

Appendix and Supplementary Materials: Impute Missing Entries with Uncertainty

A Mathematical Derivations / Proofs

A.1 Derivation of ELBO

We introduce a discrete uniform random variable α taking values on $\alpha_k = \frac{k}{K}$ for $k = 1, \dots, K$.

$$\begin{aligned}
& \log p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}; \theta, \beta) \\
= & \log \int p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta, \beta) p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta) d\mathbf{z} \\
= & \log \sum_{k=1}^K p(\alpha_k) \int p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \alpha_k, \mathbf{x}_\mathbf{r}; \theta, \beta) p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta) d\mathbf{z} \\
= & \log \sum_{k=1}^K p(\alpha_k) \int p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \alpha_k, \mathbf{x}_\mathbf{r}; \theta, \beta) \frac{p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta)}{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) d\mathbf{z} \\
\geq & \sum_{k=1}^K p(\alpha_k) \int q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) \log \left(p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \alpha_k, \mathbf{x}_\mathbf{r}; \theta, \beta) \frac{p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta)}{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \right) d\mathbf{z} \quad (\because \text{Jensen's inequality}) \\
= & \frac{1}{K} \sum_{k=1}^K \mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \left[\log p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \alpha_k, \mathbf{x}_\mathbf{r}; \theta, \beta) \right] - D_{\text{KL}} \left(q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) \middle\| p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta) \right) \\
= & \frac{1}{K} \sum_{k=1}^K \mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \left[\sum_{j \in I_C} \log p((\mathbf{x}_{1-\mathbf{r}})_j \mid \mathbf{z}, \alpha_k, \mathbf{x}_\mathbf{r}; \theta_j, \beta) + \sum_{j \in I_D} \log p((\mathbf{x}_{1-\mathbf{r}})_j \mid \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta_j, \beta) \right] \\
& - D_{\text{KL}} \left(q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) \middle\| p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta) \right) \\
= & \mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \left[\sum_{j \in I_C} \frac{1}{K} \log \frac{\alpha_k(1 - \alpha_k)}{\beta} - \rho_{\alpha_k} \left(\frac{(\mathbf{x}_{1-\mathbf{r}})_j - Q_j(\alpha_k, \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta_j)}{\beta} \right) \right] \\
& + \mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \left[\sum_{j \in I_D} \sum_{c=1}^{C_j} \mathbb{I}((\mathbf{x}_{1-\mathbf{r}})_j = c) \cdot \pi_{jc}(\mathbf{z}; \theta_j) \right] - D_{\text{KL}} \left(q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) \middle\| p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta) \right) \\
= & \mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \left[-\frac{1}{\beta} \cdot \sum_{j \in I_C} \frac{1}{K} \sum_{k=1}^K \rho_{\alpha_k} \left((\mathbf{x}_{1-\mathbf{r}})_j - Q_j(\alpha_k, \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta_j) \right) \right] + \frac{|I_C|}{K} \sum_{k=1}^K \log \alpha_k(1 - \alpha_k) - |I_C| \cdot \log \beta \\
& + \mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \left[\sum_{j \in I_D} \sum_{c=1}^{C_j} \mathbb{I}((\mathbf{x}_{1-\mathbf{r}})_j = c) \pi_{jc}(\mathbf{z}; \theta_j) \right] - D_{\text{KL}} \left(q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) \middle\| p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta) \right) \\
= & -\frac{1}{\beta} \cdot \left(\mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \left[\sum_{j \in I_C} \frac{1}{K} \sum_{k=1}^K \rho_{\alpha_k} \left((\mathbf{x}_{1-\mathbf{r}})_j - Q_j(\alpha_k, \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta_j) \right) \right] - \beta \cdot \frac{|I_C|}{K} \sum_{k=1}^K \log \alpha_k(1 - \alpha_k) + \beta \cdot |I_C| \cdot \log \beta \right. \\
& \left. - \beta \cdot \mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)} \left[\sum_{j \in I_D} \sum_{c=1}^{C_j} \mathbb{I}((\mathbf{x}_{1-\mathbf{r}})_j = c) \log \pi_{jc}(\mathbf{z}; \theta_j) \right] + \beta \cdot D_{\text{KL}} \left(q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) \middle\| p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \theta) \right) \right) \\
= & -\frac{1}{\beta} \cdot \mathcal{L}(\mathbf{x}_\mathbf{r}, \mathbf{x}_{1-\mathbf{r}}; \theta, \eta, \phi)
\end{aligned}$$

by Jensen's inequality. Note that this derivation remains valid when \mathbf{r} is replaced by any masking vector $\mathbf{m} \in \{0, 1\}^p$.

A.2 Derivation of Upper Bound

$$\begin{aligned}
& D_{\text{KL}}\left(p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}) \parallel p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}; \theta, \eta)\right) \\
&= \mathbb{E}_{p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})} \left[-\log p(\mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \theta, \eta) \right] - H(p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})) \\
&= \mathbb{E}_{p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})} \left[-\log \int p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta) p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \eta) d\mathbf{z} \right] - H(p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})) \\
&= \mathbb{E}_{p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})} \left[-\log \int \frac{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}, \mathbf{x}_{1-\mathbf{r}}; \phi)}{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}, \mathbf{x}_{1-\mathbf{r}}; \phi)} p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta) p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \eta) d\mathbf{z} \right] - H(p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})) \\
&\leq \mathbb{E}_{p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})} \left[-\int q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}, \mathbf{x}_{1-\mathbf{r}}; \phi) \log p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta) \frac{p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \eta)}{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}, \mathbf{x}_{1-\mathbf{r}}; \phi)} d\mathbf{z} \right] - H(p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})) \\
&= \mathbb{E}_{p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})} \left[\mathbb{E}_{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}, \mathbf{x}_{1-\mathbf{r}}; \phi)} \left[-\log p(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta) \right] + D_{\text{KL}}(q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}, \mathbf{x}_{1-\mathbf{r}}; \phi) \parallel p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \eta)) \right] \\
&\quad - H(p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})) \\
&= \frac{1}{\beta} \cdot \mathbb{E}_{p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})} \left[\mathcal{L}^*(\mathbf{x}_\mathbf{r}, \mathbf{x}_{1-\mathbf{r}}; \theta, \eta, \phi) \right] - \underbrace{H(p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}))}_{\text{constant}} \quad (\text{see Appendix A.1}),
\end{aligned}$$

by Jensen's inequality, where $H(\cdot)$ is entropy.

A.3 Proof of Proposition 1

Proof. First, suppose that there exist parameters ϕ and η such that, for any $\varepsilon > 0$ and observation $\mathbf{x}_\mathbf{r}$, and its missingness pattern \mathbf{r} , the following holds:

$$D_{\text{KL}}(q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) \parallel p(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \eta)) < \varepsilon.$$

We assume that such ϕ and η are given and fixed. Additionally, we define the following conditional distribution for the missing entries:

$$q(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}, \mathbf{z}; \phi) := \frac{q(\mathbf{z} \mid \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi) p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})}{q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)},$$

which is a valid distribution by the definition of the aggregated posterior. Then, we have

$$\begin{aligned}
p^*(\mathbf{r}, \mathbf{x}) q(\mathbf{z} \mid \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi) &= p^*(\mathbf{r}, \mathbf{x}) \frac{q(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}, \mathbf{z}; \phi) q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)}{p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})} \\
&= p^*(\mathbf{r} \mid \mathbf{x}) p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}) p^*(\mathbf{x}_\mathbf{r}) \frac{q(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}, \mathbf{z}; \phi) q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi)}{p^*(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r})} \\
&= p^*(\mathbf{r} \mid \mathbf{x}) p^*(\mathbf{x}_\mathbf{r}) q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) q(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}, \mathbf{z}; \phi) \\
&= p^*(\mathbf{r}, \mathbf{x}_\mathbf{r}) q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) q(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}, \mathbf{z}; \phi). \quad (\text{by MAR assumption})
\end{aligned}$$

It leads to the following objective function for imputation using U-VAE:

$$\begin{aligned}
\min_{\theta} \quad & \mathbb{E}_{p^*(\mathbf{r}, \mathbf{x}) q(\mathbf{z} \mid \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi)} \left[\sum_{j \in I_C: \mathbf{r}_j=1} \int_0^1 \rho_\alpha \left((\mathbf{x}_{1-\mathbf{r}})_j - Q_j(\alpha, \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta_j) \right) d\alpha \right] \\
&= \mathbb{E}_{p^*(\mathbf{r}, \mathbf{x}_\mathbf{r}) q(\mathbf{z} \mid \mathbf{x}_\mathbf{r}; \phi) q(\mathbf{x}_{1-\mathbf{r}} \mid \mathbf{x}_\mathbf{r}, \mathbf{z}; \phi)} \left[\sum_{j \in I_C: \mathbf{r}_j=1} \int_0^1 \rho_\alpha \left((\mathbf{x}_{1-\mathbf{r}})_j - Q_j(\alpha, \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta_j) \right) d\alpha \right].
\end{aligned}$$

Under the conditional independence assumption among the variables $\mathbf{x}_{1-\mathbf{r}}$ given $\mathbf{x}_\mathbf{r}$ and \mathbf{z} , for $j \in I_C$ such that $\mathbf{r}_j = 0$, the solution $\hat{\theta}_j$ is defined as

$$\hat{\theta}_j \in \arg \min_{\theta_j} \mathbb{E}_{p^*(\mathbf{r}, \mathbf{x}_\mathbf{r}) q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi)} \left[\int_0^1 \rho_\alpha \left((\mathbf{x}_{1-\mathbf{r}})_j - Q_j(\alpha, \mathbf{z}, \mathbf{x}_\mathbf{r}; \theta_j) \right) d\alpha q((\mathbf{x}_{1-\mathbf{r}})_j | \mathbf{x}_\mathbf{r}, \mathbf{z}; \phi) d(\mathbf{x}_{1-\mathbf{r}})_j \right].$$

Since this CRPS loss is a strictly proper scoring rule relative to the conditional CDF $F_j^*((\mathbf{x}_{1-\mathbf{r}})_j | \mathbf{z}, \mathbf{x}_\mathbf{r}; \phi)$ for all $\mathbf{x}_\mathbf{r} \in \mathbb{R}$ and $\mathbf{z} \in \mathbb{R}^d$, we have

$$\begin{aligned} Q_j^{-1}((\mathbf{x}_{1-\mathbf{r}})_j, \mathbf{z}, \mathbf{x}_\mathbf{r}; \hat{\theta}(\mathbf{x}_\mathbf{r}, \mathbf{r})_j) &= F_j^*((\mathbf{x}_{1-\mathbf{r}})_j | \mathbf{z}, \mathbf{x}_\mathbf{r}; \phi) \\ \frac{d}{d(\mathbf{x}_{1-\mathbf{r}})_j} Q_j^{-1}((\mathbf{x}_{1-\mathbf{r}})_j, \mathbf{z}, \mathbf{x}_\mathbf{m}; \hat{\theta}_j) &= q_j((\mathbf{x}_{1-\mathbf{r}})_j | \mathbf{z}, \mathbf{x}_\mathbf{r}; \phi), \quad (\text{by Assumption 1}) \end{aligned}$$

where $q_j((\mathbf{x}_{1-\mathbf{r}})_j | \mathbf{z}, \mathbf{x}_\mathbf{r}; \phi)$ is the marginal distribution of $q(\mathbf{x}_{1-\mathbf{r}} | \mathbf{z}, \mathbf{x}_\mathbf{r}; \phi)$ for the j th variable.

Therefore, it follows that

$$\begin{aligned} & D_{\text{KL}} \left(p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) \middle\| p(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}; \hat{\theta}, \eta) \right) \\ &= D_{\text{KL}} \left(p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) \middle\| \int \left(\prod_{j \in I_C: \mathbf{r}_j=0} \frac{d}{d(\mathbf{x}_{1-\mathbf{r}})_j} Q_j^{-1}((\mathbf{x}_{1-\mathbf{r}})_j, \mathbf{z}, \mathbf{x}_\mathbf{r}; \hat{\theta}_j) \right) p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta) d\mathbf{z} \right) \\ &= D_{\text{KL}} \left(p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) \middle\| \int \left(\prod_{j \in I_C: \mathbf{r}_j=0} q_j((\mathbf{x}_{1-\mathbf{r}})_j | \mathbf{z}, \mathbf{x}_\mathbf{r}; \phi) \right) p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta) d\mathbf{z} \right) \\ &= D_{\text{KL}} \left(p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) \middle\| \int q(\mathbf{x}_{1-\mathbf{r}} | \mathbf{z}, \mathbf{x}_\mathbf{r}; \phi) p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta) d\mathbf{z} \right) \quad (\text{by Assumption 2}) \\ &= D_{\text{KL}} \left(p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) \middle\| \int \frac{q(\mathbf{z} | \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi) p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r})}{q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi)} p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta) d\mathbf{z} \right) \\ &= D_{\text{KL}} \left(p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) \middle\| p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) \int q(\mathbf{z} | \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi) \frac{p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta)}{q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi)} d\mathbf{z} \right) \\ &= \mathbb{E}_{p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r})} \left[-\log \int q(\mathbf{z} | \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi) \frac{p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta)}{q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi)} d\mathbf{z} \right] \\ &\leq \mathbb{E}_{p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r})} \left[q(\mathbf{z} | \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi) \int \log \frac{q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi)}{p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta)} d\mathbf{z} \right] \\ &= \int \left(\int p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) q(\mathbf{z} | \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi) d\mathbf{x}_{1-\mathbf{r}} \right) \log \frac{q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi)}{p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta)} d\mathbf{z} \\ &= \int q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi) \log \frac{q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi)}{p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta)} d\mathbf{z} \\ &= D_{\text{KL}} \left(q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi) \| p(\mathbf{z} | \mathbf{x}_\mathbf{r}; \eta) \right) < \epsilon \end{aligned}$$

where $q(\mathbf{z} | \mathbf{x}_\mathbf{r}; \phi) = \int p^*(\mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}) q(\mathbf{z} | \mathbf{x}_{1-\mathbf{r}}, \mathbf{x}_\mathbf{r}; \phi) d\mathbf{x}_{1-\mathbf{r}}$ is aggregated posterior.

The above equations are derived by Jensen's inequality and the Fubini-Tonelli theorem. The proof is complete. \square

A.4 Proof of Proposition 2

Proof. We begin by noting that the MAR assumption implies $p(\mathbf{r} | \mathbf{x}) = p(\mathbf{r} | \mathbf{x}_\mathbf{r})$. Using $\mathbf{r} \perp\!\!\!\perp \mathbf{r} | \mathbf{x}$ and $\mathbf{m} \perp\!\!\!\perp \mathbf{x}$, we can write

$$p(\mathbf{r}, \mathbf{m} | \mathbf{x}) = p(\mathbf{r} | \mathbf{x}) \cdot p(\mathbf{m} | \mathbf{x}) = p(\mathbf{r} | \mathbf{x}) \cdot p(\mathbf{m}) = p(\mathbf{r} | \mathbf{x}_\mathbf{r}) \cdot p(\mathbf{m}) = p(\mathbf{m}, \mathbf{r} | \mathbf{x}_\mathbf{r}).$$

Therefore, it follows that $(\mathbf{r}, \mathbf{m}) \perp\!\!\!\perp \mathbf{x}_{1-\mathbf{r}} | \mathbf{x}_\mathbf{r}$. The proof is complete. \square

A.5 Quantile function

Similar to [7], for $j \in I_C$, we parameterize the function Q_j , the location parameter of ALD, by *linear spline quantile function (LSQF)* as follows:

$$Q_j(\alpha, \mathbf{z}, \mathbf{x}_m; \theta) = \gamma_j(\mathbf{z}, \mathbf{x}_m) + \sum_{l=0}^L \kappa_j^{(l)}(\mathbf{z}, \mathbf{x}_m)(\alpha - \delta^{(l)}) \cdot \mathbb{I}((\alpha - \delta^{(l)}) > 0) \quad \text{s.t.} \quad \sum_{l=0}^k \kappa_j^{(l)}(\mathbf{z}, \mathbf{x}_m) \geq 0, \quad k = 1, \dots, L,$$

where $\gamma_j(\mathbf{z}, \mathbf{x}_m) \in \mathbb{R}$, $\kappa_j(\mathbf{z}, \mathbf{x}_m) = (\kappa_j^{(1)}(\mathbf{z}, \mathbf{x}_m), \dots, \kappa_j^{(L)}(\mathbf{z}, \mathbf{x}_m)) \in \mathbb{R}^{L+1}$, $\delta = (\delta_0, \dots, \delta_L) \in [0, 1]^{L+1}$, and $0 = \delta_0 < \dots < \delta_L = 1$.

A.6 Closed-form Objective

$$\begin{aligned} & 2 \int_0^1 \rho_\alpha \left((\mathbf{x}_{1-m})_j - Q_j(\alpha, \mathbf{z}, \mathbf{x}_m; \theta_j) \right) d\alpha \\ &= (2\tilde{\alpha}_j - 1)(\mathbf{x}_{1-m})_j + (1 - 2\tilde{\alpha}_j)\gamma_j(\mathbf{z}, \mathbf{x}_m) \\ &+ \sum_{l=1}^L \kappa_j^{(l)}(\mathbf{z}, \mathbf{x}_m) \left(\frac{1 - (\delta^{(l)})^3}{3} - \delta^{(l)} - \max(\tilde{\alpha}_j, \delta^{(l)})^2 + 2 \max(\tilde{\alpha}_j, \delta^{(l)})\delta^{(l)} \right), \end{aligned}$$

where $Q_j(\tilde{\alpha}_j, \mathbf{z}, \mathbf{x}_m; \theta_j) = (\mathbf{x}_{1-m})_j$, $\tilde{\alpha}_j = \frac{(\mathbf{x}_{1-m})_j - \gamma_j(\mathbf{z}, \mathbf{x}_m) + \sum_{l'=0}^{l_0} \kappa_j^{(l')}(\mathbf{z}, \mathbf{x}_m)\delta^{(l')}}{\sum_{l'=0}^{l_0} \kappa_j^{(l')}(\mathbf{z}, \mathbf{x}_m)}$, and $Q_j(\delta^{(l_0)}, \mathbf{z}, \mathbf{x}_m; \theta_j) \leq (\mathbf{x}_{1-m})_j \leq Q_j(\delta^{(l_0+1)}, \mathbf{z}, \mathbf{x}_m; \theta_j)$.

A.7 Posterior and Prior distributions

We assume that the posterior and prior distributions are

$$\begin{aligned} q(\mathbf{z} | \mathbf{x}_r; \phi) &= N(\mu(\mathbf{x}_r; \phi), \text{diag}(\sigma^2(\mathbf{x}_r; \phi))) \\ p(\mathbf{z} | \mathbf{x}_m; \theta) &= N(\mu(\mathbf{x}_m; \theta), \text{diag}(\sigma^2(\mathbf{x}_m; \theta))), \end{aligned}$$

$\mu(\cdot, \phi) : \mathbb{R}^p \mapsto \mathbb{R}^d$, $\sigma^2(\cdot, \phi) : \mathbb{R}^p \mapsto \mathbb{R}_+^d$, $\mu(\cdot, \theta) : \mathbb{R}^p \mapsto \mathbb{R}^d$, $\sigma^2(\cdot, \theta) : \mathbb{R}^p \mapsto \mathbb{R}_+^d$ are neural networks parameterized with ϕ and θ , and $\text{diag}(a), a \in \mathbb{R}^d$ denotes a diagonal matrix with diagonal elements a .

B Comparison with Gaussian VAE

In conventional Gaussian VAE, the decoder $p(\mathbf{x}_{1-m} | \mathbf{z}, \mathbf{x}_m; \theta)$ is commonly modeled as a Gaussian distribution, primarily because the corresponding reconstruction loss reduces to the *mean squared error (MSE)*, one of the most widely used loss functions in optimization theory [12, 8, 17, 5, 1]. The MSE formulation has also been adopted in prior imputation methods, including [10, 6]. This parametric decoder assumption not only simplifies computation but also yields a closed-form reconstruction loss.

Assumption 1 (Conditional independence). *The re-masked features $\{\mathbf{x}_j : \mathbf{m}_j = 0, \mathbf{r}_j = 1\}$ are conditionally independent given the latent variable \mathbf{z} and the un-masked features $\{\mathbf{x}_j : \mathbf{m}_j = 1, \mathbf{r}_j = 1\}$.*

Under Assumption 1, the decoder distribution is assumed to be a multivariate Gaussian with a diagonal covariance matrix:

$$p(\mathbf{x}_{1-m} | \mathbf{z}, \mathbf{x}_m; \theta) = N(D(\mathbf{z}, \mathbf{x}_m; \theta), \beta \cdot \mathbf{I}_p),$$

where $D(\cdot, \cdot; \theta) : \mathbb{R}^d \times \mathbb{R}^p \mapsto \mathbb{R}^p$ denotes the mean function of the decoder, which is parameterized by a neural network with parameter θ . Here, \mathbf{I}_p is the $p \times p$ identity matrix, and $\beta > 0$ is a fixed (non-trainable) observation noise scalar [17].

Then, the objective function of the Gaussian VAE can be written as

$$\min_{\theta, \phi} \mathbb{E}_{q(\mathbf{z} | \mathbf{x}_r; \phi)} \left[\frac{1}{2} \|\mathbf{x}_{1-m} - D(\mathbf{z}, \mathbf{x}_m; \theta)\|^2 \right] + \beta \cdot \mathcal{KL}(q(\mathbf{z} | \mathbf{x}_r; \phi) \| p(\mathbf{z} | \mathbf{x}_m; \theta)), \quad (1)$$

and since $(\mathbf{x}_{1-\mathbf{m}})_j = 0$ for all j such that $\mathbf{m}_j = 1$, the loss in (1) can be equivalently expressed as:

$$\min_{\theta, \phi} \mathbb{E}_{q(\mathbf{z} | \mathbf{x}_r; \phi)} \left[\frac{1}{2} \sum_{\substack{j: \mathbf{m}_j=0, \\ \mathbf{r}_j=1}} \left((\mathbf{x}_{1-\mathbf{m}})_j - D_j(\mathbf{z}, \mathbf{x}_m; \theta) \right)^2 \right] + \beta \cdot D_{\text{KL}}(q(\mathbf{z} | \mathbf{x}_r; \phi) \| p(\mathbf{z} | \mathbf{x}_m; \theta)). \quad (2)$$

However, the objective in (2) cannot adequately capture the uncertainty associated with deciding which value to impute, as it merely estimates the conditional expectation of the missing entries. To address this limitation and better capture the uncertainty inherent in imputation, we shift from point estimation to distributional learning.

That is, we aim to estimate the full conditional distribution of the missing values given the observed ones, rather than a single conditional expectation. From a statistical perspective, imputation should not be treated as a prediction task. Whereas prediction aims to estimate the most likely value, imputation seeks to generate plausible values that preserve the joint distribution of the data and reflect the inherent uncertainty of the missingness mechanism [25, 20].

C Benchmark Datasets

We provide information about the real-world tabular datasets from UCI repository¹, and Kaggle² used in our experiment.

Table 1: Download links of dataset used in this paper.

Dataset	Download links
anuran [3]	https://archive.ics.uci.edu/dataset/406/anuran+calls+mfccs
banknote [15]	https://archive.ics.uci.edu/dataset/267/banknote+authentication
breast [27]	https://archive.ics.uci.edu/dataset/17/breast+cancer+wisconsin+diagnostic
concrete [28]	https://archive.ics.uci.edu/dataset/165/concrete+compressive+strength
default [29]	https://archive.ics.uci.edu/dataset/350/default+of+credit+card+clients
kings (CC0: Public Domain)	https://www.kaggle.com/datasets/harlfodem/housesalesprediction
letter [22]	https://archive.ics.uci.edu/dataset/59/letter+recognition
loan (CC0: Public Domain)	https://www.kaggle.com/datasets/teertha/personal-loan-modeling
redwine [4]	https://archive.ics.uci.edu/dataset/186/wine+quality
shoppers [21]	https://archive.ics.uci.edu/dataset/468/online+shoppers+purchasing+intention+dataset
whitewine [4]	https://archive.ics.uci.edu/dataset/186/wine+quality

Table 2: Description of datasets. N represents the number of observations. $|I_C|$ and $|I_D|$ represent the number of continuous variables and categorical (discrete) variables, respectively. The ‘Classification Target’ refers to the variable used as the response variable in a classification task to evaluate imputation data utility.

Dataset	N	$ I_C $	$ I_D $	Classification Target
anuran	7,195	22	3	Species
banknote	1,372	4	1	class
breast	569	30	1	Diagnosis
concrete	1,030	8	1	Age
default	30,000	14	10	default payment
kings	21,613	11	7	grade
letter	20,000	16	1	lettr
loan	5,000	5	6	Personal Loan
redwine	1,599	11	1	quality
shoppers	12,330	10	8	Revenue
whitewine	4,898	11	1	quality

¹<https://archive.ics.uci.edu/>

²<https://www.kaggle.com/datasets/>

D Reproducibility

We present all the experimental settings for reproducibility in this paper.

D.1 Experimental Settings

- We run experiments using NVIDIA RTX A6000 GPU, and our experimental codes are available with PyTorch and `scikit-learn`.

Hyperparameters This demonstrates the *generalizability of our proposed model* across various tabular datasets. For all 11 datasets, we applied the following hyperparameters uniformly, without any additional tunings:

- batch size: 1024
- β (scale parameter of asymmetric Laplace distribution): 0.1
- step (interval size of quantile levels): 0.1
- epochs: 1000 (with `AdamW` optimizer [16])
- learning rate: 0.002
- Activation function: `nn.ELU()` [2]

Training procedure The overall training procedure of U-VAE is outlined in Algorithm 1 (note that mini-batch style training is also applicable):

Algorithm 1 Training of U-VAE

Input: Dataset: $\{(\mathbf{x}^{(i)}, \mathbf{r}^{(i)})\}_{i=1}^n$

Parameter: θ, ϕ

- 1: **for** $e = 1, 2, \dots, \text{epochs}$ **do**
 - 2: $\mathbf{m}^{(i)} \sim_{i.i.d.} p(\mathbf{m})$ for $i = 1, \dots, n$
 - 3: Update θ and ϕ by the gradient descent of $\sum_{i=1}^n \mathcal{L}(\mathbf{x}^{(i)}, \mathbf{r}^{(i)}, \mathbf{m}^{(i)}; \theta, \phi)$
 - 4: **end for**
-

D.2 Missing mechanisms

Descriptions Data missingness is a common challenge in research and practical analysis, categorized into three primary missing mechanisms: 1) Missing Completely at Random (MCAR), 2) Missing at Random (MAR), and 3) Missing Not at Random (MNAR).

Under the **MCAR** mechanism, the reason for missingness is unrelated to any data, whether observed or unobserved. In other words, the likelihood of data being missing is equal across all observations. The primary advantage of MCAR is that it does not introduce bias into the data analysis. However, despite this advantage, data missingness can still reduce the statistical power of the study because of the reduced sample size.

MAR occurs when the probability of missingness is related to the observed data but not the unobserved missing data. Essentially, even though the data is missing, the mechanism assumes that the missingness can be explained by other variables in the dataset. In other words, the missingness can be modeled and imputed using the available data, allowing for more accurate analyses despite the missing values.

Lastly, if the missingness is not specified by either MCAR or MAR, it becomes **MNAR**. MNAR is the most challenging mechanism, as it implies that the missingness is related to the unobserved data itself. In this case, the missing data is systematically different from the observed data, which introduces bias if not properly accounted for. For example, patients with severe symptoms may be less likely to report their health status, resulting in missing data. MNAR requires sophisticated statistical methods to handle, as ignoring or mishandling it can lead to biased and unreliable results.

Implementations Following [19, 11, 31], we generate the missing value mask for each dataset with three mechanisms in four settings, MCAR, MAR, MNARL, and MNARQ.

In the **MCAR** setting, each value is masked according to the realization of a Bernoulli random variable with a fixed parameter.

For the **MAR** setting, a fixed subset of variables that cannot have missing values is sampled in each experiment. The remaining variables are assigned missing values based on a logistic model with random weights, where the non-missing variables serve as inputs. A bias term is optimized via line search to achieve the desired proportion of missing values.

Finally, two different mechanisms are implemented in the MNAR setting. The first, **MNARL**, is identical to the previously described MAR mechanism, but the inputs of the logistic model are then masked by an MCAR mechanism. Hence, the logistic model’s outcome depends on missing values. The second mechanism, **MNARQ**, samples a subset of variables whose values in the lower and upper p th percentiles are masked according to a Bernoulli random variable, and the values in-between are left not missing.

D.3 Details of Implementing Baseline Models

To compare our proposed method with baseline models, we performed experiments across four missing data mechanisms and five missingness rates. Our reproduced codes for the baseline models are provided in the supplementary material. Below are the detailed implementations of the baseline methods:

- Mean [14]: We employ the `SimpleImputer` package ³, with the `strategy` parameter set to ‘mean’.
- kNNI [24]: We employ the `KNNImputer` package ⁴ from `scikit-learn` for missing data imputation.
- MICE [26]: We employ the `IterativeImputer` package ⁵ from `scikit-learn`.
- missForest [23]: We employ the `missForest` module in `hyperimpute.Plugins` ⁶.
- GAIN [30]: We follow the implementations provided in the official repository⁷. As the paper does not explicitly discuss the separate handling of categorical and continuous variables, the code treats them simultaneously. Consequently, a rounding process is employed to handle categorical variables afterward.
- VAEAC [10]: We follow the implementations provided in the official repository⁸. The authors provided hyperparameters that adequately address both continuous and categorical variables, so we used these without further modification during model fitting.
- MIWAE [18]: The implemented MIWAE code in the official repository⁹ was designed for continuous variables only. To accommodate heterogeneous tabular datasets, we treat the conditional distribution of categorical columns as categorical distributions and employ cross-entropy loss for reconstruction.
- not-MIWAE [9]: The implemented not-MIWAE code in the official repository¹⁰ also focused on continuous variables exclusively. To handle categorical variables, we make the same modifications as in MIWAE.
- MIRACLE [13]: We utilize the `MIRACLE` module in `hyperimpute.Plugins` ¹¹.
- ReMasker [6]: We follow the implementations provided in official repository ¹².

³<https://scikit-learn.org/1.5/modules/generated/sklearn.impute.SimpleImputer.html>

⁴<https://scikit-learn.org/stable/modules/generated/sklearn.impute.KNNImputer.html>

⁵<https://scikit-learn.org/stable/modules/generated/sklearn.impute.IterativeImputer.html>

⁶https://github.com/vanderschaarlab/hyperimpute/blob/main/src/hyperimpute/plugins/imputers/plugin_missforest.py

⁷<https://github.com/jsyoon0823/GAIN/tree/master>

⁸<https://github.com/tigverts/vaeac>

⁹<https://github.com/pamattei/miwae>

¹⁰<https://github.com/nbip/notMIWAE>

¹¹https://github.com/vanderschaarlab/hyperimpute/blob/main/src/hyperimpute/plugins/imputers/plugin_miracle.py

¹²<https://github.com/tydusky/remasker>

D.4 Evaluation settings

Regarding **imputation data utility**, we conduct regression, classification, and feature selection tasks for post-imputation evaluation. Please refer to Table 2 for the classification target variable, and Table 3 for detailed machine learning model configuration for these tasks. The detailed evaluation procedure as follows:

Tasks	Model	Description
Classification	Random Forest	Package: <code>sklearn.ensemble.RandomForestRegressor</code> , setting: <code>random_state=0</code> , defaulted values
	Logistic Regression	Package: <code>sklearn.linear_model.LogisticRegression</code> , setting: <code>tol=0.001</code> , <code>random_state=42</code> , <code>max_iter=1000</code> , defaulted values
	<i>k</i> -Nearest Neighbors	Package: <code>sklearn.neighbors.KNeighborsClassifier</code> , setting: defaulted values
	Support Vector Machine	Package: <code>sklearn.svm.SVC</code> , setting: <code>random_state=42</code>
	Random Forest	Package: <code>sklearn.ensemble.RandomForestClassifier</code> , setting: <code>random_state=42</code> , defaulted values
	Gradient Boosting	Package: <code>sklearn.ensemble.GradientBoostingClassifier</code> , setting: <code>random_state=42</code> , defaulted values
	Adaptive Boosting	Package: <code>sklearn.ensemble.AdaBoostClassifier</code> , setting: <code>random_state=42</code> , defaulted values

Table 3: **Configuration of machine learning models (regressor and classifier) used to evaluate imputation data utility.** The names of all arguments used in the description are consistent with those defined in corresponding packages.

Regression performance (SMAPE)

1. Train an imputation method a given incomplete train dataset.
2. Impute the incomplete dataset using trained imputation model.
3. Train a machine learning model (Random Forest regressor in Table 3) using the imputed dataset, where each continuous variable serves as the response variable.
4. Assess regression prediction performance by averaging the SMAPE from the test dataset for each Random Forest regressor trained on the continuous variables.

Classification performance (Accuracy)

1. Train an imputation method using a given incomplete train dataset.
2. Impute the incomplete train dataset using trained imputation model.
3. Train 6 machine learning models in Table 3 using the imputed dataset.
4. Assess classification prediction performance by averaging the Accuracy from the complete test dataset from five different classifiers.

Feature selection performance (Feature)

1. Train an imputer using a given incomplete train dataset.
2. Impute the incomplete train dataset using trained imputer.
3. Train a Random Forest classifier using both the original complete train dataset and the imputed dataset.

4. Determine the rank-ordering of important features for both classifiers.
5. Assess feature selection performance by comparing the feature importance rank orderings of classifiers trained on the original complete train dataset and those trained on the imputed dataset using *Spearman's Rank Correlation*.

Multiple imputation We assess the effectiveness of multiple imputations by employing interval inference for the population mean, which was proposed by Rubin[20]. We report the raw bias, coverage, and confidence interval length (width), and the evaluation procedure for multiple imputations is outlined in Algorithm 2 [25].

Algorithm 2 Evaluation procedure for multiple imputation (adapted from [20, 25])

Input: Complete data set $D^* = \{x_i\}_{i=1}^n$, Number of imputations M

Output: Bias, Coverage, and Average Confidence-Interval (CI) Length

- 1: **Target estimand** $Q^* = \frac{1}{n} \sum_{i=1}^n \mathbb{I}(x_i > \bar{x})$, where $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$
- 2: **for** $s = 1, \dots, S$ **do**
- 3: Generate missing values into D^* to obtain incomplete data $D^{(s)}$
- 4: Train the imputation model on $D^{(s)}$
- 5: **for** $m = 1, \dots, M$ **do** ▷ Multiple imputations
- 6: Impute the incomplete dataset $\hat{D}_m^{(s)}$
- 7: Compute estimate $\hat{Q}_m^{(s)} = \frac{1}{n} \sum_{i=1}^n \mathbb{I}(\hat{x}_{i,m}^{(s)} > \bar{x}_m^{(s)})$
- 8: Compute imputed data variance $\hat{U}_m^{(s)} = \frac{1}{n} \hat{Q}_m^{(s)} (1 - \hat{Q}_m^{(s)})$
- 9: **end for**
- 10: **Rubin's combining rules**¹³

$$\begin{aligned}\bar{Q}^{(s)} &= \frac{1}{M} \sum_{m=1}^M \hat{Q}_m^{(s)}, & \bar{U}^{(s)} &= \frac{1}{M} \sum_{m=1}^M \hat{U}_m^{(s)} \\ B^{(s)} &= \frac{1}{M-1} \sum_{m=1}^M (\hat{Q}_m^{(s)} - \bar{Q}^{(s)})^2, & T^{(s)} &= \bar{U}^{(s)} + \left(1 + \frac{1}{M}\right) B^{(s)}\end{aligned}$$

- 11: **end for**
- 12: **Performance metrics**

$$\begin{aligned}\text{Bias} &= \frac{1}{S} \sum_{s=1}^S |\bar{Q}^{(s)} - Q^*| \\ \text{Coverage} &= \frac{1}{S} \sum_{s=1}^S \mathbb{I}(Q^* \in [\bar{Q}^{(s)} \pm 1.96\sqrt{T^{(s)}}]) \\ \text{Width} &= \frac{1}{S} \sum_{s=1}^S 2 \times 1.96\sqrt{T^{(s)}}\end{aligned}$$

¹³When M is small, one may replace 1.96 by the t -quantile with $\nu = (M-1)\left(1 + \frac{\bar{U}^{(s)}}{(1+1/M)B^{(s)}}\right)^2$ degrees of freedom, following [20].

E Detailed experimental results

E.1 Across missingness rates

Q1: Imputation fidelity (RMSE) We report the RMSE results to assess imputation fidelity across missingness rates of 0.1, 0.5, 0.7, and 0.9.

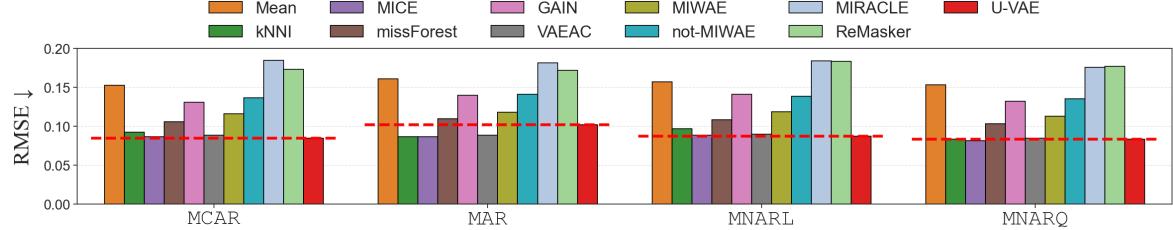


Figure 1: **Imputation fidelity (RMSE)** at 0.1 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 11 datasets and 5 random seeds are reported. ↓ denotes that lower is better.

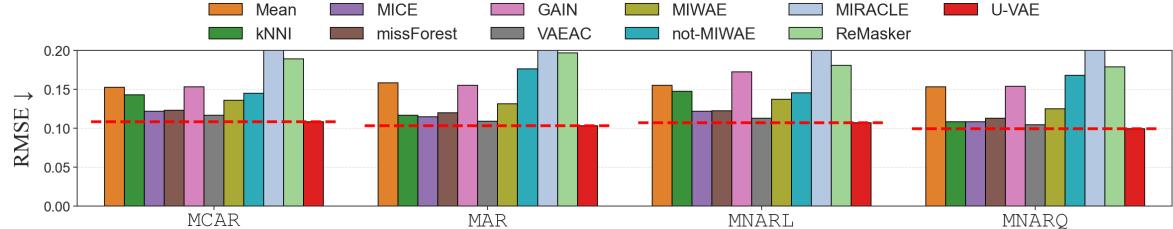


Figure 2: **Imputation fidelity (RMSE)** at 0.5 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 11 datasets and 5 random seeds are reported. ↓ denotes that lower is better.

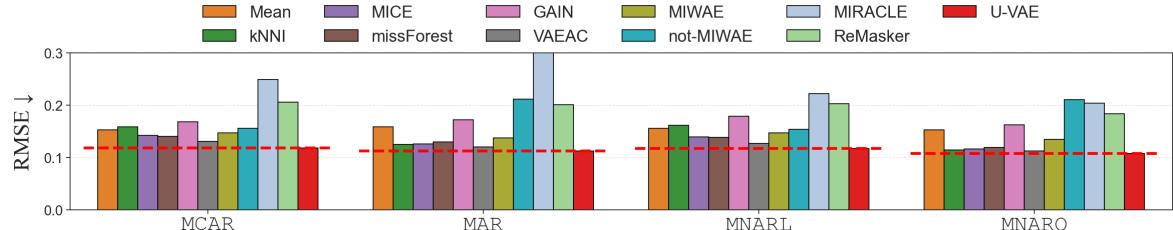


Figure 3: **Imputation fidelity (RMSE)** at 0.7 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 11 datasets and 5 random seeds are reported. ↓ denotes that lower is better.

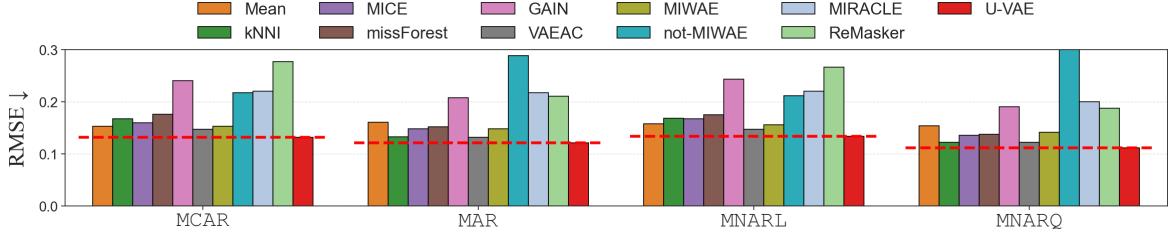


Figure 4: **Imputation fidelity (RMSE)** at 0.9 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 11 datasets and 5 random seeds are reported. ↓ denotes that lower is better.

Q1: Imputation fidelity (WD) We report the WD results to assess imputation fidelity across missingness rates of 0.1, 0.5, 0.7, and 0.9.

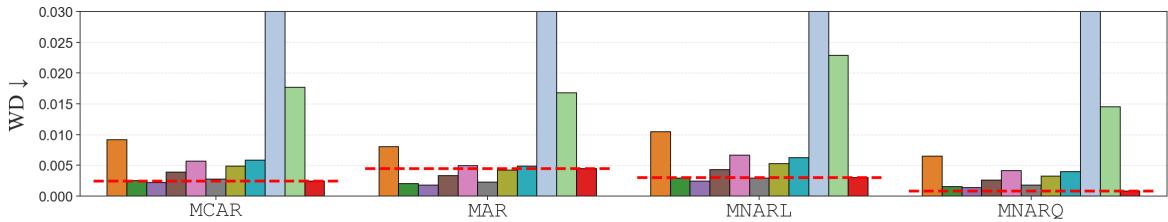


Figure 5: **Imputation fidelity (WD)** at 0.1 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 11 datasets and 5 random seeds are reported. ↓ denotes that lower is better.

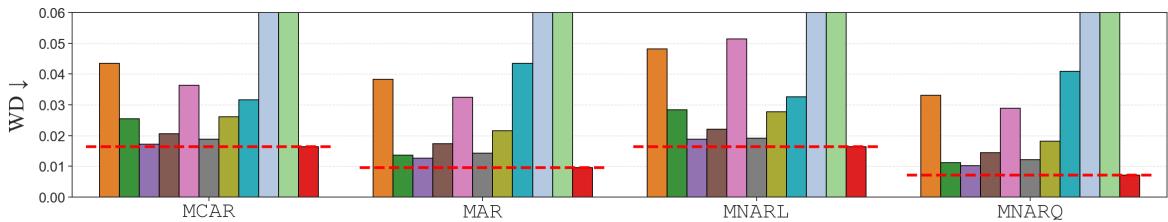


Figure 6: **Imputation fidelity (WD)** at 0.5 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 11 datasets and 5 random seeds are reported. ↓ denotes that lower is better.

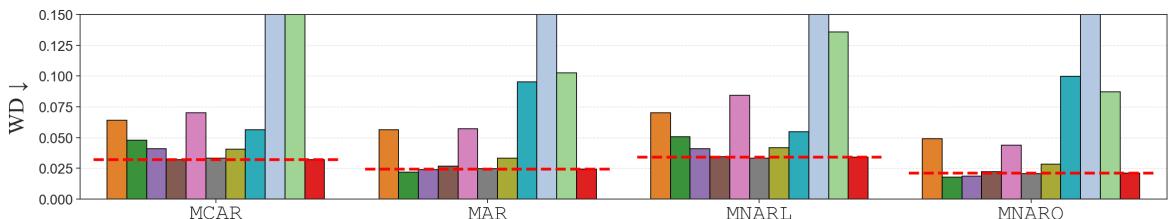


Figure 7: **Imputation fidelity (WD)** at 0.7 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 11 datasets and 5 random seeds are reported. ↓ denotes that lower is better.

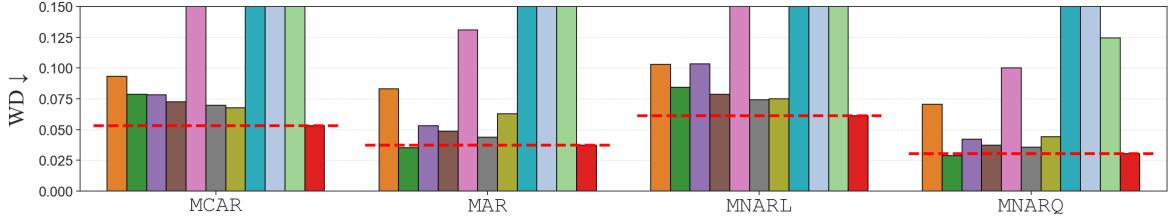


Figure 8: **Imputation fidelity (WD)** at 0.9 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 11 datasets and 5 random seeds are reported. ↓ denotes that lower is better.

Q1: Imputation fidelity result As the missingness rate increases, the RMSE of other baselines deteriorates, whereas U-VAE consistently maintains the lowest error in Figures 1–4, outperforming in 15 out of 16 settings (4 missingness rates \times 4 missingness mechanisms). This performance gap becomes more pronounced under higher missingness rates and more challenging mechanisms (e.g., MNAR), indicating U-VAE’s superior reconstruction accuracy and robustness. As shown in Figures 5–8, U-VAE again remains near the bottom in distributional divergence, indicating a closer match to the full data distribution beyond pointwise accuracy, and outperforms in 12 out of 16 settings (4 missingness rates \times 4 missingness mechanisms). Moreover, its performance degrades more gradually with increasing missingness compared to other baselines.

Q1: Imputation utility We report the SMAPE (regression), Accuracy (classification), and Feature (classification) results to assess imputation utility across missingness rates of 0.1, 0.5, 0.7, and 0.9.

model	MCAR			MAR			MNARL			MNARQ		
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑
Mean	.212 _{±.020}	.776 _{±.024}	.857 _{±.015}	.187 _{±.017}	.766 _{±.024}	.789 _{±.017}	.209 _{±.020}	.768 _{±.025}	.845 _{±.016}	.192 _{±.017}	.782 _{±.024}	.868 _{±.014}
kNNI	.175 _{±.016}	.778 _{±.024}	.859 _{±.015}	.163 _{±.015}	.767 _{±.024}	.826 _{±.015}	.174 _{±.016}	.771 _{±.024}	.851 _{±.015}	.162 _{±.015}	.783 _{±.024}	.892 _{±.013}
MICE	.188 _{±.019}	.779 _{±.024}	.869 _{±.015}	.171 _{±.016}	.773 _{±.023}	.868 _{±.014}	.187 _{±.018}	.770 _{±.024}	.850 _{±.015}	.176 _{±.016}	.785 _{±.023}	.885 _{±.012}
missForest	.199 _{±.019}	.778 _{±.024}	.865 _{±.015}	.178 _{±.016}	.767 _{±.024}	.833 _{±.014}	.198 _{±.019}	.770 _{±.024}	.854 _{±.015}	.185 _{±.017}	.783 _{±.024}	.886 _{±.012}
GAIN	.196 _{±.019}	.779 _{±.024}	.811 _{±.019}	.179 _{±.017}	.777 _{±.024}	.819 _{±.016}	.183 _{±.015}	.772 _{±.024}	.789 _{±.015}	.170 _{±.015}	.777 _{±.024}	.839 _{±.015}
VAEAC	.193 _{±.019}	.787 _{±.023}	.858 _{±.014}	.174 _{±.016}	.787 _{±.023}	.860 _{±.014}	.192 _{±.018}	.787 _{±.023}	.863 _{±.013}	.180 _{±.016}	.790 _{±.023}	.889 _{±.012}
MIWAE	.193 _{±.018}	.771 _{±.023}	.827 _{±.017}	.183 _{±.017}	.766 _{±.023}	.799 _{±.016}	.192 _{±.018}	.762 _{±.023}	.793 _{±.017}	.185 _{±.017}	.781 _{±.023}	.839 _{±.014}
not-MIWAE	.202 _{±.019}	.774 _{±.025}	.815 _{±.016}	.188 _{±.018}	.765 _{±.024}	.799 _{±.017}	.201 _{±.019}	.764 _{±.025}	.793 _{±.016}	.191 _{±.018}	.780 _{±.024}	.823 _{±.016}
MIRACLE	.196 _{±.018}	.771 _{±.025}	.808 _{±.018}	.177 _{±.015}	.764 _{±.025}	.805 _{±.016}	.196 _{±.017}	.765 _{±.025}	.792 _{±.017}	.183 _{±.015}	.779 _{±.024}	.831 _{±.017}
ReMasker	.209 _{±.020}	.769 _{±.026}	.814 _{±.017}	.188 _{±.018}	.763 _{±.025}	.793 _{±.018}	.207 _{±.019}	.762 _{±.026}	.797 _{±.017}	.195 _{±.018}	.778 _{±.024}	.832 _{±.017}
U-VAE	.174 _{±.019}	.788 _{±.025}	.895 _{±.014}	.161 _{±.018}	.785 _{±.024}	.891 _{±.015}	.174 _{±.019}	.782 _{±.025}	.885 _{±.014}	.192 _{±.018}	.793 _{±.024}	.919 _{±.013}

Table 4: **Imputation utility** at 0.1 missingness. The means and their standard errors across 11 real-world datasets and 5 random seeds are reported. ↑ (↓) denotes that higher (lower) is better. The best value is **red bolded**, and the second best is **blue bolded**.

model	MCAR			MAR			MNARL			MNARQ		
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑
Mean	.284 _{.023}	.513 _{.046}	.634 _{.027}	.261 _{.020}	.543 _{.042}	.474 _{.039}	.289 _{.023}	.518 _{.044}	.559 _{.034}	.252 _{.020}	.602 _{.043}	.619 _{.026}
kNNI	.258 _{.023}	.548 _{.040}	.682 _{.033}	.216 _{.019}	.559 _{.040}	.584 _{.030}	.261 _{.023}	.550 _{.039}	.608 _{.034}	.212 _{.019}	.615 _{.042}	.698 _{.026}
MICE	.245 _{.022}	.537 _{.042}	.664 _{.029}	.221 _{.020}	.562 _{.038}	.635 _{.028}	.248 _{.022}	.548 _{.041}	.604 _{.032}	.218 _{.019}	.626 _{.041}	.717 _{.028}
missForest	.250 _{.021}	.513 _{.045}	.655 _{.025}	.230 _{.019}	.553 _{.042}	.575 _{.031}	.251 _{.021}	.521 _{.044}	.603 _{.030}	.227 _{.019}	.608 _{.043}	.686 _{.027}
GAIN	.264 _{.022}	.627 _{.031}	.517 _{.027}	.241 _{.020}	.659 _{.031}	.578 _{.025}	.282 _{.023}	.624 _{.032}	.486 _{.032}	.231 _{.019}	.703 _{.030}	.614 _{.031}
VAEAC	.238 _{.020}	.752 _{.026}	.674 _{.026}	.216 _{.018}	.752 _{.026}	.708 _{.019}	.240 _{.021}	.746 _{.026}	.650 _{.026}	.215 _{.018}	.764 _{.026}	.713 _{.023}
MIWAE	.243 _{.020}	.394 _{.022}	.513 _{.032}	.231 _{.019}	.432 _{.020}	.538 _{.030}	.248 _{.020}	.420 _{.019}	.492 _{.041}	.223 _{.019}	.551 _{.034}	.591 _{.032}
not-MIWAE	.261 _{.022}	.492 _{.046}	.570 _{.032}	.254 _{.021}	.483 _{.039}	.522 _{.032}	.267 _{.023}	.504 _{.044}	.534 _{.033}	.245 _{.020}	.561 _{.043}	.584 _{.030}
MIRACLE	.281 _{.022}	.531 _{.041}	.592 _{.036}	.248 _{.019}	.562 _{.039}	.552 _{.036}	.282 _{.022}	.537 _{.041}	.528 _{.036}	.244 _{.018}	.614 _{.042}	.631 _{.032}
ReMasker	.276 _{.022}	.512 _{.046}	.574 _{.033}	.257 _{.021}	.539 _{.044}	.512 _{.040}	.281 _{.023}	.515 _{.045}	.531 _{.037}	.251 _{.020}	.595 _{.044}	.621 _{.037}
U-VAE	.238 _{.023}	.745 _{.044}	.687 _{.028}	.215 _{.021}	.771 _{.039}	.642 _{.034}	.240 _{.023}	.742 _{.041}	.624 _{.031}	.210 _{.020}	.808 _{.041}	.690 _{.030}

Table 5: **Imputation utility** at 0.5 missingness. The means and their standard errors across 11 real-world datasets and 5 random seeds are reported. ↑ (↓) denotes that higher (lower) is better. The best value is **red bolded**, and the second best is **blue bolded**.

model	MCAR			MAR			MNARL			MNARQ		
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑
Mean	.306 _{.025}	.435 _{.047}	.460 _{.034}	.294 _{.023}	.459 _{.047}	.359 _{.046}	.316 _{.025}	.441 _{.047}	.419 _{.039}	.275 _{.022}	.542 _{.048}	.500 _{.034}
kNNI	.293 _{.024}	.492 _{.044}	.556 _{.041}	.244 _{.021}	.481 _{.045}	.476 _{.039}	.299 _{.024}	.497 _{.044}	.475 _{.042}	.234 _{.020}	.552 _{.047}	.586 _{.035}
MICE	.280 _{.023}	.449 _{.044}	.449 _{.038}	.253 _{.022}	.490 _{.044}	.529 _{.034}	.289 _{.023}	.470 _{.045}	.416 _{.042}	.238 _{.020}	.567 _{.045}	.590 _{.037}
missForest	.269 _{.021}	.416 _{.048}	.492 _{.034}	.259 _{.021}	.470 _{.047}	.468 _{.043}	.274 _{.022}	.435 _{.048}	.451 _{.036}	.250 _{.020}	.548 _{.048}	.568 _{.035}
GAIN	.299 _{.023}	.535 _{.038}	.382 _{.039}	.282 _{.022}	.548 _{.039}	.405 _{.039}	.319 _{.024}	.532 _{.037}	.392 _{.039}	.259 _{.020}	.632 _{.036}	.482 _{.041}
VAEAC	.262 _{.022}	.719 _{.026}	.588 _{.025}	.242 _{.020}	.724 _{.027}	.600 _{.026}	.266 _{.022}	.710 _{.026}	.516 _{.038}	.233 _{.019}	.743 _{.027}	.622 _{.029}
MIWAE	.268 _{.021}	.218 _{.015}	.422 _{.037}	.259 _{.021}	.253 _{.021}	.436 _{.037}	.280 _{.022}	.248 _{.014}	.371 _{.041}	.246 _{.020}	.424 _{.042}	.462 _{.040}
not-MIWAE	.291 _{.024}	.407 _{.048}	.416 _{.040}	.297 _{.023}	.370 _{.044}	.407 _{.039}	.292 _{.024}	.419 _{.048}	.435 _{.040}	.279 _{.021}	.477 _{.049}	.445 _{.040}
MIRACLE	.312 _{.024}	.459 _{.043}	.439 _{.037}	.286 _{.021}	.490 _{.042}	.453 _{.040}	.322 _{.024}	.460 _{.043}	.408 _{.043}	.272 _{.020}	.551 _{.046}	.532 _{.040}
ReMasker	.316 _{.024}	.427 _{.047}	.404 _{.044}	.292 _{.022}	.456 _{.047}	.426 _{.047}	.330 _{.025}	.433 _{.047}	.377 _{.050}	.281 _{.022}	.531 _{.049}	.514 _{.041}
U-VAE	.260 _{.025}	.669 _{.049}	.566 _{.043}	.241 _{.024}	.733 _{.046}	.605 _{.041}	.265 _{.025}	.697 _{.048}	.524 _{.049}	.230 _{.022}	.799 _{.049}	.650 _{.038}

Table 6: **Imputation utility** at 0.7 missingness. The means and their standard errors across 11 real-world datasets and 5 random seeds are reported. ↑ (↓) denotes that higher (lower) is better. The best value is **red bolded**, and the second best is **blue bolded**.

model	MCAR			MAR			MNARL			MNARQ		
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑
Mean	.332 _{.026}	.400 _{.046}	.275 _{.045}	.331 _{.025}	.420 _{.047}	.224 _{.048}	.352 _{.027}	.402 _{.045}	.261 _{.052}	.305 _{.024}	.521 _{.048}	.331 _{.040}
kNNI	.341 _{.026}	.436 _{.046}	.334 _{.052}	.277 _{.023}	.425 _{.047}	.345 _{.045}	.348 _{.026}	.451 _{.045}	.362 _{.042}	.260 _{.022}	.523 _{.049}	.428 _{.042}
MICE	.339 _{.024}	.393 _{.047}	.272 _{.048}	.310 _{.024}	.427 _{.046}	.313 _{.048}	.357 _{.026}	.407 _{.046}	.249 _{.052}	.290 _{.022}	.515 _{.046}	.393 _{.045}
missForest	.300 _{.022}	.389 _{.048}	.217 _{.053}	.297 _{.023}	.421 _{.047}	.322 _{.047}	.312 _{.023}	.393 _{.047}	.283 _{.049}	.281 _{.022}	.519 _{.048}	.406 _{.045}
GAIN	.379 _{.025}	.445 _{.039}	.173 _{.047}	.342 _{.024}	.499 _{.036}	.279 _{.037}	.398 _{.026}	.447 _{.042}	.211 _{.043}	.323 _{.023}	.567 _{.038}	.387 _{.044}
VAEAC	.301 _{.023}	.631 _{.030}	.345 _{.039}	.283 _{.022}	.658 _{.029}	.401 _{.040}	.313 _{.025}	.610 _{.030}	.341 _{.044}	.266 _{.021}	.696 _{.029}	.480 _{.035}
MIWAE	.308 _{.024}	.180 _{.011}	.264 _{.043}	.295 _{.022}	.203 _{.020}	.286 _{.042}	.319 _{.024}	.209 _{.013}	.232 _{.046}	.283 _{.023}	.383 _{.043}	.378 _{.039}
not-MIWAE	.340 _{.024}	.384 _{.048}	.249 _{.052}	.361 _{.025}	.340 _{.044}	.293 _{.045}	.353 _{.025}	.382 _{.048}	.277 _{.046}	.337 _{.022}	.448 _{.048}	.335 _{.042}
MIRACLE	.365 _{.025}	.407 _{.046}	.265 _{.047}	.350 _{.024}	.430 _{.045}	.324 _{.043}	.379 _{.026}	.405 _{.046}	.239 _{.049}	.323 _{.022}	.504 _{.046}	.346 _{.044}
ReMasker	.420 _{.028}	.381 _{.047}	.211 _{.052}	.344 _{.024}	.419 _{.047}	.277 _{.049}	.418 _{.028}	.393 _{.047}	.260 _{.048}	.327 _{.024}	.509 _{.047}	.395 _{.041}
U-VAE	.272 _{.026}	.620 _{.050}	.417 _{.046}	.279 _{.025}	.664 _{.049}	.426 _{.047}	.317 _{.027}	.624 _{.050}	.392 _{.048}	.265 _{.025}	.772 _{.050}	.491 _{.044}

Table 7: **Imputation utility** at 0.9 missingness. The means and their standard errors across 11 real-world datasets and 5 random seeds are reported. ↑ (↓) denotes that higher (lower) is better. The best value is **red bolded**, and the second best is **blue bolded**.

Q1: Imputation utility result As shown in Tables 4–7, U-VAE consistently ranks among the top methods in downstream utility, achieving the highest or near-highest classification accuracy and feature-selection agreement, while generally excelling in SMAPE across missingness rates, outperforming in 35 out of 48 settings (4 missingness rates × 4 missingness mechanisms × 3 metrics). These results indicate that its superior imputations translate into better downstream task performance, not just improved reconstruction metrics, with only minor exceptions (e.g., a slight SMAPE drop under MNARQ at 0.1 missingness rate).

Q2: Multiple imputation We report the Bias, Coverage, and Width results to assess multiple imputation across missingness rates of 0.1, 0.5, 0.7, and 0.9.

model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓
MICE	0.005(2.0)	0.867 \pm 0.024	0.038 \pm 0.003	0.004(1.2)	0.907 \pm 0.018	0.037 \pm 0.003	0.005(2.0)	0.864 \pm 0.025	0.038 \pm 0.003	0.003(1.3)	0.915 \pm 0.017	0.037 \pm 0.003
GAIN	0.013(3.6)	0.658 \pm 0.040	0.037 \pm 0.003	0.012(2.9)	0.699 \pm 0.029	0.037 \pm 0.003	0.018(4.8)	0.550 \pm 0.042	0.038 \pm 0.003	0.008(2.2)	0.786 \pm 0.029	0.038 \pm 0.003
VAEAC	0.005(2.0)	0.871 \pm 0.024	0.038 \pm 0.003	0.004(1.3)	0.908 \pm 0.016	0.038 \pm 0.003	0.006 (2.1)	0.864 \pm 0.022	0.038 \pm 0.003	0.004(1.5)	0.910 \pm 0.016	0.038 \pm 0.003
MIWAE	0.005 (1.5)	0.912 \pm 0.016	0.041 \pm 0.003	0.004 (1.1)	0.938 \pm 0.013	0.040 \pm 0.003	0.007(1.9)	0.889 \pm 0.017	0.040 \pm 0.003	0.004(1.2)	0.926 \pm 0.014	0.040 \pm 0.003
not-MIWAE	0.005(1.6)	0.925 \pm 0.015	0.040 \pm 0.003	0.005(1.3)	0.924 \pm 0.014	0.040 \pm 0.003	0.007(1.8)	0.887 \pm 0.021	0.040 \pm 0.003	0.003 (1.0)	0.945 \pm 0.012	0.039 \pm 0.003
U-VAE	0.003 (0.9)	0.978 \pm 0.006	0.037 \pm 0.003	0.004 (1.0)	0.963 \pm 0.016	0.037 \pm 0.003	0.004 (1.1)	0.977 \pm 0.006	0.037 \pm 0.003	0.002 (0.6)	0.989 \pm 0.004	0.037 \pm 0.003

Table 8: **Multiple imputation performance** at 0.1 missingness rate. The means and their standard errors across 11 datasets and 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance. The best value is **red bolded**, and the second best is **blue bolded**.

model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓									
MICE	0.033(9.1)	0.443 \pm 0.040	0.038 \pm 0.003	0.022(5.7)	0.635 \pm 0.029	0.038 \pm 0.003	0.038(10.2)	0.394 \pm 0.040	0.038 \pm 0.003	0.020(5.8)	0.665 \pm 0.030	0.038 \pm 0.003
GAIN	0.082(20.3)	0.198 \pm 0.033	0.037 \pm 0.003	0.050(13.0)	0.464 \pm 0.025	0.037 \pm 0.003	0.091(23.7)	0.188 \pm 0.030	0.037 \pm 0.003	0.043(11.9)	0.522 \pm 0.029	0.037 \pm 0.003
VAEAC	0.033(9.0)	0.472 \pm 0.038	0.039 \pm 0.003	0.022(5.8)	0.608 \pm 0.026	0.038 \pm 0.003	0.033(8.9)	0.458 \pm 0.035	0.039 \pm 0.003	0.020(5.7)	0.652 \pm 0.029	0.038 \pm 0.003
MIWAE	0.017 (5.5)	0.707 \pm 0.039	0.046 \pm 0.004	0.014 (3.9)	0.777 \pm 0.031	0.043 \pm 0.003	0.020 (5.9)	0.703 \pm 0.038	0.045 \pm 0.004	0.011 (3.8)	0.805 \pm 0.026	0.042 \pm 0.004
not-MIWAE	0.024(6.7)	0.572 \pm 0.039	0.046 \pm 0.004	0.026(6.2)	0.626 \pm 0.025	0.042 \pm 0.004	0.026(7.0)	0.523 \pm 0.038	0.044 \pm 0.004	0.022(5.8)	0.656 \pm 0.025	0.044 \pm 0.004
U-VAE	0.010 (3.0)	0.803 \pm 0.030	0.039 \pm 0.004	0.011 (3.0)	0.798 \pm 0.024	0.037 \pm 0.004	0.016 (4.4)	0.689 \pm 0.034	0.038 \pm 0.004	0.008 (2.2)	0.857 \pm 0.021	0.037 \pm 0.004

Table 9: **Multiple imputation performance** at 0.5 missingness rate. The means and their standard errors across 11 datasets and 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance. The best value is **red bolded**, and the second best is **blue bolded**.

model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓									
MICE	0.068(16.8)	0.212 \pm 0.031	0.038 \pm 0.003	0.037(8.9)	0.515 \pm 0.026	0.038 \pm 0.003	0.072(17.5)	0.240 \pm 0.033	0.038 \pm 0.003	0.033(8.3)	0.567 \pm 0.027	0.038 \pm 0.003
GAIN	0.139(43.1)	0.135 \pm 0.030	0.036 \pm 0.003	0.072(20.5)	0.414 \pm 0.022	0.037 \pm 0.003	0.141(38.4)	0.112 \pm 0.023	0.036 \pm 0.003	0.057(17.1)	0.498 \pm 0.028	0.037 \pm 0.003
VAEAC	0.057(13.8)	0.278 \pm 0.033	0.040 \pm 0.003	0.036(8.5)	0.494 \pm 0.022	0.039 \pm 0.003	0.056(13.7)	0.296 \pm 0.032	0.040 \pm 0.003	0.033(8.2)	0.567 \pm 0.025	0.039 \pm 0.003
MIWAE	0.023 (6.6)	0.657 \pm 0.036	0.050 \pm 0.004	0.018 (5.5)	0.740 \pm 0.028	0.045 \pm 0.004	0.029 (7.9)	0.564 \pm 0.037	0.050 \pm 0.004	0.016 (5.5)	0.761 \pm 0.027	0.044 \pm 0.003
not-MIWAE	0.032(9.2)	0.506 \pm 0.038	0.048 \pm 0.004	0.038(8.8)	0.546 \pm 0.020	0.046 \pm 0.004	0.035(9.6)	0.442 \pm 0.033	0.049 \pm 0.004	0.032(8.3)	0.576 \pm 0.022	0.043 \pm 0.004
U-VAE	0.015 (4.2)	0.729 \pm 0.038	0.038 \pm 0.004	0.014 (3.7)	0.748 \pm 0.026	0.036 \pm 0.004	0.022 (6.1)	0.592 \pm 0.034	0.038 \pm 0.004	0.011 (3.1)	0.802 \pm 0.022	0.037 \pm 0.004

Table 10: **Multiple imputation performance** at 0.7 missingness rate. The means and their standard errors across 11 datasets and 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance. The best value is **red bolded**, and the second best is **blue bolded**.

model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓
MICE	0.237(123.8)	0.048 \pm 0.014	0.034 \pm 0.003	0.095(44.6)	0.396 \pm 0.021	0.037 \pm 0.003	0.227(113.5)	0.049 \pm 0.013	0.034 \pm 0.003	0.073(32.2)	0.483 \pm 0.024	0.037 \pm 0.003
GAIN	0.201(81.8)	0.058 \pm 0.012	0.033 \pm 0.003	0.099(38.0)	0.392 \pm 0.019	0.036 \pm 0.003	0.206(72.2)	0.066 \pm 0.019	0.033 \pm 0.003	0.078(30.9)	0.419 \pm 0.019	0.036 \pm 0.003
VAEAC	0.115(25.4)	0.133 \pm 0.023	0.043 \pm 0.004	0.054(11.8)	0.431 \pm 0.021	0.039 \pm 0.003	0.102(23.7)	0.152 \pm 0.021	0.041 \pm 0.004	0.048(10.9)	0.491 \pm 0.020	0.039 \pm 0.003
MIWAE	0.034 (9.6)	0.512 \pm 0.040	0.050 \pm 0.004	0.027 (7.2)	0.617 \pm 0.025	0.048 \pm 0.004	0.044 (11.7)	0.411 \pm 0.037	0.055 \pm 0.004	0.024 (7.2)	0.649 \pm 0.027	0.048 \pm 0.004
not-MIWAE	0.056(14.6)	0.308 \pm 0.026	0.050 \pm 0.004	0.047(11.0)	0.466 \pm 0.018	0.045 \pm 0.004	0.058(15.0)	0.261 \pm 0.020	0.049 \pm 0.004	0.047(11.1)	0.503 \pm 0.020	0.047 \pm 0.004
U-VAE	0.028 (7.4)	0.540 \pm 0.038	0.040 \pm 0.004	0.022 (5.8)	0.637 \pm 0.027	0.040 \pm 0.004	0.034 (9.2)	0.436 \pm 0.036	0.041 \pm 0.004	0.016 (4.3)	0.697 \pm 0.027	0.039 \pm 0.004

Table 11: **Multiple imputation performance** at 0.9 missingness rate. The means and their standard errors across 11 datasets and 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance. The best value is **red bolded**, and the second best is **blue bolded**.

Q2 result Multiple-imputation quality is evaluated via Bias (and percent Bias), Confidence Interval Coverage (target ≈ 0.95), and CI Width. As shown in Table 8-11, U-VAE achieves low Bias, near-target Coverage, and avoids unnecessary CI inflation, outperforming in 35 out of 48 settings (4 missingness rates \times 4 missingness mechanisms \times 3 metrics). Even at extreme missingness rate 0.9, it balances accuracy (Bias), reliability (Coverage), and efficiency (Width) better than others, where *many baselines show rising Bias and collapsing Coverage*.

Q3: Sensitivity Analysis – Imputation fidelity We report sensitivity analyses using additional evaluation metrics to assess imputation fidelity.

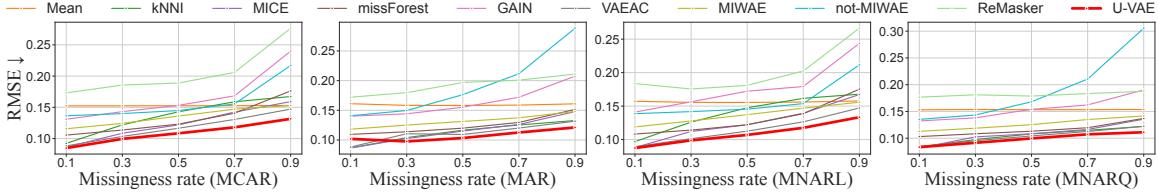


Figure 9: **Sensitivity analysis for missingness rates.** Imputation performance comparison of various imputation models under different missing data mechanisms (MCAR, MAR, MNARL, MNARQ) and missingness rates (0.1, 0.3, 0.5, 0.7, 0.9) using Bias. \downarrow denotes that lower is better

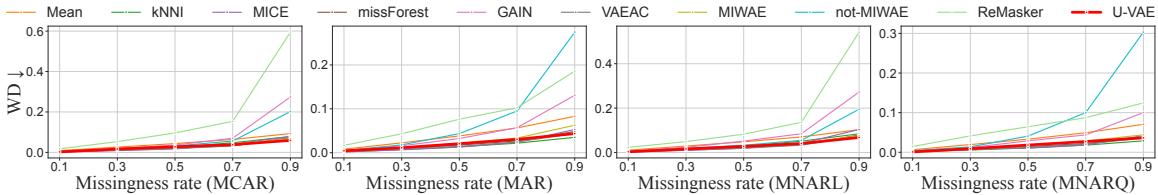


Figure 10: **Sensitivity analysis for missingness rates.** Imputation performance comparison of various imputation models under different missing data mechanisms (MCAR, MAR, MNARL, MNARQ) and missingness rates (0.1, 0.3, 0.5, 0.7, 0.9) using WD. \downarrow denotes that lower is better.

Q3: Sensitivity Analysis – Imputation utility We report sensitivity analyses using additional evaluation metrics to assess imputation utility.

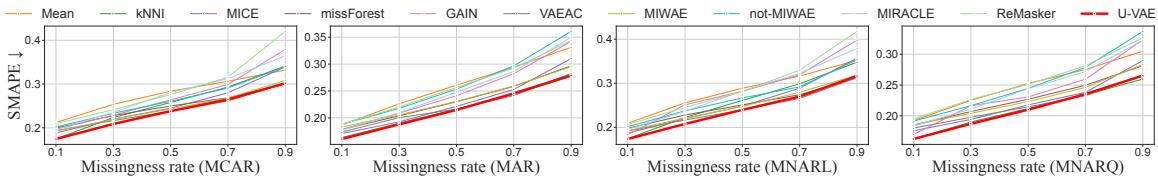


Figure 11: **Sensitivity analysis for missingness rates.** Imputation performance comparison of various imputation models under different missing data mechanisms (MCAR, MAR, MNARL, MNARQ) and missingness rates (0.1, 0.3, 0.5, 0.7, 0.9) using SMAPE (regression). \downarrow denotes that lower is better.

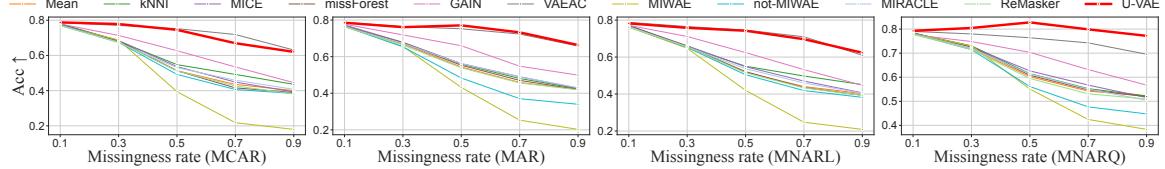


Figure 12: **Sensitivity analysis for missingness rates.** Imputation performance comparison of various imputation models under different missing data mechanisms (MCAR, MAR, MNARL, MNARQ) and missingness rates (0.1, 0.3, 0.5, 0.7, 0.9) using Accuracy (classification). ↑ denotes that higher is better.

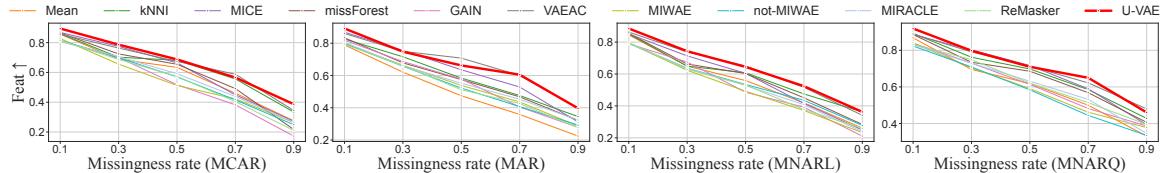


Figure 13: **Sensitivity analysis for missingness rates.** Imputation performance comparison of various imputation models under different missing data mechanisms (MCAR, MAR, MNARL, MNARQ) and missingness rates (0.1, 0.3, 0.5, 0.7, 0.9) using Feature selection. ↑ denotes that higher is better.

Q3: Sensitivity Analysis result – Imputation fidelity & utility As shown in Figure 9-13, U-VAE’s lines remain consistently at or near the top across all metrics, with performance degrading only gradually as missingness increases. This flatness reflects strong robustness, indicating that U-VAE maintains high-quality imputations even in more challenging missingness scenarios.

Q3: Sensitivity Analysis – Multiple imputation We report sensitivity analyses using additional evaluation metrics to assess multiple imputation.

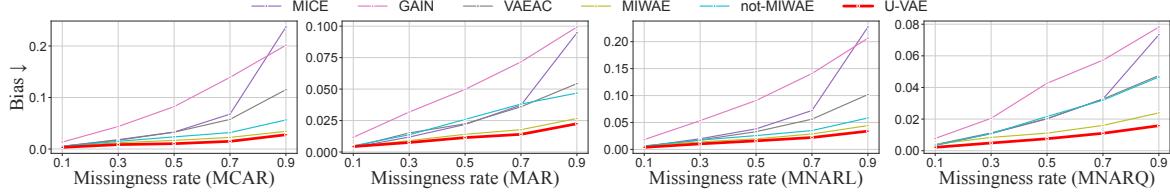


Figure 14: **Sensitivity analysis for missingness rates.** Imputation performance comparison of various imputation models under different missing data mechanisms (MCAR, MAR, MNARL, MNARQ) and missingness rates (0.1, 0.3, 0.5, 0.7, 0.9) using Width. ↓ denotes that lower is better.

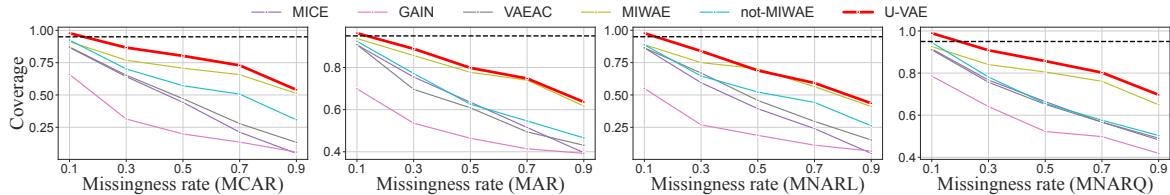


Figure 15: **Sensitivity analysis for missingness rates.** Imputation performance comparison of various imputation models under different missing data mechanisms (MCAR, MAR, MNARL, MNARQ) and missingness rates (0.1, 0.3, 0.5, 0.7, 0.9) using Coverage. Coverage close to 0.95 indicates better performance.

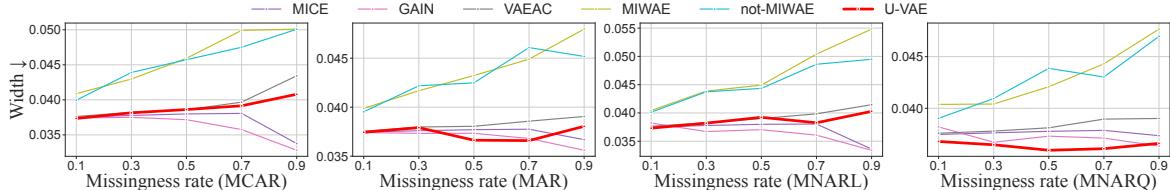


Figure 16: **Sensitivity analysis for missingness rates.** Imputation performance comparison of various imputation models under different missing data mechanisms (MCAR, MAR, MNARL, MNARQ) and missingness rates (0.1, 0.3, 0.5, 0.7, 0.9) using Width. ↓ denotes that lower is better.

Q3: Sensitivity Analysis result – Multiple imputation As shown in Figure 15 and 16, U-VAE maintains low Bias while keeping Coverage high without over-widening intervals. In other words, it achieves a good inference trade-off: accurate, trustworthy, and efficient uncertainty.

E.2 Across each dataset

Q1: Imputation fidelity (RMSE) We report RMSE results to assess imputation fidelity for each dataset at 0.3 missingness rate.

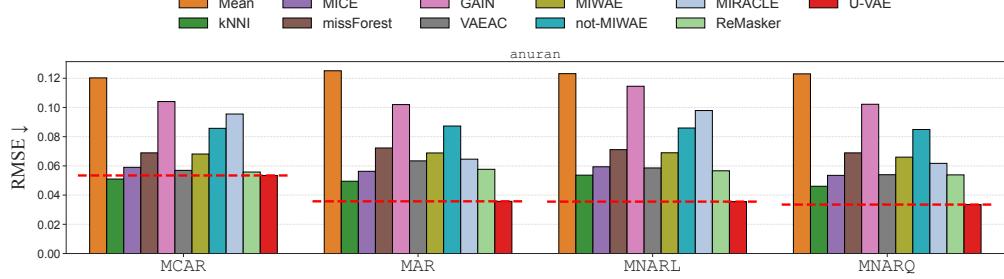


Figure 17: **Imputation fidelity (RMSE)** on the anuran dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

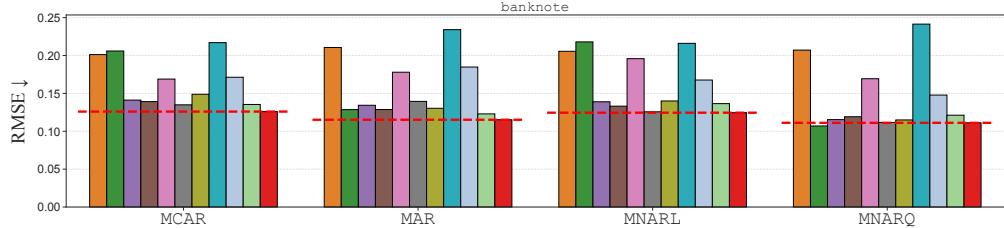


Figure 18: **Imputation fidelity (RMSE)** on the banknote dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

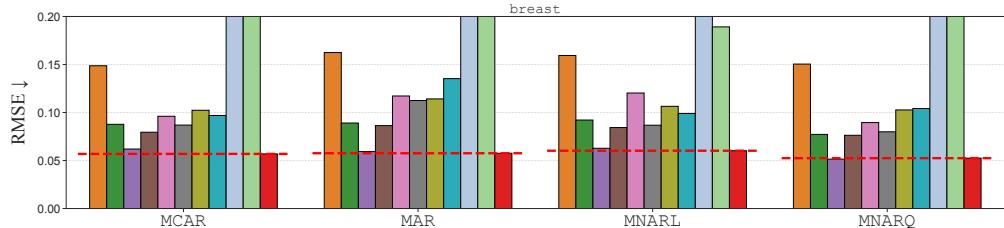


Figure 19: **Imputation fidelity (RMSE)** on the breast dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

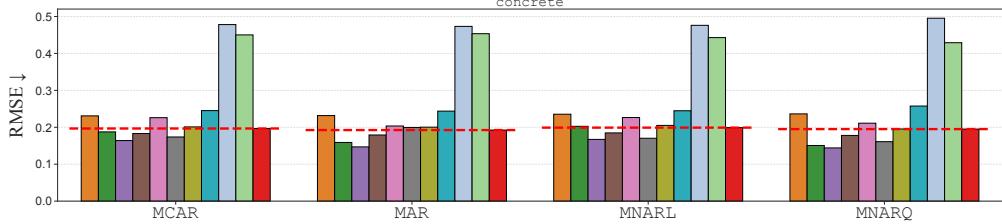


Figure 20: **Imputation fidelity (RMSE)** on the concrete dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

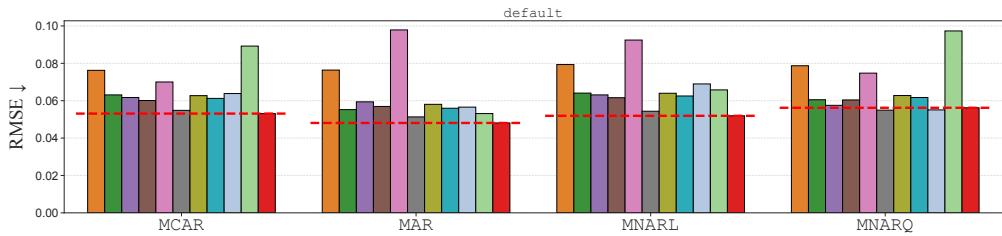


Figure 21: **Imputation fidelity (RMSE)** on the default dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

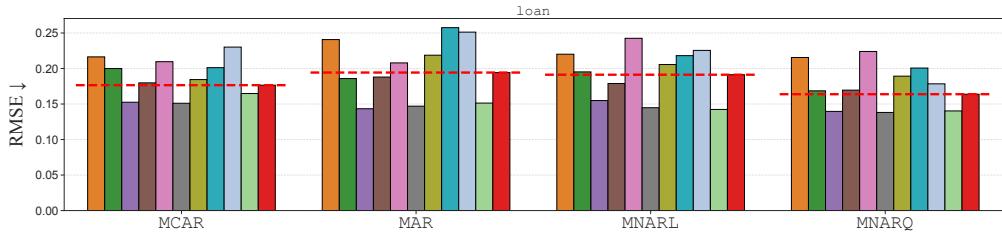


Figure 22: **Imputation fidelity (RMSE)** on the loan dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

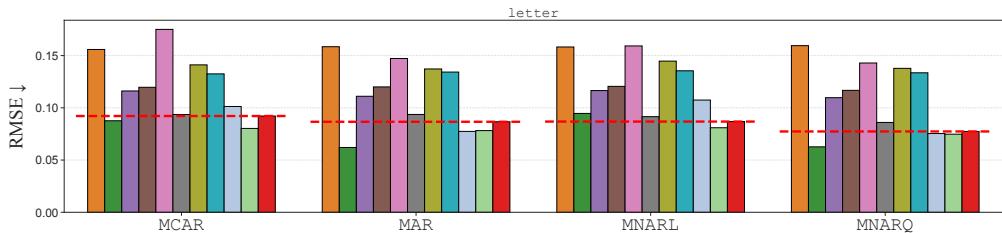


Figure 23: **Imputation fidelity (RMSE)** on the letter dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

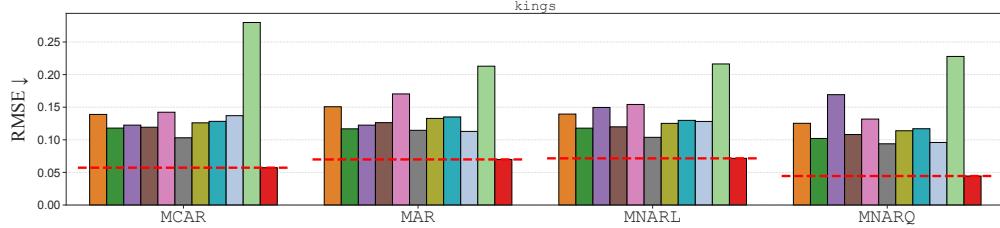


Figure 24: **Imputation fidelity (RMSE)** on the **kings** dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

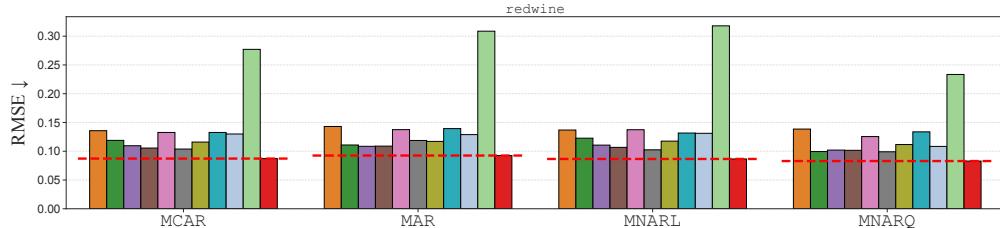


Figure 25: **Imputation fidelity (RMSE)** on the **redwine** dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.



Figure 26: **Imputation fidelity (RMSE)** on the **shoppers** dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

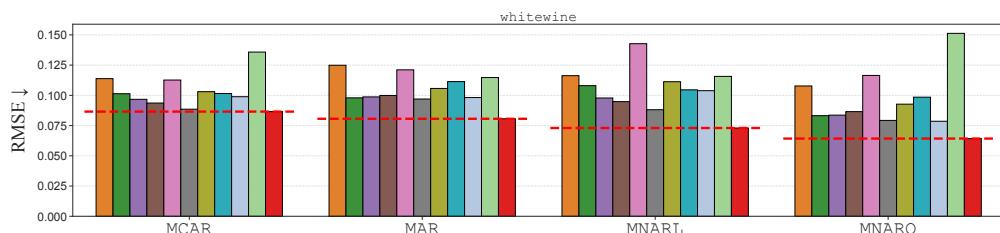


Figure 27: **Imputation fidelity (RMSE)** on the **whitewine** dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

Q1: Imputation fidelity (WD) We report WD results to assess imputation fidelity for each dataset at 0.3 missingness rate.

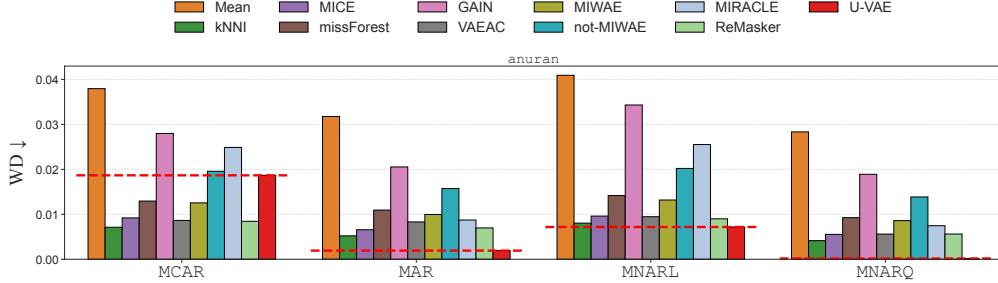


Figure 28: **Imputation fidelity (WD)** on the anuran dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

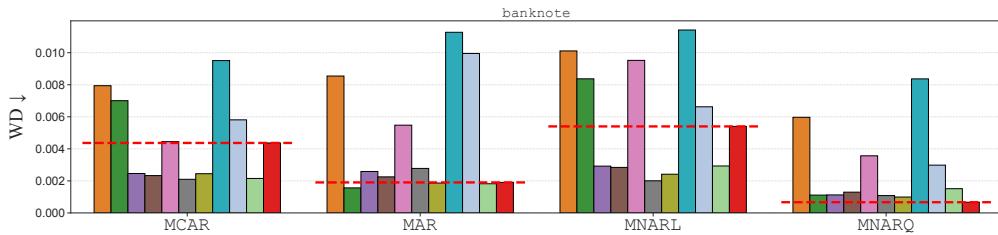


Figure 29: **Imputation fidelity (WD)** on the banknote dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

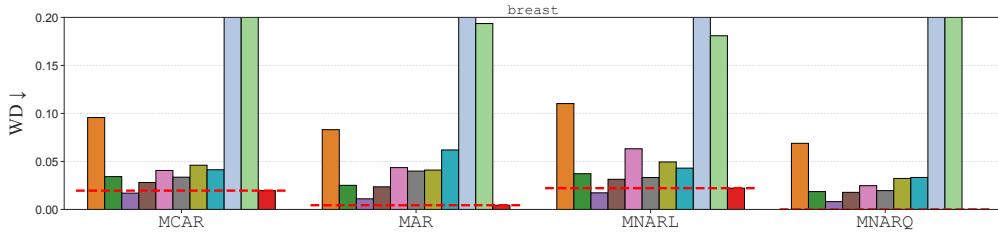


Figure 30: **Imputation fidelity (WD)** on the breast dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

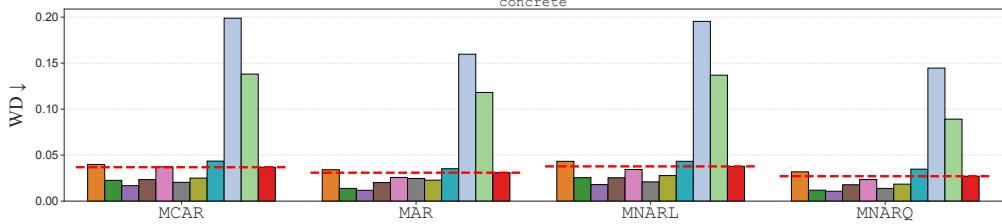


Figure 31: **Imputation fidelity (WD)** on the concrete dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

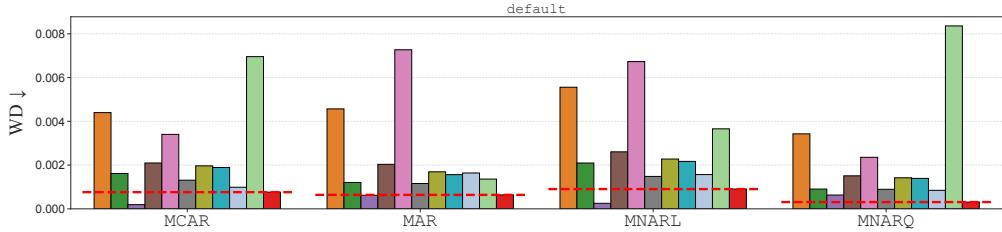


Figure 32: **Imputation fidelity (WD)** on the default dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

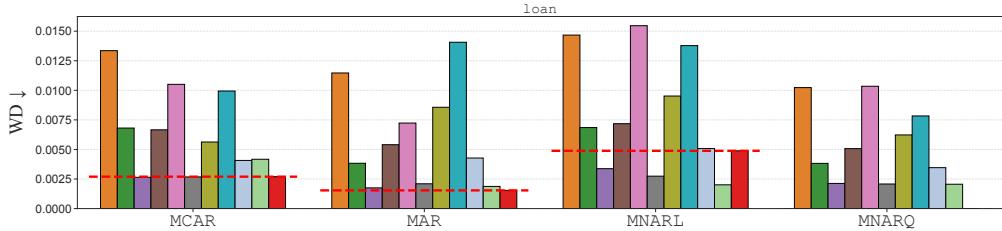


Figure 33: **Imputation fidelity (WD)** on the loan dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

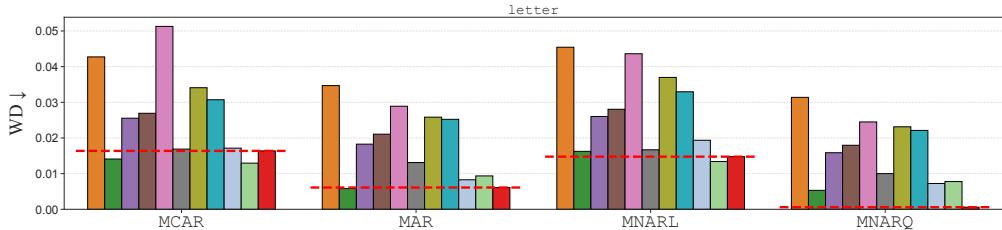


Figure 34: **Imputation fidelity (WD)** on the letter dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

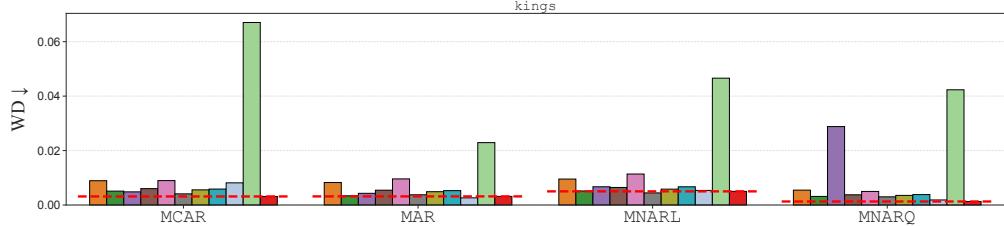


Figure 35: **Imputation fidelity (WD)** on the `kings` dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

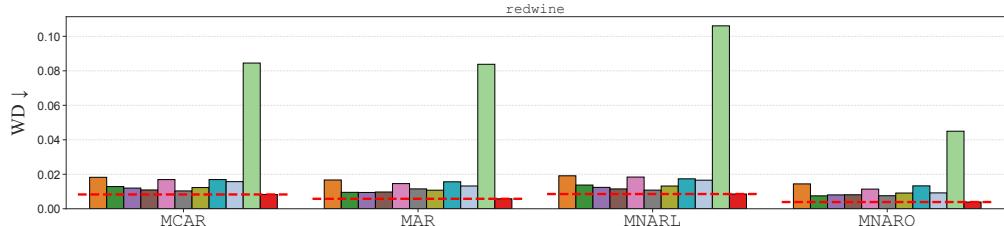


Figure 36: **Imputation fidelity (WD)** on the `redwine` dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

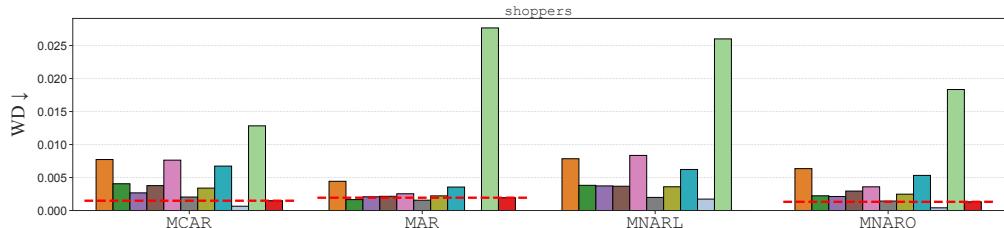


Figure 37: **Imputation fidelity (WD)** on the `shoppers` dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

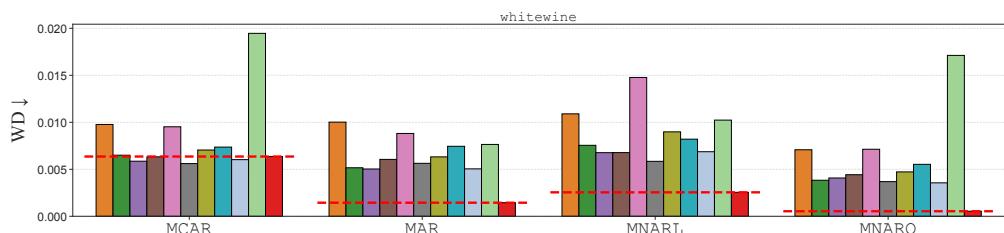


Figure 38: **Imputation fidelity (WD)** on the `whitewine` dataset at 0.3 missingness rate. The corresponding missingness mechanism is indicated below the figure. The red dashed line indicates the performance of U-VAE under each missingness mechanism. The means and their standard errors across 5 random seeds are reported. ↓ denotes that lower is better.

Q1: Imputation utility We report SMAPE (regression), Accuracy (Acc, classification), feature selection performance (Feat) results to assess imputation utility for each dataset at 0.3 missingness rate.

anuran													
model	MCAR			MAR			MNARL			MNARQ			Feat ↑
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	
Mean	0.273 \pm 0.002	0.879 \pm 0.020	0.464 \pm 0.024	0.259 \pm 0.001	0.779 \pm 0.027	0.355 \pm 0.034	0.280 \pm 0.001	0.805 \pm 0.025	0.389 \pm 0.038	0.252 \pm 0.002	0.918 \pm 0.025	0.464 \pm 0.065	
kNNI	0.208 \pm 0.001	0.850 \pm 0.018	0.376 \pm 0.027	0.204 \pm 0.001	0.765 \pm 0.025	0.447 \pm 0.024	0.209 \pm 0.001	0.769 \pm 0.015	0.340 \pm 0.068	0.202 \pm 0.002	0.940 \pm 0.027	0.495 \pm 0.050	
MICE	0.201 \pm 0.001	0.872 \pm 0.012	0.424 \pm 0.018	0.199 \pm 0.001	0.739 \pm 0.026	0.497 \pm 0.031	0.203 \pm 0.001	0.787 \pm 0.017	0.403 \pm 0.040	0.196 \pm 0.001	0.909 \pm 0.039	0.595 \pm 0.059	
missForest	0.224 \pm 0.001	0.849 \pm 0.012	0.411 \pm 0.019	0.222 \pm 0.001	0.782 \pm 0.026	0.398 \pm 0.025	0.230 \pm 0.001	0.799 \pm 0.019	0.321 \pm 0.053	0.217 \pm 0.001	0.937 \pm 0.024	0.523 \pm 0.066	
gain	0.242 \pm 0.002	0.904 \pm 0.019	0.453 \pm 0.047	0.231 \pm 0.001	0.854 \pm 0.026	0.475 \pm 0.020	0.258 \pm 0.000	0.877 \pm 0.030	0.429 \pm 0.021	0.224 \pm 0.002	0.941 \pm 0.012	0.538 \pm 0.024	
VAEAC	0.212 \pm 0.001	0.935 \pm 0.019	0.722 \pm 0.034	0.213 \pm 0.001	0.933 \pm 0.007	0.706 \pm 0.041	0.214 \pm 0.001	0.928 \pm 0.014	0.692 \pm 0.023	0.206 \pm 0.001	0.951 \pm 0.007	0.750 \pm 0.034	
MIWAE	0.220 \pm 0.001	0.885 \pm 0.005	0.361 \pm 0.032	0.216 \pm 0.001	0.787 \pm 0.027	0.457 \pm 0.015	0.224 \pm 0.001	0.793 \pm 0.012	0.360 \pm 0.022	0.213 \pm 0.001	0.922 \pm 0.023	0.462 \pm 0.082	
not-MIWAE	0.231 \pm 0.001	0.892 \pm 0.014	0.423 \pm 0.032	0.227 \pm 0.001	0.769 \pm 0.026	0.400 \pm 0.014	0.239 \pm 0.001	0.817 \pm 0.017	0.348 \pm 0.043	0.221 \pm 0.002	0.927 \pm 0.030	0.533 \pm 0.056	
MIRACLE	0.209 \pm 0.003	0.834 \pm 0.014	0.403 \pm 0.030	0.201 \pm 0.002	0.755 \pm 0.026	0.432 \pm 0.023	0.213 \pm 0.000	0.763 \pm 0.011	0.354 \pm 0.039	0.198 \pm 0.000	0.927 \pm 0.031	0.554 \pm 0.059	
ReMasker	0.217 \pm 0.002	0.859 \pm 0.007	0.334 \pm 0.020	0.214 \pm 0.001	0.765 \pm 0.028	0.409 \pm 0.021	0.219 \pm 0.002	0.773 \pm 0.021	0.332 \pm 0.042	0.211 \pm 0.002	0.925 \pm 0.031	0.495 \pm 0.082	
U-VAE	0.313 \pm 0.007	0.950 \pm 0.010	0.894 \pm 0.040	0.260 \pm 0.014	0.931 \pm 0.022	0.693 \pm 0.085	0.300 \pm 0.010	0.929 \pm 0.017	0.933 \pm 0.059	0.232 \pm 0.013	0.990 \pm 0.008	0.733 \pm 0.076	

Table 12: **Imputation utility** on the anuran dataset at 0.3 missingness. The means and their standard errors across 5 random seeds are reported. \uparrow (\downarrow) denotes that higher (lower) is better.

banknote													
model	MCAR			MAR			MNARL			MNARQ			Feat ↑
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	
Mean	0.378 \pm 0.006	0.910 \pm 0.013	0.863 \pm 0.058	0.353 \pm 0.004	0.899 \pm 0.021	0.644 \pm 0.119	0.388 \pm 0.005	0.886 \pm 0.023	0.717 \pm 0.100	0.322 \pm 0.006	0.946 \pm 0.013	0.577 \pm 0.141	
kNNI	0.399 \pm 0.010	0.896 \pm 0.016	0.653 \pm 0.085	0.278 \pm 0.009	0.911 \pm 0.024	0.758 \pm 0.115	0.387 \pm 0.008	0.842 \pm 0.013	0.580 \pm 0.061	0.263 \pm 0.008	0.957 \pm 0.008	0.909 \pm 0.033	
MICE	0.312 \pm 0.006	0.962 \pm 0.006	0.960 \pm 0.016	0.283 \pm 0.005	0.923 \pm 0.021	0.907 \pm 0.045	0.297 \pm 0.009	0.928 \pm 0.023	0.893 \pm 0.045	0.265 \pm 0.010	0.971 \pm 0.009	1.000 \pm 0.000	
missForest	0.318 \pm 0.007	0.943 \pm 0.008	0.914 \pm 0.030	0.290 \pm 0.005	0.921 \pm 0.021	0.775 \pm 0.045	0.309 \pm 0.004	0.917 \pm 0.022	0.743 \pm 0.053	0.274 \pm 0.005	0.964 \pm 0.010	0.760 \pm 0.111	
gain	0.339 \pm 0.008	0.942 \pm 0.013	0.867 \pm 0.030	0.315 \pm 0.006	0.963 \pm 0.011	0.751 \pm 0.061	0.367 \pm 0.018	0.929 \pm 0.008	0.804 \pm 0.046	0.288 \pm 0.004	0.977 \pm 0.005	0.861 \pm 0.045	
VAEAC	0.303 \pm 0.009	0.979 \pm 0.004	0.945 \pm 0.022	0.295 \pm 0.006	0.966 \pm 0.007	0.880 \pm 0.039	0.299 \pm 0.007	0.969 \pm 0.005	0.917 \pm 0.037	0.261 \pm 0.003	0.984 \pm 0.005	0.947 \pm 0.039	
MIWAE	0.322 \pm 0.011	0.864 \pm 0.013	0.748 \pm 0.057	0.293 \pm 0.010	0.800 \pm 0.020	0.806 \pm 0.075	0.317 \pm 0.005	0.810 \pm 0.010	0.695 \pm 0.073	0.272 \pm 0.003	0.929 \pm 0.026	0.855 \pm 0.083	
not-MIWAE	0.404 \pm 0.007	0.884 \pm 0.005	0.858 \pm 0.026	0.367 \pm 0.010	0.887 \pm 0.025	0.714 \pm 0.050	0.410 \pm 0.011	0.888 \pm 0.019	0.714 \pm 0.049	0.339 \pm 0.011	0.939 \pm 0.017	0.617 \pm 0.101	
MIRACLE	0.350 \pm 0.023	0.954 \pm 0.011	0.871 \pm 0.045	0.322 \pm 0.031	0.926 \pm 0.019	0.748 \pm 0.134	0.338 \pm 0.027	0.925 \pm 0.021	0.762 \pm 0.076	0.288 \pm 0.014	0.970 \pm 0.008	0.903 \pm 0.044	
ReMasker	0.302 \pm 0.008	0.956 \pm 0.014	0.938 \pm 0.043	0.290 \pm 0.003	0.926 \pm 0.020	0.793 \pm 0.060	0.308 \pm 0.009	0.925 \pm 0.018	0.747 \pm 0.086	0.272 \pm 0.005	0.967 \pm 0.010	0.890 \pm 0.090	
U-VAE	0.313 \pm 0.007	0.950 \pm 0.010	0.894 \pm 0.040	0.260 \pm 0.014	0.931 \pm 0.022	0.693 \pm 0.085	0.300 \pm 0.010	0.929 \pm 0.017	0.933 \pm 0.059	0.232 \pm 0.013	0.990 \pm 0.008	0.733 \pm 0.076	

Table 13: **Imputation utility** on the banknote dataset at 0.3 missingness. The means and their standard errors across 5 random seeds are reported. \uparrow (\downarrow) denotes that higher (lower) is better.

breast													
model	MCAR			MAR			MNARL			MNARQ			Feat ↑
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	
Mean	0.089 \pm 0.001	0.915 \pm 0.008	0.400 \pm 0.051	0.084 \pm 0.002	0.917 \pm 0.022	0.446 \pm 0.047	0.092 \pm 0.001	0.922 \pm 0.014	0.510 \pm 0.044	0.083 \pm 0.001	0.932 \pm 0.012	0.460 \pm 0.085	
kNNI	0.061 \pm 0.001	0.891 \pm 0.006	0.496 \pm 0.052	0.060 \pm 0.001	0.899 \pm 0.036	0.507 \pm 0.058	0.062 \pm 0.001	0.868 \pm 0.038	0.454 \pm 0.060	0.059 \pm 0.001	0.946 \pm 0.008	0.638 \pm 0.049	
MICE	0.056 \pm 0.001	0.908 \pm 0.006	0.529 \pm 0.050	0.054 \pm 0.001	0.911 \pm 0.036	0.545 \pm 0.038	0.056 \pm 0.001	0.898 \pm 0.035	0.429 \pm 0.031	0.054 \pm 0.001	0.942 \pm 0.008	0.675 \pm 0.062	
GAIN	0.062 \pm 0.002	0.923 \pm 0.007	0.485 \pm 0.015	0.062 \pm 0.002	0.932 \pm 0.012	0.565 \pm 0.041	0.065 \pm 0.002	0.920 \pm 0.021	0.522 \pm 0.029	0.059 \pm 0.001	0.943 \pm 0.007	0.631 \pm 0.060	
VAEAC	0.064 \pm 0.001	0.952 \pm 0.003	0.552 \pm 0.043	0.068 \pm 0.001	0.950 \pm 0.003	0.584 \pm 0.040	0.064 \pm 0.001	0.949 \pm 0.004	0.646 \pm 0.030	0.061 \pm 0.001	0.955 \pm 0.002	0.589 \pm 0.033	
MIWAE	0.067 \pm 0.001	0.818 \pm 0.011	0.474 \pm 0.017	0.065 \pm 0.001	0.798 \pm 0.040	0.508 \pm 0.064	0.067 \pm 0.001	0.769 \pm 0.014	0.468 \pm 0.026	0.063 \pm 0.001	0.904 \pm 0.030	0.615 \pm 0.030	
not-MIWAE	0.066 \pm 0.001	0.899 \pm 0.010	0.523 \pm 0.047	0.066 \pm 0.001	0.919 \pm 0.019	0.549 \pm 0.056	0.065 \pm 0.001	0.891 \pm 0.033	0.443 \pm 0.055	0.063 \pm 0.001	0.935 \pm 0.010	0.591 \pm 0.063	
MIRACLE	0.220 \pm 0.042	0.828 \pm 0.030	0.447 \pm 0.032	0.205 \pm 0.038	0.892 \pm 0.028	0.393 \pm 0.048	0.226 \pm 0.046	0.857 \pm 0.022	0.431 \pm 0.032	0.211 \pm 0.037	0.910 \pm 0.009	0.382 \pm 0.075	
ReMasker	0.090 \pm 0.008	0.921 \pm 0.009	0.366 \pm 0.045	0.088 \pm 0.010	0.885 \pm 0.035	0.442 \pm 0.047	0.090 \pm 0.010	0.928 \pm 0.007	0.423 \pm 0.039	0.095 \pm 0.005	0.915 \pm 0.014	0.449 \pm 0.068	
U-VAE	0.												

concrete													
model	MCAR			MAR			MNARL			MNARQ			
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	
Mean	0.233±0.003	0.305±0.015	0.599±0.044	0.201±0.006	0.332±0.030	0.445±0.074	0.235±0.003	0.281±0.020	0.452±0.096	0.217±0.005	0.369±0.049	0.724±0.043	
kNNI	0.215±0.004	0.322±0.010	0.786±0.064	0.158±0.002	0.372±0.038	0.719±0.053	0.215±0.004	0.316±0.014	0.467±0.071	0.174±0.008	0.418±0.044	0.788±0.040	
MICE	0.214±0.003	0.346±0.013	0.820±0.009	0.165±0.005	0.388±0.040	0.730±0.047	0.213±0.003	0.337±0.015	0.643±0.092	0.196±0.006	0.416±0.048	0.743±0.048	
missForest	0.225±0.003	0.302±0.012	0.722±0.055	0.178±0.005	0.355±0.033	0.545±0.060	0.225±0.004	0.297±0.019	0.573±0.052	0.208±0.005	0.377±0.052	0.706±0.050	
GAIN	0.230±0.003	0.423±0.017	0.724±0.044	0.191±0.008	0.443±0.021	0.736±0.041	0.227±0.003	0.427±0.016	0.643±0.062	0.210±0.004	0.462±0.024	0.657±0.075	
VAEAC	0.216±0.003	0.454±0.010	0.691±0.042	0.193±0.005	0.428±0.011	0.587±0.019	0.217±0.004	0.434±0.011	0.595±0.053	0.201±0.006	0.495±0.013	0.693±0.077	
MIWAE	0.226±0.003	0.313±0.013	0.649±0.061	0.211±0.004	0.367±0.031	0.556±0.089	0.228±0.003	0.320±0.015	0.422±0.085	0.215±0.004	0.397±0.035	0.746±0.054	
not-MIWAE	0.235±0.003	0.272±0.020	0.617±0.049	0.225±0.002	0.328±0.037	0.502±0.037	0.238±0.004	0.275±0.009	0.422±0.087	0.224±0.004	0.355±0.051	0.625±0.086	
MIRACLE	0.256±0.004	0.271±0.012	0.701±0.078	0.222±0.013	0.317±0.034	0.533±0.072	0.258±0.005	0.259±0.020	0.419±0.082	0.235±0.002	0.350±0.043	0.646±0.073	
ReMasker	0.279±0.004	0.260±0.027	0.578±0.073	0.251±0.003	0.312±0.042	0.445±0.052	0.281±0.003	0.270±0.024	0.568±0.087	0.249±0.005	0.341±0.051	0.717±0.031	
U-VAE	0.205±0.002	0.386±0.021	0.769±0.046	0.177±0.005	0.404±0.030	0.693±0.104	0.204±0.002	0.376±0.013	0.545±0.056	0.189±0.005	0.451±0.034	0.722±0.046	

Table 15: **Imputation utility** on the concrete dataset at 0.3 missingness. The means and their standard errors across 5 random seeds are reported. \uparrow (\downarrow) denotes that higher (lower) is better.

default													
model	MCAR			MAR			MNARL			MNARQ			
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	
Mean	0.392±0.001	0.802±0.003	0.853±0.012	0.354±0.007	0.801±0.004	0.842±0.004	0.391±0.002	0.801±0.004	0.847±0.012	0.334±0.009	0.809±0.005	0.842±0.012	
kNNI	0.305±0.001	0.801±0.002	0.837±0.007	0.281±0.003	0.800±0.004	0.830±0.009	0.305±0.002	0.801±0.004	0.829±0.012	0.269±0.004	0.808±0.005	0.820±0.019	
MICE	0.316±0.002	0.790±0.004	0.820±0.021	0.292±0.002	0.797±0.006	0.818±0.013	0.315±0.001	0.791±0.006	0.822±0.007	0.267±0.008	0.806±0.005	0.822±0.014	
missForest	0.339±0.000	0.802±0.003	0.841±0.014	0.311±0.004	0.802±0.005	0.831±0.004	0.339±0.002	0.801±0.004	0.822±0.012	0.293±0.008	0.809±0.005	0.829±0.019	
GAIN	0.342±0.007	0.799±0.002	0.756±0.025	0.314±0.007	0.799±0.004	0.792±0.016	0.363±0.010	0.799±0.005	0.773±0.016	0.289±0.008	0.805±0.005	0.783±0.024	
VAEAC	0.308±0.003	0.819±0.003	0.836±0.010	0.290±0.005	0.820±0.002	0.819±0.020	0.312±0.002	0.819±0.003	0.831±0.014	0.275±0.007	0.817±0.002	0.842±0.009	
MIWAE	0.307±0.001	0.767±0.003	0.830±0.017	0.297±0.001	0.709±0.029	0.826±0.006	0.307±0.002	0.713±0.026	0.813±0.007	0.289±0.003	0.796±0.013	0.836±0.013	
not-MIWAE	0.308±0.001	0.802±0.003	0.840±0.018	0.293±0.002	0.722±0.025	0.794±0.020	0.307±0.001	0.801±0.004	0.823±0.007	0.287±0.004	0.809±0.005	0.818±0.017	
MIRACLE	0.291±0.003	0.802±0.003	0.835±0.015	0.267±0.005	0.801±0.004	0.832±0.012	0.295±0.004	0.801±0.004	0.824±0.008	0.250±0.005	0.809±0.005	0.817±0.021	
ReMasker	0.392±0.014	0.803±0.002	0.823±0.014	0.319±0.005	0.800±0.004	0.836±0.006	0.342±0.012	0.800±0.004	0.826±0.008	0.358±0.018	0.808±0.006	0.830±0.017	
U-VAE	0.292±0.003	0.826±0.003	0.846±0.013	0.282±0.003	0.827±0.005	0.860±0.012	0.288±0.001	0.824±0.004	0.834±0.009	0.272±0.003	0.831±0.004	0.837±0.017	

Table 16: **Imputation utility** on the default dataset at 0.3 missingness. The means and their standard errors across 5 random seeds are reported. \uparrow (\downarrow) denotes that higher (lower) is better.

loan													
model	MCAR			MAR			MNARL			MNARQ			
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	
Mean	0.298±0.002	0.930±0.001	0.839±0.023	0.291±0.004	0.929±0.002	0.853±0.015	0.301±0.004	0.927±0.002	0.823±0.021	0.288±0.002	0.936±0.002	0.896±0.018	
kNNI	0.290±0.001	0.932±0.002	0.836±0.030	0.280±0.003	0.931±0.003	0.860±0.011	0.287±0.001	0.931±0.002	0.837±0.007	0.278±0.003	0.936±0.002	0.865±0.027	
MICE	0.276±0.001	0.933±0.001	0.854±0.020	0.271±0.002	0.929±0.002	0.852±0.013	0.280±0.003	0.924±0.002	0.811±0.011	0.273±0.001	0.937±0.001	0.890±0.016	
missForest	0.290±0.001	0.931±0.001	0.821±0.039	0.284±0.003	0.928±0.001	0.846±0.018	0.291±0.002	0.926±0.001	0.842±0.018	0.283±0.001	0.937±0.001	0.871±0.029	
GAIN	0.291±0.004	0.930±0.002	0.818±0.018	0.287±0.005	0.927±0.003	0.824±0.026	0.295±0.004	0.926±0.003	0.809±0.008	0.275±0.006	0.934±0.003	0.792±0.050	
VAEAC	0.273±0.001	0.941±0.003	0.863±0.033	0.271±0.002	0.935±0.001	0.855±0.010	0.275±0.003	0.936±0.001	0.859±0.009	0.271±0.001	0.945±0.002	0.843±0.024	
MIWAE	0.289±0.001	0.882±0.002	0.718±0.017	0.293±0.006	0.779±0.018	0.706±0.023	0.302±0.002	0.789±0.019	0.689±0.013	0.285±0.004	0.895±0.013	0.760±0.027	
not-MIWAE	0.292±0.001	0.931±0.001	0.855±0.018	0.294±0.005	0.898±0.031	0.815±0.038	0.299±0.005	0.928±0.002	0.833±0.020	0.290±0.004	0.901±0.016	0.814±0.036	
MIRACLE	0.275±0.001	0.932±0.002	0.847±0.025	0.270±0.002	0.929±0.002	0.860±0.017	0.277±0.003	0.930±0.002	0.833±0.005	0.273±0.001	0.936±0.001	0.882±0.021	
ReMasker	0.271±0.002	0.931±0.002	0.841±0.020	0.274±0.002	0.928±0.002	0.846±0.016	0.275±0.002	0.929±0.002	0.842±0.005	0.271±0.001	0.937±0.001	0.873±0.012	
U-VAE	0.266±0.001	0.943±0.002	0.870±0.022	0.261±0.004	0.941±0.002	0.879±0.015	0.270±0.002	0.938±0.001	0.859±0.017	0.256±0.002	0.945±0.001	0.883±0.023	

Table 17: **Imputation utility** on the loan dataset at 0.3 missingness. The means and their standard errors across 5 random seeds are reported. \uparrow (\downarrow) denotes that higher (lower) is better.

letter													
model	MCAR			MAR			MNARL			MNARQ			
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	
Mean	0.103±0.000	0.578±0.008	0.710±0.028	0.092±0.002	0.591±0.040	0.650±0.019	0.106±0.000	0.548±0.008	0.713±0.033	0.089±0.001	0.624±0.040	0.709±0.023	
kNNI	0.069±0.000	0.629±0.006	0.852±0.020	0.061±0.001	0.614±0.045	0.889±0.019	0.070±0.000	0.604±0.008	0.835±0.009	0.060±0.001	0.645±0.041	0.899±0.006	
MICE	0.090±0.001	0.567±0.003	0.938±0.007	0.081±0.002	0.599±0.042	0.895±0.019	0.091±0.001	0.550±0.006	0.898±0.023	0.079±0.001	0.620±0.041	0.944±0.005	
missForest	0.092±0.000	0.606±0.003	0.838±0.016	0.084±0.001	0.604±0.039	0.732±0.028	0.094±0.000	0.572±0.001	0.786±0.025	0.081±0.001	0.637±0.038	0.777±0.020	
GAIN	0.101±0.001	0.625±0.024	0.679±0.025	0.087±0.001	0.619±0.035	0.713±0.040	0.101±0.001	0.613±0.031	0.715±0.027	0.083±0.002	0.664±0.029	0.716±0.009	
VAEAC	0.081±0.000	0.770±0.007	0.906±0.013	0.077±0.001	0.760±0.004	0.897±0.014	0.081±0.000	0.762±0.003	0.921±0.005	0.072±0.001	0.780±0.006	0.918±0.002	
MIWAE	0.096±0.000	0.712±0.002	0.713±0.011	0.0									

kings												
model	MCAR			MAR			MNARL			MNARQ		
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑
Mean	0.216 \pm 0.000	0.493 \pm 0.004	0.857 \pm 0.016	0.162 \pm 0.003	0.462 \pm 0.021	0.792 \pm 0.016	0.215 \pm 0.002	0.447 \pm 0.029	0.788 \pm 0.026	0.199 \pm 0.008	0.549 \pm 0.031	0.832 \pm 0.034
kNNI	0.172 \pm 0.001	0.475 \pm 0.022	0.780 \pm 0.038	0.130 \pm 0.004	0.462 \pm 0.019	0.835 \pm 0.018	0.168 \pm 0.002	0.458 \pm 0.024	0.799 \pm 0.023	0.143 \pm 0.002	0.568 \pm 0.029	0.833 \pm 0.032
MICE	0.177 \pm 0.006	0.491 \pm 0.007	0.912 \pm 0.010	0.118 \pm 0.011	0.471 \pm 0.020	0.901 \pm 0.007	0.184 \pm 0.009	0.457 \pm 0.023	0.894 \pm 0.018	0.150 \pm 0.010	0.548 \pm 0.037	0.900 \pm 0.013
missForest	0.204 \pm 0.000	0.479 \pm 0.004	0.905 \pm 0.008	0.155 \pm 0.007	0.454 \pm 0.017	0.861 \pm 0.042	0.203 \pm 0.002	0.445 \pm 0.025	0.784 \pm 0.052	0.190 \pm 0.009	0.552 \pm 0.031	0.864 \pm 0.038
GAIN	0.151 \pm 0.003	0.523 \pm 0.020	0.850 \pm 0.016	0.130 \pm 0.008	0.535 \pm 0.014	0.851 \pm 0.034	0.182 \pm 0.012	0.530 \pm 0.013	0.828 \pm 0.040	0.156 \pm 0.012	0.577 \pm 0.023	0.853 \pm 0.047
VAEAC	0.191 \pm 0.001	0.622 \pm 0.003	0.820 \pm 0.044	0.136 \pm 0.009	0.621 \pm 0.002	0.821 \pm 0.055	0.189 \pm 0.002	0.622 \pm 0.002	0.794 \pm 0.024	0.177 \pm 0.011	0.632 \pm 0.002	0.867 \pm 0.036
MIWAE	0.150 \pm 0.003	0.521 \pm 0.002	0.807 \pm 0.030	0.120 \pm 0.007	0.525 \pm 0.008	0.855 \pm 0.035	0.151 \pm 0.002	0.528 \pm 0.007	0.809 \pm 0.024	0.138 \pm 0.002	0.581 \pm 0.019	0.832 \pm 0.025
not-MIWAE	0.190 \pm 0.005	0.479 \pm 0.006	0.838 \pm 0.035	0.139 \pm 0.009	0.449 \pm 0.023	0.863 \pm 0.005	0.197 \pm 0.002	0.443 \pm 0.029	0.788 \pm 0.026	0.177 \pm 0.010	0.556 \pm 0.030	0.862 \pm 0.023
MIRACLE	0.180 \pm 0.001	0.483 \pm 0.010	0.753 \pm 0.018	0.124 \pm 0.008	0.451 \pm 0.017	0.800 \pm 0.028	0.180 \pm 0.002	0.457 \pm 0.023	0.789 \pm 0.030	0.164 \pm 0.011	0.556 \pm 0.032	0.790 \pm 0.062
ReMasker	0.243 \pm 0.023	0.451 \pm 0.010	0.874 \pm 0.040	0.177 \pm 0.007	0.449 \pm 0.032	0.838 \pm 0.038	0.283 \pm 0.011	0.450 \pm 0.035	0.733 \pm 0.046	0.232 \pm 0.015	0.536 \pm 0.032	0.928 \pm 0.012
U-VAE	0.169 \pm 0.001	0.638 \pm 0.002	0.924 \pm 0.033	0.123 \pm 0.009	0.613 \pm 0.011	0.850 \pm 0.024	0.170 \pm 0.001	0.625 \pm 0.007	0.914 \pm 0.037	0.155 \pm 0.009	0.663 \pm 0.011	0.933 \pm 0.039

Table 19: **Imputation utility** on the **kings** dataset at 0.3 missingness. The means and their standard errors across 5 random seeds are reported. \uparrow (\downarrow) denotes that higher (lower) is better.

redwine												
model	MCAR			MAR			MNARL			MNARQ		
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑
Mean	0.098 \pm 0.001	0.432 \pm 0.019	0.680 \pm 0.033	0.092 \pm 0.002	0.413 \pm 0.041	0.475 \pm 0.093	0.099 \pm 0.002	0.370 \pm 0.011	0.507 \pm 0.085	0.090 \pm 0.001	0.498 \pm 0.037	0.719 \pm 0.044
kNNI	0.090 \pm 0.001	0.434 \pm 0.016	0.713 \pm 0.055	0.084 \pm 0.001	0.443 \pm 0.048	0.601 \pm 0.075	0.093 \pm 0.001	0.396 \pm 0.012	0.545 \pm 0.059	0.082 \pm 0.001	0.497 \pm 0.038	0.723 \pm 0.040
MICE	0.087 \pm 0.001	0.438 \pm 0.016	0.788 \pm 0.025	0.083 \pm 0.001	0.438 \pm 0.044	0.659 \pm 0.072	0.088 \pm 0.001	0.390 \pm 0.011	0.592 \pm 0.075	0.084 \pm 0.001	0.505 \pm 0.035	0.822 \pm 0.024
missForest	0.090 \pm 0.001	0.444 \pm 0.014	0.698 \pm 0.048	0.087 \pm 0.001	0.423 \pm 0.043	0.511 \pm 0.055	0.092 \pm 0.001	0.380 \pm 0.017	0.501 \pm 0.087	0.085 \pm 0.001	0.500 \pm 0.031	0.762 \pm 0.030
GAIN	0.095 \pm 0.001	0.500 \pm 0.008	0.750 \pm 0.047	0.089 \pm 0.001	0.515 \pm 0.023	0.507 \pm 0.085	0.099 \pm 0.002	0.483 \pm 0.010	0.625 \pm 0.030	0.088 \pm 0.001	0.543 \pm 0.021	0.706 \pm 0.047
VAEAC	0.087 \pm 0.001	0.595 \pm 0.014	0.775 \pm 0.031	0.088 \pm 0.001	0.587 \pm 0.013	0.620 \pm 0.046	0.088 \pm 0.001	0.594 \pm 0.007	0.568 \pm 0.035	0.083 \pm 0.001	0.597 \pm 0.011	0.734 \pm 0.033
MIWAE	0.091 \pm 0.001	0.527 \pm 0.013	0.598 \pm 0.086	0.087 \pm 0.000	0.509 \pm 0.032	0.549 \pm 0.038	0.093 \pm 0.001	0.476 \pm 0.017	0.493 \pm 0.082	0.085 \pm 0.001	0.554 \pm 0.017	0.561 \pm 0.067
not-MIWAE	0.098 \pm 0.002	0.428 \pm 0.014	0.582 \pm 0.081	0.092 \pm 0.001	0.414 \pm 0.036	0.474 \pm 0.096	0.099 \pm 0.002	0.374 \pm 0.020	0.466 \pm 0.081	0.090 \pm 0.001	0.486 \pm 0.037	0.714 \pm 0.047
MIRACLE	0.093 \pm 0.004	0.445 \pm 0.016	0.649 \pm 0.049	0.087 \pm 0.003	0.443 \pm 0.035	0.483 \pm 0.109	0.095 \pm 0.004	0.408 \pm 0.020	0.393 \pm 0.098	0.086 \pm 0.003	0.495 \pm 0.038	0.683 \pm 0.035
ReMasker	0.109 \pm 0.003	0.419 \pm 0.029	0.627 \pm 0.036	0.107 \pm 0.004	0.396 \pm 0.050	0.393 \pm 0.076	0.119 \pm 0.002	0.380 \pm 0.033	0.421 \pm 0.099	0.099 \pm 0.005	0.486 \pm 0.045	0.728 \pm 0.048
U-VAE	0.088 \pm 0.002	0.605 \pm 0.011	0.793 \pm 0.029	0.081 \pm 0.001	0.574 \pm 0.037	0.664 \pm 0.086	0.087 \pm 0.002	0.564 \pm 0.011	0.690 \pm 0.068	0.079 \pm 0.001	0.648 \pm 0.019	0.828 \pm 0.049

Table 20: **Imputation utility** on the **redwine** dataset at 0.3 missingness. The means and their standard errors across 5 random seeds are reported. \uparrow (\downarrow) denotes that higher (lower) is better.

shoppers												
model	MCAR			MAR			MNARL			MNARQ		
	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑	SMAPE ↓	Acc ↑	Feat ↑
Mean	0.624 \pm 0.006	0.874 \pm 0.004	0.802 \pm 0.017	0.518 \pm 0.018	0.873 \pm 0.004	0.763 \pm 0.017	0.618 \pm 0.008	0.873 \pm 0.004	0.761 \pm 0.030	0.528 \pm 0.015	0.882 \pm 0.005	0.807 \pm 0.014
kNNI	0.572 \pm 0.002	0.876 \pm 0.004	0.755 \pm 0.013	0.485 \pm 0.018	0.875 \pm 0.004	0.756 \pm 0.009	0.570 \pm 0.003	0.875 \pm 0.004	0.750 \pm 0.011	0.490 \pm 0.015	0.882 \pm 0.005	0.796 \pm 0.014
MICE	0.616 \pm 0.002	0.855 \pm 0.005	0.739 \pm 0.006	0.504 \pm 0.020	0.879 \pm 0.004	0.745 \pm 0.014	0.617 \pm 0.003	0.863 \pm 0.006	0.737 \pm 0.013	0.504 \pm 0.015	0.880 \pm 0.005	0.770 \pm 0.010
missForest	0.581 \pm 0.002	0.877 \pm 0.004	0.734 \pm 0.014	0.493 \pm 0.017	0.877 \pm 0.004	0.747 \pm 0.014	0.581 \pm 0.002	0.878 \pm 0.003	0.726 \pm 0.020	0.511 \pm 0.016	0.883 \pm 0.005	0.791 \pm 0.013
GAIN	0.582 \pm 0.007	0.876 \pm 0.002	0.722 \pm 0.017	0.513 \pm 0.021	0.879 \pm 0.005	0.730 \pm 0.023	0.585 \pm 0.005	0.871 \pm 0.003	0.713 \pm 0.021	0.515 \pm 0.012	0.880 \pm 0.004	0.779 \pm 0.010
VAEAC	0.573 \pm 0.002	0.888 \pm 0.003	0.759 \pm 0.014	0.499 \pm 0.019	0.888 \pm 0.003	0.762 \pm 0.010	0.569 \pm 0.003	0.886 \pm 0.004	0.729 \pm 0.011	0.498 \pm 0.015	0.889 \pm 0.003	0.799 \pm 0.017
MIWAE	0.562 \pm 0.001	0.841 \pm 0.004	0.744 \pm 0.021	0.517 \pm 0.009	0.782 \pm 0.011	0.743 \pm 0.030	0.564 \pm 0.003	0.783 \pm 0.012	0.714 \pm 0.030	0.530 \pm 0.002	0.862 \pm 0.013	0.760 \pm 0.020
not-MIWAE	0.566 \pm 0.005	0.874 \pm 0.004	0.778 \pm 0.015	0.529 \pm 0.012	0.836 \pm 0.022	0.720 \pm 0.031	0.581 \pm 0.005	0.874 \pm 0.004	0.726 \pm 0.020	0.535 \pm 0.008	0.877 \pm 0.006	0.763 \pm 0.032
MIRACLE	0.563 \pm 0.007	0.877 \pm 0.004	0.744 \pm 0.011	0.470 \pm 0.015	0.878 \pm 0.003	0.711 \pm 0.017	0.564 \pm 0.007	0.878 \pm 0.004	0.729 \pm 0.018	0.484 \pm 0.015	0.882 \pm 0.005	0.794 \pm 0.016
ReMasker	0.596 \pm 0.018	0.865 \pm 0.007	0.765 \pm 0.014	0.553 \pm 0.018	0.865 \pm 0.004	0.762 \pm 0.019	0.863 \pm 0.003	0.757 \pm 0.018	0.558 \pm 0.018	0.880 \pm 0.005	0.789 \pm 0.009	
U-VAE	0.562 \pm 0.002	0.896 \pm 0.004	0.783 \pm 0.021	0.5								

Q2: Multiple imputation We report Bias, Coverage, and Width results to assess multiple imputation for each dataset at 0.3 missingness rate.

anuran													
model	MCAR			MAR			MNARL			MNARQ			
	Bias↓	Coverage	Width↓										
MICE	0.009(1.4)	0.955±0.000	0.025±0.000	0.006(1.0)	0.936±0.011	0.025±0.000	0.010(1.6)	0.900±0.017	0.025±0.000	0.006(0.9)	0.955±0.014	0.025±0.000	
GAIN	0.027(5.3)	0.373±0.030	0.025±0.000	0.026(5.4)	0.536±0.039	0.025±0.000	0.044(8.6)	0.327±0.072	0.025±0.000	0.013(2.7)	0.682±0.043	0.025±0.000	
VAEAC	0.012(2.0)	0.809±0.022	0.025±0.000	0.011(1.9)	0.736±0.017	0.025±0.000	0.013(2.2)	0.745±0.023	0.025±0.000	0.007(1.2)	0.836±0.018	0.025±0.000	
MIWAE	0.003(0.5)	0.955±0.000	0.026±0.000	0.002(0.4)	0.973±0.011	0.026±0.000	0.003(0.6)	0.964±0.009	0.026±0.000	0.002(0.3)	0.973±0.011	0.026±0.000	
not-MIWAE	0.005(1.0)	0.927±0.023	0.027±0.000	0.007(1.3)	0.845±0.031	0.026±0.000	0.009(1.7)	0.791±0.042	0.027±0.000	0.003(0.7)	0.964±0.009	0.026±0.000	
U-VAE	0.013(5.3)	0.927±0.031	0.027±0.000	0.005(0.8)	0.964±0.017	0.026±0.000	0.006(1.0)	0.918±0.027	0.026±0.000	0.003(0.5)	0.973±0.011	0.026±0.000	

Table 23: **Multiple imputation performance** on the anuran dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

banknote													
model	MCAR			MAR			MNARL			MNARQ			
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	
MICE	0.018(3.6)	0.750±0.079	0.059±0.000	0.013(2.3)	0.850±0.061	0.059±0.000	0.018(3.5)	0.850±0.061	0.059±0.000	0.008(1.5)	0.900±0.061	0.059±0.000	
GAIN	0.040(8.2)	0.450±0.094	0.059±0.000	0.040(7.3)	0.550±0.122	0.058±0.000	0.056(12.0)	0.350±0.127	0.058±0.000	0.018(3.3)	0.800±0.094	0.058±0.000	
VAEAC	0.013(2.5)	0.950±0.050	0.059±0.000	0.016(3.0)	0.850±0.061	0.059±0.000	0.012(2.3)	1.000±0.000	0.059±0.000	0.005(1.1)	1.000±0.000	0.059±0.000	
MIWAE	0.007(1.4)	1.000±0.000	0.061±0.000	0.004(0.8)	1.000±0.000	0.060±0.000	0.007(1.3)	1.000±0.000	0.062±0.000	0.004(0.7)	1.000±0.000	0.060±0.000	
not-MIWAE	0.020(4.0)	0.850±0.061	0.063±0.000	0.013(2.5)	0.950±0.050	0.062±0.000	0.020(4.0)	0.750±0.079	0.062±0.000	0.015(3.0)	0.850±0.100	0.061±0.001	
U-VAE	0.007(1.5)	1.000±0.000	0.062±0.000	0.007(1.4)	1.000±0.000	0.061±0.000	0.010(1.9)	1.000±0.000	0.062±0.000	0.004(0.8)	1.000±0.000	0.060±0.000	

Table 24: **Multiple imputation performance** on the banknote dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

breast													
model	MCAR			MAR			MNARL			MNARQ			
	Bias↓	Coverage	Width↓										
MICE	0.008(2.0)	1.000±0.000	0.090±0.000	0.007(1.6)	1.000±0.000	0.090±0.000	0.009(2.3)	1.000±0.000	0.090±0.000	0.004(1.1)	1.000±0.000	0.090±0.000	
GAIN	0.019(4.7)	0.887±0.000	0.089±0.000	0.018(4.3)	0.900±0.000	0.089±0.000	0.022(5.7)	0.867±0.000	0.089±0.000	0.010(2.5)	0.947±0.000	0.089±0.000	
VAEAC	0.011(2.9)	0.987±0.000	0.090±0.000	0.015(3.7)	0.947±0.000	0.091±0.000	0.012(3.1)	0.993±0.000	0.090±0.000	0.007(1.8)	1.000±0.000	0.090±0.000	
MIWAE	0.009(2.3)	1.000±0.000	0.095±0.000	0.008(2.0)	0.987±0.000	0.093±0.000	0.010(2.6)	1.000±0.000	0.095±0.000	0.007(1.7)	0.993±0.000	0.093±0.000	
not-MIWAE	0.009(2.3)	1.000±0.000	0.095±0.000	0.013(3.1)	0.953±0.000	0.094±0.000	0.009(2.3)	0.993±0.000	0.095±0.000	0.007(1.7)	0.987±0.000	0.094±0.000	
U-VAE	0.007(1.8)	1.000±0.000	0.094±0.000	0.005(1.3)	1.000±0.000	0.093±0.000	0.007(1.8)	1.000±0.000	0.094±0.000	0.005(1.2)	1.000±0.000	0.093±0.000	

Table 25: **Multiple imputation performance** on the breast dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

concrete												
model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓
MICE	0.011(2.2%)	1.000 \pm 0.000	0.068 \pm 0.000	0.008(1.7%)	1.000 \pm 0.000	0.068 \pm 0.000	0.012(2.4%)	0.975 \pm 0.025	0.068 \pm 0.000	0.006(1.3%)	1.000 \pm 0.000	0.068 \pm 0.000
GAIN	0.054(11.7%)	0.475 \pm 0.100	0.067 \pm 0.000	0.031(7.0%)	0.675 \pm 0.151	0.068 \pm 0.000	0.032(7.2%)	0.625 \pm 0.056	0.068 \pm 0.000	0.012(2.9%)	0.950 \pm 0.031	0.068 \pm 0.000
VAEAC	0.013(2.7%)	0.975 \pm 0.025	0.069 \pm 0.000	0.016(3.3%)	0.850 \pm 0.025	0.069 \pm 0.000	0.014(2.9%)	0.950 \pm 0.031	0.069 \pm 0.000	0.007(1.5%)	1.000 \pm 0.000	0.068 \pm 0.000
MIWAE	0.012(2.5%)	0.975 \pm 0.025	0.073 \pm 0.000	0.008(1.6%)	1.000 \pm 0.000	0.072 \pm 0.000	0.011(2.2%)	0.950 \pm 0.031	0.073 \pm 0.000	0.006(1.1%)	1.000 \pm 0.000	0.071 \pm 0.000
not-MIWAE	0.017(3.5%)	0.900 \pm 0.047	0.075 \pm 0.001	0.013(2.8%)	0.925 \pm 0.031	0.074 \pm 0.001	0.019(4.1%)	0.900 \pm 0.047	0.074 \pm 0.001	0.010(2.0%)	0.950 \pm 0.031	0.073 \pm 0.001
U-VAE	0.010(2.2%)	1.000 \pm 0.000	0.073 \pm 0.000	0.011(2.5%)	0.925 \pm 0.050	0.072 \pm 0.000	0.015(3.3%)	0.950 \pm 0.031	0.073 \pm 0.000	0.007(1.5%)	1.000 \pm 0.000	0.071 \pm 0.000

Table 26: **Multiple imputation performance** on the concrete dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

default												
model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓
MICE	0.023(8.8)	0.443 \pm 0.014	0.012 \pm 0.000	0.017(6.2)	0.614 \pm 0.036	0.011 \pm 0.000	0.025(9.0)	0.343 \pm 0.073	0.012 \pm 0.000	0.012(4.5)	0.671 \pm 0.036	0.011 \pm 0.000
GAIN	0.036(11.7)	0.071 \pm 0.023	0.011 \pm 0.000	0.028(8.7)	0.386 \pm 0.070	0.011 \pm 0.000	0.048(14.2)	0.086 \pm 0.035	0.011 \pm 0.000	0.025(7.1)	0.500 \pm 0.051	0.011 \pm 0.000
VAEAC	0.020(7.4)	0.429 \pm 0.000	0.012 \pm 0.000	0.016(5.9)	0.500 \pm 0.060	0.012 \pm 0.000	0.020(7.6)	0.443 \pm 0.042	0.012 \pm 0.000	0.012(4.3)	0.686 \pm 0.043	0.011 \pm 0.000
MIWAE	0.012(4.6)	0.371 \pm 0.069	0.014 \pm 0.000	0.011(4.1)	0.586 \pm 0.086	0.013 \pm 0.001	0.015(5.7)	0.443 \pm 0.027	0.014 \pm 0.001	0.007(2.9)	0.671 \pm 0.036	0.012 \pm 0.000
not-MIWAE	0.015(5.9)	0.314 \pm 0.043	0.013 \pm 0.000	0.013(5.0)	0.557 \pm 0.042	0.014 \pm 0.001	0.019(6.9)	0.257 \pm 0.080	0.013 \pm 0.000	0.009(3.5)	0.600 \pm 0.036	0.012 \pm 0.000
U-VAE	0.008(3.0)	0.671 \pm 0.077	0.016 \pm 0.001	0.007(2.8)	0.686 \pm 0.058	0.014 \pm 0.001	0.006(2.5)	0.686 \pm 0.048	0.015 \pm 0.001	0.004(1.5)	0.800 \pm 0.042	0.012 \pm 0.000

Table 27: **Multiple imputation performance** on the default dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

loan												
model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓
MICE	0.016(4.1)	0.520 \pm 0.049	0.030 \pm 0.000	0.011(2.5)	0.640 \pm 0.075	0.030 \pm 0.000	0.023(5.7)	0.400 \pm 0.063	0.030 \pm 0.000	0.013(3.5)	0.600 \pm 0.063	0.030 \pm 0.000
GAIN	0.073(18.2)	0.120 \pm 0.120	0.030 \pm 0.000	0.039(9.1)	0.560 \pm 0.040	0.030 \pm 0.000	0.087(21.0)	0.040 \pm 0.040	0.030 \pm 0.000	0.035(8.7)	0.520 \pm 0.049	0.030 \pm 0.000
VAEAC	0.019(4.6)	0.480 \pm 0.120	0.031 \pm 0.000	0.009(2.2)	0.800 \pm 0.063	0.031 \pm 0.000	0.016(4.0)	0.560 \pm 0.147	0.033 \pm 0.002	0.016(4.0)	0.480 \pm 0.080	0.031 \pm 0.000
MIWAE	0.015(3.9)	0.760 \pm 0.040	0.034 \pm 0.001	0.004(1.0)	0.960 \pm 0.040	0.035 \pm 0.001	0.016(4.0)	0.720 \pm 0.049	0.037 \pm 0.001	0.015(3.7)	0.800 \pm 0.063	0.033 \pm 0.001
not-MIWAE	0.015(3.6)	0.680 \pm 0.080	0.039 \pm 0.002	0.011(2.8)	0.720 \pm 0.080	0.035 \pm 0.001	0.015(3.6)	0.720 \pm 0.136	0.039 \pm 0.001	0.014(3.5)	0.800 \pm 0.063	0.036 \pm 0.002
U-VAE	0.005(1.3)	0.960 \pm 0.040	0.034 \pm 0.001	0.004(0.9)	1.000 \pm 0.000	0.035 \pm 0.001	0.006(1.5)	0.960 \pm 0.040	0.036 \pm 0.001	0.005(1.3)	0.960 \pm 0.040	0.034 \pm 0.001

Table 28: **Multiple imputation performance** on the loan dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

letter												
model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓
MICE	0.010(2.1)	0.400 \pm 0.025	0.015 \pm 0.000	0.010(2.1)	0.550 \pm 0.012	0.015 \pm 0.000	0.014(2.9)	0.288 \pm 0.042	0.015 \pm 0.000	0.006(1.3)	0.688 \pm 0.044	0.015 \pm 0.000
GAIN	0.062(13.8)	0.175 \pm 0.023	0.015 \pm 0.000	0.045(10.1)	0.450 \pm 0.070	0.015 \pm 0.000	0.094(22.6)	0.100 \pm 0.025	0.015 \pm 0.000	0.024(5.4)	0.525 \pm 0.015	0.015 \pm 0.000
VAEAC	0.010(2.2)	0.438 \pm 0.048	0.017 \pm 0.001	0.017(3.2)	0.538 \pm 0.032	0.015 \pm 0.000	0.020(4.3)	0.462 \pm 0.051	0.016 \pm 0.001	0.007(1.6)	0.650 \pm 0.042	0.015 \pm 0.000
MIWAE	0.032(6.5)	0.575 \pm 0.091	0.039 \pm 0.006	0.028(6.4)	0.650 \pm 0.042	0.034 \pm 0.009	0.039(7.9)	0.425 \pm 0.031	0.044 \pm 0.008	0.024(4.7)	0.600 \pm 0.108	0.025 \pm 0.004
not-MIWAE	0.031(6.3)	0.475 \pm 0.076	0.039 \pm 0.008	0.039(8.8)	0.550 \pm 0.036	0.031 \pm 0.009	0.043(9.7)	0.312 \pm 0.040	0.033 \pm 0.010	0.021(4.7)	0.638 \pm 0.023	0.021 \pm 0.003
U-VAE	0.011(2.3)	0.650 \pm 0.073	0.021 \pm 0.002	0.018(4.2)	0.675 \pm 0.012	0.030 \pm 0.010	0.024(5.6)	0.600 \pm 0.064	0.022 \pm 0.004	0.006(1.4)	0.762 \pm 0.023	0.019 \pm 0.002

Table 29: **Multiple imputation performance** on the letter dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

kings												
model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓
MICE	0.030 (13.7)	0.345 \pm 0.018	0.014 \pm 0.000	0.014 (6.2)	0.582 \pm 0.046	0.013 \pm 0.000	0.030 (13.6)	0.327 \pm 0.055	0.014 \pm 0.000	0.021 (10.5)	0.582 \pm 0.055	0.014 \pm 0.000
GAIN	0.045 (15.6)	0.036 \pm 0.022	0.013 \pm 0.000	0.032 (9.1)	0.382 \pm 0.034	0.013 \pm 0.000	0.059 (18.9)	0.055 \pm 0.036	0.013 \pm 0.000	0.028 (11.3)	0.382 \pm 0.078	0.013 \pm 0.000
VAEAC	0.027 (12.7)	0.436 \pm 0.034	0.014 \pm 0.000	0.015 (6.1)	0.582 \pm 0.036	0.013 \pm 0.000	0.023 (11.6)	0.509 \pm 0.036	0.014 \pm 0.000	0.022 (10.7)	0.582 \pm 0.022	0.014 \pm 0.000
MIWAE	0.008 (5.9)	0.618 \pm 0.034	0.015 \pm 0.000	0.008 (3.2)	0.855 \pm 0.022	0.015 \pm 0.000	0.012 (6.8)	0.582 \pm 0.079	0.015 \pm 0.000	0.005 (4.1)	0.727 \pm 0.029	0.014 \pm 0.000
not-MIWAE	0.010 (6.1)	0.527 \pm 0.018	0.015 \pm 0.000	0.010 (4.0)	0.673 \pm 0.046	0.014 \pm 0.000	0.014 (6.3)	0.455 \pm 0.029	0.015 \pm 0.000	0.009 (4.4)	0.636 \pm 0.029	0.014 \pm 0.000
U-VAE	0.006 (3.7)	0.655 \pm 0.034	0.014 \pm 0.000	0.007 (2.8)	0.800 \pm 0.045	0.014 \pm 0.000	0.011 (5.5)	0.600 \pm 0.055	0.015 \pm 0.000	0.004 (2.8)	0.745 \pm 0.018	0.014 \pm 0.000

Table 30: **Multiple imputation performance** on the **kings** dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

redwine												
model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓	Bias↓	Coverage	Width↓
MICE	0.021 (5.3)	0.782 \pm 0.022	0.054 \pm 0.000	0.011 (2.7)	0.909 \pm 0.041	0.054 \pm 0.000	0.020 (5.2)	0.745 \pm 0.034	0.054 \pm 0.000	0.014 (3.8)	0.836 \pm 0.034	0.054 \pm 0.000
GAIN	0.035 (8.1)	0.473 \pm 0.067	0.054 \pm 0.000	0.028 (6.2)	0.582 \pm 0.036	0.053 \pm 0.000	0.047 (11.0)	0.291 \pm 0.034	0.054 \pm 0.000	0.018 (4.7)	0.727 \pm 0.081	0.054 \pm 0.000
VAEAC	0.020 (5.2)	0.745 \pm 0.034	0.055 \pm 0.000	0.016 (3.7)	0.764 \pm 0.074	0.054 \pm 0.000	0.021 (5.2)	0.745 \pm 0.067	0.054 \pm 0.000	0.013 (3.5)	0.818 \pm 0.050	0.054 \pm 0.000
MIWAE	0.013 (3.5)	0.891 \pm 0.034	0.061 \pm 0.001	0.008 (2.0)	0.964 \pm 0.022	0.058 \pm 0.001	0.015 (4.1)	0.855 \pm 0.022	0.062 \pm 0.001	0.009 (2.6)	0.927 \pm 0.034	0.059 \pm 0.001
not-MIWAE	0.015 (3.9)	0.927 \pm 0.034	0.062 \pm 0.001	0.009 (2.2)	0.964 \pm 0.022	0.059 \pm 0.000	0.013 (3.4)	0.909 \pm 0.029	0.062 \pm 0.000	0.010 (2.6)	0.891 \pm 0.018	0.059 \pm 0.001
U-VAE	0.009 (2.3)	0.982 \pm 0.018	0.061 \pm 0.000	0.007 (1.7)	1.000 \pm 0.000	0.058 \pm 0.001	0.011 (3.0)	0.982 \pm 0.018	0.060 \pm 0.001	0.007 (1.7)	0.945 \pm 0.022	0.057 \pm 0.000

Table 31: **Multiple imputation performance** on the **redwine** dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

shoppers												
model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓									
MICE	0.041 (17.2)	0.120 \pm 0.037	0.017 \pm 0.000	0.023 (9.5)	0.420 \pm 0.020	0.017 \pm 0.000	0.043 (17.8)	0.060 \pm 0.024	0.018 \pm 0.000	0.020 (9.0)	0.420 \pm 0.049	0.017 \pm 0.000
GAIN	0.048 (17.2)	0.120 \pm 0.037	0.017 \pm 0.000	0.028 (10.8)	0.380 \pm 0.037	0.017 \pm 0.000	0.056 (21.4)	0.014 \pm 0.014	0.017 \pm 0.000	0.025 (11.5)	0.429 \pm 0.047	0.017 \pm 0.000
VAEAC	0.029 (12.9)	0.260 \pm 0.051	0.017 \pm 0.000	0.023 (9.5)	0.420 \pm 0.049	0.017 \pm 0.000	0.029 (12.5)	0.320 \pm 0.037	0.017 \pm 0.000	0.016 (7.6)	0.560 \pm 0.024	0.017 \pm 0.000
MIWAE	0.014 (7.6)	0.440 \pm 0.075	0.018 \pm 0.000	0.010 (4.6)	0.560 \pm 0.075	0.018 \pm 0.000	0.015 (7.9)	0.420 \pm 0.086	0.018 \pm 0.000	0.008 (4.9)	0.640 \pm 0.024	0.017 \pm 0.000
not-MIWAE	0.018 (7.7)	0.220 \pm 0.058	0.019 \pm 0.000	0.013 (5.2)	0.480 \pm 0.020	0.019 \pm 0.001	0.018 (7.3)	0.260 \pm 0.068	0.020 \pm 0.001	0.013 (5.0)	0.440 \pm 0.040	0.018 \pm 0.000
U-VAE	0.006 (3.0)	0.760 \pm 0.103	0.018 \pm 0.000	0.006 (2.4)	0.800 \pm 0.045	0.018 \pm 0.000	0.008 (3.5)	0.660 \pm 0.040	0.018 \pm 0.000	0.004 (1.9)	0.860 \pm 0.024	0.018 \pm 0.000

Table 32: **Multiple imputation performance** on the **shoppers** dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

whitewine												
model	MCAR			MAR			MNARL			MNARQ		
	Bias↓	Coverage	Width↓									
MICE	0.013 (2.8)	0.745 \pm 0.045	0.031 \pm 0.000	0.010 (2.2)	0.800 \pm 0.045	0.031 \pm 0.000	0.014 (3.0)	0.655 \pm 0.060	0.031 \pm 0.000	0.008 (1.8)	0.782 \pm 0.022	0.031 \pm 0.000
GAIN	0.041 (8.8)	0.273 \pm 0.064	0.031 \pm 0.000	0.035 (7.3)	0.491 \pm 0.084	0.031 \pm 0.000	0.040 (8.5)	0.309 \pm 0.062	0.031 \pm 0.000	0.014 (3.0)	0.673 \pm 0.102	0.031 \pm 0.000
VAEAC	0.013 (3.0)	0.673 \pm 0.062	0.032 \pm 0.000	0.013 (2.7)	0.673 \pm 0.036	0.031 \pm 0.000	0.014 (3.1)	0.636 \pm 0.057	0.031 \pm 0.000	0.010 (2.2)	0.709 \pm 0.045	0.031 \pm 0.000
MIWAE	0.009 (2.0)	0.873 \pm 0.036	0.037 \pm 0.001	0.008 (1.7)	0.891 \pm 0.034	0.035 \pm 0.001	0.010 (2.2)	0.891 \pm 0.045	0.038 \pm 0.001	0.006 (1.3)	0.909 \pm 0.029	0.034 \pm 0.001
not-MIWAE	0.010 (2.2)	0.891 \pm 0.045	0.037 \pm 0.001	0.009 (1.9)	0.873 \pm 0.062	0.035 \pm 0.001	0.012 (2.8)	0.764 \pm 0.062	0.040 \pm 0.001	0.007 (1.6)	0.836 \pm 0.018	0.036 \pm 0.001
U-VAE	0.011 (2.4)	0.927 \pm 0.053	0.039 \pm 0.001	0.006 (1.4)	0.927 \pm 0.018	0.035 \pm 0.001	0.010 (2.2)	0.873 \pm 0.022	0.037 \pm 0.001	0.005 (1.1)	0.945 \pm 0.022	0.036 \pm 0.001

Table 33: **Multiple imputation performance** on the **whitewine** dataset at 0.3 missingness rate. The means and their standard errors across 5 random seeds are reported. The values in parentheses next to Bias represent the percent bias. ↓ denotes lower is better. Coverage close to 0.95 indicates better performance.

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