GRASP for the Vehicle Routing Problem with Time Windows, synchronization and precedence constraints

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***Abstract*—This article addresses a new routing problem encountered in the field of Home Health Care. Researches in this emerging area share the goal of establishing fine coordination to optimize the planning of human and material resources to provide support and quality monitoring while controlling costs. Home Health Care services allow keeping patients who are not entirely dependent at home, or to facilitate the return to normal life for people who have suffered serious illnesses by permitting them to leave traditional health institutions. They often need specific cares provided by caregivers. Usually, these services must be performed at specific times, and require the intervention of several caregivers, sometimes linked by precedence constraints. By associating the patients to customers and caregivers to vehicles, the problem introduced in this study can be seen as a particular vehicle routing problem with time windows and side timing constraints, where some patients require more than one visit simultaneously or in a given priority order. To solve this problem, a mixed integer programming model is presented and a meta-heuristic method based on a Greedy Randomized Adaptive Procedure (GRASP) is proposed. Numerical results are shown on a new benchmark derived from the literature.**

I. INTRODUCTION

Due to high complexity and size, the problem of home health care covers a wide variety of decision-making challenges in the field of Operational Research and Decision Support. It is set to become an increasingly important issue in the years ahead with the longer human life expectancy and the wish to promote the return at home for people who need long-term specific cares but who are able to leave traditional health institutions with a lighter medical follow-up. Research on this emerging problem has the intent to establish a fine coordination of human and material resources to provide an optimized planning that maximize the quality of home health care while controlling costs.

This paper proposes to adapt the classical vehicle routing problem to tackle particular features of providing home health care. Informally, the aim is to determine the routes to be performed by a fleet of vehicles (associated to a group of caregivers) available to serve a set of geographically dispersed customers (corresponding to the patients), with respect to their respective availability, and so that the activity is planned in the most effective way. The specificities relied on the fact that a Home Health Care service must often be performed at a specific time and may require the intervention of several caregivers simultaneously. In addition, sometimes a patient needs several cares linked by precedence constraints. In our study, these timing constraints impose the coordination of several caregivers and the problem is hence modeled as a variant of the Vehicle Routing Problem With Time Windows called VRPTW-PS for *Vehicle Routing Problem With Time Windows, Precedence and Synchronization constraints*.

II. LITERATURE REVIEW

The vehicle routing problem (VRP) is one among the most studied combinatorial optimization problems in Operational Research. In the field of Home Care, several variants are studied and used to model applications such as the transportation of elderly or disabled persons, delivery of drugs or medical devices to patients' homes. Many researches on problems related to the VRP in home care context have demonstrated their importance. We classify these problems into two broad categories:

Planning And Scheduling Problems: this family addresses the problem of assigning tasks to personal care and planning the schedule of visits (often daily). We can identify the works from (Bertels and al., 2006) on the allocation of nurses, (Eveborn and al., 2006), (Bredström and al., 2007, 2008), (Braysay and al., 2009) and (Rasmussen and al., 2010) on the daily planning of care services where each personal care is characterized by a set of skills (linguistic, medical certificate, sex ...) and specific work hours (part-time, full-time). All personal cares begin and end their work in a central platform (depot) where the visit reports are delivered. Each visit has a time window and a time duration and requires special skills. Generally, a patient is a person who needs special assistance such as cleaning, washing, cooking or medical attention. The patients may need also special requirements such as two assistants at a time (due to heavy loads during the bath for example) or in a given order (when a customer needs to undergo treatment before or after meals).

Pick-up And Delivery Problems: this family addresses the issues that consider a service of picking up some products or patients and then delivering them at another location. Many researches exists in this subject, we can cite for example the work of (Liu and al., 2013), (Ceselli and al., 2013) and (Coppi and al., 2013) which were primarily interested in such problems with the aim of providing medicines or medical

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devices to patients' homes and the collection of biological samples, drugs or medical devices not used by patients.

Other authors are also interested in combining these two cases such as (Rousseau and al., 2003) on *Dial A Ride* DAR problems where some patients require specific services before being transported. (Kergosien and al., 2013) have also introduced a two-level problem applied to a complex hospital located at Tours (in France). The first level concerns the routing of a fleet of vehicles serving several hospital units that deliver medicines, clean linen, meals, various supplies, patient files and picks up waste and dirty linen. The second level concerns the problem of routing employees between buildings within a large hospital unit.

From the analysis, we can deduce that the mainly used modeling method is the mixed integer programming and to solve these types of problems, exact methods are used, principally the branch-and-cut-and-price, seen in (Rasmussen and al. 2010), (Bredström and al., 2008)... Usually, the standard approximate methods like heuristics approaches for solving this category problems are based on local search and/or large neighborhood search embedded in a meta-heuristic, (Rousseau and al. 2003), (Kergosien and al. 2013).

III. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL FOR THE VRPTW-PS

This work studies a version of vehicle routing problems with time window and specific synchronization constraints applied in Home Health Care Systems. More particularly, it focuses on cases for which some patients may require a service accomplished by more than one caregiver who have not the same skills.

Formally, the studied problem can be defined on an undirected graph $G = (V, E)$ with a node-set $V = N \cup D$ where $D = \{d, f\}$ are special nodes called initial and final depots, while $N = \{1, .., n\}$ corresponds to customers, and E is an edge-set. Each edge $e = [i, j]$ is associated with a travel cost C_{ij} and a traversal duration T_{ij}. The initial depot d offers a set of possible services $S = \{1, ..., s\}$ to customers. Each customer i requires a subset of services provided by a set $S_i = \{s \in S : m_{is} = 1\}$ where m_{is} equals to 1 if customer i demands service $s \in S$, 0 otherwise. Each service s of customer i $(i = 1...n)$ is associated a known time duration D_{is}, and it must start within the customer time window $[a_i, b_i]$; where a_i and b_i are respectively the earliest start time of service and the latest start time of service at the customer i. A limited fleet of vehicles is assumed to be available at the initial depot d, let K be this set of vehicles. $[\alpha_k, \beta_k]$ is the available time for vehicle k, it means that the vehicle k can only leave the initial depot d at α_k and must return to the final depot f before β_k. Each vehicle k is associated to a real number $Pref_{ik}$ related to non preference of customer i to the vehicle (caregiver) k which is endowed to perform the requested service. In this study we assume that each vehicle $k \in K$ performs a unique service s. By defining o_{ks} $(\forall k \in K, \forall s \in S)$ equal to 1 if vehicle k offers the service s, 0 otherwise, the previous assumption let us write $\forall k \in K,\ \sum_{s \in S} o_{ks} = 1$. For simplicity, we denote $k \in K_s$ if $o_{ks} = 1$ and $s \in S_i$ if $m_{is} = 1$.

Some customers require more than one service either simultaneously or in a given order. These timing requirements are expressed for such customers $i \in N$ by defining gap_{isr} as the time between the starting times of services s and r required by customer i when service s must be provided before service r. For a simultaneous pair of services, this gap is obviously set to zero.

The problem consists in building a set of routes starting at the initial depot d and ending at the final depot f, such that each vehicle route does not exceed the maximal imposed duration, and each customer receives the required services either simultaneously or in the imposed order as stated in the data. Furthermore, each kind of service must be accomplished by the corresponding qualified vehicle. This study considers the minimization of the total traveling time and the sum of non preferences. (Labadie and al., 2014) have studied a relatively similar problem, however, they only consider the case of simultaneous synchronization. Furthermore, only the travel costs are minimized in their objective function. This works extends their model by considering two types of synchronization and a more complex optimization criterion.

The problem under consideration is NP-Hard because it includes the VRPTW which is known to be NP-Hard. It can be modeled as a mixed integer linear program (MILP) using the following decisions variables: binary variable x_{ijk} equal to 1 if and only if the vehicle $k \in K$ goes from i to j, 0 otherwise. Real variables: $tdeb_{ik}$ that indicate the starting time of service at node i if this latter is visited by vehicle k. For the sake of generalization of some timing constraints in the following formulation, we assume that all services are individually requested by the initial depot d, and that their time duration is null (i.e. $D_{ds} = 0, \forall s \in S$).

The resulting MILP is then the following:

$$\min \sum_{i \in V \setminus \{f\}} \sum_{j \in V \setminus \{d\}} \sum_{k \in K} C_{ij}.x_{ijk} \ + \sum_{i \in N} \sum_{j \in V \setminus \{d\}} \sum_{k \in K} Pref_{ik}.x_{ijk} \tag{1}$$

Subject to:

$$\forall k \in K, \sum_{j \in N} x_{djk} = 1 \tag{2}$$

$$\forall k \in K, \sum_{i \in N} x_{ifk} = 1 \tag{3}$$

$$\forall h \in N, \forall k \in K, \quad \sum_{i \in V \setminus \{f\}} x_{ihk} = \sum_{j \in V \setminus \{d\}} x_{hjk} \tag{4}$$

$$\forall i \in N, \forall s \in S, \quad \sum_{j \in V \setminus \{d\}} \sum_{k \in K_s} x_{ijk} = m_{is} \tag{5}$$

$$\forall i, j \in V, \forall s \in S : s \in S_i \cup S_j, \forall k \in K_s, \quad tdeb_{ik} + (T_{ij} + D_{is}).x_{ijk} \leq tdeb_{jk} + b_i.(1 - x_{ijk}) \tag{6}$$

$$\forall i \in N, \forall s \in S_i, \forall k \in K_s,$$
$$a_i. \sum_{j \in N} x_{ijk} \leq tdeb_{ik} \leq b_i. \sum_{j \in N} x_{ijk} \tag{7}$$

$$\forall k \in K, \quad \alpha_k \leq tdeb_{dk} \leq \beta_k \tag{8}$$
$$\forall k \in K, \quad \alpha_k \leq tdeb_{fk} \leq \beta_k \tag{9}$$

$$\forall i \in N, \forall s \in S_i, \forall r \in S_i : r \neq s,$$
$$\sum_{k \in K_s} tdeb_{ik} - \sum_{k \in K_r} tdeb_{ik} \leq gap_{isr} \tag{10}$$

$$\forall i, j \in V, \forall k \in K, \quad x_{ijk} \in \{0, 1\} \tag{11}$$
$$\forall i \in V, \forall k \in K, \quad tdeb_{ik} \geq 0 \tag{12}$$

The objective (1) consists in minimizing the sum of non preferences and the total traveling time. Constraints (2) (resp. (3)) require each vehicle to leave, (resp. return) at the depot. Constraints (4) ensure the continuity of the routes and constraints (5) ensure that all customer demands are satisfied. The constraints (6) are the scheduling constraints allowing the coherence between the periods of visits. Constraints (7)-(9) ensure that all time windows are respected. Constraints (10) are the synchronization constraints between starting time of services at customers asking for more than one service. The remaining constraints of the model fix the nature of the decision variables.

IV. RESOLUTION APPROACH

To solve the problem, a Greedy Randomized Adaptive Procedure (GRASP) is proposed. This method was introduced by (Feo and al., 1995). Its structure for the problem under consideration is given in algorithm (1). A trial solution sol is built using a greedy randomized heuristic. In our case, a randomized parallel constructive heuristic RP, detailed in the following section, intends to provide a feasible solution. However, it may fail and an insertion procedure (IP) is then called to repair the solution. Then, this solution is improved by local search (LS). This scheme is repeated a given number of times ($iterMax$) through the *For* loop. If the solution of the current iteration is better than the current best one, this later is updated.

Algorithm 1: GRASP

```
begin
    F* ← +∞
    for nbIter ← 1 to iterMax do
        sol ← RP()
        if (sol not feasible) then
            sol ← IS(sol)
        sol ← LS(sol)
        if (F(sol) ≤ F*) then
            sol* ← sol
            F* ← F
```

A. CONSTRUCTIVE HEURISTIC

A constructive heuristic called randomized parallel constructive heuristic (RP) is proposed to generate an initial solution. First, a route is opened for each available vehicle. Then, these routes are fulfilled in parallel with customers chosen randomly from a sorted list and then inserted by using a best insertion procedure.

Let i be the last node visited by a vehicle k. At each iteration, we calculate the following indicators for each not visited customer j : $Edat_{ij}$ the earliest date on which the vehicle k leaving i may be at the client j, $Ldat_{ij}$ the latest date when the vehicle k leaving customer i can be at the customer j, and LBT_{js} and UBT_{js}, that are respectively a lower bound and an upper bound that ensure the return to the depot s in their time windows.

The insertion procedure randomly selects a customer among a list of candidate customers. A customer j is candidate for a tour k under construction finishing by customer i, if it is not yet visited and if it satisfies the following conditions:

$$\boxed{\begin{array}{c} Edat_{ij} \leq b_j \\ Ldat_{ij} \geq a_j \\ LBT_{js} \leq tdeb_{jk} \leq UBT_{js} \end{array}}$$

Then, the list is sorted according to decreasing order of lower bounds and trunked to keep only the $Ncand$ first candidates. At this level, the first time a customer requiring synchronization is visited, its service start time is fixed for its twin visits.

The algorithm stops when all customers are inserted or when no more customer can be added in a vehicle (due to time windows). Thus, the resulting solution could be unfeasible. That is why the insertion procedure was developed to repair solutions that are not feasible at the end of the RP heuristic.

B. INSERTION PROCEDURE

This procedure consists to consider all customers not visited in the RP solution. Such a customer is inserted at the best possible position in any existing route. The insertion considers feasibility in terms of time constraints and minimization of the additional cost (routing to add the customer in the route, plus potential non-preference).

Time-windows constraints can be tested in O(1) by keeping in memory some additional variables (maximal possible delay at each customer in a route, maximal possible delay for a sub-route starting from any customer and including the final depot, maximal possible delay intra-route for customers to be synchronized). The technique used in this study generalizes the idea initially proposed by (Kindervater and al., 1997) and still feasible in O(1).

C. LOCAL SEARCH

The Local Search procedure used in the GRASP contains five classical neighborhood in vehicle routing problems, called in the following order:

- relocate applied within a single route, in which one customer is shifted from its current position to another position in the same route
- relocate applied on different routes, in which one customer is shifted from its current position to another position in another route
- 2-exchange moves executed within a single route, in which two customers belonging to a same route are exchanged.
- 2-exchange moves executed on different routes, in which two customers belonging to two different routes are exchanged.
- 2-opt moves executed on a single route, in which two non-consecutive edges are removed, and the route is reconnected by adding two new ones.

The moves are considered subsequently and the first feasible move is performed. The search stops when no improving move can be found. In any case, the time window constraints require a feasibility check before a move is performed (see the particular cases on time-windows and synchronization in section IV-B).

V. NUMERICAL RESULTS

To validate the proposed work, a set of instances proposed by (Bredström and al., 2008) are specifically extended. The benchmark used is grouped into three different categories according to the number of customers and vehicles, which are equal to 18 clients with using 4 vehicles in the first group, 45 clients with using 10 vehicles in the second and 80 clients with using 16 vehicles in the third. In each group, the number of synchronization ranges from two to four, and the time windows can be small, medium or large. In these new instances, the customer demands (in terms of services needed) are randomly generated but all the rest of informations remain the same as in (Bredström and al., 2008). Without loss of generality, we only treat the case when at most two services are requested.

The mathematical model is implemented using OPL Studio, Version 12.5, and the heuristic and meta-heuristic algorithms haven been coded in C language. All experiments have been carried out on a 3.20 GHz Intel(R) Core(TM) i5-3470 CPU computer with 8GB of RAM, running under Ubuntu. The method requires two parameters which are the number of iterations $iterMax$ in the GRASP and the maximal size of the candidate list $Ncand$. After several tests, these two values were set to 100 and 5 respectively. The algorithm is launched 8 times.

TABLE I. MILP RESULTS

Instance	Obj	CPU(s)
18_4 _s	57.65	0.83
18 _4 _m	55.55	1.74
18 _4 _l	43.12	2.59
45 _10 _s	-16.38	37.61
45 _10 _m	-48.63	50.63
45 _10 _l	-109.16	67.34

The results are shown in Table (I) and (II) , where columns 1 describe the used instances in both tables. In each instance name, the first value indicates the number of clients while the second represents the number of vehicles. The indexes s, m or l refer to the width of the customers time windows (small, medium or large). For example, instance 18_4_s expresses an instance with 18 clients, 4 vehicles and where the width of the time windows is small. Columns 2 represent the objective function value for the optimal solution in the first table, whereas in the second table the same column gives the values for the solutions obtained by GRASP. Columns 3 illustrate the running time (reported in seconds) consumed by Cplex solver resp. GRASP.

Column 4 of Table 2 is abtained by devinding, for each instance, the difference between the objective values obtained by the GRASP and OPL respectively by the obsolute value of the solution provided by OPL and column 5 in the same table gives the average solution cost over the 8 replications. In Table (I), the results show that only small instances can be solved to optimality within few seconds. On larger instances (up than 45 clients), the computational time grows considerably even when at most four synchronizations are considered. We also deduce that the computing time increases considerably with customers' time windows width. Note that, within the time limit of 3600 seconds, Cplex solver can not solve the problem for instances with 80 clients. For these reasons, we conducted our study to approximate methods (GRASP), which allow to obtain solutions with good qualities in acceptable time.

TABLE II. GRASP RESULTS

Instance	Grasp	CPU(s)	Gap	Av Cost
18_4 _s	70.01	1.13	0.21	70.31
18 _4 _m	68.96	1.21	0.24	69.56
18 _4 _l	43.5	2.35	0.008	43.92
45 _10 _s	98.02	4.30	6.98	130.47
45 _10 _m	28.02	8.97	1.57	34.42
45 _10 _l	-48.36	15.02	0.55	-45.32
80 _16_s	-107.62	21.9		-90.02
80 _16 _m	-111.06	21.99		-89.09
80 _16 _l	-131.54	23.09		-95.24

Table (II) shows the adequacy of the proposed method. On small instances, GRASP approaches the optimal solution and has a much smaller computation time. The improvement is not obvious. However, large instances are solved in few seconds. To improve the performance of the GRASP, it would be interesting to develop other moves with the aim to optimize the sum of preferences more adequately.

VI. CONCLUSION AND FUTURE WORKS

In this paper, a quick overview of routing problem with time-windows and synchronization constraints is given. A mixed integer programming model is proposed and a Greedy Randomized Adaptive Search Procedure is developed. The idea of handling the synchronization constraints within local search moves is novel, and seems very effective.

Computational experiments are carried out on 9 instances which are extended from the benchmark initially proposed by (Bredström and al., 2008). The experimental results obtained by solving the MIP model with Opl-Studio show that only small instances can be solved optimally. The computation time grows with the instances size since the problem is NP-Hard.

In addition to a fine testing and tuning of the algorithms, future work would be dedicated to the design of new local search moves more suitable to deal with customers' preferences. Finally, as future development, it is in our intention to hybridize the GRASP by adding a Path-Relinking procedure to improve the result, and also to study a new version of the problem where the overall waiting times at customers must be minimized.

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