Last Time:

- How to Land a Space Ship

Today:

- How to walk

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* History

  - First legged robots built in ~1960s
  - Serious research started in ~1980s

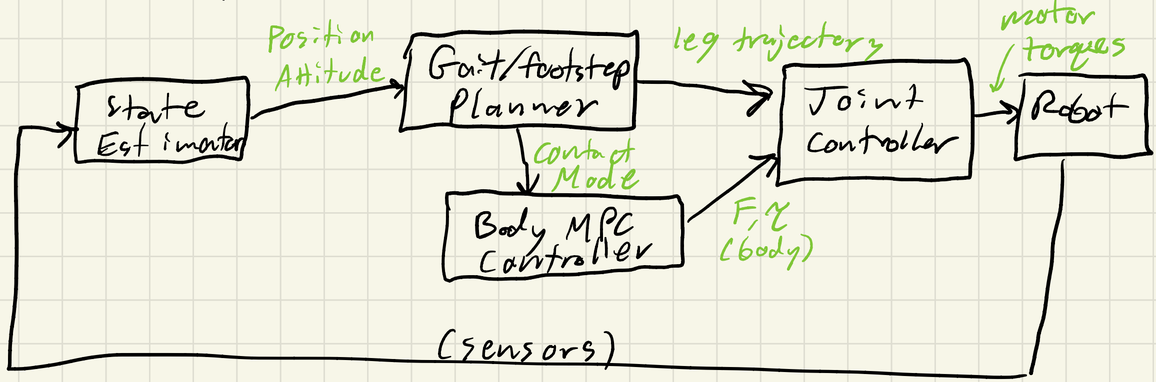
  - Different Approaches:

    Honda/Waseda: Humanoids. Control based on industrial manipulator ideas.

    Raibert/CMU/MIT: Hoppers. Control focused on floating-base dynamics

- Past ~10 years: lots of work on mechanical design (series elasticity, direct drive) and MPC for quadrupeds (ANYmal, Cheetah, Spot).

* The "Full Stack":




- Sensors: Joint encoders, IMU, contact force, vision/GPS.

- State Estimator: Some kind of EKF with extra tricks for reasoning about contact + foot slip.

- Gait/Footstep Planner: plans foot placement + swing-leg trajectories based on pre-specified gait + desired body velocity.

- MPC Controller: Treats the robot as a single rigid body and assumes given foot positions to compute contact forces.

- Joint Controller: Generates motor torque commands to produce desired forces from MPC + desired swing-leg trajectories from gait.

# * Legged Robot Dynamics:

$$x = \begin{bmatrix} q \\ v \end{bmatrix}, \quad q = \begin{bmatrix} \text{body position} \\ \text{body attitude} \\ \text{joint angles} \end{bmatrix}, \quad v = \dot{q}$$

$$\underbrace{M(q)}_{\text{Mass Matrix}}\dot{v} + \underbrace{C(q,v)}_{\text{Coriolis + Gravity}} = \underbrace{B(q)}_{\text{Input Jacobian}}\underbrace{u}_{\text{Joint torques}} + \underbrace{J(q)^T}_{\text{contact Jacobian}}\underbrace{f}_{\text{foot contact forces}}$$

$$\underbrace{\phi(q)}_{\text{signed-distance function}} \geq 0 \quad \leftarrow \text{Interpenetration constraint}$$

$$\underbrace{\|f^{2:3}\|}_{\text{tangential force}} \leq \underbrace{\mu}_{\text{friction coefficient}}\underbrace{f^1}_{\text{normal force}} \quad \leftarrow \text{Friction cone}$$

- Very messy for online control
- State dimension is large ($\sim 36$). TrajOpt scales like $n^3$
- Highly nonlinear

* Centroidal Dynamics

- Assumptions :

1) Leg mass/inertia << body (~10%)

2) Leg actuators are very fast compared to body motion

⇒ Use a lumped single rigid body model for the whole robot :

$$\underbrace{m}_{\text{total mass}}\dot{v} = -mg + \underbrace{\sum_i f_i}_{\text{sum of all contact forces}}$$

$$\underbrace{J}_{\text{total inertia}}\dot{\omega} + \underbrace{\omega \times J\omega}_{\text{coriolis}} = \overbrace{\sum_i \underbrace{r_i}_{\text{foot positions}} \times f_i}^{\text{total torque}}$$

$$\underbrace{\dot{r} = v \quad , \quad \dot{q} = \tfrac{1}{2} q * \omega}_{\text{Kinematics}}$$

- With a few more assumptions we can get a linear model:

3) body angular velocities are small

4) Body pitch + roll are small

5) Foot positions track reference (almost) exactly

$$\Downarrow$$

$$m\dot{v} = -mg + \sum_i f_i$$

$$J\dot{\omega} = \sum_i \underbrace{R_i}_{\hat{r}_i \text{ cross-product matrix using reference footsteps/ gait.}} f_i$$

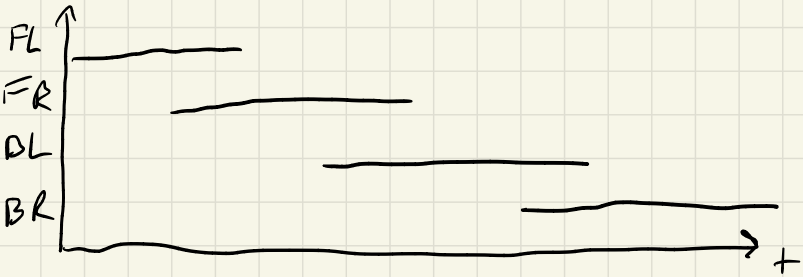
$$\dot{r} = v$$

$$\underbrace{\dot{\theta}}_{\approx \text{ roll, pitch, yaw angles}} = \omega$$

$$\underbrace{\begin{bmatrix} \dot{r} \\ \dot{\theta} \\ \dot{v} \\ \dot{\omega} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} & & 0 & \\ \hline I & I & I & I \\ R_1 & R_2 & R_3 & R_4 \end{bmatrix}}_{B} \underbrace{f}_{u}$$

## * Footstep / Gait Planner

- Specify foot contact mode timing




- Says when to put the feet down.
- Where to put the feet down?
  - standard technique: "Raibert Heuristic"

gait period

$$r_{foot} = r_{hip} + \frac{\Delta t}{2} \underbrace{V_{body}}_{\text{desired body velocity}}$$

Positions projected on the ground

- How to swing the leg between stance phases?

- Hand-designed spline curve tracked with joint-space PD control:

$$\tau = \underbrace{J_{swing}^T}_{\text{swing-leg Jacobian}} \left( K_p (r - r_{ref}) + K_d (\dot{r} - \dot{r}_{ref}) \right)$$

joint torque

foot position

- We assume this tracking is very good in the MPC controller

* MPC Controller:

  - Try to track desired body motion
  - Horizon is typically 1~2 gait periods
  - With linearized centroidal dynamics this is a convex problem:

$$\min_{\substack{x_{1:N} \\ u_{1:N-1}}} \sum_{n=1}^{N-1} \frac{1}{2}(x_n - \bar{x}_n)^T Q (x_n - \bar{x}_n) + \frac{1}{2}(u_n + \bar{u})^T R(\cdots)$$

$$\text{s.t.} \quad x_{n+1} = A x_n + B_n u_n$$

$$\underbrace{\| u_n^{2:3} \|_2}_{\text{tangential}} \leq \mu \underbrace{u_n^1}_{\text{normal}}$$

  - Friction cone is usually linearized to make this a QP
  - Difficult to strictly enforce torque limits.
  - Output is desired foot forces $u = f$
  - Joint torques can be calculated using Jacobian:

$$\tau = \underbrace{J_{(q)}^T}_{\text{stance feet}} f$$

- Many extra hacks/extensions to handle unexpected contact events, foot slip, etc.