RCUKF
Data-Driven Modeling Meets Bayesian Estimation

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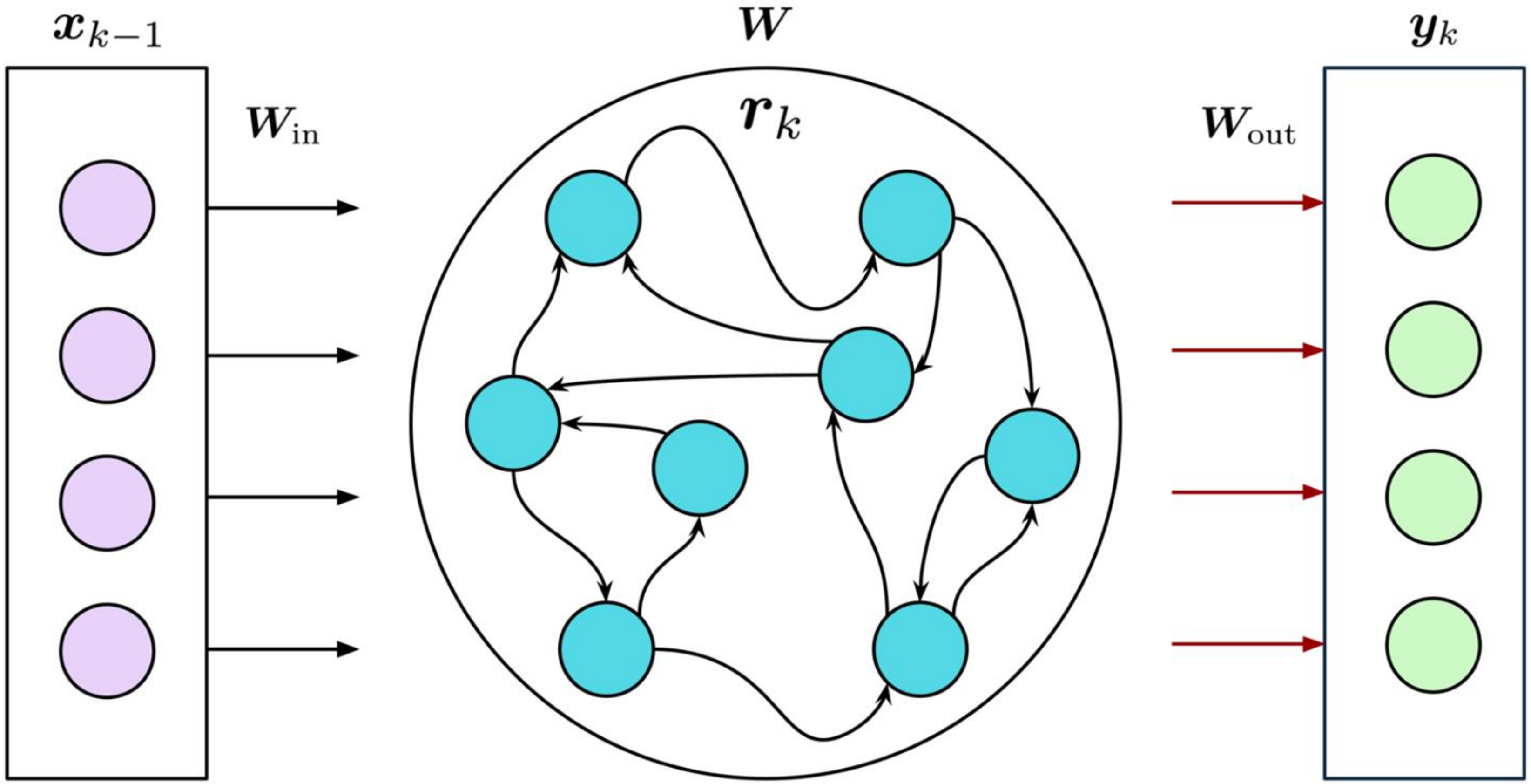
Motivation

Image Source: Freepik

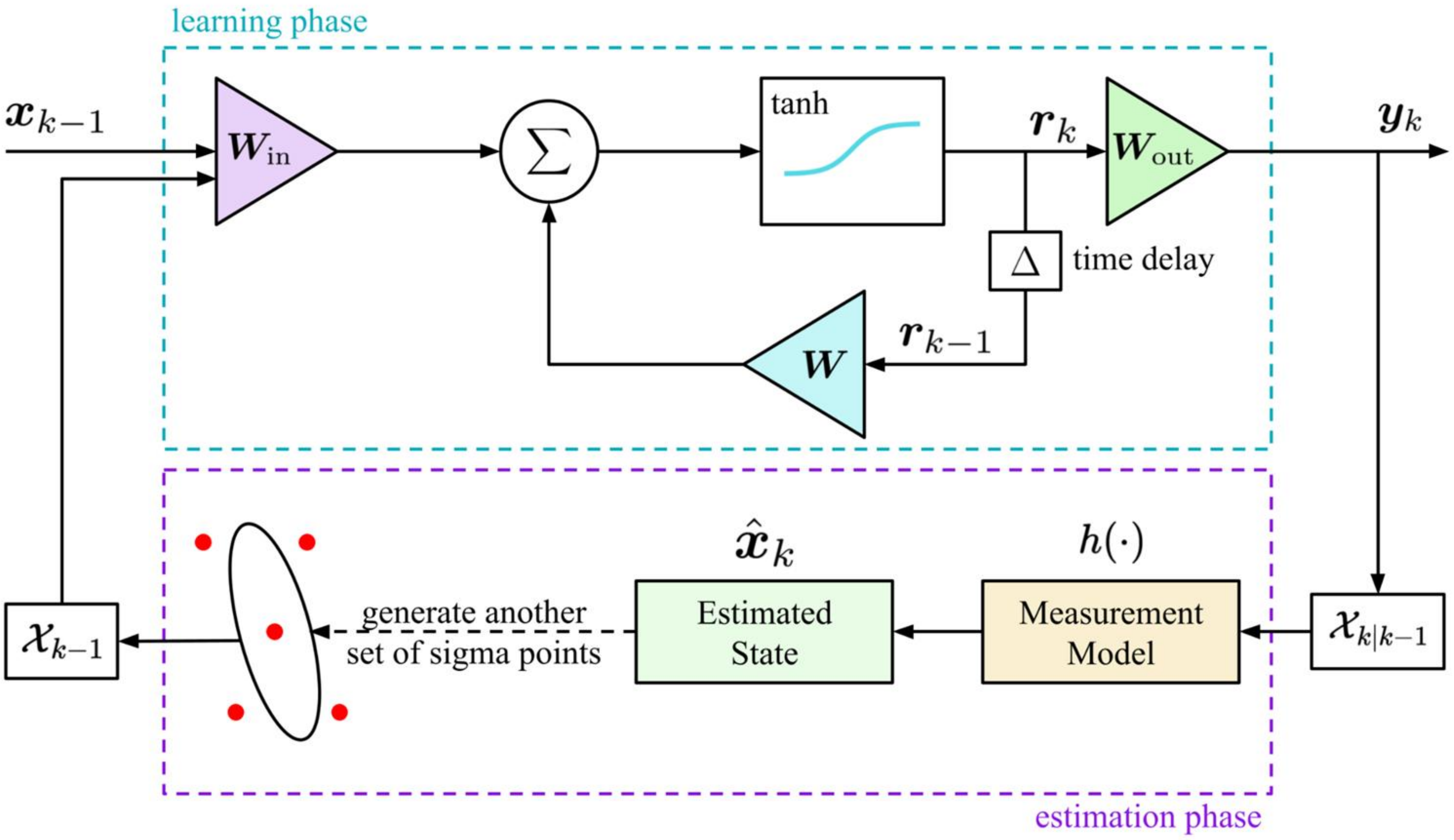
Reservoir Computer



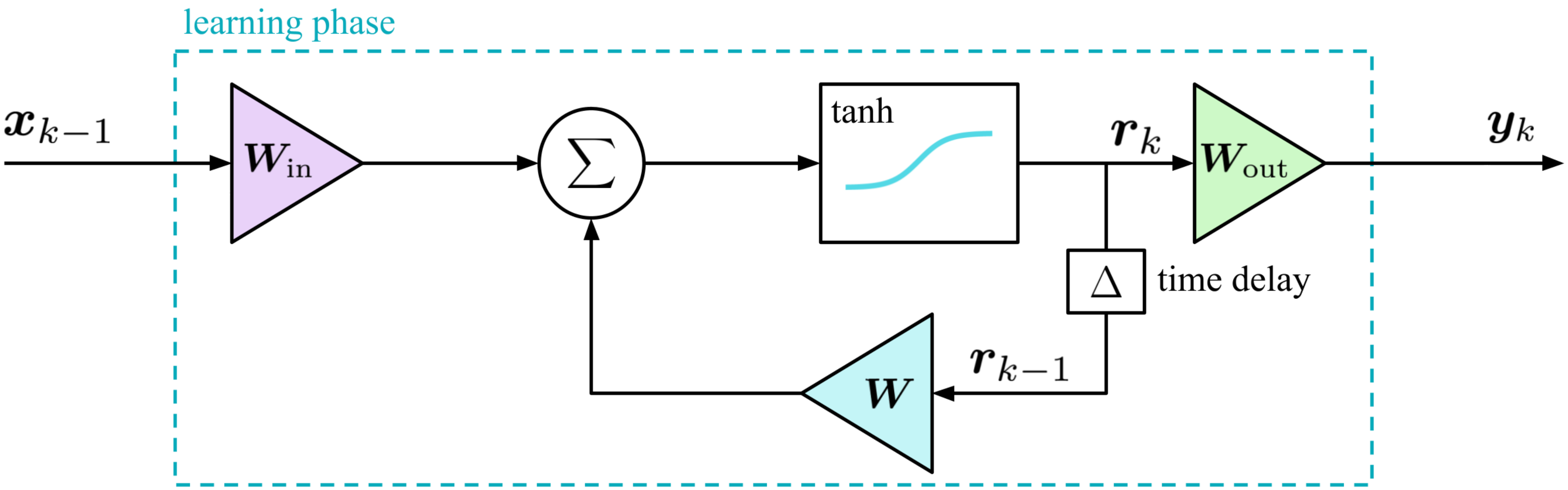
Good Quality Dataset

Attacker

RCUKF Framework



Learning Phase



Reservoir State Update & Output Equation

$$\boldsymbol{r}_k = (1-\alpha)\boldsymbol{r}_{k-1} + \alpha \tanh\big(\boldsymbol{W}\,\boldsymbol{r}_{k-1} + \boldsymbol{W}_{\text{in}}\,\boldsymbol{x}_{k-1}\big)$$

$$\boldsymbol{y}_k = \boldsymbol{W}_{\text{out}}\,\boldsymbol{r}_k$$

Learning Phase

Loss Function

$$\mathcal{L} = \underbrace{\frac{1}{N}\sum_{k=1}^{N}\left\|\boldsymbol{y}_k - \boldsymbol{y}_k^{\text{true}}\right\|_2^2}_{\text{Mean Squared Error}} + \underbrace{\delta\left\|\boldsymbol{W}_{\text{out}}\right\|_F^2}_{\text{Penalty Term}}$$

Ridge Regression

$$\boldsymbol{W}_{\text{out}} = \left(\boldsymbol{R}\boldsymbol{R}^\top + \delta\,\boldsymbol{I}\right)^{-1}\boldsymbol{R}\boldsymbol{Y}^{\text{true}}$$

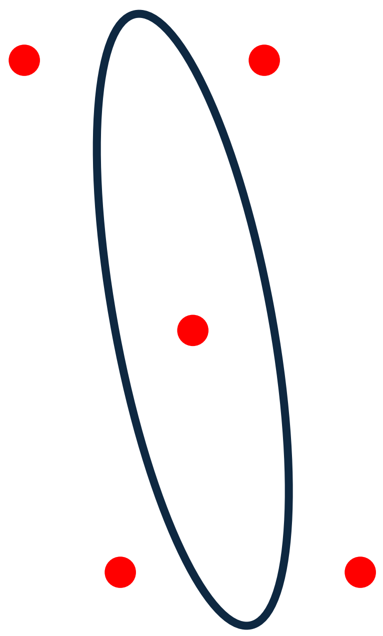
Estimation Phase

Nonlinear System Equations

$$\boldsymbol{x}_k = f\left(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}\right) + \boldsymbol{w}_{k-1}, \quad \boldsymbol{w}_{k-1} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}_{k-1})$$

$$\boldsymbol{z}_k = h\left(\boldsymbol{x}_k\right) + \boldsymbol{v}_k, \quad \boldsymbol{v}_k \sim \mathcal{N}(\mathbf{0}, \boldsymbol{R}_k)$$

Estimation Phase



Sigma Points Generation

$$\mathcal{X}_{k-1}^{(0)} = \hat{\boldsymbol{x}}_{k-1}$$

$$\mathcal{X}_{k-1}^{(i)} = \hat{\boldsymbol{x}}_{k-1} + \left(\sqrt{(n+\lambda)\,\boldsymbol{P}_{k-1}}\right)_i, \quad i = 1, \ldots, n$$

$$\mathcal{X}_{k-1}^{(i)} = \hat{\boldsymbol{x}}_{k-1} - \left(\sqrt{(n+\lambda)\,\boldsymbol{P}_{k-1}}\right)_{i-n}, \; i = n+1, \ldots, 2n$$

Estimation Phase

Sigma Points Propagation

$$\boldsymbol{r}_k^{(i)} = (1-\alpha)\,\boldsymbol{r}_{k-1}^{(i)} + \alpha\,\tanh\!\Big(\boldsymbol{W}\,\boldsymbol{r}_{k-1}^{(i)} + \boldsymbol{W}_{\text{in}}\,\mathcal{X}_{k-1}^{(i)}\Big)$$

$$\mathcal{X}_{k|k-1}^{(i)} = \boldsymbol{W}_{\text{out}}\,\boldsymbol{r}_k^{(i)}$$

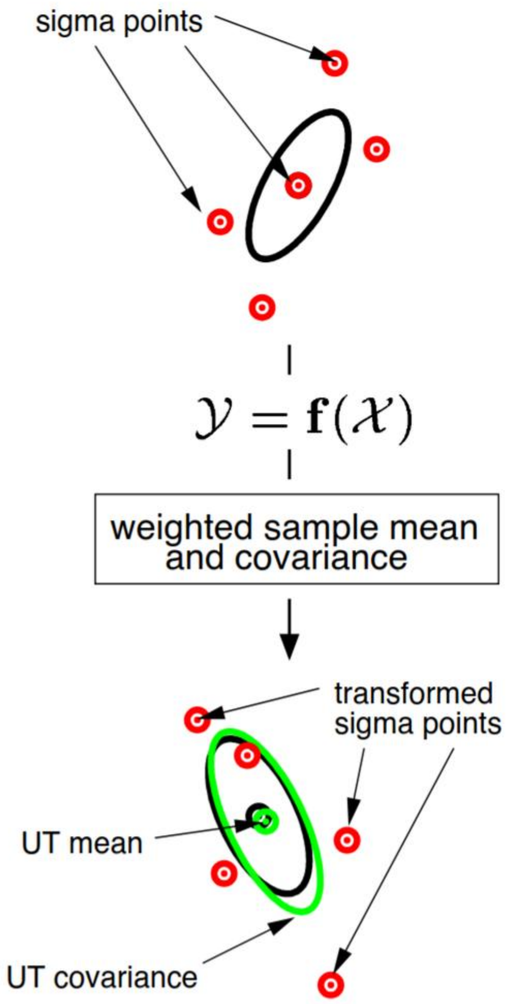


Image Source: Eric A. Wan et al., “The Unscented Kalman Filter for Nonlinear Estimation“, IEEE 2000

Estimation Phase

Predicted State and Covariance

$$\hat{\boldsymbol{x}}_{k|k-1} = \textstyle\sum_{i=0}^{2n} W_m^{(i)} \, \mathcal{X}_{k|k-1}^{(i)}$$

$$\boldsymbol{P}_{k|k-1} = \textstyle\sum_{i=0}^{2n} W_c^{(i)} \left(\mathcal{X}_{k|k-1}^{(i)} - \hat{\boldsymbol{x}}_{k|k-1}\right)\left(\mathcal{X}_{k|k-1}^{(i)} - \hat{\boldsymbol{x}}_{k|k-1}\right)^{\top} + \boldsymbol{Q}_{k-1}$$

Estimation Phase

Predicted Measurement, Measurement Covariance & Cross-Covariance

$$\mathcal{Z}_{k|k-1}^{(i)} = h\left(\mathcal{X}_{k|k-1}^{(i)}\right), \quad i = 0, \ldots, 2n; \quad \hat{\boldsymbol{z}}_{k|k-1} = \sum_{i=0}^{2n} W_m^{(i)} \, \mathcal{Z}_{k|k-1}^{(i)}$$

$$\boldsymbol{P}_{zz} = \sum_{i=0}^{2n} W_c^{(i)} \left(\mathcal{Z}_{k|k-1}^{(i)} - \hat{\boldsymbol{z}}_{k|k-1}\right) \left(\mathcal{Z}_{k|k-1}^{(i)} - \hat{\boldsymbol{z}}_{k|k-1}\right)^{\top} + \boldsymbol{R}_k$$

$$\boldsymbol{P}_{xz} = \sum_{i=0}^{2n} W_c^{(i)} \left(\mathcal{X}_{k|k-1}^{(i)} - \hat{\boldsymbol{x}}_{k|k-1}\right) \left(\mathcal{Z}_{k|k-1}^{(i)} - \hat{\boldsymbol{z}}_{k|k-1}\right)^{\top}$$

Estimation Phase

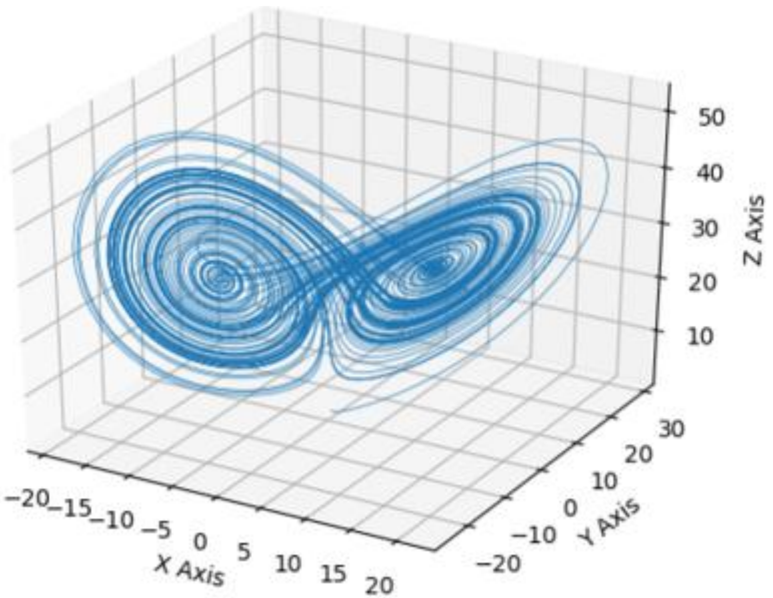
Final State Estimate

$$\boldsymbol{K}_k = \boldsymbol{P}_{xz}\,\boldsymbol{P}_{zz}^{-1}$$

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k\left(\boldsymbol{z}_k - \hat{\boldsymbol{z}}_{k|k-1}\right)$$

$$\boldsymbol{P}_k = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k\,\boldsymbol{P}_{zz}\,\boldsymbol{K}_k^{\top}$$

Lorenz System



Lorenz System Equations

$$\dot{x} = \sigma(y - x) + \epsilon_x$$

$$\dot{y} = x(\rho - z) - y + \epsilon_y$$

$$\dot{z} = xy - \beta z + \epsilon_z$$

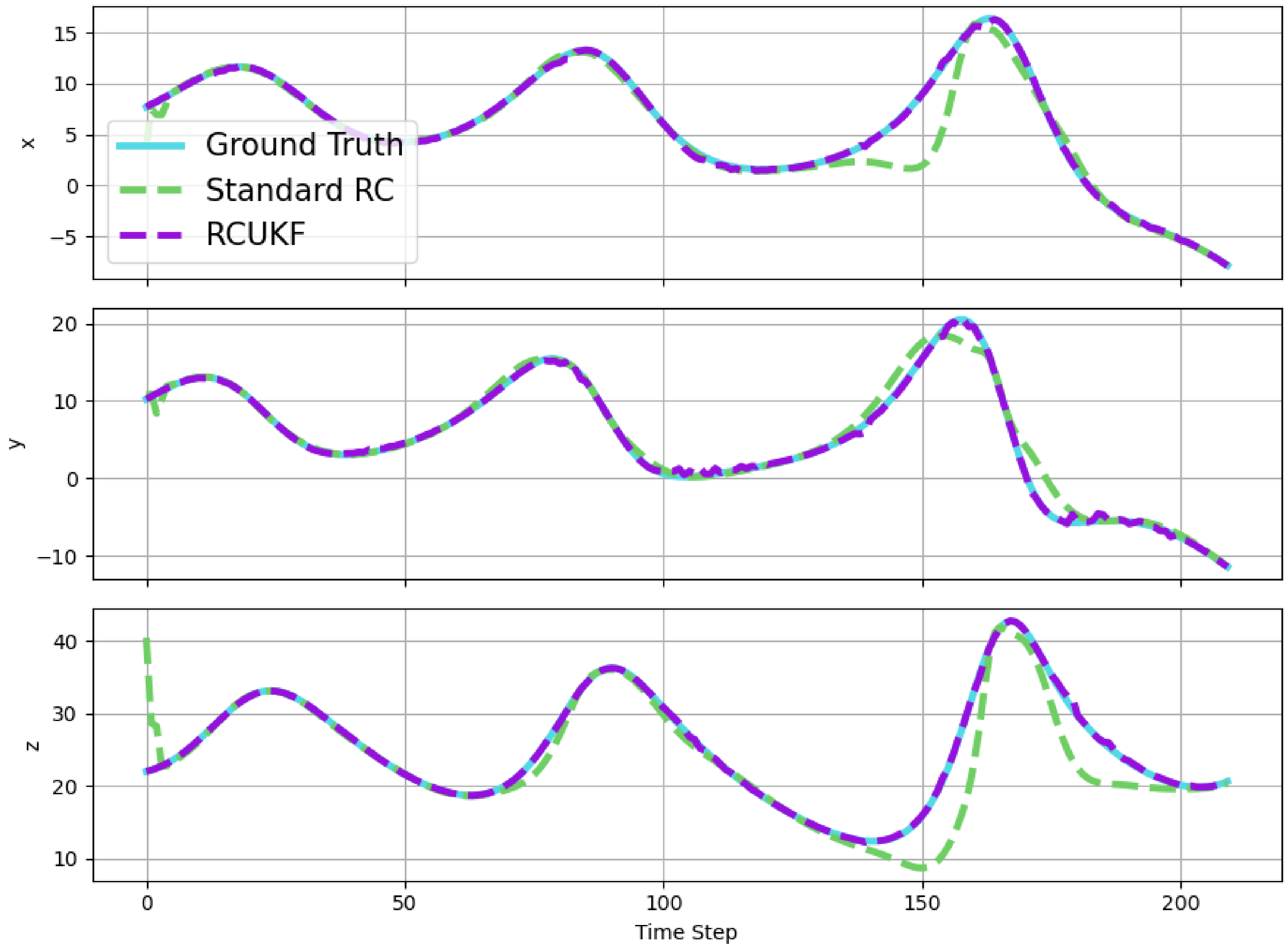
Input

X Y Z

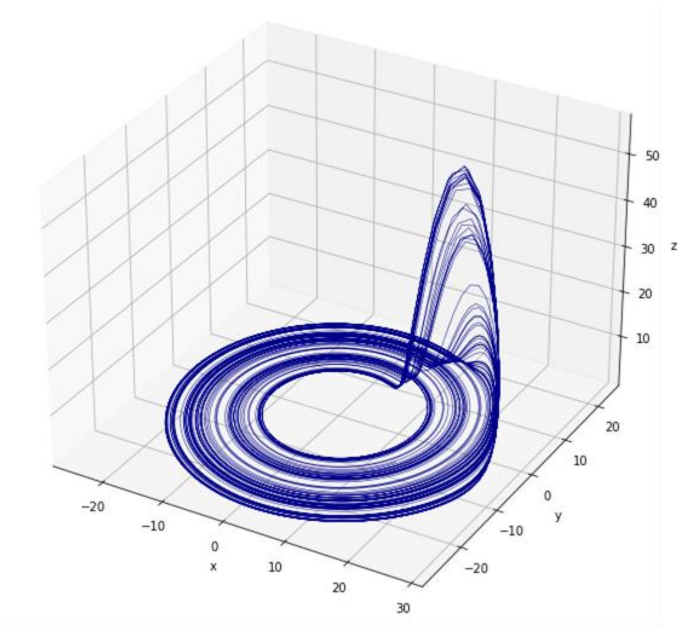
Output

X^+ Y^+ Z^+

Lorenz Prediction (700 data points)



Rössler System

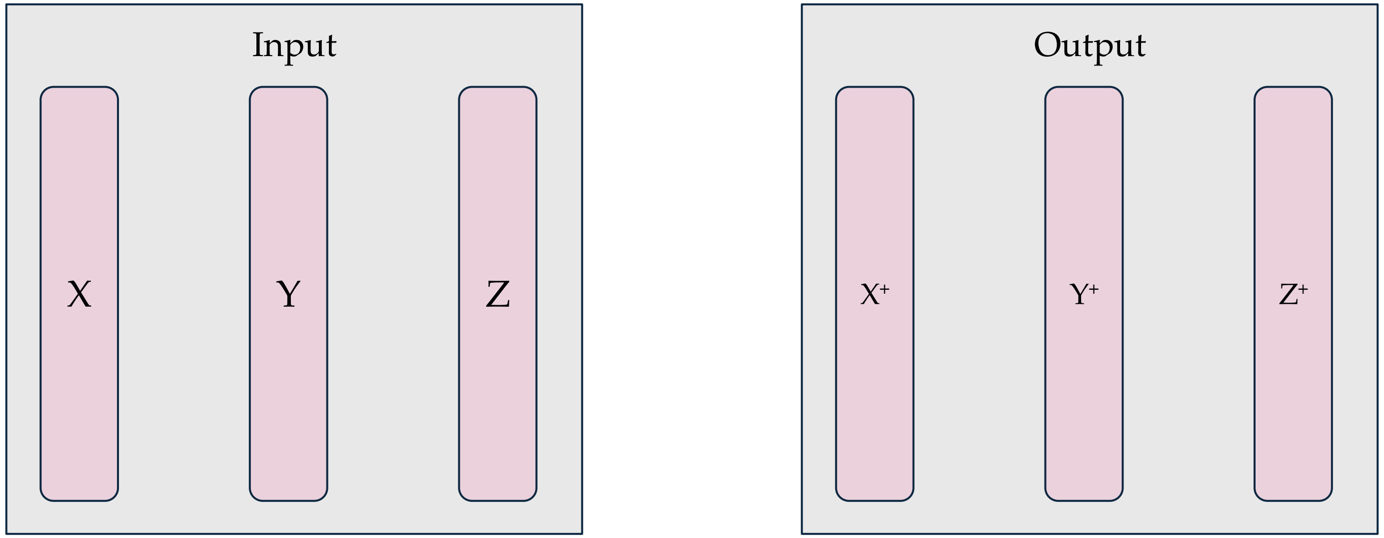


Rössler System Equations

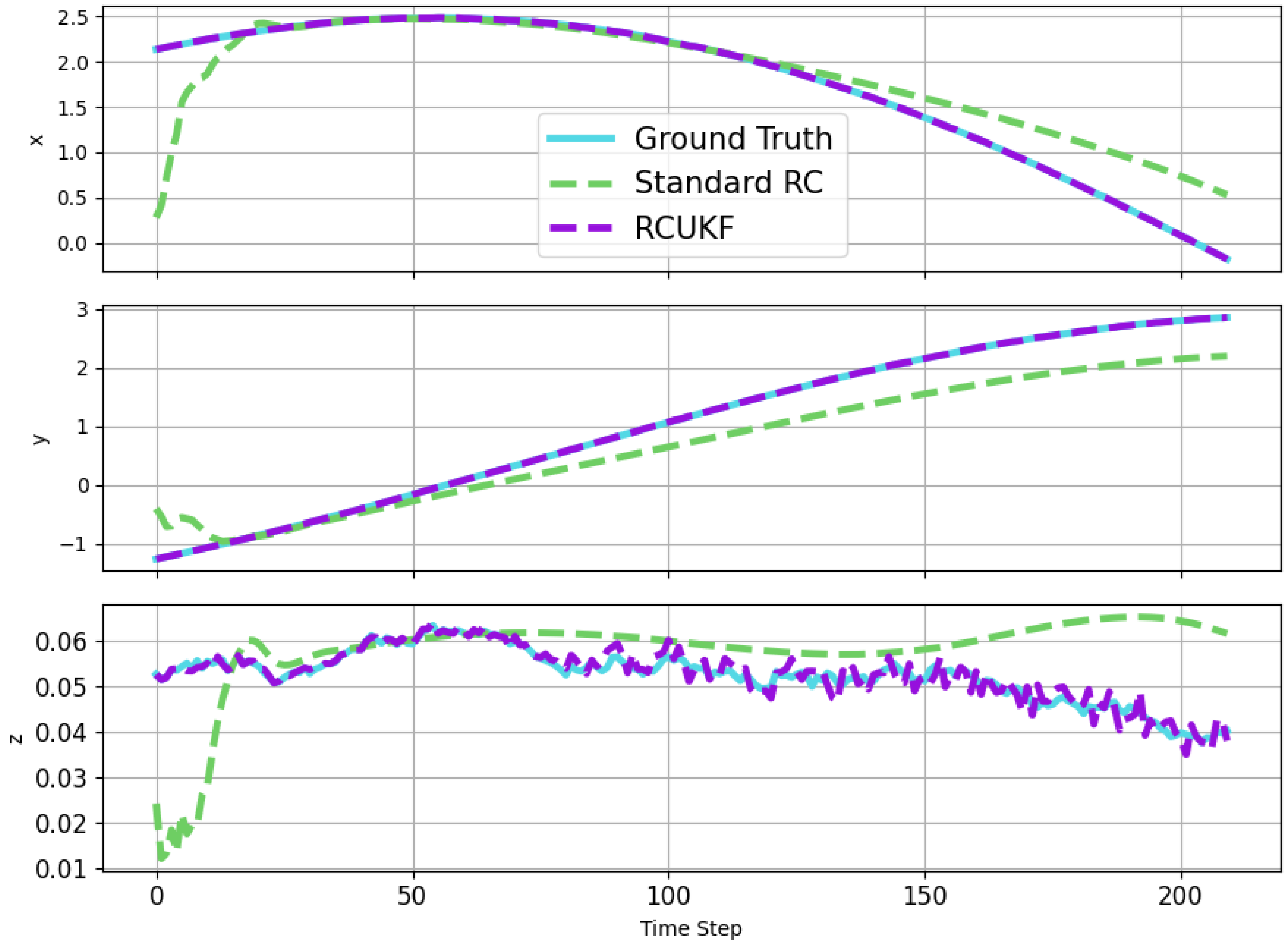
$$\dot{x} = -(y + z) + \epsilon_x$$

$$\dot{y} = x + ay + \epsilon_y$$

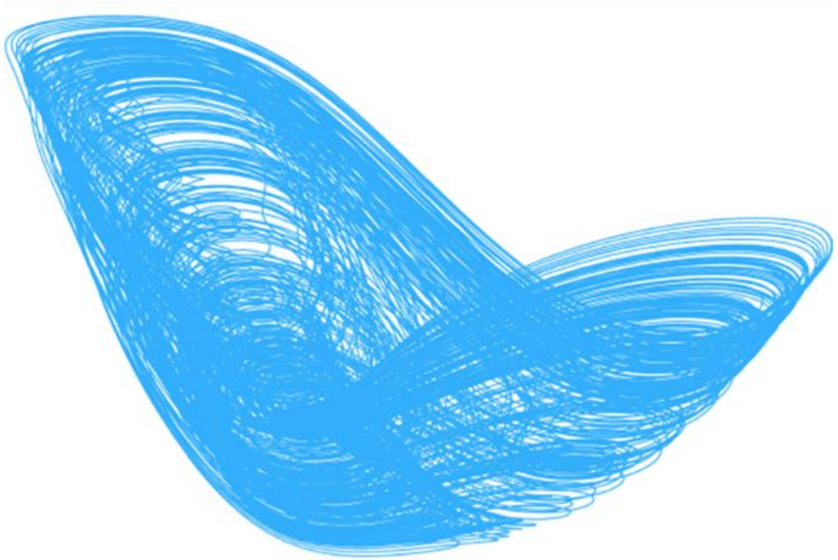
$$\dot{z} = b + z(x - c) + \epsilon_z$$



Rössler Prediction (700 data points)

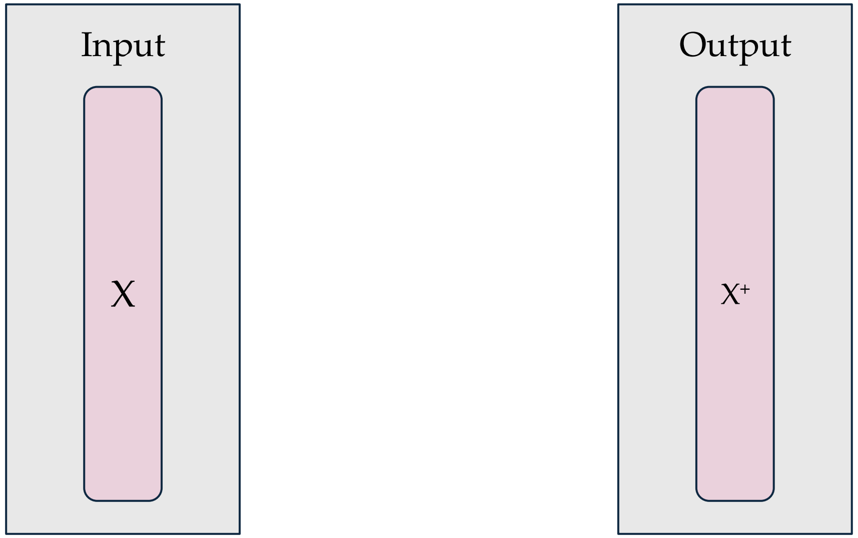


Mackey Glass Time Series

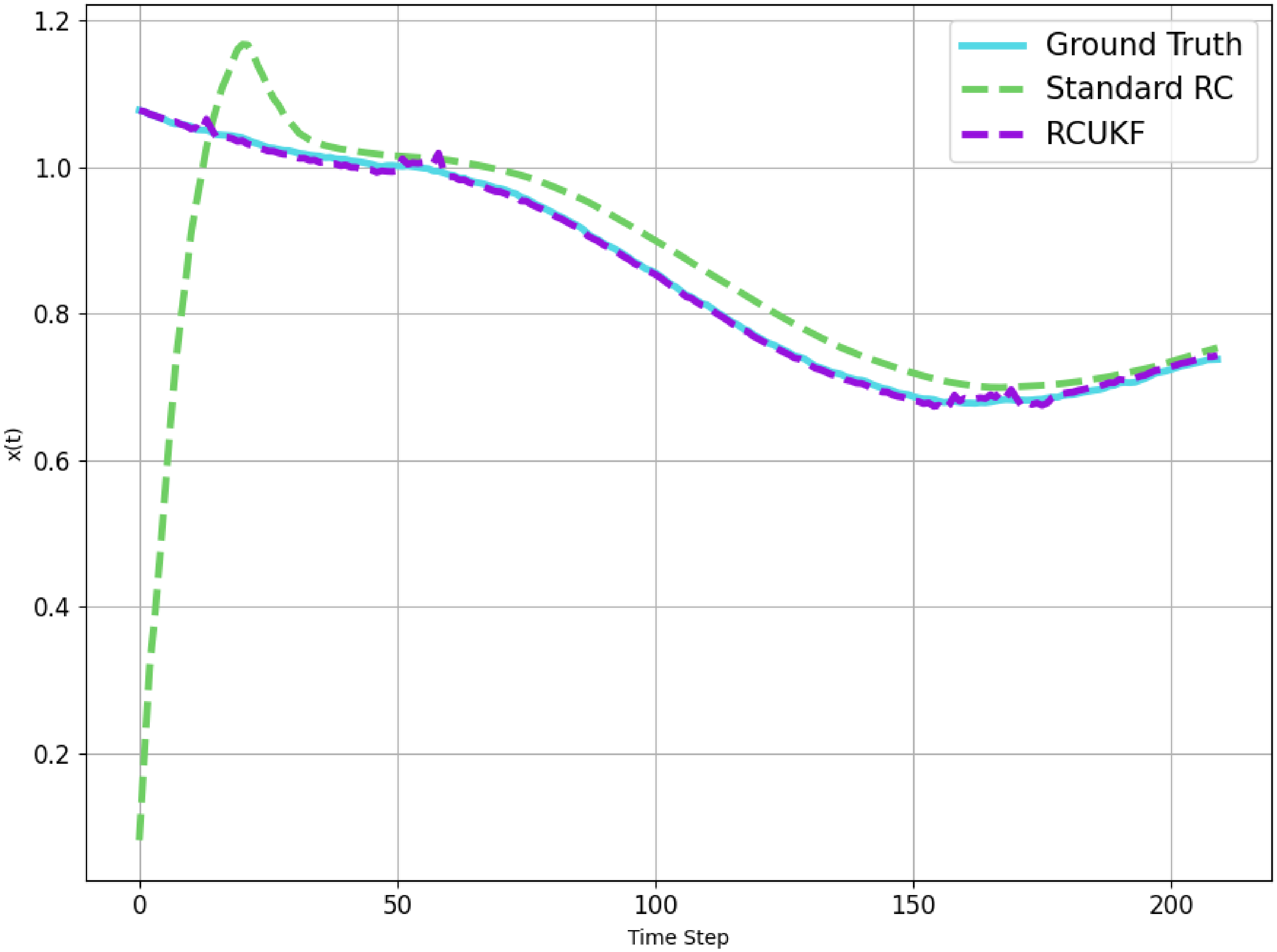


Mackey Glass Time Series Equation

$$\frac{dx}{dt} = \beta \frac{x(t-\tau)}{1+x^{n}(t-\tau)} - \gamma\, x(t) + \epsilon$$



Mackey Glass Prediction (700 data points)

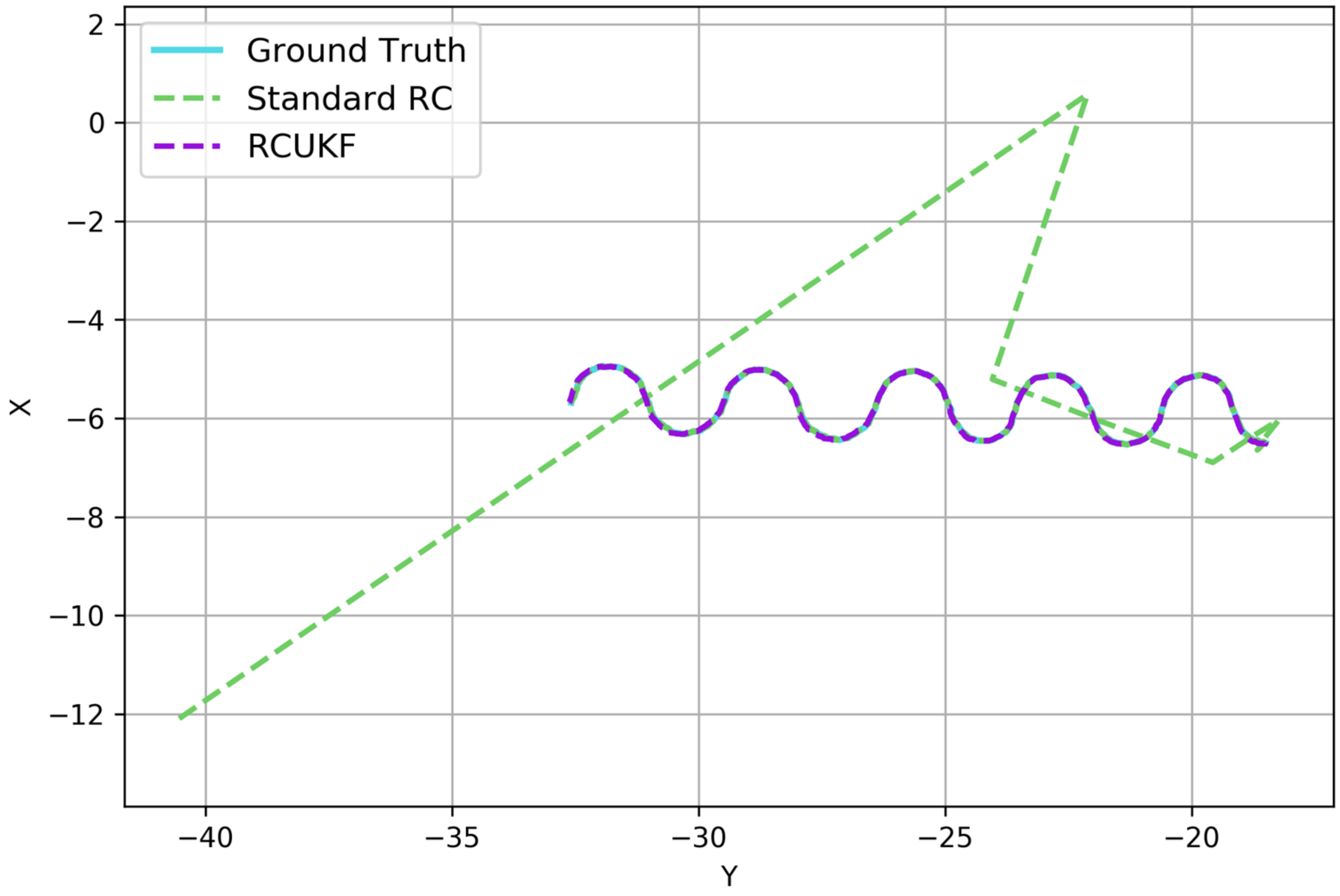


RMSE Summary

Metric	Lorenz	Rössler	Mackey–Glass
Standard RC (700 data points)			
RMSE-X	0.5829	0.3779	0.1176
RMSE-Y	0.8642	0.4494	
RMSE-Z	1.3483	0.0070	
Mean RMSE	0.9318	0.2781	0.1176
RCUKF (700 data points)			
RMSE-X	0.1628	0.1520	0.0032
RMSE-Y	0.1657	0.2888	
RMSE-Z	0.1270	0.0316	
Mean RMSE	**0.1518**	**0.1575**	**0.0032**
Standard RC (10,000 data points)			
RMSE-X	0.1569	0.1743	0.0323
RMSE-Y	0.2346	0.0448	
RMSE-Z	0.2460	0.0373	
Mean RMSE	0.2125	0.0855	0.0323
RCUKF (10,000 data points)			
RMSE-X	0.0370	0.0790	0.0014
RMSE-Y	0.0384	0.0792	
RMSE-Z	0.0504	0.0653	
Mean RMSE	**0.0419**	**0.0745**	**0.0014**

MIT FlightGoggles - Stata Ground Floor

Real time simulation with noises



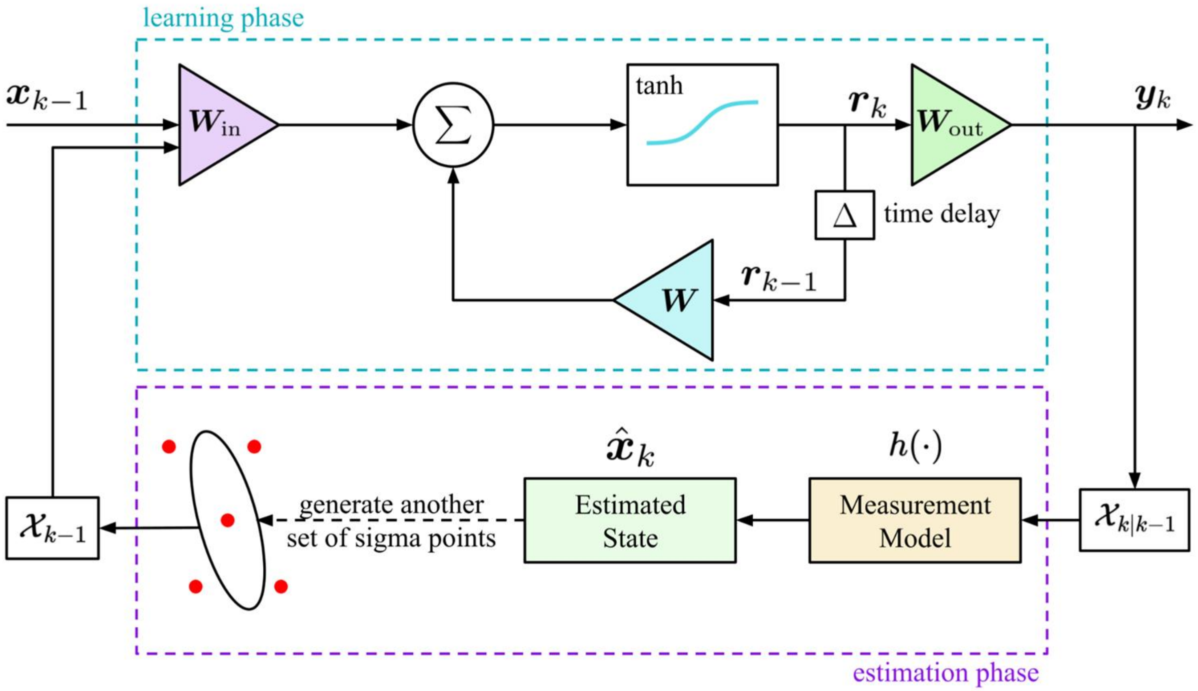
Summary

- Traditional NNs are powerful but **computationally expensive** because of backpropagation.

- RC offers high accuracy, **backpropagation-free** approach.

- **Standalone RC's** accuracy depends on:
  - Quality of datasets
  - Right initialization of $\boldsymbol{W}_{\text{in}}$ & $\boldsymbol{W}$

- We proposed a new framework **RCUKF** that both accuracy and computational efficiency.



Thank you!

Questions?

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RCUKF Project Page

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