Math40002 Analysis 1 — Unseen 0

1. Prove that every root of a *monic* polynomial with integer coefficients (i.e, a polynomial of the form $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ with $a_0, \ldots, a_{n-1} \in \mathbb{Z}$) is either irrational or an integer.

2. A subset $X \subseteq \mathbb{R}$ is *dense* if for all $a, b \in \mathbb{R}$ such that $a < b$, there is some $x \in X$ such that $a < x < b$.

   (a) Prove the irrational numbers are dense.

   (b) Prove the rational numbers are dense.