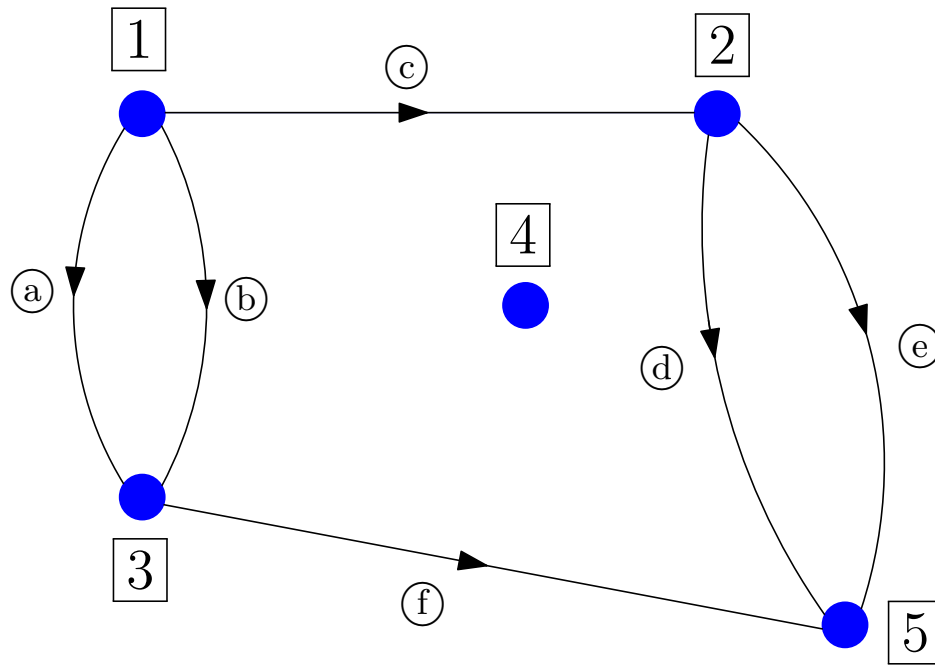


This examination consists of **two questions**. You should attempt both of them.

For maximum credit, **you must explain your answers clearly**.

1. Consider the graph shown in the figure where large blue dots indicate nodes and all lines with arrows are edges:

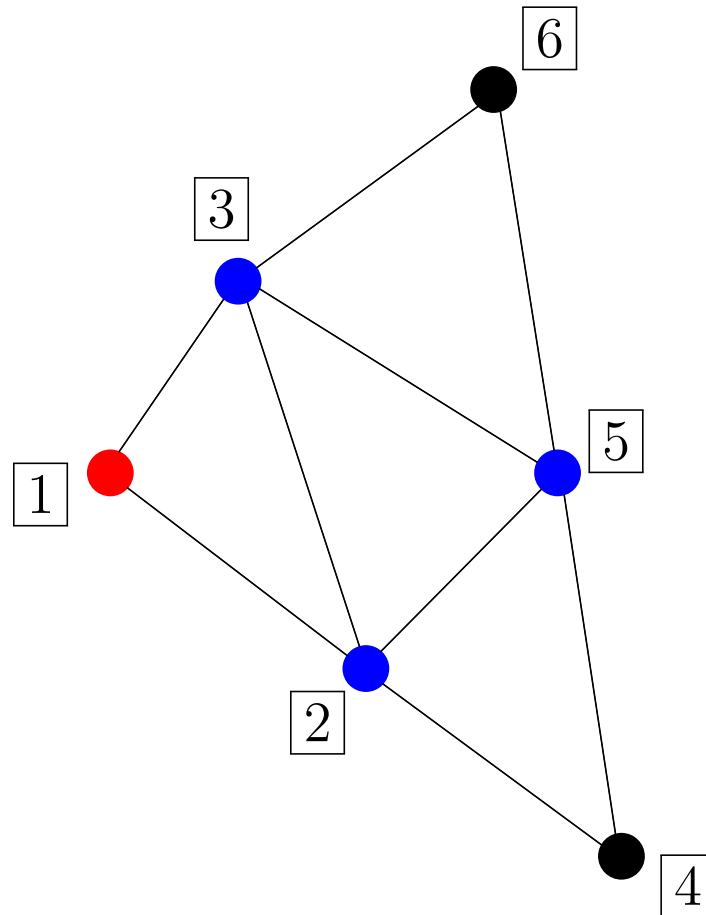


- Write down the incidence matrix \mathbf{A} for this graph (you must use the numerical and alphabetical ordering of nodes and edges on the figure, and the edge directions indicated).
- Find a basis for the right null space of \mathbf{A} .
- Find a basis for the row space of \mathbf{A} .
- Find a basis for the column space of \mathbf{A} .
- Write down as many linearly independent vectors as you can that are orthogonal to all elements of the column space of \mathbf{A} (note: two vectors \mathbf{x}_1 and \mathbf{x}_2 are said to be orthogonal if $\mathbf{x}_1^T \mathbf{x}_2 = 0$).

(12 marks)

2. The figure below shows an electric circuit in which node $\boxed{1}$ is set to unit voltage and nodes $\boxed{4}$ and $\boxed{6}$ are set to zero voltage ("grounded"). Kirchhoff's current law holds at all other nodes.

All edges have **unit conductance**.



Find the divergence of the currents at node $\boxed{6}$.

(8 marks)

THE END

(If you have extra time, use it to ensure you have explained all your answers clearly and fully).