Network Science
Spring 2024
Problem Class 3 Exercises

---

1. In Lecture 5, it is stated that:

$$\langle k_i \rangle = \sum_{G \in \Omega_N} P(G) k_i(G),$$

and

$$\langle k_i \rangle = \sum_{k=0}^{N-1} P(k_i = k) k.$$

Show that these two expressions are equivalent.

2. Show that if we let $p(N) = N^{-z}$ with $z > 3/2$ then $G \in G_{N,p}$ w.h.p. has no two edges with a common vertex (or equivalently the degree at each node is at most one). In an exercise from the last problem sheet, the assumption was that $z > 2$. A different type of argument will be needed here. Hint: consider the random variable Y_{ijk} which assigns to a graph $G \in G_{Np}$ the value 1 if between the three nodes i, j, k there are two or more edges and 0 if there is at most one such edge.