Question 1

The probability density function f for the χ^2_ν distribution is

$$f(x) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2},$$

where, as usual, $\Gamma(z)$ is the gamma function:

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} \mathrm{d}x.$$

(a) Show that the gamma function has the property $\Gamma(z+1) = z\Gamma(z)$ (Hint: use integration by parts).

(b) Show that if $Y \sim \chi^2_\nu$ then $\mathrm{E}\left(Y\right) = \nu$ (Hint: try to get the integral into a form that is a constant times 'something' that integrates to 1).

(c) Show that if $Y \sim \chi^2_\nu$ then $\mathrm{E}\left(Y^2\right) = \nu(\nu+2)$.

Question 2

Prove Boole's inequality: for a set of events A_i, $i = 1, 2, \ldots n$,

$$\mathrm{P}\left(\bigcup_{i=1}^n A_i\right) \leq \sum_{i=1}^n \mathrm{P}\left(A_i\right).$$

Hint: use induction.

Question 3 (R question)

Complete Lectures 4 and 5 first, i.e. up to end of Section 2.7 in the notes.

Download the dataset `data_ps9.csv` (link on Blackboard below problem sheet). This dataset contains 200 observations for each of the random variables X, Y and Z, where the ith row shows the simultaneous measurement of the three variables at time i. Using R, perform an exploratory data analysis to:

(a) Investigate whether or not there is any relationship between any of the variables.

(b) Guess the distributions of X, Y and Z.

Hint: Compute summary statistics for the samples and plot the samples, considering which plots might be most appropriate.