Examensarbete

Mathematical models of everything you ever wanted to have a model of, or some other very long title that is hard to say in one breath.

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LiTH - MAT - EX - - 04 / 04 - - SE

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Examensarbete: **30 hp**

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Abstract

Here is where you can write your abstract. It may be very long, or it may be very short, the reason you have an abstract is for people not to be forced to read lots of crap.

But still, they will have to read your abstract. After all, the abstract is what everyone reads...

Keywords: Keyword One, Chemostat, Another Key-Word, Key, Clé, Mot de cle, Nyckelhål, XBOX, Dagens viktigaste nyckelord, and Keywords.

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I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor...

I also have to thank, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor...

My opponent NN also deserves my thanks, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor...

Nomenclature

Most of the reoccurring abbreviations and symbols are described here.

Symbols

Y_0	The amount of the variable Y inserted into a system.
$\hat{Y}$	The unit-dimension of the variable Y, for example $\hat{t} = 1s$.
$\bar{Y}_i$	A steady state (number i) value of Y.
K_i	Constants used in kinetic expressions, for example K_I.
$\mathbf{A}$	The system matrix.

Abbreviations

CPI	Competitive Product Inhibition (or Inhibited)
CSI	Competitive Substrate Inhibition (or Inhibited)
CSTR	Continuous Stirred Tank (bio)Reactor
MMI	Michaelis-Menten Inhibition (or Inhibited)

Contents

Chapter 1

Introduction

1.1 Background

1.2 Problem description

Chapter 2

The ideal CSTR: the chemostat

In this chapter we study exponential growth, the logistic. . . .

2.1 Some simple models of biological growth

2.1.1 Exponential growth

If $\mu = \text{constant} > 0$, we get $X(t) = X_0 e^{\mu t}$.

2.1.2 The logistic equation

Let us assume that $\frac{dX}{dt} = \mu \cdot X$, with $\mu = \mu(S) = k \cdot S$ ...

$$\begin{cases} \frac{dX}{dt} = kSX & (a) \\ \frac{dS}{dt} = -\alpha kSX & (b) \end{cases}$$

$$\frac{dX}{dt} = r(1 - \frac{X}{B})X \tag{2.1}$$

An explicit solution to (2.1) is: $X(t) = \frac{X_0 B}{X_0 + (B - X_0)e^{-rt}}$, if $0 < X_0 < B$. It can be found by separating variables in equation (2.1)

2.2 The chemostat

A chemostat is made of two main parts; a nutrient reservoir, and a growth-chamber, reactor, in which the bacteria reproduces.

$$\begin{cases} \frac{dX}{dt} = \mu(S)X \overbrace{-X\frac{F}{V}}^{\text{new}} \\ \frac{dS}{dt} = -\alpha\mu(S)X \underbrace{-S\frac{F}{V} + S_0\frac{F}{V}}_{\text{new}} \end{cases} \tag{2.2}$$

$$\mathbf{A} = \begin{pmatrix} 0 & \sigma\alpha_1 \\ -\frac{1}{\alpha_1} & -\sigma - 1 \end{pmatrix}$$

The invariant line: conclusions

$$\mathbf{A} = \begin{pmatrix} \alpha_1 \frac{S}{1+S} \frac{1}{(1+\frac{X}{X_C})^2} - 1 & \alpha_1 \frac{1}{(1+S)^2} \frac{X}{1+\frac{X}{X_C}} \\ -\frac{S}{1+S} \frac{1}{(1+\frac{X}{X_C})^2} & -\frac{1}{(1+S)^2} \frac{X}{1+\frac{X}{X_C}} - 1 \end{pmatrix}.$$

Model	Monods Chemostat	CSI-CSTR
μ	$\frac{S}{1+S}$	$\frac{S}{1+S+\frac{S^2}{K_I}}$
$\frac{dX}{dt}$	$\alpha_1 \frac{S}{1+S} X - X$	$\alpha_1 \frac{S}{1+S+\frac{S^2}{K_I}} X - X$
$\frac{dS}{dt}$	$-\frac{S}{1+S} X - S + \alpha_2$	$-\frac{S}{1+S+\frac{S^2}{K_I}} X - S + \alpha_2$
XNC	$S = \frac{1}{\alpha_1 - 1}$	$S = \frac{K_I(\alpha_1-1)}{2} \pm \sqrt{\left(\frac{K_I(\alpha_1-1)}{2}\right)^2 - K_I}$
SNC	$X = \frac{(\alpha_2 - S)(1+S)}{S}$	$X = \frac{(\alpha_2 - S)(1+S+\frac{S^2}{K_I})}{S}$
limit	–	$K_I \to \infty$

The other three models, the chemostat, the MMI-CSTR and the CPI-CSTR are quite similar in comparison to the CSI-CSTR.

Monods chemostat does not "feel" this inhibition and does not care...

Here is an example of how to cite books in your bibliography. This text will be displayed at the end of chapter two. This is some kind of bibiography, according to [1], we have... And according to [1, 2] we have something else.

Bibliography

[1] Lennart Råde, Bertil Westergren, (2001), *Mathematics Handbook for Science and Engineering*, Studentlitteratur, Lund.

[2] Torkel Glad, Lennart Ljung, (1989), *Reglerteknik grundläggande teori*, Studentlitteratur, Lund.

Appendix A

The Linearized stability

A.1 The Linearization

$F(x)$, a one-variable function of x can be Taylor-expanded around a fix X. We get $F(X + x) = F(X) + F'(X)x + O(x^2)$. For small perturbations of x around X we get the linearization: $F(X + x) \approx F(X) + F'(X)x$, containing only the constant and the linear terms.

For functions of two variables $F(X + x, S + s)$ and $G(X + x, S + s)$:

$$\begin{cases} F(X + x, S + s) = F(X, S) + F'_X(X, S)x + F'_S(X, S)s + O((x + s)^2) \\ G(X + x, S + s) = G(X, S) + G'_X(X, S)x + G'_S(X, S)s + O((x + s)^2) \end{cases}$$

```
function chemostat_inhibited(alpha1, alpha2, xp0, sp0, xc)
%
%chemostat_inhibited Displays a phaseportrait, nullclines
%   and an Euler-path of an inhibited Chemostat.
%   chemostat_inhibited(alfa1, alfa2, np0, cp0, nc) will run if
%   alpha1 > 1/xc, thus there is a reproduction.
%   alpha2 > 1/(xc*alpha1-1), thus there is sufficient stock-nutrition.
%   xp0 > 0 , you can not have a nonpositive population.
%   sp0 > 0 , you can not have a nonpositive concentration.
%   xc  > 0
%
%   The blue arrows represent the vectorfield.
%   The black lines are two of the three nullclines.
%   The black dotted line is the invariant line (no solution crosses it).
%   The red line is an Eulerpath, starting in + and ending in *.
%
%   Try the following:
%   chemostat_inhibited(5, 3, 0.2, 0.3, 6)
%
%   by Per Erik Strandberg, 2003-2004.
%

% Start-condition:
%------------------
if ((alpha1>1) & (alpha2>0) & (sp0>0) & (xp0>0) & xc>0),

    if (alpha2<1/(alpha1-1)),
        disp(' ')
        disp (' (HINT: Only the trivial steady state, alpha2 is too small...)')
    else
        disp(' ')
        disp (' (HINT: Two steady states, alpha2 is quite large...)')
    end
```

```
% The illegal indata case:
%---------------------------
else
    disp('   chemostat_inhibited.m by Per Erik Strandberg, 2003-2004.')
    disp('   Did not Finish OK. (You used illegal indata.)')
    disp(' ')
    disp('   For syntax help type: help chemostat_inhibited .')
    disp(' ')
end
```