MATH70132 Mathematical Logic : Mastery Material.

Reading: The reading for the Mastery Material is from Chapter 8 of the book:

René Cori and Daniel Lascar 'Mathematical Logic: a course with exercises, Parts I and II,' (Oxford University Press, 2001).

You can download a copy of the chapter from the Leganto reading list for the module.

You should read from the start of the Chapter (from p.179), including all of sections 8.1, 8.2 and 8.4 up to and including Proposition 8.33 on p.204, omitting section 8.3.

Notation and terminology: The notation and terminology used in the book does not correspond fully to what we have been using in the lectures. Of course, by obtaining the whole book, you can check up on any unexplained or unfamiliar notation: the library will order extra copies of the books if needed. However, the following points may help.

(a) A first order language is denoted by L and always has equality (denoted by $\simeq$). All structures $\mathcal{M}$ considered are normal and the domain M of a structure $\mathcal{M}$ is called its base set.

(b) L-formulas are denoted by capital letters such as F (not Greek letters) and variables are $v_1, v_2, \ldots$. There are fewer brackets than in the lectures. The notation $F[v_1, \ldots, v_n]$ is a formula with free variables amongst $v_1, \ldots, v_n$. If $a_1, \ldots, a_n$ are in the L-structure $\mathcal{M}$ we write $\mathcal{M} \models F[a_1, \ldots, a_n]$ to mean that the formula expresses something true when a_i is substituted in for each free occurrence of v_i in F (for each $i \leq n$). This can be formalised using valuations.

(c) A *substructure* $\mathcal{N}$ of an L-structure $\mathcal{M}$ is obtained by taking a non-empty subset N of M which contains all of the constants (of $\mathcal{M}$) and is closed under all of the functions of $\mathcal{M}$. This is regarded as an L-structure by keeping the same interpretation of the constant symbols as in $\mathcal{M}$ and taking the restriction to N of all the interpretations in $\mathcal{M}$ of the relation and function symbols.

(d) At various points (but not in (g) below) we can assume that all quantifiers used are existential: we can replace $(\forall x_i)\chi$ by $(\neg(\exists x_i)(\neg\chi))$.

(e) p.184: card(X) is $|X|$ and $\aleph_0$ is $|\mathbb{N}|$

(f) p.186: a monomorphism $h : \mathcal{M} \to \mathcal{N}$ is essentially an isomorphism between $\mathcal{M}$ and a substructure of $\mathcal{N}$. (It's sometimes called an *embedding*.)

(g) An L-formula of the form $(Q_1 x_{i_1}) \ldots (Q_r x_{i_r})\psi$ where each Q_j is a quantifier ($\forall$ or $\exists$) and ψ is an L-formula without quantifiers is said to be in *prenex form.* Every L-formula is equivalent to some formula in prenex form.

If you are puzzled by what you think is other unfamiliar terminology, you can email me or, better, ask me via the Ed discussion board.

Exercises: I will produce a problem sheet, but it would also be useful if you look at the exercises in the book, particularly questions 3, 4, 14, 15 (in which $\aleph_1$ is the least uncountable ordinal).

Syllabus: You can find all of this material in other books or sources if you wish. The topics on the syllabus are:

Elementary substructures; the method of diagrams, the Tarski-Vaught test; the Löwenheim-

Skolem Theorems; Categoricity and Vaught's Test (also called Łos-Vaught); Reduced products and ultraproducts; Łos' theorem.