Robust Market Allocation – Preliminary Results

Iron and Steel Problem

Translated Excel formulation into equations and equivalent Python formulation:

$$\min_{\tau,\phi} \quad C_0 \left[1 - \left(\frac{A_0 + \gamma \sum_{i=1}^{I} \phi_i \sum_{i=1}^{I} \psi_i}{A_0} \right)^{\frac{\log(1-L_r)}{\log(2)}} \right] \tag{1}$$

$$\text{s.t.} \quad \psi_i (C_0 - \gamma \tau_i) \leq 0 \quad i = 1, \ldots, I \tag{2}$$

$$\phi_i - \frac{\psi_i}{\sum_{i=1}^{I} \psi_i} \leq 0 \quad i = 1, \ldots, I \tag{3}$$

$$0.2 - \left[1 - \left(\frac{A_0 + \gamma \sum_{i=1}^{I} \phi_i \sum_{i=1}^{I} \psi_i}{A_0} \right)^{\frac{\log(1-L_r)}{\log(2)}} \right] \leq 0 \tag{4}$$

- ψ: CO_2 emissions (t/yr)
- γ: CO_2 capture rate
- τ: Carbon Tax (\$/t)
- ϕ: Market share
- C_0: Initial cost of capture (\$/t)
- A_0: Initial CO_2 captured (t/yr)
- L_r: Learning rate

Verified by solving the 'nominal' problem (fixed parameters).

- In Excel optimal objective: **11.081714**
- In Python optimal objective: **11.08169**

Formulated as a robust problem with 36 uncertain parameters consisting of:

- Initial CO2 captured
- Learning rate
- Initial cost of capture
- CO2 capture rate
- CO2 emissions per plant

Initially let these parameters vary within upper and lower bounds. This the most conservative robust optimisation option as it assumes that **all parameters can take their worst-case values at the same time.** Figure 1 demonstrates initial results applying a basic box-uncertainty set. First varying all parameters then individual parameters independently.

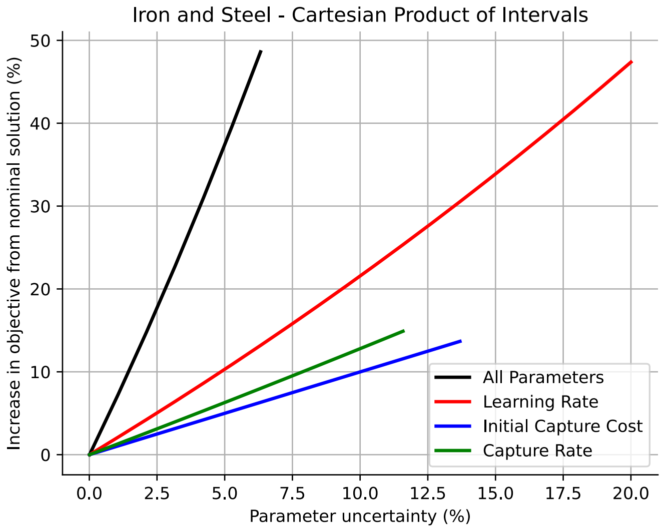

Figure 1: Results of robust optimisation of the iron and steel case study. The black line shows the optimal objective value when all parameters are allowed to vary. The red, blue, and green lines are where only the labelled parameter is allowed to vary, demonstrating learning rate has an increased effect on optimal objective value than initial capture cost of capture rate.

Subsequently investigate ellipsoid-based uncertainty. Which specifies that uncertain parameter values instead of living within a 'box' live within a 'ball'. **This provides less-conservative solutions.** I prepared this figure for my ESA to describe what this is doing.

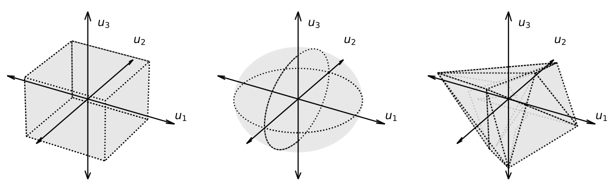

Figure 2: Three different uncertainty sets which represent the potential values that uncertain parameters can take. On the left, a box which is the most conservative as all parameters may be simultaneously at their extreme values within a corner. Centre, an ellipse which is less conservative than the box. On the right, a polygon which may be inferred from data.

Figure 3 demonstrates solutions to a number of robust optimisation problems using a variety of ball/ellipse sizes as well as a variety of initial parameter uncertainties.

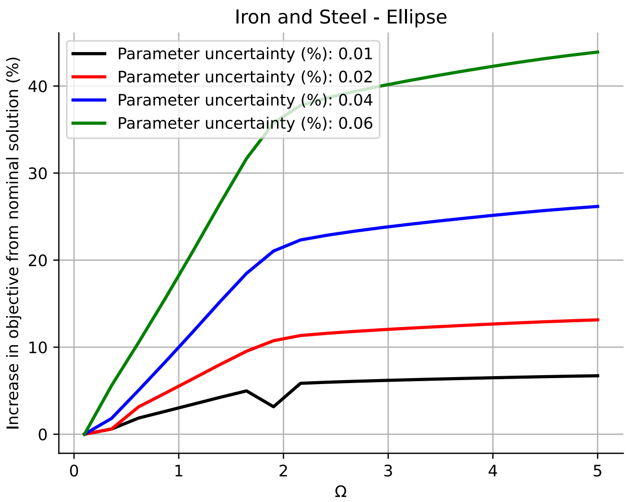

Figure 3: Varying the size of the ball (Ω), that the uncertain parameters live in. Smaller values provide solutions closer to the 'nominal' solution (fixed parameters). Larger values provide worse optimal solutions however immunise against more uncertainty.

Under some assumptions, the size of the ball Ω, can be converted into a probability of infeasibility. Figure 4 demonstrates the relationship between these two quantities.

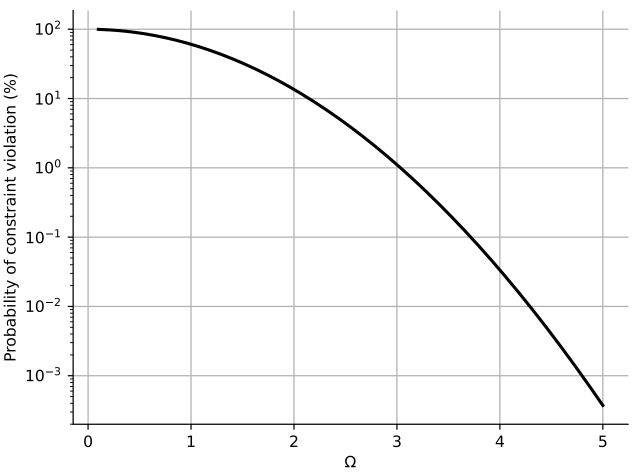

Figure 4: Relationship between Ω, and the probability of constraint violation as a percentage.

Finally, this conversion was performed, and the results detailed in Figure 5.

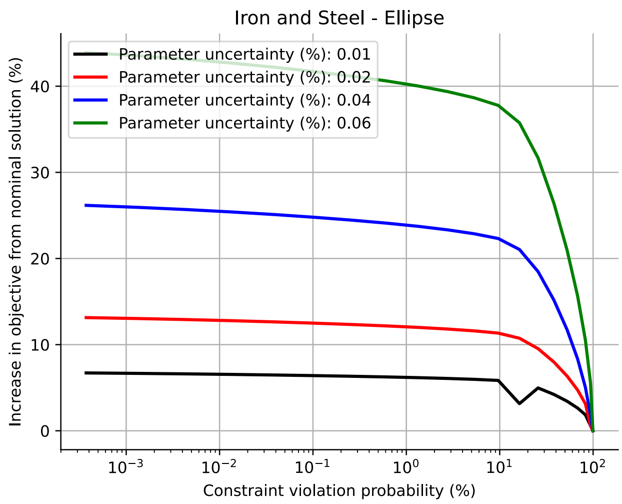

Figure 5. Results demonstrating how as constraint violation probability decreases, the optimal objective value increases. This was performed for a number of upper and lower bounds (parameter uncertainty). Note that at 100% violation probability, the nominal solution is recovered. Taking the 4% parameter uncertainty case as an example (i.e., all parameters can take values $\pm$4% of their nominal values): a solution that is violated 10% of the time increases on the nominal objective by 22% whereas a solution that is violated 0.1% of the time increases on the nominal objective by 25%.