Infinite Series

Question Bank

1. Test the convergence of the following series

(a) $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\cdots\ldots\ldots\ldots\ldots\ldots..+\left(\frac{n}{2n+1}\right)^n+\cdots\ldots\ldots\ldots..$ 5

(b) $\frac{2!}{3}+\frac{3!}{3^2}+\frac{4!}{3^3}+\cdots\ldots\ldots\ldots..$ 6

© $\sum\frac{1}{(1+n)^2}$ 5

(d) $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\cdots\ldots\ldots\ldots\ldots\ldots\ldots..+\infty$ 6

(e) $\sum_{n=2}^{\infty}\frac{1}{(\log n)^n}$ 5

(f) $\frac{2}{1}+\frac{2.5}{1.5}+\frac{2.5.8}{1.5.9}+\frac{2.5.8.11}{1.5.9.13}+\cdots\ldots\ldots\ldots\ldots..$ 6

(g) $\sum\frac{\sqrt{n}}{\sqrt{n^2+1}}x^n$ 8

2. Obtain the solution of Bessel's differential equation in the form

$y = A\, J_n(x) + B\, J_{-n}(x)$ 8

3. Prove that $J_n(-x) = (-1)^n J_n(x)$ 6

4. Prove that $J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$ 5

5. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\, sinx$ and $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\, cosx$ 6

6. Show that $4\, J_n'' = J_{n-2} - 2J_n + J_{n+2}$ 5

7. Obtain the series solution of Legendre's differential equation in the form

$P_n(x) = \sum_{k=0}^{n}\frac{(-1)^k\ (2n-2k)!}{2^n\ k!\ (n-k)!\ (n-2k)!}x^{n-2k}$ 8

8. Prove that $1 + x + x^2 = \frac{2}{3}P_0(x) + P_1(x) - \frac{2}{3}P_2(x)$ 6

9. Prove that $P_n(x) = P_{n+1}'(x) - 2x\,P_n'(x) + P_{n-1}'(x)$ 6

10. Express $J_6(x)$ in terms of $J_0(x)$ and $J_1(x)$ 8

11. Express $J_{7/2}(x)$ in terms of sine and cosine terms 8

12. Show that $J_0'(x) = -J_1(x)$ and $\frac{d}{dx}(xJ_1) = x\,J_0$ 6

13. Show that $(1-x^2)\,P_{n-1}' = n\,(x\,P_{n-1} - P_n)$ 5

14. Obtain Rodrigue's formula 8

15. Test the convergence of the following series

(a) $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots$ ……………….. 5

(b) $\sum\frac{n!\,2^n}{n^n}$ 6

© $1+\frac{x}{2}+\frac{x^2}{3^2}+\frac{x^3}{4^3}+\cdots$ ………………. (x>0) 5

(d) $\sum\frac{1}{\left(1+\frac{1}{n}\right)^2}$ 6

(e) $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\cdots$ ………… 8

(f) $\sum\sqrt{\frac{n+1}{n^3+1}}x^n$ x>0 8

(g) $\sum\frac{1}{n!}$ 5

16. Prove that $2nJ_n(x) = x[J_{n+1}(x)+J_{n-1}(x)]$ 6

17. If α and β are distinct roots of $J_n(\alpha x)=0$ then prove that

$\int_0^a x\;J_n(\alpha x)\,J_n(\beta x)dx = 0$ 8

18.Prove that $\frac{d}{dx}[x^nJ_n(x)] = x^nJ_{n-1}(x)$ 6

19. Show that $8\,J_n''(x) = J_{n-3} - 3\,J_{n-1} + 3\,J_{n+1} - J_{n+3}$ 6

20. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$ 5

21. Prove that $\frac{d}{dx}[x\,J_n(x)J_{n+1}(x)] = x[J_n^2(x)-J_{n+1}^2(x)]$ 6

22. Show that $J'_{1/2}(x)\;J_{-1/2}(x) - J'_{-1/2}(x)\;J_{1/2}(x) = \frac{2}{\pi x}$ 8

23. Prove that $(2n+1)x P_n(x) = (n+1) P_{n+1}(x) + n P_{n-1}(x)$ 6

24. Prove that $n P_n(x) = x P_n'(x) - P_{n-1}'(x)$ 6

25. Express $f(x) = x^4+3x^3-x^2+5x-2$ in terms of Legendre polynomials 6

26. Express $4x^3-2x^2-3x+8$ in terms of Legendre polynomials 5

27. Show that $(1-x^2)p_n'(x) = n[p_{n-1}(x) - xp_n(x)]$ 8

28. Obtain the Legendre polynomial $P_4(x)$ from Rodrigue's formula 6