Problems on Simple Interest and Compound Interest

Simple Interest:

1. In how many years will a sum of Rs. 4,000 yield a simple interest of Rs. 1,440 at 12% per annum?

2. Amount of Rs. 12800 was invested by Mr Rohan dividing it into two different investment schemes A and B at a simple interest rate of 11% and 14%. What was the amount in plan B if the amount of interest earned in two years was Rs. 3508.

3. A person deposits $5,000 in a bank account which pays 6% simple interest per year. Find the value of his deposit after 4 years.

4. Glen received $2,250 loan from bank. After six months, he paid back $2,295 and closed the loan. Find the rate of interest.

5. In simple interest, a sum of money amounts to $ 6200 in 2 years and $ 7400 in 3 years. Find the principal.

Compound Interest:

6. A person has taken a loan of amount of Rs.10,000 from a bank for a period of 5 years. Estimate the amount of money, the person will repay to the bank at the end of 5 years for compound interest rate of 8% per year

7. A certain sum amounts to $ 72900 in 2 years at 8% per annum compound interest, compounded annually. Find the sum.

8. A man deposited $1000 in a bank. In return he got $1331. Bank gave interest 10% per annum. How long did he kept the money in the bank?

9. Find the amount of $ 12000 after 2 years, compounded annually; the rate of interest being 5 % p.a. during the first year and 6 % p.a. during the second year. Also, find the compound interest.

10. Find the compound interest on Rs. 10000 at 12% rate of interest for 1 year, compounded half-yearly.

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Problems on Simple Interest

1 $T = ?$
$P = 4000$
$SI = 1440$
$R = 12\%$

$$SI = \frac{PTR}{100}$$

$$1440 = \frac{4000 \times T \times 12}{100}$$

$$\Rightarrow T = \frac{1440 \times 100}{4000 \times 12}$$

$$\therefore T = 3 \text{ years}$$

2 $SI = 3508$
$P = P_A + P_B$
$12800 = (12800 - x) + x$
$R_A = 11\%, \quad R_B = 14\%$
$T = 2$
$x = ?$

$$SI = \frac{PTR}{100}$$

$$3508 = \frac{(12800 - x) \times 2 \times 11}{100} + \frac{x \times 2 \times 14}{100}$$

$$3508 \times 100 = 281600 - 22x + 28x$$

$$350800 - 281600 = 6x$$

$$\therefore x = 11,533.33$$

3 $P = 5000$
$R = 6\%$
$T = 4$

$$SI = \frac{PTR}{100}$$

$$= \frac{5000 \times 4 \times 6}{100}$$

$$\Rightarrow SI = 1200$$

$$\therefore F = P + SI$$

$$= 5000 + 1200$$

$$\therefore F = 6200$$

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4 $P = 2250$ $\quad SI = F - P = 45$

$F = 2295$ $\quad SI = \frac{PTR}{100}$

$T = 1/2$

$R = ?$ $\quad 45 = \frac{2250 \times 0.5 \times R}{100}$

$\Rightarrow R = 4\%$

5 $SI = 7400 - 6200 = 1200$

$1200 \times 2 = 2400$

$P = F - SI = 6200 - 2400 = 3800$

- Problems on compound Interest

6 $P = 10000$ $\quad A = P\left[1 + \frac{R}{100}\right]^T$

$T = 5$

$A = ?$ $\quad = 10000\left[1 + \frac{8}{100}\right]^5$

$R = 8\%$

$\therefore A = 14693.28$

7 $A = 72900$ $\quad A = P\left[1 + \frac{R}{100}\right]^T$

$T = 2$

$R = 8$ $\quad 72900 = P\left[1 + \frac{8}{100}\right]^2$

$P = ?$

$\Rightarrow P = 62500$

8 $P = 1000$ $\quad A = P\left[1 + \frac{R}{100}\right]^T$

$A = 1331$

$R = 10$ $\quad 1331 = 1000\left[1 + \frac{10}{100}\right]^T$

$T = ?$

$\frac{11 \times 11 \times 11}{10 \times 10 \times 10} = \left(\frac{11}{10}\right)^T$

$\Rightarrow \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^T$

$\therefore T = 3$ years

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A = ?

9 P = 12000
T = 2
$R_1 = 5$, $R_2 = 6$
CI = ?

$$A = P\left[1 + \frac{R}{100}\right]^T$$

$$= 12000\left[1 + \frac{5}{100}\right]^1\left[1 + \frac{6}{100}\right]^1$$

$\therefore A = 13356$

$\therefore CI = A - P = 1356$

10 CI = ?
P = 10000
R = 12
Halfyearly

$$A = P\left[1 + \frac{R}{100}\right]^T$$

$$= 10000\left[1 + \frac{12}{100 \times 2}\right]^2$$

two 6 months in a year

$\Rightarrow A = 11236$

$\therefore CI = A - P = 1236$

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Problems on Nominal Interest Rate

1 Calculate the future sum of money for ₹1000 earning an interest rate ~~at~~ of 8%.
(i) compounded quarterly
(ii) compounded annually

(i) $P = 1000$

$i = 8$

$n = \frac{12}{3} = 4$

3 — three quarters in a year

$F = P(1+i)^n$

$= 1000\left[1 + \frac{8}{100 \times 4}\right]^4$

$\therefore F = 1082.43$

(ii) $P = 1000$

$i = 8$

$n = \frac{12}{12} = 1$

$F = P(1+i)^n$

$= 1000\left[1 + \frac{8}{100}\right]^1$

$\therefore F = 1080$

2 For the above first question, find the effective interest rate.

nominal interest rate

$i_{eff} = \left[1 + \left(\frac{r}{n}\right)\right]^n - 1$

$= \left[1 + \left(\frac{0.08}{4}\right)\right]^4 - 1$

$\therefore i_{eff} = 0.0824 \times 100 = 8.2\%$

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3 A finance company charge the interest rate 18% compounded monthly, what is i_{eff}?

$$i_{eff} = \left[1 + \left(\frac{r}{n}\right)\right]^n - 1 \qquad n = \frac{12}{1} = 12$$

$$= \left[1 + \frac{0.18}{12}\right]^{12} - 1$$

$\therefore i_{eff} = 0.1956 \times 100 = 19.56\%$

4 A person has to earn ₹25000 today, how much he has to invest 1 year ago when the investment earned an interest at the nominal rate of 12% compounded monthly.

$F = 25000$

$P = ?$

$i = 12$

$n = 12$

$$F = P(1+i)^n$$

$$25000 = P\left[1 + \frac{\cancel{12}}{100 \times \cancel{12}}\right]^{12}$$

$\therefore P = 22186.23$

5 Find the compound amount for $P = 5000$ at 6% for 4, 8, 12 years and compare the results. Also, check if doubling the time would double the amount of interest earned.

4

$$F = P(1+i)^n$$

$$= 5000(1+0.06)^4 = 6312.3848$$

$$CI = F - P = 1312.3848$$

Doubling 4 $F = 5000(1+0.06)^8 = 7969.2403$

$CI = F - P = 2969.2403$

8 $F = P(1+i)^n$

$= 5000(1+0.06)^8 = 7969.2403$

$CI = F - P = 2969.2403$

Doubling 8 $F = 5000(1+0.06)^{16} = 12701.7584$

$CI = F - P = 7701.7584$

12 $F = P(1+i)^n$

$= 5000(1+0.06)^{12} = 10060.9824$

$CI = F - P = 5060.9824$

Doubling 12 $F = 5000(1+0.06)^{24} = 20244.6732$

$CI = F - P = 15244.6732$

$\Rightarrow$ CI does not double as time doubles.

6 Find the i_{eff} if the rate of interest is 6% when compounded.

(a) half-yearly

(b) annually

(c) quarterly

(d) monthly

(e) daily

$$i_{eff} = \left[1 + \left(\frac{r}{n}\right)\right]^n - 1$$

(a) $i_{eff} = \left[1 + \frac{0.06}{2}\right]^2 - 1 = 0.0609 = 6.09\%$

(b) $i_{eff} = \left[1 + \frac{0.06}{1}\right]^1 - 1 = 0.06 = 6\%$

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(c) $i_{eff} = \left[1 + \frac{0.06}{4}\right]^4 - 1 = 0.0613 = 6.13\%$

(d) $i_{eff} = \left[1 + \frac{0.06}{12}\right]^{12} - 1 = 0.0616 = 6.16\%$

(e) $i_{eff} = \left[1 + \frac{0.06}{365}\right]^{365} - 1 = 0.0618 = 6.18\%$

Problems on Compound Amount Factors

1 You are considering a project that would require ₹ 300000 of investment. A viable alternative that must be considered is to do nothing and bank the money that would have been invested in the project. What is the value of 3 lakhs after 8 years assuming an interest rate of 6%?

Single Payment Compound Amount Factor

$$(F/P, i, n) = (1+i)^n$$

$P = 300000$

$i = 0.06$

$n = 8$

$$\frac{F}{P} = (1 + 0.06)^8$$

$$\therefore F = 300000 (1+0.06)^8 = 478154.42$$

2 You are writing a proposal for a science experiment that will be launched into space in 6 years. The cost of launch is expected to be 5 lakhs. How much do you need to set the site today in order to have 5 lakhs in 6 years assuming

an interest rate of 5%?

Single Payment Present Worth Factor:

$$(P/F, i, n) = \frac{1}{(1+i)^n}$$

$F = 500000$

$i = 0.05$

$n = 6$

$$\frac{P}{500000} = \frac{1}{(1+0.05)^6}$$

$$\therefore P = 373107.6983$$

3 You just got a new job and are trying to decide whether to begin saving for retirement now or in a few years. You are 25 years old and expected to retire when you are 65. You feel you can save Rs. 300 a month towards retirement by assuming an interest rate of 6%. You calculate that if you start saving today, you will have 5,97,447 rupees when you are ready to retire. ~~Find the amount to be saved.~~ (verify the end of period payment amount)

Uniform Series Sinking Fund Factor

$$(A/F, i, n) = \frac{i}{(1+i)^n - 1}$$

$F = 597447$

$i = 0.06$

$n = 65 - 25 = 40 \times 12$

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$$\frac{A}{597447} = \frac{0.06/12}{\left(1+\frac{0.06}{12}\right)^{10\times12}-1}$$

$\therefore A = 299.999 \simeq 300$

4 You do a budget after starting the new job and calculate that you have ₹230 left over each month. After paying the expenses, how much will you have after 3 years if you invest ₹230 each month assuming an early interest rate of 4%?

Uniform Series Compound Amount Factor

$$(F/A, i, n) = \frac{(1+i)^n - 1}{i}$$

$A = 230$

$n = 3 \times 12$

$i = 0.04$

$$\frac{F}{230} = \frac{\left[1+\frac{0.04}{12}\right]^{3\times12} - 1}{\frac{0.04}{12}}$$

$\therefore F = 8781.7593$

5 You borrow ₹50000 to purchase a rack mounted server which you plan to pay off in 7 years. What are the yearly payments that are to be done assuming a compound interest rate of 8%?

Uniform Series Capital Recovery Factor

$$(A/P, i, n) = \frac{i(1+i)^n}{(1+i)^n - 1}$$

$P = 50000$

$i = 0.08$

$n = 7$

$$\frac{A}{50000} = \frac{0.08(1+0.08)^7}{(1+0.08)^7 - 1}$$

$\therefore A = 9'603.62$

6 You are currently paying ₹800/month as rent; what is the amount borrowed equalled ₹800/month for 30 years at 5.5% interest rate?

Uniform Series Present Worth Factor

$$(P/A, i, n) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

$A = 800$

$n = 30 \times 12$

$i = 0.055$

$$\frac{P}{800} = \frac{\left[1 + \frac{0.055}{12}\right]^{30 \times 12} - 1}{\frac{0.055}{12}\left[1 + \frac{0.055}{12}\right]^{30 \times 12}}$$

$\therefore P = 140897.4105$

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1. A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.
2. A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

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3. A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.
4. A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

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5. A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

6. A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

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7. A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

8. A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 8,500 at the end of the first year and thereafter he wishes to deposit the amount with an annual decrease of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

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1 $P = 20{,}000$, $i = 18\% = 0.18$, $N = 10$, $F = ?$

Single Payment Compound Amount Factor

$(F/P, i\%, N) = (1+i)^N$

$$\frac{F}{P} = (1+i)^N$$

$$F = 20000(1+0.18)^{10}$$

$$\therefore F = 104676.7111$$

2 $F = 1{,}00{,}000$ $N = 10$ $P = ?$ $i = 15\% = 0.15$

Single Payment Present Worth Factor

$(P/F, i\%, N) = \frac{1}{(1+i)^N}$

$$\frac{P}{F} = \frac{1}{(1+i)^N}$$

$$P = 100000\left[\frac{1}{(1+0.15)^{10}}\right]$$

$$\therefore P = 24718.4706$$

3 $A = 10000$ $i = 20\% = 0.20$ $F = ?$

$N = 25$

Uniform Series Compound Amount Factor

$(F/A, i\%, N) = \frac{(1+i)^N - 1}{i}$

$$\frac{F}{A} = \frac{(i+1)^N - 1}{i}$$

$$F = 10000\left[\frac{(0.20+1)^{25} - 1}{0.20}\right]$$

$$\therefore F = 4719810$$

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4 $N = 15$ $F = 500000$ $i = 18\% = 0.18$ $A = ?$

Uniform series sinking Fund Factor

$$(A/F, i, N) = \frac{i}{(1+i)^n - 1}$$

$$\frac{A}{F} = \frac{i}{(1+i)^n - 1}$$

$$A = 500000 \left[\frac{0.18}{(1+0.18)^{15} - 1} \right]$$

$\therefore A = 8201.39$

5 $P = 10,00,000$ $N = 20$ $i = 15\% = 0.15$ $P = ?$

Uniform series Present Worth Factor

$$(P/A, i\%, N) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

$$\frac{P}{A} = \frac{(1+i)^n - 1}{i(1+i)^n} \qquad P = 1000000 \times \left[\frac{(1+0.15)^{20} - 1}{0.15(1+0.15)^{20}} \right]$$

$\therefore P = 62,59,331$

6 $P = 10,00,000$ $i = 18\% = 0.18$ $N = 15$, $A = ?$

~~Uniform series Present Worth Factor~~

~~P/A~~ Uniform series Capital Recovery Factor

$$(A/P, i\%, N) = \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

196,400

$$\frac{A}{P} = \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$A = 1000000 \left[\frac{0.18(0.18+1)^{15}}{(0.18+1)^{15} - 1} \right]$$

$\therefore A = 196,402$

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7 $P = 5000$ $i = 12\% = 0.12$ Quarterly $= \frac{12}{3} = 4$

$N = 10$ $F = ?$

$$F = P(1+i)^n$$

$$= 5000\left[1 + \frac{0.12}{4}\right]^{10 \times 4}$$

$$\therefore F = 16310.18$$

8 $A_1 = 8500$ $N = 10$ $i = 0.15$ $F = ?$

~~$F = 8500[1 + 0.15] \times$~~

Uniform Gradient Series Annual equivalent Amount

$$A = A_1 + G\underbrace{\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]}_{(A/G,\ i,\ N)}$$

$$A = 8500 - 500\left[\frac{(1+0.15)^{10} - 0.15 \times 10 - 1}{0.15(1+0.15)^{10} - 0.15}\right]$$

$$A = 6,808.40$$

~~$F = P(1+i)^n$~~

~~$F = 6808.40(1+0.15)^{10} =$~~

$$(F/A,\ i,\ N) = \frac{(1+i)^N - 1}{i} = 20.30371824$$

$$\therefore F = A \times 20.30371824 = 1,38,235.8353$$

Problems on present worth comparison:

1. A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10 th year of the above series.

2. There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.3,00,000, Annual operating and maintenancecost = Rs.20,000, Expected salvage value = Rs.1,25,000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.2,00,000, Annual operating and maintenancecost = Rs.35,000, Expected salvage value = Rs.70,000, Useful life = 5 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

3. **Alternative-1:** Initial purchase cost = Rs.300000, Annual operating and maintenance cost= Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost= Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by concrete mixer) from Alternative-1 and Alternative-2 are Rs.50000 and Rs.45000 respectively. Compute the equivalent present worth of the alternatives at the same rate of interest 10% per year and find out the economical alternative.

4. A construction contractor has three options to purchase a dump truck for transportation and dumping of soil at a construction site. All the alternatives have the same useful life. The cash flow details of all the alternatives are provided as follows;

Option-1: Initial purchase price = Rs.2500000, Annual operating cost Rs.45000 at theend of 1st year and increasing by Rs.3000 in the subsequent years till the end of useful life, Annual income = Rs.120000, Salvage value = Rs.550000, Useful life = 10 years.

Option-2: Initial purchase price = Rs.3000000, Annual operating cost = Rs.30000,Annual

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income Rs.150000 for first three years and increasing by Rs.5000 in the subsequent years till the end of useful life, Salvage value = Rs.800000, Useful life = 10 years.

Option-3: Initial purchase price = Rs.2700000, Annual operating cost Rs.35000 for first5 years and increasing by Rs.2000 in the successive years till the end of useful life, Annual income = Rs.140000, Expected salvage value = Rs.650000, Useful life = 10 years.

5. A material testing laboratory has two alternatives for purchasing a compression testing machine which will be used for determining the compressive strength of different construction materials. The alternatives are from two different manufacturing companies. The cash flow details of the alternatives are as follows;

Alternative-1: Initial purchase price = Rs.1000000, Annual operating cost = Rs.10000,Expected annual income to be generated from testing of different construction materials = Rs.175000, Expected salvage value = Rs.200000, Useful life = 10 years.

Alternative-2: Initial purchase price = Rs.700000, Annual operating cost = Rs.15000,Expected annual income to be generated from testing of different construction materials = Rs.165000, Expected salvage value = Rs.250000, Useful life = 5 years.

Using present worth method, find out the most economical alternative at the interest rate of 10% per year.

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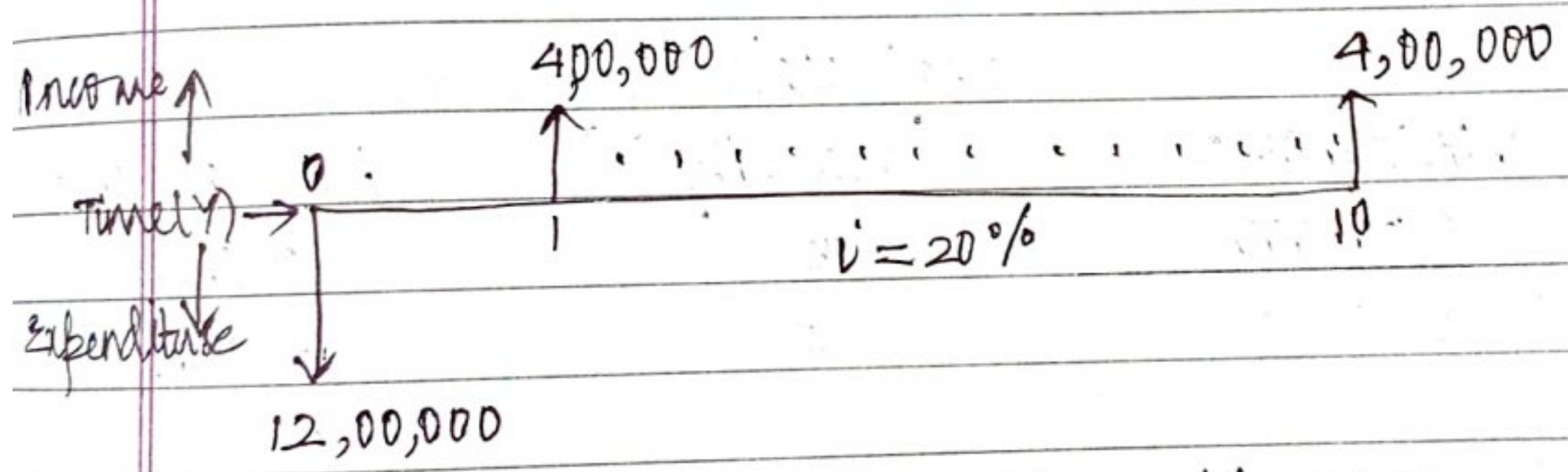
Problems on Present Worth comparison

1 Alpha industry is planning to expand its production operation. It has identified 3 different technologies for maintaining a goal. The initial outlay and annual revenue w.r.t each of the technologies are summarized in the table given. Suggest the best technology that is to be implemented based on the present worth method of comparison assuming 20% interest rate compounded annually.

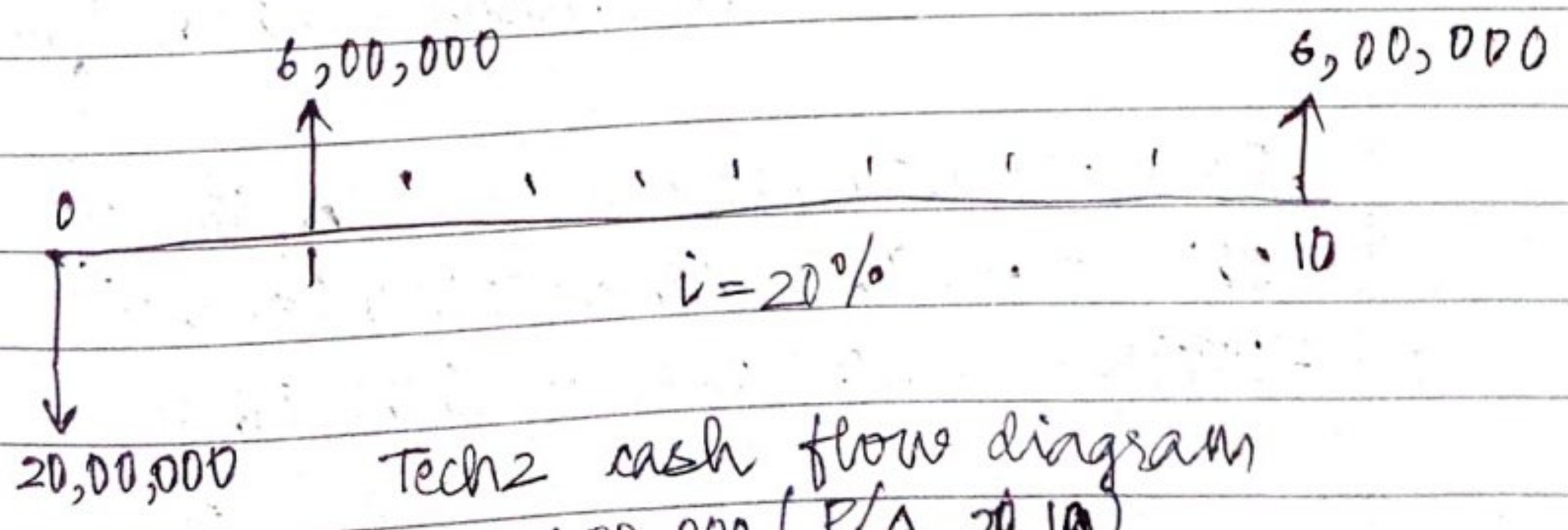
	Initial Outlay	Annual Revenue	Life (Y)
Tech1	12,00,000	4,00,000	10
Tech2	20,00,000	6,00,000	10
Tech3	18,00,000	5,00,000	10

$$PW = -P + A(P/A, i, n)$$



Tech1 cash flow diagram

$$PW = -12,00,000 + 4,00,000\,(P/A, 20, 10)$$
$$= -12,00,000 + 4,00,000\,(4.192)$$
$$\therefore PW = 4,76,800$$



Tech2 cash flow diagram

$$PW = -20,00,000 + 6,00,000\,(P/A, 20, 10)$$
$$\therefore PW = -20,00,000 + 6,00,000\,(4.192) = 5,15,200$$

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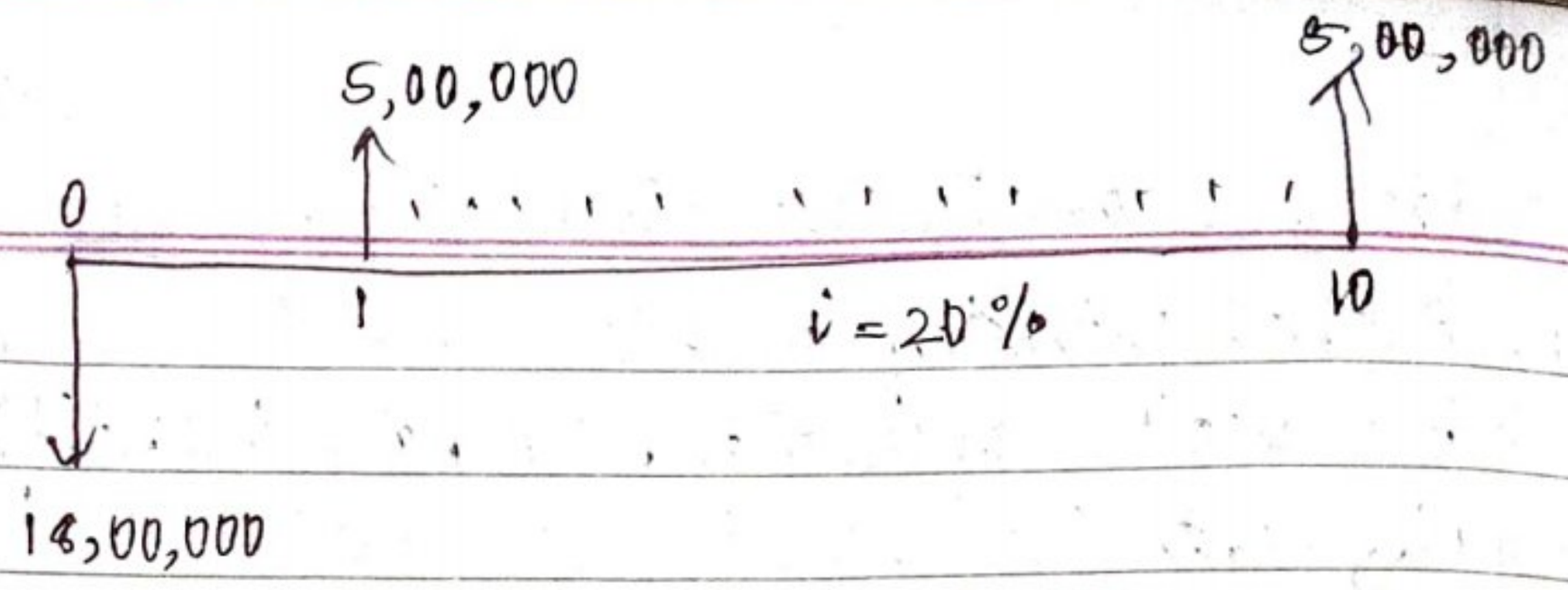


$PW = -18,00,000 + 5,00,000 (P/A, 20, 10)$

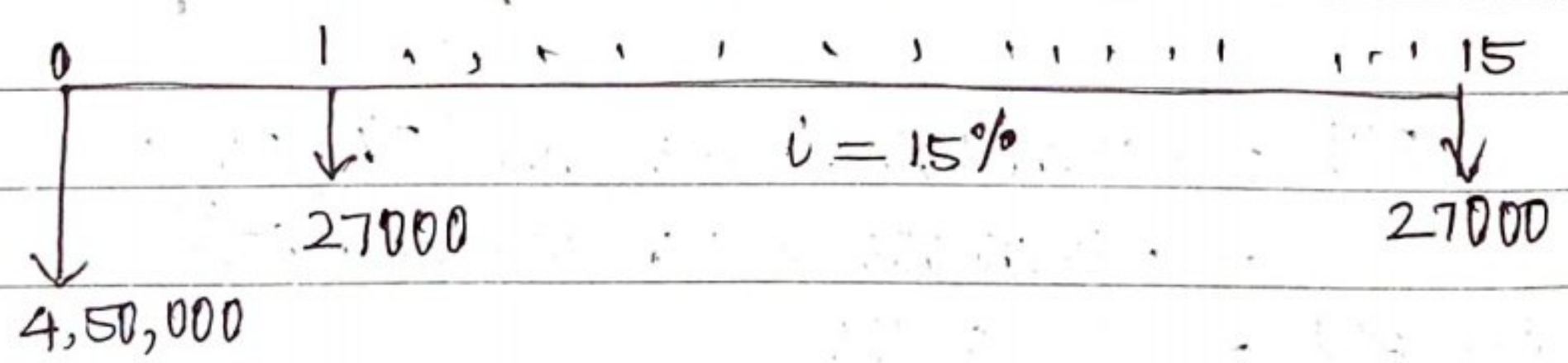
$= -18,00,000 + 5,00,000 (4.192)$

$\therefore PW = 2,96,000$

$\therefore$ Tech is the best technology

2 An engineer has 2 bids for an elevator to be installed in the new building. The details of the bids or the elevators are given below. Determine which bit should be accepted based on the present worth method of comparison assuming an interest rate of 15% compounded annually.

Bid	Engineer's estimates		
	Initial cost	Service Life (Y)	Annual cost
X-elevator	4,50,000	15	27000
B-elevator	5,40,000	15	28500



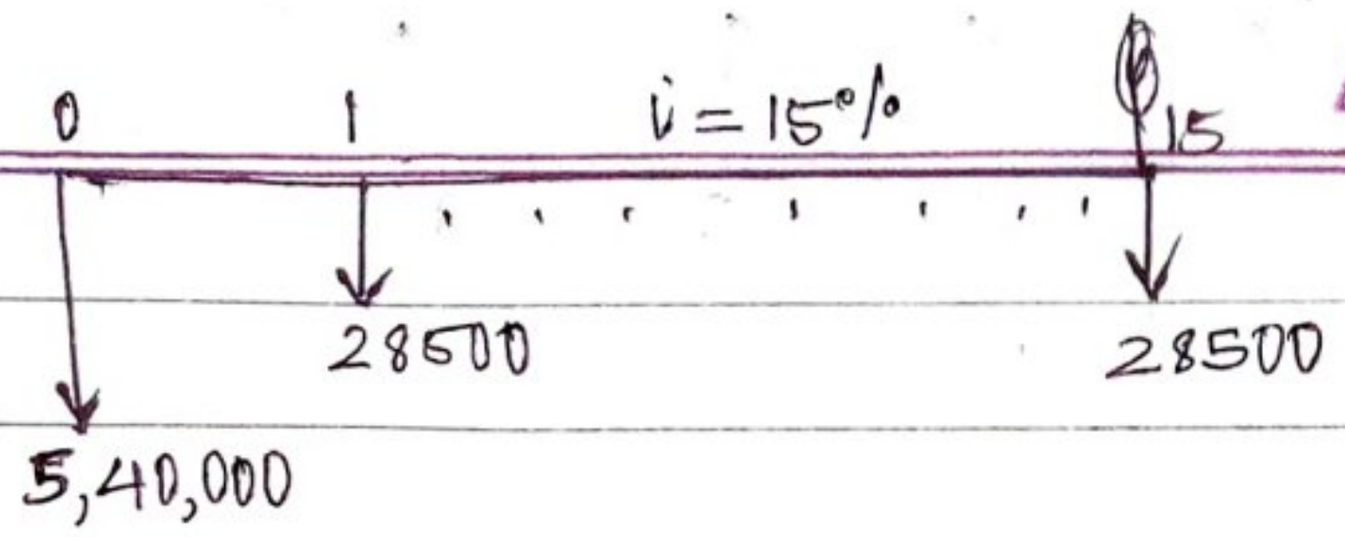
X-elevator's cash flow diagram

$PW = -4,50,000 - 27000 (P/A, 15, 15)$

$PW = -(4,50,000 + 27000 (5.847))$

$\therefore PW = -6,07,869 = 6,07,869$

β-elevator's CFD



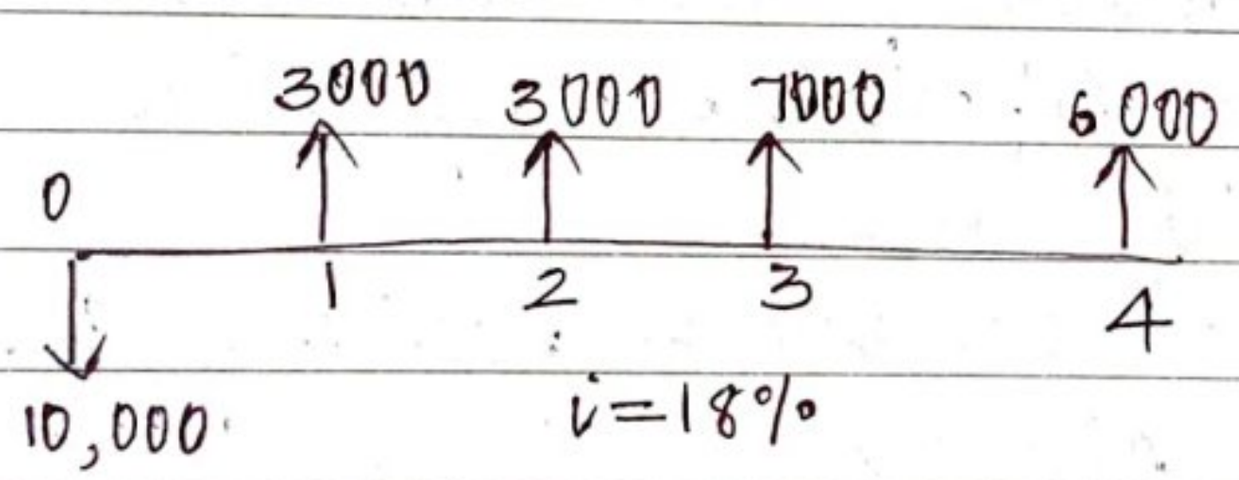
$PW = -5,40,000 - 28500(P/A, 15, 15)$ 5.847

$\therefore PW = 7,06,639.5$

⇒ α-elevator has to be installed.

3 Investment proposals A and B have net cash flows as follows. Compare the present worth of A with that of B at i=18%. Which proposal should be selected?

Proposal	End of years				
~~A(Rs)~~	0	1	2	3	4
A (Rs)	−10,000	3000	3000	7000	6000
B (Rs)	−10,000	6000	6000	3000	3000

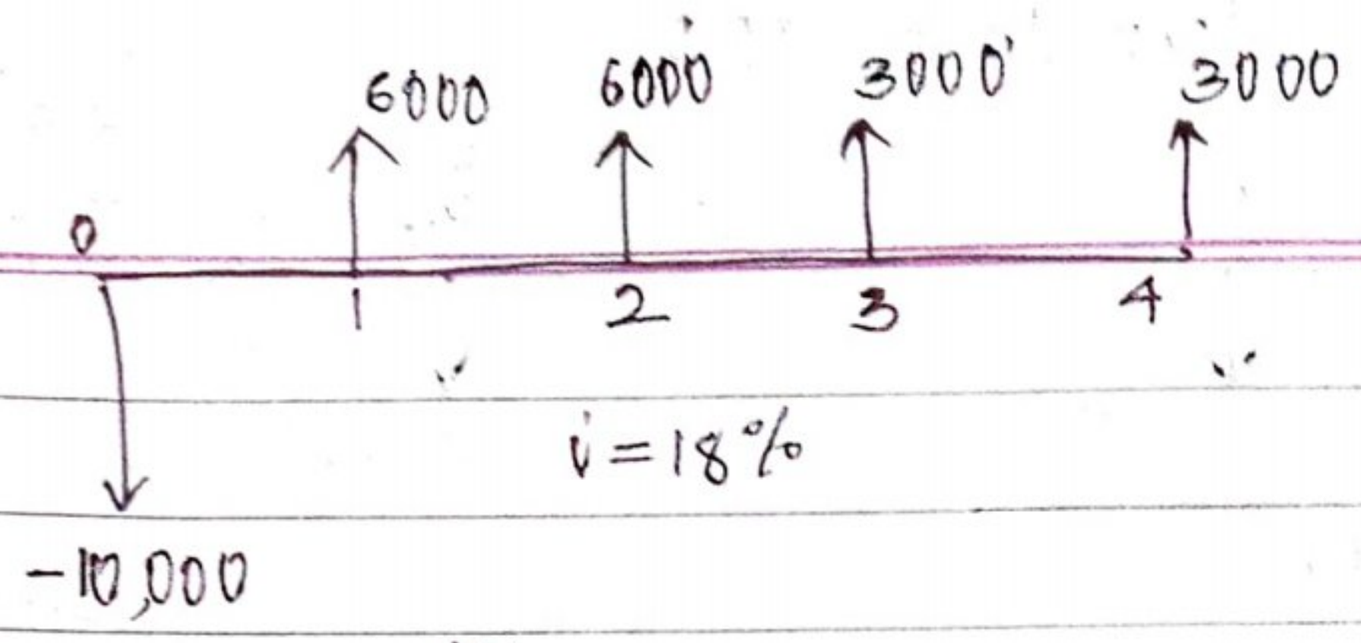


A's CFD

$PW = -10,000 + 3000(P/F, 18, 1)$
$+ 3000(P/F, 18, 2)$
$+ 7000(P/F, 18, 3)$
$+ 6000(P/F, 18, 4)$

$\therefore PW = -10,000 + 3000(0.8475) + 3000(\cancel{0.7182}\ 0.7182) + 7000(\cancel{0.8475}\ 0.6086)$
$+ 6000(\cancel{0.8475}\ 0.5158)$

$= 2052.1$



∴ B's CFD

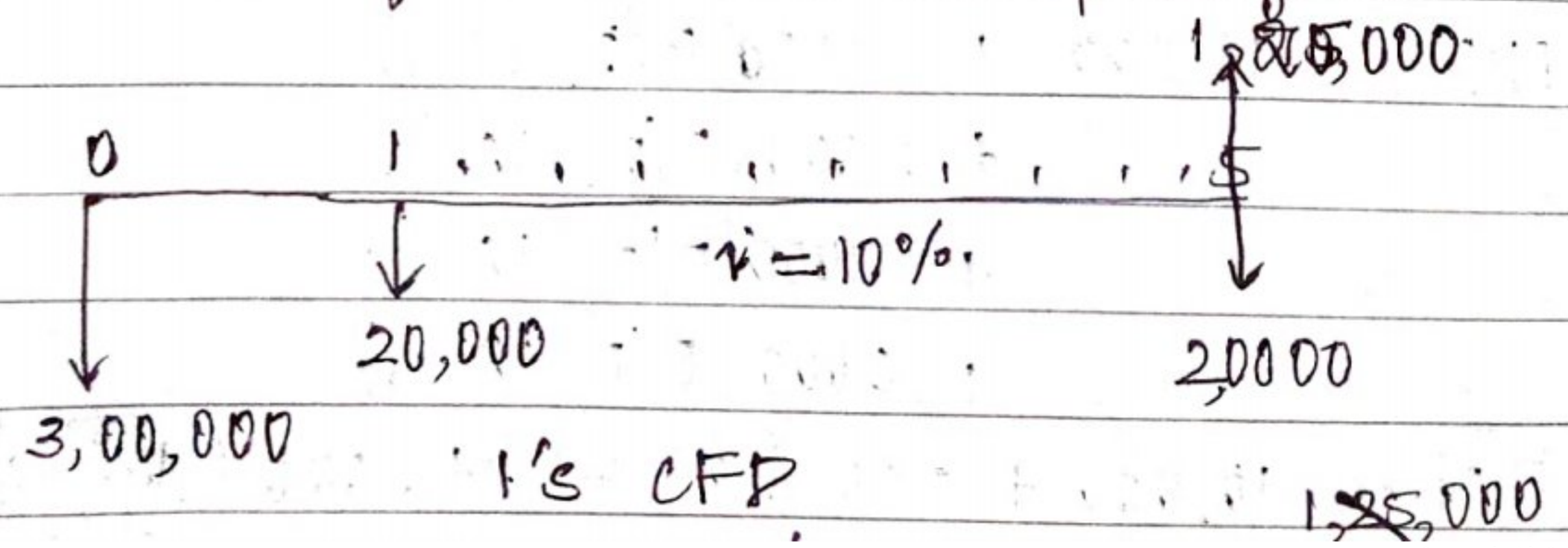
$$PW = -10,000 + 6000(P/F, 18, 1) + 6000(P/F, 18, 2) + 3000(P/F, 18, 3) + 3000(P/F, 18, 4)$$

$$= -10,000 + 6000(0.8475) + 6000(0.7182) + 3000(0.6086) + 3000(0.5158)$$

$$\therefore PW = 276.74$$

⇒ B proposal is selected.

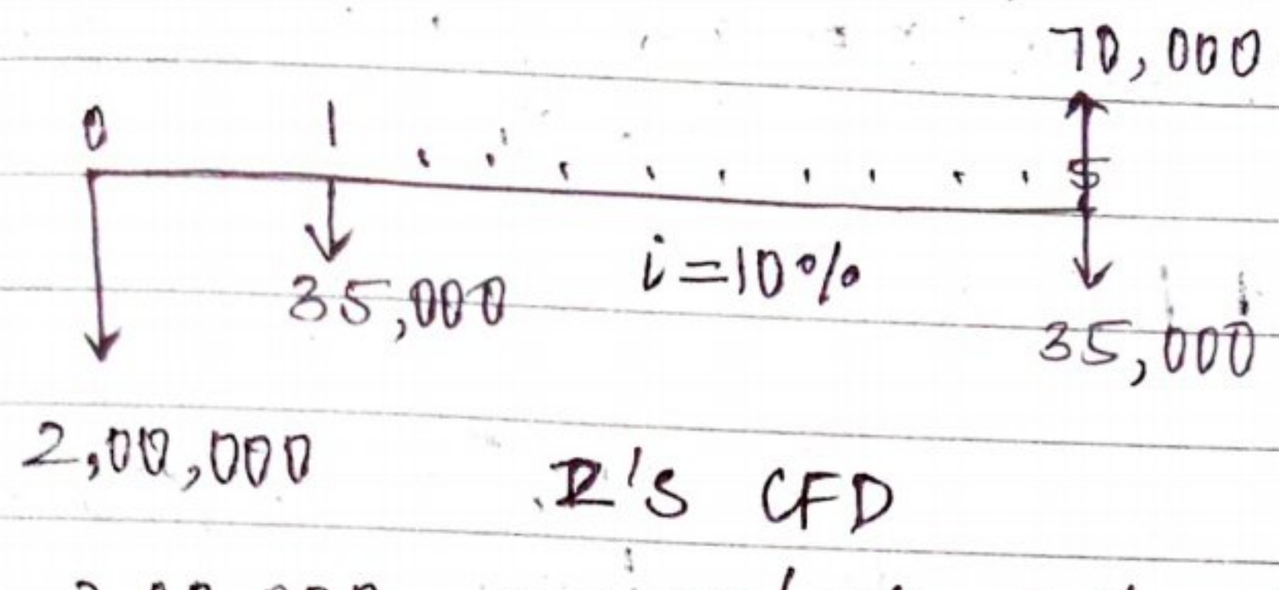
4 There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows:
Alternative-1: Initial purchase cost = Rs. 3,00,000, Annual operating and maintenance cost = Rs. 20,000, Expected salvage value = Rs. 70,000, Useful life = 5 years.
Using present worth method, find out which alternative should be selected, if the rate of interest is 10% per year.



$PW = -3,00,000 - 20,000\,(P/A, 10\%, 5) + 1,25,000\,(P/F, 10\%, 5)$

$= -3,00,000 - 20,000\,(3.7908) + 1,25,000\,(0.6209)$

$\therefore PW = -2,98,203.5 = 2,98,203.5$



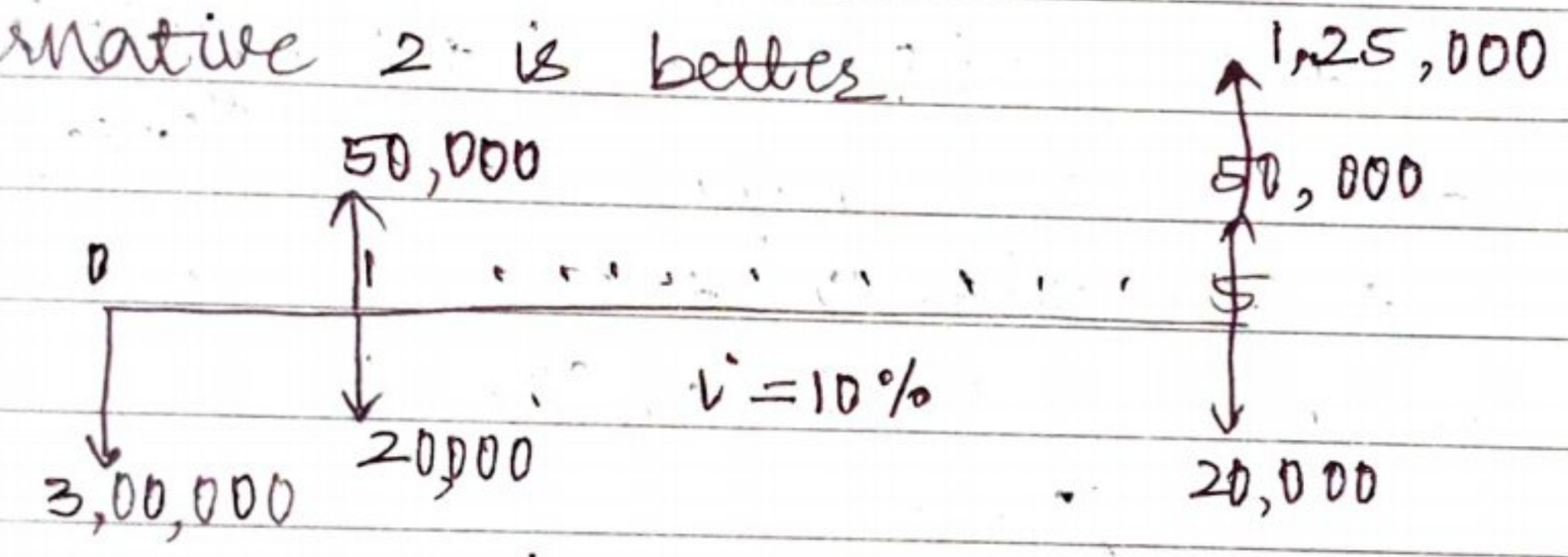
I's CFD

$PW = -2,00,000 - 35,000\,(P/A, 10\%, 5) + 70,000\,(P/F, 10\%, 5)$

$= -2,00,000 - 35,000\,(3.7908) + 70,000\,(0.6209)$

$\therefore PW = -2,89,215 = 2,89,215$

$\Rightarrow$ Alternative 2 is better

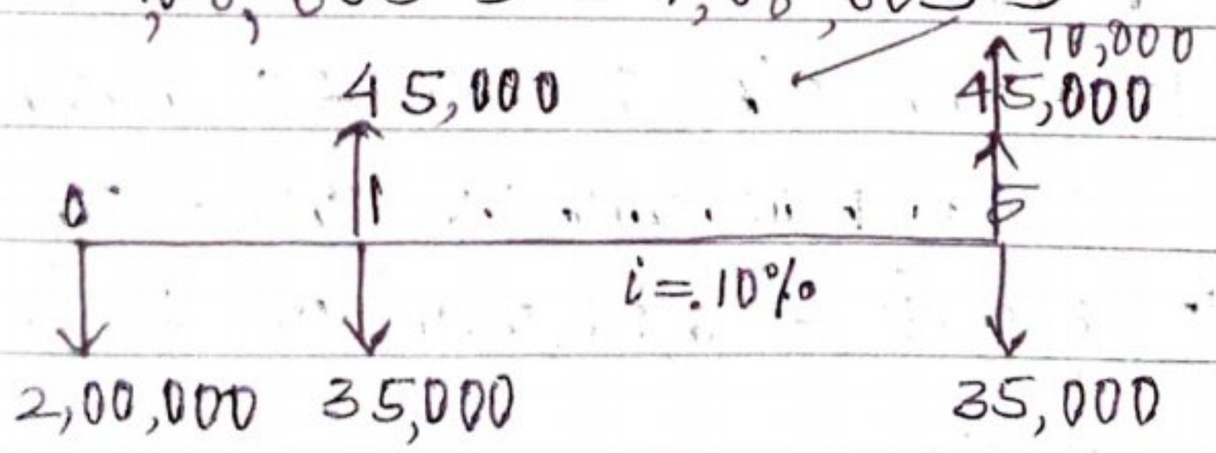


I's CFD

$PW_1(10\%) = -3,00,000 - 20,000\,(P/A, 10\%, 5) + 50,000\,(P/A, 10\%, 5)$

$+ 1,25,000\,(P/F, 10\%, 5)$ 0.6209 3.7908 ~~0.6209~~ 3.7908

$\therefore PW_1 = -108,663.5 = 1,08,663.5$



$PW_2 = -2,00,000 - 35,000\,(P/A, i, n) + 45,000\,(P/A, i, n) +$

$70,000\,(P/F, i, n)$

~~$-2,00,000$~~ 3.7908 3.7908

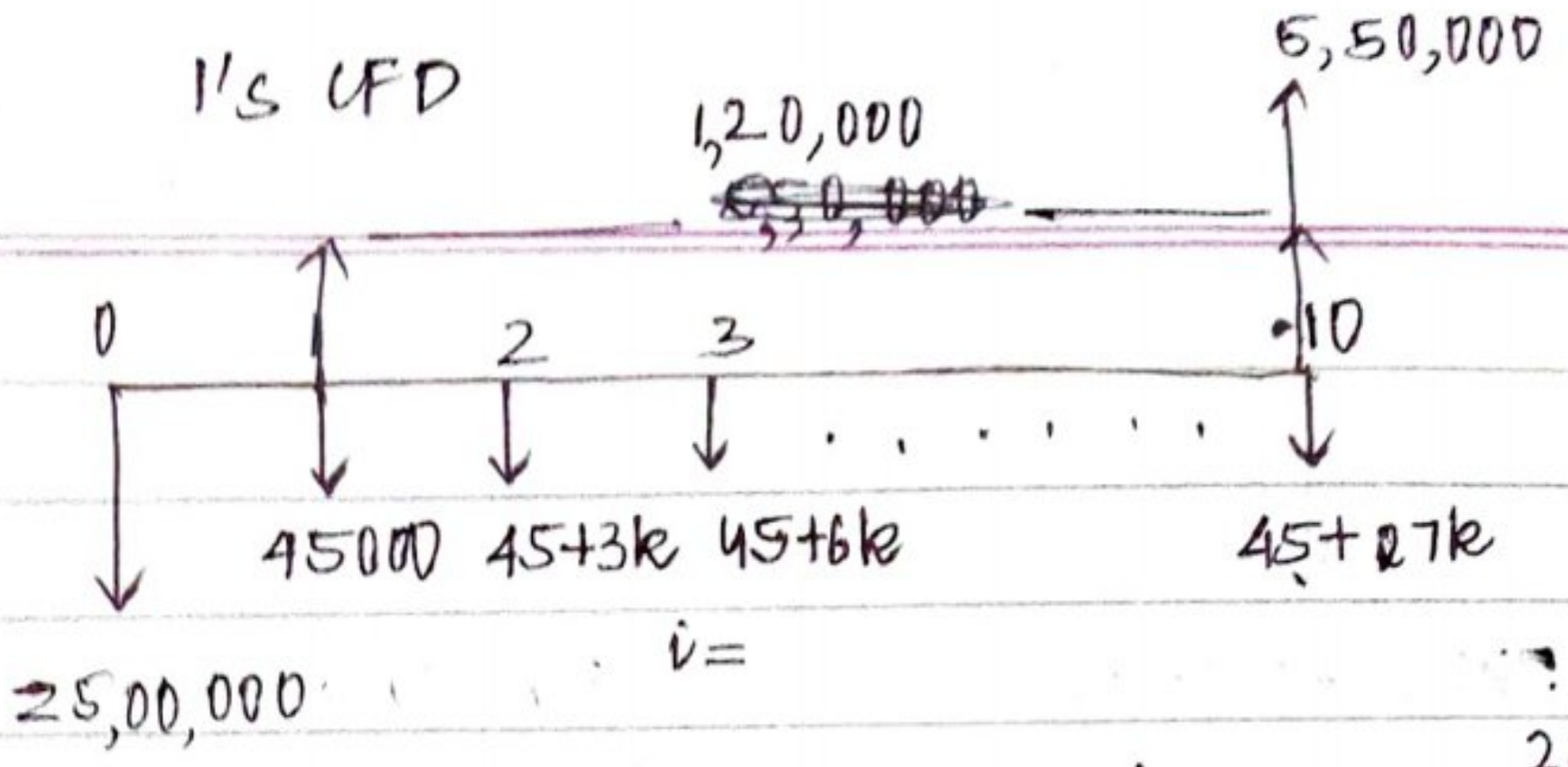
$= -2,00,000 - 35,000\,(P/A, 10\%, 5) + 45,000\,(P/A, 10\%, 5)$

$+ 70,000\,(P/F, 10\%, 5)$ — 0.6209

$\therefore PW_2 = -1,18,629 = 1,18,629$ $\therefore$ Alternative 1 is better

6

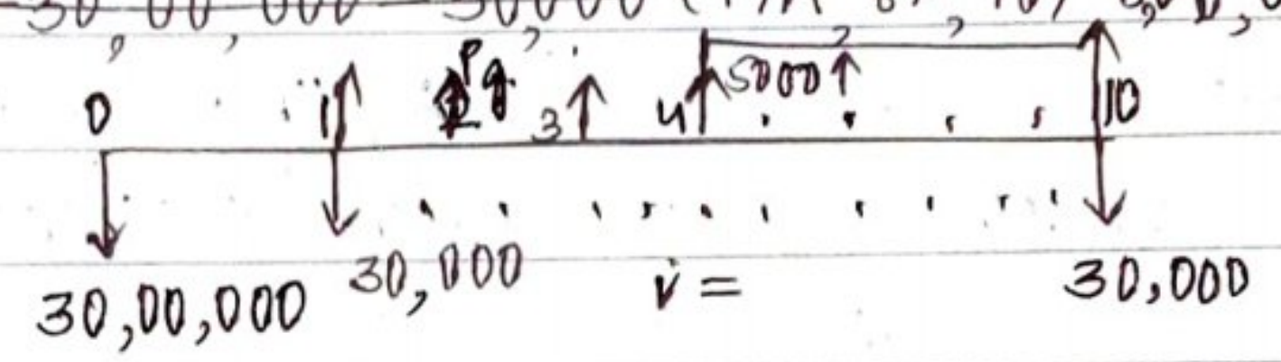
I's CFD



$$PW_1 = -25,00,000 - 45,000\,(P/A, 8\%, 10) - 3000\,(P/G, 8\%, 10) + 1,20,000\,(P/A, 8\%, 10) + 5,50,000\,(P/F, 8\%, 10)$$

(6.7101; 25.9768; 6.7101; 0.4632)

$\therefore PW_1 = 18,\ 19,912.9$

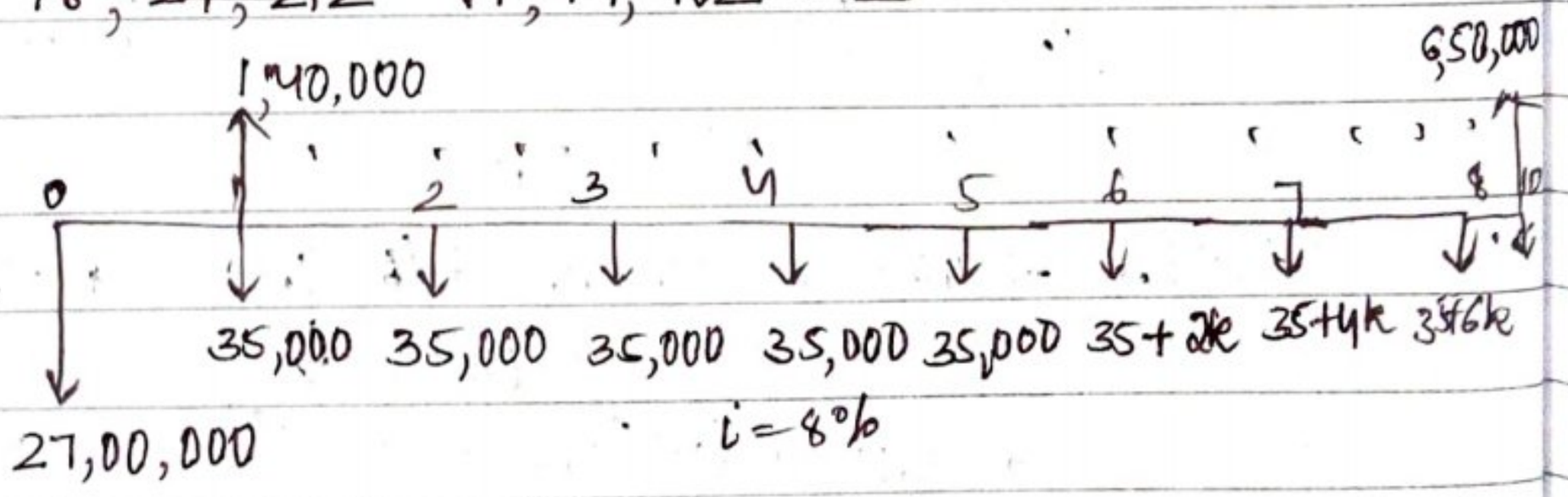


$$PW_2 = -30,00,000 - 30,000\,(P/A, 8\%, 10) + P_g\,(P/F, 8\%, 2) + 8,00,000\,(P/F, 8\%, 10) + 1,50,000\,(P/A, 8\%, 10)$$

$$= -30,00,000 - 30,000\,(6.7101) + \overset{5000}{G}\,(P/G, 8\%, 8)(P/F, 8\%, 2) + 1,50,000\,(6.7101) + 8,00,000\,(0.4632)$$

(17.8061; 0.858)

$\therefore PW_2 = \cancel{18,24,212}\ 17,47,902.152$



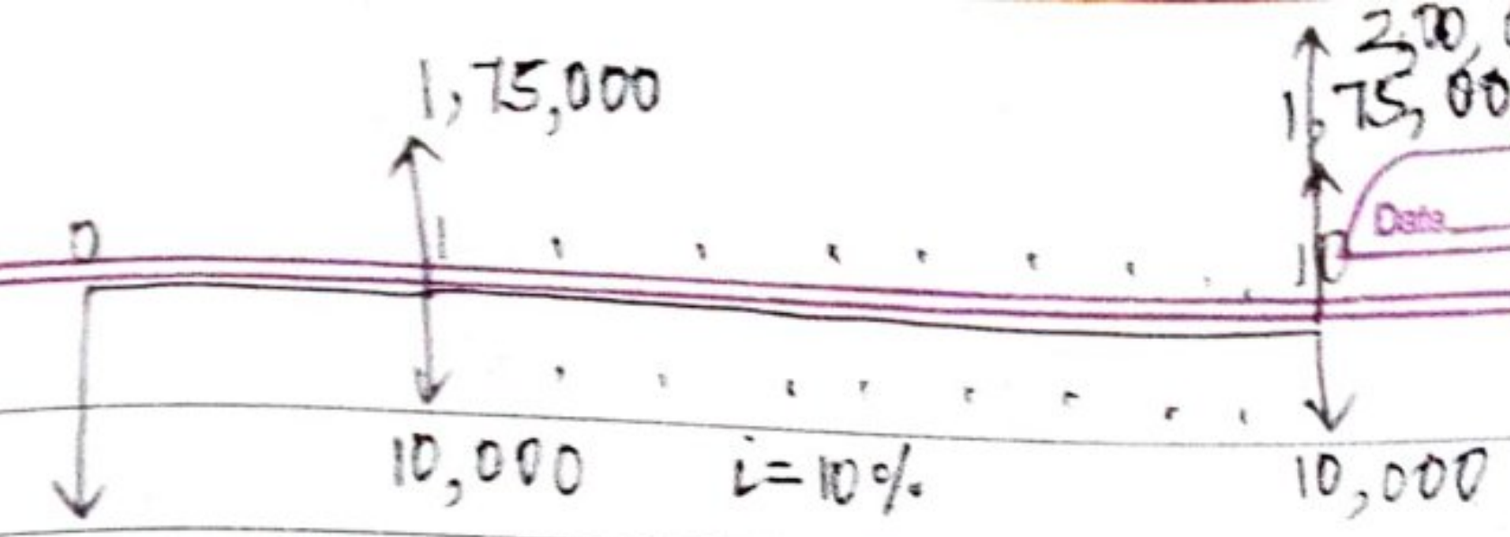
$$PW_3 = -27,00,000 - 35,000\,(P/A, 8\%, 10) + P_g\,(P/F, 8\%, 4) + 1,40,000\,(P/A, 8\%, 10) + 6,50,000\,(P/F, 8\%, 10)$$

$$= -27,00,000 + 1,05,000\,(6.7101) - 2000\,(P/G, 8\%, 6)(P/F, 8\%, 4) + 6,50,000\,(0.4632)$$

(10.5233; 0.7350)

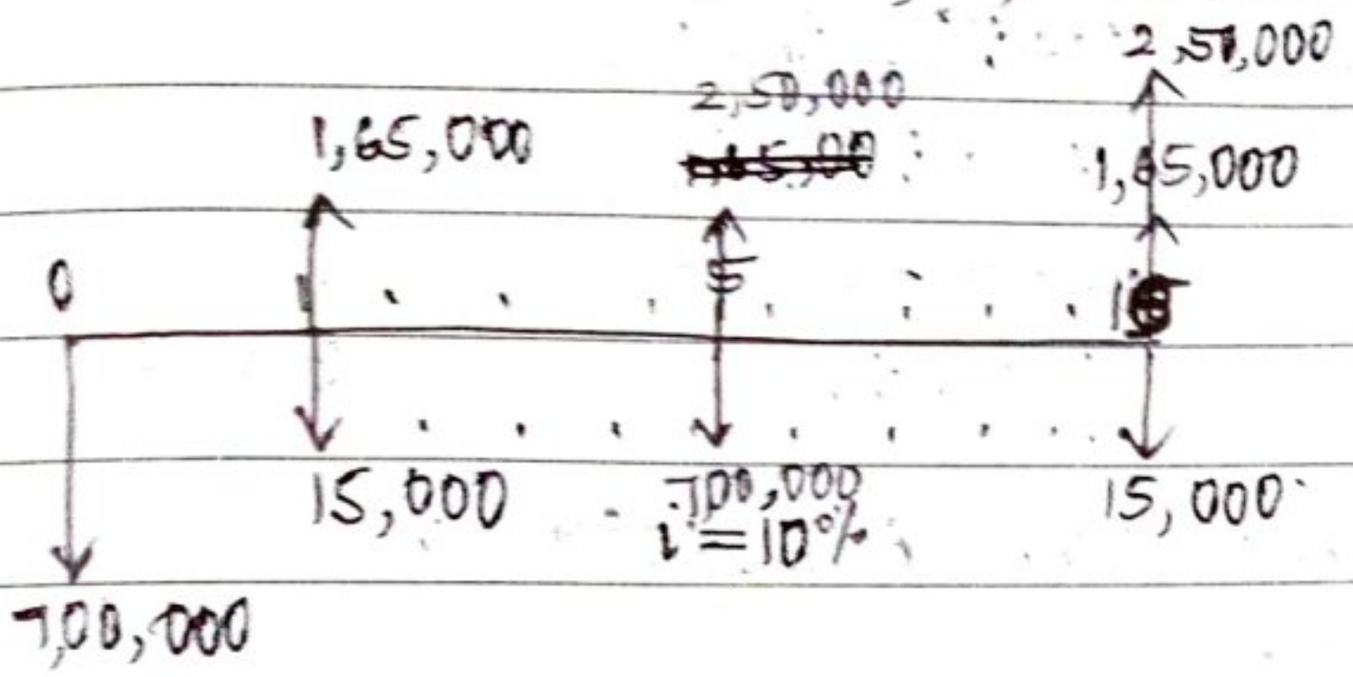
$\therefore PW_3 = 17,09,828$

$\Rightarrow PW_3$ will be selected



1's CFD

$$PW_1 = -10,00,000 + 1,65,000\,(P/A,\ 10\%,\ 10)^{6.1446} + 2,00,000\,(P/F,\ 10\%,\ 10)^{0.3855} = 90,959$$



2's CFD

~~$PW_2 = -7,00,000 + 1,50,000\,(P/A,\ 10\%,\ 10) + 2,50,000\,(P/F,\ 10\%,\ 10)$~~

~~= 2~~

$$PW_2 = -7,00,000 + (1,65,000 - 15,000)\,(P/A,\ 10\%,\ 10) + 2,50,000\,(P/F,\ 10\%,\ 5) - 7,00,000\,(P/F,\ 10\%,\ 5) + 2,50,000\,(P/F,\ 10\%,\ 10)$$

$$= -7,00,000 + 1,50,000\,(P/A,\ 10\%,\ 10)^{6.1446} - 4,50,000\,(P/F,\ 10\%,\ 5)^{0.6209} + 2,50,000\,(P/F,\ 10\%,\ 10)^{0.3855}$$

$\therefore PW_2 = 38,660$

$\Rightarrow PW_1$ will be selected.

8 $n = 10$

$A_1 = 4000$

$G = 500$

$i = 15\%$

$$A = A_1 + G \underbrace{\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]}_{(A/G,\, i,\, n)}$$

$$= 4000 + 500\,(A/G, 15\%, 10) \quad 3.383$$

$\therefore A = 5691.5$

$$F = A\,(F/A, 15\%, 10)$$

$$= 5691.5\,(20.304)$$

$\therefore F = 1,15,560.216$

Present worth equivalence — This pattern determines the present worth equivalence of a set of future transactions. A negative PWE – a negative decision towards making investment alternative. The worth is less than zero. A present worth method is used in the industry for all the future costs and revenues are transformed to equivalent monetary units. All the future cash flows are converted to the present amount at a specific rate of return.

1 A small dam and an irrigation system are expected to cost Rs. 300,000,000. Annual maintenance and operating cost – 4,00,000 at the first year and increase at the rate of 10% per year. Determine the equivalent

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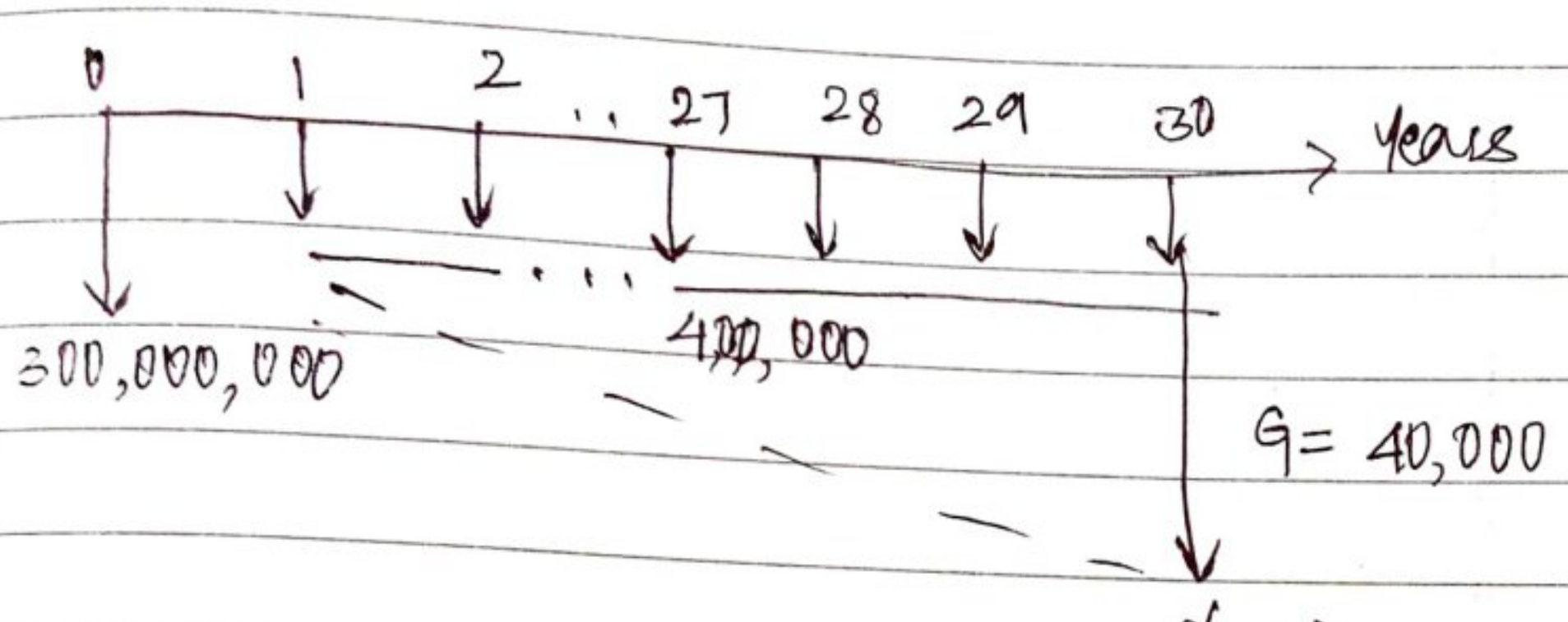
Present worth of building dam with $i = 10\%$ over 30 years life.

$\frac{10}{100} \times 4,00,000 = 40,000$



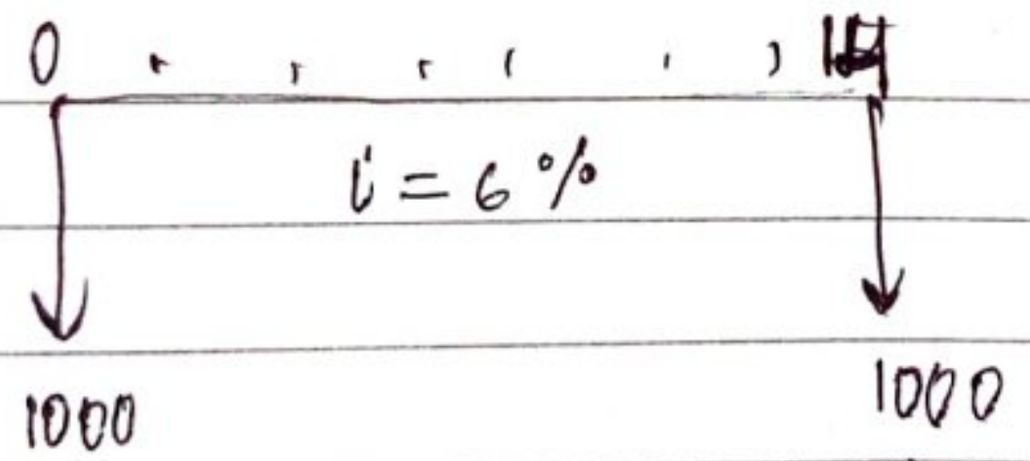
$PW = 300,000,000 + [A' + G(A/G, i, n)](P/A, i, n)$ (i = 10%, n = 30)

$= 300,000,000 + [4,00,000 + 40,000(8.1762)](9.4269)$

$\therefore PW = 306,853,808$

27,,

2 What is the present worth of a series of 15 years end payments of Rs. 1000 each, when the first payment is due today and $i = 6\%$.



Annuity due

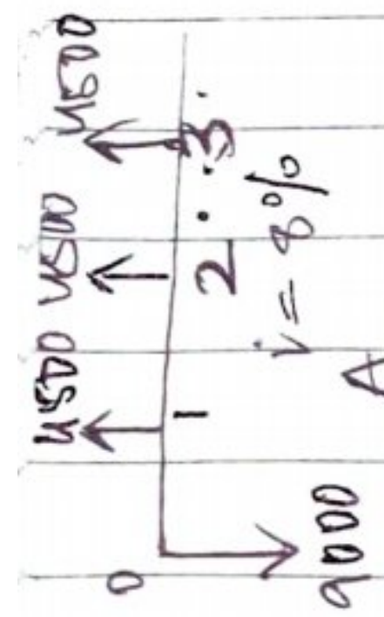
$PW = 1000 + 1000(P/A, 6\%, 14)$

$= 1000 + 1000(\cancel{9.7122})\ 9.29498$

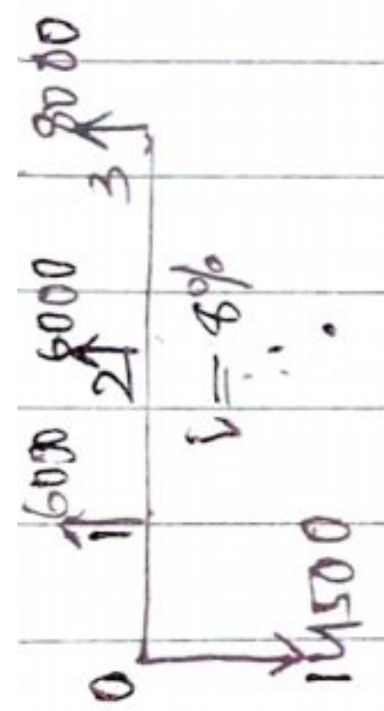
$PW = \cancel{10712.2}\ 10294.98$

Net Present Worth

1 Two devices are available to perform a necessary function for 3 years. The initial cost (negative) for each device at time 0 and subsequent annual savings (positive) is shown in the table below. The required interest rate is 8%.



	0	1	2	3
Device A	9000	4500	4500	4500
Device B	14,500	6000	6000	8000



PW of device A $= -9000 + 4500\,(P/A, 8, 3)$

$= -9000 + 4500(2.57710)$

$= \text{Rs. } 2596.95/-$

PW of device B $= -14{,}500 + 6{,}000\,(P/A, 8, 2) + 8000\,(P/F, 8, 3)$

$= -14{,}500 + 6000(1.78326) + 8000(0.79383)$

$= \text{Rs. } 2250.20/-$

$\Rightarrow$ A is preferred.

1 Two type of trucks are available for transportation use. They are needed for 10 years. Both the trucks deliver same amount of work. Assume interest rate is 7. Which truck is preferred on present worth basis? Use CFD for your analysis.

Truck/Cost details	Truck A	Truck B
First cost	10,00,000	15,00,000
Estimated annual maintenance cost	20,000	15,000
Estimated life	5 years	10 years
Estimated salvage value	2,00,000	5,00,000

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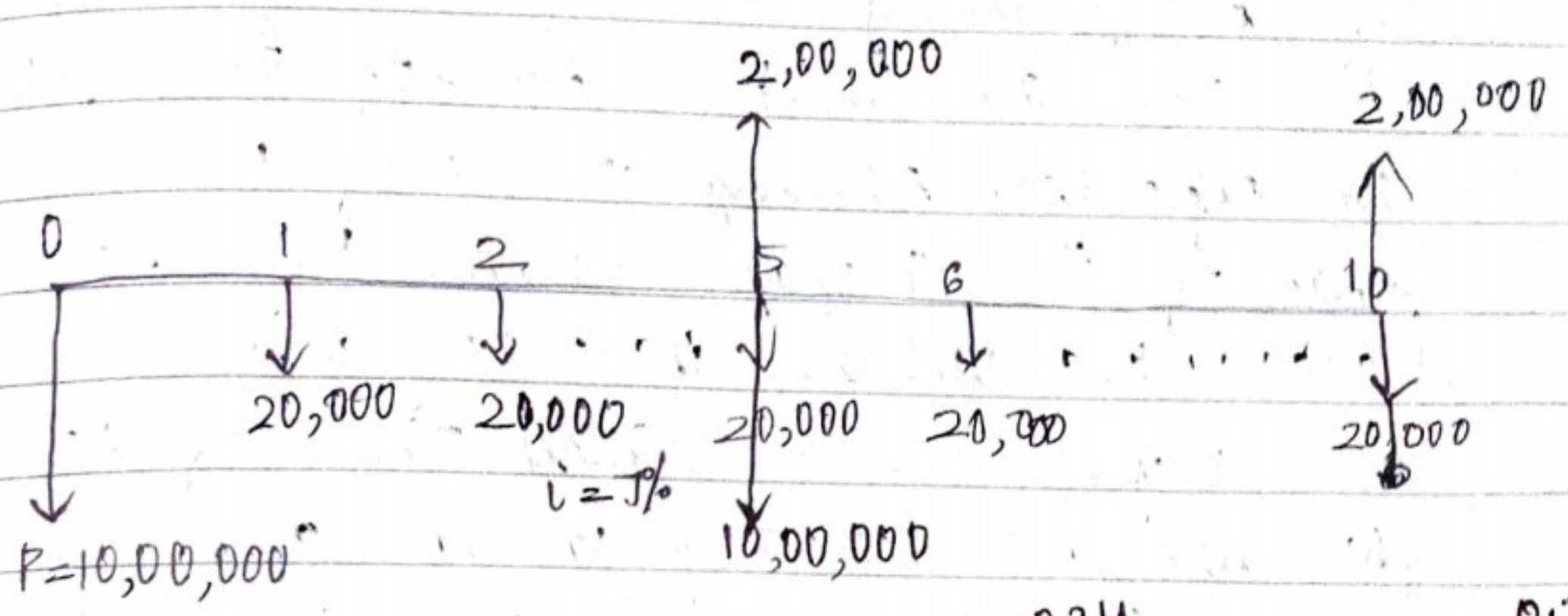
CFD for Truck A

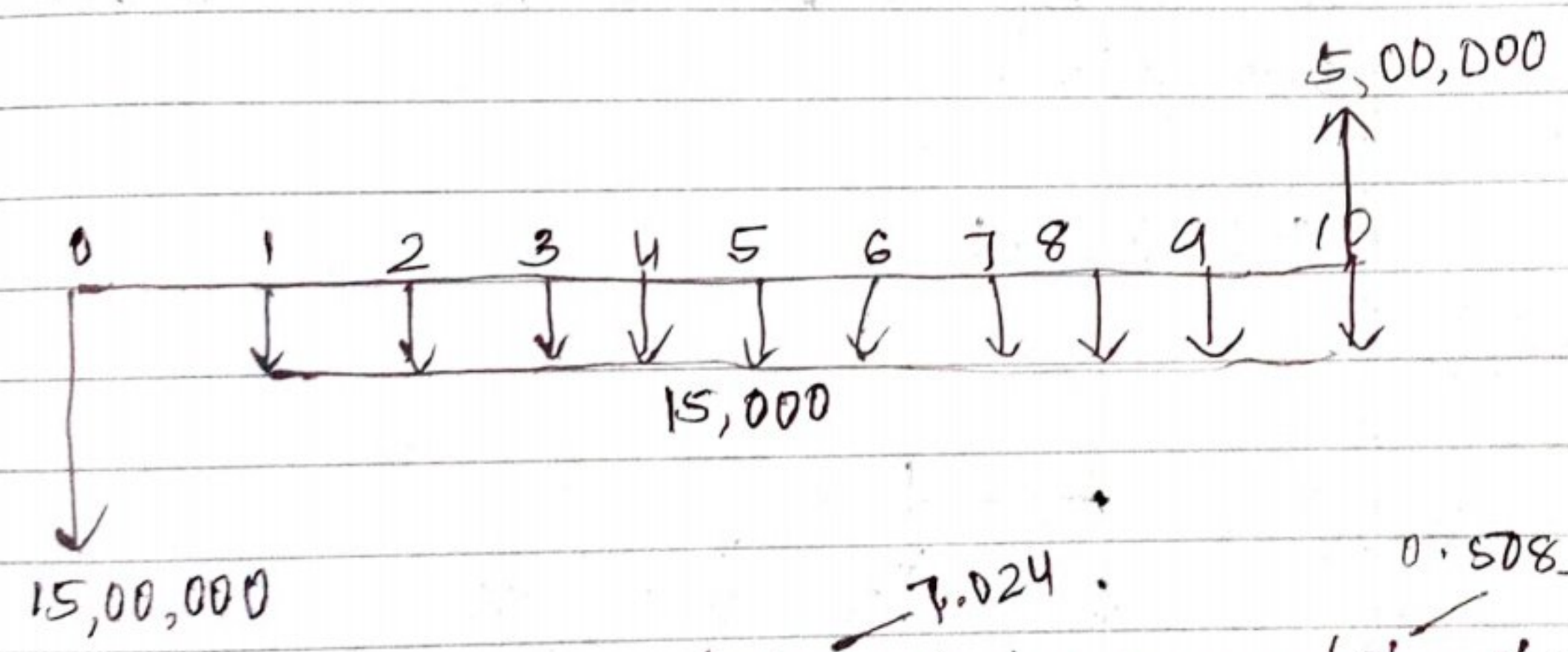


$$PW_A = -10,00,000 - 20,000\,(P/A, 7\%, 10) - 8,00,000\,(P/F, 7\%, 5) + 2,00,000\,(P/F, 7\%, 10)$$

7.024 0.7130 0.5083

$\therefore PW_A = 16,09,220$

CFD for truck B



$$PW_B = -15,00,000 - 15,000\,(P/A, 7\%, 10) + 5,00,000\,(P/F, 7\%, 10)$$

7.024 0.5083

$\therefore PW_B = 13,51,210$

$\Rightarrow$ B is preferred

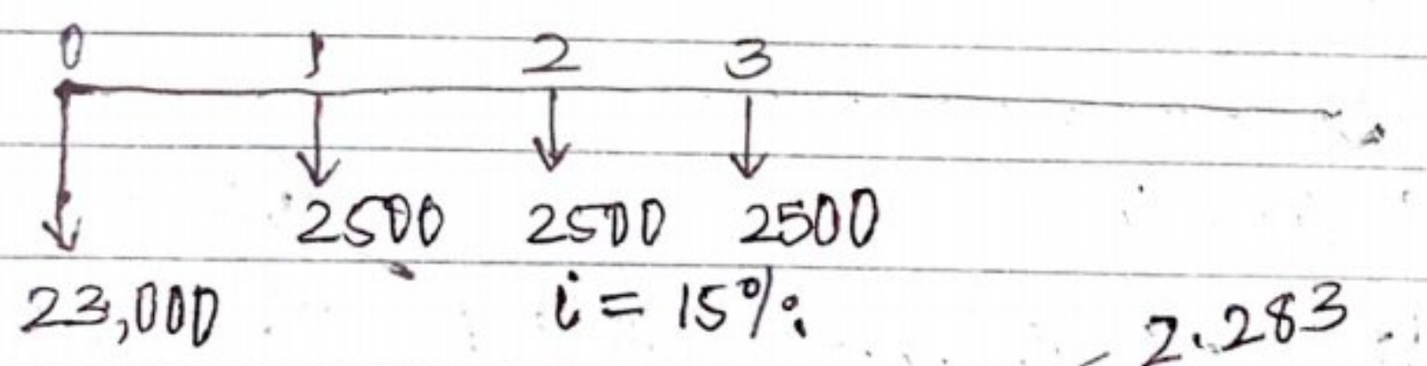
2 Assests A1 and A2 have the capability of performing a required function. Asset A2 and an initial cost of Rs. 32,000 and an expected salvage value of Rs. 4000 at the end of its 4 years service life. Asset A1 costs Rs. 9000 less initially with an economic life 1 year shorter than that of A2 but A1 has no salvage value and its annual operating cost exceeds more than A2 by Rs. 2500 and the required rate of return is 15%. State which alternative is preferred when the comparison is by

(a) repeated projects method

(b) a two year study period

Assuming the assets are needed only for two years.

CFD for A1



(a) $PW_{A1} = -23000 - 2500(P/A, 15, 3)$ 2.283

$\therefore PW_{A1} = 28707.5$

(b) $PW_{A1} = -23000 - 2500(P/A, 15, 2)$ 1.626

$\therefore PW_{A1} = 27065$

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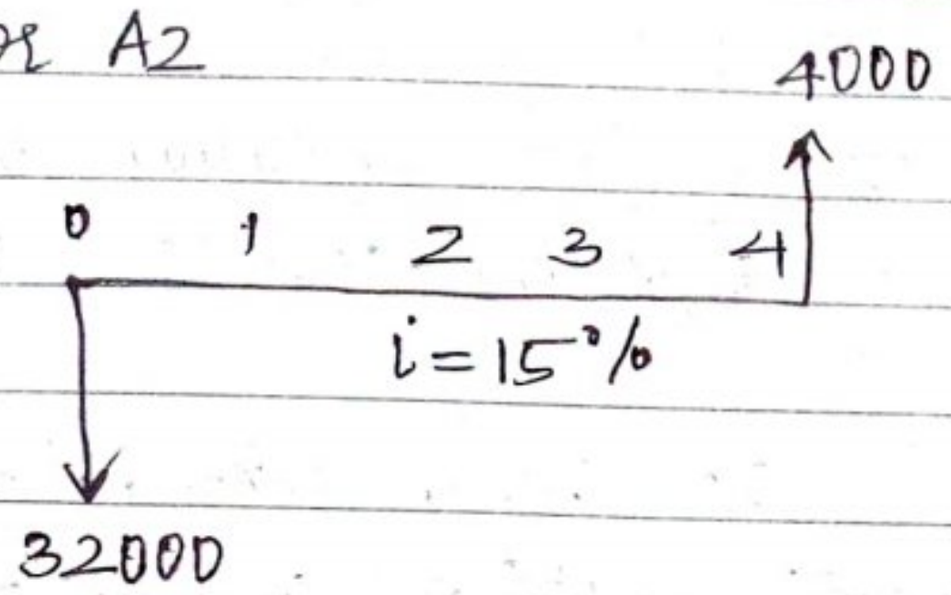
CFD for A2



(a) $PW_{A2} = -32000 + 4000\,(P/F, 15, 4)$ (0.5718) $= 29712.8$

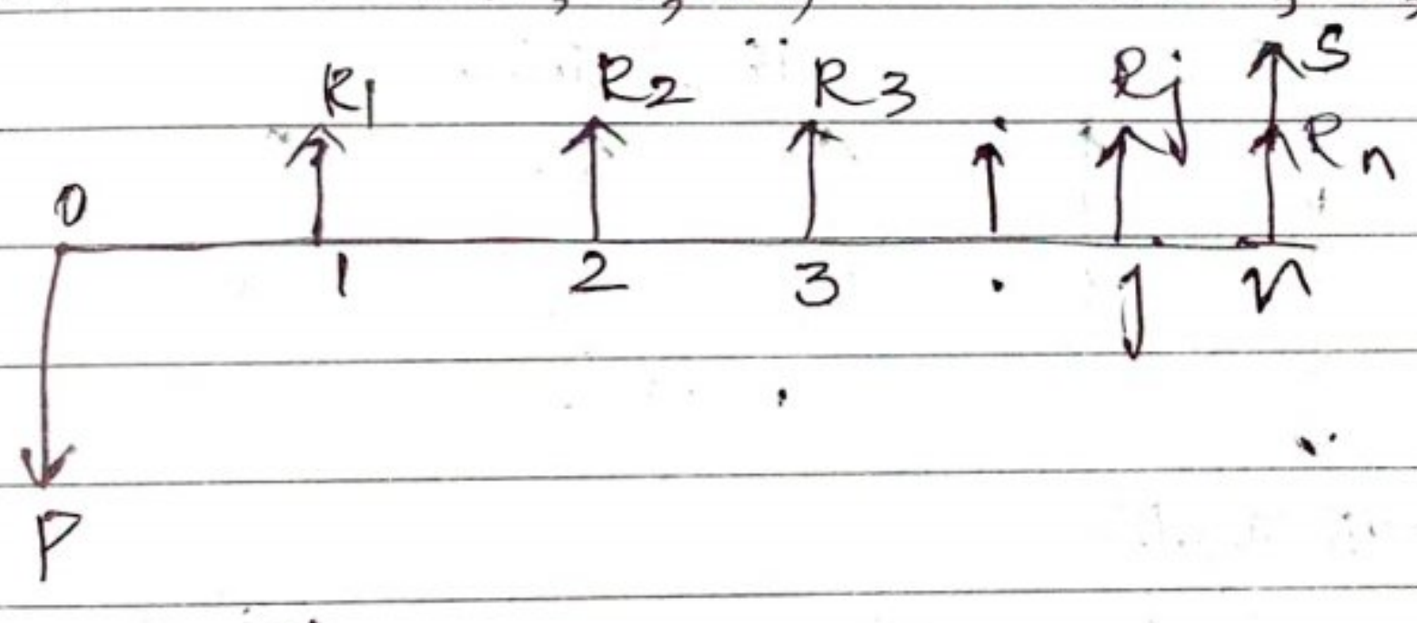
(b) $PW_{A2} = -32000 + 4000\,(P/F, 15, 2)$ (0.7561) $= 28975.6$

For (a), A1 is preferred
For (b), A1 is preferred

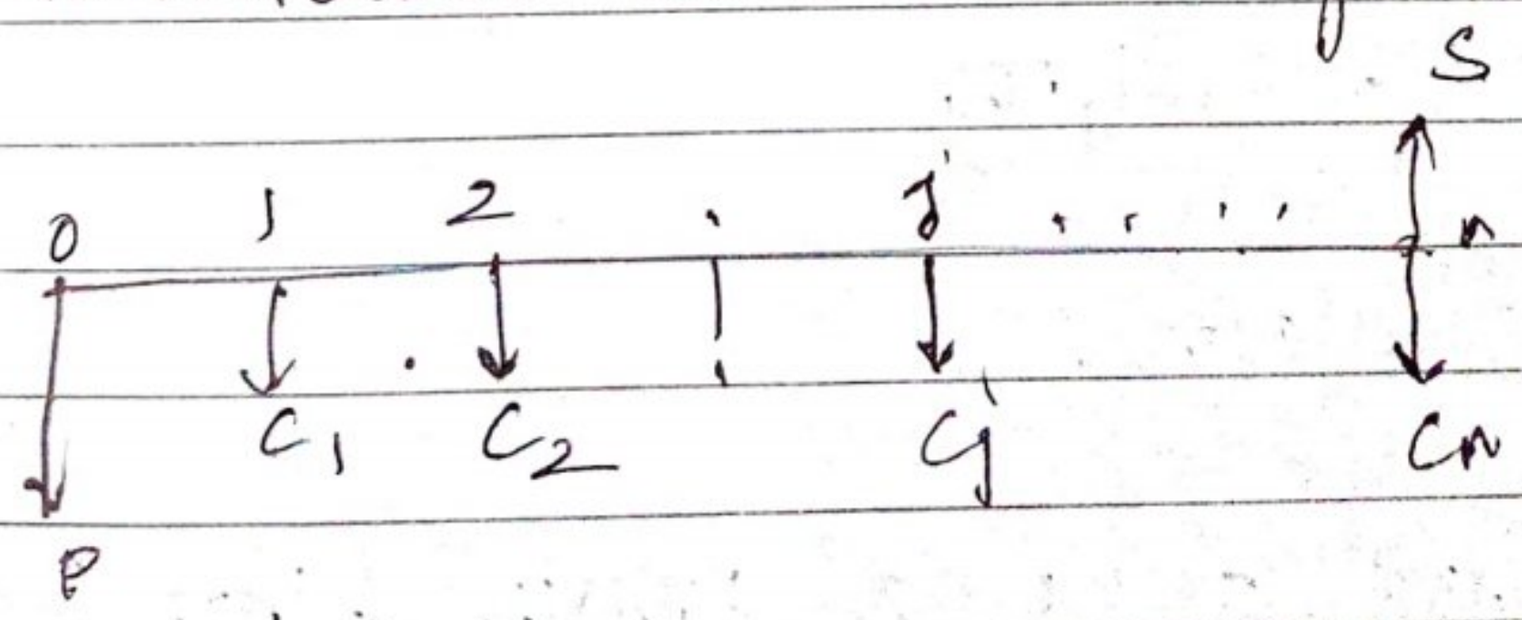
Future Worth Method

Revenue Dominated Cash Flow Diagram

$$FW(i) = -P(F/P, i, n) + A(F/A, i, n) + S$$

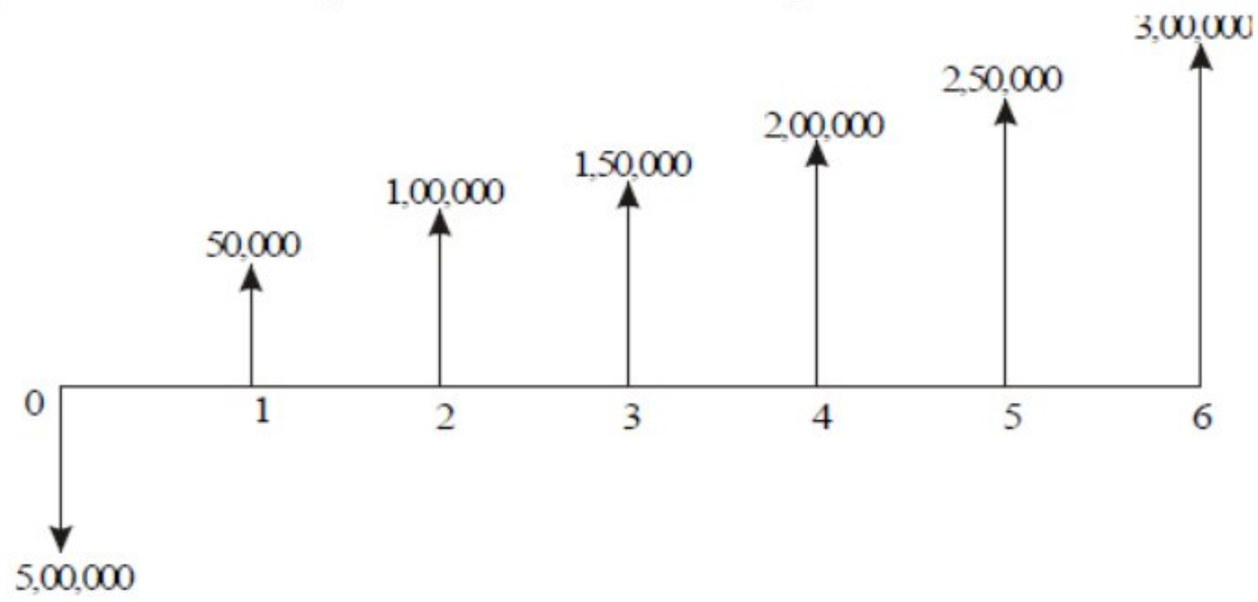


Cost Dominated Cash Flow Diagram

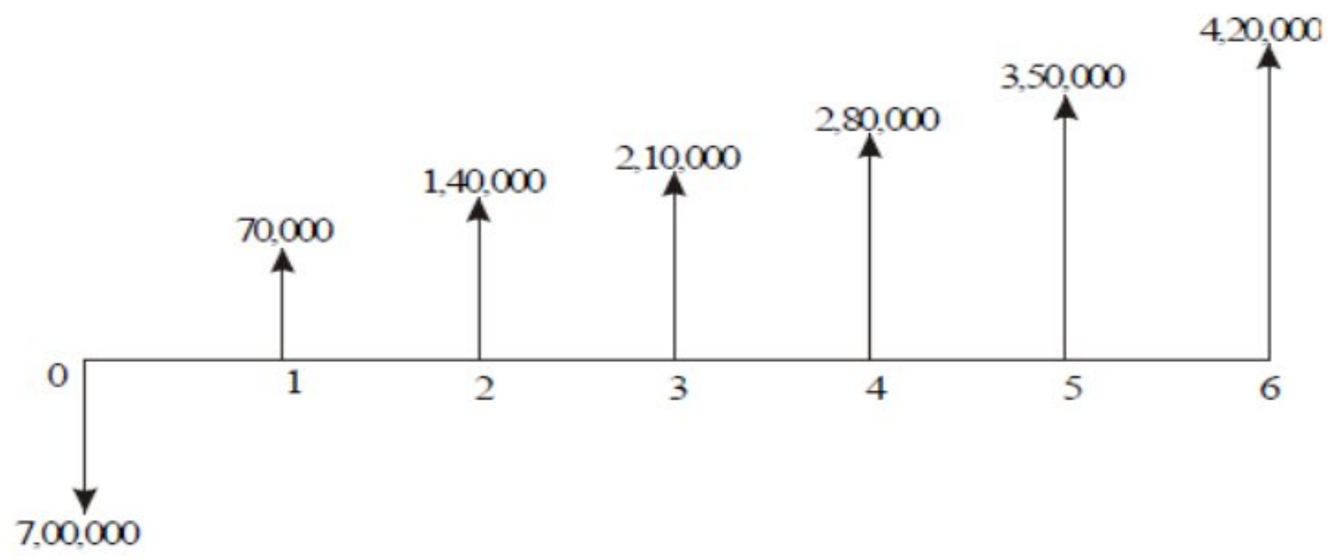


$$FW(i) = P(F/P, i, n) + A(F/A, i, n) - S$$

1. The cash flow diagram of two mutually exclusive alternatives are given:



(a)



(b)

(a) Select the best alternative based on future worth method at $i = 8\%$.
(b) Rework part (a) with $i = 9\%$ and 20%

2. M/S Krishna Castings Ltd. is planning to replace its annealing furnace. It has received tenders from three different original manufacturers of annealing furnace. The details are as follows.

	Manufacturer		
	1	2	3
Initial cost (Rs.)	80,00,000	70,00,000	90,00,000
Life (years)	12	12	12
Annual operation and maintenance cost (Rs.)	8,00,000	9,00,000	8,50,000
Salvage value after 12 years	5,00,000	4,00,000	7,00,000

Which is the best alternative based on future worth method at $i = 20\%$?

3. A company must decide whether to buy machine *A* or machine *B*:

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	Machine A	*Machine B*
Initial cost	Rs. 4,00,000	Rs. 8,00,000
Useful life, in years	4	4
Salvage value at the end of machine life	Rs. 2,00,000	Rs. 5,50,000
Annual maintenance cost	Rs. 40,000	0

At 12% interest rate, which machine should be selected? (Use future worth method of comparison).

4. A motorcycle is sold for Rs. 50,000. The motorcycle dealer is willing to sell it on the following terms:
(a) Make no down payment but pay Rs. 1,500 at the end of each of the first four months and Rs. 3,000 at the end of each month after that for 18 continuous months.
(b) Make no down payment but pay a total amount of Rs. 90,000 at the end of the 22nd month; till that time the buyer should mortgage property worth of Rs. 50,000, at present.
Based on these terms and a 12% annual interest rate compounded monthly, find the best alternative for the buyer based on the future worth method of comparison.

5. Alpha Finance Company is coming with an option of accepting Rs. 10,000 now and paying a sum of Rs. 1,60,000 after 20 years. Beta Finance Company is coming with a similar option of accepting Rs. 10,000 now and paying a sum of Rs. 3,00,000 after 25 years. Compare and select the best alternative based on the future worth method of comparison with 15% interest rate, compounded annually

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Problems on Future Worth comparison

1

(i) $i = 8\%$

(a) $P = -500,000$

$A1 = 50,000$

$G = 50,000$

$n = 6$

Equal Payment Gradient Series

$$FW(i) = -P(F/P, i, n) + \left[A1 + G(A/G, i, n)\right](F/A, i, n)$$

1.587

$$FW(8\%) = -500,000(F/P, 8\%, 6) + \left[50,000 + 50,000(A/G, 8\%, 6)\right](F/A, 8\%, 6)$$

2.276

~~4.623~~ 7.336

$$= 4,08,136.8$$

(b) $P = -7,00,000$

$A1 = 70,000$

$G = 70,000$

$n = 6$

1.587

2.276

$$FW(8\%) = -7,00,000(F/P, 8\%, 6) + \left[70,000 + 70,000(A/G, 8\%, 6)\right](F/A, 8\%, 6)$$

7.336

$$= 5,71,391.52$$

⇒ (b) is selected

(ii) $i = 9\%$

1.677

2.250

(a) $$FW(9\%) = -5,00,000(F/P, 9\%, 6) + \left[50,000 + 50,000(A/G, 9\%, 6)\right](F/A, 9\%, 6)$$

7.523

$$= \cancel{6,72,982.5}\ 3,83,987.5$$

(b) $FW(9\%) = -7,00,000\,(F/P, 9\%, 6)^{1.677} + [70,000 + 70,000\,(A/G, 9\%, 6)^{2.250} \times (F/A, 9\%, 6)^{7.523}]$

$= 5,37,582.5$

⇒ (b) is selected

(iii) $i = 20\%$

(a) $FW(20\%) = -5,00,000\,(F/P, 20\%, 6)^{2.986} + [50,000 + 50,000\,(A/G, 20\%, 6)^{1.979} \times (F/A, 20\%, 6)^{9.930}]$

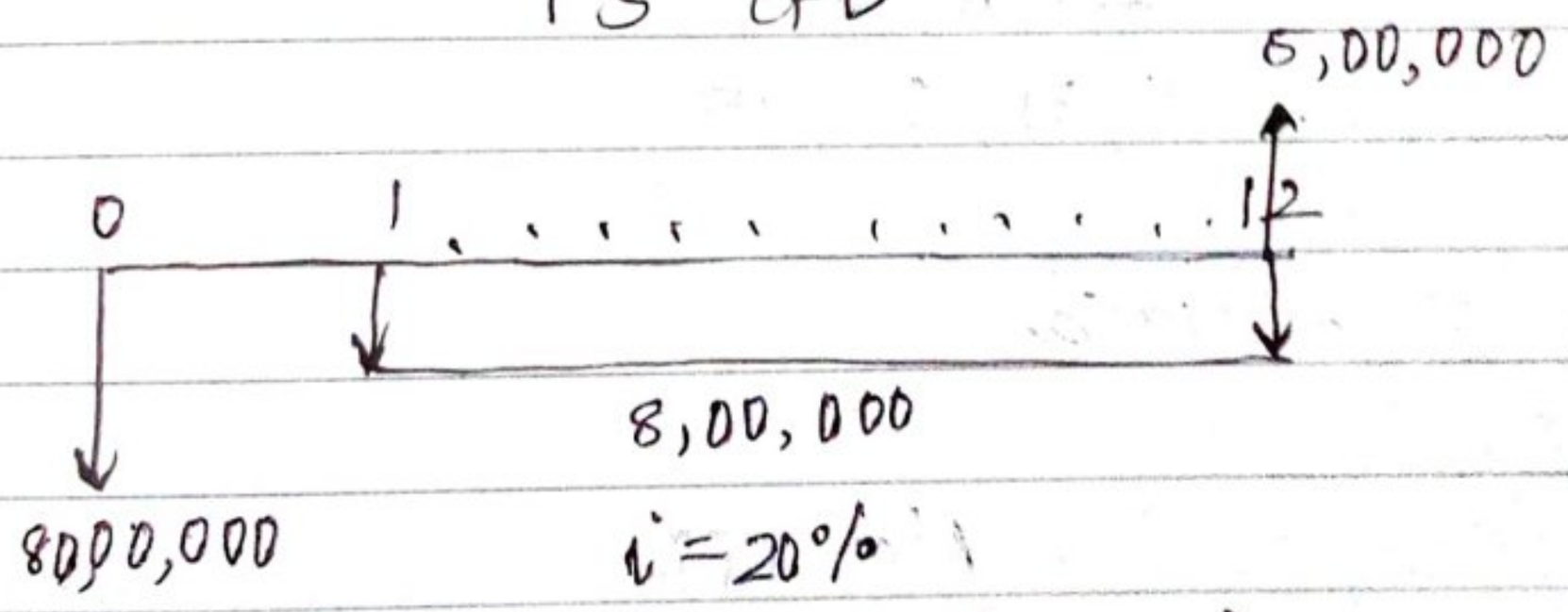
$= -13,926.5$

(b) $FW(20\%) = -7,00,000\,(F/P, 20\%, 6) + [70,000 + 70,000\,(A/G, 20\%, 6)] \times (F/A, 20\%, 6)$

$= -19,497.1$

⇒ Both incur loss, so none of them is selected

2 1's CFD

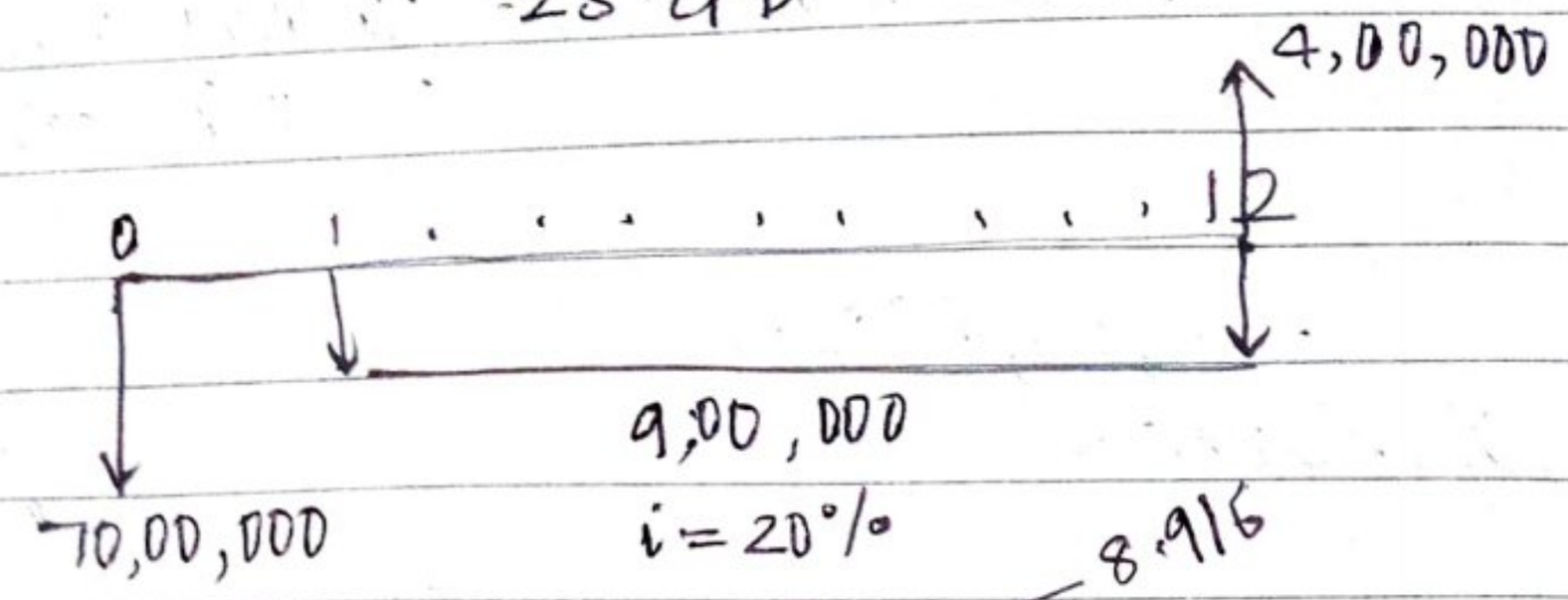


$FW(i) = P(F/P, i, n) + A(F/A, i, n) - S$

$FW(20\%) = +80,00,000\,(F/P, 20\%, 12)^{8.916} + 8,00,000\,(F/A, 20\%, 12)^{39.581} - 5,00,000$

$= 10,24,92,800$

2's CFD

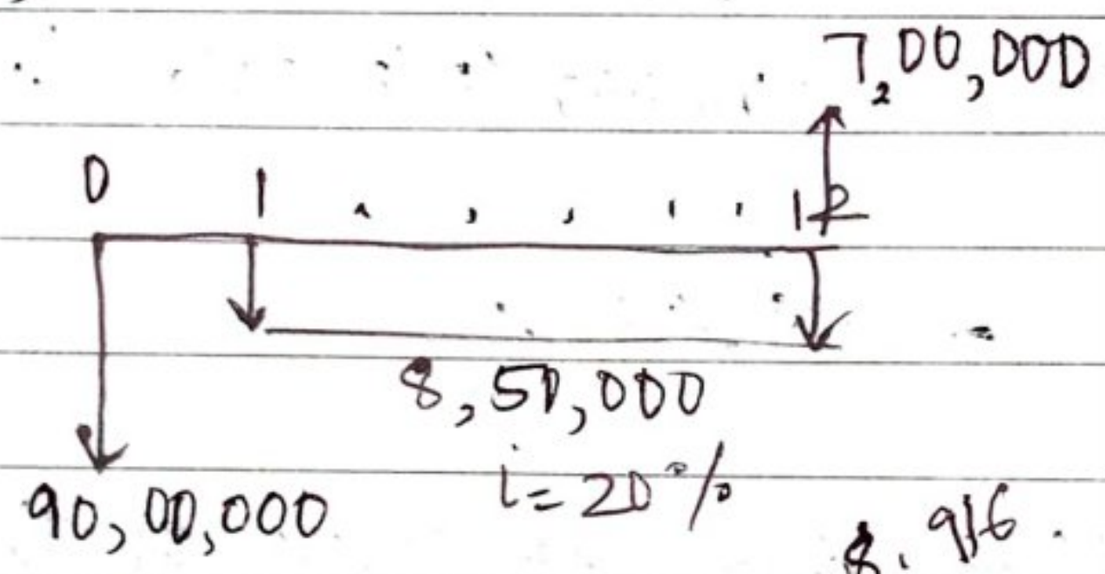


$$FW(20\%) = 70,00,000(F/P, 20\%, 12) + 9,00,000(F/A, 20\%, 12) - 4,00,000$$

8.916 39.581

$$= 9,76,34,900$$

3's CFD



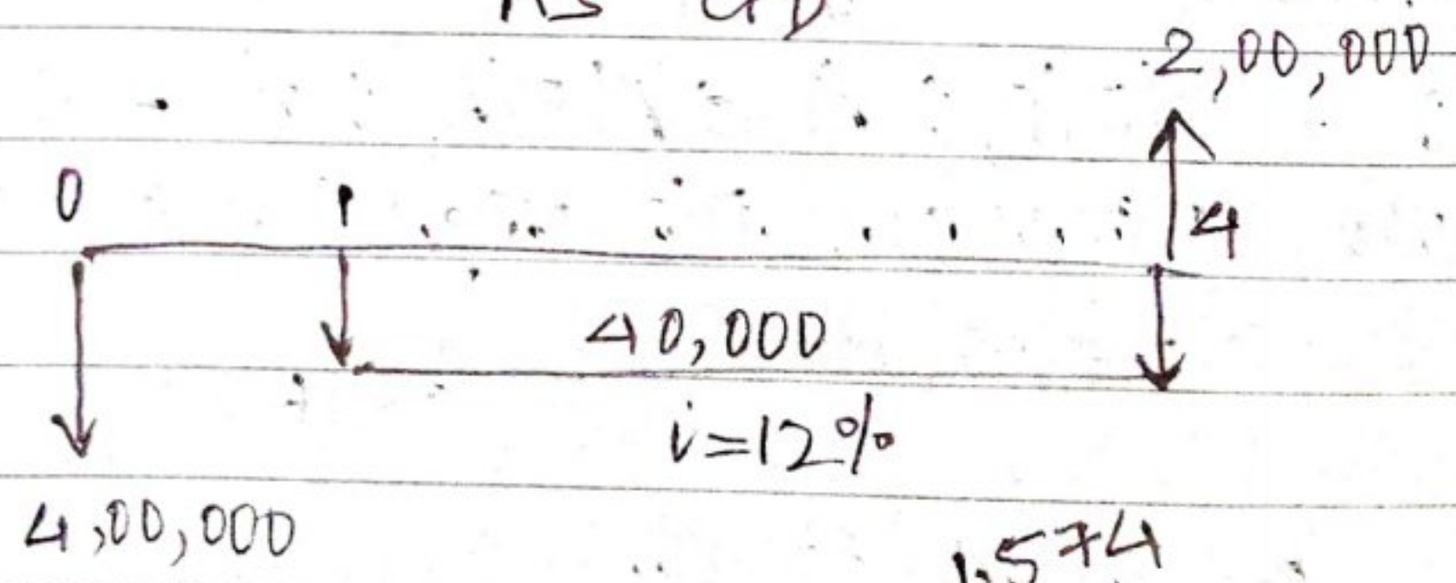
$$FW(20\%) = 90,00,000(F/P, 20\%, 12) + 8,50,000(F/A, 20\%, 12) - 7,00,000$$

8.916 39.581

$$= 11,31,87,850$$

⇒ 2 is selected

3 A's CFD



$$FW(12\%) = 4,00,000(F/P, 12\%, 4) + 40,000(F/A, 12\%, 4) - 2,00,000$$

1.574 4.779

$$= 6,20,760$$

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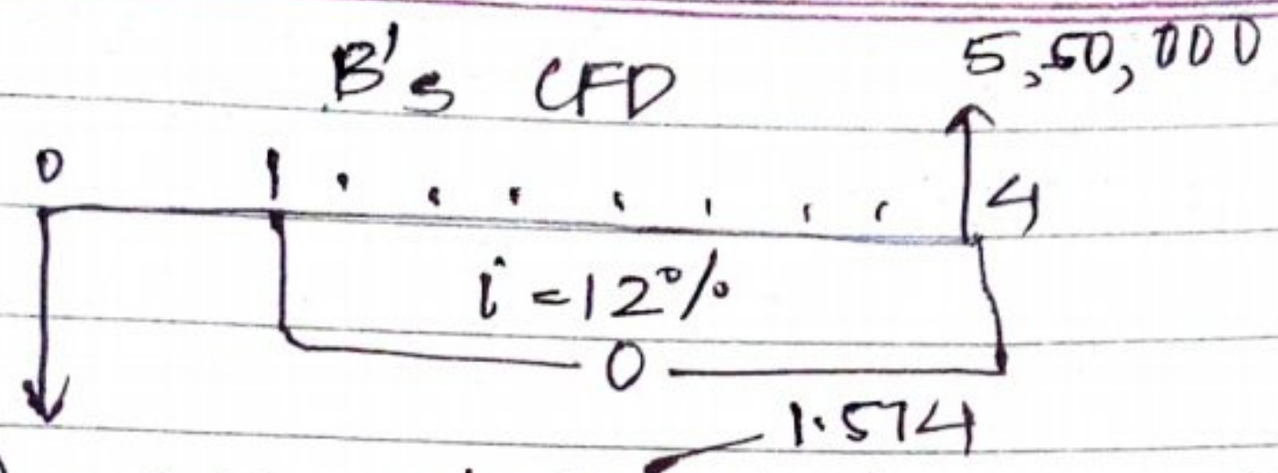
B's CFD



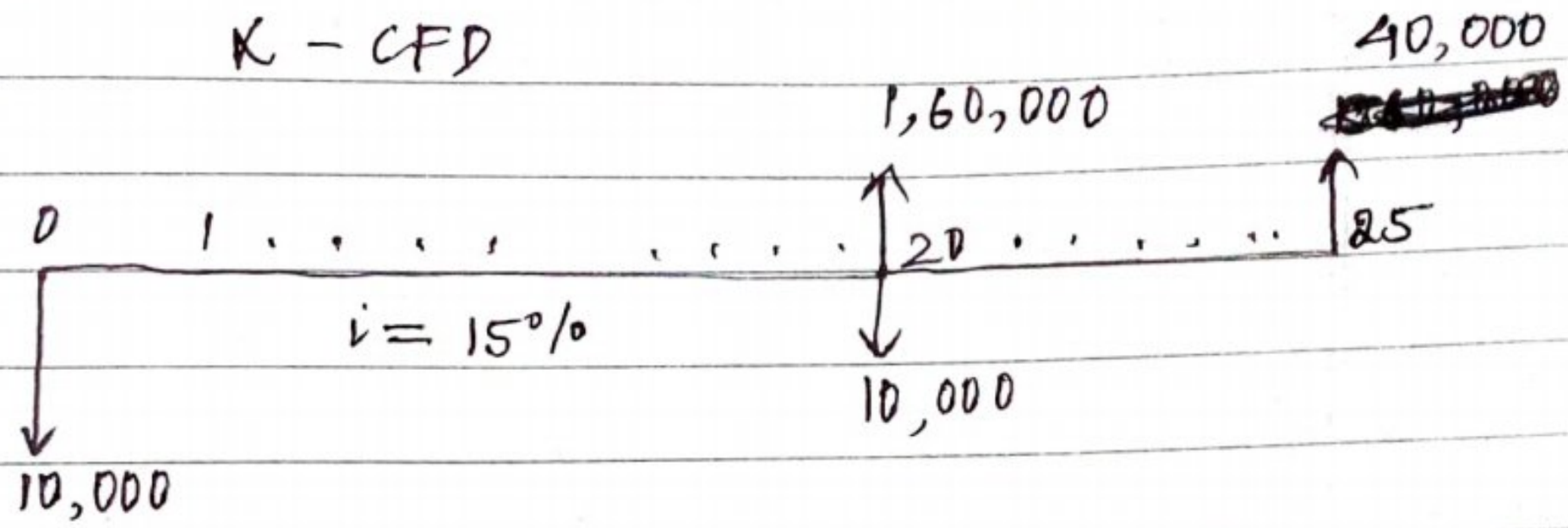
1.574

$FW(12\%) = 8,00,000(F/P, 12\%, 4) - 5,50,000$

$= 7,09,200$

⇒ A is selected

5 K - CFD



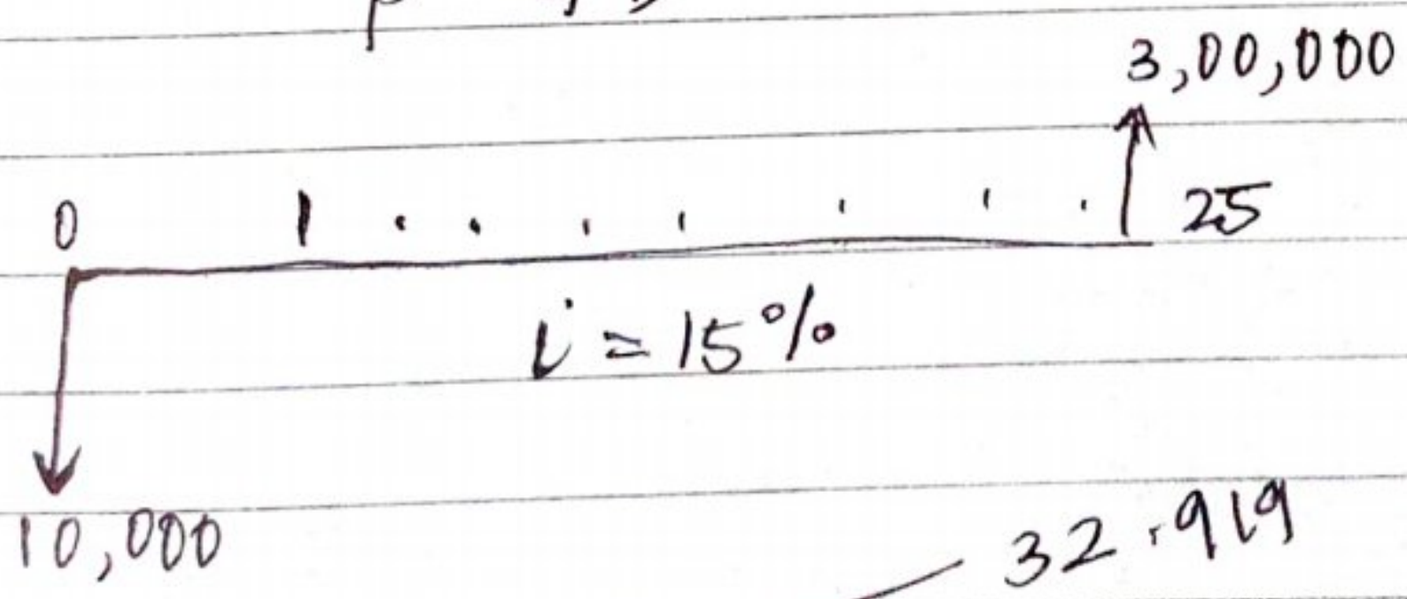
$\cancel{FW(15\%) = -10,000(F/P, 15\%, 25) - 10,000(F/P, 15\%, 5)}$

$\cancel{+1,60,000 + 1,60,000}$ 40,000 32.919

$FW(15\%) = -10,000(F/P, 15\%, 25) + 1,60,000 + 40,000$

$= -1,29,190$

β - CFD



32.919

$FW(15\%) = -10,000(F/P, 15\%, 25) + 3,00,000$

$= -29190$

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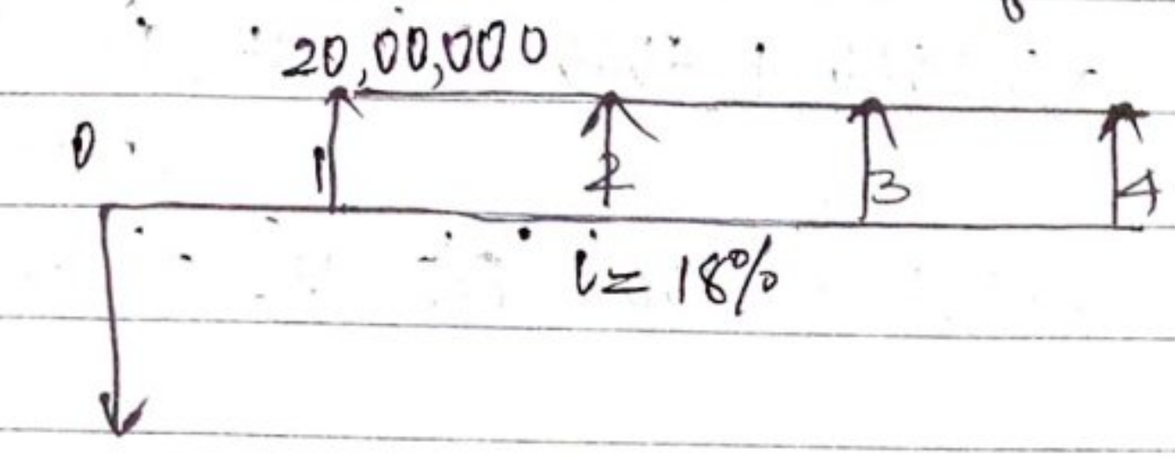
1

Alternative	End of year 0	1	2	3	4
A(Rs)	-50,00,000	20,00,000	20,00,000	20,00,000	20,00,000
B(Rs)	-45,00,000	18,00,000	18,00,000	18,00,000	18,00,000

At $i = 18\%$, select the best alternative based on future worth method of comparison.



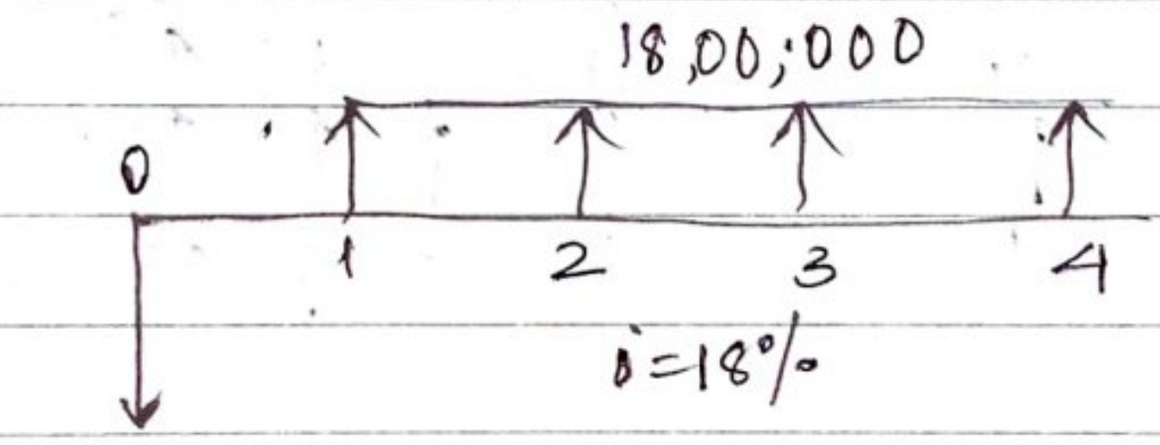
-50,00,000 A's CFD

$$FW(i) = -P(F/P, i, n) + A(F/A, i, n) + S$$

$$= -50,00,000\,(F/P, 18\%, 4) + 20,00,000\,(F/A, 18\%, 4)$$

1.9388 5.2154

$$= 7,36,800$$



-45,00,000

B's CFD

1.9388 5.2154

$$FW(18\%) = -45,00,000\,(F/P, 18\%, 4) + 18,00,000\,(F/A, 18\%, 4)$$

$$= 6,61,800$$

⇒ A is selected

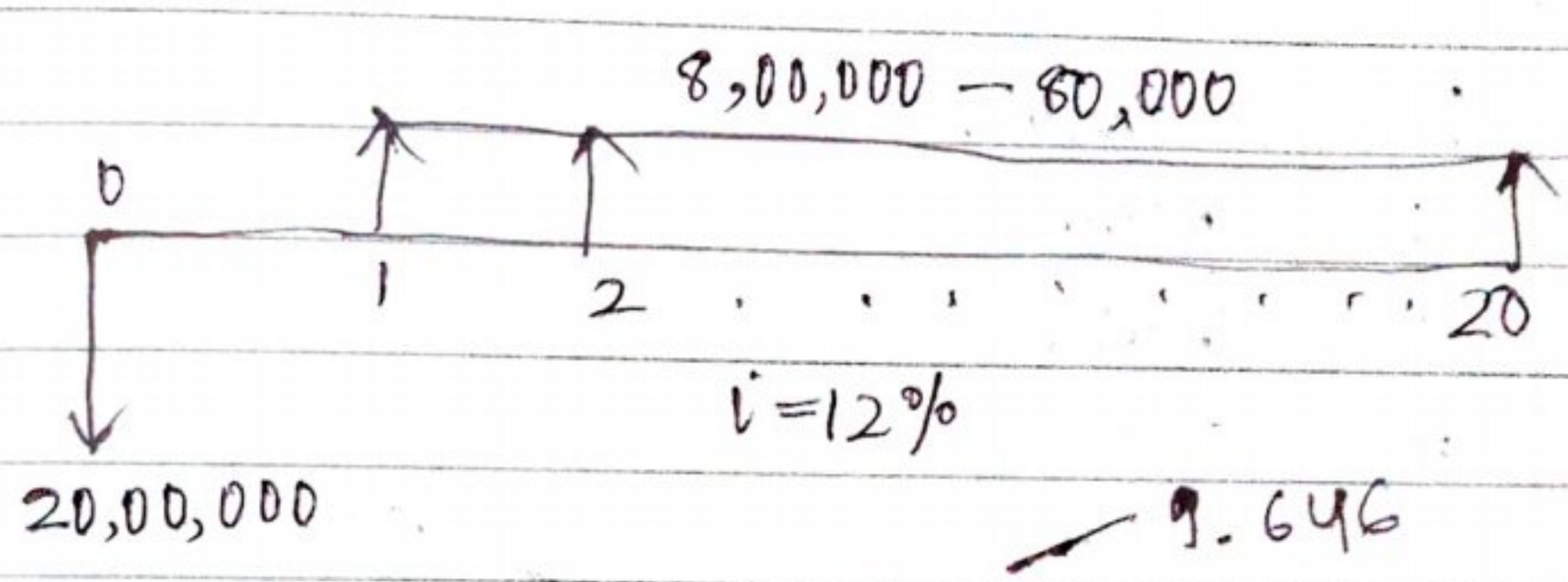
2 A man owns a corner plot. He must decide which of the several alternatives to select in trying to obtain a desirable return on his investment, after much study and calculation, he decides that the two best alternatives are as given

as the following table.

	Build gas station	Build soft ice-cream stand
First cost (Rs)	20,00,000	36,00,000
Annual property tax (Rs)	80,000	1,50,000
Annual income (Rs)	8,00,000	9,80,000
Life of building (years)	20	20
Salvage value (Rs)	0	0

Evaluate the alternative based on the future worth method at $i = 12\%$.

1 gas station's CFD



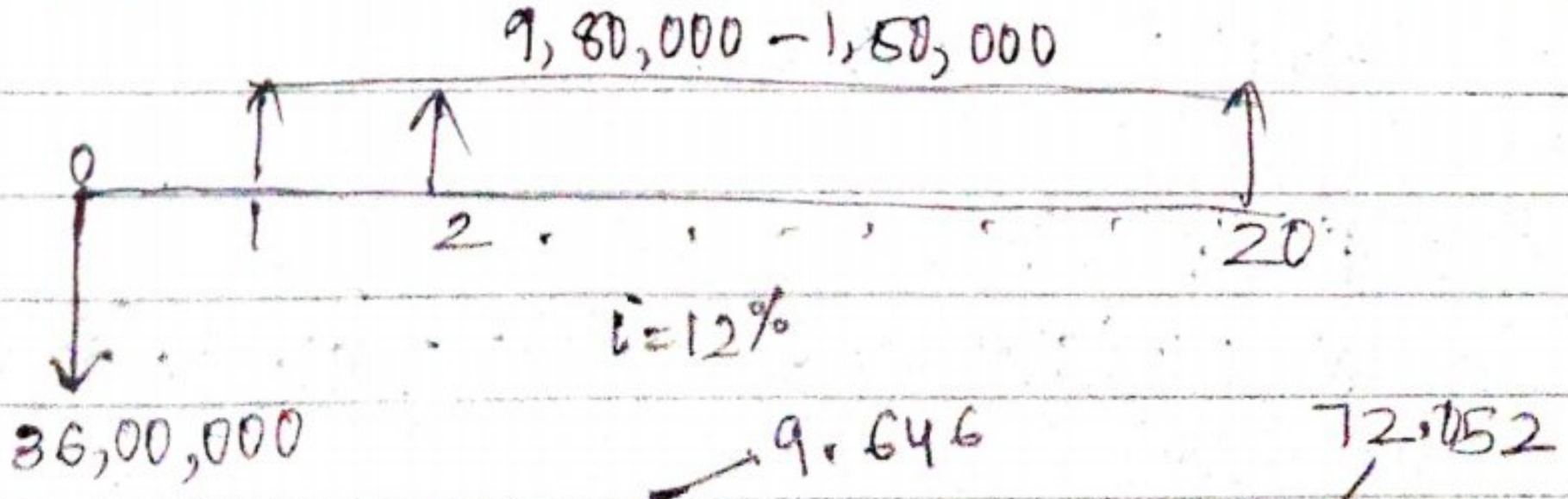
$$FW(12\%) = -20,00,000(F/P, 12\%, 20) + 720000(F/A, 12\%, 20)$$

9.646

72.052

$$= 3,25,85,440$$

2 ice-cream stand's CFD



9.646

72.052

$$FW(12\%) = -36,00,000(F/P, 12\%, 20) + 830000(F/A, 12\%, 20)$$

$$= 2,50,77,560$$

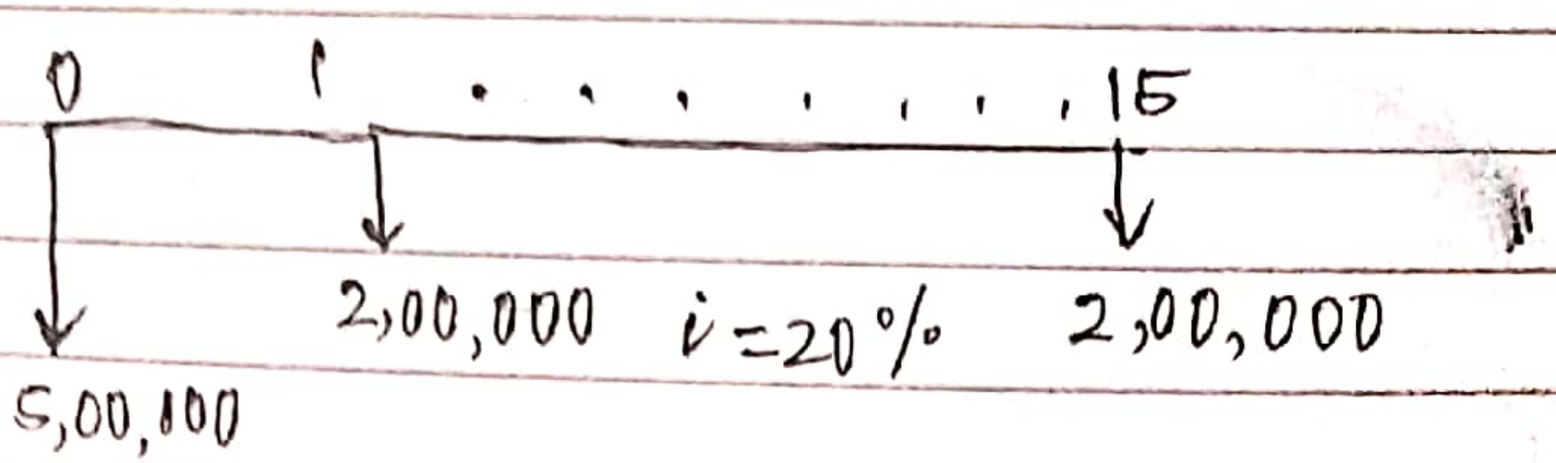
⇒ 1 is selected

Annual Equivalent Method Problems

6-SM1 A company is planning to purchase an advanced machine centre. Three original manufacturers have responded to its tender whose particulars are tabulated as follows. Determine the best alternative based on the annual equivalent method by assuming $i = 20\%$, compounded annually.

Manufacturers	Down payment (Rs)	Yearly equal installment (Rs)	No. of installments
1	5,00,000	2,00,000	15
2	4,00,000	3,00,000	15
3	6,00,000	1,50,000	15

1



$$AE(i) = P(A/P, i, n) + A - S(A/F, i, n)$$

$$AE(20\%) = 5,00,000\,(A/P, 20\%, 15) + 2,00,000 - 0$$

0.2139

$$= 3,06,950$$

for rest

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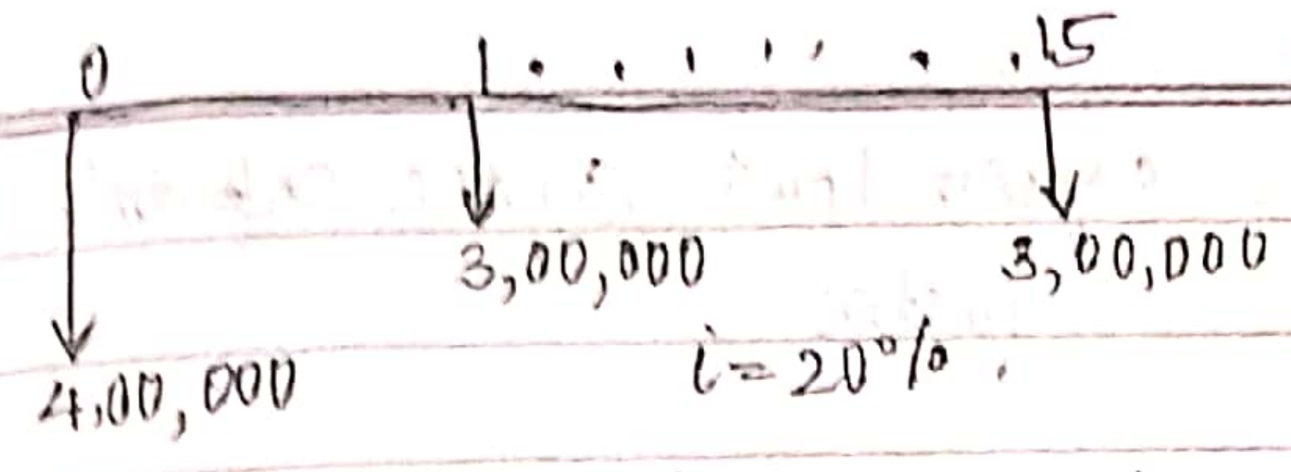
2

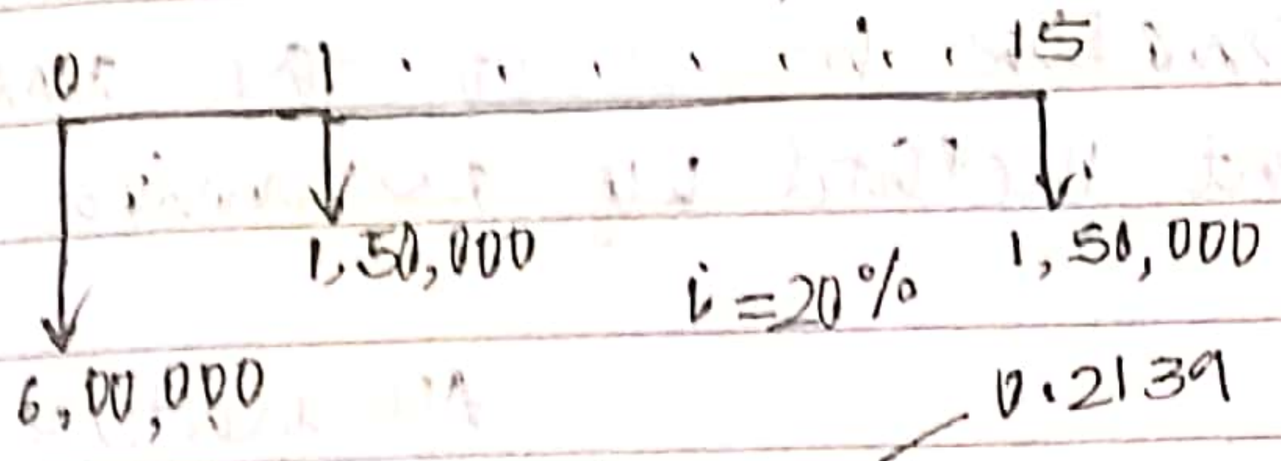


$AE(20\%) = 4,00,000\,(A/P, 20\%, 15) + 3,00,000$

0.2139

$= 3,85,560$ //

3



0.2139

$AE(20\%) = 6,00,000\,(A/P, 20\%, 15) + 1,50,000$

$= 2,18,340$ //

⇒ 3 is best alternative

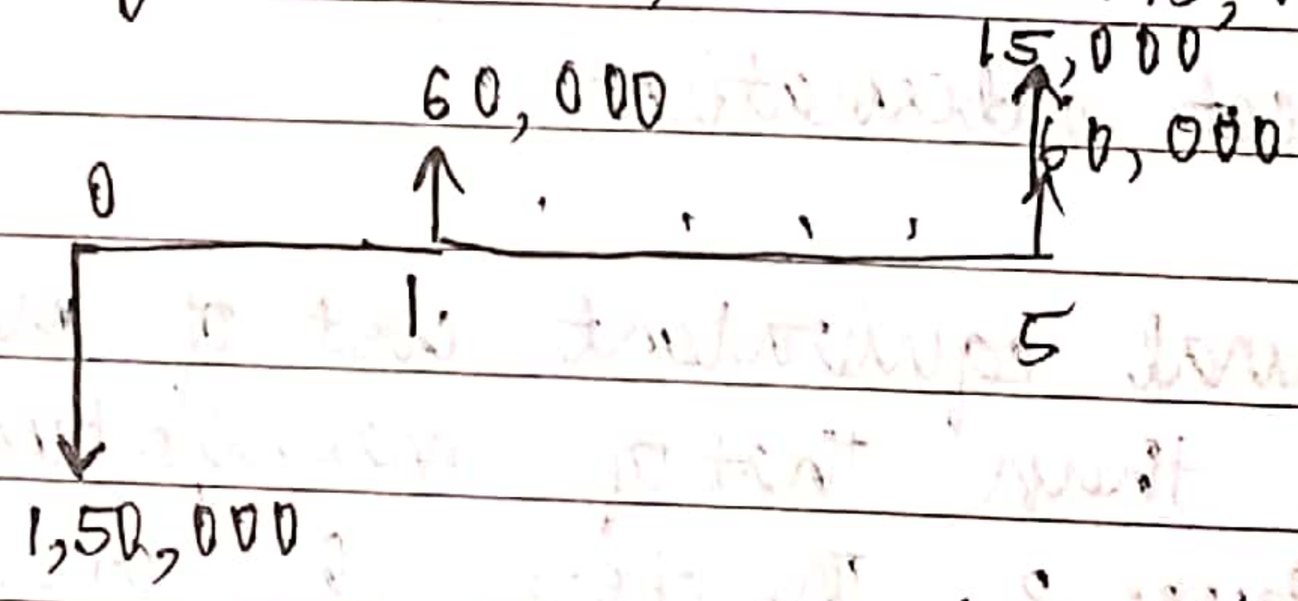
The annual equivalent cost of manufacturer 3 is less than that of manufacturer 1 and manufacturer 2. Therefore, the company should buy the advanced machine centre from manufacturer 3.

(A/P, i, n) = equivalent series capital recovery factor

2 A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values. Determine the best alternative based on the annual equivalent method by assuming i = 25%.

	Alternative A	Alternative B
Investment (Rs.)	-1,50,000	-1,75,000
Annual equal return (Rs)	+60,000	+70,000
Salvage value (Rs)	+15,000	+35,000

A



for revenue

$AE(i) = -P(A/P, i, n) + A + S(A/F, i, n)$

$AE(25\%) = -1,50,000\,(A/P, 25\%, 5) + 60,000 + 15,000\,(A/F, 25\%, 5)$

0.3718

0.1218

$= 6057$

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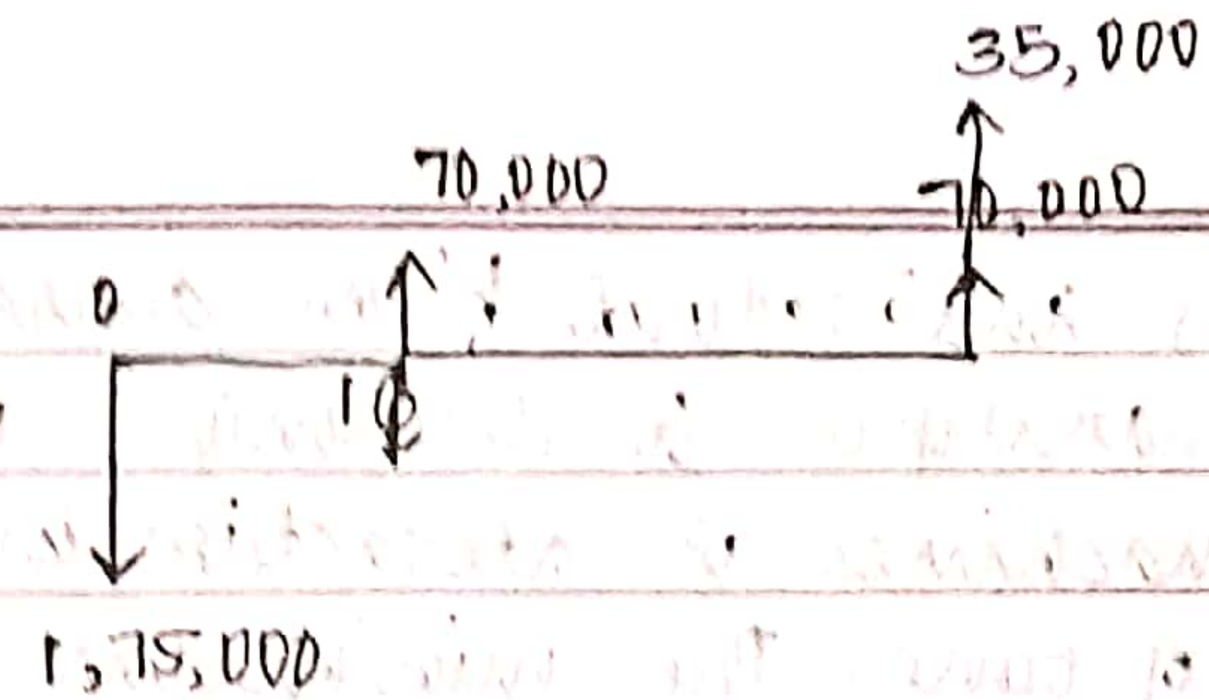
B



$$AE(25\%) = -1,75,000\,(\overset{0.3718}{A/P,\ 25\%,\ 5}) + 35,000\,(\overset{0.1218}{A/F,\ 25\%,\ 5}) + 70,000$$

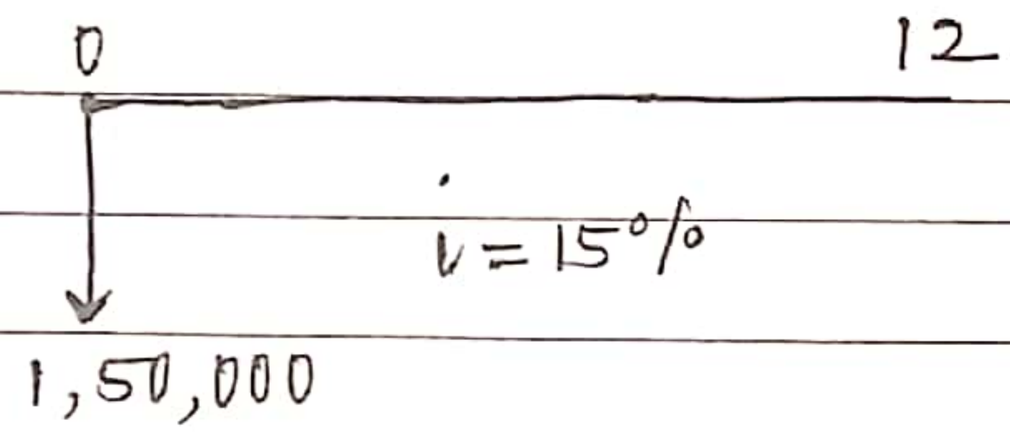
$$= 9198$$

⇒ B is better alternative

3 A certain individual firm desires an economic analysis to determine which of the two machines is attractive in a given interval of time. The minimum attractive rate of return for the firm is 15%. The following data are to be used in the analysis. Which machine would you choose? Base your answer on annual equivalent cost.

	Machine X	Machine Y
First cost	1,50,000	2,40,000
Estimated life	12 years	12 years
Salvage value	0	6,000
Annual maintenance cost	0	4,500

Machine X

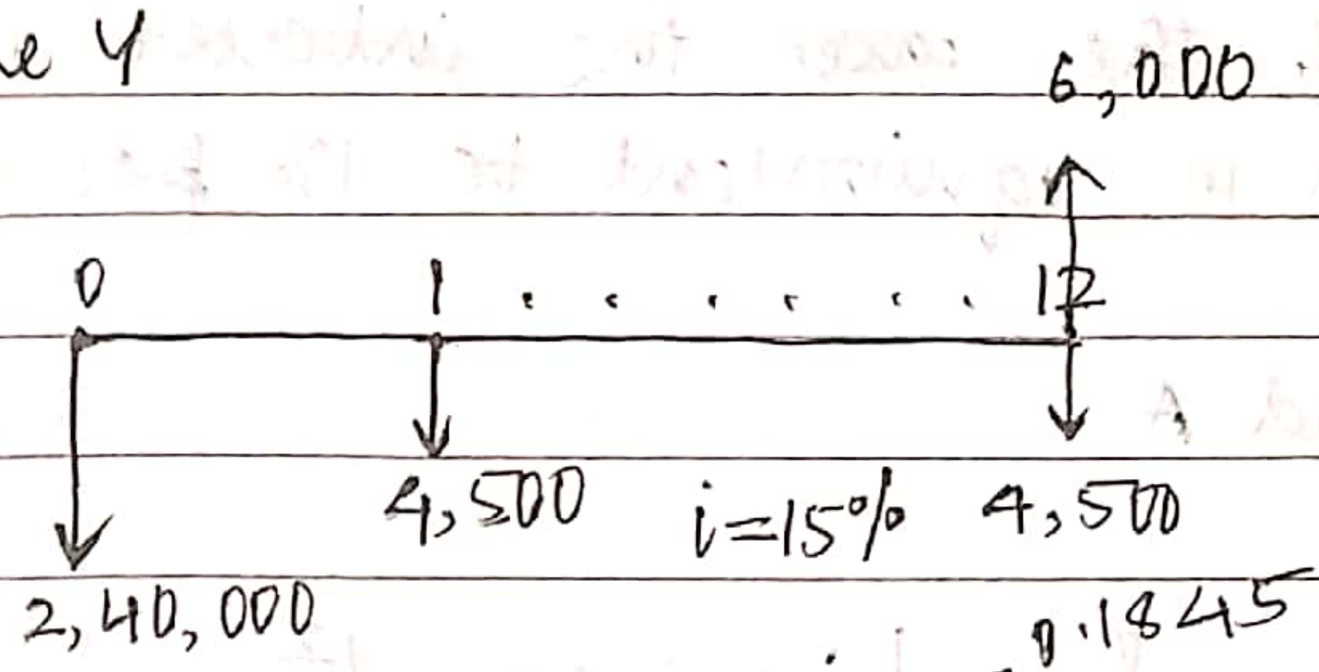


0.1845

AE(15%) = 1,50,000 (A/P, 15%, 12)

= 27,675 //

Machine Y



$$AE(15\%) = 2,40,000\,(A/P, 15\%, 12)^{0.1845} + 4,500 - 6000\,(A/F, 15\%, 12)^{0.0345}$$

$$= 48,573$$

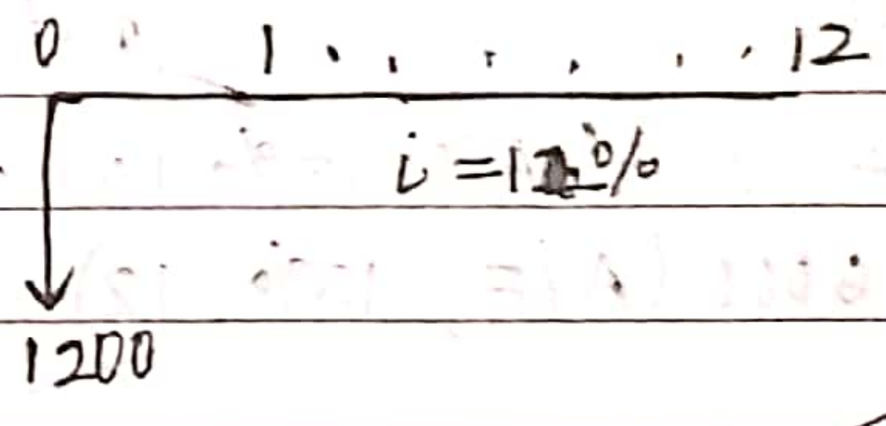
⇒ Machine X is better

4 A transport company has been looking for a new tyre for its truck and has located the following alternatives. If the company feels that the warranty period is a good estimate of the tyre life and that a nominal interest rate (compounded annually) of 12% is appropriate, which tyre should it buy?

Brand	Tyre warranty (months)	Price per tyre
A	12	1,200
B	24	1,600
C	36	2,100
D	48	2,700

In all the cases, the interest rate is 12%, which is equivalent to 1% per month.

Brand A



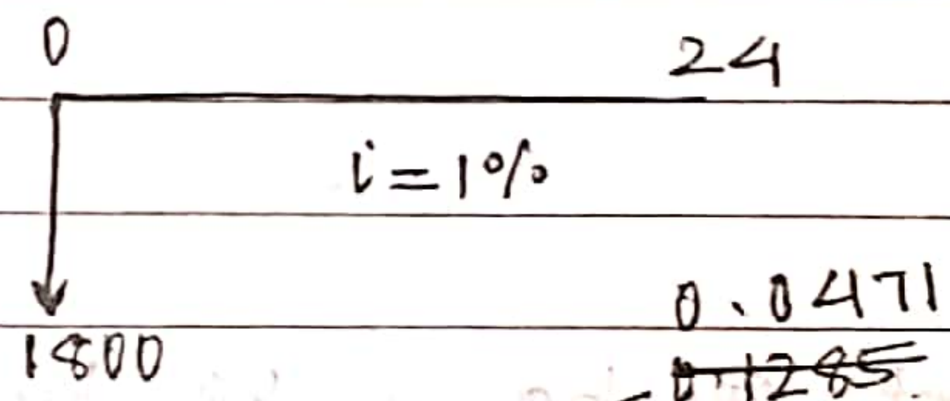
~~0.1614~~
0.0888

$AE(\cancel{12\%}\ 1\%) = 1200\,(A/P, \cancel{12\%}\ 1\%, 12)$ $\quad 1200\,(A/P, 12\%, 1)$

$= \cancel{193.68}$

$= 106.56$ //

Brand B

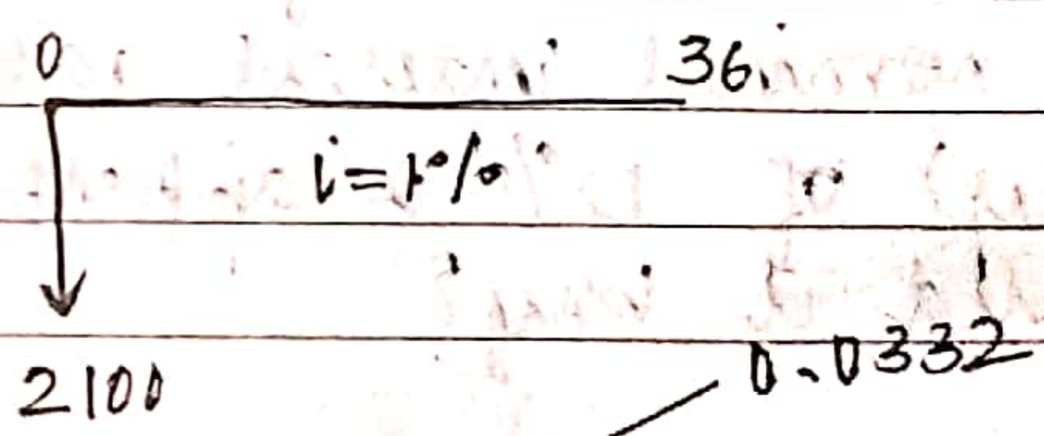


0.0471
~~0.1285~~

$AE(\cancel{12\%}\ 1\%) = 1800\,(A/P, \cancel{12\%}\ 1\%, 24)$

$= \cancel{231.3}\ 84.78$

Brand C

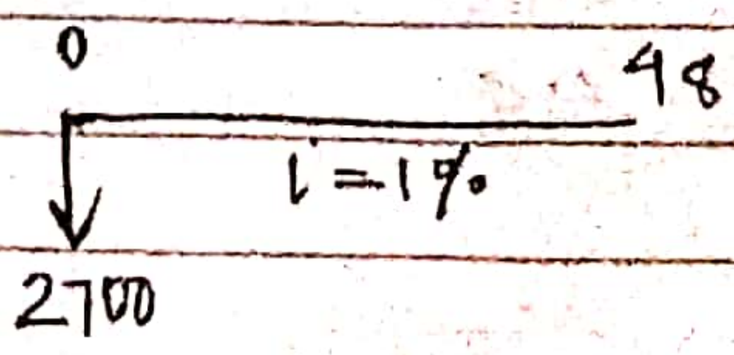


0.0332

$AE(1\%) = 2100\,(A/P, 1\%, 36)$

$= 69.72$ //

Brand D



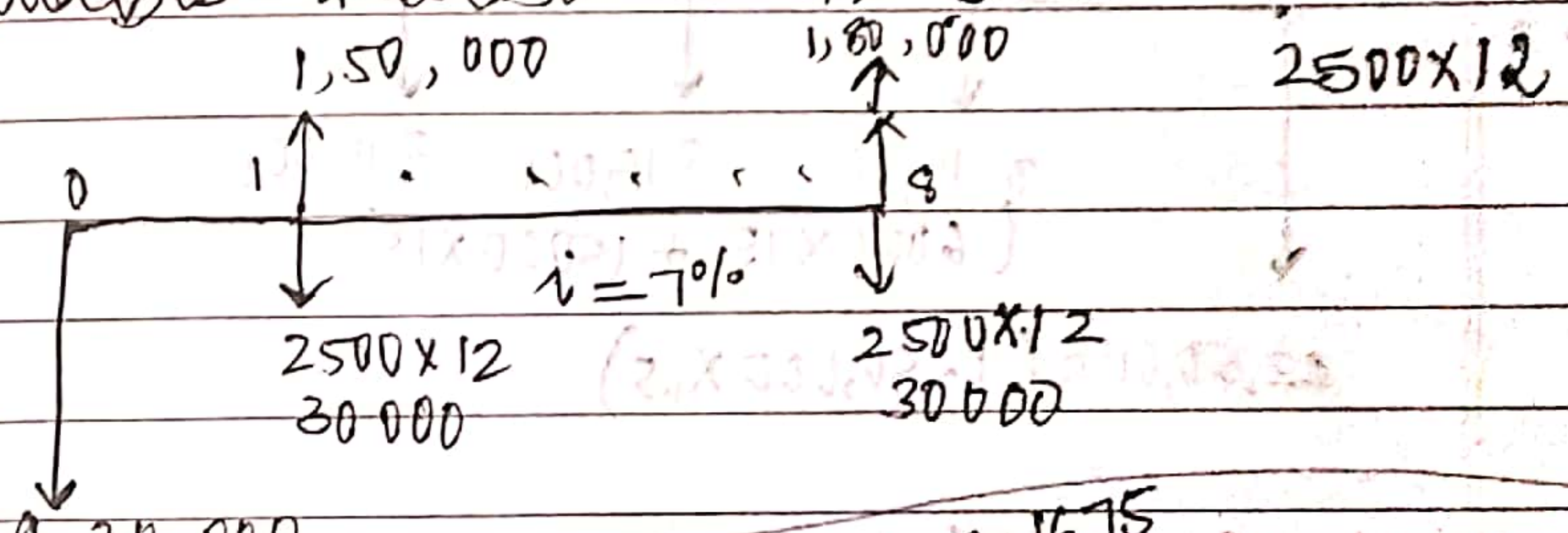
0.0262

$AE(1\%) = 2700\,(A/P,\ 1\%,\ 48)$

$= 71.01$ //

5 A consulting firm pro

5 A company plans to purchase a truck without an operators platform that will reduce labour cost for signed installations by Rs. 1,50,000 per year. The price of the truck is Rs. 1,30,000 and its operating cost will exceed those of present equipment by Rs. 2500 per month. The resell value is expected to be Rs. 1,80,000 in 8 years. Should the truck be purchased when the current available interest is 7%?



0.1675

$AE(7\%) = 9,30,000\,(A/P,\ 7\%,\ 8) + (1,50,000 - \cancel{2500}\ 30,000)$
$\times\ - 180,000\,(A/F,\ 7\%,\ 8)$

0.0975

$= 2,85,725$ //

for revenue

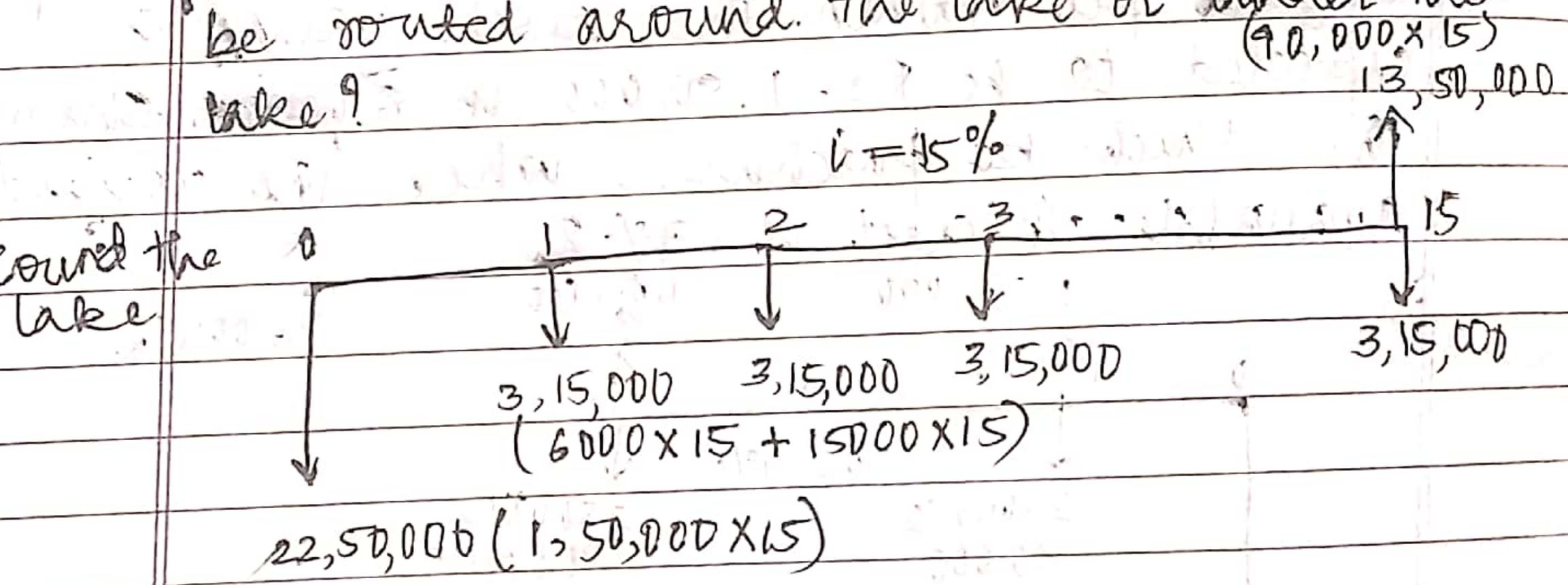
$AE(7\%) = -P(A/P,\ i,\ n) + A + S(A/F,\ i,\ n)$
$= -9,30,000\,(0.1675) + (150,000 - 30,000)$
$+ 180,000\,(0.0975)$
$= -18,225$ //

6 Two possible routes for laying a power line are under study. Data on the routes are as follows.

	Around the lake	Under the lake
Length	15 km	5 km
First cost (Rs)	1,50,000/km	7,50,000/km
Useful life (years)	15	15
Maintenance cost (Rs)	6,000/km/yr	12,000/km/yr
Salvage value (Rs)	90,000/km	1,50,000/km
Yearly power loss (Rs)	15,000/km	15,000/km

If 15% interest is used, should the power line be routed around the lake or under the lake?

Around the lake



$$AE(15\%) = P(A/P, i, n) + A - S(A/F, i, n)$$

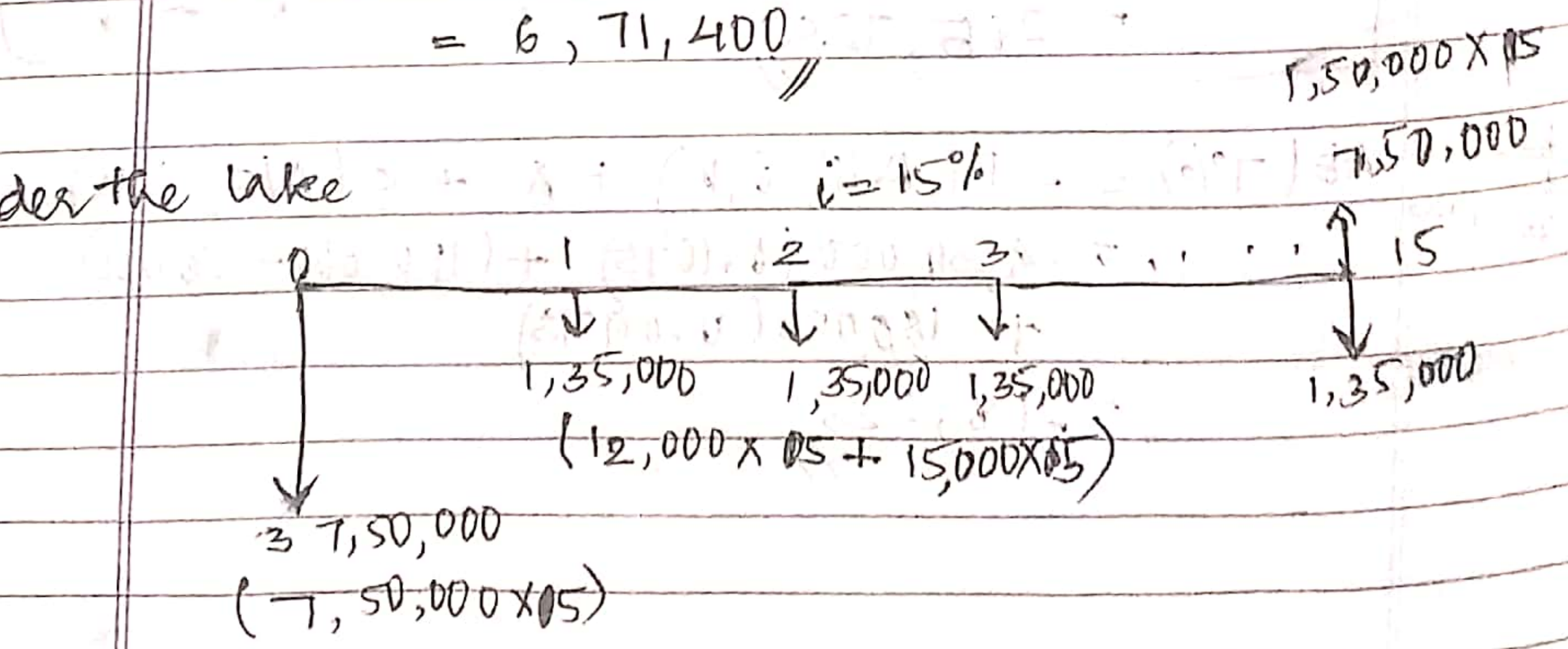
0.1710

$$= 22,50,000(A/P, 15\%, 15) + 3,15,000 - 13,50,000(A/F, 15\%, 15)$$

0.0210

$$= 6,71,400$$

Under the lake



$AE(15\%) = P(A/P, i, n) + A - S(A/F, i, n)$

$= 3,750,000(A/P, 15\%, 15) + 1,35,000$

$- 7,50,000(A/F, 15\%, 15)$

0.1710

0.0210

$= 7,60,500$ //

⇒ Alternative 1 is better

Equivalent Annual Cost — Derivation

7 Stand by lighting generator is required for a shop. Two types are available. If both the generators have a life of 4 years and interest rate is 15% per year which offers the lowest equivalent annual cost?

cost	Type 1	Type 2
First cost	5000	3200
Salvage value	1000	Nil
Annual operating cost	780	950

Type 1 $EAC = -5000(A/P, 15, 4) - 780 + 1000(A/F, 15, 4)$

0.3503 0.2003

$= -2331.2$ //

Type 2 $EAC = -3200(A/P, 15, 4) \cancel{+950(A/F, 15, 4)} - 950$

0.3503 ~~0.2003~~

~~= 930.675~~ //

$= -2070.96$ //

⇒ Type 2 is better

$$CRF = \frac{i(1+i)^n}{(1+i)^n - 1}$$

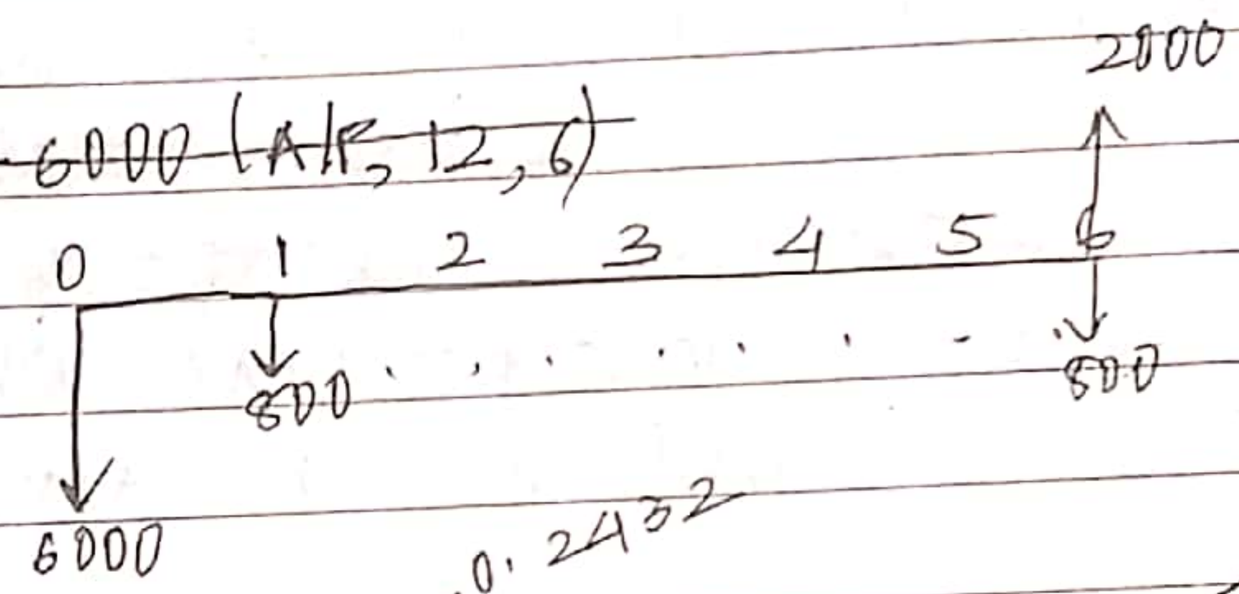
$$SFF = \frac{i}{(1+i)^n - 1}$$

8 Following alternatives can perform the same function. At an annual interest rate of 12%, rank the alternatives as per the equivalent annual cost.

Alternative	First cost (Rs)	Life (years)	Salvage (Rs)	Annual cost (Rs)
A	6000	6	2000	800
B	3000	3	1000	1000
C	2000	3	Nil	1200

A

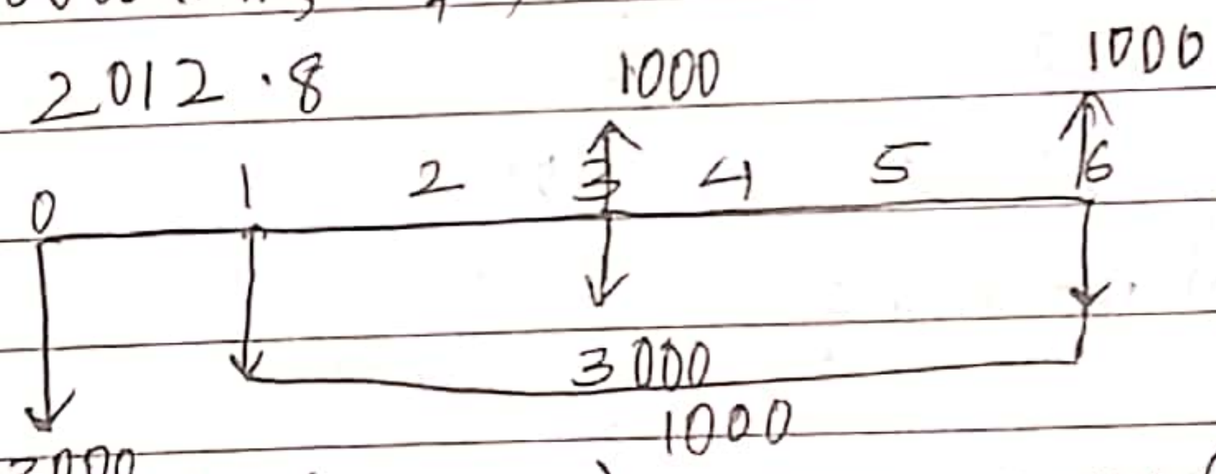
~~EAC = 6000 (A/P, 12, 6)~~



$$EAC = 6000(A/P, \underbrace{12}_{0.2432}, 6) + 800 - 2000(A/F, \underbrace{12}_{0.1232}, 6)$$

$$= 2012.8$$

B



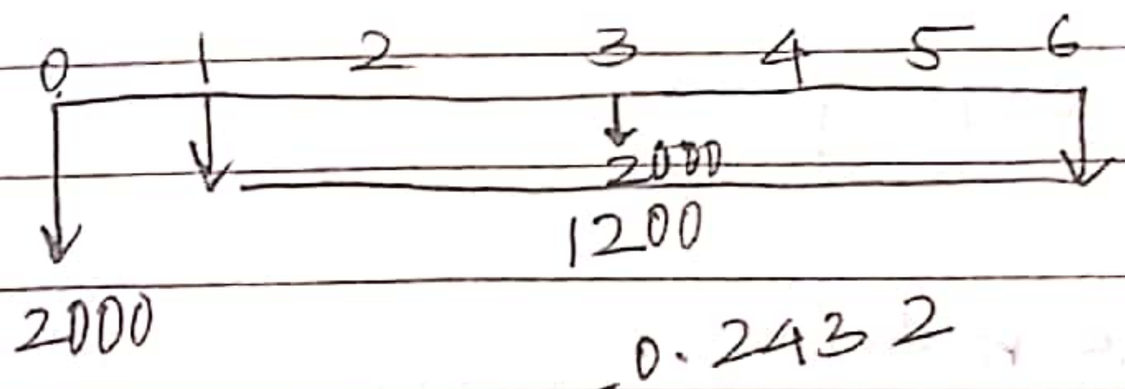
~~EAC = 3000 (A/P, 12, 6) + 1000 − 1000 (A/F, 12, 3)~~

~~− 1000 (A/F, 12, 3)~~

$$EAC = 3000(A/P, \underbrace{12}_{0.4163}, 3) + 1000 + 2000(A/P, 12, 3) - 1000(A/F, \underbrace{12}_{0.1232}, 6)$$

$$= 2958.3$$

C



$$EAC = 2000(A/P, \underbrace{12}_{0.2432}, 6) + 1200$$

$$= 1686.4$$

⇒ C, A and B

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Sinking Fund Factor

9 A firm borrows Rs 10 lakhs at 9% simple interest hoping that it can earn double the interest rate by utilizing the amount within the firm. This amount is borrowed by issuing 20 years bond, with a promise of a sinking fund to be setup. The firm will set aside a payment each year and put it in an external account, which pays 6% annually compounded interest. Find the annual payments into the sinking fund and total debt repayment.

$A = 10,00,000 (A/F, 6, 20)$

sinking fund factor

$$(A/F, 6, 20) = \frac{i}{[(1+i)^N] - 1}$$

$$= \frac{6}{(1+6)^{20} - 1}$$

$$= 0.02718$$

$A = 10,00,000 (0.02718)$

$= 27,184$ //

Total annual debt repayment is

$= 27,184 + 10,00,000 (0.09)$

$= 1,17,184.56$ //

Rate of Return

It is the loss or gain of an investment over a certain period, expressed as a percentage of the initial cost of the investment.

A positive ROR means the position has made a profit, while a negative ROR means a loss. There will be a rate of return on any investment you make.

A rate of return can give traders and investors key information for future trades/investments

Internal Rate of Return

It is a metric used in financial analysis to estimate the profitability of potential investments.

Minimum Acceptable Rate of Return

MARR/Hurdle Rate is the interest rate chosen such that under local circumstances and assumptions, a project having positive worth is attractive.

It is the minimum profit an investor expects to make from an investment, taking into account the risks of investment and the opportunity cost of undertaking it instead of other investments.

External Rate of Return

It is the rate of return on a project where any "excess" cash from a project is assumed to earn interest at a pre-determined explicit rate - usually the

Rate of Return Formula

$IRR = A + \{(C-O)/(C-D)\} * (B-A)$

A = discount factor of lower trial rate

B = discount factor of higher trial rate

C = present value of cashinflow at lower trial rate

D = present value of cashinflow at higher trial rate

O = original outlay or the initial cash outlay.

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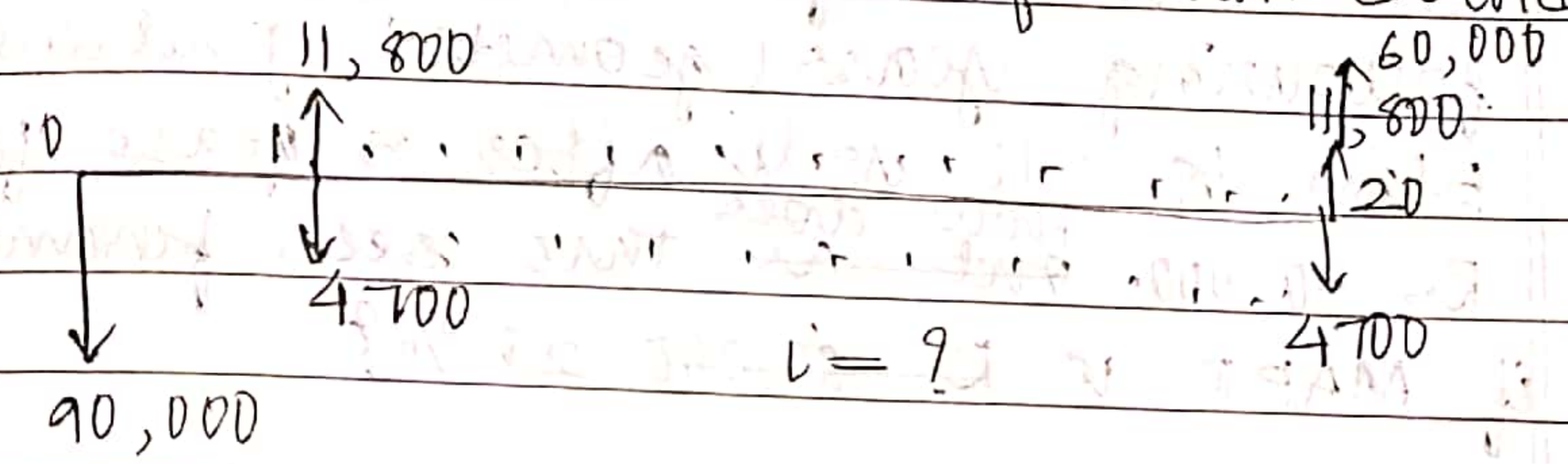
Problems on rate of return

1. A farm house can be purchased for Rs. 90,000 and expected resale value after 20 years is 60000. If the annual rental income is 11,800 and expenses of Rs. 4700, what will be the rate of return earned?



$$PW(i) = -P + A(P/A, i, n) + S(P/F, i, n)$$

~~Trial and Error method~~

$$PW(10\%) = -90,000 + 7100(P/A, 10, 20) + 60000(P/F, 10, 20)$$

8.514 0.3855

$$= \cancel{173,579.4} \; -6420.6$$

$$PW(20\%) = -90,000 + 7100(P/A, 20, 20) + 60000(P/F, 20, 20)$$

4.870 0.0261

$$= \cancel{1,26,143}$$

$$PW(30\%) = -90,000 + 7100(P/A, 30, 20) + 60000(P/F, 30, 20)$$

3.316 0.0052

$$= 1,13,855.6$$

Trial and Error Method

$$PW(10\%) = -90,000 + 7100(P/A, 10, 20) + 60000(P/F, 10, 20)$$

8.514 0.3855

$$= -6420.6$$

$$PW(8\%) = -90,000 + 7100(P/A, 8, 20) + 60000(P/F, 8, 20)$$

9.818 0.2145

$$= -7422.2$$

$$PW(5\%) = -90,000 + 7100(P/A, 5, 20) + 60000(P/F, 5, 20)$$

12.462 0.3769

$$= 21094.2$$

Interpolation Formula

$$I = A + \left[\frac{C - 0}{C - D}\right] \times (B - A)$$

$$= 8\% + \left[\frac{-7422.2 - 0}{-7422.2 - 21094.2}\right] \times (5 - 8)$$

$$\therefore I = 7\%$$

2 A person is planning a new business. The initial outlay and cash flow pattern for the new business are as listed below. The expected life of the business is five years. Find the rate of return for the new business.

Period	0	1	2	3	4	5
Cash (Rs) flow	−1,00,000	30,000	30,000	30,000	30,000	30,000

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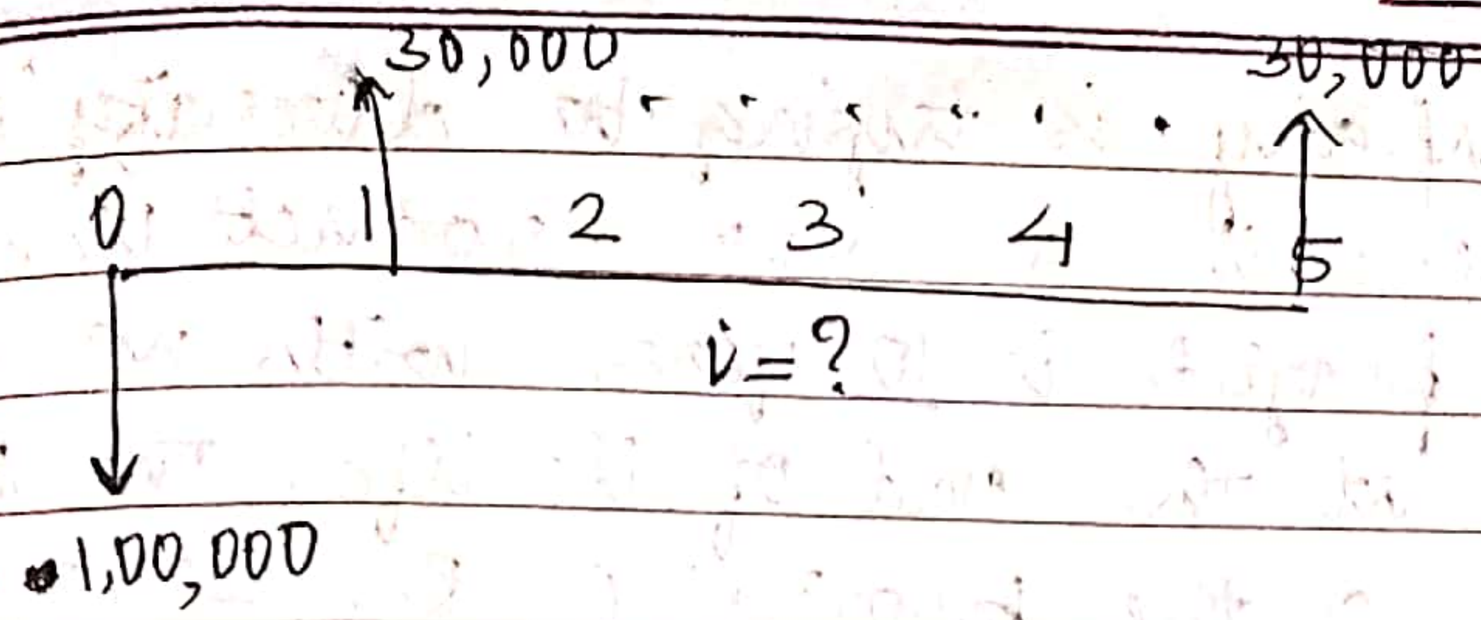


~~PW (i) = −P + A(P/A, i, n) + S(P/F, i, n)~~

~~PW (i) = −1,00,000 + 30,000(P/A, i, 5)~~

~~When i = 10%,~~

~~PW (10%) = −1,00,000 + 30,000 (P/A, 10%, 5)~~

PW(i) = −P + A(P/A, i, n) + S(P/F, i, n)

Trial and Error Method,

PW(10%) = −1,00,000 + 30,000(P/A, 10%, 5) = 13730 (3.791)

PW(15%) = −1,00,000 + 30,000 (P/A, 15%, 5) = 560 (3.352)

PW(18%) = −1,00,000 + 30,000 (P/A, 18%, 5) = −6190 (3.127)

Interpolation formula,

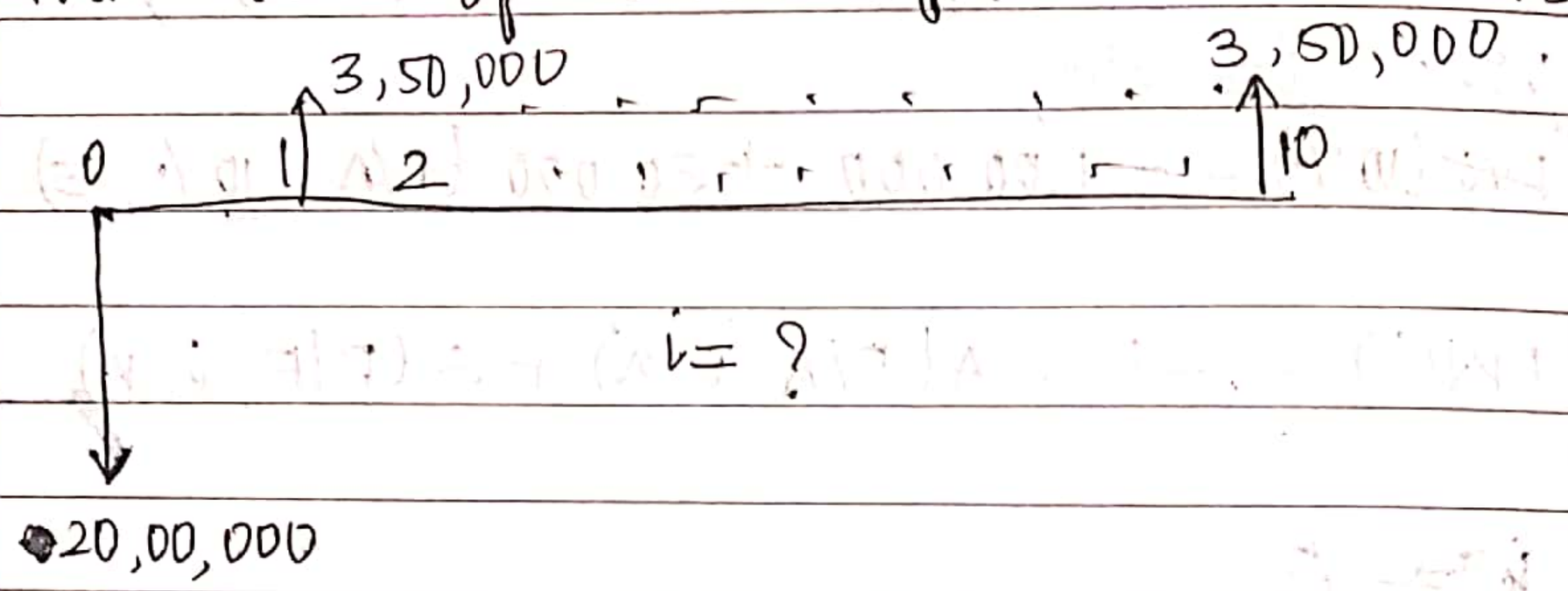
$$IRR = A + \left[\frac{C - 0}{C - D}\right] \times (B - A)$$

$$= 15\% + \left[\frac{560 - 0}{560 - (-6190)}\right] \times (18\% - 15\%)$$

$$= 15\% + 0.248\%$$

$$= 15.248\%$$

3 A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is Rs. 20,00,000. The annual net profit is Rs. 3,50,000. Find the rate of return for the new business.



$PW(i) = -P + A(P/A, i, n) + S(P/F, i, n)$

$PW(10\%) = -20,00,000 + 3,50,000(P/A, 10\%, 10) = 150750$ 6.145

$PW(15\%) = -20,00,000 + 3,50,000(P/A, 15\%, 10) = -243,350$ 5.019

$$I = A + \left[\frac{C-D}{C-D}\right] \times (B-A)$$

$$= 10\% + \left[\frac{(1,50,750-0)}{(1,50,750-(-2,43,350))}\right] \times (15-10)\%$$

$$= 10\% + 1.9125\%$$

$$= 11.91\%$$

4 A firm has identified three mutually exclusive investment proposals whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%. Find the best alternative based on the rate of return.

Imp

less than this should not be selected

	A1	A2	A3
Investment	1,50,000	2,10,000	2,55,000
Annual Net	45,570	58,280	69,000

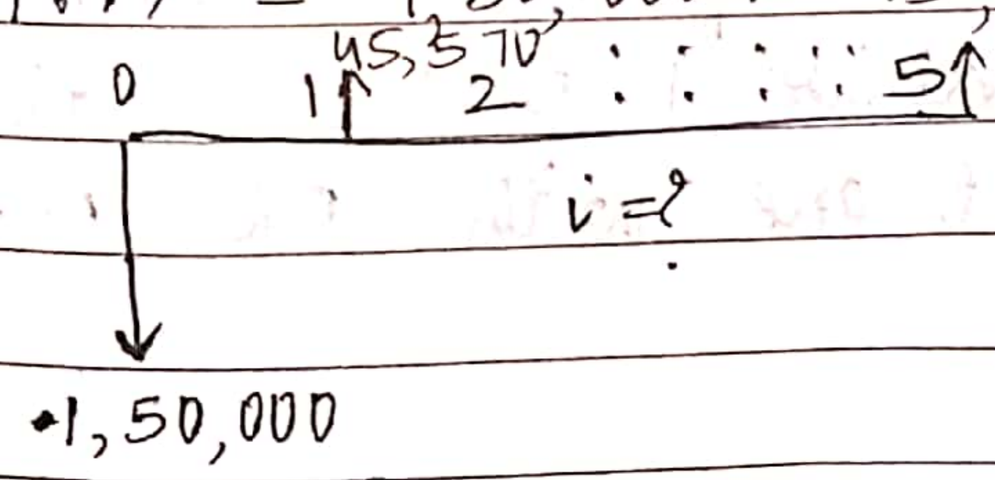
$PW(i) = -P + A(P/A, i, n) + S(P/F, i, n)$

Alternative 1

$PW(10\%) = -1,50,000 + 45,570(P/A, 10\%, 5) = 22755.87$ (3.791)

$PW(15\%) = -1,50,000 + 45,570(P/A, 15\%, 5) = 2750.64$ (3.352)

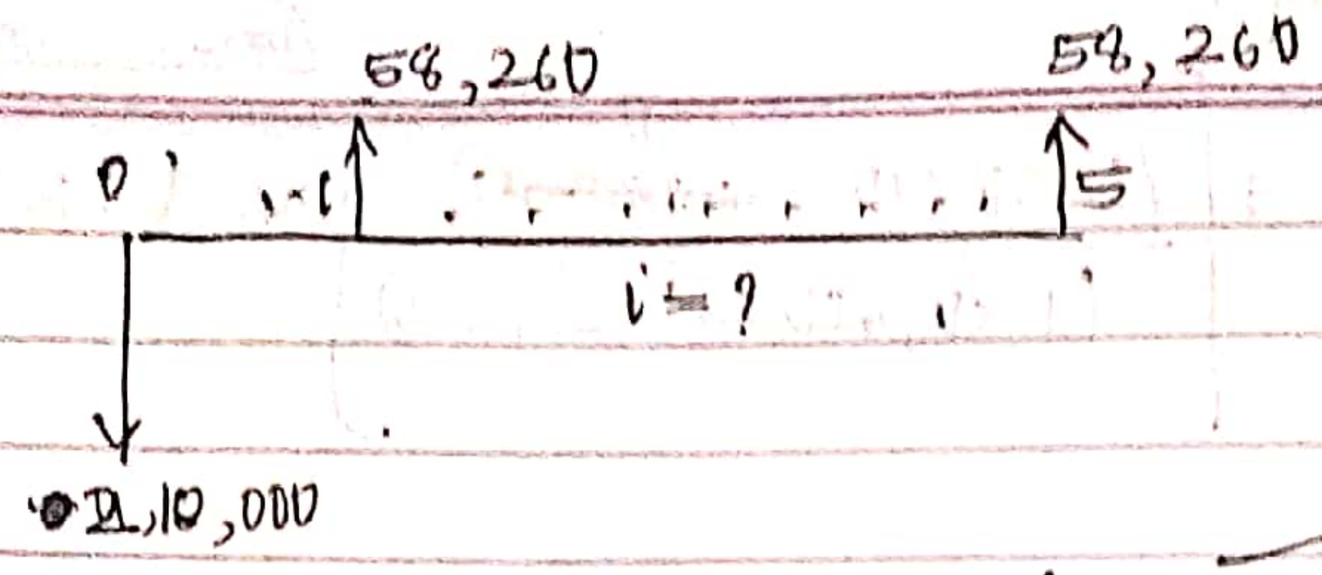
$PW(18\%) = -1,50,000 + 45,570(P/A, 18\%, 5) = -7502.61$ (3.127)



$$I = 15 + \left[\frac{2750.64 - 0}{2750.64 - (-7502.61)}\right] \times (18 - 15)$$

$= 15.80\%$

Alternative 2



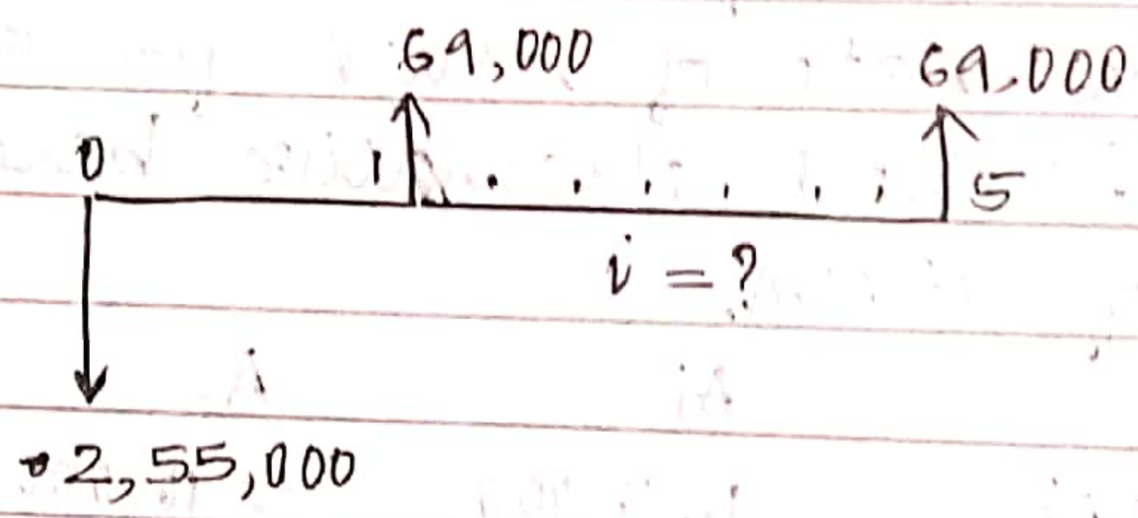
$PW(10\%) = -2,10,000 + 58,260\,(P/A, 10\%, 5) = 10,863.66$ (3.791)

$PW(15\%) = -2,10,000 + 58,260\,(P/A, 15\%, 5) = -14,712.48$ (3.352)

$$I = 10 + \left[\frac{10,863.66 - 0}{10,863.66 - (-14,712.48)}\right] \times (15 - 10)$$

$= 12.12\%$

Alternative 3



$PW(10\%) = -2,55,000 + 69,000\,(P/A, 10\%, 5) = 6579$ (3.791)

$PW(15\%) = -2,55,000 + 69,000\,(P/A, 15\%, 5) = -23712$ (3.352)

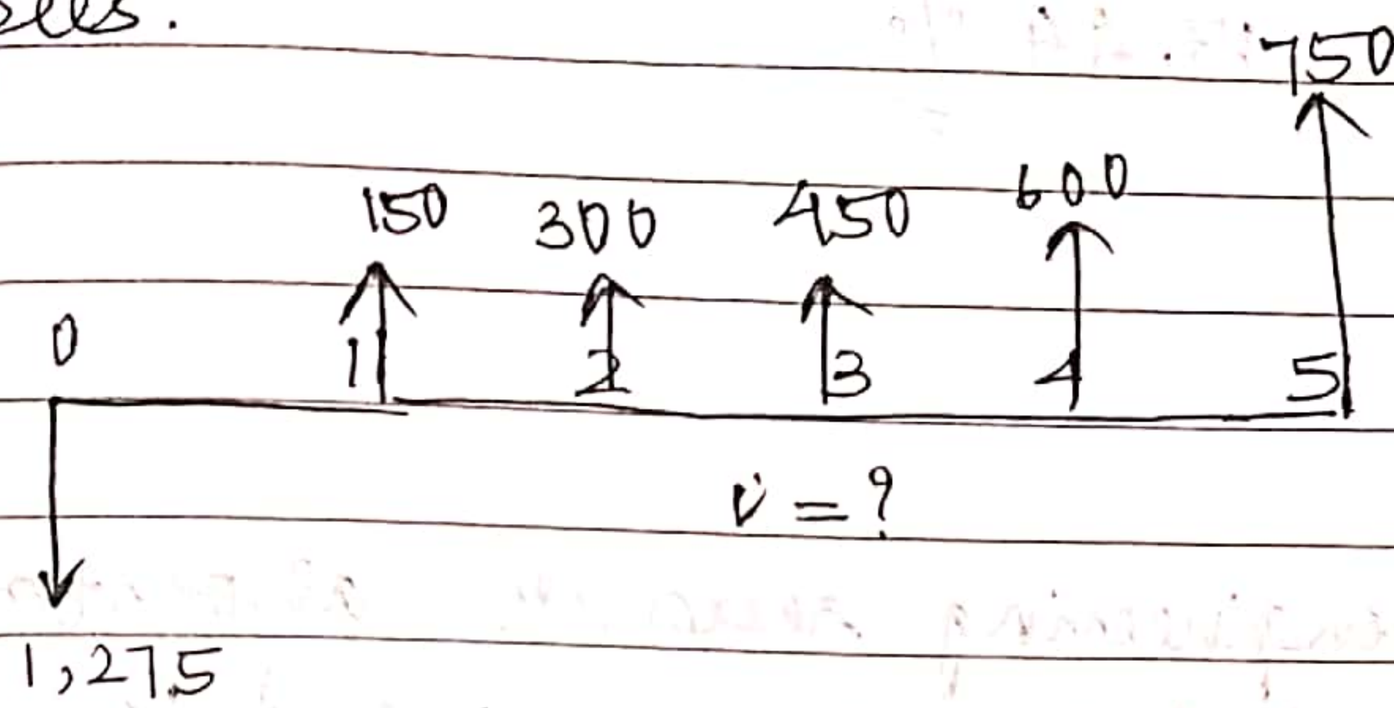
$$I = 10 + \left[\frac{6579 - 0}{6579 - (-23712)}\right] \times (15 - 10)$$

$= 11\%$

∴ Alternative 3 is rejected ∵ MARR = 12%
Out of alternative 1 and 2, alternative 1 is the best one with IRR = 15.8%

$PW(10\%) = -1275 + 150(P/A, 10\%, 5) + 150(P/G, 10\%, 5)$
(Another method)

5 For the cash flow diagram shown, compute the rate of return. The amounts are in rupees.



For the positive cash flows,
$A1 = Rs.150$, $G = Rs.150$

The annual equivalent of the positive cash flows of the uniform gradient series is given by $A = A1 + G(A/G, i, n)$

$A = 150 + 150(A/G, i, 5)$

The formula for the present worth is
$-1,275 + [150 + 150(A/G, i, 5)](P/A, i, 5)$

$i = 5\%$ $-1275 + [150 + 150(1.902)](4.329) = 609.4137$

$i = 10\%$ $-1275 + [150 + 150(4.099)](7.722) = 4631.1717$

$i = 2\%$ $-1275 + [150 + 150(0.488)](1.859) = -860.0712$

$I = A +$

$PW(10\%) = -1275 + [150 + 150(A/G, 10\%, 5)](P/A, 10\%, 5)$ (1.810, 3.791)
$= 322.9065$

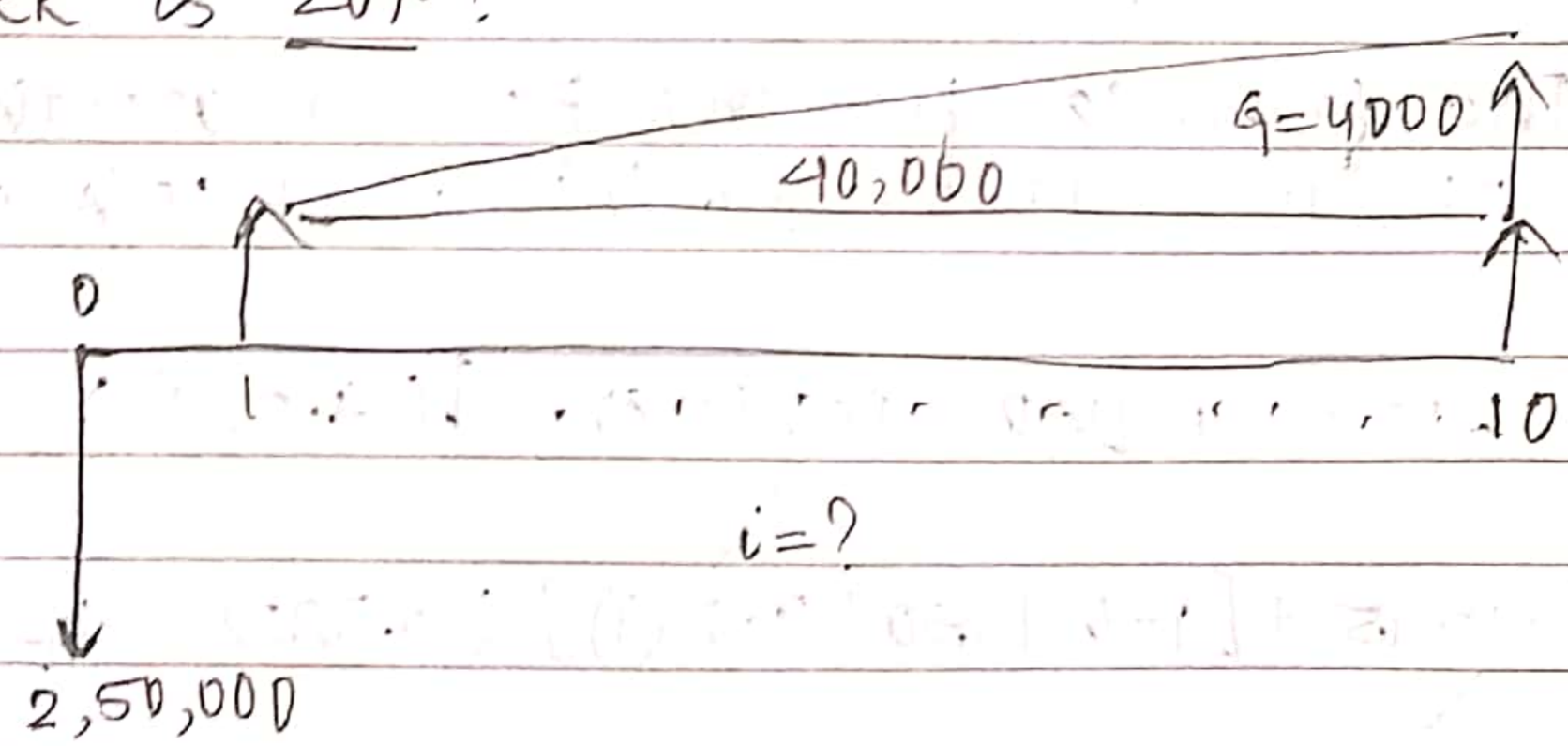
$PW(15\%) = -1275 + [150 + 150(A/G, 15\%, 5)](P/A, 15\%, 5)$ (1.723, 3.352)
$= 94.1244$

$PW(18\%) = -1275 + [150 + 150(A/G, 18\%, 5)](P/A, 18\%, 5)$ (1.673, 3.127)
$= -21.2293$

$$I = 15 + \left[\frac{94.1244 - 0}{94.1244 - (-21.2293)}\right] \times (18-15)$$

$$= 17.44\ \%$$

6 A bio engineering research laboratory has a patent that is considered for 10 years at Rs. 40,000 for the first year with increments of Rs. 4000 per year (arithmetic gradient) for the following 9 years. The accounting office says the company has Rs. 2,50,000 in development investment costs that are considered to be first cost. Is the lease reasonable if the laboratory's MARR is 20%?



$A_1 = 40,000$

$G = 4000$

$A_t = A_1 + G(A/G, i, n)$

$= 40,000 + 4000(A/G, i, 10)$

$PW(i) = -2,50,000 + [40,000 + 4000(A/G, i, 10)](P/A, i, 10)$

$PW(10\%) = -2,50,000 + [40,000 + 4000(A/G, 10\%, 10)](P/A, 10\%, 10)$

$= 87360.5$

3.725 6.145

$PW(15\%) = -250000 + [40,000 + 4000(A/G, 15\%, 10)](P/A, 15\%, 10)$ (3.383, 5.019)
$= 18677.108$

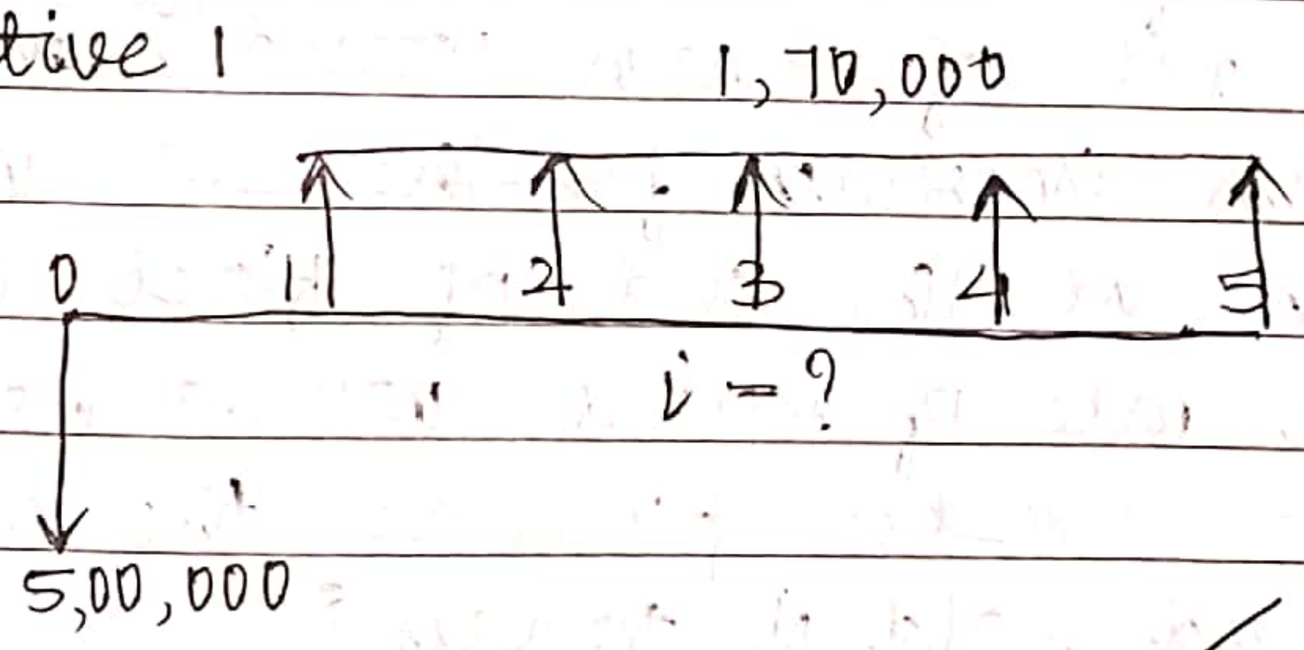
$PW(18\%) = -250000 + [40000 + 4000(A/G, 18\%, 10)](P/A, 18\%, 10)$ (3.194, 4.494)
$= -12824.656$

$I = 15 + \left[\frac{18677.108 - 0}{18677.108 - (-12824.656)}\right] \times (18 - 15)$
$= 16.77\%$ (Not lease reasonable ∵ MARR=20%)

7 A company is planning to expand its present business activity. It has two alternatives for the expansion program. The corresponding alternatives are given. Each alternative has a life of 5 years and a negligible salvage value. The MARR is 12%. Suggest the best alternative to the company.

Rs →	Initial Investment	Yearly Revenue
Alternative 1	5,00,000	1,70,000
Alternative 2	8,00,000	2,70,000

Alternative 1



$PW(10\%) = -5,00,000 + 1,70,000(P/A, 10\%, 5) = 144470$ (3.791)

$PW(15\%) = -5,00,000 + 1,70,000(P/A, 15\%, 5) = 69840$ (3.352)

$PW(18\%) = -5,00,000 + 1,70,000(P/A, 18\%, 5) = 31590$ (3.127)

$PW(20\%) = -5,00,000 + 1,70,000(P/A, 20\%, 5) = 8470$ (2.991)

$PW(25\%) = -5,00,000 + 1,70,000(P/A, 25\%, 5) = -42870$ (2.689)

$I = 20 + \left[\frac{8470 - 0}{8470 - (-42870)}\right] \times (25 - 20) = 20.82\%$

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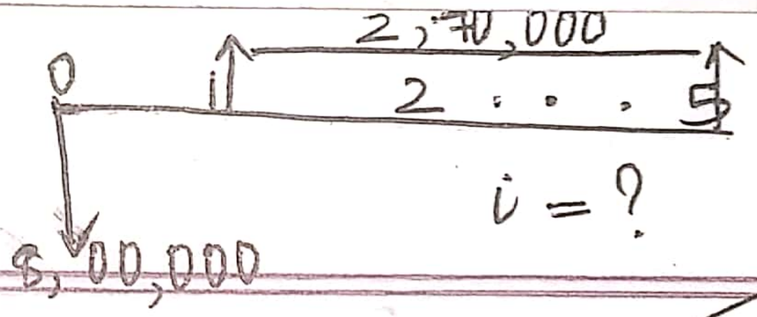
Alternative – 2



$PW(10\%) = -8,00,000 + 2,70,000\,(P/A, 10\%, 5) = 2,23,570$ (3.791)

$PW(20\%) = -8,00,000 + 2,70,000\,(P/A, 20\%, 5) = 7570$ (2.991)

$PW(25\%) = -8,00,000 + 2,70,000\,(P/A, 25\%, 5) = -73970$ (2.689)

$$I = 20 + \left[\frac{7570 - 0}{(7570 - (-73970))}\right] \times (25 - 20)$$

$$= 20.46\%$$

$\Rightarrow$ A1 is selected

Depreciation

$A = (P - F) * (A/F, 12\%, 8)$

Problems on depreciation

1. A CNC machine costing Rs. 22,00,000 is estimated to serve for 5 years after which its salvage value is estimated to be Rs. 2,00,000. Find
   (a) depreciation during third year by fixed percentage method
   (b) book value of the machine after 2 years by sum-of-years-digits method

2. A CNC machine costs Rs. 30,00,000 and is estimated to serve for 8 years after which its salvage value is estimated to be Rs. 2,50,000. Find
   (a) depreciation fund at the end of fifth year by fixed percentage method and declining balance method.
   (b) book value of the machine after 4th year and 6th year by declining balance method.

$K = 1 - (S/P)^{\wedge(1/N)}$

a fixed percentage

3. The original assets of a company are Rs. 5,80,000. The life of a plant is 9 years. If the scrap value at the time is expected to be Rs. 80,000. Calculate the depreciation at the end of each year by sum-of-years-digits method.

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4 An asset cost was Rs. 400 when purchased 4 years ago. A scrap value of Rs. 50 was expected at the end of 7 year useful life. Determine the depreciation charge during the coming year and the assets current book value by

(a) straight line method of depreciation
(b) declining balance depreciation

5 An asset has a tax line of 10 years. The asset was purchased 6 years ago ~~when~~ and the owner wants to sell it for now whose book value based on declining balance method is Rs. 20,000. If the asset was purchased for Rs. 1,80,000 what is the minimum price that the owner should accept? What is the salvage value of the asset?

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solutions

1 $P = 22,00,000$

$n = 5$

F or $S = 2,00,000$

(a) Fixed Percentage Method or Straight Line Method

$D_t = (P-F)/n$ $\qquad t = 3$

$= (22,00,000 - 2,00,000)/5$

$= 4,00,000$

$D_3 = 4,00,000$

D_t is same every year in this method of depreciation.

(b) Sum-of-years-digits method $\qquad t = 2$

$$B_t = (P-F)\frac{(n-t)}{n}\frac{(n-t+1)}{(n+1)} + F$$

$$= (2200000 - 200000) \times \frac{(5-2)}{5} \times \frac{(5-2+1)}{(5+1)} + 200000$$

$= 10,00,000$

2 $P = 30,00,000$

$n = 8$

F or $S = 2,50,000$

(a) Fixed Percentage Method $\qquad t = 5$

$D_t = (P-F)/n$

$= (30,00,000 - 2,50,000)/8 = 3,43,750$

$D_5 = 3,43,750$

D_t is same every year in this method of depreciation.

Declining Balance Method $\qquad t = 5$

$K = 1 - (S/P)^{(1/n)} = 1 - (250000/3000000)^{(1/8)}$

$= 0.26$

$D_t = K(1-K)^{t-1} \times P$

$D_5 = 0.26(1-0.26)^{5-1} \times 30,00,000$

$= 2,33,895.29$

(b) $t = 4$ and $t = 6$ by Declining Balance Method

$D_4 = 0.26(1-0.26)^{4-1} \times 30,00,000 = 3,16,074.72$

$D_6 = 0.26(1-0.26)^{6-1} \times 30,00,000 = 1,73,082.51$

3 $P = 5,80,000$

$n = 9$ $\qquad P - F = 5,00,000$

F or $S = 80,000$

Sum-of-years-digits Method

End of year (t)	D_t
0	—
1	$9/45 \times 500000 = 100000$
2	$8/45 \times 500000 = 88888.88$
3	$7/45 \times 500000 = 77777.77$
4	$6/45 \times 500000 = 66666.66$
5	$5/45 \times 500000 = 55555.55$
6	$4/45 \times 500000 = 44444.44$
7	$3/45 \times 500000 = 33333.33$
8	$2/45 \times 500000 = 22222.22$
9	$1/45 \times 500000 = 11111.11$

$D_t = \dfrac{n-t+1}{n(n+1)/2} \times (P-F)$

$D_t = \text{Rate} \times (P-F)$

$\text{Sum} = n(n+1)/2 = 9(9+1)/2 = 45$

The rates for years 1 to 9 are
9/45, 8/45, 7/45, 6/45, 5/45, 4/45, 3/45, 2/45, 1/45
respectively

~~4 P = 400~~
~~n = 7~~
~~S = 50~~

4 $P = 400$
$n = 7$
F $S = 50$
$t = 5$

(a) Straight Line Method
$D_t = (P - F)/n = (400 - 50)/7 = 50$
$D_5 = 50$ as D_t is same every year in this method

(b) Declining Balance Method
$K = 1 - (S/P)^{(1/n)}$
$= 1 - (50/400)^{(1/7)} = 0.25$
$D_t = K(1-K)^{t-1} \times P$
$D_5 = 0.25(1-0.25)^{5-1} \times 400 = 31.64$

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