

Assignment 4

Part 1:

1.

a. False. The function g should receive a type T1, but in the statement itself it receives a which is a number. Unless T1 represents a number the claim is False.

b. False. The function f should receive a type T2, but in the statement itself it receives x which is of type T1.

c. True. X's type is T1, thus the type of (f x) is T2, thus the type of the expression (lambda () (f x)) is T2.

d. True. In the expression (f x y), x receives the type of T1, y is indeed T2, so the expression (f x y) returns T3 as it should. Thus the lambda receives T1 and returns T3 which is the type of the (f x y).

2.

a.

Expression	Type
((lambda (f x1) (f 1 x1)) + #t)	T0
(lambda (f x1) (f 1 x1))	T1
(f 1 x1)	T2
f	Tf
1	Tnum1
x1	Tx
+	T+
#t	Tt

Type variables equations:

Expression	Equation
((lambda (f x1) (f 1 x1)) + #t)	$T1 = [T_p \ X \ T_t \rightarrow T_0]$
(lambda (f x1) (f 1 x1))	$T1 = [T_f \ X \ T_x \rightarrow T_2]$
(f 1 x1)	$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$

Primitive types equations:

Expression	Equation
1	$T_{num1} = \text{Number}$
+	$T_+ = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$
#t	$T_t = \text{Boolean}$

Equations	Substitution
$T1 = [T_+ X T_t \rightarrow T_0]$	$\{\}$
$T1 = [T_f X T_x \rightarrow T_2]$	
$T_f = [T_{num1} X T_x \rightarrow T_2]$	
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_t = \text{Boolean}$	

Equations	Substitution
$T1 = [T_p X T_e \rightarrow T_0]$	$T1 = [T_+ X T_t \rightarrow T_0]$
$T1 = [T_f X T_x \rightarrow T_2]$	
$T_f = [T_{num1} X T_x \rightarrow T_2]$	
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_t = \text{Boolean}$	

$$T1 = [T_f X T_x \rightarrow T_2] \circ \text{Substitution} \rightarrow (T_f X T_x \rightarrow T_2) = (T_+ X T_t \rightarrow T_0)$$

Thus, we will split the substitution to 3 different equations:

Equations	Substitution
$T1 = [T_p X T_e \rightarrow T_0]$	$T1 = [T_+ X T_t \rightarrow T_0]$
$T1 = [T_f X T_x \rightarrow T_2]$	
$T_f = [T_{num1} X T_x \rightarrow T_2]$	
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_t = \text{Boolean}$	
$T_f = T_+$	
$T_x = T_t$	
$T_2 = T_0$	

Equations	Substitution
$T1 = [T_p X T_e \rightarrow T_0]$	$T1 = [T_+ X T_t \rightarrow T_0]$
$T1 = [T_f X T_x \rightarrow T_2]$	$T_f = [T_{num1} X T_x \rightarrow T_2]$
$T_f = [T_{num1} X T_x \rightarrow T_2]$	
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_t = \text{Boolean}$	
$T_f = T_+$	
$T_x = T_t$	
$T_2 = T_0$	

Equations	Substitution
$T1 = [T_p \ X \ T_\epsilon \rightarrow T_0]$	$T1 = [T_+ \ X \ T_t \rightarrow T_0]$
$T1 = [T_f \ X \ T_x \rightarrow T_2]$	$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$
$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$	
$T_t = \text{Boolean}$	
$T_f = T_+$	
$T_x = T_t$	
$T_2 = T_0$	

Equations	Substitution
$T1 = [T_p \ X \ T_\epsilon \rightarrow T_0]$	$T1 = [T_p \ X \ T_t \rightarrow T_0]$
$T1 = [T_f \ X \ T_x \rightarrow T_2]$	$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$
$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_+ = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$
$T_p = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$	
$T_t = \text{Boolean}$	
$T_f = T_+$	
$T_x = T_t$	
$T_2 = T_0$	

Equations	Substitution
$T1 = [T_p \ X \ T_\epsilon \rightarrow T_0]$	$T1 = [T_p \ X \ T_t \rightarrow T_0]$
$T1 = [T_f \ X \ T_x \rightarrow T_2]$	$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$
$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_+ = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$
$T_p = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$	$T_t = \text{Boolean}$
$T_\epsilon = \text{Boolean}$	
$T_f = T_+$	
$T_x = T_t$	
$T_2 = T_0$	

Equations	Substitution
$T1 = [T_p \ X \ T_\epsilon \rightarrow T_0]$	$T1 = [T_p \ X \ T_t \rightarrow T_0]$
$T1 = [T_f \ X \ T_x \rightarrow T_2]$	$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$
$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_+ = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$
$T_p = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$	$T_t = \text{Boolean}$
$T_\epsilon = \text{Boolean}$	$T_x = \text{Boolean}$
$T_x = T_\epsilon$	
$T_f = T_+$	
$T_2 = T_0$	

Equations	Substitution
$T_1 = [T_p \ X \ T_e \rightarrow T_0]$	$T_1 = [T_p \ X \ T_t \rightarrow T_0]$
$T_1 = [T_f \ X \ T_x \rightarrow T_2]$	$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$
$T_f = [T_{num1} \ X \ T_x \rightarrow T_2]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_+ = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$
$T_p = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$	$T_t = \text{Boolean}$
$T_e = \text{Boolean}$	$T_x = \text{Boolean}$
$T_x = T_e$	
$T_f = T_+$	
$T_2 = T_0$	

Now we notice that there is a contradiction :

$T_f = T_+$, meaning

$[T_{num1} \ X \ \text{Boolean} \rightarrow T_2] = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$, meaning

$\text{Boolean} = \text{Number}$.

b.

Expression	Type
$((\text{lambda } (f \ x1) (f \ x1 \ 1)) + *)$	T0
$(\text{lambda } (f \ x1) (f \ x1 \ 1))$	T1
$(f \ x1)$	T2
f	T_f
x1	T_x
1	T_{num1}
+	T_+
*	T_*

Type variables equations:

Expression	Equation
$((\text{lambda } (f \ x) (f \ x \ 1)) + *)$	$T_1 = [T_+ \ X \ T_* \rightarrow T_0]$
$(\text{lambda } (f \ x) (f \ x \ 1))$	$T_1 = [T_f \ X \ T_x \rightarrow T_2]$
$(f \ x1)$	$T_f = [T_x \ X \ T_{num1} \rightarrow T_2]$

Primitive types equations:

Expression	Equation
1	$T_{num1} = \text{Number}$
+	$T_+ = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$
*	$T_* = [\text{Number} \ X \ \text{Number} \rightarrow \text{Number}]$

Equation	Substitution
$T_1 = [T_+ X T_* \rightarrow T_0]$	$\{\}$
$T_1 = [T_f X T_x \rightarrow T_2]$	
$T_f = [T_x X T_{num1} \rightarrow T_2]$	
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_* = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	

Equation	Substitution
$T_1 = [T_+ X T_* \rightarrow T_0]$	$T_1 = [T_+ X T_* \rightarrow T_0]$
$T_1 = [T_f X T_x \rightarrow T_2]$	
$T_f = [T_x X T_{num1} \rightarrow T_2]$	
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_* = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	

Equation	Substitution
$T_1 = [T_+ X T_* \rightarrow T_0]$	$T_1 = [T_+ X T_* \rightarrow T_0]$
$T_1 = [T_f X T_x \rightarrow T_2]$	
$T_f = [T_x X T_{num1} \rightarrow T_2]$	
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_* = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_f = T_+$	
$T_x = T_*$	
$T_2 = T_0$	

Equation	Substitution
$T_1 = [T_+ X T_* \rightarrow T_0]$	$T_1 = [T_+ X T_* \rightarrow T_0]$
$T_1 = [T_f X T_x \rightarrow T_2]$	$T_f = [T_x X T_{num1} \rightarrow T_2]$
$T_f = [T_x X T_{num1} \rightarrow T_2]$	
$T_{num1} = \text{Number}$	
$T_+ = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_* = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_f = T_+$	
$T_x = T_*$	
$T_2 = T_0$	

Equation	Substitution
$T_{\pm} = [T_{+} X T_{*} \rightarrow T_0]$	$T_1 = [T_{+} X T_{*} \rightarrow T_0]$
$T_{\pm} = [T_f X T_x \rightarrow T_z]$	$T_f = [T_x X T_{num1} \rightarrow T_2]$
$T_f = [T_x X T_{num1} \rightarrow T_z]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	
$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_f = T_{+}$	
$T_x = T_{*}$	
$T_2 = T_0$	

Equation	Substitution
$T_{\pm} = [T_{+} X T_{*} \rightarrow T_0]$	$T_1 = [T_{+} X T_{*} \rightarrow T_0]$
$T_{\pm} = [T_f X T_x \rightarrow T_z]$	$T_f = [T_x X T_{num1} \rightarrow T_2]$
$T_f = [T_x X T_{num1} \rightarrow T_z]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_f = T_{+}$	
$T_x = T_{*}$	
$T_2 = T_0$	

Equation	Substitution
$T_{\pm} = [T_{+} X T_{*} \rightarrow T_0]$	$T_1 = [T_{+} X T_{*} \rightarrow T_0]$
$T_{\pm} = [T_f X T_x \rightarrow T_z]$	$T_f = [T_x X T_{num1} \rightarrow T_2]$
$T_f = [T_x X T_{num1} \rightarrow T_z]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_f = T_{+}$	
$T_x = T_{*}$	
$T_2 = T_0$	

Equation	Substitution
$T_{\pm} = [T_{+} X T_{*} \rightarrow T_0]$	$T_1 = [T_{+} X T_{*} \rightarrow T_0]$
$T_{\pm} = [T_f X T_x \rightarrow T_z]$	$T_f = [T_x X T_{num1} \rightarrow T_2]$
$T_f = [T_x X T_{num1} \rightarrow T_z]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	
$T_f = T_{+}$	
$T_x = T_{*}$	
$T_2 = T_0$	
$T_x = \text{Number}$	
$T_2 = \text{Number}$	

Equation	Substitution
$T_{\pm} = [T_{+} X T_{*} \rightarrow T_0]$	$T_1 = [T_{+} X T_{*} \rightarrow T_0]$
$T_{\pm} = [T_f X T_x \rightarrow T_z]$	$T_f = [T_x X T_{num1} \rightarrow T_2]$
$T_f = [T_x X T_{num1} \rightarrow T_z]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	$T_x = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_f = T_{+}$	
$T_x = T_{*}$	
$T_2 = T_0$	
$T_x = \text{Number}$	
$T_2 = \text{Number}$	

Equation	Substitution
$T_{\pm} = [T_{+} X T_{*} \rightarrow T_0]$	$T_1 = [T_{+} X T_{*} \rightarrow T_0]$
$T_{\pm} = [T_f X T_x \rightarrow T_z]$	$T_f = [T_x X T_{num1} \rightarrow T_2]$
$T_f = [T_x X T_{num1} \rightarrow T_z]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	$T_x = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_f = T_{+}$	$T_2 = T_0$
$T_x = T_{*}$	
$T_z = T_0$	
$T_x = \text{Number}$	
$T_2 = \text{Number}$	

Equation	Substitution
$T_{\pm} = [T_{+} X T_{*} \rightarrow T_0]$	$T_1 = [T_{+} X T_{*} \rightarrow T_0]$
$T_{\pm} = [T_f X T_x \rightarrow T_z]$	$T_f = [T_x X T_{num1} \rightarrow T_2]$
$T_f = [T_x X T_{num1} \rightarrow T_z]$	$T_{num1} = \text{Number}$
$T_{num1} = \text{Number}$	$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{+} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_{*} = [\text{Number} X \text{Number} \rightarrow \text{Number}]$	$T_x = [\text{Number} X \text{Number} \rightarrow \text{Number}]$
$T_f = T_{+}$	$T_2 = T_0$
$T_x = T_{*}$	
$T_z = T_0$	
$T_x = \text{Number}$	
$T_2 = \text{Number}$	

Now we notice that $T_x = \text{Number}$, meaning
 $[\text{Number} X \text{Number} \rightarrow \text{Number}] = \text{Number}$, which is a contradiction.

Part 3:

3.1:

1. Typing rule **SetExp**:
For every type environment $_Tenv$ and type expression $_S$:
 $_Tenv \vdash SetExp : [VarRef * _S \rightarrow void]$
2. Typing rule **LitExp**:
For every type environment $_Tenv$ and literal expression $_q$:
 $_Tenv \vdash _q : LitExp$

3.2.2:

Typing rule **define-type**:

For every: type environment $_Tenv$,

tokens $_r1, \dots, _rn, n \geq 0$,

Type expressions $_S(i,1), \dots, _S(i,m_i)$ for $i \leq n$ and $m_i \leq m$:

Type with **records**:

$_Tenv \vdash define_type : [Token * _S(1,1) * \dots * _S(1,m_1) * \dots * Token * _S(n,1) * \dots * _S(n,m_n) \rightarrow void]$

Record-less type ($n = 0$):

$_Tenv \vdash define_type : [Empty \rightarrow void]$

Typing rule **type-case**:

For every: type environment $_Tenv$,

user defined type id,

with component records $_r1, \dots, _rn, n \geq 0$,

with fields $_fij$ for $i \leq n, j \leq R_i$,

type expression $_S$,

expressions $_v, _e1, \dots, _en, n \geq 0$:

If $_Tenv \vdash _ei : _S$ for all $i \leq n$ and $_ri$ sub-type of $_T$,

Then $_Tenv \vdash (type_cast_T_v (_r1 (_f11 \dots _f1R1) _e1) \dots (_rn (_fn1 \dots _fnRn) _en)) : _S$