the motion is aperiodic or oscillatory, (ii) the value of the resisting force which will make the motion critically damped, (iii) the value of the mass for which the given forces will make the motion critically damped, (iv) suppose that, the mass is given an initial impulse of 20 gm-cm/sec, while at rest, obtain an expression for the displacement x at a time t. **(5) [BL5]**

5. A mass-spring system consists of a mass m=0.5 kg and a spring with a spring constant k=200 N/m. The system is subject to a damping force $F_d = -bv$ where b = 3 kg/s. Additionally, the system is driven by an external force $F(t)$ = 10cos(3t) N.

(i) Determine the steady-state amplitude of the system.

(ii) Evaluate the effect of damping on the amplitude by comparing the result with the undamped system.

(iii) Analyze the system's response if the driving frequency is altered to the natural frequency of the undamped system. **(10) [BL5]**

Module – 3: Optics

Interference of Light

Definition

The phenomenon in which alternate bright and dark bands or fringes are produced as a result of superposition of two monochromatic light waves of same wavelength, equal or nearly equal amplitude and having constant phase difference proceeding in the same direction is called interference of light.

Conditions for sustained interference

(i) The light sources must be coherent. The light waves must have constant phase difference.

(ii) The two sources must emit monochromatic light waves. The amplitude should also be equal or nearly equal.

(iii) The two sources should be very narrow.

(iv) The sources must lie very close to each other.

Intensity of the light at the point of superposition

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

Where a_1 and a_2 are the amplitudes of the light waves and δ is the phase difference between them.

Conditions for bright and dark fringes path difference between the light waves of wavelength λ

In terms of phase difference

Bright fringes: $\cos\delta = 1$ or $\delta = 2n\pi$ where n = 0, 1, 2, 3…..etc.

Dark fringes: $\cos\delta = -1$ or $\delta = (2n+1)\pi$ where n = 0, 1, 2, 3…..etc.

In terms of path difference

Bright fringes: $\frac{2\pi}{\lambda}\times\Delta = 2n\pi$ or $\Delta = 2n\frac{\lambda}{2}$ where n = 0, 1, 2, 3…..etc.

Dark fringes: $\frac{2\pi}{\lambda}\times\Delta = (2n+1)\pi$ or $\Delta = (2n+1)\frac{\lambda}{2}$ where n = 0, 1, 2, 3…..etc.

Position of nth bright fringe (x_n) and nth dark fringe (x_n) on the screen

Distance between the slits = d, Distance of the screen and the slits = D
Wavelength of the light waves = λ

For bright fringe $x_n = \frac{n\lambda D}{d}$ and for Dark fringe $x_n = \frac{D}{d}(2n+1)\frac{\lambda}{2}$

Fringe width (β) of bright and dark fringes

$\beta = \frac{\lambda D}{d}$ Therefore fringe widths of bright and dark fringes are equal.

Newton's Rings

Radius of curvature of the plano-convex lens = R, Wavelength of light = λ and Refractive index of the thin film = μ

Radius of the nth dark ring (r_n) $r_n = \sqrt{\frac{Rn\lambda}{\mu}}$ Diameter of the nth dark ring (D_n) $D_n = \sqrt{\frac{4n\lambda R}{\mu}}$

Radius of the nth bright ring (r_n) $r_n = \sqrt{\frac{(2n+1)\lambda R}{2\mu}}$

Diameter of the n^{th} bright ring $D_n = \sqrt{\dfrac{2(2n+1)\lambda R}{\mu}}$

Relation between the radius of curvature (R) of the plano-convex lens and the wavelength of the light

$$R = \frac{\mu(D_{m+n}^2 - D_n^2)}{4m\lambda}$$

Diffraction of Light

Diffraction is a phenomenon of bending of light around the corner of a sharp obstacle. Diffraction of light is noticeable when the dimension of obstacle is close to the wavelength of light. Light enters into the geometrical shadow region deviating from linear path.

Single slit diffraction

Intensity $I = I_o \dfrac{\sin^2\alpha}{\alpha^2}$ where $I_o = (CAa)^2$ and $\alpha = \dfrac{\pi a \sin\theta}{\lambda}$

a – Slit width, λ – wavelength of the light, θ – angle of diffraction

Condition of minima: $\alpha = m\pi$ where $m = \pm1, \pm2, \pm3$…..

Condition of maxima: $\alpha = \tan\alpha$ where $\alpha = 0, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}$

Double slit diffraction

Intensity $I = 4I_0 \dfrac{\sin^2\alpha}{\alpha^2}\cos^2\beta$ where $I_o = (CAa)^2$, $\alpha = \dfrac{\pi a \sin\theta}{\lambda}$ and $\beta = \dfrac{\pi d \sin\theta}{\lambda}$

a – Slit width, d- separation between the slits (d = a + b) ,λ – wavelength of the light and θ – angle of diffraction.

Condition of diffraction minima: $\alpha = m\pi$ where $m = \pm1, \pm2, \pm3$…..

Condition of diffraction maxima: $\alpha = \tan\alpha$ where $\alpha = 0, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}$

Condition of interference minima: $\beta = (2s+1)\dfrac{\lambda}{2}$ where $s = 0, \pm1, \pm2, \pm3$….

Condition of interference maxima: $\beta = p\pi$, where $p = 0, \pm1, \pm2, \pm3$,…..

Diffraction grating

Plane diffraction grating consists of a number of parallel and equidistant lines ruled on an optically plane parallel glass plate by a fine diamond point. The number of such ruled lines per mm is of the order of 100. Each ruled line behaves as an opaque line while the transparent portion between two consecutive ruled lines behaves as a slit. If a be the width of a clear space and b be the width of a ruled line, then the distance (a+b) is called grating element or grating constant. In optics, a diffraction grating is an optical component with a periodic structure that diffracts light into several beams travelling in different directions. The directions or diffraction angles of these beams depend on the incident angle to the diffraction grating, the spacing or distance between adjacent diffracting elements (e.g., parallel slits for a transmission grating) on the grating, and the wavelength of the incident light. The grating acts as a dispersive element. Because of this, diffraction gratings are commonly used in spectrometers.

$$\text{Intensity} \quad I = I_0 \frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}$$

where N = no of slits, $I_o = (CAa)^2$, $\alpha = \dfrac{\pi a \sin\theta}{\lambda}$ and $\beta = \dfrac{\pi d \sin\theta}{\lambda}$

Grating equation: $(a+b)\sin\theta = n\lambda$ where n = 0, ±1, ±2, ±3….

Angular dispersive power of a Grating $\dfrac{d\theta}{d\lambda}$

$$d\sin\theta = n\lambda$$
$$d\cos\theta\frac{d\theta}{d\lambda} = n$$
$$\frac{d\theta}{d\lambda} = \frac{n}{d\cos\theta}$$

Maximum number of Grating spectra (n_{max})

$$d\sin\theta = n\lambda$$
$$(a+b)\sin\theta = n\lambda$$
$$n = \frac{(a+b)\sin\theta}{\lambda}$$
$$n_{max} = \frac{(a+b)}{\lambda}$$

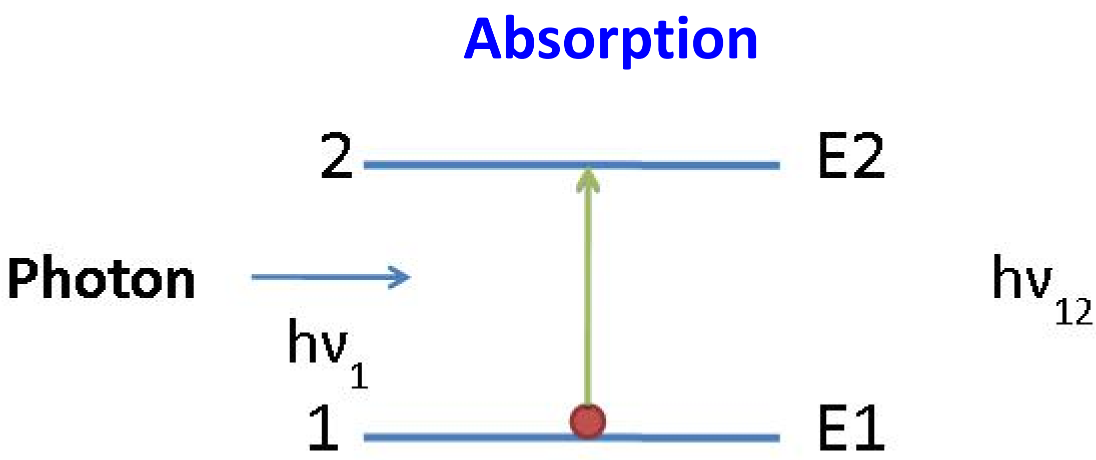
Resolving power of a Grating

$$\frac{\lambda}{d\lambda} = nN \qquad \frac{\lambda}{d\lambda} = \frac{d\sin\theta N}{\lambda}$$

LASER

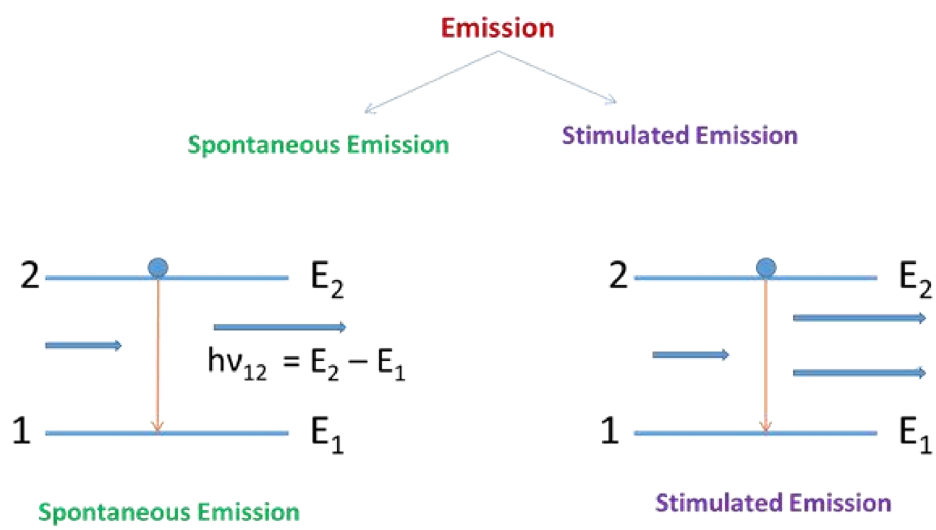
The word LASER is an acronym for**Light Amplification by Stimulated Emission of Radiation.**

Absorption and Emission of Radiation



The atom will absorb the photon and will go to the upper excited state E_2. This phenomenon is called Absorption.
The atom cannot stay in the excited state forever. It can stay there for a certain period of time which is known as the lifetime of the excited state.

Emission
The atom will come down to the lower excited state E1 and release the amount of energy
$h\nu_{12} = E_2 - E_1$



Population Inversion
- Let us consider N_1 and N_2 are the number of atoms in the lower energy state E_1 and higher energy state E_2 respectively

- According to Boltzmann distribution, under thermal equilibrium

$$N_2/N_1 = e^{-(E_2 - E_1)/KT}$$

At room temperature, most of the atoms remain in lower energy state E_1.
Hence in this condition, stimulated emission is negligible due to fewer amounts of atoms in level E_2.

- To achieve sufficient stimulated emission more atoms are required in level 2, ie, $N_2 > N_1$. This is known as population inversion.

Population inversion can be attained by optical pumping.

Einstein's A, B Coefficients

Rate of absorption: $-\frac{dN_1}{dt} = B_{12} N_1 u_\nu$ Rate of spontaneous emission: $-\frac{dN_2}{dt} = A_{21} N_2$

Rate of stimulated emission: $-\frac{dN_2}{dt} = B_{21} N_2 u_\nu$

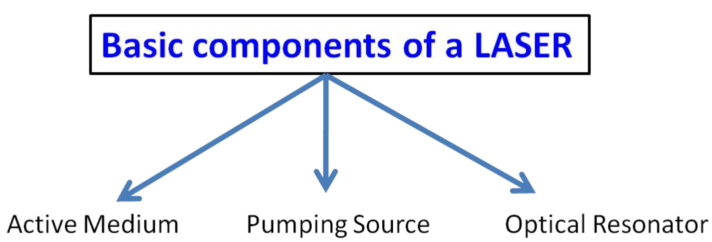
Here A_{21}, B_{12}, B_{21} are known as Einstein's A, B coefficients.

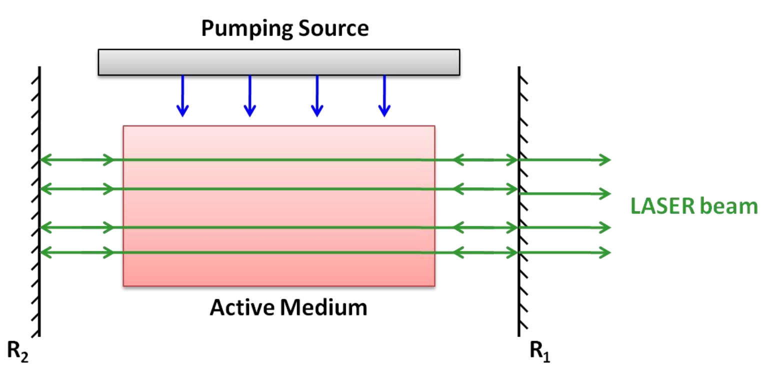
The radiation energy density (u_ν)

$$u_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Relation between Einstein's A, B Coefficients

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad B_{12} = B_{21}$$





R_2 = 100 % Reflecting R_1 = Partially transparent

Active medium is placed between a pair of mirror . Such a closed system is called Optical Resonator

Frequency of cavity modes: $\nu_m = \frac{mc}{2L\mu}$ L – Length of the cavity, μ – refractive index of the active medium.

Here m = 1, 2, 3,……… represents cavity modes of different frequencies.

The separation between frequencies of two consecutive modes is: $\Delta\nu = \nu_{m+1} - \nu_m = \frac{c}{2L\mu}$

The Quality factor Q of the cavity

$$Q = 2\pi \times \frac{\text{Maximum energy stored per cycle in the mode}}{\text{Energy dissipated per cycle in the mode}}$$

It can also be expressed as $Q = \frac{\omega}{\Delta\omega}$ where Δω is the linewidth.

Problems

1. (i) In the context of Young's Double Slit Experiment, consider a scenario where the distance between the slits is increased. Explain how this change affects the resulting interference pattern on the screen. Provide a detailed analysis of the impact on fringe separation, fringe visibility, and overall pattern characteristics. Additionally, discuss the underlying wave nature of light and the key factors influencing the observed pattern. Finally, evaluate the practical implications of these changes in the experiment on the precision and accuracy of measurements. Justify your evaluation based on the fundamental principles of wave optics and the mathematical expressions governing interference in the double-slit configuration. **(5) [BL 5]**

   [**Hint:** This question requires students to recall and understand the basic principles of Young's Double Slit Experiment, apply their knowledge to predict the effects of changing parameters, analyze the consequences in terms of interference patterns, and ultimately evaluate the broader implications of such changes on experimental outcomes.]

(ii) A film of oil (refractive index – 1.7) is formed between a plane glass plate and an equi-convex lens (refractive index of both may be taken as 1.5). The focal length of the lens is 1 m. Estimate the radius of 10^{th} dark ring when light of wavelength 600 nm falls normally on the combination **(5) [BL 5]**

[**Hints:** Use $r_m^2 = \frac{m\lambda R}{n}$. Here m = 10, $\lambda = 600 \times 10^{-9}$ m, n = 1.7. Find R from $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$. Therefore, $R_1 = R_2 = R = f = 1$ m. Thus $r_m = 1.879$ mm]

2. (i) The refractive indices of a glass for wavelengths 656.3 nm and 527.0 nm are respectively 1.6545 and 1.6635. Calculate the length of the base of a 60 degrees prism of this glass which can just resolve sodium lines of wavelengths 589.0 nm and 589.6 nm. t-length of base of prism. [**Hint:** $\frac{\lambda}{d\lambda} = t\frac{dn}{d\lambda}$] **(5) [BL4]**

   ii) Fraunhofer double slit diffraction pattern is observed in the focal plane of a lens of focal length 0.5 m. The wavelength of incident light is 500 nm. The distance between two maxima adjacent to the maximum of zero order is 5mm and the fourth order maximum is missing. Find the width of each slit and the distance between their centres. (**5) [BL 4]**

3. (i)An oil film (r.i. n=1.2) on water (r.i. n=1.33) is viewed from directly above with light of wavelength 600 nm in air. The film appears circular and has a central thickness 1μm decreasing to zero thickness at the edge. Evaluate whether the edge will appear bright or dark. How many dark rings appear in the fringe? **(5) [BL 5]**

[**Hints:** The incident rays suffer identical phase changes at the upper and lower surface of the film. So the condition for the mth bright fringe at the centre is

$2.n.d = m\lambda$ or $m = \frac{2nD}{\lambda}$. At the edge optical path difference = 0. So it appears bright (m = 0).]

(ii) Lights of wavelengths 580 nm and 450 nm are used in Young's double slit experiment. Determine the least distance from the central fringe where the bright fringe of the two wavelengths coincides. The separation between the slits (d) is 3 mm and the distance of the screen from the slits is 1 m. **(5) [BL5]**

[**Hints**: Consider nth bright fringe of wavelength 580 nm coincides with the $(n+1)^{th}$ bright fringe of wavelength 450 nm. Position of the nth bright fringe $x_n = \frac{n\lambda D}{d}$.

Calculate the value of n. Then determine the least distance from the central maxima using the expression $x_n = \frac{n\lambda D}{d}$]

4. (i) Light of wavelength 500 nm and 520 nm falls on a grating having 5000 lines/cm. If a lens of focal length 2 metres is used to form spectra on a screen, evaluate the distance between the lines (a) in the first order, (b) in the third order. **(5)[BL5]**

   [**Hints:** $a + b = \frac{1}{5000}$, $\sin\theta_1 = \frac{m\lambda_1}{(a+b)}$, $\sin\theta_2 = \frac{m\lambda_2}{(a+b)}$

   Angular separation $\Delta\theta = \theta_2 - \theta_1$

   Hence, the linear separation of the lines will be $\Delta y = f\Delta\theta$]

(ii) A star is viewed by eye at night. How large is the image formed on the retina? Assume diameter of the pupil is 5 mm and distance between the pupil and the retina is 3 cm. Given λ = 550 nm**. (5) [BL5]**

[**Hints**: $2\theta = 2\times\frac{1.22\lambda}{d}$ if L is distance between the pupil and the retina and D is diameter of the pupil, Then the diameter of the image of the star on the retina is $d_s = 2\theta\times L$]

5. (i) Consider a case of double slit diffraction, where slit width is 0.0088 cm, separation between the slits is 0.07cm and wavelength of light used 632.8 nm. Find the number of interference minima occurring between the two diffraction minima on either side of the central maxima. **(5) [BL4]**

   [**Hints:** Condition for diffraction minima: $a\sin\theta = m\lambda$, where m =1

   Condition for interference minima: $d\sin\theta = (2n+1)\frac{\lambda}{2}$]

(ii) A parallel beam of light of wavelength 500 nm is incident normally on a narrow slit of width 0.2 mm. The Fraunhofer diffraction pattern is observed on screen which is placed at the focal plane of a convex lens (placed very closed to the slit) of focal length 20 cm. Calculate the distance between the first two maxima on the screen. **(5) [BL4]**

[Hints: In single slit diffraction, condition for first order maxima is $a \sin\theta_1 = 1.43\lambda$ condition for second order maxima is $a \sin\theta_2 = 2.46\lambda$ .Calculate θ_1 and θ_2 using these conditions. The distance between first two maxima $= (\theta_2 - \theta_1)f$ where f is the focal length of convex lens.]

6. (i) Using the principles of quantum mechanics and the characteristics of a Helium-Neon (He-Ne) laser, derive an expression for the population inversion necessary for laser action. Assume a four-level laser system and consider relevant energy levels. Include a step-by-step explanation of the mathematical expressions involved. **(5) [BL5]**
[**Hints:** Begin by describing the energy levels involved in the He-Ne laser system.

Apply the principles of statistical mechanics to establish the population distribution among these energy levels.Utilize the rate equations to express the rate of change of population in each energy level. Equate the rates of change to zero to find the conditions for population inversion. Finally, derive an expression for the population inversion in terms of relevant parameters such as spontaneous and stimulated emission rates, and pumping rate.This question requires a deep understanding of quantum mechanics, statistical mechanics, and the specific characteristics of a He-Ne laser.]

(ii) A 20 mW He-Ne laser has efficiency of 2 %. Consider that all input energy is utilized to pump the atoms from the ground state to an excited state which is 15 eV above the ground state. Determine how many atoms are pumped to the excited state in one second. **(5) [BL6]**

[**Hints**: Power input = Power output/Efficiency. Number of atoms excited/second = Input power/energy difference between ground and excited states]

7. (i) Consider an optical resonator of length 15 cm. Determine the number of modes present in the wavelength range 6 nm about a central wavelength of 520 nm. **(5)** [**BL5]**

[Hint: Frequency separation between two consecutive modes = c/2L]

(ii) The energy gap between two levels corresponds to the wavelength λ = 600 nm. Calculate the ratio of populations of the two states in thermal equilibrium at room temperature. **(5) [BL5]**

[Hint: $N_2/N_1 = e^{-(E_2 - E_1)/KT}$]