UNIT - 3

PRESENT WORTH COMPARISON: Conditions for present worth comparisons, Basic Present worth comparisons, Present worth equivalence, Net Present worth, Assets with unequal lives, infinite lives, Future worth comparison, Pay – back comparison, Exercises, Discussions and problems.

3.1 Conditions for present worth comparisons

Present worth: In a present worth comparison of alternatives, the costs associated with each alternative investment are all converted to a present sum of money, and the least of these values represents the best alternative. Annual costs, future payments, and gradients must be brought to the present. Converting all cash flows to present worth is referred to as discounting.

In present worth analysis, the *P* value, now called *PW*, is calculated at the MARR for each alternative. This converts all future cash flows into present dollar equivalents.This makes it easy to determine the economic advantage of one alternative over another. The PW comparison of alternatives with equal lives is straightforward. If both alternatives are used in identical capacities for the same time period, they are termed *equal-service* alternatives. For mutually exclusive alternatives the following guidelines are applied:

One alternative- Calculate PW at the MARR. If PW _ 0, the alternative is financially viable.

Two or more alternatives- Calculate the PW of each alternative at the MARR. Select the alternative with the PW value that is numerically largest, that is, less negative or more positive. The second guideline uses the criterion of numerically largest to indicate a lower PW of costs only or larger PW of net cash flows. Numerically largest is not the absolute value because the sign matters here. The selections below correctly apply this guideline.

Conditions for using the present worth comparisons:While evaluating alternatives ,the following assumptions are cosidered for better results and smooth evaluation process.

1. Cash flows are known (no risks taken)
2. Cash flows are in constant value dollars (no inflation)
3. Interest rate is known
4. Taxes are not included in calculations
5. Intangibles are not considered
6. Funds are considered to exist at all time

3.2 Basic Present worth comparisons:

The present worth of a cash flow overtime is its value today is represented as time zero (0) in a cahs flow diagram.Two general pattrens are apparent in present worth calculations:present worth equivallience and net present worth.

3.3 Present worth equivalence:

One pattren detremines the present worth equivalence of a series of future transactions. The purpose is to secure one figure that represent all the transactions for instance,a series of expenses that will occur in the future can be discounted to obtain its PW and decision can be made about whether an investment of the PW amount should be made now to avoid the expenses**.**

Assets with unequal lives:

Unequal lives among feasible alternatives somewhat complecate their analysis and comparison.To make engineering economy studies in such cases thw assumptions are emplyed

1.The repeatability assumptions

2. The co-terminated assumptions

In repeatability assumption or common multiple method , least common multiple is taken as life of the asste,based on that LCM the assent as to be repalced many times.

Problem -1

There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.3,00,000, Annual operating and maintenance cost = Rs.20,000, Expected salvage value = Rs.1,25,000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.2,00,000, Annual operating and maintenance cost = Rs.35,000, Expected salvage value = Rs.70,000, Useful life = 5 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

Solution:

Since both alternatives have the same life span i.e. 5years, the present worth of the alternatives will be compared over a period of 5 years. The cash flow diagram of Alternative-1 is shown in Fig. 3.1.

The cash outflows i.e. costs or expenditures are represented by vertically downward arrows

whereas the cash inflows i.e. revenue or income are represented by vertically upward arrows. The same convention is adopted here.

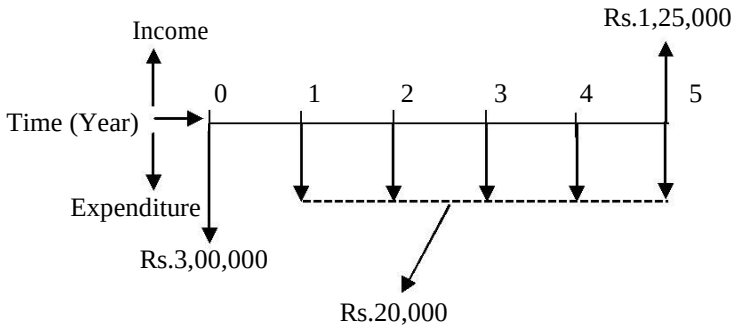


Fig. 3.1 Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows; The initial cost, P = Rs.3,00,000 (cash outflow),

Annual operating and maintenance cost, A = Rs.20,000 (cash outflow), Salvage value, F = Rs.1,25,000 (cash inflow).

PW_1 = - 3,00,000 – 20,000($P/A, i, n$) + 1,25,000($P/F, i, n$)

PW_1 = - 3,00,000 – 20,000($P/A, 10\%, 5$) + 1,25,000($P/F, 10\%, 5$)

Now putting the mathematical expressions of different compound interest factors in the above expression for PW_1 (in Rs.) results in the following;

$$PW_1 = -3,00,000 - 20,000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 1,25,000 \times \frac{1}{(1+i)^n}$$

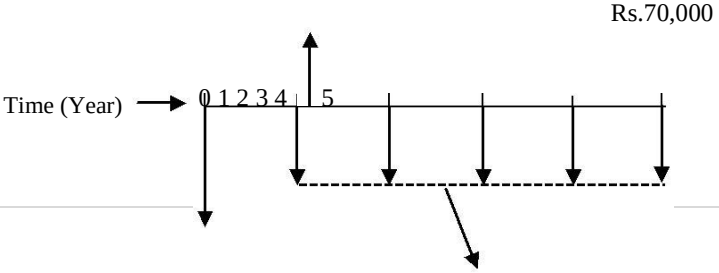
$$PW_1 = -3,00,000 - 20,000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 1,25,000 \times \frac{1}{(1+0.1)^5}$$

$$PW_1 = -3,00,000 - 20,000 \times 3.7908 + 1,25,000 \times 0.6209$$

PW_1 = - 3,00,000 -75,816 + 77,613

***PW₁* = - Rs.2,98,203**

The cash flow diagram of Alternative-2 is shown in Fig. 3.2.



Rs.2,00,000

Rs.35,000

Fig.3.2 Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is calculated as follows;

The initial cost, P = Rs.2,00,000 (cash outflow),

Annual operating and maintenance cost, A = Rs.35,000 (cash outflow), Salvage value, F = Rs.70,000 (cash inflow).

PW_2 = - 2,00,000 – 35,000(*P/A, i, n*) + 70,000(*P/F, i, n*)

PW_2 = - 2,00,000 – 35,000(*P/A, 10%, 5*) + 70,000(*P/F, 10%, 5*)

$$PW_2 = -2{,}00{,}000 - 35{,}000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 70{,}000 \times \frac{1}{(1+i)^n}$$

$$PW_2 = -2{,}00{,}000 - 35{,}000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 70{,}000 \times \frac{1}{(1+0.1)^5}$$

$$PW_2 = -2{,}00{,}000 - 35{,}000 \times 3.7908 + 70{,}000 \times 0.6209$$

PW_2 = - 2,00,000 – 1,32,678 + 43,463

***PW2* = - Rs.2,89,215**

Comparing the equivalent present worth of both the alternatives, it is observed that Alternative-2 will be selected as it shows lower negative equivalent present worth compared to Alternative-1 at the interest rate of 10% per year.

The equivalent present worth of both the alternatives can also be calculated by using the values of compound interest factors from interest tables. The equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows;

PW_1 = - 3,00,000 – 20,000(*P/A, i, n*) + 1,25,000(*P/F, i, n*)

PW_1 = - 3,00,000 – 20,000(*P/A, 10%, 5*) + 1,25,000(*P/F, 10%, 5*)

The values of compound interest factors i.e. *(P/A, i, n)* and *(P/F, i, n)* can be obtained from the interest tables . By referring to the interest table for 10% interest rate, the values

of compound interest factors i.e. *(P/A, 10%, 5)* and *(P/F, 10%, 5)* at interest rate („*i'*) of 10% and for interest period („*n'*) of 5 years are obtained at the intersection of these factors and interest period „*n*‟ equal to 5 i.e. the values are obtained from *P/A* column and*P/F* column at „*n*‟ equal to 5 from the interest table (discrete compounding) corresponding to 10% interest rate. The obtained values of *(P/A, 10%, 5)* and *(P/F, 10%, 5)* are 3.7908 and 0.6209 respectively (same as those obtained using mathematical expressions of these factors).

Now putting the values of compound interest factors in the above expression, the equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows;

$PW_1 = -3,00,000 - 20,000 \times 3.7908 + 1,25,000 \times 0.6209$

$PW_1 = -\ 3,00,000 - 75,816 + 77,613$

***PW1* = - Rs.2,98,203**

Now the calculation of equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is presented below.

$PW_2 = -\ 2,00,000 - 35,000(P/A, i, n) + 70,000(P/F, i, n)$

$PW_2 = -\ 2,00,000 - 35,000(P/A, 10\%, 5) + 70,000(P/F, 10\%, 5)$

Now putting the values of compound interest factors in the above expression (same as above) the equivalent present worth of Alternative-2 i.e. PW_2 is calculated as follows;

$PW_2 = -2,00,000 - 35,000 \times 3.7908 + 70,000 \times 0.6209$

$PW_2 = -\ 2,00,000 - 1,32,678 + 43,463$

***PW2* = - Rs.2,89,215**

It may be noted that in the above example only cost components and the salvage value of the alternatives were considered for comparison.

Problem 2:

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by concrete mixer) from Alternative-1 and Alternative-2 are Rs.50000 and Rs.45000 respectively. Compute the equivalent present worth of the alternatives at the same rate of interest 10% per year and find out the economical alternative.

Solution:

The cash flow diagram of Alternative-1 is shown in Fig. 3.3.

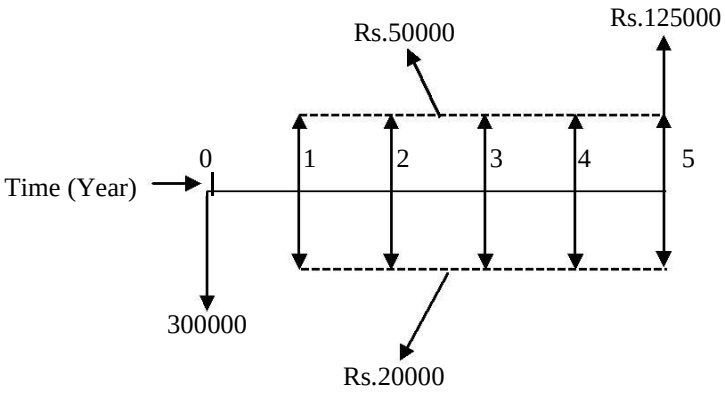


Fig. 3.3 Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 is calculated as follows;

PW_1 = - 300000 - 20000(*P/A, i, n*) + 50000*(P/A, i, n)* + 125000(*P/F, i, n*)

PW_1 = - 300000 - 20000(*P/A, 10%, 5*) + 50000(*P/A, 10%, 5*) + 125000(*P/F, 10%, 5*)

PW_1 = - 300000 + (50000 – 20000) (*P/A, 10%, 5*) + 125000(*P/F, 10%, 5*)

PW_1 = - 300000 + 30000(*P/A, 10%, 5*) + 125000(*P/F, 10%, 5*)

$$PW_1 = -300000 + 30000 \times \frac{(1+i)_n - 1}{i(1+i)^n} + 125000 \times \frac{1}{(1+i)^n}$$

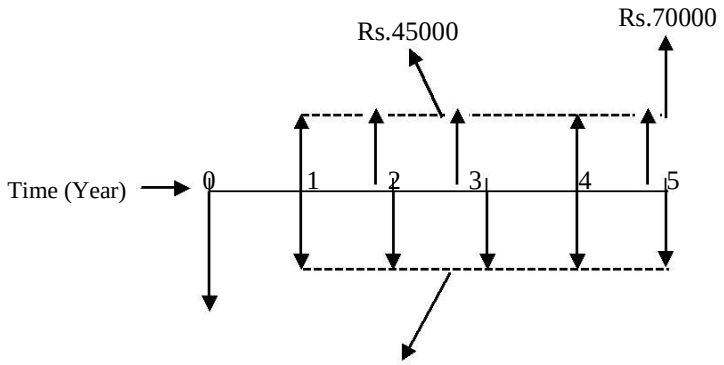
$$PW_1 = -300000 + 30000 \times \frac{(1+0.1)_5 - 1}{0.1(1+0.1)^5} + 125000 \times \frac{1}{(1+0.1)^5}$$

$$PW_1 = -300000 + 30000 \times 3.7908 + 125000 \times 0.6209$$

PW_1 = - 300000 + 113724 + 77613

PW_1 = - Rs.108663

The cash flow diagram of Alternative-2 is shown in Fig. 2.4.



200000

Rs.35000

Fig. 2.4 Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is calculated as follows;

PW_2 = - 200000 - 35000(*P/A, i, n*) + 45000*(P/A, i, n)* + 70000(*P/F, i, n*)

PW_2 = - 200000 - 35000(*P/A, 10%, 5*) + 45000*(P/A, 10%, 5)* + 70000(*P/F, 10%, 5*)

PW_2 = - 200000 + (45000 – 35000) (*P/A, 10%, 5*) + 70000(*P/F, 10%, 5*)

PW_2 = - 200000 + 10000(*P/A, 10%, 5*) + 70000(*P/F, 10%, 5*)

$$PW_2 = -200000 + 10000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 70000 \times \frac{1}{(1+i)^n}$$

$$PW_2 = -200000 + 10000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 70000 \times \frac{1}{(1+0.1)^5}$$

$$PW_2 = -200000 + 10000 \times 3.7908 + 70000 \times 0.6209$$

PW_2 = - 200000 + 37908 + 43463

***PW2* = - Rs.118629**

Comparing the equivalent present worth of the both the alternatives, it is observed that Alternative-1 will be selected as it shows lower cost compared to Alternative-2.

Problem 3:

A construction contractor has three options to purchase a dump truck for transportation and dumping of soil at a construction site. All the alternatives have the same useful life. The cash flow details of all the alternatives are provided as follows;

Option-1: Initial purchase price = Rs.2500000, Annual operating cost Rs.45000 at the end of 1st year and increasing by Rs.3000 in the subsequent years till the end of useful life, Annual income = Rs.120000, Salvage value = Rs.550000, Useful life = 10 years.

Option-2: Initial purchase price = Rs.3000000, Annual operating cost = Rs.30000, Annual income Rs.150000 for first three years and increasing by Rs.5000 in the subsequent years till the end of useful life, Salvage value = Rs.800000, Useful life = 10 years.

Option-3: Initial purchase price = Rs.2700000, Annual operating cost Rs.35000 for first 5 years and increasing by Rs.2000 in the successive years till the end of useful life, Annual income = Rs.140000, Expected salvage value = Rs.650000, Useful life = 10 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 8% per year.

Solution:

The cash flow diagram of Option-1 is shown in Fig. 3.4

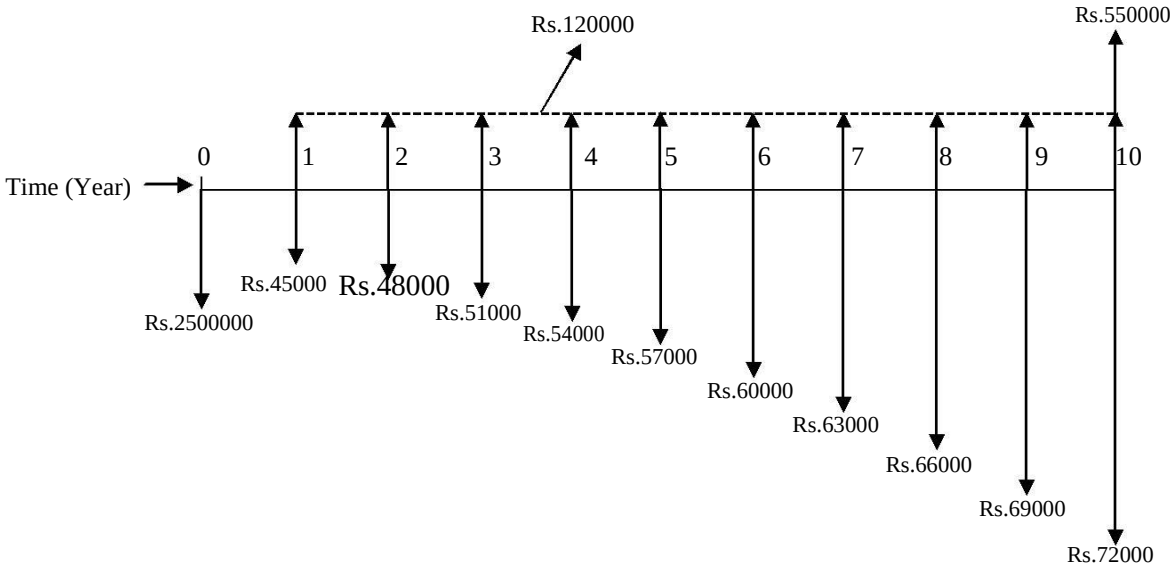
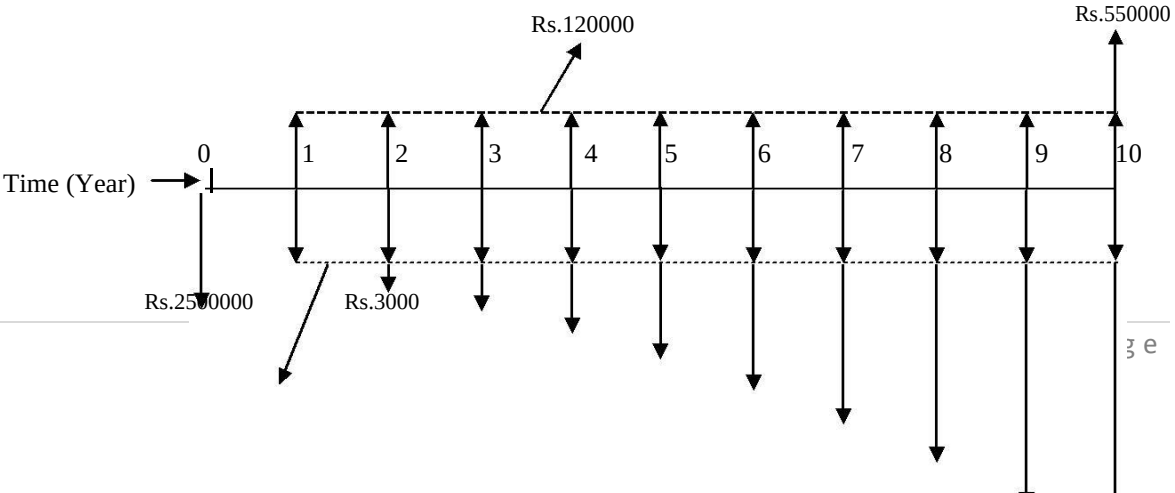


Fig. 3.4 Cash flow diagram of Option-1

For Option-1, the annual operating cost is in the form of a positive uniform gradient series with gradient starting from end of year „2". The operating cost at the end of different years can be split into the uniform base amount of Rs.45000 and the gradient amount in multiples of Rs.3000 as shown in Fig. 3.5.



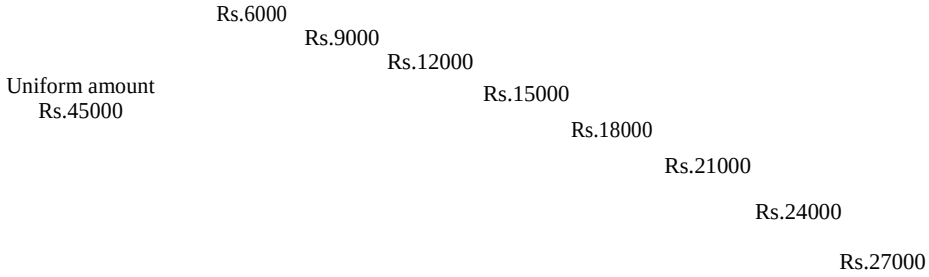


Fig. 3.5 Cash flow diagram of Option-1
with annual operating cost split into uniform base amount and gradient amount

The present worth of the uniform gradient series will be located at the beginning i.e. in year „0" i.e. 2 years before the commencement of the uniform gradient.

Now the equivalent present worth (in Rs.) of Option-1 is calculated as follows;

PW_1 = - 2500000 - 45000(*P/A, i, n*) - 3000*(P/G, i, n)* + 120000*(P/A, i, n)* + 550000(*P/F, i, n*)

PW_1 = - 2500000 - 45000*(P/A, 8%, 10)* - 3000*(P/G, 8%, 10)* + 120000*(P/A, 8%, 10)* + 550000*(P/F, 8%, 10)*

PW_1 = - 2500000 + (120000 - 45000) *(P/A, 8%, 10)* - 3000*(P/G, 8%, 10)* + 550000*(P/F, 8%, 10)*

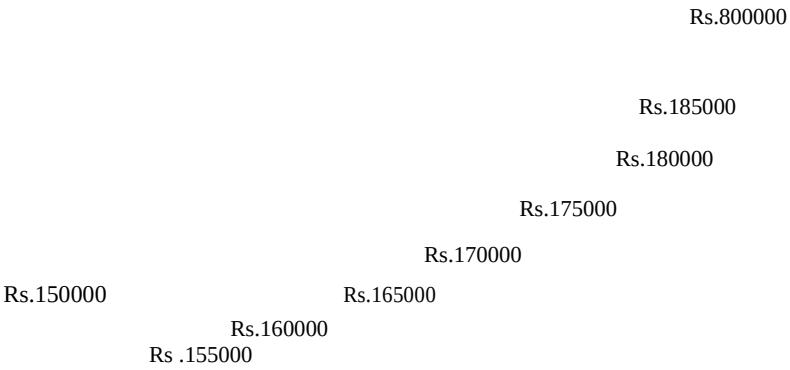
Now putting the values of different compound interest factors (the expressions in terms of *'i'* and *'n'* in the above expression for PW_1 results in the following;

$$PW_1 = -2500000 + 75000 \times 6.7101 - 3000 \times 25.9768 + 550000 \times 0.4632$$

PW_1 = - 2500000 + 503258 - 77930 + 254760

PW_1 = - Rs.1819912

The cash flow diagram of Option-2 is shown in Fig. 3.6



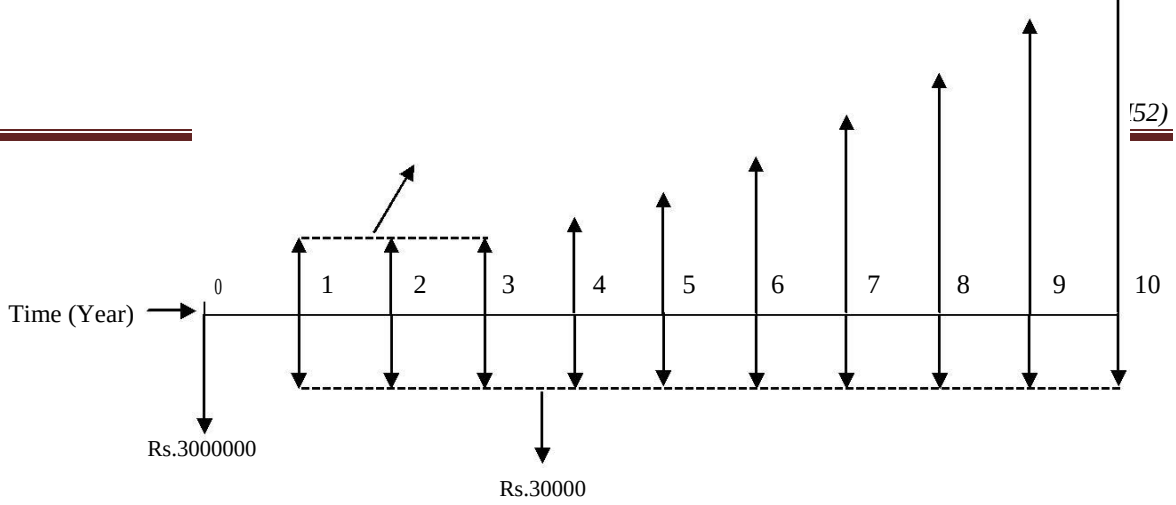


Fig. 3.6 Cash flow diagram of Option-2

For Option-2, the annual income is in the form of a positive uniform gradient series with gradient starting from end of year „4". The annual income can be split into the uniform base amount of Rs.150000 and the gradient amount in multiples of Rs.5000 starting from end of year „4" and is shown in Fig3.7

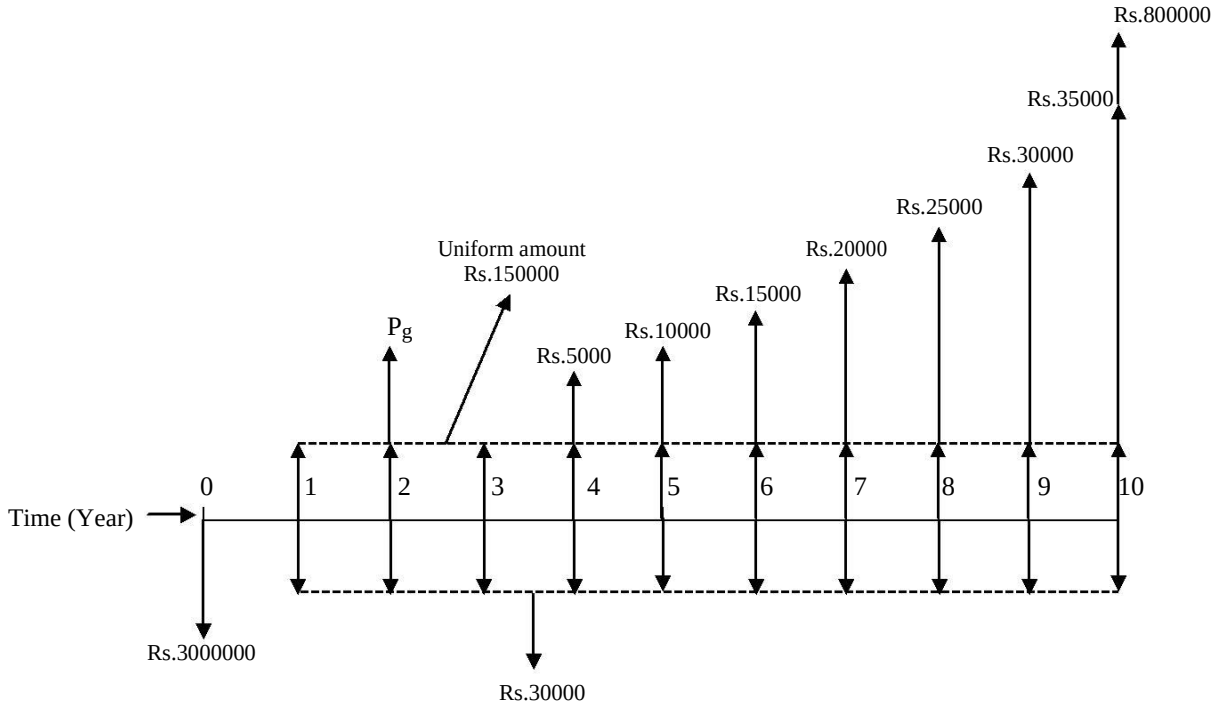


Fig.3.7 Cash flow diagram of Option-2 with
annual income split into uniform base amount and gradient amount

The equivalent present worth of the gradient series (of the annual income) starting from end of year „4" will be located at the end of year „2" i.e. 2 years before the start of the gradient. Further the present worth of this amount at beginning i.e. at time „0" will be obtained by multiplying the equivalent present worth „P_g' (shown in Fig. 3.7) at the end of year „2" (which is a future amount) with the single payment present worth factor *(P/F, i, n)*.

Now the equivalent present worth (in Rs.) of Option-2 is determined as follows;

PW_2 = - 3000000 - 30000(*P/A, 8%, 10*) + 150000*(P/A, 8%, 10)* + P_g *(P/F, 8%, 2)* + 800000(*P/F, 8%, 10*)

Now in the above expression, P_g will be replaced by *G (P/G, i, n)* i.e. 5000(*P/G, 8%, 8*).

PW_2 = - 3000000 - 30000(*P/A, 8%, 10*) + 150000*(P/A, 8%, 10)* + 5000*(P/G, 8%, 8) (P/F, 8%, 2)* + 800000(*P/F, 8%, 10*)

PW_2 = - 3000000 + (150000 - 30000) *(P/A, 8%, 10)* + 5000*(P/G, 8%, 8) (P/F, 8%, 2)* + 800000(*P/F, 8%, 10*)

Now putting the values of different compound interest factors in the above expression for PW_2 results in the following;

$PW_2 = -3000000 + 120000 \times 6.7101 + 5000 \times 17.8061 \times 0.8573 + 800000 \times 0.4632$

PW_2 = - 3000000 + 805212 + 76326 + 370560

PW_2 = - Rs.1747902

The cash flow diagram of Option-3 is shown in Fig. 3.8

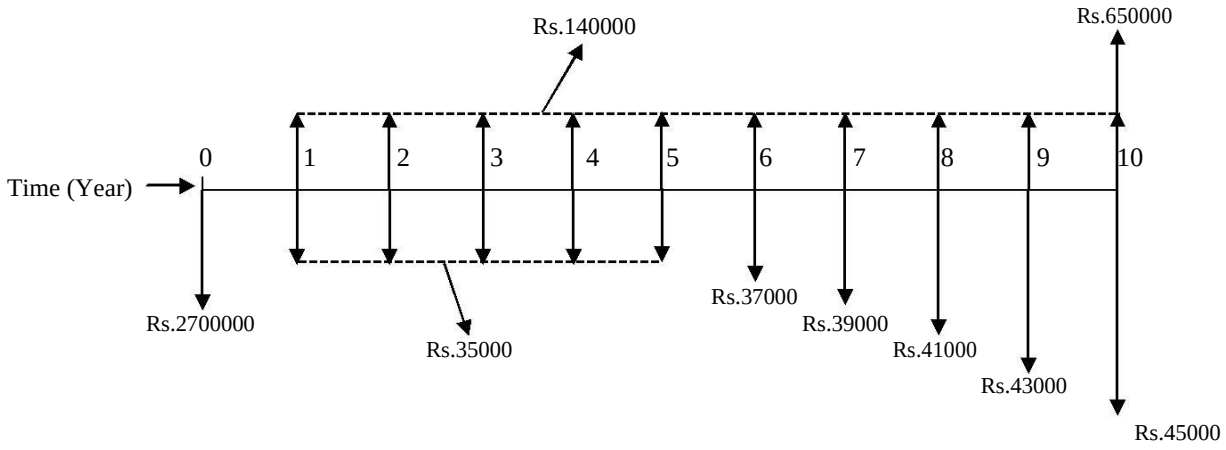


Fig. 3.8 Cash flow diagram of Option-3

For Option-3, the annual operating cost is in the form of a positive uniform gradient series with gradient starting from end of year „6". The annual operating cost can thus be split into the uniform base amount of Rs.35000 and the gradient amount in multiples of Rs.2000 starting from end of year „6" .

The equivalent present worth of the gradient series for the annual operating cost starting from

end of year „6" will be located at the end of year „4". Further the present worth of this amount at time „0" will be determined by multiplying the equivalent present worth „P_g' (shown in Fig. 2.10) at the end of year „4" with the single payment present worth factor *(P/F, i, n)*.

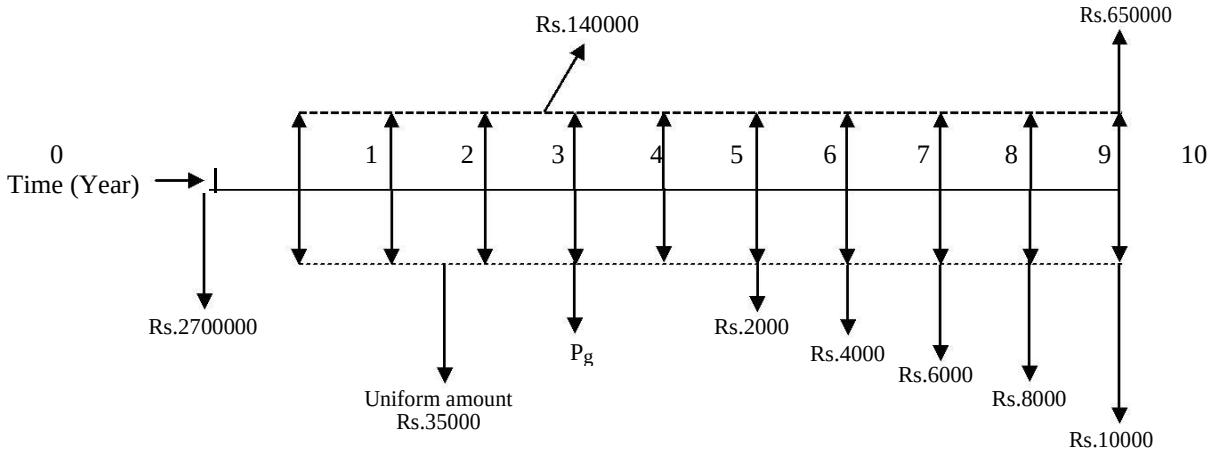


Fig. 3.9 Cash flow diagram of Option-3 with annual operating cost split into uniform base amount and gradient amount

The equivalent present worth (in Rs.) of Option-3 is obtained as follows;

PW_3 = - 2700000 - 35000(*P/A, 8%, 10*) - P_g *(P/F, 8%, 4)* + 140000*(P/A, 8%, 10)* + 650000(*P/F, 8%, 10*)

Now in the above expression, P_g will be replaced by *G (P/G, i, n)* i.e. 2000(*P/G, 8%, 6*).

PW_3 = - 2700000 - 35000(*P/A, 8%, 10*) - 2000*(P/G, 8%, 6) (P/F, 8%, 4)* + 140000*(P/A, 8%, 10)* + 650000(*P/F, 8%, 10*)

PW_3 = - 2700000 + (140000 - 35000) *(P/A, 8%, 10)* - 2000*(P/G, 8%, 6) (P/F, 8%, 4)* + 650000(*P/F, 8%, 10*)

Now putting the values of different compound interest factors in the above expression, the value of PW_3 is given by;

$$PW_3 = -2700000 + 105000 \times 6.7101 - 2000 \times 10.5233 \times 0.7350 + 650000 \times 0.4632$$

PW_3 = - 2700000 + 704561 - 15469 + 301080

PW_3 = - Rs.1709828

From the comparison of equivalent present worth of all the three mutually exclusive alternatives, it is observed that Option-3 shows lowest negative equivalent present worth as compared to other options. Thus Option-3 will be selected for the purchase of the dump truck.

Problem 4:A material testing laboratory has two alternatives for purchasing a compression testing machine which will be used for determining the compressive strength of different construction materials. The alternatives are from two different manufacturing companies. The cash flow details of the alternatives are as follows;

Alternative-1: Initial purchase price = Rs.1000000, Annual operating cost = Rs.10000, Expected annual income to be generated from testing of different construction materials = Rs.175000, Expected salvage value = Rs.200000, Useful life = 10 years.

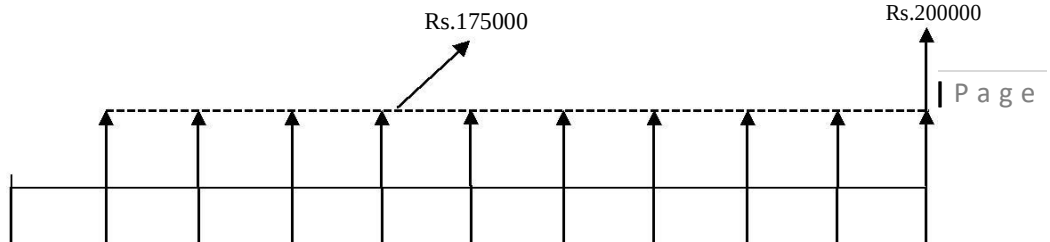
Alternative-2: Initial purchase price = Rs.700000, Annual operating cost = Rs.15000, Expected annual income to be generated from testing of different construction materials = Rs.165000, Expected salvage value = Rs.250000, Useful life = 5 years.

Using present worth method, find out the most economical alternative at the interest rate of 10% per year.

Solution:

The alternatives have different life spans i.e. 10 years and 5 years. Thus the comparison will be made over a time period equal to the least common multiple of the life spans of the alternatives. In this case the least common multiple of the life spans is 10 years. Thus the cash flow of Alternative-1 will be analyzed for one cycle (duration of 10 years) whereas the cash flow of Alternative-2 will be analyzed for two cycles (duration of 5 years for each cycle). The cash flow of the Alternative-2 for the second cycle will be exactly same as that in the first cycle.

The cash flow diagram of Alternative-1 is shown in Fig. 3.10



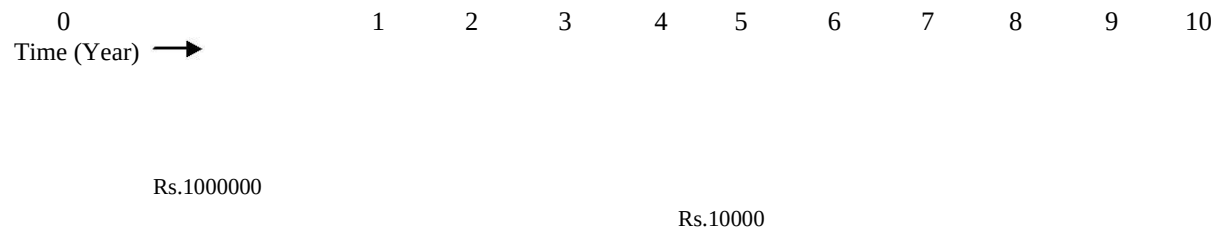


Fig. 3.10 Cash flow diagram of Alternative-1

The equivalent present worth PW_1 (in Rs.) of Alternative-1 is calculated as follows;

PW_1 = - 1000000 - 10000(*P/A, i, n*) + 175000*(P/A, i, n)* + 200000(*P/F, i, n*)

PW_1 = - 1000000 - 10000(*P/A, 10%, 10*) + 175000*(P/A, 10%, 10)* + 200000(*P/F, 10%, 10*)

PW_1 = - 1000000 + (175000 - 10000) (*P/A, 10%, 10*) + 200000(*P/F, 10%, 10*)

Putting the values of different compound interest factors in the above expression for PW_1;

$$PW_1 = -1000000 + 165000 \times 6.1446 + 200000 \times 0.3855$$

PW_1 = - 1000000 + 1013859 + 77100

PW_1 = Rs.90959

The cash flow diagram of Alternative-2 is shown in Fig. 3.11. As the least common multiple of the life spans of the alternatives is 10 years, the cash flow of Alternative-2 is shown for two cycles with each cycle of duration 5 years.

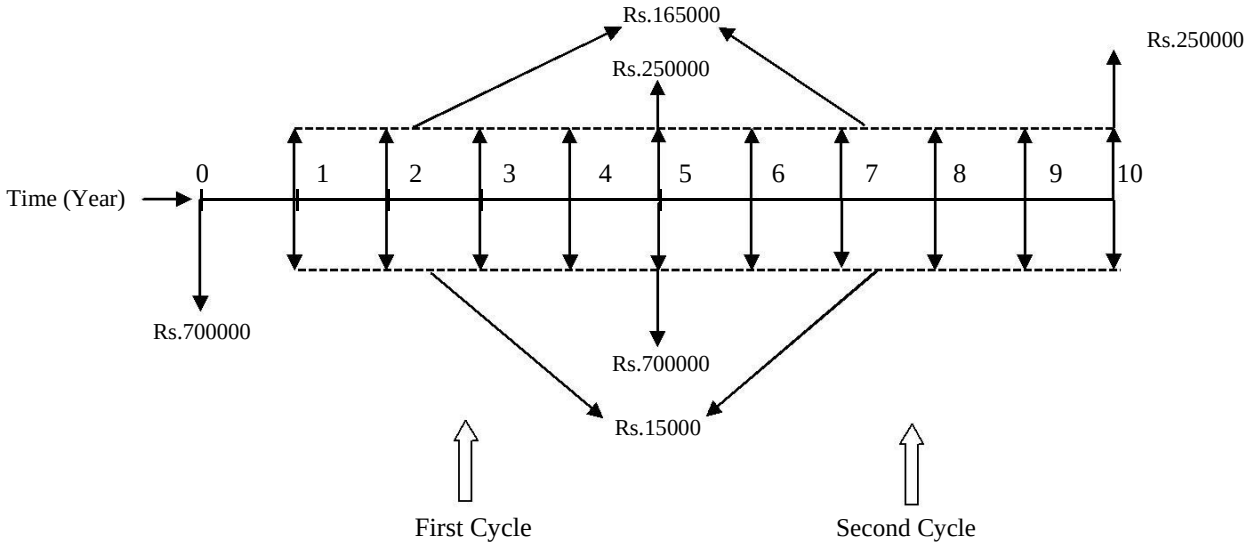


Fig. 3.11 Cash flow diagram of Alternative-2 for two cycles

In the cash flow diagram of Alternative-2, the initial purchase price of Rs.700000 is again

located at the end of year „5" i.e. at the end of first cycle or the beginning of the second cycle. In addition the annual operating cost and the annual income are also repeated in the second cycle from end of year „6" till end of year „10". Further the salvage value of Rs.250000 is also located at end of year „10" i.e. at the end of second cycle.

The equivalent present worth PW_2 (in Rs.) of Alternative-2 is determined as follows;

PW_2 = - 700000 - 15000(*P/A, 10%, 10*) + 165000*(P/A, 10%, 10)* + 250000(*P/F, 10%, 5*) - 700000*(P/F, 10%, 5)* + 250000*(P/F, 10%, 10)*

PW_2 = - 700000 + (165000 - 15000) (*P/A, 10%, 10*) - (700000 – 250000) (*P/F, 10%, 5*) + 250000*(P/F, 10%, 10)*

Putting the values of different compound interest factors in the above expression for PW_2 results in the following;

$PW_2 = -700000 + 150000 \times 6.1446 - 450000 \times 0.6209 + 250000 \times 0.3855$

PW_2 = - 700000 + 921690 - 279405 + 96375

PW_2 = Rs.38660

Thus from the comparison of equivalent present worth of the alternatives, it is evident that Alternative-1 will be selected for purchase of the compression testing machine as it shows the higher positive equivalent present worth.

Problem 5:

A construction firm has decided to purchase a dozer to be employed at a construction site. Two different companies manufacture the dozer that will fulfill the functional requirement of the construction firm. The construction firm will purchase the most economical one from one of these companies. The alternatives have different useful lives. The cash flow details of both alternatives are presented as follows;

Company-A Dozer: Initial purchase cost = Rs.3050000, Annual operating cost Rs.40000 at end of 1st year and increasing by Rs.2000 in the subsequent years till the end of useful life, Annual income = Rs.560000, Expected salvage value = Rs.1050000, Useful life = 6 years.

Company-B Dozer: Initial purchase cost = Rs.4000000, Annual operating cost = Rs.55000, Annual revenue to be generated Rs.600000 at the end of 1st year and increasing by Rs.5000 in the subsequent years till the end of useful life, Expected salvage value = Rs.1000000, Useful life = 12 years.

Using present worth method, find out the most economical alternative at the interest rate of 7% per year.

Solution:

Since the alternatives have different life spans i.e. 6 and 12 years, the comparison will be made over a time period equal to the least common multiple of the life spans of the alternatives i.e. 12 years. The cash flow of Company-A Dozer will be analyzed for two cycles i.e. duration of 6 years for each cycle. The cash flow of Company-B Dozer will be analyzed for one cycle i.e. duration of 12 years.

The cash flow diagram of Company-A Dozer is shown in Fig. 3.12. Since the least common multiple of the life spans of the alternatives is 12 years, the cash flow is shown for two cycles.

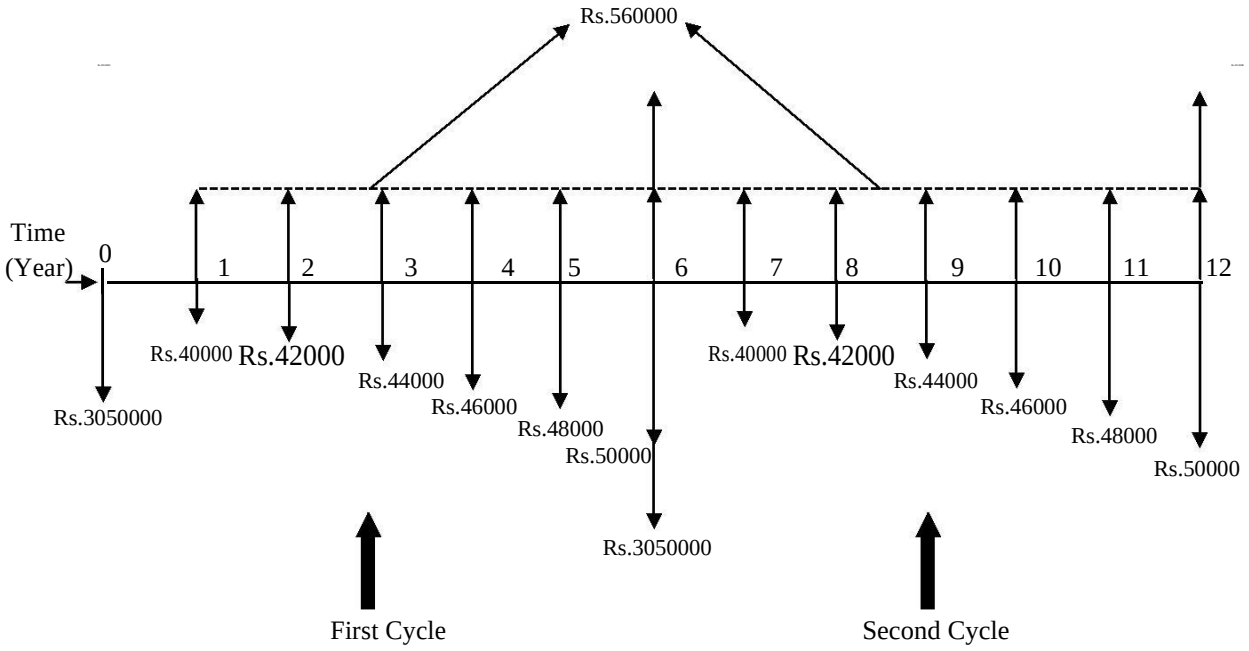


Fig. 3.12 Cash flow diagram of Company-A Dozer for two cycles

For Company-A Dozer, the annual operating cost is in the form of a positive uniform gradient series which can be split into the uniform base amount of Rs.40000 and the gradient amount in multiples of Rs.2000 starting from end of year „2" for first cycle as shown in Fig. 3.12. The equivalent present worth of this gradient for cycle one will be

located at the beginning i.e. in year „0". However for second cycle, the equivalent present worth of the gradient for the annual operating cost starting from end of year „8" (shown in Fig. 3.12) will be located at the end of year „6". Further the present worth of this amount at time „0" will be determined by multiplying the equivalent present worth of the gradient at the end of year „6" with the single payment present worth factor *(P/F, i, n)*.

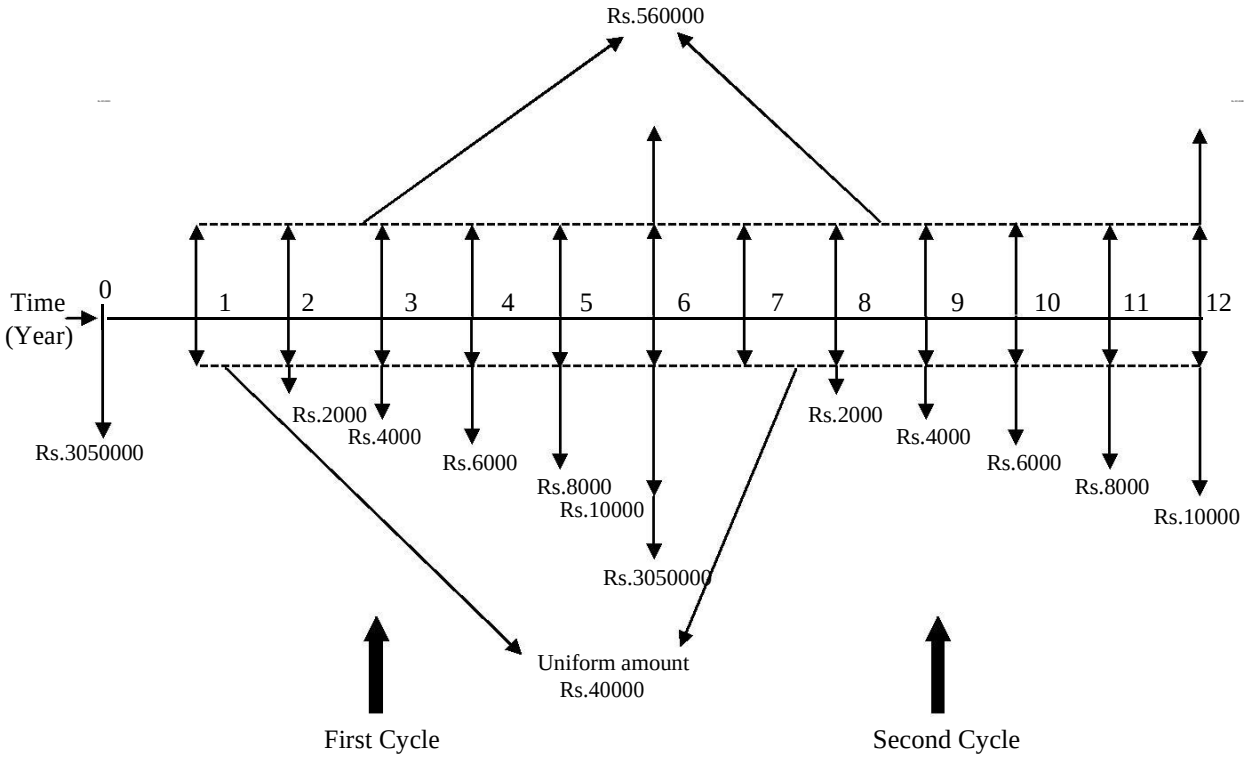


Fig. 3.13 Cash flow diagram of Company-A Dozer for two cycles
with annual operating cost split into uniform base amount and gradient amount

The equivalent present worth PW_A (in Rs.) of Company-A Dozer is calculated as follows;

PW_A = - 3050000 - 40000*(P/A, 7%, 12)* - 2000*(P/G, 7%, 6)* + 560000*(P/A, 7%, 12)* + 1050000*(P/F, 7%, 6)* - 3050000*(P/F, 7%, 6)* - 2000*(P/G, 7%, 6) (P/F, 7%, 6)* + 1050000*(P/F, 7%, 12)*

PW_A = - 3050000 + (560000 - 40000) *(P/A, 7%, 12)* - 2000*(P/G, 7%, 6)* - (3050000 - 1050000) *(P/F, 7%, 6)* - 2000*(P/G, 7%, 6) (P/F, 7%, 6)* + 1050000*(P/F, 7%, 12)*

Putting the values of different compound interest factors in the above expression;

$$PW_A = -3050000 + 520000 \times 7.9427 - 2000 \times 10.9784 - 2000000 \times 0.6663 - 2000 \times 10.9784 \times 0.6663 + 1050000 \times 0.4440$$

PW_A = - 3050000 + 4130204 - 21957 - 1332600 - 14630 + 466200

PW_A = Rs.177217

The cash flow diagram of Company-B Dozer is shown in Fig. 3.14.

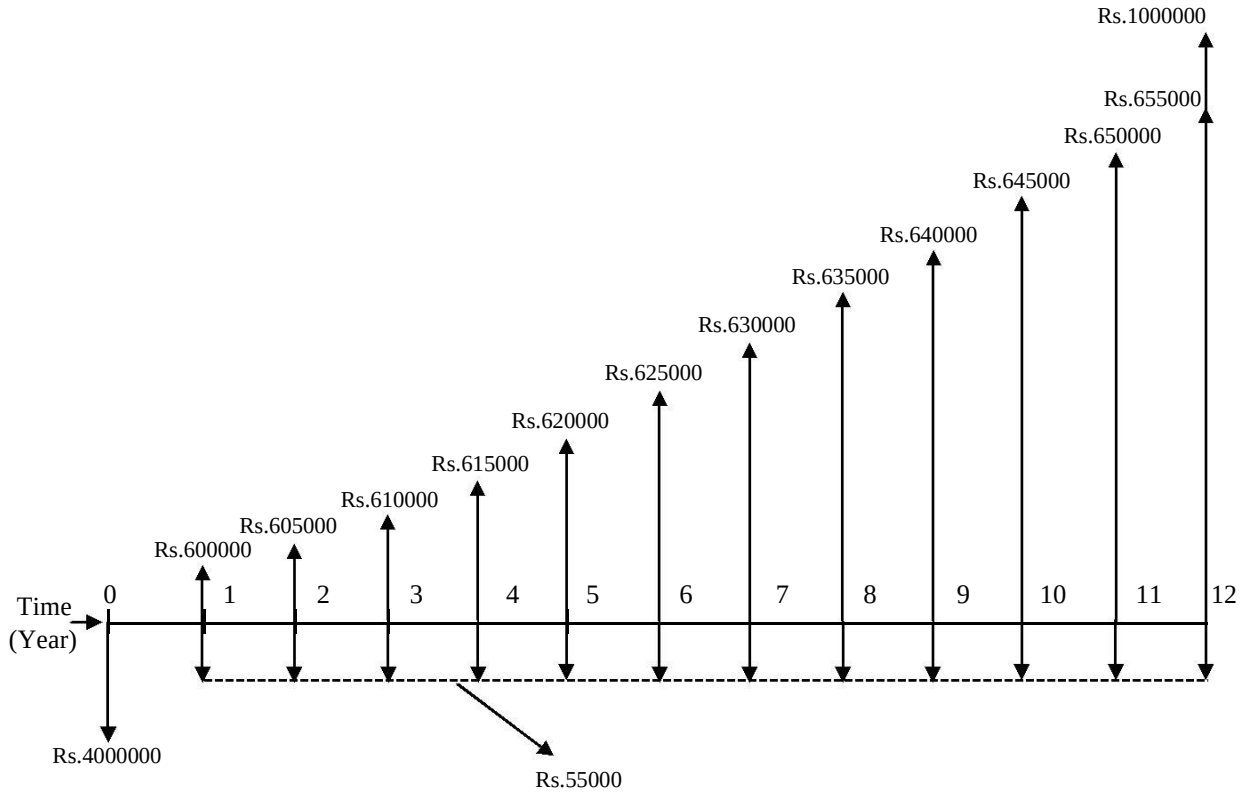


Fig. 3.14 Cash flow diagram of Company-B Dozer

For Company-B Dozer, the annual revenue is in the form of a positive uniform gradient series that can be split into the uniform base amount of Rs.600000 and gradient amount in multiples of Rs.5000 as shown in Fig. 3.14. The equivalent present worth of this gradient amount will be located at the beginning i.e. in year „0".

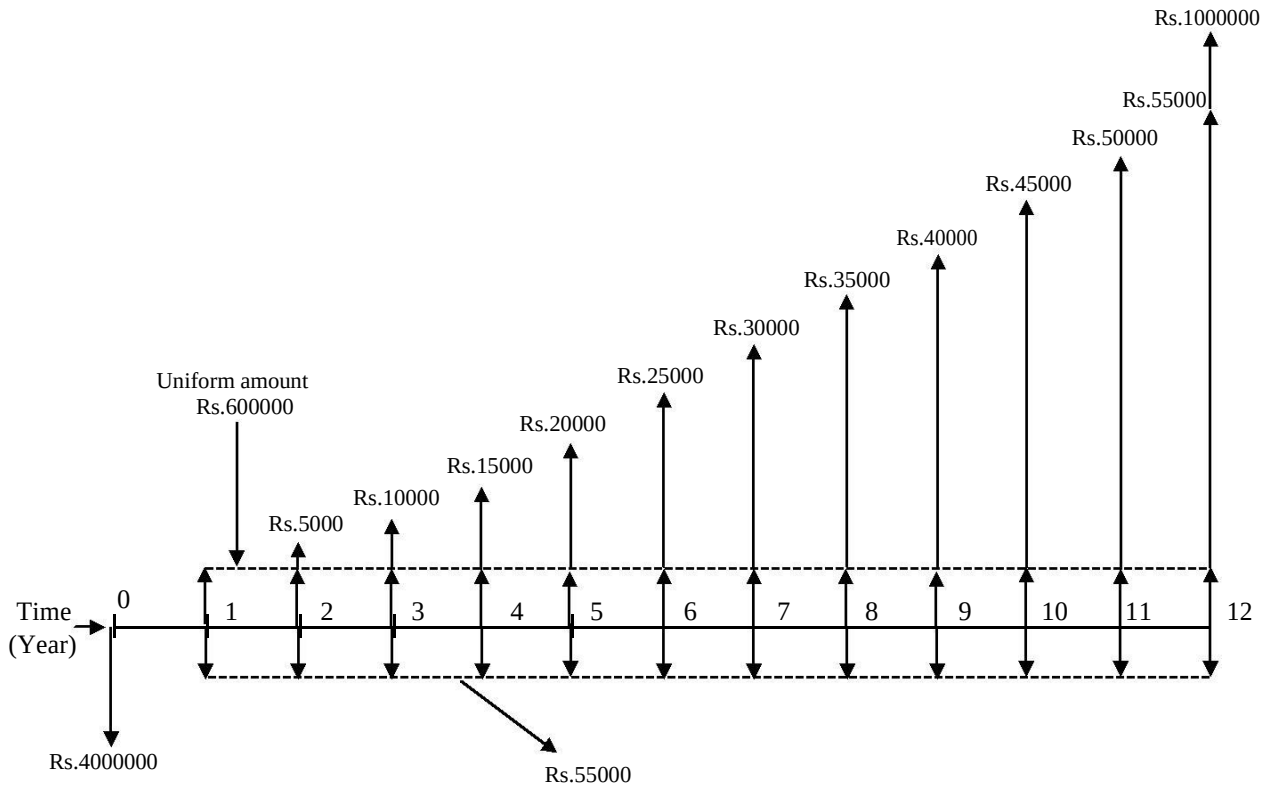


Fig. 3.15 Cash flow diagram of Company-B Dozer
with annual revenue split into uniform base amount and gradient amount

The equivalent present worth PW_B (in Rs.) of Company-B Dozer is determined as follows;

PW_B = - 4000000 - 55000*(P/A, 7%, 12)* + 600000*(P/A, 7%, 12)* + 5000*(P/G, 7%, 12)* + 1000000*(P/F, 7%, 12)*

PW_B = - 4000000 + (600000 - 55000) *(P/A, 7%, 12)* + 5000*(P/G, 7%, 12)* + 1000000*(P/F, 7%, 12)*

Now putting the values of different compound interest factors in the above expression for PW_B results in the following;

PW_B =– 4000000+545000×7.9427 +5000×37.3506+1000000×0.4440

PW_B = - 4000000 + 4328772 + 186753 + 444000

PW_B = Rs.959525

Thus from the comparison of equivalent present worth of the alternatives, it is evident that the construction firm should select Company-B Dozer over Company-A Dozer, as it shows higher positive equivalent present worth i.e. $PW_B > PW_A$.

Comparison by future worth method:-

In the following example, the comparison of three mutually exclusive alternatives by future worth method will be illustrated.

Problem 6: A construction contractor has three options to purchase a dump truck for transportation and dumping of earth at a construction site. All the alternatives have the same useful life. The cash flow details of all the alternatives are presented as follows;

Option-1: Initial purchase price = Rs.2500000, Annual operating cost Rs.45000 at the end of 1^{st} year and increasing by Rs.3000 in the subsequent years till the end of useful life, Annual income = Rs.120000, Salvage value = Rs.550000, Useful life = 10 years.

Option-2: Initial purchase price = Rs.3000000, Annual operating cost = Rs.30000, Annual income Rs.150000 for first three years and increasing by Rs.5000 in the subsequent years till the end of useful life, Salvage value = Rs.800000, Useful life = 10 years.

Option-3: Initial purchase price = Rs.2700000, Annual operating cost Rs.35000 for first 5 years and increasing by Rs.2000 in the successive years till the end of useful life, Annual income = Rs.140000, Expected salvage value = Rs.650000, Useful life = 10 years.

Using future worth method, find out which alternative should be selected, if the rate of interest is 8% per year.

Solution:

The cash flow diagram of Option-1 is shown in figure 3.16.

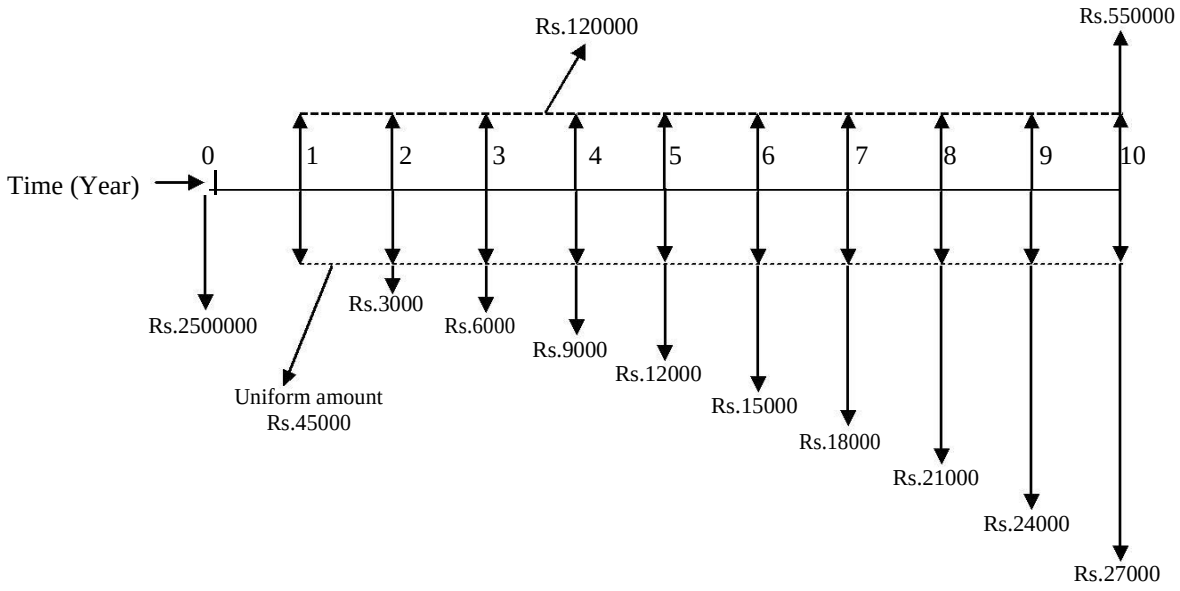


Fig. 3.16 Cash flow diagram of Option-1 with annual operating cost split into uniform base amount and gradient amount (shown for ready reference)

The equivalent future worth (in Rs.) of Option-1 is determined as follows;

$$FW_1 = -2500000(F/P,8\%,10) - 45000(F/A,8\%,10) - 3000(F/G,8\%,10) + 120000(F/A,8\%,10) + 550000$$

$$FW_1 = -2500000(F/P,8\%,10) + (120000 - 45000)(F/A,8\%,10) - 3000(F/G,8\%,10) + 550000$$

Now putting the values of different compound interest factors in the above expression for FW_1 results in the following;

$$FW_1 = -2500000 \times 2.1589 + 75000 \times 14.4866 - 3000 \times 56.0820 + 550000$$

$$FW_1 = -5397250 + 1086495 - 168246 + 550000$$

FW_1 = - Rs.3929001

The cash flow diagram of Option-2 is shown in figure 3.17

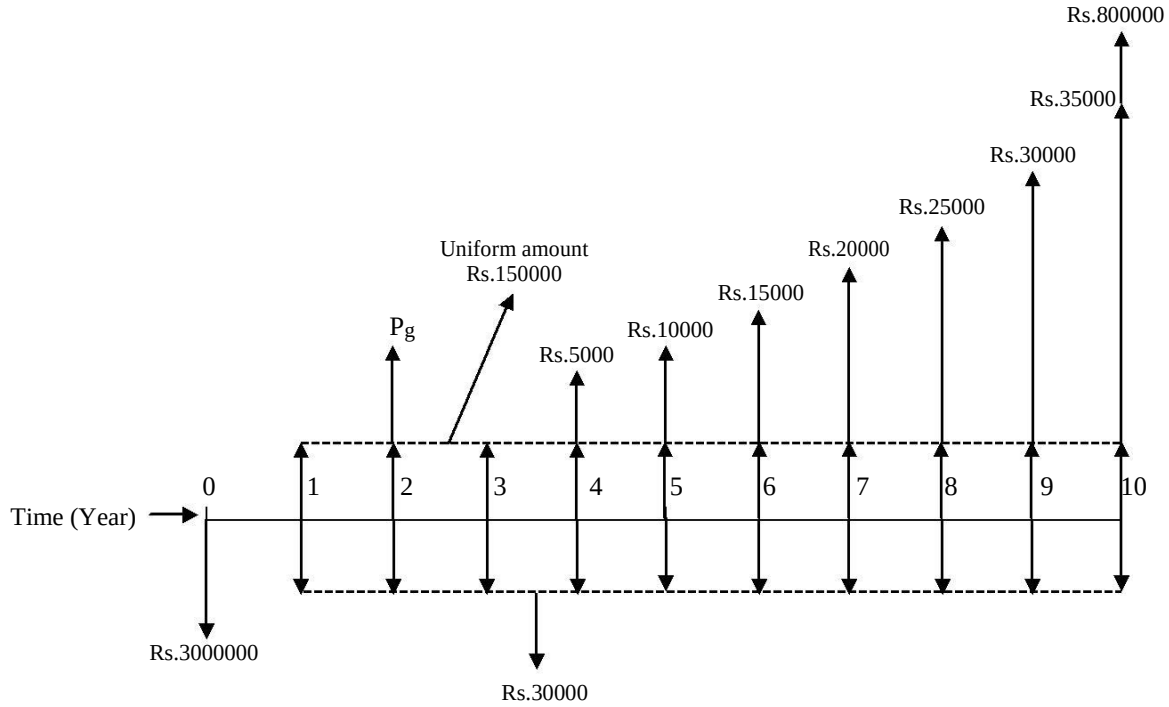


Fig.3.17 Cash flow diagram of Option-2 with annual income split into uniform base amount and gradient amount (shown for ready reference)

The equivalent present worth of the gradient series (of the annual income) starting from end of year „4" will be located at the end of year „2". The future worth of this amount at end of year „10" will be obtained by multiplying the equivalent present worth „P_g' (shown in Fig. 2.8) at the end of year „2" with the single payment compound amount factor *(F/P, i, n)*.

The equivalent future worth (in Rs.) of Option-2 is determined as follows;

$$FW_2 = -3000000(F/P,8\%,10) - 30000(F/A,8\%,10) + 150000(F/A,8\%,10) + P_g(F/P,8\%,8) + 800000$$

Now replacing P_g with *G (P/G, i, n)* i.e. 5000(*P/G, 8%, 8*) in the above expression;

$$FW_2 = -3000000(F/P,8\%,10) + (150000 - 30000)(F/A,8\%,10) + 5000(P/G,8\%,8)(F/P,8\%,8) + 800000$$

It may be noted here that, in the above expression, 5000*(P/G, 8%, 8) (F/P, 8%, 8)* can be replaced by 5000*(F/G, 8%, 8)* and will result in the same value.

Now putting the values of different compound interest factors in the above expression;

$$FW_2 = -3000000 \times 2.1589 + 120000 \times 14.4866 + 5000 \times 17.8061 \times 1.8509 + 800000$$

FW_2 = - 6476700 + 1738392 + 164787 + 800000

FW_2 = - Rs.3773521

The cash flow diagram of Option-3 is shown in figure 3.18

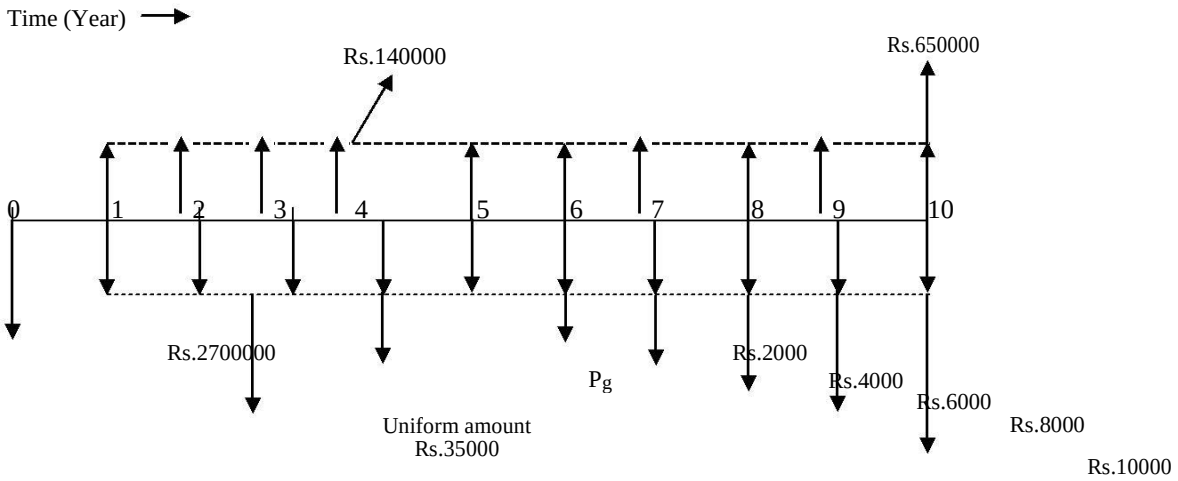


Fig. 3.18 Cash flow diagram of Option-3 with annual operating cost split into uniform base amount and gradient amount (shown for ready reference)

For the annual operating cost, the equivalent present worth of the gradient series starting from end of year „6" will be located at the end of year „4". The future worth of this amount at end of year „10" will be determined by multiplying the equivalent

present worth „P_g' (shown in Fig. 2.10) at the end of year „4" with the single payment compound amount factor *(F/P, i, n).*

The equivalent future worth (in Rs.) of Option-3 is determined as follows;

$$FW_3 = -2700000(F/P,8\%,10) - 35000(F/A,8\%,10) - P_g(F/P,8\%,6) + 140000(F/A,8\%,10) + 650000$$

Now replacing P_g with *G (P/G, i, n)* i.e. 2000(*P/G, 8%, 6*) in the above expression;

$$FW_3 = -2700000(F/P,8\%,10) + (140000 - 35000)(F/A,8\%,10) - 2000(P/G,8\%,6)(F/P,8\%,6) + 650000$$

In the above expression, 2000*(P/G, 8%, 6) (F/P, 8%, 6)* can also be replaced by 2000*(F/G, 8%, 6).*

Now putting the values of different compound interest factors in the above expression;

$$FW_3 = -2700000 \times 2.1589 + 105000 \times 14.4866 - 2000 \times 10.5233 \times 1.5869 + 650000$$

$$FW_3 = -5829030 + 1521093 - 33399 + 650000$$

FW_3 = - Rs.3691336

Comparing the equivalent future worth of all the three alternatives, it is evident that Option-3 shows lowest negative equivalent future worth as compared to other options. Thus Option-3 will be selected for the purchase of the dump truck. This outcome obtained by future worth method is same as that obtained from the present worth method i.e. Option-3 is the most economical alternative.

Pay-back comparision:

payback method does not consider the present value of cash flows. Under this method, an investment project is accepted or rejected on the basis of payback period. Payback period means the period of time that a project requires to recover the money invested in it. The payback period of a project is expressed in years and is computed using the following formula:

Formula of payback period:

$$\text{Payback period} = \frac{\text{Investment required for a project}}{\text{Net annual cash inflow}}$$

Pay back peroid=(Required investment/Annual receipts-annual disbursements)

Pay back=(investment-salvage) / Operating Advantage/year=(I-S)/OA/year

Problem1: The lake city bank is considering a purchase of a data processing storage unit which will cost Rs 20,000 and will last 20 years and then have a guaranteed salvage value of Rs 2000.It will generate savings of Rs 4000/year before depreciation but necessiates that Rs 1000 of the savings must be paid in taxes.If Management insists on a 5 year pay off period.Does this investment qualify.?

Solution:

Given: Investment I=Rs 20,000

Expected life =20 years

Salvage value=Rs 2000

O.A/year= Rs 4000

Taxes paid=Rs 1000/year

OA/year after taxes=OA/year-taxes=4000-1000=Rs 3000

Pay off period= 5 years

Payback period is given by…. Paypack period = (I-S)/OA per year= (20000-2000)/3000= 6 years.

Conclusion: The investment doesnot meet the management criteria as the payback period is more than the required one.

Engineering Economics

Module -2

Present Worth Comparisons

Present worth(PW)

In this method of **comparison**, the cash flows of each alternative will be reduced to time zero by assuming an interest rate i. In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives.

There are 5 basic methods to have results of selected alternative or to be analysed

Equivalent worth :

i) Present worth (PW)
ii) Annual worth (AW)
iii) Future worth (FW)

Rate of returns

i) Internal rate of return(IRR)
ii) External rate of return(ERR)

Conditions for PW comparisions

1. Cash flows are known
2. Cash flows are in constant value Rs.
3. The interest rate is known
4. Comparisions are made with before-tax cash flows
5. Comparisions do not include intangible considerations
6. Comparisions do not include consideration of the availability of funds to implement alternatives

REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in

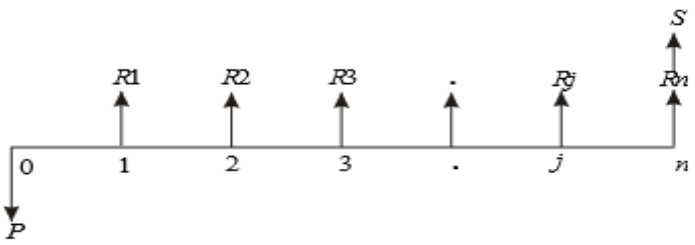


Fig. 4.1 Revenue-dominated cash flow diagram.

To find the present worth of the above cash flow diagram for a given interest rate, the formula is

$PW(i) = -P + R1[1/(1+i)^1] + R2[1/(1+i)^2] + ... + Rj[1/(1+i)^j] + Rn[1/(1+i)^n] + S[1/(1+i)^n]$

In Fig. 4.1,

P represents an initial investment

Rj the net revenue at the end of the jth year. The interest rate is i, compounded annually.

S is the salvage value at the end of the nth year.

Expenditure is assigned a negative sign and revenues are assigned a positive sign.

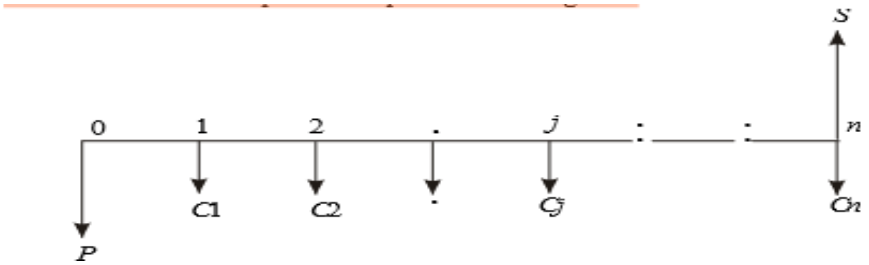
If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared.

Finally, the alternative with the maximum present worth amount should be selected as the best alternative.

COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.

To compute the present worth amount of the above cash flow diagram for a given interest rate *i*, we have the formula



$PW(i) = P + C1[1/(1+i)^1] + C2[1/(1+i)^2] + ... + Cj[1/(1+i)^j] + Cn[1/(1+i)^n] - S[1/(1+i)^n]$

In Fig,

P represents an initial investment

Cj the net cost of operation and maintenance at the end of the jth year

S the salvage value at the end of the nth year

Expenditure is assigned a positive sign and the revenue a negative sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared.

Finally, the alternative with the minimum present worth amount should be selected as the best alternative.

Basic PW comparisions

The present worth of a cash flow overtime is its value today is represented as time '0' in a cash flow diagram.

Two general patterns are,

i) Present worth Equivalence

Example problem

1. The lease on a warehouse amounts to Rs.5000 per month for 5yrs. If payments are made on the first of each month,what is the present worth of the agreement at a nominal annual interest rate of 12% compounded annually.
   A=5000
   N=5*12=60months i=12%=12/12=1%
   PW=A(P/A,1%,60)
   = 5000(P/A,1%,60)
   =5000 * 44.955
   PW=2,24,775

ii) Net Present Worth

It has an initial outlay at time 0 followed by a series of receipts and disbursements.

Net present-worth = PW (benefits) – PW (costs)

Example problem

1. A piece of new equipments was proposed by engineers to increase the production of a certain manual welding operation. The investment is Rs.25000 and the equivalent will have salvage value of Rs.5000 at the end of 5yrs. Increased productivity will gain Rs.8000 per year after extra operating costs have been subtracted from the additional production, with rate of interest 20%.Use present worth method.
   Ans:
   Total PW=PW of cash receipts – PW of cash outlays
   = 8000(P/A,20%,5)+5000(P/F,20%,5) - 25000

   =8000(2.991) + 5000(0.4019) – 25000 using table

   Total PW=934.5

Problems based on the Concept of PW method of comparison applied to the selection of best alternative.

Problems

1. Alpha industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized in Table below. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

	Initial outlay (Rs.)	Annual revenue (Rs.)	Life (years)
Technology 1	12,00,000	4,00,000	10
Technology 2	20,00,000	6,00,000	10
Technology 3	18,00,000	5,00,000	10

In all technologies, the initial outlay is assigned negative sign and annual revenues are assigned positive sign

Technology 1

P=12,00,000

A=4,00,000

i=20%

n=10 years

The cash flow diagram of this technology is as shown in Fig. 4.3.

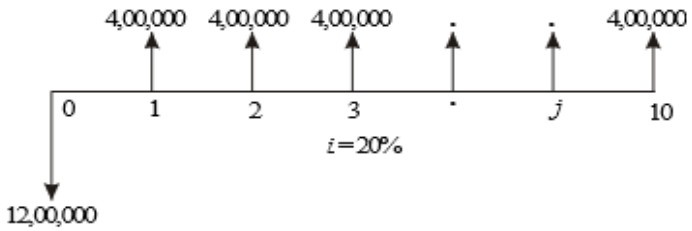


Fig. 4.3 Cash flow diagram for technology 1.

PW (20%) = -12,00,000 + 4,00,000(P/A,20%,10)

= -12,00,000 + 4,00,000 * 4.192

PW(20%) = Rs. 4,76,800

Technology 2

P=20,00,000 , A=6,00,000 i=20% , n=10

The cash flow diagram of this technology is shown in Fig. 4.4.

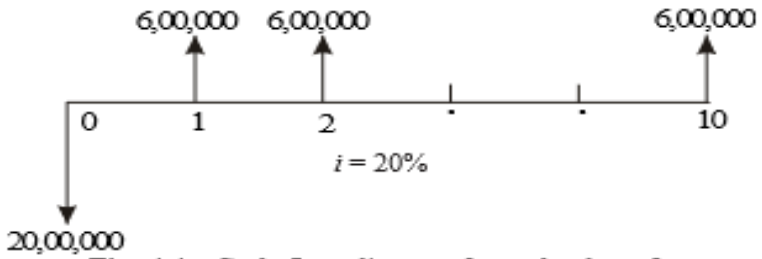


Fig. 4.4 Cash flow diagram for technology 2.

PW(20%)= -20,00,000 + 6,00,000(P/A,20%,10)

=-20,00,000 + 6,00,000 * 4.192

PW(20%)=5,15,200

Technology 3

P=18,00,000 A=5,00,000 , i=20%, n=10

The cash flow diagram of this technology is shown in Fig. 4.5.

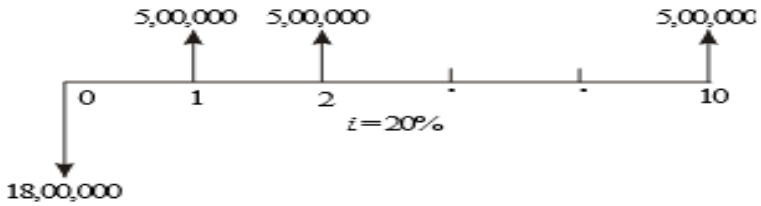


Fig. 4.5 Cash flow diagram for technology 3.

PW(20%)= -18,00,000 + 5,00,000(P/A,20%,10)

=-18,00,000 + 5,00,000 * 4.1925

PW(20%)=2,96,250

Technology 2 should be considered.

2. An engineer has two bids for an elevator to be installed in a new building. The details of the bids for the elevators are as follows:

Bid	Engineers estimates		
	Initial cost (Rs.)	Service life (years)	Annual operations and maintainance costs(Rs)
Alpha elevator	4,50,000	15	27000
Beta elevator	5,40,000	15	28500

Determine which bid should be accepted, based on the present worth method of comparison assuming 15% interest rate, compounded annually.

Bid 1 : Alpha elevator

P=4,50,000 A=27,000 n=15 i=15%

PW(15%) = 4,50,000 + 27,000 (P/A,15%,15)

=4,50,000 + 27000 * 5.8474

PW(15%)=6,07,879.80

The cash flow diagram of bid 1 is shown in Fig. 4.6.

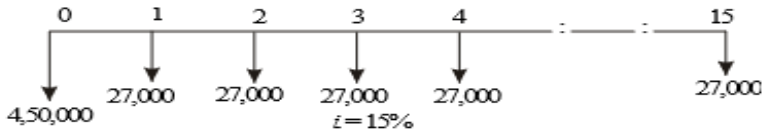


Fig. 4.6 Cash flow diagram for bid 1.

Bid 2 : Beta elevator

P=5,40,000 A=28,500 n=15 i=15%

PW(15%)=5,40,000 + 28,500 (P/A,15%,15)

=5,40,000 + 28,500 * 5.8474

PW(15%)=7,06,650.90

The cash flow diagram of bid 2 is shown in Fig. 4.7.

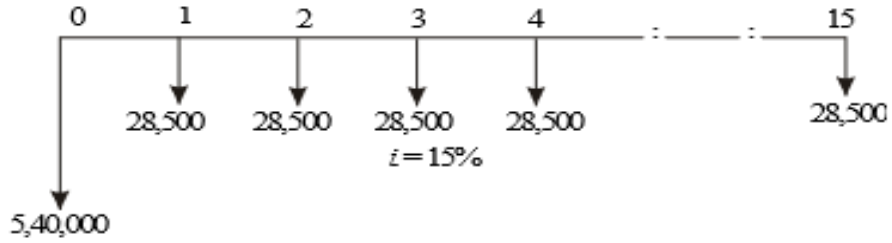


Fig. 4.7 Cash flow diagram for bid 2.

Alpha elevator should be implemented.

3. Investment proposals A and B have the net cash flows as follows:

Proposals	End of years				
	0	1	2	3	4
A(Rs.)	-10,000	3000	3000	7000	6000
B(Rs.)	-10,000	6000	6000	3000	3000

Compare the present worth of A with that of B at i = 18%. Which proposal should be selected?

Proposal A

Present worth of A at $i = 18\%$. The cash flow diagram of proposal A is shown in Fig. 4.8.

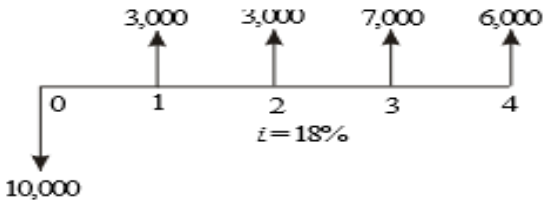


Fig. 4.8 Cash flow diagram for proposal A.

PW(18%)=-
10,000+3000(P/F,18%,1)+3000(P/F,18%,2)+7000(P/F,18%,3)+6000(P/F,18%,4)

=-10,000 + 3000 (0.8475) + 3000(0.7182) + 7000(0.6086) + 6000(0.5158)

PW(18%)= Rs. 2052

Proposal B

Present worth of B at $i = 18\%$. The cash flow diagram of the proposal B is shown in Fig. 4.9.

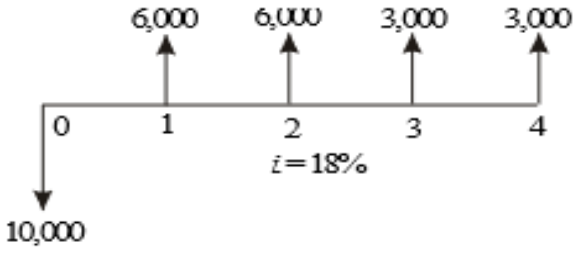


Fig. 4.9 Cash flow diagram for proposal B.

PW(18%)= -10,000 + 6000(P/F,18%,1) + 6000(P/F,18%,2) + 3000(P/F,18%,3) + 3000(P/F,18%,4)

= -10000 + 6000 (0.8475) + 6000 (0.7182) +3000(0.6086) + 3000(0.5158)

PW(18%) = Rs.2766.2

The PW of proposal B is higher than proposal A . so select proposal B.

4. A granite company is planning to buy a fully automated granite cutting machine. If it is purchased under down payment, the cost of the machine is Rs. 16,00,000. If it is purchased under installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of Rs. 2,00,000 each. Suggest the best alternative for the company using the present worth basis at i = 18%, compounded annually.

Two alternatives,

i) Down payment = **Rs.16,00,000**

ii) Down payment of Rs.4,00,000 and 10 annual equal installments of 2,00,000 each.

PW(18%)=4,00,000 + 2,00,000 (P/A,18%, 10)

=4,00,000 + 2,00,000 * 4.494

PW(18%)=12,98,800

Present worth calculation of the second alternative. The cash flow diagram of the second alternative is shown in Fig. 4.10.

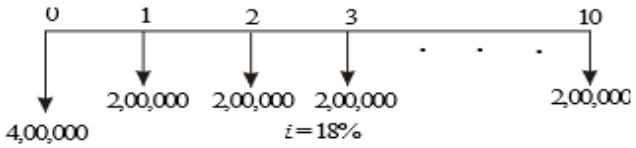


Fig. 4.10 Cash flow diagram for the second alternative.

Second alternative should be selected.

5. A small business with an initial outlay of Rs. 12,000 yields Rs. 10,000 during the first year of its operation and the yield increases by Rs. 1,000 from its second year of operation up to its 10th year of operation. At the end of the life of the business, the salvage value is zero. Find the present worth of the business by assuming an interest rate of 18%, compounded annually.

The cash flow diagram for the small business is depicted in Fig. 4.15.

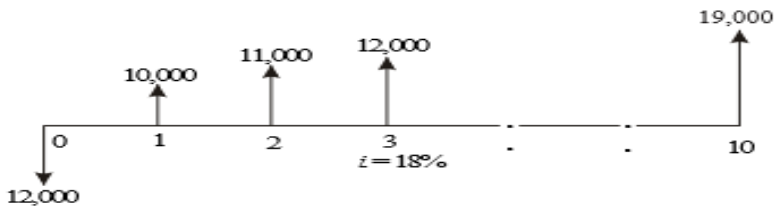


Fig. 4.15 Cash flow diagram for the small business.

A= A1 + G (A/G,18%,10)

=10,000 + 1000 (3.194)

A=13,194

PW(18%) = -12,000 +A (P/A,18%,10)

= -12,000 + 13,194 (4.494)

PW(18%) = 47,293

6. A company borrowed 1,00,000 to finance a new product the loan was for 20 years at a nominal interest rate of 8% compounded semiannually. It was to be repaid in 40 equal payments. After one half (1/2) the payments were made, the company decided to pay the remaining balance in one final payment at the end of the 10th year how much was owed?

P=1,00,000 N=20years = 20 * 2 = 40 installments

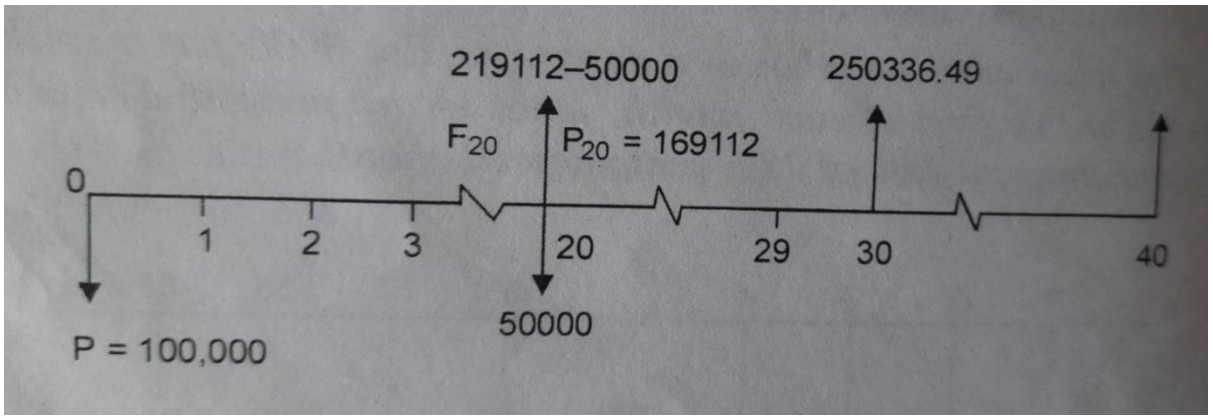
i=8% Compounded semi-annually = 8/2=4%

i) F20 = P (F/P, 4% , 20)
= 1,00,000 (2.19112)
F20 = 2,19,112

P=50,000 – 2,19,112
P=1,69,112

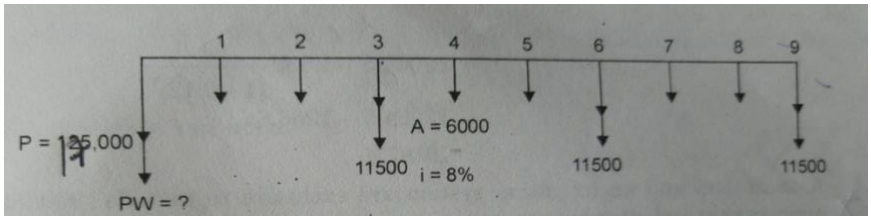
ii) F30=P(F/P , 4% , 10)
=1,69,112 * 1.4803
F30=2,50,336.49

The company owed to finance is 2,50,336.49



7. A proposed improvement in an assembly line will have an initial purchase and installation cost of Rs.1,75,000. The annual maintenance cost will be Rs.6000. Periodic overhauls once every 3yrs excluding the last year of use, will cost Rs.11,500 each. The improvement will have a usefull life of 9 years at which time it will have no salvage value, what is the present worth of the 9 year costs of the improvement at i=8% ?

PW= A(P/A,i%,N) + F(P/F,i%,N) + F(P/F,i%,N) + F(P/F,i%,N)
=6000(P/A,8%,9) + 11500(P/F,8%,3) + 11500(P/F,8%,6) + 11500 (P/F,8%,9) + Initial investment
= 6000 (6.247) + 11500(0.793) +11500 (0.630) +11500(0.5002) + 1,75,000
PW= 2,34,598.8



8. A bakery is thinking of purchasing a small delivery truck that has a first cost of Rs.18,000 and is to be kept in service for 6year. At what time the salvage value is expected to be 2500. Maintenance and operating costs are estimated at Rs.2500 the first year and will increase at a rate of Rs.200/year. Determine the PW of this vehicle using interest rate of 12%.

A= A1 + G (A/G,12%,6)

A=2500 + 200 (2.17205)

A=2934.4

PW= A(P/A,i%,N) – F(P/F,i%,N)

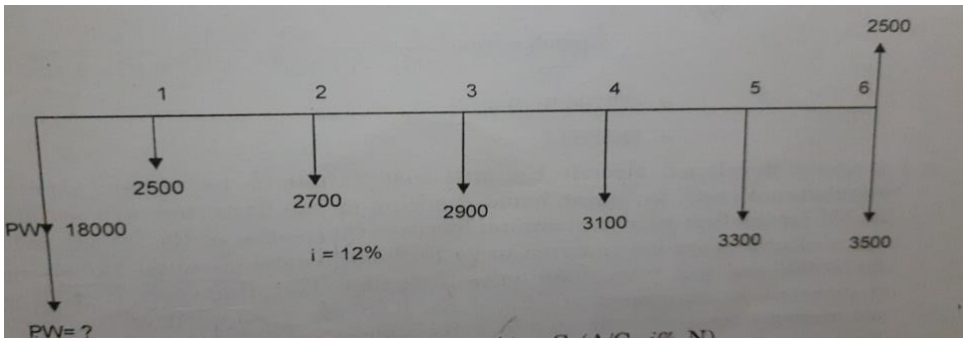
=2934.4 (P/A,12%,6) – 2500 (P/F,12%,6)

=2934.4 (4.111) – 2500 (0.5066)

PW=10,797.9

PW of the vehicle = 10,797.9 + 18,000

=28,797.9



9. A small dam and an irrigation system are exclusive to cost Rs.3,00,000. Annual maintenance and operating costs are executed to be Rs.40000 the first year and will increase at a rate of 10% per year. Determine the equivalent PW of building and operating the system with interest of 10% over a 30 year life.

i=10% N=30

A= A1 + G (A/G,i%,N)

=40000 + 4000 (A/G,10%,30)

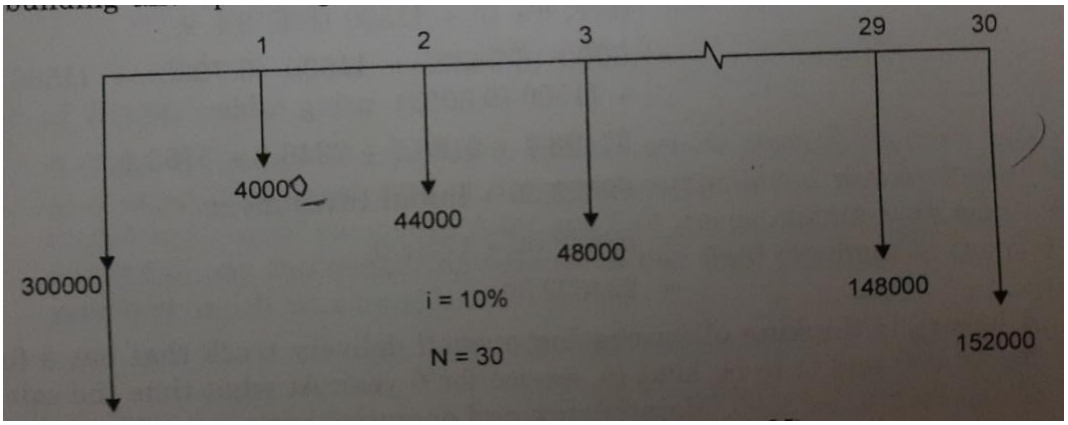
A=72,704.9

PW= A(P/A,i%,N)

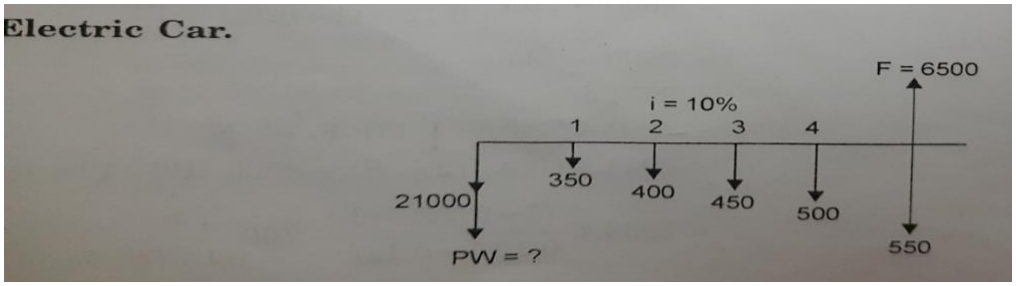
=72,704.9 * (P/A,10%,30)

=72,704.9 * 9.427

PW=6,85,389.1 + Initial investment
PW=6,85,389 + 3,00,000
PW=9,85,389



10. A newly developed electric car will cost 21000 to purchase. Operating and maintaenance costs is Rs.350 for the first year with annual increase of Rs.50 per year. Salvage value after 5years is Rs.6500. A new gasoline run will cost Rs.16,000 with average 30 miles per gallon. Gasoline costs Rs.1.26 per gallon is expected to increase at a rate of Rs. 0.05 per year each of the next 4years maintaenance costs are estimated to be Rs.300 per year.Salvage value is Rs.1500 after 5years of service.If the vehicles are expected to be driven for 20,000 miles per year, determine which option will have the lower cost over 5 years. Use PW analysis with 10% rate of interest.

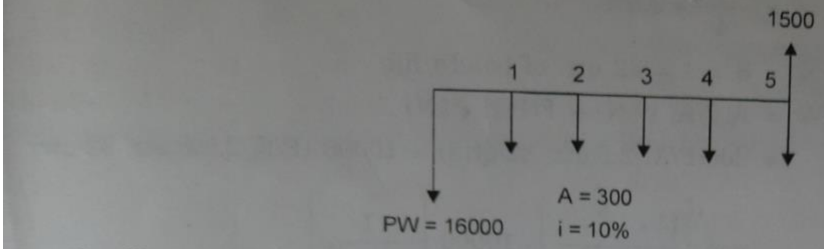


PRESENT WORTH COMPARISONS

PW = –21000 –[350 + 50(A/G, 10%, 5) (P/A, 10%5)] + 6500 (P/F, 10% 5) using table

PW = –18633.40

(*b*) *New gasoline run about*



Gasoline cost/year
Expected to run
20000 miles per year
30 miles per gallon

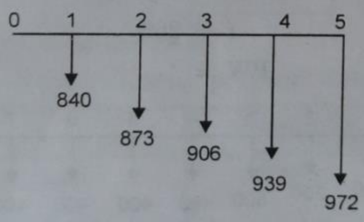
$\therefore$ No. of gallons $= \frac{20000}{30}$

$= 666.67$

$\therefore$ Cost of gasoline per year is $= 666.67 \times 1.26$

$= 840$

It is expected to increae by Rs. 0.05 per litre/year



PW gasoline = [A + G(A/G, *i*% N] (P/A *i*% N)

= [840 + 33(A/G, 10%5)] (P/A, 10%5)

= [840 + 33 × (1.810)] (3.791)

= 899.7 × 3.791

PW gasoline = 3410.9

PW = –16000 – 300(P/A, 10%5) – 3410.9 + 1500(P/F, 10%5)

= –16000 – 300(3.791) – 3410.9 + 931.38

= –19616.82

Electric car is having lowest cost.

11. A company is considering constructing a plant to manufacture a proposed new product. The land costs 3,00,000 the building costs 6,00,000. The equipment costs 2,50,000 and 1,00,000 working capital is required. It is expected that the product will result in sales of 7,50,000 per year for 10 years at which time the land can be sold for 4,00,000 the building for 3,50,000 the equipment for 50,000 and all of the working capital recovered. The annual out-of-pocket expenses for

labour,materials and all other items are estimated to be total 4,75,000. If the company requires a minimum return of 25% on projects of comparable risk, determine if it should invert in the new product line use PW methods.

Ans :

Investment = 3,00,000 + 6,00,000 + 2,50,000 + 1,00,000=12.5 lakhs
Profit (A)= 7,50,000 / year for 10 years
Salvage value = 4,00,000 + 3,50,000+50,000+100000=9,00,000
Annual expenses = 4,75,000 / year for 10 years
i=25% , N=10 years

PW= -P + A (P/A,i%,N) – A (P/A,i%,N) + F (P/F,i%,N)
= - 12,50,000 + 7,50,000 (P/A,25%,10) – 4,75,000 (P/A,25%,10) + 9,00,000 (P/F,25%,10)
= -12,50,000 + 7,50,000 (3.571) – 4,75,00 (3.571) + 9,00,000 (0.108)
PW= -1,70,775

12. Determine the PW of the following proposal when MARR is 15%.

	Proposal A
First cost (P)	10,000
Expected lift(N)	5 years
Salvage value(F)	-1000
Annual receipts(+A)	8000
Annual expenses(-A)	4000

PW= - P + A(P/A,i%,N) – A (P/A,i%,N) – F(P/F,i%,N)
= -10,000 + 8000(P/A,15%,5) – 4000(P/A,15%,5) – 1000(P/F,15%,5)
= -10,000 + 8000(3.352) – 4000(3.352) – 1000(0.497)
PW=2910.8

13. Evaluate machine XYZ on the basis of the present worth method when MARR is 12% pertinent cost data is as follows:

	Machine XYZ
First cost	13,000
Useful life	15 years
Salvage value	3000
Annual operating cost	100
Overhaul end of 5^{th} year	200
Overhaul end of 10^{th} year	550

PW= -P –A(P/A,i%,N) – F(P/F,i%,N) – F(P/F,i%,N) +F(P/F,i%,N)
= -13,000 – 100 (P/A,12%,15) – 200 (P/F,12%,5) – 550(P/F,12%,10) + 3000(P/F,12%,15)
= -13,000 – 100 (6.811) – 200 (0.567) – 550 (0.322) + 3000(0.1827)
PW= -13,422

Assests with Unequal Lives (Present Worth Comparisions with Unequal lives)

1. Repeatability assumptions(LCM method)
2. Co-terminated assumptions/Study period method

Problems

1. Two holidays cottages are under consideration. Compare the present worth of the cost of 24 years service, at an interest rate of 5% when no salvage value.

	Cottage 1	Cottage 2
First cost	Rs. 4500	Rs. 10,000
Estimated life	12 years	24 years
Annual maintenance cost	Rs. 1000	Rs. 720

LCM of 12 and 24 = 24

Cottage 1

N=12

24/12 = 2

PW = P + A (P/A,i%,N) + F1(P/F,i%,N)

= 4500 + 1000 (P/A,5%,24) + 4500(P/F,5%,12)

=4500 + 1000 (13.79864) + 4500 (0.55684)

PW=20,804.42

Cottage 2

PW= P + A(P/A,i%,N)

=10,000 + 720 (P/A,5%,24)

=10,000 + 720 (13.79864)

PW=19,935.02

Cottage 2 is selected.

2. Two types of trucks are available for transportation use. They are needed for 10 years. i=7% , PW=?

	Truck A	Truck B
First cost	Rs. 10,00,000	Rs. 15,00,000
Estimated annual maintenance cost	Rs. 20,000	Rs. 15,000
Estimated life	5 years	10 years
Estimated salvage value	Rs. 2,00,000	Rs. 5,00,000

LCM of 5 and 10 = 10

Truck A

10/5 = 2

PW = -P – A (P/A,i%,N) + S1(P/F,i%,N) – F1(P/F,i%,N + S2(P/F,i%,N)

= -10,00,000 – 20,000 (P/A,7%,10) + 2,00,000(P/F,7%,5) – 10,00,000 (P/F,7%,5) + 2,00,000 (P/F,7%,10)

= -10,00,000 – 20,000 (7.024) + 2,00,000(0.7130)- 10,00,000(0.7130) + 2,00,000(0.5083)

PW= -16,09,220

Truck B

PW = - P – A (P/A,i%,N) + F (P/F,i%,N)

= -15,00,000 – 15,000 (P/A,7%,10) + 5,00,000 (P/F,7%,10)

= -15,00,000 – 15,000 (7.024) + 5,00,000 (0.5083)

PW = -13,51,210

Truck B will be selected.

3. The following alternatives are available to accomplish an objective of 12 years duration. Compare the PW of the alternatives using i=7% per year.

	Plan A	Plan B	Plan C
Life cycle (in yrs)	6	3	4
First cost ($)	2000	8000	10,000
Annual cost ($)	3200	700	500

LCM of 6,3,4 = 12

Plan A

12/6=2

PW = P + A(P/A,i%,N) + F(P/F,i%,N)

=2000 + 3200 (P/A,7%,12) + 2000(P/F,7%,6)

=2000 + 3200 (7.943) + 2000 (0.6663)

PW= 28,750.2

Plan B

12/3 = 4

PW = P +A(P/A,i%,N) + F1(P/F,7%,3) + F2(P/F,7%,6) + F3(P/F,7%,9)

= 8000 + 700 (P/A,7%,12) + 8000(P/F,7%,3) + 8000(P/F,7%,6) + 8000 (P/F,7%,9)

= 8000 + 700 (7.943) + 8000 (0.8163) + 8000(0.6663) + 8000(0.5439)

PW = 29772.4

Plan C

12/4=3

PW= P + A(P/A,i%,N) + F1(P/F,i%,N) + F2(P/F,i%,N)

= 10,000 + 500(P/A,7%,12) + 10,000(P/F,7%,4) + 10,000(P/F,7%,8)

= 10,000 + 500(7.943) + 10,000(0.7629) + 10,000(0.5820)

PW= 27420.4

Plan C will be selected.

EXAMPLE 2: PRESENT WORTH ANALYSIS OF DIFFERENT-LIFE ALTERNATIVES

GAMA-H Construction, Inc., plans to purchase new cut-and-finish equipment. Two manufacturers offered the estimates below.

	Vendor A	Vendor B
Initial Cost	P 2, 268,000	P 2,721,600
Annual M&O cost	P 340,200	P 301,320
Salvage value	P 205,200	P 302,400
Life, years	6	9

1. Determine which vendor should be selected on the basis of a present worth comparison, if the MARR is 15% per year.
2. GAMA-H Construction, Inc has a standard practice of evaluating all options over a 5-year period. If a study period of 5 years is used and the salvage values are not expected to change, which vendor should be selected?

The PW of the alternatives must be compared over the **same number of years** and must end at the same time to satisfy the equal-service requirement

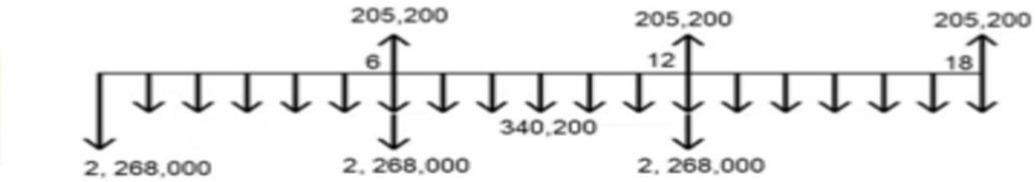
4.

LCM of 6 and 9 is 18

SOLUTION: $P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ $P = F(1+i)^{-n}$

1.

Vendor A:



$$P.W.(A) = -2{,}268{,}000 - A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] + F_6(1+i)^{-n} + F_{12}(1+i)^{-n} + F_{18}(1+i)^{-n}$$

$$P.W.(A) = -2{,}268{,}000 - 340{,}200\left[\frac{(1+0.15)^{18}-1}{0.15(1+0.15)^{18}}\right] + (-2{,}268{,}000 + 205{,}200)(1+0.15)^{-6}$$

$$+ (-2{,}268{,}000 + 205{,}200)(1+0.15)^{-12} + (205{,}200)(1+0.15)^{-18}$$

$$P.W.(A) = -2{,}268{,}000 - 2{,}084{,}734 - 891{,}805.4 - 385{,}552.10 + 16{,}581.20$$

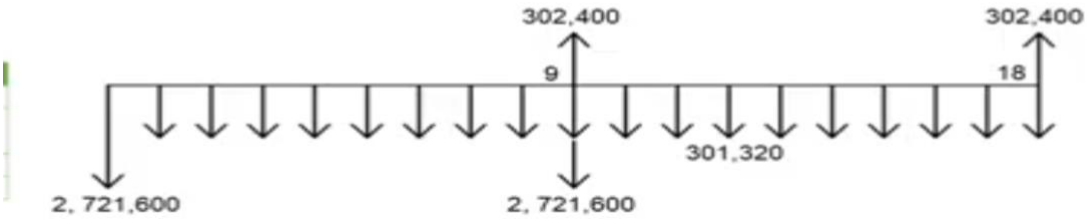
P.W.(A)= - 5,613,510.3

PW= - P – A(P/A, 15%,18) + S1 (P/F,15%,6) + S2(P/F,15%,12) + S3(P/F,15%,18) – F1 (P/F,15%,6) – F2(P/F,15%,12)

SOLUTION:

1.

Vendor B:



$$P.W.(B) = -2{,}721{,}600 - A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] + F_9(1+i)^{-n} + F_{18}(1+i)^{-n}$$

$$P.W.(B) = -2{,}721{,}600 - 301{,}320\left[\frac{(1+0.15)^{18}-1}{0.15(1+0.15)^{18}}\right] + (-2{,}721{,}600 + 302{,}400)(1+0.15)^{-9}$$

$$+ (302{,}400)(1+0.15)^{-18}$$

$$P.W.(B) = -2{,}721{,}600 - 1{,}846{,}478.70 - 687{,}687.6 + 24{,}435.50$$

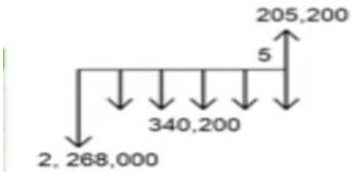
P.W.(B)=- 5,231,330.10

Proposal B (Vendor B) will be selected.

SOLUTION:

2.

Vendor A:



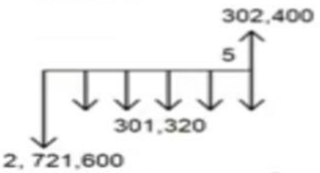
$$P.W.(A) = -2{,}268{,}000 - A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + F_5(1+i)^{-n}$$

$$P.W.(A) = -2{,}268{,}000 - 340{,}200\left[\frac{(1+0.15)^5 - 1}{0.15(1+0.15)^5}\right] + 205{,}200(1+0.15)^{-5}$$

$$P.W.(A) = -2{,}268{,}000 - 1{,}140{,}403.2 + 102{,}020.7$$

P.W.(A)= - 3,306,382.5

Vendor B:



$$P.W.(B) = -2{,}721{,}600 - A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + F_5(1+i)^{-n}$$

$$P.W.(B) = -2{,}721{,}600 - 301{,}320\left[\frac{(1+0.15)^5 - 1}{0.15(1+0.15)^5}\right] + 302{,}400(1+0.15)^{-5}$$

$$P.W.(B) = -2{,}721{,}600 - 1{,}010{,}071.4 + 150{,}346.2$$

P.W.(B)= - 3,581,325.2

Vendor A will be selected

Problems on PW comparisions of Infinite lives / Comparison of assets having Infinite lives

3.8 COMPARISON OF ASSETS HAVING INFINITE LIVES

The special variation of the present worth method involves the determination of the worth of all receipts and expenses over an infinite length of time. This is known as the capitalized worth method. If only expenses are considered the output obtained by this method can be expressed as capitalized cost.

This method is convenient for convenient mutually exclusive alternatives when the period is indefinitely long or when the common multiple of the lives is very long and the repeatability assumption is applicable.

Capitalized cost is calculated in the same way as in a present-worth comparison, where N equals infinity.

$$(P/A, i\%N) = \frac{1}{i}\frac{(1+i)^N - 1}{(1+i)^N}$$

The limit of (P/A, i%, N) as N approaches infinity is

$$(P/A, i\%, \infty) = \frac{1}{i}$$

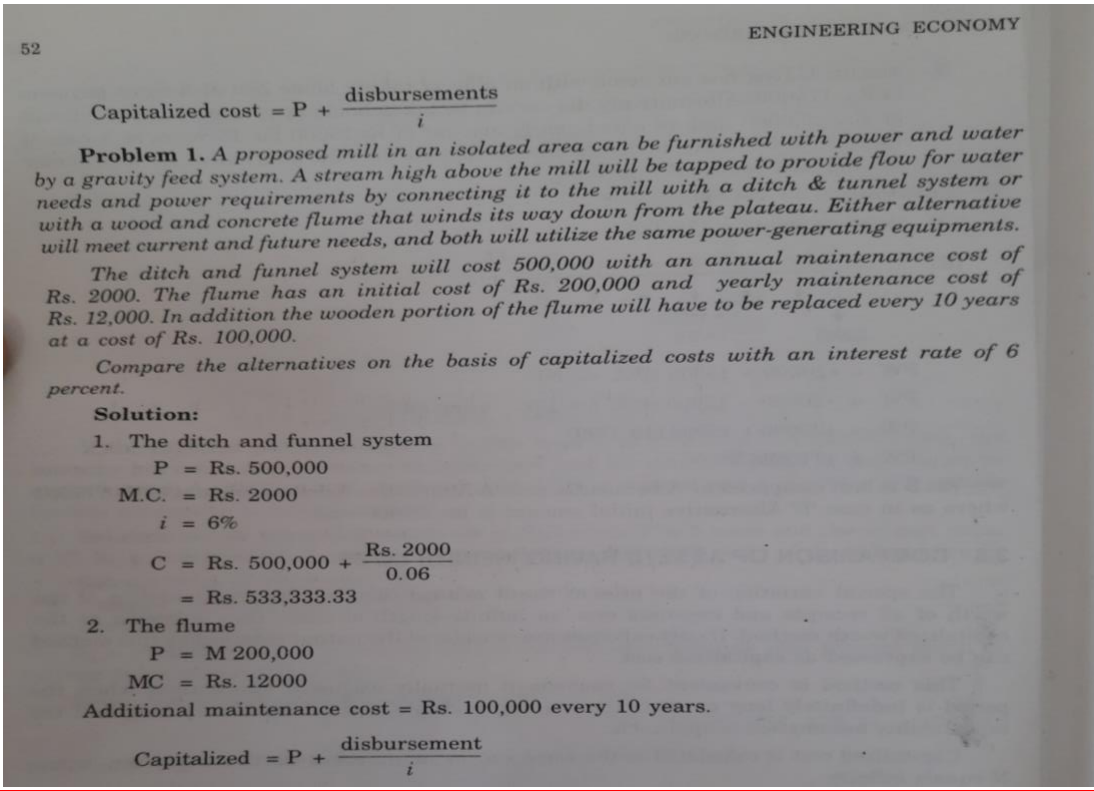
If P represents the first cost.

Capitalized worth $= P + A(P/A, i, \infty)$

$= P + A(1/i)$

$= P + A/i$

Where A is the uniform difference between annual receipts and disbursements. When there is no revenue,



52 ENGINEERING ECONOMY

Capitalized cost = P + $\frac{\text{disbursements}}{i}$

Problem 1. *A proposed mill in an isolated area can be furnished with power and water by a gravity feed system. A stream high above the mill will be tapped to provide flow for water needs and power requirements by connecting it to the mill with a ditch & tunnel system or with a wood and concrete flume that winds its way down from the plateau. Either alternative will meet current and future needs, and both will utilize the same power-generating equipments.*

The ditch and funnel system will cost 500,000 with an annual maintenance cost of Rs. 2000. The flume has an initial cost of Rs. 200,000 and yearly maintenance cost of Rs. 12,000. In addition the wooden portion of the flume will have to be replaced every 10 years at a cost of Rs. 100,000.

Compare the alternatives on the basis of capitalized costs with an interest rate of 6 percent.

Solution:

1. The ditch and funnel system

P = Rs. 500,000

M.C. = Rs. 2000

i = 6%

C = Rs. 500,000 + $\frac{\text{Rs. 2000}}{0.06}$

= Rs. 533,333.33

2. The flume

P = M 200,000

MC = Rs. 12000

Additional maintenance cost = Rs. 100,000 every 10 years.

Capitalized = P + $\frac{\text{disbursement}}{i}$

Capitalized cost = 2,00,000 + [12,000 +1,00,000(A/F,6%,10) /0.06]

=2,00,000 + [12,000 + 1,00,000 (0.07587) / 0.06]

Capitalized cost = 5,26,500

Problem 2 : Capitalised cost = 5 crore

Annual M C = 10 lakhs

Additional maintenance cost every 10 yrs = 8 lakhs

i=9%

First cost (P) =?

Capitalised cost = first cost (P) +[disbursement(A) / i]

First cost= capitalised cost – Total annual costs / i

= 50000000 – 10,00,000 + [8,00,000 (A/F,9%,10)]/0.09

First cost= 3,83,03,822.22

Future Worth method [Refer Paneerselvam text book pg. No. 55 to 66] And Naidu text book pg.53 to 60 (includes Pay back comparison method) [both the documents are shared in GCR]

Problems to be solved

1. A finance company advertises two investment plans. In plan 1, the company pays Rs. 12,000 after 15 years for every Rs. 1,000 invested now. In plan 2, for every Rs. 1,000 invested, the company pays Rs. 4,000 at the end of the 10th year and Rs. 4,000 at the end of 15th year. Select the best investment plan from the investor's point of view at i = 12%, compounded annually.
   Ans : Plan 1 PW=1,192 Plan 2 PW=1018 Plan 1 is the best one.
2. Novel Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 8,00,000 at the end of the 20th year. Innovative Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 15,00,000 at the end of the 25th year. Which is the best investment alternative? Use present worth base with i = 12%.
   Ans: Novel investment ltd. PW(12%)=Rs.8266 Innovative Investment ltd. PW(12%) = Rs.13,506 . Innovative investment ltd plan is the best one for the investor.

3. Autocon company is evaluating three robots for possible use in its assembly operations (only one robot will be purchased). Data associated with these robots are as follows,

	Robot A	**Robot B**	**Robot C**
First cost Rs.	**55,000**	**58,000**	**53,000**
Operating & maintenance costs Rs.	**3000/year**	**4500/year**	**4000/year**
Expected income	**40,000/year**	**44,000/year**	**38,000/year**
Estimated salvage value Rs.	**4000**	**6000**	**4000**

Assuming a technological life of 3 years and a desired interest rate of 12% , which robot seems to be preferable assuming all other factors are equal. Use a net PW evaluation.
PW(Robot A) = 36,721 PW(Robot B)=41,149 PW(Robot C) = 31,515

4. The following data presents for two feasible alternatives A and B for which revenues and costs are known and which have different lives. If the minimum alternative rate of return is 10% , show which feasible alternative is more desirable by using PW.

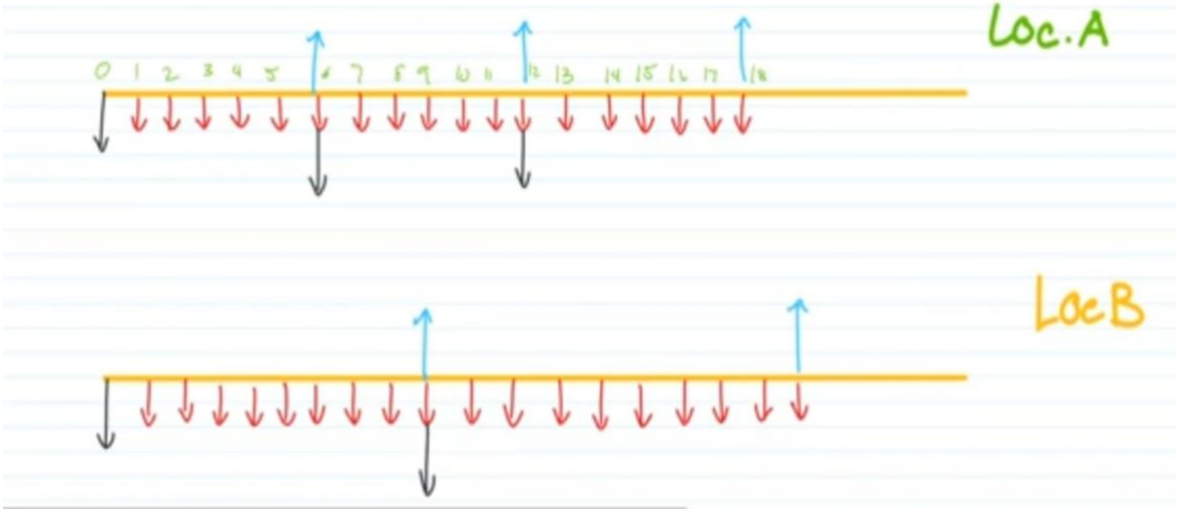
	A	**B**
Investment (first) cost	**3500**	**5000**
Annual revenue	**1900**	**2500**

Annual cost	**645**	**1383**
Useful life	**4**	**8**

5. Determine which lease option should be selected on the basis of a present worth comparison using the LCM

	Location A	Location B
F First cost, $	−15,000	−18,000
A Annual lease cost, $ per year	−3,500	−3,100
Deposit return, $	1,000	2,000
Lease term, years	6	9

LCM = 18



PW(location A) = -45,036.36

PW(location B) = -41,383.28