

# MATH40005 Probability and Statistics

Midterm [20 points]

24 November 2022

Please remember to define your notation and justify all your answers.

**Question 1:** Consider the experiment, where we first flip a fair coin and then roll a fair die.

- (a) (2 points) Find the sample space for this experiment.
- (b) (2 points) What is the probability that one obtains heads followed by either a 2 or a 4?

**Question 2:** If the letters I, I, I, I, I, N, N, C, O, M, P, R, E, E, H, S, S, B, L, T are arranged at random, what is the probability that

- (a) (2 points) the arrangements spell the word INCOMPREHENSIBILITIES?
- (b) (2 points) the arrangements have three adjacent E's?

**Question 3:** (4 points) Consider a standard well-shuffled 52-card deck comprising of 13 ranks in the four suits clubs, diamonds, hearts and spades. Suppose that four players play a game, where, at the start, each player gets 13 cards. What is the probability that each player has one ace?

**Question 4:** (4 points) Consider a standard domino game. Each domino is a rectangular tile, usually with a line dividing its face into two square ends. Each of the two ends is marked with a number of  $n$  spots, where  $n \in \{0(\text{blank}), 1, 2, 3, 4, 5, 6\}$ . In a standard set of dominos, each possible combination of spots occurs on exactly one domino, e.g. there is exactly one domino that has an end with 5 spots and the other end with 3 spots, and exactly one domino that has 6 spots on each end. How many dominos are in a standard set of dominos?

**Question 5:** (4 points) Let  $\Omega$  be a non-empty set and let  $\mathcal{F}$  and  $\mathcal{G}$  denote  $\sigma$ -algebras on  $\Omega$ . Is  $\mathcal{H} := \mathcal{F} \cup \mathcal{G}$  a  $\sigma$ -algebra on  $\Omega$ ? If so, prove it, otherwise, provide a counterexample.