Imperial College London

MATH97010 MATH70003

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
May-June 2022

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Introduction to Geophysical Fluid Dynamics

Date: 06 June 2022

Time: 09:00 – 11:30 (BST)

Time Allowed: 2:30 hours

Upload Time Allowed: 30 minutes

This paper has 5 Questions.

Candidates should start their solutions to each question on a new sheet of paper.

Each sheet of paper should have your CID, Question Number and Page Number on the top.

Only use 1 side of the paper.

Allow margins for marking.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Each question carries equal weight.

SUBMIT YOUR ANSWERS AS ONE PDF TO THE RELEVANT DROPBOX ON BLACKBOARD WITH COMPLETED COVERSHEETS WITH YOUR CID NUMBER, QUESTION NUMBERS ANSWERED AND PAGE NUMBERS PER QUESTION.

1. Shallow-water equations and Kelvin waves

Consider the linearized shallow-water model describing a single layer of inviscid fluid on the f-plane, with the Coriolis parameter f_0. The fluid has constant density ρ_0, the rest depth is H, and the constant gravity acceleration is g. Use conventional notations for velocity (u, v, w), deviation of the free surface η, and pressure p.

(i) (3 marks)

Write down the horizontal momentum equations, continuity equation, and vertically integrated hydrostatic balance, which relates p and η.

(ii) (7 marks)

Derive the prognostic equations for u and v, and show that η satisfies the equation:

$$\nabla^2\eta - \frac{1}{c_0^2}\frac{\partial^2\eta}{\partial t^2} - \frac{f_0^2}{c_0^2}\,\eta = 0\,,$$

where $c_0^2 = gH$.

(iii) (2 marks)

Consider the domain corresponding to the upper half-plane with a solid boundary at $y = 0$. Write down the corresponding boundary conditions at $y = 0$.

(iv) (8 marks)

Near the boundary obtain the Kelvin wave solution and its dispersion relation.

(Total: 20 marks)

2. Two-layer QG phase speed and stability

Consider the 2-layer QG model on the β-plane and with the equal layer depths (in the usual notation):

$$\frac{D_n \Pi_n}{Dt} = 0\,, \qquad n = 1, 2\,,$$

$$\Pi_1 = \nabla^2 \psi_1 - S\,(\psi_1 - \psi_2) + \beta y + f_0\,, \qquad \Pi_2 = \nabla^2 \psi_2 - S\,(\psi_2 - \psi_1) + \beta y + f_0\,.$$

Here $\psi_n(x, y, t)$ is a layer-wise streamfunction; S, β and f_0 are constants.

(i) (6 marks)

Linearize the model about a horizontally uniform, zonal background flow (U_1, U_2).

(ii) (10 marks)

Look for wave solutions in the form,

$$\psi_1 = A_1 \exp[i(k(x - ct) + ly)]\,, \qquad \psi_2 = A_2 \exp[i(k(x - ct) + ly)]\,,$$

and find the phase speed c, as function of the isotropic wavenumber K, such that $K^2 = k^2 + l^2$.

(iii) (4 marks)

Prove that the wave solutions always decay for sufficiently small and large K.

(Total: 20 marks)

3. 2D homogeneous turbulence

Consider the forward enstrophy cascade in 2D homogeneous and stationary turbulence in a doubly periodic domain. Assume enstrophy input rate η at large scales, inertial range with inviscid dynamics at intermediate scales, and viscous dissipation at small scales.

(i) (4 marks)

Prove that if relative vorticity $\zeta = \partial v/\partial x - \partial u/\partial y$ is materially conserved, then local changes of the enstrophy ζ^2 are driven by diverging enstrophy flux.

(ii) (4 marks)

Prove that total enstrophy in the domain is conserved over the inertial range.

(iii) (4 marks)

By using physical dimensionalities of the energy spectral density $E(k)$ and isotropic wavenumber k in the inertial range, find the corresponding scalings for advective vorticity $\zeta(k)$ and time $\tau(k)$.

(iv) (4 marks)

By using scalings derived in (iii), find the power law dependencies of $E(k)$ on η and k.

(v) (4 marks)

Explain by words and a sketch, what is the physical mechanism corresponding to the forward enstrophy cascade?

(Total: 20 marks)

4. Stokes drift

Consider 2D linear gravity wave characterized by the horizontal coordinate x and vertical coordinate z, and propagating in the constant-density layer of the rest depth H. Assume that the flow solution is given in terms of the velocity potential $\phi(t,x,z)$ that satisfies the kinematic boundary condition on the free surface:

$$\eta(x,t) = h_0 \cos(kx - \omega t)\,,$$

and the no-flow-through boundary condition on the bottom:

$$w|_{z=-H} = 0\,.$$

(i) (8 marks)

Assuming that the flow is non-divergent find the velocity potential $\phi(t,x,z)$ satisfying both boundary conditions.

(ii) (2 marks)

From the velocity potential $\phi(t,x,z)$ find the corresponding Eulerian velocity field: $\mathbf{u}(t,x,z) = (u,w)$.

For the rest of the problem assume that the fluid layer is infinitely deep: $H \to \infty$.

(iii) (4 marks)

In the vicinity of a point (x,z) solve for a Lagrangian trajectory emanating from this point.

(iv) (6 marks)

By Taylor expansion of the velocity field around (x,z) find the horizontal velocity component of a Lagrangian particle moving along the Lagrangian trajectory found in (iii). By time averaging over one time period prove that the difference u_s between the average horizontal Eulerian and Lagrangian velocity components scales as A^2.

(Total: 20 marks)

5. Dynamics in the limit of small Rossby number

(i) (6 marks)

Define the Rossby number ϵ, and assume that it is small. Consider the shallow-water model on the midlatitude β-plane. Expand in powers of ϵ the horizontal momentum equation to obtain its geostrophic and ageostrophic components. Find the leading-order balance and explain its physical meaning. What is the proper scaling for pressure in this balance?

(ii) (7 marks)

Write down the continuity equation and expand it in powers of ϵ. Assume that to leading order the density anomaly scales as ϵ^2. Find the leading-order and of the order of ϵ continuity equations.

(iii) (7 marks)

From the order of ϵ horizontal momentum equations, obtain the corresponding geostrophic vorticity equation. What determines evolution of the absolute vorticity?

(Total: 20 marks)

BSc and MSci EXAMINATIONS (MATHEMATICS)

May 2022

This paper is also taken for the relevant examination for the Associateship.

M3A28, M4A28, M5A28

Introduction to Geophysical Fluid Mechanics (Solutions)

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1. (i) $\boxed{\text{seen} \Downarrow}$

$$\frac{\partial u}{\partial t} - f_0 v = -g\,\frac{\partial \eta}{\partial x}\,, \qquad \frac{\partial v}{\partial t} + f_0 u = -g\,\frac{\partial \eta}{\partial y}\,, \qquad p = -\rho_0 g\,(z-\eta)\,, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$\boxed{3,\ \text{A}}$

(ii) Integrate continuity equation using the boundary condition, $w(z{=}H{+}\eta) = \partial\eta/\partial t$: $\boxed{\text{unseen} \Downarrow}$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0\,. \qquad (*)$$

Take u-momentum equation, differentiate it with respect to time, and add it to v-momentum equation multiplied by f_0; and, similarly, obtain the other equation:

$$\frac{\partial^2 u}{\partial t^2} + f_0^2 u = -g\left(\frac{\partial^2 \eta}{\partial x \partial t} + f_0\,\frac{\partial \eta}{\partial y}\right), \qquad \frac{\partial^2 v}{\partial t^2} + f_0^2 v = -g\left(\frac{\partial^2 \eta}{\partial y \partial t} - f_0\,\frac{\partial \eta}{\partial x}\right).$$

Take curl of the momentum equations, and use the velocity divergence from $(*)$ to replace the Coriolis term:

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{f_0}{H}\,\frac{\partial \eta}{\partial t} = 0\,.$$

Take divergence of the momentum equations, and use the velocity divergence from $(*)$ to substitute into the tendency term:

$$\frac{1}{H}\,\frac{\partial^2 \eta}{\partial t^2} + f_0\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - g\nabla^2\eta = 0\,.$$

From the above two equations obtain:

$$\frac{\partial}{\partial t}\left[\nabla^2\eta - \frac{1}{c_0^2}\,\frac{\partial^2 \eta}{\partial t^2} - \frac{f_0^2}{c_0^2}\,\eta\right] = 0\,, \qquad c_0^2 \equiv gH \qquad \Longrightarrow \qquad \nabla^2\eta - \frac{1}{c_0^2}\,\frac{\partial^2 \eta}{\partial t^2} - \frac{f_0^2}{c_0^2}\,\eta = 0 \qquad (**)$$

$\boxed{7,\ \text{A}}$

(iii) On the boundary: $v = 0$; therefore, the last equation yields boundary condition for η : $\boxed{\text{unseen} \Downarrow}$

$$\frac{\partial^2 \eta}{\partial y \partial t} - f_0\,\frac{\partial \eta}{\partial x} = 0\,. \qquad (***)$$

$\boxed{2,\ \text{A}}$

(iv) Let's look for the wave solution, $\boxed{\text{unseen} \Downarrow}$

$$\eta = \tilde{\eta}(y)\,e^{i(kx-\omega t)}\,,$$

of $(**)$ and $(***)$:

$$\frac{d^2\tilde{\eta}}{dy^2} + \left[\frac{\omega^2}{c_0^2} - \frac{f_0^2}{c_0^2} - k^2\right]\tilde{\eta} = 0\,, \qquad -\frac{\omega}{f_0}\,\frac{d\tilde{\eta}}{dy}(0) - k\,\tilde{\eta}(0) = 0\,.$$

The main equation can be written as:

$$\frac{d^2\tilde{\eta}}{dy^2} = \lambda^2\tilde{\eta}\,, \qquad \lambda^2 = -\frac{\omega^2}{c_0^2} + \frac{f_0^2}{c_0^2} + k^2\,.$$

Kelvin wave corresponds to solution decaying away from the boundary (real λ).
Let's consider

$$\lambda = l\,, \qquad \tilde{\eta} = Ae^{-ly}\,, \qquad boundary: \; l = \frac{f_0 k}{\omega} \qquad (****)$$

With $(****)$ the Kelvin wave dispersion relation becomes:

$$(\omega^2 - f_0^2)\left(1 - \frac{c_0^2}{\omega^2}\,k^2\right) = 0\,.$$

The Kelvin wave is given by the second root, with

$$\omega = c_0 k\,, \quad l = \frac{f_0}{c_0} \qquad \Longrightarrow \qquad \eta = Ae^{-yf_0/c_0}\,e^{i(kx - c_0 kt)}\,,$$

and this wave propagates to the right. $\boxed{8,\text{ D}}$

2. (i) The background flow potential-vorticity gradients are $\boxed{\text{unseen} \Downarrow}$

$$\frac{d\Pi_1}{dy} = \beta + S\,(U_1 - U_2)\,, \qquad \frac{d\Pi_2}{dy} = \beta + S\,(U_2 - U_1)\,,$$

and the linearized equations are

$$\begin{aligned}
\left(\frac{\partial}{\partial t} + U_1\,\frac{\partial}{\partial x}\right)\left(\nabla^2\psi_1 - S\,(\psi_1 - \psi_2)\right) + \frac{\partial\psi_1}{\partial x}\,\frac{d\Pi_1}{dy} &= 0\,,\\
\left(\frac{\partial}{\partial t} + U_2\,\frac{\partial}{\partial x}\right)\left(\nabla^2\psi_2 - S\,(\psi_2 - \psi_1)\right) + \frac{\partial\psi_2}{\partial x}\,\frac{d\Pi_2}{dy} &= 0\,.
\end{aligned}$$

$\boxed{6,\ \text{A}}$

(ii) After substituting the wave solutions, the linearized problem in the matrix form becomes: $\boxed{\text{unseen} \Downarrow}$

$$A_1\left[(c-U_1)\,(K^2+S)+\beta+S\,(U_1-U_2)\right] \qquad +A_2\left[-(c-U_1)\,S\right] \qquad = 0$$

$$A_1\left[-(c-U_2)\,S\right] \qquad +A_2\left[(c-U_2)\,(K^2+S)+\beta-S\,(U_1-U_2)\right] = 0\,.$$

For nontrivial solutions the matrix determinant must be zero. This leads to quadratic equation for the phase speed, and the corresponding roots are

$$c = \frac{U_1+U_2}{2} - \frac{\beta\,(K^2+S)}{K^2(K^2+2S)} \quad \pm \frac{\left[4\beta^2S^2 - K^4(U_1-U_2)^2\,(4S^2-K^4)\right]^{1/2}}{2K^2(K^2+2S)}\,.$$

$\boxed{10,\ \text{B}}$

$\boxed{\text{unseen} \Downarrow}$

(iii) Unstable solutions correspond to the negative values inside the square root, that is:

$$K^4\left(4S^2 - K^4\right) > \frac{4\beta^2S^2}{(U_1-U_2)^2}\,.$$

The lhs is a hyperparabola facing down and passing through $K=0$ and some positive K. The rhs sets some positive threshold, hence, solutions are unstable within the range of K between short-wave and long-wave cutoffs but otherwise decay. $\boxed{4,\ \text{D}}$

3. (i) Prove this by time differentiation and using the material conservation law: $\boxed{\text{seen} \Downarrow}$

$$\frac{\partial}{\partial t}\zeta^2 = 2\zeta\,\frac{\partial \zeta}{\partial t} = -2\zeta\,\mathbf{u}\cdot\nabla\zeta = -\mathbf{u}\cdot\nabla\zeta^2 = -\nabla\cdot(\mathbf{u}\,\zeta^2) + \zeta^2\nabla\cdot\mathbf{u}\,,$$

and note that the last term is zero, because 2D flow is nondivergent. $\boxed{4,\ \mathrm{A}}$

(ii) Integrate enstrophy over the domain area and use divergence theorem: $\boxed{\text{sim. seen} \Downarrow}$

$$\frac{\partial}{\partial t}\int_A \zeta^2\,dA = \int_A \frac{\partial}{\partial t}\zeta^2\,dA = -\int_A \nabla\cdot(\mathbf{u}\,\zeta^2)\,dA = -\int_S \mathbf{u}\,\zeta^2\,d\mathbf{S} = 0\,,$$

note that the final integral is zero, because of the doubly periodic conditions. $\boxed{4,\ \mathrm{A}}$

$\boxed{\text{sim. seen} \Downarrow}$

(iii) The dimensionality analysis yields

$$\zeta(k) = [k^3E(k)]^{1/2}\,, \qquad \tau(k) = [\zeta(k)]^{-1} = [k^3E(k)]^{-1/2}\,.$$

$\boxed{4,\ \mathrm{B}}$

$\boxed{\text{seen} \Downarrow}$

(iv) The dimensionality analysis yields

$$\eta \sim \frac{\zeta(k)^2}{\tau(k)} = k^{9/2}E(k)^{3/2}$$

$$E(k) \sim \eta^{2/3}k^{-3}\,.$$

$\boxed{4,\ \mathrm{D}}$

$\boxed{\text{seen} \Downarrow}$

(v) The forward enstrophy cascade occurs through stretching, filamentation, folding and chaotic stirring of materially conserved vorticity injected at large scales (drawing a blob of fluid being deformed and stirred by advection would be helpful). $\boxed{4,\ \mathrm{C}}$

4. (i) Let's look for solution in the form $\phi(t,x,z)=f(z)\sin(kx-\omega t)$ and notice that divergence of $\mathbf{u}=\nabla\phi$ is zero: $\boxed{\text{unseen} \Downarrow}$

$$\nabla^2\phi = 0$$
$$\frac{\partial^2 f}{\partial z^2} - k^2 f = 0$$
$$\phi(t,x,z) = (Ae^{kz} + Be^{-kz})\,\sin(kx-\omega t)\,.$$

Boundary conditions for

$$w = \frac{\partial\phi}{\partial z} = (Ak\,e^{kz} - Bk\,e^{-kz})\sin(kx-\omega t)$$

at the top and bottom yield:

$$w|_{z=-H} = 0 = Ak\,e^{-kH} - Bk\,e^{kH}$$
$$B = A\,e^{-2kH}$$
$$w|_{z=0} = \frac{\partial\eta}{\partial t} = \omega h_0 \sin(kx-\omega t) = (A+B)\,k\sin(kx-\omega t)$$
$$A+B = \frac{\omega h_0}{k}$$
$$A = \frac{\omega h_0}{k}\,\frac{1}{1+e^{-2kH}}\,, \qquad B = \frac{\omega h_0}{k}\,\frac{e^{-2kH}}{1+e^{-2kH}}\,.$$

$\boxed{8,\ \text{C}}$

(ii) $\boxed{\text{unseen} \Downarrow}$

$$u = \frac{\partial\phi}{\partial x} = k\,(Ae^{kz}+Be^{-kz})\,\cos(kx-\omega t)\,, \qquad w = \frac{\partial\phi}{\partial z} = k\,(Ae^{kz}-Be^{-kz})\,\sin(kx-\omega t)$$

$\boxed{2,\ \text{A}}$

(iii) Since $H\to\infty$: $B\to 0$, and $A\to\omega h_0/k$, therefore, coordinates of the Lagrangian particle evolve according to: $\boxed{\text{sim. seen} \Downarrow}$

$$\frac{\partial(\xi_x,\xi_z)}{\partial t} = \mathbf{u}(\xi_x,\xi_z,t)$$
$$\xi_x = x + \int u\,dt\,, \qquad \xi_z = z + \int w\,dt$$
$$\xi_x - x = -\frac{k}{\omega}\,A\,e^{kz}\,\sin(kx-\omega t)\,, \qquad \xi_z - z = \frac{k}{\omega}\,A\,e^{kz}\,\cos(kx-\omega t)$$

$\boxed{4,\ \text{A}}$

(iv) By retaining the first terms of Taylor expansion and by time averaging, we obtain the difference between the Lagrangian and Eulerian velocity components: $\boxed{\text{sim. seen} \Downarrow}$

$$u_S = \overline{u(\xi_x,\xi_z,t)} - \overline{u(\mathbf{x},t)} \approx \overline{(\xi_x-x)\,\frac{\partial u(\mathbf{x},t)}{\partial x} + (\xi_z-z)\,\frac{\partial u(\mathbf{x},t)}{\partial z}}$$
$$\sim \overline{[-A\,e^{kz}\sin(kx-\omega t)]\,[-A\,e^{kz}\sin(kx-\omega t)]} + \overline{[A\,e^{kz}\cos(kx-\omega t)]\,[A\,e^{kz}\cos(kx-\omega t)]}$$
$$= A^2e^{2kz}\overline{[\sin^2(kx-\omega t)+\cos^2(kx-\omega t)]} \sim A^2 \sim h_0^2$$

Hence, the Stokes drift u_S scales quadratically with amplitude of the wave. $\boxed{6,\ \text{B}}$

5. (i) Rossby number is the ratio of the material-derivative and Coriolis-forcing scalings: $\boxed{\text{seen} \Downarrow}$

$$\epsilon = \frac{U^2/L}{fU} = \frac{U}{fL}\,.$$

Expand the velocity, as well as the dynamical anomalies of pressure and density:

$$\mathbf{u} = \mathbf{u}_g + \epsilon\,\mathbf{u}_a\,, \qquad p' = p'_g + \epsilon\,p'_a\,, \qquad \rho' = \rho'_g + \epsilon\,\rho'_a\,.$$

For the β-plane and $mesoscales$:

$$T = \frac{L}{U} = \frac{L}{\epsilon f_0 L} = \frac{1}{\epsilon f_0}\,, \qquad L/R_0 \sim \epsilon \quad \Longrightarrow \quad [\beta y] \sim \frac{f_0}{R_0}\,L \sim \epsilon f_0\,.$$

In the Rossby-number-expanded horizontal momentum equations only the pressure gradient can balance the Coriolis force:

$$\begin{array}{llll}
\dfrac{Du_g}{Dt} - f_0\,(v_g + \epsilon v_a) - \beta y\,v_g + \epsilon^2[...] & = & -\dfrac{1}{\rho_0}\,\dfrac{\partial p_g}{\partial x} & -\dfrac{\epsilon}{\rho_0}\,\dfrac{\partial p_a}{\partial x} \\
\dfrac{Dv_g}{Dt} + f_0\,(u_g + \epsilon u_a) + \beta y\,u_g + \epsilon^2[...] & = & -\dfrac{1}{\rho_0}\,\dfrac{\partial p_g}{\partial y} & -\dfrac{\epsilon}{\rho_0}\,\dfrac{\partial p_a}{\partial y} \\
\epsilon f_0 U \qquad f_0 U \qquad \epsilon f_0 U \qquad \epsilon^2 f_0 U & & [p']/(\rho_0 L) & \epsilon\,[p']/(\rho_0 L)
\end{array}$$

The leading-order balance is geostrophic:

$$f_0 v_g = \frac{1}{\rho_0}\,\frac{\partial p_g}{\partial x}\,, \qquad f_0 u_g = -\frac{1}{\rho_0}\,\frac{\partial p_g}{\partial y}\,. \qquad\qquad (*)$$

The Coriolis force balances the pressure gradient, and the flow velocity is perpendicular to the pressure gradient. Proper scaling for pressure must be $[p_g] \sim \rho_0 f_0 U L$. $\boxed{6,\ \text{M}}$

(ii) Start with the continuity equation and ϵ-expand it, recalling that $[\rho_g] = \rho_0 \epsilon^2$: $\boxed{\text{seen} \Downarrow}$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0\,,$$

$$\rho = \rho_0 + \rho_g, \quad u = u_g + \epsilon\,u_a, \quad v = v_g + \epsilon\,v_a, \quad w = w_g + \epsilon\,w_a$$

$$\frac{\partial \rho_g}{\partial t} + (\rho_0 + \rho_g)\Big(\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y}\Big) + u_g\,\frac{\partial \rho_g}{\partial x} + v_g\,\frac{\partial \rho_g}{\partial y} + \epsilon\rho_0\,\Big(\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y}\Big) + \epsilon^2\,[...]$$
$$+\frac{\partial}{\partial z}\,(w_g\rho_0 + \epsilon w_a\rho_0 + w_g\rho_g + \epsilon w_a\rho_g) = 0\,.$$

Horizontal geostrophic velocity is nondivergent [from $(*)$], hence, $w_g = 0$. At the ϵ-order:

$$\frac{\partial(w_a\rho_0)}{\partial z} + \rho_0\,\Big(\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y}\Big) = 0\,.$$

$\boxed{7,\ \text{M}}$

(iii) Vorticity equation is obtained by going to the ϵ-order in the momentum equations: $\boxed{\text{seen} \Downarrow}$

$$\frac{D_g u_g}{Dt} - (\epsilon f_0 v_a + v_g \beta y) = -\epsilon\,\frac{1}{\rho_0}\,\frac{\partial p_a}{\partial x}\,, \qquad \frac{D_g v_g}{Dt} + (\epsilon f_0 u_a + u_g \beta y) = -\epsilon\,\frac{1}{\rho_0}\,\frac{\partial p_a}{\partial y}\,,$$

$$\frac{D_g}{Dt} \equiv \frac{\partial}{\partial t} + u_g\,\frac{\partial}{\partial x} + v_g\,\frac{\partial}{\partial y}\,.$$

By taking curl of these equations and using result from problem (2), obtain:

$$\frac{D_g \zeta_g}{Dt} + \beta v_g = \frac{D_g}{Dt}\,[\zeta_g + \beta y] = \epsilon\,f_0\,\frac{\partial w_a}{\partial z}\,, \qquad\qquad \zeta_g \equiv \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}$$

$\boxed{7,\ \text{M}}$

Review of mark distribution:

Total A marks: 32 of 32 marks
Total B marks: 20 of 20 marks
Total C marks: 12 of 12 marks
Total D marks: 16 of 16 marks
Total marks: 100 of 80 marks
Total Mastery marks: 20 of 20 marks

If your module is taught across multiple year levels, you might have received this form for each level of the module. You are only required to fill this out once for Please record below, some brief but non-trivial comments for students about how well (or otherwise) the questions were answered. For example, you may wish to comment on common errors and misconceptions, or areas where students have done well. These comments should note any errors in and corrections to the paper. These comments will be made available to students via the MathsCentral Blackboard site and should not contain any information which identifies individual candidates. Any comments which should be kept confidential should be included as confidential comments for the Exam Board and Externals. If you would like to add formulas, please include a separate pdf file with your email.

ExamModuleCode	QuestionNumber	Comments for Students
Introduction to Geophysical Fluid Dynamics_MATH60003_MATH97010_MATH70003	1	The exam was properly balanced, and the students did rather well
Introduction to Geophysical Fluid Dynamics_MATH60003_MATH97010_MATH70003	2	The exam was properly balanced, and the students did rather well
Introduction to Geophysical Fluid Dynamics_MATH60003_MATH97010_MATH70003	3	The exam was properly balanced, and the students did rather well
Introduction to Geophysical Fluid Dynamics_MATH60003_MATH97010_MATH70003	4	The exam was properly balanced, and the students did rather well