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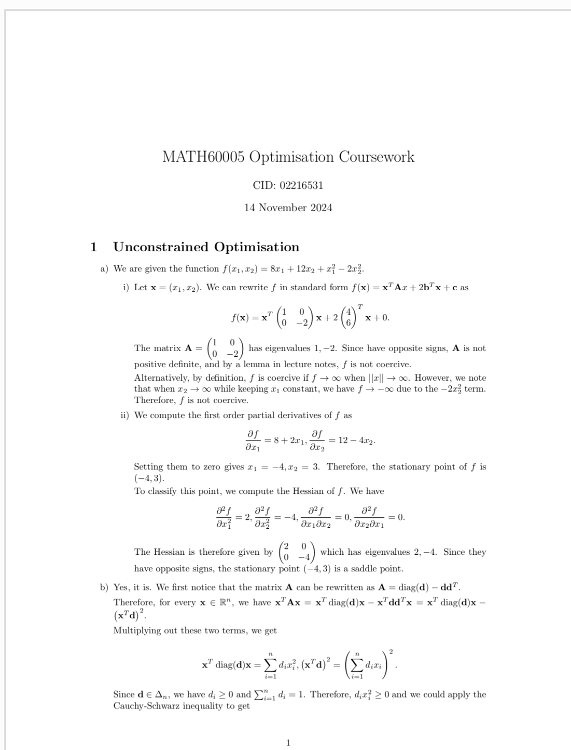

MATH60005 Optimisation Coursework

CID: 02216531

14 November 2024

1 Unconstrained Optimisation

a) We are given the function $f(x_1, x_2) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$.

i) Let $\mathbf{x} = (x_1, x_2)$. We can rewrite f in standard form $f(\mathbf{x}) = \mathbf{x}^T\mathbf{A}x + 2\mathbf{b}^T\mathbf{x} + \mathbf{c}$ as

$$f(\mathbf{x}) = \mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 4 \\ 6 \end{pmatrix}^T \mathbf{x} + 0.$$

The matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ has eigenvalues $1, -2$. Since have opposite signs, $\mathbf{A}$ is not positive definite, and by a lemma in lecture notes, f is not coercive.
Alternatively, by definition, f is coercive if $f \to \infty$ when $||x|| \to \infty$. However, we note that when $x_2 \to \infty$ while keeping x_1 constant, we have $f \to -\infty$ due to the $-2x_2^2$ term. Therefore, f is not coercive.

ii) We compute the first order partial derivatives of f as

$$\frac{\partial f}{\partial x_1} = 8 + 2x_1, \frac{\partial f}{\partial x_2} = 12 - 4x_2.$$

Setting them to zero gives $x_1 = -4, x_2 = 3$. Therefore, the stationary point of f is $(-4, 3)$.
To classify this point, we compute the Hessian of f. We have

$$\frac{\partial^2 f}{\partial x_1^2} = 2, \frac{\partial^2 f}{\partial x_2^2} = -4, \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0, \frac{\partial^2 f}{\partial x_2 \partial x_1} = 0.$$

The Hessian is therefore given by $\begin{pmatrix} 2 & 0 \\ 0 & -4 \end{pmatrix}$ which has eigenvalues $2, -4$. Since they have opposite signs, the stationary point $(-4, 3)$ is a saddle point.

b) Yes, it is. We first notice that the matrix $\mathbf{A}$ can be rewritten as $\mathbf{A} = \operatorname{diag}(\mathbf{d}) - \mathbf{d}\mathbf{d}^T$.
Therefore, for every $\mathbf{x} \in \mathbb{R}^n$, we have $\mathbf{x}^T\mathbf{A}\mathbf{x} = \mathbf{x}^T \operatorname{diag}(\mathbf{d})\mathbf{x} - \mathbf{x}^T\mathbf{d}\mathbf{d}^T\mathbf{x} = \mathbf{x}^T \operatorname{diag}(\mathbf{d})\mathbf{x} - (\mathbf{x}^T\mathbf{d})^2$.
Multiplying out these two terms, we get

$$\mathbf{x}^T \operatorname{diag}(\mathbf{d})\mathbf{x} = \sum_{i=1}^n d_i x_i^2, (\mathbf{x}^T\mathbf{d})^2 = \left(\sum_{i=1}^n d_i x_i\right)^2.$$

Since $\mathbf{d} \in \Delta_n$, we have $d_i \geq 0$ and $\sum_{i=1}^n d_i = 1$. Therefore, $d_i x_i^2 \geq 0$ and we could apply the Cauchy-Schwarz inequality to get

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