CPC Framework - Nonlinear Curve Fitting

Peicong Cheng[1*] Makoto Yamashita[1]

[1]School of Mathematical and Computer Science, Institute of Science Tokyo, Japan

1 Introduction

This document presents a collection of challenging optimization problems under the CPC Framework, specifically targeting on solving nonlinear curve fitting problem. All problems included here are newly proposed and designed to test the robustness and effectiveness of global optimization algorithms.

Users are encouraged to experiment with these problems and share their results. If a significantly improved and correct solution is discovered, please feel free to contact the author. Verified updates will be recorded and reflected in future versions of this document.

The benchmark set will continuous to expand with more difficult and diverse problems. For detailed information about each problem, please refer to the corresponding individual folders.

All figures in this document were generated using GNUplot.

If you use this benchmark in your research or publications, please cite the CPC Framework accordingly. Citation information can be found from README.

*Corresponding author: `cheng.p.b2bf@m.isct.ac.jp`

2 Overview of Nonlinear Curve Fitting

The brief overview of the characteristics of all benchmark functions for curve fitting is provided in Table 1. It summarizes five key properties including number of fitting parameters (#FP), number of observations (#Obs.). Further details are presented in section 3.

Table 1: Overview of characteristics for CPC Benchmark Functions

Fun ID	#FP	#Obs.	Fun ID	#FP	#Obs.
CPC-CF1	2	6	CPC-CF20	4	11
CPC-CF2	6	10	CPC-CF21	6	12
CPC-CF3	6	17	CPC-CF22	6	11
CPC-CF4	6	6	CPC-CF23	6	12
CPC-CF5	6	10	CPC-CF24	5	11
CPC-CF6	6	7	CPC-CF25	6	9
CPC-CF7	6	11	CPC-CF26	6	9
CPC-CF8	5	5	CPC-CF27	5	7
CPC-CF9	6	7	CPC-CF28	6	18
CPC-CF10	7	15	CPC-CF29	5	11
CPC-CF11	7	10	CPC-CF30	6	14
CPC-CF12	8	16	CPC-CF31	7	8
CPC-CF13	5	11	CPC-CF32	5	15
CPC-CF14	5	19	CPC-CF33	5	5
CPC-CF15	6	11	CPC-CF34	5	10
CPC-CF16	6	7	CPC-CF35	5	11
CPC-CF17	5	12	CPC-CF36	5	10
CPC-CF18	6	8	CPC-CF37	5	8
CPC-CF19	5	6	CPC-CF38	5	8

3 Detailed Formulations, Fitting Data and Visualizations

CPC-CF1

Fitting Formula

$$y = b_1 \cdot x^{b_2} \qquad \text{(CPC-CF1)}$$

Fitting Data:
x = 20, 25, 27, 30, 35, 38.5;
y = 4.62e-8, 1.02e-7, 1.27e-7, 2.14e-7, 3.56e-7, 4.69e-7;
Number of Fitting Parameters: 2
Number of Observations: 6
Best Known Solution:

$$b_1 = 1.59002016210314E-12$$
$$b_2 = 3.45494898292865$$

Corresponding SSE: 5.93866286357495E-16
Figure: See Figure 1.

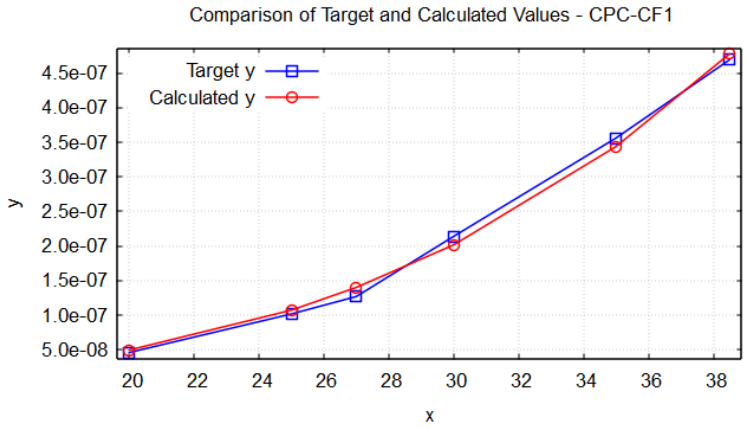

Figure 1: Illustration of CPC-CF1

CPC-CF2

Fitting Formula

$$y = b_1 - b_2 \cdot \exp\left(b_3 \cdot x + \frac{1}{b_4 \cdot x^{b_5} + b_6 + 1}\right) \qquad \text{(CPC-CF2)}$$

Fitting Data:
x = 0.9, 1.5, 13.8, 19.8, 24.1, 28.2, 35.2, 60.3, 74.6, 81.3;
y = 455.2, 428.6, 124.1, 67.3, 43.2, 28.1, 13.1, -0.4, -1.3, -1.5;
Number of Fitting Parameters: 6
Number of Observations: 10
Best Known Solution #1:

$$b_1 = -1.51915803043969 \qquad b_2 = -499.722104508364$$
$$b_3 = -0.099985832892196 \qquad b_4 = -592291.565458672$$
$$b_5 = -2.4441176458665 \qquad b_6 = 11.2101279187105$$

Corresponding SSE #1: 0.000745812770809716
Best known Solution #2:

$$b1 = -1.51915802763208 \qquad b2 = 542.371607095582$$
$$b3 = -0.0999858329115727 \qquad b4 = 0.000251712278624172$$
$$b5 = 2.44411786504324 \qquad b6 = -13.210126186887$$

Corresponding SSE #2: 0.000745812770810948
Figure: See Figure 2.

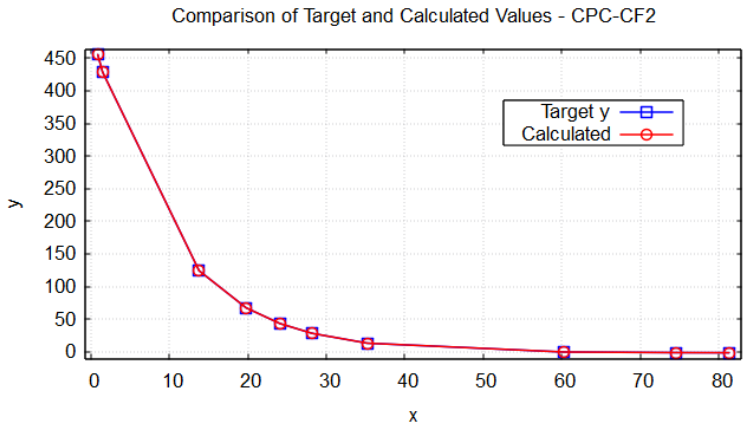

Figure 2: Illustration of CPC-CF2

CPC-CF3

Fitting Formula

$$y = \frac{b_1}{1 + b_2 \cdot \exp\left(-b_3 \cdot x - b_4 \cdot x^2\right)} + \frac{2 \cdot b_4}{\left(1 + b_5 \cdot \exp\left(-b_6 \cdot x\right)\right)^{b_4}} \qquad \text{(CPC-CF3)}$$

Fitting Data:

x=-6.7, -6.3, -5.9, -5.5, -5.1, -4.7, -4.3, -3.7, -3.3, -2.3, -1.9, -1.5, -1.2, -0.7, -0.2, 0.3, 0.7;

y=0.06, 0.1, 0.14, 0.22, 0.34, 0.5, 0.62, 0.72, 0.75, 0.87, 0.95, 1.05, 1.15, 1.35, 1.45, 1.5, 1.54;

Number of Fitting Parameters: 6
Number of Observations: 17
Best Known Solution:

$$b_1 = 0.698457049712715 \qquad b_2 = 6.14345552284075E - 10$$
$$b_3 = 6.31959826250225 \qquad b_4 = 0.420540567382403$$
$$b_5 = 0.172043333824482 \qquad b_6 = 2.43316659528823$$

Corresponding SSE: 0.000916735191780284
Figure: See Figure 3.

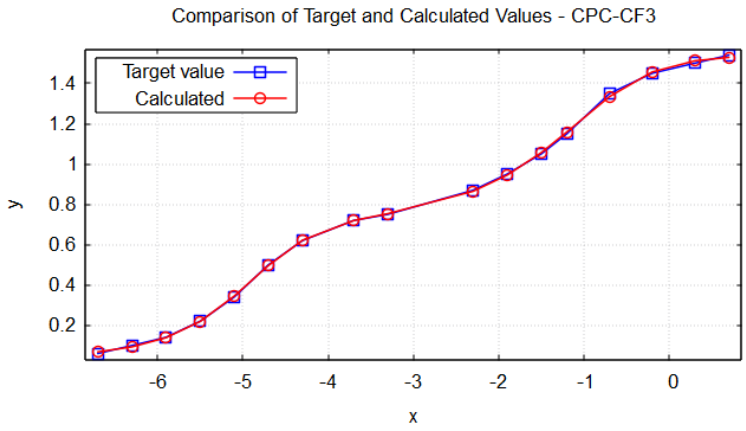

Figure 3: Illustration of CPC-CF3

CPC-CF4

Fitting Formula

$$y = p_1 \cdot p_2^x \cdot (x + p_3)^{p_4} + \frac{p_5 \cdot x}{1 + p_6 \cdot x} \qquad \text{(CPC-CF4)}$$

Fitting Data:
x=0, 2, 4, 6, 8, 10;
y=0.0242, 0.2792, 0.1386, 0.0238, 0.0034, 0.002;
Number of Fitting Parameters: 6
Number of Observations: 6
Best Known Solution:

$$b_1 = 0.0022116342798846 \qquad b_2 = \pm 0.12826238840171$$
$$b_3 = 1.3857145262211 \qquad b_4 = 7.33447028271487$$
$$b_5 = 8.57023065004798E-5 \qquad b_6 = -0.053696335809848$$

Corresponding SSE: 6.64220506077396E-26
Figure: See Figure 4.

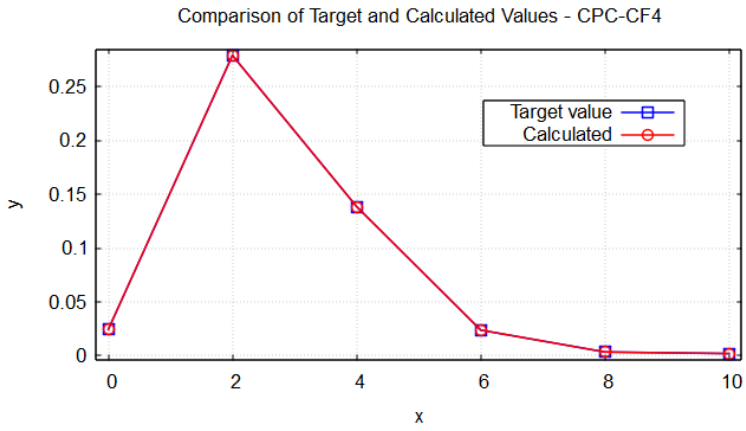

Figure 4: Illustration of CPC-CF4

CPC-CF5

Fitting Formula

$$y = \frac{p_1}{1 + p_2 \cdot \exp\left(-p_3 \cdot x\right) + p_4 \cdot x^{p_5}} + p_6 \cdot x \qquad \text{(CPC-CF5)}$$

Fitting Data:

x=1, 2, 3, 4, 5, 6, 7, 8, 9, 10;

y=161.68, 170.99, 179.42, 185.64, 189.29, 192.58, 196.32, 205.40, 240.02, 293.98;

Number of Fitting Parameters: 6

Number of Observations: 10

Best Known Solution:

$$b_1 = 153.267512886014 \qquad b_2 = 0.000593206672503047$$
$$b_3 = -0.957451012183078 \qquad b_4 = -4.79985248441563E-9$$
$$b_5 = 9.2625305177603 \qquad b_6 = 9.03913523103294$$

Corresponding SSE: 2.75159063173681

Figure: See Figure 5.

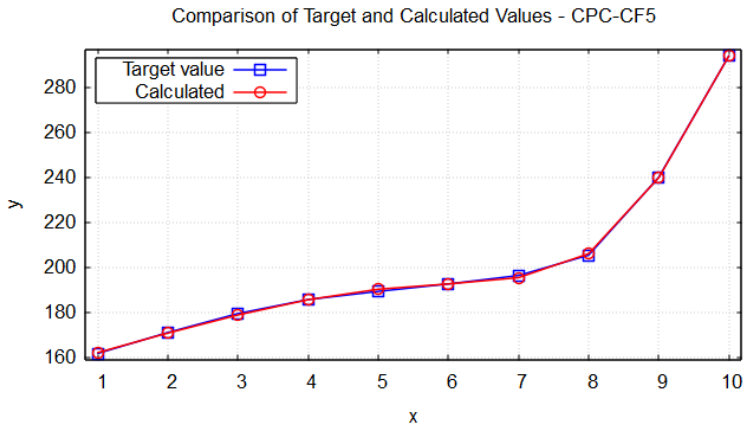

Figure 5: Illustration of CPC-CF5

CPC-CF6

Fitting Formula

$$y = b_1 \cdot \exp\left(\frac{b_2}{x} - b_3 \cdot x + b_4 \cdot x^{b_5}\right) + b_6 \cdot x \qquad \text{(CPC-CF6)}$$

Fitting Data:
x=0.17, 0.33, 0.83, 1.67, 2.5, 3.33, 5;
y=6.62, 5.58, 4.63, 5.08, 5.42, 5.82, 6.78;
Number of Fitting Parameters: 6
Number of Observations: 7
Best Known Solution:

$b_1 = -1.10349462824345E-7$ $\quad b_2 = -0.062912903053405$
$b_3 = 0.0103969360287926$ $\quad b_4 = 21.4556813933589$
$b_5 = 0.0451212897891041$ $\quad b_6 = 217.976039884328$

Corresponding SSE: 0.0071283690038996
Figure: See Figure 6.

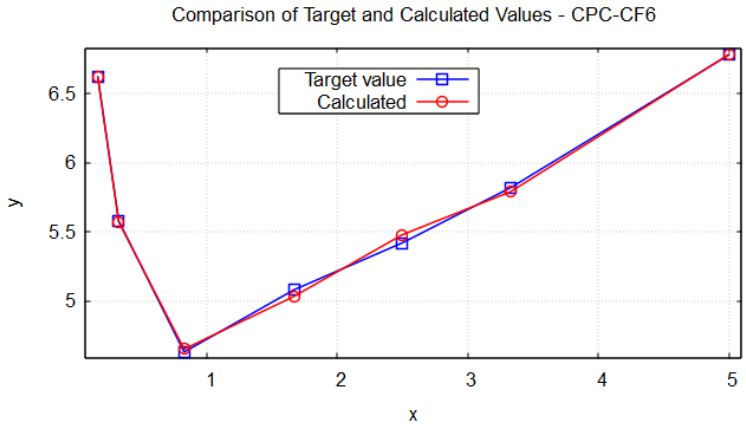

Figure 6: Illustration of CPC-CF6

CPC-CF7

Fitting Formula

$$y = p_1 + p_2 \cdot \exp\left(p_3 \cdot \exp\left(p_4 \cdot x\right) + p_5 \cdot x\right) + p_6 \cdot x \qquad \text{(CPC-CF7)}$$

Fitting Data:
x = 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30;
y = 56.7, 59.8, 64.8, 66.7, 66.2, 65.2, 64.2, 63.5, 63.0, 62.7, 62.5;
Number of Fitting Parameters: 6
Number of Observations: 11
Best Known Solution:

$$b_1 = 48.6652527718107 \qquad b_2 = 3385.10838634086$$
$$b_3 = -7438272163.60286 \qquad b_4 = -1.05231544302434$$
$$b_5 = -0.248420824416755 \qquad b_6 = 0.396466004146532$$

Corresponding SSE: 0.0038932836085615
Figure: See Figure 7.

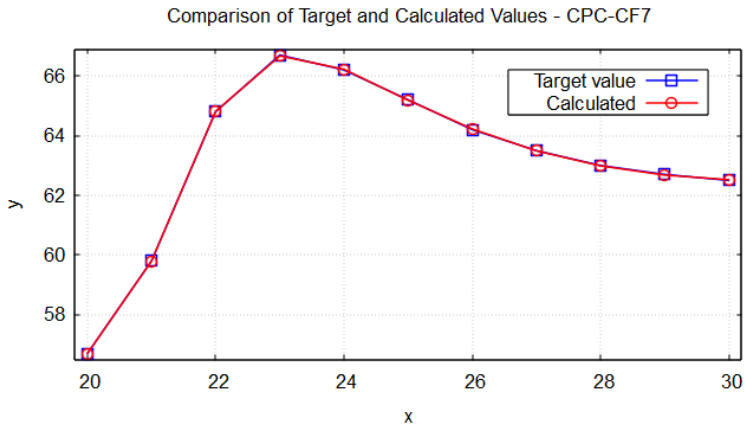

Figure 7: Illustration of CPC-CF7

CPC-CF8

Fitting Formula

$$y = b_1 \cdot \exp(b_2 \cdot x) + b_3 \cdot \exp\left(\frac{b_4}{x}\right) + b_5 \qquad \text{(CPC-CF8)}$$

Fitting Data:
x=250, 275, 300, 325, 350;
y=2, 6, 15, 19, 37;
Number of Fitting Parameters: 5
Number of Observations: 5
Best Known Solution:

$b_1 = 1.11507876865241E-7$ $b_2 = 0.0657510708243728$
$b_3 = -4246745414532.71$ $b_4 = -7737.47845772536$
$b_5 = 0.618599549954133$

Corresponding SSE: 6.83633006184019E-23
Figure: See Figure 8.

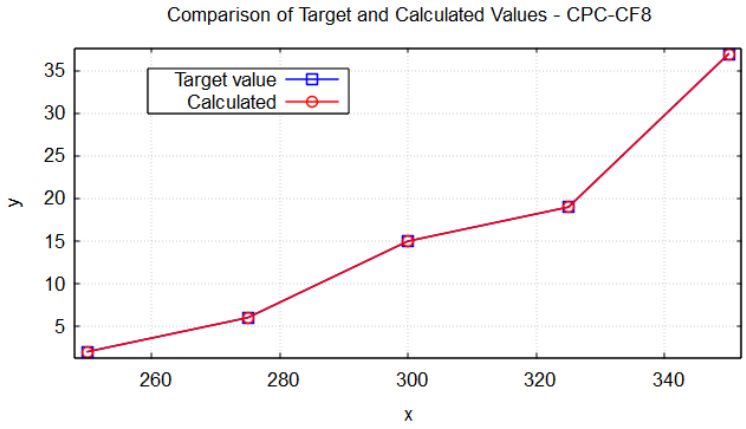

Figure 8: Illustration of CPC-CF8

CPC-CF9

Fitting Formula

$$y = \frac{p_3}{1 + \exp(p_1 \cdot x + p_5)} + \frac{p_3}{1 + p_4 \cdot \exp(p_2 \cdot x)} + p_6 \cdot x + p_5 \qquad \text{(CPC-CF9)}$$

Fitting Data:
x=1.75, 2.25, 2.493, 2.5, 2.563, 2.686, 2.885;
y=260, 320, 381, 440, 460, 500, 570;
Number of Fitting Parameters: 6
Number of Observations: 7
Best Known Solution:

$$b_1 = 1365.39708276845 \qquad b_2 = 0.00810989182971912$$
$$b_3 = -58.7912362517774 \qquad b_4 = -1.01059935225442$$
$$b_5 = -3409.9964336119 \qquad b_6 = 789.317040718249$$

Corresponding SSE: 0.366692001219814
Figure: See Figure 9.

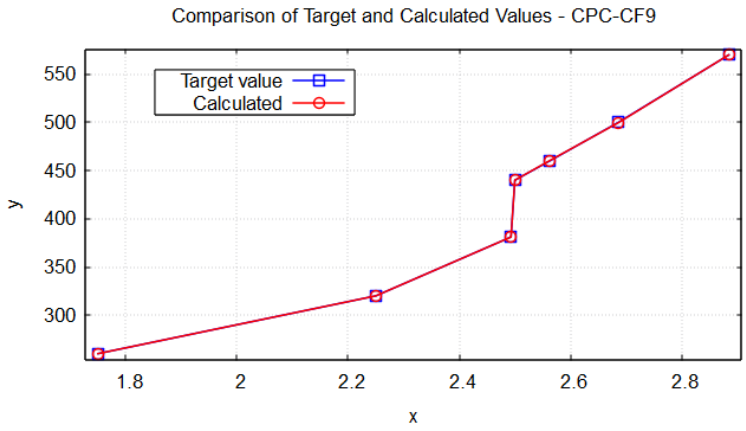

Figure 9: Illustration of CPC-CF9

CPC-CF10

Fitting Formula

$$y = b_1 + b_2 \cdot \exp\left(b_3 \cdot (x - b_4)^2\right) + b_5 \cdot \exp\left(b_6 \cdot (x - b_7)^2\right) \qquad \text{(CPC-CF10)}$$

Fitting Data:
x=1, 2, 4, 6, 8, 10, 15, 20, 30, 40, 60, 90, 120, 150, 180;
y=0, 0, 0, 0, 0.20640, 0.06304, 0.08540, 0.56381, 0.94651, 1.46233, 1.73185, 1.70072, 1.82921, 2.09236, 2.26539;
Number of Fitting Parameters: 7
Number of Observations: 15
Best Known Solution:

$$b_1 = 2.33075927498162 \qquad b_2 = -2.25324442913129$$
$$b_3 = -0.000959463568279876 \qquad b_4 = 3.07215726604237$$
$$b_5 = -0.634591095156797 \qquad b_6 = -0.000285778972517449$$
$$b_7 = 91.2302020325827$$

Noted that (b_2, b_3, b_4) and (b_5, b_6, b_7) can be swapped equally and correspondingly.
Corresponding SSE: 0.0665420273579987
Figure: See Figure 10.

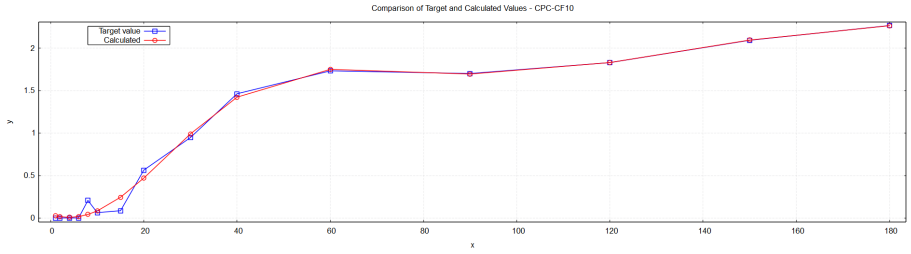

Figure 10: Illustration of CPC-CF10

CPC-CF11

Fitting Formula

$$y = b_1 + b_2 \cdot \exp\left(b_3 \cdot (x - b_4)^2\right) + b_5 \cdot \exp\left(b_6 \cdot (x - b_7)^2\right) \qquad \text{(CPC-CF11)}$$

Fitting Data:
x=0, 10, 20, 30, 40, 50, 60, 70, 80, 90;
y=0, 4.808, 18.367, 37.920, 58.726, 74.756, 81.454, 79.025, 71.244, 61.629;
Number of Fitting Parameters: 7
Number of Observations: 10
Best Known Solution:

$$b_1 = -274.11200871452 \qquad b_2 = 25.4564055048359$$
$$b_3 = 274.151543080075 \qquad b_4 = -0.0989450431603676$$
$$b_5 = 19.2328889871382 \qquad b_6 = 0.0174006444455463$$
$$b_7 = -0.670531583929066$$

Noted that (b_2, b_3, b_4) and (b_5, b_6, b_7) can be swapped equally and correspondingly.
Corresponding SSE: 0.00704568539168775
Figure: See Figure 11.

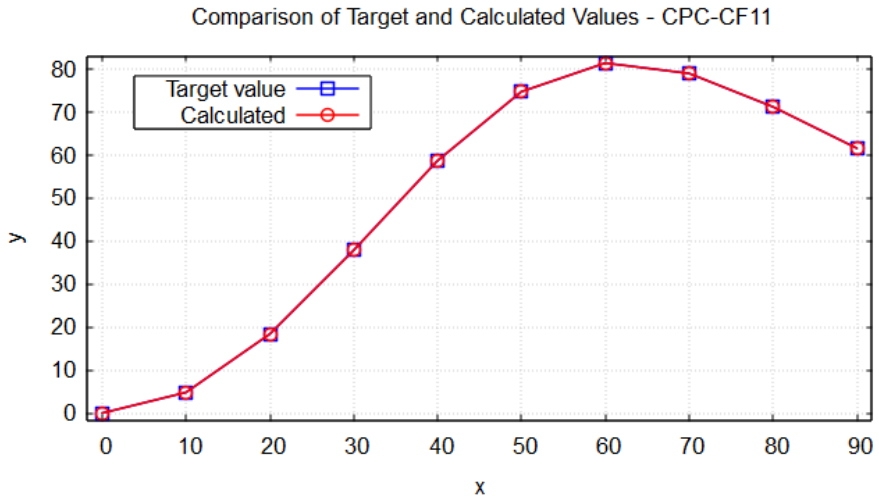

Figure 11: Illustration of CPC-CF11

CPC-CF12

Fitting Formula

$$y = b_1 + b_2 \cdot x + b_3 \cdot \exp\left(-\left(b_4 \cdot (x - b_5)\right)^2\right) + b_6 \cdot \exp\left(-\left(b_7 \cdot (x - b_8)\right)^2\right) \quad \text{(CPC-CF12)}$$

Fitting Data:

x = 0.975, 0.985, 0.995, 1.015, 1.025, 1.03, 1.035, 1.04, 1.045, 1.05, 1.055, 1.06, 1.07, 1.09, 1.1, 1.13;

y = 306, 323, 438, 828, 1068, 1195, 1267, 1257, 1243, 1246, 1235, 1126, 889, 474, 327, 237;

Number of Fitting Parameters: 8

Number of Observations: 16

Best Known Solution:

$$\begin{aligned} b_1 &= 466.131697605529 & b_2 &= -205.72868822854 \\ b_3 &= 1082.88349997631 & b_4 &= \pm 27.64683371532 \\ b_5 &= 1.04396273654461 & b_6 &= -95.6472460398537 \\ b_7 &= \pm 148.590514483893 & b_8 &= 1.04468788399754 \end{aligned}$$

Noted that (b_3, b_4, b_5) and (b_6, b_7, b_8) can be swapped equally and correspondingly.

Corresponding SSE: 1391.50187843587

Figure: See Figure 12.

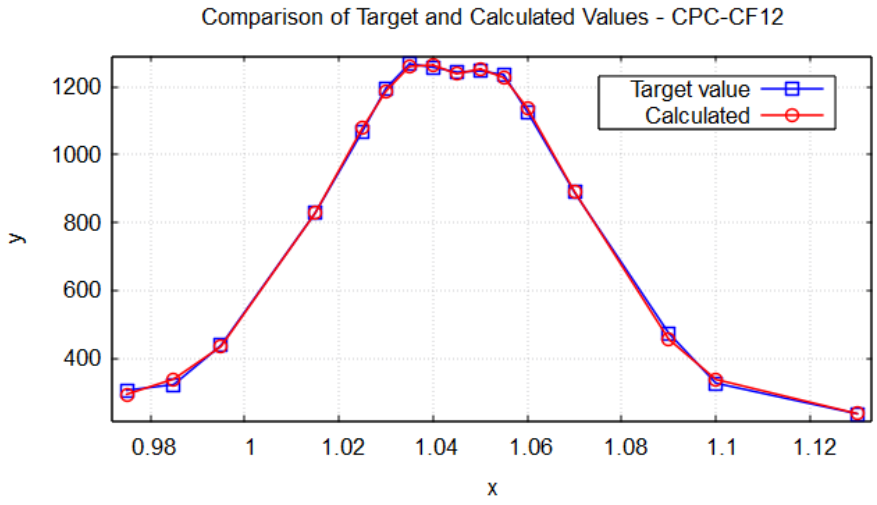

Figure 12: Illustration of CPC-CF12

CPC-CF13

Fitting Formula

$$y = b_1 + b_2 \cdot \left(\frac{b_3}{1 + \exp\left((x - b_4) \cdot b_5\right)} + \frac{1 - b_3}{1 + \exp\left((x - b_5) \cdot b_4\right)} \right) \quad \text{(CPC-CF13)}$$

Fitting Data:
x = -14, -12.45, -9.36, -6.26, -3.17, 0.07, 3.03, 6.12, 9.22, 12.31, 14;
y = 166, 123, 47.5, -8.85, -30.6, 0.842, 30.6, 10.6, -44.5, -116, -165;
Number of Fitting Parameters: 5
Number of Observations: 11
Best Known Solution:

$$b_1 = -5781.61197398192 \qquad b_2 = 11584.3739131452$$
$$b_3 = 0.0389953445407397 \qquad b_4 = 0.00995153809375899$$
$$b_5 = -0.363412878455352$$

Corresponding SSE: 24.8301836379401
Figure: See Figure 13.

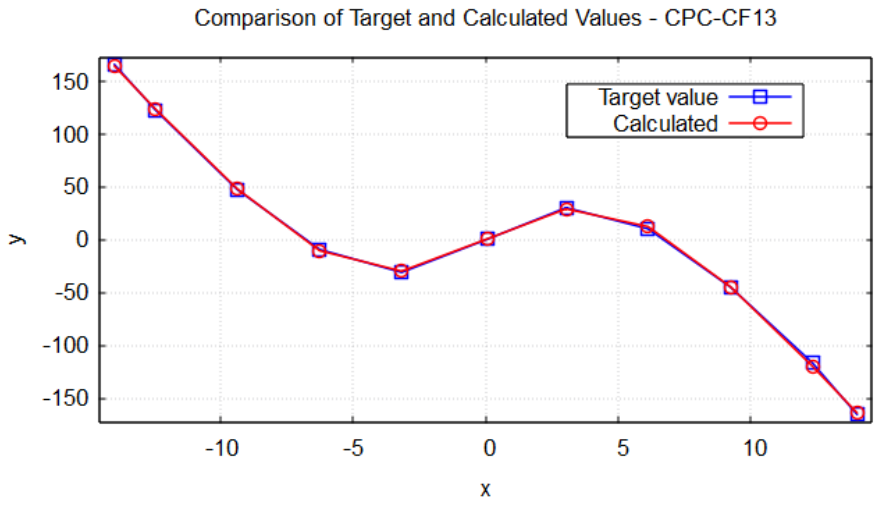

Figure 13: Illustration of CPC-CF13

CPC-CF14

Fitting Formula

$$y = b_1 + b_2 \cdot x^{b_3} + b_4 \cdot x^{b_5} \qquad \text{(CPC-CF14)}$$

Fitting Data:
x=5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95;
y=3.3, 6.5, 9.2, 11.9, 14.5, 17.0, 19.4, 21.7, 23.9, 25.9, 27.6, 29.2, 30.0, 30.3, 30.0, 29.2, 26.0, 21.0, 12.0;
Number of Fitting Parameters: 5
Number of Observations: 19
Best Known Solution:

$$b_1 = -1.49544302130361 \qquad b_2 = 1.35116693752419$$
$$b_3 = 0.769997162759275 \qquad b_4 = -1.28601055506628E-13$$
$$b_5 = 7.274073072499$$

Noted that (b_2, b_3) and (b_4, b_5) can be swapped equally and correspondingly.
Corresponding SSE: 0.909227033679008
Figure: See Figure 14.

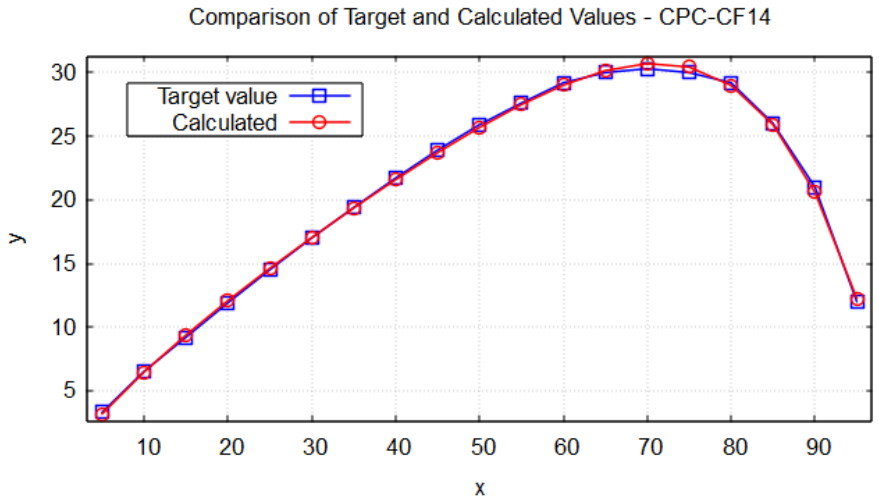

Figure 14: Illustration of CPC-CF14

CPC-CF15

Fitting Formula

$$y = \frac{b_1}{1 + b_2 \cdot \exp(b_3 \cdot x)} + \frac{b_4}{1 + b_5 \cdot \exp(b_6 \cdot x)} \qquad \text{(CPC-CF15)}$$

Fitting Data:

x=0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100;

y=92, 93, 94.5, 96, 98, 100, 101, 100, 93, 83, 74;

Number of Fitting Parameters: 6

Number of Observations: 11

Best Known Solution:

$$b_1 = 89.6413167283419 \qquad b_2 = 0.00034899783066984$$
$$b_3 = 0.0892308859926423 \qquad b_4 = 115.20449521865$$
$$b_5 = 45.9036855279558 \qquad b_6 = -0.0353460628497896$$

Noted that (b_1, b_2, b_3) and (b_4, b_5, b_6) can be swapped equally and correspondingly.

Corresponding SSE: 0.299364969714065

Figure: See Figure 15.

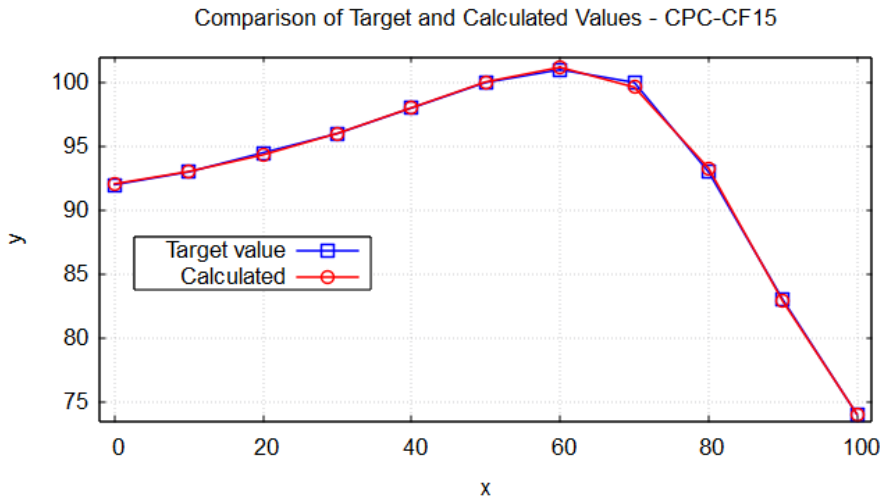

Figure 15: Illustration of CPC-CF15

CPC-CF16

Fitting Formula

$$y = b_1 \cdot \exp\left(\frac{b_2}{x} - b_3 \cdot x + b_4 \cdot x^{b_5}\right) + b_6 \cdot x \qquad \text{(CPC-CF16)}$$

Fitting Data:
x=0.17, 0.33, 0.83, 1.67, 2.5, 3.33, 5;
y=6.62, 5.58, 4.63, 5.08, 5.42, 5.82, 6.78;
Number of Fitting Parameters: 6
Number of Observations: 7
Best Known Solution:

$b_1 = -1.10400041286257E-7$ $b_2 = -0.0629140576286658$
$b_3 = 0.0103971755417879$ $b_4 = 21.4552086780548$
$b_5 = 0.0451222560592873$ $b_6 = 217.972654785621$

Corresponding SSE: 0.00712836900559349
Figure: See Figure 16.

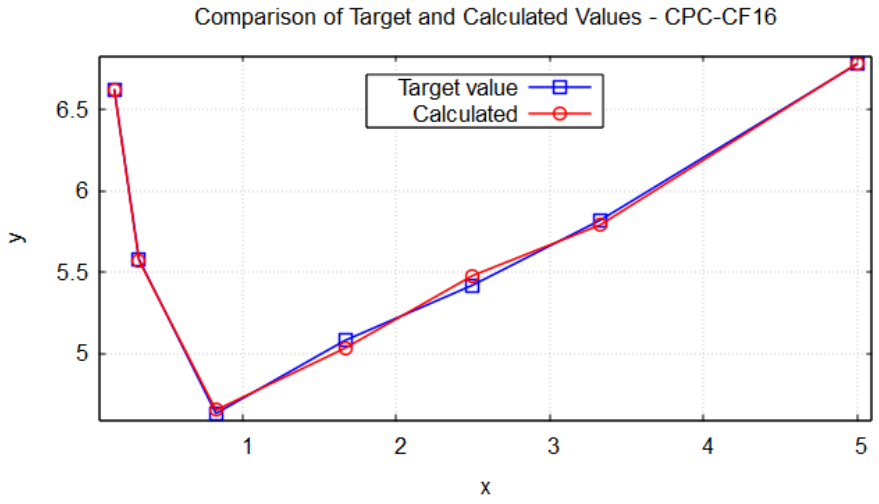

Figure 16: Illustration of CPC-CF16

CPC-CF17

Fitting Formula

$$y = b_1 \cdot \left(\left(\left(\frac{x - b_2}{(1 - b_2) \cdot x + b_3} \right)^2 \right) + b_4 \cdot x + 2.1 \right)^{b_5} \qquad \text{(CPC-CF17)}$$

Fitting Data:
x=1, 0.963, 0.940, 0.810, 0.639, 0.564, 0.495, 0.456, 0.374, 0.324, 0.289, 0.248;
y=0, 7.642, 84.369, 730.794, 943.654, 1203.544, 1489.304, 1814.872, 2541.681, 3399.595, 4385.219, 5469.341;
Number of Fitting Parameters: 5
Number of Observations: 12
Best Known Solution:

$b_1 = 978429605337.302$ $\quad b_2 = 0.856032240531499$
$b_3 = -0.148796029582726$ $\quad b_4 = 101.198417866751$
$b_5 = -4.71601411789011$

Corresponding SSE: 34105.9360420192
Figure: See Figure 17.

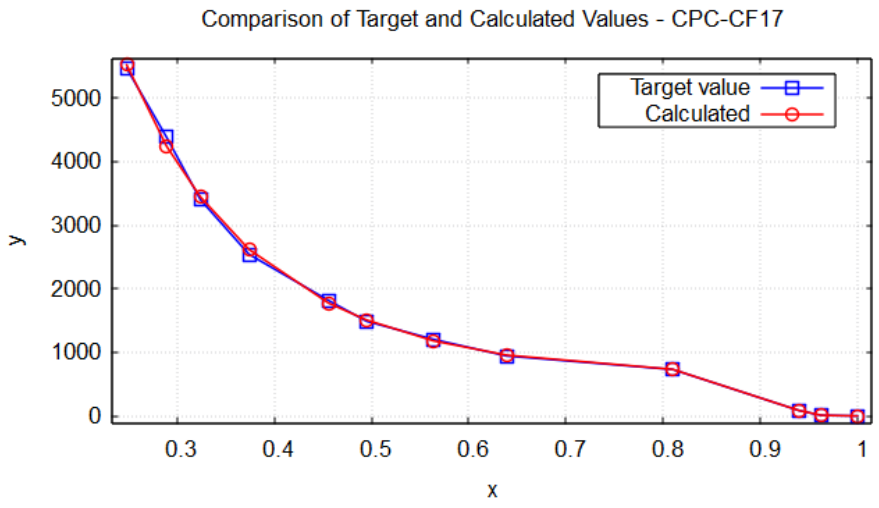

Figure 17: Illustration of CPC-CF17

CPC-CF18

Fitting Formula

$$y = b_1 \cdot \exp(b_2 \cdot x) + b_3 \cdot \exp\left(b_4 \cdot x^{0.5} + \frac{b_5}{x}\right) + b_6 \qquad \text{(CPC-CF18)}$$

Fitting Data:
x=0.392, 0.793, 1.189, 2.39, 4.787, 7.191, 9.385, 11.784;
y=1.63, 6.79, 6.79, 3.88, 1.83, 1.25, 0.86, 0.624;
Number of Fitting Parameters: 6
Number of Observations: 8
Best Known Solution:

$$b_1 = -114.314492876071 \qquad b_2 = -3.80969022465523$$
$$b_3 = 65.5059383843122 \qquad b_4 = -0.00138626190894187$$
$$b_5 = 0.136120736633647 \qquad b_6 = -65.3153025898425$$

Corresponding SSE: 0.00551217990403637
Figure: See Figure 18.

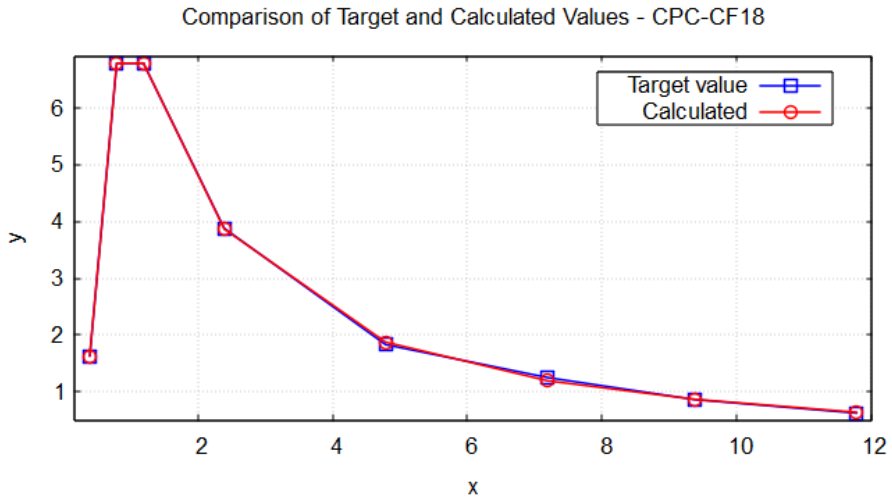

Figure 18: Illustration of CPC-CF18

CPC-CF19

Fitting Formula

$$y = \frac{b_1}{1 + b_2 \cdot x + b_3 \cdot x^2} + b_4 \cdot \exp\left(b_5 \cdot x\right) \qquad \text{(CPC-CF19)}$$

Fitting Data:
x=1.5, 2.5, 3.5, 4.5, 5.5, 6.5;
y=1.69, 1.636, 1.387, 1.254, 1.444, 0.1631;
Number of Fitting Parameters: 5
Number of Observations: 6
Best Known Solution:

$$b_1 = 2.80048435154845 \qquad b_2 = -0.252724837453752$$
$$b_3 = 0.0171406613968031 \qquad b_4 = -1.16778637281153$$
$$b_5 = 0.51958596410434$$

Corresponding SSE: 0.00125250718130954
Figure: See Figure 19.

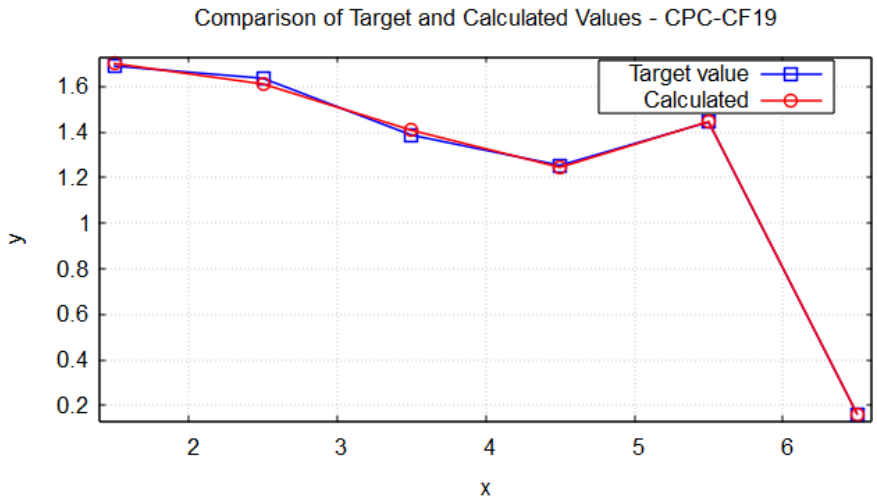

Figure 19: Illustration of CPC-CF19

CPC-CF20

Fitting Formula

$$y = b_1 \cdot \exp\left(-b_2 \cdot \exp\left(b_3 \cdot x\right)\right) + b_4 \qquad \text{(CPC-CF20)}$$

Fitting Data:
x=11.95, 12.22, 12.88, 13.50, 13.96, 14.25, 14.32, 14.52, 14.78, 15.04, 15.30;
y=7.66, 7.66, 7.52, 7.10, 6.50, 6.01, 5.90, 5.61, 5.37, 5.28, 5.25;
Number of Fitting Parameters: 4
Number of Observations: 11
Best Known Solution:

$$b_1 = 2.45379459644838 \qquad b_2 = 2.61800795774347E-12$$
$$b_3 = 1.88303559146416 \qquad b_4 = 5.26129573148535$$

Corresponding SSE: 0.000864905914490896
Figure: See Figure 20.

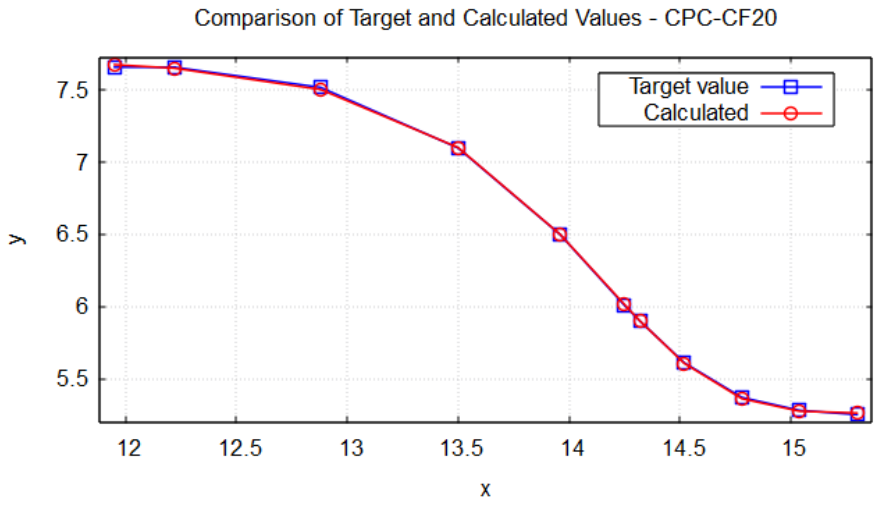

Figure 20: Illustration of CPC-CF20

CPC-CF22

Fitting Formula

$$y = b_1 + b_2 \cdot \exp\left(-\exp\left(0.03 \cdot (x - b_3)^{b_4}\right) - b_5 \cdot (x - b_3) + 1\right) + b_6 \cdot x \quad \text{(CPC-CF22)}$$

Fitting Data:
x=0, 0.017, 0.034, 0.068, 0.102, 0.136, 0.203, 0.237, 0.305, 0.373, 0.441, 0.492;
y=3.33, 5, 7.4, 8.61, 7.72, 5.98, 3.16, 2.4, 1.15, 0.56, 0.33, 0.26;
Number of Fitting Parameters: 6
Number of Observations: 12
Best Known Solution:

$$b_1 = 1.71260037290387 \qquad b_2 = 272701.106397653$$
$$b_3 = -0.567674292750943 \qquad b_4 = -6.93866083502803$$
$$b_5 = 15.0025007621966 \qquad b_6 = -3.26254244023447$$

Corresponding SSE: 0.181494338924873
Figure: See Figure 22.

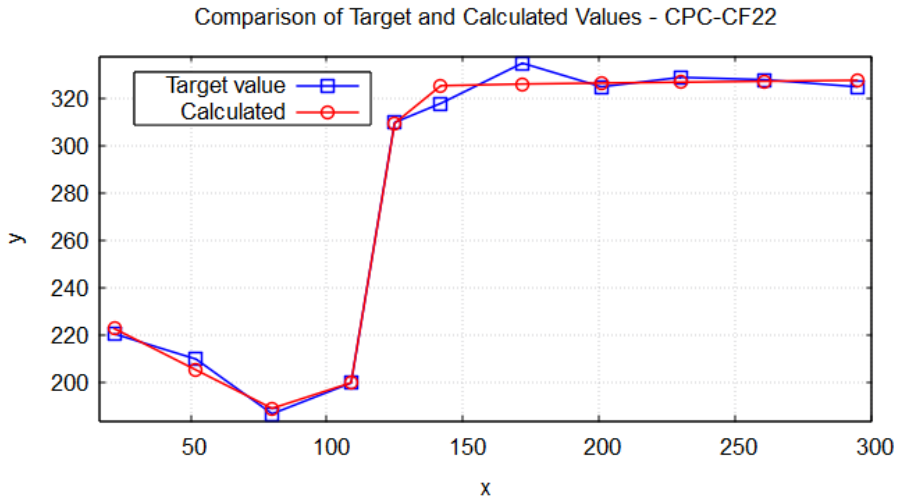

Figure 21: Illustration of CPC-CF22

CPC-CF22

Fitting Formula

$$y = \frac{b_1 + b_2 \cdot x + b_3 \cdot \exp(b_4 \cdot x)}{1 + b_5 \cdot \exp(b_6 \cdot x)} \qquad \text{(CPC-CF22)}$$

Fitting Data:

x=22, 52, 80, 109, 125, 142, 172, 201, 230, 261, 295;

y=220.7, 210.1, 187, 200, 310, 318, 335, 325, 329, 328, 325;

Number of Fitting Parameters: 6

Number of Observations: 11

Best Known Solution:

$b_1 = 235.742027737864$ $\quad b_2 = -0.581391364002116$

$b_3 = 6.95718844154625E-10$ $\quad b_4 = 0.232692391487199$

$b_5 = 2.14684970177838E-12$ $\quad b_6 = 0.232653896144278$

Corresponding SSE: 179.333517780975

Figure: See Figure 22.

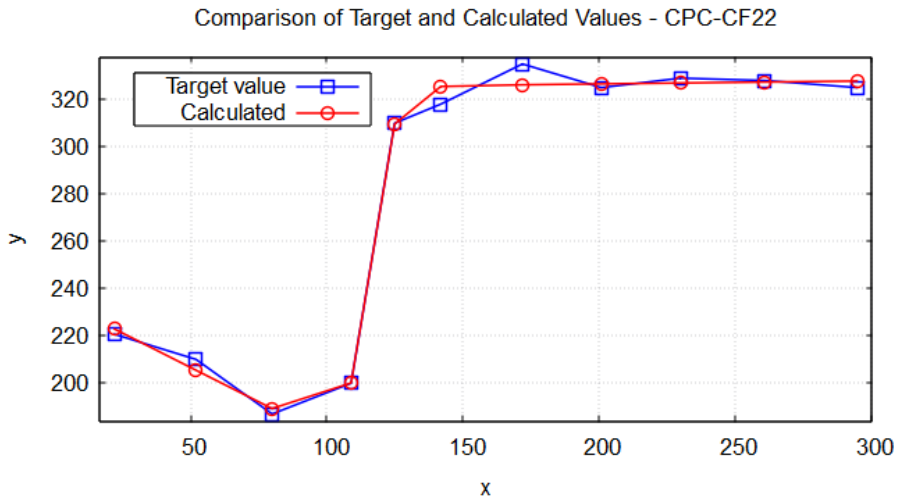

Figure 22: Illustration of CPC-CF22

CPC-CF23

Fitting Formula

$$y = \frac{b_1}{1 + b_2 \cdot \exp(-b_3 \cdot x) + b_4 \cdot x} + b_5 \cdot x^{b_6} \qquad \text{(CPC-CF23)}$$

Fitting Data:

x=2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13;

y=0.3, 2.1, 7.7, 13.8, 12.9, 12.5, 9.8, 9, 9.1, 9.7, 9.2, 10.8;

Number of Fitting Parameters: 6

Number of Observations: 12

Best Known Solution:

$$b_1 = -50.7626837382563 \qquad b_2 = -3665.0456051457$$
$$b_3 = 1.67897487690146 \qquad b_4 = -0.780818160788405$$
$$b_5 = 4.61799432069399E-5 \qquad b_6 = 4.53154208599279$$

Corresponding SSE: 1.97895314545278

Figure: See Figure 23.

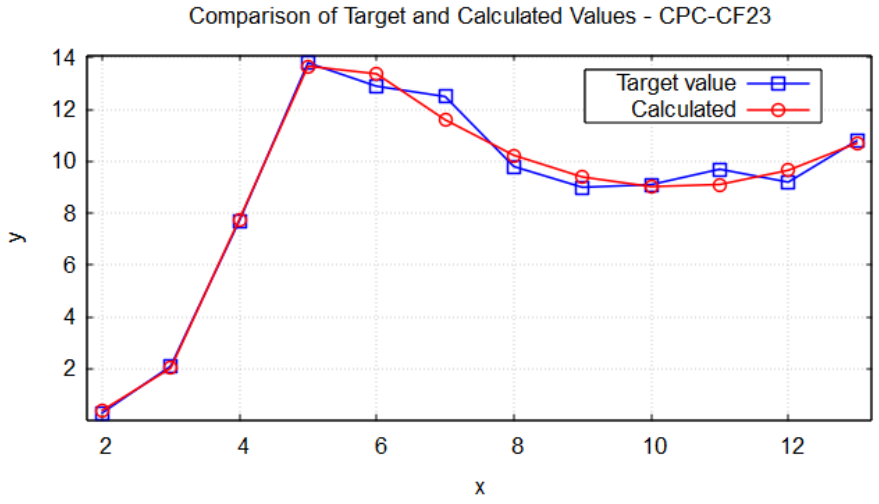

Figure 23: Illustration of CPC-CF23

CPC-CF24

Fitting Formula

$$y = b_1 + b_2 \cdot x + b_3 \cdot x^{b_4} + \frac{b_5}{b_2 + x} \qquad \text{(CPC-CF24)}$$

Fitting Data:
x=140, 150, 180, 220, 260, 280, 290, 298.15, 360, 600, 1000;
y=118.73, 125.55, 144.14, 164.31, 181.26, 188.78, 192.55, 195.18, 213.97, 252.4, 288.67;
Number of Fitting Parameters: 5
Number of Observations: 11
Best Known Solution:

$$b_1 = -372.566668683015 \qquad b_2 = -1989.48819080734$$
$$b_3 = 1992.97238185162 \qquad b_4 = 0.999746861630241$$
$$b_5 = -652020.137923665$$

Corresponding SSE: 0.520071142252271
Figure: See Figure 24.

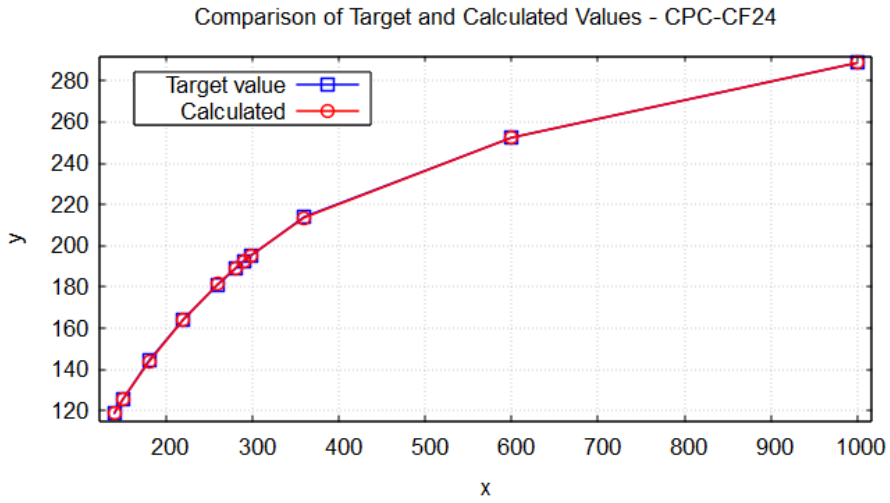

Figure 24: Illustration of CPC-CF24

CPC-CF25

Fitting Formula

$$y = b_1 + \frac{b_2}{1 + \exp(b_3 \cdot x) \cdot b_4 + b_5 \cdot \exp(\exp(b_6 \cdot x))} \qquad \text{(CPC-CF25)}$$

Fitting Data:
x=-7, -6.5, -6, -5.5, -5, -4.5, -4, -3.5, -3.0;
y=1, 5, 14, 19, 23, 26, 26.1, 27, 27.1;
Number of Fitting Parameters: 6
Number of Observations: 9
Best Known Solution:

$b_1 = 0.860857804834604$ $b_2 = 26.390917537766$
$b_3 = -4.2085466850132$ $b_4 = -4.62050597015436E-11$
$b_5 = 0.00031008121054579$ $b_6 = -0.379433202323263$

Corresponding SSE: 0.444992321594586
Figure: See Figure 25.

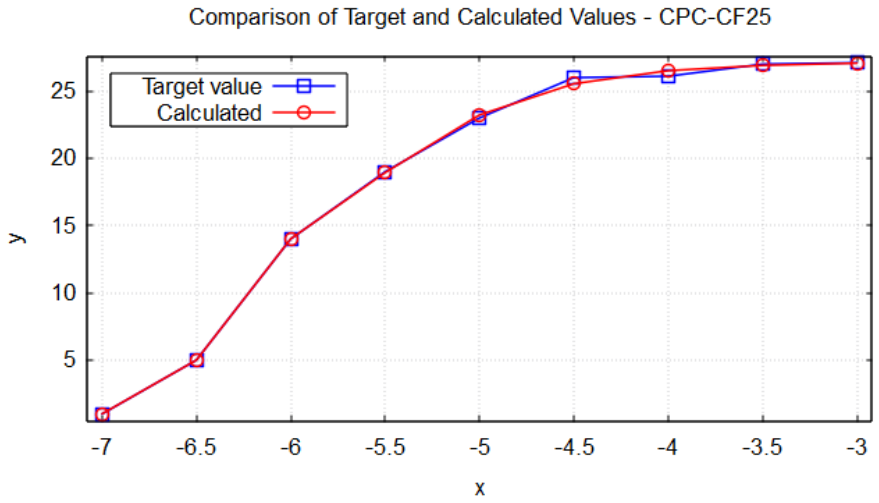

Figure 25: Illustration of CPC-CF25

CPC-CF26

Fitting Formula

$$y = b_1 \cdot \ln(x + b_2) + b_3 \cdot \exp(-b_4 \cdot x) + b_5 \cdot \exp(-b_6 \cdot x) \qquad \text{(CPC-CF26)}$$

Fitting Data:
x=300, 600, 900, 1200, 1800, 3600, 5400, 7200, 27000;
y=39.059, 23.585, 18.418, 15.825, 13.208, 10.363, 8.812, 7.285, 2.007;
Number of Fitting Parameters: 6
Number of Observations: 9
Best Known Solution:

$$b_1 = -13.8111305997956 \qquad b_2 = -191.201968760269$$
$$b_3 = 127.099803685777 \qquad b_4 = -4.3222612821104E-6$$
$$b_5 = -26.3906786572059 \qquad b_6 = 0.00039558333908046$$

Corresponding SSE: 0.00526041261382463
Figure: See Figure 26.

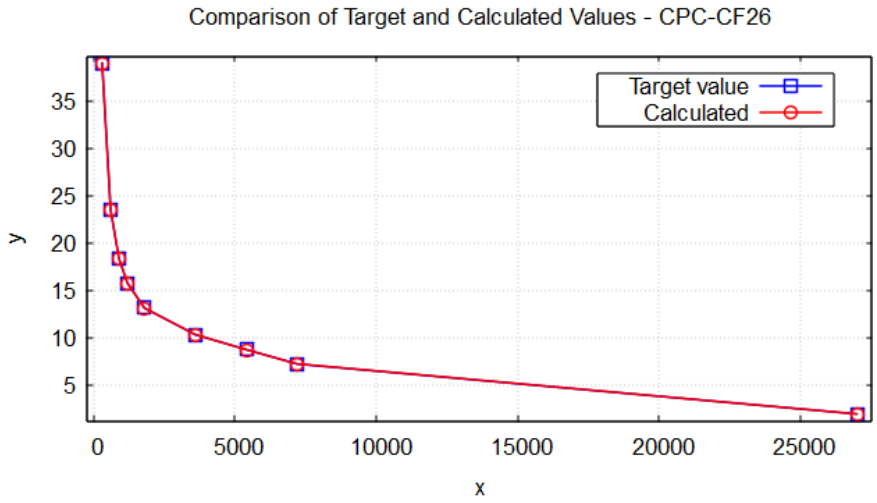

Figure 26: Illustration of CPC-CF26

CPC-CF27

Fitting Formula

$$y = b_1 + b_2 \cdot \exp\left(-0.5 \cdot (b_3 \cdot (x - b_4))^2\right) - b_2 \cdot \exp\left(-0.5 \cdot (2 \cdot b_3 \cdot (x - b_5))^2\right) \quad \text{(CPC-CF27)}$$

Fitting Data:
x=0, 54, 174, 250, 282, 330, 355;
y=98, 17.95, 46.51, 48.37, 11.16, 74.4, 100;
Number of Fitting Parameters: 5
Number of Observations: 7
Best Known Solution:

$$b_1 = 127.98329219402 \qquad b_2 = -394.857267645677$$
$$b_3 = 0.0129256539981901 \qquad b_4 = 177.293233349716$$
$$b_5 = 200.396790405798$$

Corresponding SSE: 6.68197831825399
Figure: See Figure 27.

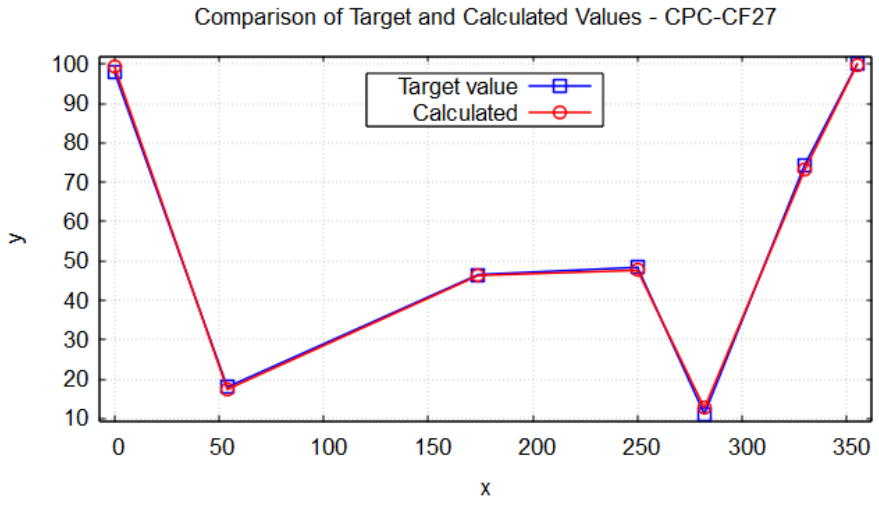

Figure 27: Illustration of CPC-CF27

CPC-CF28

Fitting Formula

$$y = b_1 \cdot \exp\left(b_2 \cdot (x - b_3)^2\right) + b_4 \cdot \exp\left(b_5 \cdot (x - b_6)^2\right) \qquad \text{(CPC-CF28)}$$

Fitting Data:
x = 2015, 2016, 2017, 2018, 2019, 2021, 2023, 2025, 2027, 2028, 2029, 2030, 2031, 2033, 2035, 2037, 2039, 2040;
y = 23.268, 24.46, 24.873, 24.959, 24.925, 24.775, 24.621, 24.484, 24.374, 24.335, 24.311, 24.304, 24.318, 24.417, 24.621, 24.931, 25.338, 25.571;
Number of Fitting Parameters: 6
Number of Observations: 18
Best Known Solution:

$$b_1 = 24.3066786132578 \qquad b_2 = 0.00039439916816074$$
$$b_3 = 2028.80268295086 \qquad b_4 = -0.00480262674653017$$
$$b_5 = 0.0175520648706556 \qquad b_6 = 2034.10918281322$$

Noted that (b_1, b_2, b_3) and (b_4, b_5, b_6) can be swapped equally and correspondingly.
Corresponding SSE: 0.0249395757906404
Figure: See Figure 28.

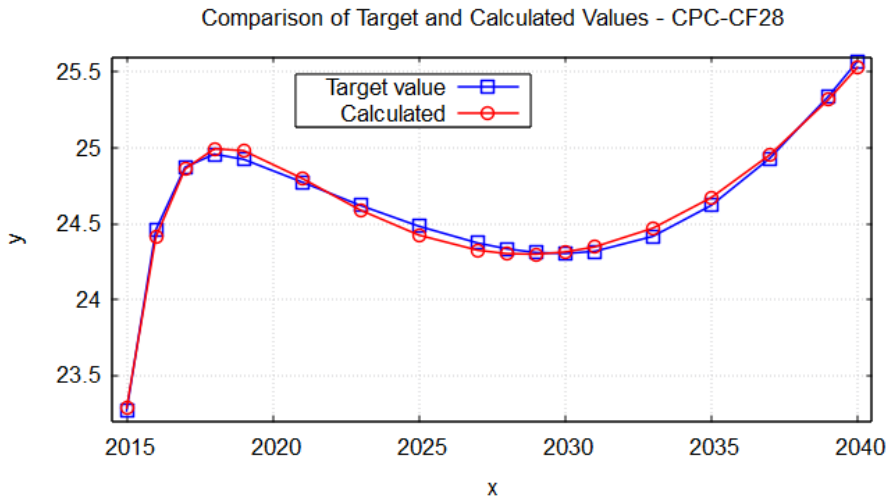

Figure 28: Illustration of CPC-CF28

CPC-CF29

Fitting Formula

$$y = b_1 \cdot \exp\left(-\exp\left(-b_2 \cdot (x - b_3)\right) - b_1\right) + b_4 \cdot \exp\left(b_5 \cdot x - b_3\right) \quad \text{(CPC-CF29)}$$

Fitting Data:
x=12.64, 13.344, 14.048, 14.752, 15.104, 15.2, 15.232, 15.456, 15.808, 16.864, 17.76;
y=-0.040, -0.033, -0.035, -0.031, 0.076, 0.406, 0.624, 0.965, 1.001, 0.998, 0.997;
Number of Fitting Parameters: 5
Number of Observations: 11
Best Known Solution:

$$b_1 = -0.568509227190211 \qquad b_2 = -18.310446341601$$
$$b_3 = 15.2308257368052 \qquad b_4 = 3600481.20221626$$
$$b_5 = 0.00756081517124649$$

Corresponding SSE: 0.00073778796484665
Figure: See Figure 29.

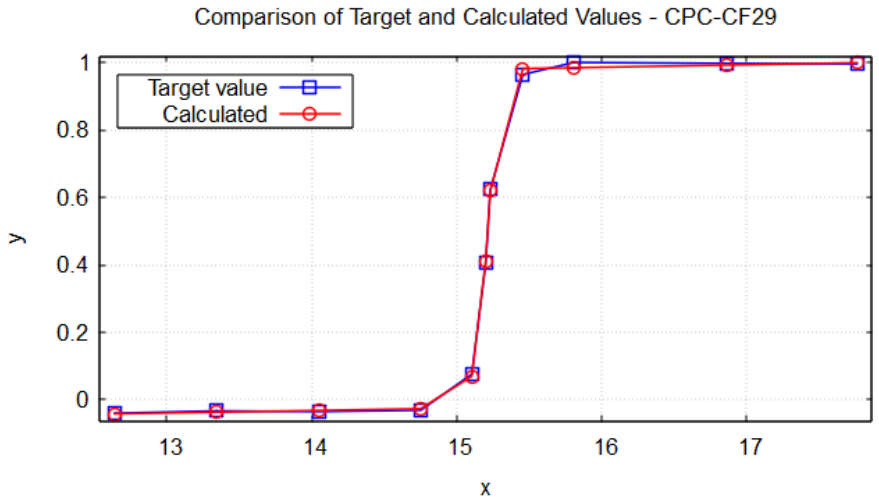

Figure 29: Illustration of CPC-CF29

CPC-CF30

Fitting Formula

$$y = \frac{b_1 + b_2 \cdot \exp(b_3 \cdot x)}{1 + b_4 \cdot x + b_5 \cdot \exp(b_6 \cdot x)} \qquad \text{(CPC-CF30)}$$

Fitting Data:

x=23.5, 64.8, 145.3, 185.2, 267.1, 389.7, 513.7, 638.2, 763.3, 887.5, 1013, 1138.6, 1263.8, 1305.8;

y=55, 122, 187, 190, 173, 138, 111, 92, 78, 68, 60, 53, 48, 47;

Number of Fitting Parameters: 6

Number of Observations: 14

Best Known Solution:

$$b_1 = 75.0385091353405 \qquad b_2 = -75.9037869681941$$
$$b_3 = 0.00144624919976626 \qquad b_4 = 0.00209539856446374$$
$$b_5 = -1.06385957075926 \qquad b_6 = 0.00190958574103846$$

Corresponding SSE: 0.461594701729558

Figure: See Figure 30.

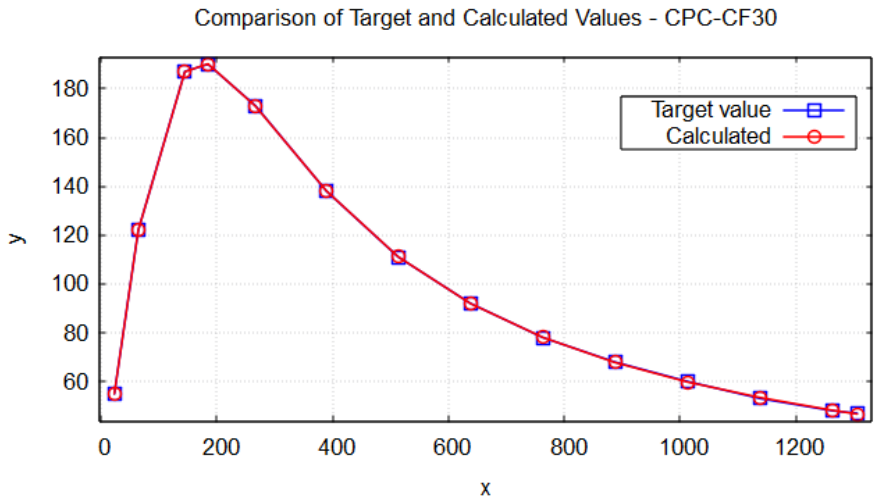

Figure 30: Illustration of CPC-CF30

CPC-CF31

Fitting Formula

$$y = \frac{1 + b_1 \cdot x^{b_2}}{b_3 + \exp\left(b_4 + b_5 \cdot x + b_6 \cdot x^{b_7}\right)} \qquad \text{(CPC-CF31)}$$

Fitting Data:
x=0.1001, 0.1602, 0.2288, 0.3079, 0.4003, 0.5094, 0.6403, 0.8002;
y=87.7, 89.8, 100.2, 121.2, 131.3, 138.9, 135.4, 121.4;
Number of Fitting Parameters: 7
Number of Observations: 8
Best Known Solution:

$$b_1 = 153836.892287962 \qquad b_2 = 8.41350026261178$$
$$b_3 = 0.0113552636458642 \qquad b_4 = 10.5335264025218$$
$$b_5 = 6.18101949191871 \qquad b_6 = -9.29641485995377$$
$$b_7 = -0.420105546979591$$

Corresponding SSE: 2.24560392654908
Figure: See Figure 31.

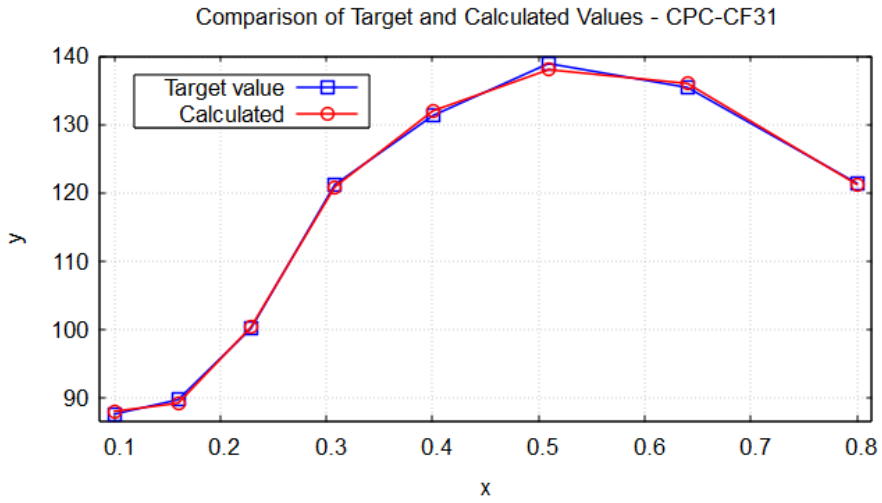

Figure 31: Illustration of CPC-CF31

CPC-CF32

Fitting Formula

$$y = b_1 + \exp\left(-b_1 - b_2 \cdot b_3 \cdot x\right) + b_3 \cdot x^{b_4 + b_5 \cdot x} \qquad \text{(CPC-CF32)}$$

Fitting Data:
x=0.25, 0.5, 0.75, 1, 1.5, 2.5, 3.5, 4.5, 5, 6, 8, 10, 12, 14, 16;
y=15, 34, 37.5, 41, 41, 34, 29, 25, 20.5, 19, 14, 9, 6, 3.5, 2;
Number of Fitting Parameters: 5
Number of Observations: 15
Best Known Solution:

$$b_1 = -5.60698772204911 \qquad b_2 = -0.00106695489688122$$
$$b_3 = -178.657613343298 \qquad b_4 = -0.19318011483806$$
$$b_5 = -0.0653441171012468$$

Corresponding SSE: 8.31641750282762
Figure: See Figure 32.

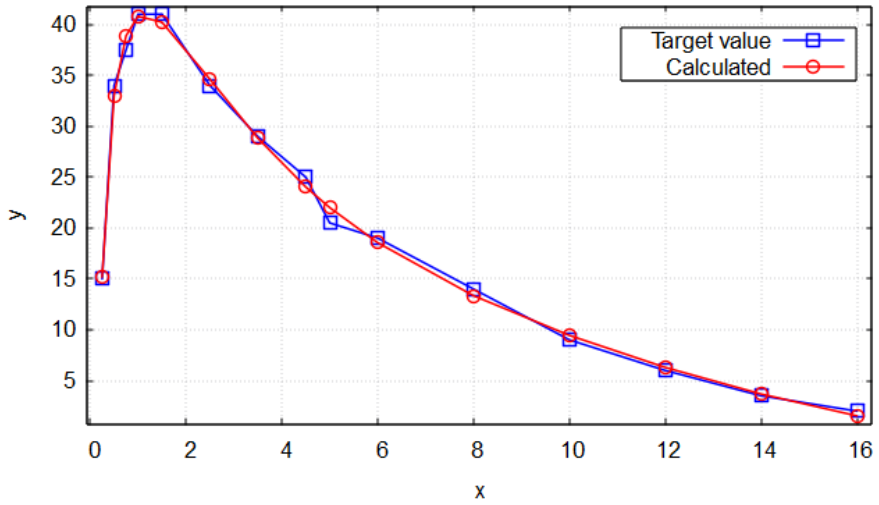

Figure 32: Illustration of CPC-CF32

CPC-CF33

Fitting Formula

$$y = \left(b_1 \cdot b_2 - b_3 \cdot \exp\left(b_1 - b_4 \cdot x\right) + \frac{b_2 \cdot b_3}{x} \right)^{b_5} \qquad \text{(CPC-CF33)}$$

Fitting Data:
x=1, 2, 4, 6, 8;
y=100, 140, 160, 170, 175;
Number of Fitting Parameters: 5
Number of Observations: 5
Best Known Solution:

$$b_1 = 17.752587104156 \qquad b_2 = 27534771.2014361$$
$$b_3 = -34.5700075359545 \qquad b_4 = 1.25216863019635$$
$$b_5 = 0.261789631090794$$

Corresponding SSE: 9.92390552864888E-23
Figure: See Figure 33.

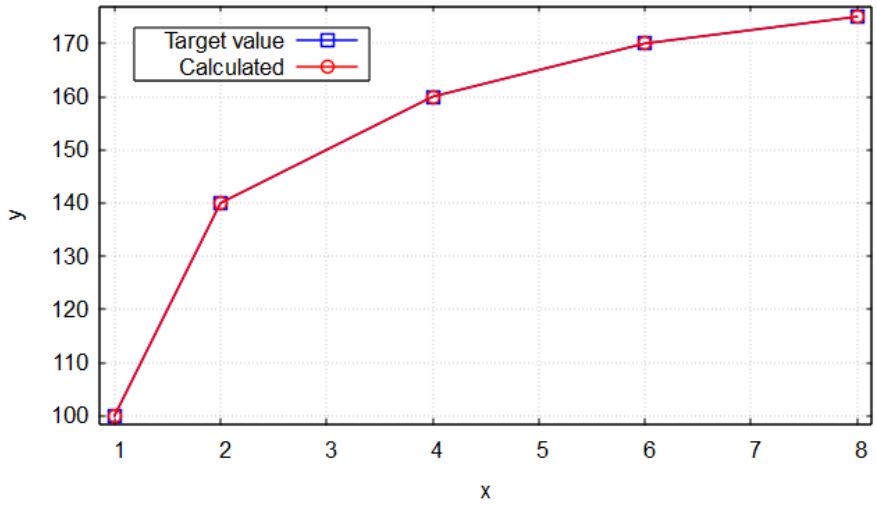

Figure 33: Illustration of CPC-CF33

CPC-CF34

Fitting Formula

$$y = \frac{b_1 \cdot b_2 + b_3 \cdot \exp(b_4 \cdot x)}{b_2 + \exp(b_4 \cdot x) + \frac{b_5}{x + b_1}} \qquad \text{(CPC-CF34)}$$

Fitting Data:
x=0, 10, 20, 30, 40, 50, 60, 70, 80, 90;
y=22.260, 21.526, 20.589, 19.092, 17.312, 15.425, 13.854, 12.541, 11.697, 11.294;
Number of Fitting Parameters: 5
Number of Observations: 10
Best Known Solution:

$$b_1 = 514.22844217014 \qquad b_2 = 0.00131837275252952$$
$$b_3 = 23.2697482304989 \qquad b_4 = -0.0594755563915843$$
$$b_5 = 38.5710450719004$$

Corresponding SSE: 0.0108081427953313
Figure: See Figure 34.

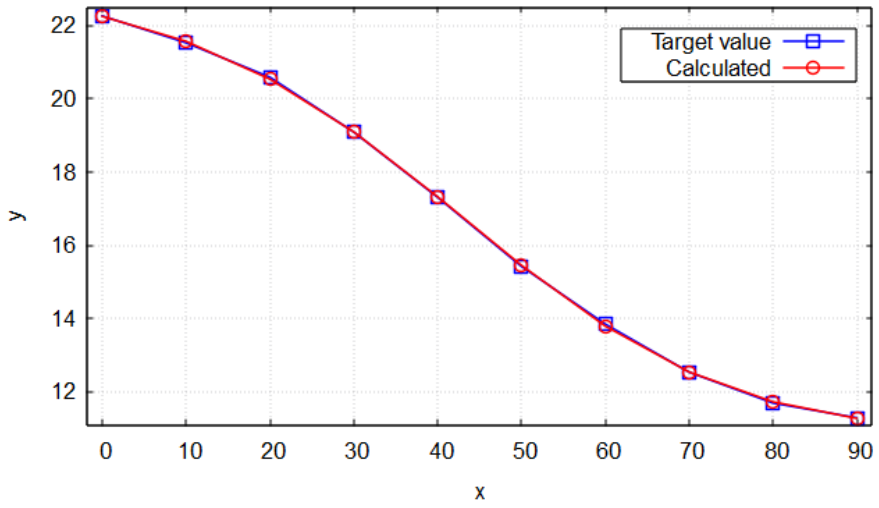

Figure 34: Illustration of CPC-CF34

CPC-CF35

Fitting Formula

$$y = b_1 + b_2 \cdot \exp\left(x \cdot b_3 + b_4 \cdot x^{b_5}\right) \qquad \text{(CPC-CF35)}$$

Fitting Data:

x=204.11, 219.64, 243.34, 281.01, 312.05, 331.71, 341.49, 349.16, 379.08, 420.91, 430.99;

y=64.33, 69.17, 71.18, 78.55, 82.57, 81.23, 77.37, 73.13, 60.59, 23.04, 2.98;

Number of Fitting Parameters: 5

Number of Observations: 11

Best Known Solution:

$$b_1 = 406.47077418954 \qquad b_2 = -406.674717454503$$
$$b_3 = -0.00085920356574541 \qquad b_4 = 7.50850085490799E-18$$
$$b_5 = 6.33124962653917$$

Corresponding SSE: 30.0258546096148

Figure: See Figure 35.

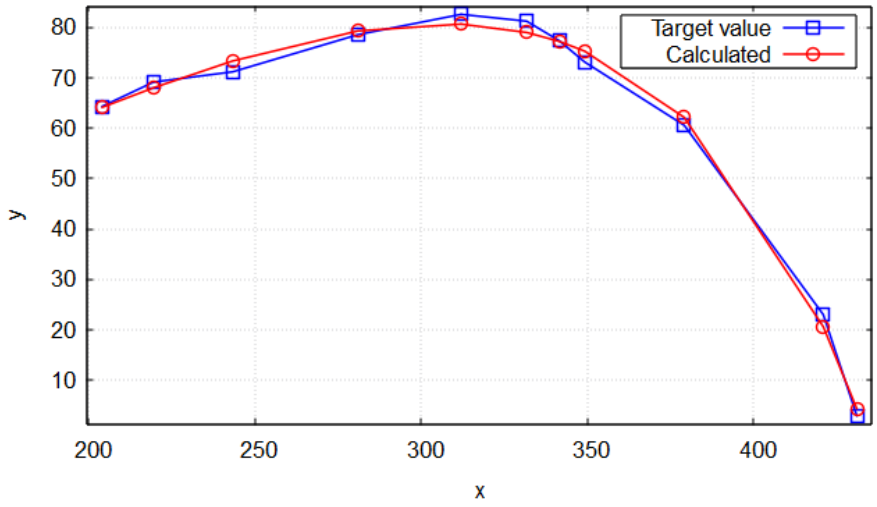

Figure 35: Illustration of CPC-CF35

CPC-CF36

Fitting Formula

$$y = b_1 \cdot \exp\left(\frac{b_2}{(b_3 + x)^2} - b_4 \cdot x\right) + b_5 \qquad \text{(CPC-CF36)}$$

Fitting Data:

x=298.15, 303.15, 308.15, 313.15, 315.5, 318.15, 323.15, 328.15, 333.15, 338.15;

y=114.6, 89.68, 68.43, 50.21, 3.95, 40.09, 33.42, 27.01, 21.59, 17.53;

Number of Fitting Parameters: 5

Number of Observations: 10

Best Known Solution:

$$b_1 = 714295163.326678 \qquad b_2 = -0.496314098155443$$
$$b_3 = -315.526295076198 \qquad b_4 = 0.0525820331903116$$
$$b_5 = 3.98232757766595$$

Corresponding SSE: 0.813367180953699

Figure: See Figure 36.

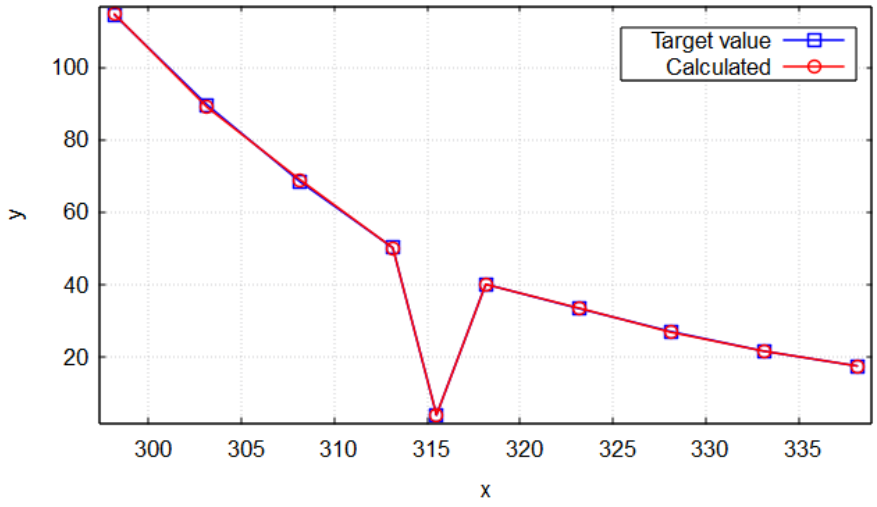

Figure 36: Illustration of CPC-CF36

CPC-CF37

Fitting Formula

$$y = b_1 + b_2 \cdot \exp(b_3 \cdot x) + b_4 \cdot \exp\left(\frac{b_5}{x}\right) \qquad \text{(CPC-CF37)}$$

Fitting Data:
x=73.1, 94, 100, 130, 150, 184, 192, 200;
y=0.25, 0.6, 0.65, 0.9, 0.91, 0.94, 0.95, 1;
Number of Fitting Parameters: 5
Number of Observations: 8
Best Known Solution:

$$b_1 = -2.35387255927895 \qquad b_2 = 1.43051346849043$$
$$b_3 = 0.00841358104669811 \qquad b_4 = -63.1154736511059$$
$$b_5 = -535.261375790585$$

Corresponding SSE: 0.000696726236787323
Figure: See Figure 37.

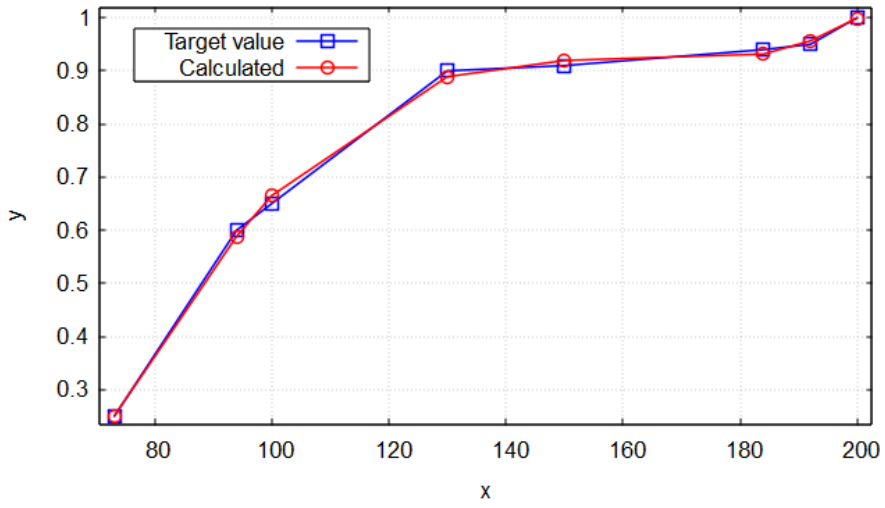

Figure 37: Illustration of CPC-CF37

CPC-CF38

Fitting Formula

$$y = b_1 + \frac{b_2}{1 + b_3 \cdot x + b_4 \cdot \exp(b_5 \cdot x)} \qquad \text{(CPC-CF38)}$$

Fitting Data:
x=-24536, -26603, -27424, -28573, -29204, -29428, -29822, -30065;
y=0, 107, 216, 414, 643, 785, 1044, 1285;
Number of Fitting Parameters: 5
Number of Observations: 8
Best Known Solution:

$b_1 = -239.519997242835$ $b_2 = 48.431102070892$
$b_3 = 3.24782287496463E-5$ $b_4 = 5.48680860090078E-19$
$b_5 = -0.00123899152096207$

Corresponding SSE: 676.714297334315
Figure: See Figure 38.

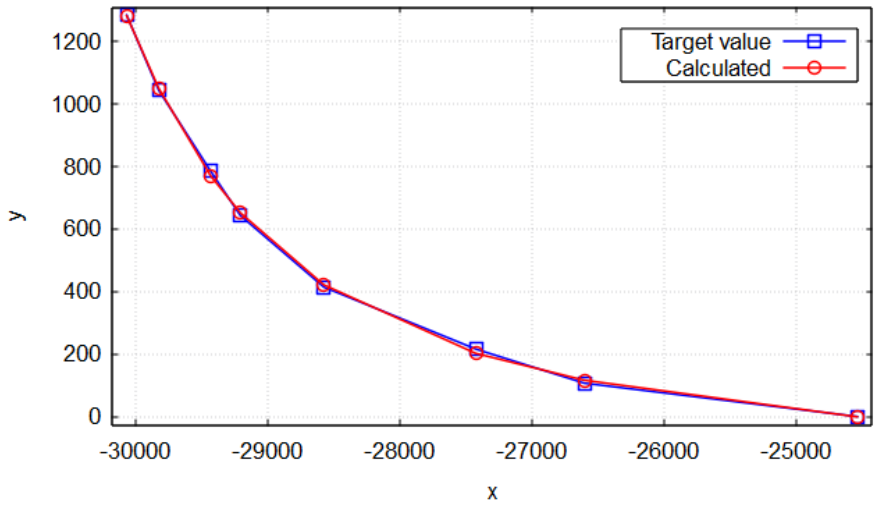

Figure 38: Illustration of CPC-CF38