SOUTHERN UNIVERSITY
OF SCIENCE AND TECHNOLOGY

Homework #4

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Course: *Modern Control and Estimation (MEE 5106)* – Professor: *Prof. Wei Zhang*
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Question 1

Probabilistic modeling of uncertainties: Two outstanding students S_A and S_B are deciding whether to accept the offer for the SUSTech PhD program.

(a) Suppose you believe (i) S_A and S_B make decisions independently; (ii) the chance that S_A accepts the offer is 0.8; (iii) the chance S_B accepts the offer is 0.6. Please construct a probability space to represent your uncertain belief, namely, find the sample space Ω (all the possible outcomes) and the probability mass function (probability mass for each outcome) so that the above three conditions are satisfied.

(b) Now assume the two students are good friends, you know that (i) if S_B accepts the offer, then S_A will for sure accept the offer ; (ii) if S_B does not accept the offer, then S_A only has 30% chance to accept the offer (also implies there is 70% chance S_A will not accept the offer given the fact that S_B does not accept the offer); (iii) the chance that neither of them accepts the offer is 35%. Please construct a probability space to represent your uncertain knowledge in this case.

Answer.

(a) From the Question 1(a), we have the $P(S_A) = 0.8$ and $P(S_B) = 0.6$, the S_A and S_B make decisions independently means that $P(S_A S_B) = P(S_A)P(S_B)$, So we have all the possible outcomes are $S_A S_B, \bar{S_A} S_B, S_A \bar{S_B}, \bar{S_A}\bar{S_B}$, the notation $\bar{S_A}$ means S_A does not accept the offer. And the probability mass function of those outcomes are :

$$
\begin{aligned}
P(S_A S_B) &= P(S_A)P(S_B) = 0.48 \\
P(S_A \bar{S_B}) &= P(S_A)P(\bar{S_B}) = 0.32 \\
P(\bar{S_A} S_B) &= P(\bar{S_A})P(S_B) = 0.12 \\
P(\bar{S_A}\bar{S_B}) &= P(\bar{S_A})P(\bar{S_B}) = 0.08
\end{aligned}
$$

(b) From the Question 1(b), we can get the probability of $P(S_A|S_B) = 1$, $P(S_A|\bar{S_B}) = 0.3$ and $P(\bar{S_A}\bar{S_B}) = 0.35$. From the **Conditional probability's** definition

$$
P(A|B) = \frac{P(AB)}{P(B)}
$$

Question 2

Conditional Probability and Expectation: Suppose X and Y are discrete random variables. X is uniformly distributed on the set $\{0, 1, \ldots n\}$, while Y is conditionally uniform on 0 through i given $X = i$, for each $i = 0, \ldots, n$.

(a) Compute the conditional mean $E(Y|X = i)$ for a general $i \leq n$.

(b) Compute $E(Y)$ by conditioning on the values of X, namely, using the formula $E(Y) = \sum_{i=0}^{n} E(Y|X = i)p_X(i)$, where $p_X(i) = Prob(X = i)$.

(c) Find the joint probability mass function (pmf) $p(i, j) \triangleq Prob(X = i, Y = j)$, for $i = 0, \ldots, n$ and $j = 0, \ldots, n$. (hint: for some pair (i, j) the joint pmf is zero. Make sure you clearly identify those).

(d) Compute the marginal $p_Y(j) = Prob(Y = j)$ for $j = 0, \ldots n$.

(e) Write a matlab function to compute the mean $E(Y)$ of Y using p_Y for $n = 100$, and compare the result with (b).

(f) Assume $n \geq 1$. Let $g(X) = 2$, if $X = 1$ or n, and $g(X) = 0$ otherwise. Compute $E(g(X)Y)$ through conditional expectation.

Answer.

(a) ssssIdentify the author of Equation below and briefly describe it in Latin.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{1}$$

Question 3

Conditional Density and Expectation

(a) Suppose that (X, Y) is uniformly distributed on the triangle $S = \{(x, y) : -6 < y < x < 6\}$. Find $E(Y|X = x)$.

(b) Let (X, Y) be two random variables with joint density function:

$$f(x, y) = \begin{cases} \frac{1}{4}(2x + y) & x \in [0, 1], y \in [0, 2] \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X)$ and $E(X|Y = 1/2)$.

(c) Let X is an arbitrary 3D random vector with density $f(x_1, x_2, x_3)$. Show that if X_1 is independent of both X_2 and X_3, then $X_1|X_3$ is independent of $X_2|X_3$. (hint: show$f(x_1, x_2|x_3) = f(x_1|x_3)f(x_2|x_3)$).

Answer.

(a) sss

Question 4

Random Vectors Let $X = \begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix}^T \in \Re^3$ be a random vector with mean $E(X) = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ and covariance $Cov(X) = \begin{bmatrix} 6 & 2 & 5 \\ 2 & 9 & 3 \\ 5 & 3 & 6 \end{bmatrix}$. Let $W = \begin{bmatrix} X_1 \\ X_3 \end{bmatrix}$.

(a) Compute $E(W)$, and $Cov(W)$

(b) Compute $Cov(W, X_2)$

(c) Let $V = \begin{bmatrix} X_2 - 1 \\ X_1 + X_3 \end{bmatrix}$. Compute $Cov(V, V)$

Answer.

(a) sss

The table below shows the nutritional consistencies of two sausage types. Explain their relative differences given what you know about daily adult nutritional recommendations.

Per 50g	Pork	Soy
Energy	760kJ	538kJ
Protein	7.0g	9.3g
Carbohydrate	0.0g	4.9g
Fat	16.8g	9.1g
Sodium	0.4g	0.4g
Fibre	0.0g	1.4g

Question 5

Listing 1: Luftballons Perl Script

```perl
#!/usr/bin/perl

use strict;
use warnings;

for (1..99) { print $_." Luftballons\n"; }

# This is a commented line

my $string = "Hello World!";

print $string."\n\n";

$string =~ s/Hello/Goodbye Cruel/;

print $string."\n\n";

finale();

exit;

sub finale { print "Fin.\n"; }
```

1. How many luftballons will be output by the Listing above?
2. Identify the regular expression in Listing and explain how it relates to the anti-war sentiments found in the rest of the script.

Answer.

1. 99 luftballons.
2. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Praesent porttitor arcu luctus, imperdiet urna iaculis, mattis eros. Pellentesque iaculis odio vel nisl ullamcorper, nec faucibus ipsum molestie. Sed dictum nisl non aliquet porttitor. Etiam vulputate arcu dignissim, finibus sem et, viverra nisl. Aenean luctus congue massa, ut laoreet metus ornare in. Nunc fermentum nisi imperdiet lectus tincidunt vestibulum at ac elit. Nulla mattis nisl eu malesuada suscipit.