16-745: Optimal Control and Reinforcement Learning

Spring 2023

Course Description

This is a course about how to make robots move through and interact with their environment with speed, efficiency, and robustness. We will survey a broad range of topics from nonlinear dynamics, linear systems theory, classical optimal control, numerical optimization, state estimation, system identification, and reinforcement learning. The goal is to provide students with hands-on experience applying each of these ideas to a variety of robotic systems so that they can use them in their own research.

Prerequisites: Strong linear algebra skills, experience with a high-level programming language like Python, MATLAB, or Julia, and basic familiarity with ordinary differential equations.

Instructors

Prof. Zac Manchester

TA: Kevin Tracy

TA: Jeong Hun (JJ) Lee

TA: Swaminathan (Swami) Gurumurthy

TA: Mrinal Verghese

Email: zacm@cmu.edu

Email: ktracy@cmu.edu

Email: jeonghunlee@cmu.edu

Email: sgurumur@andrew.cmu.edu

Email: mverghes@andrew.cmu.edu

Logistics

- Lectures will be held Tuesdays and Thursdays 5:00–6:20 PM Eastern time in Room 160 in Hall of Arts. Lectures will also be live streamed on zoom and recorded for later viewing.
- Office hours will be TODO: based on survey.
- Homework assignments will be due by 11:59 PM Eastern time on Wednesdays. Two weeks will be given to complete each assignment.
- GitHub will be used to distribute assignments and GradeScope will be used for submissions.
- Piazza will be used for general discussion and Q&A outside of class and office hours.
- There will be no exams. Instead, students will form groups of up to five to complete a project on a topic of their choice.

Learning Objectives

By the end of this course, students should be able to do the following:

- 1. Analyze the stability of dynamical systems
- 2. Design LQR controllers that stabilize equilibria and trajectories
- 3. Use offline trajectory optimization to design trajectories for nonlinear systems
- 4. Use online convex optimization to implement model-predictive control
- 5. Understand the effects of stochasticity and model uncertainty
- 6. Directly optimize feedback policies when good models are unavailable

Learning Resources

There is no textbook required for this course. Video recordings of lectures and lecture notes will be posted online. Additional references for further reading will be provided with each lecture.

Homework

Four homeworks will be assigned during the semester. Students will have at least two weeks to complete each assignment. All homework will be distributed and collected using GitHub. Solutions and grades will be returned within one week of homework due dates.

Project Guidelines

Students should work in groups of 1–5 to complete a substantial final project. The goal is for students to apply the course content to their own research. Project proposals will be solicited on the first homework assignment, and topics will be selected in consultation with the instructors.

Project grades will be based on a short presentation given during the last week of class and a final report submitted by May 10. Reports should be written in the form of a 6–8 page conference paper using the standard two-column IEEE format.

Grading

Grading will be based on:

- 50% Project
- 40% Homework
- 10% Participation

Attendance during lectures is not required to earn a full participation grade. Students can also participate through any combination of office hours, Piazza discussions, project presentations, and by offering constructive feedback about the course to the instructors.

Course Policies

Late Homework: Students are allowed a budget of 6 late days for turning in homework with no penalty throughout the semester. They may be used together on one assignment, or separately on multiple assignments. Beyond these six days, no other late homework will be accepted.

Accommodations for Students with Disabilities: If you have a disability and are registered with the Office of Disability Resources, I encourage you to use their online system to notify me of your accommodations and discuss your needs with me as early in the semester as possible. I will work with you to ensure that accommodations are provided as appropriate. If you suspect that you may have a disability and would benefit from accommodations but are not yet registered with the Office of Disability Resources, I encourage you to contact them at access@andrew.cmu.edu.

Statement of Support for Students' Health & Well-Being: Take care of yourself. Do your best to maintain a healthy lifestyle this semester by eating well, exercising, avoiding drugs and alcohol, getting enough sleep, and taking some time to relax. This will help you achieve your goals and cope with stress.

If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, we strongly encourage you to seek support. Counseling and Psychological Services (CaPS) is here to help: call 412-268-2922 and visit http://www.cmu.edu/counseling. Consider reaching out to a friend, faculty, or family member you trust for help getting connected to the support that can help.

If you or someone you know is feeling suicidal or in danger of self-harm, call someone immediately, day or night:

CaPS: 412-268-2922

Re:solve Crisis Network: 888-796-8226

If the situation is life threatening, call the police:

On campus: CMU Police: 412-268-2323

Off campus: 911

If you have questions about this or your coursework, please let me know. Thank you, and have a great semester.

Tentative Schedule

Jan 17	Week	Dates	Topics	Assignments
Jan 19 Stability, Discrete-Time Dynamics HW0 Out Jan 24 Optimization Intro Numerical Optimization Pt. 1 HW1 Out Jan 31 Numerical Optimization Pt. 2 & Optimal Control Intro Feb 2 Pontryagin, Shooting Methods, & LQR Intro LQR as a QP & Riccati Equation HW 1 Due Feb 9 No Class HW 2 Out Feb 14 Dynamic Programming & Intro to Convexity Feb 16 Convex Model-Predictive Control Feb 21 Intro to Trajectory Optimization, Iterative LQR, & DDP HW2 Due Feb 23 DDP with Constraints and Free Final Time HW3 Out Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions No Class Mar 9 No Class No Class Mar 14 Optimizing with Attitude Mar 16 LQR with Attitude, Quadrotors, & Contact Intro Mar 21 Trajectory Optimization for Hybrid Systems Mar 23 Data-Driven Methods & Iterative Learning Control Mar 24 RL from an Optimal Control & LQG HW4 Due Robust Control & Minimax DDP Apr 4 RL from an Optimal Control Perspective Practical Tips & Tricks, Control History Apr 10 Apr 11 Case Study: How to Drive a Car Case Study: How to Walk Project Presentations	1	Jan 17	Course Overview, & Dynamics Intro	Survey
Jan 26		Jan 19		
Jan 26 Numerical Optimization Pt. 1 HW1 Out Jan 31 Numerical Optimization Pt. 2 & Optimal Control Intro Pontryagin, Shooting Methods, & LQR Intro Feb 7 LQR as a QP & Riccati Equation HW 1 Due Feb 9 No Class Feb 14 Dynamic Programming & Intro to Convexity Feb 16 Convex Model-Predictive Control Feb 23 DDP with Constraints and Free Final Time HW3 Out Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions Mar 9 No Class Mar 9 No Class Mar 9 No Class Mar 14 Optimizing with Attitude Mar 16 LQR with Attitude, Quadrotors, & Contact Intro Mar 21 Trajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control Mar 23 Data-Driven Methods & Iterative Learning Control Mar 30 Robust Control & Minimax DDP Apr 4 RL from an Optimal Control Perspective Practical Tips & Tricks, Control History Apr 11 Case Study: How to Land a Rocket Apr 20 Case Study: How to Drive a Car Case Study: How to Drive a Car Case Study: How to Walk Project Presentations	2	Jan 24	Optimization Intro	HW0 Due
Feb 2 Pontryagin, Shooting Methods, & LQR Intro 4 Feb 7 LQR as a QP & Riccati Equation HW 1 Due Feb 9 No Class HW 2 Out 5 Feb 14 Dynamic Programming & Intro to Convexity Feb 16 Convex Model-Predictive Control 6 Feb 21 Intro to Trajectory Optimization, Iterative LQR, & DDP HW2 Due Feb 23 DDP with Constraints and Free Final Time HW3 Out 7 Feb 28 Direct Trajectory Optimization, Collocation, & SQP Attitude Intro: SO(3) & Quaternions 8 Mar 7 No Class 9 Mar 14 Optimizing with Attitude HW3 Due Mar 16 LQR with Attitude, Quadrotors, & Contact Intro HW4 Out 10 Mar 21 Trajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control 11 Mar 28 Stochastic Optimal Control & LQG HW4 Due Mar 30 Robust Control & Minimax DDP 12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 20 Case Study: How to Drive a Car Case Study: How to Walk 14 Apr 25 Project Presentations		Jan 26	Numerical Optimization Pt. 1	HW1 Out
Feb 2 Pontryagin, Shooting Methods, & LQR Intro 4 Feb 7 LQR as a QP & Riccati Equation HW 1 Due Feb 9 No Class HW 2 Out 5 Feb 14 Dynamic Programming & Intro to Convexity Convex Model-Predictive Control 6 Feb 21 Intro to Trajectory Optimization, Iterative LQR, & DDP HW2 Due Feb 23 DDP with Constraints and Free Final Time HW3 Out 7 Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions 8 Mar 7 No Class 9 Mar 14 Optimizing with Attitude HW3 Due Mar 16 LQR with Attitude, Quadrotors, & Contact Intro HW4 Out 10 Mar 21 Trajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control 11 Mar 28 Stochastic Optimal Control & LQG HW4 Due Mar 30 Robust Control & Minimax DDP 12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 13 Case Study: How to Drive a Car Case Study: How to Walk 14 Apr 25 Project Presentations	3	Jan 31	Numerical Optimization Pt. 2 & Optimal Control Intro	
Feb 9 Feb 14 Dynamic Programming & Intro to Convexity Feb 16 Convex Model-Predictive Control Feb 21 Intro to Trajectory Optimization, Iterative LQR, & DDP Feb 23 DDP with Constraints and Free Final Time HW3 Out Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions No Class No Class No Class Mar 9 Mar 14 Optimizing with Attitude HW3 Due HW4 Out Arrajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control Mar 23 Data-Driven Methods & Iterative Learning Control Mar 30 Robust Control & Minimax DDP Apr 4 Apr 6 Practical Tips & Tricks, Control History Apr 13 Apr 11 Case Study: How to Land a Rocket Apr 20 Case Study: How to Drive a Car Case Study: How to Walk Project Presentations		Feb 2	Pontryagin, Shooting Methods, & LQR Intro	
Feb 9 Feb 14 Feb 16 Feb 16 Convex Model-Predictive Control 6 Feb 21 Intro to Trajectory Optimization, Iterative LQR, & DDP Feb 23 DDP with Constraints and Free Final Time HW3 Out 7 Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions 8 Mar 7 No Class No Class No Class 9 Mar 14 Optimizing with Attitude HW3 Due HW4 Out 10 Mar 21 Trajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control 11 Mar 28 Stochastic Optimal Control & LQG Mar 30 Robust Control & Minimax DDP 12 Apr 4 Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 20 Case Study: How to Drive a Car Case Study: How to Walk 14 Apr 25 Project Presentations	4	Feb 7	LQR as a QP & Riccati Equation	HW 1 Due
Feb 16		Feb 9	No Class	HW 2 Out
Feb 16 Convex Model-Predictive Control Feb 21 Intro to Trajectory Optimization, Iterative LQR, & DDP HW2 Due Feb 23 DDP with Constraints and Free Final Time HW3 Out Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions No Class Mar 9 No Class Mar 9 No Class Mar 16 LQR with Attitude, Quadrotors, & Contact Intro HW4 Out Mar 21 Trajectory Optimization for Hybrid Systems Mar 23 Data-Driven Methods & Iterative Learning Control Mar 28 Stochastic Optimal Control & LQG HW4 Due Mar 30 Robust Control & Minimax DDP Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History Apr 11 Case Study: How to Land a Rocket Apr 13 No Class 14 Apr 18 Case Study: How to Drive a Car Case Study: How to Walk Project Presentations	5	Feb 14	Dynamic Programming & Intro to Convexity	
Feb 23 DDP with Constraints and Free Final Time HW3 Out Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions Mar 7 No Class Mar 9 Optimizing with Attitude HW3 Due Mar 16 LQR with Attitude, Quadrotors, & Contact Intro HW4 Out Mar 21 Trajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control Mar 23 Data-Driven Methods & Iterative Learning Control Mar 30 Robust Control & Minimax DDP Apr 4 RL from an Optimal Control Perspective Practical Tips & Tricks, Control History Apr 11 Case Study: How to Land a Rocket Apr 13 No Class Apr 18 Case Study: How to Drive a Car Case Study: How to Walk Project Presentations		Feb 16	Convex Model-Predictive Control	
Feb 23 DDP with Constraints and Free Final Time HW3 Out Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions Mar 7 No Class Mar 9 No Class Mar 16 LQR with Attitude, Quadrotors, & Contact Intro HW4 Out Mar 21 Trajectory Optimization for Hybrid Systems Mar 23 Data-Driven Methods & Iterative Learning Control Mar 28 Stochastic Optimal Control & LQG HW4 Due Mar 30 Robust Control & Minimax DDP Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History Apr 13 Case Study: How to Land a Rocket Apr 13 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk Project Presentations	6	Feb 21	Intro to Trajectory Optimization, Iterative LQR, & DDP	HW2 Due
Mar 2 Attitude Intro: SO(3) & Quaternions No Class Mar 9 No Class Mar 14 Optimizing with Attitude Mar 16 LQR with Attitude, Quadrotors, & Contact Intro Mar 21 Trajectory Optimization for Hybrid Systems Mar 23 Data-Driven Methods & Iterative Learning Control Mar 28 Stochastic Optimal Control & LQG HW4 Due Mar 30 Robust Control & Minimax DDP Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History Apr 11 Case Study: How to Land a Rocket No Class 14 Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk Project Presentations		Feb 23	DDP with Constraints and Free Final Time	HW3 Out
Mar 2 Attitude Intro: SO(3) & Quaternions Mar 7	7	Feb 28	Direct Trajectory Optimization, Collocation, & SQP	
Mar 9 Mar 14 Optimizing with Attitude Mar 16 LQR with Attitude, Quadrotors, & Contact Intro Mar 21 Trajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control Mar 28 Stochastic Optimal Control & LQG Mar 30 Robust Control & Minimax DDP Apr 4 Apr 6 Practical Tips & Tricks, Control History Apr 11 Case Study: How to Land a Rocket Apr 13 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk Apr 25 Project Presentations		Mar 2	Attitude Intro: SO(3) & Quaternions	
9 Mar 14 Optimizing with Attitude HW3 Due 10 Mar 21 Trajectory Optimization for Hybrid Systems 10 Mar 23 Data-Driven Methods & Iterative Learning Control 11 Mar 28 Stochastic Optimal Control & LQG HW4 Due 12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 13 No Class 14 Apr 25 Case Study: How to Walk 15 Apr 25 Project Presentations	8	Mar 7	No Class	
Mar 16 LQR with Attitude, Quadrotors, & Contact Intro Mar 21 Trajectory Optimization for Hybrid Systems Mar 23 Data-Driven Methods & Iterative Learning Control Mar 28 Stochastic Optimal Control & LQG Mar 30 Robust Control & Minimax DDP Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History Apr 11 Case Study: How to Land a Rocket Apr 13 No Class Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk Project Presentations		Mar 9	No Class	
Mar 16 LQR with Attitude, Quadrotors, & Contact Intro Mar 21 Trajectory Optimization for Hybrid Systems Mar 23 Data-Driven Methods & Iterative Learning Control Mar 28 Stochastic Optimal Control & LQG Mar 30 Robust Control & Minimax DDP 12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 13 No Class 14 Apr 20 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk Project Presentations	9	Mar 14	Optimizing with Attitude	HW3 Due
10 Mar 23 Data-Driven Methods & Iterative Learning Control 11 Mar 28 Stochastic Optimal Control & LQG HW4 Due Robust Control & Minimax DDP 12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 13 No Class 14 Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk 14 Apr 25 Project Presentations		Mar 16	LQR with Attitude, Quadrotors, & Contact Intro	HW4 Out
Mar 23 Data-Driven Methods & Iterative Learning Control 11 Mar 28 Stochastic Optimal Control & LQG Mar 30 Robust Control & Minimax DDP 12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 13 No Class 14 Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk 14 Apr 25 Project Presentations	10	Mar 21	Trajectory Optimization for Hybrid Systems	
11 Mar 30 Robust Control & Minimax DDP 12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 13 No Class 14 Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk 14 Apr 25 Project Presentations		Mar 23	Data-Driven Methods & Iterative Learning Control	
Mar 30 Robust Control & Minimax DDP 12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket Apr 13 No Class 14 Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk 14 Apr 25 Project Presentations	11	Mar 28	Stochastic Optimal Control & LQG	HW4 Due
Apr 6 Practical Tips & Tricks, Control History Apr 11 Case Study: How to Land a Rocket No Class Apr 13 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk Project Presentations	11	Mar 30	Robust Control & Minimax DDP	
Apr 6 Practical Tips & Tricks, Control History 13 Apr 11 Case Study: How to Land a Rocket No Class 14 Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk Project Presentations	12	Apr 4	RL from an Optimal Control Perspective	
13 Apr 13 No Class 14 Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk 14 Apr 25 Project Presentations		Apr 6	Practical Tips & Tricks, Control History	
Apr 13 No Class Apr 18 Case Study: How to Drive a Car Apr 20 Case Study: How to Walk Apr 25 Project Presentations	13	Apr 11	Case Study: How to Land a Rocket	
Apr 20 Case Study: How to Walk Apr 25 Project Presentations		Apr 13	No Class	
Apr 20 Case Study: How to Walk Project Presentations	14	Apr 18	Case Study: How to Drive a Car	
		Apr 20	Case Study: How to Walk	
Apr 27 Project Presentations	14	Apr 25	Project Presentations	
		Apr 27	Project Presentations	

Project Guidelines

Students should work in groups of 1–5 to complete a substantial final project. The goal is for students to apply the coarse content to their own research. Project proposals will be solicited on the first homework and topics will be selected in consultation with the instructors.

Project grades will be based on a short presentation given during the last week of class and a final report submitted via Google drive by May 10 Anywhere on Earth. Reports should be written in the form of a 6 page (plus references) ICRA or IROS conference paper using the standard two-column IEEE format. Sections should include an abstract, introduction and/or background to motivate your problem, 2–3 main technical sections on your contributions, conclusions, and references. Grading will be based on the following criteria:

10%	Class presentation	
10%	Adherence to IEEE formatting and length requirements	
10%	Innovation & Creativity: Is what you did new/cool/interesting? Convince me.	
30%	Clarity of presentation: Can I understand what you did from your writing + plots?	
40%	Technical correctness: Are your results reasonable? Is your code correct?	