UNIT - 1

INTRODUCTION: Engineering Decision- Makers, Engineering and Economics, Problem solving and Decision making, Intuition and Analysis, Tactics and Strategy, Law of supply and demand.

Engineers are planners and builders. They are also problem solvers, managers, and decision makers. Engineering economics touches each of these activities. Plans and production must be financed. Problems are eventually defined by monetory dimensions, and decisions are evaluated by their monetary consequences. Much of the management function is directed toward economic oiectives and is monitored by economic measures.

Engineering economics is closely aligned with conventional microeconomics, but it has a history and a special flavor of its own. It is devoted to problem solving and decision making at the operations level. It can lead to suboptirnization-a condition in which a solution satisfies tactical objectives at the expense of strategic effectiveness—but careful attention to the collection and analysis of data minimizes the danger.

An engineering economist draws upon the accumulated knowledge of engineenng and econemics to identify alternative uses of limited resources and to select the prefered course of action. Evaluations rely mainly on mathematical models and cost data, but judgment and experience are pivotal inputs. Many accepted models are available for analyses of short-range projects when the time value of money is not relevant, and of long-range proposals when discounting is required for input data assumed to be known or subject to risk.

1.1 Engineering Decision- Makers:

The following questions might an engineer has to answer

- Which one of several competing engineering designs should be selected?
- Should the machine now in use be replaced with a new one?
- With limited capital available, which investment alternative should be funded?
- Would it be preferable to pursue a safer conservative course of action or to follow a riskier one that offers higher potential returns?
- How many units of production have to be sold before a profit can be made? This area is commonly called break-even analysis
- Among several proposals for funding that yield substantially equivalent worthwhile results but have different cash flow pattern. Which is preferable?

• Are the benefits expected from public service project large enough to make its implementation costs acceptable?

Two characteristics of the questions above should be apparent. First, each deals with a choice among alternatives; second, all involve economic considerations. Less obvious are the requirements of adequate data and awareness of technological constraints to define the problem and to identify legitimate solutions. These considerations are embodied in the decision-making role of engineering economists to

1. Identify alternative uses for limited resources and obtain appropriate data
2. Analyze the data to determine the preferred alternative

The breadth of problems, depth of analysis, and scope of application that a practicing engineer encounters vary widely.

Newly graduated engineers are regularly assigned to cost reduction projects and are expected to be cost-conscious in all their opcration8. As they gain more experience, they may become specialists in certain application areas or may undertake more general responsibilities as managers. Beginners are usually restricted to short-range decisions for low-budget operations, whereas engineering managers are confronted with policy decisions that involve large sums and are influenced by many factors with long-range consequences. Both situations arc served by the principles and practices of engineering economics.

1.2 Engineering and Economics

In earlier days, engineers were mainly concerned with

- The design,
- Construction,
- Operation of machines,
- Structures and processes.

They gave less attention to the resources, human and physical, that produced the final products. Many factors have since contributed to an expansion of engineering responsibilities and concerns. Besides the traditional work with scientists to develop discoveries about nature into useful products; engineers are now expected not only to generate novel technological solutions but also to make skillful financial analyses of the effects of implementation. In today’s close and tangled relations among industry the public, and government, cost and value analyses are expected to be more detailed and inclusive (e.g., worker safety, environmental effects, consumer protection, resource conservation) than ever before, Without these analyses, an entire project can easily become more of a burden than a benefit, Most definitions of engineering suggest that the mission of engineers is to transform the resources of nature for the benefit of the human race. The types of resources susceptible to engineering enrichment indude everything from ores and crops to information and energy.

1.3 Problem solving and Decision making:

Problem solving

An engineering economist draws upon the accumulated knowledge of engineering and economics to fashion and employ tools to identify a preferred course of action. The tools developed so far are not perfect. There is still considerable debate about their theoretical bases and how they should be used. This concern is wholesome because it promises improved procedures, but the variety of analysis techniques can frustrate practitioners, especially inexperienced ones.

There are many aspects to consider and many ways to consider them.The fundamental approach to economic problem solving is to elaborate on the time-honored scientific method.The method has link with two worlds a) Real world b) symblic word. The figure1.1 shows the detailed steps to be followed in problem solving process.

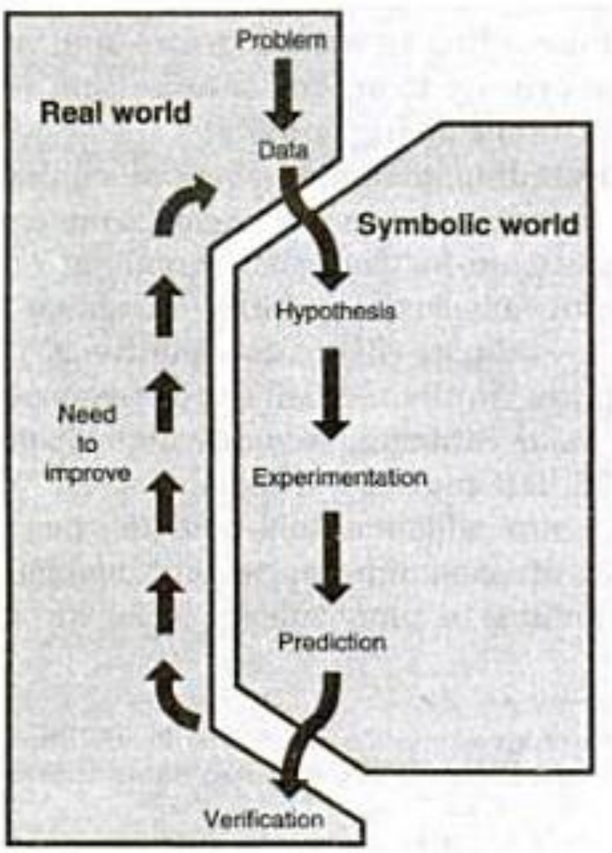


Figure1.1: Problem solving process

1. Problems in engineering and managerial economy originate in the real world of economic planning, management, and control
2. The problem is confined and clarified by data from the real world.
3. The information is combined with scientific principles supplied by the analyst to formulate a hypothesis
4. The symbolic language aids the digestion of data.
5. By manulating and experimenting with the abstractions of the real world, the analyst can simulate multiple configurations of reality.
6. The predicted behavior is converted back to reality for testing in the form of hardware, designs, or commands.
7. If it is valid, the problem is solved, if not the cycle is repeated with added information.

Decision making

The standard decision making process is shown in the figure 1.2

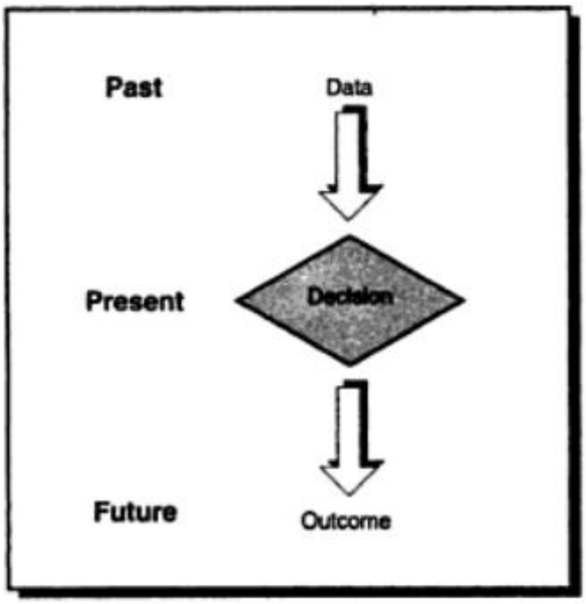


Figure 1.2: decision making process

1. Recognize the problem
2. Define the goal or objective
3. Assemble relevant data
4. Identify feasible alternatives
5. Select the criterion to determine the best alternative
6. Construct a model
7. Predict each alternative's outcomes or consequenses

8. Choose the best alternative
9. Audit the results

1.3 Intuition and Analysis:

Generally engineers generally attack practical problems with solution deadlines instead of engaging in esoteric issues for long-term enlightenment, their mission might appear relatively simple. Engineering economic evaluations could even seem mundane, since they usually rely on data from the marketplace and technology from the shelf-simply grab prices from a catalog, plug them into a handy formula, and grind out an answer. Occasionally, such a routine works. Spectacular workbench discoveries and overnight fortunes attest to the fact that plungers sometimes win. There are also innumerable instances where rule-of-thumb, skin-deep evaluat ions are absolutely unsatisfactory.

A decision made now is based on data from past performances and establishes a course of action that will result in some future outcome. When the decision is shallow and the outcomes are not of much consequence. A reflex response based upon intuition is feasible. Instinctive judgments are often formalized by standard operating procedures (SOPs). In economic analyses. SOPs often take the form of works heets for the justification of investments. Such short-form justifications are typically limited to smaller investments, say, less than Rs10, 000,00, that can be recaptured from savings generated by the investment within 6 months or 1 year. These SOP forms represent collective intuition derived from experience. They have a secure place in economic evaluations, but their use should be tempered by economic principles and a continuing audit to verify that previous judgments are appropriate for current decisions.

Most decision makers informally set boundaries for routine responses to noncritical problems of a personal and professional nature. Three possible parameters to identify routine responses are shown in Figure.1.2.The level that separates an automatic decision from a problem that requires more investigation among decision makers.

Since there are limits to a decision maker’s time and energy whereas the reservoir of problems often seems infinite, guidelines are necessary to confine involvement. SOPs do save time. An intuitive response is quick. Both draw upon experience to yield a reasonable solution. However, handy answers may mask better solutions than could have been exposed by analysis.

1.4 Tactics and Strategy

Strategy and tactics historically are military terms associated with broad plans from the high command and specific schedules from lower eche lons, respectively.

Strategy sets ultimate objectives, and the associated tactics define the multiple maneuvers required to achieve the objectives. Strategic and tactical considerations have essentially the same meaning for economic studies. There are usually several strategies available to an organization. A strategic decision ideally selects the overall plan that makes the best use of the organization’s resources in accordance with its long-range objectives. A strategic industrial decision could be a choice from several different product designs to develop or products to promote. In government, strategic evaluations could take the form of benefit-cost analyses to select the preferred method of flood control or development of recreational sites. The measure of merit for strategic alternatives is effectiveness-the degree to which a plan meets economic targets.

A strategic plan can normally be implemented in a number of ways. For example, each industrial design or product has tactical alternatives, such as which kind of machine to employ or materials to use; tactics for flood control might involve choices among dams, levees, dredging, etc. The relative values of tactical choices are rated according to their efficiency- the degree to which an operation accomplishes a mission within economic expectations.

The relationship between strategies and tactics offers some construct ive insights. The effectiveness of each strategy is initially estimated from the effect it will have on system objectives. It thus serves as a guide to the area in which tactics will produce the highest efficiency. The actual effic iency of each tactic is determined from a study of the activities required to conduct the tactical operation.An aerospace engine manufacturer is considenng job shop automation. Two strategies, each with three apparent means of accomplishment, are depicted in Figure. 1.3. Strategy 2 is to automate the impeller line, which will automate about 65 percent of the total shop. Strategy 1 is to automate both impeller and gear lines, which will affect about 80 percent of the facility. The company has decided that the effectiveness measure is the percentage of shop automation. The tactics shown in Figure. 1.3 represents approaches to automation. DNC refers to centralized, integrated direct numerical control by computer, CNC is local computer numerical control for each machine, and trace refers to template tracing and duplication for multiple-part setups. The efficiency, or means by which each approach is measured, in this case will be the dollar benefits from installing the new equipment. In this case, we have a situation where the best dollar return is not realized by the most effective strategy. Possibly, the cost of automating the gear line is relatively higher than for the impeller portion of the shop. The select ion of a tactic policy must be evaluated in relation to the strategic objectives and the resources required for implementation.

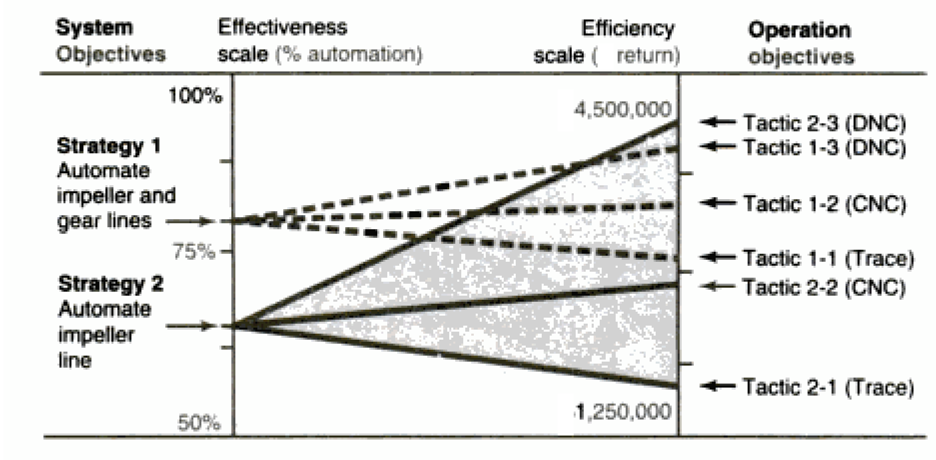


Figure 1.3 Realtionship between Strategy and Tactics

Example 1: Planning a Trip

People use strategy and tactics in very simple ways every day without realizing it. For example, if we are planning a trip from Bangalore to New Delhi, we need a strategy to get there.

Strategy involves answering many questions, such as: How do we plan to travel (airplane, boat, etc.)? What resources do we have? When do we need to get there?

If we decide to travel by airplane, the tactics of purchasing that plane ticket can be to:

- Call a travel agent
- Use the website of a specific airline
- Use a website that compares the rates of different airlines.

Law of supply and demand:

The demand and supply of a product are interdependent and they are sensitive with respect to the price of that product. The interrelationships between them are shown in the figure 1.3

From the figure it is clear that when there is a decrease in the price of a product, the demand for the product increases and its supply decreases. Also, the product is more in demand and hence the demand of the product increases. At the same time, lowering of the price of the product makes the producers restrain from releasing more quantities of the product in the market. Hence, the supply of the product is decreased. The point of intersection of the supply curve and the demand curve is known as the equilibrium point. At the price corresponding to this point, the quantity of supply is equal to the quantity of demand. Hence, this point is called the equilibrium point.

Factors influencing demand

The shape of the demand curve is influenced by the following factors:

- Income of the people
- Prices of related goods
- Tastes of consumers

If the income level of the people increases significantly, then their purchasing power will naturally improve. If, for instance, the price of television sets is lowered drastically its demand would naturally go up. As a result, the demand for its associated product, namely VCDs would also increase. Hence, the prices of related goods influences the demand of a product.

Over a period of time, the preference of the people for a particular product may increase, which in turn, will affect its demand. For instance, diabetic people prefer to have sugar-free products. If the incidence of diabetes rises naturally there will be increased demand for sugar-free products.The demand and supply curve is shown in the figure.1.3

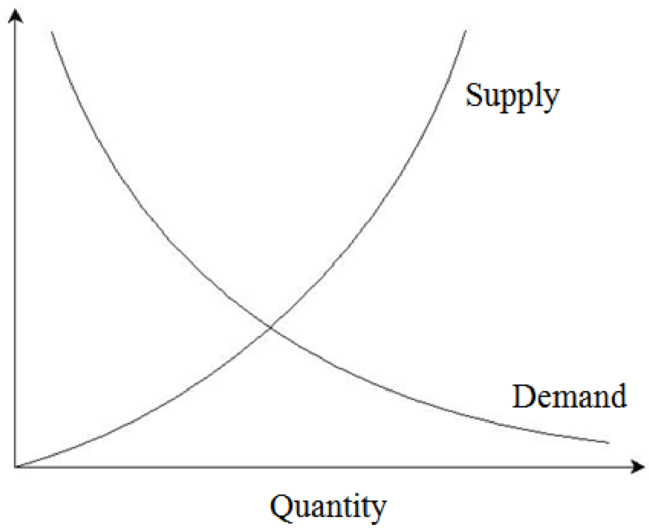


Figure 1.3: Demand and supply curve.

Factors influencing supply

The shape of the supply curve is affected by the following factors:

- Cost of the inputs
- Technology

- Weather
- Prices of related goods

If the cost of inputs increases, then naturally, the cost of the product will go up. In such a situation, at the prevailing price of the product the profit margin per unit will be less. The producers will then reduce the production quantity, which in turn will affect the supply of the product. For instance, if the prices of fertilizers and cost of labour are increased significantly, in agriculture, the profit margin per bag of paddy will be reduced. So, the farmers will reduce the area of cultivation, and hence the quantity of supply of paddy will be reduced at the prevailing prices of the paddy.

If there is advancement in technology used in the manufacture of the product in the long run, there will be a reduction in the production cost per unit. This will enable the manufacturer to have a greater profit margin per unit at the prevailing price of the product. Hence, the producer will be tempted to supply more quantity to the market.

Weather also has a direct bearing on the supply of products. For example, demand for woollen products will increase during winter. This means the prices of woollen goods will be incresed in winter. So, naturally, manufacturers will supply more volume of woollen goods during winter.

Again, take the case of television sets. If the price of TV sets is lowered significantly, then its demand would naturally go up. As a result, the demand for associated products like VCDs would also go up. Over a period of time, this will lead to an increase in the price of VCDs, which would result in more supply of VCDs.

Science is a field of study where the basic principles of different physical systems are formulated and tested. Engineering is the application of science. It establishes varied application systems based on different scientific principles.It is clear that price has a major role in deciding the demand and supply of a product. Hence, from the organization’s point of view, efficient and effective functioning of the organization would certainly help it to provide goods/services at a lower cost which in turn will enable it to fix a lower price for its goods or services.

Module 1 (Part 2)

INTEREST AND INTEREST FACTORS: Interest rate, simple interest, Compound interest, Cash- flow diagrams, Exercises and Discussion.

Interest: A fee paid for the use of another party's money. To the borrower it is the cost of renting money, to the lender the income from lending it.

2.1 Interest rate

An interest rate is the rate at which interest is paid by borrowers for the use of money that they borrow from a lender. Specifically, the interest rate (I/m) is a percent of principal (P) paid a certain amount of times (m) per period (usually quoted per annum). For example, a small company borrows capital from a bank to buy new assets for its business, and in return the lender receives interest at a predetermined interest rate for deferring the use of funds and instead lending it to the borrower. Interest rates are normally expressed as a percentage of the principal for a period of one year.

2.2 Simple interest

The interest is said to simple, when the interest is charged only on the principal amount for the interest period. No interest is charged on the interest amount accrued during the preceding interest periods. In case of simple interest, the total amount of interest accumulated for a given interest period is simply a product of the principal amount, the rate of interest and the number of interest periods. It is given by the following expression.

$I_T = P \times n \times i$

Where

I_T = total amount of interest

P = Principal amount
n = number of interest periods i = rate of interest

Simple interest reflects the effect of time value of money only on the principal amount.

2.3 Compound interest:

The interest is said to be compound, when the interest for any interest period is charged on principal amount plus the interest amount accrued in all the previous interest periods. Compound interest takes into account the effect of time value of money on both principal as well as on the accrued interest also.

If an investor invests a sum of Rs. 100 in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of every year will be as shown in the following table 2.1

	Compound Amounts (amount of deposit = Rs. 100.00)	
Year end	Interest	Compound amount
	(Rs.)	(Rs.)
0		100.00
1	15.00	115.00
2	17.25	132.25
3	19.84	152.09
4	22.81	174.90
5	26.24	201.14

Table 2.1 Time value of money

The formula to find the future worth in the third column is

$$F = P\,(1 + i)^n$$

where

P = principal amount invested at time 0, F = future amount,

i = interest rate compounded annually, n = period of deposit.

The maturity value at the end of the fifth year is Rs. 201.14. This means that the amount Rs. 201.14 at the end of the fifth year is equivalent to Rs. 100.00 at time 0 (i.e. at present). This is diagrammatically shown in the following figure 2.1. This explanation assumes that the inflation is at zero percentage.

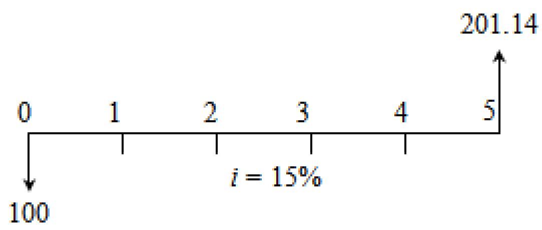


Figure 2.1 Time value of money

Alternatively, the above concept may be discussed as follows: If we want Rs. 100.00 at the end of the *n*th year, what is the amount that we should deposit now at a given interest rate, say 15%? A detailed working is shown in the following table 2.2

Table 2.2 Present worth Amounts (rate of interest = 15%)

End of year	Present worth	Compound amount
(n)		after n year(s)
0		100
1	86.96	100
2	75.61	100
3	65.75	100
4	57.18	100
5	49.72	100
6	43.29	100
7	37.59	100
8	32.69	100
9	28.43	100
10	24.72	100

The formula to find the present worth in the second column is $P = F(1 + i)^n$

From the table 2.2, it is clear that if we want Rs. 100 at the end of the fifth year, we should now deposit an amount of Rs. 49.72. Similarly, if we want Rs. 100.00 at the end of the 10th year, we should now deposit an amount of Rs. 24.72.Also, this concept can be stated as ,A person has received a prize from a finance company during the recent festival contest. But the prize will be given in either of the following two modes.

1. Spot payment of Rs. 24.72 or
2. Rs. 100 after 10 years from now (this is based on 15% interest rate compounded annually).

If the prize winner has no better choice that can yield more than 15% interest rate compounded annually, and if 15% compounded annually is the common interest rate paid in all the finance companies, then it makes no difference whether he receives Rs. 24.72 now or Rs. 100 after 10 years.

On the other hand, let us assume that the prize winner has his own business wherein he can get a yield of 24% interest rate (more than 15%) compounded annually, it is better for him to receive the prize money of Rs. 24.72 at present and utilize it in his business. If this option is followed, the equivalent amount for Rs. 24.72 at the end of the 10th year is Rs. 212.45. This example clearly demonstrates the time value of money.

2.4 Cash flow Diagrams:

The purpose of cash flow diagrams is to show the *net* amount of money received or spent during various time periods for a project. The time periods are typically given in months or years, depending on the scope of the project.

Cash flow diagrams can be drawn from the perspective of the borrower or the lender. The two perspectives make the diagram flip upside down, since a gain for the borrower is a loss for the lender, and vice versa. For each time period, calculate the net amount of money gained or lost during each time period.

Net cash flow = gains - losses = income - expenses

Gains or income include anything that gives the borrower money, such as taking out a loan, receiving interest from an investment, receiving benefits from an operational system, etc.

Losses or expenses include any money leaving the borrower, such as loan payments, buying equipment for cash, etc.

The cash flow diagram is a set of orthogonal arrows. The X axis is a timeline arrow pointing to the right, labeled with the number of each time period (0, 1, 2, etc.). Each time period is evaluated at the end of that time period, so pretend that all transactions in time period 2 are accounted for at the end of time period 2. The end of Period 0 is the present time.A cash flow diagram is shown in the figure 2.2

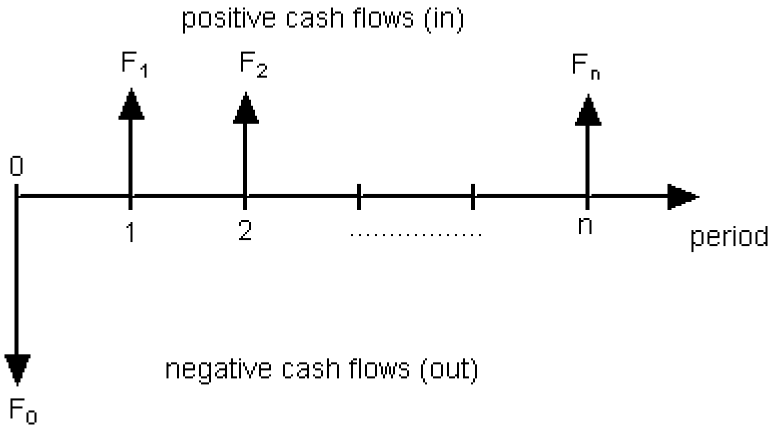


Figure 2.2: Cash flow diagram

Interest Formulas

While making investment decisions, computations will be done in many ways. To simplify all these computations, it is extremely important to know how to use interest formulas more effectively. Before discussing the effective application of the interest formulas for investment-decision making, the various interest formulas are presented first.Interest rate can be classified into simple interest rate and compound interest rate.

In simple interest, the interest is calculated, based on the initial deposit for every interest period. In this case, calculation of interest on interest is not applicable. In compound interest, the interest for the current period is computed based on the amount (principal plus interest up to the end of the previous period) at the beginning of the current period.

The notations which are used in various interest formulae are as follows:

P = principal amount

n = No. of interest periods

i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F = future amount at the end of year n

A = equal amount deposited at the end of every interest period

G = uniform amount which will be added/subtracted period after period to/ from the amount of deposit A1 at the end of period 1

Single-Payment Compound Amount

The objective is to find the single future sum (F) of the initial payment (P) made at time 0 after n periods at an interest rate i compounded every period. The cash flow diagram of this situation is shown in Figure 2.3

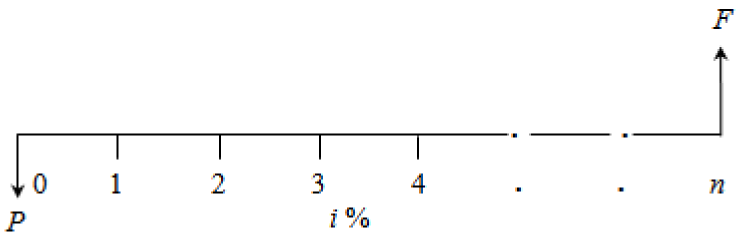


Figure 2.3 Cash flow diagram for Single-Payment Compound Amount

The formula to obtain the single-payment compound amount is

$$P(1+i)^n$$
$$F = P(F/P, i, n)$$

Where $(F/P, i, n)$ is called as single-payment compound amount factor.

Problem 1*:*A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution

P = Rs. 20,000

i = 18% compounded annually n = 10 years

$= P(1 + i)^n = P(F/P, i, n)$

=20,000 $(F/P$, 18%, 10)

=20,000 x5.234 = Rs. 1, 04,680

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1, 04,680 after 10 years.

Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount (*P*) of a single future sum (*F*) which will be received after n periods at an interest rate of i compounded at the end of every interest period.The corresponding cash flow diagram is shown in the following Figure 2.4

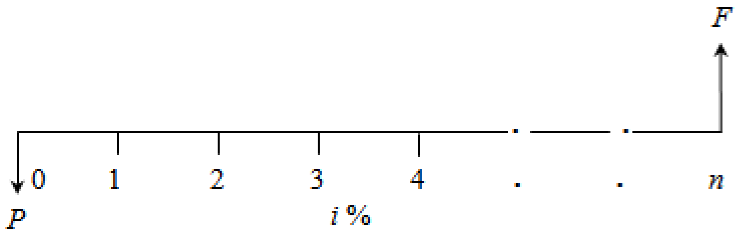


Figure 2.4 Cash flow diagram of single-payment present worth amount

The formula to obtain the present worth is

$$P = \frac{F}{(1+i)^n} = F(P/F\,, i, n)$$

Problem 2: A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution

F = Rs. 1,00,000

i = 15%, compounded annually n = 10 years

$P = F/(1 + i)^n = F(P/F, i, n)$

= 1,00,000 (P/F, 15%, 10)

= 1,00,000 ´x 0.2472

= Rs. 24,720

Q The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the nth interest period at an interest rate of i compounded at the end of each interest period. The corresponding cash flow diagram is shown in Figure 2.5

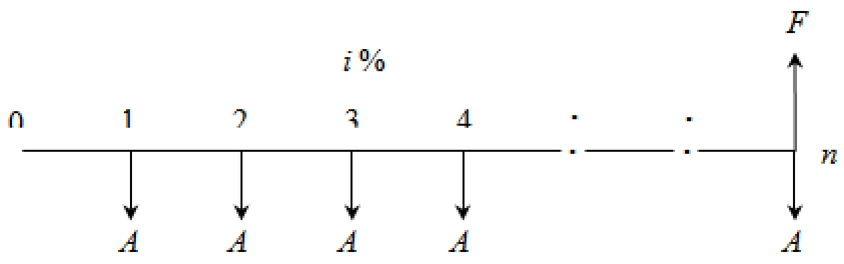


Figure 2.5 Cash flow diagram of equal-payment series compound amount.

In the above Figure

A = equal amount deposited at the end of each interest period

n= No. of interest periods i = rate of interest

F = single future amount

The formula to get F is

$$F = A\frac{(1+i)^n - 1}{i} = A(F/A, i\%, n)$$

Where (F/A, i, n) is termed as equal-payment series compound amount factor.

Problem 3: A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution

A = Rs. 10,000

n = 25 years

i = 20%

F = ?

The corresponding cash flow diagram is shown in the figure 2.6

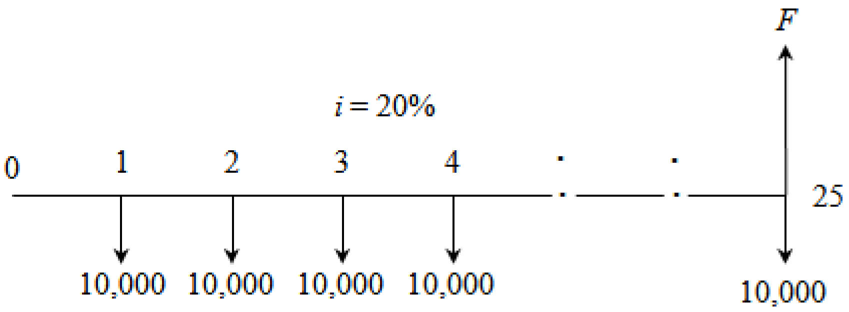


Figure 2.6 Cash flow diagram of equal-payment series compound amount.

$$F = A\frac{(1+i)^n - 1}{i}$$

=A (F/A, i, n)

=10,000(F/A, 20%, 25)

=10,000 x 471.981

= Rs. 47, 19,810

The future sum of the annual equal payments after 25 years is equal to Rs. 47,19,810.

Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i.

The corresponding cash flow diagram is shown in the figure 2.7

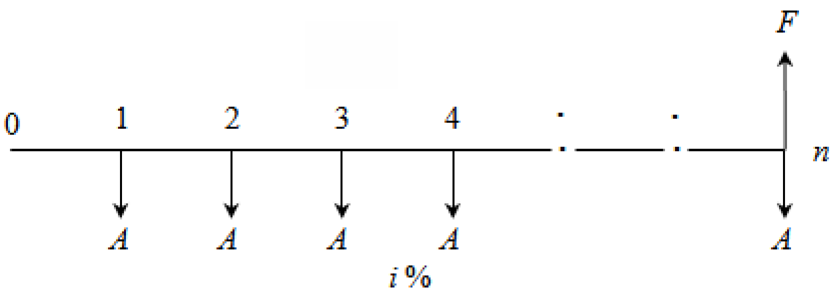


Figure 2.7 Cash flow diagram of equal-payment series sinking fund.

Where (A/F, i, n) is called as equal-payment series sinking fund factor.

Problem 4: A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

Solution

F = Rs. 5,00,000 *n* = 15 years

i = 18% *A* = ?

The corresponding cash flow diagram is shown in Figure 2.8.

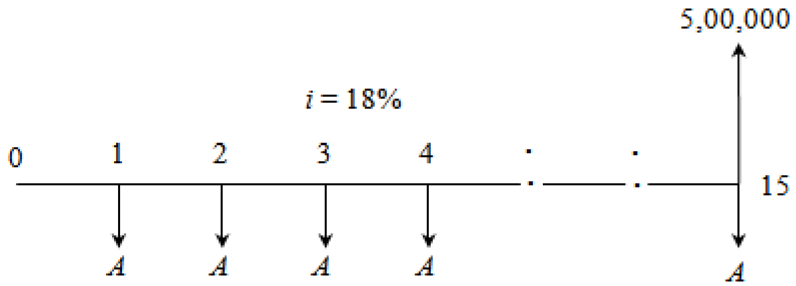


Figure 2.8. Cash flow diagram of equal-payment series sinking fund.

$$A = F\frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

=500000(A/F, 18%, 15)

=5,00,000 x 0.0164

=Rs 8200

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal payment made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Figure 2.9

Here,
P= Present worth
A=Annual equavalent worth
n= Duration
i= Interest rate
The formula to compute P is

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

=A (P/A I, n)

where (P/A, i, n) is called equal-payment series present worth factor

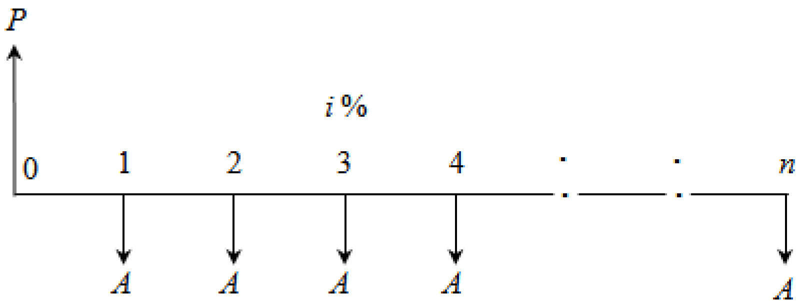


Figure 2.9 Cash flow diagram of equal-payment series present worth amount

Problem 5: A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

Solution

A = Rs. 10,00,000 i = 15%

n = 20 years P = ?

The corresponding cash flow diagram is illustrated in Figure. 2.10

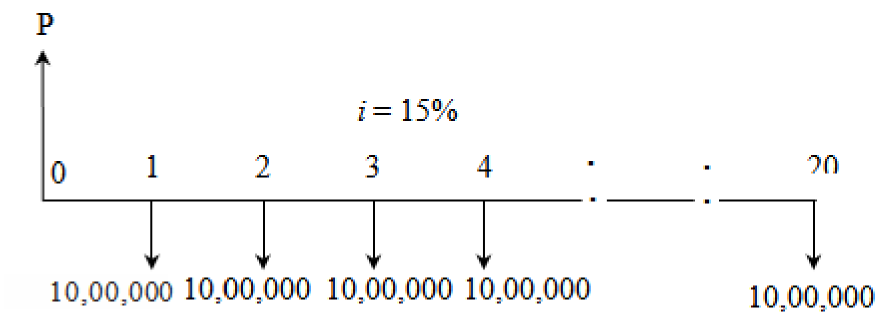


Figure 2.10: Cash flow diagram of equal-payment series present worth amount

$$\frac{(1+i)^n - 1}{i(1+i)^n}$$

=A(P/A,i%,n)

= 10,00,000 (*P*/*A*, 15%, 20)

= 10,00,000 x 6.2593

= Rs. 62,59,300

The amount of reserve which must be set-up now is equal to Rs. 62, 59,300.

Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period. Cash flow diagram of equal-payment series capital recovery amount is shown in figure

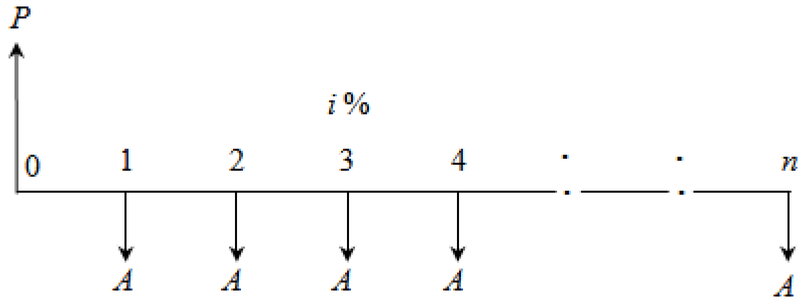


Figure 2.11. Cash flow diagram of equal-payment series capital recovery amount.

In the above figure,

P= present worth (loan amount)
A=Annual equvallent worth
i= Interest rate
n= no. of interest periods
The formula to compute P is as follows

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}$$

=P (A/P, i, n)

Where, (A/P, i, n) is called equal-payment series capital recovery factor.

Problem 6: A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

Solution

P = Rs. 10, 00,000

i = 18%

n = 15 years

A =?

The corresponding cash flow diagram is shown in the figure 2.12

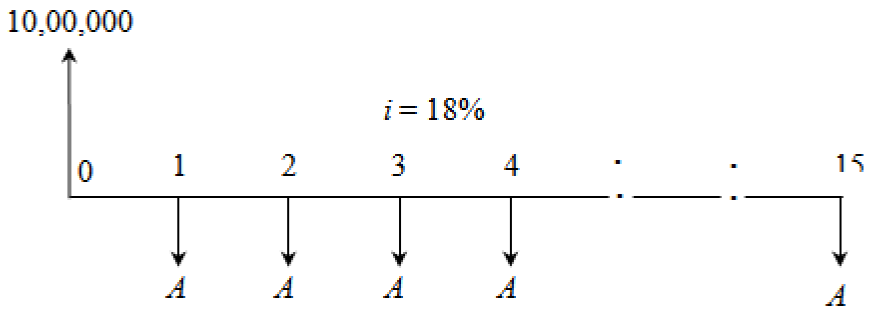


Figure 2.12 cash flow diagram

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}$$

=P (A/P, i, n)

= 10, 00,000 (A/P, 18%, 15)

= 10, 00,000 (0.1964)

= Rs. 1, 96,400

The annual equivalent installment to be paid by the company to the bank is Rs. 1, 96,400.

Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n - 1$ year with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in the figure 2.13

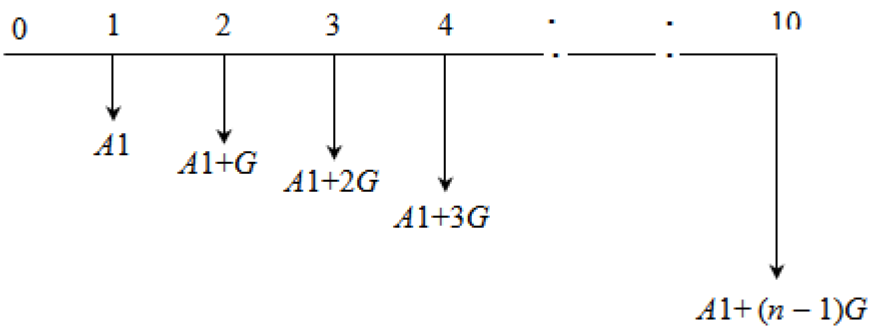


Figure 2.13 Cash flow diagram of uniform gradient series annual equivalent amount.

The formula to compute A under this situation is

$$A = A1 + G\,\frac{(1+i)^n - 1}{i(1+i)^n - i}$$

=A1+G(A/G, I ,n)

where(A/G, i, n) is called uniform gradient series factor

Problem 7: A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10 th year of the above series.

Solution

A1= Rs 4000

G=Rs 500

i= 15%

n= 10 years

A= ?

F= ??

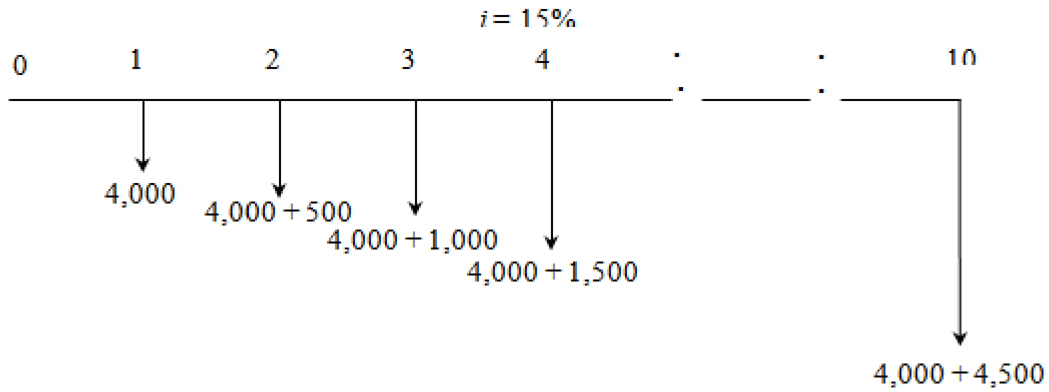


Figure 2.14 cash flow diagrams

$$A = A1 + G\,\frac{(1+i)^n - 1}{i(1+i)^n - 1}$$

$$A1 + G(A/G, i, n)$$

=4,000 + 500(*A*/*G*, 15%, 10)

=4,000 + 500 X 3.3832

=Rs. 5,691.60

This is equivalent to paying an equivalent amount of Rs. 5,691.60 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th year is obtained as follows:

F= A (F/A, i, n)

=A (F/A, 15%, 10)

=5,691.60(20.304)

Rs. 1, 15,562.25

At the end of the 10th year, the compound amount of all his payments will be

Rs.1, 15,562.25.

Problem 8*:* A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 8,500 at the end of the first year and thereafter he wishes to deposit the amount with an annual decrease of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

Solution:

A1=Rs 8500

G=-Rs 500

I=15%

N=10 Years

A=? F=?

The cash flow diagram is shown in the figure 2.15

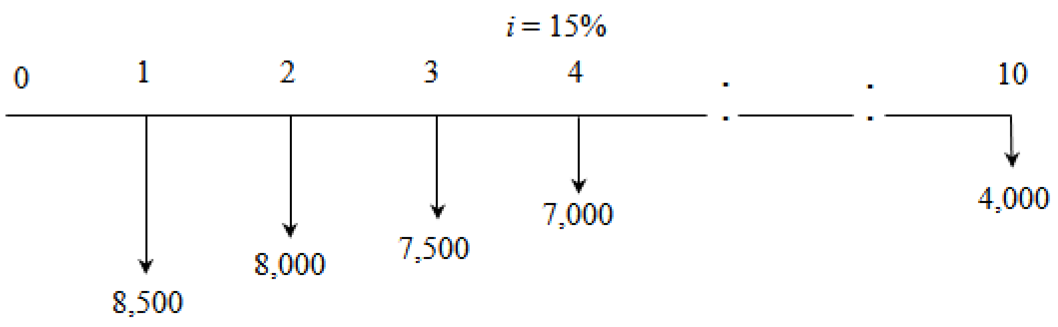


Figure 2.15 Cash flow diagram

$$A = A1 + G\,\frac{(1+i)^n - 1}{i(1+i)^n - 1}$$

$$A = A1 + G(A/G, i, n)$$

A=8500-500(A/G, i, n)

= 8500-500(A/G,15%, 10)

= 8500-500(3.3832)

Rs 6808.4

This is equivalent to paying an equivalent amount of Rs. 6,808.40 at the end of every year for the next 10 years.

The future worth sum of this revised series at the end of the 10th year is obtained as follows:

F= A (F/A, i, n)

=A (F/A, 15%, 10)

=6,808.40(20.304)

=Rs. 1, 38,237.75

At the end of the 10th year, the compound amount of all his payments is Rs. 1, 38,237.75.

Effective Interest Rate

Let *i* be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year. For example, compounding may be monthly, quarterly, or semi-annually. Compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in a year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is

Problem 6. A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

Solution

P = Rs. 5,000
n = 10 years
i = 12% (Nominal interest rate)
F = ?

Method 1

No. of interest periods per year = 4

No. of interest periods in 10 years = 10 x4 = 40

Revised No. of periods (No. of quarters), N = 40

Interest rate per quarter, r = 12%/4

= 3%, compounded quarterly.

F= $P(1 + r)^N = 5{,}000(1 + 0.03)^{40}$

= Rs. 16,310.19

Method 2

No. of interest periods per year, C = 4

Effective interest rate, $R = (1 + i/C)^C - 1$

$= (1 + 12\%/4)^4 - 1$

=12.55%, compounded annually.

F= $P(1 + R)^n$

$= 5{,}000(1 + 0.1255)^{10}$ = Rs. 16,308.91

Engineering Economics

- <u>7 principles of EE</u>

1. Among alternatives

2. Differences in future outcomes

3.Outcomes of feasible alternatives

4.Common unit of measurement

5.Selection preferred alternative

6.Uncertaininity

7.Improvement/revisit the decision

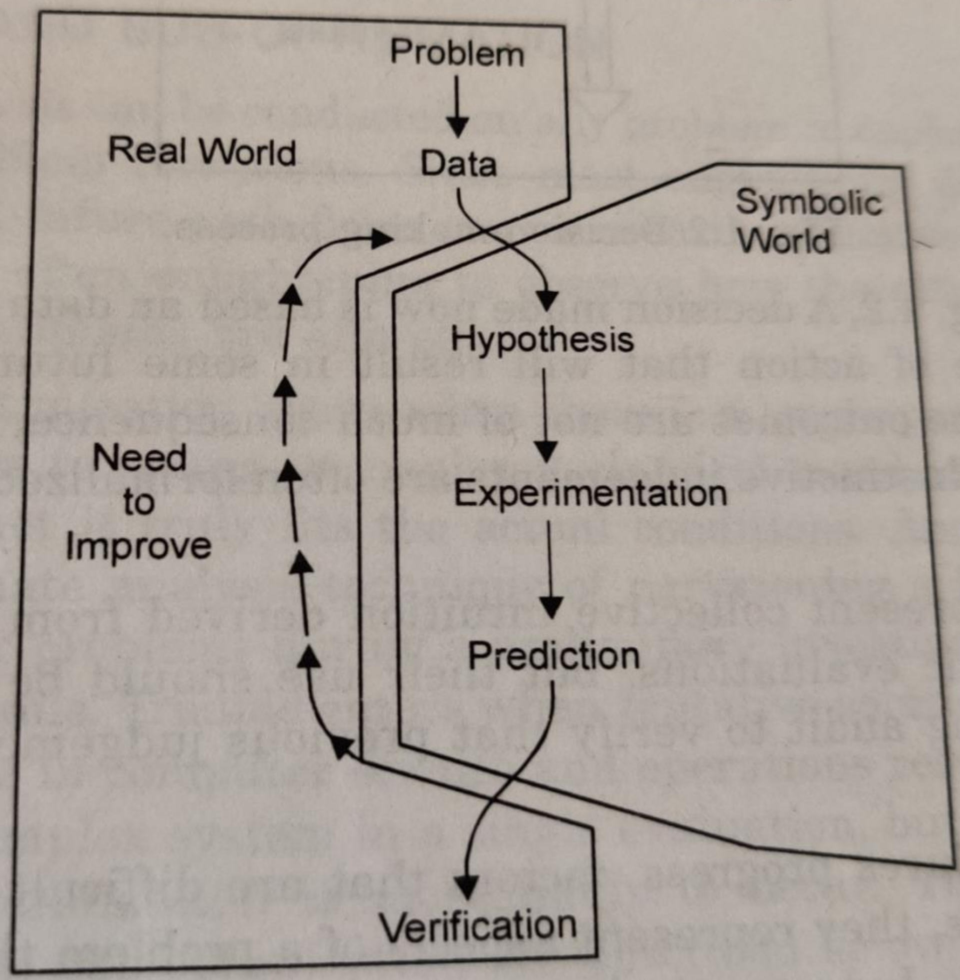


Fig. 1.1 Problem solving process.

- Interest and interest factors

Interest rate, simple / compound interest factors, Cash flow diagrams, Numerical excercises.

Interest – is a rent paid for money borrowed.

Two types

1. Simple interest
2. Compound interest ---

a)Nominal interest rates

b)Effective interest rates

c)Continuous compounding

Simple Interest (SI)

$$I = P \times N \times i$$

I = Interest earned for particular time periods

P = Principal amount lent / borrowed.

N = No. of interest periods (Eg. years/months)

i = Interest rate per interest period.

If P is a fixed value, annual interest charged is constant.

$$F = P + I$$

$$F = P + PiN$$

$$\boxed{F = P(1 + iN)}$$

$F \rightarrow$ Future sum of money to be paid.

Effective Interest rate (i_{eff})

$$i_{eff} = \left(1 + \frac{r}{N}\right)^N - 1$$

Compound Amount, $\boxed{F = P(1 + i)^N}$

Problems

1. The rental cost of money is a loan of Rs. 1000 for 2months @ 10%. Use simple interest.

Solution : P=1000

i=10%

N=2/12 years

F=P(1+i.N)

=1000(1+0.1*0.167)

=1016.67

F=1000(1+0.1(31+28/365))=1016.16

2.What sum must be loaned at 8% simple interest to earn rs.350 in 4years.

When N is a full year

I = P*i*N

350=P*0.08*4

P=350/(0.08*4)

P= 1093.75

i=8%

N=4

I=350

P=?

3.How long will it take rs.800 to yield rs.72 in simple interest at 4%.

N=?

P=800

I=72

i=4%

I=P*i*N

72=800*0.04*N

N=2.25years

4.At what rate will 65.07 yield Rs. 8.75 in simple interest in 3 years 6 months?

i=?

P=65.07

I=8.75

N=3.6years

I=P*i*N

8.75=65.07*i*3.5

i=3.8%

5.How long will it take any sum to triple itself at 5% simple interest rate? Asssume P=100

Answer : i=5%, I=300,N=60yrs

6. Determine the effective interest rate for a nominal annual rate of 6% that is compounded

i)Semi-annually – n=2

ii)Quarterly n=4

iii)Monthly n=12

iv)Daily n=365

Ans: r=6%=0.06

1) $i=(1+0.06/2)^2 -1$

i=6.09%

ii) i=6.13%

iii)i=6.16%

iv)6.18%

7. A personal loan of Rs.1000 is made for a period of 18 months at an interest rate of 1 ½ percent per month on the unpaid balance. If the entire amount owed is repaid in a lump sum at the end of that time, determine the effective annual interest rate.

Ans: P=1000

N=18 months

i= 1 ½ * 12=18%

i=(1+0.18/18)^18 - 1

i=19.61%

8. Find the compound amount of Rs.100 for 4yrs at 6% compounded annually.

Ans: P=100

N=4 , i=6%

F=P(1+i)^n

=100(1+0.06)^4

F=126.24

9. A loan of Rs.2000 , interest rate is 10% per year. If interest had not been paid each year but, had been allowed to compound, how much interest would be due to the lender as a lump sum at the end of 6yrs?

Ans: P=2000

i=10% , N=6

$F=2000(1+0.1)^6$

$F=3543.12$

Interest = F – P =3543.12-2000=1543.12

10. Accumulate a principle of Rs.1000 for

5yrs 9months at a nominal rate of 12% compounded monthly. How much interest is earned?

Ans: P=1000 , N=69months , i=12%=1% per month

$F=1000(1+0.01)^{69}$

$=1986.89$

Notation and Cash flow Diagrams

A cash flow is the difference between

total cash inflows(receipts)and

cash outflow(expenditures) for a specified period of time.

Interest formulas for Discrete Compounding and Cash flows

There are 6 most common discrete compound interest factors,

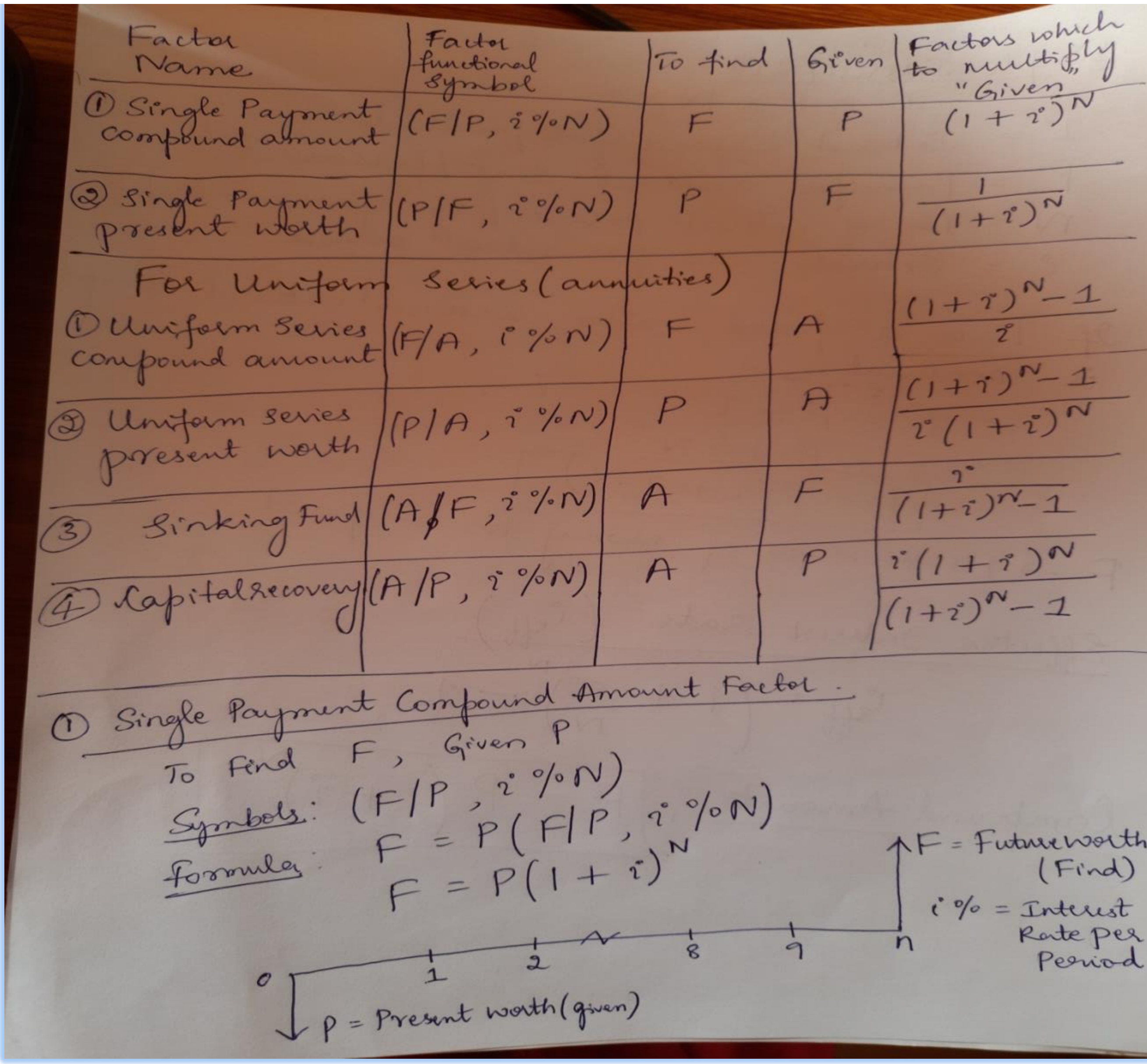
For Single Cash flows

1. Single payment compound amount P, F?

2. Single payment present worth F , P?

For Uniform series

1. Uniform series compound amount
2. Uniform series present worth
3. Equal payment series sinking fund
4. Equal payment series annual equivalent amount
5. Arithmetic gradient conversion factor (to uniform series).



Factor Name	Factor functional symbol	To find	Given	Factors which to multiply "Given"
① Single Payment compound amount	(F/P, i%N)	F	P	$(1+i)^N$
② Single Payment present worth	(P/F, i%N)	P	F	$\frac{1}{(1+i)^N}$
For Uniform Series (annuities)				
① Uniform Series compound amount	(F/A, i%N)	F	A	$\frac{(1+i)^N-1}{i}$
② Uniform series present worth	(P/A, i%N)	P	A	$\frac{(1+i)^N-1}{i(1+i)^N}$
③ Sinking Fund	(A/F, i%N)	A	F	$\frac{i}{(1+i)^N-1}$
④ Capital recovery	(A/P, i%N)	A	P	$\frac{i(1+i)^N}{(1+i)^N-1}$

① Single Payment Compound Amount Factor.

To Find F, Given P

Symbols: (F/P, i%N)

Formula: $F = P(F/P, i\%N)$

$F = P(1+i)^N$

1. <u>single payment compound amount</u>

   To find F , given P

   1. A person deposits a sum of rs.20000 at the interest rate of 18% compounded annually for 10yrs. Find the maturity value after 10yrs?

   P=20000

   i=18%

   F=P(F/P,i%N)

F=P(1+i)^n
=20000(1+0.18)^10
F=1,04,680

2. A person deposits a sum of rs.10,000 in a bank at a nominal rate of interest of 12% for 10yrs.Find the maturity amount of the deposit after 10yrs. if the compounding is done quarterly.

P=10,000
i=12%(compounded quarterly)
N=10
F=?

Method-1

No. of interest periods per year=4
No. of interest periods for 10yrs=10*4=40
Revised n=40
Rate of interest =12/4=3% =0.03

F=10,000(1+0.03)^40
F=32620.37

Method-2

R=(1+i/N)^n – 1
=(1+0.12/4) ^4 – 1
R=12.55%
Hence R replaces 'i' in the formula,
F=P(1+i)^n
=10000(1+0.1255)^10

F=32,620

3.Suppose that 10,000 is borrowed now at 15% interest per annum. A partial repayment of 3000 is made four yrs from now. The amount that will be remain to be paid then is most nearly (a) 7000 (b)8050 (c)8500 (d)13000 (e)14490.

P=10,000

i=15%

N=4

F =P(1+i)^n

=10,000(1+0.15)^4

=17490

The amount to be paid is

=17490-3000=14490

2.Single Payment present worth

To find P given F

1. An investor has an option to purchase a land that will be worth Rs.10,000 in 6yrs. If the value of land increases at 8% each year,how much should the investor be willing to pay now for this property.

F=10,000

N=6yrs

i=8%

P=?

P=F(P/F,i%N)

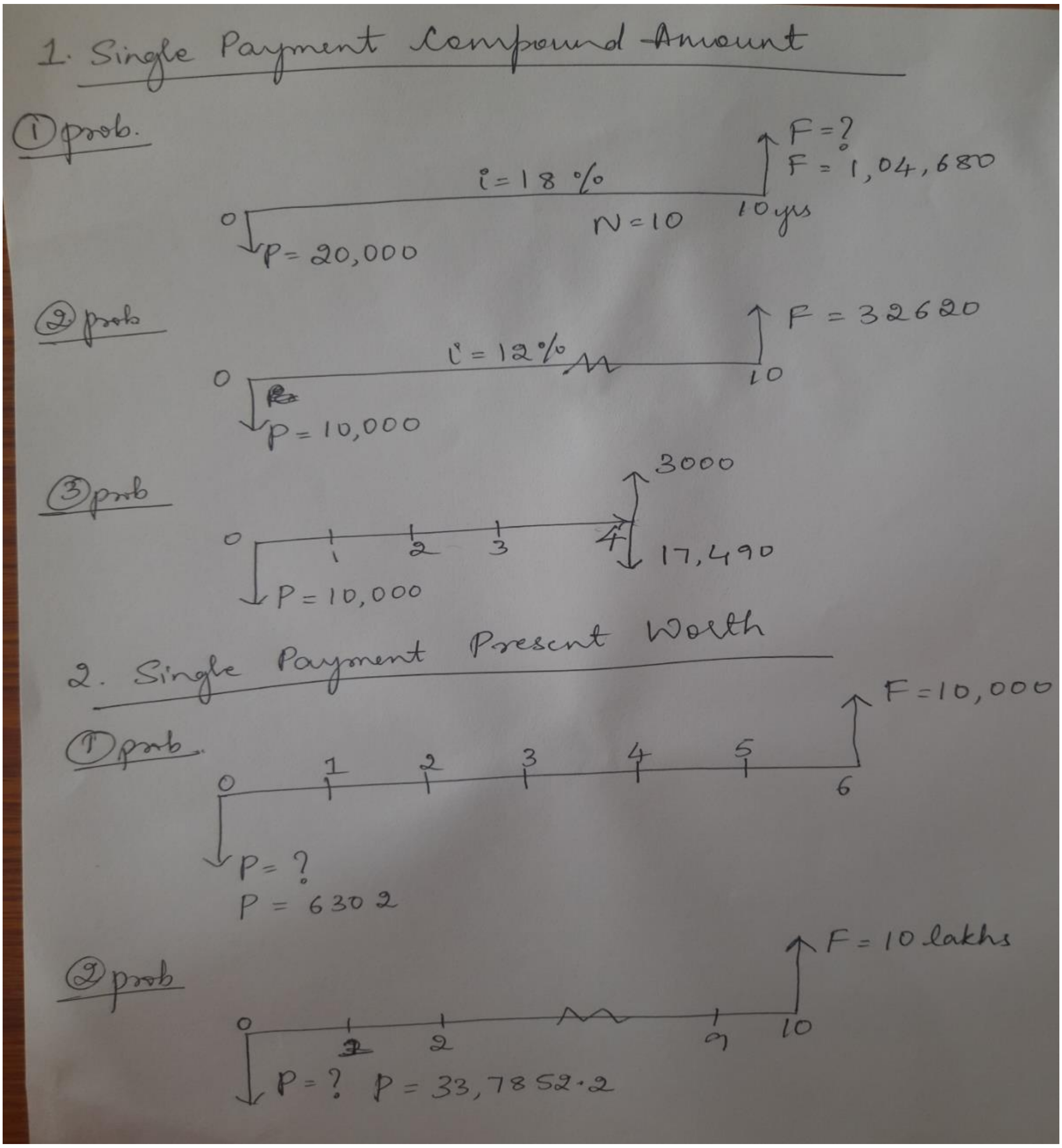
P=10,000(P/F,8%,6)

P=10,000/(1+0.08)^6

P=6302

2. If a person wishes to invest in a private bank that pays rate of interest 11% but compounded quarterly. So that he gets 10,00,000 for 10yrs from now.
   F=10 lakhs
   i=11%
   No. of interest period per year =4
   Total no. of interest periods for 10yrs=4*10=40
   Interest quarterly=0.11/4
   P=F(P/F,0.11/4,40)
   P=10,00,000/(1+0.11/4)^40
   P=3,37,852.22

Cash flow diagrams for the problems solved on single payment compound amount and present worth.



Problems to be solved

1. A future amount F , is equivalent to Rs. 1500. Now when eight years separates the amounts and the annual interest is 12%. What is the value of F? **Ans** : **3713.9**
2. Suppose you borrow Rs.8000 now, with the promise to repay the loan principal plus accumulated interest in 4years at i=10% per year. How much would you owe at the end of 4years? **Ans: 11,713**
3. A person wishes to have a future sum of Rs.10 lakhs for his daughter's engineering education in 15 years from now, what is the single payment

that he should deposit now. So, that he gets the desired amount after 10years? The banks gives 12% rate of interest compounded annually. **Ans: 3,21,973.3**

4. A person wishes to have a future sum of Rs.1,00,000 for his son's marriage after 10years from now. What is the single payment that he should deposit now. So, that he gets the desired amount after 10years? The banks gives 15% interest rate compounded annually. **Ans: 24,720**

Probem 1 .

A person invests a sum of Rs.50,000 in a bank at a nominal interest rate of 18% for 15yrs. The compounding is monthly. Find the maturity amount of the deposit after 15yrs.

P=50,000

i=18% = 18/12=1.5% (compounding monthly)

N=15 * 12=180

F=?

F=P(1+i)^n

F=50000(1+1.5/100) ^180

F=7,29,218.4

(3) Series Compound-Amount Factor (Uniform Series)

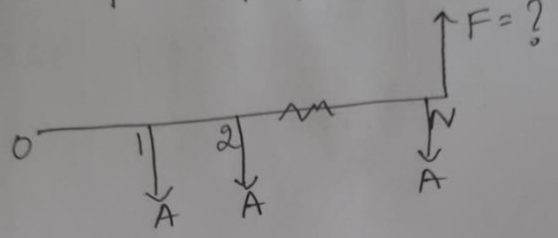
To find F, given A

Symbols: $(F/A, i\% N)$

Formula:

$$F = A\,\frac{(1+i)^N - 1}{i}$$

$$F = A(F/A, i\% N)$$



(4) Sinking Fund Factor (Uniform Series)

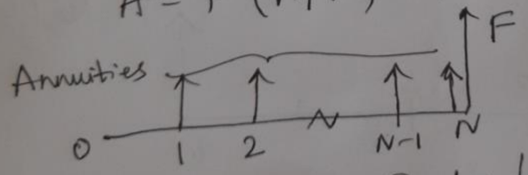
To find A, given F.

Symbols: $(A/F, i\% N)$

formula:

$$A = F\left(\frac{i}{(1+i)^N - 1}\right)$$

$$A = F(A/F, i\% N)$$



(5) Series Present Worth Factor (Uniform Series)

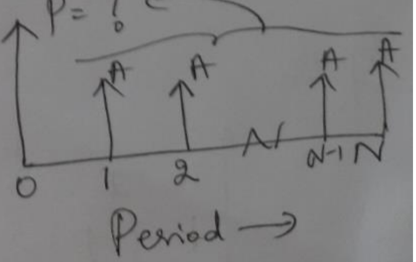
To Find P, given A

Symbols: $(P/A, i\% N)$

formula:

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$P = A(P/A, i\% N)$$



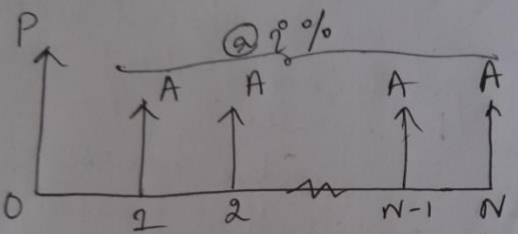
⑥ Capital Recovery Factor (Uniform Series)

To Find A, given P

Symbols: $(A/P, i\% N)$

Formula: $$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$$A = P(A/P, i\% N)$$



End of Time Period

⑦ Arithmetic Gradient Conversion Factor (Uniform Series)

To find A, Given G

Symbols: $(A/G, i\% N)$

Formula: $$A = A_1 + G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

$$A = A_1 + G(A/G, i, N)$$

G = Gradient Increase in the cash flow receipts/disbursements

3. Series Compound amount factor (uniform series)

To find F, given A

1. A 45 year old person is planning for his retired life. He plans to divert Rs. 30,000 from his bonus as investment every year for the next 15yrs. The banks gives 10%

interest rate compounded annually. Find the maturity value of his account when he is 60yrs old.

A=30,000 , N=15 , i=10% , F=?

F=A(F/A,i%N)

F=30,000[(1+0.1)^15 – 1] / 0.1

F=9,53,174

2. An automobile company recently advertised its car for a down payment of Rs.1,50,000. Alternatively, the car can be taken home by customers without making any payment,but they have to pay an equal yearly amount of Rs.25,000 for 15yrs at an interest rate of 18% compounded annually.

Ans: Car value=P=1,50,000

A=25,000 , N=15 , i=18%

F=A(F/A,i%,N)

F=25000[(1+0.18)^15 – 1]/0.18

F=15,24,131.6

4. Sinking fund factor (Uniform series)

1. A person estimates an expenditure of Rs. 5 lakh for his daughter's wedding about 10yrs from now. He plans to deposit an equal amount at the end of every year for the next 10yrs at the rate of interest of 10% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 10yrs.

F=5,00,000

N=10

A=? i=10%

A=500000(A/F,i%,N)

A=500000 * 0.1 / [(1+0.1)^10 – 1]

A=31,372.7 is the annual amount to be paid.

2.A company has to replace a present facility after 15yrs at an outlay of Rs.5 L . If he plans to deposit an equal amount at the end of every year for the next 15yrs at the rate of interest of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15yrs.

F=5,00,000 , N=15 , i=18% , A=?

A=F(i/(1+i)^n – 1)

A=500000 (A/F,18%,15)

A=500000 * 0.0164

A=8200

5. Series Present Worth factor (Uniform series)

1. A company wants to set up an reserve which will help the company to have an annual equivalent amount of Rs.10,00,000 for the next 20yrs towards its employees welfare measures.The reserve is assumed to grow at the rate of 15% annually. Find the single payment that must be made now as the reserve amount.

   A=10,00,000 , N=20 , i=15% , P=?

   P=A(P/A,i%,N)

   P=[1000000(1+0.15)^20 – 1] / 0.15(1+0.15)^20

P=62,59,300

2. Suppose that installation of low-loss thermal windows in your area is expected to save Rs.150 a year on your home heating bill for next 18yrs. If you can earn 8% a year on other investments, how much could you afford to spend now for these windows?

   A=150 , N=18 , i=8% , P=?

   P=A(P/A,8%,18)

   P=150 * (1+0.08)^18 – 1 / 0.08(1+0.08)^18

   P=1405.8

6. Capital Recovery Factor (Uniform series)

   1. A proposed product modification to avoid production difficulties will require an immediate expenditure of Rs.14,000 to modify certain dies. What annual savings must be realised to recover this expenditure in 4yrs with interest at 10% ?

      i=10% , N=4 , P=14000

$$A = P * \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$A=14000 * \frac{0.1(1+0.1)^4}{(1+0.1)^4 - 1}$$

A=4416.6

2. ICICI bank is offering Rs. 30 lakhs home loan to a person to buy a new apartment at a interest rate of 7.5% compounded annually. This amount should be repaid in 15yrs equal instalments. Find the annual instalment amount the person has to pay to the bank.
   P=30,00,000 N=15 i=7.5% A=?

$$A= P * \frac{i(1+i)^n}{(1+i)^n-1}$$

$$A=3000000 * \frac{0.075(1+0.075)^{15}}{(1+0.075)^{15}-1}$$

A=3,39,861 per annum

If the bank decides to compound the rate of interest **monthly rather than annually**,how much less money the person has to pay annually?

No. of interest periods per year = N=12

Total no. of interest periods n=12*15=180

Annual rate of interest=7.5%

Rate of interest per month = 0.075/12=0.00625

$$A=3000000 * \frac{0.00625(1+0.00625)^{180}}{(1+0.00625)^{180}-1}$$

A=27,810

He has to pay 3,39,861 – (27810*12)

=6141 less/year.

3. A company 3yrs ago borrowed 40,000 to pay for a new machine tool, agreeing to repay the loan in 100 monthly payments at an annual nominal interest rate of 12% compounded monthly. The company now wants to pay off the loan. How much would this payment be, assuming no penalty costs for early payout?

   P=40,000 i=12% =12/12=1% N=100months A=?

$$A = P * \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$A = 40000 * \frac{0.01(1+0.01)^{100}}{(1+0.01)^{100} - 1}$$

A=634.6

F=P(F/P,1%,36)

=P(1+i)^n

=40000 (1+0.01)^36

F=57230.75

The worth of 40000 for 3yrs is 57,230.75 and installement payed for 3yrs = 634.6*36=22845.6

The pay off amount = 57320.75 – 22845.6 = 34385.2

7.Arithmetic Gradient Conversion Factor (Uniform series)

To find annual equivalent amount of a series with an amount A1 @ end of first year and with an equal increment (G) at the end of (n-1) yrs with i% interest.

1. Assume that an endowment was originally set up to provide a Rs. 10000 as first payment with payments decreasing by 1000 each year during the 10 year endowment life. What constant annual payment for 10yrs would be equivalent to the original endowment plan if i=8% ?

   A1=10,000 , i=8% , G=1000 , N=10

$$A = A1 + G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$A = 10000 - 1000\left[\frac{1}{0.08} - \frac{10}{(1+0.08)^{10} - 1}\right]$$

   A=10000 – 3871.31
   A=6128.69

2. A person is planning for his retired life. He has 10 more yrs of service. He would like to deposit 20% of his salary, which is Rs.4000 at the end of 1^{st} year and thereafter he wishes to deposit the amount with an annual increase of Rs.500 for the next 9yrs with an interest rate of 15%. Find the total amount at the end of the 10^{th} year of above series.

   A1=4000 , G=500 , i=15% , N=10 , A=? , F=?

$$A = A1 + G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$A = 4000 + 500\left[\frac{1}{0.15} - \frac{10}{(1+0.15)^{10} - 1}\right]$$

   A=4000+500(3.3832)
   A=5,691.60
   F=A(F/A,15%,10)

F=5691.60 * 20.304

F=1,15,562.25

3. A film star is at the height of his carrier. He wants to invest Rs. 10 lakhs from the end of this year and follow it up with 9 lakhs, 8 lakhs and so on for the next five yrs, when his income would go on diminishing. Find the maturity amount 6 yrs later if a film producer agrees to pay him 15% rate of interest,compounded annually.

   A1=10,00,000 i=15% N=6 G=100000 F=? A=?

   A=A1 – G (A/G,15%,6)

   A=1000000 – 100000(2.09719)

   A=7,90,281

   F=A(F/A,15%,6)

   F=7,90,281 (8.75374) from discrete series table

   F=69,17,914

Problems to be solved

1. A person deposits a sum of Rs.1,00,000 in a bank for his son's education who will be admitted to a professional course after 6yrs. The bank pays 15% interest rate, compounded annually.Find the future amount of the deposited money at the time of admitting his son in the professional course.
 Ans: F=2,31,306
2. A person needs a sum of Rs. 2,00,000 for his daughter's marriage which will take place 15yrs from now. Find the amount of money that he should deposit now in a bank if the bank gives 18% interest,compounded annually.**Ans: P=16703.2**
3. A person who is now 35yrs old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25yrs. The banks gives 20% interest rate compounded annually. Find the maturity value of his account when he is 60yrs old. **Ans: F=47,19,810**
4. A woman wishes to have Rs. 1,00,000 in her retirement savings plan after working for 25yrs. She will accomplish this by depositing Rs. A each year in a savings acoount that earns 6% per year. How much must she save each year. **Ans : A=1822**
5. A financial institution introduces a plan to pay a sum of Rs.15,00,000 after 10yrs at the rate of 18% compounded annually. Find the annual equivalent amount that a person should invest at the end of every year for the next 10yrs to receive Rs.15,00,000 after 10yrs from the institution.**Ans:A=63771.96**
6. A person wants to give scholarship to poor students of Rs.25,000 every year. He wants to deposit a lumpsum in the bank which makes him receive the required amount every year for next 20yrs. The reserve is assumed to grow annually at the rate of 9%. Find the single payment that must be made now as the reserve amount?
Ans: P=2,28,213

7. It is estimated that a certain piece of equipment car save Rs.6000 per year in labour and materials costs. The equipment has an expected life to 5yrs. If the company must earn a 20% rate of return on such investments,how much could be justified now for the purchase of this piece of equipment? Draw a cash flow diagram. **Ans:P=17,943.7**

8. If Rs.25000 is deposited now into a savings account that earns 12% per year, what uniform annual amount could be withdrawn at the end of each year for 10yrs so that nothing would be left in the account after the tenth withdraw? **Ans : A=4422.5**

9. A bank gives a loan to a company to purchase an equivalent worth Rs.10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15yearly equal instalments. Find the instalment amount that the company has to pay to the bank.
   Ans: A=1,96,400

10. A person is planning for his retired life. He has 10 more yrs of service. He would like to deposit 20% of his salary, which is Rs.10,000 at the end of 1^{st} year and thereafter he wishes to deposit the amount with an annual increase of Rs.2000 for the next 9yrs with an interest rate of 20%. Find the total amount at the end of the 10^{th} year of above series.
    Ans : A=16147.7 and F=419173.01