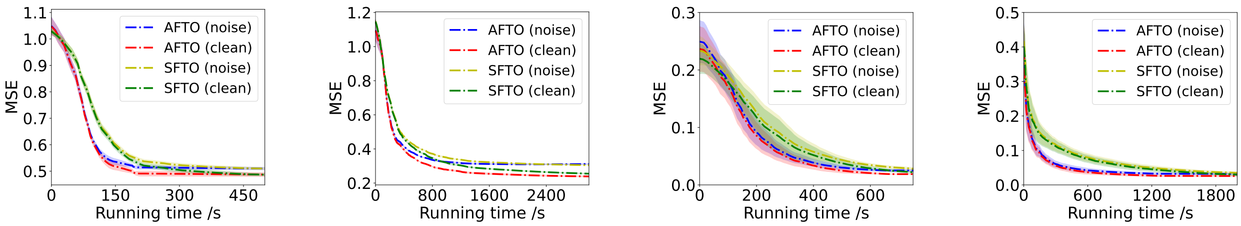
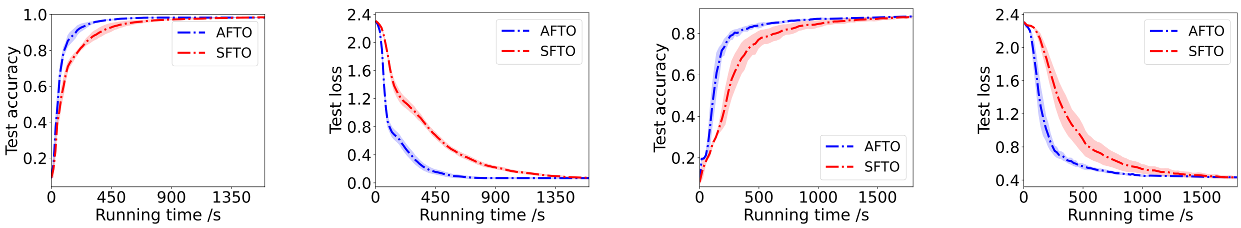


(a) Diabetes (b) Boston (c) Red-wine (d) White-wine

Figure 1: MSE of clean test data and test data with Gaussian noise on (a) Diabetes, (b) Boston, (c) Red-wine quality, and (d) White-wine quality datasets. All experiments are repeated five times, and the shaded areas represent the standard deviation.



(a) Test accuracy (b) Test loss (c) Test accuracy (d) Test loss

Figure 2: (a) Test accuracy and (b) test loss vs running time when SVHN is utilized to pretrain the model. (c) Test accuracy and (d) test loss vs running time when MNIST is utilized to pretrain the model. All experiments are repeated five times.

	N	S	Stragglers	τ
Diabetes	4	3	1	10
Boston	4	3	1	10
Red-wine	4	3	1	10
White-wine	6	4	1	10
SVHN (finetune)	4	3	1	5
SVHN (pretrain)	6	3	2	15

Table 1: Experimental setting in distributed robust hyperparameter optimization and distributed domain adaptation.

distributed manner and converges much faster than SFTO since the master can update its variables once it receives updates from a subset of workers instead of all workers in AFTO. Furthermore, we compare the proposed method with the state-of-the-art distributed bilevel optimization methods ADBO (**?**) and FEDNEST (**?**). It is shown in Table 2 that the proposed AFTO can achieve superior performance, which demonstrates the effectiveness of the proposed method.

Distributed Domain Adaptation

Pretraining/finetuning paradigms are increasingly adopted recently in self-supervised learning (**?**). In (**?**), a domain adaptation strategy is proposed, which combines data reweighting with a pretraining/finetuning framework to automatically decrease/increase the weight of pretraining samples that cause negative/positive transfer, and can be formulated as trilevel optimization (**?**). The corresponding distributed trilevel optimization problem is given as follows,

$$\begin{aligned} &\min \textstyle\sum_j L_{FT,j}(\boldsymbol{\varphi},\boldsymbol{v},\boldsymbol{w}) \text{ s.t.} \%removedVspace\ \boldsymbol{v} = \arg\min_{\boldsymbol{v}'} \textstyle\sum_j \big(L_{FT} \\ &\boldsymbol{w} = \arg\min_{\boldsymbol{w}'} \textstyle\sum_j \frac{1}{\mathcal{D}_j} \sum_{x_{i,j}\in\mathcal{D}_j} \mathcal{R}(x_{i,j},\boldsymbol{\varphi}) \cdot L^i_{PT,j}(\boldsymbol{\varphi},\boldsymbol{v}',\boldsymbol{w}') \\ &\text{var.} \qquad \boldsymbol{\varphi},\boldsymbol{v},\boldsymbol{w}, \end{aligned} \tag{32}$$

where $\boldsymbol{\varphi}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ respectively denote the parameters for pretraining, finetuning, and reweighting networks. $x_{i,j}$ and $L^i_{PT,j}$ represent the i^{th} pretraining sample and loss in worker j, $L_{FT,j}$ represents the finetuning loss in worker j. $\mathcal{R}(x_{i,j},\boldsymbol{\varphi})$ denotes the importance of pretraining sample $x_{i,j}$, and λ is the proximal regularization parameter. To evaluate the performance of the proposed method, the multiple domain digits recognition task in (**??**) is considered. There are two benchmark datasets for this task: MNIST (**?**) and SVHN (**?**). In the experiments, we utilize the same image resize strategy as in (**?**) to make the format consistent, and LeNet-5 is used for all pretraining/finetuning/reweighting networks. We summarize the experimental setting in Table 1 and Appendix H. Following (**?**), we utilize the test accuracy/test loss vs running time to evaluate the proposed AFTO. It is seen from Figure 2 that the proposed AFTO can effectively solve the distributed trilevel optimization problem and exhibits superior performance, which achieves a faster convergence rate than SFTO with a maximum acceleration of approximately 80%.

Conclusion

Existing trilevel learning works focus on the non-distributed setting which may lead to data privacy risks, and do not provide the non-asymptotic analysis. To this end, we propose an asynchronous federated trilevel optimization method for TLO problems. To our best knowledge, this work takes an initial step that aims to solve the TLO problems in an asynchronous federated manner. The proposed μ-cuts are utilized

Method	Diabetes	Boston	Red-wine	White-wine
FEDNEST	0.5293 0.0229	0.3509 0.0177	0.0339 0.0014	0.0268 0.0010
ADBO	0.5284 0.0074	0.3243 0.0046	0.0336 0.0018	0.0277 0.0013
AFTO	**0.5124 0.0068**	**0.3130 0.0037**	**0.0321 0.0026**	**0.0248 0.0021**

Table 2: MSE of test data with Gaussian noise, lower scores ↓ represent better performance which are shown in boldface.

to construct the hyper-polyhedral approximation for TLO problems, and it is demonstrated that they are applicable to a wide range of non-convex functions that meet the μ-weakly convex assumption. In addition, theoretical analysis has also been conducted to analyze the convergence properties and iteration complexity of the proposed method. maxime quas, nulla delectus modi quisquam eveniet quos tenetur veniam sapiente.Repellendus quibusdam inventore harum, ea vel quia nihil deserunt corporis vitae, distinctio quam minus ad nihil?Commodi perferendis minima aspernatur suscipit rerum voluptatum culpa magnam asperiores consectetur, dignissimos molestias voluptates, numquam vero quia mollitia eius fugit quis totam recusandae earum officia, corrupti odit deserunt et quia harum, aspernatur repellat nostrum amet ullam sapiente.Unde doloremque vel asperiores veritatis ex expedita quibusdam exercitationem, eos cupiditate repudiandae corporis nulla mollitia consequuntur ipsa incidunt sunt, quo adipisci tempore fugit voluptatibus quam voluptatum consectetur iusto facere, hic rerum voluptates distinctio deserunt ex impedit ab expedita, recusandae laudantium enim sunt provident et?Neque 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