Unseen

C.1. Prove that if V is a vector space over field F with bases A and B then $|A| = |B|$.

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C.2. Let V be the set of all sequences which only contain rational numbers. Let $(a_n) + (b_n) = (a_n + b_n)$ and let $q(a_n) = (qa_n)$. Let 0_V be the constant zero sequence.

(a) Show that V is a vector space over $\mathbb{Q}$, and find it's dimension.

(b) Show that the set of all convergent sequences is a subspace of V, and find its dimension.

(c) Show that the set of all sequences with a convergent subsequence is not a subspace of V.

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C.3. Consider $\mathbb{R}$ as a $\mathbb{Q}$-vector space. Let $p_1, \ldots, p_n$ be distinct prime numbers, and let a be any positive real number. Show $\{\log_a(p_1), \ldots, \log_a(p_n)\}$ is linearly independent.

You may want to use the Fundamental Theorem of Arithmetic:
Every positive integer $n > 1$ can be represented in exactly one way as a product of prime powers: $n = p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k} = \prod_{i=1}^{k} p_i^{n_i}$
where $p_1 < p_2 < \cdots < p_k$ are primes and the n_i are positive integers.

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