

# RCUKF

## Data-Driven Modeling Meets Bayesian Estimation

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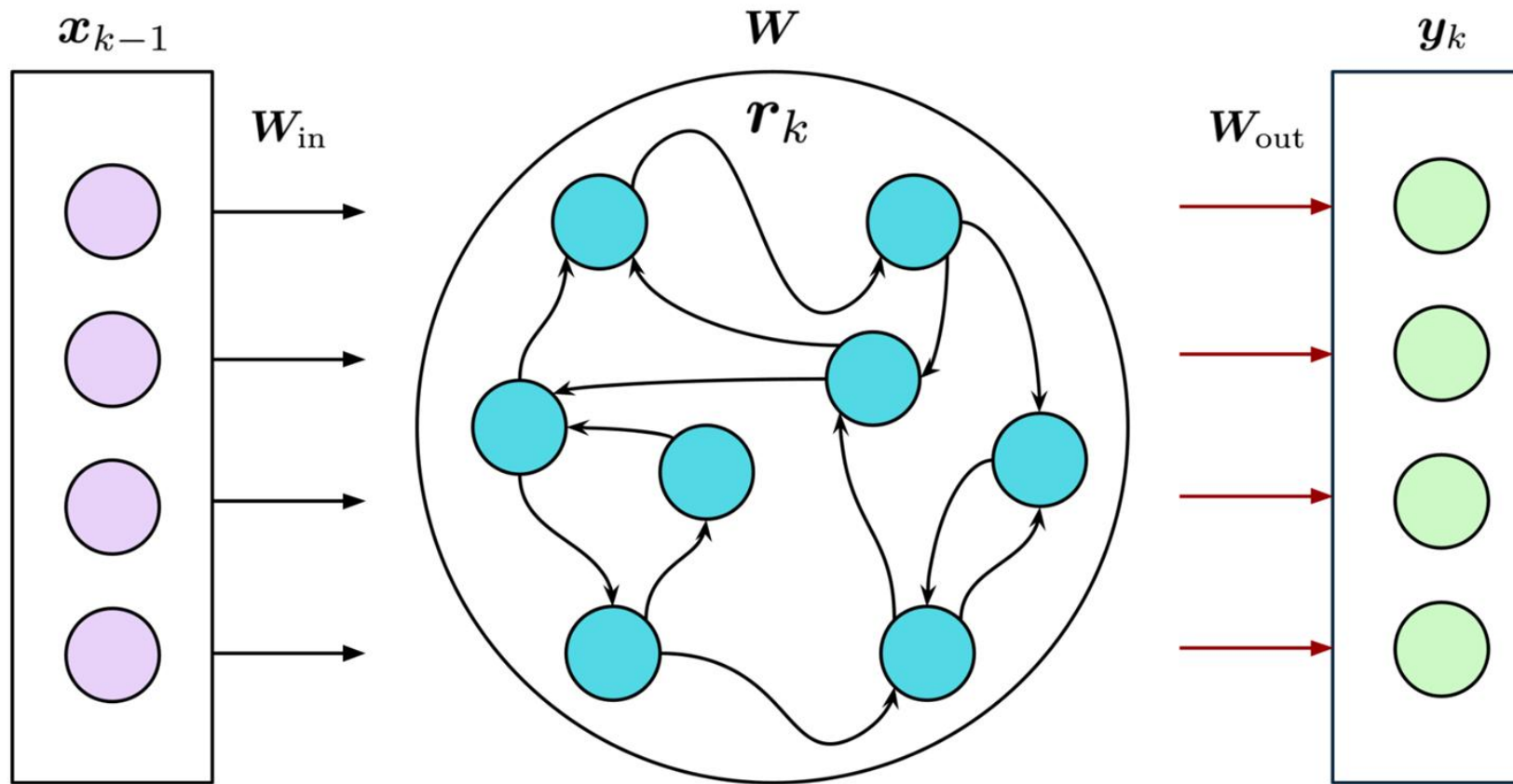
# Motivation



Image Source: Freepik



# Reservoir Computer



# Unscented Kalman Filter

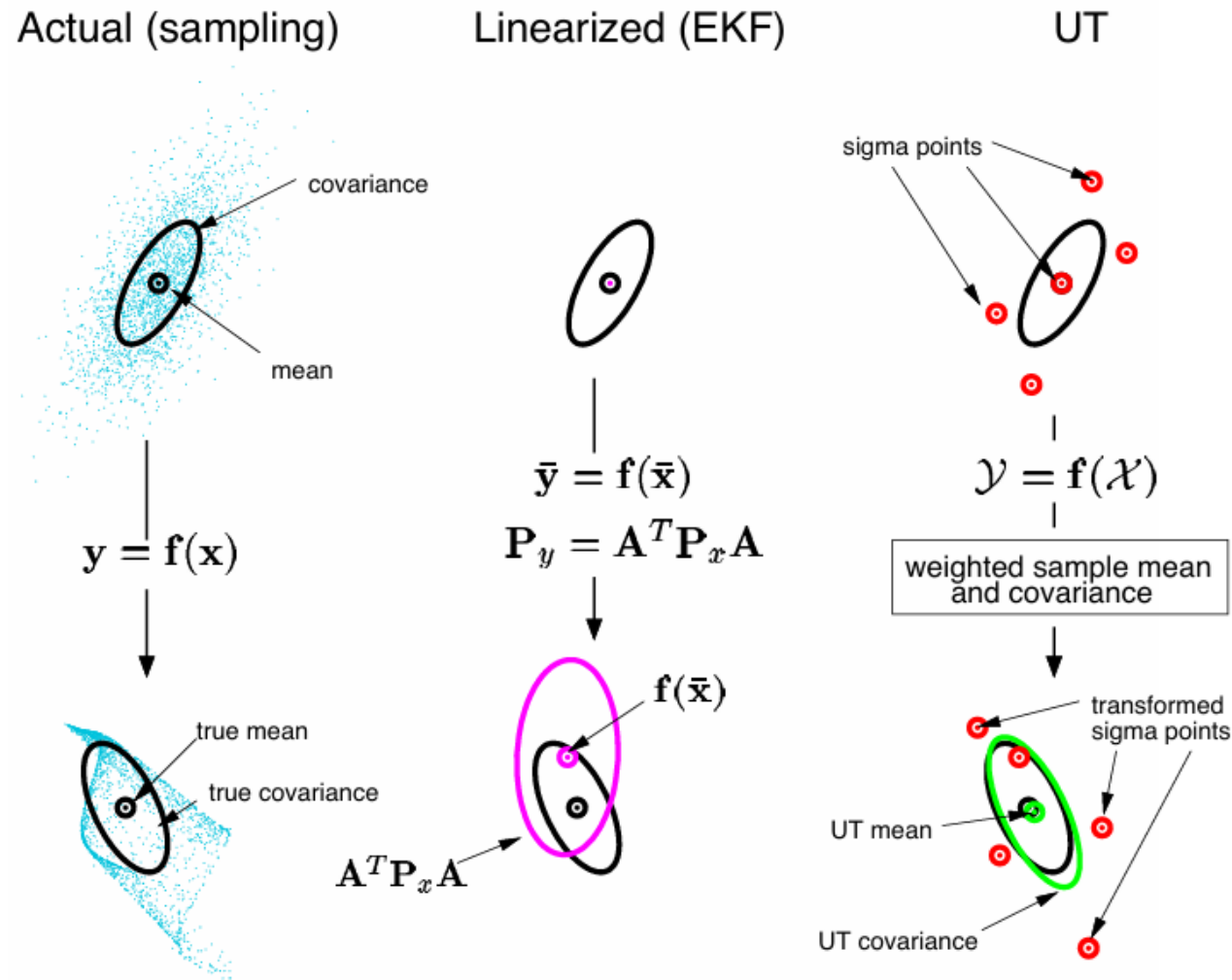
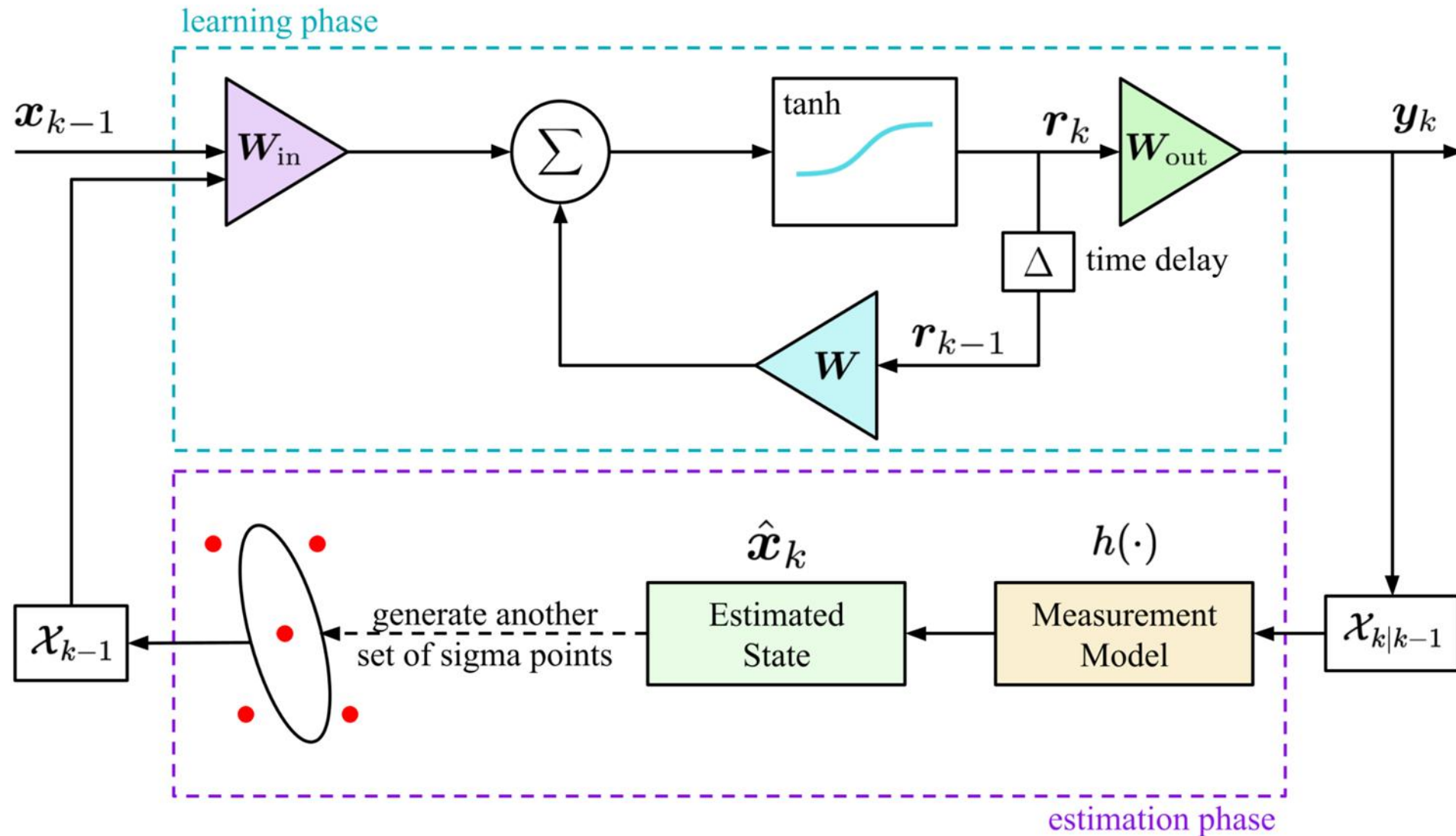
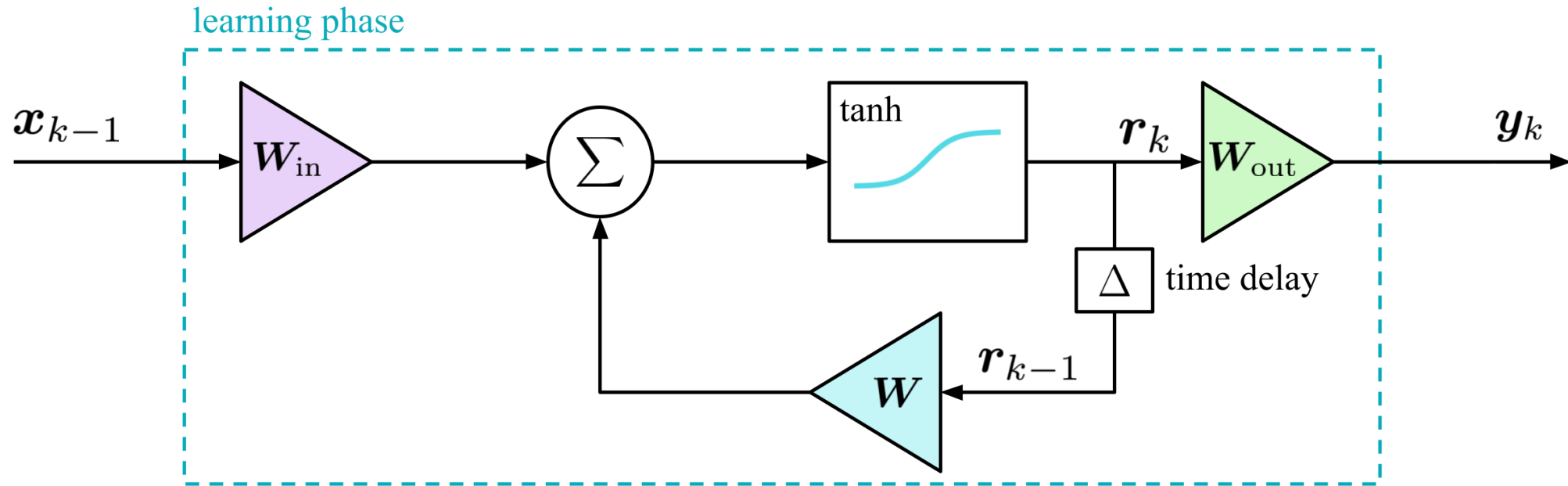


Image Source: Eric A. Wan et al., "The Unscented Kalman Filter for Nonlinear Estimation", IEEE 2000

# RCUKF Framework



# Learning Phase



Reservoir State Update & Output Equation

$$\mathbf{r}_k = (1 - \alpha)\mathbf{r}_{k-1} + \alpha \tanh(\mathbf{W} \mathbf{r}_{k-1} + \mathbf{W}_{in} \mathbf{x}_{k-1})$$

$$\mathbf{y}_k = \mathbf{W}_{out} \mathbf{r}_k$$

# Learning Phase

## Loss Function

$$\mathcal{L} = \underbrace{\frac{1}{N} \sum_{k=1}^N \|\mathbf{y}_k - \mathbf{y}_k^{\text{true}}\|_2^2}_{\text{Mean Squared Error}} + \underbrace{\delta \|\mathbf{W}_{\text{out}}\|_F^2}_{\text{Penalty Term}}$$

## Ridge Regression

$$\mathbf{W}_{\text{out}} = (\mathbf{R} \mathbf{R}^\top + \delta \mathbf{I})^{-1} \mathbf{R} \mathbf{Y}^{\text{true}}$$

# Estimation Phase

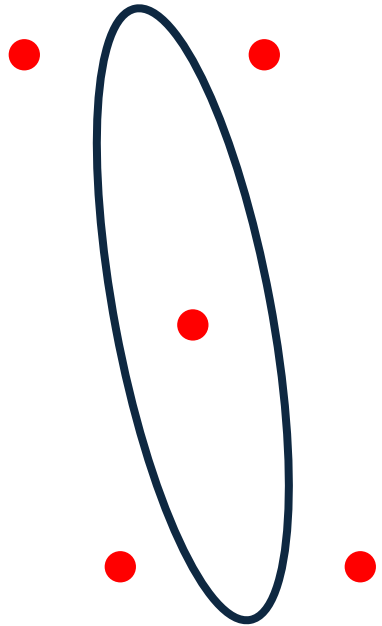
## Nonlinear System Equations

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1}, \quad \mathbf{w}_{k-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{k-1})$$

$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k, \quad \mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k)$$



# Estimation Phase



## Sigma Points Generation

$$\mathcal{X}_{k-1}^{(0)} = \hat{\mathbf{x}}_{k-1}$$

$$\mathcal{X}_{k-1}^{(i)} = \hat{\mathbf{x}}_{k-1} + \left( \sqrt{(n + \lambda) \mathbf{P}_{k-1}} \right)_i, \quad i = 1, \dots, n$$

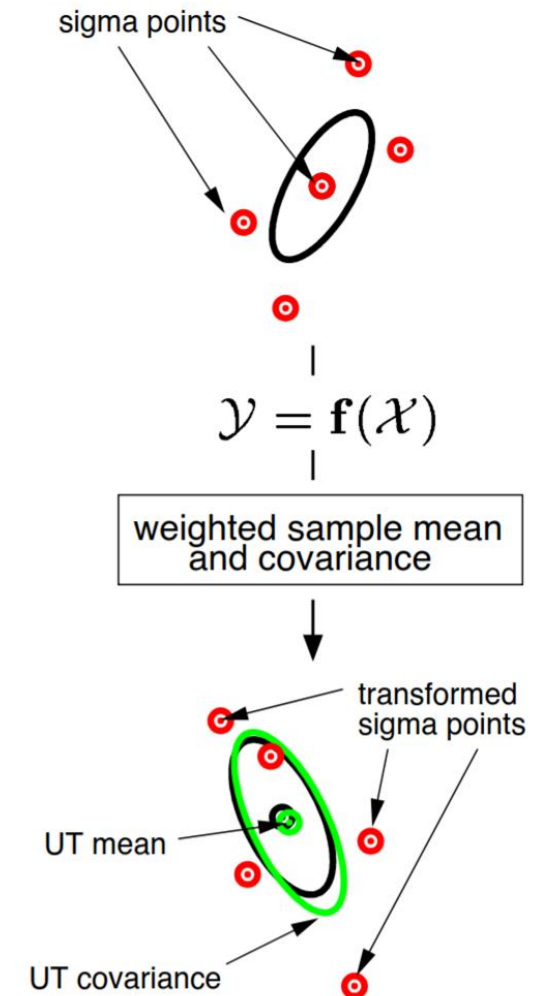
$$\mathcal{X}_{k-1}^{(i)} = \hat{\mathbf{x}}_{k-1} - \left( \sqrt{(n + \lambda) \mathbf{P}_{k-1}} \right)_{i-n}, \quad i = n + 1, \dots, 2n$$

# Estimation Phase

## Sigma Points Propagation

$$\mathbf{r}_k^{(i)} = (1 - \alpha) \mathbf{r}_{k-1}^{(i)} + \alpha \tanh\left(\mathbf{W} \mathbf{r}_{k-1}^{(i)} + \mathbf{W}_{\text{in}} \mathbf{x}_{k-1}^{(i)}\right)$$

$$\mathbf{x}_{k|k-1}^{(i)} = \mathbf{W}_{\text{out}} \mathbf{r}_k^{(i)}$$



# Estimation Phase

## Predicted State and Covariance

$$\hat{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2n} W_m^{(i)} \mathbf{x}_{k|k-1}^{(i)}$$

$$\mathbf{P}_{k|k-1} = \sum_{i=0}^{2n} W_c^{(i)} \left( \mathbf{x}_{k|k-1}^{(i)} - \hat{\mathbf{x}}_{k|k-1} \right) \left( \mathbf{x}_{k|k-1}^{(i)} - \hat{\mathbf{x}}_{k|k-1} \right)^\top + \mathbf{Q}_{k-1}$$

# Estimation Phase

## Predicted Measurement, Measurement Covariance & Cross-Covariance

$$\mathcal{Z}_{k|k-1}^{(i)} = h(\mathcal{X}_{k|k-1}^{(i)}), \quad i = 0, \dots, 2n; \quad \hat{\mathbf{z}}_{k|k-1} = \sum_{i=0}^{2n} W_m^{(i)} \mathcal{Z}_{k|k-1}^{(i)}$$

$$\mathbf{P}_{zz} = \sum_{i=0}^{2n} W_c^{(i)} \left( \mathcal{Z}_{k|k-1}^{(i)} - \hat{\mathbf{z}}_{k|k-1} \right) \left( \mathcal{Z}_{k|k-1}^{(i)} - \hat{\mathbf{z}}_{k|k-1} \right)^\top + \mathbf{R}_k$$

$$\mathbf{P}_{xz} = \sum_{i=0}^{2n} W_c^{(i)} \left( \mathcal{X}_{k|k-1}^{(i)} - \hat{\mathbf{x}}_{k|k-1} \right) \left( \mathcal{Z}_{k|k-1}^{(i)} - \hat{\mathbf{z}}_{k|k-1} \right)^\top$$

# Estimation Phase

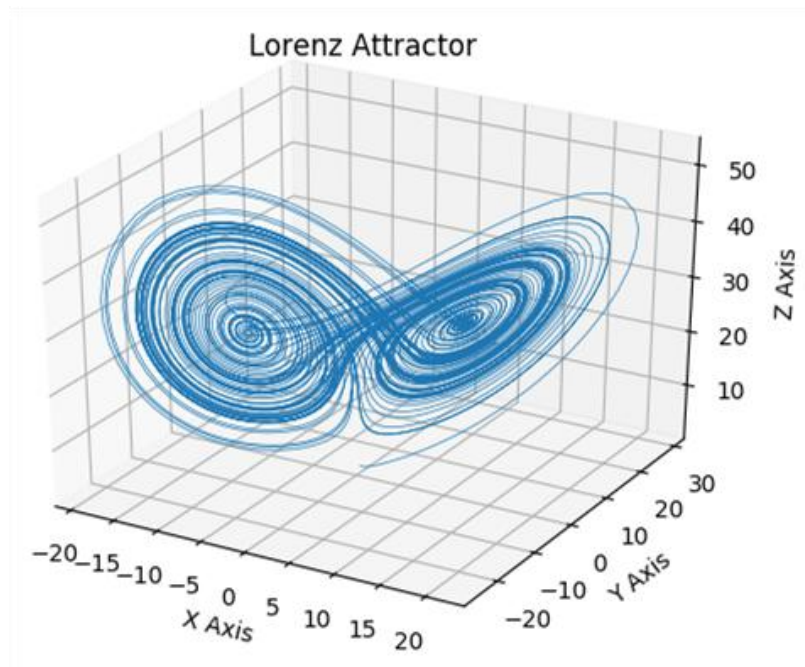
## Final State Estimate

$$\mathbf{K}_k = \mathbf{P}_{xz} \mathbf{P}_{zz}^{-1}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})$$

$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{P}_{zz} \mathbf{K}_k^\top$$

# Lorenz System

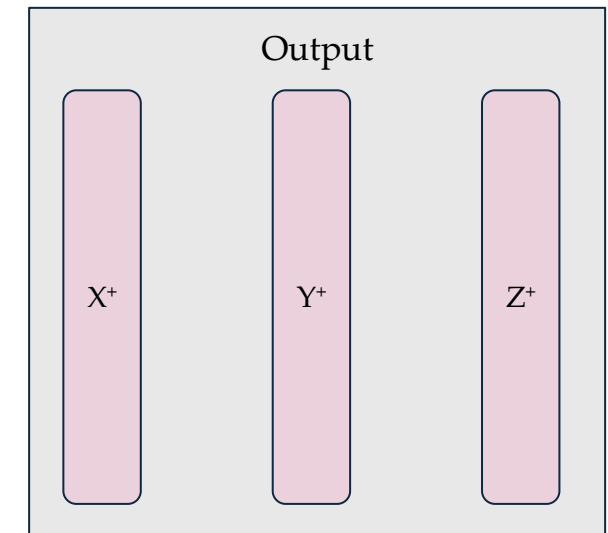
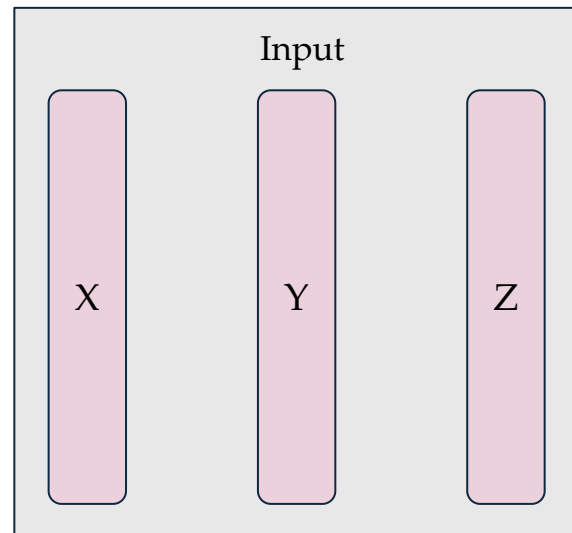


## Lorenz System Equations

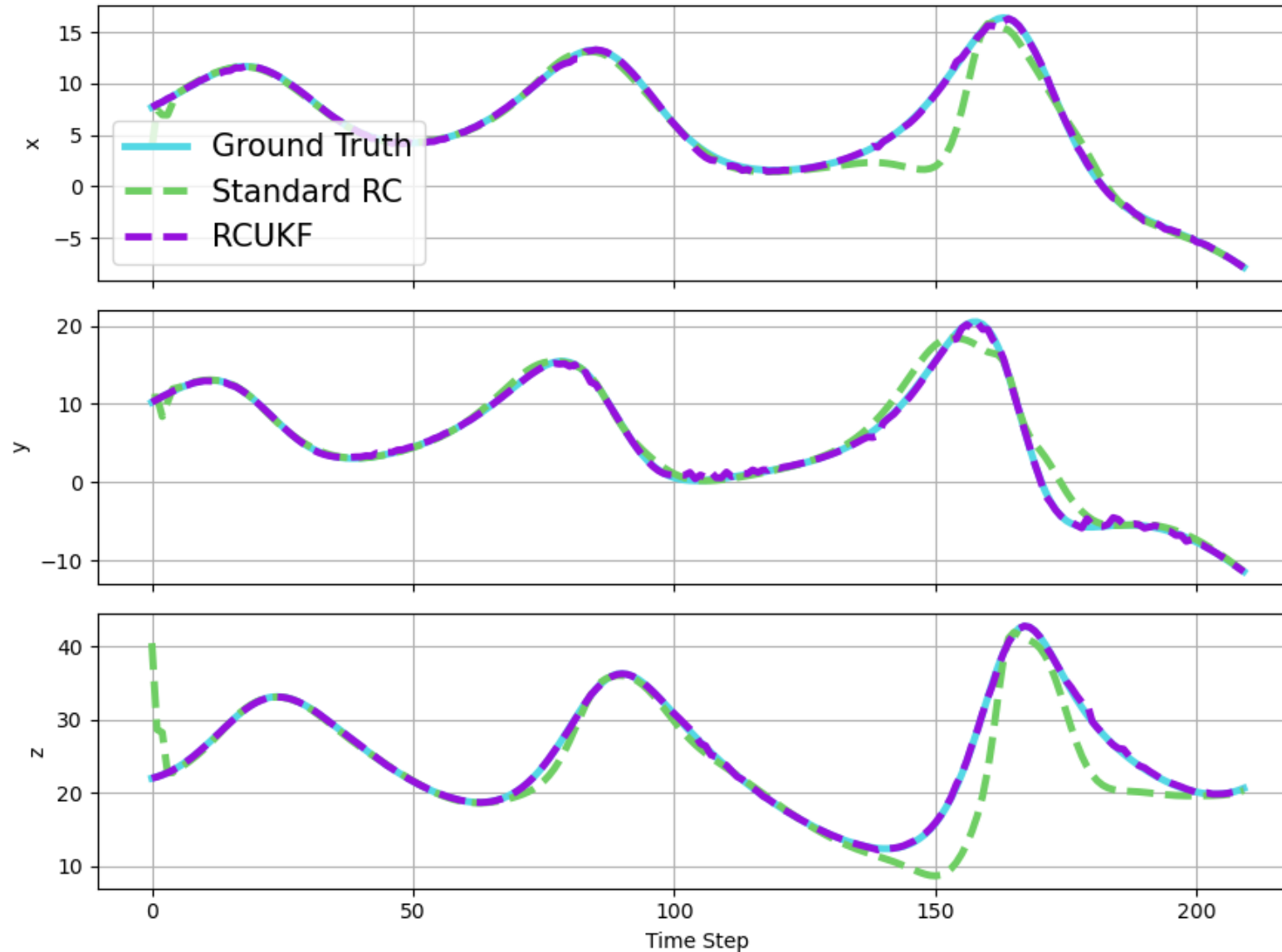
$$\dot{x} = \sigma(y - x) + \epsilon_x$$

$$\dot{y} = x(\rho - z) - y + \epsilon_y$$

$$\dot{z} = xy - \beta z + \epsilon_z$$



# Lorenz Prediction (700 data points)



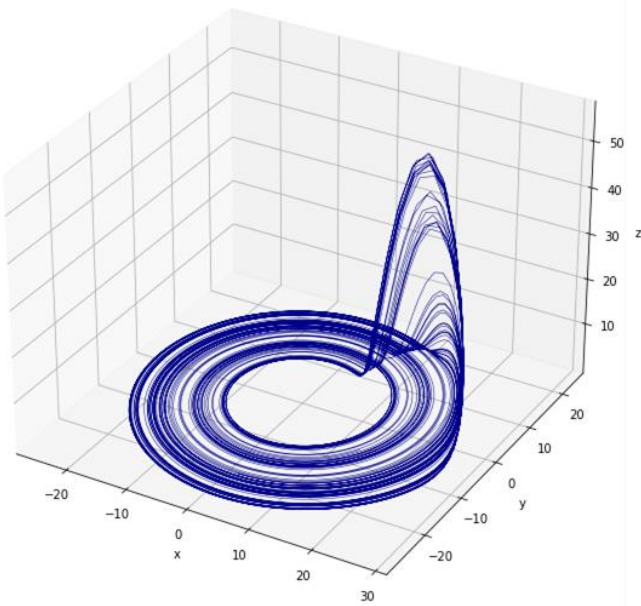
# Rössler System

## Rössler System Equations

$$\dot{x} = -(y + z) + \epsilon_x$$

$$\dot{y} = x + ay + \epsilon_y$$

$$\dot{z} = b + z(x - c) + \epsilon_z$$



Input

X

Y

Z

Output

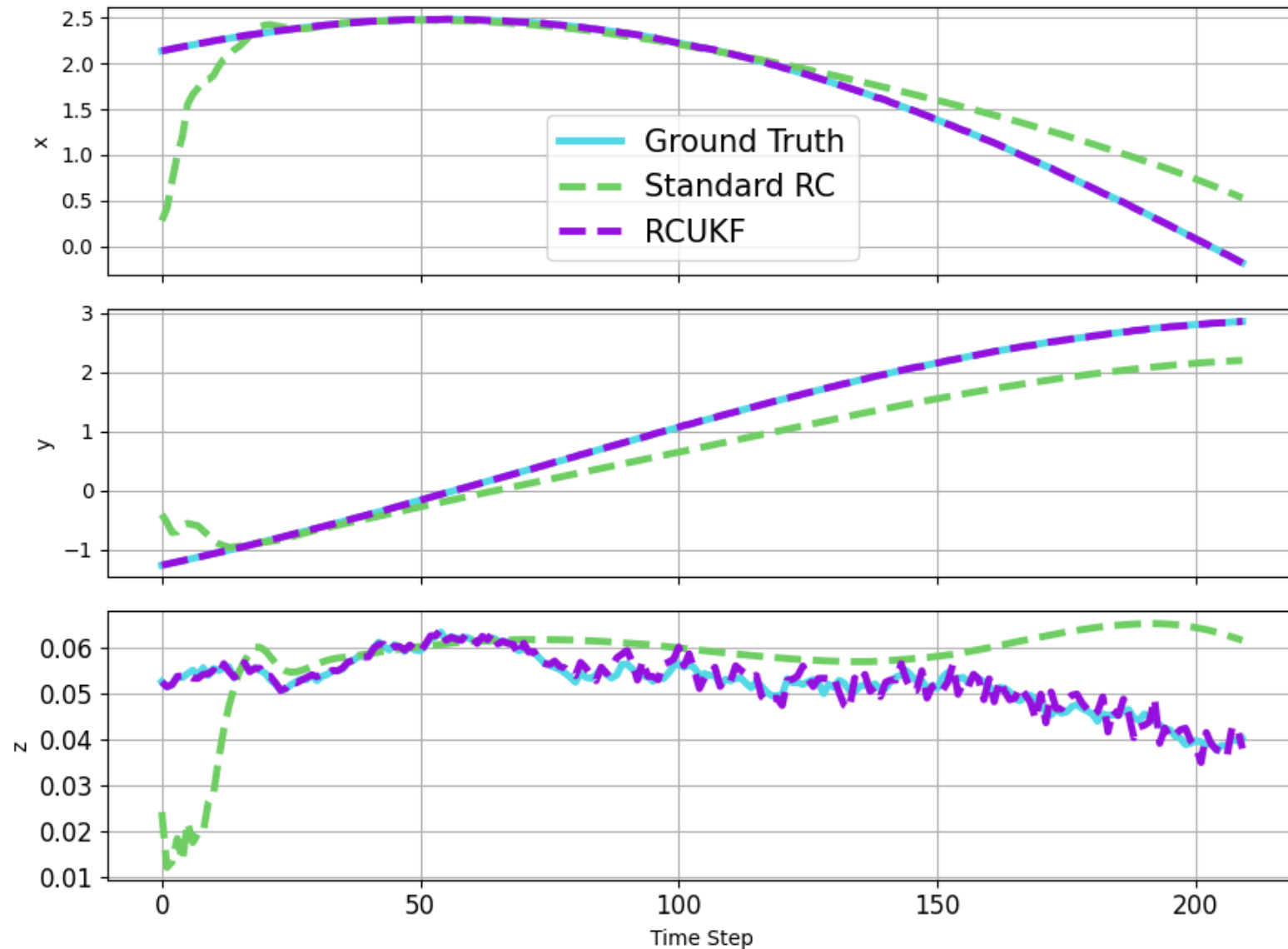
X<sup>+</sup>

Y<sup>+</sup>

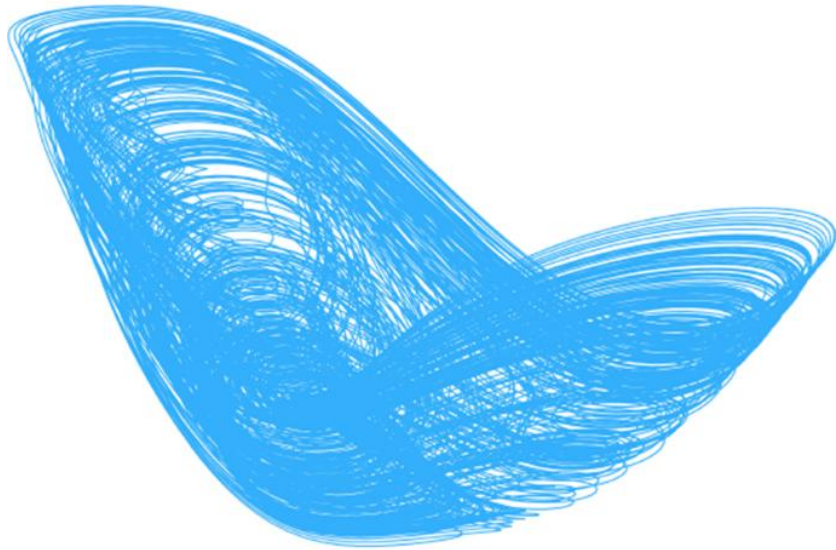
Z<sup>+</sup>



# Rössler Prediction (700 data points)

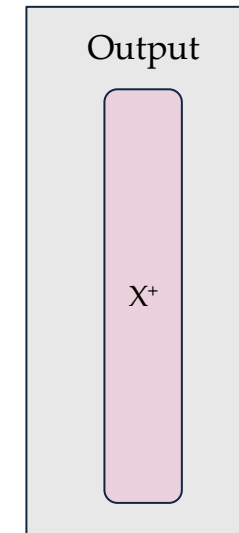
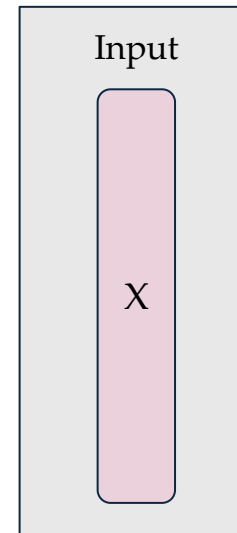


# Mackey Glass Time Series

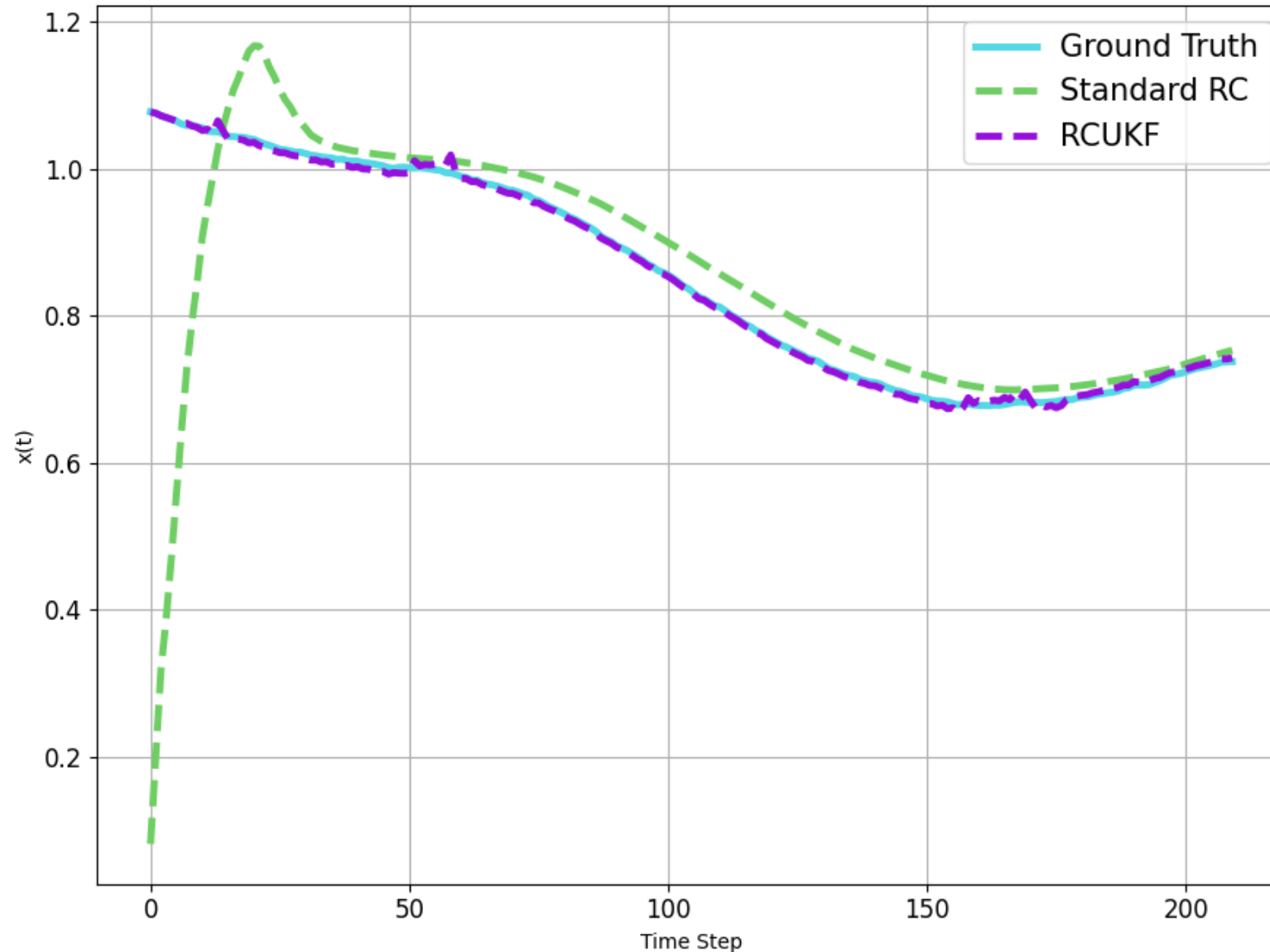


## Mackey Glass Time Series Equations

$$\frac{dx}{dt} = \beta \frac{x(t-\tau)}{1+x^n(t-\tau)} - \gamma x(t) + \epsilon$$



# Mackey Glass Prediction (700 data points)



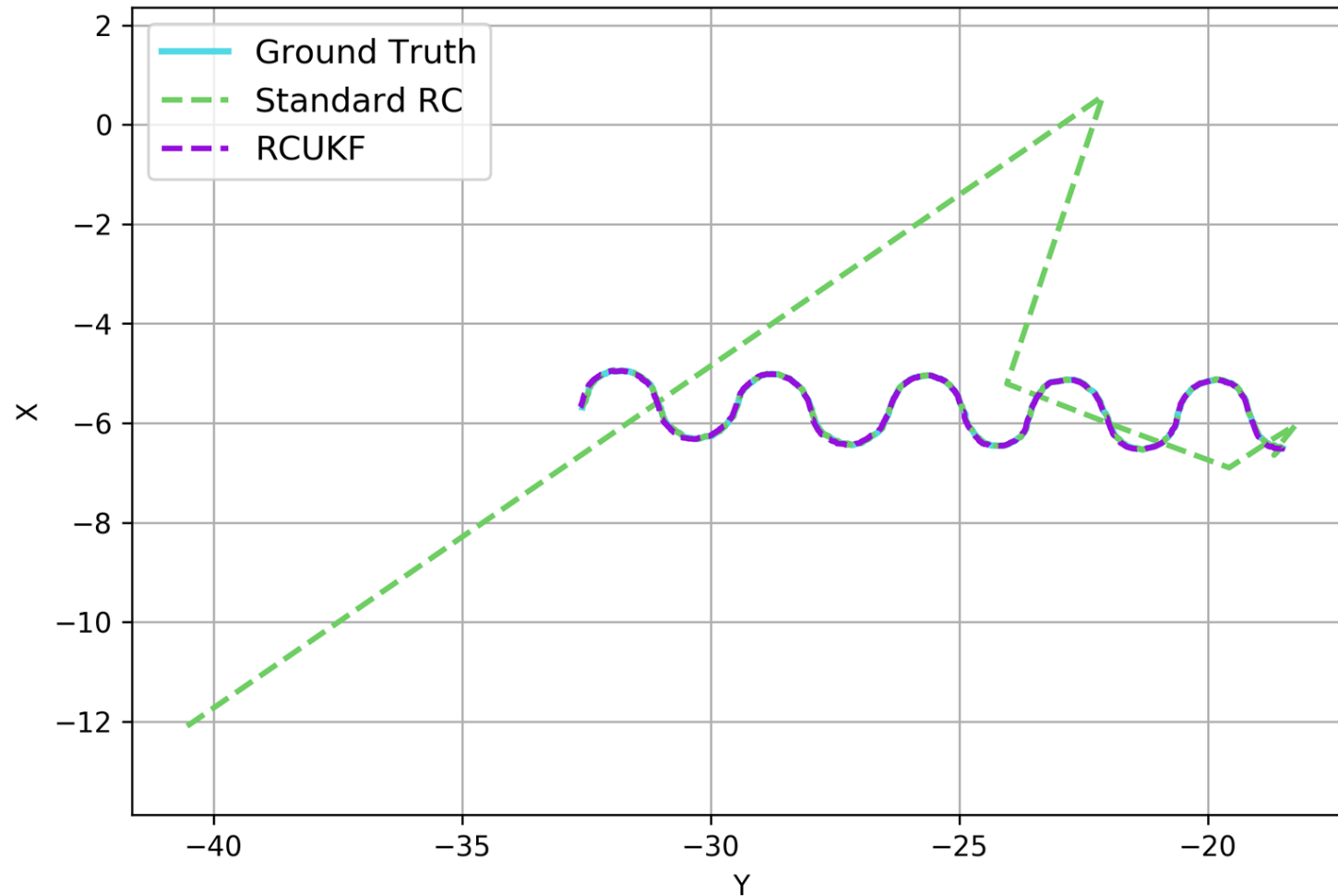
# RMSE Summary

Metric	Lorenz	Rössler	Mackey–Glass
<i>Standard RC (700 data points)</i>			
RMSE-X	0.5829	0.3779	0.1176
RMSE-Y	0.8642	0.4494	
RMSE-Z	1.3483	0.0070	
Mean RMSE	0.9318	0.2781	0.1176
<i>RCUKF (700 data points)</i>			
RMSE-X	0.1628	0.1520	0.0032
RMSE-Y	0.1657	0.2888	
RMSE-Z	0.1270	0.0316	
Mean RMSE	<b>0.1518</b>	<b>0.1575</b>	<b>0.0032</b>
<i>Standard RC (10,000 data points)</i>			
RMSE-X	0.1569	0.1743	0.0323
RMSE-Y	0.2346	0.0448	
RMSE-Z	0.2460	0.0373	
Mean RMSE	0.2125	0.0855	0.0323
<i>RCUKF (10,000 data points)</i>			
RMSE-X	0.0370	0.0790	0.0014
RMSE-Y	0.0384	0.0792	
RMSE-Z	0.0504	0.0653	
Mean RMSE	<b>0.0419</b>	<b>0.0745</b>	<b>0.0014</b>

# MIT FlightGoggles - Stata Ground Floor



# Real time simulation with noises



# Thank you!

Questions?



RCUKF Project Page



ONE Lab Homepage



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