Appendix

A Numerically solving the planner's problem

The full planner's problem on page 8 consists of a very large number of choice variables and hence requires vast computation efforts when solved directly. Fortunately, Fajgelbaum and Schaal (2017) provide guidance on how to transform this *primal* problem into its much simpler *dual* representation. The following section illustrates how to use their derivation to numerically solve my version of the model.

To show how a unique global optimum exists, first note that every constraint of the social planner's problem is convex but potentially for the *Balanced Flows Constraint*. However, the introduction of congestion causes even the *Balanced Flows Constraint* to be convex if $\beta > \gamma$. To see this, note that every part of the lengthy constraint is linear, but for the interaction term $Q^n_{i,k}\tau^n_{i,k}(Q^n_{i,k}, I_{i,k})$ representing total trade costs. Since $\tau^n_{i,k}$ was parameterised as in (1), this expands to

$$Q^n_{i,k}\tau^n_{i,k}(Q^n_{i,k}, I_{i,k}) = \delta^\tau_{i,k}\frac{(Q^n_{i,k})^{1+\beta}}{I^\gamma_{i,k}} \tag{A.1}$$

which is convex if $\beta > \gamma$. Under this condition, the social planner's problem is to maximise a concave objective over a convex set of constraints, guaranteeing that any local optimum is indeed a global maximum.[A.1] $\beta > \gamma$ describes a notion of congestion dominance: increased infrastructure expenditure might alleviate the powers of congestion, but it can never overpower it. It precludes corner solutions in which all available concrete is spent on one link, all but washing away trade costs and leading to overwhelming transport flows on this one edge. If $\beta > \gamma$, geography always wins.

Consider first the full Lagrangian of the primal planner's problem

$$\begin{aligned}
\mathcal{L} = &\sum_i L_i u(c_i) - \sum_i \lambda^C_i \left[L_i c_i - \left(\sum_{n=1}^{N} (C^n_i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right] \\
&- \sum_i \sum_n \lambda^P_{i,n}\left[C^n_i + \sum_{k\in N(i)} Q^n_{i,k}(1+\tau^n_{i,k}(Q^n_{i,k}, I_{i,k})) - Y^n_i - \sum_{j\in N(i)} Q^n_{j,i}\right] \\
&- \lambda^I\left[\sum_i \sum_{k\in N(i)} \delta^i_{i,k} I_{i,k} - K\right] - \sum_i \sum_{k\in N(i)} \zeta^S_{i,k}\left[I_{i,k} - I_{k,i}\right] \\
&+ \sum_i \sum_{k\in N(i)} \sum_n \zeta^Q_{i,k,n} Q^n_{i,k} + \sum_i \sum_n \zeta^C_{i,n} C^n_i + \sum_i \sum_n \zeta^c_i c_i - \sum_i \sum_{k\in N(i)} \zeta^I_{i,k}\left[4 - I_{i,k}\right]
\end{aligned} \tag{A.2}$$

This is a function of the choice variables $(C^n_i, Q^n_{i,k}, c_i, I_{i,k})$ in all dimensions $\langle i,k,n\rangle$ and the Lagrange multipliers $(\boldsymbol{\lambda^C}, \boldsymbol{\lambda^P}, \boldsymbol{\lambda^I}, \boldsymbol{\zeta^Q}, \boldsymbol{\zeta^C}, \boldsymbol{\zeta^c}, \boldsymbol{\zeta^I})$ also in $\langle i,k,n\rangle$. Standard optimisation yields

[A.1] This is Fajgelbaum and Schaal Proposition 1.

first-order conditions which can be collapsed to the following set of equations

$$\begin{aligned}
c_i &= \left(\frac{1}{\alpha}\Big(\sum_{n'}(\lambda^P_{i,n'})^{1-\sigma}\Big)^{\frac{1}{1-\sigma}}\right)^{\frac{1}{\alpha-1}} \\
C^n_i &= \left[\frac{\lambda^P_{i,n}}{(\sum_{n'}(\lambda^P_{i,n'})^{1-\sigma})^{\frac{1}{1-\sigma}}}\right]^{-\sigma} L_i c_i \\
Q^n_{i,k} &= \left[\frac{1}{1+\beta}\frac{I^\gamma_{i,k}}{\delta^\tau_{i,k}}\max\Big\{\frac{\lambda^P_{k,n}}{\lambda^P_{i,n}}-1,0\Big\}\right]^{\frac{1}{\beta}} \\
I_{i,k} &= \max\Bigg\{\Bigg[\frac{\kappa}{\lambda^I(\delta^I_{i,k}+\delta^I_{k,i})}\Bigg(\sum_n \max\Big\{(\delta^\tau_{i,k})^{-\frac{1}{\beta}}\lambda^P_{i,n}\Big(\frac{\lambda^P_{k,n}}{\lambda^P_{i,n}}-1\Big)^{\frac{1+\beta}{\beta}},0\Big\} \\
&\quad + \sum_n \max\Big\{(\delta^\tau_{k,i})^{-\frac{1}{\beta}}\lambda^P_{k,n}\Big(\frac{\lambda^P_{i,n}}{\lambda^P_{k,n}}-1\Big)^{\frac{1+\beta}{\beta}},0\Big\}\Bigg)\Bigg]^{\frac{\beta}{\beta-\gamma}},4\Bigg\}
\end{aligned} \tag{A.3}$$

These directly follow the more general framework outlined in the technical appendix of Fajgelbaum and Schaal applied to my version of the model. In the final equation denoting optimal infrastructure supply, $\kappa = \gamma(1+\beta)^{-\frac{1+\beta}{\beta}}$, and the multiplier λ^I is such that adherence to the *Network Building Constraint* is ensured. Note that there is a typo in the original authors' paper which prints one of the exponents as $(\delta^\tau_{i,k})^{\frac{1}{\beta}}$ when it should be $(\delta^\tau_{i,k})^{-\frac{1}{\beta}}$. Through these algebraic manipulations, I have expressed all choice variables as functions of merely the Lagrange parameters $\boldsymbol{\lambda^P}$ over dimensions $\langle i,k,n\rangle$. I can hence recast the entire Lagrangian in much simpler form as

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\lambda}, x(\boldsymbol{\lambda})) &= \sum_i L_i u(c_i(\boldsymbol{\lambda})) \\
&- \sum_i\sum_n \lambda^P_{i,n}\Big[C^n_i(\boldsymbol{\lambda}) + \sum_{k\in N(i)} Q^n_{i,k}(\boldsymbol{\lambda})(1+\tau^n_{i,k}(Q_{i,k}(\boldsymbol{\lambda})^n, I_{i,k}(\boldsymbol{\lambda}))) - Y^n_i - \sum_{j\in N(i)} Q^n_{j,i}(\boldsymbol{\lambda})\Big]
\end{aligned} \tag{A.4}$$

where $x(\boldsymbol{\lambda})$ denote the choice variables as functions of the Lagrange parameters as derived above. Fajgelbaum and Schaal note that thanks to complementary slackness, all other constraints can be readily dropped from consideration and only the *Balanced Flows Constraint* remains part of the problem.

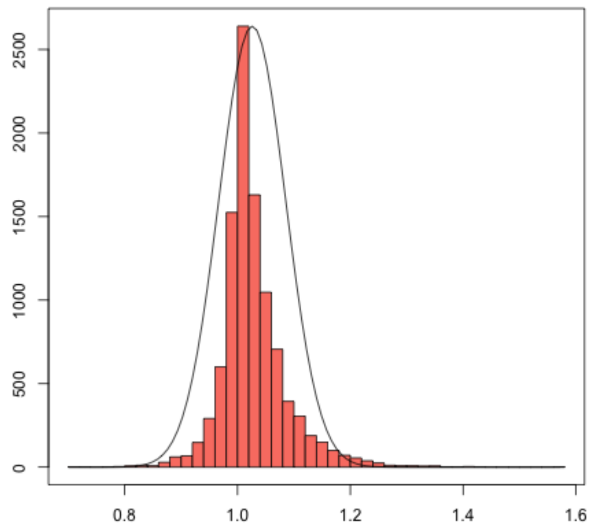
As Fajgelbaum and Schaal further explain, the dual of this problem can now be conceived as the minimisation of

$$\min_{\boldsymbol{\lambda}\geq 0} \mathcal{L}(\boldsymbol{\lambda}, x(\boldsymbol{\lambda}))$$

which is an optimisation problem over merely $\|\boldsymbol{\lambda^P}\| = I \times N$ variables. Fajgelbaum and Schaal interpret $\boldsymbol{\lambda^P}$ as a field of prices varying over goods and locations. I am left only to minimise equation (A.4) to obtain the price-field $\boldsymbol{\lambda^P}$. I implement constrained optimisations within the `fmincon` environment in MATLAB and achieve fairly fast convergence. Solving for smaller networks (like Rwanda or Djibouti) is a matter of seconds, yet the largest countries (Algeria, Angola, DRC, and Sudan) each take about a day of computation time (on a five-year old device, nonetheless). Plugging the derived $\boldsymbol{\lambda^P}$ parameters into the various FOCs in (A.3) yields the optimal transport network $I_{i,k}$, trade flows between locations $Q^n_{i,k}$, and consumption patterns C^n_i and c_i.

B Additional figures and tables

Figure A.1: Histogram of Λ_i



Histogram displaying the frequency distribution of Λ_i over all 10,158 grid cells. Plot also shows the respective PDF of a normal distribution with mean and standard deviation matching Λ_i.

Table A.1: Correlations of Λ_i with the various control sets

	Dependent variable: Λ_i		
	Geo	+ FE	+ Simulation
Altitude	0.00001	−0.00001	−0.00001
	(0.00001)	(0.00001)	(0.00001)
Average Yearly Temperature	0.001	−0.003**	−0.003**
	(0.001)	(0.001)	(0.001)
Average Land Suitability	0.008	0.007	0.007
	(0.007)	(0.007)	(0.007)
Malaria Transmission Index	0.001***	0.0005*	0.0005*
	(0.0002)	(0.0003)	(0.0003)
Biome 1	0.018	0.022**	0.022*
	(0.020)	(0.011)	(0.011)
Biomes 2 & 3	−0.027	−0.020	−0.020
	(0.022)	(0.014)	(0.014)
Biome 5	−0.049**	−0.036**	−0.036**
	(0.024)	(0.016)	(0.016)
Biomes 7 & 9	0.017	0.025**	0.025**
	(0.020)	(0.011)	(0.011)
Biome 10	0.015	0.022**	0.022**
	(0.020)	(0.011)	(0.011)
Biome 12	−0.028***	−0.026**	−0.025*
	(0.022)	(0.013)	(0.014)
Biome 13	0.005	0.009	0.008
	(0.020)	(0.010)	(0.011)
Biome 14	−0.005***	−0.002	−0.002
	(0.035)	(0.033)	(0.032)
< 25KM to Natural Harbour	−0.019***	−0.017	−0.014
	(0.012)	(0.012)	(0.013)
< 25KM to Navigable River	−0.019***	−0.005	−0.001
	(0.007)	(0.005)	(0.005)
< 25KM to Lake	−0.001***	0.007	0.006
	(0.009)	(0.009)	(0.008)
Yearly Growing Days	−0.00001	−0.0001	−0.0001
	(0.00003)	(0.00004)	(0.00004)
Average Precipitation	0.00002***	0.00005	0.00004
	(0.0001)	(0.0001)	(0.0001)
Border Cell	0.001***	0.001**	0.001**
	(0.001)	(0.001)	(0.001)
Longitude	0.001***	−0.001	−0.001
	(0.0003)	(0.001)	(0.001)
Longitude^2	0.00003	−0.00002	−0.00002
	(0.00002)	(0.00003)	(0.00003)
Longitude^3	−0.00000***	0.00000	0.00000
	(0.00000)	(0.00000)	(0.00000)
Longitude^4	0.00000**	−0.00000	−0.00000
	(0.00000)	(0.00000)	(0.00000)
Latitude	−0.001	−0.001	−0.001
	(0.0003)	(0.001)	(0.001)
Latitude^2	0.00001	0.00001	0.00001
	(0.00002)	(0.00003)	(0.00003)
Latitude^3	0.00000	0.00000	0.00000
	(0.00000)	(0.00000)	(0.00000)
Latitude^4	0.000	−0.000	−0.000
	(0.00000)	(0.00000)	(0.00000)
Terrain Ruggedness			−0.00000
			(0.00000)
Average Night Lights			−0.001***
			(0.0003)
Total Population			0.000
			(0.000)
Urban Grid Cell			0.006
			(0.003)
Country FE		Yes	Yes
Observations	10,158	10,158	10,158
R^2	0.062	0.122	0.124
Note:			*p<0.1; **p<0.05; ***p<0.01