MATH50004/MATH50015/MATH50019 Differential Equations
Spring Term 2023/24
Hints for Problem Sheet 6

Exercise 26.

(i) is straightforward, and (ii) is a standard computational exercise, and the variation of constant formula can be applied here after the matrix exponential function for the homogeneous part is calculated. An elegant solution for (ii), however, does not require this formula, and relies on the constant solution computed in (i) for the inhomogeneous system. To see this elegant solution, look at the proof of Proposition 3.10, in particular at the part which proves that the sum of a solution to the homogeneous system and a solution to the inhomogeneous system is a solution to the inhomogeneous system.

Exercise 27

For (i), note that attractivity means in this one-dimensional context (where we have monotonicity of solutions) that solutions starting in a certain neighbourhood of the equilibrium approach the equilibrium montonically (and it is not possible that they move temporarily away such as in the two-dimensional example demonstrated in Example 4.3 (ii)). For (ii), determine the sign of the right hand side in a neighbourhood of the equilibrium, which depends on the first non-vanishing Taylor coefficient of the Taylor expansion of the right hand side in the equilibrium.

Exercise 28

As suggested in the hint, use polar coordinates, and note that for global attractivity, it is sufficient to show that the radial component converges to 0 forward in time. For this, show that the radial component decreases faster than the solution to the differential equation $\dot{r} = -r$, and use Exercise 4.

Exercise 29

The solution to this exercise is very similar to the proof of Theorem 4.5, and the proof also involves Propositions 3.8 and 3.9.

Exercise 30

Proof first that the improper integral used in the hint exists, and show that this function solves the inhomogeneous system. For (ii), consider taking the difference of two different bounded solutions and note that this difference solves the homogeneous problem. Consider the question whether the homogeneous differential equation can have non-trivial bounded solutions. Note that the difference of two solutions is also important for solving (iii).