system of particles. **(5) [BL4]**

[*Hints:* Use the concept of moment of inertia.]

4. If **A** and **B** are irrotational, analyze that **A × B** is solenoidal. **(5) [BL4]**

[*Hints:* Use the concept of divergence and curl of a vector.]

5. A particle of mass *m* moves in the X-Y plane and the position of the particle is given by $\vec{r} = \hat{\imath}\, a\, Cos\, \omega t + \hat{\jmath}\, b\, Sin\, \omega t$ where *a, b* and *ω* are constants.

Evaluate that

(a) The force acting on the particle is always directed to towards the origin.

(b) Calculate the torque and angular momentum about the origin.

(c) Is the force field conservative? **(10) [BL5]**

Module-2: Oscillations

Equation of motion of free vibration:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Solution: $x = C\, Sin\, (\omega t + \varphi)$

Equation of motion of damped vibration:

$$\frac{d^2x}{dt^2} = -2b\frac{dx}{dt} - \omega^2 x$$

Solution: $x = e^{-bt}\left(A\, e^{\sqrt{b^2-\omega^2}t} + B\, e^{-\sqrt{b^2-\omega^2}t}\right)$

Case 1 (Low damping b <ω): $x = Ce^{-bt}\, Sin\left(\sqrt{b^2-\omega^2}t + \varphi\right)$, **where** $C = \sqrt{A^2+B^2}$

Case 2 (Critical damping b ~ ω): $x = e^{-bt}[(A+B) + (A-B)t]$

Case 3 (Large damping b >ω): $x = e^{-bt}\left(A\, e^{\sqrt{b^2-\omega^2}t} + B\, e^{-\sqrt{b^2-\omega^2}t}\right)$

Equation of motion of forced vibration:

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = f\, e^{ipt}$$

p is frequency of external periodic force.

Questions:

1. A particle of mass m is attached to a horizontal spring of force constant k, initially at rest. The spring is compressed by a distance x_0 and then released. As the particle oscillates, it encounters a horizontal rough surface with coefficient of kinetic friction μ. Determine the maximum compression x_{max} of the spring such that the particle does not come to a complete stop during its motion. **(5) [BL5]**

   [**Hint:**To determine the maximum compression x_{max} of the spring where the particle does not stop, analyze the energy considerations and effect of friction.]

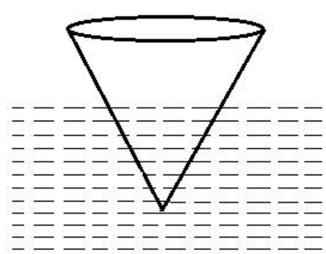
2. Using the conservation of energy principle analyze that angular velocity of a simple pendulum of string length L is

$$\omega = \left[\frac{2}{mL^2}\left(E - mgL(1 - cos\theta)\right)\right]^{1/2}$$

Where θ is angle made by the string with the vertical axis at a particular time. **(5) [BL4]**

   [**Hint:**Use the conservation of energy: Total energy = Kinetic energy + Potential energy**]**

3. A conical buoy floats with its axis vertical and its apex points downwards in a big vessel containing water. If the weight of the buoy is 300 lbs and 1 cu.ft of water weights 64 lbs, find the time period of vertical oscillations, if the diameter of the base is 4 ft and the height of the cone is 5 ft. **(5)** [**BL4]**



   [**Hints:** From the concept of Archimrdes' principle deduce the equation of motion and hence calculate the time period.]

4. In one dimensional motion of a mass of 10 gm, it is acted on by a restoring force and a resisting force. Spring Constant = 10 dyne/cm and damping coefficient =2 dyne-sec/cm. (i) Find whether

the motion is aperiodic or oscillatory, (ii) the value of the resisting force which will make the motion critically damped, (iii) the value of the mass for which the given forces will make the motion critically damped, (iv) suppose that, the mass is given an initial impulse of 20 gm-cm/sec, while at rest, obtain an expression for the displacement x at a time t. **(5) [BL5]**

5. A mass-spring system consists of a mass m=0.5 kg and a spring with a spring constant k=200 N/m. The system is subject to a damping force $F_d = -bv$ where b = 3 kg/s. Additionally, the system is driven by an external force $F(t)$ = 10cos(3t) N.

(i) Determine the steady-state amplitude of the system.

(ii) Evaluate the effect of damping on the amplitude by comparing the result with the undamped system.

(iii) Analyze the system's response if the driving frequency is altered to the natural frequency of the undamped system. **(10) [BL5]**

Module – 3: Optics

Interference of Light

Definition

The phenomenon in which alternate bright and dark bands or fringes are produced as a result of superposition of two monochromatic light waves of same wavelength, equal or nearly equal amplitude and having constant phase difference proceeding in the same direction is called interference of light.

Conditions for sustained interference

(i) The light sources must be coherent. The light waves must have constant phase difference.

(ii) The two sources must emit monochromatic light waves. The amplitude should also be equal or nearly equal.

(iii) The two sources should be very narrow.

(iv) The sources must lie very close to each other.