MATH60005/70005: Optimisation (Autumn 24-25)

Chapter 5: exercises

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1. Show the convexity of the following functions:

   - the quad-over-lin function

     $$f(x_1, x_2) = \frac{x_1^2}{x_2}$$

     defined over $\mathbb{R} \times \mathbb{R}_{++} = \{(x_1, x_2) : x_2 > 0\}$.

   - the generalized quad-over-lin function

     $$g(\mathbf{x}) = \frac{\|\mathbf{A}\mathbf{x} + \mathbf{b}\|^2}{\mathbf{c}^\top \mathbf{x} + d} \qquad \left(\mathbf{A} \in \mathbb{R}^{m\times n}, \mathbf{b} \in \mathbb{R}^m, \mathbf{c} \in \mathbb{R}^n, d \in \mathbb{R}\right)$$

     is convex over $D = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{c}^\top \mathbf{x} + d > 0\}$.

   - $f(x_1, x_2) = -\log(x_1 x_2)$, over $\mathbb{R}^2_{++}$.

   - $h(\mathbf{x}) = e^{\|\mathbf{x}\|^2}$.

2. Show that $\sqrt{1 + \mathbf{x}^\top Q \mathbf{x}}$ is convex for Q positive definite.

3. Find the optimal solution of

$$\begin{aligned}
\max_{\mathbf{x}\in\mathbb{R}^3} \quad & 2x_1^2 + x_2^2 + x_3^2 + 2x_1 - 3x_2 + 4x_3 \\
\text{subject to} \quad & x_1 + x_2 + x_3 = 1 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$