Course 02249, DTU Compute
Carsten Witt

Computationally Hard Problems – Fall 2016
Assignment 4

Date: 20.09.2016, **Due date**: 26.09.2016, 21:00

Exercise 4.1: Consider the following problem:

Problem: [IP-4] The input is an undirected graph $G = (V, E)$. The objective is to decide if the graph contains an *independent set* of size 4 in G, i. e., four distinct vertices $v_1, v_2, v_3, v_4 \in V$ such no edge between them is present: $\{v_i, v_j\} \notin E$ for $1 \leq i < j \leq 4$.

We propose the following randomized algorithm for IP-4. Let $n = |V|$ and let $p(n)$ denote a polynomial.
The algorithm sets a counter c to 0. It then repeatedly performs the following steps.

1. Increment the counter c by 1.
2. The algorithm picks four (not necessarily distinct) vertices from V at random according to the uniform distribution. It then checks whether they form an independent set of size 4. If this is the case, the algorithm answers YES and stops.
3. If $c > p(n)$ then the algorithm answers NO and stops.

Show that there is a choice for the polynomial $p(n)$ such that the algorithm is an $\mathcal{RP}$-algorithm.
Hint: To estimate success probabilities, the inequality $(1 - 1/x)^x \leq 1/2$ for $x \geq 1$ may be useful.

End of Exercise 1

Exercise 4.2: Prove for each of the following problems that they are in the class $\mathcal{NP}$:

a) Road Maintenance,

b) Glasses in a cupboard,

c) Satisfiability.

You need not (and should not try to) prove that the problems are $\mathcal{NP}$-complete.

End of Exercise 2

Continued on next page.

Exercise 4.3: Consider the following problem:

Problem: [MinimumCliqueCover]
Input: An undirected graph $G = (V, E)$ and a natural number k.
Output: YES if there is clique cover for G of size at most k. That is, a collection $V_1, V_2, \ldots, V_k$ of not necessarily disjoint subsets of V such that each V_i induces a complete subgraph of G and such that for each edge $\{u, v\} \in E$ there is some V_i that contains both u and v. NO otherwise.
For a subset $V' \subset V$ of the nodes, the *induced* subgraph has node set V' and edge set E', where $e \in E' \iff e \in E$.

Show that MinimumCliqueCover is in $\mathcal{NP}$.
You need not show that the problem is $\mathcal{NP}$-complete!

End of Exercise 3

The following exercise is **mandatory**:

Exercise 4.4: Recall the following problem, which was already defined on Exercise Sheet 2.

Problem: [MinimumTestSet]
Input: You want to test an electronic circuit. The circuit can have n different errors, $e_1, \ldots, e_n$. You can perform m different tests $T_1, \ldots, T_m$. Every test can detect some, but not necessarily all, errors.
Output for the optimizing version: A minimum set of tests that can detect all possible errors.

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1. Formulate the above problem in an appropriate way as a decision problem. In particular, one should be able to use a polynomial-time algorithm for this decision problem as a black box to solve the optimization problem as stated above in polynomial time. You need not prove that your decision problem has this property if you feel that the teachers can easily see this.

2. Show that this decision problem is in $\mathcal{NP}$. You need not show that the problem is $\mathcal{NP}$-complete.

End of Exercise 4