Imperial College London
Department of Mathematics

Question Sheet 0

MATH40003 Linear Algebra and Groups — Term 2, 2022/23

The first problem class will be on Monday of week 2. The following questions are about Lecture 1 and the last few topics from Term 1. The questions are to revise Term 1 material (including the basis change formula). There are few more question on Sheet 1 that can be solved after Lecture 1. Those will be solved in week 3 together with the rest of Sheet 1.

Question 1 Compute the determinant of the identity matrix I_n ($n \in \mathbb{N}$).

Question 2 Prove that the determinant is linear on the rows. This is Theorem 5.1.5 from the notes.

Question 3 If $A \in M_n(F)$ and $1 \leq i, j \leq n$, write down a formula for the (ℓ, m)-entry of A_{ij} (for $1 \leq \ell, m \leq n-1$).

Question 4 Prove that a lower triangular matrix has determinant equal to the product of the elements on the diagonal. (A matrix is said to be lower-triangular if all its entries above the diagonal are 0.)

Question 5 Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the map defined by

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ x_1 \end{pmatrix}.$$

(i) Prove T is a linear transformation.

Let

$$B = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}, \quad C = \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}.$$

Let also E be the standard basis of $\mathbb{R}^3$.

(ii) Calculate ${}_E[T]_B$ and ${}_E[T]_C$.

(iii) Does it make sense to write ${}_B[T]_E$?

(iv) If you have not used it to solve (ii), write the change of basis matrix from C to B.