MATH40004/MATH40011 - Calculus and Applications

Midterm – 15 February 2022

Please write down how you worked out the solutions for each question step by step as correct final solutions without clear demonstration of your methodology will not get marks.

Question 1. As we saw in the overview lectures, the Fourier transform of the Heaveside function:

$$H(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x < 0. \end{cases}$$

is $\hat{H}(\omega) = \frac{1}{i\omega} + \pi\delta(\omega)$. Use convolution theorem to obtain the Fourier transform of $f(x) = \int_{-\infty}^{x} g(t)dt$ in terms of $\hat{g}(\omega)$. Simplify your result if $\int_{-\infty}^{\infty} g(x)dx = 0$.

(6 marks)

Question 2. Solve the following ordinary differential equation:

$$y'' = \frac{1}{2}e^{y}$$

for $y(0) = 0$ and $y'(0) = 1$.

(5 marks)

Question 3. (a) Solve the following second order linear ODE:

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + \frac{1}{4}x = te^{\frac{-t}{2}} + 1.$$

(5 marks)

(b) Write an equivalent system of first order linear ODEs. Use your solution in part (a) or otherwise to obtain a particular integral and complementary function for this system of ODE.

(4 marks)