MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Quiz 6

This is an assessed quiz. The quiz will open on Friday, 1 March at 10am
and close on Saturday, 2 March at 10.30am.

Question 1 (Lyapunov exponents).

Is the following statement true? Consider two matrices $A, B \in \mathbb{R}^{d \times d}$. Then for any Lyapunov exponent σ_A of $\dot{x} = Ax$ and Lyapunov exponent σ_B of $\dot{x} = Bx$, the sum $\sigma_A + \sigma_B$ is a Lyapunov exponent of $\dot{x} = (A + B)x$.

  (a) The statement is true.

  (b) The statement is false.

Question 2 (Stability of equilibria of one-dimensional differential equations).

Consider the one-dimensional differential equation $\dot{x} = -x + x^2 + x^3 + x^4$. Which of the following two statements is true?

  (a) The equilibrium $x^* = 0$ is stable.

  (b) The equilibrium $x^* = 0$ is unstable.

Question 3 (Attractive equilibria).

Consider the differential equation $\dot{x} = f(x)$, where $f : \mathbb{R} \to \mathbb{R}$ is locally Lipschitz continuous with an equilibrium $x^* \in \mathbb{R}$. Is the following statement true? The equilibrium x^* of $\dot{x} = f(x)$ is attractive if and only if the equilibrium $-x^*$ of $\dot{x} = -f(-x)$ is attractive.

  (a) The statement is true.

  (b) The statement is false.

Question 4 (Attractive and repulsive equilibria).

Consider a continuously differentiable function $f : \mathbb{R} \to \mathbb{R}$ and a differential equation $\dot{x} = f(x)$ with one equilibrium $x^* \in \mathbb{R}$. Is the following statement true? If x^* is attractive, then x^* is not repulsive.

  (a) The statement is true.

  (b) The statement is false.

Question 5 (Heteroclinic orbits).

Consider a continuously differentiable function $f : \mathbb{R}^2 \to \mathbb{R}^2$ and a differential equation $\dot{x} = f(x)$ with two equilibria $x_1^*, x_2^* \in \mathbb{R}^d$. Suppose that there exists a heteroclinic orbit $O(x)$ connecting these two equilibria. Which of the following three statements is true?

  (a) At least one of the equilibria x_1^* and x_2^* is unstable.

  (b) Both equilibria x_1^* and x_2^* are stable.

  (c) It depends on the right hand side f whether (a) or (b) holds.