Sheet 2, question 5a (revised)

5. Suppose that f is analytic in some (concentric) annulus D centred on some point z_0, and has an even Laurent expansion about z_0, i.e.,

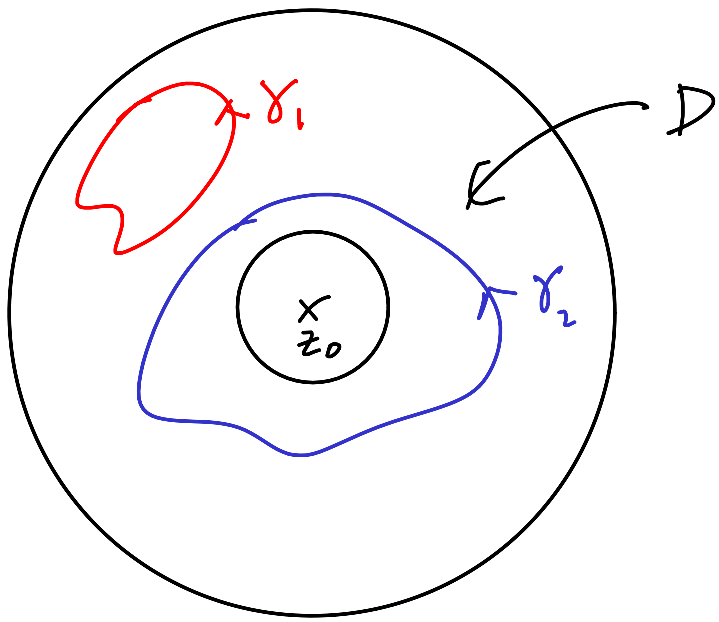
$$f(z) = \sum_{j=-\infty}^{\infty} a_{2j}(z - z_0)^{2j}.$$

(a) For a closed contour $\gamma \subset D$, show that

$$\int_{\gamma} f(z)dz = 0$$

Solution

a). We have



It follows from Cauchy's Theorem that $\int_\gamma f(z)\,dz = 0$ for any closed contour γ that lies wholly in D and does not surround z_0, eg. γ_1 in the sketch above. But furthermore, the same is true if γ does surround z_0, eg., γ_2 above. This follows from the residue theorem and the fact that the residue of f at z_0 is 0.