

丘维声《高等代数》上册习题整理

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1 线性方程组

1.1 解线性方程组的矩阵消元法

习题1.1(1) 解线性方程组

$$\begin{cases} x_1 - 3x_2 - 2x_3 = 3, \\ -2x_1 + x_2 - 4x_3 = -9, \\ -x_1 + 4x_2 - x_3 = -7. \end{cases}$$

解答:

$$\begin{aligned} & \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ -2 & 1 & -4 & -9 \\ -1 & 4 & -1 & -7 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow R_2 + 2R_1 \\ R_3 \leftarrow R_3 + R_1}} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ 0 & -5 & -8 & -3 \\ 0 & 1 & -3 & -4 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & -5 & -8 & -3 \end{array} \right] \\ & \xrightarrow{R_3 \leftarrow R_3 + 5R_2} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & -23 & -23 \end{array} \right] \xrightarrow{R_3 \leftarrow -\frac{1}{23}R_3} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 3 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow R_2 + 3R_3 \\ R_1 \leftarrow R_1 + 2R_3}} \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right] \\ & \xrightarrow{R_1 \leftarrow R_1 + 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right]. \end{aligned}$$

故

$$(x_1, x_2, x_3)^T = (2, -1, 1)^T. \square$$

习题1.1(2) 解线性方程组

$$\begin{cases} x_1 + 3x_2 + 2x_3 = 1, \\ 2x_1 + 5x_2 + 5x_3 = 7, \\ 3x_1 + 7x_2 + x_3 = -8, \\ -x_1 - 4x_2 + x_3 = 10. \end{cases}$$

解答:

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 2 & 5 & 5 & 7 \\ 3 & 7 & 1 & -8 \\ -1 & -4 & 1 & 10 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - 3R_1 \\ R_4 \leftarrow R_4 + R_1}} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -1 & 1 & 5 \\ 0 & -2 & -5 & -11 \\ 0 & -1 & 3 & 11 \end{array} \right] \xrightarrow{R_2 \leftarrow -R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & -1 & -5 \\ 0 & -2 & -5 & -11 \\ 0 & -1 & 3 & 11 \end{array} \right]$$

$$\begin{aligned}
& \xrightarrow{\substack{R_3 \leftarrow R_3 + 2R_2 \\ R_4 \leftarrow R_4 + R_2}} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & -7 & -21 \\ 0 & 0 & 2 & 6 \end{array} \right] \xrightarrow{R_3 \leftarrow -\frac{1}{7}R_3} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 6 \end{array} \right] \xrightarrow{\substack{R_4 \leftarrow R_4 - 2R_3 \\ R_2 \leftarrow R_2 + R_3}} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right] \\
& \xrightarrow{\substack{R_1 \leftarrow R_1 - 2R_3 \\ R_1 \leftarrow R_1 - 3R_2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right].
\end{aligned}$$

故

$$(x_1, x_2, x_3)^T = (1, -2, 3)^T. \square$$

习题1.1(3) 解线性方程组

$$\begin{cases} x_1 - 3x_2 - 2x_3 - x_4 = 6, \\ 3x_1 - 8x_2 + x_3 + 5x_4 = 0, \\ -2x_1 + x_2 - 4x_3 + x_4 = -12, \\ -x_1 + 4x_2 - x_3 - 3x_4 = 2. \end{cases}$$

解答:

$$\begin{aligned}
& \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 3 & -8 & 1 & 5 & 0 \\ -2 & 1 & -4 & 1 & -12 \\ -1 & 4 & -1 & -3 & 2 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow R_2 - 3R_1 \\ R_3 \leftarrow R_3 + 2R_1 \\ R_4 \leftarrow R_4 + R_1}} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & -5 & -8 & -1 & 0 \\ 0 & 1 & -3 & -4 & 8 \end{array} \right] \\
& \xrightarrow{\substack{R_3 \leftarrow R_3 + 5R_2 \\ R_4 \leftarrow R_4 - R_2}} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 27 & 39 & -90 \\ 0 & 0 & -10 & -12 & 26 \end{array} \right] \xrightarrow{R_3 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & -10 & -12 & 26 \\ 0 & 0 & 27 & 39 & -90 \end{array} \right] \\
& \xrightarrow{R_3 \leftarrow -\frac{1}{2}R_3} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 5 & 6 & -13 \\ 0 & 0 & 27 & 39 & -90 \end{array} \right] \xrightarrow{R_4 \leftarrow R_4 - \frac{27}{5}R_3} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 5 & 6 & -13 \\ 0 & 0 & 0 & \frac{33}{5} & -\frac{99}{5} \end{array} \right] \xrightarrow{R_4 \leftarrow \frac{5}{33}R_4} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 5 & 6 & -13 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right] \\
& \xrightarrow{\substack{R_3 \leftarrow R_3 - 6R_4 \\ R_3 \leftarrow \frac{1}{5}R_3}} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 7 & 8 & -18 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow R_2 - 8R_4 \\ R_2 \leftarrow R_2 - 7R_3}} \left[\begin{array}{cccc|c} 1 & -3 & -2 & -1 & 6 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right] \xrightarrow{\substack{R_1 \leftarrow R_1 + R_4 \\ R_1 \leftarrow R_1 + 2R_3 \\ R_1 \leftarrow R_1 + 3R_2}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right].
\end{aligned}$$

故

$$(x_1, x_2, x_3, x_4)^T = (2, -1, 1, -3)^T. \square$$

习题1.1(4) 解线性方程组

$$\begin{cases} x_1 + 3x_2 - 7x_3 = -8, \\ 2x_1 + 5x_2 + 4x_3 = 4, \\ -3x_1 - 7x_2 - 2x_3 = -3, \\ x_1 + 4x_2 - 12x_3 = -15. \end{cases}$$

解答:

$$\begin{aligned} & \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 2 & 5 & 4 & 4 \\ -3 & -7 & -2 & -3 \\ 1 & 4 & -12 & -15 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 + 3R_1 \\ R_4 \leftarrow R_4 - R_1}} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & -1 & 18 & 20 \\ 0 & 2 & -23 & -27 \\ 0 & 1 & -5 & -7 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_4} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & 1 & -5 & -7 \\ 0 & 2 & -23 & -27 \\ 0 & -1 & 18 & 20 \end{array} \right] \\ & \xrightarrow{\substack{R_3 \leftarrow R_3 - 2R_2 \\ R_4 \leftarrow R_4 + R_2}} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & -13 & -13 \\ 0 & 0 & 13 & 13 \end{array} \right] \xrightarrow{R_3 \leftarrow -\frac{1}{13}R_3} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 13 & 13 \end{array} \right] \xrightarrow{R_4 \leftarrow R_4 - 13R_3} \left[\begin{array}{ccc|c} 1 & 3 & -7 & -8 \\ 0 & 1 & -5 & -7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \\ & \xrightarrow{\substack{R_2 \leftarrow R_2 + 5R_3 \\ R_1 \leftarrow R_1 + 7R_3 \\ R_1 \leftarrow R_1 - 3R_2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]. \end{aligned}$$

故

$$(x_1, x_2, x_3)^T = (5, -2, 1)^T. \square$$

习题1.1(5) 解线性方程组

$$\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = 4, \\ x_1 + x_2 - x_3 + x_4 = -11, \\ x_1 + 3x_2 + x_4 = 1, \\ -7x_2 + 3x_3 + x_4 = -3. \end{cases}$$

解答:

$$\begin{aligned} & \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 1 & 1 & -1 & 1 & -11 \\ 1 & 3 & 0 & 1 & 1 \\ 0 & -7 & 3 & 1 & -3 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - R_1}} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 3 & -4 & 5 & -15 \\ 0 & 5 & -3 & 5 & -3 \\ 0 & -7 & 3 & 1 & -3 \end{array} \right] \xrightarrow{R_2 \leftarrow \frac{1}{3}R_2} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\ 0 & 5 & -3 & 5 & -3 \\ 0 & -7 & 3 & 1 & -3 \end{array} \right] \\ & \xrightarrow{\substack{R_3 \leftarrow R_3 - 5R_2 \\ R_4 \leftarrow R_4 + 7R_2}} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\ 0 & 0 & \frac{11}{3} & -\frac{10}{3} & 22 \\ 0 & 0 & -\frac{19}{3} & \frac{38}{3} & -38 \end{array} \right] \xrightarrow{\substack{R_3 \leftarrow 3R_3 \\ R_4 \leftarrow 3R_4}} \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -\frac{4}{3} & \frac{5}{3} & -5 \\ 0 & 0 & 11 & -10 & 66 \\ 0 & 0 & -19 & 38 & -114 \end{array} \right] \end{aligned}$$

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4