MATH60005/70005: Optimisation (Autumn 24-25)

Chapter 3: exercises

Dr Dante Kalise & Dr Estefanía Loayza-Romero
Department of Mathematics
Imperial College London, United Kingdom
{dkaliseb,kloayzar}@imperial.ac.uk

Polynomial fit and denoising

We will replicate the linear regression and regularized linear least squares examples from this week, with a different model. First, we will generate a noisy dataset of 200 samples coming from

$$v_i = u_i^2 + \mathcal{N}(0, 0.04)\,, \quad i = 1, \ldots, 200\,,$$

where the u_i's are uniformly sampled in $[-1, 1]$, and $\mathcal{N}(0, 0.04)$ means adding Gaussian noise of mean 0 and variance 0.04 for **each** sample.

1. Generate the pairs (u_i, v_i) using suitable random generators. Make a plot illustrating (u_i, v_i). What is the model you can identify to express v as a function of u?
2. Write a linear regression problem for finding the optimal parameters in a model

$$v(u) = au^2 + bu + c$$

3. Compute the least squares solution and compute the total least squares error in the ℓ_2 norm.
4. Now, instead of solving a regression problem, use the v_i values to recover a denoised signal using regularized least squares using the same total variation regularization described in the lecture notes (denoising part).