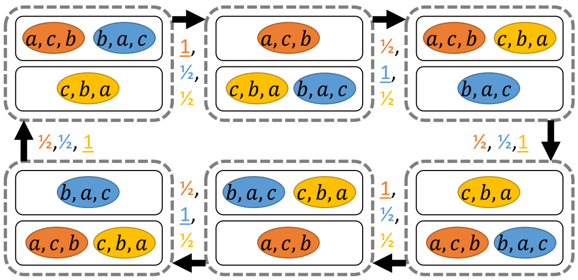

Figure 4: A cycle occurs, assuming districts are fractional and agents are globalist. On each edge is the score vector for the candidates induced by this move.

3. *If* $b^+ - b_- = 1$ *when districts are fractional and voters are globalist.* **[bound of Theorem 1]**

Proof. For item 1, when districts are deterministic, Figure 2 shows a cycle when tie-breaking t in each district is $a \succ b$, and overall, $\tilde{t}$ is $b \succ a$.

For item 2, when districts are fractional, Figure 3 shows a cycle, when agents are lexicographic.

For item 3, when districts are fractional and voters globalist, Figure 4 shows a cycle. □

Simulations

While the results on convergence are tight, we are interested to see the effects of the decentralized iterative dynamic on the overall welfare of the system. Moreover, as we have multiple parameters of the type of districts and types of agents, we are interested in exploring how (and if) are these parameters affect the efficiency of the process and how this affects the overall social welfare.

While the examples above demonstrated a relatively small number of candidates, when trying to simulate more of them, the question arises of determining the preference order of the voters. We have chosen to run each simulation both with randomized preferences as well as with single-peaked preferences. Single-peaked preferences are relevant when there is an agreed upon ordering of candidates on some axis (e.g., political right to left; location of public parks along a street; etc.). Every voter has a particular location on the axis which is its most favored location, and the farther away an option is from that location, the lower it is in the voter's preference order. Unlike randomized preferences, which can create quite unrealistic preference orders, there is a case to be made regarding these preferences (see (**?**)) and resembling realistic preferences (as (**?**) note, elections are rarely purely single peaked, but they are closer than purely random models, as (**?**) indicates).

In order to calculate social welfare, in each simulation, we measure the percentage of voter that prefer the final state over the initial state.[5]

In order to simplify data analysis and comparison, we ran a few experiments with a large variety of voter numbers, but

[5]In a case where a cycle occurred, we averaged over all the states of the cycle.

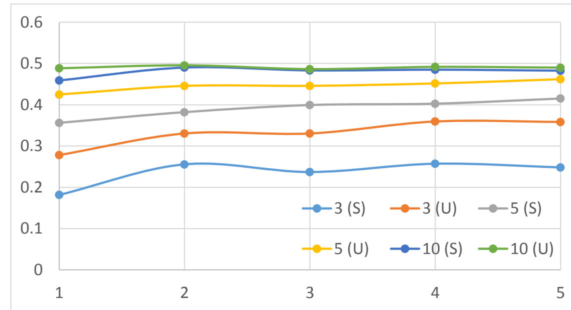

Figure 5: The average proportion of agents that prefer the final position over the opening position for non-strategic agents with deterministic tie-breaking, lexicographic utility, according to the gap constraint (the x-axis) and number of districts for elections with 8 candidates. (S) marks single-peaked preferences vs. uniformly random ones (U) Generally, as the gap constraints are loosened, agents can increase their welfare.

here we will present the extensive simulations we have done of the iterative dynamic with 53 voters (the number was useful due to the district size). We examined the effects of the number of districts (we ran simulations with 2,3,5 and 10), and the size of the gap between maximal and minimal group size – $b^+ - b_-$ (we ran simulations with 1,2,3,4,5).

A simulation setting included a choice of number of the district, a choice of the number of candidates, a choice of the size of gap, whether voter preferences were uniformly randomly generated or single-peaked, whether districts were fractional or deterministic, whether agents' utilities were global or lexicographic, and whether agents were vote-strategic or not. For each of these 320 potential settings, we ran 1,000 scenarios, each beginning in the truthful state (for vote-strategic agents), and advancing from there.

Results

Despite the theoretical convergence results, the most striking of the simulation results was that the rate of non-convergence was so small. It was, overall, slightly less than 0.66%. While no setting had a particularly large amount of cycles, the number of districts did slightly increase them, as did using lexicographical utilities and using fractional districts. On average, 57% of initial states were already in equilibrium, but this was highly volatile, and mostly appeared in settings with a small number of districts.

Convergence happened in almost all cases within 13 steps. Single-peaked preferences took markedly less time (presumably, thanks to their more structured form), and global utility converged faster than lexicographic utility.

The flexibility that is given to players to manipulate – the maximal/minimal bounds over district sizes – affects the outcome, though it was not necessarily monotonic. As can be seen in Figure 5 (the x-axis is $b^+ - b_-$) the effect of the gap on all cases is almost identical – even when the "amplitude" of the agents' social welfare is different (mostly dependent on district size and on distribution), the effect of

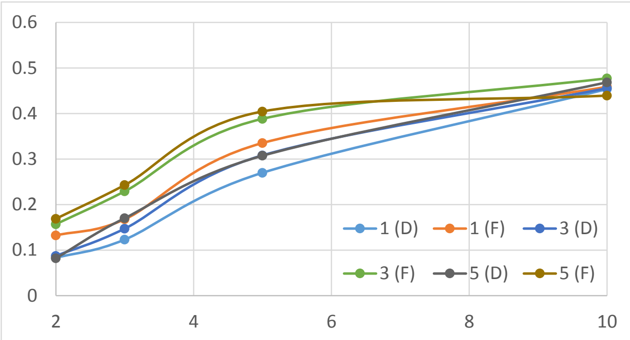

Figure 6: The average proportion of agents that prefer the final position over the opening position for non-strategic agents with lexicographic utility and 5 candidates, according to number of districts (the x axis) and various gaps. (F) marks fractional tie-breaking vs. deterministic one (D).

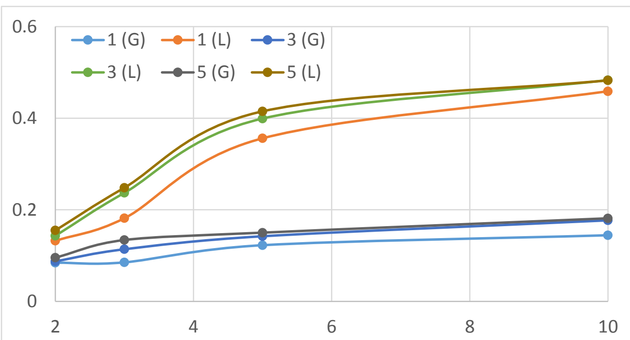

Figure 7: The average proportion of agents that prefer the final position over the opening position for strategic agents with deterministic tie-breaking and 8 candidates, according to number of districts (the x axis) and various gaps. (L) marks lexicographic preferences vs. globalist ones (G).

changing the constraints is fairly consistent across all settings. This is also true when comparing the social welfare as a function of the number of district for fractional vs. deterministic tie-breaking. As can be seen in Figure 6, the utility increase as the number of districts grows (the x-axis), and generally speaking under fractional settings, the agents tend to prefer the final state more than under deterministic tie-breaking settings.

The difference in utility between globalists and lexicographic agents is quite significant (almost 250% more), as can be seen in Figure 7, an advantage that is consistent (though with different magnitudes) when changing number of candidates, tie-breaking system, and whether agents are strategic or not. This has to do with the greater ability of lexicographic agents to be partially satisfied – several agents, with opposing views can be satisfied. However, the growth is quite dramatic. Furthermore, notice that smaller gaps produce less utility for the agents, while larger gaps presumably allow greater flexibility to the manipulating agents.

Beyond lexicographic agents' greater utility, global

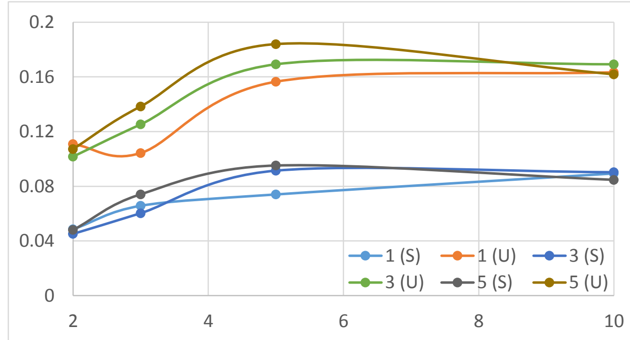

Figure 8: The average proportion of agents that prefer the final position over the opening position for non-strategic agents with deterministic tie-breaking, global utility and 5 candidates, according to number of districts (the x axis) and various gaps. (S) marks single-peaked preferences vs. uniformly random ones (U).

agents' distribution can effect their utility significantly. As can be seen in Figure 8, single-peaked agents were almost half the utility compared those whose preferences were allocated uniformly at random. Notice that, as before, in general, higher gap agents were more successful.

We should note that one of the most surprising outcomes is that strategic agents did not, ultimately, have a meaningfully better utility than non-strategic agents. In a sense, all agents could save themselves the effort, and just not bother with strategizing.

Discussion

The "reverse gerrymandering" setting, presented in this paper, may sound slightly unnatural at first blush, since people do not usually get to jump between voting districts (though (**?**) worked on such a setting). However, we believe that when viewed from the perspective of people participating in workplace committees, with their overlapping organizational influence, they do indeed strategize on where they could be more influential, and they move if they find a better position. In a more futuristic outlook, as autonomous systems become more common, the issue of these agents will need to be finding on their own where they are pivotal to help, and when will it be wrong to move. We presented here both theorems on the properties of this dynamic, and also explored it empirically (including for cases which we showed could not converge, hence an empirical examination is the main tool for analysis), discovering some key issues on the effect of changing the agent preference model, and the effect of district size on the agent. There is still much to discover – what other preference models work well with this setting; understanding better the effect of the gap constraints on social welfare; and combining various different types of agents in the same simulation, examining how their differences interact.

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