## 16-745: Optimal Control and Reinforcement Learning

Spring 2023

### Course Description

This is a course about how to make robots move through and interact with their environment with speed, efficiency, and robustness. We will survey a broad range of topics from nonlinear dynamics, linear systems theory, classical optimal control, numerical optimization, state estimation, system identification, and reinforcement learning. The goal is to provide students with hands-on experience applying each of these ideas to a variety of robotic systems so that they can use them in their own research.

**Prerequisites:** Strong linear algebra skills, experience with a high-level programming language like Python, MATLAB, or Julia, and basic familiarity with ordinary differential equations.

#### Instructors

Prof. Zac Manchester

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TA: Jeong Hun (JJ) Lee

TA: Swaminathan (Swami) Gurumurthy

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## Logistics

- Lectures will be held Tuesdays and Thursdays 5:00–6:20 PM Eastern time in Room 160 in Hall of Arts. Lectures will also be live streamed on zoom and recorded for later viewing.
- Recitation will be held Fridays at TODO: based on survey.
- Office hours will be TODO: based on survey.
- Homework assignments will be due by 11:59 PM Eastern time on Wednesdays. Two weeks will be given to complete each assignment.
- GitHub will be used to distribute assignments and GradeScope will be used for submissions.
- Piazza will be used for general discussion and Q&A outside of class and office hours.
- There will be no exams. Instead, students will form groups of up to five to complete a project on a topic of their choice.

### Learning Objectives

By the end of this course, students should be able to do the following:

- 1. Analyze the stability of dynamical systems
- 2. Design LQR controllers that stabilize equilibria and trajectories
- 3. Use offline trajectory optimization to design trajectories for nonlinear systems
- 4. Use online convex optimization to implement model-predictive control
- 5. Understand the effects of stochasticity and model uncertainty
- 6. Directly optimize feedback policies when good models are unavailable

### Learning Resources

There is no textbook required for this course. Video recordings of lectures and lecture notes will be posted online. Additional references for further reading will be provided with each lecture.

#### Homework

Four homeworks will be assigned during the semester. Students will have at least two weeks to complete each assignment. All homework will be distributed and collected using GitHub. Solutions and grades will be returned within one week of homework due dates.

### **Project Guidelines**

Students should work in groups of 1–5 to complete a substantial final project. The goal is for students to apply the course content to their own research. Project proposals will be solicited on the first homework assignment, and topics will be selected in consultation with the instructors.

Project grades will be based on a short presentation given during the last week of class and a final report submitted by May 10. Reports should be written in the form of a 6–8 page conference paper using the standard two-column IEEE format.

## Grading

Grading will be based on:

- 50% Project
- 40% Homework
- 10% Participation

Attendance during lectures is not required to earn a full participation grade. Students can also participate through any combination of office hours, Piazza discussions, project presentations, and by offering constructive feedback about the course to the instructors.

#### Course Policies

**Late Homework:** Students are allowed a budget of 6 late days for turning in homework with no penalty throughout the semester. They may be used together on one assignment, or separately on multiple assignments. Beyond these six days, no other late homework will be accepted.

Accommodations for Students with Disabilities: If you have a disability and are registered with the Office of Disability Resources, I encourage you to use their online system to notify me of your accommodations and discuss your needs with me as early in the semester as possible. I will work with you to ensure that accommodations are provided as appropriate. If you suspect that you may have a disability and would benefit from accommodations but are not yet registered with the Office of Disability Resources, I encourage you to contact them at access@andrew.cmu.edu.

Statement of Support for Students' Health & Well-Being: Take care of yourself. Do your best to maintain a healthy lifestyle this semester by eating well, exercising, avoiding drugs and alcohol, getting enough sleep, and taking some time to relax. This will help you achieve your goals and cope with stress.

If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, we strongly encourage you to seek support. Counseling and Psychological Services (CaPS) is here to help: call 412-268-2922 and visit <a href="http://www.cmu.edu/counseling">http://www.cmu.edu/counseling</a>. Consider reaching out to a friend, faculty, or family member you trust for help getting connected to the support that can help.

If you or someone you know is feeling suicidal or in danger of self-harm, call someone immediately, day or night:

CaPS: 412-268-2922

Re:solve Crisis Network: 888-796-8226

If the situation is life threatening, call the police:

On campus: CMU Police: 412-268-2323

Off campus: 911

If you have questions about this or your coursework, please let me know. Thank you, and have a great semester.

# Tentative Schedule

Jan 17	Week	Dates	Topics	Assignments
Jan 19 Stability, Discrete-Time Dynamics HW0 Out  Jan 24 Optimization Intro  Numerical Optimization Pt. 1 HW1 Out  Jan 31 Numerical Optimization Pt. 2 & Optimal Control Intro  Feb 2 Pontryagin, Shooting Methods, & LQR Intro  LQR as a QP & Riccati Equation HW 1 Due  Feb 9 No Class HW 2 Out  Feb 14 Dynamic Programming & Intro to Convexity  Feb 16 Convex Model-Predictive Control  Feb 21 Intro to Trajectory Optimization, Iterative LQR, & DDP HW2 Due  Feb 23 DDP with Constraints and Free Final Time HW3 Out  Feb 28 Direct Trajectory Optimization, Collocation, & SQP  Mar 2 Attitude Intro: SO(3) & Quaternions  No Class  Mar 9 No Class  No Class  Mar 14 Optimizing with Attitude  Mar 16 LQR with Attitude, Quadrotors, & Contact Intro  Mar 21 Trajectory Optimization for Hybrid Systems  Mar 23 Data-Driven Methods & Iterative Learning Control  Mar 24 RL from an Optimal Control & LQG HW4 Due  Robust Control & Minimax DDP  Apr 4 RL from an Optimal Control Perspective  Practical Tips & Tricks, Control History  Apr 10 Apr 11 Case Study: How to Drive a Car  Case Study: How to Walk  Project Presentations	1	Jan 17	Course Overview, & Dynamics Intro	Survey
Jan 26		Jan 19		
Jan 26 Numerical Optimization Pt. 1 HW1 Out  Jan 31 Numerical Optimization Pt. 2 & Optimal Control Intro Pontryagin, Shooting Methods, & LQR Intro  Feb 7 LQR as a QP & Riccati Equation HW 1 Due Feb 9 No Class Feb 14 Dynamic Programming & Intro to Convexity Feb 16 Convex Model-Predictive Control  Feb 23 DDP with Constraints and Free Final Time HW3 Out  Feb 28 Direct Trajectory Optimization, Collocation, & SQP Mar 2 Attitude Intro: SO(3) & Quaternions  Mar 9 No Class Mar 9 No Class Mar 9 No Class Mar 14 Optimizing with Attitude Mar 16 LQR with Attitude, Quadrotors, & Contact Intro  Mar 21 Trajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control  Mar 23 Data-Driven Methods & Iterative Learning Control  Mar 30 Robust Control & Minimax DDP  Apr 4 RL from an Optimal Control Perspective Practical Tips & Tricks, Control History  Apr 11 Case Study: How to Land a Rocket Apr 20 Case Study: How to Drive a Car Case Study: How to Drive a Car Case Study: How to Walk  Project Presentations	2	Jan 24	Optimization Intro	HW0 Due
Feb 2 Pontryagin, Shooting Methods, & LQR Intro  4 Feb 7 LQR as a QP & Riccati Equation HW 1 Due Feb 9 No Class HW 2 Out  5 Feb 14 Dynamic Programming & Intro to Convexity Feb 16 Convex Model-Predictive Control  6 Feb 21 Intro to Trajectory Optimization, Iterative LQR, & DDP HW2 Due Feb 23 DDP with Constraints and Free Final Time HW3 Out  7 Feb 28 Direct Trajectory Optimization, Collocation, & SQP Attitude Intro: SO(3) & Quaternions  8 Mar 7 No Class  9 Mar 14 Optimizing with Attitude HW3 Due Mar 16 LQR with Attitude, Quadrotors, & Contact Intro HW4 Out  10 Mar 21 Trajectory Optimization for Hybrid Systems Data-Driven Methods & Iterative Learning Control  11 Mar 28 Stochastic Optimal Control & LQG HW4 Due Mar 30 Robust Control & Minimax DDP  12 Apr 4 RL from an Optimal Control Perspective Apr 6 Practical Tips & Tricks, Control History  13 Apr 11 Case Study: How to Land a Rocket Apr 20 Case Study: How to Drive a Car Case Study: How to Walk  14 Apr 25 Project Presentations		Jan 26	Numerical Optimization Pt. 1	HW1 Out
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		Apr 27	Project Presentations	

### **Project Guidelines**

Students should work in groups of 1–5 to complete a substantial final project. The goal is for students to apply the coarse content to their own research. Project proposals will be solicited on the first homework and topics will be selected in consultation with the instructors.

Project grades will be based on a short presentation given during the last week of class and a final report submitted via Google drive by May 10 Anywhere on Earth. Reports should be written in the form of a 6 page (plus references) ICRA or IROS conference paper using the standard two-column IEEE format. Sections should include an abstract, introduction and/or background to motivate your problem, 2–3 main technical sections on your contributions, conclusions, and references. Grading will be based on the following criteria:

10%	Class presentation	
10%	Adherence to IEEE formatting and length requirements	
10%	Innovation & Creativity: Is what you did new/cool/interesting? Convince me.	
30%	Clarity of presentation: Can I understand what you did from your writing + plots?	
40%	Technical correctness: Are your results reasonable? Is your code correct?	