IMPERIAL

MATH70002

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May 2024

This paper is also taken for the relevant examination for the
Associateship of the Royal College of Science

Fluid Dynamics 2

Date: Monday, May 20, 2024

Time: 14:00 – 16:30 (BST)

Time Allowed: 2.5 hours

This paper has 5 Questions.

Please Answer All Questions in 1 Answer Booklet

Candidates should start their solutions to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

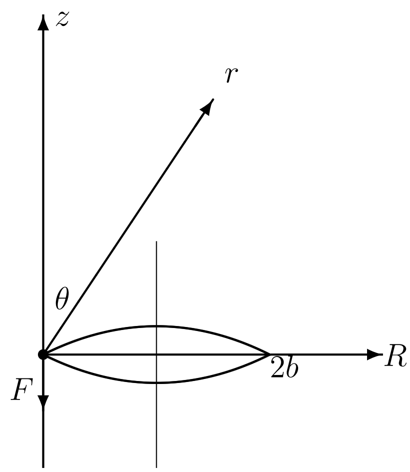
Any required additional material(s) will be provided.

Allow margins for marking.

Credit will be given for all questions attempted.

Each question carries equal weight.

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO



1. (a) Define the stress tensor σ_{ij} for a Newtonian fluid with velocity $\mathbf{u}$, pressure p and viscosity μ and show that the Stokes equations $\nabla p = \mu \nabla^2 \mathbf{u}$ with $\nabla \cdot \mathbf{u} = 0$ imply

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0.$$

(2 marks)

(b) A three-dimensional body of arbitrary shape moves through unbounded fluid at low Reynolds number. Express the drag on the body in terms of the stress tensor and explain why it can be calculated by looking at the flow on a sphere of very large radius. (3 marks)

In particular, the low-Reynolds-number flow due to a solid sphere of radius a moving with speed U is given by the Stokes streamfunction,

$$\psi = \frac{Ua^2}{4} \sin^2\theta \left(\frac{3r}{a} - \frac{a}{r} \right) \qquad \text{with} \qquad p = \frac{3a\mu U}{2} \frac{\cos\theta}{r^2}$$

in terms of spherical polar coordinates $(r,\, \theta,\, \phi)$. It is known that the drag on the sphere is $D = -6\pi a \mu U$. Explain how this leads to the Stokeslet flow due to a point force F at the origin

$$\psi = \frac{F}{8\pi\mu} r \sin^2\theta \qquad p = \frac{F}{4\pi} \frac{\cos\theta}{r^2}.$$

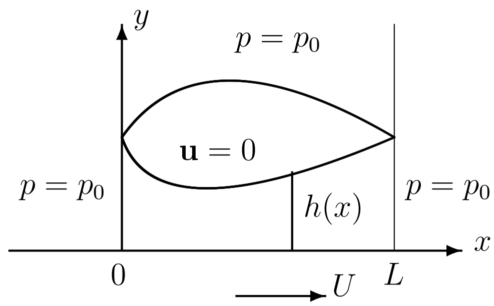
(4 marks)

(c) Rewrite the Stokeslet in terms of cylindrcal polar coordinates $(R,\, \phi,\, z)$ and find the z-component of the velocity for that flow, $w = \frac{1}{R}\frac{\partial \psi}{\partial R}$. (4 marks)

(d) A circular ring of radius b, negligible thickness and weight W lies in a horizontal plane $z = 0$ and falls vertically, as shown in the figure. By treating the ring as a distribution of point forces, find the velocity along the symmetry axis of the ring as a function of z. (4 marks)

(e) Show that this velocity is maximum when $z = \pm b/\sqrt{2}$. (3 marks)

(Total: 20 marks)



2. The lower surface of a long, fairly flat, solid body has shape $y = h(x)$ in $0 < x < L$ and rests on a cloth on a flat, horizontal table. The tablecloth is pulled with constant velocity $(U, 0, 0)$, as illustrated in the figure. This question investigates whether the body can remain stationary, supported vertically by a thin layer of fluid with viscosity μ between it and the cloth.

Assume there is no variation or flow in the z-direction and that the pressure is constant p_0 in $x < 0$, $x > L$ and above the body. Sideways forces and gravity within the fluid may be neglected.

(a) Explaining your assumptions, use lubrication theory to deduce that the pressure gradient in the gap is given by

$$\frac{\partial p}{\partial x} = 6\mu\left(\frac{C}{h^2} - \frac{U}{h^3}\right),$$

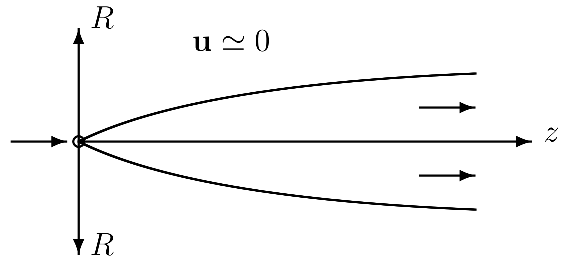
where C is a constant you should define in terms of definite integrals. (15 marks)

(b) Show that the weight W of the body can be supported provided

$$W = 6\mu\int_0^L x\left(\frac{U}{h^3} - \frac{C}{h^2}\right)\,dx.$$

(5 marks)

(Total: 20 marks)



3. Fluid is blown through a small circular hole in the plane $z = 0$. In terms of cylindrical polar coordinates $(R,\, \phi,\, z)$, a steady, axisymmetric jet develops in $z > 0$ which is thin in the R-direction, as shown in the figure. The full axisymmetric Navier-Stokes equations for the flow $(u,\, 0,\, w)$ and pressure p are

$$u\frac{\partial u}{\partial R} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial R} + \nu\left(\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial u}{\partial R}\right) - \frac{u}{R^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$u\frac{\partial w}{\partial R} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial w}{\partial R}\right) + \frac{\partial^2 w}{\partial z^2}\right)$$

$$\frac{1}{R}\frac{\partial (Ru)}{\partial R} + \frac{\partial w}{\partial z} = 0.$$

(a) Briefly explain the arguments which reduce these equations to the boundary layer equations

$$\frac{1}{R}\frac{\partial (Ru)}{\partial R} + \frac{\partial w}{\partial z} = 0 \qquad u\frac{\partial w}{\partial R} + w\frac{\partial w}{\partial z} = \frac{\nu}{R}\frac{\partial}{\partial R}\left(R\frac{\partial w}{\partial R}\right).$$

(5 marks)

(b) As $R \to \infty$ we have $w \to 0$, while regularity requires $u = 0$ at $R = 0$. We need a 3rd condition to determine the solution.

Show that the momentum flux

$$J = \int_0^\infty R w^2 \, dR$$

is independent of z. (6 marks)

(c) Defining the Stokes streamfunction so that $\frac{\partial \psi}{\partial R} = Rw$ and $\frac{\partial \psi}{\partial z} = -Ru$, seek a similarity solution of the form

$$\psi = zF(\eta), \qquad \text{where} \quad \eta = R/z.$$

Show that a solution of this form requires

$$\nu\left(F'' - \frac{F'}{\eta}\right)' + \left(\frac{FF'}{\eta}\right)' = 0.$$

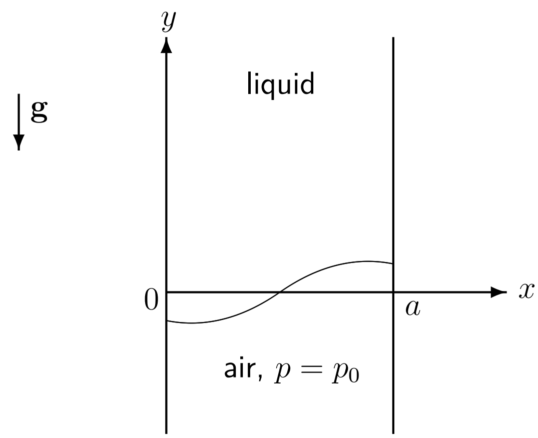
(7 marks)

(d) You are given that this problem has the solution

$$F = \frac{12\nu J\eta^2}{32\nu^2 + 3J\eta^2}.$$

At large R, is the fluid pushed away from the jet or sucked into it? (2 marks)

(Total: 20 marks)



4. An inviscid liquid of density ρ occupies $y > 0$, $0 < x < a$, between two solid, vertical plates at $x = 0$ and $x = a$. Dynamically insignificant air occupies $y < 0$ and gravity acts in the negative y-direction as shown in the figure.

(a) The interface $y = 0$ is perturbed to

$$y = \varepsilon \cos kx \, e^{st}$$

where $0 < \varepsilon \ll 1$, k is real and positive while s can be complex (in which case the real part is to be understood where appropriate). Given that a surface tension γ acts, show that the liquid pressure at the perturbed surface is to $O(\varepsilon)$

$$p = p_0 - \varepsilon \gamma k^2 \cos kx \, e^{st}.$$

(5 marks)

(b) Assuming the perturbed flow is irrotational and takes the form

$$\mathbf{u} = \varepsilon \nabla V \qquad \text{where} \quad V = f(y) \cos kx \, e^{st},$$

find $f(y)$ and using boundary conditions determine the possible values of k. (5 marks)

(c) Use the time-dependent Bernoulli equation to deduce that

$$s^2 = gk - \frac{\gamma k^3}{\rho}.$$

(7 marks)

(d) Deduce the largest value of a for which the flat surface is stable. (3 marks)

(Total: 20 marks)

5. Discuss the influence of the Reynolds number on the incompressible flow of a Newtonian fluid. (20 marks)

(Total: 20 marks)

BSc and MSci EXAMINATIONS (MATHEMATICS)

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MATH60002/70002

Fluid Dynamics 2 (Solutions)

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1. (a) The Newtonian stress tensor is $\boxed{\text{seen} \Downarrow}$

$$\sigma_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

Then

$$\frac{\partial \sigma_{ij}}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \mu\frac{\partial}{\partial x_i}\frac{\partial u_j}{\partial x_j} = (-\nabla p + \mu\nabla^2 \mathbf{u})_i,$$

using $\nabla \cdot \mathbf{u} = 0$. $\boxed{2, \text{A}}$

(b) The force on a body with surface B is $D = \int_B \sigma_{ij} n_j \, dS$ where the normal is directed out of the body. Integrating over the region between the body B and a sphere of large radius S and using the divergence theorem, we have $\boxed{\text{seen} \Downarrow}$

$$0 = \int \frac{\partial \sigma_{ij}}{\partial x_j} \, dV = \oint_B \sigma_{ij}(-n_j) \, dS + \oint_S \sigma_{ij} n_j \, dS$$

allowing for the direction of the normal. It follows that

$$D = \int_S \sigma_{ij} n_j \, dS$$

and so the drag force can be calculated on the large sphere as required. $\boxed{3, \text{D}}$ For the given case of the sphere we find that the flow consists of two terms, one a factor of r^2 smaller than the other. Only terms in σ_{ij} which scale as $1/r^2$ will contribute to the integral over a large sphere, and we conclude that the second term in ψ does not contribute to the drag. At infinity the flow looks like

$$\psi = \frac{3Ua}{4} r \sin^2\theta \qquad \text{with} \qquad p = \frac{3a\mu U}{2}\frac{\cos\theta}{r^2}$$

and we are told that this velocity gives rise to a drag $D = -6\pi a\mu U$. so that we can rewrite it as

$$\psi = -\frac{D}{8\pi\mu} r \sin^2\theta, \qquad p = -\frac{D}{4\pi}\frac{\cos\theta}{r^2}.$$

We note that a has disappeared – viewed from infinity details of the shape canot be seen, all that is felt is a point force near the origin. Rewriting $F = -D$ for the force on the fluid not the body, we obtain the stated Stokeslet flow. $\boxed{4, \text{C}}$

(c) We have $R = r\sin\theta$ and $z = r\cos\theta$, so that the Stokeslet is $\boxed{\text{unseen} \Downarrow}$

$$\psi = \frac{F}{8\pi\mu}\frac{R^2}{(R^2 + z^2)^{1/2}}, \qquad p = \frac{F}{4\pi}\frac{z}{(R^2 + z^2)^{3/2}},$$

so that

$$w = \frac{1}{R}\frac{\partial \psi}{\partial R} = \frac{F}{8\pi\mu}\frac{2z^2 + R^2}{(R^2 + z^2)^{3/2}}.$$

$\boxed{1, \text{A}}$

(d) Consider a uniform ring of point forces, which sum to the total weight W. Each of these points gives rise to the same z-velocity w along the axis of the ring, for which $R = b$ so that superposing all these forces gives rise to $\boxed{3, \text{B}}$ $\boxed{\text{unseen} \Downarrow}$

$$w = \frac{W}{8\pi\mu}\frac{2z^2 + b^2}{(b^2 + z^2)^{3/2}}.$$

Each point force also gives rise to a horizontal velocity. However, all these horizontal velocities must cancel on summation, as by symmetry, the velocity on the ring axis must be in the z-direction. $\boxed{4, \text{A}}$

(e) By differentiation,

$\boxed{\text{unseen} \Downarrow}$

$$\frac{dw}{dz} = \frac{W}{8\pi\mu}\left(4z(b^2+z^2)^{-3/2} - 3z(2z^2+b^2)(b^2+z^2)^{-5/2}\right)$$

$$= \frac{W}{8\pi\mu}\frac{z}{(b^2+z^2)^{5/2}}(b^2-2z^2),$$

so that we have a stationary value at $z = \pm b/\sqrt{2}$, which must be a maximum. $\boxed{3, \text{B}}$

2. (a) We assume that the length scale of variation in the x-direction is much larger than that in the y direction, which we write as $h_0 \ll L_0$. It therefore follows that $\nabla^2 \simeq \frac{\partial^2}{\partial y^2}$. We infer from mass conservation that the scale of normal velocity, V_0, is $O(h_0/L_0)$ times that of tangential velocities, U_0, i.e. $V_0 \simeq (h_0/L_0)U_0$. We assume that inertia is negligible compared with the viscous terms on the small length-scale, $\boxed{\text{seen} \Downarrow}$

$$\frac{\rho U_0^2}{L_0} \ll \frac{\mu U_0}{h_0^2}.$$

Balancing the pressure gradient in the x-direction with the viscous term, we then find that

$$\frac{\partial p}{\partial y} \gg \frac{\partial p}{\partial x} \simeq \mu \frac{\partial^2 u}{\partial y^2} \gg \mu \frac{\partial^2 v}{\partial y^2}.$$

We therefore find that nothing can balance p_y, and so we obtain the lubrication equations,

$$u_x + v_y = 0, \qquad p_y = 0, \qquad p_x = \mu u_{yy}.$$

$\boxed{\text{sim. seen} \Downarrow}$

Since $p_y = 0$, we have p_x is independent of y and so $\boxed{5,\ \text{A}}$

$$u = \frac{p_x}{2\mu}\left(y^2 + Ay + B\right),$$

where A and B are functions of x only. Now we have that $u = v = 0$ on $y = h$ while on $y = 0$, $u = U$ and $v = 0$. This implies

$$u = \frac{p_x}{2\mu}\left(y^2 - yh\right) + U\left(1 - \frac{y}{h}\right).$$

Now $\boxed{2,\ \text{B}}$

$$0 = \int_0^h v_y\, dy = -\int_0^h u_x\, dy = -\frac{d}{dx}\int_0^h u\, dy,$$

$\boxed{1,\ \text{A}}$

since $u = 0$ when $y = h$. Combining the last two equations

$$\frac{d}{dx}\left(\frac{p_x}{2\mu}(-\tfrac{1}{6}h^3) + U\tfrac{1}{2}h\right) = 0,$$

or integrating

$$\left(\frac{h^3 p_x}{12\mu}\right) - \tfrac{1}{2}Uh = \tfrac{1}{2}C, \qquad \text{or} \quad p_x = 6\mu\left(\frac{C}{h^3} + \frac{U}{h^2}\right)$$

for some constant C. Now since $p = p_0$ at both ends, $\boxed{4,\ \text{A}}$

$$0 = \int_0^L p_x\, dx \qquad \Longrightarrow \qquad C = -U\frac{\int_0^L h^{-2}dx}{\int_0^L h^{-3}dx}.$$

$\boxed{3,\ \text{B}}$

(b) Now the total upwards vertical force exerted is $\boxed{\text{unseen} \Downarrow}$

$$F = \int_0^L (p - p_0)\, dx = \Big[x(p - p_0)\Big]_0^L - \int_0^L x p_x\, dx.$$

by integration by parts. If F balances the weight W it follows that

$$W = -6\mu \int_0^L x\left(\frac{U}{h^2} + \frac{C}{h^3}\right) dx.$$

$\boxed{2,\ \text{C}}$

 $\boxed{3,\ \text{D}}$

3. (a) We assume (1) that the lengthscale for R is much less than that for z, from which it follows that $w \gg u$. (2) As a result the scale for the radial pressure gradient is larger than the axial one, yet all other terms in the R-momentum equation are smaller than the corresponding terms in z-momentum equation. We conclude that the radial momentum reduces to $\partial p/\partial R = 0$. (3) Exiting the layer as R increases, we match with the inviscid flow, which defines the axial pressure gradient. For this problem we have $w \to 0$ and so there is no driving pressure gradient. sim. seen ⇓ 2, A

(b) We have 3, B

$$\frac{dJ}{dz} = \int_0^\infty R\,2w\frac{\partial w}{\partial z}\,dR.$$

sim. seen ⇓

Multiplying by R and integrating the boundary layer equation, we have

$$\int_0^\infty \left(Ru\frac{\partial w}{\partial R} + Rw\frac{\partial w}{\partial z}\right)\,dR = \int_0^\infty \nu\frac{\partial}{\partial R}\left(R\frac{\partial w}{\partial R}\right)\,dR = \left[\nu R\frac{\partial w}{\partial R}\right]_0^\infty = 0,$$

using the boundary conditions. Also,

$$\int_0^\infty Ru\frac{\partial w}{\partial R}\,dR = \Big[Ruw\Big]_0^\infty - \int_0^\infty \frac{\partial (Ru)}{\partial R}w\,dR = \int_0^\infty \left(R\frac{\partial w}{\partial z}\right)w\,dR$$

integrating by parts and using mass conservation. Putting all this together we have $dJ/dz = 0$ so that J is a constant of the flow. 6, A

(c) Writing $\psi = zF(\eta)$ and $\eta = R/z$, we have sim. seen ⇓

$$Rw = F' \qquad -Ru = (F - \eta F')$$

so that

$$J = \int_0^\infty \frac{(F')^2}{\eta}\,d\eta.$$

Also

$$\left[w\frac{\partial w}{\partial z} + u\frac{\partial w}{\partial R}\right] = \frac{F'(F'')}{R^2}\left(\frac{-R}{z^2}\right) - \frac{(F-\eta F')}{R}\left(\frac{-F'}{R^2} + \frac{F''}{Rz}\right)$$

$$= \frac{1}{R^2 z}\left[-\eta F'F'' + \eta F'F'' - FF'' - (F')^2 + \frac{FF'}{\eta}\right]$$

Meanwhile

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial w}{\partial R}\right) = \frac{1}{R}\frac{\partial}{\partial R}\left[R\left(\frac{-F'}{R^2} + \frac{F''}{Rz}\right)\right] = \frac{1}{R}\left(\frac{F'}{R^2} - \frac{F''}{Rz} + \frac{F'''}{z^2}\right)$$

Combining these, we obtain the ODE

$$-FF'' - (F')^2 + \frac{FF'}{\eta} = \nu\left[\frac{F'}{\eta} - F'' + \eta F'''\right].$$

Dividing through by η we see that this is equation is integrable

$$\left(\frac{-FF'}{\eta}\right)' = \nu\left(F'' - \frac{F'}{\eta}\right)'.$$

As R and z do not appear explicitly in either this equation or the integral constraint, we conclude a similarity solution of the given form may be possible. The boundary conditions are [not required] 5, D 2, C

$$F(0) = 0 \qquad \lim_{\eta\to\infty} F' = 0, \qquad \int_0^\infty \frac{(F')^2}{\eta}\,d\eta = J.$$

(d) Large R corresponds to large η, when $F \to 4\nu + O(1/\eta^2)$, so that $u \sim -4\nu/R < 0$ so the flow is inwards,and fluid is entrained into the jet. Alternatively, as $R \to \infty$ $\psi \to 4\nu z$ which increases with z, so the mass flux increases and fluid must be sucked in from infinity at a constant rate. $\boxed{2,\ \mathrm{D}}$

4. (a) The curvature, $K = \nabla \cdot \widehat{\mathbf{n}}$ where $\widehat{\mathbf{n}}$ is the unit normal. A normal vector is seen ⇓

$$\widehat{\mathbf{n}} = \nabla(y - \varepsilon \cos kx\, e^{st}) = (\varepsilon k \sin kx\, e^{st},\ 1,\ 0)$$

and as this has modulus $1+O(\varepsilon^2)$, it will serve as the unit normal. We note it points into the liquid, so the pressure there is $p = p_0 - \gamma K$. Thus $K = \varepsilon k^2 \cos kx\, e^{st}$, and the normal stress condition is as required

$$p = p_0 - \varepsilon\gamma k^2 \cos kx\, e^{st}. \tag{1}$$

(Alternatively quote the curvature formula $y''/(1+y'^2)^{3/2}$.) 5, A

(b) As $\mathbf{u} = \varepsilon\nabla V$ and $\nabla^2 V = 0$, with $V = f(y)\cos kx\, e^{st}$ we have $f'' - k^2 f = 0$ so that $f = Ae^{ky} + Be^{-ky}$. Imposing a zero velocity as $y \to \infty$ we have $A = 0$. The kinematic condition on the interface, which can be evaluated on $y = 0$ to leading order, gives sim. seen ⇓

$$0 = \frac{D}{Dt}(y - \varepsilon \cos kx\, e^{st}) = -\varepsilon s \cos kx\, e^{st} + \varepsilon \frac{\partial V}{\partial y} + O(\varepsilon^2)$$

which gives $s + kB = 0$ so that

$$V = -\frac{s}{k} \cos kx\, e^{-ky+st}.$$

Now we also have the impermeability condition on the solid walls, $\frac{\partial V}{\partial x} = 0$ on $x = 0, a$. This requires $\sin ka = 0$ or $ka = m\pi$ for positive integer m.

2, A

(c) Now the Bernoulli equation is 3, C

$$p + \rho\frac{\partial V}{\partial t} + \tfrac{1}{2}\rho|\mathbf{u}|^2 + \rho g y = \text{constant}.$$

As $\mathbf{u} = O(\varepsilon)$ we can neglect $|\mathbf{u}|^2$. Evaluating on the surface, this gives to $O(\varepsilon)$

$$p + \rho\varepsilon s\frac{-s}{k} \cos kx\, e^{st} + \rho g\varepsilon \cos kx\, e^{st} = \text{constant}. \tag{2}$$

Combining (1) and (2) we have

$$\frac{\rho s^2}{k} = \rho g - \gamma k^2 \qquad \Longrightarrow \qquad s^2 = k\left(g - \frac{\gamma k^2}{\rho}\right),$$

for $ka = m\pi$.

6, B

(d) For (neutral) stability, we need $s^2 \leqslant 0$ for all permissible values of k. The worst case is $m = 1$, for which $ka = \pi$. We conclude the equilibrium will be stable if 1, C

sim. seen ⇓

$$a < \pi\left(\frac{\gamma}{\rho g}\right)^{1/2}.$$

3, D

5. This is an essay question, basically testing overall comprehension of the entire course - any pertinent material will earn credit. 20, M

Review of mark distribution:
Total A marks: 32 of 32 marks
Total B marks: 20 of 20 marks
Total C marks: 12 of 12 marks
Total D marks: 16 of 16 marks
Total marks: 100 of 80 marks
Total Mastery marks: 20 of 20 marks

MATH60002 Fluid Dynamics 2

Question	Marker's comment
1	Mainly elementary manipulation required for this question, but some understanding of simple physical arguments. 3D geometry. and symmetry. The MSci/MSc students did noticeably better on this question than the BSc, who had more difficulty transforming from spherical to cylindrical coordinates.
2	Everyone should have expected to be asked to derive the lubrication equations. The argument leading to the cross-layer equation reducing to p_y=0 was often not well made.In lectures we considered the case with a stationary table and and a moving body. Some did not apply the boundary conditions correctly, perhaps having the lectures in mind.Unfortunately, there was a mistake on the paper, which I didn't notice until marking the , exam and which snuck past the checker and External examiner also. I a very sorry about this.Most people unfortunately did not get close to the flawed equation. I dealt with the issue as follows: Firstly I altered the mark scheme so that fewer marks were at stake for that portion of the question. Then anyone who made an attempt at that portion of the question was awarded full marks for their attempt. Anyone who got within shouting distance of that part but not very close was awarded partial credit for it. My marking will be reviewed by others. A corrected question and solution will appear online. Sorry for this.The final part of the question was essentially unchanged by the error. The majority did not think of integrating by parts and gave an argument which would only work in the trivial case where p_x was constant.
3	This was the best answered regular question. In lectures we covered the 2-D jet; the axisymmetric version is a little messier, as usual, and so I gave the solution rather than requiring it be derived in full.The boundary layer derivation was done well, but the arguments leading to the neglect of both components of the pressure gradient were flawed. Some found the similarity solution manipulations troublesome and some felt that because Fgt;0 for large R the fluid must be being pushed away, whereas looking at the radial velocity makes it clear that fluid is sucked in, just as it is for the 2-D jet.
4	This question appeared in full on the problem sheet so I was a little surprised more didn't do well. A few people ignored the fact that the air was dynamically insignificant, and tried to solve a 2-fluid model. A fundamental mistake was not realising that the velocity was O(epsilon) and so u^2 and u (epsilon h)' were both negligible. Many who realised that k=n(pi)/a did not seem to appreciate that stability must hold for all k, ie all integers ngt;0 and that the most testing case was n=1.

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3	This was the best answered regular (1-4) question. In lectures we covered the 2-D jet; the axisymmetric version is a little messier, as usual, and so I gave the solution rather than requiring it be derived in full.The boundary layer derivation was done well, but the arguments leading to the neglect of both components of the pressure gradient were flawed. Some found the similarity solution manipulations troublesome and some felt that because Fgt;0 for large R the fluid must be being pushed away, whereas looking at the radial velocity makes it clear that fluid is sucked in, just as it is for the 2-D jet.
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5	Not much to be said about this - an essay question testing overaall comprehension of the course. Credit was given for anything pertinent. A few people gave overbrief answers, but on the whole people did well.