MATH60005/70005 - Optimisation

Welcome Lecture, Autumn Term 24/25

Dr Dante Kalise (dkaliseb@ic.ac.uk)
Dr Estefanía Loayza-Romero (kloayzar@ic.ac.uk)

Department of Mathematics, Imperial College London

IMPERIAL

We'll begin by introducing ourselves:

- I'm Dante and together with Estefanía we'll be your Optimisation instructors.

We'll begin by introducing ourselves:

- I'm Dante and together with Estefanía we'll be your Optimisation instructors.
- Estefanía is not here but she'll come to say hi next week.

We'll begin by introducing ourselves:

- I'm Dante and together with Estefanía we'll be your Optimisation instructors.
- Estefanía is not here but she'll come to say hi next week.
- I am Reader in Computational Optimisation and Control at the Applied Mathematics Section, also course director of the Applied Mathematics MSc, and part of the EDI network of our Department.

We'll begin by introducing ourselves:

- I'm Dante and together with Estefanía we'll be your Optimisation instructors.
- Estefanía is not here but she'll come to say hi next week.
- I am Reader in Computational Optimisation and Control at the Applied Mathematics Section, also course director of the Applied Mathematics MSc, and part of the EDI network of our Department.
- Estefanía is Chapman Fellow in Applied Mathematics (we're both in AMMP),
- we are applied mathematicians working on Optimisation, Control Theory, and Scientific Computation.

Module details

Our live sessions

Live sessions: Thursdays 13:00-14:00, Fridays 12:00-14:00, from week 2 until week 10, and Thursday Dec. 12 13:00-14:00 for the mastery topic.

All lectures will be held in the Clore.

Dante will teach weeks 2-5 and Estefanía will teach weeks 6 onwards.

These live sessions will consist of:

1. Brief overview of the material of the week,
2. Several illustrative examples,
3. Discussion of selected problems and algorithms from a weekly problem sheet.

Our live sessions

- All live sessions will be recorded in Panopto and available via Blackboard.
- Office hours: right after lectures or by appointment. I am in Huxley 742.
- Module assessment: 90% exam based on contents from weeks 2 to 10 (or until week 11 for those following the Year 4/MSc version with mastery topic), and a 10% group coursework ($0 < n \leq 3$) evaluating exercises and algorithmic aspects up to week. Coursework will be released on November 1st and due on November 14th.

Module material

Content structure: the module material consists of:

1. Lecture notes and pre-recorded videos,
2. Live lectures and notes,
3. Weekly exercise sheet with solutions,
4. Additional material such as code, external resources, extra exercises, and the reading list.
5. Past year exams and coursework.

Content is released two weeks in advance (Weeks 1-3 al already live!).

Do I need to watch videos, read the lecture notes, do the exercises all at once? NO. You need to attend the lectures and attempt some of the exercises, lecture notes and videos are there as a complement from the live sessions.

Past experiences

From past surveys:

- *Plus: Problem sheet difficulty, structure of the course, good mix between theory and applications, Dante's enthusiasm, helpful live sessions, and interesting CW. I am very pleasantly surprised by the high quality of this module, especially considering it was run for the first time.*
- *Best module in terms of quality of teaching, feedback, and learning materials I have taken in the last three years. Well done! Only tiny thing could be improved to get a 100% (instead of 95%) is typed solutions for problem sheets.*
- *This module was taught exceptionally well, and Dr Kalise has been excellent. I haven't heard a single bad word about his teaching or this module in the MLC or MCR.*

Module overview

Topics part 1:

- **Mathematical preliminaries**. Basic subspaces in optimization, linear algebra, and multivariable calculus.
- **Unconstrained optimization**. Classification of maxima and minima, saddle points, necessary and sufficient optimality conditions, quadratic forms.
- **Linear least square problems**. Problem statement, optimality conditions, solution algorithms, and applications.
- **Gradient descent methods**. Algorithmic formulation, convergence, stepsize choice, quasi-Newton methods, SGD.

Module overview

Topics part 2:

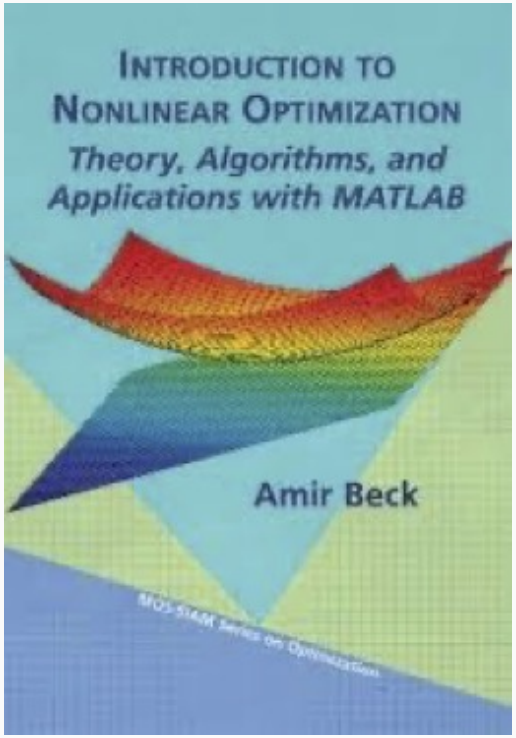
- **Convexity**. Convex sets and functions, implications in optimization.
- **Convex optimization**. Stationarity conditions, projection operators, and projected gradient descent.
- **Optimality conditions**. Necessary and sufficient optimality conditions, KKT conditions, nonlinear constraints.
- **Duality**. Weak and strong duality, applications, ADMM.
- (mastery) **Optimal control**. Introduction to dynamic optimization, optimality conditions, the linear-quadratic case.

Module overview

Prerequisites: mathematical preliminaries notes devoted to discussing mathematical preliminaries required for this module. Basic knowledge of linear algebra, real analysis, and multivariable calculus is required. Please ask for advice if you struggle studying this material.

A part of the coursework involves a light computational component. For this part, coding proficiency will not be assessed. You can work in your preferred language as the evaluation will be focused on your results and analysis. Adequate support will be provided in Matlab and Python. Computational aspects are excluded from the final exam.

Reading list



Netiquette

E-mails:

- Use it for personal matters. Check in Blackboard first for info. Use EdForum for maths-related questions. **Please include both lecturers in your communications**.

Netiquette

E-mails:

- Use it for personal matters. Check in Blackboard first for info. Use EdForum for maths-related questions. **Please include both lecturers in your communications**.
- include Optimization in the subject and a good description.

Netiquette

E-mails:

- Use it for personal matters. Check in Blackboard first for info. Use EdForum for maths-related questions. **Please include both lecturers in your communications**.
- include Optimization in the subject and a good description.
- We normally observe Mon-Fri 9:00am to 6:00pm working hours for e-mail communications.

Netiquette

E-mails:

- Use it for personal matters. Check in Blackboard first for info. Use EdForum for maths-related questions. **Please include both lecturers in your communications**.
- include Optimization in the subject and a good description.
- We normally observe Mon-Fri 9:00am to 6:00pm working hours for e-mail communications.
- if our reply is brief, don't assume anything personal. Every day we receive dozens of e-mails from people with the most diverse requests, we're just trying to navigate around that.

Netiquette

E-mails:

- Use it for personal matters. Check in Blackboard first for info. Use EdForum for maths-related questions. **Please include both lecturers in your communications**.
- include Optimization in the subject and a good description.
- We normally observe Mon-Fri 9:00am to 6:00pm working hours for e-mail communications.
- if our reply is brief, don't assume anything personal. Every day we receive dozens of e-mails from people with the most diverse requests, we're just trying to navigate around that.
- in exchange, your have our utmost attention and we'll try our best to help you.

Netiquette

E-mails:

- Use it for personal matters. Check in Blackboard first for info. Use EdForum for maths-related questions. **Please include both lecturers in your communications**.
- include Optimization in the subject and a good description.
- We normally observe Mon-Fri 9:00am to 6:00pm working hours for e-mail communications.
- if our reply is brief, don't assume anything personal. Every day we receive dozens of e-mails from people with the most diverse requests, we're just trying to navigate around that.
- in exchange, your have our utmost attention and we'll try our best to help you.
- don't disregard these e-mail principles, they'll also help with other modules and with your professional life.

What you need to do achieve the glory in Optimization:

- Read all the official info posted in Blackboard.

What you need to do achieve the glory in Optimization:

- Read all the official info posted in Blackboard.
- Every week, for the next 11 weeks, have a quick read at the lecture notes and watch the videos (not essential) before Thursday.

What you need to do achieve the glory in Optimization:

- Read all the official info posted in Blackboard.
- Every week, for the next 11 weeks, have a quick read at the lecture notes and watch the videos (not essential) before Thursday.
- Also try to have a look at the week's problem sheet.

What you need to do achieve the glory in Optimization:

- Read all the official info posted in Blackboard.
- Every week, for the next 11 weeks, have a quick read at the lecture notes and watch the videos (not essential) before Thursday.
- Also try to have a look at the week's problem sheet.
- Attend the live sessions to reinforce your learning and see exam-type problems.

What you need to do achieve the glory in Optimization:

- Read all the official info posted in Blackboard.
- Every week, for the next 11 weeks, have a quick read at the lecture notes and watch the videos (not essential) before Thursday.
- Also try to have a look at the week's problem sheet.
- Attend the live sessions to reinforce your learning and see exam-type problems.
- Do the week self-assessment checklist and study until it's been completed.
- If you want to make further progress on your own, check the reading list for proofs, material, and discussion.
- Make good use of office hours or other opportunities we have to interact.

NOW, LET'S DISCUSS OPTIMIZSATION!

The Legend of Queen Dido

In the 9th century B.C., Phoenician Queen Dido fled to North Africa (nowadays Tunisia) and asked the local leader for a piece of land.

Yarb: How much land do you want?

Dido: As much land as it can be enclosed by the hide of a bull!

Yarb: sure, that doesn't seem too much land...

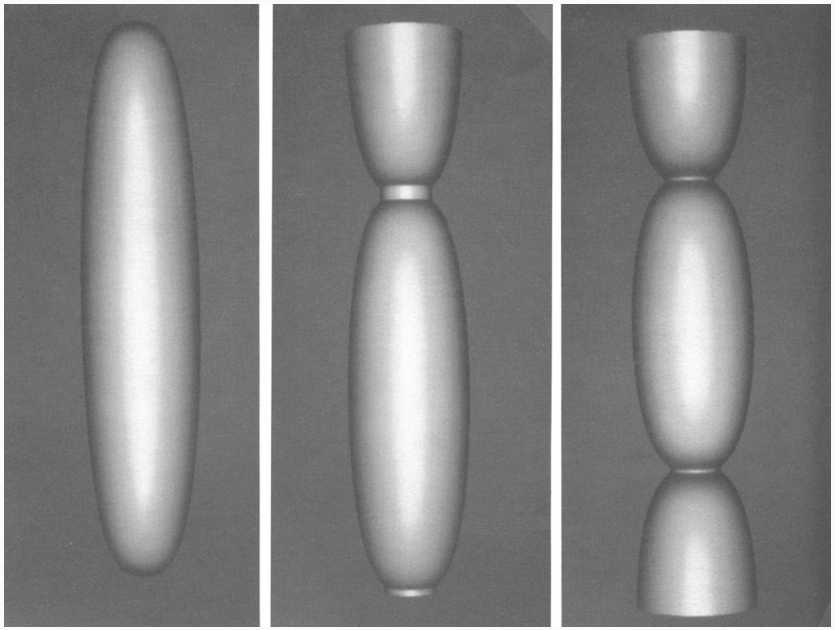
What is the largest area that can be enclosed by the hide of a bull?

The Legend of Queen Dido

The Legend of Queen Dido

The Legend of Queen Dido

Optimal Shapes



Somes shapes minimize a physical quantity: stress, surface tension.

What is Optimization?

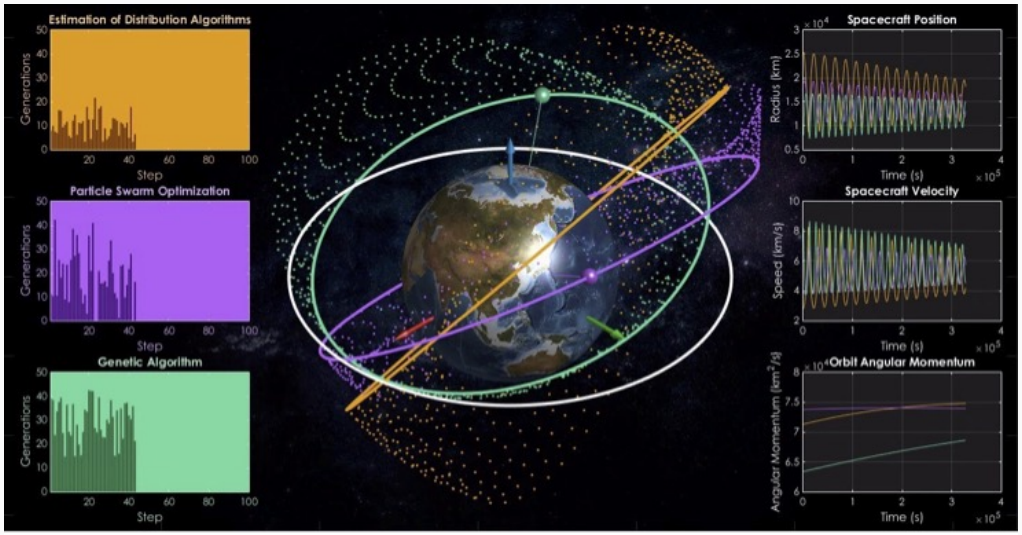
Optimization: an act, process, or methodology of making something (such as design, system, or decision) as fully perfect, functional, or effective as possible.

From an official report of the European Mathematical Society:

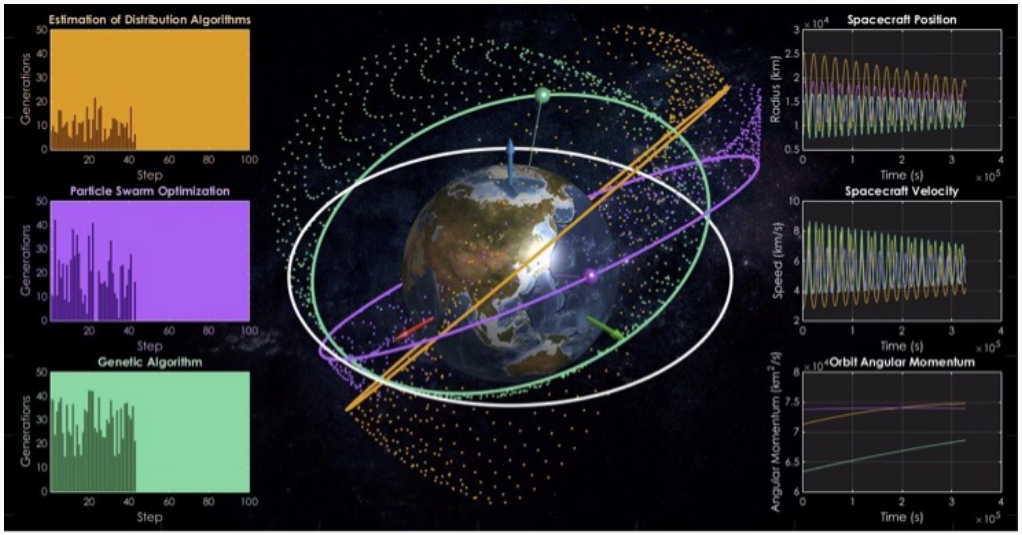
" Together with theory and experimentation, a third pillar of scientific inquiry of complex systems has emerged in the form of a combination of **modelling**, **simulation**, **optimization** *and* **visualisation.**"

A Sample of Relevant Applications

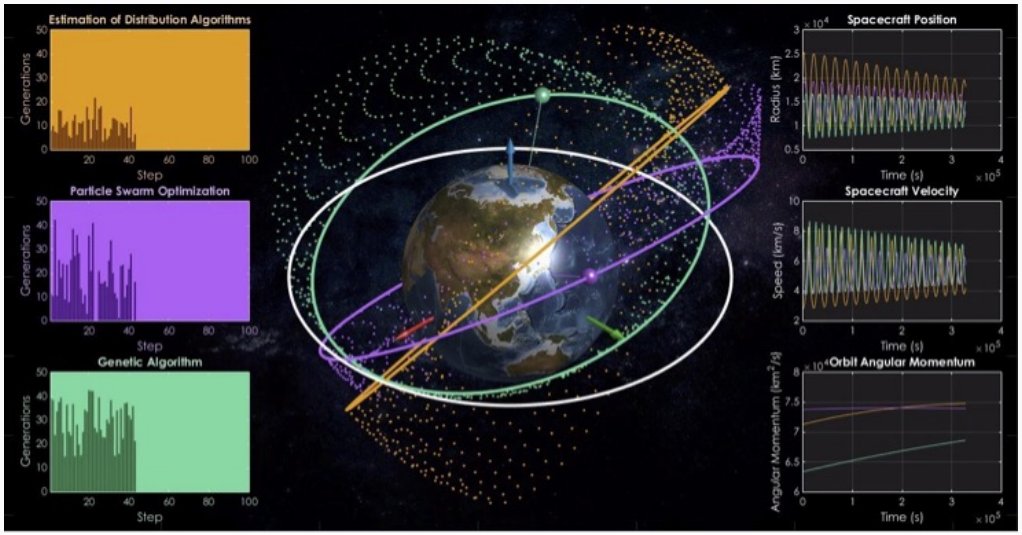
A Sample of Relevant Applications

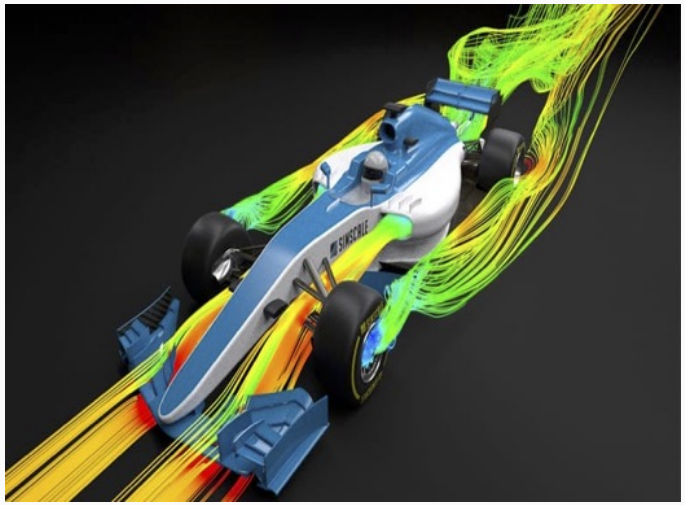


A Sample of Relevant Applications



A Sample of Relevant Applications





What is Optimization?

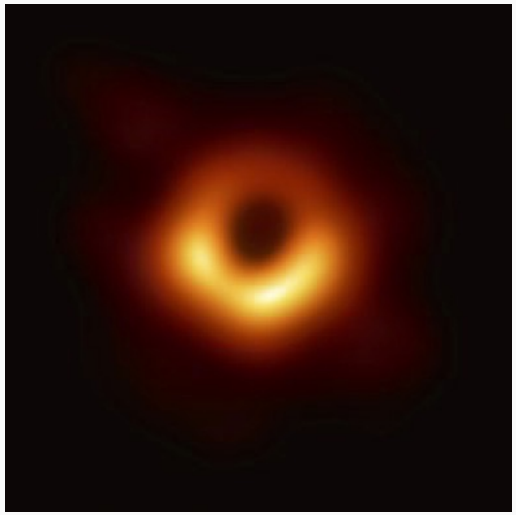
A little poll: Where do you think optimization is extensively used?

1. In making the weather forecast.
2. In autopilots and self-driving cars.
3. In machine learning and deep neural networks.
4. Timetabling.
5. In the discovery of black holes.

What is Optimization?

A little poll: Where do you think optimization is extensively used?

1. In making the weather forecast.
2. In autopilots and self-driving cars.
3. In machine learning and deep neural networks.
4. Timetabling.
5. In the discovery of black holes.



Weather Forecast as an Optimization Problem

A Very Complex Problem

CNN Weather Climate Storm Tracker Video LIVE TV Edi

Weather forecasts are less accurate because of Covid-19, a new study reveals

By Allison Chinchar and Virginia Langmaid, CNN Meteorologists

Updated 0700 GMT (1500 HKT) July 18, 2020

Delta Air Lines commercial planes are seen parked at Birmingham-Shuttlesworth International Airport, Alabama, because of flight reductions due to Covid-19.

(CNN) — As the US heads into peak hurricane season, a reduction in commercial airline flights due to Covid-19 has significantly impacted our ability to accurately forecast the weather.

A study out this week by Dr. Ying Chen, a senior research associate at Lancaster University's Environment Centre, highlights this problem.

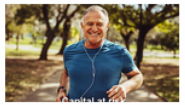
News & buzz

Trump campaig
fourth president
citing...

Opinion: This is
most unpatriotic
Trump's...

Paid Content by

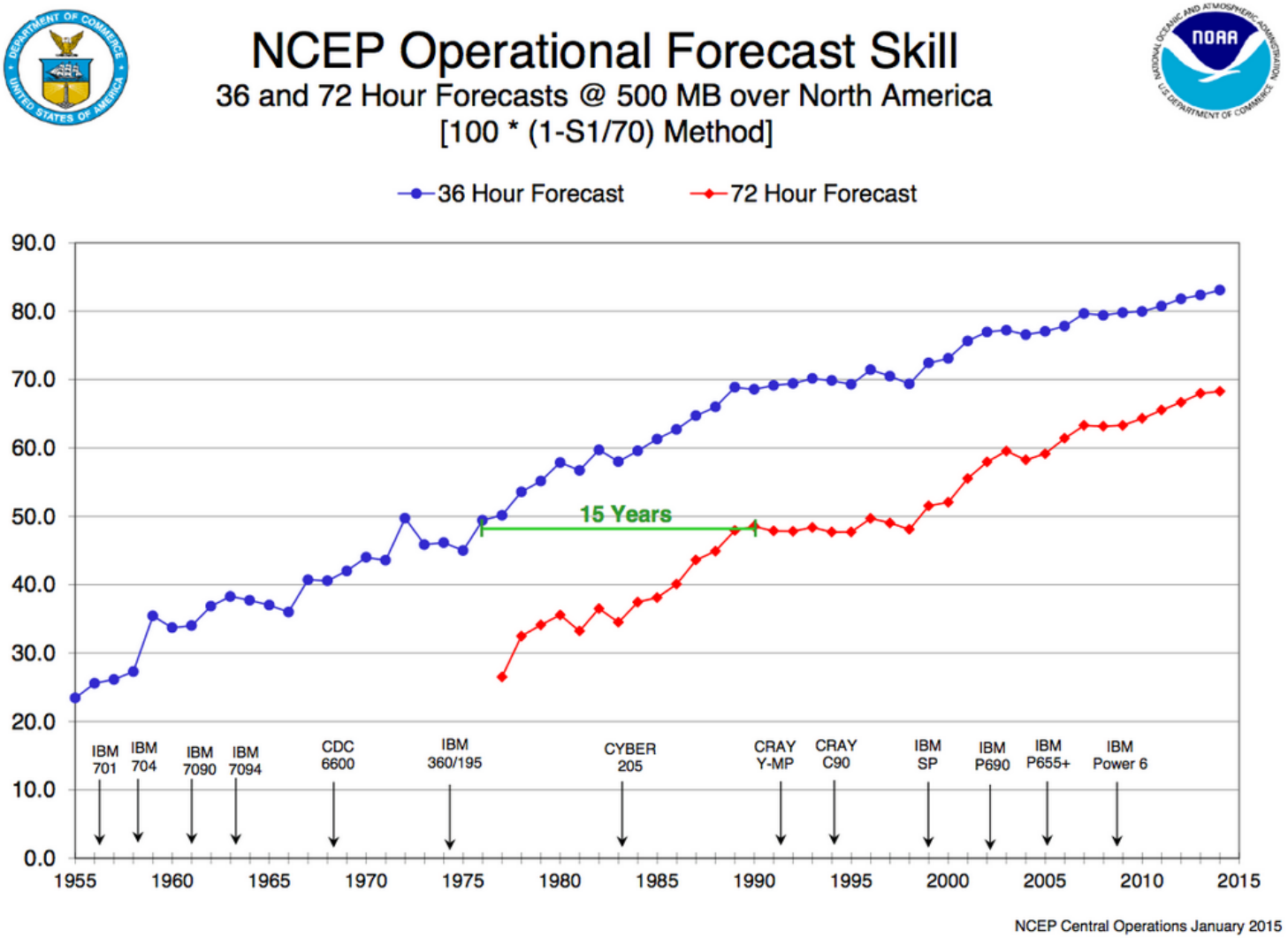
Julia Roberts Spends $8.3
Century-Old San Francisco
Mansion Global



Read our 7
essential

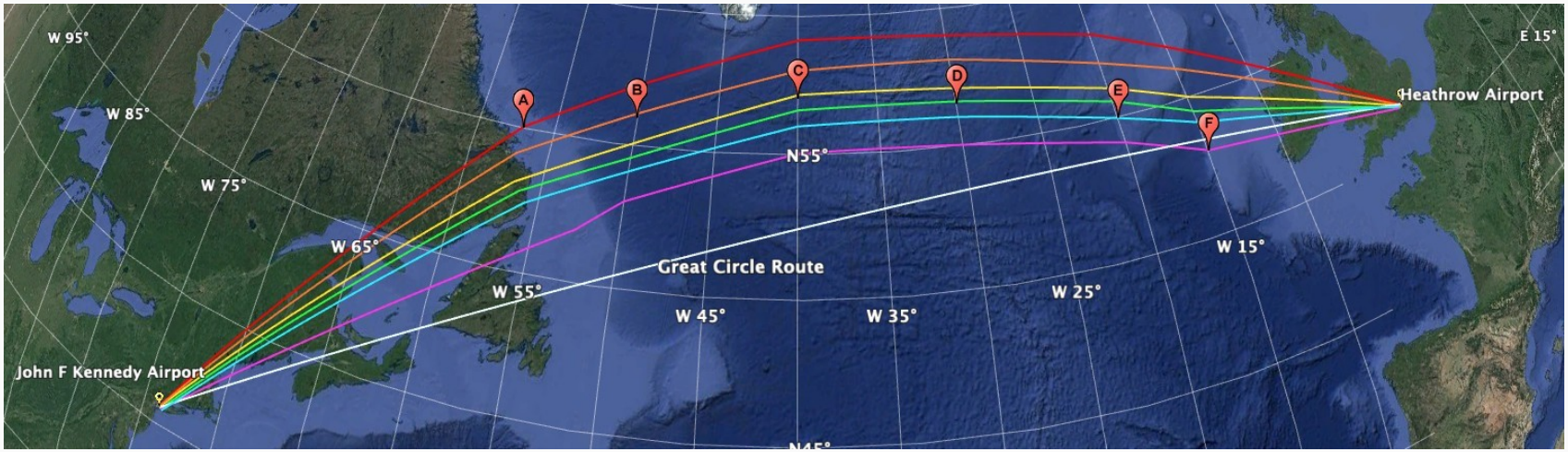
UK R
1966

HPC, Optimization, and Forecasts



Histogram of computing power & operational 500-mb forecast skill, 1955 through 2014

Minimizing Transatlantic Flight Emissions



was developed in labs in Oxford and is being administered by the British Army,

ists are on course to claim a majority. The party has said that if a majority of

Flights should 'coast' on jet stream to reduce emissions

By Olivia Rudgard
ENVIRONMENT CORRESPONDENT

FLIGHTS between New York and London should coast on the jet stream to cut journey times and emissions, a study has suggested.

Academics at the University of Reading argued that cutting-edge satellite data will allow each plane to prioritise the fastest route to save emissions and cut journey times.

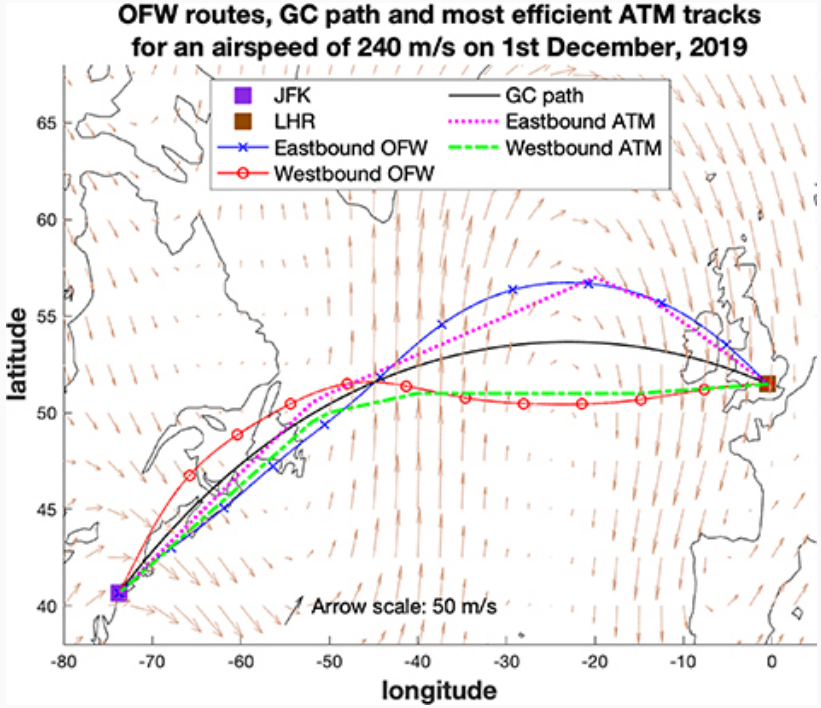
Analysis of 35,000 transatlantic flights between December 2019 and February 2020 found they could have used up to 16 per cent less fuel if they had made better use of the air currents, and could have also cut journey times by eight to nine minutes on average.

tracked using ground-based radar, which does not work over oceans. This means they must report their positions to air traffic control every 10 to 15 minutes while out of range. But networks of low-earth orbit satellites will allow them to be constantly tracked, meaning flight plans can be more dynamic.

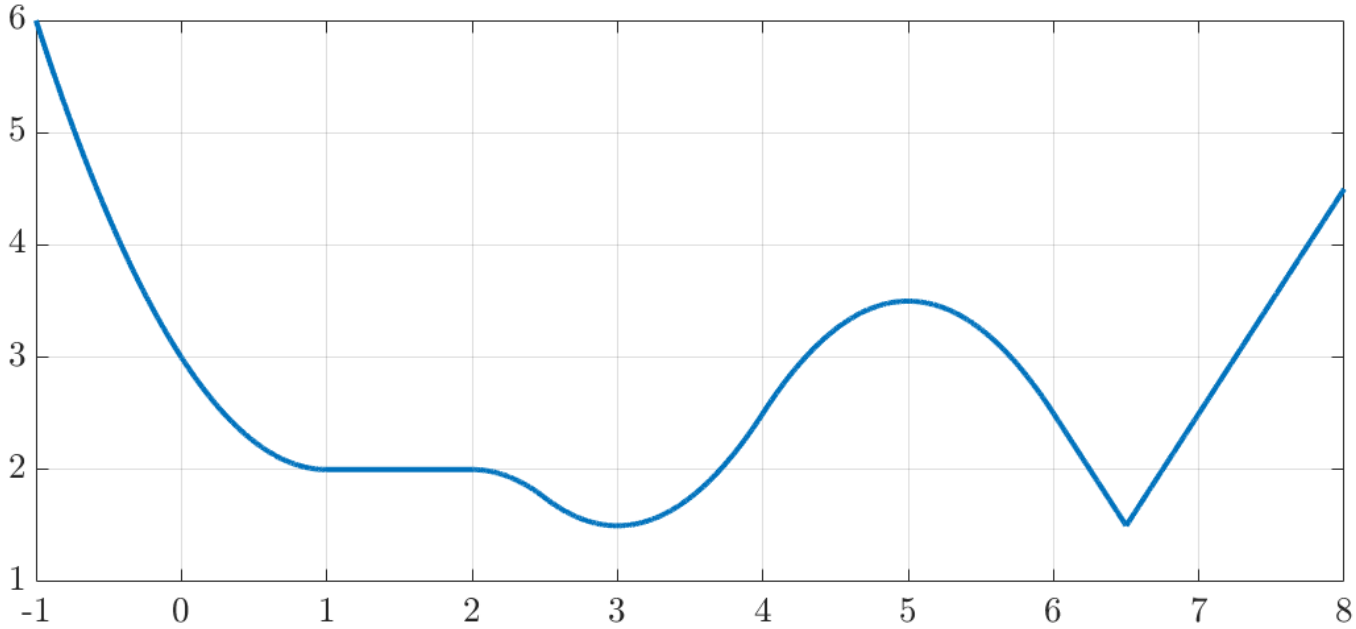
One such network, created by US firm Aireon, began flight-tracking trials in 2019, with satellites bussed to space by Elon Musk's SpaceX rocket Falcon 9.

Such technology has been in development for more than a decade, but the desire to constantly track planes was boosted by the high-profile disappearance of passenger plane MH370 over the Indian Ocean in March 2014.

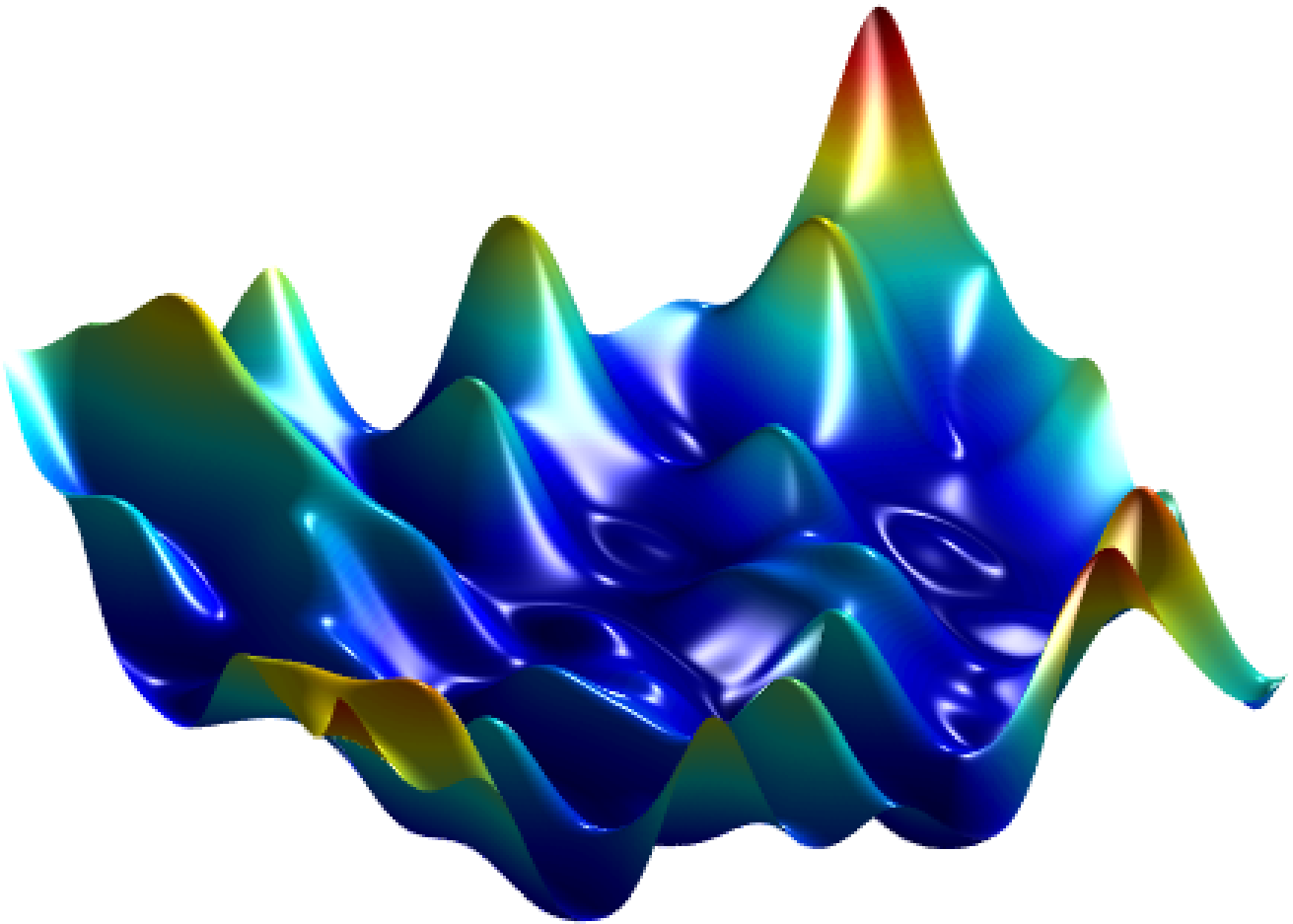
Previous studies have suggested pri-



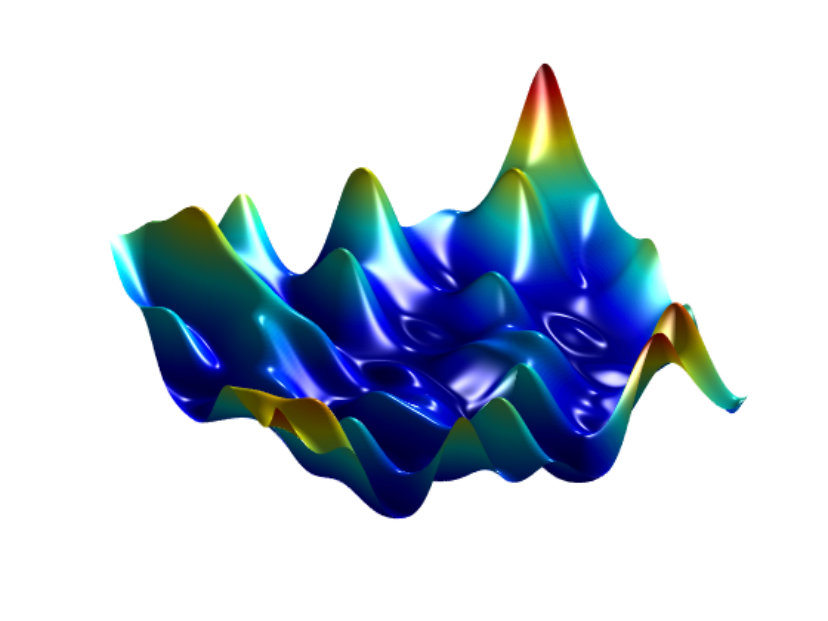
Looking for Optima: What do you See?

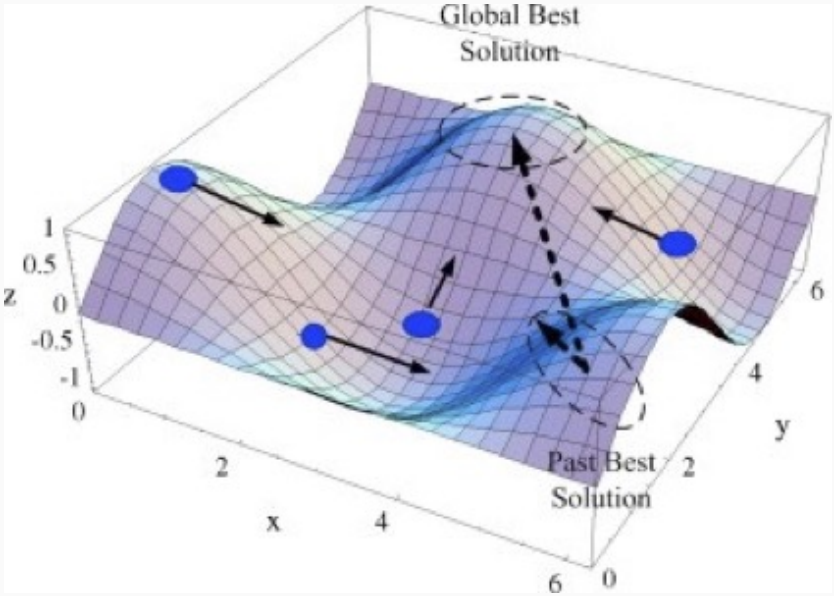


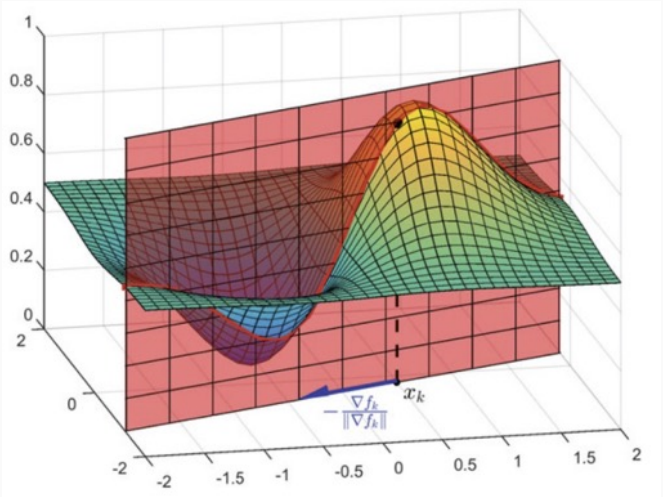
Looking for Optima in Several Variables



Looking for Optima in Several Variables



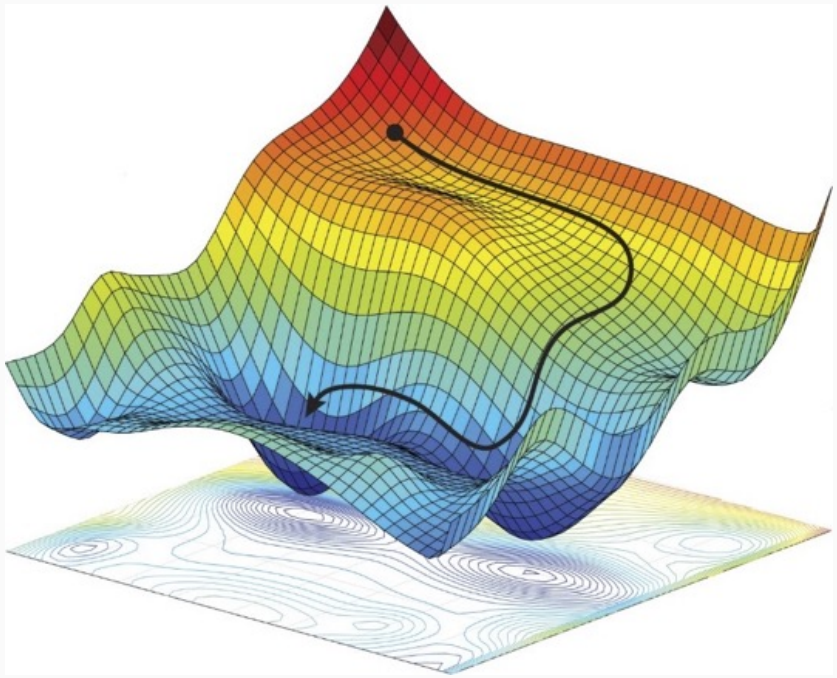




Looking for Optima in Several Variables

```
%====================================
% x ......... optimsl solution (up to a tolerance)
%             of min f(x)
% fun_val ... optimal function value
x=x0;
grad=g(x);
iter=0;
hist=x0;
error=100;
xold=x0;
while (error>epsilon)
    iter=iter+1;
    xnew=proj(xold-t*grad);
    error=norm(xold-xnew);
    xold=xnew;
    fun_val=f(xold);
    grad=g(xold);
    hist=[hist xold];
    fprintf('iter_number = %3d error = %2.6f fun_val = %2.6f \n',iter,error,fun_val);
end
end
```

```
iter_number =   1 error = 17.320508 fun_val = 0.000000
iter_number =   2 error = 1.145644 fun_val = -7.000000
iter_number =   3 error = 1.038328 fun_val = -9.906250
iter_number =   4 error = 0.455007 fun_val = -10.453125
iter_number =   5 error = 0.133500 fun_val = -10.540527
iter_number =   6 error = 0.111858 fun_val = -10.569580
iter_number =   7 error = 0.016688 fun_val = -10.570946
iter_number =   8 error = 0.013982 fun_val = -10.571400
iter_number =   9 error = 0.002086 fun_val = -10.571421
iter_number =  10 error = 0.001748 fun_val = -10.571428
iter_number =  11 error = 0.000261 fun_val = -10.571428
iter_number =  12 error = 0.000218 fun_val = -10.571429
iter_number =  13 error = 0.000033 fun_val = -10.571429
```



Final Remarks

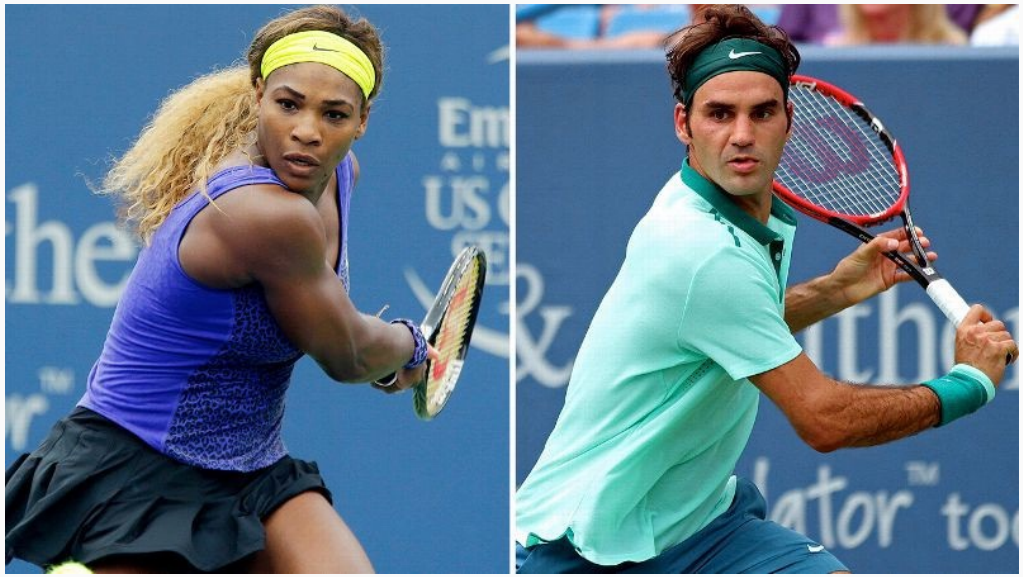
- Optimization is everywhere.
- It's a challenging but beautiful subject.
- Throughout this term you'll see many other relevant examples.
- Our inbox is always open to you, or just come for a chat after the lectures!

A single warning:

Final Remarks

- Optimization is everywhere.
- It's a challenging but beautiful subject.
- Throughout this term you'll see many other relevant examples.
- Our inbox is always open to you, or just come for a chat after the lectures!

A single warning:



Final Remarks

- Optimization is everywhere.
- It's a challenging but beautiful subject.
- Throughout this term you'll see many other relevant examples.
- Our inbox is always open to you, or just come for a chat after the lectures!

A single warning:

- You have to do your share, but you'll reap the benefits.

WELCOME TO OPTIMIZATION!

MATH60005/70005 - Optimisation

Welcome Lecture, Autumn Term 24/25

Dr Dante Kalise (dkaliseb@ic.ac.uk)
Dr Estefanía Loayza-Romero (kloayzar@ic.ac.uk)

Department of Mathematics, Imperial College London

IMPERIAL