

# Heuristic Voting as Ordinal Dominance Strategies

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## Abstract

Decision making under uncertainty is a key component of many AI settings, and in particular of voting scenarios where strategic agents are trying to reach a joint decision. The common approach to handle uncertainty is by maximizing expected utility, which requires a cardinal utility function as well as detailed probabilistic information. However, often such probabilities are not easy to estimate or apply.

To this end, we present a framework that allows for “shades of gray” of likelihood without probabilities. Specifically, we create a hierarchy of sets of world states based on a prospective poll, with inner sets contain more likely outcomes. This hierarchy of likelihoods allows us to define what we term ordinally-dominated strategies. We use this approach to justify various known voting heuristics as bounded-rational strategies.

## Introduction

The question of how agents – human or artificial – choose a strategy when facing a choice, has been at the center of attention in artificial intelligence since its inception. Approaches to decision making often rely on two primary components: the epistemic state of the agent (her beliefs on how her actions will affect the world), and her innate preferences (the utility or cost associated with each outcome).

In voting scenarios, agents’ actions are aggregated to reach a shared result. Voters can make strategic choices once they know what the state of the world is (what other agents are voting), following their own utility function (in most voting settings, an ordinal preference over possible outcomes is assumed). This voting decision may either be applied once based on the current beliefs of the voters, or in an iterative fashion so that voters have several opportunities to observe the world and change their action. When the votes of others are unknown, the epistemic state might depend on some prior knowledge and/or signals from the environment.

The most common way to address this lack of knowledge is to assign probabilities to each state of the world and assume that agents each maximize their expected

utility over all possible states (see Related Work). However, in many situations human agents may not have the ability to determine precise probabilities of each state of the world, or to act according to them (Tversky and Kahneman 1974; Chater, Tenenbaum, and Yuille 2006). There is no reason to believe that in voting scenarios people will perform differently in this respect.

**Voting without Probabilities** Alternative approaches, focusing on decision making in face of strict uncertainty (defined in terms of possible or impossible states) have been formulated and applied in various AI and economic settings (Dow and Werlang 1994; Boutilier 1994; Halpern 1997; Matt, Toni, and Vaccari 2009), and more recently, in the context of voting (Conitzer, Walsh, and Xia 2011; Reijngoud and Endriss 2012; Meir, Lev, and Rosenschein 2014). The idea at the core of this approach is that, for any given voter, a vote  $a'$  locally dominates another vote  $a$  if  $a'$  is at least as good as  $a$  for this voter in any voting profile that she considers to be possible.

In fact, voting behavior can also be defined without any explicit epistemic model. Indeed, several recent papers suggest various heuristics that are specific to a voting rule and/or context. Some of these heuristics have been shown to be empirically consistent with voters’ behavior in lab experiments (Laslier 2010), and others guarantee desirable convergence and/or welfare properties when applied by all voters in a group (Grandi et al. 2013).

**Contribution** We extend the framework suggested in the strict uncertainty papers mentioned above, by allowing gradual levels of uncertainty. Specifically, we build on the idea of having a likelihood hierarchy – a sequence of sets of states of the world, where each next set is a superset of the previous set in the sequence, so that the states in inner sets are considered by the voter to be more likely. An undominated vote in this setting is one which is not dominated at any level of the hierarchy.

Using this hierarchy of likelihood, we suggest an alternative representation for information structures in voting. We show how the relevant information can be

boiled down to what we call a pivot-graph, which succinctly captures all situations where the voter may be pivotal. We then show that the information structure allows us to justify several existing voting heuristics as rational decisions for an appropriate epistemic model (a specific hierarchy of pivot-graphs). This observation enables us to generalize existing convergence results in the literature on iterative voting, by showing how convergence follows from topological properties of the pivot-graphs.

**Related Work** For an up-to-date coverage of iterative voting, heuristics and uncertainty-based models, see (Meir 2017). In particular, Conitzer et al. (2011) consider a voter facing an arbitrary information set, and Reijngoud and Endriss (2012) study partial information settings where, for example, only the candidates' scores or only the identity of the leader are known. Closest to our paper is the local dominance model (Meir, Lev, and Rosenschein 2014), in which all voters base their belief on a shared prospective state. It has been shown that in an iterative voting setting where voters play possible actions that dominate their current action, they are guaranteed to converge to an equilibrium under certain assumptions on the distance metric (Meir, Lev, and Rosenschein 2014; Meir 2015).

Voting heuristics do not explicitly define voters' beliefs; instead, they specify a (typically) simple function that dictates a vote in every given state, aiming to capture realistic voting behaviors (Reijngoud and Endriss 2012; Grandi et al. 2013). In particular, some models suggest that a non-pivotal voter either votes truthfully (Dutta and Laslier 2010) or abstains (Desmedt and Elkind 2010).

These models stand in contrast with the expected utility models, such as, for example, the calculus of voting (Riker and Ordeshook 1968; Myerson and Weber 1993), where a voter computes the probability for each action (vote) to be pivotal in every pairwise tie. We see our model as a way to capture a similar line of reasoning in identifying the influential ties, albeit without using probabilities. A more fundamental difference with the calculus of voting approach is that the latter assumes a common knowledge of rationality and the preference distribution, from which an equilibrium is derived.

In the more general AI literature, there are other non-probabilistic models of uncertainty, where two of the most prominent ones are the possibility theory (Dubois, Fargier, and Prade 1996) and Dempster-Shafer theory (Shafer 1976). These models attribute a cardinal possibility measure to states and develop calculus rules for belief updates and comparisons. The closest to our work is the plausibility measure approach (Halpern 1997), that allows for a partial order of plausibility. Our hierarchical ordinal dominance concept is even more strict, and relies on the structure of the problem where uncertainty is essentially about the accuracy of a single point estimate (a poll).

While we take our distance-based epistemic assumptions from the aforementioned local dominance voting model, an earlier precursor of this idea is the logic for inexact knowledge based on margin-of-error (Williamson 1992).

## Model

An election is composed of a set  $V$  of  $n$  voters and a set  $C$  of  $m$  candidates. Each voter  $i \in V$  has a weak preference relation  $\succsim^i \subseteq C \times C$  over the candidates, that is, for each two candidates  $x, y \in C$ ,  $x \succsim^i y$  or  $y \succsim^i x$ , and if both are true, they are equivalent. Moreover, the relation is transitive (so, for  $x, y, z \in C$ ,  $x \succsim^i y$  and  $y \succsim^i z$ , then  $x \succsim^i z$ ).

The voting rules we shall focus on are the score-based voting (SBV) rules. An SBV  $(\hat{f}, A)$  is defined by a set  $A \subseteq \mathbb{N}^m$  of allowed votes, and a function  $\hat{f} : A^n \rightarrow C$ . For example, the set  $A$  under Plurality contains all vectors that have exactly one non-zero element, which is 1; Approval allows all binary vectors; Borda allows all permutations of  $(0, \dots, m-1)$ ; etc. We denote by  $\vec{a} = (a_i)_{i \in V}$  the voting profile; by  $a_i(c) \in \mathbb{N}$  the absolute score given to  $c$  by agent  $i$  in vote  $a_i \in A$ ; and by  $s_{\vec{a}}(c) = \sum_{i \in V} a_i(c)$  the total score given to candidate  $c$ . The winner is  $\hat{f}(\vec{a}) = \arg \max_{c \in C} s_{\vec{a}}(c)$ , breaking ties lexicographically.

For each voter  $i$ , the outcome (and thus, her utility) depends on her own vote, as well as on the state of the world  $s = s_{\vec{a}_{-i}}$ , that encompasses the votes of all other participants. We separate these two arguments by writing the outcome function as  $f(s, a_i) = \hat{f}(s + a_i)$ . For every state  $s$  and any two actions  $a_i, b_i \in A$ , we write  $f(s, a_i) \succ^i f(s, b_i)$  when voter  $i$  prefers action  $a_i$  over  $b_i$  at state  $s$ .

**Example 1.** There are 100 voters and 5 candidates –  $w, b, c, d, e$  – using the Plurality voting system. The voters have access to a poll where votes are  $\vec{s} = (29, 26, 22, 17, 6)$ . A voter  $i$  currently voting for  $b$  sees the state  $s = s_{\vec{a}_{-i}} = (29, \mathbf{25}, 22, 17, 6)$ . For any action  $a'_i$  of  $i$ ,  $f(s, a'_i) = w$ , which means that voter  $i$  is indifferent between her actions.

Note that the voters never explicitly reason about the preferences of other individuals – only about their (aggregated) actions. We will return to this example later in the paper.

## Information Structures

An information set is a set of states  $S' \subseteq S$ . An information structure of agent  $i$  is a collection of information sets  $\mathcal{S}^i = (S_j^i)_{j=1}^k$ , where  $S_j^i \subseteq S_{j+1}^i$  for all  $j$ . That is, each information set contains the sets with a lower index.

An agent does not assign probabilities to states or to information sets, but an intuitive interpretation of the model is that agent  $i$  believes any state in  $S_j^i$  to be substantially more likely than all states outside  $S_j^i$ .

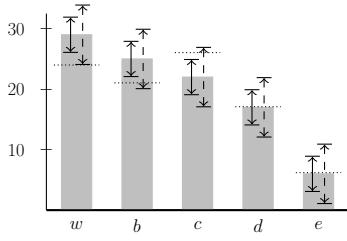


Figure 1: Information structure of voter  $i$  at state  $s_{\bar{a}-i} = (29, 25, 22, 17, 6)$  (gray pillars). The set  $S_1^i$  contains all states that result from changing the score of any candidate in  $s_{\bar{a}-i}$  by at most 3 votes (solid arrows). The set  $S_2^i$  allows for a variation of 5 votes (dashed arrows). The dotted lines indicate state  $s'$  from Example 2, where  $c$  wins.

An information structure can either be shared by all agents, or be agent-specific.

Example 2. Consider voter  $i$  from Example 1 and assume she has an information structure  $\mathcal{S}^i = (S_1^i, S_2^i)$  as in Figure 1. In particular,  $i$  believes that candidate  $c$  may win (as, e.g.,  $s' = (24, 21, 26, 17, 6) \in S_2^i$ ); however, this is far less likely than a victory of  $w$  or  $b$ , as there is no state  $s'' \in S_1^i$  where  $c$  wins.

### Ordinal Dominance

Following (Conitzer, Walsh, and Xia 2011; Reijngoud and Endriss 2012; Meir, Lev, and Rosenschein 2014), for any information set  $S_j^i$  and actions  $a, b \in A$ , we say that action  $a$   $S_j^i$ -dominates action  $b$  (denoted  $a \succ_j^i b$ ) if  $f(s, a) \succeq^i f(s, b)$  for all  $s \in S_j^i$  and  $f(s, a) \succ^i f(s, b)$  for at least one  $s \in S_j^i$ . Agent  $i$  is indifferent between actions  $a, b$  at  $S_j^i$  (denoted  $a \sim_j^i b$ ) if  $f(s, a) \sim^i f(s, b)$  for all  $s \in S_j^i$ . Note that  $S_j^i$ -dominance is a partial order over actions  $A$  (transitive, antisymmetric and irreflexive relation).

Definition 1. Action  $a$  ordinally dominates action  $b$  (in structure  $\mathcal{S}^i = (S_j^i)_{j \in [k]}$ ) if there is some  $j \in [k]$  such that action  $a$   $S_j^i$ -dominates action  $b$ .

The next lemma guarantees that it is not possible that  $a \succ_j b$  and  $b \succ_{j'} a$  for some  $j' \neq j$ .

Lemma 3. Ordinal dominance is a partial order.

Proof. Transitivity: Suppose action  $a$  ordinally dominates action  $b$  and  $b$  ordinally dominates action  $c$ , due to  $S_j^i$  and  $S_{j'}^i$ , respectively. W.l.o.g.  $j' \leq j$ , then  $S_{j'}^i \subseteq S_j^i$ . There is a state  $s' \in S_{j'}^i$ , where  $f(s', b) \succ^i f(s', c)$ , and since  $s' \in S_{j'}^i \subseteq S_j^i$ , we also have  $f(s', a) \succeq^i f(s', b)$ , and so  $f(s', a) \succ^i f(s', c)$ . Similarly, for any  $s \in S_{j'}^i$ ,  $f(s, a) \succeq^i f(s, b) \succeq^i f(s, c)$ . Thus  $a \succ_{j'}^i c$  which means that  $a$  ordinally dominates  $c$ .

Antisymmetry: Suppose action  $a$  ordinally dominates action  $b$  due to  $S_j^i$ . For every  $j' \leq j$ , there cannot be a

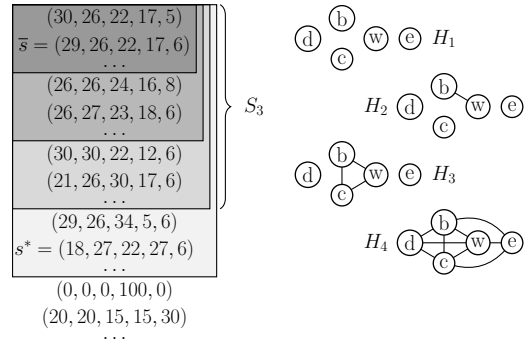


Figure 2: A schematic example of the information structure  $\mathcal{S}_{EMD, \bar{r}}(\bar{s})$ , and the induced pivot graph structure  $\mathcal{H}_{EMD, \bar{r}}(\bar{s})$ . E.g., graph  $H_4$  contains the edge  $(b, d)$  due to state  $s^* \in S_4$ . States below  $S_4$  are considered impossible. All the pivot graphs are upward closed w.r.t. the lexicographic order on  $C$ , but they are not always a clique ( $(d, e) \notin H_4$ ).

state  $s \in S_{j'}^i \subseteq S_j^i$  where  $f(s, b) \succ^i f(s, a)$ . Similarly, for any  $j' > j$ , there is a state  $s' \in S_j^i \subseteq S_{j'}^i$ , where  $f(s', b) \prec^i f(s', a)$ . Thus,  $b$  does not  $S_{j'}^i$ -dominate  $a$ . Since this is true for any  $j'$ ,  $b$  does not ordinally dominate  $a$ .  $\square$

### Distance-Based Uncertainty

Following Meir et al. (2014), we consider the following way to derive information sets and information structures. Given a metric  $d : S \times S \rightarrow [0, 1]$  and a parameter  $r \in [0, 1]$ , every state  $s \in S$  explicitly defines an information set  $S_{d,r}(s) = \{s' : d(s, s') \leq r\}$ . In general, the metric  $d$  can be completely arbitrary and the induced set is meaningless.<sup>1</sup> However, in the context of voting there are several natural metrics: For example,  $d(s, s')$  may reflect what fraction of votes has changed between  $s$  and  $s'$ . In Meir et al. (2014) and Meir (2015), the distances between candidate score vectors were defined by different  $\ell$ -norms and the Earth Mover distance (EMD), which is essentially the  $\ell_1$  norm with the additional constraint that the total number of votes remains the same. Thus,  $S_{d,r}(s)$  may reflect a range of possible candidates' scores given a poll or a current state  $s$ .

### Pivot Graphs

A pair of actions  $(a', a'')$  is pivotal for a pair of candidates  $c', c'' \in C$  in state  $s \in S$ , if  $f(s, a') = c'$  and  $f(s, a'') = c''$ . An agent  $i$  is pivotal for the pair of candidates  $c', c'' \in C$  in information set  $S_j^i$ , if there are  $s \in S_j^i$  and actions  $a'_i, a''_i \in A$  that are pivotal for  $c', c''$  in  $s$ .

Information set  $S_j^i$  then induces a pivot-graph  $H_j^i = (C, E)$ , which contains a vertex for every candidate, and

<sup>1</sup>In fact, any set  $S' \subseteq S$  can be derived from  $s$  for some carefully designed metric  $d$ .

an edge  $(c', c'')$  if agent  $i$  is pivotal for the pair  $c', c''$  in  $S_j^i$ .

Every information structure  $\mathcal{S}^i$  induces a pivot graph structure  $\mathcal{H}^i = (H_j^i)_{j=1}^k$ , where each  $H_j^i$  is a subgraph of  $H_{j+1}^i$  (since adding more states can only add edges to the graph). The set  $\mathcal{H}(C)$  contains all pivot graph structures.

### Epistemic Models and OD Equilibrium

An epistemic model of agent  $i$  maps any state  $s$  to an information structure  $\mathcal{S}^i(s) = (S_1^i(s), S_2^i(s), \dots, S_k^i(s))$ , and thus also to a pivot graph structure  $\mathcal{H}^i(s)$ .

We define the set  $OD_{\succsim^i}(\mathcal{S}, a)$  that contains all actions that ordinally dominate  $a$  in  $\mathcal{S}$  according to preferences  $\succsim^i$ , and a set  $UOD(\mathcal{S}, a)$  containing all actions  $a'$  that ordinally dominate  $a$  and are not ordinally dominated themselves. Naturally, this leads to a definition of an OD-equilibrium – when for every agent  $OD_{\succsim^i}(\mathcal{S}^i(s_{\bar{a}-i}), a_i)$  is empty (and hence,  $UOD_{\succsim^i}(\mathcal{S}^i(s_{\bar{a}-i}), a_i) = \emptyset$ , too).

Observation 4. For a “full information” epistemic model where  $\mathcal{S}^i(s) = (\{s\})$ , the set  $OD(\mathcal{S}^i(s), a_i)$  coincides with the set of better-responses to  $(s, a_i)$ ; the set  $UOD(\mathcal{S}^i(s), a_i)$  coincides with best-responses; and OD equilibrium coincides with a pure strategy Nash equilibrium.

An epistemic model is cliqued if its mapping to a pivot graph structure  $\mathcal{H}(s)$  at every state  $s \in S$ ,  $H_j(s)$ , is a clique. The epistemic model is upward closed if the pivot graph structure  $\mathcal{H}(s)$  at every state  $s$  has an order  $L$  over candidates such that if  $(c, c') \in H_j(s)$  and  $c'' >_L c'$  then  $(c, c'') \in H_j(s)$ . Note that any cliqued epistemic model is upward closed (where  $L$  may be an arbitrary order where all candidates in  $H_j(s)$  precede all others). More generally,  $L$  can be roughly thought of as an order of likelihood of states. For simplicity of notation, we denote both the pivot graph structure and the epistemic model (which is a function from states to structures) by  $\mathcal{H}$ .

Distance metrics provide us with a simple way to define an information structure: given a metric  $d$  and an increasing sequence of distances  $\vec{r} = (r_1, r_2, \dots, r_k)$ , we get an epistemic model  $\mathcal{H}_{d, \vec{r}}(s) = (H_{d, r_1}(s), H_{d, r_2}(s), \dots, H_{d, r_k}(s))$  that is induced by  $S_{d, \vec{r}}(s)$ . We later show how the topological properties of  $\mathcal{H}_{d, \vec{r}}(s)$  are related to the properties of the metric  $d$ .

Example 5. We expand Example 1 where candidates’ scores (in % of total) are (29, 26, 22, 17, 6). We do not specify the number of voters in the poll. We consider a voter with a concentric information structure, based on the radii  $\vec{r} = \{1\%, 3\%, 7\%, 17\%\}$  and the EMD metric. These information sets induce pivot-graphs as illustrated in Figure 2.

In Example 5, consider a Plurality voter whose preferences are  $e \succ d \succ c \succ b \succ w$ . Then the action “ $c$ ” (a shorthand for  $(0, 0, 1, 0, 0)$ ) ordinally dominates action

“ $e$ ” (due to  $H_3$ ) and “ $b$ ” ordinally dominates everything else due to  $H_2$ .

### Sharp Pivot Property in Large Populations

Note that structures  $\mathcal{S}^i$  and  $\mathcal{H}^i$  are two different ways to represent the information of an agent. In general,  $\mathcal{H}^i$  contains less information than  $\mathcal{S}^i$ , since there may be states where some action pairs are pivotal, whereas others are not.

Yet, it seems plausible that in a large population, a voter is unlikely to make such fine distinctions about the possible outcomes: voters do not know the exact score of each candidate, but only have a rough idea of what it is (each candidate’s share of the votes). As a result, it is reasonable to assume that if a voter considers herself pivotal in some possible tie, she will consider any change in her vote as possibly pivotal. We capture this intuition in the following formal definition.

Definition 2 (Sharp Pivot Property (SPP)). An information structure  $\mathcal{S}^i$  satisfies the Sharp Pivot Property if: for all  $j$  and all  $c', c'' \in C$ , an edge  $(c', c'') \in H_j^i$  entails that any pair of actions  $(a'_i, a''_i)$  such that  $a'_i(c') - a'_i(c'') > a''_i(c') - a''_i(c'')$  is pivotal for  $(c', c'')$  in  $S_j^i$ .

That is, if there is some action pair that makes  $c'$  the winner instead of  $c''$ , then any move that increases the gap in favor of  $c'$  might make  $c'$  beat  $c''$  and become the winner (in some  $s \in S_j^i$ ).

Working with pivot-graph structures is much more convenient than working with arbitrary information sets, and their meaning in the context of voting is clear. We will assume throughout the paper that all information structures have SPP, which means that  $\mathcal{H}^i$  contains all the relevant information in  $\mathcal{S}^i$ .

Justifying SPP While SPP is plausible, we would like to show that at least in some cases it provably holds. The argument is a rather fine one, that relies on viewing the (finite) poll as approximating some underlying real-valued distribution  $p \in \Delta(C)$ . Distribution  $p$  defines a unique score vector  $s_p^n$  for every population size  $n$  (using some fixed rounding of  $p \cdot n$ ), and we argue that for a sufficiently large population  $n$ , the information structure  $\mathcal{S}_{d, \vec{r}}(s_p^n)$  satisfies SPP.

Intuitively, consider the leader  $w$  in Example 5 with the Plurality rule, some other candidate (say  $b$ ), and some information set  $S_{d, r_j}(s)$ . If  $s(w) - s(b)$  is much lower than  $r_j \cdot n$ , then any move where the voter deserts  $w$  and/or joins  $b$  might be pivotal for  $(b, w)$ . In contrast, if  $s(w) - s(b)$  is much higher than  $r_j \cdot n$ , then  $b$  can never win or even tie with  $w$ . Only if  $s(w) - s(b) \cong r_j \cdot n$ , then moving from  $w$  to  $b$  makes  $b$  the winner, but (say) moving from  $d$  to  $b$  does not. This extreme case becomes unlikely as the population gets larger.

Theorem 6. For the EMD metric  $d$ , any SBV, and any radii vector  $\vec{r}$ , the following holds for almost all<sup>2</sup> distri-

<sup>2</sup>Except a 0-measure set of distributions.

butions  $p \in \Delta(C)$ : There is  $n_0$  such that for all  $n > n_0$ , the information structure  $\mathcal{S}_{d,r}(s_p^n)$  satisfies SPP.

### Computing Dominance Relations

We show that strategies can be efficiently compared according to ordinal dominance.

**Proposition 7.** Given a pivot graph structure  $\mathcal{H}^i = (H_1^i, \dots, H_k^i)$  and any SBV  $(\hat{f}, A)$ , voter  $i$  can check in time  $O(m^2k)$  if vote  $a'_i \in A$  ordinally dominates vote  $a_i \in A$ .

Intuitively, Algorithm 1 checks (for each uncertainty level  $j$ ), whether the new vote  $a'_i$  is “safe” (not worse than  $a_i$  in any possible tie), and whether it is “pivotal” (better than  $a_i$  in at least one tie).

$I[X] \in \{-1, 1\}$  is an indicator variable for statement  $X$ , and we use  $a_i(c)$  to indicate candidate  $c$ ’s score when voter  $i$  vote is  $a_i$ . The complexity of checking whether a given vote  $a_i$  is undominated is left as an open question.

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#### Algorithm 1: $\text{OD}(a'_i, a_i \in A, \succsim^i, \mathcal{H}^i \in \mathcal{H})$

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for  $c, c' \in C$  do
   $\text{diff}(c, c') \leftarrow a'_i(c) + a_i(c') - a_i(c) - a'_i(c')$ ;
   $\text{effect}(c, c') \leftarrow \text{sign}(\text{diff}(c, c')) \cdot I[c \succ^i c']$ ;
for  $j \leq k$  do
   $\text{safe}(j) \leftarrow \min_{(c, c') \in H_j^i} \text{effect}(c, c')$ ;
   $\text{pivot}(j) \leftarrow \max_{(c, c') \in H_j^i} \text{effect}(c, c')$ ;
   $\text{dom}(j) \leftarrow I[\text{pivot}(j) + \text{safe}(j) \geq 1]$ ;
if  $\exists j \leq k$  s.t.  $\text{dom}(j) = 1$  then
  return TRUE
else
  return FALSE

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**Proof.** Suppose  $a'$  ordinally dominates  $a$ . Then there is some level  $j \leq k$  such that  $a' \succ_j^i a$ . This means that for any pair of candidates  $(c, c')$  that can be tied in  $H_j^i$ , either  $c$  is preferred to  $c'$  and  $a'$  weakly reduced  $c'$ ’s score, or  $c'$  is preferred to  $c$  and  $a'$  weakly adds to  $c'$ ’s score (thus  $\text{effect}(c, c') \geq 0$ ). Hence, in particular  $\text{safe}(j) \geq 0$ . In addition, there must be a pair for which the gain is strict, and  $\text{effect}(c, c') = 1$ , which means  $\text{pivot}(j) = 1$ . In total,  $\text{dom}(j) \geq 1 + 0 = 1$  so the algorithm returns TRUE.

Otherwise, in every level  $j$ , either  $a'_i, a_i$  have the same outcome in all states, or there is a pair  $(c, c') \in H_j^i$  such that  $f(s, a_i) = c, f(s, a'_i) = c'$ , and  $c \succ^i c'$ .

In the latter case, since  $f$  is a scoring rule this means that  $a'_i(c) - a_i(c) < a'_i(c') - a_i(c')$ , i.e. that  $c'$  gained strictly more score than  $c$  when changing from  $a_i$  to  $a'_i$ . Thus  $\text{diff}(c, c') = a'_i(c) + a_i(c') - a_i(c) - a'_i(c') < 0$ , and  $\text{effect}(c, c') = -1$ . The algorithm then computes  $\text{safe}(j) = -1$ . Therefore  $\text{dom}(j) \leq 1 - 1 = 0$ .

In the first case,  $\text{effect}(c, c') = 0$  for all pairs, and thus  $\text{safe}(j) = \text{pivot}(j) = 0$ , and  $\text{dom}(j) = 0$ .  $\square$

### Justifying Voting Heuristics with OD

Many heuristics have been suggested to analyze how voters behave and change their vote. Most heuristics are derived from a single “prospective state”  $s$ , which is assumed to be the current voting profile or poll. Formally, a set heuristics is a function  $h : S \times A \rightarrow 2^A$  that maps the prospective state and the current action to a set of new possible actions. We say that  $h$  is a point heuristics if  $|h(s, a)| \leq 1$  for every  $s, a$ . To be consistent with previous definitions, we always omit  $a$  from the set  $h(s, a)$ , and assume that when  $h(s, a) = \emptyset$  the voter simply keeps her current vote.

**Definition 3.** We say that an epistemic model  $\mathcal{H}$  justifies heuristic  $h$ , if for any state  $s \in S$  and current action  $a \in A$ : (I)  $h(s, a) = \emptyset$  if and only if  $UOD(\mathcal{S}(s), a) = \emptyset$ ; and (II)  $h(s, a) \subseteq UOD(\mathcal{S}(s), a)$ .  $\mathcal{H}$  strongly justifies  $h$  if (II) holds with equality.

This means that the heuristic  $h$  always recommends undominated ordinal-dominance moves under the epistemic model  $\mathcal{H}$ , and only keeps the current action if no such move exists.

As a simple example, consider the Plurality rule and the heuristic  $h^{\text{not-last}}(s, a)$  that is empty except when action  $a$  is the least preferred candidate  $\hat{a}_i$ , and then it moves to an arbitrary other candidate. Consider the epistemic model  $\mathcal{H}^{\text{all}}(s) = (H_1^{\text{all}})$  where  $H_1^{\text{all}}$  is the complete graph.

**Observation 8.**  $\mathcal{H}^{\text{all}}$  strongly justifies  $h^{\text{not-last}}$ .

This is since (I) suppose that  $a \neq \hat{a}_i$ . Then no candidate ordinally dominates  $a$  and thus  $UOD(\mathcal{H}^{\text{all}}(s), a) = \emptyset = h^{\text{not-last}}(s, a)$ ; (II) when  $a = \hat{a}_i$ , any other candidate  $c$  is undominated but globally dominates  $a$  (since there is a possible state where  $i$  is pivotal for  $c$  against  $a$ ), in which case  $UOD(\mathcal{H}^{\text{all}}(s), a) = C \setminus \{\hat{a}_i\} = h^{\text{not-last}}(s, a)$ .

### Local Dominance

Local dominance (Meir, Lev, and Rosenschein 2014) heuristic with metric  $d$  and parameter  $r$  explicitly define a set  $S_{d,r}(s) = \{s' : d(s, s') \leq r\}$ . The heuristic action  $h_{d,r}^{LD}(s, a_i)$  is defined for the Plurality rule as follows: Let  $D \subseteq C$  be the set of candidates that  $S_{d,r}(s)$ -dominate  $a_i$ ; If  $D$  is non-empty, then vote for the most preferred candidate in  $D$ .

We define an epistemic model  $\mathcal{H}_{d,r}^{LD}$  where  $\mathcal{H}_{d,r}^{LD}(s)$  contains a single pivot graph  $H_1$  which is the pivot graph induced by  $S_{d,r}(s)$ . Note that our definition applies for any voting rule, unlike the one in Meir et al. (2014). In Plurality,  $\mathcal{H}_{d,r}^{LD}$  justifies  $h_{d,r}^{LD}$  (straightforward proof omitted due to space constraints).

**Truth/lazy-bias** Denote the top candidate of  $i$  by  $q_i \in C$ , and denote by  $\perp$  an “abstain” action that adds no score to candidates. We adopt the suggested variations in Dutta and Laslier (2010) and Desmedt and Elkind (2010), where the voter prefers the truthful/abstain action if this does not affect the outcome.

However, this naïve modification alone may lead to unreasonable behaviors, e.g., where no-one votes (Elkind et al. 2015), even under full information.<sup>3</sup>

For  $r_2 > r_1$ , the “truth bias” heuristics  $h_{d,r_1,r_2}^{LD+TB}(s, a_i)$  is as follows (Meir, Lev, and Rosenschein 2014): (1) agent  $i$  performs a local-dominance move at radius  $r_1$ , if exists. If such a move does not exist,  $i$  examines if  $f(s', a_i) \succ^i f(s', q_i)$  for some  $s' \in S_{d,r_2}(s)$ . (2a) If so, agent  $i$  keeps the current vote  $a_i$ , (2b) otherwise,  $i$  moves to  $q_i$ .

While the behavior seems to maintain the reason behind truth bias, the definition of  $h$  is cumbersome. Instead, we can use  $r_1, r_2$  to define an epistemic model  $\mathcal{H}_{d,r_1,r_2}^{LD+TB}$  as follows. We let  $H_1(s)$  be as in  $\mathcal{H}_{d,r_1,r_2}^{LD}$  above. We similarly compute  $H_2(s)$  from  $S_{d,r_2}(s)$ , but taking only edges between the current vote  $a_i$  and candidates less preferred than  $a_i$ . Let  $\mathcal{H} = \mathcal{H}_{d,r_1,r_2}^{LD+TB}(s) = (H_1(s), H_1(s) \cup H_2(s))$ .

Proposition 9.  $\mathcal{H}_{d,r_1,r_2}^{LD+TB}$  justifies  $h_{d,r_1,r_2}^{LD+TB}$  in Plurality.

Proof. First, if  $H_1(s)$  is nonempty (at least one tie) then  $h_{d,r_1,r_2}^{LD+TB}(s, a_i) = a_i^* \in UOD(\mathcal{H}(s), a_i)$  as in a standard LD move. Otherwise, there are two cases.

If  $H_2(s)$  contains some edge  $(a_i, b)$ , then by SPP for any  $a' \neq a_i$  there is a state  $s' \in S_{d,r_2}(s)$  where  $f(s', a_i) = a_i$  and  $f(s', a') = b$  (think of  $s'$  as state where a single additional vote for  $a_i$  is critical). Since  $a_i$  is preferred to  $b$  by the definition of  $H_2(s)$ , we conclude that no candidate  $a'$  dominates  $a_i$  in  $H_2(s)$  (thus  $UOD(\mathcal{H}(s), a_i) = \emptyset$ ); and that  $f(s', a_i) = a_i \succ^i b = f(s', q_i)$  (thus  $h_{d,r_1,r_2}^{LD+TB}(s, a_i) = \emptyset$ ).

In the second case, there is no such edge, then  $H_2(s)$  is empty. This means that no action of  $i$  can change the outcome whatsoever, and thus by the slight truth-bias  $q_i$  is strictly preferred to any other action. In particular, it ordinarily dominates  $a_i$  and is undominated so  $UOD(\mathcal{H}(s), a_i) = \{q_i\}$ . Finally, since  $i$  is non-pivotal then in particular there is no state  $s' \in S_{d,r_2}(s)$  such that  $f(s', a_i) \succ^i f(s', q_i)$ . Thus  $h_{d,r_1,r_2}^{LD+TB}(s, a_i) = q_i \in UOD(\mathcal{H}(s), a_i)$ , as required.  $\square$

The statement for lazy-bias is similar, and uses the same information structure but with a slight preference to abstain instead of voting truthfully.

## T-Pragmatist

The  $T$ -pragmatist (point) heuristic (Brams and Fishburn 1978; Reijngoud and Endriss 2012) considers the leading  $T$  candidates in  $s$  (denoted  $\mathbf{T}$ ), and sets a new action  $a'_i = h^{T-prag}(s, a_i)$  where  $a'_i$  is identical to  $a_i$  except the favorite candidate in  $\mathbf{T}$  is given maximal score (in any SBV). E.g., in Example 1, if  $e \succ^i c \succ^i b \succ^i w \succ^i d$ , then  $h^{2-prag}(s, a_i) = b$  and  $h^{3-prag}(s, a_i) = h^{4-prag}(s, a_i) = c$ .

<sup>3</sup>To be completely formal, the preference relation  $\succ^i$  has to be extended to preferences over pairs (winner, action).

Consider a single-level epistemic model  $\mathcal{H}^{T,i-star}(s)$ , which contains a star graph, in which the center node is the most preferred candidate by voter  $i$  in the top  $T$  candidates, and it is tied with all other  $T - 1$  candidates in the top  $T$ .

It is possible to show that (I)  $h^{T-prag}(s, a_i) = \emptyset \iff OD(\mathcal{H}^{T,i-star}(s), a_i) = \emptyset$ ; and (II)  $h^{T-prag}(s, a_i) \subseteq OD(\mathcal{H}^{T,i-star}(s), a_i)$ . This shows a connection between the heuristic and the epistemic model, but it is not a sufficient justification since  $h^{T-prag}(s, a_i)$  may be dominated. A closer look reveals that the actions dominating it are quite plausible: ranking the other candidates in  $\mathbf{T}$  at the bottom can only benefit the voter! We conclude that the  $T$ -pragmatist heuristic could be improved.

We define the  $h^{T*}$  heuristic similarly to  $h^{T-prag}$ , with the following difference: all other candidates in  $\mathbf{T}$  get minimal score (i.e., ranked at the bottom of  $a'_i$ ) while maintaining the same order among themselves as in  $a_i$ .

Proposition 10.  $\mathcal{H}^{T,i-star}$  strongly justifies  $h^{T*}$  in any SBV.

Proof. Denote by  $q'_i$  the favorite candidate of  $i$  in  $\mathbf{T}$ . For  $T = 1$ , the graph has no edges, which means the heuristic advises doing nothing as well. Otherwise, the graph contains all  $T - 1$  edges  $(q'_i, x)$  for  $x \in \mathbf{T} \setminus \{q'_i\}$ .

Let  $a'_i = h^{T*}(s, a_i)$  be any vote s.t.  $a'_i(q'_i) \geq a'_i(y) \geq a'_i(x)$  for all  $x \in \mathbf{T} \setminus \{q'_i\}$ ,  $y \notin \mathbf{T}$ , and  $a_i(x) > a_i(x') \Rightarrow a'_i(x) > a_i(x')$ .

- If  $a_i$  has the same inequalities then  $h^{T*}(\vec{s}, a_i) = \emptyset$ .
- We have that for all  $x \in \mathbf{T} \setminus \{q'_i\}$ ,  $a'_i(q'_i) - a'_i(x) > a_i(q'_i) - a_i(x)$  so in a tie  $(q'_i, x)$  vote  $a_i$  never beats  $a'_i$ .
- If  $a_i$  does not rank  $q'_i$  at the top, then consider  $x'$  ranked lowest by  $a'_i$ . We have that  $a'_i(q'_i) - a'_i(x') > a_i(q'_i) - a_i(x')$ , thus there is a near-tie state  $\vec{s}'$  of  $(q'_i, x')$  where  $f(\vec{s}', a'_i) = q'_i \succ^i x' = f(\vec{s}', a_i)$ .
- Similarly, if  $a_i$  does not rank some  $x' \in \mathbf{T} \setminus \{q'_i\}$  at the bottom, then in particular  $a'_i(x') < a_i(x')$  and thus in a near-tie of  $(q'_i, x')$ , vote  $a'_i$  beats  $a_i$ .

Finally, let  $b$  be any other vote. We need to show that  $a'_i$  is not ordinarily dominated by  $b$ .  $b$  can only beat  $a'_i$  in some  $s' \in H$  if there is a possible (near-)tie  $(q'_i, x)$ , and  $b(x) < a'_i(x)$ . Consider  $x'$  that is ranked at the bottom by  $a'_i$ . It must be that  $b(x') > a'_i(x)$ . Thus in a near-tie of  $(a'_i, x')$ , vote  $a'_i$  beats  $b$ .  $\square$

## Leader Rule (Approval voting)

Assume candidates  $c_1, \dots, c_m$  are sorted in decreasing score order in a state  $s$ . In Approval voting the allowed actions are  $A = 2^C$ . The Leader rule (Laslier 2009)  $a' = h^{LR}(s, a_i)$  is a strategy approving all candidates strictly preferred to the leader of  $s$ , and approves the leader of  $s$  (candidate  $c_1$ ) if and only if it is preferred to the runner-up  $c_2$  (i.e., exactly one of  $c_1, c_2$  is being approved in  $a'$ ).

We consider the epistemic model where  $\mathcal{H}^{LR}(s)$  consists of two nested pivot graphs. The inner graph  $H_1$

contains a single edge between  $c_1$  and  $c_2$ . The outer graph  $H_2$  is a star connecting  $c_1$  to all candidates.

Proposition 11.  $a' = h^{LR}(s, a_i)$  ordinally dominates all other actions according to  $\mathcal{H}^{LR}$ . In particular,  $\mathcal{H}^{LR}$  strongly justifies  $h^{LR}$ .

Proof. Let  $a''$  be any alternative vote to  $a'$ . We will show that  $a'$  dominates  $a''$  in at least one of the tie graphs  $H_1$  or  $H_2$ .

Consider  $a''$  that differs from  $a'$  on (at least)  $c_1$  or  $c_2$  or both. On the graph  $H_1$ , the voter is pivotal for  $c_1, c_2$  and thus there is a state  $s$  where  $f(s, a'') = c_2 \prec^i c_1 = f(s, a')$ , or  $f(s, a'') = c_1 \prec^i c_2 = f(s, a')$ . Thus  $a'$  dominates  $a''$  on  $H_1$ .

Next, consider  $a''$  that approves  $c_1, c_2$  iff  $a'$  approves them, but differs in (at least) some other candidate  $c'$ . If  $c_1 \succ c'$ ,  $c'$  is not approved in  $a'$  and thus approved in  $a''$  (this is regardless of whether  $c_1$  is approved). Since there is a state  $s$  in  $H_2$  where  $c_1$  and  $c'$  are tied,  $f(s, a'') = c' \prec c_1 = f(s, a')$ . If  $c_1 \prec c'$ ,  $c'$  is approved in  $a'$  but not in  $a''$ . Again, since there is a state  $s$  where they are tied,  $f(s, a') = c' \prec c_1 = f(s, a'')$ . Thus  $a' \succ_2^i a''$  and therefore  $a'$  ordinally dominates  $a''$ .  $\square$

## OD and Iterative Voting

Since ordinal-dominance induces a natural concept of OD-response, we are interested in its implications on iterative voting with multiple strategic voters. In iterative voting, voters proceed from some initial state  $s^0$ , and in each iteration an arbitrary voter changes her vote, a process that may either converge to an equilibrium or reach a cycle. Our convergence results depend on the structure of the pivot graphs in the epistemic model.

We first show that both cliqued and upward-closed epistemic structures are the result of distance-based uncertainty with natural assumptions on the distance function.

Proposition 12.1. Any neutral distance metric  $d$  on scoring vectors induces an upward-closed epistemic model.

2. Any candidate-wise distance metric<sup>4</sup>  $d$  on scoring vectors induces a cliqued epistemic model.

Proof of 1. Assume for contradiction that there is a state  $s$  in which there are  $c_1, c_2, c_3 \in C$  such that  $s(c_1) \geq s(c_2) \geq s(c_3)$ ,  $c_2, c_3$  are tied in a state  $s'$  within a distance  $r$  from  $s$ , but  $c_1, c_3$  are not tied within the same distance. Let  $\vec{w}$  be the score vector for  $s$ , and let  $\vec{w}'$  be scoring vector for  $s'$ . From the triangle inequality,  $d(\vec{w} - \vec{w}', 0) = d((c'_1, \dots, c'_m), 0) \leq r$ . We now examine the  $\vec{w}(c'_2 + w_1 - w_2, 0, c'_3, c'_4, \dots, c'_m)$ . The first element has to be smaller than  $c'_1$ , and since  $\sum_{i=1}^m c'_i = 0$ , we

<sup>4</sup>This is a metric on a scoring vector, composed of a singleton metric  $D : \mathbb{R}^2 \rightarrow [0, 1]$ , where  $d(s, s') = \max_{c \in C} D(s(c), s'(c))$ . This includes, for example, the  $\ell_\infty$  norm.

now begin reducing  $c'_4, \dots, c'_m$  until we create  $a = \vec{w}'$ , such that its elements sum up to 0 as well. Since every dimension in the new vector is less than before,  $d(\vec{w}', 0) \leq d(\vec{w} - \vec{w}', 0) \leq r$ , and  $\vec{w}' + \vec{w}$  has a tie between  $c_1$  and  $c_3$ . Proof of 1: Assume that there is a state  $s = (s_1, \dots, s_m)$  in which there are  $c_1, c_2, c_3 \in C$  such that  $s_1 \geq s_2 \geq s_3$ ; and another state  $s'$  within a distance  $r$  from  $s$  where  $c_2, c_3$  are tied. We construct a (non-normalized) vector  $s''$  where  $c_1, c_3$  are tied, such that  $|s''_j - s_j| \leq |s'_j - s_j|$  for all  $j$  (hence  $s''$  is closer to  $s$  than  $s'$ ) or one where  $s''$  is such that  $s''_j = s'_j$  for  $j > 2$  and  $|s''_1 - s_1| \leq |s'_1 - s_1|$  and  $|s''_2 - s_2| \leq |s'_2 - s_2|$ .

W.l.o.g we have  $s''_3 \geq s_3$  and  $s''_j \leq s_j$  for all  $j \neq 2, 3$ . Denote by  $w = s'_2 = s'_3$  be the winning score in  $s'$ . There are several cases: (I) if  $w \geq s_1$ , define  $s''_1 = s_1, s''_2 = s_2 \leq s'_1, s''_3 = s_1$ ; (II) if  $s_2 < w < s_1$ , define  $s''_1 = w \in [s'_1 + 1, s_1], s''_2 = s_2, s''_3 = w$ . It is easy to check that  $s''$  holds both conditions, thus  $d(\frac{s''}{\|s''\|}, s) \leq d(s', s) \leq r$  as required. If (III)  $w \leq s_2$ , it is quite simple to see that by setting  $s''_1 = w, s''_2 = w - 1, s''_3 = w$  we are closer to  $s$  than  $s'$ .

Proof of 2: any two candidates which are tied with the score leader of  $s - c_1$  - at states at distance  $r$  from  $s$  are also tied for the leadership in a state within the same distance  $r$  from  $s$ . Since if there is any tie between candidates  $c', c''$ , either one of them is  $c_1$ , or both of them are tied with  $c_1$  in the radius  $r$  (as the difference in the score of  $c_1$  in  $s$  and the state where it isn't tied for the victory is larger than when it is tied for the win), all candidates which are tied with some other candidate in radius  $r$ , are tied with  $c_1$ , and hence "can be tied with  $c_1$  in  $S_{d,r}(s)$ " is a transitive relation. Since any candidate-wise metric is in particular neutral,  $H_{d,r}$  is upward closed by the first part.

Let  $x$  be the lowest-ranked candidate participating in any tie. By upward-closeness,  $(y, c_1) \in H_{d,r}$  for all  $y$  ranked weakly above  $x$ . Then by transitivity, any edge  $(y, z)$  where  $y, z$ , are ranked weakly above  $x$  is also in  $H_{d,r}$ , which means that  $H_{d,r}$  is a clique.  $\square$

Proof of 2 is similar to Meir (2015), Lemma 2.

Theorem 13. Suppose agents each have a cliqued epistemic model (not necessarily the same one). Then, iterative voting under Plurality must converge to OD equilibrium, from any initial state.

Proof. For contradiction, let us assume the theorem is wrong and there is a cycle. That is, there is a sequence of scoring vectors (states)  $s^1, \dots, s^q$  such that  $s^{j+1}$  is the outcome of an agent  $i$  making an OD move in  $s^j$ , and  $s^1$  is the result of an agent making an OD move in  $s^q$ . Let  $B$  be the set of candidates whose score changes throughout the cycle, and let  $z \in B$  be the candidate with the lowest score in the cycle (if there are multiple such candidates, let  $z$  be the lowest ranked in the tie-breaking rule).

Let  $s^q$  was be a state where  $z$  is at their lowest score, and in which an agent  $j$  makes a move, changing their

If in  $s^{q'}$   $x$ 's score was larger than  $z$ 's, this means there was a tie between  $x$  and  $a$  was in the pivot-graph for agent  $j$ , and by moving to  $z$ , this indicates  $x \succ^j a$ . If  $x$ 's score was smaller than  $z$ 's in  $s^{q'}$ , the score of  $b$  in  $s^{\bar{q}}$  is larger than that of  $x$  (since all scores are larger than that of  $z$  in  $s^{q'}$ ), and since  $b \succ_j a$ , agent  $j$  should have preferred to stay with agent  $b$ .  $\square$

Proof. Assume, for contradiction, that the process does not converge. Let  $R$  be the set of candidates whose score changes an infinite number of times, and let  $z \in R$  be the candidate which has the lowest score in the cycle (breaking ties using the tie-breaking rule), and let  $\bar{s}^z$  be the state where it reaches this abysmal score. That is, some voter  $j$  moves from vetoing candidate  $a$  to vetoing candidate  $z$ . Candidate  $a$ 's (and any other  $c \in R$ ) score is above  $z$ 's, as otherwise its own vetoing before would give it a lower score than  $z$ . Since this is a cliqued epistemic model, leaving  $a$  means it is the favorite candidate of voter  $j$  over all candidates with scores above  $z$ , in particular, for any  $c \in R$ ,  $a \succ^j c$ .

Other convergence results from (Meir, Lev, and Rosenschein 2014) could be similarly extended for any upward-closed information structure. Theorem 14 with Prop. 12 imply the first non-Plurality result for local dominance.

## Discussion and Future Directions

of the world. Moreover, they do not even have an exact understanding of their uncertainty of the world’s state. Hence, their understanding is modeled in a coarser way – as “shades of likelihood” of various voting outcomes, derived from a prospective poll. This framework is robust enough so as to allow us to capture many previously suggested heuristics and strategies of voter behavior under uncertainty. That is, we are able to express these heuristics as rational strategies under particular information structure known to players.

A natural and important use of our model is to reformulate heuristics from various game-theoretic domains – not limited to social choice – as ordinaly-dominant strategies. This might offer an insight into the built-in assumptions inherent in these heuristics, and allow, perhaps, novel formulations of new heuristics and methods, tailored to particular uncertainty structures.

Another promising direction is exploring possible connections between ordinal information structures and existing theories of qualitative uncertainty such as (Halpern 1997).

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