MATH60005/70005: Optimization (Autumn 22-23)

Week 9: Exercises

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1. Solve the problem

$$\begin{aligned} \min \quad & x_1^2 + 2x_2^2 + 4x_1x_2 \\ \text{s.t.} \quad & \mathbf{x} \in \Delta_2 \,. \end{aligned}$$

2. **Orthogonal regression**. Suppose we have $\mathbf{a}_1, \ldots, \mathbf{a}_m \in \mathbb{R}^n$. For a given $\mathbf{0} \neq \mathbf{x} \in \mathbb{R}^n$ and $y \in \mathbb{R}$, we define the hyperplane:

$$H_{\mathbf{x},y} := \left\{ \mathbf{a} \in \mathbb{R}^n : \mathbf{x}^\top \mathbf{a} = y \right\}$$

In the orthogonal regression problem, we seek to find a nonzero vector $\mathbf{x} \in \mathbb{R}^n$ and $y \in \mathbb{R}$ such that the sum of squared Euclidean distances between the points $\mathbf{a}_1, \ldots, \mathbf{a}_m$ to $H_{\mathbf{x},y}$ is minimal:

$$\min_{\mathbf{x},y} \left\{ \sum_{i=1}^{m} d\left(\mathbf{a}_i, H_{\mathbf{x},y}\right)^2 : \mathbf{0} \neq \mathbf{x} \in \mathbb{R}^n, y \in \mathbb{R} \right\}$$

Solve this problem using KKT conditions.

3. Consider the problem

$$\begin{aligned} \min \quad & x_1^2 - x_2 \\ \text{s.t.} \quad & x_2 = 0 \,, \end{aligned}$$

and its equivalent formulation

$$\begin{aligned} \min \quad & x_1^2 - x_2 \\ \text{s.t.} \quad & x_2^2 \leq 0 \,. \end{aligned}$$

Determine KKT conditions for both problems, are they equivalent and solvable?