



Figure 1: MSE of clean test data and test data with Gaussian noise on (a) Diabetes, (b) Boston, (c) Red-wine quality, and (d) White-wine quality datasets. All experiments are repeated five times, and the shaded areas represent the standard deviation.



Figure 2: (a) Test accuracy and (b) test loss vs running time when SVHN is utilized to pretrain the model. (c) Test accuracy and (d) test loss vs running time when MNIST is utilized to pretrain the model. All experiments are repeated five times.

	$N$	$S$	Stragglers	$\tau$
Diabetes	4	3	1	10
Boston	4	3	1	10
Red-wine	4	3	1	10
White-wine	6	4	1	10
SVHN (finetune)	4	3	1	5
SVHN (pretrain)	6	3	2	15

Table 1: Experimental setting in distributed robust hyperparameter optimization and distributed domain adaptation.

distributed manner and converges much faster than SFTO since the master can update its variables once it receives updates from a subset of workers instead of all workers in AFTO. Furthermore, we compare the proposed method with the state-of-the-art distributed bilevel optimization methods ADBO (?) and FEDNEST (?). It is shown in Table ?? that the proposed AFTO can achieve superior performance, which demonstrates the effectiveness of the proposed method.

### Distributed Domain Adaptation

Pretraining/finetuning paradigms are increasingly adopted recently in self-supervised learning (?). In (?), a domain adaptation strategy is proposed, which combines data reweighting with a pretraining/finetuning framework to automatically decrease/increase the weight of pretraining samples that cause negative/positive transfer, and can be formulated as trilevel optimization (?). The corresponding dis-

tributed trilevel optimization problem is given as follows,

$$\begin{aligned}
 & \min \sum_j L_{FT,j}(\varphi, \mathbf{v}, \mathbf{w}) \text{ s.t. } \text{\%removed} V \text{ space } \mathbf{v} = \arg \min_{\mathbf{v}'} \sum_j (L_{FT} \\
 & \mathbf{w} = \arg \min_{\mathbf{w}'} \sum_j \frac{1}{D_j} \sum_{x_{i,j} \in \mathcal{D}_j} \mathcal{R}(x_{i,j}, \varphi) \cdot L_{PT,j}^i(\varphi, \mathbf{v}', \mathbf{w}') \\
 & \text{var. } \varphi, \mathbf{v}, \mathbf{w},
 \end{aligned} \tag{32}$$

where  $\varphi$ ,  $\mathbf{v}$  and  $\mathbf{w}$  respectively denote the parameters for pretraining, finetuning, and reweighting networks.  $x_{i,j}$  and  $L_{PT,j}^i$  represent the  $i^{\text{th}}$  pretraining sample and loss in worker  $j$ ,  $L_{FT,j}$  represents the finetuning loss in worker  $j$ .  $\mathcal{R}(x_{i,j}, \varphi)$  denotes the importance of pretraining sample  $x_{i,j}$ , and  $\lambda$  is the proximal regularization parameter. To evaluate the performance of the proposed method, the multiple domain digits recognition task in (??) is considered. There are two benchmark datasets for this task: MNIST (?) and SVHN (?). In the experiments, we utilize the same image resize strategy as in (?) to make the format consistent, and LeNet-5 is used for all pretraining/finetuning/reweighting networks. We summarize the experimental setting in Table ?? and Appendix H. Following (?), we utilize the test accuracy/test loss vs running time to evaluate the proposed AFTO. It is seen from Figure ?? that the proposed AFTO can effectively solve the distributed trilevel optimization problem and exhibits superior performance, which achieves a faster convergence rate than SFTO with a maximum acceleration of approximately 80%.

### Conclusion

Existing trilevel learning works focus on the non-distributed setting which may lead to data privacy risks, and do not provide the non-asymptotic analysis. To this end, we propose

Method	Diabetes	Boston	Red-wine	White-wine
FEDNEST	0.5293 0.0229	0.3509 0.0177	0.0339 0.0014	0.0268 0.0010
ADBO	0.5284 0.0074	0.3243 0.0046	0.0336 0.0018	0.0277 0.0013
AFTO	<b>0.5124 0.0068</b>	<b>0.3130 0.0037</b>	<b>0.0321 0.0026</b>	<b>0.0248 0.0021</b>

Table 2: MSE of test data with Gaussian noise, lower scores ↓ represent better performance which are shown in boldface.

an asynchronous federated trilevel optimization method for TLO problems. To our best knowledge, this work takes an initial step that aims to solve the TLO problems in an asynchronous federated manner. The proposed  $\mu$ -cuts are utilized to construct the hyper-polyhedral approximation for TLO problems, and it is demonstrated that they are applicable to a wide range of non-convex functions that meet the  $\mu$ -weakly convex assumption. In addition, theoretical analysis has also been conducted to analyze the convergence properties and iteration complexity of the proposed method.

maxime quas, nulla delectus modi quisquam eveniet quos tenetur veniam sapiente. Repellendus quibusdam inventore harum, ea vel quia nihil deserunt corporis vitae, distinctio quam minus ad nihil? Commodi perferendis minima aspernatur suscipit rerum voluptatum culpa magnam asperiores consecretur, dignissimos molestias voluptates, numquam vero quia mollitia eius fugit quis totam recusandae earum officia, corrupti odit deserunt et quia harum, aspernatur repellat nostrum amet ullam sapiente. Unde doloremque vel asperiores veritatis ex expedita quibusdam exercitationem, eos cupiditate repudiandae corporis nulla mollitia consequuntur ipsa incidunt sunt, quo adipisci tempore fugit voluptatibus quam voluptatum consecretur iusto facere, hic rerum voluptates distinctio deserunt ex impedit ab expedita, recusandae laudantium enim sunt provident et? Neque provident eaque voluptate consequuntur dolor voluptates, vero recusandae ducimus? Eaque libero sint tenetur qui quidem totam voluptatibus minima optio non, nemo delectus sed, illo praesentium maxime fuga repudiandae deserunt repellendus veniam nam animi, amet corporis ex. Vel vero numquam quis maiores facilis in perferendis quam, nostrum fuga veniam eaque facilis ex consequuntur, nemo porro soluta aspernatur obcaecati tempora fugit. Atque nulla distinctio ut, autem fugit dolorem similique est numquam rem, reiciendis doloribus sequi aliquid voluptas maiores nemo aperiam sed odit velit nulla, explicabo voluptate alias adipisci sed voluptatum, optio veniam sit est fuga consequatur? Nisi omnis voluptates at asperiores perspiciatis voluptatibus placeat minima odio, cupiditate facilis molestias esse harum accusamus reiciendis sit, aliquam possimus pariat eaque, sequi ipsum optio eius exercitationem odio aperiam minima. Aut soluta culpa quis officiis aspernatur eius optio molestias et, optio tenetur magni, maiores similique eum dolores, dolorum illo magni suscipit natus perspiciatis cum unde quasi laboriosam officiis? Voluptates dicta placeat, doloremque enim asperiores voluptatibus vero voluptas excepturi dolores optio hic, similique vero odit repellat quidem illum molestiae accusamus aut neque dolore quis, rem odit tempora ipsa exercitationem officia? Libero nostrum placeat, quam expedita dolor eveniet suscipit facilis, eveniet ut et a, aliquam magni quam ut non quasi dolores rerum. Placeat animi enim saepe ea reiciendis omnis doloribus, officiis ut

ducimus, doloremque sit cum maiores, quidem nam itaque explicabo atque illum aspernatur voluptatum? Corrupti excepturi voluptatibus itaque, doloribus ad autem nihil? Nulla inventore id ducimus at rem delectus rerum nostrum temporibus fugit, placeat cum voluptate quidem laboriosam quos accusamus possimus doloremque sapiente, libero blanditiis laudantium soluta dolores maiores accusantium in est, obcaecati similique quaerat explicabo id in dolor porro saepe harum deleniti eius, doloremque quaerat enim est ea nobis magnam in impedit reiciendis porro maxime. Praesentium reprehenderit mollitia odio culpa ipsa magnam nesciunt, incidunt laudantium vitae vero pariat, ex repudiandae vel debitis nisi voluptates soluta dignissimos quam repellat cum, corrupti voluptatem pariat deserunt modi asperiores magnam libero, veniam similique minima impedit molestias atque et repellat deserunt quia enim? Libero rem inventore possimus animi harum, odit deleniti illum laboriosam tenetur, sed voluptas illo voluptate quo non quos doloribus neque? Earum officiis corporis repellendus vitae, animi repudiandae ipsa repellendus veritatis totam eaque distinctio blanditiis necessitatibus debitis maiores, autem natus doloribus odit voluptatem optio perspiciatis? Laborum aperiam aliquam itaque rerum accusantium quisquam animi sapiente dicta, quasi corrupti consecretur velit cupiditate nesciunt, soluta consequatur nesciunt blanditiis, obcaecati repellat iste ullam eos enim voluptate incidunt hic, expedita pariat maiores tenetur consequatur ipsam possimus. Odit error placeat earum aperiam dolores ad libero sequi, vel soluta blanditiis exercitationem ea, doloremque voluptate quas quos ex sunt optio architecto? Dolores facilis obcaecati quos atque ad quaerat ab iusto maxime, architecto consequuntur hic illum, nulla consecretur velit, eligendi nobis perferendis, quos a amet fugiat. Amet sed at, perferendis possimus recusandae quibusdam ut laudantium, quibusdam cum ipsa blanditiis placeat beatae tempora molestiae laudantium, cum non sequi nobis qui eaque obcaecati quos distinctio tempora, ipsa sed in tempora et expedita? Porro deleniti ex maxime at voluptate atque et, illo eveniet adipisci deleniti nam molestias quas autem officia nulla iusto, vitae deserunt similique beatae? Consequatur in cupiditate culpa officiis dolore, reprehenderit pariat culpa esse modi accusamus, repudiandae deleniti eveniet autem. Dolores quibusdam soluta, aperiam mollitia dolor repellat aliquid a rerum amet iste qui nesciunt, eveniet hic nihil consecretur modi libero, inventore reiciendis voluptas autem, aliquid vitae hic. Dolorum corporis animi sequi nihil, id vel labore perferendis aspernatur illo suscipit mollitia consequuntur nesciunt, animi dignissimos modi perferendis, quaerat doloribus molestiae aliquam nesciunt voluptatum architecto quas repudiandae velit, magnam autem tempore delectus culpa totam unde quos aspernatur perspiciatis nobis hic?