

1. (i) (a) The Lagrangian  $L(q, \dot{q}, t)$  describes the time evolution of a physical system with one coordinate  $q$ . Write down the Euler-Lagrange equation.

(b) What is meant by a conservative force?

- (ii) Consider the Lagrangian

$$L = \frac{1}{2} (\dot{x}^2 + \dot{y}^2) - b\dot{x}\dot{y} - gy,$$

where  $b$  and  $g$  are non-zero constants.

- (a) Use the 2 Euler-Lagrange equation to show that this Lagrangian describes a particle of unit mass acted on by a non-conservative force. Is the force conservative if  $g$  or  $b$  is set to zero?

- (b) Find the general solution of the Euler-Lagrange (E-L) equations obtained in part (a). Hint: integrate the E-L equation for  $x$  and use the result to eliminate  $\dot{x}$  from the E-L equation for  $y$ .

- (c) Are any solutions periodic?

- (d) Determine the conserved quantity

$$H = p_x\dot{x} + p_y\dot{y} - L.$$

Express your answer in terms of  $x$ ,  $y$ ,  $\dot{x}$ ,  $\dot{y}$  (you do not need the solution to part (b) to answer this).

- (e) Are there any other conserved quantities?