Question 1

Recall from Term 1 that the probability density function of the uniform distribution on the interval (a, b) is

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

We write $X \sim \mathrm{U}(a, b)$ to indicate that the random variable X follows this distribution.

(a) If $X \sim \mathrm{U}(a, b)$, compute $\mathrm{E}(X)$.

(b) If $X \sim \mathrm{U}(a, b)$, compute $\mathrm{Var}(X)$.

Question 2

Suppose X is uniformly distributed on the interval $[0, 4]$, i.e. $X \sim \mathrm{Unif}(0, 4)$.

(a) Compute $\mathrm{P}(|X - 2| \geq 1)$.

(b) Use Chebyshev's inequality to bound the probability that $|X - 2| \geq 1$.

(c) Is the bound in (b) informative?

(d) For which values $\epsilon > 0$ can Chebyshev's inequality be used to obtain a nontrivial bound for $\mathrm{P}(|X - 2| \geq \epsilon)$?

Question 3

Suppose that a population is taking part in a vote and an unknown proportion p of the voters supports a particular option, labelled A. Suppose it is possible to interview a sample of n randomly selected voters and record $\widehat{p}$, the proportion of that sample that supports option A. What value of n should be chosen so that with high confidence (confidence at least 95%) $\widehat{p}$ is within 0.01 of p?

Question 4

Consider the probability space $(\Omega, \mathcal{F}, \mathrm{P})$. Recall from Term 1 the definition of an indicator variable for an event $A \in \mathcal{F}$, denoted $\mathbb{I}_A$ (or $\mathbb{I}(A)$) and defined for $\omega \in \Omega$ by

$$\mathbb{I}_A(\omega) = \begin{cases} 1, & \text{if } \omega \in A, \\ 0, & \text{if } \omega \notin A. \end{cases}$$

(a) Is $\mathbb{I}_A$ a discrete random variable or a continuous random variable?

(b) If $\mathbb{I}_A$ is discrete, write down its probability mass function, or if it is continuous, write down its probability density function.

(c) Compute $\mathrm{E}(\mathbb{I}_A)$.