

Master thesis

**Work Distribution of a Heterogeneous Library Staff - A
Personnel Task Scheduling Problem**

Claes Arvidson, Emelie Karlsson

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Optimeringslära, Linköpings Universitet

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Supervisor: **T. Larsson**,
Optimeringslära, Linköpings Universitet

Examiner: **E. Rönnberg**,
Optimeringslära, Linköpings Universitet

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Abstract

Here is where you can write your abstract. It may be very long, or it may be very short, the reason you have an abstract is for people not to be forced to read lots of crap.

But still, they will have to read your abstract. After all, the abstract is what everyone reads. . .

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My opponent NN also deserves my thanks, I would like to thank my supervisor, I would like to thank my supervisor, I would like to thank my supervisor...

Nomenclature

Most of the reoccurring abbreviations and symbols are described here.

Symbols

Y_0 The amount of the variable Y inserted into a system.
 \hat{Y} The unit-dimension of the variable Y , for example $\hat{t} = 1s$.
 \bar{Y}_i A steady state (number i) value of Y .

K_i Constants used in kinetic expressions, for example K_I .

\mathbf{A} The system matrix.

Abbreviations

CPI Competitive Product Inhibition (or Inhibited)
CSI Competitive Substrate Inhibition (or Inhibited)
CSTR Continuous Stirred Tank (bio)Reactor
MMI Michaelis-Menten Inhibition (or Inhibited)

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Chapter 1

Introduction

1.1 Background

At a library absence can cause problems, since the qualifications required to perform tasks varies. If a worker were to be unavailable a day due to a meeting or simply being ill it would require for a stand-in to fill the vacancy. Therefore, it is of great interest to have a schedule with as many skilled stand-ins as possible to overcome such disturbances.

1.2 Problem description

The goal of this thesis is to distribute given tasks to the heterogeneous workforce at the library of Norrköping. Each task is either classified as an outer or an inner task where an outer task is when a librarian needs to interact with visitors. Inner services can in some rare cases require a predetermined person to be assigned to a specified time or day.

Demands and requests are to be fulfilled to the furthest extent possible. Weekends are included in the scheduling problem, which adds more constraints regarding the number of contiguous working days. However, the librarians are permitted a few exceptions from these laws regarding days of rest.

The main purpose of the thesis is to create a schedule robust enough to withstand absence, such that outer services always are assigned to a qualified and available worker. This is visualized as having a list of available stand-ins for each shift.

There are a limited number of workers at the library and they make the resources that are to be distributed. Each individual has a set of *egenskaper* and *kompetenser*. Competences refer to the capability of being assigned the different outer tasks; Expedition, Norpan, Information desk, Bookbus and Hageby as well as different inner tasks. The set of skills an individual can possess are described in Table 1.1. In total there are 39 workers available.

De yttre och inre uppgifterna kan ses som behov av personal som måste täckas av den personal som finns att tillgå. De olika yttre uppgifterna som behöver utföras inkluderar arbete vid olika stationer vid olika tidpunkter och datum. Varje uppgift har en bestämd längd och återkommer regelbundet inom ett

Table 1.1: Personal

skills	Beskrivning
Arbetsomfattning	0-100 %
Anställningsform	Bibliotikarie/Assistent
Kompetens	Inre och yttre tjänster som personalen klarar av.
Veckovila	Hur personalen tar ut sin veckovila efter helgarbete.
Övriga önskemål	Jobbar ej kväll etc

10-veckorsintervall, vilket gör att ett rullande schema kan skapas med perioden tio veckor.

Varje uppgift kan, liksom personalen, karaktäriseras av vissa egenskaper som finns representerade i tabell 1.2.

Table 1.2: Yttre och inre uppgifter

Yttre uppgift	Egenskap
	Starttid, sluttid, vecka och tidsåtgång
	Station
	Krav på antal bibliotikarier av rätt kompetens.
	Krav på antal assistenter av rätt kompetens.
	Krav på totala antalet personal.
Inre uppgift	Egenskap
	Vecka, Tidsåtgång
	Typ
	Krav på antal bibliotikarier av rätt kompetens.
	Krav på antal assistenter av rätt kompetens.

Utöver de ovan nämnda resurserna och behoven, finns ett antal krav som ställs på hur schemat får utformas. Dessa kan delas upp i arbetsvillkor, robusthetskrav samt övriga krav och finns representerade i tabell 1.3.

Table 1.3: Krav

Arbetskrav	Beskrivning
	En person ska bli tilldelad högst en yttre eller inre uppgift under sin arbetstid.
	Övrig arbetstid distribueras självständig med uppgifter såsom exempelvis bokpl
	Helgarbete ska fördelas rättvist mellan de i personalen som är tillgängliga för he
	Helgarbete innefattar arbete under fredag kväll, påföljande lördag och söndag.
	Högst ett kvällspass per personal i veckan bortsett från den vecka helgarbete sk
	Schemat ska upprepa sig var 10e vecka.
	Varje arbetsvecka ska ha liknande struktur i största möjliga mån.
Robusthetskrav	Beskrivning
	För varje yttre uppgift ska det finnas minst en reserv.
	Reserverna ska vara av rätt kompetens för uppgiften de är reserver till.
	Prioritet ligger i att det lägsta antalet reserver för en uppgift ska vara så hög so
Övriga krav	Beskrivning
	Stormöte och avdelningsmöten ska vardera hållas en gång var femte vecka.

Chapter 2

Literature review

2.1 Tour scheduling

blabla

Chapter 3

Implementation insights

Chapter 4

The ideal CSTR: the chemostat

In this chapter we study exponential growth, the logistic. . . .

4.1 Some simple models of biological growth

4.1.1 Exponential growth

If $\mu = \text{constant} > 0$, we get $X(t) = X_0 e^{\mu t}$.

4.1.2 The logistic equation

Let us assume that $\frac{dX}{dt} = \mu \cdot X$, with $\mu = \mu(S) = k \cdot S \dots$

$$\begin{cases} \frac{dX}{dt} = kSX & (a) \\ \frac{dS}{dt} = -\alpha kSX & (b) \end{cases}$$
$$\frac{dX}{dt} = r\left(1 - \frac{X}{B}\right)X \quad (4.1)$$

An explicit solution to (4.1) is: $X(t) = \frac{X_0 B}{X_0 + (B - X_0)e^{-rt}}$, if $0 < X_0 < B$. It can be found by separating variables in equation (4.1)

4.2 The chemostat

A chemostat is made of two main parts; a nutrient reservoir, and a growth-chamber, reactor, in which the bacteria reproduces.

$$\begin{cases} \frac{dX}{dt} = \mu(S)X - \overbrace{X \frac{F}{V}}^{\text{new}} \\ \frac{dS}{dt} = -\alpha \mu(S)X - \underbrace{S \frac{F}{V} + S_0 \frac{F}{V}}_{\text{new}} \end{cases} \quad (4.2)$$

$$\mathbf{A} = \begin{pmatrix} 0 & \sigma\alpha_1 \\ -\frac{1}{\alpha_1} & -\sigma - 1 \end{pmatrix}$$

The invariant line: conclusions

Model	Monods Chemostat	CSI-CSTR
μ	$\frac{S}{1+S}$	$\frac{S}{1+S+\frac{S^2}{K_I}}$
$\frac{dX}{dt}$	$\alpha_1 \frac{S}{1+S} X - X$	$\alpha_1 \frac{S}{1+S+\frac{S^2}{K_I}} X - X$
$\frac{dS}{dt}$	$-\frac{S}{1+S} X - S + \alpha_2$	$-\frac{S}{1+S+\frac{S^2}{K_I}} X - S + \alpha_2$
XNC	$S = \frac{1}{\alpha_1 - 1}$	$S = \frac{K_I(\alpha_1 - 1)}{2} \pm \sqrt{\left(\frac{K_I(\alpha_1 - 1)}{2}\right)^2 - K_I}$
SNC	$X = \frac{(\alpha_2 - S)(1+S)}{S}$	$X = \frac{(\alpha_2 - S)(1+S+\frac{S^2}{K_I})}{S}$
limit	—	$K_I \rightarrow \infty$

The other three models, the chemostat, the MMI-CSTR and the CPI-CSTR are quite similar in comparison to the CSI-CSTR.

Monods chemostat does not “feel” this inhibition and does not care...

Here is an example of how to cite books in your bibliography. This text will be displayed at the end of chapter two. This is some kind of bibliography, according to [1], we have... And according to [1, 2] we have something else.

Bibliography

- [1] Lennart Råde, Bertil Westergren, (2001), *Mathematics Handbook for Science and Engineering*, Studentlitteratur, Lund.
- [2] Torkel Glad, Lennart Ljung, (1989), *Reglerteknik grundläggande teori*, Studentlitteratur, Lund.

Appendix A

The Linearized stability

A.1 The Linearization

$F(x)$, a one-variable function of x can be Taylor-expanded around a fix X . We get $F(X + x) = F(X) + F'(X)x + O(x^2)$. For small perturbations of x around X we get the linearization: $F(X + x) \approx F(X) + F'(X)x$, containing only the constant and the linear terms.

For functions of two variables $F(X + x, S + s)$ and $G(X + x, S + s)$:

$$\begin{cases} F(X + x, S + s) = F(X, S) + F'_X(X, S)x + F'_S(X, S)s + O((x + s)^2) \\ G(X + x, S + s) = G(X, S) + G'_X(X, S)x + G'_S(X, S)s + O((x + s)^2) \end{cases}$$

```
function chemostat_inhibited(alpha1, alpha2, xp0, sp0, xc)
%
%chemostat_inhibited Displays a phaseportrait, nullclines
% and an Euler-path of an inhibited Chemostat.
% chemostat_inhibited(alfa1, alfa2, np0, cp0, nc) will run if
% alpha1 > 1/xc, thus there is a reproduction.
% alpha2 > 1/(xc*alpha1-1), thus there is sufficient stock-nutrition.
% xp0 > 0 , you can not have a nonpositive population.
% sp0 > 0 , you can not have a nonpositive concentration.
% xc > 0
%
% The blue arrows represent the vectorfield.
% The black lines are two of the three nullclines.
% The black dotted line is the invariant line (no solution crosses it).
% The red line is an Eulerpath, starting in + and ending in *.
%
% Try the following:
% chemostat_inhibited(5, 3, 0.2, 0.3, 6)
%
% by Per Erik Strandberg, 2003-2004.
%

% Start-condition:
%-----
if ((alpha1>1) & (alpha2>0) & (sp0>0) & (xp0>0) & xc>0),

    if (alpha2<1/(alpha1-1)),
        disp(' ')
        disp (' (HINT: Only the trivial steady state, alpha2 is too small...)')
    else
        disp(' ')
        disp (' (HINT: Two steady states, alpha2 is quite large...)')
    end
end
```

```
% The illegal indata case:
%-----
else
    disp('  chemostat_inhibited.m by Per Erik Strandberg, 2003-2004.')
```

Did not Finish OK. (You used illegal indata.)'

```
    disp(' ')
    disp(' For syntax help type: help chemostat_inhibited .')
```

disp(' ')

```
end
```

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