2) <u>Displacement</u>

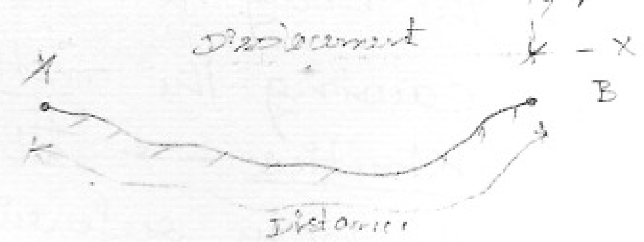
It is defined as the change in position (or) the linear distance b/w the two positions of the body in the begining & @ the end of the time interval. It is a vector quantity.

∴ Displacement = Final position − Initial position

3) <u>Distance</u>

It is the total length of the path covered by the particle. It is a +ve scalar quantity.

- Consider a particle moving from A to B & Let t be the time taken by the body to move from A to B.



- Then the distance is the length measured along the hatched line AB.
- The displacement is the linear distance AB along X axis.

(or)

- Let x_1 travel from origin O @ a time t_1 & Let it reach x_2 @ time t_2 & then again reach x_3 @ time $\cancel{t_3}$, then

Displacement $= x_3 - x_1$ (Final position − Initial pos)

Distance $= (x_2 - x_1) + (x_3 - x_2)$ (total travelled leng)

4) Velocity

The rate of change of displacement w.r.t time is "velocity". It is a vector quantity. The SI unit of velocity is m/sec. It may be (+ve) or (-ve)

→ If s is displacement & time interval t, then the "average velocity" is given by

$$v = \frac{s}{t}$$

→ The velocity of a particle @ a given instant is called "instantaneous velocity". It is given by limiting value of the ratio s/t when both s & t are very small. Let δs & δt be small displacement & time, then instantaneous velocity is given by $v = \lim_{\delta t \to 0} \frac{\delta s}{\delta t}$

$$v = \frac{ds}{dt}$$

5) Speed

The rate of change of distance w.r.t time is defined as speed. It is a scalar quantity.

6) Acceleration

The rate of change of velocity w.r.t time is "acceleration". It is a vector quantity. The SI unit of acceleration is m/sec^2.

It may be (+ve) or (-ve)

→ If v is velocity & time interval t, then "average acceleration" is given by

$$\boxed{a = \frac{v}{t}}$$

→ The acceleration of a particle @ a given instant is called "instantaneous accⁿ". It is given by limiting value of the ratio v/t when both v & t are very small. Let δv & δt be small velocity & time, then instantaneous accⁿ is given by $a = \lim_{\delta t \to 0} \frac{\delta v}{\delta t}$

$$\boxed{a = \frac{dv}{dt}}$$

- Here w.k.t $v = \frac{ds}{dt} \longrightarrow (1)$

By substituting (1) in accⁿ, we get

$$a = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \boxed{\frac{d^2s}{dt^2}}$$

- The above equⁿ can also be written as

$$a = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds} v$$

$$\boxed{a = v\frac{dv}{ds}}$$

→ The negative accⁿ is reffered as "retardation" or "deceleration". This is due to decrease in the magnitude of velocity.

→ General principles in dynamics

1) Newton's first law → Every body continues in its state of rest or of uniform motion, unless it is acted by some external agency (Hence forces are acted to form a balanced system & thus no accⁿ)

2) <u>Newton's second law</u> → The rate of change of momentum is directly proportional to impressed force, & takes in the direction, in which the force acts.

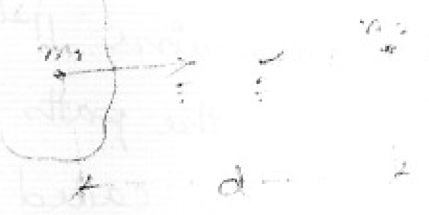
i.e, Force = mass × accⁿ

$$F = m \times a$$

3) <u>Newton's third law</u> → It states that for every action there is an equal & opposite reaction.

4) <u>Newton's law of gravitation</u> → This law state that any two particles of masses m_1 & m_2 seperated by a distance d, attract each other with a force directly proportional to their masse & inversely proportional to square of their distance. Thus the force is given by

$$F \propto \frac{m_1 m_2}{d^2}$$



$$F = G\frac{m_1 m_2}{d^2}$$

where G is constant of proportionality & is called as universal constant of gravitation.

→ The force of attraction on a body by the earth is called as "<u>weight</u>" of the body.

- Let m be the mass & W be the weight

Then substituting in above eqⁿ

$F = W$; $m_1 = m$; $m_2 = M$; $r = R$

where; M is mass of the earth & R is its radiu

∴ By substituting in the above eqn, we get

$$W = \frac{m M G}{R^2}$$

Note: The radius of the earth along polar axis is 63,56400m & along equatorial axis is [illegible]8875m. Hence weight of the body varies from place to place. However the variation is small.

w.k.t $W = mg$

$$\therefore g = \frac{MG}{R^2}$$

where; g is the accn due to gravity ($9.81 \, m/sec^2$)

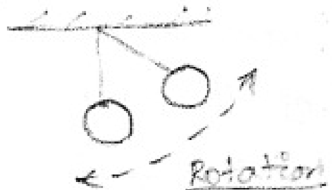
⇒ Types of motion

⇝ **Motion** → The movement of a body in any direction in space is called as motion.

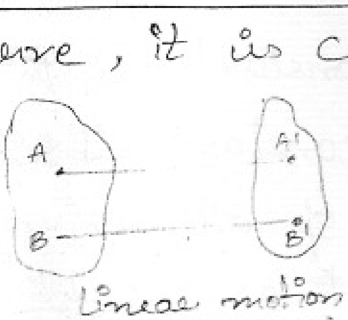
⇝ **Plane motion** → It is the motion considered along the single plane. It may be classified into

1) **Translation**

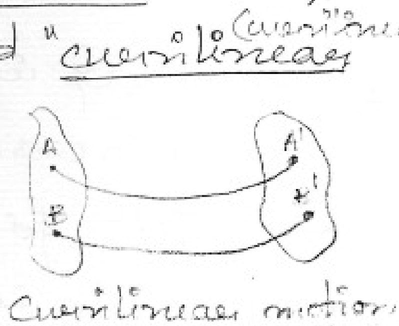
A motion is said to be in translation, if a straight line drawn on the moving body remains $\parallel^{el}$ to its original position @ any time. If the path traced by a point is straight line, it is called "rectilinear translation" (linear motion) & if the path is a curve, it is called "curvilinear translation" (curvilinear motion).



Rotation



Linear motion



Curvilinear motion

2) **Rotation**

A motion is said to be rotation if all particles of a rigid body move in a concentric

3) General plane motion

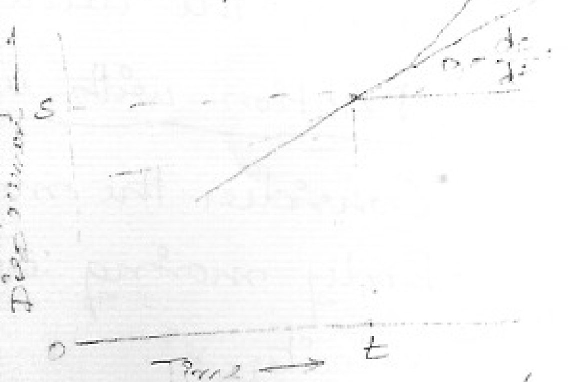
It is a combination of both translation & rotation. Ex:- points on wheel of moving vehicles ; a ladder sliding down from its position against wall etc

→ Motion curves

It is the graphical representation of the displacement, velocity & acc^n with time.

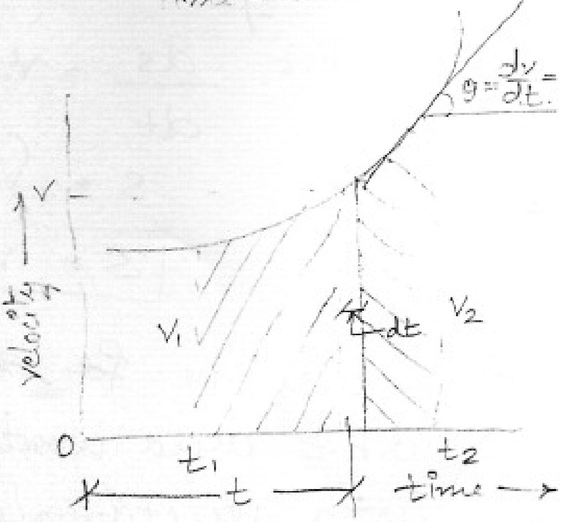
1) Displacement - Time curve (s - t curve)

It is the curve with time as abscissa & displacement as ordinate. The velocity @ any time may be found from the slope (angle 'θ') s-t curve



2) Velocity-time curve (v-t curve)

It is the curve with time as abscissa & velocity as ordinate. The acc^n @ any time may be found from the slope v-t curve



- w.k.t $v = \frac{ds}{dt}$

$ds = v\,dt$

(or) $s = \int v\,dt$

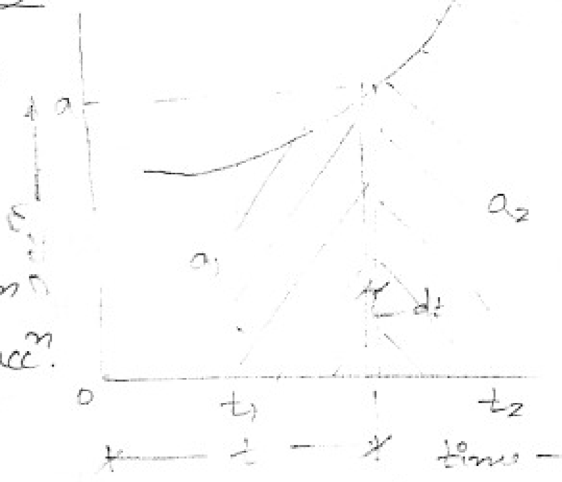
- From fig $v\,dt$ is the elemental curve @ time t in the interval dt

- Thus in v-t curve :-

* Slope of the curve represents accⁿ &
* Area under the curve represents displacement

3) <u>Accⁿ-time curve (a-t curve)</u>

It is a curve with time as abscissa & accⁿ as ordinate. This curve is useful in studying the motion of a body under varying accⁿ.



w.k.t $a = \frac{dv}{dt}$

(or) $dv = a\,dt$

$v = \int a\,dt$

Hence the area under the curve represents velocity

→ <u>Motion with uniform velocity</u>

- Consider the motion of a body moving with uniform velocity 'v'.

- w.k.t $\frac{ds}{dt} = v$

$s = \int v\,dt$

$\boxed{s = vt}$ (since v is constant)

- v-t curve for such motion is given in the fig. w.k.t area under curve with uniform velocity is a rectangle. Hence $\boxed{s = vt}$

⇒ Motion with uniform accⁿ

- Consider the motion of a body with uniform accⁿ. Let u = initial velocity
  v = final velocity
  t = time taken for change of velocity from u to v.

- w.k.t accⁿ is rate of change of velocity

$$\therefore a = \frac{v-u}{t}$$

(or) $$\boxed{v = u + at} \longrightarrow (1)$$

- w.k.t displacement is avg velocity × time

$$\therefore s = \left(\frac{u+v}{2}\right)t \longrightarrow (2)$$

- By substituting (1) & (2), we get

$$s = \left(\frac{u+u+at}{2}\right)t$$

$$= \frac{2ut}{2} + \frac{at^2}{2}$$

$$\boxed{s = ut + \frac{1}{2}at^2} \longrightarrow (3)$$

- From equⁿ (1)
  w.k.t $$t = \frac{v-u}{a} \longrightarrow (4)$$

- By substituting (4) in (2), we get

$$s = \frac{u+v}{2} \times \frac{v-u}{a} \qquad a^2-b^2 = (a+b)(a-b)$$

$$s = \frac{v^2-u^2}{2a}$$

(or) $$\boxed{v^2-u^2 = 2as} \longrightarrow (5)$$

- Thus equ^ns^ of motion of a body moving with constant accn is given by ①, ③, ⑤

(OR)

⟹ By the method of integration

- w.k.t $a = \frac{dv}{dt}$

$$dv = a\,dt$$

$$\int dv = \int a\,dt$$

since a is a constant

$$v = at + C_1$$ (where C_1 is constant of integⁿ)

- when $t = 0$; velocity 'v' = initial velocity 'u'

∴ the above equⁿ becomes

$$u = 0 + C_1 \quad (or) \quad u = C_1$$

Hence $\boxed{v = u + at} \longrightarrow ①$

- w.k.t $v = \frac{ds}{dt}$

$$ds = v\,dt$$

$$\int ds = \int v\,dt$$

$$\int ds = \int (u + at)\,dt \longrightarrow ②$$

$$s = ut + \frac{1}{2}at^2 + C_2$$ (where C_2 is const. of integⁿ)

- when $t = 0$; $s = 0$

∴ the above equⁿ becomes

$$0 = 0 + C_2 \quad (or) \quad C_2 = 0$$

Hence $\boxed{s = ut + \frac{1}{2}at^2} \longrightarrow ③$

- w.k.t $a = \frac{dv}{dt}$

$$a = \frac{dv}{ds} \times \frac{ds}{dt}$$

$$a = \frac{dv}{ds} v \qquad \left(\because \frac{ds}{dt} = v\right)$$

$$\therefore a\,ds = v\,dv \longrightarrow (4)$$

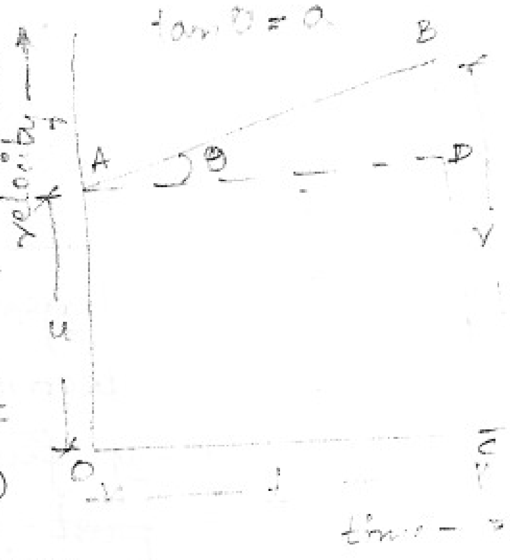
$$a \int ds = \int_u^v v\,dv$$

$$as = \left(\frac{v^2}{2}\right)_u^v$$

$$as = \frac{v^2}{2} - \frac{u^2}{2}$$

$$\boxed{v^2 - u^2 = 2as} \longrightarrow (5)$$

(OR)



⟹ By referring to v-t curve

- Since accⁿ is uniform, the slope of the curve is constant i.e, it is a straight line (AB)
- w.k.t a = slope of the diagram

$$a = \tan\theta$$

$$a = \frac{BD}{AD}$$

$$a = \frac{BC - DC}{AD} = \frac{BC - OA}{OC}$$

$$a = \frac{v - u}{t}$$

$$\boxed{v = u + at} \longrightarrow (1)$$

- w.k.t S = Area of AOCB

S = Area of □ˡᵉ AODC + Area of Δˡᵉ ABD ⟶ (2)

$$S = AO \times OC + \frac{1}{2} \times AD \times BD$$

$$= u \times t + \frac{1}{2} \times AD \times AD \tan\theta \qquad \left(\tan\theta = \frac{BD}{AD}\right)$$

$$= u \times t + \frac{1}{2} \times t \times t \times a$$

$$\boxed{s = ut + \frac{1}{2}at^2} \longrightarrow (3)$$

- w.k.t s = area of trapezium $AOCB \longrightarrow (4)$

$$= \frac{1}{2}(AO + BC)\, OC$$

$$= \frac{1}{2}(u + v)t$$

- w.k.t $t = \frac{v-u}{a}$

$$\therefore s = \frac{1}{2}(u+v)\frac{(v-u)}{a}$$

$$\boxed{2as = v^2 - u^2} \longrightarrow (5)$$

⟹ Exercise problems

1) Three telegraph poles A, B & C are spaced @ 50m intervals along a straight road. A car starting from rest accelerates uniformly, passes post A & then takes 8 secs to reach post B & further 7 secs to reach post C. Calculate :- (a) accⁿ of the car ; (b) velocity of the car @ A, B, C ; (c) The distance of post A from the starting point of the car.

- Let O be the starting point of the car &
- v_a, v_b, v_c be the velocities of points A, B, C, resp[ectively]

- Let x be the distance from starting point to A & a be the accⁿ.
- w.k.t $s = ut + \frac{1}{2}at^2$

a) For motion from A to B

$$50 = v_a \times 8 + \frac{1}{2}a\, 8^2$$

$50 = 8 v_a + 32 a \longrightarrow (1)$

b) For motion from A to C

$100 = v_a \times 15 + \frac{1}{2} a \, 15^2$

$100 = 15 v_a + 112.5 a \longrightarrow (2)$

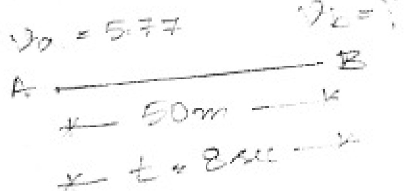
- By substituting (1) & (2)

$\underline{v_a = 5.77 \text{ m/s}}$

$\underline{a = 0.119 \text{ m/s}^2}$

- w.k.t $v = u + at$

a) For motion from A to B



$v_b = 5.77 + 0.119 \times 8$

$\underline{v_b = 6.72 \text{ m/s}}$

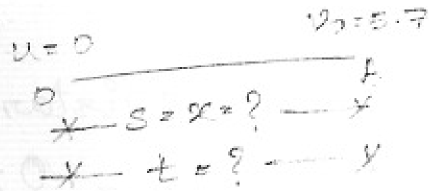
b) For motion from B to C

$v_c = 6.72 + 0.119 \times 7$

$\underline{v_c = 7.55 \text{ m/s}}$

- w.k.t $2as = v^2 - u^2$

∴ For motion from O to A



$2 \times 0.119 \, s = 5.77^2 - 0^2$

$\underline{s = 139.8 \text{ m}}$

- w.k.t $s = ut + \frac{1}{2} a t^2$

∴ For motion from O to A

$139.8 = 0 \times t + \frac{1}{2} \times 0.119 \, t^2$

$139.8 = 0.0595 \, t^2$

$\underline{t = 48.47 \text{ sec}}$

2) Elevators A & B in the adjoining shafts start moving in opposite directions with constant acc^{ns} $\overset{(a_A)}{0.3\ m/s^2}$ & $\overset{(a_B)}{0.6\ m/s^2}$ resp^{ly}. 'A' moves down while 'B' moves up. If they were $\overset{(S_A+S_B)}{150m}$ apart @ $\overset{(u_A=u_B=0)}{t=0}$, after what time they will be opposite to each other? How far each one has travelled?

$\Rightarrow$ When they are opposite to each other

$$S_A + S_B = 150$$

i.e, $$\left(u_A t + \frac{1}{2} a_A t^2\right) + \left(u_B t + \frac{1}{2} a_B t^2\right) = 150$$

$u_A = 0$, $a_A = 0.3m$, $S_A = ?$ ↓A

– w.k.t $u_A = u_B = 0$

$$\therefore \frac{1}{2} a_A t^2 + \frac{1}{2} a_B t^2 = 150$$

$S_A + S_B = 150$, $t = ?$

$$\frac{1}{2} \times 0.3 \times t^2 + \frac{1}{2} \times 0.6 \times t^2 = 150$$

↑B $u_B = 0$, $a_B = 0.6$, $S_B = ?$

$$\underline{\underline{t = 18.25\ sec}}$$

– $\therefore$ Distance travelled by A

$$S_A = 0 \times 18.25 + \frac{1}{2} \times 0.3 \times 18.25^2$$

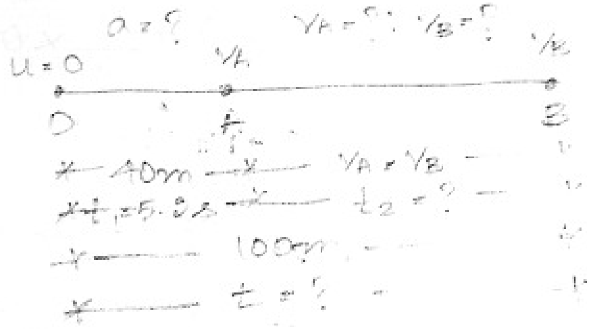
$$\underline{\underline{S_A = 50m}}$$

– $\therefore$ Distance travelled by B

$$S_B = 0 \times 18.25 + \frac{1}{2} \times 0.6 \times 18.25^2$$

$$\underline{\underline{S_B = 100m}}$$

3) A sprinter in a 100m race accelerates uniformly for the first 40m & then runs with constant velocity. If the sprinter's time for the first 40m is 5.2s, determine his time for the race. (15)



- w.k.t $S = ut + \frac{1}{2} a t^2$

$\therefore$ For first 40m (OA)

$$40 = 0 \times 5.2 + \frac{1}{2} a (5.2)^2$$

$$\underline{\underline{a = 2.95 \ m/s^2}}$$

- w.k.t $V_A = u_A + a t_1$

$$V_A = 0 + 2.95 \times 5.2$$

$$\underline{\underline{V_A = 15.34 \ m/s}}$$

- Here $V_A = V_B$

w.k.t $V_B = \frac{S_2}{t_2}$

$$15.34 = \frac{60}{t_2}$$

$$\therefore \underline{\underline{t_2 = 3.91 \ s}}$$

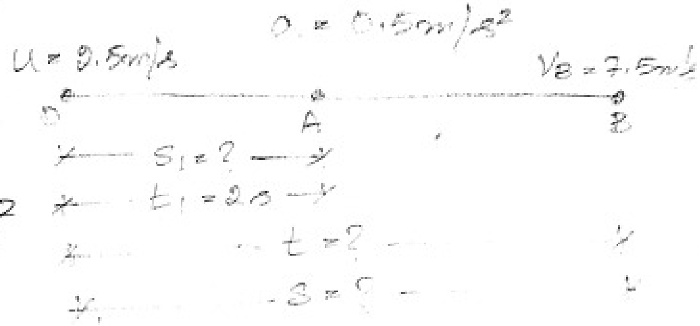
- $\therefore$ total time $= t = 5.2 + 3.91$

$$\underline{\underline{t = 9.11 \ s}}$$

---

4) A particle starts with an initial velocity of 2.5 m/s & uniformly accelerates @ the rate 0.5 m/s². Determine the displacement in 2s, time reqd. to attain the velocity of 7.5m/s

and the distance travelled when it attain a velocity of 7.5 m/s.



→ w.k.t $S = ut + \frac{1}{2}at^2$

For t = 2s (OA)

$S_1 = 2.5 \times 2 + \frac{1}{2} \times 0.5 \times 2^2$

$\underline{S_1 = 6m}$

- For velocity = 7.5 m/s (OB)
- w.k.t $v = u + at$

$7.5 = 2.5 + 0.5 \times t$

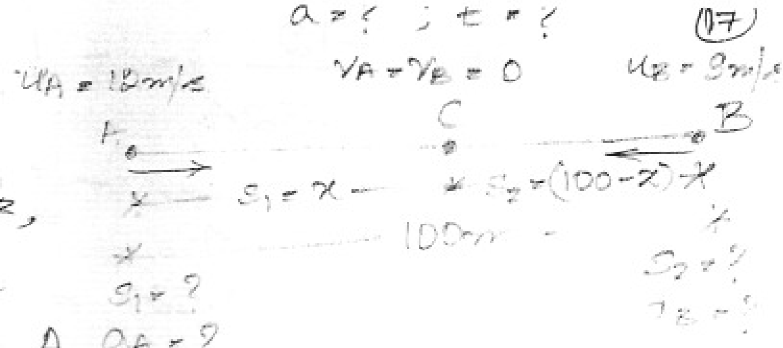
$\therefore \underline{t = 10 \text{ sec}}$

- For velocity = 7.5 m/s (OB)
- w.k.t $v^2 - u^2 = 2as$

$7.5^2 - 2.5^2 = 2 \times 0.5 \times s$

$\therefore \underline{s = 50m}$

5) Two cars are travelling towards each other on a single lane road @ the velocities 12 m/s & 9 m/s respty. When 100m apart, both drivers realise the situation & apply their brakes. They succeed in stopping simultaneous & just short of colliding. Assume constant retardation for each case & determine :- (a) time reqd. for cars to stop (b) retardation for each car & (c) distance travelled by each car while slowing down.



→ Let A & B be the cars 100m apart

- After applying break, Let them meet @ C @ a distance x from A

→ Consider the motion of car A.

- w.k.t $v_A = u_A + a_A t$

$$0 = 12 + a_A t$$

$$\therefore a_A = \frac{-12}{t} \longrightarrow (1)$$

{a is -ve ∵ retardation}

- w.k.t $v_A^2 - u_A^2 = 2 a_A S_1$

$$0^2 - 12^2 = 2\left(\frac{-12}{t}\right) x$$

$$x = 6t \longrightarrow (2)$$

→ Consider the motion of car B

- w.k.t $v_B = u_B + a_B t$

$$0 = 9 + a_B t$$

$$\therefore a_B = -\frac{9}{t} \longrightarrow (3)$$

{a is -ve ∵ retardation}

- w.k.t $v_B^2 - u_B^2 = 2 a_B S_2$

$$0^2 - 9^2 = 2\left(-\frac{9}{t}\right)(100 - x)$$

$$100 - x = 4.5t \longrightarrow (4)$$

- By substituting (2) & (4), we get

$$100 - 6t = 4.5t$$

$$\underline{\underline{t = 9.524 \text{ sec}}}$$

$$\therefore a_A = \frac{-12}{9.524} = \underline{\underline{-1.26\,m/s^2}}$$

$$a_B = \frac{-9}{9.524} = \underline{\underline{-0.945\,m/s^2}}$$

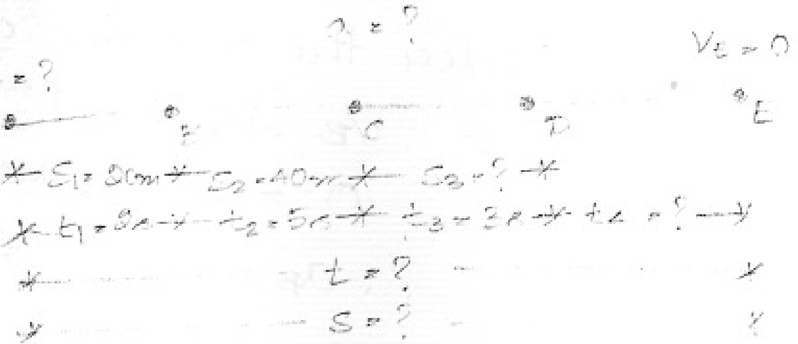
$$S_1 = x = 6 \times 9.524 = \underline{\underline{57.14\,m}}$$

$$S_2 = 100 - x = \underline{\underline{42.86\,m}}$$

---

6) A particle under constant retardation is moving in a straight line & covers a distance of 20m in first two secs & 40m in the next 5 secs. Calculate the distance it covers @ ~~a distance of 20m in first two secs & 40m in the next 5 secs.~~ Calculate the distance it covers in the subsequent 3 secs & the total distance covered, before it comes to rest.

→ Consider the motion b/w A & B



– w.k.t $S_1 = u_A t_1 + \frac{1}{2} a t_1^2$

$$\therefore 20 = u_A \times 2 + \frac{1}{2} a\, 2^2$$

$$20 = 2u_A + 2a$$

$$10 = u_A + a \longrightarrow (1)$$

→ Consider the motion b/w A & C

– w.k.t $60 = u_A \times 7 + \frac{1}{2} a\, 7^2$

$$60 = 7u_A + 24.5a \longrightarrow (2)$$

→ By substituting (1) & (2)

$u_A = 10.57\ m/s$

$a = -0.57\ m/s^2$

→ Consider the motion b/w A & D

- w.k.t $S_{AD} = u_A t_{AD} + \frac{1}{2} a t_{AD}^2$

$= 10.57 \times 10 + \frac{1}{2}(-0.57) \times 10^2$

$\underline{S_{AD} = 77.2m}$

- ∴ $S_3 = 77.2 - 40 - 20$

$\underline{S_3 = 17.2m}$

→ Consider the motion b/w A & E

- w.k.t $v^2 - u^2 = 2as$

$0^2 - (+10.57)^2 = 2 \times (-0.57)\ s$

$\underline{S = 98m}$

- w.k.t $v = u + at$

$0 = 10.57 + (-0.57) t$

$\underline{t = 18.54\ s}$

- ∴ $t_4 = 18.54 - 2 - 5 - 3$

$\underline{t_4 = 8.54\ s}$

---

7) A motorist is travelling @ 80km/h, when he observes a traffic light 200m ahead of him turns red. The traffic light is timed to stay red for 10s. If the motorist wishes to pass the light without stopping, just as it turns green, determine (a) the reqd. uniform deceleration of the motor & (b) the speed of the motor as it passes the light

$u = 80\,kmph \quad a = ? \quad v = ?$

$s = 200\,m$

$t = 10\,s$

→ Initial velocity

$u = 80\,km/h$

$= \dfrac{80 \times 1000}{60 \times 60}$

$\underline{\underline{u = 22.22\,m/s}}$

→ w.k.t $s = ut + \frac{1}{2} a t^2$

$200 = 22.22 \times 10 + \frac{1}{2} a \times 10^2$

$200 = 222.2 + 50a$

$\underline{\underline{a = -0.44\,m/s^2}}$

→ Final velocity (speed @ which motor passes the signal)

w.k.t $v = u + at$

$v = 22.22 + (-0.44) \times 10$

$v = 17.8\,m/s$

or $v = \dfrac{17.8 \times 10^{-3}}{1/(60 \times 60)}$

$\underline{\underline{v = 64.1\,kmph}}$

---

8) The greatest possible acc^n or dec^n that a train may have is a & its max speed is v. Find the min time in which the train can get from one station to the next, if the total distance is s.

→ To have min travel time, the train must accelerate @ max acc^n to reach max velocity then run @ that velocity & finally decelerate @ max dec^n so that the time is kept least.

→ Let t_1, t_2 & t_3 be the time for acc^n, uniform motion & dec^n resptly.

→ ∴ total time of travel

$t = t_1 + t_2 + t_3 \rightarrow (1)$

→ w.k.t $\tan\theta = a$

∴ $\tan\theta = \frac{v}{t_1}$

$v = at_1 \rightarrow (2)$

$|||^{ly}$ $\tan\theta = \frac{v}{t_3}$

$v = at_3 \rightarrow (3)$

→ Here $t_1 = t_3$

→ Let s_1, s_2 & s_3 be the distance travelled while accelerating, uniform velocity & while decelerating resptly.

– w.k.t area under v-t curve gives displacem

∴ $s = s_1 + s_2 + s_3$

$s = \frac{1}{2} v t_1 + v t_2 + \frac{1}{2} v t_3$

– since $t_1 = t_3$

$s = \frac{1}{2} v t_1 + v t_2 + \frac{1}{2} v t_1$

$s = v t_1 + v t_2$

$s = v (t_1 + t_2)$

$\frac{s}{v} = t_1 + t_2 \longrightarrow (4)$

→ From eqn (1), (3) & (4), we get

$$t = \frac{s}{v} + \frac{a}{v}$$

9) Two trains A & B leave the same station in ||el lines. Train A starts with uniform accⁿ of 0.15 m/s² & attains a speed of 27 kmph when the steam is reduced to keep speed constant. Train B leaves 40s later with uniform accⁿ of 0.3 m/s² to attain max speed of 54 kmph. When & where will B overtake A? (22)

→ Constant speed of A

$= 27$ kmph

$= \frac{27 \times 1000}{60 \times 60}$

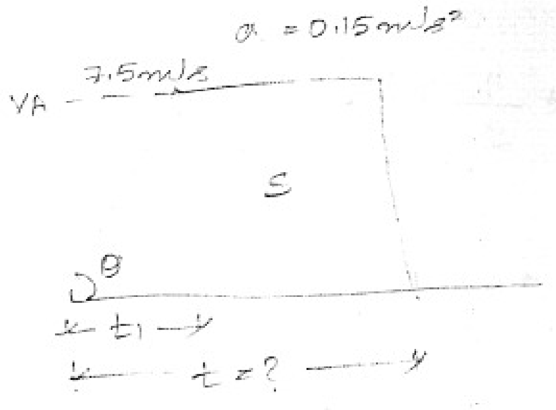
$\underline{V_A = 7.5 \text{ m/s}}$

→ Constant speed of B

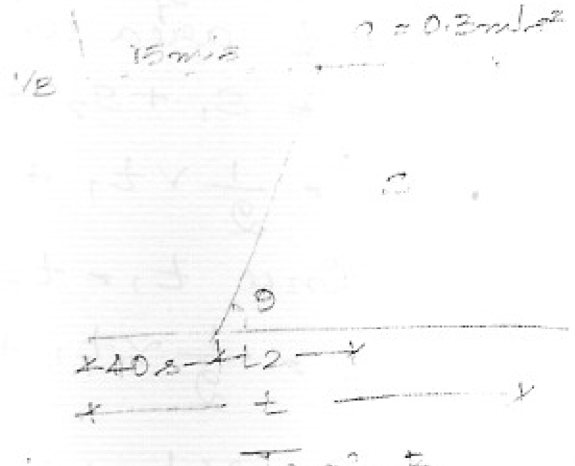
$= 54$ kmph

$= \frac{54 \times 1000}{60 \times 60}$

$\underline{V_B = 15 \text{ m/s}}$



Train A



Train B

→ Consider motion of train A

∴ time taken to attain uniform accⁿ be t_1

— w.k.t $V_A = u_A + a_A t_1$

$7.5 = 0 + 0.15 t_1$

$\underline{t_1 = 50 \text{ s}}$

→ Let t & s be the time taken & distance travelled where B overtakes A.

∴ For train A

s = area under v-t curve of train A

$$s = \frac{1}{2} t_1 v_A + v_A (t - t_1)$$

$$s = \frac{1}{2} \times 50 \times 7.5 + 7.5 (t - 50)$$

$$s = 187.5 + 7.5t - 375$$

$$s = 7.5t - 187.5 \longrightarrow (1)$$

→ Consider motion of train B

∴ time taken to attain uniform accⁿ be t_2

- w.k.t $v_B = u_B + a_B t_2$

$$15 = 0 + 0.3 t_2$$

$$\underline{\underline{t_2 = 50 s}}$$

→ Let t & s be the time taken & distance travelled when B overtakes A

∴ For train B

s = area under v-t curve of train B

$$s = \frac{1}{2} t_2 v_B + v_B (t - t_2 - 40)$$

$$s = \frac{1}{2} \times 50 \times 15 + 15 (t - 50 - 40)$$

$$s = 375 + 15t - 750 - 600$$

$$s = 15t - 975 \longrightarrow (2)$$

→ By substituting eqⁿ (1) & (2), we get

$$\underline{s = 600 \text{ m}}$$

$$\underline{\underline{t = 105 s}}$$

→ Motion under gravity

The accⁿ due to gravity can be solved by using eqⁿˢ for motion with constant accⁿ. Its value is found to be 9.81 m/s^2 & is always directed towards the centre of the earth (i.e, vertically downwards). Hence we use the sign convention :- upward motion (+ve) & downward motion (-ve).

∴ upward accⁿ $= +a = -g = -9.81\ m/s^2 \uparrow$

downward accⁿ $= +g = +9.81\ m/s^2 \downarrow$

→ **Note** :- While solving the problems, the starting position will be taken as origin. If final position is above initial position, the displacement will be (+ve) (or) vice versa.

→ Exercise problems

1) A baseball is thrown downward from a 15m tower with an initial speed of 5m/s. Determine the speed @ which it hits the ground & the time of travel.

$u = 5\ m/s \downarrow$

$s = -15m$

$a = +g$

$t = ?$

$v = ?$

→ w.k.t $v^2 - u^2 = 2as$

$$v^2 - 5^2 = 2 \times (+9.81) \times (-15)$$

$$v^2 - 25 = -294.3$$

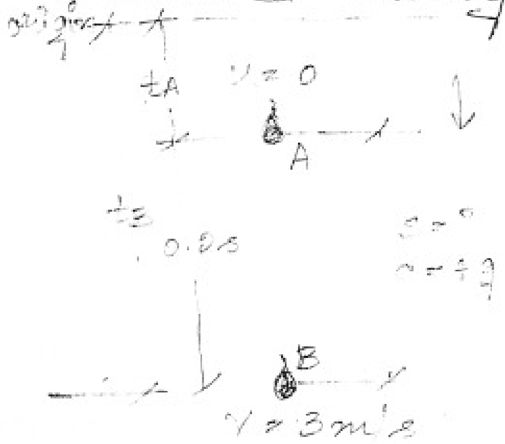
$$\underline{v = 17.86\ m/s}$$

→ w.k.t $v = u + at$

$$-17.86 = -5 + (+9.81)t$$

$$\underline{t = 1.31s}$$

2) Water drips from a tap @ the rate of 5 drops per second. Determine vertical seperation b/w 2 consecutive drops when lower drop of the two has attained velocity of 3m/s.



→ The time difference b/w 2 consecutive drops A & B

$$= \frac{1}{5} = \underline{0.2s}$$

→ For drop B

:- w.k.t $-v = u + at_B$

$$-3 = -0 + (+9.81)\, t_B$$

$$\underline{t_B = 0.3058\, s}$$

→ ∴ $t_A = t_B - 0.2$

$$\underline{t_A = 0.1058\, s}$$

→ w.k.t $s = ut + \frac{1}{2}at^2$

∴ For drop A

$$S_A = 0 \times 0.1058 + \frac{1}{2}(+9.81)(0.1058)^2$$

$$\underline{S_A = -0.0549\, m} \quad (-\because s\downarrow)$$

∴ For drop B

$$S_B = 0 \times 0.3058 + \frac{1}{2}(+9.81)(0.3058)^2$$

$$\underline{S_B = -0.459\, m} \quad (-\because s\downarrow)$$

→ vertical seperation b/w A & B

$$s = S_A - S_B = \underline{0.404\, m}$$

3) A stone is dropped into a well. After 4 secs the sound of splash is heard. If the velocity of the sound is 330 m/s, find the depth of the well upto the water surface (26)

→ Let s be the depth of well,
t_1 be the time taken by stone to strike water &
t_2 be the time taken by sound to reach initial position.

→ For downward motion of stone
w.k.t $s = ut_1 + \frac{1}{2} a t_1^2$

$$-s = 0 \times t_1 + \frac{1}{2} (+9.81) t_1^2$$

$$-s = 4.905 t_1^2 \longrightarrow (1)$$

→ w.k.t the total time taken is 4 s

$$\therefore t_1 + t_2 = 4 \longrightarrow (2)$$

→ For upward motion of sound
w.k.t distance = velocity × time

$$s = v \times t_2$$

$$s = 330 \times (4 - t_1) \quad (\text{from eqn } (2))$$

$$s = 1320 - 330 t_1 \longrightarrow (3)$$

→ From eqn (1) & (3)

$$-4.905 t_1^2 = 1320 - 330 t_1$$

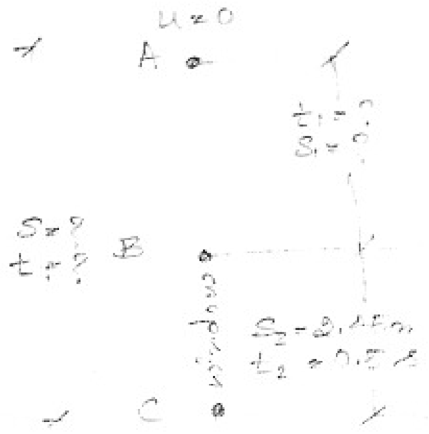
$$t_1 = 3.78 s$$

→ $\therefore s = 1320 - 330 \times 3.78 = 72.6 m$

$t_2 = 4 - 3.78 = 0.22 s$

4) If a stone falls past a window of 2.45m height in half a sec, find the height from which the stone fell. (27)

→ Let the stone be dropped from A @ a height S_1 from top of window BC (i.e, B)



→ For motion from A to B

- w.k.t $S_1 = ut_1 + \frac{1}{2} a t_1^2$

$$-S_1 = 0t_1 + \frac{1}{2}(+9.8)t_1^2$$

Doubt!

$$\cancel{+}S_1 = \cancel{+}4.905 t_1^2 \longrightarrow (1)$$

→ For motion from A to C

- w.k.t $S = ut + \frac{1}{2} a t^2$

$$-S = 0 \times t + \frac{1}{2}(+9.8)(t_1 + 0.5)^2$$

$$-(S_1 + S_2) = -4.905\left(t_1^2 + (0.5)^2 + 2 \times t_1 \times 0.5\right)$$

$$-(4.905t_1^2 + 2.45) = -4.905t_1^2 - 1.226 - 4.905t_1$$

$$-\cancel{4.905t_1^2} - 2.45 + \cancel{4.905t_1^2} + 1.226 + 4.905t_1 = 0$$

$$-1.224 + 4.905t_1 = 0$$

$$\underline{t_1 = 0.249\,s} \longrightarrow (2)$$

→ By substituting (2) in (1)

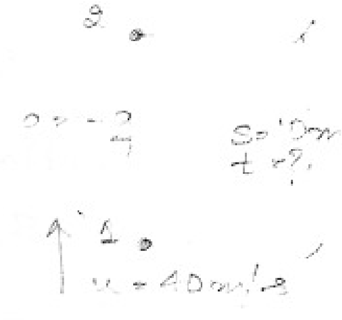
$S_1 = 4.905 \times 0.249^2 = \underline{0.305m}$

$\therefore S = S_1 + S_2 = \underline{2.755m}$

$t = t_1 + t_2 = \underline{0.749s}$

5) A stone is thrown upward with a velocity of 40m/s. Determine the time of the stone when it is @ a height of 10m & is moving downwards.

→ w.k.t $\vec{S} = \vec{u}t + \frac{1}{2}at^2$



$$10 = 40 \times t + \frac{1}{2}(-9.81)t^2$$

$$10 = 40t - 4.905t^2$$

$$\therefore t = 0.258\,s \text{ (or) } 7.89\,s$$

→ The smaller value of t is for upward motion & larger value for downward motion

$$\underline{\therefore t = 7.89\,s}$$

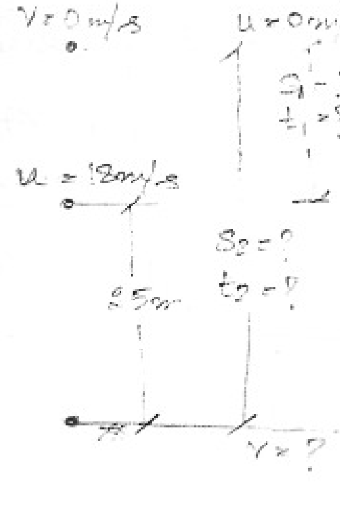
---

6) A small steel ball is shot vertically upwards from the top of a building 25m above the ground with an initial velocity of 18m/s.

a) In what time, it will reach the max height

b) How high above the building will the ball rise

c) Compute the velocity with which it will strike the ground & the total time it is in motion.



→ For upward motion

– w.k.t $v = u + at_1$

$$0 = 18 - 9.81t_1$$

$$\underline{t_1 = 1.83\,s}$$

- ∴ w.k.t $v^2 - u^2 = 2as$

$$0^2 - (18)^2 = 2 \times -9.81 \times s$$

$$\therefore \underline{s_1 = -16.51\,m} \quad (- \because \text{ground is final position})$$

→ ∴ Total height from the ground

$$s_2 = 25 + 16.51 = \underline{41.51\,m}$$

→ For downward motion

- w.k.t $v^2 - u^2 = 2as_2$

$$v^2 - 0^2 = 2 \times (+9.81) \times 41.51$$

$$\underline{v = 28.53\,m/s}$$

- w.k.t $v = u + at_2$

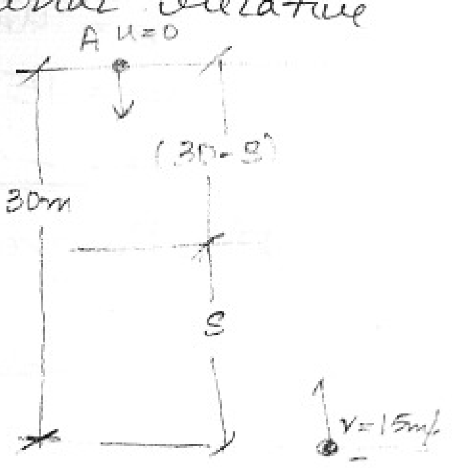
$$28.53 = 0 + (+9.81)t_2$$

$$\underline{t_2 = 2.91\,s}$$

→ ∴ Total time during which the body was in motion $= t = t_1 + t_2 = \underline{4.73\,s}$

---

7) A ball is dropped from top of a tower 30m high. At the same instant a second ball is thrown upward from the ground with an initial velocity of 15m/s. When & where do they cross & with what relative velocity?



→ For downward motion of ball A

- w.k.t $\overset{(-)}{s} = ut + \frac{1}{2}\overset{(+)}{a}t^2 \downarrow$

$$-(30-s) = 0 \times t + \frac{1}{2}(+9.81)t^2$$

$$-30+s = +4.905t^2 \longrightarrow (1)$$

→ For upward motion of ball B

- w.r.t $\bar{s} = \overset{(+)}{\bar{u}}t + \frac{1}{2}\bar{a}t^2$

$$-s = 15t \mp \frac{1}{2} \times 9.81 t^2$$

(∵ final is down)

$$-S = 15t \mp 4.905t^2 \longrightarrow (2)$$

→ From equ^n (1) & (2)

$$+30 - 15t + \cancel{4.905t^2} = +\cancel{4.905t^2}$$

$$30 - 15t = 0$$

$$\underline{t = 2s}$$

→ $\therefore -S = 15 \times 2 \mp 4.905 \times 2^2$

$$\underline{-S = 10.38m} \downarrow$$

→ @ $t = 2s$

- downward velocity of first ball

$$v_A = u_A + at$$

$$v_A = 0 + 9.81 \times 2$$

$$v_A = \underline{19.62 \, m/s}$$

- upward velocity of second ball

$$v_B = u_B + at$$

$$v_B = 15 + (-9.81) \times 2$$

$$v_B = \underline{-4.62 \, m/s} \downarrow$$

→ Relative velocity $= 19.62 - 4.62$

$$= \underline{15 \, m/s}$$

② ⟹ Motion with variable accⁿ

A vehicle is normally not accelerated uniformly. Initially it starts with zero accⁿ, then the rate of accⁿ is ↑sed & when desired speed is nearing, the rate of accⁿ is reduced. By the time desired speed is picked up accⁿ is brought to zero.

If the variation of accⁿ or velocity or displacement w.r.t time is known, such problems can be solved using the differential equⁿˢ: → $v = \frac{ds}{dt}$

$$\rightarrow a = \frac{dv}{dt} = \frac{d\left(\frac{ds}{dt}\right)}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$$

⟹ Exercise problems

1) The motion of a particle moving in a straigh line is given by the expression $s = t^3 - 3t^2 + 2t + 5$, where s is the dispˡᵗ in mts & t is the time in secs. Determin (a) velocity & accⁿ after 4 secs; (b) max (or) min velocity & corresponding dispˡᵗ and (c) time @ which velocity is zero.

⟹ ∴ $s = t^3 - 3t^2 + 2t + 5$

w.k.t $v = \frac{ds}{dt}$

$v = 3t^2 - 6t + 2$

- w.k.t $a = \frac{d^2s}{dt^2}$

$a = 6t - 6$

a) velocity & acc^n after 4 secs

$v = 3\times4^2 - 6\times4 + 2 = \underline{26 \ m/s}$

$a = 6\times4 - 6 = \underline{18 \ m/s^2}$

b) max or min velocity & corresponding disp^t

The velocity is max or min when

$a = \frac{dv}{dt} = 0$

$\therefore 0 = 6t - 6$

$\underline{t = 1 \ sec}$

- $\therefore v = 3\times1^2 - 6\times1 + 2 = \underline{-1 \ m/sec}$ (min)

- corresponding disp^t

$\therefore S = 1^3 - 3\times1^2 + 2\times1 + 5 = \underline{5m}$

c) time @ which velocity is zero.

- w.k.t $v = 0$

$\therefore 0 = 3t^2 - 6t + 2$

$t = \underline{1.57 s}$ & $\underline{0.42 s}$

---

2) The velocity of a particle moving in a straight line is given by the expression $v = t^3 - t^2 - 2t + 2$. The particle is found to be @ a distance 4m from station A after 2 secs. Determine: (a) acc^n & disp^t after 4 secs. (b) max/min acc^n.

- w.k.t $v = t^3 - t^2 - 2t + 2$

(a) acc^n & disp^t after 4 secs.

- w.k.t $a = \frac{dv}{dt} = 3t^2 - 2t - 2$

$\therefore a = 3\times4^2 - 2\times4 - 2 = \underline{38 m/s^2}$

- w.k.t $\frac{ds}{dt} = v = t^3 - t^2 - 2t + 2$

$\therefore s = \frac{t^4}{4} - \frac{t^3}{3} - \frac{\cancel{2}t^2}{\cancel{2}} + 2t + C$

where; C = constant of integration

- $\therefore$ w.k.t $s = 4m$ & $t = 2s$

$\therefore 4 = \frac{2^4}{4} - \frac{2^3}{3} - 2^2 + 2\times2 + C$

$\underline{C = 2.67}$

- $\therefore$ s when 4 secs

$S = \frac{4^4}{4} - \frac{4^3}{3} - 4^2 + 2\times4 + 2.67$

$\underline{S = 37.33 m}$

(b) max/min acc^n

- when $\frac{da}{dt} = 0$

$\frac{da}{dt} = 6t - 2$

$\therefore 6t - 2 = 0$

$\underline{t = 0.33 \text{ sec}}$

- $\therefore$ min acc^n

$a = 3\times(0.33)^2 - 2\times0.33 - 2$

$\underline{a = -2.33 m/s^2}$

---

3) A particle starts with an initial velocity of 8m/s & moves along a straight line.

Its accⁿ @ any time t after start, is given by the expⁿ $\lambda - \mu t$ where λ & μ are constants. Determine the eqⁿ for dispt, if the particle covers a distance of 40m in 5 secs & stops.

- w.k.t $a = \lambda - \mu t = \frac{dv}{dt}$

$$\therefore v = \lambda t - \frac{\mu t^2}{2} + C_1$$

- Here $v = 8$ m/s when $t = 0$

$$\therefore 8 = 0 - 0 + C_1$$

$$\therefore C_1 = 8$$

$$\therefore v = \lambda t - \frac{\mu t^2}{2} + 8 = \frac{ds}{dt}$$

$$\therefore s = \frac{\lambda t^2}{2} - \frac{\mu t^3}{2 \times 3} + 8t + C_2$$

- Here $s = 0$m when $t = 0$

$$\therefore 0 = 0 - 0 + 0 + C_2$$

$$\therefore C_2 = 0$$

$$\therefore s = \frac{\lambda t^2}{2} - \frac{\mu t^3}{6} + 8t$$

→ Here $s = 40$m & $t = 5$ secs

$$\therefore 40 = \frac{\lambda 5^2}{2} - \frac{\mu 5^3}{6} + 8 \times 5$$

$$\cancel{40} = 12.5\lambda - 20.83\mu + \cancel{40}$$

$$\lambda = \frac{20.83\mu}{12.5}$$

$$\lambda = 1.66\mu \longrightarrow (1)$$

→ when $t = 5$ secs & stops i.e, $v = 0$

$$\therefore \quad 0 = \lambda \times 5 - \frac{\mu \, 5^2}{2} + 8$$

$$0 = 5\lambda - 12.5\mu + 8 \longrightarrow (2)$$

→ From equ^n (1) & (2), we get.

$$0 = 5 \times 1.66\mu - 12.5\mu + 8$$

$$0 = 8.332\mu - 12.5\mu + 8$$

$$0 = -4.168\mu + 8$$

$$\underline{\mu = 1.92}$$

$$\therefore \quad \lambda = 1.66 \times 1.92 = \underline{3.18}$$

→ The equ^n for disp^t

$$S = \frac{3.18t^2}{2} - \frac{1.92t^3}{6} + 8t$$

$$\underline{S = 1.59t^2 - 0.32t^3 + 8t}$$

---

4) A car is moving with a velocity of 72 km/h. After seeing a child on the road, the breaks are applied & the vehicle is stoppe in a distance of 15m. If the retardation produced is proportional to distance from the point where breaks are applied, find the expression for retardation.

→ Here retardation $= -a$

& $-a$ is $\propto$ to S

i.e, $\dfrac{-a}{S} = k$

where k = constant

$\therefore a = -ks$

w.k.t $a = v\frac{dv}{ds}$

$\therefore v\frac{dv}{ds} = -ks$

$v\,dv = -ks\,ds$

$\frac{v^2}{2} = -\frac{ks^2}{2} + C_1$

- when the breaks were applied,

$S = 0$ & $v = 72\,km/h$

$v = \frac{72\,km}{1}$

$= \frac{72 \times 1000}{60 \times 60}$

$v = 20\,m/s$

$\therefore \frac{20^2}{2} = -0 + C_1$

$\therefore C_1 = 200$

$\therefore \frac{v^2}{2} = -\frac{ks^2}{2} + 200$

- when vehicle stops @ a distance $S = 15m$

$v = 0$

$\therefore 0 = -\frac{k\,15^2}{2} + 200$

$0 = -112.5k + 200$

$\therefore k = 1.778$

- Hence the expression for retardation

$a = -1.778\,s$

---

5) The accⁿ of point A is defined by the relation $a = 600x(1 + kx^2)$ where a and x are expressed in m/s^2 & m resp. k is

constant. Knowing the velocity of A is (3)
7.5 m/s when $x = 0$ & 15 m/s when $x = 0.45$ m
determine the value of k.

→ w.k.t $a = 600x(1 + kx^2)$

$$a = v\frac{dv}{dx}$$

$$\therefore v\frac{dv}{dx} = 600x(1 + kx^2)$$

$$\int_{7.5}^{15} v\,dv = \int_{0}^{0.45} 600x(1 + kx^2)\,dx$$

$$\int_{7.5}^{15} v\,dv = \int_{0}^{0.45} (600x + k\,600\,x^3)\,dx$$

$$\left[\frac{v^2}{2}\right]_{7.5}^{15} = \left[\frac{600x^2}{2} + \frac{600\,kx^4}{4}\right]_0^{0.45}$$

$$\frac{15^2}{2} - \frac{7.5^2}{2} = \frac{600 \times 0.45^2}{2} + \frac{600\,k\,(0.45)^4}{4}$$

$$112.5 - 28.12 = 60.75 + 6.15k$$

$$\underline{\underline{k = 3.84}}$$

---

(6) A particle travels in a straight line such that for a short time $t = 2$s to 6s. Its motion is described by $v = \frac{4}{a}$ m/s where a is in m/s². If $v = 6$ m/s when $t = 2$s, determine the particle accⁿ when $t = 3$s.

→ w.k.t $v = \frac{4}{a}$ m/s.

$$\left\{\begin{array}{l} @\ t = 3s \\ v = ? \\ a = ? \end{array}\right\}$$

$$a = \frac{4}{v} = \frac{dv}{dt}$$

$\therefore v \cdot dv = 4\, dt$

- For velocity @ 3 secs

$$\int_6^v v\,dv = 4\int_2^3 dt$$

$$\left[\frac{v^2}{2}\right]_6^v = 4\,[t]_2^3$$

$$\frac{v^2}{2} - \frac{6^2}{2} = 4(3-2)$$

$$0.5v^2 - 18 = 4$$

$$\underline{\underline{v = 6.63\ m/s}} \quad @\ t = 3s.$$

- $\therefore$ a @ 3 secs

$$a = \frac{4}{v} = \frac{4}{6.63} = \underline{\underline{0.603\ m/s^2}}$$

---

7) Rectilinear motion of a particle is described by equ^n $a = -0.4v$ where a & v are in mm/s^2 & mm/s respty. If $v = 30$ mm/s @ $t = 0$, find the distance travelled by particl before coming to rest.

→ w.k.t $a = -0.4v$

$$a = v\frac{dv}{dx}$$

$$\therefore v\frac{dv}{dx} = -0.4v$$

$$\cancel{v}\,dv = -0.4\cancel{v}\,dx$$

→ when particle comes to rest $v = 0$; $x = ?$

$$\therefore \int_{30}^{0} dv = -0.4\int_0^x dx$$

$$[v]_{30}^{0} = -0.4\,[x]_0^x$$

$$0 - 30 = -0.4x - 0$$

$$x = 75\,mm$$

---

8) A particle starts from rest @ the origin & is given an accⁿ $a = \dfrac{K}{(x+10)^2}$ where a & x are expressed in m/s² & m respectively & K is a constant. If the velocity of a particle is $v = 10\,m/s$ when $x = 20\,m$, determine the value of K & the max velocit[y]

→ w.k.t $a = \dfrac{K}{(x+10)^2}$

$$a = \frac{v\,dv}{dx}$$

$$\therefore v\frac{dv}{dx} = \frac{K}{(x+10)^2}$$

$$v\,dv = \frac{K}{(x+10)^2}\,dx$$

→ when particle is @ $v = 10\,m/s$ & $x = 20\,m$

$$\int_0^{10} v\,dv = K\int_0^{20} \frac{dx}{(x+10)^2}$$

$$\left[\frac{1}{x^2} = -\frac{1}{x}\right]$$

$$\left[\frac{v^2}{2}\right]_0^{10} = K\left[\frac{-1}{x+10}\right]_0^{20}$$

$$\frac{0^2}{2} - \frac{10^2}{2} = K\left[\frac{-1}{0+10} - \frac{-1}{20+10}\right]$$

$$-50 = K\left[-0.1 + 0.033\right]$$

$$K = 750$$

→ For max velocity

$$\frac{K}{(x+10)^2} = 0$$

$\therefore x = \infty$

& $K = 750$; $v = v_{max}$

$\therefore$ By substituting these values in above eqn

$$\frac{v_{max}^2}{2} = 750\left[\frac{1}{10} - \frac{1}{\infty + 10}\right]$$

$$\frac{v_{max}^2}{2} = 750\left[\frac{1}{10} - 0\right]$$

$$v_{max} = 12.24 \text{ m/s}$$

---

9) A particle moves along a straight path in a viscous medium with accn $a = -\frac{2}{x^2}$ where a is m/s² & x in m. Knowing that $x = 1$m, $v = 2$ m/s @ $t = 1$s, determine the position & velocity of a particle @ $t = 4$s.

$\rightarrow$ w.k.t $a = -\frac{2}{x^2} = v\frac{dv}{dx}$

$$\therefore v\,dv = \frac{-2}{x^2}\,dx$$

$$\int_2^v v\,dv = -2\int_1^x \frac{1}{x^2}\,dx$$

$$\left[\frac{v^2}{2}\right]_2^v = -2\left(\frac{-1}{x}\right)_1^x$$

$$\frac{v^2}{2} - \cancel{\frac{2^2}{2}} = \frac{2}{x} - \cancel{\frac{2}{1}}$$

$$\frac{v^2}{2} = \frac{2}{x}$$

$$v^2 = \frac{4}{x}$$

$$v = \frac{2}{\sqrt{x}}$$

- w.k.t $v = \frac{dx}{dt}$

$\therefore \frac{dx}{dt} = \frac{2}{\sqrt{x}}$

$\sqrt{x}\, dx = 2\, dt$

- to determine position @ $t = 4s$.

$\int_1^x \sqrt{x}\, dx = 2\int_1^4 dt$

$\left[\frac{x^{3/2}}{3/2}\right]_1^x = 2[t]_1^4$

$\frac{2}{3}\left[x^{3/2} - 1\right] = 2[4-1]$

$x^{3/2} - 1 = 9$

$x^{3/2} = 10$

$\underline{\underline{x = 4.74\,m}}$

- to determine velocity @ $t = 4s$.

w.k.t $v = \frac{2}{\sqrt{x}} = \frac{2}{\sqrt{4.74}} = \underline{0.92\,m/s}$

---

(10) The motion of a particle starting from rest is defined by $a = 10t - t^2$. Find the displacement before it starts in reverse direction of motion & velocity when acc^n changes its direction.

→ w.k.t $a = 10t - t^2 = \frac{dv}{dt}$

$\therefore \frac{dv}{dt} = 10t - t^2$

$dv = (10t - t^2)\, dt$

$\int_0^v dv = \int_0^t (10t - t^2)\, dt$

$[v]_0^v = \left[\frac{10t^2}{2} - \frac{t^3}{3}\right]_0^t$

$\therefore v = 5t^2 - \frac{t^3}{3}$

— w.k.t $v = \frac{dx}{dt}$

$$\therefore \frac{dx}{dt} = 5t^2 - \frac{t^3}{3}$$

$$dx = \left(5t^2 - \frac{t^3}{3}\right) dt$$

$$\int_0^x dx = \int_0^t \left(5t^2 - \frac{t^3}{3}\right) dt$$

$$[x]_0^x = \left[\frac{5t^3}{3} - \frac{t^4}{3\times4}\right]_0^t$$

$$x = \frac{5t^3}{3} - \frac{t^4}{12}$$

— when particle reverses the direction, $v = 0$

$$\therefore \frac{5t^2}{1} - \frac{t^3}{3} = 0$$

$$\frac{15t^2 - t^3}{3} = 0$$

$$15t^2 - t^3 = 0$$

$$\therefore t = 15s$$

— $\therefore$ displacement before it starts in reverse directi

$$x = \frac{5\times15^3}{3} - \frac{15^4}{12}$$

$$x = 1406.25m$$

— when accⁿ changes its direction, $a = 0$

$$\therefore 0 = 10t - t^2$$

$$t = 10s$$

– velocity when acc^n changes its direction

$$v = 5\times10^2 - \frac{10^3}{3}$$

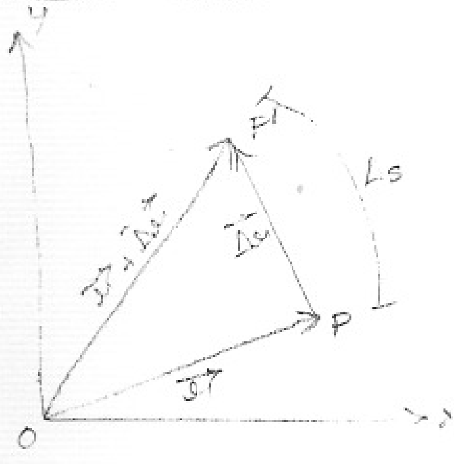
$$\underline{v = 166.67\ m/s}$$

CURVILINEAR MOTION

⟹ Kinematics of Curvilinear motion

The motion of a particle is said to be curvilinear when it moves along a curved path. It can be either in two or three dimensions. This motion can be analysed using $\square^{lar}$ components, normal & tangential components (or) radial & transverse components. {In this chapter, discussions is made only on two dimension problems using $\square^{lar}$ components only}

→ Position, velocity & acc^n in curvilinear motion



- For a particle moving along a curved path in a plane, the position, velocity & acc^n are vector quantities.
- Consider a particle moving along a curved path as shown in the fig.
- If the particle is @ point P @ time t, then its position is defined by vector $\vec{r}$ which is directed from O to P
- Let the particle be @ position P' @ a time $t + \Delta t$. The position vector @ P' is $\vec{r} + \vec{\Delta r}$ where $\vec{\Delta r}$ is the change in position.

i.e, PP' is the displacement & Δs is the distan

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

where $v_x = \frac{dx}{dt}$ & $v_y = \frac{dy}{dt}$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2}$$

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

→ Differentiating again w.r.t t

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

where $a_x = \frac{dv_x}{dt}$ & $a_y = \frac{dv_y}{dt}$

$$\therefore\ a = \sqrt{a_x^2 + a_y^2}$$

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

⇒ Exercise problems

1) The motion of particles A & B is defined by their position vectors $\vec{r_A} = (2t^2 + 4t - 6)\hat{i} + (3t^2 + 6t + 1)\hat{j}$ & $\vec{r_B} = (t^2 + 2t + 2)\hat{i} + (11t + 3)\hat{j}$ where r & t are in m & sec ~~resp~~. Determine the point where particles collide & their speed just before collision. At what time do they collide?

→ The two particles collide when $\vec{r_A} = \vec{r_B}$

i.e, $2t^2 + 4t - 6 = t^2 + 2t + 2$

$$t^2 + 2t - 8 = 0$$

$$(t - 2)(t + 4) = 0$$

$$t = 2 \text{ (or) } t = -4$$

travelled (Length of arc PP')

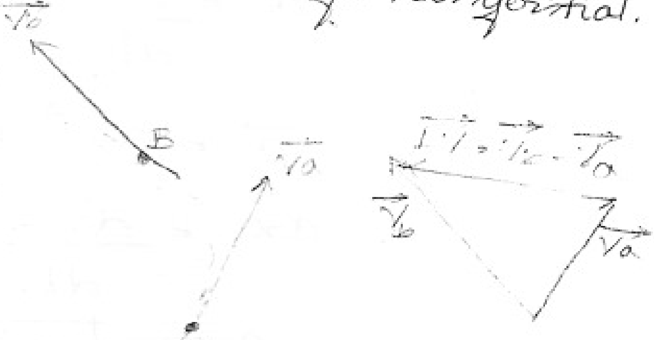
→ ∴ avg velocity = $\vec{v}_{av} = \dfrac{\Delta \vec{r}}{\Delta t}$

& instantaneous velocity = $\vec{v} = \lim_{\Delta t \to 0} \dfrac{\Delta \vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$

- The magnitude of $\vec{v}$ is speed.

→ The velocity of the particle is always tangential.

- Consider velocity vectors @ two points A & B as shown in the figure



- The change in velocity is given by

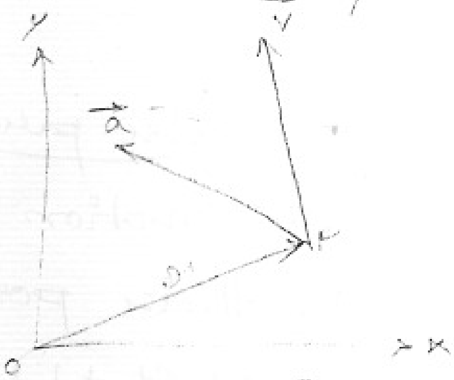
$\vec{\Delta v} = \vec{v_B} - \vec{v_A}$ (redrawn from common point)

→ The avg acc^n is given by

$\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$

& instantaneous acc^n is

$\vec{a} = \lim_{\Delta t \to 0} \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{d\vec{v}}{dt}$



{Velocity is tangential but acc^n is not}

→ Rectangular components

→ The position vector $\vec{r}$ of a particle

$\vec{r} = x\hat{i} + y\hat{j}$

where x & y are the function of t

→ Differentiating w.r.t t, we get

$\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j}$

& $3t^2 + 6t + 1 = 11t + 3$

$3t^2 - 5t - 2 = 0$

$3t^2 - 6t + t - 2 = 0$

$3t(t-2) + 1(t-2) = 0$

$(3t+1)(t-2) = 0$

$t = -1/3$ or $t = 2$

- As time cannot be -ve ; $\underline{t = 2 \text{ sec}}$ (from 1st equn)

- $\therefore \vec{r_A} = (2\times 2^2 + 4\times 2 - 6)\hat{i}$
$+ (3\times 2^2 + 6\times 2 + 1)\hat{j}$

$$\underline{\vec{r_A} = 10\hat{i} + 25\hat{j}}$$

- The point where they collide is (10, 25)

$\rightarrow \vec{V_A} = \dfrac{d\vec{r_A}}{dt} = (4t+4)\hat{i} + (6t+6)\hat{j}$

$\therefore$ @ $t = 2s$

$\vec{V_A} = (4\times 2 + 4)\hat{i} + (6\times 2 + 6)\hat{j}$

$\underline{\vec{V_A} = 12\hat{i} + 18\hat{j}}$

$V_A = \sqrt{12^2 + 18^2} = \underline{21.63 \text{ m/s}}$

$\rightarrow \vec{V_B} = \dfrac{d\vec{r_B}}{dt} = (2t+2)\hat{i} + 11\hat{j}$

$\therefore$ @ $t = 2s$

$\vec{V_B} = (2\times 2 + 2)\hat{i} + 11\hat{j}$

$\underline{\vec{V_B} = 6\hat{i} + 11\hat{j}}$

$V_B = \sqrt{6^2 + 11^2} = \underline{12.53 \text{ m/s}}$

2) A particle moves along the path $y = 4x^2 + 8x + 10$ starting with an initial velocity $\vec{v_0} = 5\hat{i} + 3\hat{j}$. If v_x is constant determine v_y & a_y @ $x = 3m$. (5)

$\Rightarrow$ w.k.t $\vec{v_0} = 5\hat{i} + 3\hat{j}$

i.e, $v_{x0} = 5m/s$ & $v_{y0} = 3m/s$

& v_{x0} is constant ; $\therefore$ $\underline{v_x = 5m/s}$

– w.k.t $v_x = \frac{dx}{dt}$ & $a_x = \frac{dv_x}{dt}$

& $v_y = \frac{dy}{dt}$ & $a_y = \frac{dv_y}{dt}$

& $y = 4x^2 + 8x + 10$

– Differentiate y w.r.t t

$\therefore$ $v_y = \frac{dy}{dt} = (8x + 8)\frac{dx}{dt}$

$v_y = 8x\frac{dx}{dt} + \frac{8\,dx}{dt}$

$\underline{v_y = 8x\,v_x + 8\,v_x}$

– $\therefore$ @ $x = 3m$ & $v_x = 5m/s$

$v_y = 8 \times 3 \times 5 + 8 \times 5 = \underline{160m/s}$

– Differentiate v_y w.r.t t

$\therefore$ $a_y = \frac{dv_y}{dt} = 8\,v_x\frac{dx}{dt} + 0$ {since v_x is const

$a_y = 8\,v_x\,v_x = 8v_x^2$

– $\therefore$ @ $v_x = 5m/s$

$a_y = 8 \times 5^2 = \underline{200m/s^2}$

3) The y-coordinate of a particle moving along a curve is $y = t^3 - 6t + 3$ where y & t are in m & secs resp. Its accⁿ in x-direction is given by $a_x = 4t + 3$ m/s². If the velocity of the particle in x-direcⁿ is 2 m/s when t = 0, calculate the magnitude & direction of velocity & accⁿ of the particle when t = 1 s. (6)

→ w.k.t $y = t^3 - 6t + 3$

$$\therefore v_y = \frac{dy}{dt} = 3t^2 - 6$$

$$\text{(III)}^{ly}\ a_y = \frac{dv_y}{dt} = 6t$$

→ In x-direction.

$$a_x = 4t + 3 = \frac{dv_x}{dt}$$

$$\therefore v_x = \frac{4t^2}{2} + 3t + C_1 \quad \{\text{By integrating}\}$$

– @ t = 0 & v_x = 2 m/s

$$2 = \frac{4\times 0^2}{2} + 3\times 0 + C_1$$

$$\underline{C_1 = 2}$$

$$\therefore \underline{v_x = 2t^2 + 3t + 2}$$

→ when t = 1 s

$$v_x = 2\times 1^2 + 3\times 1 + 2 = \underline{7 \text{ m/s}}$$

$$v_y = 3\times 1^2 - 6 = \underline{-3 \text{ m/s}}$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{7^2 + (-3)^2} = \underline{7.62 \text{ m/s}}$$

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-3}{7}\right) = 23.2^\circ\ (\measuredangle\theta_v)$$

- (iii) $a_x = 4 \times 1 + 3 = \underline{7\ m/s^2}$

$a_y = 6 \times 1 = \underline{6\ m/s^2}$

$\therefore a = \sqrt{a_x^2 + a_y^2} = \sqrt{(7)^2 + (6)^2} = \underline{9.22\ m/s^2}$

$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{6}{7}\right) = 40.6^\circ$ (θ_a)

---

4) The velocity of a particle moving in x-y plan is given by $\vec{v} = 3\hat{i} - 2\hat{j}$ m/s @ t = 2s. Its avg acc^n during the next 2s is $\hat{i} + 2\hat{j}$ m/s. Determine the velocity @ t = 4s & the angle b/w the avg acc^n vector & the velocity vector @ t = 4s?

→ w.k.t $\vec{v_2} = 3\hat{i} - 2\hat{j}$

& $\vec{a_{av}} = \hat{i} + 2\hat{j}$

- $\vec{a_{av}} = \frac{\vec{v_4} - \vec{v_2}}{\Delta t}$ [$\Delta t = 4 - 2 = 2$]

$\therefore \vec{v_4} = \vec{v_2} + \vec{a_{av}} \times \Delta t$

$\vec{v_4} = 3\hat{i} - 2\hat{j} + (\hat{i} + 2\hat{j}) \times 2$

$\underline{\vec{v_4} = 5\hat{i} + 2\hat{j}}$

- $\therefore \theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{2}{5}\right) = \underline{21.8^\circ}$ (θ_v)

- $\therefore \theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{2}{1}\right) = \underline{63.43^\circ}$ (θ_a)

- Angle b/w avg acc^n vector & velocity vector @ t = 4s is 63.43 − 21.8

$= \underline{41.63^\circ}$

---

(8)

5) A particle moves in a curved path so that $a_x = 40 - 12t$ m/s² & $y = t^3$ m, where t is in secs. If the particle starts from rest, @ the origin, determine the velocity of the particle @ t = 2s.

→ w.k.t $y = t^3$

$\therefore v_y = 3t^2$

$a_y = 6t$

→ (III)^{ly} $a_x = 40 - 12t$

$v_x = 40t - \frac{12t^2}{2} + C_1$

- when $t = 0$; $v_x = 0$ @ origin.

$\therefore 0 = 0 - 0 + C_1$

$C_1 = 0$

$\therefore V_x = 40t - 6t^2$

→ @ t = 2s

$v_y = 3 \times 2^2 = 12$ m/s

$v_x = 40 \times 2 - 6 \times 2^2 = 56$ m/s

$\therefore v = \sqrt{v_y^2 + v_x^2} = \sqrt{12^2 + 56^2} = 57.27$ m/s

$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{12}{56}\right) = 12.1°$ (∠θ_v)

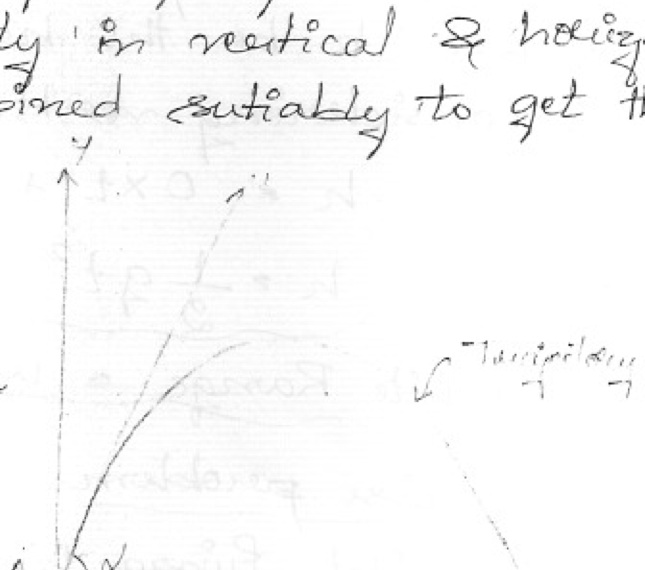
UNIT - II

PROJECTILES

→ **Projectiles**

The particle moves along a curved path if it is freely projected in the air in the direction other than vertical. These freely projected particles which are having the combined effect of a vertical & a horizontal motion are called "projectiles".

The motion of a projectile has a vertical & a horizontal component. The vertical component of the motion is subjected to gravitational accⁿ/retardation while horizontal component remains constant, if air resistance is neglected. The motion of a projectile can be analysed independently in vertical & horizontal directions & then combined suitably to get the total effect.



→ **Velocity of projection**

It is the velocity with which the particle is projected "u" m/s.

→ **Angle of projection**

It is the angle b/w the direction of projection & horizontal direction (α)

→ **Trajectory**

It is the path traced by the projectile.

⟹ Horizontal range / Range

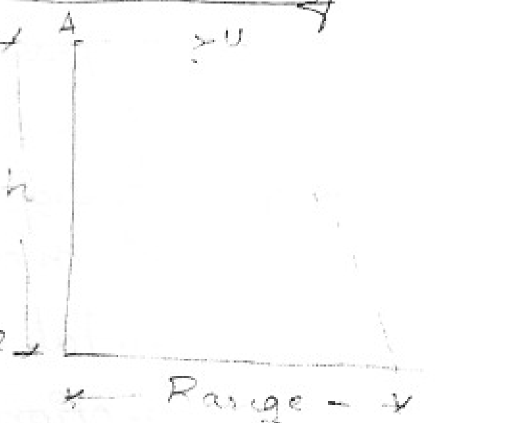
It is the horizontal distance through which the projectile travels in its flight.

⟹ Time of flight

It is the time interval during which the projectile is in motion.

⟹ Motion of body projected horizontally

- Consider a particle thrown horizontally from point A with a velocity u m/sec as shown in the fig.



- At any instant, the particle is subjected to :-
  a) Horizontal motion with constant velocity u m/s
  b) Vertical motion with initial velocity zero & moving with acc^n due to gravity 'g'.
- Let h be the height of A from the ground

Considering vertical motion,

$$h = 0 \times t + \frac{1}{2} g t^2$$

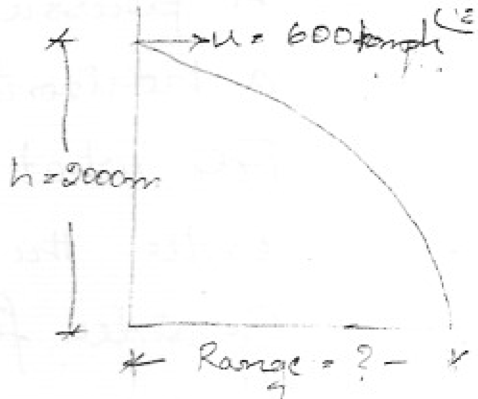
$$h = \frac{1}{2} g t^2$$

while Range = ut

⟹ Exercise problems

1) A pilot flying his bomber @ a height of 2000m with a uniform horizontal velocity of 600 kmph wants to strike a target. At what distance from the target, he should release the bomb?

$\rightarrow$ $h = 2000\ m$



$u = 600\ kmph$

$= \dfrac{600 \times 1000}{60 \times 60}\ m/s$

$\underline{u = 166.67\ m/s}$

- w.k.t initial velocity is zero

& $h = 0 \times t + \frac{1}{2} g t^2$

$\therefore 2000 = \frac{1}{2} \times 9.81\ t^2$

$\underline{t = 20.2\ s}$

- Range $= ut = 166.67 \times 20.2$

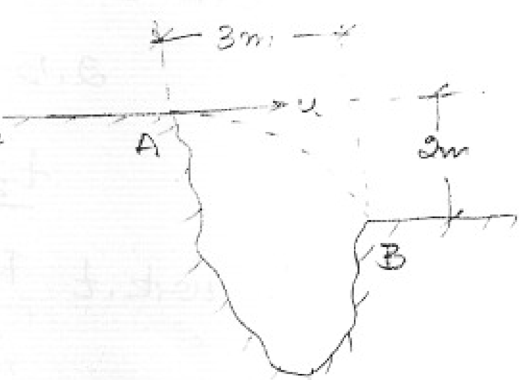
$= \underline{3365.5\ m}$

---

2) A person wants to jump over a ditch as shown in the fig. Find the minimum velocity with which he should jump.

$\rightarrow$ Here $h = 2m$

$R = 3m$



- $\therefore$ w.k.t $h = 0 \times t + \frac{1}{2} g t^2$

$\therefore 2 = \frac{1}{2} \times 9.81\ t^2$

$\underline{t = 0.638\ s}$

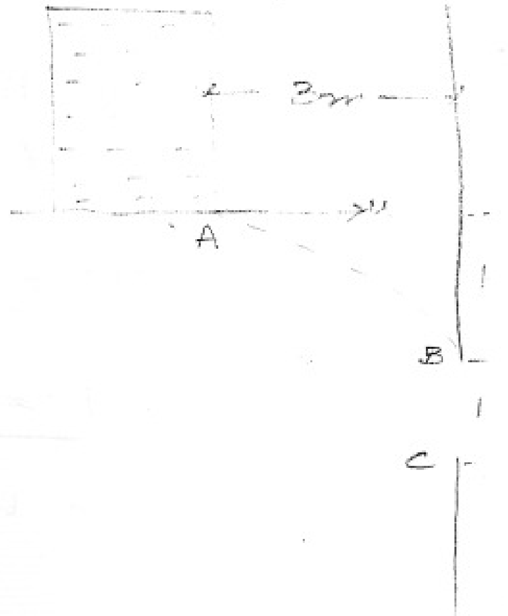
- w.k.t $R = ut$

$3 = u \times 0.638$

$\therefore \underline{u = 4.69\ m/s}$

3) A pressure tank issues water @ A with a horizontal velocity u as shown in the fig. For what range of values of u, water will enter the opening BC? (4

→ Consider for point B



$h = 1m$

$\therefore$ w.k.t $h = 0 \times t_1 + \frac{1}{2} g t_1^2$

$$1 = \frac{1}{2} \times 9.81 t_1^2$$

$$\underline{t_1 = 0.45 s}$$

$\therefore$ w.k.t $R_1 = u_1 t_1$

$$3 = u_1 \times 0.45$$

$$\underline{u_1 = 6.67 m/s}$$

→ Consider for point C

$h = 1 + 1.5 = 2.5m$

$\therefore$ w.k.t $h = 0 \times t_2 + \frac{1}{2} g t_2^2$

$$2.5 = \frac{1}{2} \times 9.81 t_2^2$$

$$\underline{t_2 = 0.713 s}$$

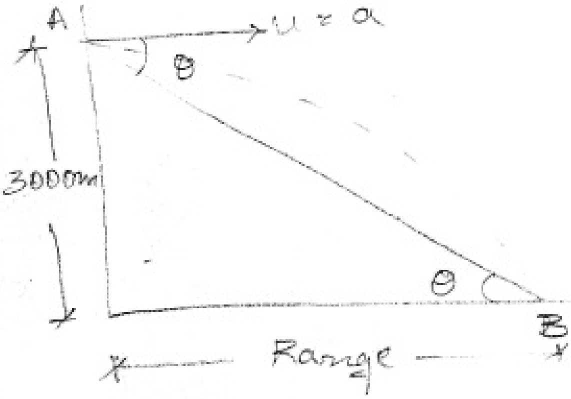
$\therefore$ w.k.t $R_2 = u_2 t_2$

$$3 = u_2 \times 0.713$$

$$\underline{u_2 = 4.202 m/s}$$

→ $\therefore$ the range of velocity for which the jet can enter the opening BC is 4.202 m/s to 6.67 m/s.

4) A rocket is released from a jet fighter flying horizontally @ 1200 kmph @ an altitude of 3000m above its target. The rocket thrust gives it a constant horizontal accⁿ of $6\ m/s^2$. At what angle below the horizontal should pilot see the target @ the instant of releasing the rocket in order to score a hit?



→ In vertical direction

$h = 3000m$

$u = 0$

$\therefore$ w.k.t $h = 0 \times t + \frac{1}{2} g t^2$

$\therefore 3000 = \frac{1}{2} \times 9.81 t^2$

$\underline{t = 24.73s}$ (time of flight)

→ In horizontal direction

$u = 1200$ kmph & $a = 6\ m/s^2$

$\therefore$ w.k.t $s = ut + \frac{1}{2} at^2$ (here s = range)

$\therefore \quad u = \frac{1200 \times 1000}{60 \times 60} = \underline{333.3\ m/s}$

$\therefore \quad S = 333.3 \times 24.73 + \frac{1}{2} \times 6 \times 24.73^2$

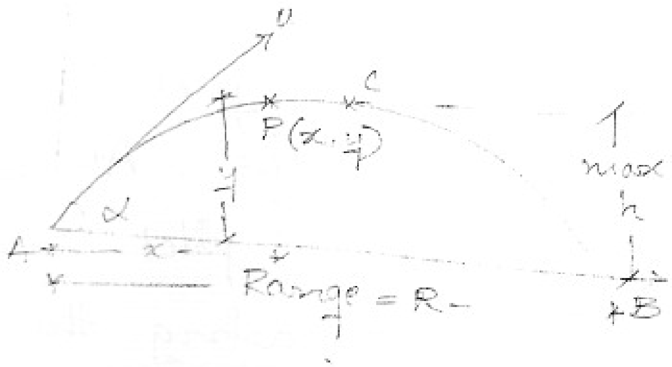
$\underline{S = 10,078.1m}$

→ $\therefore \tan\theta = \frac{h}{s}$

$\theta = \tan^{-1} \frac{3000}{10,078.1} = \underline{16.57^\circ}$

⇒ Inclined projection on level ground

→ Consider the motion of a projectile projected from point A.



→ In horizontal motion

$u_x = u \cos \alpha$

$a_x = 0$

$S_x = x$

Final velocity = v_x

$\therefore$ $v_x = u \cos \alpha$

$x = (u \cos \alpha) t$ $\quad (\because a = 0$ & $x = ut + \frac{1}{2} a t^2)$

$v_x^2 = u^2 \cos^2 \alpha$

→ In vertical motion

$u_y = u \sin \alpha$

$a_y = -g$

$S_y = y$

Final velocity = v_y

$\therefore$ $v_y = u \sin \alpha - gt$

$y = (u \sin \alpha) t - \frac{1}{2} g t^2$

$v_y^2 = u^2 \sin^2 \alpha - 2gy$

→ $\therefore$ velocity @ any time 't' is :-

$\vec{v} = v_x \hat{i} + v_y \hat{j}$

→ Note :- Initially particle moves upward with velocity $u \sin \alpha$ & retardation $9.81\ m/s^2$ and

$\therefore$ w.k.t $v^2 - u^2 = 2as$ (Here, $s = h$)

$\therefore 0 - u^2 \sin^2\alpha = -2gh$

$$\therefore h = \frac{u^2 \sin^2\alpha}{2g}$$

- $\therefore$ time reqd. to reach max height

- w.k.t $v = u + at$

$$0 = u\sin\alpha - gt$$

$$\therefore t = \frac{u\sin\alpha}{g}$$

⇒ <u>Time of flight & Range.</u>

- At the final point, $y = 0$

$\therefore$ w.k.t $y = (u\sin\alpha)t - \frac{1}{2}gt^2$

$$0 = (u \cdot \sin\alpha)t - \frac{1}{2}gt^2$$

$$0 = t\left(u\sin\alpha - \frac{1}{2}gt\right)$$

$\therefore \underline{t = 0}$ → initial position

(or)

$t = \dfrac{2u\sin\alpha}{g}$ → final position

- w.k.t $R = ut$

$R = (u\cos\alpha)\dfrac{2u\sin\alpha}{g}$ $(\because u_x = u\cos\alpha)$

$R = \dfrac{u^2 \sin 2\alpha}{g}$ $(\because \sin 2\alpha = 2\cos\alpha\sin\alpha)$

velocity becomes zero after sometime (@ C) & then the particle starts moving downward with gravitational accⁿ.

⇒ <u>Equation of Trajectory</u>

→ w.k.t $x = (u\cos\alpha)t$

$$\therefore t = \frac{x}{u\cos\alpha} \longrightarrow (1)$$

→ w.k.t $y = (u\sin\alpha)t - \frac{1}{2}gt^2$

- By substituting equⁿ (1) in above equⁿ,

$$y = (u\sin\alpha)\frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

$$= \frac{\cancel{u}\sin\alpha}{\cancel{u}\cos\alpha}x - \frac{1}{2}\frac{g\,x^2}{u^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

(or)

- w.k.t $\frac{1}{\cos^2\alpha} = \sec^2\alpha = (1+\tan^2\alpha)$

$$\therefore y = x\tan\alpha - \frac{g}{2}\frac{x^2}{u^2}(1+\tan^2\alpha)$$

- The above equⁿ represents a parabola, which is the equⁿ of the path (or) trajectory

⇒ <u>Maximum height</u>

- When particle reaches max height, (i.e, @ C)

Initial velocity = $u\sin\alpha$

Final velocity = 0

$a = -g$

$\therefore$ w.k.t $v^2 - u^2 = 2as$ (Here, $s = h$)

$\therefore 0 - u^2 \sin^2\alpha = -2gh$

$\therefore h = \dfrac{u^2 \sin^2\alpha}{2g}$

- $\therefore$ time reqd. to reach max height

- w.k.t $v = u + at$

$0 = u \sin\alpha - gt$

$\therefore t = \dfrac{u \sin\alpha}{g}$

⟹ Time of flight & Range.

- At the final point, $y = 0$

$\therefore$ w.k.t $y = (u \sin\alpha) t - \frac{1}{2} g t^2$

$0 = (u \sin\alpha) t - \frac{1}{2} g t^2$

$0 = t \left(u \sin\alpha - \frac{1}{2} g t \right)$

$\therefore t = 0 \xrightarrow{\text{(or)}}$ initial position

$t = \dfrac{2u \sin\alpha}{g} \longrightarrow$ final position

- w.k.t $R = ut$

$R = (u \cos\alpha) \dfrac{2u \sin\alpha}{g}$ ($\because u_x = u \cos\alpha$)

$R = \dfrac{u^2 \sin 2\alpha}{g}$ ($\because \sin 2\alpha = 2 \cos\alpha \sin\alpha$)

→ Maximum Range

- To get max value of Range in

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$\sin 2\alpha$ should have a max value of 1

$\therefore R_{max} = \frac{u^2}{g}$

{ since $\sin 2\alpha = 1$
$2\alpha = \sin^{-1} 1$
$\alpha = 45^\circ$ }

- $\therefore \alpha_{max} = 45^\circ$

- However w.k.t $\sin 2\alpha = \sin(180 - 2\alpha)$

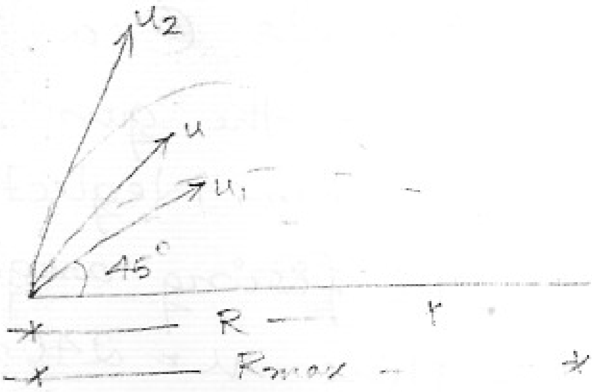
$\therefore \alpha$ has 2 values

i.e, $2\alpha_1 = 2\alpha$

$\alpha_1 = \alpha$

& $2\alpha_2 = 180 - 2\alpha$

$\alpha_2 = 90 - \alpha$



But $\alpha_1 + \alpha_2 = 90^\circ$

$\therefore$ If $\alpha_1 = 45 + \theta$

$\alpha_2 = 45 - \theta$

→ Exercise problems

1) A body is projected @ an angle such that its horizontal range is 3 times the max height. Find the angle of projection.

→ $3 h_{max} = R \longrightarrow (1)$

$\therefore h = \frac{u^2 \sin^2 \alpha}{2g}$ & $R = \frac{u^2 \sin 2\alpha}{g}$

- By substituting in eqn (1)

$$\frac{3\,u^2 \sin^2 \alpha}{2g} = \frac{u^2 \sin 2\alpha}{g}$$

$$\frac{3}{2}\sin^2\alpha = \sin 2\alpha$$

$$\frac{3}{2}\sin^2\alpha = 2\sin\alpha\cos\alpha$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{4}{3}$$

$$\tan\alpha = \frac{4}{3}$$

$$\underline{\alpha = 53.13^\circ}$$

---

2) A projectile is fired with initial velocity 240m/s @ a target 'B' located 600m above the gun & a horizontal distance of 3.6 km. Neglecting air resistance, determine the firing angles.

→ Here $u = 240\,m/s$; $y = 600\,m$; $x = 3.6\,km$

∴ w.k.t equ^n of trajectory is given by

$$y = x\tan\alpha - \frac{g x^2}{2u^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{g}{2}\frac{x^2}{u^2}(1+\tan^2\alpha)$$

$$\therefore 600 = 3600\tan\alpha - \frac{9.81}{2}\times\frac{3600^2}{240^2}(1+\tan^2\alpha)$$

$$600 = 3600\tan\alpha - 1103.5 - 1103.5\tan^2\alpha$$

$$-1103.6\tan^2\alpha + 3600\tan\alpha - 1703.6 = 0$$

$$\tan\alpha = 0.57 \text{ or } 2.68$$

$$\therefore \alpha = \underline{29.68^\circ} \text{ or } \underline{69.53^\circ}$$

3) Horizontal distance of a target to be hit by a projectile is 8 km. Projectile leaves the cannon with a velocity of 550 m/s. What must be the angle of elevation of the cannon? If the projectile has to clear a 700m high hill midway b/w target & cannon, what should be the angle of elevation of cannon?

→ Here $u = 550$ m/s ; $x = 8$ km

a) @ $y = 0$; $\alpha = ?$

w.k.t $y = x \tan\alpha - \frac{g}{2}\frac{x^2}{u^2}(1+\tan^2\alpha)$

$0 = 8000 \tan\alpha - \frac{9.81}{2} \times \frac{8000^2}{550^2}(1+\tan^2\alpha)$

$0 = 8000 \tan\alpha - 1037.7 - 1037.7 \tan^2\alpha$

$\therefore \tan\alpha = 0.132$ or 7.57

$\alpha = \underline{\underline{7.52^\circ}}$ or $\underline{82.47^\circ}$

b) The projectile has to clear 700m high hill

$\therefore$ w.k.t $h = \frac{u^2 \sin^2\alpha}{2g}$

$\therefore h = \frac{550^2 \sin^2 7.52}{2 \times 9.81} = \underline{12264.2\,m}$

$h = \frac{550^2 \sin^2 82.47}{2 \times 9.81} = \underline{-9639.7\,m}$

$\therefore \alpha = 7.52^\circ$

4) A projectile is aimed @ a target on the horizontal plane & falls 12m short when the angle of projection is 15°, while it

overshoots by 24m when the angle is 45°.
Find the angle of projection to hit the target.

→ Let s be the distance of the target from point of projection.

– w.k.t $R = \dfrac{u^2 \sin 2\alpha}{g}$

∴ a) $s - 12 = \dfrac{u^2}{g} \sin(2 \times 15)$

$s - 12 = \dfrac{u^2}{g} \times \dfrac{1}{2}$

$s - 12 = \dfrac{u^2}{2g} \longrightarrow (1)$

b) $s + 24 = \dfrac{u^2}{g} \sin(2 \times 45)$

$s + 24 = \dfrac{u^2}{g} \longrightarrow (2)$

– From eqns (1) & (2)

$s - 12 = \dfrac{1}{2}(s + 24)$

$s - 12 = 0.5s + 12$

$s - 12 - 0.5s - 12 = 0$

$\underline{\underline{s = 48m}}$

– ∴ To find α

w.k.t $s = \dfrac{u^2 \sin 2\alpha}{g}$

$48 = \dfrac{u^2}{g} \sin 2\alpha$

→ Here $\frac{u^2}{g} = S + 24$

$\therefore \frac{u^2}{g} = 48 + 24$

$\frac{u^2}{g} = 72$

$\therefore 48 = 72 \times \sin 2\alpha$

$2\alpha = 41.81^\circ$

$\underline{\alpha = 20.9^\circ}$

---

5) The horizontal component of the velocity of a projectile is twice its initial vertical component. Find the range of the horizontal plane, if the projectile passes through a point 18m horizontally & 3m vertically above the point of projection.

→ Here $u_x = 2u_y$

w.k.t $u_x = u\cos\alpha$ & $u_y = u\sin\alpha$

$\therefore \cancel{u}\cos\alpha = 2\cancel{u}\sin\alpha$

$\tan\alpha = 1/2$

$\underline{\alpha = 26.6^\circ}$

→ Here $R = ?$ when $y = 3m$ & $x = 18m$

$\therefore$ w.k.t $y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$

$3 = 18\tan 26.6 - \frac{9.81 \times 18^2}{2 \times u^2}(1 + \tan^2 26.6)$

$3 = 9.013 - \frac{1589.22}{u^2} + \frac{350.5}{u^2}$

$\underline{u = 18.19\ m/s}$

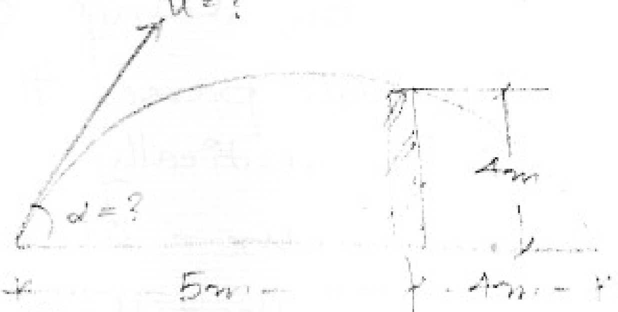
$$\therefore R = \frac{u^2 \sin 2\alpha}{g}$$

$$= \frac{18.19^2 \sin 2 \times 26.56}{9.81}$$

$$\underline{R = 27m}$$

6) Find the least initial velocity with which a projectile has to be projected so that it clears a wall 4m height located @ a distance of 5m, & strikes the ground @ a distance 4m beyond the wall. The point of projection is @ the same level as the foot of the wall.

→ Here $R = 9m$



w.k.t $R = \frac{u^2 \sin 2\alpha}{g}$

$$9 = \frac{u^2 \sin 2\alpha}{g}$$

$$u^2 = \frac{9 g}{\sin 2\alpha} \rightarrow (1)$$

→ w.k.t equ^n of trajectory is given by

$$y = x \tan\alpha - \frac{g x^2}{2u^2 \cos^2\alpha}$$

{ Here $y = 5m$ & $x = 4m$

$$4 = 5 \tan\alpha - \frac{g \times 5^2}{2 \times \frac{9g}{\sin 2\alpha} \cos^2\alpha}$$

w.k.t $\sin 2\alpha = 2 \sin\alpha \cos\alpha$

$$\therefore 4 = 5 \tan\alpha - \frac{25}{\frac{18 \cos^2\alpha}{2 \sin\alpha \cos\alpha}}$$

$$4 = 5 \tan\alpha - \frac{25 \times 2 \sin\alpha}{18 \cos\alpha}$$

30° upwards. Neglecting air resistance, find (6)

a) total time of flight
b) horizontal range of the bullet
c) max height reached by the bullet
d) final velocity of the bullet just before touching the ground.

→ Here $y_0 = +120m$

$u = 360\,kmph = \frac{360 \times 1000}{60 \times 60} = \underline{100\,m/s}$

$\alpha = 30°$

a) <u>Time of flight</u>

- w.k.t $-y_0 = u \sin\alpha\, t - \frac{1}{2} g t^2$

$-120 = 100 \sin 30 \times t - \frac{1}{2} \times 9.81 t^2$

$-120 = 50t - 4.905t^2$

$t = -2s$ or $12.19s$

$\therefore t = 12.19s$

b) <u>Horizontal range</u>

- w.k.t $R = u\cos\alpha \times t$

$= 100 \cos 30 \times 12.19$

$R = 1055.6m$

c) <u>Max height reached by the bullet</u>

$h = \frac{u^2 \sin^2\alpha}{2g}$

$= \frac{100^2 \sin^2 30}{2 \times 9.81}$

$h = 127.42m$ (above point A)

$\therefore 120 + 127.42 = \underline{247.42m}$ (above the ground)

$$4 = 5\tan\alpha - 2.77\tan\alpha$$

$$4 = 2.22\tan\alpha$$

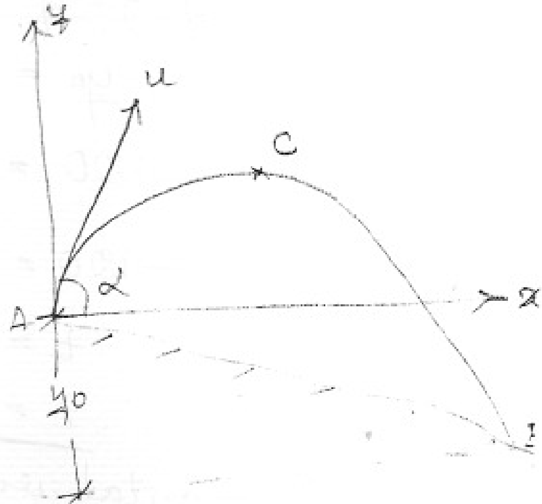
$$\tan\alpha = 1.8$$

$$\alpha = 60.94^\circ$$

- ∴ From equn ①

$$u^2 = \frac{9 \times 9.81}{\sin 2 \times 60.94} = 10.2\,m/s$$

⇒ Inclined projection with point of projection & point of strike @ different levels



- Consider the point of projection is @ a height y_0 above the point of strike as shown in the figure.
- w.k.t $S = ut - \frac{1}{2}gt^2$

Here $S = y = -y_0$
$u = u_y = u\sin\alpha$ } → in vertical direction

- ∴ time of flight is obtained by

$$-y_0 = u\sin\alpha\, t - \frac{1}{2}gt^2$$

- The horizontal range is given by

$$R = V_x t$$

$$R = u\cos\alpha \times t$$

{∵ velocity = $\frac{\text{distance}}{\text{time}}$

⇒ Exercise problems

1) A bullet is fired from a height of 120m @ a velocity of 360kmph @ an angle of

d) Final velocity

- w.k.t Horizontal component of velocity

$v_x = u \cos \alpha$

$= 100 \cos 30$

$v_x = 86.6$ m/s

- w.k.t Vertical component of velocity

$v_y = u \sin \alpha - gt$

$= 100 \sin 30 - 9.81 \times 12.19$

$v_y = -69.58$ m/s (or)

$v_y = 69.58$ m/s downward

- $\therefore$ velocity $= v = \sqrt{v_x^2 + v_y^2}$

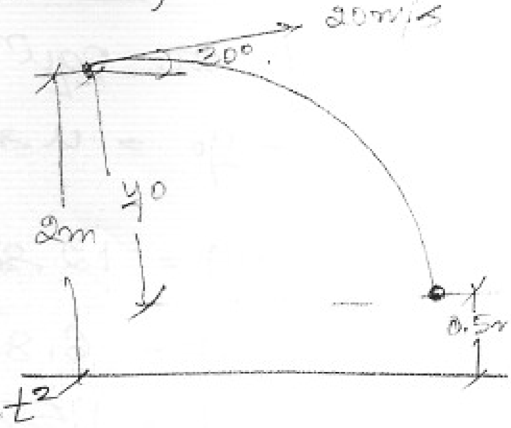
$= \sqrt{(86.6)^2 + (69.58)^2}$

$v = 111.1$ m/s

- $\therefore \theta_r = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{69.58}{86.6}\right) = 38.78°$

---

2) A cricket ball thrown by a fielder from a height of 2m, @ an angle of 30° to the horizontal, with an initial velocity of 20m/s, hits the wickets @ a height of 0.5m from the ground. How far was the fielder from the wicket



→ Here $u = 20$ m/s

$\therefore y_0 = 2 - 0.5 = 1.5$ m

- w.k.t time of flight is calculated by

$-y_0 = u \sin \alpha t - \frac{1}{2} g t^2$

$-1.5 = 20 \sin 30 t - \frac{1}{2} \times 9.81 t^2$

$-1.5 = 10t - 4.905 t^2$

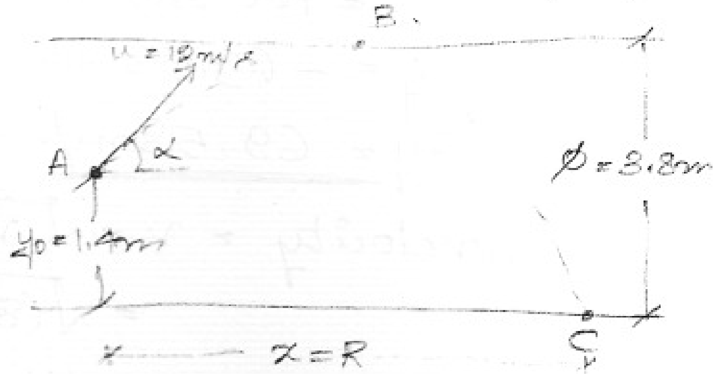
$t = 2.17$ s

$\therefore$ Range = $u\cos\alpha \times t$

$= 20\cos 30 \times 2.17$

$R = 37.58m$

3) A man standing in a horizontal tunnel of 3.8m diameter throws a stone from a point A as shown in the fig. Determine max possible distance x knowing that initial velocity of projection is 12m/s



⇒ Here max height reached by projectile

$$h = \frac{u^2 \sin^2\alpha}{2g}$$

$$\therefore (3.8 - 1.4) = \frac{12^2 \sin^2\alpha}{2 \times 9.81}$$

$$2.4 = 7.33 \sin^2\alpha$$

$$\sin^2\alpha = 0.327$$

$$\sin\alpha = \sqrt{0.327}$$

$$\sin\alpha = 0.572$$

$$\alpha = 34.9°$$

- w.k.t the eqn of trajectory is given by

$$-y_0 = u\sin\alpha t - \frac{1}{2}gt^2$$

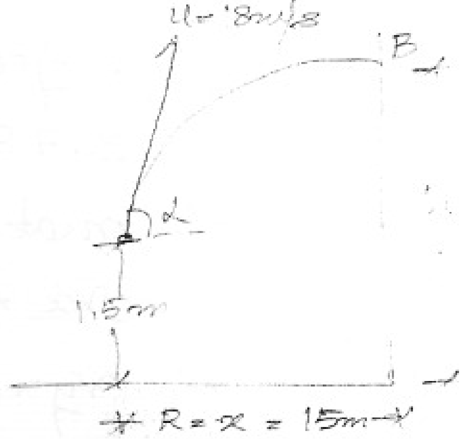
$$-1.4 = 12\sin 34.9 t - \frac{1}{2} \times 9.81 t^2$$

$$-1.4 = 6.86t - 4.9t^2$$

$$t = 1.58s$$

- $\therefore$ $R = u\cos\alpha \times t = 12\cos 34.9 \times 1.58 = 15.5m$

4) A player throws a ball from point A as shown in the figure with a velocity 18m/s. Determine max height h @ which ball can strike the wall & corresponding angle α.



⟹ Here $u = 18m/s$; $R = 15m$

- w.r.t the eqn of path

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$h - 1.5 = 15\tan\alpha - \frac{9.81\times15^2}{2\times18^2\cos^2\alpha}$$

$$h = 1.5 + 15\tan\alpha - 3.4\sec^2\alpha$$

If we substitute $y_0 = u\sin\alpha t - \frac{1}{2}gt^2$, then y, h & t become unknowns

- To obtain max h, differentiate the above eqn w.r.t α

$$\therefore \frac{dh}{d\alpha} = 0$$

$$0 = \frac{d}{d\alpha}(1.5 + 15\tan\alpha - 3.4\sec^2\alpha)$$

$$0 = 15\sec^2\alpha - 3.4(2\sec\alpha \cdot \sec\alpha\tan\alpha)$$

$$0 = \sec^2\alpha(15 - 6.81\tan\alpha)$$

- we have two solutions
i.e, $\sec\alpha \neq 0$ &

$$15 - 6.81\tan\alpha = 0$$

$$\therefore \tan\alpha = \frac{15}{6.81}$$

$$\underline{\alpha = 65.57^\circ}$$

- $\therefore h = 1.5 + 15\tan 65.57 - 3.4\sec^2 65.57$

$\underline{h = 14.61m}$

5) A projectile is launched from a gun. After 3.78 secs, the velocity of the projectile is observed to make an angle of 30° with the horizontal & at 4.79 s it reaches its max height. Calculate the initial velocity & angle of projection.

⇒ Let $t_1 = 3.78$ secs be the time while the projectile makes an angle of 30°.

- w.k.t $v_x = u\cos\alpha$

$$\& \quad v_y = u\sin\alpha - gt_1$$

$$\therefore \quad v_y = u\sin\alpha - 9.81 \times 3.78$$

- w.k.t $\tan\theta = \frac{v_y}{v_x}$

$$\tan 30 = \frac{u\sin\alpha - 37.1}{u\cos\alpha} \longrightarrow (1)$$

- When the projectile reaches max height @ $t_2 = 4.79$ secs, then

$$v_y = 0$$

i.e, $u\sin\alpha - 9.81 \times 4.79 = 0$

$$u\sin\alpha = 46.9 \longrightarrow (2)$$

- By substituting (2) in (1)

$$\tan 30 = \frac{46.9 - 37.1}{u\cos\alpha}$$

$$u\cos\alpha = 17.12 \longrightarrow (3)$$

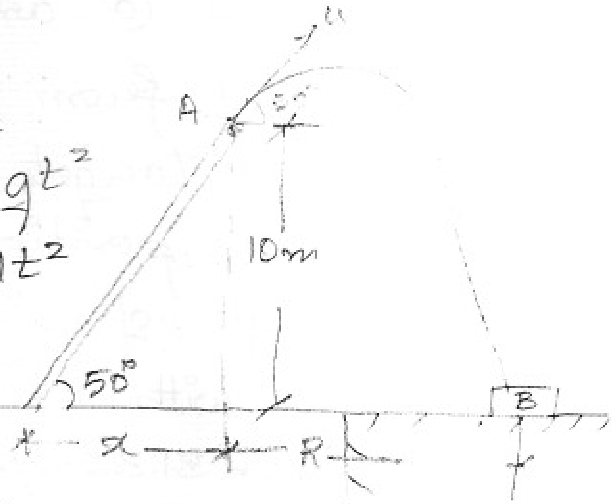
- By dividing equⁿ (2) & (3)

$$\frac{\cancel{u}\sin\alpha}{\cancel{u}\cos\alpha} = \frac{46.9}{17.12}$$

$$\underline{\alpha = 70^\circ} \; ; \; \underline{u = 50\,m/s}$$

6) Gravel is thrown into a bin from the top of a conveyor with a velocity of 5m/s. Determine :- a) time it takes the gravel to hit the bottom of the bin. b) the horizontal distance from the start of the conveyor to the bin where the gravel strikes the bin & c) the velocity @ which the gravel strikes the bin. (2)

⟹ Here $u = 5$ m/s ; $\alpha = 50°$



a) <u>To find time of flight</u>

- w.k.t $-y_0 = u \sin\alpha\, t - \frac{1}{2} g t^2$

$\therefore -10 = 5 \sin 50 t - \frac{1}{2} \times 9.81 t^2$

$-10 = 3.83t - 4.905t^2$

$\underline{t = 1.87\,s}$

b) <u>To find total distance (i.e, from conveyor to bin)</u>

- w.k.t $R = u \cos\alpha \times t$

$= 5 \cos 50 \times 1.87$

$\underline{R = 6.01m}$

- w.k.t $\cos 50° = \frac{10}{x}$ {In Δ^{le}}

$\therefore \underline{x = 15.55m}$

- $\therefore$ total distance $= 6.01 + 15.55 = \underline{21.56m}$

c) <u>To find velocity</u>

- w.k.t $V_x = u \cos\alpha$

$= 5 \cos 50$

$\underline{V_x = 3.21 m/s}$

$V_y = u \sin\alpha - gt$

$= 5 \sin 50 - 9.81 \times 1.87$

$\underline{V_y = -14.51 m/s}$ {-ve means downward}

$\therefore v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{3.21^2 + 14.51^2}$

$v = \underline{14.86\ m/s}$

$- \theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.51}{3.21}\right) = \underline{77.8^\circ}$

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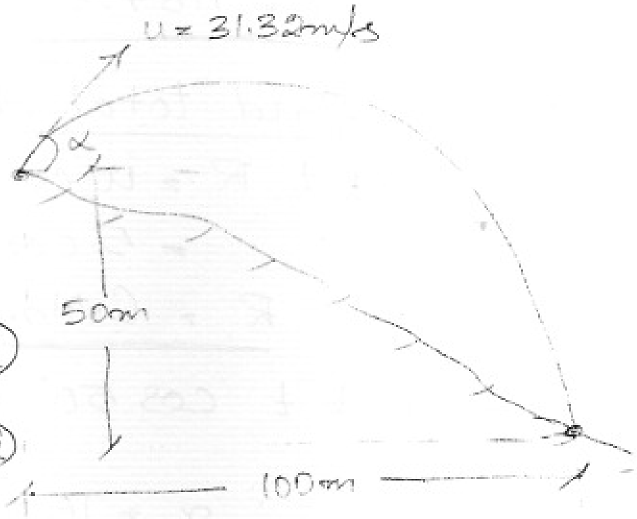
7) A soldier fires a bullet with a velocity of 31.32 m/s @ an angle α upwards from the horizontal from his position on a hill to strike a target which is 100m away & 50m below his position as shown in the fig. Find the angle of projection α. Find also the velocity with which the bullet strikes the object

→ Here $u = 31.32\ m/s$

$y = -50m$

$x = R = 100m$

{ $\alpha = ?$ & t is unknown }



$- \therefore$ w.k.t eqn of trajectory

$y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$

$\therefore -50 = 100\tan\alpha - \frac{9.81 \times 100^2}{2 \times 31.32^2}(1+\tan^2\alpha)$

$-\cancel{50} = 100\tan\alpha - \cancel{50} - 50\tan^2\alpha$

$0 = 100\tan\alpha - 50\tan^2\alpha$

$0 = 2\tan\alpha - \tan^2\alpha$

$0 = \tan\alpha(2 - \tan\alpha)$

$\therefore \tan\alpha = 0$ & $2 - \tan\alpha = 0$

$\underline{\alpha = 0}$ $\tan\alpha = 2$

$\underline{\alpha = 63.43^\circ}$

a) When $\alpha = 0$

$$-y_0 = u \sin\alpha\, t - \frac{1}{2} g t^2$$

$$-50 = 31.32 \sin 0 \times t - \frac{1}{2} \times 9.81 t^2$$

$$-50 = -4.905 t^2$$

$$t = 3.19\,s$$

- ∴ w.k.t $v_x = u \cos\alpha$

$$= 31.32 \cos 0$$

$$v_x = 31.32\,m/s$$

$$v_y = u \sin\alpha - g t$$

$$= 31.32 \sin 0 - 9.81 \times 3.19$$

$$v_y = 31.32\,m/s$$

- ∴ $v = \sqrt{(v_x)^2 + (v_y)^2}$

$$= \sqrt{31.32^2 + 31.32^2}$$

$$v = 44.3\,m/s$$

- $\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{31.32}{31.32}\right) = 45°$

b) When $\alpha = 63.43°$

$$-y_0 = u \sin\alpha\, t - \frac{1}{2} g t^2$$

$$-50 = 31.32 \sin 63.43 \times t - \frac{1}{2} \times 9.81 t^2$$

$$-50 = 28.01 t - 4.905 t^2$$

$$t = 7.13\,s$$

- w.k.t $v_x = u \cos\alpha$

$$= 31.32 \cos 63.43$$

$$v_x = 14\,m/s$$

$$v_y = u \sin\alpha - g t$$

$$= 31.32 \sin 63.43 - 9.81 \times 7.13$$

$v_y = -41.93\,m/s$ {-ve means downward}

$\therefore v = \sqrt{v_x^2 + v_y^2}$

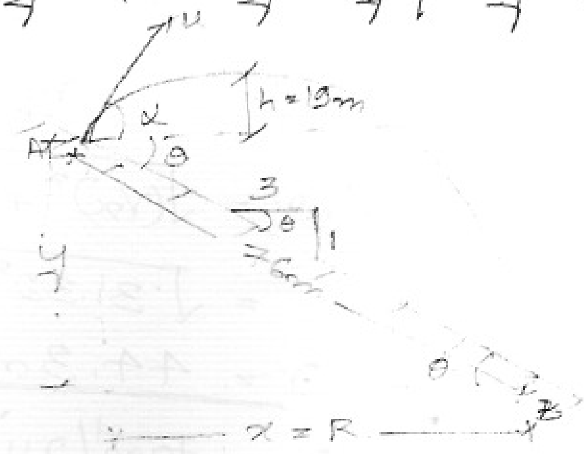
$= \sqrt{(14)^2 + (41.93)^2}$

$v = 44.2\ m/s$

$\theta_r = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{41.93}{14}\right) = 71.53°$

---

8) A ball rebounds @ A & strikes the inclined plane @ point B @ a distance 76m as shown in the fig. If the ball rises to a max height h = 19m above the point of projection, compute the initial velocity & angle of projectio



a) To calculate x & y.

$\tan\theta = \frac{1}{3}$

$\theta = 18.43°$

$\therefore \sin\theta = \frac{y}{76}$

$y = \sin 18.43 \times 76$

$y = 24.03m$

$\therefore \cos\theta = \frac{x}{76}$

$x = \cos 18.43 \times 76$

$x = 72.1m$

- w.k.t equ^n of trajectory is given by

$-y_0 = u\sin\alpha \times t - \frac{1}{2}gt^2$

$-24.03 = (u\sin\alpha)t - \frac{1}{2} \times 9.81t^2 \longrightarrow (1)$

- w.k.t when the projectile reaches max height

$v_y = 0$

i.e, $u\sin\alpha - gt = 0$

$u \sin\alpha = gt \longrightarrow (2)$

- By substituting (2) in (1)

$$-24.03 = (9.81t)t - 4.905t^2$$

$$-24.03 = 4.905t^2$$

{t = math error}

- ∴ w.k.t $v^2 - u^2 = 2as$

$$v_y^2 - u_y^2 = 2(-g)h$$

$$0 - (u\sin\alpha)^2 = -2 \times 9.81 \times 19$$

$$(u\sin\alpha)^2 = 372.78$$

$$u\sin\alpha = 19.3 \longrightarrow (2)$$

- By substituting (2) in (1)

$$-24.03 = 19.3t - 4.905t^2$$

$$\underline{t = 4.92s}$$

- w.k.t $R = u\cos\alpha \times t$

$$72.1 = u\cos\alpha \times 4.92$$

$$u\cos\alpha = 14.65 \longrightarrow (3)$$

- By substituting (3) & (2)

$$\frac{\cancel{u}\sin\alpha}{\cancel{u}\cos\alpha} = \frac{19.3}{14.65}$$

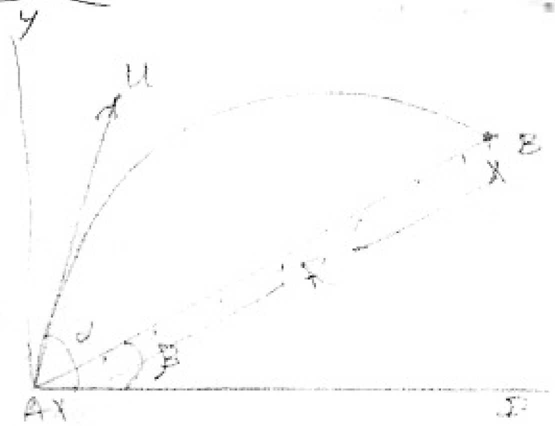
$$\tan\alpha = 1.31$$

$$\underline{\alpha = 52.8^\circ}$$

- ∴ $u\sin 52.8 = 19.3$

$$\underline{u = 24.2 \text{ m/s}}$$

⇒ Projection on inclined plane



- Let AB be the plane inclined @ an angle β to the horizontal as shown in the fig.
- Let a projectile be fired up from point A with initial velocity u m/s @ an angle α for a range AB along inclined plane.

∴ w.k.t $\cos\beta = \frac{AD}{R} \Rightarrow AD = R\cos\beta = x$

$\sin\beta = \frac{BD}{R} \Rightarrow BD = R\sin\beta = y$

- w.k.t eqn of trajectory is given by

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$\therefore \quad R\sin\beta = R\cos\beta\tan\alpha - \frac{g\,R^2\cos^2\beta}{2u^2\cos^2\alpha}$$

$$R\sin\beta - R\cos\beta\tan\alpha = -R^2\frac{g\cos^2\beta}{2u^2\cos^2\alpha}$$

$$\cancel{R}(\sin\beta - \cos\beta\tan\alpha) = -\frac{R\,g\cos^2\beta}{2u^2\cos^2\alpha}$$

$$R = (\cos\beta\tan\alpha - \sin\beta)\frac{2u^2\cos^2\alpha}{g\cos^2\beta}$$

$$R = \left(\cos\beta\frac{\sin\alpha}{\cos\alpha} - \sin\beta\right)\frac{2u^2\cos^2\alpha}{g\cos^2\beta}$$

$$R = (\cos\beta\sin\alpha - \sin\beta\cos\alpha)\frac{2u^2\cos^2\alpha}{g\cos^2\beta}$$

$$R = \sin(\alpha-\beta)\,\frac{2u^2\cos\alpha}{g\cos^2\beta} \longrightarrow (*)$$

$$R = 2\underbrace{\cos\alpha}_{A}\underbrace{\sin(\alpha-\beta)}_{B}\,\frac{u^2}{g\cos^2\beta}$$

- w.k.t $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$$\therefore R = \sin(\alpha+\alpha-\beta) - \sin(\cancel{\alpha}-\cancel{\alpha}-\beta)\,\frac{u^2}{g\cos^2\beta}$$

$$R = \sin(2\alpha-\beta) - \sin\beta\,\frac{u^2}{g\cos^2\beta}$$

- Let t be the time of flight

w.k.t $R = u\cos\alpha\, t$

$$AD = u\cos\alpha\, t$$

$$t = \frac{AD}{u\cos\alpha} = \frac{R\cos\beta}{u\cos\alpha}$$

$$t = \frac{2u^{\cancel{2}}\cancel{\cos\alpha}}{g\cos^{\cancel{2}}\beta}\,\frac{\sin(\alpha-\beta)\cancel{\cos\beta}}{\cancel{u}\cancel{\cos}\alpha} \quad \{\text{from eqn (}$$

$$t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

- For the given values u & β, R is max when

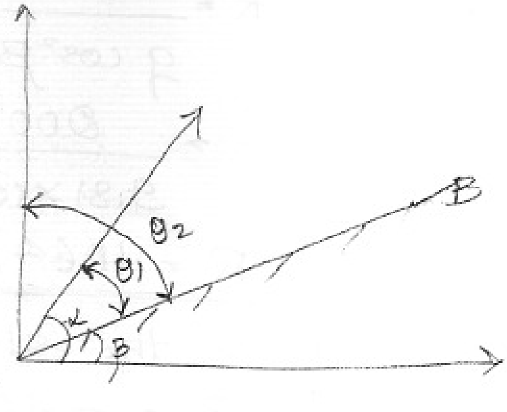
$$\sin(2\alpha-\beta) = 1$$

$$2\alpha-\beta = \sin^{-1}1$$

$$2\alpha-\beta = 90$$

$$2\alpha = 90+\beta$$

$$\alpha = \frac{90+\beta}{2}$$



- From fig, $\theta_1 = \alpha - \beta$

$$= \frac{90+\beta}{2} - \beta$$

$$\theta_1 = \frac{90-\beta}{2}$$

$\theta_2 = 90 - \beta$

$\therefore \theta_1 = \dfrac{\theta_2}{2}$ or $\theta_2 = 2\theta_1$

<u>Note</u> :- If the projection is down the plane, the values of β should be taken as -ve

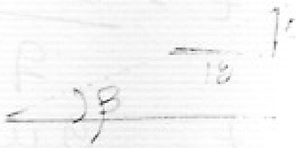
$\Rightarrow$ <u>Exercise problems</u>

1) A plane has a slope of 5 in 12. A shot is projected with a velocity of 200 m/s @ an upward angle of $30°$ to horizontal. Find the range on the plane if ; a) The shot is fired up the plane ; b) the shot is fired down the pl

$\Rightarrow$ Here $\alpha = 30°$; $u = 200$ m/s

- Inclination of plane

$\beta = \tan^{-1}\left(\dfrac{5}{12}\right)$



$\beta = \underline{22.62°}$

a) When the shot is fired up the plane

$\beta = +22.62°$

$R = \dfrac{u^2}{g\cos^2\beta}\left[\sin(2\alpha - \beta) - \sin\beta\right]$

$= \dfrac{200^2}{9.81 \times \cos^2 22.62}\left[\sin(2\times 30 - 22.62) - \sin 22.62\right]$

$\underline{R = 1064.6\,m}$

b) When the shot is fired down the plane

$\beta = -22.62°$

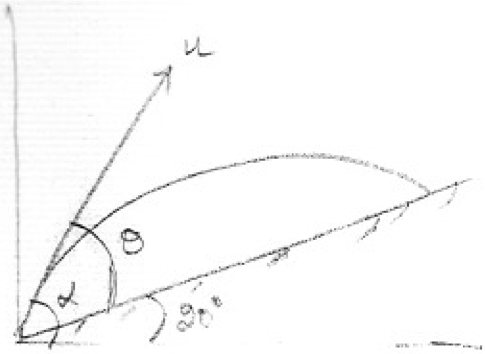
$R = \dfrac{200^2}{9.81 \times \cos^2(-22.62)}\left[\sin(2\times 30 + 22.62) - \sin -22.6\right]$

$R = 6586.27m$

2) A person can throw a ball @ a max velocity of 30m/s. If he wants to get max range on the plane inclined @ 20° to horizontal, @ what ∠le should the ball be projected & what would be the max range: a) up the plane & b) down the plane.

⟹ Here $u = 30m/s$; $\beta = 20°$

a) up the plane



$$- \theta = \frac{90 - \beta}{2}$$

$$\theta = \frac{90 - 20}{2} = 35°$$

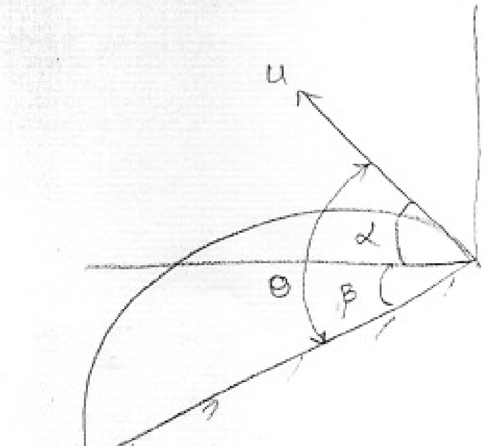
$$- \alpha = \theta + 20 = 55°$$

$$- R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha - \beta) - \sin\beta\right]$$

$$= \frac{30^2}{9.81\cos^2 20}\left[\sin(2\times55 - 20) - \sin 20\right]$$

$$R = 68.36m$$

b) down the plane



- For max projection, direction of projection bisects the ∠le b/w the plane & vertical.

$$\therefore \theta = \frac{1}{2}(90 + 20)$$

$$\theta = 55°$$

$$\alpha = \theta - \beta = 55 - 20 = 35°$$

$$R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha-\beta) - \sin\beta\right]$$

$$= \frac{30^2}{9.81\cos^2(-20)}\left[\sin(2\times 35+20) - \sin(-20)\right]$$

$$\underline{\underline{R = 139.48\,m}}$$

UNIT-10

KINEMATICS

⇒ **Dynamics**

It is a branch of mechanics that deals with the body in motion.

Two branches of mechanics :-

a) Kinematics → It deals with motion of the body without referring to the forces causing the motion of the body.

b) Kinetics → It deals with the motion of the body referring to the forces causing the motion of the body.

→ **Variables in Rectilinear Kinematics**

Rectilinear kinematics deals with the foll variab

* [illegible] — motion along straight [illegible]
* [illegible] — change of position [illegible]

1) **Position :**

It is defined as a point occupied by a body in space. The position is indicated w.r.t co-ordinate system. Position is a vector quantity as it has both magnitude & direction.