Computationally Hard Problems

Student name and id: Anders H. Opstrup (s160148)

Collaborator name(s) and id(s): Salik Lennert Pedersen (s134416)

Hand-in for week: 2

Exercise 1

Given is a disjunctive form consisting of k monomials $m_1, ..., m_k$ over n boolean variables $x_1, ..., x_n$. The task is to decide if there is a truth assignment to the variables such that the truth value of the disjunctive form is false.

You are given a decision algorithm A_d that solves this problem, i. e., for each instance to REFUTATION, A_d answers YES if there is an assignment that makes the truth value of the disjunctive form false; otherwise it answers NO.

a)

Describe an algorithm A_o which solves the optimization problem, that is, which finds a truth assignment making the disjunctive form false if one exists. The running time has to be polynomial in the input size and A_o may make calls to A_d. Such calls count as one basic computational step.

b)

Argue that your algorithm is correct.

c)

Prove that the running time of the algorithm is bounded from above by a polynomial. Any polynomial is sufficient; you need not look for a polynomial of minimal degree. Recall that a call to A_d counts one step.

Note: The input of REFUTATION is disjunctive form of monomials over n boolean variables, nothing else. In particular, a legal input cannot specify specific settings of variables.