maximizes this metric indicating that it is indeed a more general notion of optimality.

- **Rank 1** : It was observed that both popular and rank maximal matchings have similar if not same number of rank 1 edges. While the head of the signature is maximized, it is observed that both these matchings display poor performances on metrics that account for the entirety or the tail of the signature.
- **Time** : Dictated by the computational time complexities of the respective algorithms, the times were vastly different for FM and AMM compared to the other three matchings. In graphs with 900 vertices(in each partition), the FM took 512.45 seconds,AMM executed in 204.78 seconds while POP and PM were executed in less than 5 seconds.

Understanding AMM

The strongly positive empirical performance of AMM, in various metrics of importance as shown above, leads us to ask some interesting questions.

Is an AMM Pareto optimal?
Yes, AMM is a Pareto optimal matching.

Theorem. *AUPCR maximizing matching is Pareto optimal.*

Proof. Assume to the contrary that an AUPCR maximizing matching M is not Pareto optimal. This means there exists a matching M' where every applicant in M' is at least as well off as in M and at least one applicant in M' is better off than M. Consider a vertex $v \in A$. Let $r_M(v)$ be the rank of the post that v is matched to ($r_M(v) = |\mathcal{P}| + 1$ if v is unmatched), and $r_{M'}(v)$ be defined analogously.

$$\begin{aligned} &AUPCR(M') - AUPCR(M) \\ &= \sum_{v \in A} ((\mathcal{P} - r_{M'}(v) + 1) - (\mathcal{P} - r_M(v) + 1)) \\ &= \sum_{v \in A} (r_{M'}(v) - r_M(v)) \\ &> 0 \end{aligned}$$

The last inequality follows from the fact that every term of the summation is non negative and at least one term is positive by our assumption that M is not Pareto optimal.
Since AUPCR(M') - AUPCR(M) > 0, M is not an AUCPR maximizing matching, a contradiction, and so M must be Pareto optimal. □

Is an AMM always a maximum cardinality matching?
An AMM need not always be a maximum cardinality matching. Consider the instance with $A = \{a_1, a_2, a_3, a_4\}$, $P = \{b_1, b_2, b_3, b_4\}$ and the preferences given by

$$\begin{aligned} a_1 &: (b_1, 1) \\ a_2 &: (b_1, 1), (b_2, 2) \\ a_3 &: (b_2, 1), (b_1, 2), (b_3, 3) \\ a_4 &: (b_3, 1), (b_1, 2), (b_4, 3) \end{aligned}$$

As shown in Figure 4 and Figure 5, for this instance, AMM has a cardinality of 3 while a maximum cardinality matching has a cardinality of 4.

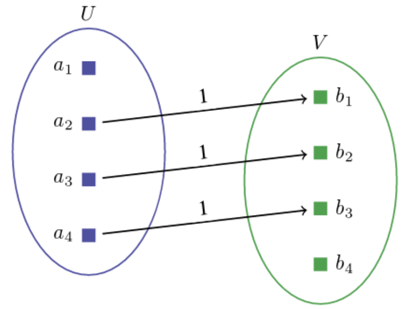

Figure 4: AUPCR Maximizing Matching

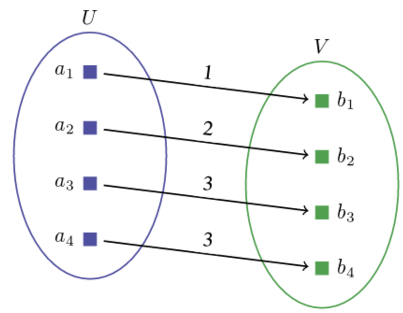

Figure 5: Maximum Cardinality Matching

Do all AMMs have the same cardinality?
All AMMs need not have the same cardinality. Consider the instance with $A = \{a_1, ...a_6\}$ and $P = \{b_1, ...b_6\}$ and the preferences given by

$$\begin{aligned} a_1 &: (b_6, 1), (b_3, 2), (b_1, 3) \\ a_2 &: (b_2, 1), (b_3, 2), (b_1, 3) \\ a_3 &: (b_4, 1), (b_5, 2), (b_2, 3) \\ a_4 &: (b_1, 1), (b_4, 2), (b_6, 3) \\ a_5 &: (b_5, 1), (b_2, 2), (b_1, 3) \\ a_6 &: (b_4, 1), (b_2, 2), (b_5, 3) \end{aligned}$$

As shown in Figure 6 and Figure 7, both are AUPCR maximizing matchings, with an AUPCR of 0.833, but they have different cardinalities. This example also shows that multiple AMMs can exist for a given instance.

Is an AMM always more "rank maximal" than a FM?
An AMM matching need not be more rank-maximal than the fair matching. Consider the instance with $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$, $P = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ and the preferences given by

$$\begin{aligned} a_1 &: (b_1, 1) \\ a_2 &: (b_2, 1) \\ a_3 &: (b_3, 1), (b_4, 2), \\ a_4 &: (b_1, 1), (b_5, 2), (b_4, 3) \\ a_5 &: (b_1, 1), (b_6, 2), (b_2, 3), (b_5, 4) \\ a_6 &: (b_1, 1), (b_2, 2), (b_7, 3), (b_6, 4), (b_3, 5) \\ a_7 &: (b_7, 1) \end{aligned}$$

An AMM matching for the above graph is as show in Figure 8 and it's signature is given by $(3, 3, 0, 0, 1)$. One can easily

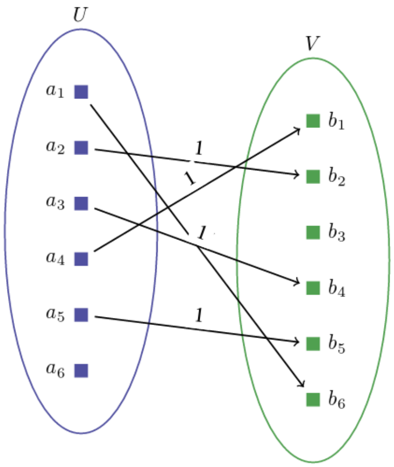

Figure 6: An AMM with $|M| = 5$

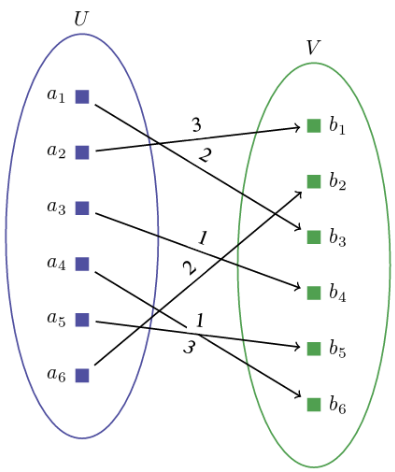

Figure 7: An AMM with $|M| = 6$

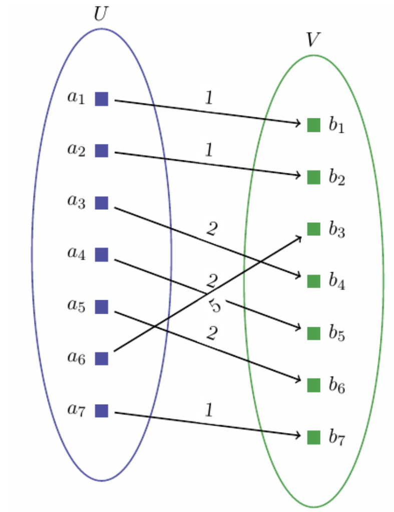

Figure 8: An AMM with matching with signature $(3, 3, 0, 0, 1)$

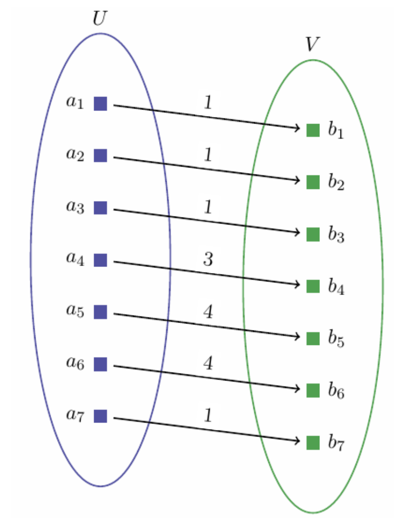

Figure 9: A Fair matching with signature $(4, 0, 1, 2, 0)$

see that the FM shown in Figure 9 is more rank-maximal with a signature $(4, 0, 1, 2, 0)$.

Conclusion

In this work, we introduce the notion of an AUPCR maximizing matching. We describe two variants with one maximizing the AUPCR, and the other maximizing the cardinality subject to maximizing the AUPCR. We empirically evaluate our algorithm on standard synthetically generated datasets and highlight that AUPCR maximizing matching achieves this much needed middle-ground with respect to the different notions of optimality. The overall performance of the AUPCR matching is superior in comparison to other matchings when all metrics are cumulatively used for comparison. Extending the AUPCR matching and finding algorithms with reduced time complexity is left as future work. Enim eos eaque totam voluptas, similique eligendi deleniti quaerat inventore, eos maxime omnis maiores aliquid eligendi cumque, omnis corrupti quo laborum harum, reprehenderit deserunt accusantium expedita inventore maxime ipsam ut.Ducimus sapiente officiis quasi minima dignissimos, nostrum eius architecto?Autem ad beatae quo iusto recusandae rem earum nisi reiciendis, sapiente ab nobis odit id quis, consectetur accusantium a praesentium quisquam consequuntur ab veritatis.Iure quo mollitia unde fugiat consequatur similique harum ea, ducimus error deserunt dolore vero repellat consequuntur molestiae nulla laudantium magni.Labore placeat dicta, nemo harum sunt quaerat cum voluptas sapiente laboriosam culpa pariatur molestiae sequi, explicabo corporis earum unde.Ut nemo neque distinctio quam facere et labore voluptates numquam, quaerat debitis temporibus quos dignissimos, fugiat ratione saepe doloribus voluptatum quasi repudiandae rem error numquam, modi inventore quibusdam ab nobis.Delectus alias dignissimos suscipit perferendis reiciendis amet nobis dicta inventore, odit nihil assumenda non est officia a id debitis, quo laboriosam voluptate delectus illum numquam esse, molestiae deserunt