Last Time:

- Robust Control
- Minimax DDP

Today:

- How to land a space ship

---

* The Rocket Soft-Landing Problem

- Go from some initial state x_0 to some final position r_f with $z_f = 0$ and $v_f = 0$

  ("Landing" $\Rightarrow z_f = 0$, "soft" $\Rightarrow v_f = 0$)

- Minimize some combination of fuel consumption and landing position error $\|r_f - r_g\|$

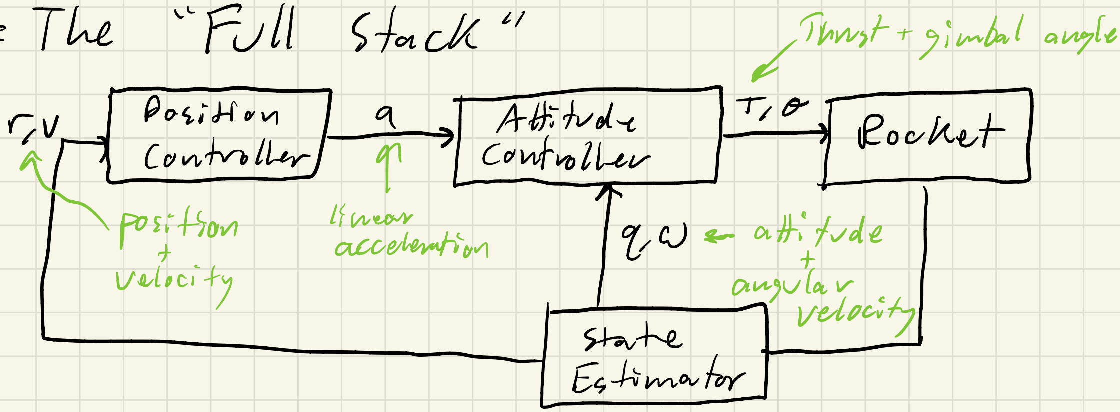
- Respect thrust limits + safety constraints

* Examples:

  - NASA Curiosity "Sky Crane" (2012)
  - SpaceX Falcon 9 + Starship
  - NASA Perseverance w/ TRN (2021)

  - Lots of tricks to make this work in practice.

* The "Full Stack"



- State Estimation:

  SpaceX: GPS + IMU, with good filtering accurate to < 1 m position, < 1 cm/sec velocity, < 1 deg attitude. Enables precision landing.

  Mars: No GPS. IMU + Radar Altimiter + Vision. ~ 30 meter accuracy. Avoid Boulders.

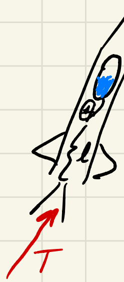
- Decoupled Control Loop:

  High-Level Position Controller: Uses a point-mass model. Reasons about safety, thrust and fuel constraints. Generates acceleration/thrust commands. Runs at ~ Hz.

  Low-Level Attitude Controller: Reasons about attitude, flexible modes, fluid slosh, generate thrust + gimble commands to track desired acceleration. Runs at 10s of Hz.

* Rocket Dynamics

- Rigid-Body Model:



Position controller (slow)

$$\dot{v} = -g + \frac{T}{m}$$ ← point mass

$$\dot{m} = -\alpha T$$ ← fuel burn

attitude controller (fast)

$$J\dot{\omega} + \omega \times J\omega = l \times T$$ ← attitude

- Fuel can be 80%+ of initial vehicle mass. Total mass can change by 2-5x. Have to account for this.

- Fluid slosh: Highly nonlinear, hard to model. Standard model:

← pendulum with l_p and m_p fit to data.

- Flexible modes: Rockets are built to be very light ⇒ not stiff ⇒ low-frequency bending modes. First bending mode ~Hz. Dealt with by adding notch filter to the attitude controller at the bending frequency to avoid exciting it.

- Aerodynamics Forces: Mostly ignored. Velocity constraints in the position controller can make sure these aren't too big.

- Lots of model uncertainty. Linear robust control ideas (e.g. H_∞ loop shaping) are used in the attitude controller.

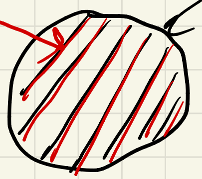
---

* Background: Convex Relaxation

- Sometimes we can have a non-convex constraint that can be expressed as the boundary of a larger convex set:

Enlarged set

$$S_2 = \{x \mid \|x\| \leq 1\}$$



original set S_1

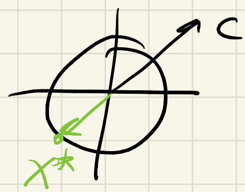
$$S_1 = \{x \mid \|x\| = 1\}$$

$$S_1 = \partial S_2$$

- Replacing the original constraint with the larger convex one is called "convex relaxation"

- Sometimes if the cost is "nice" we can still get the answer to the original problem by solving the relaxed version:

$$\min \ c^T x$$
$$\text{s.t.} \quad \|x\| = 1$$
$$\Downarrow$$
$$\|x\| \leq 1$$



- When this happens we call it a "tight relaxation"

* Convex Relaxation of Thrust Constraints

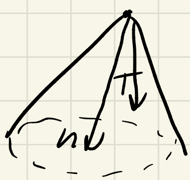
- Maximum thrust constraint:

$$T \in \mathbb{R}^3 \ , \ \|T\| \leq T_{max}$$

(convex)

- Thrust angle constraint:

$$n^T \frac{T}{\|T\|} \leq \cos(\theta_{max})$$

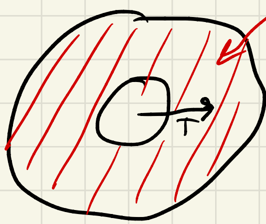


(convex)

- Rocket engine also has minimum thrust constraint once turned on:

$$T_{min} \leq \|T\| \leq T_{max}$$

Non Convex!



- Let's add a new "slack variable" $\Gamma \in \mathbb{R}$ that equals the thrust magnitude!

(1) $\|T\| = \Gamma$ ← Boundary of a convex set (sphere)

(2) $T_{min} \leq \Gamma \leq T_{max}$

(3) $n^T T \leq \Gamma \cos(\theta_{max})$

convex

- Now we can convexify the constraints by relaxing (1):

(1) $\|T\| \leq \Gamma$

(2) $T_{min} \leq \Gamma \leq T_{max}$

(3) $n^T T \leq \Gamma \cos(\theta_{max})$

All convex!

- The paper proves that this relaxation is tight using Pontryagin's minimum principle.