

Master thesis

**Work Distribution of a Heterogeneous Library Staff - A
Personnel Task Scheduling Problem**

Claes Arvidson, Emelie Karlsson

LITH - MAT - EX - - 04 / 04 - - SE

Work Distribution of a Heterogeneous Library Staff - A Personnel Task Scheduling Problem

Optimeringslära, Linköpings Universitet

Claes Arvidson, Emelie Karlsson

LiTH - MAT - EX - - 04 / 04 - - SE

Exam work: **30 hp**

Level: **A**

Supervisor: **T. Larsson**,
Optimeringslära, Linköpings Universitet

Examiner: **E. Rönnberg**,
Optimeringslära, Linköpings Universitet

Linköping: **June 2016**

Abstract

Here is where you can write your abstract. It may be very long, or it may be very short, the reason you have an abstract is for people not to be forced to read lots of crap.

But still, they will have to read your abstract. After all, the abstract is what everyone reads. . .

Keywords: Keyword One, Chemostat, Another Key-Word, Key, Clé, Mot de cle, Nyckelhål, XBOX, Dagens viktigaste nyckelord, and Keywords.

URL for electronic version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-77777>

Acknowledgements

I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor...

I also have to thank, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor...

My opponent NN also deserves my thanks, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor...

Nomenclature

Most of the reoccurring abbreviations and symbols are described here.

Symbols

Y_0 The amount of the variable Y inserted into a system.
 \hat{Y} The unit-dimension of the variable Y , for example $\hat{t} = 1s$.
 \bar{Y}_i A steady state (number i) value of Y .

K_i Constants used in kinetic expressions, for example K_I .

\mathbf{A} The system matrix.

Abbreviations

CPI Competitive Product Inhibition (or Inhibited)
CSI Competitive Substrate Inhibition (or Inhibited)
CSTR Continuous Stirred Tank (bio)Reactor
MMI Michaelis-Menten Inhibition (or Inhibited)

Contents

1	Introduction	1
1.1	Background	1
1.2	Problem description	1
1.2.1	Main objectives	1
1.2.2	Requirements on weekday activities	1
1.2.3	Requirements on weekend activities	1
1.2.4	Diversity in skill	2
1.2.5	Personnel availability and task assignment limitations . .	2
1.2.6	Inner work	2
1.3	Problem categorization	3
2	Literature review	5
2.1	Tour scheduling problem with a heterogenous work force (TSP) .	5
2.2	Personnel task scheduling problem (PTSP)	6
2.3	Shift minimisation personnel task scheduling problem (SMPTSP)	6
2.4	Fixed/flexible job scheduling problem (FJSP)	6
2.5	Work load allocation and worker satisfaction	7
3	Implementation insights	9
4	The ideal CSTR: the chemostat	11
4.1	Some simple models of biological growth	11
4.1.1	Exponential growth	11
4.1.2	The logistic equation	11
4.2	The chemostat	11
A	The Linearized stability	15
A.1	The Linearization	15

List of Figures

List of Tables

1.1	Personnel	3
1.2	Outer and inner services	3
1.3	Requirements	3

Chapter 1

Introduction

1.1 Background

At a library absence can cause problems, both due to lack of personnel as well as due to the qualifications required to perform a task varies. If a worker were to be unavailable a day because of a meeting or being ill it would require for a stand-in to fill the vacancy. Therefore, it is of great interest to have a schedule with as many skilled stand-ins as possible to overcome such disturbances. Furthermore, the library personnel have certain demands and preferences as to how a satisfactory working schedule should be. For instance, it is neither preferable to work more than one evening each week nor work more weekends than required.

1.2 Problem description

1.2.1 Main objectives

Main objective is to create a schedule with as many stand-ins as possible, so that all days have a maximum amount of stand-ins. Diversity during the week and repetivity of the schedule each week is also desired.

1.2.2 Requirements on weekday activities

During weekdays, there is a worker demand at the stations Exp, Info, Plocklista and Library on wheels. \downarrow TABLE OF DEMAND \downarrow . Plocklista is unique in the sense that its duration is longer than one shift. It is modeled as the three first shifts of a weekday.

In addition there is the Library on wheels, which has a different demand of workers depending on odd and even weeks.

1.2.3 Requirements on weekend activities

There are three different weekend stations: Exp, Info and Hageby. Working a weekend also means working friday unless you are scheduled to work in Hageby.

1.2.4 Diversity in skill

There are essentially two types of workers in a library. Librarians and assistants. The competence of the workers looks as follows: ¡TABLE OF DIFFERENT COMPETENCES¿

A subset of the librarians can handle the Library on wheels and all are expected to take shifts at Hageby.

1.2.5 Personnel availability and task assignment limitations

The staff at the library have certain times of availability. Every day, a person can perform at maximum one task. Plocklistan is limited to once a week.

Personnel is only allowed to work one weekend out of five. The following week, the worker is free on the times requested by the worker. Also, some workers have specific

1.2.6 Inner work

The goal of this thesis is to distribute given tasks to the heterogeneous workforce at the library of Norrköping. Each task is either classified as an outer or an inner service where an outer service is when a librarian needs to interact with visitors. Inner services can in some rare cases require a predetermined person to be assigned to a specified time or day.

Demands and requests are to be fulfilled to the furthest extent possible. Weekends are included in the scheduling problem, which adds more constraints regarding the number of contiguous working days. However, the librarians are permitted a few exceptions from these laws regarding days of rest.

The main purpose of the thesis is to create a schedule robust enough to withstand absence, such that outer services always are assigned to a qualified and available worker. This is visualized as having a list of available stand-ins for each shift.

There are a limited number of workers at the library and they make the resources that are to be distributed. Each individual has a set of *skills* and *competences*. Competences refer to the capability of being assigned the different outer services; Expedition, Norpan, Information desk, Library on wheels and Hageby as well as different inner services. The set of skills an individual can possess are described in Table 1.1. In total there are 39 workers available.

The outer services can be seen as assignments which requires available workers to be assigned to them. Each outer service is specified to a certain station, time and date. They also have a fix length and occur on a regular basis every ten weeks, which makes it possible to create a periodic schedule with a period of ten weeks.

Furthermore, outer and a few inner services can be characterized by different properties, which are represented in Table 1.2.

In addition to the properties mentioned above, there are several requirements that have to be met. These can be divided into job, robust and other requirements and are listed in Table 1.3 below.

Table 1.1: Personnel

Skills	Description
Work degree	0-100 %
Type of employment	Librarian/Assistant
Competence	Inner and outer services the worker is qualified for
Weekly rest	Which days the worker has requested after working a weekend
Other requests	Does not work evenings etc.

Table 1.2: Outer and inner services

Outer service	Property
	Start time, end time, week and duration
	Station
	Number of qualified librarians demanded
	Number of qualified assistants demanded
Inner service	Property
	Start time, end time, week and duration
	Type
	Number of qualified librarians demanded
	Number of qualified assistants demanded

Table 1.3: Requirements

Job requirements	Description
1	A maximum of one outer service is to be distributed to each person and day.
2	Remaining work time is individually distributed on assignments such as reshelving books.
3	Weekend work are to be evenly distributed between the workers available on weekend.
4	Working a weekend includes work on saturday and sunday the same week.
5	One evening shift on a weekday per person each week except when weekend work is requested.
6	Every ten weeks the schedule is to be repeated.
7	It is recommended for each week to be as similar as possible.
Robust requirements	Description
1	Each outer service require at least one stand-in.
2	The stand-ins have to be qualified for the tasks they are stand-in for.
3	Focus is to maximize the lowest number of stand-ins of any task.
Other requirements	Description
1	Department and general meetings are to be held once per five weeks.

There are also additional requirements of the resulting schedule made by the workers at the library. Two examples would be that a handful staff members are unable to work weekends as well as some personnel are unable to work in the evenings.

1.3 Problem categorization

The problem can be formulated as a personnel tasks scheduling problem for a heterogeneous workforce since the main objective is to distribute tasks to

workers during their available times. The workforce is heterogeneous as certain tasks can only be performed by librarians or is restricted to a certain subset of the personnel. Another aspect of the problem is the cyclic nature of the personnel schedules, which gives a degree of freedom in availability of personnel.

Chapter 2

Literature review

The scheduling problem has been studied since the 1950's as a mathematical optimization problem and concerns the ability of creating a satisfactory schedule, considering a number of constraints and objectives. According to Ernst et al. the complexity of the rostering problem has not in itself become more advanced since then. However, the mathematical models used to solve these scheduling problems have become more realistic and refined. This together with more powerful computational methods, makes it possible today to solve scheduling problems in a more satisfactory way [1].

In the paper [1] the scheduling problem is classified into different subcategories. A few relevant areas for our work include task based demand scheduling, days off scheduling, shift scheduling, tour scheduling and task assignment. Task based demand scheduling involves the process of finding the demand from a list of tasks which need to be performed. This problem resembles our problem since also in our problem we require a number of tasks to be performed. However, in our case the pool of workers is fixed and thus the objective differs.

Days off scheduling involves scheduling staff and assigning a day off. This problem is often found together with shift scheduling which involves choosing the most suitable shifts for a workforce. The combination of the two is called tour scheduling and will be discussed more further down in this report. The big difference between our problem and tour scheduling is that in our problem we are not allowed to choose the free days of the workers, only in what week to put them.

The problem which is most similar to our problem is however task assignment. This problem and different variations of it will be discussed in section 2.2 forward.

2.1 Tour scheduling problem with a heterogeneous work force (TSP)

The tour scheduling problem involves creating work shifts with days off for a work force. In the paper review of different scheduling problem written by Ernst et al. (2004) one can see a number of references to tour scheduling problems. However, there are significantly lower number of references regarding heterogeneous workforce.

Papers of interest: "Task assignment and tour scheduling": Loucks and Jacobs, 1991 — Inhomogeneous work force

"Scheduling Restaurant Workers to Minimize Labor Cost and Meet Service Standards" Choi, Hwang and Park, 2009

"An integer linear programming-based heuristic for scheduling heterogeneous, part-time service employees" Heterogeneous work force, tour scheduling. Using two objective functions Hojati and Patil, 2010

2.2 Personnel task scheduling problem (PTSP)

Most likely our problem. Definition is "in which a set of tasks with fixed start and finish times have to be allocated to a heterogeneous workforce". The objective of these problems is to minimise the overall cost of personnel required to perform all tasks.

Papers of interest: "The Personnel Task Scheduling Problem", Mohan Krishnamoorthy, Andreas T. Ernst (2001) - probably the most fundamental article

"Task assignment for maintenance personnel": Roberts and Escudero, 1983a, 1983b

"A stochastic programming model for scheduling maintenance personnel" Duffuaa and Al-Sultan, 1999

2.3 Shift minimisation personnel task scheduling problem (SMPTSP)

Difference: "The only cost incurred is due to the number of personnel (shifts) that are used."

Papers of interest: "Algorithms for large scale Shift Minimisation Personnel Task Scheduling Problems" Krishnamoorthy, Ernst, Baatar (2011)

"The shift minimisation personnel task scheduling problem: A new hybrid approach and computational insights" Smet, Wauters, Mihaylov, Berghe (2014)

"Fast local search and guided local search and their application to British Telecom's workforce scheduling problem" Tsang and Voudouris, 1997 - also with travelling costs, investigates two methods.

"A Triplet-Based Exact Method for the Shift Minimisation Personnel Task Scheduling Problem" Baatar et al., 2015

2.4 Fixed/flexible job scheduling problem (FJSP)

Identical skill of the workers/machines and identical skill requirements of the operations to execute.

Problem defined in: "Algorithms for large scale Shift Minimisation Personnel Task Scheduling Problems" M. Krishnamoorthy <http://www.sciencedirect.com/science/article/pii/S037>

Problem: "A metaheuristic for the fixed job scheduling problem under spread time constraints" André Rossi, <http://www.sciencedirect.com/science/article/pii/S0305054809002251> (Fixed job)

Cemal Özgüven <http://www.sciencedirect.com/science/article/pii/S0307904X11004173> (Flexible job) Processors with a ready time, due date etc. (??)

2.5 Work load allocation and worker satisfaction

Trötthet och uttråkad. Något vi borde ta med i litteraturen enligt Torbjörn, fast inte leta källor på det.

Source: "Employee positioning and workload allocation", Eiselt, Marianov, 2006 "Scheduling part-time and mixed-skilled workers to maximize employee satisfaction" Mohammad Akbari 2012

"Scheduling part-time personnel with availability restrictions and preferences to maximize employee satisfaction" Srimathy Mohan 2008

Chapter 3

Implementation insights

Chapter 4

The ideal CSTR: the chemostat

In this chapter we study exponential growth, the logistic...

4.1 Some simple models of biological growth

4.1.1 Exponential growth

If $\mu = \text{constant} > 0$, we get $X(t) = X_0 e^{\mu t}$.

4.1.2 The logistic equation

Let us assume that $\frac{dX}{dt} = \mu \cdot X$, with $\mu = \mu(S) = k \cdot S \dots$

$$\begin{cases} \frac{dX}{dt} = kSX & (a) \\ \frac{dS}{dt} = -\alpha kSX & (b) \end{cases}$$
$$\frac{dX}{dt} = r\left(1 - \frac{X}{B}\right)X \quad (4.1)$$

An explicit solution to (4.1) is: $X(t) = \frac{X_0 B}{X_0 + (B - X_0)e^{-rt}}$, if $0 < X_0 < B$. It can be found by separating variables in equation (4.1)

4.2 The chemostat

A chemostat is made of two main parts; a nutrient reservoir, and a growth-chamber, reactor, in which the bacteria reproduces.

$$\begin{cases} \frac{dX}{dt} = \mu(S)X - \overbrace{X \frac{F}{V}}^{\text{new}} \\ \frac{dS}{dt} = -\alpha \mu(S)X - \underbrace{S \frac{F}{V} + S_0 \frac{F}{V}}_{\text{new}} \end{cases} \quad (4.2)$$

$$\mathbf{A} = \begin{pmatrix} 0 & \sigma\alpha_1 \\ -\frac{1}{\alpha_1} & -\sigma - 1 \end{pmatrix}$$

The invariant line: conclusions

Model	Monods Chemostat	CSI-CSTR
μ	$\frac{S}{1+S}$	$\frac{S}{1+S+\frac{S^2}{K_I}}$
$\frac{dX}{dt}$	$\alpha_1 \frac{S}{1+S} X - X$	$\alpha_1 \frac{S}{1+S+\frac{S^2}{K_I}} X - X$
$\frac{dS}{dt}$	$-\frac{S}{1+S} X - S + \alpha_2$	$-\frac{S}{1+S+\frac{S^2}{K_I}} X - S + \alpha_2$
XNC	$S = \frac{1}{\alpha_1 - 1}$	$S = \frac{K_I(\alpha_1 - 1)}{2} \pm \sqrt{\left(\frac{K_I(\alpha_1 - 1)}{2}\right)^2 - K_I}$
SNC	$X = \frac{(\alpha_2 - S)(1+S)}{S}$	$X = \frac{(\alpha_2 - S)(1+S+\frac{S^2}{K_I})}{S}$
limit	—	$K_I \rightarrow \infty$

The other three models, the chemostat, the MMI-CSTR and the CPI-CSTR are quite similar in comparison to the CSI-CSTR.

Monods chemostat does not “feel” this inhibition and does not care...

This document is an example of BibTeX using in bibliography management. Three items are cited: *The L^AT_EX Companion* book [2], the Einstein journal paper [3], and the Donald Knuth’s website [4]. The L^AT_EX related items are [2, 4].

Bibliography

- [1] Ernst et al. An annotated bibliography of personnel scheduling and rostering. 2004.
- [2] Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The L^AT_EX Companion*. Addison-Wesley, Reading, Massachusetts, 1993.
- [3] Albert Einstein. Zur Elektrodynamik bewegter Körper. (German) [On the electrodynamics of moving bodies]. *Annalen der Physik*, 322(10):891–921, 1905.
- [4] Donald Knuth. Knuth: Computers and typesetting.

Appendix A

The Linearized stability

A.1 The Linearization

$F(x)$, a one-variable function of x can be Taylor-expanded around a fix X . We get $F(X + x) = F(X) + F'(X)x + O(x^2)$. For small perturbations of x around X we get the linearization: $F(X + x) \approx F(X) + F'(X)x$, containing only the constant and the linear terms.

For functions of two variables $F(X + x, S + s)$ and $G(X + x, S + s)$:

$$\begin{cases} F(X + x, S + s) = F(X, S) + F'_X(X, S)x + F'_S(X, S)s + O((x + s)^2) \\ G(X + x, S + s) = G(X, S) + G'_X(X, S)x + G'_S(X, S)s + O((x + s)^2) \end{cases}$$

```
function chemostat_inhibited(alpha1, alpha2, xp0, sp0, xc)
%
%chemostat_inhibited Displays a phaseportrait, nullclines
% and an Euler-path of an inhibited Chemostat.
% chemostat_inhibited(alfa1, alfa2, np0, cp0, nc) will run if
% alpha1 > 1/xc, thus there is a reproduction.
% alpha2 > 1/(xc*alpha1-1), thus there is sufficient stock-nutrition.
% xp0 > 0 , you can not have a nonpositive population.
% sp0 > 0 , you can not have a nonpositive concentration.
% xc > 0
%
% The blue arrows represent the vectorfield.
% The black lines are two of the three nullclines.
% The black dotted line is the invariant line (no solution crosses it).
% The red line is an Eulerpath, starting in + and ending in *.
%
% Try the following:
% chemostat_inhibited(5, 3, 0.2, 0.3, 6)
%
% by Per Erik Strandberg, 2003-2004.
%

% Start-condition:
%-----
if ((alpha1>1) & (alpha2>0) & (sp0>0) & (xp0>0) & xc>0),

    if (alpha2<1/(alpha1-1)),
        disp(' ')
        disp (' (HINT: Only the trivial steady state, alpha2 is too small...)')
    else
        disp(' ')
        disp (' (HINT: Two steady states, alpha2 is quite large...)')
    end
end
```

```
% The illegal indata case:
%-----
else
    disp('  chemostat_inhibited.m by Per Erik Strandberg, 2003-2004.')
```

Did not Finish OK. (You used illegal indata.)'

```
    disp(' ')
    disp(' For syntax help type: help chemostat_inhibited .')
```

disp(' ')

```
end
```

Copyright

The publishers will keep this document online on the Internet - or its possible replacement - for a period of 25 years from the date of publication barring exceptional circumstances. The online availability of the document implies a permanent permission for anyone to read, to download, to print out single copies for your own use and to use it unchanged for any non-commercial research and educational purpose. Subsequent transfers of copyright cannot revoke this permission. All other uses of the document are conditional on the consent of the copyright owner. The publisher has taken technical and administrative measures to assure authenticity, security and accessibility. According to intellectual property law the author has the right to be mentioned when his/her work is accessed as described above and to be protected against infringement. For additional information about the Linköping University Electronic Press and its procedures for publication and for assurance of document integrity, please refer to its WWW home page: <http://www.ep.liu.se/>

Upphovsrätt

Detta dokument hålls tillgängligt på Internet - eller dess framtida ersättare - under 25 år från publiceringsdatum under förutsättning att inga extraordinära omständigheter uppstår. Tillgång till dokumentet innebär tillstånd för var och en att läsa, ladda ner, skriva ut enstaka kopior för enskilt bruk och att använda det oförändrat för ickekommersiell forskning och för undervisning. Överföring av upphovsrätten vid en senare tidpunkt kan inte upphäva detta tillstånd. All annan användning av dokumentet kräver upphovsmannens medgivande. För att garantera äktheten, säkerheten och tillgängligheten finns det lösningar av teknisk och administrativ art. Upphovsmannens ideella rätt innefattar rätt att bli nämnd som upphovsman i den omfattning som god sed kräver vid användning av dokumentet på ovan beskrivna sätt samt skydd mot att dokumentet ändras eller presenteras i sådan form eller i sådant sammanhang som är kränkande för upphovsmannens litterära eller konstnärliga anseende eller egenart. För ytterligare information om Linköping University Electronic Press se förlagets hemsida <http://www.ep.liu.se/>

© 2016, Claes Arvidson, Emelie Karlsson