

We want to minimize  $J_{C_1 \cap C_2}(\gamma) + \frac{1}{2} \|\gamma - \tilde{\gamma}\|^2$  (\*)

where  $\tilde{\gamma}$  is point to compute the projection

$$C_1 = \{ \gamma \in \mathbb{R}_+^{M \times N} \mid \forall i \in \{1, \dots, M\} \quad \sum_j \gamma_{ij} = m_i \}$$

$$C_2 = \{ \gamma \in \mathbb{R}_+^{M \times N} \mid \forall j \in \{1, \dots, N\} \quad \sum_i \gamma_{ij} = m_j \}$$

Remark:  $P_{C_1}$  and  $P_{C_2}$  are available on the prox-repository website.

Remark: You proposed to use Algo 3.2 from article of Combettes, Dung and Vũ.

Algo to solve (\*)

let  $\tilde{\gamma} \in \mathbb{R}^{M \times N}$  (provided by the gradient step of your FB algo)

$u_{1,0} = \tilde{\gamma}$  and  $u_{2,0} = \tilde{\gamma}$

$\delta = 1.9$

for  $k=0, 1, \dots$

$\gamma_k = \tilde{\gamma} - \frac{1}{2} (u_{1,k} + u_{2,k})$

$u_{1,k+1} = u_{1,k} + \delta (\gamma_k - P_{C_1}(\delta^{-1} u_{1,k} + \gamma_k))$

$u_{2,k+1} = u_{2,k} + \delta (\gamma_k - P_{C_2}(\delta^{-1} u_{2,k} + \gamma_k))$

Remark: You need to find "smart" initialisations for  $u_1$  and  $u_2$

Remark: You need to have a "smart" stopping criteria (i.e. check that constraints are satisfied)