between them. Let $G_I = (I, E^I)$ be the random subgraph of G induced by I. Letting $d_i^I(C)$ be agent i's degree in the subgraph of G_I induced by a coalition C, agent i approves C if $d_i^I(C) \geq \frac{|C|}{2}$, i.e., if i is connected to at least $\frac{|C|}{2}$ of the vertices in C. First, we characterize the distribution of both the coalitions and social welfare of *perfect* partitions (Theorems 11-12), on which we elaborate in Appendices N–O. Then, we discuss the performance of perfect outcomes by providing upper and lower bounds on their *Price of Anarchy* (**?**).

By Theorem 14 in (**?**), without loss of generality, a perfect partition consists of edges and triangles. Thus, we let M_n^I and T_n^I be the random variables which represent the number of edges and triangles in G_I (resp.). Let $\mathbb{T}$ be the set of all triplets (i, j, k) $(i < j < k)$ that form a triangle in G. Accordingly, the following theorem fully characterizes the social welfare of a *perfect* partition π, as well as the coalitions comprising π, for various values of $(p_i)_{i \in N}$.

Theorem 11. *For each $i \in N$ and $n \in \mathbb{N}$, let $p_i(n) = \frac{q_i(n)}{n}$ for some $q_i : \mathbb{N} \to \mathbb{R}$, $q^{\max}(n) = \max_{i \in N} q_i(n)$ and $q^{\min}(n) = \min_{i \in N} q_i(n)$. Given a perfect partition π, we infer:* **(1)** A perfect partition comprises of singletons w.h.p. (with high probability): *If $q^{\max}(n) \to 0$ as $n \to \infty$, then $T_n^I = 0$, $M_n^I = 0$ and $\mathcal{SW}_I(\pi) = 0$;* **(2)** Triangles and edges reside in perfect partitions a.s. (almost surely): *If $|\mathbb{T}| = 1$ and $\frac{n}{q^{\min}(n)} \to 1$ as $n \to \infty$, then $T_n^I \geq 1$ a.s. Otherwise, if $|\mathbb{T}| \geq 2$ and $q^{\min}(n) \to \infty$ as $n \to \infty$, then $T_n^I \geq 1$ and $M_n^I \geq 1$ a.s.;* **(3)** *If $\frac{q^{\max}(n)}{q^{\min}(n)} \to 1$ as $n \to \infty$, then $\frac{q^{\min}(n)}{n} \leq \mathbb{E}[|I|] \leq q^{\max}(n)$, thus yielding that $|I| \geq 1$ (i.e.,* at least one agent remains*) a.s.;* **(4)** *If $q_i(n) \equiv c_i$ for $c_i > 0$ $\forall i$, then $\mathbb{E}[\mathcal{SW}_I(\pi)] \leq c_{\max}^2$, where $q^{\max}(n) \equiv c_{\max}$.*

Further, we can model agents' *uncertainty about their mutual friendships*. Formally, let $(p_{ij})_{i,j \in N} \in [0, 1]^{n \times n}$ with $p_{ij} = p_{ji}$ for every $i, j \in N$. Let $\mathcal{E} \subseteq N \times N$ be a random variable, where $(i, j) \in \mathcal{E}$ with probability p_{ij} and different pairs of indices are independent, thus yielding a *Erdös-Rényi random graph* $\tilde{G} = (N, \mathcal{E})$ (**?**) whose set of edges is $\mathcal{E}$. The majority game on the resulting random graph satisfies 1-3 in Theorem 11 with minor adjustments (See Appendix O.1), yet gives rise to an additional property which extends property 4 (proved in Appendix O.2):

Theorem 12. *Let $p_{ij}(n) = c/n$ for some constant $c > 0$. Let π be a perfect partition. Then, T_n^I converges in distribution to a Poisson random variable with parameter $c^3/6$, $\mathbb{E}[M_n^I] = (n-1)c/2$ and $\mathbb{E}[\mathcal{SW}(\pi)] \leq (n-1)c$.*

Let $\mathbb{P}_I$ be the set of all perfect partitions for a random set of players I and let $\pi^\star$ be an welfare-optimal partition. Inspired by the *Price of Anarchy* (**?**), we put forth the *Price of Perfection* (**PP**) of a RDHG $\mathcal{G}'$, defined as the worst-case ratio between the social welfare of $\pi^\star$ and that of a perfect partition, i.e., $\text{PP}(\mathcal{G}') = \max_{\pi \in \mathbb{P}_I} \frac{\mathcal{SW}_I(\pi^\star)}{\mathcal{SW}_I(\pi)}$. Similarly, we define the *Expected Price of Perfection* (**EPP**) by $\text{EPP}(\mathcal{G}') = \max_{\pi \in \mathbb{P}_I} \frac{\mathbb{E}[\mathcal{SW}_I(\pi^\star)]}{\mathbb{E}[\mathcal{SW}_I(\pi)]}$. Using Theorem 11, we devise upper and lower bounds on both variants of the price of perfection, where Corollary 2 is clearly a direct outcome of (3)-(4) in Theorem 11.

Lemma 3. *Under the assumptions of Theorem 11 and: (1) in Theorem 11, PP$(\mathcal{G}')$ = EPP$(\mathcal{G}') = 0$ w.h.p.; (2) in Theorem 11, PP$(\mathcal{G}') \leq |I|/2$ a.s.; (3)-(4) in Theorem 11, EPP$(\mathcal{G}') \leq q^{\max}(n) = c_{\max}/2$.*

Proof. For (1), the claim clearly stems since $M_n^I = 0$ w.h.p. For (2), $\pi^\star$ clearly satisfies $\mathcal{SW}_I(\pi^\star) \leq |I|$. If we were to consider each connected component of G separately, we may assume that G is connected and does not consists of any isolated vertices. Hence, if there exists a perfect partition in G, then a perfect partition consisting of edges and triangles exists (Theorem 14 in (**?**)). However, G_I might contain isolated vertices, even if G does not. Since $M_n^I \geq 1$ a.s., we infer that $\mathcal{SW}_I(\pi) \geq 2$ at the very least, thus yielding that $\text{PP}(\mathcal{G}') \leq |I|/2$. For (3), we observe that $\mathbb{E}[\mathcal{SW}_I(\pi^\star)] \leq \mathbb{E}[|I|] \leq q^{\max}(n)$. Combined with the proof for (2), we conclude that $\text{EPP}(\mathcal{G}') \leq q^{\max}(n)$. ☐

Corollary 2. *Under the assumptions of (3)-(4) in Theorem 11, if $\mathcal{SW}(\pi^\star) \geq 1$, then EPP$(\mathcal{G}') \geq 1/c_{\max}^2$. Alternately, if $\mathcal{SW}_I(\pi^\star) = |I|$, we infer that EPP$(\mathcal{G}') \geq \frac{c_{\min}}{nc_{\max}^2}$.*

Proof. The first lower bound is a direct outcome of (4) in Theorem 11. For the second part, from (3) in Theorem 11 we infer that $\frac{q^{\min}(n)}{n} \leq \mathbb{E}[\mathcal{SW}_I(\pi^\star)] \leq q^{\max}(n)$. Combined with (4) in Theorem 11, we conclude the desired bounds. ☐

Conclusions and Future Work

Our work contributes significantly to the study of hedonic games, as the first one to explore the complexity of probabilistically inferring solution concepts in uncertain domains. The main complexity results are summarized in Table 1. Our study opens the way for many future works, including the investigation of other classes of hedonic games and other solution concepts. Further, our probabilistic setting arises several intriguing questions, among those: For an outcome satisfying a solution concept β, what is the maximum number of players whose withdrawal from the game still preserves β in the outcome induced by the remaining players? Another direction is *robustness* (**?**): A probabilistic withdrawal of players upon an outcome satisfying a solution concept β (e.g., stability) should preserve β.

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