.

A crucial feature of mathematics is that it is a *practical* subject. One only learns by doing mathematics, solving problems, checking details of proofs, finding counterexamples. Throughout the notes, exercises are incorporated to test your understanding, develop intuition and establish results that are required later on. You should not treat these exercises as optional [1]. You will not understand the material in this course unless you spend some time trying to solve the problems! You will have to think hard, and you may have to come back to some of the problems more than one session to master them. Persistence is very important in mathematics.

Please make sure that you check your college email regularly, as I may send you messages about the module, quizzes, tests, etc.

The most important thing is that you do not fall behind.

[1] The exception here are those questions marked $(*)$, which go beyond the scope of the course, but are hopefully nevertheless interesting.