

Master thesis

**Work Distribution of a Heterogeneous Library Staff - A
Personnel Task Scheduling Problem**

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Abstract

Here is where you can write your abstract. It may be very long, or it may be very short, the reason you have an abstract is for people not to be forced to read lots of crap.

But still, they will have to read your abstract. After all, the abstract is what everyone reads. . .

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Nomenclature

Most of the reoccurring definitions, symbols and abbreviations are described here.

Definitions

Plocklista	Text
Library on wheels	Text
Rostering	Text
Matheuristic	Text
Initial schedule (?) Basic schedule (?)	

Symbols

Abbreviations

Exp	Text
Info	Text
PL	Text
PTSP	Text
SMPTSP	Text

Contents

1	Introduction	1
1.1	Background	1
1.2	Problem description	1
1.2.1	Description of the daily tasks at the library	1
1.2.2	Personnel attributes	3
1.2.3	Scheduling objectives: stand-in maximization and schedule variation	5
1.3	Method	6
1.4	Topics Covered	6
2	Literature review	7
2.1	Personnel Task Scheduling Problem	7
2.1.1	Applications	9
2.2	Shift Minimisation Personnel Task Scheduling Problem	9
2.3	Tour Scheduling Problem	10
2.4	Other similar problems	12
2.4.1	Fixed Job Schedule Problem	12
2.4.2	Tactical Fixed Interval Scheduling Problem	12
2.4.3	Operational Fixed Interval Scheduling Problem	13
2.4.4	Stochastic job problems	13
2.5	Modeling soft constraints	13
2.6	Summary	15
2.7	Relevance to our problem	15
3	The mathematical model	17
3.1	Set and variable definitions	17
3.2	Objective function	19
3.3	Constraints	19
3.3.1	Demand and assignment constraints	19
3.3.2	Weekend and rotation constraints	20
3.3.3	Objective function constraints	22
3.3.4	Meeting constraints	22
4	Results from mathematical model	25
4.1	Problem studied	25
4.2	Evaluation of results	25

5	Heuristic implementations	27
5.0.1	Block creation	27
5.0.2	Block percolation	29
5.0.3	Rotation assignment	30
5.0.4	Assignment of Library on Wheels	31
5.0.5	Initial solution	32
5.0.6	Costs	32
5.0.7	Destroy	34
5.0.8	Repair	34
5.0.9	Evaluation of solution	34
5.0.10	Final phase	35
6	Task distribution approach	37
6.1	Introduction	37
6.2	Costs	37
6.3	Weekend phase	39
6.4	Weekday phase	41
6.5	Simplifications of Mathematical Model	41
6.6	Implementation	41
A	Problem definitions	45
A.1	Sets	45
A.2	Variables	46
A.3	Parameters	47
B	Flow charts	49
B.1	Weekly scheduling approach	49
B.2	Task distribution approach	50
C	Weekblock table	53

List of Figures

5.1	Illustration of how the Heuristic models have been designed . . .	27
5.2	A flow chart of appearing costs when a PL is assigned a block on a Monday, third week relative to the library schedule.	33
B.1	A flow chart of the implemented heuristic with weekly scheduling	49
B.2	Algorithm for distributing weekends.	50
B.3	Algorithm for distributing weekday tasks.	51

List of Tables

1.1	Outer tasks can be performed exclusively by librarians or by both librarians and assistants.	2
1.2	Demand of staff for the three daily tasks.	2
1.3	Demand of staff at Hageby	3
1.4	Demand of staff Library on Wheels	3
1.5	Availability schedule for a sample worker. Yellow signifies that the worker is available. In parenthesis, the weekend shift. The week rotation is 4	4
1.6	Example of a feasible first week for the sample worker.	5
2.1	PTSP variants	8
3.1	Resulting table of the function $\text{mod}_{10}(w - v + 10) + 1$	21
5.1	Soft and relaxed constraints in the task distribution approach. .	28
5.2	A generalized weekblock with all existing tasks	28
5.3	Illustration of one of the unique block appearances	29
5.4	Typical availability for a generalized worker. Yellow signifies that the worker is available. In parenthesis, the weekend shift hours. .	30

5.5	A weekend block with Desk tasks preventing any other tasks on Fridays.	30
5.6	An iteration in the destroy/repair loop showing a swap of weekends when three workers are destroyed	31
5.7	A block example to be evaluated using costs	32
5.8	Library demand at a shift and solution qualities.	33
5.9	List of all costs with description.	34
6.1	Worker costs. Costs are incremented when a worker gets a task which breaks a soft constraint.	37
6.2	Library objective functions and their cost components. Weekend and weekday objective functions are to be maximized while the worker objective function is to be minimized.	38
6.3	Worker availability placing only weekends.	40
6.4	Worker availability after placing weekends as well as evening tasks and BokB for the same week.	40
6.5	Worker demand during a week when entering weekday phase. . .	42
C.1	Number of assignable unique blocks for the workers based on their availability and qualification.	53

Chapter 1

Introduction

1.1 Background

At a library, staff absence can cause problems and result in a shortage of staff with a special competence. If a worker is unavailable a certain day due to a meeting or illness it would require a stand-in to fill the vacancy. Therefore, it is of great interest to create schedules with as many skilled stand-ins as possible to overcome such disturbances, when they occur. Furthermore, the library personnel have certain demands and preferences as to how a satisfactory working schedule should be. For instance, it is neither preferable to work more than one evening per week nor work more weekends than required.

The problem addressed in this thesis work concerns the library staff at the Central Library of Norrköping. The library has more than 40 employees and the culturally important building from the 1950's is known to all inhabitants of Norrköping. The library is open weekdays between 08:00 and 20:00 and during weekends between 11:00 and 16:00. The generous opening times also creates a challenge for the library as it requires a large pool of well coordinated personnel to keep it running. In addition, the library also provides its services to one other smaller library. This creates a challenge for the library schedulers.

1.2 Problem description

In this section, the most important features of the worker scheduling problem studied in this thesis work will be explained.

1.2.1 Description of the daily tasks at the library

The most important activity at a library is the activity directed towards the public. This includes book loan services as well as providing customers with helpful information about the resources at the disposal of the library. These activities are referred to as "outer tasks". In addition, the book collection must be maintained, the returned books must be sorted and put back to the shelves, the web page must be updated and so on. Such work is often referred to as "inner work" and this non-public work is an equally important part of the everyday activities at the library.

Three main outer task types can be identified at the library of Norrköping, working in the service counter (sv. expeditionsdiken), working in the information counter (sv. informationsdiken) and assembling books which are to be sent to other libraries according to the "fetch list" (sv. plocklista). The fetch list is a task type for which the worker is scheduled during a whole day, while the other task types have two to five hour shifts. The outer tasks can be performed by either librarians or both, as described in Table 1.1.

Table 1.1: Outer tasks can be performed exclusively by librarians or by both librarians and assistants.

Task	Description	Qualification
Service counter (Exp)	Administiring loans, library cards and the loaning machine	Librarian or assistant
Information counter (Info)	Handling questions about the library's resources.	Librarian
Fetch list (PL)	Fetching books that are to be sent to other libraries.	Librarian or assistant
Hageby (HB)	Handling librarian tasks at the filial Hageby during weekends.	Librarian
Library on Wheels (BokB)	Driving the Library on Wheels to different areas of town.	Librarian

Since the number of visitors at the library varies throughout the day and also during different days of the week, so does the demand for personnel for the three tasks. The demand of personnel for the different task types is illustrated in Table 1.2, according to figures given by the library.

Table 1.2: Demand of staff for the three daily tasks.

Day	Time	Exp demand	Info demand	PL demand ¹
Mon-Fri	08:00-10:00	2	2	1
Mon-Fri	10:00-13:00	3	3	1
Mon-Fri	13:00-16:00	3	3	1
Mon-Fri	16:00-20:00	3	3	-
Sat	11:00-16:00	3	3	-
Sun	11:00-16:00	3	3	-

As in the case with most libraries, the Central Library of Norrköping also has responsibilities that fall outside of its normal daily activities. One such responsibility is the running of a smaller library filial in Hageby, situated in a suburban area of Norrköping, during weekends. It is decided that only librarians are qualified for this task, as it implies some tasks that need certain knowledge.

Table 1.3 shows the demand of staff at Hageby. Worth noting is that it is the same person working at Hageby a weekend due to a couple of reasons. Since a worker is supposed to work both Saturday and Sunday when due for a weekend, it is more desirable to let the worker focus on the same task both days. Furthermore, since Hageby is located in a suburban, some travel distance is required to reach it. This is compensated by letting the worker at Hageby be free from work Friday evening, which otherwise is included in the weekend work.

Table 1.3: Demand of staff at Hageby

Day	Time	HB demand
Sat	11:00-16:00	1
Sun	11:00-16:00	1

Similarly, only a handful librarians are qualified for the task known as the "Library on Wheels" (sv. Bokbussen), which is a library bus that provides citizens in remoter areas of the city with books and other library services. The Library on Wheels only operates a few times a week and the schedule differs between odd and even weeks.

Table 1.4: Demand of staff Library on Wheels

Odd Week	Mon	Tue	Wed	Thu	Fri
08:00-10:00	1	0	1	1	1
16:00-20:00	1	0	1	1	0
Even Week	Mon	Tue	Wed	Thu	Fri
08:00-10:00	1	0	1	1	0
16:00-20:00	0	0	1	1	0

Apart from the outer work described above, there is also inner work at the library which sometimes need to be scheduled. One such type of inner work is meetings, as it concerns a large number of staff. Both library meetings, scheduled for the whole work force, and department meetings, scheduled only a subset of workers, exist. The library meetings are fixed in time and take place every fifth weeks on Monday mornings 8:15-10:00 while the department meetings are more flexible and take place every fifth week. For this, all department members must be available. Only three departments have scheduled meetings: the child department, the adult department and the media department.

1.2.2 Personnel attributes

At a library, the main group of workers are librarians and assistants (also janitors, cleaners, security guards etc.). The librarians make sure the library's resource collection is up to date, that the visitors find what they're looking for and perform a larger number of inner tasks. Assistants can perform most of these tasks, while others require librarian competence.

The Central Library of Norrköping currently has 39 workers, 23 of which are librarians and 16 of which are assistants. All staff have different availability for performing tasks, depending on their working hours and the amount of inner work they are in charge of. In the standard case, each worker is assigned one evening per week and once per five weeks he/she is assigned to work during the weekend. The following week after a weekend is compensated by two free days, placed according to the wishes of the worker.

Let us consider a sample worker who is qualified for the same tasks as a librarian, works full time and is also assigned to work on Wednesday evenings. The worker is also assigned to work weekend on the fourth week and has chosen to take out its days off on Thursdays and Fridays on week five. The workers availability is thus illustrated in table 1.5. The schedule repeats itself after

five weeks and illustrates only the availability for tasks but says nothing about whether the worker has been assigned any tasks or not.

All workers have a five week schedule in the same manner as the sample worker. However, in order to meet the weekend demands illustrated by Tables 1.2, 1.4 and 1.3, it is evident that not all workers can be assigned for weekend work at week 4. Instead, a worker's schedule should be seen as a *relative schedule*, which can be shifted by whole weeks. We refer to this as the *week rotation*.

The relative schedule is relative to the *overall schedule* for the library. If the sample worker for example has week rotation 1, this means the weekend week in his relative schedule would be shifted to week 1 in the overall schedule. In Table 1.5, the weekend rotation is 4.

Table 1.5: Availability schedule for a sample worker. Yellow signifies that the worker is available. In parenthesis, the weekend shift. The week rotation is 4

Week 1	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							
Week 2	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							
Week 3	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							
Week 4	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							
Week 5	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							

Considering again the sample worker's schedule, it may look as if the worker can be scheduled for tasks at any yellow shift. However, there are regulations set by the library controlling how the outer tasks are to be distributed among the workers. The purpose of the regulations is to guarantee that the worker schedules are not too unbalanced or uncomfortable.

There four basic demands of the resulting schedule. Firstly, workers are only allowed to take at a maximum one task per day. Secondly, workers only work

at most weekend per five weeks. During this weekend it is customary that you work Friday evening shift, Saturday and Sunday shift consecutively, unless you are scheduled for Hageby, which is far away and is thus compensated by not entailing Friday evening work. Thirdly, weekend work is to be compensated by days off. There are a few possible variations of the weekend compensation, but usually the worker takes one and a half or two days off the week following upon weekend work. Lastly, a worker is only allowed to work one evening per week, excluding the Friday which belongs to the weekend week.

An example of a feasible schedule for the sample worker is provided in Table 1.6. It should be noted that this schedule is created for a general worker and does not necessarily apply to all workers. Among workers, special availability restrictions due to odd-and even weeks exist and only a subset of the librarians are available for the Library on Wheels and Hageby. Also, some workers never work evening or weekend shifts.

Table 1.6: Example of a feasible first week for the sample worker.

Week 1	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)		Exp		PL			
10:00-13:00	Exp			PL			
13:00-16:00				PL	Info		
16:00-20:00			Info				

1.2.3 Scheduling objectives: stand-in maximization and schedule variation

What is investigated in this thesis is not primarily how to distribute the tasks according to the rules given above, but rather how to do this in a way so that the number of stand-in personnel is maximized. The emergency back-up system of stand-ins is crucial for the library in order to be able to keep the library desks open and fully staffed. Thus, the highest priority is to maximize the number of stand-in personnel.

A stand-in is defined as a worker who is available for outer tasks during the first three shifts, as well as not scheduled for any shift that day. Inner work sometimes prevents personnel from being scheduled as a stand-in for the outer tasks, and therefore also information about such work must be known to the scheduler. Both librarians and assistants can be scheduled as a stand-in, but only librarians are qualified to take emergency shifts at the Info desk. Since the regular library activity is the most crucial activity, there is no need for stand-ins during evening and weekends. Similarly, there are no assigned stand-ins for the Library on Wheels or for Hageby.

Apart from maximizing the number of stand-ins for each day at the library, another important measurement of a good schedule is the level of variation. It is desirable for the sake of the personnel to create schedules in which the weeks resemble each other or are repeated according to some pattern. On the other hand, since some shifts are more attractive than others and in order to avoid too much repetitiveness it is desirable that the days during a work week are not too similar. The example schedule in Table 1.6 is an example of a sufficiently varied weekly schedule.

1.3 Method

1.4 Topics Covered

Chapter 2

Literature review

The scheduling problem is a mathematical optimization problem which has been studied since the 1950s and concerns creating a feasible and satisfactory schedule for workers or machines performing tasks. One of the most extensive overviews in the area is provided by Ernst et al. (2004). They state that, although the complexity of the scheduling problem has not increased in recent years, the mathematical models used to solve the scheduling problems have become more realistic and refined. Modern scheduling problems often concern the distribution of tasks and the creation of worker shifts, also taking softer values into account, such as worker satisfaction and worker fatigue. Due to this modeling refinement as well as the development of more powerful computational methods, it is possible today to solve scheduling problems in a more satisfactory way than before.

In this chapter, the scheduling problem is classified into different subcategories which are areas related to the thesis work presented in this paper. The relevant areas for our work include Personnel Task Scheduling Problems (PTSP), Shift Minimization Task Scheduling Problems (SMTSP), Tour Scheduling Problems (TSP) and a few variations of these. Also problems featuring soft constraints will be studied in an additional section. Models and solution methods of these problems will be discussed under each headline and a summary will be provided at the end together with a discussion about the relevance of the studied articles to our thesis work.

2.1 Personnel Task Scheduling Problem

In many real life situations production managers will face the Personnel Task Scheduling Problem (PTSP) while scheduling service operations. Krishnamoorthy and Ernst (2001) writes in their article that the PTSP occurs when the rosterer or shift supervisor need to allocate tasks with specified start and end times to available personnel who have the required qualifications. It also occurs in situations where tasks of fixed times shall be assigned to machines. Decisions will then have to be made regarding the amount of maintenance workers needed and which machine the workers are assigned to look after.

There are numerous variants to the PTSP. Studies on these have been made by Krishnamoorthy and Ernst (2001) who gives a list of attributes that com-

monly appear in a PTSP, which are listed in Table 2.1. Furthermore, there are traits that always appear in a PTSP; tasks with fixed start and end time are to be distributed to staff members that possess certain skills, allowing them to perform only a subset of the available tasks. Start and end time of their shifts are also predetermined for each day.

One variant, which also is the most simple, is mentioned in Krishnamoorthy and Ernst (2001) and is called the *Feasibility Problem* where the aim is to merely find a feasible solution. This requires that each task is allocated to a qualified and available worker. It is also required that a worker cannot be assigned more than one task simultaneously as well as tasks cannot be pre-empted, meaning that each task has to be completed by one and the same worker.

In Table 2.1 attributes of PTSP variants are shown. The nomenclature of the attributes T, S, Q and O refer to the *Task type*, *Shift type*, *Qualifications* and *Objective function* respectively.

Table 2.1: PTSP variants

Attribute	Type	Explanation
T	F	Fixed contiguous tasks
	V	Variable task durations
	S	Split (non-contiguous) tasks
	C	Changeover times between consecutive tasks
S	F	Fixed, given shift lengths
	I	Identical shifts which are effectively of infinite duration
	D	Maximum duration without given start or end times
	U	Unlimited number of shifts of each type available
Q	I	Identical qualification for all staff (homogeneous workforce)
	H	Heterogeneous workforce
O	F	No objective, just find a feasible schedule
	A	Minimise assignment cost
	T	Worktime costs including overtime
	W	Minimise number of workers
	U	Minimise unallocated tasks

Many of the most basic and a few more complex problems can be described with this definition of PTSP attributes. It is, however, not possible to describe all of the numerous types of PTSP using these nomenclatures according to Krishnamoorthy and Ernst (2001).

By combining attributes it is possible to obtain more complex variants of the PTSP. An example would be the PTSP[F;F;H;A-T-W] mentioned in Krishnamoorthy and Ernst (2001), where multiple objectives are used. This problem has fixed contiguous tasks, fixed shift lengths, heterogeneous workforce and three objective functions (A-T-W), which represent assignment costs, work time with overtime included and requirements to minimize the number of workers respectively. The objective function for this problem is then a linear combination with parameters used to weigh (prioritize) them against each other.

Given the nomenclature above, our problem would be most related to the PTSP[F;F;H;F]. The difference is that the objective function is not empty. We are looking to maximize the number of qualified stand-ins each day as well as maximize employee satisfaction by meeting their recommendations. We have

a fixed number of workers, no costs and no unallocated tasks when a feasible solution is found. Therefore, none of the objective function types given in Table 2.1 are relevant in our case.

Some variants of the PTSP are given names in the literature, which is stated by Krishnamoorthy and Ernst (2001). An example is when the shifts and qualifications are identical ($S=I$, $Q=I$) and the objective function is to minimize the number of workers that are used ($O=W$). This variant, $PTSP[F;I;I;W]$, has been published as the Fixed Job Schedule Problem and is described in Section 2.4.

2.1.1 Applications

An example where the PTSP can be found is when developing a rostering solution for ground personnel at an airport. This is mentioned in the article by Krishnamoorthy and Ernst (2001). The problem can be dealt with by first assigning the workers to days in order to satisfy all the labour constraints, followed by assigning the tasks to the scheduled workers.

In the article mentioned above, three further problems of type PTSP, related to airplanes are mentioned. They occur when scheduling for either airport maintenance staff, planes to gates or staff that do not stay in one location, such as airline stewards. Scheduling for airport maintenance staff can lead to either $PTSP[F;I;H;U-A]$ or $PTSP[F;I-U;H;W]$, which are similar problems but given two different names: Operational Fixed Interval Scheduling Problem and Tactical Fixed Interval Scheduling Problem respectively. These are both described further in Section 2.4.

Another application, which has been frequently studied, is classroom assignments and is discussed in Krishnamoorthy and Ernst (2001). Based on specifications such as the amount of students in a class or the duration of a class, different classrooms have to be considered. Requirements of equipment, e.g. for a laboratory, may also greatly limit the available classrooms to choose from. A majority of the complications of this problem is due to the fact that lessons can span over multiple periods.

Worth noting for classroom assignment problems is that there are no start or end times for the shifts, as they represent the rooms. The aim in the present problem would be to simply find a feasible assignment of classrooms. Therefore, the nomenclature of the problem would be $PTSP[S;I;H;F]$, with the possibility of adding preferences to the objective function. An example of a preference would be to assign the lessons as close to each other as possible on a day, preventing travel distances for teachers and students.

2.2 Shift Minimisation Personnel Task Scheduling Problem

A close relative to the PTSP is the Shift Minimisation Personnel Task Scheduling Problem (SMPTSP), which is a variant where the aim is to minimize the cost occurring due to the number of personnel that are used. The same common traits for this problem are mentioned in the article Krishnamoorthy et al. (2012) as in the PTSP; workers with fixed work hours are to be assigned tasks with specified start and end times, for which they are qualified.

In the same article as above the authors "... concentrate mainly on a variant of the PTSP in which the number of personnel (shifts) required is to be minimised.". In doing so, it is possible to determine the lowest number of staff and the mix of staff a company need in order to be able to complete the tasks and still be operational. They also presumed that the pool of workers is unlimited for each skill group, which is not the case in the present problem due to the limitations of librarians and assistants.

Furthermore, it is said in Krishnamoorthy et al. (2012) that SMPTSP can be applied when there is a large number of workers available with different qualifications and it is needed to ensure that the tasks for that day are performed. The PTSP and SMPTSP are therefore useful day-to-day operational management tools and commonly occur in practical instances where tasks are allocated on a daily basis.

It is shown in Kroon et al. (1995) that SMPTSP is a complex problem even if the preemption constraint were to be removed. However, it is stated in Krishnamoorthy et al. (2012) that if the qualifications of the workers were identical it would be an easily solvable problem. The difference in publication year between the articles indicates that this problem has become less challenging in recent years.

During the last decade, a couple heuristics have been implemented to deal with the SMPTSP. One method introduced by Krishnamoorthy et al. (2012) is a Lagrangean relaxation approach that combines two problem specific heuristics: Volume Algorithm (VA) and Wedelin's Algorithm (WA). These heuristics exploit the special structure of the SMPTSP by relaxing some of the harder constraints into the objective function, thus being a problem specific heuristic. What remains after the relaxation is a problem decomposed into several independent problems which can be solved independently. A solution to these decomposed problems is discussed further in Krishnamoorthy et al. (2012).

Another way to solve the SMPTSP is to use a very large-scale neighbourhood search algorithm, as was presented by Smet and Vanden Berghe (2012). The main purpose of their implemented hybrid local search is to repeatedly fix and optimize to find neighbouring solutions. Using this method on 137 benchmark instances introduced by Krishnamoorthy et al. (2012), Smet and Vanden Berghe managed to find 81 optimal solutions compared to Krishnamoorthy et al. who only managed to find 67. Though, both methods found feasible solutions for 135 out of the 137 problem instances. This comparison is presented in Smet et al. (2014).

Smet et al. (2014) introduced a third and most effective method to solve the benchmark instances related to the SMPTSP. By using a versatile two-phase matheuristic approach, solutions to all 137 benchmark instances could be found for the first time. The procedure used in their implementation is to first generate an initial solution by using a constructive heuristic, followed by improving the solution using an improvement heuristic. Which constructive and improvement heuristic they considered can be seen in their article.

2.3 Tour Scheduling Problem

The Tour Scheduling Problem (TSP) is described by Loucks and Jacobs (1991) as a combination of shift scheduling and days-off scheduling. Shift scheduling

refers to creating sets of contiguous hours during which a worker is assigned for work. The need for days off scheduling typically occurs when the time horizon for scheduling is weekly or more and when weekend staff is needed. Using the notation used described in Section 2.1, this would be classified as a variant of the PTSP[F;D;I;W] or PTSP[F;D;H;W], depending on if the workforce is homogeneous or heterogeneous.

According to Loucks and Jacobs (1991), the vast majority of all tour scheduling problems up to 1991 involve a homogeneous workforce, that is, any worker can perform any assigned task. One such early study of the scheduling problem often mentioned in literature is provided by Thompson (1988). The problem studied in this PhD thesis concern only homogeneous workforces and the task assignment part is not considered.

In the article by Loucks and Jacobs (1991), the authors study a tour scheduling problem with a heterogeneous workforce. The problem both involves tour scheduling and task assignment, where the latter part is most interesting to us. The problem is studied in the context of fast food restaurants, where certain personnel is qualified only for certain stations in the restaurant. In such industries, the demand of staff differs between weekdays and times of the day. Two worker attributes are considered, their availability for work and their qualifications to perform different tasks. The problem concerns finding shifts for all workers which are to be assigned a length between a minimum and maximum number of hours per day.

The representative problem studied in the article involves creating a one-week schedule for 40 workers in a fast food restaurant, available for eight different tasks with a seven-day and 128-hour workweek. Several synthetic problems are studied in the article; all, however, with a minimum shift length of three hours, a maximum shift length of eight hours and a maximum of five work days.

A similar problem to the one described by Loucks and Jacobs is studied by Choi et al. (2009). They focus on a particular fast food restaurant in Seoul, which is made a representative of fast food chains in general. In this study, only two types of workers are available, fulltime and part time workers, with no other reference to difference in skill. The different shifts are already given by the restaurant managers and the task is to combine them into a tour. The task assignment aspect is lacking in this article.

In both articles the main objective is to minimize both overstaffing and understaffing, which will both have economical consequences for the fast food chain. This is done by reducing or increasing the workforce. For a problem with a fixed workforce, such as ours, this objective function is not relevant. In the example studied by Loucks and Jacobs there is also a goal to meet staff demand on total working hours. This is modeled as a secondary goal and is similar to our goal and somehow models a "soft" value, which is of interest to us.

A more recent TSP concern monthly tour scheduling, as opposed to most literature which concerns only weekly scheduling. Such a study was done by Rong (2010). The main advantage of monthly scheduling over shorter time periods, as stated in the article, is the possibility to plan a schedule with respect to fairness and balance over a longer period of time. The problem concerns workers with different skills, where each worker also can possess multiple skills. This is referred to as a mixed skill problem. Thus, the problem is similar to our problem, where mixed skill is also present. In the study, workers have individual weekend-off requirements. The problem does not involve task assignment, which

makes it less relevant for us.

The solution methods used to solve the TSP differs greatly between the articles studied. In the older articles, such as Thompson (1988) and Loucks and Jacobs (1991), custom made algorithms very similar to the methods used in manual scheduling are proposed to solve the problem. These solution methods involve classifying staff and distributing them according to some rule (for example, the staff with the most scarce skill is assigned first). General commercial solvers are not proposed, due to lack of efficiency during these times.

In Hojati and Patil (2011), the same model and data is used as in Loucks and Jacobs (1991). Also this article states that commercial solvers are insufficient for solving the problem. Instead two methods are proposed, one which decomposes the problem into two problems solvable by commercial solvers and one customized heuristic method based on the Lagrangian relaxation method. This method solves the problem with more satisfactory results than in Loucks and Jacobs (1991) as explained in the article. In Choi et al. (2009), a pure integer programming method is used.

2.4 Other similar problems

In this section a couple of problems, similar to our own, will be described. The focus will not be on the variety of methods used to solve these problem; instead, the focus will be to give clarity to how closely related many of these problems are.

2.4.1 Fixed Job Schedule Problem

Variations of the task assignment problem relevant to our problem include for example the Fixed Job Schedule Problem (FJSP). The FJSP has been studied since the 1970s in the context of task assignment in processors. According to Krishnamoorthy et al. (2012), the problem concerns the distribution of tasks with fixed starting and ending times over a workforce with identical skills, such as processing units. Such problems have been solved by Gertsbakh and Stern (1978) and Fischetti et al. (1992).

In the article by Gertsbakh and Stern (1978) a situation is studied where n jobs need to be scheduled over an unlimited number of processors. The objective function of such a problem becomes to minimize the number of machines needed to perform all tasks. Fischetti solves a similar problem, but adds time constraints, saying that no processor is allowed to work for more than a fixed time T during a day as well as a spread time constraint forcing tasks to spread out with time gap S over a processor.

2.4.2 Tactical Fixed Interval Scheduling Problem

The Tactical Fixed Interval Scheduling Problem (TFISP) is a problem very closely related to the SMPTSP with the sole difference that the TFISP concerns workers which always are available, such as industrial machines or processors. The problem is studied in Kroon et al. (1995). A typical TFISP can be expressed using the nomenclature in Table 2.1 and written as PTSP[F;I-U;H;W].

Opposed to the FJSP, the TFISP deals with a heterogeneous workforce. Two different contexts are studied by Kroon et al. (1995). One of them concerns the handling of arriving aircraft passengers at an airport. Two modes of transport from the airplane to the airport are investigated, directly by gate or by bus. The two transportation modes thus correspond to two processing units, which can only handle a number of jobs at the same time.

2.4.3 Operational Fixed Interval Scheduling Problem

The Operational Fixed Interval Scheduling Problem (OFISP) is a close relative to TFISP, where both types are restricted by the following: Each machine (worker) cannot handle more than one job at a time, can only handle a subset of the jobs and preemption of jobs is not allowed. The difference between them occurs in the objective function, as described in Kroon et al. (1995). TFISP tries to minimize the number of workers, while OFISP tries to minimize the operational costs and the number of unallocated tasks using priority indices. In the present nomenclature this would give rise to the problem PTSP[F;I;H;U-A].

Given the problem definition above, working shifts are to be created for the workers and tasks have to be allocated on a day-to-day basis. OFISP, studied by Kroon et al. (1995), can therefore be seen both as a job scheduling problem and a task assignment problem.

2.4.4 Stochastic job problems

What differs mostly between the problem types described above and the problem studied in this thesis work, is the difference in objective function. The main objective function is often to minimize staff for a fixed number of jobs, not taking stand-in assignment into account.

An area where the need for stand-in personnel appears is in the maintenance industry, where some jobs can be foreseen and other (emergency) jobs are of a stochastic nature, that is, there is a probability that such jobs will occur a certain hour. The problem combining both foreseen and stochastic maintenance worker scheduling was studied by Duffuaa and Al-Sultan (1999), as a continuation of the work by Roberts and Escudero (1983).

In the article, a fixed, heterogeneous workforce consisting of electricians, plumbers and mechanics is studied. The shifts of the personnel are predetermined by their given work times and thus the problem becomes a pure task assignment problem. An objective function is used where the goal is to maximize the number of planned and unplanned jobs performed by the workers, by taking into account the probability of unplanned work to occur. Thus, certain workers will be left at the station as stand-ins in the case an unplanned job arises. The commercial solver LINDO was used to solve the problem.

2.5 Modeling soft constraints

For most scheduling problems, the main objective is to minimize worker-related costs by reducing the number of workers needed to perform a task, or by reducing the working time for part-time employees. Recently, however, many studies have started to focus more on softer values such as worker satisfaction as an objective

function. Such values are usually considered when scheduling is done manually, but have been forgotten or set aside in mathematical modeling.

In an article by Akbari et al. (2013) a scheduling problem for part-time workers with different preferences, seniority level and productivity is investigated. In this article, these aspects are reflected in the objective function and weighted against each other. A similar problem was also studied by Mohan (2008), but for a workforce of only part-time workers.

Other factors which may affect worker satisfaction, and in the long run efficiency and presence at work are fatigue, fairness and boredom. These are discussed by Eiselt and Marianov (2008). Repetitiveness of a job as well as the level of challenge can cause boredom of workers. Increasing variance in the schedule is done by Eiselt and Marianov (2008) through providing an upper bound of how many tasks can be performed in a given time span. The article suggests a sort of measurement of the distance between the task requirements and the worker abilities used. This will then be minimized in the objective function.

Another modeling method which is relevant specifically for scheduling problems featuring soft constraints is fuzzy goal programming. The method is discussed by Shahnazari-Shahrezaei et al. (2013), who model soft constraints as "fuzzy goals". These goals can become contradictory, for example could a preference of high seniority level workers come in conflict with a preference in working hours by an employee. The article uses fuzzy set theory, and uses a solution approach involving Li's two-phase method (Li (1990)). Soft constraints are modeled as trapezoid functions and an optimal solution with the best average value of all functions is found.

The solution methods proposed by Akbari et al. (2013) to solve a scheduling problem with soft constraints are two metaheuristics: Simulated Annealing (SA) and Variable Neighbourhood Search (VNS). According to Akbari et al. (2013), SA has been studied as a solution method for the scheduling problem since the 1990s and many studies have shown that it is capable of providing near-optimal solutions in a short time compared to optimal integer-programming models for a variety of problems. An exponential cooling time was used for the algorithm and it was concluded that it was faster than the commercial solver LINGO in finding a solution.

VNS is the other proposed method by Akbari et al. (2013). A big difference between this and other methods is that VNS requires very little parameter tuning while often providing good solutions. Larger movement sizes are used at higher temperatures, and smaller in cooler temperatures. The method uses both a random and a systematic phase. In the random phase, worker schedules are regenerated randomly and better solutions are saved. In the systematic phase, two shifts are swapped. In the article, it was concluded that VNS could solve the problem faster than the implemented SA heuristic.

In the article by Eiselt and Marianov (2008), a commercial solver was used. This was also the case in Mohan (2008), although the article compares these results with the results obtained from a branch-and-cut algorithm. Also Shahnazari-Shahrezaei et al. (2013) uses a commercial solver.

2.6 Summary

In order to get an overview over the problems discussed in this chapter it is a good idea to look at general historical trends among the problems. One clear trend is the shift from problems concerning homogeneous to heterogeneous workforces. As stated by Krishnamoorthy et al. (2012), the problem of heterogeneous workforces was trivial at the publishing year, while in Loucks and Jacobs (1991), it is introduced as a rather new and challenging concept. Furthermore, the articles concerning the different PTSPs are mainly from the 1980s and 1990s and were challenging in their structure during these times. Some of them, such as FJSP, are related to the scheduling in processors, which was a hot topic at the start of the computer era. All the articles about modeling of soft constraints are presented after the year 2000, probably as a result of better computational powers.

The various solution methods found in the studied articles include commercial solvers, heuristics and matheuristics. Heuristics which have been studied in the articles include SA, VNS and Lagrangian relaxation, with and without VA. Also some local search-based heuristics have been used. Commercial solvers are used increasingly in newer articles such as Hojati and Patil (2011) and Mohan (2008), probably due to the improvement in performance in such solvers. Similarly, matheuristics are also discussed mostly in more recent articles, such as Akbari et al. (2013).

2.7 Relevance to our problem

As described in Section 2.1, many different types of personnel tasks scheduling problems exist. Only a few of them have been discussed in this chapter, of which some are closer related to our problem than others. In order to get a better understanding for how they are related to our problem, a few connections between them will be presented in this section.

Using Table 2.1, the closest classification of the librarian scheduling problem studied in this thesis work would be the $PTPS[F;F;H;F]$. This describes a personnel task scheduling problem with fixed tasks, fixed shift lengths, a heterogeneous work force and an empty objective function. As stated in Section 2.1, the main difference lies in the objective function, as ours is not empty. Thus, many problem types discussed in this chapter concern problems which are similar to ours but with a different objective function. This includes the SMPTSP, the TSP, the FJSP, the TFISP and the OFISP. The first two are relevant to our problem as they concern task assignment and, in the case of TSP, days off scheduling. However, both involve shift scheduling while we have fixed shifts. The other three problems concern only task assignment, but are otherwise further from our problem since the tasks types are different from ours.

As stated in Section 2.1, our problem has an objective function in which the number of stand-in personnel is to be maximized. Such problems often arise in the maintenance job scheduling problem, as is discussed in Section 2.4.4. As a secondary objective, we are also interested in making the schedule varied and with a repeating pattern at an interval of five weeks. Modeling of such constraints is discussed in Section 2.5. The two sections are complementary to the previous sections, as they are less relevant in the overall problem formulation

but more relevant concerning the objective function.

Chapter 3

The mathematical model

In this chapter the mathematical model implemented to solve this problem will be presented. Prior to the objective function and constraints, the most significant sets and variables will be provided to give the reader some basic knowledge of the implemented model. Section 3.2 presents the objective function and gives a short description of what it represents. In Section 3.3 the essential constraints will be presented and explained. A complete model with all definitions and the full set of constraints can be found in Appendix A.

3.1 Set and variable definitions

To solve the problem many sets and variables had to be declared. There are many unique and individual requirements, given by the library, that have to be met. An example is that some personnel want a day free from outer tasks to be able to attend meetings or perform inner tasks. Another example is that some have two alternating schedules for odd and even weeks. These specific cases have to be modeled and result in a variety of set and variable definitions. Hence, only the most important ones are listed below. A complete list of the definitions can be found in Appendix A.

I	Set of workers
I_{lib}	Set of librarians ($I_{lib} \subseteq I$)
I_{ass}	Set of assistants ($I_{ass} \subseteq I$)
W	Set of all ten weeks
W_5	Set of first five weeks
D	Set of all days in a week
D_5	Set of all weekdays
S_d	Set of shifts available day d
S_3	Set of first three shifts on a weekday
J_d	Set of task types available day d

In order to further define the problem we introduce the following variables:

Let,

$$x_{iwdsj} = \begin{cases} 1, & \text{if worker } i \text{ is assigned in week } w, \text{ day } d, \text{ shift } s \text{ to a task } j \\ 0, & \text{otherwise} \end{cases} \quad (3.1)$$

$$H_{iwh} = \begin{cases} 1, & \text{if worker } i \text{ works weekend } h (= 1, 2) \text{ in week } w \\ 0, & \text{otherwise} \end{cases} \quad (3.2)$$

$$r_{iw} = \begin{cases} 1, & \text{if worker } i \text{ has its schedule rotated } w-1 \text{ steps} \\ 0, & \text{otherwise} \end{cases} \quad (3.3)$$

$$l_{iwd} = \begin{cases} 1, & \text{if librarian } i \text{ is a stand-in week } w, \text{ day } d \\ 0, & \text{otherwise} \end{cases} \quad (3.4)$$

$$a_{iwd} = \begin{cases} 1, & \text{if assistant } i \text{ is a stand-in week } w, \text{ day } d \\ 0, & \text{otherwise} \end{cases} \quad (3.5)$$

$$y_{iwd} = \begin{cases} 1, & \text{if worker } i \text{ works week } w, \text{ day } d, \text{ shift } s \text{ at task type E, I or P} \\ 0, & \text{otherwise} \end{cases} \quad (3.6)$$

$$W_{iwd} = \begin{cases} 1, & \text{if a worker } i \text{ is working a shift week } w, \text{ day } d \\ 0, & \text{otherwise} \end{cases} \quad (3.7)$$

$$b_{iw} = \begin{cases} 1, & \text{if worker } i \text{ works at H week } w \\ 0, & \text{otherwise} \end{cases} \quad (3.8)$$

$$f_{iw} = \begin{cases} 1, & \text{if worker } i \text{ is assigned to work friday evening week } w \\ 0, & \text{otherwise} \end{cases} \quad (3.9)$$

$$M_{wds} = \begin{cases} 1, & \text{if a big meeting is placed on week } w, \text{ day } d, \text{ shift } s \\ 0, & \text{otherwise} \end{cases} \quad (3.10)$$

$$m_{wdsD} = \begin{cases} 1, & \text{if a meeting is placed on week } w, \text{ day } d, \text{ shift } s \text{ at department } D \\ 0, & \text{otherwise} \end{cases} \quad (3.11)$$

$$d_{iwd} = \begin{cases} 1, & \text{if there is a difference in assignment of tasks at a shift } s, \text{ for a worker } i, \text{ day } d \text{ between week } w \text{ and } w+5 \\ 0, & \text{otherwise} \end{cases} \quad (3.12)$$

$$l^{min} = \text{lowest number of stand-in librarians found (integer)} \quad (3.13)$$

$$a^{min} = \text{lowest number of stand-in assistants found (integer)} \quad (3.14)$$

$$s^{min} = \text{weighted sum of number of stand-in librarians and assistants} \quad (3.15)$$

Based on the variables defined above it has been possible to solve our scheduling problem. l^{min} and a^{min} are the variables of most significance as they represent the number of stand-ins found after a run.

3.2 Objective function

Due to multiple objective functions it has been necessary to weigh them against each other using parameters. These are shown in Equation 3.16 below as M and N .

$$\text{maximize } M \cdot s^{\min} - N \cdot \sum_{i \in I} \sum_{w \in W_5} \sum_{d \in D_5} \sum_{s \in S_3} d_{iws} \quad (3.16)$$

The two first objective functions represent the lowest amount of stand-in librarians and assistants found. The third objective function is a preference from the library that two weeks with a five-week interval should be as similar as possible (e.g. week 1 and 6, 2 and 7 etc.).

The parameter N prioritizes the similarity of weeks compared to the number of stand-ins. Based on the information given by the library it is a much higher priority to have many stand-ins. Hence, $N \ll L, A$ in our case.

If $L < A$ in the model the solver would prioritize assistants over librarians as stand-ins. Librarians are, however, more desired as stand-ins, due to their ability to perform all task types. Therefore, it is desired to set $L \geq A$.

3.3 Constraints

To model the present problem it has been of relevance to divide many of the constraints into weekend- and weekday constraints. Several help constraints have also been added to avoid multiplication of two variables, making the problem non-linear.

3.3.1 Demand and assignment constraints

The most crucial constraint is to ensure that the demand of workers is met each day. This can be modeled as:

$$\sum_{i \in I} x_{iwsj} = \text{demand}_{wsj}, \forall w \in W, d \in D, s \in S, j \in J_d \quad (3.17)$$

demand_{wsj} is an integer representing the number of workers required week w , day d , shift s for a task j .

The following constraint says whether a worker is assigned a task during a weekday where the evening shifts and task type *Library on Wheels* are excluded:

$$y_{iws} = \sum_{j \in J_d \setminus \{B\}} x_{iwsj}, \forall i \in I, w \in W, d \in D_5, s \in S_3 \quad (3.18)$$

The variable assignment above is used to simplify a couple of constraints.

To ensure that no worker is assigned more than one task the following constraint is implemented:

$$\sum_{s \in S} \sum_{j \in J_d} x_{iwsj} \leq 1, \forall i \in I, w \in W, d \in D \quad (3.19)$$

However, if we allow a person to have two shifts at the Library on Wheels on a day, Equation 3.19 has to be slightly modified, which is shown in the complete model in Appendix A.

It is preferred to allow only one PL per week and a maximum of three PL per ten weeks. These are easily modeled with the following constraints:

$$\sum_{s \in S_d} \sum_{d \in D} x_{iwsP} \leq 1, \forall i \in I, w \in W \quad (3.20)$$

$$\sum_{w \in W} \sum_{s \in S_d} \sum_{d \in D} x_{iwsP} \leq 3, \forall i \in I \quad (3.21)$$

The duration of a PL (in the model named 'P'), which is from 08:00-16:00, is the cause of these preferences. Some workers are required to have some allotted time to perform inner tasks.

Another preference is to have various start times of the assigned tasks each week, so that the more and less desired shifts are evenly distributed. The following equation is implemented to meet such requirements:

$$\sum_{d \in D_5} y_{iws} \leq 2, \forall i \in I, w \in W, s \in S_3 \quad (3.22)$$

Equation 3.22 allows a worker to have at most two tasks starting at the same hour. Worth noting is that task type *Library on Wheels* is disregarded in the constraint as the variable y_{iws} is used; see Equation 3.18 for the definition.

It is desirable to avoid assigning too many tasks to a worker. Reason for this is to let the worker have some time to allot for inner services during weekdays. The equation below models this preference:

$$\sum_{d \in D_5} \sum_{s \in S_d} \sum_{j \in J_d} x_{iwsj} \leq 4, \forall i \in I, w \in W \quad (3.23)$$

Equation 3.23 allows a worker at most four shifts per week.

3.3.2 Weekend and rotation constraints

To further model the problem weekends and week rotations have to be considered. The following three constraints are the most basic constraints regarding weekends H_{iwh} and week rotations r_{iw} :

$$\sum_{w \in W} r_{iw} = 1, \forall i \in I \quad (3.24)$$

$$\sum_{w \in W} H_{iwh} \leq 1, \forall i \in I, h = 1, 2 \quad (3.25)$$

$$r_{iw} \geq H_{iw1}, \forall i \in I, w \in W \quad (3.26)$$

Equation 3.24 provides all workers with a rotation of their schedule regardless if they are working weekends or not. Equation 3.25 allows a worker a maximum of two weekends ($h = 1, h = 2$) per ten weeks. Lastly, Equation 3.26 in combination with Equation 3.24 ensures that the schedule rotation is aligned with the first weekend, if the worker is due for weekend work. In case the worker is not due for weekend work, the rotation of the schedule is free.

A worker is supposed to work weekends with a five-week interval. However, in case when there are enough workers to satisfy the demand on weekends it

may be enough to work one weekend per ten weeks for some. To avoid problems with the rotation when only the second weekend is assigned to a worker, the following equation is implemented:

$$r_{i(10(w+4)+1)} \geq H_{iw2}, \forall i \in I, w \in W \quad (3.27)$$

Worth noting is that if a worker is assigned both weekends then Equation 3.27 provides the same information as Equation 3.26.

Every workers schedule is able to rotate up to nine times, where the decision variable r_{iv} decides the rotation. Therefore, the parameter $qualavail_{iwsdj}$ has to align with the rotation so that all workers are assigned tasks only when available. This is provided in the following equation:

$$x_{iwsdj} \leq \sum_{v \in V} r_{iv} * qualavail_{i(mod_{10}(w-v+10)+1)dsj}, \forall i \in I, w \in W, d \in D, s \in S_d, j \in J_d \quad (3.28)$$

Table 3.1 gives a better understanding of how the modulus function in the parameter $qualavail_{iwsdj}$ is used. w represent the current week in the relative schedule and v represent $v-1$ rotations to the right from the relative schedule. The resulting week after rotations can be seen in the cells.

Table 3.1: Resulting table of the function $mod_{10}(w - v + 10) + 1$

		w =									
		1	2	3	4	5	6	7	8	9	10
v =	1	1	2	3	4	5	6	7	8	9	10
	2	10	1	2	3	4	5	6	7	8	9
	3	9	10	1	2	3	4	5	6	7	8
	4	8	9	10	1	2	3	4	5	6	7
	5	7	8	9	10	1	2	3	4	5	6
	6	6	7	8	9	10	1	2	3	4	5
	7	5	6	7	8	9	10	1	2	3	4
	8	4	5	6	7	8	9	10	1	2	3
	9	3	4	5	6	7	8	9	10	1	2
	10	2	3	4	5	6	7	8	9	10	1

An example to better understand the function: Imagine that we are looking at an unrotated relative schedule where everyone is said to work weekend the first week. Say that we look at a workers first week, $w = 1$. If this schedule is rotated two times to the right, $v = 3$, then the first week in the new rotated schedule represent the previous ninth week, $w = 9$ from the old schedule.

A worker is supposed to work with the same task both Saturday and Sunday when working a weekend. This can be modeled with the following constraints:

$$\sum_{j \in J_d} x_{iw61j} + \sum_{j \in J_d} x_{iw71j} = 2 * \sum_{h=1}^2 H_{iwh}, \forall i \in I, w \in W \quad (3.29)$$

$$x_{iw61j} = x_{iw71j}, \forall i \in I, w \in W, j \in J_d \quad (3.30)$$

Friday evening is also included in extension to working Saturday and Sunday, unless the worker is assigned to HB. It is, however, not a necessity to perform

the same task Friday evenings as during the weekend, thus Fridays not included in Equation 3.30. Equation 3.31 below adds Fridays to the weekend:

$$\sum_{j \in J \setminus \{L\}} x_{iw54j} = f_{iw}, \forall i \in I, w \in W \quad (3.31)$$

'L' in the equation refers to the task Library on Wheels. The variable f_{iw} is, as mentioned in the variable declaration, equal to one if and only if a worker is working weekend as well as not being assigned to HB. The help constraints implemented to avoid multiplication of the two variables b_{iw} and H_{iwh} are left out of this section and can instead be seen in Appendix A.

It is of interest to implement a constraint preventing workers being assigned to HB more than once every ten weeks to avoid unfairness. The constraint can be modeled as:

$$\sum_{w \in W} \sum_{d=6}^7 x_{iwd1B} \leq 2, \forall i \in I \quad (3.32)$$

'B' in this equation refers to HB. The combination of Equations 3.30 and 3.32 ensure that once a worker is assigned to HB it will be for two consecutive weekend days, which is the amount of days the last mentioned equation allow a worker every ten weeks. Why this is preferable is described in Section 1.2.1.

3.3.3 Objective function constraints

To calculate the variables s^{min} used in the objective function, 3.16, the following equation is implemented:

$$s^{min} \leq L \cdot \sum_{i \in I_{lib}} l_{iwd} + A \cdot \sum_{i \in I_{ass}} a_{iwd}, \forall w \in W, d \in D_5 \quad (3.33)$$

l_{iwd} and a_{iwd} are binary variables stating, for a day d , whether a worker is a stand-in or not. s^{min} is being maximized in the objective function; therefore, it will assume the lowest value of the sum of stand-in librarians and assistants for any day during the ten weeks. Just as with previous equation, help constraints have been left out for simplicity reasons. In this case regarding how l_{iwd} and a_{iwd} are determined.

As stated in Section 3.2, two weeks with a five-week interval should be as similar as possible. Equation 3.34 in combination with the objective function equation, 3.16, provides this preference:

$$d_{iws} = |y_{iws} - y_{i(w+5)s}|, \forall i \in I, w \in W_5, d \in D_5, s \in S_3 \quad (3.34)$$

The decision variable d_{iws} in the previous equation states if there is a difference in assignment between two tasks at the same hour and day for two weeks: w and $w + 5$. Thus, a variable minimized in the objective function.

3.3.4 Meeting constraints

At the library there are both library meetings as well as department meetings, where both occur with a five-week interval. Library meetings are set to take place from 08:00-10:00 on Mondays, whereas department meetings are more

freely distributed. A few workers are not assigned library meetings, since they are needed at the stations to keep the library running. The set I_{big} in the equation below consists of all workers who are to be assigned library meetings. The constraints modeling library meetings are as follows:

$$\sum_{w \in W_5} M_{w11} = 1 \quad (3.35)$$

$$M_{(w+5)11} = M_{w11}, \forall w \in W_5 \quad (3.36)$$

$$\sum_{s=2}^3 \sum_{j \in J_d \setminus \{L\}} x_{iwsj} \leq 1 - M_{w11}, \forall i \in I \setminus I_{big}, w \in W \quad (3.37)$$

Equation 3.35 and 3.36 assign two library meetings with a five-week interval during the ten-week scheduling period. Equation 3.37 makes sure that the workers that do not attend library meetings are assigned any other task the day of the meeting.

The constraints added to model department meetings are somewhat similar to the library meeting constraints. Just as for library meetings they take place two times during the ten weeks with a five-week interval, which is described by Equation 3.38 and 3.39.

$$\sum_{w \in W_5} \sum_{d \in D_5} \sum_{s \in S_3} m_{wdsD} = 1 \quad (3.38)$$

$$m_{(w+5)dsD} = m_{wdsD}, \forall D = 1, \dots, 3, w \in W_5, d \in D_5, s \in S_3 \quad (3.39)$$

$$m_{wdsD} + x_{iwsj} \leq 1, \forall D = 1, \dots, 3, i \in I_D, w \in W, d \in D_5, s \in S_3, j \in J_d \quad (3.40)$$

$$m_{wdsD} \leq \sum_{v \in V} r_{iv} * qualavail_{i(mod_{10}(w-v+10)+1)dsE}, \forall D = 1, \dots, 3, i \in I_D, w \in W, d \in D_5, s \in S_3 \quad (3.41)$$

Equation 3.40 prohibits a worker from multitasking, i.e. attend a meeting as well as work with a task. Lastly, Equation 3.41 enables department meetings only when everyone in that department is available for a task; the rotation is taken into the account.

Chapter 4

Results from mathematical model

4.1 Problem studied

An adaptation of the problem described in the problem description was modeled and run in CPLEX. The special case studied involved the specific demands of the library on the personnel such as who is not particularly good at a task. For this reason, many extra constraints were added, see Appendix A. The features of the problem studied involve:

- A schedule of 10 weeks was to be generated.

Running the mathematical model in CPLEX 12.5.0.0 finds an optimum solution in approximately 17 minutes (1031). In the objective function, the parameters $L = 4000$, $A = 2000$ and $N = 5$ were used, thus having twice as much priority on finding librarian stand-ins than assistant stand-ins.

4.2 Evaluation of results

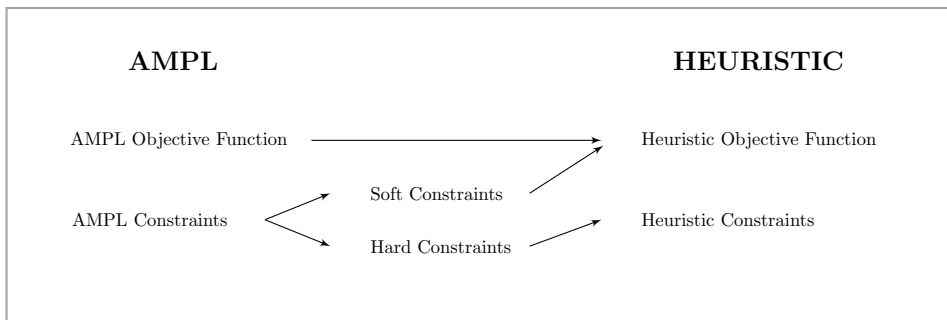
Chapter 5

Heuristic implementations

Apart from the implementation of the AMPL model we have also created two separate solvers of the problem using heuristics. One method was decided to be based on creating a pool of all week appearances that exists. Each week appearance contains a unique set of tasks during all seven days. After creating the pool of blocks these are assigned to the workers based on costs, one for each week. The second method creates a schedule through distributing individual tasks.

The difference in objective function and constraints between the model used in AMPL and the model used in the heuristics is described by Figure 5.1. Since different constraints are modeled as soft and hard in the two heuristics, how they differ from the AMPL model is described in Tables 5.1 and TODO. Some constraints have not been implemented in the heuristics due to factors such as lack of time. These are referred to as relaxed constraints.

Figure 5.1: Illustration of how the Heuristic models have been designed



5.0.1 Block creation

A big part of this heuristic is to create the big pool of unique week appearances. These are then filtered for each of the worker based on its availability. The workers availability are generalized into three categories: *Weekend week*,

Table 5.1: Soft and relaxed constraints in the task distribution approach.

Soft Constraints	
Affecting constraints	Constraint description
3.19	One task per day restriction.
3.23	Four tasks per week restriction.
3.20	One PL per week restriction.
3.21	Number of PL per 10 weeks restriction.
3.22	Not more than two tasks at the same shift in a week restriction.
Relaxed Constraints	
Affecting constraints	Constraint alteration
Several. $W = W_5$ in all constraints.	5 week scheduling instead of 10 week scheduling.
BokB-constraints	BokB placed according to constraints, but the schedule is fixed.
Availability data	Even and odd week workers have availability at each shift according to the stricter of the two weeks.
3.35 - 3.41	Meetings not implemented.

weekrest week and *weekday week*, where weekday week occurs three times during a five week period, see Table 1.5.

One week block contains seven days and up to four shifts where each shift can contain up to three task types. Table 5.2 below is a representation of a general week block with all possible tasks for each day d and shift s . I in the table represents "No task" is assigned that day, D represents "Desk task" meaning either Exp or Info, PL represents "Fetch list" and HB represents "Hageby".

Table 5.2: A generalized weekblock with all existing tasks

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00	I,D,PL	I,D,PL	I,D,PL	I,D,PL	I,D,PL	I,D,HB	I,D,HB
10:00-13:00	D	D	D	D	D		
13:00-16:00	D	D	D	D	D		
16:00-20:00	D	D	D	D	D		

Every day must contain exactly *one* task from either of the four shifts when creating a week block. The tasks I and PL are ranging more than one shift. The duration of a PL is three shift and the I refers to the entire day. Hence, they are both placed in the first shift to simplify the complete task representation. When creating the combinations of block appearances there are additional conditions that have to be met. These are:

1. At most two tasks can be assigned the same shift and week.
2. No more than two evenings are allowed each week, one of which is required to be a Friday.
3. At most one PL is allowed in a weekblock.

4. Saturday and Sunday shall always contain the same task type.
5. If Saturday and Sunday contain Desk tasks, then so shall Friday afternoon (fourth shift).
6. No more than four tasks are allowed during the weekdays leaving at least one day without tasks.

To see the growth of the problem when more tasks are added, consider the following example: If item 1, 2, 3, 5 and 6 are disregarded there exists $6^5 * 3 = 23,328$ unique weekblocks. In contrast, if Exp and Info were to be considered separately, instead of the combination of the two, the possible combinations would be $10^5 * 4 = 400,000$. By applying all conditions the total amount of unique block appearances for this implementation are 4,175.

An illustration of one of the 4,175 existing block can be seen in Table 5.3 below.

Table 5.3: Illustration of one of the unique block appearances

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00		PL	I		I	HB	HB
10:00-13:00							
13:00-16:00				D			
16:00-20:00	D						

This block contains five tasks; two of them are weekend tasks and three are weekday tasks. Which week this block can be assigned to is dependent on the worker's rotation. Since Hageby is assigned to the weekblock one can conclude that only a librarian can have this block assigned to itself. Due to this fact, the Desk tasks can imply either Exp or Info desk work as librarians are qualified for both.

5.0.2 Block percolation

After creating all existing week combinations they are percolated to each of the workers based on their availability in each of the three categories mentioned in Section 5.0.1. Table C.1 in Appendix C shows the results after this percolation has been made for all workers.

All of the values in the table are a subset of the total amount of 4,175 blocks. Knowing the structure of the problem, one can deduce that the amount of available weekrest blocks are always less than or equal to the available weekday blocks, as it should be. The only difference between the two mentioned block categories is when a worker is free from work due to its weekrest. Therefore, using this information one can interpret that they are equal when the worker is never working weekends.

Looking at a generalized worker's availability shown in Table 5.4 one might think that there shall be more weekend blocks available than weekday blocks as the availability, almost in all cases, are higher for weekend blocks. This is not the case as all blocks without weekend tasks are removed in the percolation. This means that all combinations with "No task" on weekends are removed, as well as the case that is shown in Table 5.5. The case in Table 5.5 occurs when

Table 5.4: Typical availability for a generalized worker. Yellow signifies that the worker is available. In parenthesis, the weekend shift hours.

Weekend week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							
Weekrest week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							
Weekday week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							
Weekday week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							
Weekday week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00 (11:00-16:00)							
10:00-13:00							
13:00-16:00							
16:00-20:00							

a worker is assigned weekend Desk tasks and therefore can not be assigned any other task that Friday.

Table 5.5: A weekend block with Desk tasks preventing any other tasks on Fridays.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00	X	X	X	X		D	D
10:00-13:00							
13:00-16:00							
16:00-20:00					D		

"X" in Table 5.5 can represent any task and shifts colored in black means that no other task can be assigned that shift.

5.0.3 Rotation assignment

There are 35 weekend workers available of which 21 are librarians and 14 are assistants. The demand of weekend workers each week is seven, i.e. the demand for five weeks is exactly $7 * 5 = 35$ workers. Another requirement in excess to

seven workers each weekend is that at least four of them have to be librarians due to three librarians are needed in Information desks and one in Hageby. Therefore, it deems reasonable to swap rotations between workers in the destroy so that it always remains feasible.

A random generator is used in the assignment of rotations that always makes sure that the two mentioned requirements are met. Furthermore, all of the LoW-workers have fixed weekends and hence are not given new rotations in the destroy/repair loop and is more thoroughly described in Section 5.0.4 below.

Table 5.6 shows a destroy/repair iteration regarding rotation assignments. The amount of workers being destroyed in each iteration is three.

Table 5.6: An iteration in the destroy/repair loop showing a swap of weekends when three workers are destroyed

Initial assignment:					
Week	1	2	3	4	5
Librarians	4	4	5	4	4
Assistants	3	3	2	3	3
After destroy:					
Week	1	2	3	4	5
Librarians	3	4	4	4	4
Assistants	3	2	2	3	3
After repair:					
Week	1	2	3	4	5
Librarians	4	5	4	4	4
Assistants	3	2	3	3	3

Yellow indicates that a worker, either librarian or assistant, has been destroyed that week and green indicates that a worker has been repaired. Comparing the "Initial assignment" with "After repair" a swap can be seen between week 2 and 3. Worth noting is that a swap can occur even if the amount of qualified workers remains the same after a repair. To understand this, imagine that two librarians with different rotations are destroyed, then two cases can occur: Either they are assigned the same rotation as before or they swap weekends.

5.0.4 Assignment of Library on Wheels

In order to avoid creating enormous amounts of block combinations, LoW tasks are assigned manually. To assign LoW manually also lead to a fix weekend rotation for the five LoW-workers. This slightly reduces the degrees of freedom of the problem as the LoW-workers' rotation will remain unchanged. However, as there are only a couple different set of feasible LoW assignments it shall not be the deciding factor for the quality of the solution.

Without a manual assignment of LoW the number of existing tasks in a weekblock would increase significantly. The number of tasks would increase from 36 to 43, as there are seven LoW tasks during a week, resulting in a lot more than 4,175 unique week appearances.

5.0.5 Initial solution

The initial solution is created in a similar fashion as the repair function in this heuristic. Based on a greedy heuristic the best weekblock for a random worker and week is found and inserted. This is done until every worker have one weekend block, one weekrest block and three weekday blocks assigned to them.

To find the best weekblock several costs have been introduced to measure whether a block is good or bad to assign a worker. Say, if the library demands two librarians at the Information desk Monday at 08:00 and currently there are one assigned, then it would be good to assign another one. Such an assignment will, therefore, be rewarded using a cost. Good assignments will provide negative costs and bad assignments will provide positive costs to the objective function value.

Table 5.7 together with Figure 5.2 shows an increment where different cost parameters have to be considered when evaluating a block before assigning it to a worker. In order to calculate demand costs for the PL in the library, assume that this block which is being evaluated is to be inserted at week three.

Table 5.7: A block example to be evaluated using costs

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
08:00-10:00	PL	I		I		I	I
10:00-13:00					D		
13:00-16:00			D				
16:00-20:00							

Task to be evaluated

	Mon
08:00-10:00	PL
10:00-13:00	
13:00-16:00	

5.0.6 Costs

In order to find a feasible solution many of the implemented costs must be carefully chosen. Presently, there are 16 existing costs where some of them are correlated with each other. The solution behaves differently depending on the relation between the costs changes.

Six of the costs are presented in Figure 5.2. They have all been assigned unique values so that, for example, positive assistant demand cost differs from negative assistant demand cost in absolute value. To explain why, one can imagine a case where four workers are demanded of which two have to be librarians, see Table 5.8. The first case, where one assistant and three librarians have been assigned, is more desirable than the case with three assistants and one librarian. In the first case it is still a feasible solution as the qualification of assistants are a subset of the librarians. The second case is feasible, however, not optimal due to the exceeding use of librarians. The optimal solution is shown in green as Case 3.

The complete list of costs with description can be seen in Table 5.9.

Figure 5.2: A flow chart of appearing costs when a PL is assigned a block on a Monday, third week relative to the library schedule.

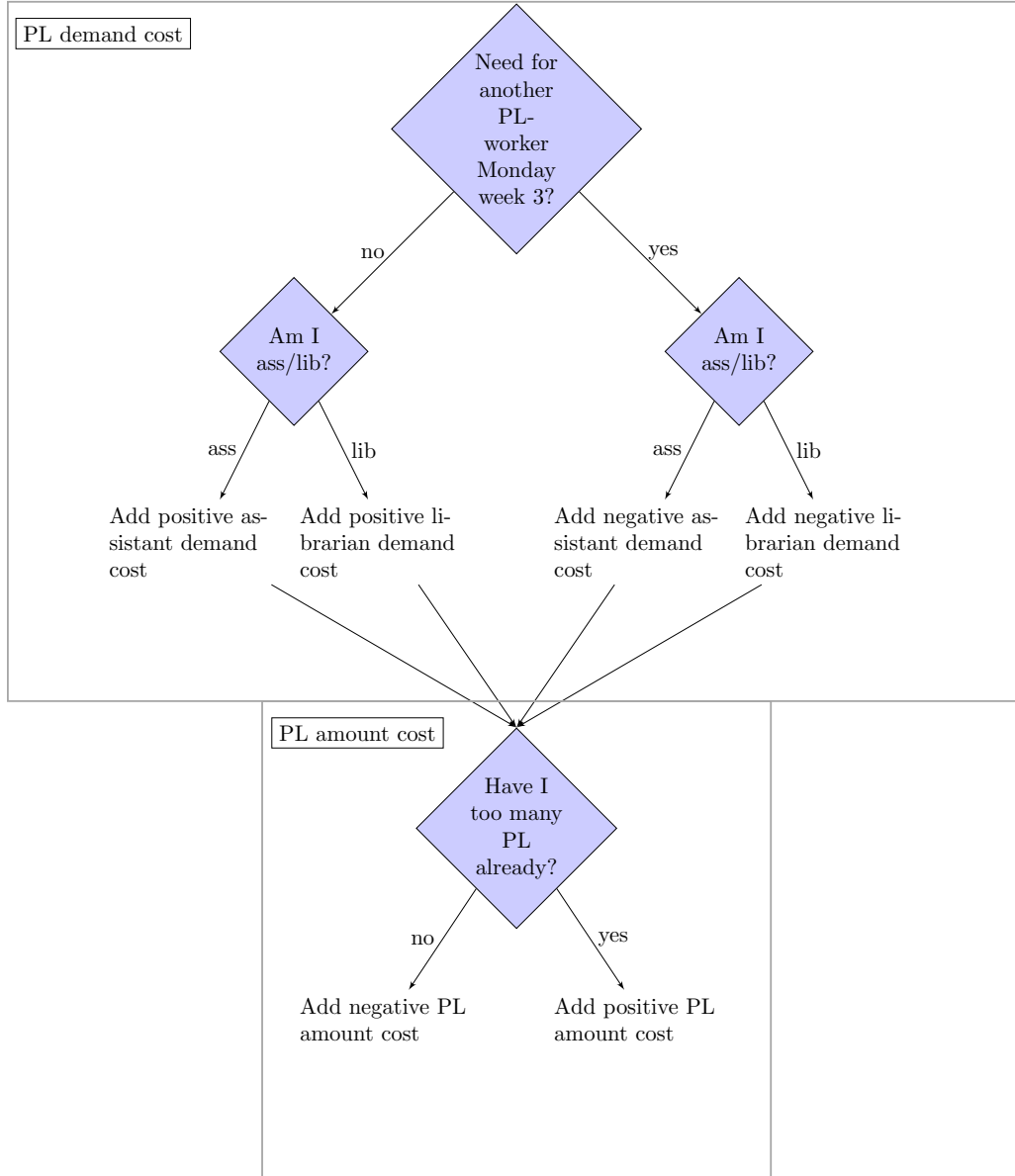


Table 5.8: Library demand at a shift and solution qualities.

Demand:	4 workers (≥ 2 librarians)	
Case 1:	3 assistants, 1 librarian	(infeasible)
Case 2:	1 assistant, 3 librarians	(feasible)
Case 3:	2 assistants, 2 librarians	(optimal)

Table 5.9: List of all costs with description.

Cost name	Description
Demand costs	
Demand_few_assistants	In need of more assistants to fill quota.
Demand_few_librarians	In need of more librarians to fill quota.
Demand_many_assistants	More assistants assigned than needed (redundancy).
Demand_many_librarians	More librarians assigned than reference value.
Demand_few_total	Incorrect amount of workers assigned a task.
Demand_many_total	Incorrect amount of workers assigned a task.
Demand_evening_cost	Incorrect amount of workers assigned an evening task (more crucial).
Demand_PL_good_assistant	Assigning a PL to assistant, empty before assignment.
Demand_PL_good_librarian	Assigning a PL to librarian, empty before assignment.
Demand_PL_bad_assistant	Assigning a PL to assistant, others already assigned.
Demand_PL_bad_librarian	Assigning a PL to librarian, others already assigned.
PL amount costs	
PL_good_amount	Assigning PL when in need of more for feasibility.
PL_violate_amount	Assigning more PL than allowed to that worker.
Weekend costs	
HB_amount	No or more than one HB workers assigned the same weekend.
No_weekend	
Stand-in costs	
Stand_in_cost	Ocurring when a possible stand-in is ruined due to assignment.

5.0.7 Destroy

The destroy consists of two contiguous steps: Choosing three workers to destroy and then destroying their assigned blocks. The LoW tasks remain unchanged even if a worker assigned those tasks is destroyed.

5.0.8 Repair

Initially in the repair function the destroyed workers are assigned new rotations, see Table 5.6, followed by five new week blocks. Which order the blocks are inserted in is randomly generated. For each iteration is one of the three workers randomly chosen followed by one block that has not been reassigned until all 15 destroyed weeks have been repaired. Which week block that is chosen is based on the costs listed in Table 5.9.

5.0.9 Evaluation of solution

After each destroy/repair loop the new solution is evaluated. It is common for the new solution to be worse than the previous one. However, all new solutions are accepted regardless.

All costs are regarded in the evaluation function. If there are any demand differences in the library or any differences in amount of PL assigned to a worker these are given the respective costs. Furthermore, the amount of stand-ins is evaluated here provided with a negative cost showing that these are desired.

If there are no infeasibilities in the solution the evaluation function will return the value zero, unless there are stand-ins. In case there are stand-ins a negative value can be returned.

5.0.10 Final phase

At the end of a run there are mostly a few infeasibilities left that do not get solved. To fix this, a final phase was implemented. Whenever a run is "relatively close" to a solution and it is "theoretically possible" to find a solution the final phase is initiated. "Relatively close" means in this case that the only infeasibility is the lack of workers at a handful of weekday shifts. "Theoretically possible" means that there are more stand-ins available all days where the infeasibility occurs.

The final phase destroys one random week for a random worker and immediately repairs it, resulting in the same or better solution until it is feasible.

Due to well chosen cost parameter values the run will always eventually get relatively close and reach the final phase. If there are fewer stand-ins available a day where workers are lacking the solution will be discarded before entering the final phase.

Chapter 6

Task distribution approach

6.1 Introduction

The second heuristic approach tested distributes individual tasks to workers. This method greatly resembles the process of manually placing tasks as is typically done in many practical situations. The heuristic undergoes two phases. In the first phase, weekends, as well as some weekday tasks, are distributed and in the second phase the rest of the weekday tasks are distributed.

The primary method used in this approach is a large neighbourhood search (LNS) together with a simulated annealing (SA) accept function. Destroying and repairing the solution, as is customary in LNS, helps leading the solution out of local optima or plateaus. Similarly, SA is used in order to allow the solution to move in a less favourable directions to avoid these local optima.

6.2 Costs

The search in this approach is guided using costs. A worker gets has a number of costs, as displayed in Table 6.1. Each cost represents a misplaced task, resulting in an infeasible schedule or unused stand in potential. In the library, different costs are weighed together in three different ways. These are referred to as objective functions, see Table 6.2.

Table 6.1: Worker costs. Costs are incremented when a worker gets a task which breaks a soft constraint.

Stand in cost	
C_{SI_min}	Cost of day with minimum number of stand ins.
Infeasibility costs	
C_{Task_Day}	Cost from all tasks exceeding the daily limit.
C_{Task_Week}	Cost from all tasks exceeding the weekly limit.
C_{PL_Day}	Cost from all PL exceeding the weekly limit.
C_{PL_Tot}	Cost from all PL exceeding the total limit.
C_{SShift_Week}	Cost from the number of task performed at the same shift in a week exceeding the weekly limit.

Table 6.2: Library objective functions and their cost components. Weekend and weekday objective functions are to be maximized while the worker objective function is to be minimized.

Weekend Objective Function	
Cost	Cost description
C_{SI_min}	Cost of day with minimum number of stand ins.
C_{Shift_min}	Cost of shift with minimum number of workers.
C_{Day_min}	Cost of day with minimum number of workers.
C_{SI_aver}	Average number of stand ins at a day.
C_{Shift_aver}	Average number of workers at a shift.
C_{Day_aver}	Average number of workers at a day.
Worker Objective Function	
Cost	Cost description
C_{Task_Day}	Cost from all tasks exceeding the daily limit in all workers.
C_{Task_Week}	Cost from all tasks exceeding the weekly limit in all workers.
C_{PL_Week}	Cost from all PL exceeding the weekly limit in all workers.
C_{PL_Tot}	Cost from all PL exceeding the total limit in all workers.
C_{SShift_Week}	Cost from the number of task performed at the same shift in a week exceeding the weekly limit in all workers.
Weekday Objective Function	
Cost	Cost description
C_{SI_min}	Cost of day with minimum number of stand ins.

The weekday objective function is associated with the weekend distribution phase of the problem. There are three types of costs in the objective function, measuring a certain aspect of a good weekend schedule. Since we want to increase the number of stand ins at the most critical days, the worst days of all aspects are considered, referred to as the min cost. Also, the average of each aspect is considered, in order to distinguish solutions which for which the worst day is identical. All costs in this objective function are calculated using the formula:

$$C_{type} = W_{lib} * num_of_lib + W_{ass} * num_of_ass \quad (6.1)$$

The weights are constants used in all weekend objective function costs. Typically W_{lib} is larger than W_{ass} since librarians can perform a greater number of tasks.

After maximizing weekends the weekday distribution phase is entered. Here, the worker objective function is used to identify infeasibilities in the worker schedules and is simply a sum of all the worker costs over all workers. When this cost is zero, the schedule is feasible. Then, in order to compare feasible schedules, the weekday objective function is used, containing only the minimum stand in cost, calculated in the same way as in the weekend objective function. This objective function is corresponding to the objective function in the Mathematical Model.

6.3 Weekend phase

The weekend phase is the first phase the algorithm enters and is described in detail in Appendix TODO. The LNS component is described by feasibility loop, while the simulated annealing step is marked as a decision to accept solutions which are worse than the current one.

The reason for implementing the weekend phase separate from the distribution of other tasks was because of the realization that the minimum number of stand ins depended to a very large extent on the weekend structure. In particular, it is the weekend rest associated with the weekend week which decides how many people are present at a particular shift or day. This can be seen in for example Table 1.5.

Measuring the goodness of a schedule is done through using the weekend objective function.

In order to measure what a good distribution of weekends is, the weekend objective function described in the previous section is used during the search process. Although the overall objective is to maximize the number of stand ins at the most critical day in the schedule, that is, even out the stand ins over the days, it is not trivial to measure this in a schedule with no tasks placed. Evening work reduces potential stand ins as well as tasks distributed during the days. Furthermore, limits on how many tasks per week a person is allowed to take further complicates the measurement of stand ins.

Because of the difficulty in measuring stand ins, certain weekday tasks are placed already in the weekend phase. This includes all Library on Wheel tasks and all evening tasks. For the Library on Wheels, there is not much choice or variability, and thus a fixed schedule is applied for the workers. Similarly, the workers who are available at the evening tasks are in most part, equal to the number of workers needed, thus leaving little choice in scheduling evenings. This is illustrated in tables 6.3 and 6.4 which show the number of available workers before and after placing the task mentioned.

The algorithm for the distribution of weekends is illustrated in Figure B.2. Two loops exist, an outer weekend distribution loop and an inner feasibility loop. The outer loop is performed a number of iterations and performs a search among neighbouring solutions provided by the inner loop through destroying and repairing a proportion of the solution. The decisions directing the search are coloured pink and can be described as:

Accept anyway? : All feasible solutions are pass from the feasibility loop to the weekend loop. If the solution is better or equally good as the previous one, it is accepted. However, if it is not it can still be accepted with the probability P , as described in Equations 6.4 to 6.3. The delta energy is the differens in objective function value between two solutions. T is the temperature of the SA accept function, which has an initial value of T_0 and which cools down with the iteration count t at a rate α ($0 < \alpha < 1$).

$$\Delta E = Wend_0 - Wend_1 \quad (6.2)$$

$$T = T_0 \alpha^t \quad (6.3)$$

$$P = \exp(-\Delta E/kT) \quad (6.4)$$

Table 6.3: Worker availability placing only weekends.

	Num available assistants						
	Mo	Tu	We	Th	Fr	Sa	Su
Shift 1:	8	10	10	10	6	0	0
Shift 2:	8	9	9	9	6	0	0
Shift 3:	9	8	9	7	5	0	0
Shift 4:	3	2	2	3	0	0	0

	Num available librarians						
	Mo	Tu	We	Th	Fr	Sa	Su
Shift 1:	16	15	16	13	12	0	0
Shift 2:	18	15	17	14	13	0	0
Shift 3:	17	14	18	18	13	0	0
Shift 4:	3	4	4	3	1	0	0

	Num available BB-librarians						
	Mo	Tu	We	Th	Fr	Sa	Su
Shift 1:	2	0	1	1	1	0	0
Shift 2:	0	0	0	0	0	0	0
Shift 3:	0	0	0	0	0	0	0
Shift 4:	1	0	2	2	0	0	0

Table 6.4: Worker availability after placing weekends as well as evening tasks and BokB for the same week.

	Num available assistants						
	Mo	Tu	We	Th	Fr	Sa	Su
Shift 1:	7	10	9	9	8	0	0
Shift 2:	7	9	8	8	8	0	0
Shift 3:	7	7	8	6	6	0	0
Shift 4:	0	0	0	0	0	0	0

	Num available librarians						
	Mo	Tu	We	Th	Fr	Sa	Su
Shift 1:	14	11	13	11	12	0	0
Shift 2:	14	11	14	11	12	0	0
Shift 3:	13	11	14	13	13	0	0
Shift 4:	0	0	0	1	1	0	0

	Num available BB-librarians						
	Mo	Tu	We	Th	Fr	Sa	Su
Shift 1:	0	0	0	0	0	0	0
Shift 2:	0	0	0	0	0	0	0
Shift 3:	0	0	0	0	0	0	0
Shift 4:	0	0	1	0	0	0	0

Solution feasible? : The feasibility check in the algorithm corresponds to checking whether the number of workers available at each shift equals to the demand at each shift. A worker is counted as available if the worker is qualified and available for the task and has no other tasks scheduled during that day.

The infeasibility loop checks the necessary condition of having enough workers, but is not sufficient in order to prove that the final schedule is feasible. If other constraints cannot be fulfilled in the final assignment of tasks, the schedule would be infeasible.

Better than saved? :

Best solution? :

Iteration > max? :

Current better than best? :

Destroying and repairing tasks in the feasibility loop accounts for the LNS part of the problem. Destroying is always random, while the redistribution of weekends is done with priority on worker qualification, meaning that tasks with librarian qualification are placed first. Also, HB preferences are taken into account in the repair function.

6.4 Weekday phase

When entering the weekday phase, all weekends, evenings and BokB tasks are already placed. In this phase, the rest of the tasks, referred to as "weekday" tasks are also placed. The demand is illustrated in tabl 6.5. All week demand now looks the same for all weeks.

The weekday distribution phase is similar to the weekend distribution phase, since it is divided into two loops.

Evenings, weekends and BokB already placed.

Concept: destroy worst worker until all workers have feasible schedules. Record the library cost of the solution.

Destroy: weekday tasks for workers with highest cost. Repair: 1. qualification, 2. avail demand diff. Place cheapest worker.

Infeasibility: when a feasible worker cost is not found for a large number of iterations.

6.5 Simplifications of Mathematical Model

-10 Week scheduling -Objective function term about similar weeks -BokB fixed weeks for every other week workers. -Even odd weeks. How to handle? (-lower limit PL)

6.6 Implementation

C++, object orientation, run on a linux operating system. Reading availability of workers into the program and outputting a result file, which can be read by Excel. Results are to be visualized in Excel (write this in another part?)

Table 6.5: Worker demand during a week when entering weekday phase.

	Exp	Info	PL	HB	BokB
Monday					
Shift 1:	2	2	1	0	0
Shift 2:	3	3	0	0	0
Shift 3:	3	3	0	0	0
Shift 4:	0	0	0	0	0
Tuesday					
Shift 1:	2	2	1	0	0
Shift 2:	3	3	0	0	0
Shift 3:	3	3	0	0	0
Shift 4:	0	0	0	0	0
Wednesday					
Shift 1:	2	2	1	0	0
Shift 2:	3	3	0	0	0
Shift 3:	3	3	0	0	0
Shift 4:	0	0	0	0	0
Thursday					
Shift 1:	2	2	1	0	0
Shift 2:	3	3	0	0	0
Shift 3:	3	3	0	0	0
Shift 4:	0	0	0	0	0
Friday					
Shift 1:	2	2	1	0	0
Shift 2:	3	3	0	0	0
Shift 3:	3	3	0	0	0
Shift 4:	0	0	0	0	0
Saturday					
Shift 1:	0	0	0	0	0
Sunday					
Shift 1:	0	0	0	0	0

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Appendix A

Problem definitions

A.1 Sets

I	Set of workers
I_{lib}	Set of librarians ($I_{lib} \subseteq I$)
I_{ass}	Set of assistants ($I_{ass} \subseteq I$)
W	Set of weeks
D	Set of days in a week
S_d	Set of shifts day d
J_d	Set of task types day d
I_{LOW}	Set of librarians available to work in library on wheels
I_{free_day}	Set of workers that shall be assigned a free weekday per week
I_{odd_even}	Set of all workers with odd or even weeks
$I_{weekend_avail}$	Set of workers available for weekend work

A.2 Variables

$$x_{i w d s j} = \begin{cases} 1, & \text{if worker } i \text{ is assigned in week } w, \text{ day } d, \text{ shift } s \text{ to a task } j \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.1})$$

$$H_{i w h} = \begin{cases} 1, & \text{if worker } i \text{ works weekend } h \text{ in week } w \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.2})$$

$$r_{i w} = \begin{cases} 1, & \text{if worker } i \text{ has its scheduled rotated } w-1 \text{ steps} \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.3})$$

$$lib_{i w d} = \begin{cases} 1, & \text{if librarian } i \text{ is a stand-in week } w \text{ day } d \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.4})$$

$$ass_{i w d} = \begin{cases} 1, & \text{if assistant } i \text{ is a stand-in week } w \text{ day } d \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.5})$$

$$y_{i w d s} = \begin{cases} 1, & \text{if worker } i \text{ is working } w \text{ day } d \text{ regardless of task type} \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.6})$$

$$hb_{i w} = \begin{cases} 1, & \text{if assistant } i \text{ is a stand-in week } w \text{ day } d \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.7})$$

$$friday_evening_{i w} = \begin{cases} 1, & \text{if assistant } i \text{ is a stand-in week } w \text{ day } d \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.8})$$

$$lib_min = \text{lowest number of stand-in librarians found (integer)} \quad (\text{A.9})$$

$$ass_min = \text{lowest number of stand-in assistants found (integer)} \quad (\text{A.10})$$

A.3 Parameters

$$N1l = \text{a value to prioritize the amount of stand-in librarians} \quad (\text{A.11})$$

$$N1a = \text{a value to prioritize the amount of stand-in assistants} \quad (\text{A.12})$$

$$N2 = \text{a value to prioritize similar weeks} \quad (\text{A.13})$$

$$avail_day_{iwd} = \begin{cases} 1, & \text{if worker } i \text{ is available for work week } w, \text{ day } d \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.14})$$

$$task_demand_{dsj} = \text{number of workers required day } d, \text{ shift } s \text{ for task type } j \quad (\text{A.15})$$

$$qualavail_{iwsj} = \begin{cases} 1, & \text{if worker } i \text{ is qualified and available week } w, \text{ day } d, \text{ shift } s \text{ for task type } j \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.16})$$

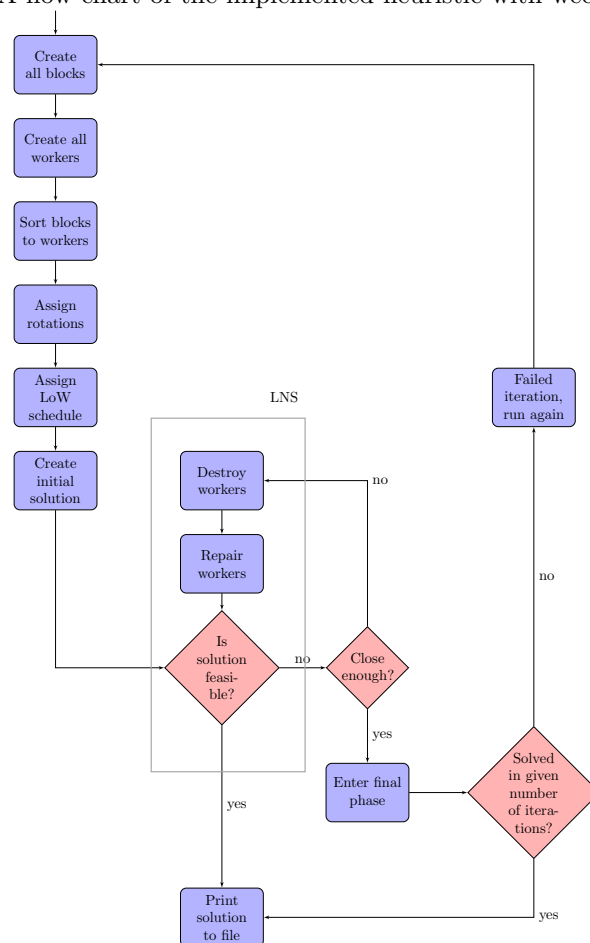
$$LOW_demand_{wds} = \text{number of workers required day } d, \text{ shift } s \text{ at the library on wheels} \quad (\text{A.17})$$

Appendix B

Flow charts

B.1 Weekly scheduling approach

Figure B.1: A flow chart of the implemented heuristic with weekly scheduling



B.2 Task distribution approach

Figure B.2: Algorithm for distributing weekends.

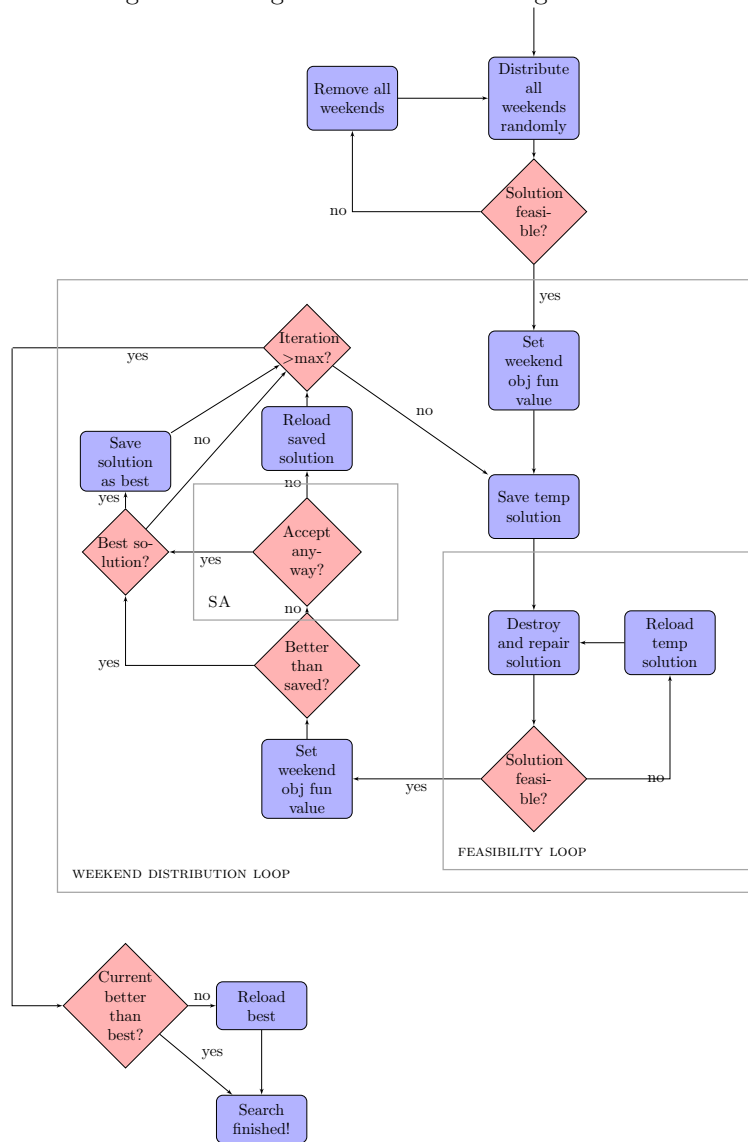
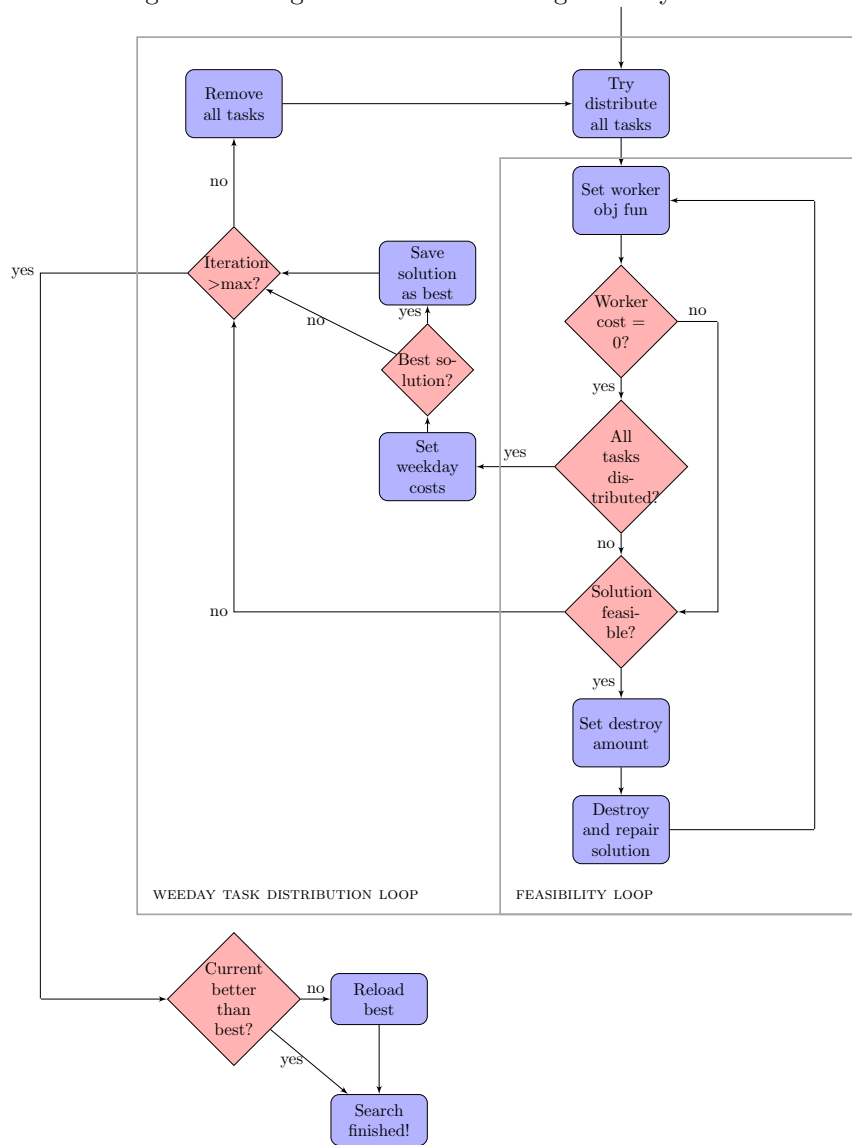


Figure B.3: Algorithm for distributing weekday tasks.



Appendix C

Weekblock table

Table C.1: Number of assignable unique blocks for the workers based on their availability and qualification.

Worker	Weekend	Weekrest	Weekday
1	532	347	1580
2	1580	1580	1580
3	1063	347	1580
4	557	165	779
5	261	130	531
6	532	130	1580
7	261	130	531
8	261	29	531
9	115	12	247
10	532	130	1580
11	9	8	8
12	1063	347	1580
13	771	92	1190
14	265	29	489
15	51	18	120
16	495	130	843
17	237	69	267
18	532	279	1580
19	495	47	843
20	532	130	1580
21	227	227	227
22	2	1	1
23	3	2	2
24	11	5	47
25	1063	279	1580
26	5	4	4
27	213	106	425
28	2	2	2
29	426	106	1281
30	127	126	455
31	495	130	843
32	261	29	531
33	72	45	306
34	425	425	425
35	91	20	221
36	55	27	101
37	1063	347	1580
38	3	1	1
39	2	1	1

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