Network Science
Spring 2024
Problem sheet 1

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1. Barabasi Section 2.12 Konigsberg Problem
Solution: a) Can be drawn, exactly 2 nodes have odd degree; b) Cannot be drawn, more than 2 nodes have odd degree; c) Can be drawn, all nodes have even degree; d) Can be drawn, exactly 2 nodes have odd degree.

2. Consider a path graph with N nodes, so the links are 1-2,2-3,3-4,...,$(N-1)$-N. Find the average distance for the graph.
Solution: a) The graph has $N-i$ shortest paths of length i. Then,

$$\overline{d} = \frac{1}{\binom{N}{2}} \sum_{i=1}^{N-1} (N-i)(i) = \frac{N+1}{3}.$$

3. Consider a path graph with N nodes where N is odd and greater than one. Add a link between an end-node (a node with degree=1) and the node in the center of the graph.

   i) What is the diameter of the new graph?
Solution: Number the nodes from 1 to N and say the added link connects nodes 1 and $(N+1)/2$. A path whose length is the diameter must connect nodes N and $(N+1)/2$ which are separated by $(N-1)/2$ links. Then, a further $floor((N+1)/4)$ links must be traversed, so $D = (N-1)/2 + floor((N+1)/4)$.

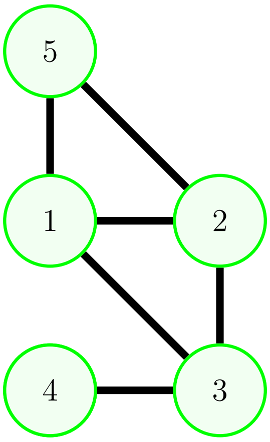
   ii) What are the local clustering coefficients C_i of this new graph?
Solution: The only case where a node has non-zero clustering is $N = 5$. Say the links are $1-2, 2-3, 3-4, 4-5, 1-3$. Then, nodes 1 and 2 will have $C_i = 1$. Node 3 will have $C_i = 1/3$, and nodes 4 and 5 will have $C_i = 0$.

4. Consider an undirected graph with N nodes where each node has degree 4 (and there are no self-loops and a maximum of 1 link between a pair of nodes).

   i. Sketch a graph with $N = 6$ which satisfies this condition.
Solution: Draw a 6-node cycle graph with the following links: 1-2,2-3,3-4,4-5,5-6,6-1, and then add the following links: 1-3,2-4,3-5,4-6,5-1,6-2.

   ii. Compute the clustering coefficient for each node when $N = 6$. (You may assume that the graph you found in (i) is the only possible one.)
Solution: Each node will have the same clustering coefficient, and we know that each node has 4 neighbors. So there are $\binom{4}{2} = 6$ possible links between these neighbors, and 4 links actually exist so $C_i = 4/6 = 2/3$ for each node in the graph.

5. Create the graph shown below in NetworkX. Compute the average clustering coefficient, $\overline{C}$ in NetworkX and the global clustering coefficient, C_g by hand.



Solution:
The graph has 2 triangles and 10 connected triples so $C_g = 2{*}3/10 = 3/5$. Probably the easiest way to compute connected triples is by counting the number of pairs of edges which share a vertex.