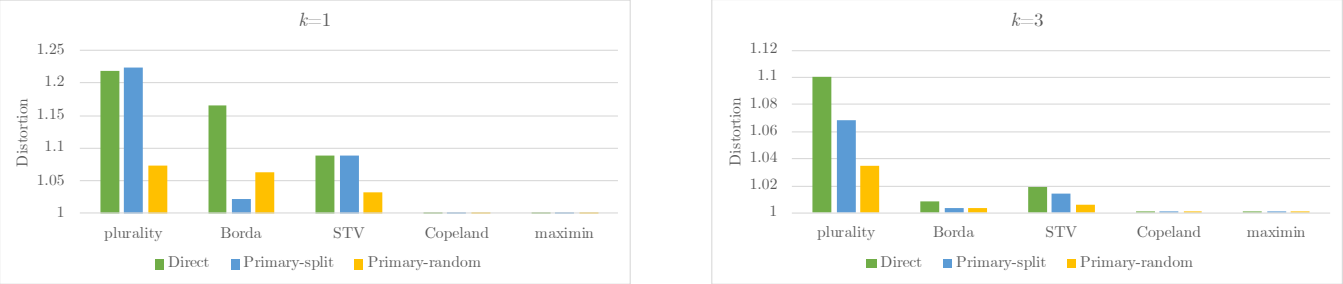

Figure 1: The distortion values for various settings (Direct, Primary-split and Primary-random) and voting rules for $k \in \{1, 3\}$. Each bar represents the average distortion over 1000 instances. Within each figure, all of the bars represent average distortion over the same set of voter and candidate locations, but not necessarily party affiliations. ANOVA on all elections returned $p < 10^{-9}$ when $k = 1$ and $p < 10^{-5}$ when $k = 3$

on one side of the hyperplane, and $\frac{m}{2} = 10$ candidates (call this set A_1) uniformly at random on the other side.

Once the locations of the voters and candidates are fixed, we create two instances. In one instance ("split"), we assign $V_{-1} \cup A_{-1}$ to party -1, and assign $V_1 \cup A_1$ to party 1. This instance belongs to $\mathcal{I}^{0.5}_{m,\text{sep-}\mathbb{R}^k}$. In the other instance ("random"), we assign half of the voters and half of the candidates chosen uniformly at random to party -1, and the rest to party 1. This instance belongs to $\mathcal{I}^{0.5}_{m,\mathbb{R}^k}$, but not necessarily to $\mathcal{I}^{0.5}_{m,\text{sep-}\mathbb{R}^k}$. This allows us to directly compare the effect that separability has on the distortion. Finally, we run five voting rules — plurality, Borda, STV, Copeland and maximin — on both instances under the direct and primary systems, and measure the distortion. Note that their distortion under the direct system would be identical for both instances because the two instances only differ in party affiliations. Thus, for each rule, we obtain three numbers: Direct, tion numbers across the 1000 instances.

See Figure 1 for selected simulation results. The results for $k > 3$ resemble those for $k = 3$ in their comparison of the direct versus the primary system; the only difference is that the overall distortions are lower for higher k. To compare the three distortion numbers in each case, we ran a repeated measures ANOVA comparing the distortion values, and in all but 2 of 25 cases, had a p value under 0.05 (the two cases had $k = 9$).

Perhaps the most important observation is that our simulation results stand in direct contrast to our worst-case bounds. In almost all of our settings, the distortion under the primary system (split and random) is better than that the distortion under its direct counterpart, and often shows a significant improvement. This is especially noticeable in the non-Condorcet consistent rules (plurality, Borda and STV), as in all but 3 of 15 cases the distortion significantly improves the primary system in both cases. This effect is most pronounced with plurality. With Condorcet consistent rules, the distortion values are very low, regardless of whether the direct system is used or the primary system. In general, as we increase the dimension k, the groups become more homogenous and the p values grow, while distortions approach 1.

Overall, the simulation results show a distortion that is far below our theoretical worse-case results. We suspect that the reason for this difference might have to do with our choice of uniform voter and candidate distributions, and distortion numbers might differ under different distributions.

Discussion

Our paper initiates the novel quantitative study of multi-stage elections (and their comparison to single-stage elections), but leaves plenty to explore. Some directions are fairly straightforward extensions of our results. The most straightforward question is to tighten our bounds. There is also the question of explaining the trends we observe in the average case, which sometimes differ from our worst-case results. A next step would be to study realistic distributional models of voter preferences and candidate positions in the political spectrum, and analyze their effect on distortion.

Other extensions are seemingly more involved. Extending our framework to more than two parties requires the use of a ranked voting rule in the general election, which may significantly affect the analysis. Interestingly, such an extension would also incorporate independent candidates because one can imagine an independent candidate to be a party of their own. Examining the use of multiple and different voting rules as **?** (**?**) do for two-step voting (though without candidate elimination between stages) is an enticing direction. For example, in a multi-party direct system, we may use plurality, whereas in the primary system, the parties may use STV. It is also reasonable to consider that each party has its own voting rule. It would be interesting as well to examine party manipulation techniques in primary systems. Similarly, it is reasonable to believe that candidates may also strategically shift, to some extent, their location following the primaries, to make themselves more appealing to the general electorate.

We believe that the study of multi-stage elections and party mechanisms can not only contribute novel theoretical challenges to tackle, but can also bring research on computational social choice closer to reality and increase its impact.

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