IMPERIAL

MATH70002

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
Summer 2025

This paper is also taken for the relevant examination for the Associateship of the Royal College of Science

Fluid Dynamics 2

Date: Monday, May 19, 2025

Time: Start time 14:00 – End time 16:30 (BST)

Time Allowed: 2.5 hours

This paper has 5 Questions.

Please Answer All Questions in 1 Answer Booklet

This is a closed book examination.

Candidates should start their solutions to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Allow margins for marking.

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO DO SO

1. (a) In a Hele-Shaw cell, fluid with viscosity μ and velocity $(u,\, v,\, w)$ fills a thin layer between two stationary solid plates at $z = 0$ and $z = h$.
State the two main assumptions of lubrication theory leading to the equations (2 marks)

$$p_x = \mu u_{zz}, \qquad p_y = \mu v_{zz}, \qquad p_z = 0, \qquad u_x + v_y + w_z = 0,$$

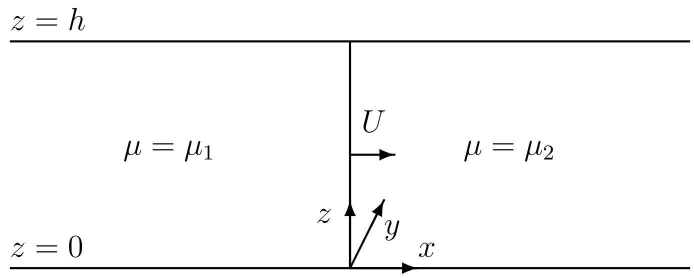
which you need not derive. Deduce that

$$(\overline{u},\, \overline{v},\, 0) = -\frac{h^2}{12\mu}\nabla p, \qquad \nabla^2 p = 0, \tag{1}$$

where $p(x,\, y)$ is the pressure field and

$$\overline{u} = \frac{1}{h}\int_0^h u\, dz, \qquad \overline{v} = \frac{1}{h}\int_0^h v\, dz.$$ (5 marks)

(b) The region $x < Ut$ in a Hele-Shaw cell contains fluid 1 of viscosity μ_1 while $x > Ut$ contains fluid 2 of viscosity μ_2, as shown in the sketch. All the fluid in $0 < z < h$ may be assumed to move with the velocity $(\overline{u},\, \overline{v},\, 0)$, as given by relation (1) with $\mu = \mu_1$ or $\mu = \mu_2$ as appropriate. In equilibrium, both fluids move with uniform velocity $(\overline{u},\, \overline{v},\, w) = (U,\, 0,\, 0)$.



The interface $x = Ut$ is perturbed to $x = Ut + \varepsilon e^{iky+st}$ where $0 < \varepsilon \ll 1$, k is real and s is the growth rate. Assuming that the pressure in fluid j for $j = 1, 2$ is perturbed as

$$-\frac{h^2}{12\mu_j}\, p = U\xi + \varepsilon P_j(\xi) e^{iky+st}, \qquad \text{where } \xi = x - Ut,$$

derive the kinematic condition at the interface within each fluid. (3 marks)

Assuming that the velocity perturbation vanishes as $x \to \pm\infty$, and that the pressure is continuous at the perturbed interface, show that

$$P_1 = \frac{s}{k}\, e^{k\xi}$$

and that

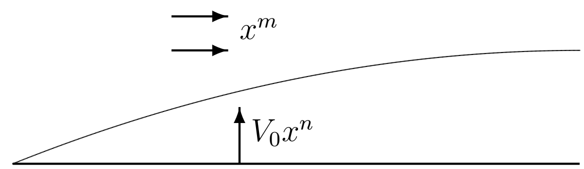
$$s = kU\, \frac{\mu_2 - \mu_1}{\mu_1 + \mu_2}.$$ (8 marks)

Is the moving interface stable? (2 marks)

[Note: surface tension may be neglected and the Bernoulli theorem does not apply.]

(Total: 20 marks)

2. (a) Fluid passes through a stationary flat plate at $y = 0$, $x > 0$ with velocity $(u, v) = (0, V_0 x^n)$. Simultaneously, a free stream passes over the plate with a slip velocity $U = x^m$, as in the figure, where V_0, m and n are given constants.



Seek a similarity solution to the non-dimensional boundary layer equation

$$uu_x + vu_y = UU' + u_{yy}$$

with a streamfunction of the form

$$\psi = x^a f(\eta) \qquad \text{where} \quad \eta = yx^b$$

for suitable constants a and b. Show that such a similarity solution is only possible if m and n obey a certain relation. (4 marks)

Show further that $f(\eta)$ must satisfy the equation

$$f''' + \tfrac{1}{2}(m+1)ff'' + m(1 - f'^2) = 0. \qquad \text{(5 marks)}$$

Give the appropriate boundary conditions at $\eta = 0$ and as $\eta \to \infty$. (3 marks)

(b) If $m = -\frac{1}{3}$, integrate the ODE for $f(\eta)$ twice. (2 marks)

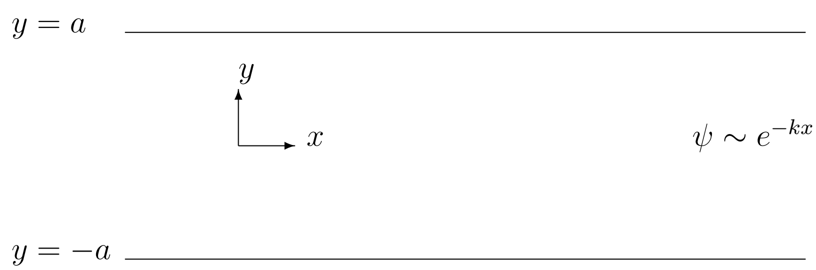
By considering conditions at $\eta = 0$ and $\eta \to \infty$, show that if the solution behaves as $\eta \to \infty$ like $f \sim \eta +$ Constant + Exponentially Small Terms, then $V_0^2 > \frac{2}{3}$. (4 marks)

Do you expect $V_0 > 0$ or $V_0 < 0$ to be more likely to give a viable solution? (2 marks)

(Total: 20 marks)

3. (a) Briefly summarise the distinguishing features of flow at low Reynolds number. (5 marks)

(b) Viscous fluid occupies the space between two stationary plates at $y = \pm a$. Stokes flow is driven by some localized activity near $x = 0$. There is no variation in the z-direction. Consider the resultant motion at large values of $x > 0$ by seeking a solution for which the streamfunction $\psi = f(y)e^{-kx}$ where $k > 0$ and $f(y)$ is symmetric (even) about $y = 0$.



Find the general form of $f(y)$ and show that for an even solution satisfying the solid boundary conditions, it is necessary that

$$\sin 2ka + 2ka = 0.$$ (6 marks)

Show that the only real solution to this equation is $2ka = 0$. (2 marks)

(c) You are given that there is a complex solution $ka = c + id$, with $c > 0$, $d > 0$. For this value of k, consider the behaviour of the solution

$$\psi = \Re e \left[\left(\cos ky - y \sin ky \, \frac{\cos ka}{a \sin ka} \right) e^{-kx} \right]$$

where $\Re e$ denotes the real part. Show that ψ has infinitely many local maxima and minima along $y = 0$. What is the ratio between successive values of these extrema? Give a rough sketch of the flow pattern. (5 marks)

(d) Consider now the similar problem for irrotational flow rather than Stokes flow with the same x-behaviour. What is the main difference between the two flows? (2 marks)

(Total: 20 marks)

4. (a) The vorticity equation for steady two-dimensional flow can be written

$$\mathbf{u} \cdot \nabla\omega = \nu\nabla^2\omega$$

where $(0,\, 0,\, \omega) = \nabla \times \mathbf{u}$ and $\mathbf{u} = \nabla \times (0, 0, \psi)$. Infer that at high Reynolds number the vorticity is almost constant on streamlines.

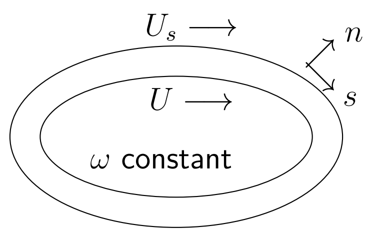
Suppose the flow has closed streamlines. Show that the line integral around a closed streamline

$$\oint_{\psi=c} \nabla^2\mathbf{u} \cdot d\mathbf{l} = 0, \qquad \text{where } c \text{ is constant.}$$

Deduce the Prandtl-Batchelor theorem, that within a closed streamline the vorticity should be constant. (6 marks)

(You are reminded that $\mathbf{u} \cdot \nabla\mathbf{u} = \nabla(|\mathbf{u}|^2/2) - \mathbf{u} \times \omega$ and that $\nabla^2\mathbf{u} = -\nabla \times \omega$.)

(b) Fluid flows inside a closed curve in 2-D which has tangential coordinate s and normal coordinate n. On the boundary the normal velocity is zero, while the tangential velocity $u = U_s(s)$ is given. Assuming there is a thin boundary layer, the flow on the edge of the constant vorticity core is $u = U(s)$ as shown in the diagram.



The boundary layer equations can be written

$$uu_s + vu_n = UU' + u_{nn} \qquad \text{where} \quad u = \psi_n, \quad v = -\psi_s.$$

Transform $(s,\, n)$ to the von Mises coordinates $(\theta,\, \psi)$ where $\theta = s$ is the periodic tangential coordinate and show that

$$\frac{\partial}{\partial\theta}\left(u^2\right) = \frac{\partial}{\partial\theta}\left(U^2\right) + u\frac{\partial^2}{\partial\psi^2}\left(u^2\right). \qquad \text{(7 marks)}$$

(c) Show that for constants a and b the streamfunction

$$\psi(x,\, y) = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2},$$

represents flow with constant vorticity inside an ellipse. (2 marks)

(d) Tangential flow $U_s = 2 - x^2$ is driven on the boundary of the ellipse $x^2 + 2y^2 = 1$. Show that there is no need for a boundary layer in this case and calculate the constant vorticity in the steady state. (5 marks)

(Total: 20 marks)

5. Write a description of each of the following topics. If you use any results from outside this module, make sure you describe them clearly. It is possible to exceed the stated mark allocation for each question part, but no more than 20 marks are available in total.

(a) Explain the mechanisms involved in the flight of machines and/or animals.

(13 marks)

(b) Discuss the effects of surface tension at the interface between two fluids. (7 marks)

(Total: 20 marks)

BSc and MSci EXAMINATIONS (MATHEMATICS)

May 2025

This paper is also taken for the relevant examination for the Associateship.

MATH60002/70002

Fluid Dynamics 2 (Solutions)

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1. (a) In Lubrication Theory, we assume (1) that x and y vary on a scale $L \gg h$, the scale that z varies on. It follows from continuity that if w has scale W while u and v have scale U, that seen $\Downarrow$

$$W = \frac{h}{L} U \ll U.$$

We also assume (2) that inertia is negligible compared to viscous forces on the short length scale,

$$\rho \frac{U^2}{L} \ll \mu \frac{U}{h^2} \qquad \Longrightarrow R_e \left(\frac{h}{L}\right)^2 \ll 1.$$

This leads to the equations 2, A

$$p_z = 0, \qquad p_x = \mu u_{zz}, \qquad p_y = \mu v_{zz}, \qquad u_x + v_y + w_z = 0.$$

As the pressure is independent of z, by integration

$$u = \frac{p_x}{2\mu} z^2 + Az + B, \qquad v = \frac{p_y}{2\mu} z^2 + Cz + D,$$

or imposing the no slip conditions

$$u = \frac{p_x}{2\mu}(z^2 - hz), \qquad v = \frac{p_y}{2\mu}(z^2 - hz).$$

Thus

$$\overline{u} \equiv \frac{1}{h}\int_0^h u\,dz = \frac{p_x}{2\mu}\left(-\tfrac{1}{6}h^2\right) \qquad \text{and} \qquad \overline{v} = -\frac{h^2 p_y}{12\mu}.$$

Now integrating the incompressibility condition, we have

$$\int_0^h (u_x + v_y)\,dz = -\int_0^h w_z\,dz = 0 \qquad \Longrightarrow \qquad \overline{u}_x + \overline{v}_y = 0 \qquad \Longrightarrow \qquad \nabla^2 p = 0.$$

3, A

(b) The kinematic condition is 2, B

$$0 = \frac{D}{Dt}(x - Ut - \varepsilon e^{iky+st}) = \overline{u} - U - \varepsilon s e^{iky+st} + O(\varepsilon^2),$$ unseen $\Downarrow$

noting that $\overline{v} = O(\varepsilon)$. From equation (1), we have 3, D

$$\overline{u} = U + \varepsilon \frac{dP_j}{dx} e^{iky+st}.$$

So combining these we have

$$s = P_j' \qquad \text{for } j = 1, 2.$$

To leading order this condition may be applied on the unperturbed interface, $x = Ut$. We also have $P_2 \to 0$ as $x \to +\infty$ and $P_1 \to 0$ as $x \to -\infty$. Since $\nabla^2 p = 0$, we have

$$P_j'' - k^2 P_j = 0 \qquad \Longrightarrow \qquad P_j = A_j e^{kx} + B_j e^{-kx},$$

for spatially constant A_j and B_j and so

$$P_1 = \frac{s}{k} e^{k(x-Ut)} \qquad \text{and} \qquad P_2 = -\frac{s}{k} e^{-k(x-Ut)}.$$

We also have that the pressure p is continuous across the perturbed boundary, which requires

$$\mu_1(U + P_1) = \mu_2(U + P_2) \qquad \text{on } x = Ut,$$

or

$$\mu_1\left(U + \frac{s}{k}\right) = \mu_2\left(U - \frac{s}{k}\right)$$

so that $\boxed{4, \text{A}}$

$$s = kU\,\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}.$$

$\boxed{4, \text{D}}$

We deduce that the motion is stable if the more viscous fluid pushes into the less viscous fluid ($\mu_1 > \mu_2$), but unstable the other way round. $\boxed{2, \text{A}}$

Note: this is known as the Saffman-Taylor instability.

2. (a) [Seen only with $V_0 = 0$.] By differentiation, we have $\boxed{\text{seen} \Downarrow}$

$$u = \psi_y = x^{a+b} f'(\eta), \qquad v = -\psi_x = -x^{a-1}(af + b\eta f').$$

As $\eta \to \infty$ we need $a + b = m$ and $f' \to 1$. On $\eta = 0$, we need $f'(0) = 0$ and critically $a - 1 = n$ and $af(0) = -V_0$.
Continuing the calculus,

$$u_x = x^{a+b-1}\left[(a+b)f' + b\eta f''\right], \qquad u_y = x^{a+2b} f'', \qquad u_{yy} = x^{a+3b} f'''.$$

Thus the LHS of the boundary layer equation is

$$uu_x + vu_y = x^{2a+2b-1}\left[f'((a+b)f' + b\eta f'') - (af + b\eta f')f''\right]$$

$$= x^{2a+2b-1}[(a+b)f'^2 - aff”].$$

The RHS is

$$UU' + u_{yy} = mx^{2m-1} + x^{a+3b} f'''.$$

For the powers of x to balance we must have $a + b = m$ (again) and $a - 1 = b$. Thus

$$a = \tfrac{1}{2}(m+1), \qquad b = \tfrac{1}{2}(m-1) \qquad \text{and the ODE becomes}$$

$$mf'^2 - \tfrac{1}{2}(m+1)ff'' = m + f'''.$$

However we must also require from the normal flow at $\eta = 0$ that

$$n = \tfrac{1}{2}(m-1),$$

The boundary conditions are

$$f'(0) = 0 \qquad f'(\infty) = 1 \qquad \text{and} \qquad f(0) = -\frac{2V_0}{m+1}.$$

$\boxed{8, \text{B}}$

(b) When $m = -\frac{1}{3}$, we have $\boxed{4, \text{A}}$

$$f''' + \tfrac{1}{3}(ff'' + f'^2) = \tfrac{1}{3} \qquad \Longrightarrow \qquad f' + \tfrac{1}{6}f^2 = \tfrac{1}{6}\eta^2 + A\eta + B.$$ $\boxed{\text{unseen} \Downarrow}$

At $\eta = 0$, we therefore have $\boxed{2, \text{A}}$

$$f(0)^2 = 9V_0^2 = 6B.$$

As $\eta \to \infty$, we have $f' \to 1$ or $f \sim \eta + C$. This requires

$$\tfrac{1}{3}C = A, \qquad 1 + \tfrac{1}{6}C^2 = B$$

It follows that $B > 1$ so $f(0)^2 > 6$. or $V_0^2 > \frac{2}{3}$. $\boxed{4, \text{C}}$
In general, we expect the boundary layer is more likely to remain attached if it is sucked towards the wall, rather than blown away from it, especially with an adverse pressure gradient. So we expect $V_0 < 0$. $\boxed{2, \text{B}}$

3. (a) Low-Reynolds-number flows are linear. Forces must always balance, so drag is proportional to velocity and terminal velocities are attained instantaneously. The flows are reversible in time and also in space in the sense that velocities can be reversed. They are unique and have the least dissipation for given kinematics. Sometimes no solution exists, as inertia cannot be ignored everywhere, cf Stokes' paradox. [Not all these observations required.] seen ⇓

5, A

(b) In 2-D Stokes flows are given by the biharmonic equation, or unseen ⇓

$$\nabla^2\omega = 0, \qquad \nabla^2\psi = -\omega.$$

Seek solutions with $\psi,\ \omega \propto e^{-kx}$. Then writing $\omega = F(y)e^{-kx}$ and $\psi = f(y)e^{-kx}$, we have

$$F'' + k^2F = 0 \qquad \Longrightarrow \qquad F = A\sin ky + B\cos ky.$$

Even solutions have $F = B\cos ky$. Then

$$f'' + k^2 f = -B\cos ky \qquad \Longrightarrow \qquad f = C\sin ky + D\cos ky + \widehat{B}y\sin ky.$$

Even solutions are

$$f = D\cos ky + \widehat{B}y\sin ky.$$

(People who find solving the ODE difficult should pick up a hint from the given solution in the question.)

The boundary conditions are $f(\pm a) = f'(\pm a) = 0$. This gives

$$D\cos ka + \widehat{B}a\sin ka, \qquad -kD\sin ka + \widehat{B}(\sin ka + ka\cos ka) = 0.$$

Then a non-zero solution is possible if

$$ka(\sin^2 ka + \cos^2 ka) + \sin ka\cos ka = 0 \qquad \Longrightarrow \qquad 2ka + \sin 2ka = 0.$$

If we consider $f(t) = \sin t + t$ then $f'(t) = 1 + \cos t \geqslant 0$ We note $f(0) = 0$ and so it is not possible that there should be another real root. 3, B

3, C

(c) We consider the solution for $k = c + id$, 2, B

$$\psi = \Re e\left[\left(\cos ky - y\sin ky\,\frac{\cos ka}{a\sin ka}\right)e^{-kx}\right].$$

Along $y = 0$, $\psi_y = 0$ by symmetry, while $\psi = e^{-cx}\cos dx$. So $\psi_x = 0$ when $c\cos dx + d\sin dx = 0$. There are therefore local maxima and minima of ψ separated by π/d. The value of $|\psi|$ at these successive extrema is reduced by a factor $e^{-\pi c/d}$. We see there is an infinite sequence of closed streamlines, with the vorticity reducing exponentially in strength from one to the next. 3, D

(d) Now we consider irrotational flows $\propto e^{-kx}$. Then the solutions to $\nabla^2\psi = 0$ which are even in y are 2, A

$$\psi = A\cos ky\, e^{-kx}.$$

Imposing the condition $\psi_x = 0$ on $y = \pm a$ defines $ka = (n + \frac{1}{2})\pi$. All these values are real and so there is monotonic exponential decrease. It is not possible to have simply connected regions with closed streamlines in irrotational flow, because the vorticity must be non-zero inside a closed streamline. 2, C

4. (a) At high Reynolds number we expect the viscous term to be negligible except in boundary layers, so we have $\mathbf{u} \cdot \nabla\omega = 0$ and ω does not vary along streamlines. Hence we have $\omega = f(\psi)$ for some function f. We now write the Navier Stokes equation in the form seen ⇓

$$\nabla\left(\tfrac{1}{2}|\mathbf{u}|^2\right) - \mathbf{u} \times \omega = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u}.$$

We consider the line integral of this equation along the streamline $\psi = \psi_0$. The cross product is perpendicular to the streamline and so we have

$$\oint_{\psi=\psi_0} \nabla\left(\frac{p}{\rho} + \tfrac{1}{2}|\mathbf{u}|^2\right) \cdot d\mathbf{l} = \nu \oint_{\psi=\psi_0} \nabla^2\mathbf{u} \cdot d\mathbf{l}.$$

Now the LHS is an exact differential so integrates to zero (or use Stokes' theorem). We therefore have, using $\nabla^2\mathbf{u} = -\nabla \times \omega$, 3, B

$$0 = \oint_{\psi=\psi_0} \nabla \times (0,\, 0,\, \omega) \cdot d\mathbf{l} = \oint \frac{d\omega}{d\psi} \nabla \times (0,\, 0,\, \psi) \cdot d\mathbf{l} = \frac{d\omega}{d\psi} \oint \mathbf{u} \cdot d\mathbf{l},$$ 3, C

since ω is a function of ψ and so $d\omega/d\psi$ is constant on $\psi = \psi_0$. Now $\oint \mathbf{u} \cdot d\mathbf{l}$ is the total vorticity contained in a streamline and will not vanish in general. We conclude that $d\omega/d\psi = 0$ inside closed streamlines and hence that ω is constant. seen ⇓

(b) Using the chain rule for partial derivatives as we transform $(s,\, n) \to (\theta, \psi)$, where $\theta = s$, $u = \psi_n$, $v = -\psi_s$, we have

$$\frac{\partial}{\partial s} = \frac{\partial \theta}{\partial s}\frac{\partial}{\partial \theta} + \frac{\partial \psi}{\partial s}\frac{\partial}{\partial \psi} = \frac{\partial}{\partial \theta} - v\frac{\partial}{\partial \psi},$$

and

$$\frac{\partial}{\partial n} = \frac{\partial \theta}{\partial n}\frac{\partial}{\partial \theta} + \frac{\partial \psi}{\partial n}\frac{\partial}{\partial \psi} = u\frac{\partial}{\partial \psi}.$$

The pde becomes

$$u\left(\frac{\partial u}{\partial \theta} - v\frac{\partial u}{\partial \psi}\right) + vu\frac{\partial u}{\partial \psi} = U\frac{\partial U}{\partial \theta} + u\frac{\partial}{\partial \psi}\left(u\frac{\partial u}{\partial \psi}\right),$$

or

$$\frac{\partial(u^2)}{\partial \theta} = \frac{\partial(U^2)}{\partial \theta} + u\frac{\partial^2(u^2)}{\partial \psi^2},$$

as required. 3, A

(c) We can see that the streamlines are $x^2/a^2 + y^2/b^2 =$ constant, a nested family of ellipses around the origin. We further note that $\nabla^2\psi$ is constant, so that this flow describes a constant vorticity flow inside an ellipse. 4, D unseen ⇓

(d) A boundary layer will be needed unless $U = U_s$. Inside the ellipse $x^2 + 2y^2 = 1$ consider the streamfunction 2, A unseen ⇓

$$\psi = k(1 - x^2 - 2y^2) \qquad \text{which has the vorticity} \quad \omega = 6k.$$

On the boundary, $x^2 + 2y^2 = 1$ the velocity has magnitude U where

$$U^2 = \psi_x^2 + \psi_y^2 = k^2(4x^2 + 16y^2) = 4k^2(x^2 + 2 - 2x^2) = 4k^2(2 - x^2)$$

Thus $U = U_s$ if $k = 1/2$ for which the constant vorticity is $\omega = 3$. 3, A

2, D

5. Essay question. Anything relevant will earn credit. It is possible to earn 20 marks just for part (a), but I doubt more than 10 will be possible for part (b). The total cannot exceed 20. $\boxed{20, \mathrm{M}}$

Review of mark distribution:
Total A marks: 32 of 32 marks
Total B marks: 20 of 20 marks
Total C marks: 12 of 12 marks
Total D marks: 16 of 16 marks
Total marks: 100 of 80 marks
Total Mastery marks: 20 of 20 marks

MATH70002 Fluid Dynamics 2 Markers Comments

Question 1

This question combined ideas from the Lubrication and Surface stability sections of the module, which some may have found confusing. People stumbled in two surprising places; firstly, a number failed to integrate the lubrication equations twice to obtain quadratic behaviour in z which is characteristic of all such problems. Secondly, in the stability part, many forgot that $\nabla^2 p=0$, even when the expression for P_1 was explicitly given, and most stability problem we looked at had exponential decay. Possibly the moving interface was found confusing.

Question 2

This question was the best answered in the exam. People were well prepared for a similarity solution of this ilk, and most proved able to mesh with the new factor of the problem namely the flow through $y=0$. However few were able to deal with the last part correctly, mainly because they didn't manage to square eta+constant correctly. Overall in this exam, people were let down by lack of manipulative practice rather than lack of fluid dynamics.

Question 3

This question was found hard, mainly because people found solving ODEs difficult, especially $f''+k^2f = \cos ky$. This was despite the fact that the solution form (involving $y\sin ky)$ was given in the question. Instead some people assumed that $sin 2ky +Ay$ must be the solution, for no obvious reason. Again, this question proved harder than I expected for reasons of calculus rather than fluid dynamics.

Some tried to solve the biharmonic equation directly rather than in two steps, as we always did in lectures.

I was surprised that many did not pick up the 2 marks for proving sin x + x had only one real root.

Question 4

This question was found very difficult, even though the first two portions were given in the notes. With hindsight, I wish I had given more pointers in the first part. Still, it was disappointing that few were able to transform the partial derivatives correctly in part b. This is not harder than in question 2, where people scored much more highly. In the unseen part I was surprised that few people understood that there was only a need for a boundary layer if there was a mismatch between the velocities on either side. I altered the marks scheme to give more marks to the parts that (some) people managed to do, but even so this question will have depressed the overall scores on the module.

Question 5 The essay question was done well on the whole. few managed to find very much to say about surface tension - I was hoping people would at least comment that it resisted short wavelength disturbances more strongly than long waves. But most people found lots to say about flying. More emphasis could have been given to the importance of circulation around the wing and different methods of obtaining it.