CPC Framework - Nonlinear Equation System

Peicong Cheng[1*] Makoto Yamashita[1]

[1]School of Mathematical and Computer Science, Institute of Science Tokyo, Japan

1 Introduction

This document presents a collection of challenging optimization problems under the CPC Framework, specifically targeting on solving nonlinear equation system. All problems included here are newly proposed and designed to test the robustness and effectiveness of global optimization algorithms.

Users are encouraged to experiment with these problems and share their results. If a significantly improved and correct solution is discovered, please feel free to contact the author. Verified updates will be recorded and reflected in future versions of this document.

The benchmark set will continuous to expand with more difficult and diverse problems. For detailed information about each problem, please refer to the corresponding individual folders.

All figures in this document were generated using GNUplot.

If you use this benchmark in your research or publications, please cite the CPC Framework accordingly. Citation information can be found from README.

*Corresponding author: cheng.p.b2bf@m.isct.ac.jp

2 Overview of Nonlinear Equation System

This section is currently under development and will be updated in future revisions.

In the meantime, we would like to present two challenging problems from this category in advance without disclosing the known solutions we have found so far.

We warmly invite interested readers to attempt these problem and, if willing, share their results.

CPC-ES1

Equation system:

$$
\begin{aligned}
0.0448 - x_1 \cdot \left(\frac{x_2 - 0.0633}{x_3 - 0.0633} \right)^{x_5} &= 0; \\
0.0471 - x_1 \cdot \left(\frac{x_2 - 0.0333}{x_4 - 0.0333} \right)^{x_5} &= 0; \\
0.4714 - x_1 \cdot \left(\frac{x_2 + 0.3333}{x_3 + 0.3333} \right)^{x_5} &= 50 + x_5; \qquad \text{(CPC-ES1)} \\
0.5715 - x_1 \cdot \left(\frac{x_2 + 0.7 - 1500}{0.5946 \cdot x_4 + 0.5 \cdot x_3 + 0.7} \right)^{x_5} &= 0; \\
0.5657 - x_1 \cdot \left(\frac{x_2 + 0.8}{x_4 + 0.8} - x_3 \right)^{x_5} &= 1000 - x_1
\end{aligned}
$$

Dimension: 5
Domain: $x_i \in \mathbb{R}, \quad i = 1, 2$
Global minimum:

$$
\begin{aligned}
x_1 &= ? \\
x_2 &= ? \\
x_3 &= ? \\
x_4 &= ? \\
x_5 &= ?
\end{aligned}
$$

CPC-ES2

Equation system:

$$
\begin{aligned}
x_1 - x_8 \cdot x_5^2 &= x_3 \cdot x_2 + x_5 \\
x_3 &= (130 \cdot x_7 + 0.5) \cdot \left(\frac{(x_1 - x_8)^2}{19.62} \right) \\
x_4 &= 0.001963495 \cdot x_2 \\
x_3 &= 87794 \cdot (x_1 - x_8) \\
x_6 &= 43897 \cdot x_2 \\
x_7 &= \left(-1.8 \cdot \ln \left(\frac{6.9}{x_5} \right) \right)^{-2 \cdot x_3} \\
x_8 &= \left(-1.8 \cdot \ln \left(\frac{6.9}{x_6} \right) \right)^{-2 \cdot x_4} \\
x_3 &= 18 - \frac{(x_2)^2}{19.62}
\end{aligned}
\tag{CPC-ES2}
$$

Dimension: 8
Domain: $x_i \in \mathbb{R}, \quad i = 1, 2$
Global minimum:

$$
\begin{aligned}
x_1 &= ? \\
x_2 &= ? \\
x_3 &= ? \\
x_4 &= ? \\
x_5 &= ? \\
x_6 &= ? \\
x_7 &= ? \\
x_8 &= ?
\end{aligned}
$$