Classical Dynamics **Autumn Term 2022**

Test 2

1. (a) The Lagrangian of a physical system with one generalised coordinate q is

$$L = \frac{1}{2}e^{2q}\left(\dot{q}^2 - 1\right).$$

(i) Show that the corresponding Hamiltonian is

$$H = \frac{1}{2}e^{-2q}p^2 + \frac{1}{2}e^{2q}.$$

(ii) Write down Hamilton's equations for this system.

(iii) Consider the time-independent canonical transformation

$$Q = e^q, \qquad P = e^{-q}p.$$

Write down the new Hamiltonian $K(Q,P)$ and solve Hamilton's equations for the new system.

(iv) Use your results from part (iii) to obtain $q(t)$ and $p(t)$. Are any solutions periodic?

[14 marks]

(b) The time evolution of a physical system is governed by the Hamiltonian

$$H = xyp_z + yzp_x + zxp_y.$$

Here the phase space is six-dimensional with coordinates x, y, z and momenta p_x, p_y, p_z.

(i) Write down (six) Hamilton's equations.

(ii) Show that $x^2 - y^2$ and $x^2 - z^2$ are constants of the motion.

Hint: compute the Poisson bracket $\{x^2, H\}$.

(iii) Find $x(t)$ and $z(t)$ assuming that $x = y$ and $x^2 - z^2 = 1$.

Hint: solve Hamilton's equation for $\dot{z}$ and substitute the solution $z(t)$ into Hamilton's equation for $\dot{x}$.

[11 marks]

[Total: 25 marks]