Computationally Hard Problems

Student name and id: Anders H. Opstrup (s160148)

Hand-in for week: 5

Exercise 1

Problem: [SAT-TWO-THIRDS]
Input: A set of clauses $C = \{c_1, ..., c_k\}$ over n boolean variables $x_1, ..., x_n$.
Output: YES if there is a truth assignment to the variables such that at least $(2/3)k$ many clauses are satisfied. NO otherwise.

Show that this problem is NP-complete. You may use any problem stated as $NP-$ complete in the lecture notes for this course. You may also assume that SAT-TWO-THIRDS is in NP.

a)

We use the 3-SAT problem to prove that SAT-TWO-THIRDS is NP-complete. We thereby reduce 3-SAT to SAT-TWO-THIRDS.

b)

b.1)

By putting each variable in an clause with two helper variables which always is true, we can control the outcome of the algorithm by a third helper variable by $\wedge$ it on a clause.

(< 3)-clauses By always $\wedge$ the $\neg y_3$ to $2/3$ of the clauses we will satisfy the SAT-TWO-THIRD problem.

1. $c'_j, 1 = z_1 \vee y_1 \vee y_2$
2. $c'_j, 2 = y_1 \vee z_1 \vee y_2$
3. $c'_j, 3 = y_1 \vee y_2 \vee z_1 \wedge \neg y_3$

(> 3)-clauses By always $\wedge$ the $\neg y_3$ to $2/3$ of the clauses we will satisfy the SAT-TWO-THIRD problem.

1. $c'_j, 1 = z_1 \vee y_1 \vee y_2$
2. $c'_j, 2 = z_2 \vee y_1 \vee y_2$
3. $c'_j, 3 = z_3 \vee y_1 \vee y_2$
4. $c'_j, 4 = z_4 \vee y_1 \vee y_2$

5. $c'_j, 5 = z_5 \vee y_1 \vee y_2 \wedge \neg y_3$
6. $c'_j, 6 = z_6 \vee y_1 \vee y_2 \wedge \neg y_3$

b.2)

NA

b.3)

NA