MVC Quiz 1 (2023) Answers

1. $\underline{A} = yz^3\hat{\imath} + x^2y^2\hat{\jmath} + yz\hat{k}$

$$\Rightarrow \text{Curl}\,\underline{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz^3 & x^2y^2 & yz \end{vmatrix}$$

$$= \hat{\imath}(z) - \hat{\jmath}(-3yz^2) + \hat{k}(2xy^2 - z^3)$$

$$\hat{\underline{n}} = \pm\underline{\nabla}(x^2+y^2+z^2)/|\underline{\nabla}(\quad)|$$

$$= \pm(2x\hat{\imath} + 2y\hat{\jmath} + 2z\hat{k})/\sqrt{(4x^2+4y^2+4z^2)}$$

$$= (x\hat{\imath} + y\hat{\jmath} + z\hat{k})/5 \quad \text{since } \hat{n}\cdot\hat{k} > 0 \text{ (given)}$$

$\& \; z > 0$ on S

$$\therefore (\text{Curl}\,\underline{A})\cdot\hat{\underline{n}} = (zx + 3y^2z^2 + 2xy^2z - z^4)/5 \quad \boxed{C}$$

2. S can be parametrized by writing

$$x = 5\sin\theta\cos\varphi, \; y = 5\sin\theta\sin\varphi, \; z = 5\cos\theta$$

with $0 \leqslant \varphi \leqslant 2\pi$ & $\frac{3}{5} \leqslant \cos\theta \leqslant 1$

Then, setting z to zero we have $x^2+y^2 = 25\sin^2\theta$

with $\frac{4}{5} \geqslant \sin\theta \geqslant 0$

& hence $x^2+y^2 \leqslant 16$ $\boxed{D}$

3. $(\text{Curl}\,\underline{A})\cdot\hat{\underline{n}}/|\hat{n}\cdot\hat{k}| = x + 3y^2z + 2xy^2 - z^3$

$$\therefore I = \int_{\Sigma_z} x + 3y^2\sqrt{(25-x^2-y^2)} + 2xy^2 - (25-x^2-y^2)^{3/2}\, d\Sigma_z$$

$$\left(\text{Substituting } z = (25-x^2-y^2)^{1/2}\right)$$

$\boxed{C}$

4. $d\Sigma_z = r\,dr\,d\theta \quad 0 \leqslant r \leqslant 4 \quad 0 \leqslant \theta \leqslant 2\pi$

$$I = \int_0^{2\pi}\int_0^4 r^2\cos\theta + 3(25-r^2)^{1/2} r^3 \sin^2\theta + 2r\cos\theta\, r^3 \sin^2\theta - r(25-r^2)^{3/2}\,dr\,d\theta$$

$\boxed{D}$

5. 1st & 3rd terms integrate to zero

Then $I = 3\pi\int_0^4 (25-r^2)^{1/2} r^3\,dr - 2\pi\int_0^4 (25-r^2)^{3/2} r\,dr$

first integral can be solved by substitution $r = 5\sin v$

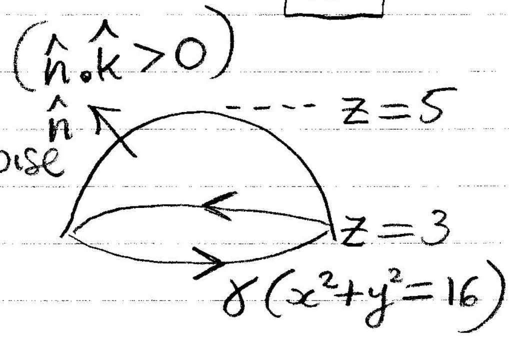
second integral use $u = r^2$

After some algebra:

$I = 3604\pi/5 - 5764\pi/5$

$= -432\pi$ $\boxed{D}$

6. Using right hand rule γ is traversed anti-clockwise $\boxed{A}$



7. γ has $z=3$ & hence $x^2+y^2=16$ $\boxed{A}$

8. $\underline{A}\cdot d\underline{r} = yz^3\,dx + x^2y^2\,dy + yz\,dz$ $\boxed{D}$

On γ we have $dz=0$ & $z=3$

9. $\underline{A}\cdot d\underline{r} = 27y\,dx + x^2y^2\,dy$ on γ

$= [-(27)(4\sin\theta)^2 + (4\cos\theta)^2(4\sin\theta)^2(4\cos\theta)]\,d\theta$

$\therefore I = \int_0^{2\pi} 1024\cos^3\theta\sin^2\theta - 432\sin^2\theta\,d\theta$ $\boxed{B}$

10. $\rightarrow$

10. Now $\cos^3\theta \sin^2\theta$

$$\equiv \frac{1}{4}\cos\theta\left(\sin^2 2\theta\right) \equiv \frac{1}{8}\left(\cos\theta - \cos 4\theta \cos\theta\right)$$

$$\equiv \frac{1}{8}\cos\theta - \frac{1}{16}\cos 5\theta - \frac{1}{16}\cos 3\theta$$

$$\therefore \int_0^{2\pi} \cos^3\theta \sin^2\theta \, d\theta = 0$$

This leaves $I = -432\int_0^{2\pi} \sin^2\theta \, d\theta = -432\pi$

$\boxed{C}$