Module 5 - Elementary Numerical Methods

<table>
<tr><th>Q.NO</th><th>Question</th></tr>
<tr><td>1.</td><td>a) From the following table estimate the number of students who have obtained the marks in between 40 and 45 using Forward Interpolation formula
<table>
<tr><td>Marks</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td><td>70-80</td></tr>
<tr><td>No. of students</td><td>31</td><td>42</td><td>51</td><td>35</td><td>31</td></tr>
</table>
b) Using Forward interpolation and Backward interpolation formula find the value of f(1) and f(10).
<table>
<tr><td>x</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>f(x)</td><td>4.8</td><td>8.4</td><td>14.5</td><td>23.6</td><td>36.2</td><td>52.8</td><td>73.9</td></tr>
</table></td></tr>
<tr><td>2.</td><td>a) The area of a circle (A) corresponding to the diameter (D) is given below; Find the area corresponding to diameter 105 using an appropriate interpolation formula.
<table>
<tr><td>D</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr>
<tr><td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr>
</table>
b) Given the following table find the value of f(42) and f(85) using forward interpolation and backward interpolation formula respectively
<table>
<tr><td>X</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr>
<tr><td>f(x)</td><td>184</td><td>204</td><td>226</td><td>250</td><td>276</td><td>304</td></tr>
</table></td></tr>
<tr><td>3.</td><td>a) The population of a town is given by the table
<table>
<tr><td>Years</td><td>1951</td><td>1961</td><td>1971</td><td>1981</td><td>1991</td></tr>
<tr><td>Population in thousands</td><td>19.96</td><td>39.65</td><td>58.81</td><td>77.21</td><td>94.61</td></tr>
</table>
Using Newton’s forward interpolation and backward interpolation formula. Calculate the increase in the population from the year 1955 to 1985.
b) Find f(3.5) by using Newton’s backward interpolation formula. Given that f(0) = 7.4720 , f(1) = 7.5854 , f(2) = 7.6922 , f(3) = 7.8119 , f(4) = 7.9252.</td></tr>
<tr><td>4.</td><td>a) Construct interpolating polynomial for the data given below by using Newton-Divided difference formula.
<table>
<tr><td>X</td><td>2</td><td>4</td><td>5</td><td>6</td><td>8</td><td>10</td></tr>
<tr><td>f(x)</td><td>10</td><td>96</td><td>196</td><td>350</td><td>868</td><td>1746</td></tr>
</table>
b) Use Newton’s-divided difference formula to find f(9),given data
<table>
<tr><td>X</td><td>5</td><td>7</td><td>11</td><td>13</td><td>17</td></tr>
<tr><td>f(x)</td><td>150</td><td>392</td><td>1452</td><td>2366</td><td>5202</td></tr>
</table></td></tr>
<tr><td>5.</td><td>a) If f(- 4) = 1245, f(-1) = 33, f(0) = 5, f(2) = 9 and f(5) = 1335 then find f(3) by using Newton’s-divided Difference formula.
b) Using Newton’s divided difference formula find f(8) , f(15) from the following table.
<table>
<tr><td>x</td><td>4</td><td>5</td><td>7</td><td>10</td><td>11</td><td>13</td></tr>
<tr><td>f(x)</td><td>48</td><td>100</td><td>294</td><td>900</td><td>1210</td><td>2028</td></tr>
</table></td></tr>
</table>

<table>
<tr><td>6.</td><td>a) Using Newton's divided difference formula find f(4) from the following table.
<table>
<tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr>
<tr><td>f(x)</td><td>- 4</td><td>2</td><td>14</td><td>158</td></tr>
</table>
b) If y(1) = 3 , y(3) = 9 , y(4) = 30 , y(6) = 132 find the interpolation polynomial using Lagrange's Interpolation formula</td></tr>
<tr><td>7.</td><td>a) Using Lagrange's formula, find the interpolating polynomial that approximate the function described by the following table. Hence find f (3).
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>5</td></tr>
<tr><td>f(x)</td><td>2</td><td>3</td><td>12</td><td>147</td></tr>
</table>
b) Using Lagrange's Interpolation formula, find the value of y when $x = 2$ from the following data
<table>
<tr><td>x</td><td>1</td><td>3</td><td>4</td><td>6</td></tr>
<tr><td>y</td><td>4</td><td>40</td><td>85</td><td>259</td></tr>
</table></td></tr>
<tr><td>8.</td><td>a) The following are the measurement T made on a curve recorded by oscilograph representing a change of current I due to a change in the conditions of an electric current.
<table>
<tr><td>T</td><td>1.2</td><td>2</td><td>2.5</td><td>3</td></tr>
<tr><td>I</td><td>1.36</td><td>0.58</td><td>0.34</td><td>0.20</td></tr>
</table>
Using Lagrange's formula, find I at T = 1.6.
b) Using Lagrange's formula find the interpolating polynomial and find 'y' when x = 4 from the data given below
<table>
<tr><td>x</td><td>0</td><td>2</td><td>3</td><td>5</td></tr>
<tr><td>y</td><td>2</td><td>10</td><td>17</td><td>37</td></tr>
</table></td></tr>
<tr><td>9.</td><td>a) Using Newton - Raphson method, find a real root of
$x\ sinx + cosx = 0$ nearer to x = π , (Here x is in radians)
b) Using Newton - Raphson method, find a real root of the equation $x^3 - 3x - 5 = 0$.</td></tr>
<tr><td>10.</td><td>a) Using Newton - Raphson method, find a real root of the equation $xe^x = 2$ correct to four decimal places.
b) Find the approximate value of the real root of the equation $e^x - 4x = 0$ by Newton - Raphson method</td></tr>
<tr><td>11.</td><td>a) Find the approximate value of the real root of the equation $tanx = x$ by Newton - Raphson method by taking x = 4.5
b) Find the real root of the equation $x^3 + x^2 + 3x + 4 = 0$ applying Newton – Raphson method carryout two iterations.</td></tr>
<tr><td>12.</td><td>a) Using Regula–Falsi method, find the root of the equation $cosx = 3x - 1$ that lies between 0.5 & 1.0 (Here x is in radians) correct to four place of decimals
b) Using Regula–Falsi method, find the root of the equation $xe^x = cosx$ that lies between 0.4 &0.6, carry out four iterations.</td></tr>
<tr><td>13.</td><td>a) Using Regula–Falsi method, find the root of the equation tanx + tanhx = 0 lies between 2 and 3, carry out four approximation.</td></tr>
</table>

	b) Using Regula–Falsi method, find the root of the equation $x\log_{10}x - 1.2 = 0$, correct to four place of decimal
14.	a) Using Regula–Falsi method, find the root of the equation $x^3 - 4x - 9 = 0$ b) Find the approximate value of the real root of the equation $x^3 - 3x + 4 = 0$ using the method of Regula-Falsi.(carryout three iterations)
15.	a) Evaluate $\int_0^5 \frac{dx}{4x+5}$ using Simpson's 1/3rd rule. by using 10 equal parts. Hence find $\log_e 5$. b) By using Simpson's 1/3rd rule , evaluate $\int_0^{0.6} e^{-x^2}\,dx$ taking 7 ordinates
16.	a) By using Simpson's 1/3rd rule , evaluate $\int_1^5 f(x)dx$ given X: 1, 2, 3, 4, 5 f(x): 13, 50, 70, 80, 100 b) The Velocity 'v' (km\min) of a moped which starts from rests, is given at fixed intervals of time 't'(min) as follows: T: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 V: 10, 18, 25, 29, 32, 20, 11, 5, 2, 0 Estimate approximately the distance covered in 20 minutes using Simpson's 1/3rd rule.
17.	a) Use Simpson's 3/8th rule to compute the area bounded by the curve y = f(x), x-axis and extreme ordinates from the following table . X: 0, 1, 2, 3, 4, 5, 6 f(x): 0, 2, 2.5, 2.3, 2, 1.7, 1.5 b) Evaluate $\int_4^{5.2} \log_e x\,dx$ using Simpson's 3/8th rule by taking 7 ordinates.
18.	a) Evaluate $\int_0^{\frac{\pi}{2}} e^{\sin x}dx$ by Simpson's 3/8th rule and Weddle's rule by taking 7 ordinates b) Evaluate $\int_0^1 \frac{1}{1+x}$dx dividing the interval into six equal parts by applying Simpson's 3/8th rule and deduce an approximate value of $\log_e 2$.
19.	a) Evaluate $\int_0^{1.2} e^x dx$ using Weddle's rule. Taking six equal sub intervals compare the results with exact value. b) Using Weddle's rule, estimate the area bounded by the curve the x-axis and the line x=1 & x=4 X: 1, 1.5, 2, 2.5, 3, 3.5, 4 f(x): 2, 2.4, 2.7, 2.8, 3, 2.6, 2.1
20.	a) Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ using Weddle's rule. By using 6 equal parts. Hence find $\log_e 2$. b) Evaluate $\int_0^6 3x^2$ dx dividing the interval into six equal parts by applying (i) Simpson's 1/3rd rule (ii) Simpson's 3/8th rule (iii) Weddle's rule and verify with exact integral.