Math40002 Analysis 1 **Unseen 1**

1. For the following sets, determine whether they are finite, countable, or uncountable.

   Prove your answer.

   (a) The set of all finite subsets of $\mathbb{R}$.

   (b) The set of all co-finite subsets of $\mathbb{R}$, that is, $\{ A \subseteq \mathbb{R} \mid \mathbb{R} \setminus A \text{ is finite } \}$.

   (c) The set of all finite subsets of $\mathbb{Q}$.

   (d) The set of all co-finite subsets of $\mathbb{Q}$, that is, $\{ A \subseteq \mathbb{Q} \mid \mathbb{Q} \setminus A \text{ is finite } \}$.

   (e) The set of all open intervals with endpoints in $\mathbb{R}$: $\{ (a, b) \mid a, b \in \mathbb{R} \}$.

   (f) The set of all open intervals with endpoints in $\mathbb{Q}$: $\{ (a, b) \mid a, b \in \mathbb{Q} \}$.

   (g) The set of all finite unions of open intervals with endpoints in $\mathbb{Q}$: The set of all sets of the form $\bigcup_{i=1}^{n}(a_i, b_i)$.

   (h) The set of all countable unions of open intervals with endpoints in $\mathbb{Q}$: The set of all sets of the form $\bigcup_{i=1}^{\infty}(a_i, b_i)$.

   (i) The set of all finite intersections of open intervals with endpoints in $\mathbb{Q}$: The set of all sets of the form $\bigcap_{i=1}^{n}(a_i, b_i)$.

   (j) The set of all countable intersections of open intervals with endpoints in $\mathbb{Q}$: The set of all sets of the form $\bigcap_{i=1}^{\infty}(a_i, b_i)$.

(k) The set of all non-increasing functions from $\mathbb{N}$ to $\mathbb{N}$.

2. The elements of $\mathbb{N} \times \mathbb{N}$ may be arranged in the form of a simple sequence as follows:

$$(1,1), (2,1), (1,2), (1,2), (3,1), (2,2), (1,3), (4,1), (3,2), (2,3), (1,4), \ldots$$

Show that (p, q) is the $(\frac{1}{2}(p+q-1)(p+q-2)+q)$-th element of the sequence. Note that this is an explicit formula of a bijection between $\mathbb{N}$ and $\mathbb{N} \times \mathbb{N}$.

Hint: If you arrange $\mathbb{N} \times \mathbb{N}$ *as a grid in* $\mathbb{R}^2$ *(e.g. see the figure at the beggining of the proof of Theorem 2.19 from Week 1 Lecture Notes) then the sequence corresponds to counting the elements along diagonals of increasing length.*