Question 1

(a) Prove that for any random variable X and any constant $a \in \mathbb{R}$,

$$\mathrm{Cov}\,(X, a) = 0.$$

(b) Prove that for any random variable X, Y and Z, and constants $a, b \in \mathbb{R}$,

$$\mathrm{Cov}\,(aX + bY, Z) = a\mathrm{Cov}\,(X, Z) + b\mathrm{Cov}\,(Y, Z)\,.$$

(c) For any random variables X and Y, and constants $a, b \in \mathbb{R}$, find an expression for

$$\mathrm{Cov}\,(aX + b, Y)$$

in terms of $\mathrm{Cov}\,(X, Y)$.

Question 2

Suppose you are tracking the value of two companies listed on the London Stock Exchange over the course of one week. Rather than record the actual values of the share prices, you record the increase or decrease in each share price value at the daily close to the nearest pound. You record the following table:

	Monday	Tuesday	Wednesday	Thursday	Friday
Company X	5	4	8	6	2
Company Y	3	2	7	4	-1

Table 1: Daily change in share price (£)

Do the following calculations (there is no need to use a calculator):

(a) Compute the sample covariance between the two sequences to two decimal places.

(b) Compute the sample correlation between the two sequences. You may leave your answer as a fraction.

(c) Compute the sample correlation between the two sequences to two decimal places.

(d) Are the two sequences significantly correlated?

Please turn over for Questions 3 and 4.

Question 3

Suppose that X and Y are two normally distributed random variables that are neither independent nor identically distributed. In fact, suppose it is known that $X \sim \mathrm{N}(1, 8)$ and $Y \sim \mathrm{N}(5, 2)$, and their correlation is $\mathrm{Cor}(X, Y) = \frac{1}{9}$. Defining the new random variable $Z = 2X + Y$, compute the correlation $\mathrm{Cor}(Y, Z)$.

Question 4

The figures below show X, the amount of uranium stored in US power plants and Y, the number of PhD degrees awarded in mathematics in the US for the years 1996 to 2008. The graphs appear to be very similar, and if one computes the sample correlation between the two data sets for X and Y, one obtains a value of $r = 0.952$. Taking this sample correlation value into account, can we conclude that the two quantities X and Y are related and have an influence on each other? Provide justification for your answer.