

# MATH60005/70005: Optimisation (Autumn 24-25)

## Chapter 6: exercises

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1. Derive the orthogonal projection formula for a closed ball centered at  $\mathbf{x}_0 \in \mathbb{R}^n$ ,  $B[\mathbf{x}_0, r]$ .
2. Show that the stationarity condition over the unit ball in  $\mathbb{R}^n$ , that is,

$$\min\{f(\mathbf{x}) : \|\mathbf{x}\| \leq 1\}$$

is given by  $\nabla f(\mathbf{x}^*) = 0$ , or  $\|\mathbf{x}^*\| = 1$  and there exists  $\lambda \leq 0$  such that  $\nabla f(\mathbf{x}^*) = \lambda \mathbf{x}^*$ .

3. Consider the minimization problem

$$\begin{aligned} \min \quad & 2x_1^2 + 3x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 - 8x_1 - 4x_2 - 2x_3 \\ \text{subject to} \quad & x_1, x_2, x_3 \geq 0. \end{aligned}$$

- Show that the vector  $(\frac{17}{7}, 0, \frac{6}{7})^\top$  is an optimal solution.
- Implement a projected gradient method with constant stepsize  $\frac{1}{L}$ , where  $L$  is the Lipschitz constant of the gradient of the function.

