Problems on discrete Random variables.
(Geometric & Poisson)

1.

a. $P(x) \geq 0$, $\sum P(x) = 1$ (P-d is valid with these two condⁿ)

(i) $K + 3k + 5k + 7k + 9k + 11k = 1$

$36k = 1$

$k = 1/36$

(ii) $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$= K + 3k + 5k$

$= 9k$

$= 9(1/36) = 1/4$

(iii) $P(3 < x \leq 5) = P(x=4) + P(x=5)$

$= 9k + 11k$

$= 20(1/36)$

$= 0.555$

(iv) Variance $(\sigma^2) = \sum x_i^2 P(x_i) - \mu^2 = \sum (x_i - \mu)^2 P(x_i)$

Mean $(\mu) = \sum x_i P(x_i)$

$= 0(k) + 1(3k) + 2(5k) + 3(7k) + 4(9k) + 5(11k)$

$= 125k$

$= 125(1/36)$

$\boxed{\mu = 3.472}$

Variance $(\sigma^2) = (0-3.472)^2 k + (1-3.472)^2 3k + (2-3.472)^2 5k$
$+ (3-3.472)^2 7k + (4-3.472)^2 9k + (5-3.472)^2 11k$

$\boxed{\sigma^2 = 1.971}$

6. We must have $P(x) \geq 0$ & $\sum P(x) = 1$

$\sum P(x) = 1$

(i) $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) = 1$

$k = 1/10$ & $k = -1$

$k = -1$, condⁿ fails. $\therefore$ $k = 1/10$.

$\therefore$ $\boxed{k = 1/10}$

(ii) $P(x < 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= 0 + 1/10 + 1/5 + 1/5 + 3/10 + 1/100$

$= 0.81$

(iii) $P(x > 6) = P(7) = 17/100 = 0.17$

(iv) Mean $(\mu) = \sum x_i P(x_i)$

$= 0(0) + 1(1/10) + 2(1/5) + 3(1/5) + 4(3/10) + 5(1/100)$
$+ 6(1/50) + 7(17/100)$

$\boxed{\mu = 3.66}$

Prob distⁿ is

x	0	1	2	3	4	5	6	7
$P(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

2 a. $\sum P(x) = 1$

(i) $k + 2k + 3k + 4k + 3k + 2k + k = 1 \Rightarrow \boxed{k = 1/16}$

(ii) $P(x \leq 1) = P(-3) + P(-2) + P(-1) + P(0) + P(1) = 13/16$

(iii) $P(x > 1) = P(2) + P(3) = 3/16$

(iv) $P(-1 < x \leq 2) = P(0) + P(1) + P(2) = 9/16$.

b) $\sum P(x) = 1$

(i) $0.1 + k + 0.2 + 2k + 0.3 + k = 1 \Rightarrow \boxed{k = 0.1}$

(ii) $P(x < 1) = P(-2) + P(-1) + P(0) = 0.4$

(iii) $P(x > -1) = P(0) + P(1) + P(2) + P(3) = 0.8$

3.

b. $P(x) = p\, q^{x-1}$

$p = \frac{3}{100} = 0.03$

$q = 1 - p = 0.97$

(i) $P(x=5) = 0.03\,(0.97)^{5-1} = 0.0265$

(ii) $P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$= 0.03 + 0.0291 + 0.0282 + 0.027 + 0.0265$

$= 0.1408$

(iii) mean $(\mu) = 1/p = 33.33$

(iv) Variance $(v) = q/p^2 = 1077.77$

4. a. $1 - p = \boxed{0.8 = q}$

$\boxed{p = 0.2}$

(i) More than 6 people $[P(x) > 6] = 1 - P(x \leq 6)$

$= 1 - [P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)]$

$= 1 - [0.2 + 0.16 + 0.128 + 0.1024 + 0.0819 + 0.0655]$

$= 0.2622$

(ii) Six people $P(x=6) = 0.0655$

6. $p = 0.5 \quad q = 0.5$

$P(x) = 0.5\,(0.5)^{x-1}$

(i) Sixth attempt $P(x=6) = 0.0156$

(ii) More than 6 attempt $P(x>6) = 1 - P(x \leq 6)$

$= 1 - [P(1) + P(2) + P(3) + P(4) + P(5) + P(6)]$

$= 1 - [0.5 + 0.25 + 0.125 + 0.0625$
$0.0312 + 0.0156]$

(iii) $\mu = 1/p = 2$

(IV) $V(\sigma^2) = 2$

5 a. $p = 0.9 \qquad q = 0.1$

$P(x) = 0.9\,(0.1)^{x-1}$

$P(x=3) = 0.9\,(0.1)^{3-1} = 0.009$.

b. $p = \frac{65}{100} = 0.65 \qquad q = 0.35$

$P(x) = 0.65\,(0.35)^{x-1}$

(i) $P(x=3) = 0.65\,(0.35)^{3-1} = 0.0796$

(ii) Two OR three $P(x=2) + P(x=3) = 0.2275 + 0.0796$

$= 0.3071$

6.

b. Poisson distn $P(x) = \frac{m^x e^{-m}}{x!}$

$p = \frac{2}{100} = 0.02$

$\mu = m = np = 200 \times 0.02 = 4$.

$P(x) = \frac{4^x e^{-4}}{x!} = 0.0183\,\frac{4^x}{x!}$

(i) $P(x=0) = 0.0183$

(ii) 3 or More defective $= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - 0.0183\left[1 + \frac{4^1}{1!} + \frac{4^2}{2!}\right]$

$= 0.7621$

(iii) Atleast 1 defective $= 1 - P(0)$

$= 1 - 0.0183$

$= 0.9817$.

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a. $p = 2/100 = 0.02$

$m = np = 500 \times 0.02 = 10$

$P(x) = \frac{10^x e^{-10}}{x!}$

(i) $P(X=3) = \frac{10^3 e^{-10}}{3!} = 0.00756$

(ii) Atleast 1 defective $= 1 - P(x=0) = 1 - e^{-10} = 0.999$

b. $m = 0.5$ $P(x) = \frac{0.5^x e^{-0.5}}{x!}$

(i) less than 2 $= P(0) + P(1)$

$= 0.606 + 0.3032$

$= 0.909$

(ii) More than 2 $= 1 - P(x \leq 2)$

$= 1 - \{P[x=0] + P(x=1) + P(x=2)]$

$= 1 - [0.606 + 0.3032 + 0.0758]$

$= 0.015$

(iii) $P(x=0) = 0.606$.

For three week $(0.606)^3 = 0.222$

8

a. $m = 1.2$

$P(x) = \frac{1.2^x e^{-1.2}}{x!}$

(i) $P(x=2) = 0.2168$

(ii) $P(x<3) = P(0) + P(1) + P(2)$

$= 0.3011 + 0.3614 + 0.2168$

$= 0.8793$.

b. $m = 3$.

$$P(x) = \frac{3^x e^{-3}}{x!}$$

(i) atleast 3 $= 1 - P(x \leq 3) = 1 - [P(0) + P(1) + P(2) + \cancel{P(3)}]$

$=$

(ii) Atmost 7 $= P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)$.

9.

a $m = 4.5$

$$P(x) = \frac{4.5^x e^{-4.5}}{x!}$$

(i) $P(x=4) = \frac{4.5^4 e^{-4.5}}{4!} =$

(ii) atleast 3 $= 1 - P(x < 3) = 1 - [P(0) + P(1) + P(2)]$

b. $m = 3.6$

$$P(x) = \frac{3.6^x e^{-3.6}}{x!}$$

(i) 4 OR less $= P(0) + P(1) + P(2) + P(3) + P(4)$

$=$

(ii) $P(x=2) =$

10 a. $m = 3$, $P(x) = 3^x e^{-3}/x!$

Atmost 10 $= P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10)$

$=$

Multiply ans with 100.

6.

$m = 9.5$

$P(x) = \frac{9.5^x e^{-9.5}}{x!}$

(i) demand to exceed supply $P(x > 12)$

$= 1 - P(x \leq 12)$

(ii) Idle $P(x=0) = \frac{9.5^0 e^{-9.5}}{0!} = e^{-9.5} =$

(iii) $P(x=3) = \frac{9.5^3 e^{-9.5}}{3!} =$

11.

a. $p = \frac{1}{500} = 0.002$

$n = 25$

$m = np = 0.05$

$P(x) = \frac{0.05^x e^{-0.05}}{x!}$

$P(x=0) = \frac{0.05^0 e^{-0.05}}{0!} = 0.95122$

$\therefore$ $10{,}000 \times 0.95122 = 9512$

20.

b. $p = 2/100$

(i) $P(x \geq 3) = 1 - P(x < 3)$

(ii) $P(x \leq 1) = 1 - P(x=0)$