Intro to Multivariate Calculus - Applications

Quiz: Obtain the area of a circle of radius R by using element of area in polar coordinates.

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Partial Differential Equations (PDEs)

Find $f(\vec{x})$, $\vec{x} \in \mathbb{R}^n$ satisfying:

$$f(x_1, \cdots, x_n, f, \frac{\partial f}{\partial x_1}, \cdots, \frac{\partial f}{\partial x_n}, \frac{\partial^2 f}{\partial x_i x_j}, \cdots) = 0$$

Examples:

- Laplace Equation
- Wave Equation

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Transforming a PDE under a change of coordinates

$$u(x, y) \quad \longleftrightarrow \quad u(r, \theta)$$

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Example: Laplace Equation in polar coordinates

Laplace equation in Cartesian coordinates:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

In polar coordinates we have:

$$\frac{\partial}{\partial x}[u] = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta}\frac{\partial \theta}{\partial x}$$

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$$\frac{\partial^2}{\partial x^2}[u] = \cos^2\theta \frac{\partial^2 u}{\partial r^2} + \frac{2\cos\theta\sin\theta}{r^2}\frac{\partial u}{\partial \theta} - \frac{2\cos\theta\sin\theta}{r}\frac{\partial^2 u}{\partial r \partial \theta} + \frac{\sin^2\theta}{r}\frac{\partial u}{\partial r} + \frac{\sin^2\theta}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

$$\frac{\partial^2}{\partial y^2}[u] = \sin^2\theta \frac{\partial^2 u}{\partial r^2} - \frac{2\cos\theta\sin\theta}{r^2}\frac{\partial u}{\partial \theta} + \frac{2\cos\theta\sin\theta}{r}\frac{\partial^2 u}{\partial r \partial \theta} + \frac{\cos^2\theta}{r}\frac{\partial u}{\partial r} + \frac{\cos^2\theta}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

Find function $u(r)$ fulfilling the Laplace equation.

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Quiz: D'Alembert solution of the Wave equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

Show $u(x,t)$ having the D'Alembert form:

$$u(\xi); \quad \xi = x - ct$$

is a general solution.