



# OptTrot

## Optimized Trotterized circuit library

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### Abstract

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## 1 Introduction

Trotterization is a standard method used to implement a time evolution operator by combining several local Hamiltonian evolution operators. By using the method, we can expect the approximated operator closed to the original operator, even the local terms did not commute with each other.

$$\lim_{n \rightarrow \infty} (e^{A/2} e^{B/2})^n = e^{A+B} \quad (1)$$

In quantum computing, the method has an advantage of preserving the local structure of the Hamiltonian on a dynamic circuit [1]. However, Trotterization method increases circuit depth with linear order by number of Pauli terms. If the time evolution was an ultimate goal to achieve in quantum circuit, it could be meaningful, but in the most algorithms and applications, time evolution is just a part of the whole process. Thus, reducing techniques are significant to apply the quantum computer to general tasks. In addition, increased circuit depth for reducing Trotter error yields inefficient costs in NISQ era, which makes the algorithm into less practical one.

By the limitation, there are many alternative methods to implement a time evolution operator with shorter depth circuit than Trotterization, such as linear combination of unitary(LCU) method[2], Qubitization[3], Taylorization[4], and Fractional query[5]. Such methods make the evolution circuit more practical, however, they loose identity of the given system, especially the cases, when the given hamiltonian is nearly commute or local observable was a dominant feature[1].

### 1.1 Trotter Error by applying order

It is well known that the exponential mapping error is represented with Baker Campbell Hausdorff formula. Usually, the formula is not written with commutator form, Childs et al proved that the error term as a function of sequential commutator of local terms.

$$O(\alpha t^2) \quad (2)$$

The results of Childs et al allow us to calculate the error boundary more precisely including a physical structure of the given Hamiltonian.

For example, let a given Hamiltonian was  $H = c_i P_i + c_j P_j$ .

$$\exp(-it(c_i)P_i) \exp(-it(c_j)P_j) = \exp(-it(c_i P_i + c_j P_j)) + O(\alpha_{com} t^2) \quad (3)$$



then the leading coefficient becomes  $\alpha_{com} = \begin{cases} c_i + c_j & \text{if } [P_i, P_j] = 0 \\ c_i - c_j & \text{if } [P_i, P_j] \neq 0 \end{cases}$ .

It is affected by coefficients, their size, and sign, and applying order and commutation property. In the above example, we cannot observe the commutation and anti-commutation effect, since, if they were commuting to each other, the  $O(\alpha_{com}t^2) = 0$ . Let us expand the system to more general case. Suppose that the given Hamiltonian has two representations,

$$H = H_1 + H_2 + H_3 \quad (4)$$

$$H = c_1P_1 + c_2P_2 + c_3P_3 + c_4P_4 + c_5P_5 \quad (5)$$

$$H_1 = c_1P_1 + c_3P_3 \quad (6)$$

$$H_2 = c_2P_2 \quad (7)$$

$$H_3 = c_4P_4 + c_5P_5 \quad (8)$$

where,  $[H_i, H_j] \neq 0$ , and  $[P_k, P_l] \neq 0$  if  $P_k \in H_i, P_l \in H_j, i \neq j$ .

$$\Pi_{l=1}^5 \exp(-it(c_l P_l)) = \exp(-itH) + O(\alpha_{com1}t^2) \quad (9)$$

$$\Pi_{k=1}^3 \exp(-it(H_k)) = \exp(-itH) + O(\alpha_{com2}t^2) \quad (10)$$

$$(11)$$

Following the  $q = 1$  order expansion, then in the first order the bounded error order is reduced to

$$\alpha_{com1} = 2(|c_1c_2[P_1, P_2]| + |c_1c_4[P_1, P_4]| + |c_1c_5[P_1, P_5]|) \quad (12)$$

$$+ |c_2c_3[P_2, P_3]| + |c_2c_4[P_2, P_4]| + |c_2c_5[P_2, P_5]| \quad (13)$$

$$+ |c_3c_4[P_3, P_4]| + |c_3c_5[P_3, P_5]| \quad (14)$$

$$\alpha_{com2} = 2(|[H_1, H_2]| + |[H_1, H_3]| + |[H_2, H_3]|) \quad (15)$$

$$(16)$$

$$\alpha_{com1} = |c_1c_2| + |c_3c_2| + |c_1c_4| + |c_2c_4| + |c_1c_5| + |c_2c_5| + |c_2c_4| + |c_2c_5| \quad (17)$$

$$\alpha_{com2} = |c_1c_2 + c_3c_2| + |c_1c_4 + c_2c_4 + c_1c_5 + c_2c_5| + |c_2c_4 + c_2c_5| \quad (18)$$

Therefore, by the distribution of  $\{c_i\}$  and commutation relationship of the local terms, the constructed error rate vary.

### Observation 1 *Group commuting order*

## 2 Optimizing a circuit with commuting pairs

Clique: optimal condition:  $\sum_i c_i \approx 0$ .



### 3 Pauli Frame method

### 4 Conclusion

In the report, we overlook the trotter error affected by order and hamiltonian structure in  $n$ -th order Suzuki-Trotter formula.

### References

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