CPC Framework - Nonlinear and Discontinuous Function Minimization

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1   Introduction

This document presents a collection of challenging optimization problems under the CPC Framework, specifically targeting the minimization of nonlinear and discontinuous functions. All problems included here are newly proposed and designed to test the robustness and effectiveness of global optimization algorithms.

Users are encouraged to experiment with these problems and share their results. If a significantly improved and correct solution is discovered, please feel free to contact the author. Verified updates will be recorded and reflected in future versions of this document.

The benchmark set will continuous to expand with more difficult and diverse problems. For detailed information about each problem, please refer to the corresponding individual folders.

All figures in this document were generated using GNUplot.

If you use this benchmark in your research or publications, please cite the CPC Framework accordingly. Citation information can be found from README.

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2 Overview of Nonlinear and Discontinuous Function Minimization

The brief overview of the characteristics of all 25 benchmark functions is provided in Table 1. It summarizes five key properties including dimensionality (Dim.), search range, separability (Sep.), feasible ratio (FR), and number of global optima (#Optima). Further details are presented in section 3.

Table 1: Overview of characteristics for CPC Benchmark Functions

Fun ID	Dim.	Search Space	Sep.	FR(%)	#Optima
CPC-DF1	2	$[-100, 100]^2$	No	48.603052	1
CPC-DF2	n=5	$[-5, 5]^n$	Yes	0.011532	$2^5=32$
CPC-DF3	2	$[-100, 100]^2$	No	0.05724	1
CPC-DF4	n=5	$[-512, 512]^n$	No	0.000484	1
CPC-DF5	n=5	$[-6, 6]^n$	Yes	0.000036	$2^5=32$
CPC-DF6	n=4	$[-100, 100]^n$	No	0.000544	1
CPC-DF7	n=3	$[-10, 10]^n$	Yes	0.007868	1
CPC-DF8	2	$[0, 14]^2$	No	15.886572	1
CPC-DF9	2	$[-10, 10]^2$	No	37.250012	1
CPC-DF10	2	$[-10, 10]^2$	No	35.671264	2
CPC-DF11	2	$[-6, 6]^2$	No	0.00114	$2^2=4$
CPC-DF12	n=5	$[-100, 100]^n$	No	0.010204	$2^4=16$
CPC-DF13	2	$[-100, 100]^2$	No	0.000116	1
CPC-DF14	2	$[-10, 10]^2$	No	29.208808	1
CPC-DF15	2	$[-5, 5]^2$	No	0.000228	1
CPC-DF16	n=5	$[-512, 512]^n$	No	0.00002	$2^1=2$
CPC-DF17	2	$[-100, 100]^2$	No	0.554864	1
CPC-DF18	4	$[-5000, 5000]^4$	No	≤ 0.000001	1
CPC-DF19	6	$[-100, 100]^6$	No	0.014292	1
CPC-DF20	n=5	$[-10, 10]^n$	No	0.023148	1
CPC-DF21	n=5	$[-30, 30]^n$	No	0.036648	1
CPC-DF22	2	$[-100, 100]^2$	No	0.06192	1
CPC-DF23	n=5	$[-100, 100]^n$	No	0.76602	1
CPC-DF24	2	$[-100, 100]^2$	No	0.486596	Inf
CPC-DF25	n=3	$[-10, 10]^n$	No	6.763992	1

3 Detailed Formulations and Visualizations

CPC-DF1

Function expression:

$$f(\mathbf{x}) = (\sin(f1 + f2) + \ln((f1 - f2)^2 - 1.5 * f1 + 2.5 * f2 + 1))^{0.6} + \sqrt{\sin(f1^2 + f2^2)}$$

$$\text{(CPC-DF1)}$$

Where:

$$\begin{cases} f_1 = x_1 - 0.125 \\ f_2 = x_2 + 0.25 \end{cases}$$

Dimension: 2
Domain: $x_i \in [-100, 100], \quad i = 1, 2$
Feasible Ratio: 48.603052%
Best Known Minimum:

$$x_1 = 0.125$$
$$x_2 = -0.25$$
$$f(x^*) = 0$$

Figure: See Figure 1.
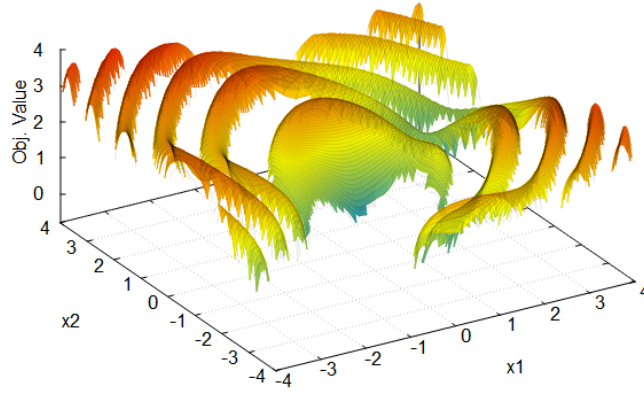
Figure 1: Illustration of CPC-DF1

CPC-DF2

Function expression:

$$f(\mathbf{x}) = \sum_{i=1}^{n=5} \left(\sqrt{10 \cdot x_i \cdot \sin(x_i) - (10 \cdot \cos(2 \cdot \pi \cdot x_i \cdot (i-1)) - 8)^{0.03}} \right)$$

(CPC-DF2)

Dimension: $n = 5$
Domain: $x_i \in [-5, 5], \quad i = 1, \ldots, n$
Feasible Ratio: 0.011532%
Best Known Minimum:

$$x_1 = \pm 0.322320916824258$$
$$x_2 = \pm 3.10153425481943$$
$$x_3 = \pm 0.45210108997646$$
$$x_4 = \pm 0.32169325871902$$
$$x_5 = \pm 0.47751943350732$$
$$f(x^*) = 2.69460833912395$$

Figure: See Figure 2.
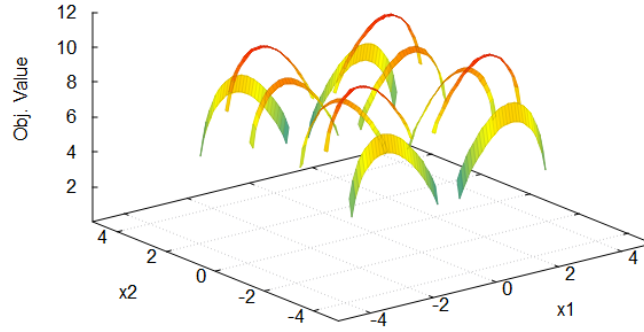
Figure 2: Illustration of CPC-DF2

4

CPC-DF3

Function expression:

$$
\begin{aligned}
f(\mathbf{x}) =& -\ln\left(x_1 - x_2 \cdot \ln\left(\cos\left(-\frac{2}{3} \cdot x_1^3 - 8 \cdot x_1^2\right)\right)\right) - \\
& \ln\left(33 \cdot x_1 - x_1 \cdot x_2 + 5 - \left((x_1 - 4)^2 + (x_2 - 5)^2 - 4\right)^2\right)
\end{aligned}
\quad \text{(CPC-DF3)}
$$

Dimension: 2
Domain: $x_i \in [-100, 100], \quad i = 1, 2$
Feasible Ratio: 0.05724%
Best Known Minimum:

$$
\begin{aligned}
x_1 &= 6.1828121298816 \\
x_2 &= 6.49031991565847 \\
f(x^*) &= -10.503674524476093
\end{aligned}
$$

Figure: See Figure 3.
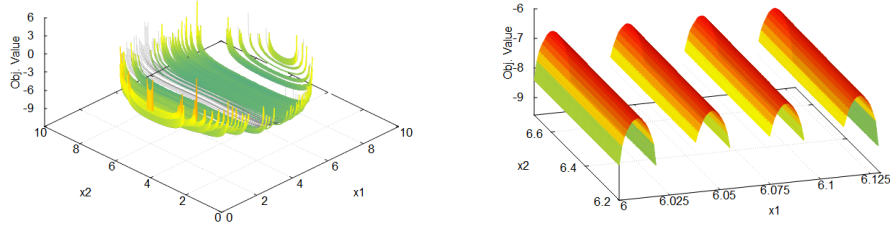
Figure 3: Illustrations of CPC-DF3. The plot on the right is a local zoom of the plot on the left, highlighting the discontinuity in the middle region

CPC-DF4

Function expression:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1=4} \Big(- (x_{i+1} + 47) \cdot \sin \Big(\sqrt{x_{i+1} + \frac{x_i}{2} + 47} \Big) + \ln \Big(\sin \Big(\sqrt{x_i - x_{i+1} - 47} \Big) \Big) \cdot (-x_i) \Big) \qquad \text{(CPC-DF4)}$$

Dimension: $n = 5$
Domain: $x_i \in [-512, 512], \quad i = 1, \ldots, n$
Feasible Ratio: 0.000484%
Best Known Minimum:

$$x_1 = 508.460866086038$$
$$x_2 = 459.060139792734$$
$$x_3 = 409.75137155371$$
$$x_4 = 168.331188302594$$
$$x_5 = 62.2686261244202$$
$$f(x^*) = -1035.99247951216$$

Figure: See Figure 4.
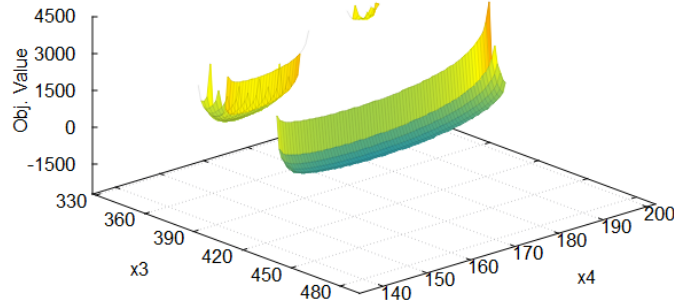
Figure 4: Illustrations of CPC-DF4

CPC-DF5

Function expression:

$$f(\mathbf{x}) = -\ln\left(5 - \sum_{i=1}^{n=5}\left(\sqrt{i \cdot x_i^2 - \left(\frac{x_i^2}{i} - 10 \cdot \cos\left(2 \cdot \pi \cdot i \cdot x_i\right)\right)^{\frac{i}{5}}}\right) + 1\right)$$

(CPC-DF5)

Dimension: $n = 5$
Domain: $x_i \in [-6, 6], \quad i = 1, \ldots, n$
Feasible Ratio: 0.000036%
Best Known Minimum:

$$x_1 = \pm 0.75815313559746$$
$$x_2 = \pm 0.627823153749116$$
$$x_3 = \pm 1.16738971896674$$
$$x_4 = \pm 0.826039501323027$$
$$x_5 = \pm 0.0500382556682677$$
$$f(x^*) = -1.791759028336902$$

Figure: See Figure 5.
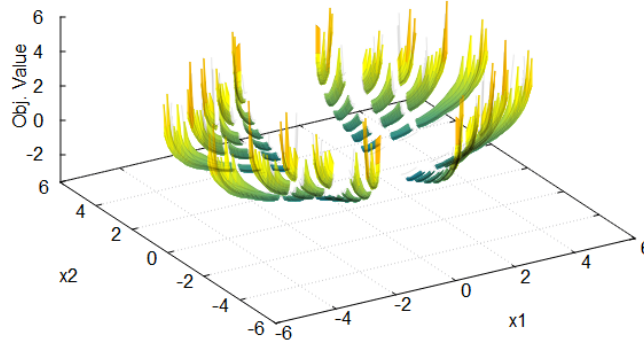
Figure 5: Illustrations of CPC-DF5

CPC-DF6

Function expression:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1=3} \left(\sin^2\left(\sqrt{x_{i+1}^2 - 20 \cdot x_i^2} \right) - 0.5 \cdot x_i \right.$$
$$\left. + 0.001 \cdot \left(x_{i+1}^2 - 2 \cdot x_i \cdot x_{i+1} + x_i^2 \right)^{0.2} + 1.5 \right) \qquad \text{(CPC-DF6)}$$

Dimension: $n = 4$
Domain: $x_i \in [-100, 100], \quad i = 1, \ldots, n$
Feasible Ratio: 0.000544%
Best Known Minimum:

$$x_1 = 1.10653652103725$$
$$x_2 = 4.94886207314733$$
$$x_3 = 22.360415331491$$
$$x_4 = 99.9988173726853$$
$$f(x^*) = -9.69243034582198$$
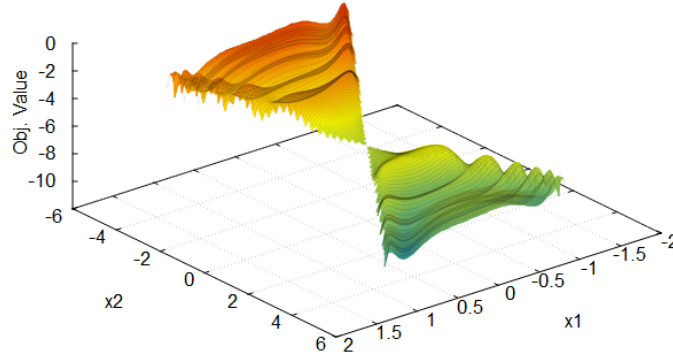
Figure: See Figure 6.

Figure 6: Illustrations of CPC-DF6

CPC-DF7

Function expression:

$$f(\mathbf{x}) = -\sum_{i=1}^{n=3} \left(\sum_{j=1}^{m=5} \left(\ln \left(0.1 - j \cdot \sin \left(\frac{(j+1) \cdot x_i^i}{j} + j \right) \right) \right) \right) \qquad \text{(CPC-DF7)}$$

Dimension: $n = 3$
Domain: $x_i \in [-10, 10], \quad i = 1, \ldots, n$
Feasible Ratio: 0.007868%
Best Known Minimum:

$$x_1 = 5.44819855994141$$
$$x_2 = -3.28078041187348$$
$$x_3 = -8.39826894144372$$
$$f(x^*) = -11.413034896524696$$
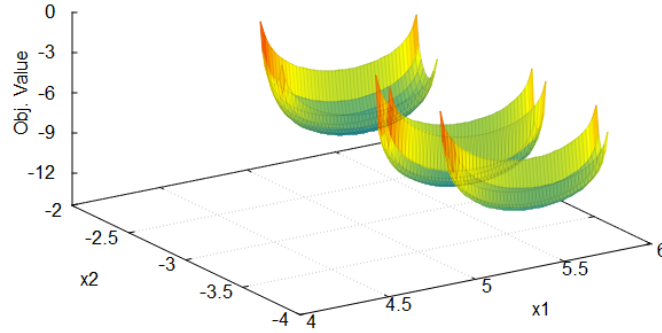
Figure: See Figure 7.

Figure 7: Illustrations of CPC-DF7

CPC-DF8

Function expression:

$$f(\mathbf{x}) = 1 - \left(\frac{\sin\left(\pi \cdot (x_1 - 2)\right) \cdot \sin\left(\pi \cdot (x_2 - 2)\right)}{\pi^2 \cdot x_1 \cdot (x_1 - 2) \cdot (x_2 - 2)} \right)^{1.03} + \left(2 + (x_1 - 7)^2 - 2 \cdot (x_2 - 7)^2 \right)^{0.65} \qquad \text{(CPC-DF8)}$$

Dimension: 2
Domain: $x_i \in [0, 14], \quad i = 1, 2$
Feasible Ratio: 15.886572%
Best Known Minimum:

$$x_1 = 1.23746138046895$$
$$x_2 = 2.80435634606639$$
$$f(x^*) = 0.9521806056544282$$
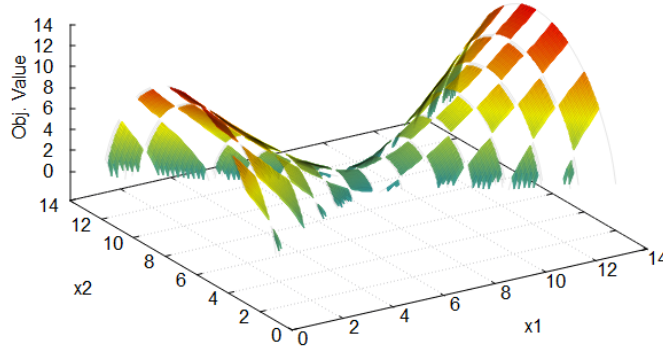
Figure: See Figure 8.

Figure 8: Illustrations of CPC-DF8

CPC-DF9

Function expression:

$$f(\mathbf{x}) = \left(x_1^2 + x_2 - 10\right)^{1.02 \cdot \cos(x_1)} +$$
$$\left(x_1 + x_2^2 - 7\right)^{1.02 + \sin(x_2)} \cdot \left(x_1^2 + x_2^3 - 1\right)^{1.02 + \cos(x_1 \cdot x_2)} \qquad \text{(CPC-DF9)}$$

Dimension: 2
Domain: $x_i \in [-10, 10], \quad i = 1, 2$
Feasible Ratio: 37.250012%
Best Known Minimum:

$$x_1 = -9.48209811742485$$
$$x_2 = -4.05981503487842$$
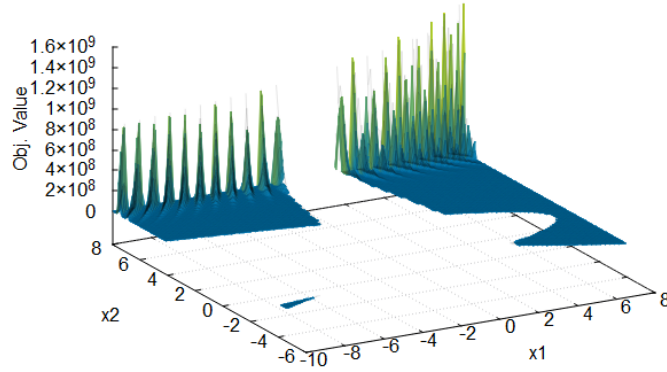$$f(x^*) = 0.0121784619770848$$

Figure: See Figure 9.

Figure 9: Illustrations of CPC-DF9

CPC-DF10

Function expression:

$$f(\mathbf{x}) = \left(x_1^2 + \frac{x_2^2}{x_1} - 5\right)^{0.01} + 25 \cdot \left((\sin(x_1 + 1))^2 - (\sin(x_2 - 1))^2\right)^{0.25}$$

$$\text{(CPC-DF10)}$$

Dimension: 2
Domain: $x_i \in [-10, 10], \quad i = 1, 2$
Feasible Ratio: 35.671264%
Best Known Minimum:

$$x_1 = 1.79128784747792$$
$$x_2 = -1.79128784747792$$
$$or$$
$$x_1 = -2.79128784747792$$
$$x_2 = 2.79128784747792$$
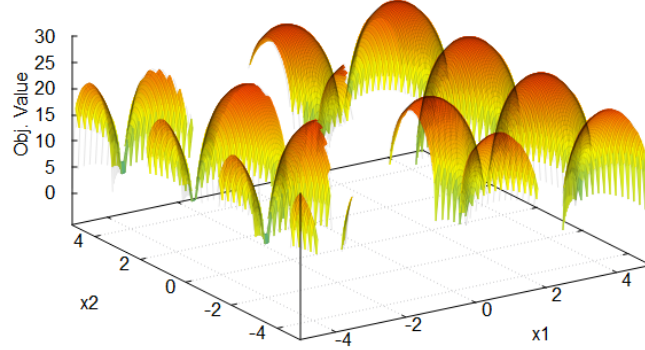$$f(x^*) = 0$$

Figure: See Figure 10.

Figure 10: Illustrations of CPC-DF10

CPC-DF11

Function expression:

$$f(\mathbf{x}) = \cos{(x_1)} \cdot \left(|\sin{(x_2)}| - 8.0 \cdot \left| \cos{\left(\frac{x_1}{x_2^2 + 1} \right)} \right| \right)^{1+\sqrt{0.001 - |\sin(x_1 \cdot x_2)|}}$$

(CPC-DF11)

Dimension: 2
Domain: $x_i \in [-6, 6], \quad i = 1, 2$
Feasible Ratio: 0.00114%
Best Known Minimum:

$$x_1 = \pm 3.14209263378032$$
$$x_2 = \pm 1.00015913597542$$
$$f(x^*) = -0.8415568426048948$$
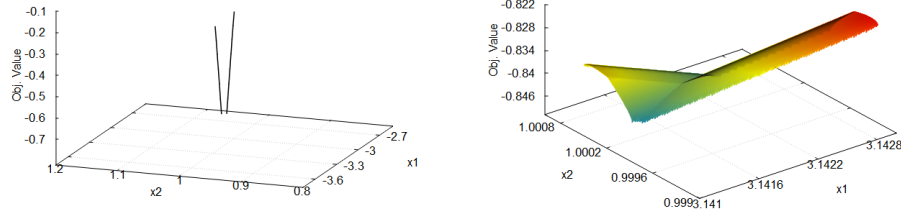
Figure: See Figure 11.

Figure 11: Illustrations of CPC-DF11. In the larger range shown on the left, the function appears visually indistinguishable from two straight lines. The plot on the right shows a magnified view, which reveals the actual surface structure of the function.

CPC-DF12

Function expression:

$$f(\mathbf{x}) = -\sum_{i=1}^{n-1=4} \left(\frac{\sqrt{\sin\left(\sqrt{x_{i+1}^2 - x_i^2} - 0.5\right) - 0.5}}{\left(10 \cdot \left(x_{i+1}^2 + x_i^2\right) - 0.85\right)^{0.2}} + 0.5 \right) \qquad \text{(CPC-DF12)}$$

Dimension: $n = 5$
Domain: $x_i \in [-100, 100], \quad i = 1, \ldots, n$
Feasible Ratio: 0.010204%
Best Known Minimum:

$$x_1 = 0$$
$$x_2 = \pm 1.66286508893592$$
$$x_3 = \pm 2.49574965913224$$
$$x_4 = \pm 3.17542943879395$$
$$x_5 = \pm 3.77273562655558$$
$$f(x^*) = -3.10636194099529$$
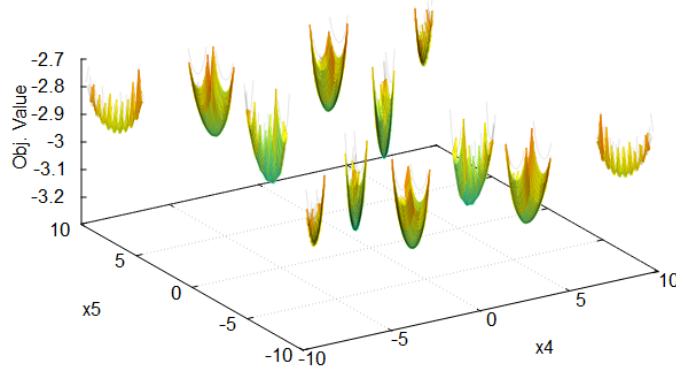
Figure: See Figure 12.

Figure 12: Illustrations of CPC-DF12

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CPC-DF13

Function expression:

$$f(\mathbf{x}) = \sqrt{x_1 - 2 \cdot x_2^2 - \exp\left(x_2 - x_1^2\right) -}$$
$$\left(0.5 \cdot \cos\left(3 \cdot \pi \cdot x_1 + 4 \cdot \pi \cdot x_2 + 5\right) - 0.495 \cdot x_1\right)^{0.2} \qquad \text{(CPC-DF13)}$$

Dimension: 2
Domain: $x_i \in [-100, 100], \quad i = 1, 2$
Feasible Ratio: 0.000116%
Best Known Minimum:

$$x_1 = 0.689254127755344$$
$$x_2 = 0.0828214502156397$$
$$f(x^*) = -0.691928486927794$$
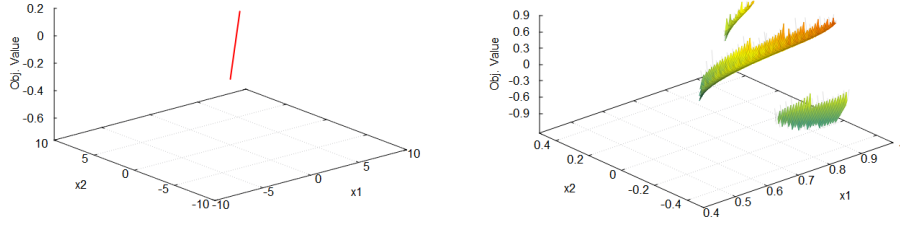
Figure: See Figure 13.

Figure 13: Illustrations of CPC-DF13. In the larger range shown on the left, the function appears visually indistinguishable from a straight line. The plot on the right shows a magnified view, which reveals the actual surface structure of the function.

CPC-DF14

Function expression:

$$
\begin{aligned}
f(\mathbf{x}) =& (1.5 - x_1 + x_1 \cdot x_2)^{0.2} + \\
& \left(2.5 - x_1 + x_1 \cdot x_2^2\right)^{x_1 - 0.5} + \left(2.625 - x_1 + x_1 \cdot x_2^3\right)^{0.2}
\end{aligned}
\tag{CPC-DF14}
$$

Dimension: 2
Domain: $x_i \in [-10, 10], \quad i = 1, 2$
Feasible Ratio: 29.208808%
Best Known Minimum:

$$
\begin{aligned}
x_1 &= 3 \\
x_2 &= 0.5 \\
f(x^*) &= 0.03125
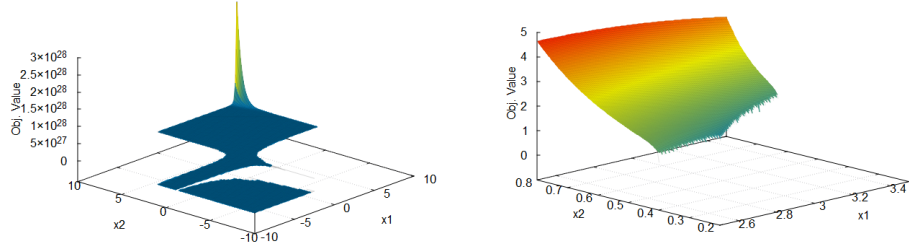\end{aligned}
$$

Figure: See Figure 14.

Figure 14: Illustrations of CPC-DF14. In the larger range shown on the left, the function appears nearly flat, making its structure indistinguishable. The plot on the right shows a magnified view around the minimum, which demonstrates the irregular boundary between feasible and infeasible regions.

16

CPC-DF15

Function expression:

$$
\begin{aligned}
f(\mathbf{x}) =& 10 - \left(0.01 - \sqrt{\cos\left(12 \cdot \sqrt{x_1^{x_2} - x_2^{x_1}} + 5 \right) - \frac{x_1}{x_2}} \right)^{1.35} \\
& + \left(0.5 \cdot \left(x_1^2 - x_2^2 \right) \cdot \cos\left(x_1 - x_2 \right) + 2 \right)^{1.5}
\end{aligned}
\tag{CPC-DF15}
$$

Dimension: 2
Domain: $x_i \in [-5, 5], \quad i = 1, 2$
Feasible Ratio: 0.000228%
Best Known Minimum:

$$
\begin{aligned}
x_1 &= 3.17233243403426 \\
x_2 &= 4.15125018443042 \\
f(x^*) &= 9.998005563035544
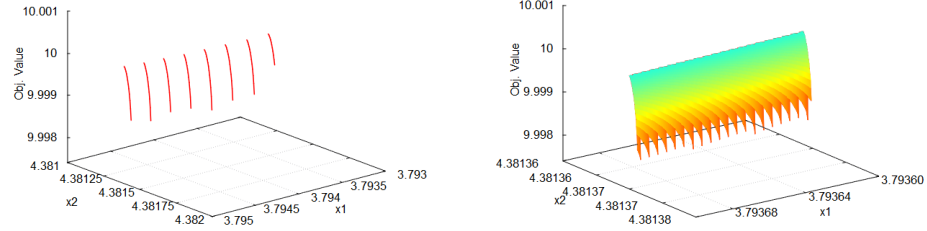\end{aligned}
$$

Figure: See Figure 15.

Figure 15: Illustrations of CPC-DF15. In the larger range shown on the left, the function appears visually indistinguishable from several straight lines. The plot on the right shows a magnified view, which reveals the actual surface structure of the function.

CPC-DF16

Function expression:

$$f(\mathbf{x}) = \frac{10^{-7}}{2} \cdot \sum_{i=1}^{n-1=4} \left(\left(-\left(i \cdot x_{i+1} + 27\right) \cdot \sin\left(\sqrt{x_{i+1} - \frac{x_i}{2} + 27}\right) + \right. \right.$$
$$\left. \left. \sin\left(\sqrt{x_i - i \cdot x_{i+1} - 27}\right) \cdot (-x_i)\right)^i \right)$$

(CPC-DF16)

Dimension: $n = 5$
Domain: $x_i \in [-512, 512], \quad i = 1, \ldots, 5$
Feasible Ratio: 0.00002%
Best Known Minimum:

$$x_1 = 478.157812829594$$
$$x_2 = 216.36680384591$$
$$x_3 = 94.5514197828485$$
$$x_4 = 21.8404273498408$$
$$x_5 = -3.36983292634916 \quad or -15.6821332526741$$
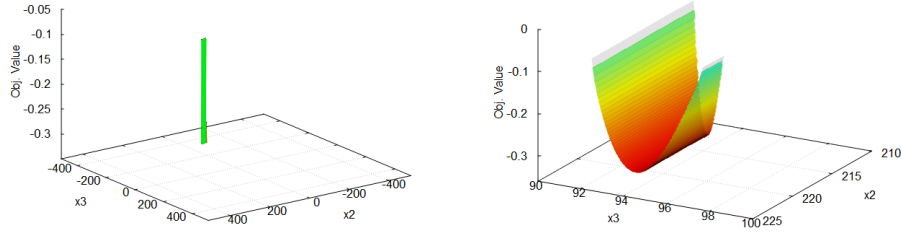$$f(x^*) = 0.29834364506683275$$

Figure: See Figure 16.

Figure 16: Illustrations of CPC-DF16. The plot on the right provides a magnified view around the minimum.

CPC-DF17

Function expression:

$$f(\mathbf{x}) = 1 - \ln\Bigg(\left(x_1^2 - x_2^2 + 0.25\right)^{0.5} -$$

$$\left(0.3 \cdot \cos\left(3 \cdot \pi \cdot x_1 - x_2 + 1\right) \cdot \cos\left(4 \cdot \pi \cdot x_2 + 2 \cdot x_1 - 1\right) - 0.29\right)^{0.5}\Bigg)$$
$$\text{(CPC-DF17)}$$

Dimension: 2
Domain: $x_i \in [-100, 100], \quad i = 1, 2$
Feasible Ratio: 0.554864%
Best Known Minimum:

$$x_1 = -99.9999999999997$$
$$x_2 = 0.974564895523551$$
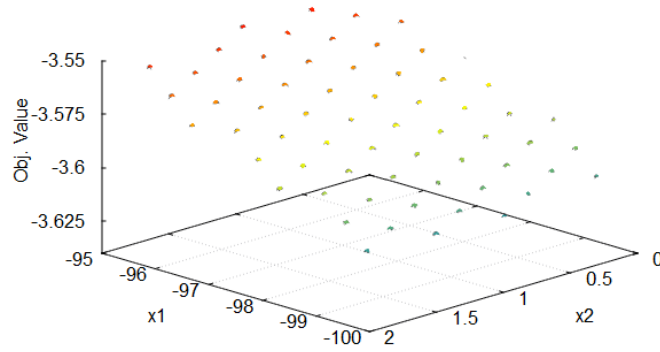$$f(x^*) = -3.60513519536451$$

Figure: See Figure 17.

Figure 17: Illustrations of CPC-DF17

CPC-DF18

Function expression:

$$\min \quad \sum_{i=1}^{4}(f_i^{2+\frac{i}{100}}) \qquad\qquad \text{(CPC-DF18)}$$

Where:

$$
\begin{cases}
f_1 = 2x_1 - (x_2 - x_3 + x_4)^{(10.3-x_1)} \\[2mm]
f_2 = 2 - \ln(-2x_3 - x_4 - 2x_2) - (-x_2\ln(x_1 - \exp(\dfrac{x_1}{x2} - \dfrac{x_3}{x_4})))^{0.75} \\[2mm]
f_3 = 0.0463x_3 - \dfrac{2x_1^2}{x_2\ln(x_3 + x_4 - x_2 - x_1)} \\[2mm]
f_4 = 0.651 - (\dfrac{x_4x_2^5}{x_3})(x_4x_1 - \dfrac{2}{\ln(x_3 + x_4 - x_2 - x_1)})^{0.5}
\end{cases}
$$

Dimension: 4
Domain: $x_i \in [-5000, 5000], \quad i = 1, \ldots, 4$
Feasible Ratio: ¡0.000001%
Best Known Minimum:

$$
\begin{aligned}
x_1 &= 9.95678574505792 \\
x_2 &= -0.276603739268777 \\
x_3 &= -2034.04049530503 \\
x_4 &= 4065.00844459407 \\
f(x^*) &= 3.80984596609911E - 24
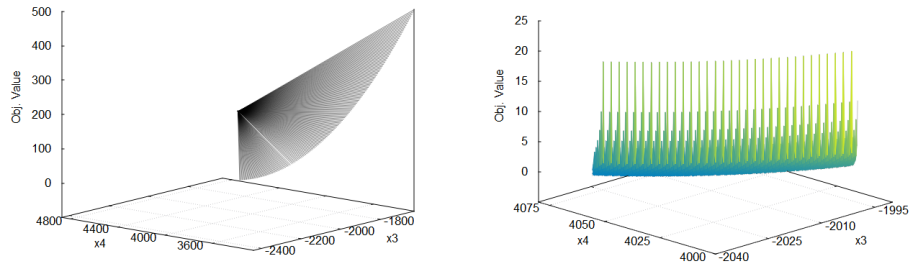\end{aligned}
$$

Figure: See Figure 18.

Figure 18: Illustrations of CPC-DF18. The plot on the right provides a magnified view around the minimum.

CPC-DF19

Function expression:

$$\min \quad \sum_{i=1}^{6}(f_i^2) \tag{CPC-DF19}$$

Where:

$$
\begin{cases}
f_1 = -(1.29)\ln(x_1^2 - x_2) - 1.1(50^2 - x_2^2)^{0.25} \\
f_2 = -(1.59)\ln(x_3^2 - x_4) - 1.2\ln(-x_2^2 + x_4^2) \\
f_3 = -(1.99)\ln(x_5^2 - x_6) - 1.3\ln(x_4^2 - 50^2) \\
f_4 = (x_6 + x_5 - x_1)^{0.1} - \dfrac{x_3}{x_4} \\
f_5 = (-20 + x_1 - x_3)^{0.2} - 1.2 \\
f_6 = (-40 + x_3 - x_5)^{0.3} - 1.3
\end{cases}
$$

Dimension: 6
Domain: $x_i \in [-100, 100], \quad i = 1, \dots, 6$
Feasible Ratio: 0.014292%
Best Known Minimum:

$$
\begin{aligned}
x_1 &= -7.13550946334904 \\
x_2 &= 49.9999988507609 \\
x_3 &= -29.6238293288547 \\
x_4 &= -50.0000000209737 \\
x_5 &= -72.0216194398939 \\
x_6 &= 64.8914397889634 \\
f(x^*) &= 6.64951691932561E - 14
\end{aligned}
$$

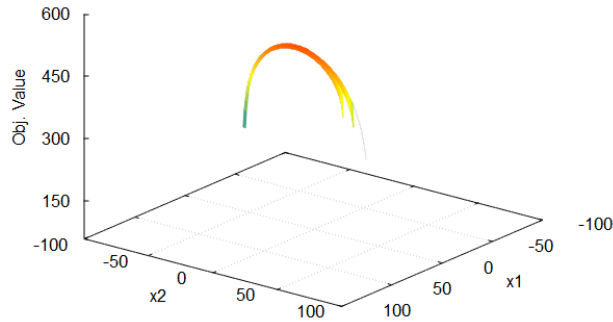**Figure:** See Figure 19.

Figure 19: Illustrations of CPC-DF19

CPC-DF20

Function expression:

$$f(\mathbf{x}) = -\sum_{i=1}^{n-1=4} \left(\sqrt{\ln\left(\sin\left(x_i \cdot x_{i+1} - i\right) + 0.1\right) + x_i \cdot \left(\cos\left(x_i - i \cdot x_{i+1}\right)\right)^{\frac{1}{n}}} \right)$$

$$\text{(CPC-DF20)}$$

Dimension: $n = 5$
Domain: $x_i \in [-10, 10], \quad i = 1, \ldots, n$
Feasible Ratio: 0.023148%
Best Known Minimum:

$$x_1 = 9.93144285039616$$
$$x_2 = 9.75601384188703$$
$$x_3 = 8.10882586659496$$
$$x_4 = 9.08244323594653$$
$$x_5 = -5.6045744273385$$
$$f(x^*) = -12.1737390146446$$
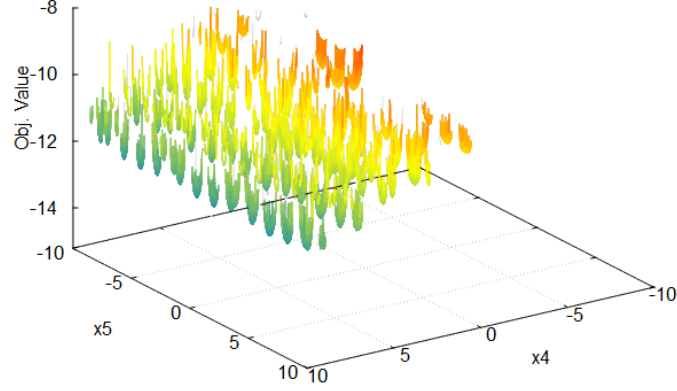
Figure: See Figure 20.

Figure 20: Illustrations of CPC-DF20

CPC-DF21

Function expression:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1=4} \left(-\ln\left(\sin(x_{i+1})^2 - \sin(x_i)^2 + 0.02 \right) + (x_i - 0.25)^{0.15} \right) - \ln(x_n)$$

$$\text{(CPC-DF21)}$$

Dimension: $n = 5$
Domain: $x_i \in [-30, 30], \quad i = 1, \ldots, n$
Feasible Ratio: 0.036648%
Best Known Minimum:

$$x_1 = 0.25$$
$$x_2 = 0.538612607157351$$
$$x_3 = 0.777475105568925$$
$$x_4 = 1.03362860526369$$
$$x_5 = 29.8498508967094$$
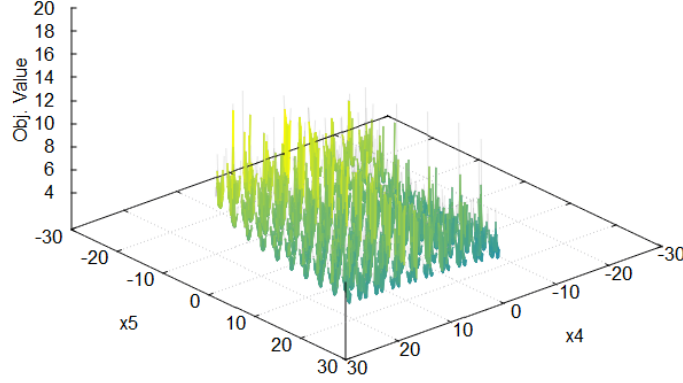$$f(x^*) = 4.79275837921165$$

Figure: See Figure 21.

Figure 21: Illustrations of CPC-DF21

23

CPC-DF22

Function expression:

$$Min.f_1^{1.01} + f_2^{1.01} \qquad\qquad \text{(CPC-DF22)}$$

Where:

$$\begin{cases} f_1 = xy - \left(\dfrac{1}{y} - \dfrac{cos(-x+y)}{x}\right)^{x-y} - 0.01x \\[3mm] f_2 = (-x-y)^{\frac{x}{y}} - \left(\dfrac{1}{x} - x\ln\left(x + y\sin\left(yx\right)\right)\right) + 0.02y \end{cases}$$

Dimension: 2
Domain: $x, y \in [-100, 100]$
Feasible Ratio: 0.06192%
Best Known Minimum:

$$x = -0.71754015858574$$
$$y = -25.2691379447078$$
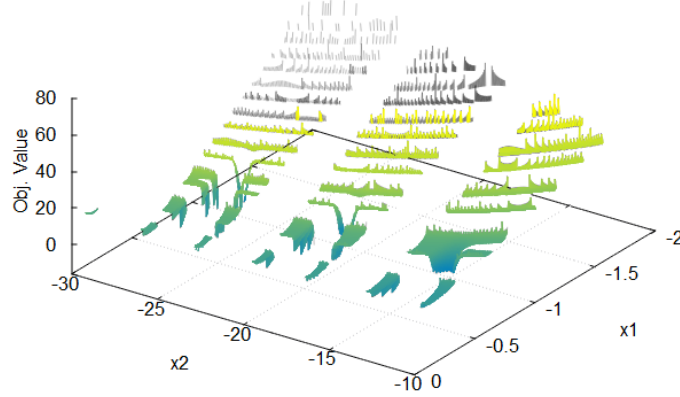$$f(x^*) = 6.92344940900275E - 13$$

Figure: See Figure 22.

Figure 22: Illustrations of CPC-DF22

CPC-DF23

Function expression:

$$f(\mathbf{x}) = -20 \cdot \exp\left(-0.2 \cdot \sqrt{\frac{1}{n} \sum_{i=1}^{n-1=4} \left(\frac{x_i^2}{i} - x_{i+1}^i\right)}\right)$$
$$- \ln\left(\frac{1}{n} \sum_{i=1}^{n-1=4} \left(\sin\left(\sqrt{\cos\left(i \cdot x_i\right) - \sin\left(i \cdot x_{i+1}\right)}\right)\right)\right) \tag{CPC-DF23}$$

Dimension: $n = 5$
Domain: $x_i \in [-100, 100], \quad i = 1, \ldots, n$
Feasible Ratio: 0.76602%
Best Known Minimum:

$$x_1 = 0.0023230141235744$$
$$x_2 = -2.74973369219868$$
$$x_3 = 2.20680567853693$$
$$x_4 = 1.5709301388649$$
$$x_5 = -0.392676764497066$$
$$f(x^*) = -19.740173705628983$$
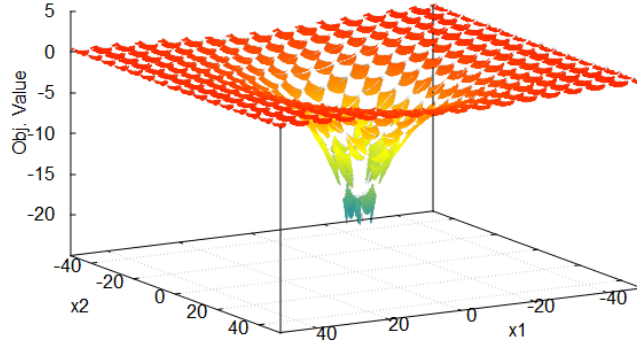
Figure: See Figure 23.

Figure 23: Illustrations of CPC-DF23

CPC-DF24

Function expression:

$$f(\mathbf{x}) = 1 + \frac{\ln\left(2 \cdot \left(\sin\left(\sqrt{100 - x_1^2 - x_2^2}\right)\right)^{0.25} - 1\right)}{\left(1 + 0.001 \cdot (x_1^2 + x_2^2 - 5)^{0.25}\right)^2} \qquad \text{(CPC-DF24)}$$

Dimension: 2
Domain: $x_i \in [-100, 100], \quad i = 1, 2$
Feasible Ratio: 0.486596%
Best Known Minimum: The function admits infinitely many global minima, all lying on the circle of radius $r = 9.514170455044628$ in the (x_1, x_2) plane. Three representative minimizers are provided below as examples:

$$x_1 = 7.11536959763038$$
$$x_2 = -6.31592866780186$$
$$or$$
$$x_1 = -0.302459295102638$$
$$x_2 = -9.5093615885847$$
$$or$$
$$x_1 = -7.41653900436549$$
$$x_2 = 5.95939498979296$$
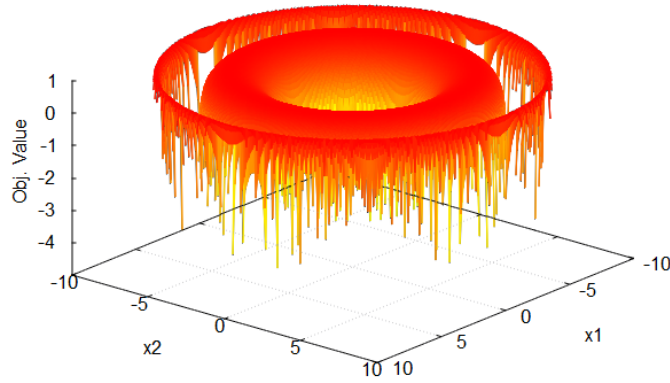$$f(x^*) = -33.4475306875937$$

Figure: See Figure 24.

Figure 24: Illustrations of CPC-DF24

CPC-DF25

Function expression:

$$f(\mathbf{x}) = - \sum_{i=1}^{n-1=2} \left(\frac{\ln\left(\sqrt{\sin\left(x_i - \frac{\pi}{2}\right) + \cos\left(x_{i+1} - \pi\right)} + 0.25 \cdot x_i + 1\right)}{\sqrt{\ln\left(\sin\left(x_i \cdot \pi\right) - \cos\left(2 \cdot x_i - x_{i+1} + \frac{\pi}{2}\right) - \frac{x_i}{i}\right) + 0.1}} \right) \tag{CPC-DF25}$$

Dimension: $n = 3$
Domain: $x_i \in [-10, 10], \quad i = 1, \ldots, n$
Feasible Ratio: 6.763992%
Best Known Minimum:

$$x_1 = 1.68949261280718$$
$$x_2 = 3.2400558621679$$
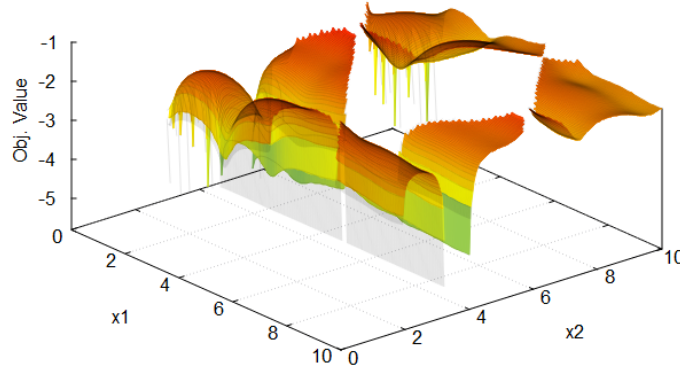$$x_3 = 3.4032112805143$$
$$f(x^*) = -20.737029881233$$

Figure: See Figure 25.

Figure 25: Illustrations of CPC-DF25