Master thesis

Work Distribution of a Heterogeneous Library Staff - A Personnel Task Scheduling Problem

Claes Arvidson, Emelie Karlsson

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Optimeringslära, Linköpings Universitet

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Supervisor: **T. Larsson**,
Optimeringslära, Linköpings Universitet

Examiner: **E. Rönnberg**,
Optimeringslära, Linköpings Universitet

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Abstract

Here is where you can write your abstract. It may be very long, or it may be very short, the reason you have an abstract is for people not to be forced to read lots of crap.

But still, they will have to read your abstract. After all, the abstract is what everyone reads...

Keywords: Keyword One, Chemostat, Another Key-Word, Key, Clé, Mot de cle, Nyckelhål, XBOX, Dagens viktigaste nyckelord, and Keywords.

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My opponent NN also deserves my thanks, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor...

Nomenclature

Most of the reoccurring abbreviations and symbols are described here.

Symbols

Y_0	The amount of the variable Y inserted into a system.
$\hat{Y}$	The unit-dimension of the variable Y, for example $\hat{t} = 1s$.
$\bar{Y}_i$	A steady state (number i) value of Y.
K_i	Constants used in kinetic expressions, for example K_I.
$\mathbf{A}$	The system matrix.

Abbreviations

CPI	Competitive Product Inhibition (or Inhibited)
CSI	Competitive Substrate Inhibition (or Inhibited)
CSTR	Continuous Stirred Tank (bio)Reactor
MMI	Michaelis-Menten Inhibition (or Inhibited)

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Chapter 1

Introduction

1.1 Background

At a library absence can cause problems, since the qualifications required to perform tasks varies. If a worker were to be unavailable a day due to a meeting or simply being ill it would require for a stand-in to fill the vacancy. Therefore, it is of great interest to have a schedule with as many skilled stand-ins as possible to overcome such disturbances.

1.2 Problem description

The goal of this thesis is to distribute given tasks to the heterogeneous workforce at the library of Norrköping. Each task is either classified as an outer or an inner service where an outer service is when a librarian needs to interact with visitors. Inner services can in some rare cases require a predetermined person to be assigned to a specified time or day.

Demands and requests are to be fulfilled to the furthest extent possible. Weekends are included in the scheduling problem, which adds more constraints regarding the number of contiguous working days. However, the librarians are permitted a few exceptions from these laws regarding days of rest.

The main purpose of the thesis is to create a schedule robust enough to withstand absence, such that outer services always are assigned to a qualified and available worker. This is visualized as having a list of available stand-ins for each shift.

There are a limited number of workers at the library and they make the resources that are to be distributed. Each individual has a set of *skills* and *competences*. Competences refer to the capability of being assigned the different outer services; Expedition, Norpan, Information desk, Library on wheels and Hageby as well as different inner services. The set of skills an individual can possess are described in Table 1.1. In total there are 39 workers available.

The outer services can be seen as assignments which requires available workers to be assigned to them. Each outer service is specified to a certain station, time and date. They also have a fix length and occur on a regular basis every ten weeks, which makes it possible to create a periodic schedule with a period of ten weeks.

Table 1.1: Personnel

Skills	**Description**
Work degree	0-100 %
Type of employment	Librarian/Assistant
Competence	Inner and outer services the worker is qualified for
Weekly rest	Which days the worker has requested after working a weekend
Other requests	Does not work evenings etc.

Furthermore, outer and a few inner services can be characterized by different properties, which are represented in Table 1.2.

Table 1.2: Outer and inner services

Outer service	**Property**
	Start time, end time, week and duration
	Station
	Number of qualified librarians demanded
	Number of qualified assistants demanded
Inner service	**Property**
	Start time, end time, week and duration
	Type
	Number of qualified librarians demanded
	Number of qualified assistants demanded

In addition to the properties mentioned above, there are several requirements that have to be met. These can be divided into job, robust and other requirements and are listed in Table 1.3 below.

Table 1.3: Requirements

Job requirements	**Description**
1	A maximum of one outer service is to be distributed to each person and d
2	Remaining work time is individually distributed on assignments such as re
3	Weekend work are to be evenly distributed between the workers available
4	Working a weekend includes work on saturday and sunday the same week
5	One evening shift on a weekday per person each week except when weeker
6	Every ten weeks the schedule is to be repeated.
7	It is recommended for each week to be as similar as possible.
Robust requirements	**Description**
1	Each outer service require at least one stand-in.
2	The stand-ins have to be qualified for the tasks they are stand-in for.
3	Focus is to maximize the lowest number of stand-ins of any task.
Other requirements	**Description**
1	Department and general meetings are to be held once per five weeks.

There are also additional requirements of the resulting schedule made by the workers at the library. Two examples would be that a handful staff members

are unable to work weekends as well as some personnel are unable to work in the evenings.

Chapter 2

Literature review

The scheduling problem has been studied since the 1960's as a mathematical optimization problem. (The Use of Mathematical Models in Plant Maintenance Decision Making, 1967) In surveys such as "An Annotated Bibliography of Personnel Scheduling and Rostering" Ernst et al. 2004, covers papers from 1954. The bibliography covers the most important contributions to the area of scheduling up to the year 2004 and divides the area into different subcategories. Below, we have identified the most relevant areas of scheduling in relation to the problem at hand.

2.1 Tour scheduling problem (TSP)

In the paper review of different scheduling problem written by Ernst et al. (2004) one can see a number of references to tour scheduling problems. However, there are significantly lower number of references regarding heterogeneous workforce.

Papers of interest: "Task assignment and tour scheduling": Loucks and Jacobs, 1991

"Scheduling Restaurant Workers to Minimize Labor Cost and Meet Service Standards" Choi, Hwang and Park, 2009

"An integer linear programming-based heuristic for scheduling heterogeneous, part-time service employees" Heterogenous work force, tour scheduling. Using two objective functions Hojati and Patil, 2010

2.2 Personnel task scheduling problem (PTSP)

Most likely our problem. Definition is "in which a set of tasks with fixed start and finish times have to be allocated to a heterogeneous workforce". The objective of these problems is to minimise the overall cost of personnel required to perform all tasks.

Papers of interest: "The Personnel Task Scheduling Problem", Mohan Krishnamoorthy, Andreas T. Ernst (2001) - probably the most fundamental article

"Task assignement for maintenance personnel": Roberts and Escudero, 1983a, 1983b

"A stochastic programming model for scheduling maintenance personnel" Duffuaa and Al-Sultan, 1999

2.3 Shift minimisation personnel task scheduling problem (SMPTSP)

Difference: "The only cost incurred is due to the number of personnel (shifts) that are used."

Papers of interest: "Algorithms for large scale Shift Minimisation Personnel Task Scheduling Problems" Krishnamoorthy, Ernst, Baatar (2011)

"The shift minimisation personnel task scheduling problem: A new hybrid approach and computational insights" Smet, Wauters, Mihaylov, Berghe (2014)

"Fast local search and guided local search and their application to British Telecom's workforce scheduling problem" Tsang and Voudouris, 1997 - also with travelling costs, investigates two methods.

"A Triplet-Based Exact Method for the Shift Minimisation Personnel Task Scheduling Problem" Baatar et al., 2015

2.4 Fixed/flexible job scheduling problem (FJSP)

Identical skill of the workers/machines and indentical skill requirements of the operations to execute.

Problem defined in: "Algorithms for large scale Shift Minimisation Personnel Task Scheduling Problems" M. Krishnamoorthy http://www.sciencedirect.com/science/article/pii/S037

Problem: "A metaheuristic for the fixed job scheduling problem under spread time constraints" André Rossi, http://www.sciencedirect.com/science/article/pii/S0305054809002251 (Fixed job)

Cemal Özgüven http://www.sciencedirect.com/science/article/pii/S0307904X11004173 (Flexible job) Processors with a ready time, due date etc. (??)

2.5 Work load allocation and worker satisfaction

Trötthet och uttråkad. Något vi borde ta med i litteraturen enligt Torbjörn, fast inte leta källor på det.

Source: "Employee positioning and workload allocation", Eiselt, Marianov, 2006 "Scheduling part-time and mixed-skilled workers to maximize employee satisfaction" Mohammad Akbari 2012

"Scheduling part-time personnel with availability restrictions and preferences to maximize employee satisfaction" Srimathy Mohan 2008

Chapter 3

Implementation insights

Chapter 4

The ideal CSTR: the chemostat

In this chapter we study exponential growth, the logistic. . . .

4.1 Some simple models of biological growth

4.1.1 Exponential growth

If $\mu = \text{constant} > 0$, we get $X(t) = X_0 e^{\mu t}$.

4.1.2 The logistic equation

Let us assume that $\frac{dX}{dt} = \mu \cdot X$, with $\mu = \mu(S) = k \cdot S$ ...

$$\begin{cases} \frac{dX}{dt} = kSX & (a) \\ \frac{dS}{dt} = -\alpha kSX & (b) \end{cases}$$

$$\frac{dX}{dt} = r(1 - \frac{X}{B})X \tag{4.1}$$

An explicit solution to (4.1) is: $X(t) = \frac{X_0 B}{X_0 + (B - X_0)e^{-rt}}$, if $0 < X_0 < B$. It can be found by separating variables in equation (4.1)

4.2 The chemostat

A chemostat is made of two main parts; a nutrient reservoir, and a growth-chamber, reactor, in which the bacteria reproduces.

$$\begin{cases} \frac{dX}{dt} = \mu(S)X \overbrace{-X\frac{F}{V}}^{\text{new}} \\ \frac{dS}{dt} = -\alpha\mu(S)X \underbrace{-S\frac{F}{V} + S_0\frac{F}{V}}_{\text{new}} \end{cases} \tag{4.2}$$

$$\mathbf{A} = \begin{pmatrix} 0 & \sigma\alpha_1 \\ -\frac{1}{\alpha_1} & -\sigma - 1 \end{pmatrix}$$

The invariant line: conclusions

Model	Monods Chemostat	CSI-CSTR
μ	$\frac{S}{1+S}$	$\frac{S}{1+S+\frac{S^2}{K_I}}$
$\frac{dX}{dt}$	$\alpha_1 \frac{S}{1+S} X - X$	$\alpha_1 \frac{S}{1+S+\frac{S^2}{K_I}} X - X$
$\frac{dS}{dt}$	$-\frac{S}{1+S} X - S + \alpha_2$	$-\frac{S}{1+S+\frac{S^2}{K_I}} X - S + \alpha_2$
XNC	$S = \frac{1}{\alpha_1 - 1}$	$S = \frac{K_I(\alpha_1-1)}{2} \pm \sqrt{\left(\frac{K_I(\alpha_1-1)}{2}\right)^2 - K_I}$
SNC	$X = \frac{(\alpha_2 - S)(1+S)}{S}$	$X = \frac{(\alpha_2 - S)(1+S+\frac{S^2}{K_I})}{S}$
limit	–	$K_I \to \infty$

The other three models, the chemostat, the MMI-CSTR and the CPI-CSTR are quite similar in comparison to the CSI-CSTR.

Monods chemostat does not “feel” this inhibition and does not care...

Here is an example of how to cite books in your bibliography. This text will be displayed at the end of chapter two. This is some kind of bibiography, according to [1], we have... And according to [1, 2] we have something else.

Bibliography

[1] Lennart Råde, Bertil Westergren, (2001), *Mathematics Handbook for Science and Engineering*, Studentlitteratur, Lund.

[2] Torkel Glad, Lennart Ljung, (1989), *Reglerteknik grundläggande teori*, Studentlitteratur, Lund.

Appendix A

The Linearized stability

A.1 The Linearization

$F(x)$, a one-variable function of x can be Taylor-expanded around a fix X. We get $F(X+x) = F(X) + F'(X)x + O(x^2)$. For small perturbations of x around X we get the linearization: $F(X+x) \approx F(X) + F'(X)x$, containing only the constant and the linear terms.

For functions of two variables $F(X+x, S+s)$ and $G(X+x, S+s)$:

$$\begin{cases} F(X+x,S+s) = F(X,S) + F'_X(X,S)x + F'_S(X,S)s + O((x+s)^2) \\ G(X+x,S+s) = G(X,S) + G'_X(X,S)x + G'_S(X,S)s + O((x+s)^2) \end{cases}$$

```
function chemostat_inhibited(alpha1, alpha2, xp0, sp0, xc)
%
%chemostat_inhibited Displays a phaseportrait, nullclines
%   and an Euler-path of an inhibited Chemostat.
%   chemostat_inhibited(alfa1, alfa2, np0, cp0, nc) will run if
%   alpha1 > 1/xc, thus there is a reproduction.
%   alpha2 > 1/(xc*alpha1-1), thus there is sufficient stock-nutrition.
%   xp0 > 0 , you can not have a nonpositive population.
%   sp0 > 0 , you can not have a nonpositive concentration.
%   xc  > 0
%
%   The blue arrows represent the vectorfield.
%   The black lines are two of the three nullclines.
%   The black dotted line is the invariant line (no solution crosses it).
%   The red line is an Eulerpath, starting in + and ending in *.
%
%   Try the following:
%   chemostat_inhibited(5, 3, 0.2, 0.3, 6)
%
%   by Per Erik Strandberg, 2003-2004.
%

% Start-condition:
%------------------
if ((alpha1>1) & (alpha2>0) & (sp0>0) & (xp0>0) & xc>0),

    if (alpha2<1/(alpha1-1)),
        disp(' ')
        disp (' (HINT: Only the trivial steady state, alpha2 is too small...)')
    else
        disp(' ')
        disp (' (HINT: Two steady states, alpha2 is quite large...)')
    end
```

```
% The illegal indata case:
%--------------------------
else
    disp('   chemostat_inhibited.m by Per Erik Strandberg, 2003-2004.')
    disp('   Did not Finish OK. (You used illegal indata.)')
    disp(' ')
    disp('   For syntax help type: help chemostat_inhibited .')
    disp(' ')
end
```