IMPERIAL

MATH70037

BSc, MSci and MSc EXAMINATIONS (MATHEMATICS)
Summer 2025

This paper is also taken for the relevant examination for the
Associateship of the Royal College of Science

# Galois Theory

**Date:** Friday, May 23, 2025

**Time:** Start time 14:00 – End time 16:30 (BST)

**Time Allowed**: 2.5 hours

**This paper has 5 Questions.**

***Please Answer All Questions in 1 Answer Booklet***

This is a closed book examination.

Candidates should start their solutions to each question on a new sheet of paper.

Supplementary books may only be used after the relevant main book(s) are full.

Any required additional material(s) will be provided.

Credit will be given for all questions attempted.

Allow margins for marking.

**DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO DO SO**

1. (a) Give the definition of a *splitting field* of a polynomial $f(x) \in K[x]$, where $K$ is a field. (2 marks)

In parts (b), (c), (d) you can use all results from lectures if you state them clearly.

(b) Let $K \subset L$ be a finite field extension. Prove that there is a polynomial $f(x) \in K[x]$ such that $L$ can be embedded $K$-linearly into a splitting field of $f(x)$ over $K$. (4 marks)

(c) Determine the degrees of splitting fields of polynomials $x^4 - 4x^2 + 2$ and $x^4 - 2$ over $\mathbb{Q}$. (6 marks)

(d) Let $K \subset L \subset M$ be finite field extensions. Suppose that $L$ is a splitting field of a polynomial $f(x) \in K[x]$, and $M$ is a splitting field of a polynomial $g(x) \in L[x]$. Does there exist a polynomial $h(x) \in K[x]$ such that $M$ is a splitting field of $h(x)$? Give a proof or a counterexample. (8 marks)

(Total: 20 marks)

2. (a) Give the definition of a *separable* polynomial in $K[x]$, where $K$ is a field. (1 mark)

(b) State and prove the criterion of separability of a polynomial in terms of its derivative. (5 marks)

(c) Let $\mathbb{F}_p$ be the field with $p$ elements, where $p$ is a prime, $p > 2$. Let $K = \mathbb{F}_p(t)$ be the field of rational functions in the variable $t$ with coefficients in $\mathbb{F}_p$. Determine which of the following polynomials in $K[x]$ are separable: $x^p - t$, $x^p - x$, $x^{p-1} - t$, $x^{p-1} - x$, $x^{p^2} + x^p + t$. (5 marks)

(d) Let $f(x) \in \mathbb{Q}[x]$ be an irreducible polynomial of degree $p$, where $p$ is prime, $p > 2$. Let $G$ be the Galois group of $f(x)$.

(i) Suppose that $f(x)$ has exactly $p - 2$ real roots. Prove that $G$ is isomorphic to the symmetric group $S_p$. (Justify your answer. You can use all other results from lectures if you state them clearly.) (4 marks)

(ii) Is it possible that $G$ is abelian when $f(x)$ has a non-real root? (Justify your answer. You can use any results from lectures if you state them clearly.) (5 marks)

(Total: 20 marks)

3. (a) State Dedekind's theorem on linear independence of characters. (No proof is required.) (2 marks)

(b) Showing your working, determine the Galois groups of the following polynomials in $\mathbb{Q}[x]$:

(i) $x^3 + 6x + 3$;

(ii) $x^4 + 2x + 1$;

(iii) $x^5 - x - 1$

In part (iii) you can use without proof that the irreducible quadratic polynomials over $\mathbb{F}_3$ are $x^2 + 1$, $x^2 + x - 1$, $x^2 - x - 1$. (14 marks)

(c) Let $K$ be a finite field. Let $f(x) \in K[x]$ be a separable polynomial of degree $5$. List all possible Galois groups of $f(x)$. (Justify your answer. You can use all results from lectures if you state them clearly.) (4 marks)

(Total: 20 marks)

4. (a) State the primitive element theorem. (No proof is required.) (2 marks)

(b) Without using the primitive element theorem, prove that each of the following extensions can be generated by one element:

(i) $\mathbb{Q} \subset \mathbb{Q}(\sqrt{2}, \sqrt{3})$;

(ii) $\mathbb{F}_2 \subset \mathbb{F}_{256}$, where $\mathbb{F}_{2^r}$ is a finite field with $2^r$ elements;

(iii) a splitting field of $x^n - 1$ over the field $K = \mathbb{F}_2(t)$, where $n$ is an odd positive integer. (12 marks)

(c) Does there exist a field extension $K \subset M$ of degree $[M : K] = 4$ such that there is no subextension $K \subset L \subset M$ with $K \neq L \neq M$? Give an example or prove that such an extension $K \subset M$ does not exist. (6 marks)

(Total: 20 marks)

5. (a) State the fundamental theorem of Galois theory. (No proof is required.) (2 marks)

(b) Let $K$ be a field of characteristic zero. Let $f(x) \in K[x]$ be a polynomial of degree 4 with a splitting field $L$ and Galois group $G = \mathrm{Gal}(L/K)$. Give necessary and sufficient conditions for $G = S_4$ in terms of the roots of $f(x)$ in $L$. (5 marks)

(c) Let $f(x) = \sum_{i=0}^{4} a_i x^i \in \mathbb{Q}[x]$ be an irreducible polynomial such that $a_0 = a_4$ and $a_1 = a_3$. Prove that the order of the Galois group of $f(x)$ divides $8$. (5 marks)

(d) Let $f_a(x) = x^4 + x^3 + x^2 + x + a \in \mathbb{Q}[x]$ and let $G_a$ be the Galois group of $f_a(x)$ over $\mathbb{Q}$. Find four integer values of $a$ such that the Galois groups $G_a$ are pairwise non-isomorphic. (You are not asked to compute the groups $G_a$ explicitly.) (8 marks)

(Total: 20 marks)

BSc and MSci EXAMINATIONS (MATHEMATICS)

May 2025

This paper is also taken for the relevant examination for the Associateship.

# 60037/70037

# Galois theory (Solutions)

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1. (a) *Give the definition of a* splitting field *of a polynomial $f(x) \in K[x]$, where $K$ is a field.*

seen ⇓

A finite field extension $K \subset L$ is a splitting field of $f(x)$ if $f(x)$ completely splits over $L$, and the field $L$ is generated by the roots of $f(x) = 0$ over $K$.

2, A

In parts (b), (c), (d) you can use all results from lectures if you state them clearly.

seen ⇓

(b) *Let $K \subset L$ be a finite field extension. Prove that there is a polynomial $f(x) \in K[x]$ such that $L$ can be embedded $K$-linearly into a splitting field of $f(x)$ over $K$.*

Let $\alpha_1, \ldots, \alpha_n$ be a basis of the $K$-vector space $L$. Let $f_i(x) \in K[x]$ be the minimal polynomial of $\alpha_i$ over $K$, for $i = 1, \ldots, n$. Write $f(x) = f_1(x) \ldots f_n(x)$ and let $M$ be a splitting field of $f(x)$ over $L$. In particular, we have a $K$-linear embedding of $L$ into $M$. Clearly, $f(x)$ is completely split in $M$. The field $M$ is generated by the roots of $f(x)$ over $L$, but $L$ itself is generated by some of the roots of $f(x)$ over $K$, so $M$ is generated by the roots of $f(x)$ over $K$ and thus is a splitting field of $f(x)$ over $K$.

4, A

(c) *Determine the degrees of splitting fields of polynomials $x^4 - 4x^2 + 2$ and $x^4 - 2$ over $\mathbb{Q}$.*

seen/sim.seen ⇓

The first polynomial is $x^4 - 2ax^2 + c$, where $a = 2$, $c = 2$, $b = a^2 - c = 2$. By the theorem about biquadratic extensions, the Galois group of this polynomial is $C_4$. Thus a splitting field is a degree 4 extension of $\mathbb{Q}$.

The splitting field of the second polynomial is $\mathbb{Q}(\sqrt{-1}, \sqrt[4]{2})$, where $\sqrt[4]{2}$ is a real 4th root of 2. We have $\mathbb{Q}(\sqrt{-1}) \cap \mathbb{Q}(\sqrt[4]{2}) = \mathbb{Q}(\sqrt{-1}) \cap \mathbb{R} = \mathbb{Q}$. By the tower law we have $[\mathbb{Q}(\sqrt{-1}, \sqrt[4]{2}) : \mathbb{Q}] = [\mathbb{Q}(\sqrt{-1}, \sqrt[4]{2}) : \mathbb{Q}(\sqrt[4]{2})][\mathbb{Q}(\sqrt[4]{2}) : \mathbb{Q}] = 8$.

6, B

(d) *Let $K \subset L \subset M$ be finite field extensions. Suppose that $L$ is a splitting field of a polynomial $f(x) \in K[x]$, and $M$ is a splitting field of a polynomial $g(x) \in L[x]$. Does there exist a polynomial $h(x) \in K[x]$ such that $M$ is a splitting field of $h(x)$? Give a proof or a counterexample.*

unseen ⇓

8, C

This is false in general. For a counterexample, let $K = \mathbb{Q}$, $f(x) = x^2 - 2$, then $L = \mathbb{Q}(\sqrt{2})$. Take $g(x) = x^2 - (1 + \sqrt{2})$, then $M = \mathbb{Q}(\sqrt{1 + \sqrt{2}})$. Consider the polynomial $p(x) = x^4 - 2x^2 - 1 \in \mathbb{Q}[x]$. By the theorem about biquadratic extensions, since $a = 1$, $c = -1$, $b = a^2 - c = 2$, so that $b$, $c$, and $bc$ are not squares in $\mathbb{Q}$, the Galois group of $p(x)$ is $D_4$, in particular, $p(x)$ is irreducible. By the main theorem about normal extensions, if $M$ is a splitting field of some polynomial in $\mathbb{Q}[x]$, then $p(x)$ must be completely split in $M$ since $p(x)$ is irreducible and has a root $\sqrt{1 + \sqrt{2}}$ in $M$. But this is not the case, because the Galois group of $p(x)$ over $M$ is the subgroup of $D_4$ that stabilises a root, but the stabiliser of a point in $D_4 \subset S_4$ is the cyclic group $C_2$, which is non-trivial.

2. (a) *Give the definition of a* separable *polynomial in $K[x]$, where $K$ is a field.* seen ⇓

A polynomial is called separable if its roots in its splitting field are distinct. 1, A

(b) *State and prove the criterion of separability of a polynomial in terms of its derivative.* seen ⇓

A polynomial $f(x) \in K[x]$ is separable if and only if $f$ and its derivative $f'$ are coprime in $K[x]$, that is, gcd$(f, f') = 1$. Proof: The ring $K[x]$ is Euclidean. We can use Euclid's algorithm to compute gcd$(f, f')$. This calculation gives the same answer in any field extension of $K$, thus we can replace $K$ by a splitting field $L$ of $f$. A root of $f(x) = 0$ in $L$ has multiplicity at least 2 if and only if it is a common root of $f$ and $f'$. Thus $f(x)$ is separable if and only if gcd$(f, f') = 1$. 5, A

(c) *Let $\mathbb{F}_p$ be the field with $p$ elements, where $p$ is a prime, $p > 2$. Let $K = \mathbb{F}_p(t)$ be the field of rational functions in the variable $t$ with coefficients in $\mathbb{F}_p$. Determine which of the following polynomials in $K[x]$ are separable:* meth seen ⇓

*$x^p - t$, $x^p - x$, $x^{p-1} - t$, $x^{p-1} - x$, $x^{p^2} + x^p + t$.*

The first and last polynomials have zero derivatives, so they are inseparable. The second polynomial has derivative 1, so it is separable. We have $(x^{p-1} - t)' = -x^{p-2}$ which has only one root $0$, which is not a root of $x^{p-1} - t$, so this polynomial is separable. We have $(x^{p-1} - x)' = -x^{p-2} - 1$. Substituting $x^{p-2} = -1$ into the original polynomial we get $x = 0$, but this is not a root of the derivative, so the polynomial is separable. 5, A

(d) *Let $f(x) \in \mathbb{Q}[x]$ be an irreducible polynomial of degree $p$, where $p$ is prime, $p > 2$. Let $G$ be the Galois group of $f(x)$.*

(i) *Suppose that $f(x)$ has exactly $p - 2$ real roots. Prove that $G$ is isomorphic to the symmetric group $S_p$. (Justify your answer. You can use all other results from lectures if you state them clearly.)* seen ⇓

Let $L$ be a splitting field of $f(x)$ and let $\alpha \in L$ be a root of $f(x)$. Then $\mathbb{Q} \subset \mathbb{Q}(\alpha) \subset L$, hence $[\mathbb{Q}(\alpha) : \mathbb{Q}] = p$ divides $[L : \mathbb{Q}] = |G|$. Since $p$ is prime, by Cauchy's theorem, $G \subset S_p$ contains an element of order $p$, which must be a $p$-cycle. We have $\mathbb{Q} \subset L \subset \mathbb{C}$. Since $L$ is normal over $\mathbb{Q}$, any automorphism of $\mathbb{C}$ sends $L$ to $L$. This gives a homomorphism $\mathrm{Gal}(\mathbb{C}/\mathbb{R}) \to G$. The complex conjugation switches the two non-real roots, thus $G$ contains a transposition. By a result from lectures, since $p$ is prime, a $p$-cycle and a transposition generate $S_p$. Thus $G = S_p$. 4, C

(ii) *Is it possible that $G$ is abelian when $f(x)$ has a non-real root? (Justify your answer. You can use any results from lectures if you state them clearly.)*

No. The above argument shows that $G$ contains a $p$-cycle, and also contains a non-trivial permutation that is a product of independent transpositions. Since $p$ is odd, it fixes at least one point. A $p$-cycle is transitive, hence it cannot commute with a non-trivial permutation that fixes a point. Thus $G$ cannot be abelian. unseen ⇓

5, D

3. (a) *State Dedekind's theorem on linear independence of characters. (No proof is required.)* seen ⇓

2, A

Let $S$ be a semigroup and let $K$ be a field. Any set of distinct nonzero characters $S \to K$ is a linearly independent subset of the $K$-vector space of functions $S \to K$. meth seen ⇓

(b) *Showing your working, determine the Galois groups of the following polynomials in $\mathbb{Q}[x]$:*

(i) $x^3 + 6x + 3$

The polynomial is irreducible by Eisenstein's criterion with $p = 3$. Since $p$ is prime, the Galois group contains a 3-cycle. Modulo 2 we get $x^3 + 1 = (x+1)(x^2+x+1)$, where the second factor is irreducible, so the Galois group contains a transposition. Hence the Galois group is $S_3$. 4, A

(ii) $x^4 + 2x + 1$

We calculate the resolvent cubic using a formula from lectures $g(t) = t^3 - 4t - 4$. Working modulo 3 we check that $t^3 - t - 1$ has no roots in $\mathbb{F}_3$ hence is irreducible. Thus $g(t)$ is irreducible over $\mathbb{Q}$. Using a formula from lectures we find that the discriminant is negative, hence not a square in $\mathbb{Q}$. Thus $g(t)$ has Galois group $S_3$. Note that the original polynomial is reducible: $x^4 + 2x + 1 = (x+1)(x^3 - x^2 + x + 1)$, so its Galois group is $S_3$. 5, B

(iii) $x^5 - x - 1$

*(Hint: the irreducible quadratics over $\mathbb{F}_3$ are $x^2 + 1$, $x^2 + x - 1$, $x^2 - x - 1$.)* Modulo 2 this polynomial is reducible: $x^5 + x + 1 = (x^2 + x + 1)(x^3 + x^2 + 1)$, where both factors are irreducible over $\mathbb{F}_2$. Thus the Galois group contains the product of independent 2- and 3-cycles. The cube of this permutation is a transposition. Modulo 3, $x^5 - x - 1$ is irreducible because it has no roots and is not divisible by the irreducible quadratic polynomials over $\mathbb{F}_3$. Thus the Galois group has a 5-cycle. By a result from lectures, since 5 is prime, we obtain that the Galois group is $S_5$. 5, D

(c) *Let $K$ be a finite field. Let $f(x) \in K[x]$ be a separable polynomial of degree 5. List all possible Galois groups of $f(x)$. (Justify your answer. You can use all results from lectures if you state them clearly.)* unseen ⇓

By lectures, the Galois group of an extension of finite fields is cyclic. The possible degrees of irreducible factors of $f(x)$ are $1,1,1,1,1$; $1,1,1,2$; $1,1,3$; $1,4$; $1,2,2$; $2,3$; $5$. The respective Galois groups are the trivial group, $C_2$, $C_3$, $C_4$, $C_2$ (note that all quadratic extensions of a finite field are isomorphic), $C_2 \times C_3 \cong C_6$, $C_5$. 4, A

4. (a) *State the primitive element theorem. (No proof is required.)* seen ⇓

Let $F/K$ be a finite extension of fields. Suppose that $F = K(\alpha_0, \ldots, \alpha_n)$ where the minimal polynomials of $\alpha_1, \ldots, \alpha_n$ over $K$ are separable. Then $F = K(\gamma)$ for some $\gamma \in F$. (OK to state this for a finite separable extension $F/K$.) 2, A

(b) *Without using the primitive element theorem, prove that each of the following extensions can be generated by one element.* meth seen ⇓

(i) $\mathbb{Q} \subset \mathbb{Q}(\sqrt{2}, \sqrt{3})$.
$\alpha = \sqrt{2} + \sqrt{3}$ works. We have $\alpha^2 = 5 + 2\sqrt{6}$, so $\sqrt{6} \in \mathbb{Q}(\alpha)$. We have $\sqrt{6}\alpha = 3\sqrt{2} + 2\sqrt{3}$, so $\sqrt{2}, \sqrt{3} \in \mathbb{Q}(\alpha)$. 3, A

(ii) $\mathbb{F}_2 \subset \mathbb{F}_{256}$, *where* $\mathbb{F}_{2^r}$ *is a finite field with* $2^r$ *elements.*
Since $256 = 2^8$, the subfields of $\mathbb{F}_{256}$ are $\mathbb{F}_2$, $\mathbb{F}_4$, $\mathbb{F}_{16}$, which are all contained in $\mathbb{F}_{16}$. Thus any element of $\mathbb{F}_{256}$ not contained in $\mathbb{F}_{16}$ generates $\mathbb{F}_{256}$. 4, B

(iii) *A splitting field of* $x^n - 1$ *over the field* $K = \mathbb{F}_2(t)$, *where* $n$ *is an odd positive integer.*
Since $n$ is odd, the polynomial $x^n - 1 \in K[x]$ is separable. Let $L$ be its splitting field. The roots of $x^n = 1$ in $L$ form a finite group of order $n$, which must be cyclic like any finite subgroup of $L^\times$. A generator of this cyclic group generates $L$ over $K$. 5, B

(c) *Does there exist a field extension* $K \subset M$ *of degree* $[M : K] = 4$ *such that there is no subextension* $K \subset L \subset M$ *with* $K \neq L \neq M$*? Give an example or prove that such an extension* $K \subset M$ *does not exist.* unseen ⇓

In the course, we have seen examples of irreducible polynomials $f(x) \in \mathbb{Q}[x]$ of degree 4 with Galois group $S_4$. Let $M$ be a splitting field of $f(x)$ and let $\alpha$ be a root of $f(x)$ in $M$. Then $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 4$ and $\mathrm{Gal}(M/\mathbb{Q}(\alpha)) = S_3 \subset S_4$ is the stabiliser of the root $\alpha$. If there is a subextension $\mathbb{Q} \subsetneq L \subsetneq \mathbb{Q}(\alpha)$, then by the Galois correspondence there is a subgroup $G \subset S_4$ such that $S_3 \subsetneq G \subsetneq S_4$. Then $G$ index 2 in $S_4$, so $G$ must be equal to the alternating group $A_4$, but this is false because $S_3$ contains transpositions which are not in $A_4$. Thus a subextension $L$ does not exist. 6, D

5. (a) *State the fundamental theorem of Galois theory. (No proof is required.)* $\boxed{\text{seen} \Downarrow}$

Let $K \subset L$ be a Galois extension. There is a bijection (Galois correspondence) between the subgroups $G \subset \mathrm{Gal}(L/K)$ and the subfields $K \subset F \subset L$ given by

$$G \mapsto L^G, \qquad F \mapsto \mathrm{Aut}_F(L).$$

Under this bijection, the normal subgroups of $\mathrm{Gal}(L/K)$ correspond to the subextensions $K \subset F \subset L$ such that $F$ is Galois over $K$. $\boxed{2, \mathrm{M}}$

(b) *Let $K$ be a field of characteristic zero. Let $f(x) \in K[x]$ be a polynomial of degree 4 with a splitting field $L$ and Galois group $G = \mathrm{Gal}(L/K)$. Give necessary and sufficient conditions for $G = S_4$ in terms of the roots of $f(x)$ in $L$.* $\boxed{\text{seen/sim.seen} \Downarrow}$

Let $\alpha_i$, $i = 1, 2, 3, 4$, be the roots of $f(x)$ in $L$. We have $G = S_4$ if and only if $G$ is not contained in one of the proper maximal subgroups of $S_4$, which are $A_4$, $D_4$, $S_3$ (the stabiliser of a point). We have $G \subset A_4$ if and only if the discriminant of $f(x)$ is a square in $K$, i.e. $\prod_{i<j}(\alpha_i - \alpha_j) \in K$. We have $G \subset D_4$ if and only if the cubic resolvent of $f(x)$ has a root in $K$, i.e. $\alpha_i\alpha_j + \alpha_k\alpha_l \in K$ for some $\{i, j, k, l\} = \{1, 2, 3, 4\}$. Finally, $G \subset S_3$ if and only if $\alpha_i \in K$ for some $i$. $\boxed{5, \mathrm{M}}$

(c) *Let $f(x) = \sum_{i=0}^4 a_i x^i \in \mathbb{Q}[x]$ be an irreducible polynomial such that $a_0 = a_4$ and $a_1 = a_3$. Prove that the order of the Galois group of $f(x)$ divides $8$.* $\boxed{\text{unseen} \Downarrow}$

We observe that $f(\alpha) = 0$ implies $f(\alpha^{-1}) = 0$. The function $\alpha \mapsto \alpha^{-1}$ is a permutation of roots which has order 2. If $\alpha = \alpha^{-1}$, then $\alpha = \pm 1$ which is impossible since $f(x)$ is irreducible. Thus the permutation $\alpha \mapsto \alpha^{-1}$ has no fixed points, hence is a double transposition, say $(12)(34)$. The Galois group is contained in the centraliser of $(12)(34)$ which is $D_4$. $\boxed{5, \mathrm{M}}$

(d) *Let $f_a(x) = x^4 + x^3 + x^2 + x + a \in \mathbb{Q}[x]$ and let $G_a$ be the Galois group of $f_a(x)$ over $\mathbb{Q}$. Find four integer values of $a$ such that the Galois groups $G_a$ are pairwise non-isomorphic. (You are not asked to compute the groups $G_a$ explicitly.)* $\boxed{\text{unseen} \Downarrow}$

$f_0(x) = x(x + 1)(x^2 + 1)$ has Galois group $G_0 \cong C_2$.

$f_1(x) = x^4 + x^3 + x^2 + x + 1$ divides $x^5 - 1$, so its roots are non-trivial 4th roots of unity. From the course we know that $G_1 \cong (\mathbb{Z}/5)^\times \cong C_4$.

$f_3(x) = x^4 + x^3 + x^2 + x + 3$. Modulo 2 we get $x^4 + x^3 + x^2 + x + 1$ which is irreducible as it has no roots in $\mathbb{F}_2$ and is not the square of $x^2 + x + 1$. So $G_3$ contains a 4-cycle. Modulo 3 we get $x(x + 1)(x^2 + 1)$, where the last factor is irreducible over $\mathbb{F}_3$. Thus $G_3$ contains a transposition. This implies that $G_3$ is either $D_4$ or $S_4$.

$f_{-4}(x) = x^4 + x^3 + x^2 + x - 4 = (x - 1)(x^3 + 2x^2 + 3x + 4)$. Modulo 3 the second factor reduces to $x^3 - x^2 + 1$, which has no roots in $\mathbb{F}_3$ and so is irreducible. Thus $G_{-4}$ contains an element of order 3, and so is either $S_3$ or $C_3$.

The above four groups are not isomorphic because their cardinalities are different. $\boxed{8, \mathrm{M}}$

**Review of mark distribution:**

Total A marks: 32 of 32 marks
Total B marks: 20 of 20 marks
Total C marks: 12 of 12 marks
Total D marks: 16 of 16 marks
Total marks: 100 of 80 marks
Total Mastery marks: 20 of 20 marks

## MATH6/70037 Galois Theory Markers Comments

Question 1 Very few students gave a clean proof of Q1 (b) thought it was essentially book work.

Question 2 Many students found Q2 (d) (ii) was a bit hard to explain in a clean way.

Question 3 A more straightforward question.

Question 4 Q4 (c) was answered by very few students, though it is similar to Q1 (d).

Question 5 Q5 (d) was not finished by anyone, probably for lack of time.