UNIT – 7

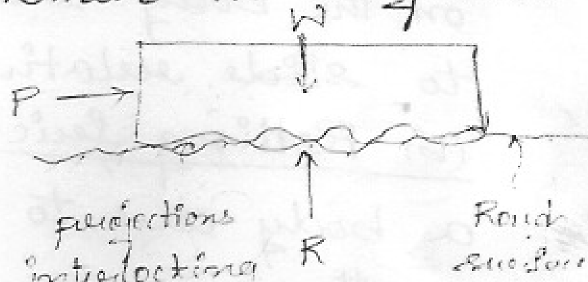
[redacted]

FRICTION

⇒ Introduction

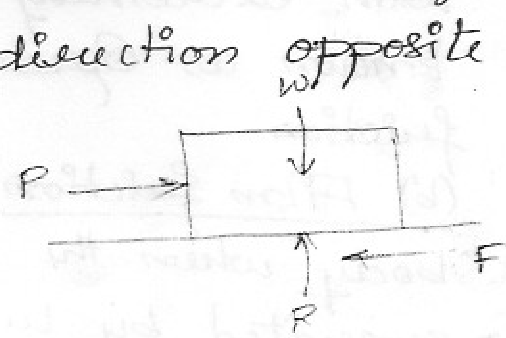
It was assumed that the surfaces in contact were smooth & @ these points of contact there will be only a normal (⊥er) reaction acting to the surface.

In practice, every surface have minute projecting particles. When the surfaces are in contact, then the projections interlock each other @ the surfaces in contact. When such surfaces tend to move, the interlocking of projections resists the movement in tangential direction. More the surface is rough, more is the resistance.



⇒ Friction

When one body tends to move in contact over other body, a resistance to movement is set-up. This resistance to movement is called Friction (or) Force of friction (or) Frictional force. This force always act in the direction opposite to the motion trend.



Frictional force is due to:

a) interlocking of irregularities

b) molecular forces of attraction to a small extent

→ Types of friction

1) Static friction → The friction acting on a body which is @ rest

2) Limiting friction → The max friction developed just before the movement of an object.

3) Dynamical friction → The friction acting on a body which is actually in motion. Also known as Kinetic friction.

4) Dry friction → The friction acting on a body when the contact surfaces are dry (or) unlubricated & there is a tendency of relative motion. Also known as Coulomb friction.

(a) Solid / Sliding friction → The friction acting on the body when two surfaces have tendency to slide relative to eachother.

(b) Rolling friction → The friction acting on a body due to rolling of one surface over another.

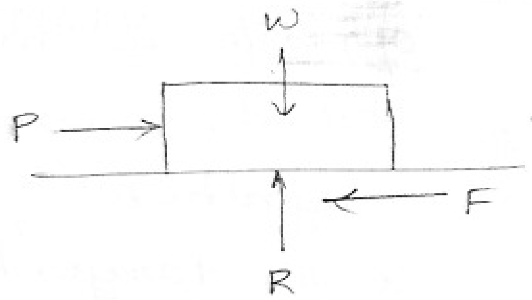
5) Fluid friction → The friction acting on a body when the contact surfaces are lubricated.

(a) Skin friction → The friction acting on a body when the contact surfaces are lubricated with extremely thin layer of lubricant. Also known as Greasy (or) Non-viscous (or) Boundary friction.

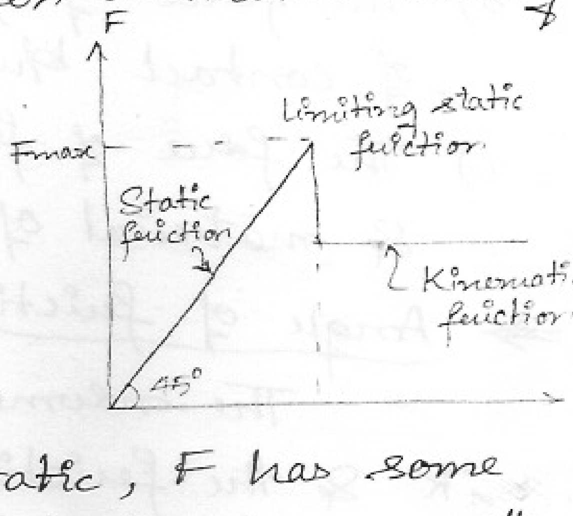
(b) Film friction → The friction acting on the body when the contact surfaces are completely seperated by lubricant. Also known as Viscous

⇒ Motion trend of a block

- Consider a block of weight 'W' on a rough horizontal surface & subjected to a horizontal force 'P' as shown in the figure.



- When force 'P' is small, the block does not move. This is ∵ of the frictional force which balances 'F'
- If force 'P' is increased, F increases & remains equal to 'P' as long as object is static.
- With further increase in 'P', beyond certain value (F_{max}), the object starts moving. The frictional force decreases & then remains nearly constant.
- The variation in magnitude of frictional force with the applied forces P is shown in the graph.

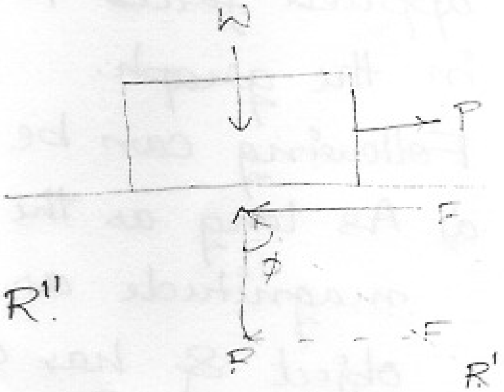


- Following can be concluded :-
  a) As long as the object is static, F has same magnitude as the net force trying to move the object & has opposite direction.
  b) Motion impends when net force trying to move the object becomes equal to the F_{max}
  c) When object starts moving, the frictional force is constant, independent of net applied force. [i.e, $\frac{F}{R} = \mu$ (constant)]

→ Laws of dry friction (Coulomb's laws of dry friction)

1) The frictional force always acts in a direction opposite to movement of the body. [$\overleftarrow{F}$ to $\overrightarrow{P}$]
2) Till the limiting value is reached, the magnitude of frictional force is exactly equal to the tangential force which tends to move the body. [F = tangential force upto lim value]
3) The magnitude of limiting friction bears a constant ratio to the normal reaction b/w two contacting surfaces. [$\mu = \frac{F}{R}$]
4) The force of friction depends upon the roughness of the surfaces. [F ∝ roughness]
5) The force of friction is independent of the area of contact b/w the two surfaces. [F indi of A]
6) The force of friction depends upon the nature & material of the two contact surfaces. [F ∝ mat

→ Angle of friction



The normal reaction 'R' & the frictional force 'F' can be combined to give a resultant reaction 'R''. The angle made by this resultant 'R'' with normal reaction 'R' is called the angle of friction 'ϕ'. ∴ $\tan\phi = F/R$

→ Angle of limiting friction

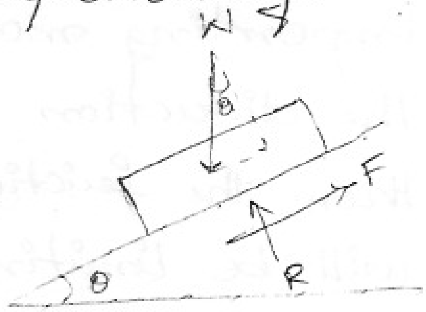
As frictional force ↑ses, the angle ϕ also ↑ses & it can reach maximum value α (i.e, when limiting value of friction is reached).

Thus when motion is impending

$$\tan \alpha = \frac{F_{max}}{R}$$

This value of α is called angle of limiting friction. Hence the angle of limiting friction can be defined as the angle b/w the resultant reaction & the normal to the plane on whic the motion of the body is impending.

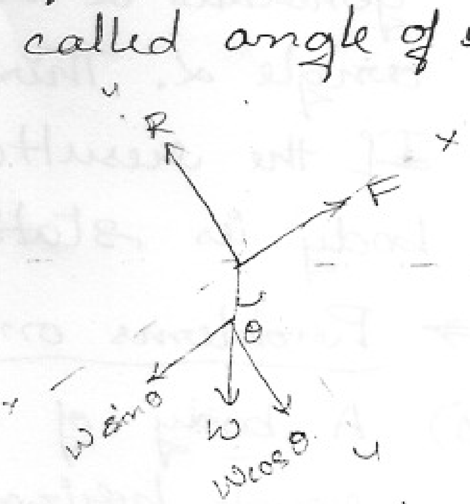
→ Angle of repose



The limiting value upto which the body repose (sleep) is called angle of repose.

- Consider the block of weight "W" resting on an inclined plane that makes an angle "θ" w.r. horizontal. When θ is small, the block rests on inclined plane. If θ is ↑sed, the block will start sliding. Thus, the max inclinat of the plane on which the body, free from external forces, can repose is called angle of re

- By resolving the forces



$\Sigma V = 0$

$R - W\cos\theta = 0$

$R = W\cos\theta \longrightarrow (1)$

$\Sigma H = 0$

$F - W\sin\theta = 0$

$F = W\sin\theta \longrightarrow (2)$

- By ($\div$)ing (1) & (2)

$$\frac{R}{F} = \frac{W\cos\theta}{W\sin\theta}$$

$\therefore \frac{F}{R} = \tan\theta$

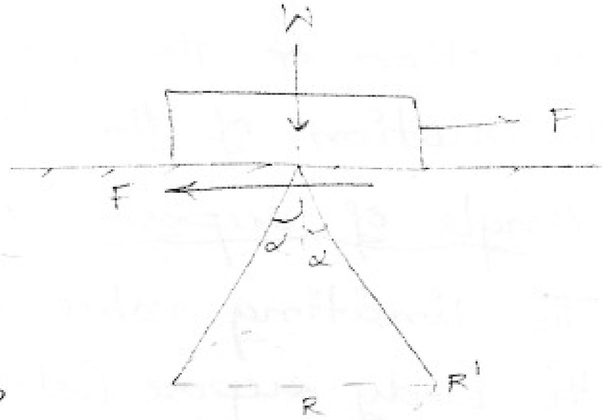
happen or occur
∗ repose = rest ; sleep.

- Since θ value will be max,

$\theta = \alpha$

Thus, the value of angle of repose is same as the value of limiting angle of friction.

⟹ Cone of friction

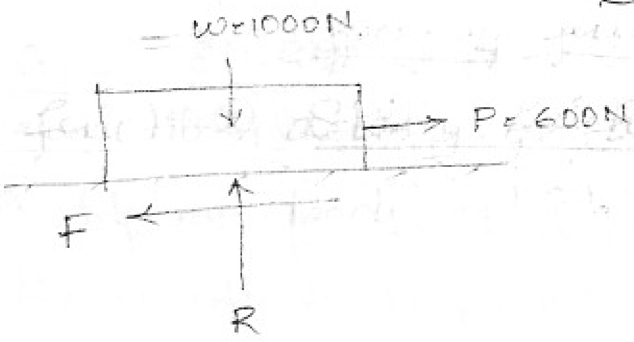


- When a body is having impending motion in the direction of P, then the frictional force will be limiting friction & the resultant reaction R' will make limiting frictional angle α w.r.t R
- If body is having impending motion in any other direction, R' makes an angle α w.r.t R
- Thus if direction of force P is gradually changed through 360°, the resultant reaction i generates a right circular cone with semi-central angle α. This is called cone of friction.
- If the resultant R' is within the cone, the body is stationary.

⟹ Problems on blocks on horizontal plane

1) A body of weight 1000N is placed on a rough horizontal plane. Determine co-efficient of friction due to a force of 600N in horizontal direction just causes the body to slide.

(7)



⇒ $W = 1000N$

$P = 600N$

- w.k.t $\mu = \frac{F}{R}$

- Resolving the forces

$\Sigma V = 0$

$R - 1000 = 0$

$\underline{R = 1000N} \longrightarrow (1)$

$\Sigma H = 0$

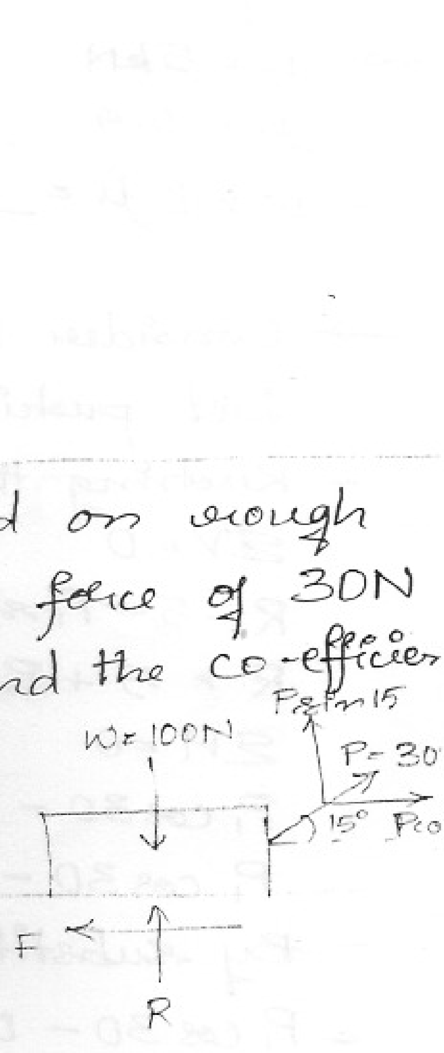
$600 - F = 0$

$\underline{F = 600N} \longrightarrow (2)$

- From (1) & (2)

$\mu = \frac{600}{1000} = \underline{0.6}$

2) A body weighing 100N is placed on rough horizontal plane is pulled by a force of 30N inclined @ 15° with horizontal. Find the co-efficient of friction.



⇒ $W = 100N$

$P = 30N$ @ $15°$

- w.k.t $\mu = \frac{F}{R}$

- Resolving the forces

$\Sigma V = 0$

$30 \cos 15 - F = 0$

$F = 30 \cos 15 = \underline{28.97N} \longrightarrow (1)$

$\Sigma H = 0$

$R - 100 + 30 \sin 15 = 0$

$R = \underline{92.24N} \longrightarrow (2)$

- From ① & ②

$$\mu = \frac{28.98}{92.24} = \underline{\underline{0.314}}$$

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3) A block of weight 5kN rests on a rough horizont surface & the co-efficient of friction b/w them is 0.4. Show that the magnitude of force reqd to pull is less than the magnitude of push, if the angle made by both the forces (pull & push) is 30

→ $W = 5kN$

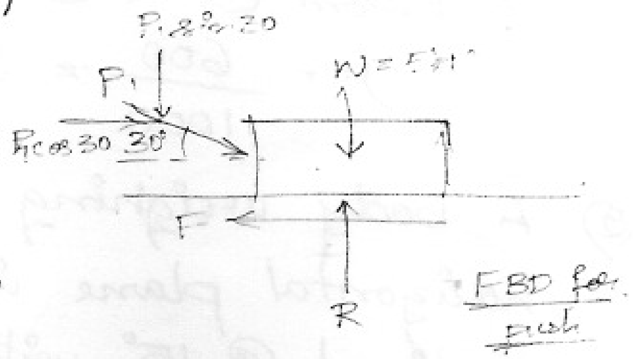
$\mu = 0.4$

- w.k.t $\mu = \frac{F}{R}$ (or) $F = \mu R$

→ Consider FBD of body for push

Let pushing force be P_1

- Resolving the forces



$\Sigma V = 0$

$R - 5 - P_1 \sin 30$

$R = 5 + P_1 \sin 30 \longrightarrow$ ①

$\Sigma H = 0$

$P_1 \cos 30 - F = 0$

$P_1 \cos 30 - 0.4 R = 0 \longrightarrow$ ②

- By substituting ① & ②

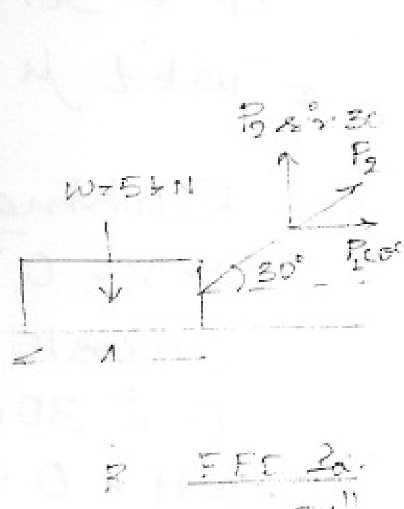
$P_1 \cos 30 - 0.4(5 + P_1 \sin 30) = 0$

$P_1 \cos 30 - 2 \cancel{+} P_1 \sin 30 \times 0.4 = 0$

$0.86 P_1 - 2 \cancel{+} 0.2 P_1 = 0$

$\overset{0.66}{\cancel{1.06}} P_1 - 2 = 0$

$\underline{\underline{P_1 = 3.1 kN}}$



→ Consider FBD of body for pull

Let pulling force be P_2

- Resolving the forces

$\Sigma V = 0$

$R - 5 + P_2 \sin 30 = 0$

$R = 5 - P_2 \sin 30 \longrightarrow (3)$

$\Sigma H = 0$

$P_2 \cos 30 - F = 0$

$P_2 \cos 30 - 0.4R = 0 \longrightarrow (4)$

- By substituting (3) & (4)

$P_2 \cos 30 - 0.4(5 - P_2 \sin 30) = 0$

$P_2 \cos 30 - 2 + 0.4 P_2 \sin 30 = 0$

$0.86 P_2 - 2 + 0.2 P_2 = 0$

$1.06 P_2 - 2 = 0$

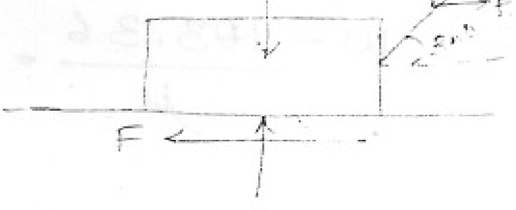
$\underline{P_2 = 1.88 kN}$

- Here $P_1 > P_2$; Hence the magnitude of pull is less than magnitude of push. ∴ pull is prefere than push.

4) A body resting on a horizontal plane reqd. a pull of 100N included @ 30° to horizontal just to move it. It was also found that a push of 110N included @ 20° to the plane just moved the body. Determine the weight of the body & co-efficient of friction.

→ Consider FBD of body for pull

Let W & μ be the weight of the body & co-efficient of friction respt.



- Resolving the forces

$\Sigma V = 0$

$R - W + P\sin 30 = 0$

$R - W + 100\sin 30 = 0 \longrightarrow (1)$

$\Sigma H = 0$

$P\cos 30 - F = 0$

$100\cos 30 - \mu R = 0$

$\therefore R = \dfrac{100\cos 30}{\mu} \longrightarrow (2)$

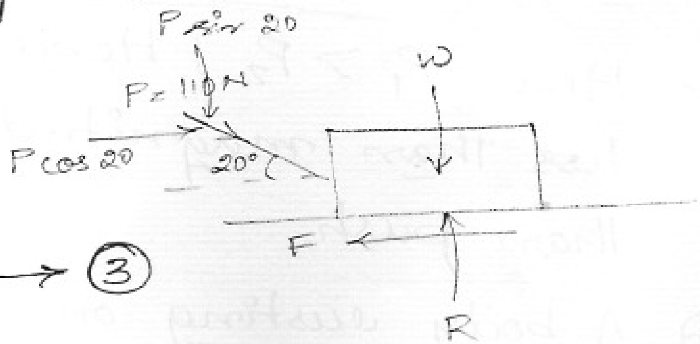
- By substituting (1) & (2)

$\dfrac{86.6}{\mu} - W + 50 = 0$

$W - \dfrac{86.6}{\mu} = 50. \longrightarrow (a)$

→ Consider FBD of a body for pull

- Resolving the forces



$\Sigma V = 0$

$R - W - P\sin 20 = 0$

$R - W - 110\sin 20 = 0 \longrightarrow (3)$

$\Sigma H = 0$

$P\cos 20 - F = 0$

$110\cos 20 - \mu R = 0$

$\therefore R = \dfrac{110\cos 20}{\mu} \longrightarrow (4)$

- By substituting (3) & (4)

$\dfrac{103.36}{\mu} - W - 37.62 = 0$

$W - \dfrac{103.36}{\mu} = -37.62 \longrightarrow (b)$

→ Solving eqns ⓐ & ⓑ

$$W - \frac{86.6}{\mu} = 50$$

$$W - \frac{103.36}{\mu} = -37.62$$

$$\frac{16.76}{\mu} = 87.62$$

$$\mu = \frac{16.76}{87.62}$$

$$\underline{\mu = 0.19} \longrightarrow ⓒ$$

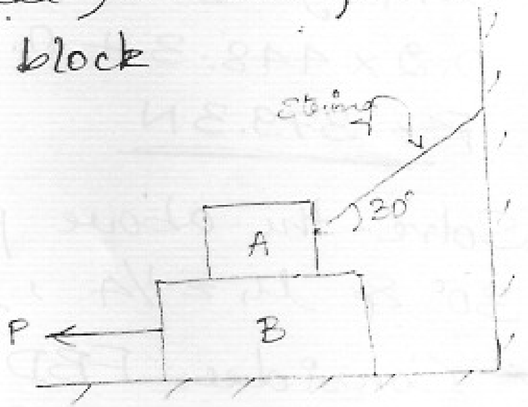
- By substituting ⓒ in ⓐ

$$W - \frac{86.6}{0.19} = 50$$

$$W = 50 + 455.78$$

$$\underline{W = 505.78 \text{ kN}}$$

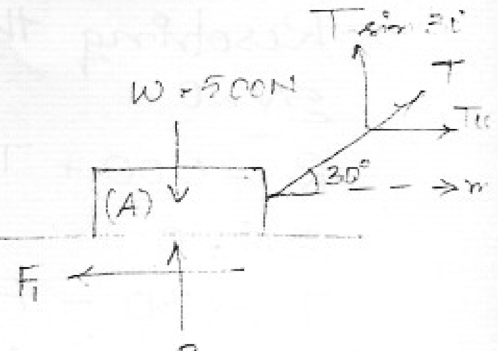
5) Find the force P just reqd. to slide the block B in the arrangement shown in sketch. Find also the tension in the string. Take $\mu = 0.2$ for all contact surfaces; Weight of block A & B = 500N & 1000N



→ Consider FBD of block A

Due to pull of Block B towards left, the Block A tends to move right

Let T be the tension in string.



- Resolving the forces

$\Sigma V = 0$

$R_1 - 500 + T\sin 30 = 0 \longrightarrow ①$

$\Sigma H = 0$

$T\cos 30 - F = 0$

$T\cos 30 - \mu R_1 = 0$

$0.86T - 0.2R_1 = 0$

$R_1 = 4.33T \longrightarrow (2)$

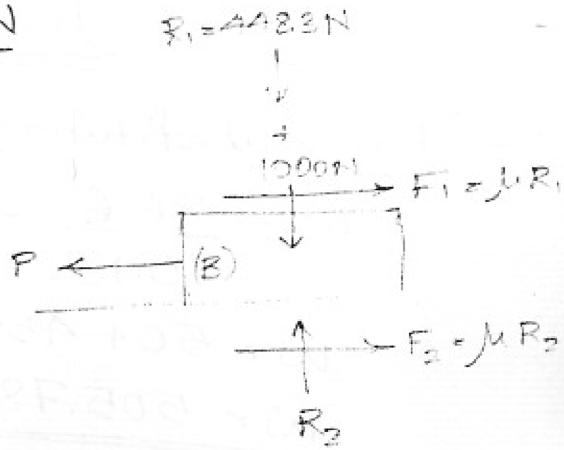
- By substituting (2) & (1)

$4.33T - 500 + T\sin 30 = 0$

$4.83T - 500 = 0$

$\underline{T = 103.5\,N}$

$\therefore R_1 = 4.33 \times 103.5 = \underline{448.3\,N}$



→ Consider FBD of block B

- Resolving the forces

$\Sigma V = 0$

$R_2 - 1000 - 448.3 = 0$

$\underline{R_2 = 1448.3}$

$\Sigma H = 0$

$F_1 + F_2 - P = 0$

$\mu R_1 + \mu R_2 - P = 0$

$0.2 \times 448.3 + 0.2 \times 1448.3 - P = 0$

$\underline{P = 379.3\,N}$

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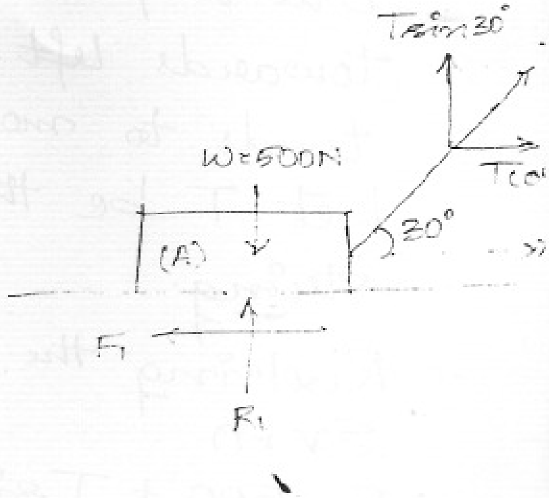
3) Solve the above problem if P acts @ an angle $30°$ & $\mu_1 = 1/4$, $\mu_2 = 1/3$



→ Consider FBD of block A

- Resolving the forces

$\Sigma V = 0$

$R_1 - 500 + T\sin 30 = 0 \longrightarrow (1)$

$\Sigma H = 0$

$T\cos 30 - F_1 = 0$

$T\cos 30 - \mu_1 R_1 = 0$

$0.86T - 1/4 R_1 = 0$

$\therefore R_1 = 3.44T \longrightarrow$ (2)

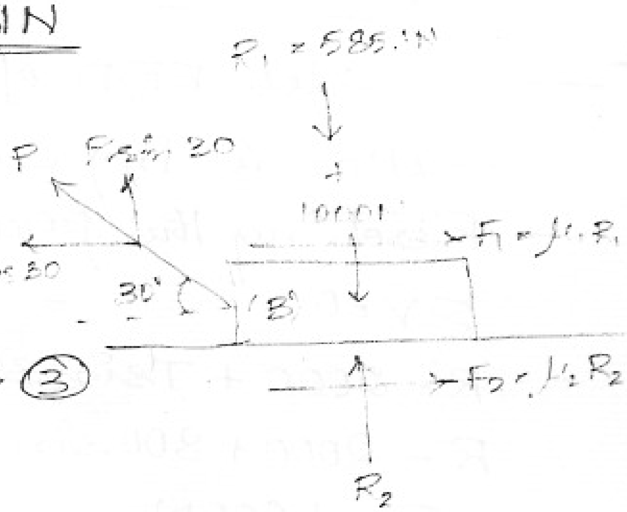
- By substituting (1) & (2)

$3.44T - 500 - T\sin 30 = 0$

$2.94T - 500 = 0$

$\underline{T = 170.1N}$

$\therefore R_1 = 170.1 \times 3.44 = \underline{585.1N}$



→ Consider FBD of block B

- Resolving the forces

$\Sigma V = 0$

$R_2 - 1000 - 585.1 + P\sin 30 = 0$

$R_2 + P\sin 30 - 1585.1 = 0 \longrightarrow$ (3)

$\Sigma H = 0$

$F_1 + F_2 - P\cos 30 = 0$

$\mu_1 R_1 + \mu_2 R_2 - P\cos 30 = 0$

$\frac{1}{4} \times 585.1 + \frac{1}{3} \times R_2 - P\cos 30 = 0$

$0.33R_2 - P\cos 30 + 146.27 = 0 \longrightarrow$ (4) $\left(\frac{\div}{\div}\right)$ by 0.33

- By substituting (3) & (4)

$R_2 + P\sin 30 - 1585.1 = 0$

$\underset{(-)}{R_2} \underset{(+)}{- P2.6} \quad \underset{(-)}{+ 443.24} = 0$

$3.1P - 2028.3 = 0$

$\underline{P = 654.3N} \longrightarrow$ (a)

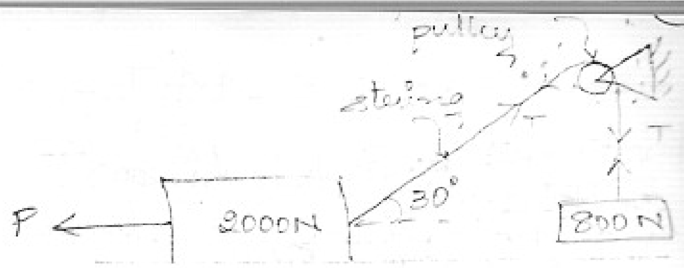
- By substituting (a) in (3)

$R_2 + 654.3\sin 30 - 1585.1 = 0$

$\underline{R_2 = 1258.1N}$

7) A block A weighing 2000N is attached to one en of chord which passes around a frictionless pulley as shown, carries a load of 800N @ other end. Find the value of P. (a) If the motion is impending towards left. (b) Impending towards

→ Let T be the tension in the string

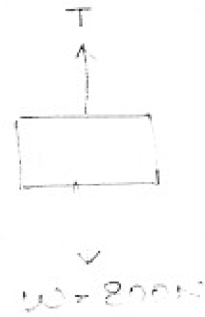


→ Consider FBD of 800N

- Resolving the forces

$T - W = 0$

$\underline{T = 800N}$



→ Consider FBD of the block when motion is impending towards left

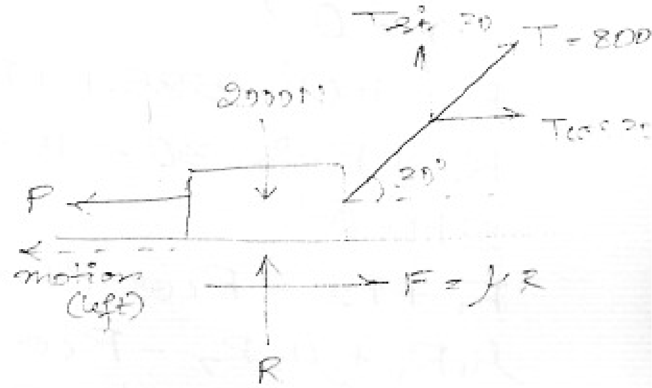
- Resolving the forces

$\Sigma V = 0$

$R - 2000 + T\sin 30 = 0$

$R - 2000 + 800\sin 30 = 0$

$\underline{R = 1600N}$



$\Sigma H = 0$

$T\cos 30 + F - P = 0$

$800\cos 30 + 0.35 \times 1600 - P = 0$

$\underline{P = 1252.8N}$

→ Consider FBD of the block when motion is impending towards right.

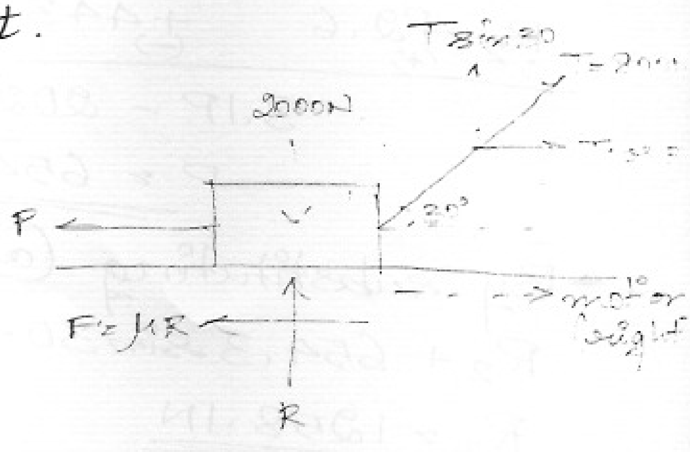
- Resolving the forces

$\Sigma V = 0$

$R - 2000 - T\sin 30 = 0$

$R - 2000 + 800\sin 30 = 0$

$\underline{R = 1600N}$



$\Sigma H = 0$

$T\cos 30 - F - P = 0$

$800\cos 30 - 0.35 \times 1600 - P = 0$

$\underline{P = 132.82N}$

Problems on blocks placed on inclined planes

1) Find the value of P so that the body will impend down the plane. Also find the value of P for the body to impend up the plane. $\mu = 0.$

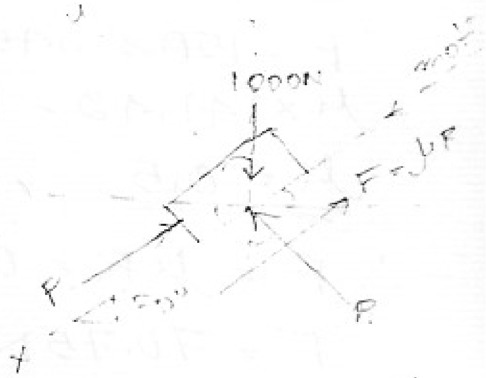
⟹ Consider FBD when motion trend is impending down.

- Resolving the forces

$\Sigma V = 0$

$R - 100 \cos 30 = 0$

$\underline{R = 86.6N}$

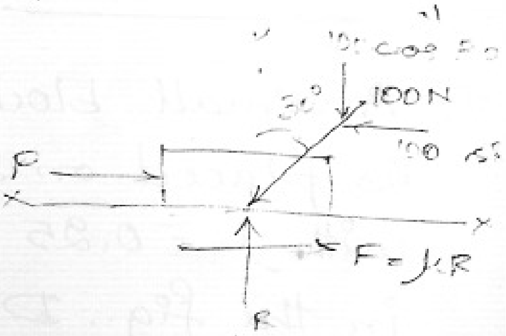


$\Sigma H = 0$

$F - 100 \sin 30 + P = 0$

$\mu R - 100 \sin 30 + P = 0$

$\underline{P = 24.2N}$



⟹ Consider FBD when motion trend is impending up

- Resolving the forces

$\Sigma V = 0$

$R - 100 \cos 30 = 0$

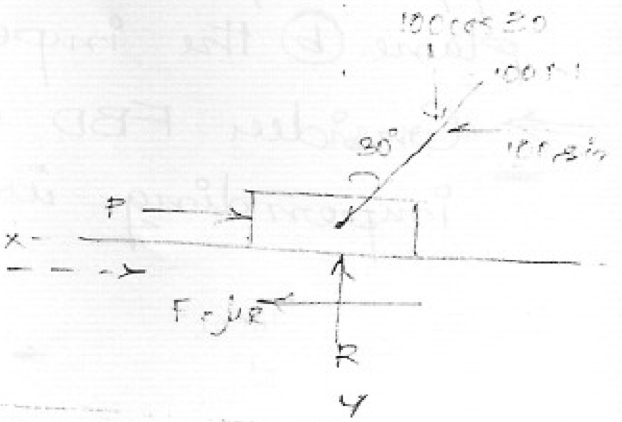
$\underline{R = 86.6N}$

$\Sigma H = 0$

$P - F - 100 \sin 30 = 0$

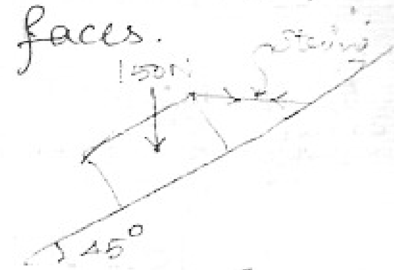
$P - \mu R - 100 \sin 30 = 0$

$\underline{P = 75.9N}$



2) A block is resting on a rough inclined plane as shown in the figure. The block is tied up by a horizontal string which has a tension of 5[illegible]

Find: ⓐ The friction force on the block & the normal reaction in the inclined plane. (b) the co-efficient of friction b/w contact faces.



$\Rightarrow$ Consider the FBD of body.

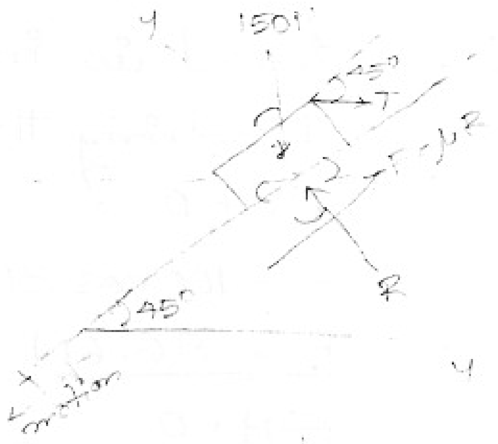
Let T be the force in string

- Resolving forces

$\Sigma V = 0$

$R - 150\cos 45 - 50\sin 45 = 0$

$\underline{R = 141.42\,N}$



$\Sigma H = 0$

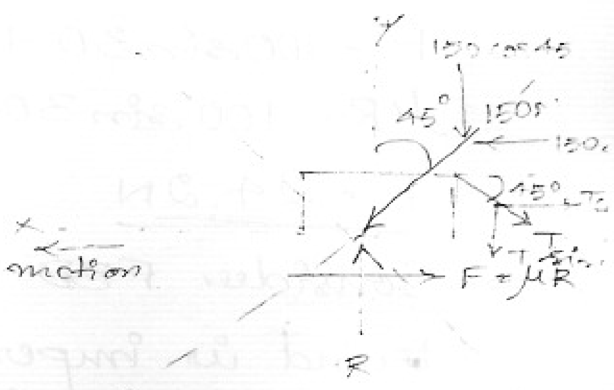
$F - 150\sin 45 + 50\cos 45 = 0$

$\mu \times 141.42 - 106.1 + 35.35 = 0$

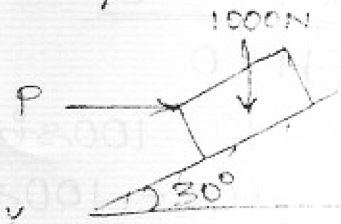
$\underline{\mu = 0.5}$

$\therefore F = \mu R = 0.5 \times 141.42$

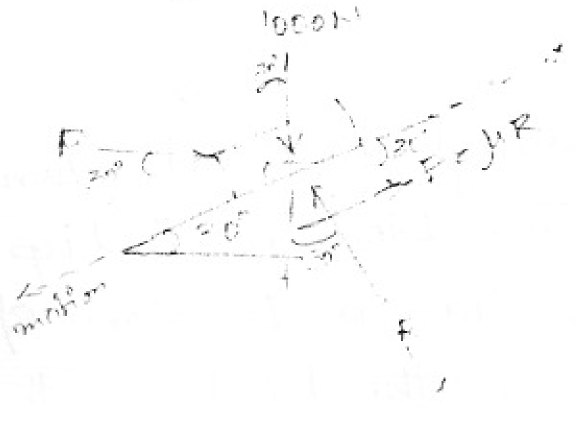
$\underline{F = 70.75\,N}$

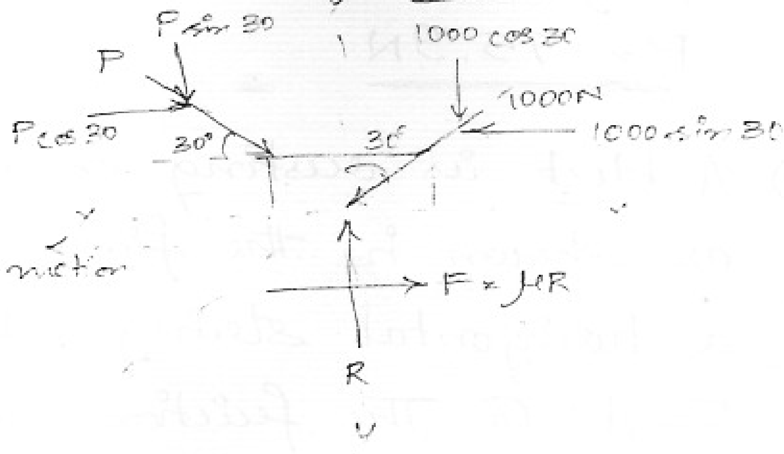


3) A small block of 1000N is placed on 30° inclined with $\mu = 0.25$ as shown in the fig. Determine the horizontal force to be applied if: (a) the impending motion down the plane. (b) the impending motion up the plane.



$\Rightarrow$ Consider FBD when motion impending is down.





- Resolving the forces

$\Sigma V = 0$

$R - 1000 \cos 30 - P \sin 30 = 0$

$R = P \sin 30 + 1000 \cos 30$

$R = P \sin 30 + 866 \longrightarrow (1)$

$\Sigma H = 0$

$F + P\cos 30 - 1000 \sin 30 = 0$

$0.25R + P\cos 30 - 500 = 0 \longrightarrow (2)$

- By substituting (1) in (2)

$0.25\left(P\sin 30 + 866\right) + P\cos 30 - 500 = 0$

$0.125P + 216.5 + 0.86P - 500 = 0$

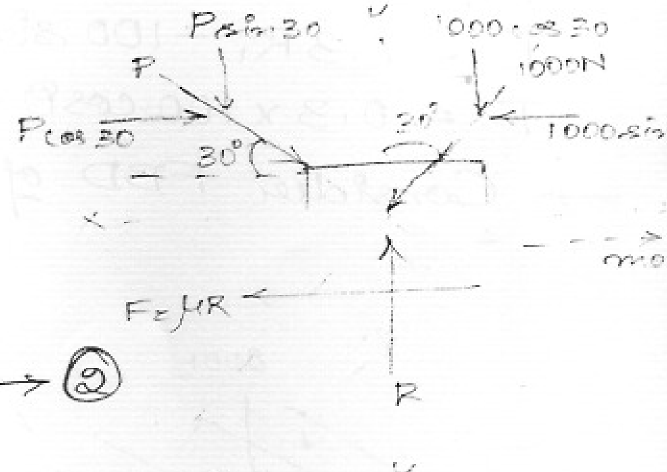
$\underline{P = 286.3N}$

- $\therefore R = 286.3 \sin 30 + 866$

$\underline{R = 1009.15N}$

⇒ Consider FBD when motion impending is up

- Resolving the forces



$\Sigma V = 0$

$R - 1000 \cos 30 - P \sin 30 = 0$

$R = P \sin 30 + 866 \longrightarrow (1)$

$\Sigma H = 0$

$P\cos 30 - 1000 \sin 30 - F = 0$

$P\cos 30 - 500 - 0.25R = 0 \longrightarrow (2)$

- By substituting (1) in (2)

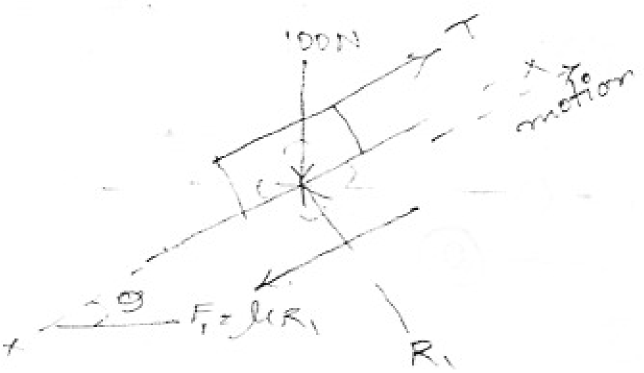
$P\cos 30 - 500 - 0.25\left(P\sin 30 + 866\right)$

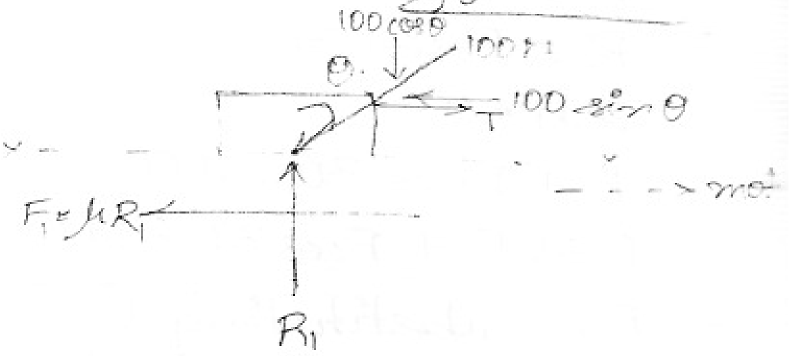
$0.86P - 500 - 0.125P - 216.5 = 0$

$\underline{P = 974.8N}$

4) What should be the value of inclination of plane so that motion of the block 200N impends down the plane? Take $\mu = 0.3$ for all contact surfaces.

⇒ Let θ be the angle of inclined plane. Consider FBD of 100N





- Resolving the forces
- Let T be the tension in the string

$\Sigma V = 0$

$R_1 - 100\cos\theta = 0$

$R_1 = 100\cos\theta \longrightarrow (1)$

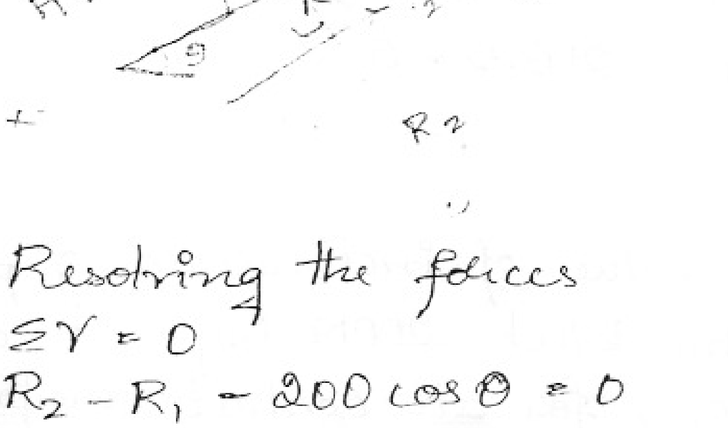
$\Sigma H = 0$

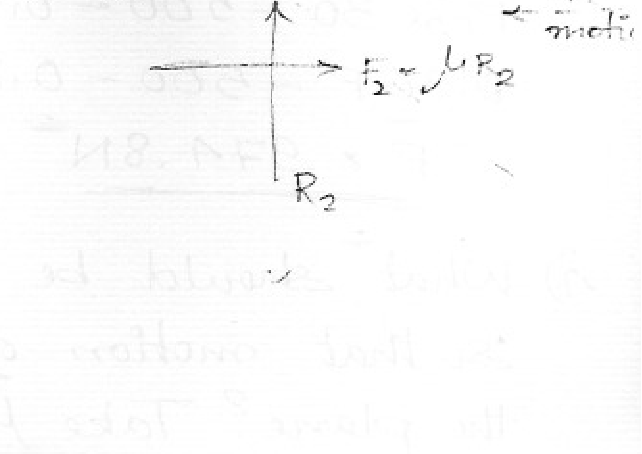
$-F_1 - 100\sin\theta + T = 0$

$T - 0.3R_1 - 100\sin\theta = 0$

$T = 0.3\times 100\cos\theta - 100\sin\theta \longrightarrow (2)$

→ Consider FBD of 200N





- Resolving the forces

$\Sigma V = 0$

$R_2 - R_1 - 200\cos\theta = 0$

$R_2 - 100\cos\theta - 200\cos\theta = 0$

$\therefore R_2 = 300\cos\theta \longrightarrow (3)$

$\Sigma H = 0$

$F_2 - 200\sin\theta + F_1 = 0$

$\mu R_2 + \mu R_1 - 200\sin\theta = 0$

$0.3 \times 300\cos\theta + 0.3 \times 100\cos\theta - 200\sin\theta = 0$

$90\cos\theta + 30\cos\theta - 200\sin\theta = 0$

$120\cos\theta - 200\sin\theta = 0$

$120\cos\theta = 200\sin\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{120}{200}$$

$\tan\theta = 0.6$

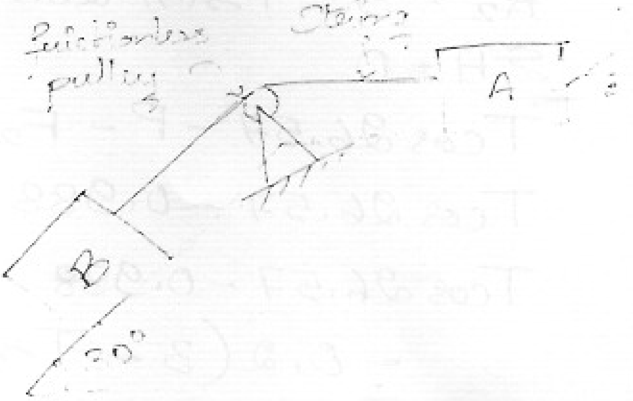
$\theta = 30.93^\circ \longrightarrow (4)$

- By substituting (4) in (2)

$T = 0.3 \times 100\cos 30.93 - 100\sin 30.93$

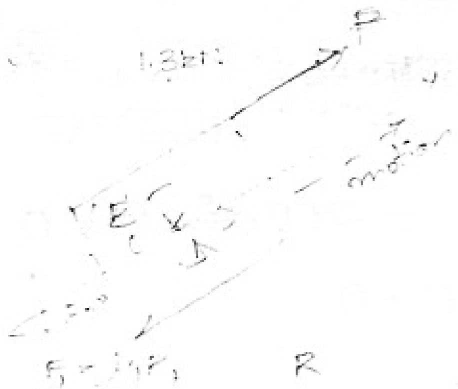
$T = 77.14\,N$

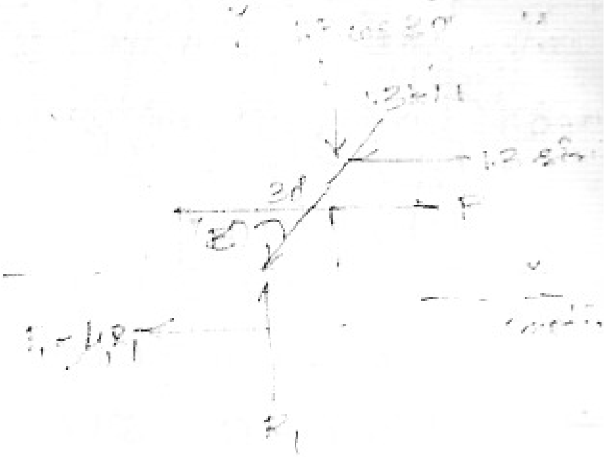
5) Two blocks A & B weighing 3kN & 1.3kN resp, are connected up by a string over a frictionless pulley as shown in the figure. Find the minimum value of force T to generate an impending motion to the right. Let μ be 0.2 & 0.3 for block A & B resptly.



$\Rightarrow$ The force T tends to move the block A towards right. Since the block B is connected with string. it tends to slide the block B up the plane.

→ Consider FBD for Block B





→ Resolving the forces

$\Sigma V = 0$

$R_1 - 1.3\cos 30 = 0$

$\underline{R_1 = 1.125\,kN}$

$\Sigma H = 0$

$P - 1.3\sin 30 - F_1 = 0$

$P - 1.3\sin 30 - 0.3 \times 1.125 = 0$

$\underline{P = 0.988\,kN}$

$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{4}{8}$

$\underline{\theta = 26.57°}$

→ Consider FBD for Block A

- Resolving the forces

$\Sigma V = 0$

$R_2 - 3 + T\sin 26.57 = 0$

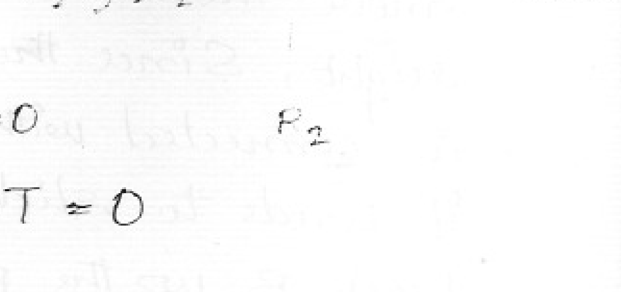
$R_2 = 3 - T\sin 26.57 \longrightarrow (1)$

$\Sigma H = 0$

$T\cos 26.57 - P - F_2 = 0$

$T\cos 26.57 - 0.988 - 0.2 R_2 = 0$ $\quad F_2 = \mu_2 R_2$

$T\cos 26.57 - 0.988$
$\quad - 0.2(3 - T\sin 26.57) = 0$

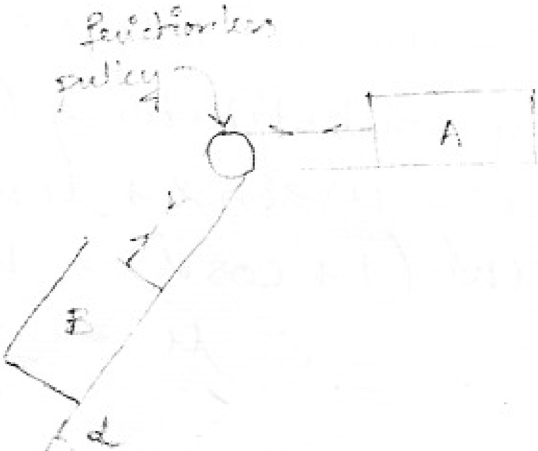


$0.89T - 0.988 - 0.6 + 0.447T = 0$

$1.337T = 1.588$

$\underline{T = 1.18\,kN}$ $\qquad \underline{R_2 = 2.47\,kN}$

6) Two blocks A & B weighing W_1 & W_2 are connected as shown in the fig. If $W_1 = W_2$ & if μ is the co-efficient of friction for all contact surfaces. Find the angle of inclination of inclined plane α @ which the motion of the system will impend.



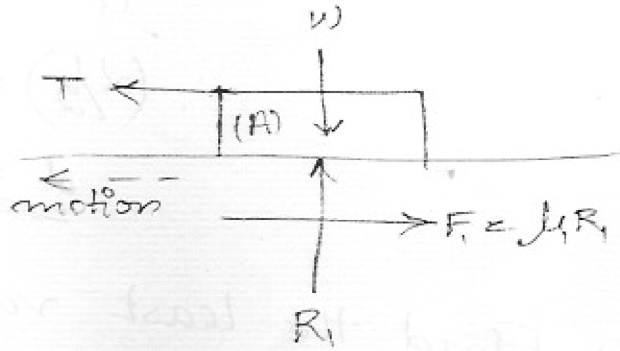
⇒ Let the angle be α made by the inclined plane

- Let T be the tension in the string.

- Here $W_1 = W_2 = W$.

⇒ Consider FBD of block A

- Resolving the forces



$\Sigma V = 0$

$R_1 - W = 0$

$R_1 = W \longrightarrow (1)$

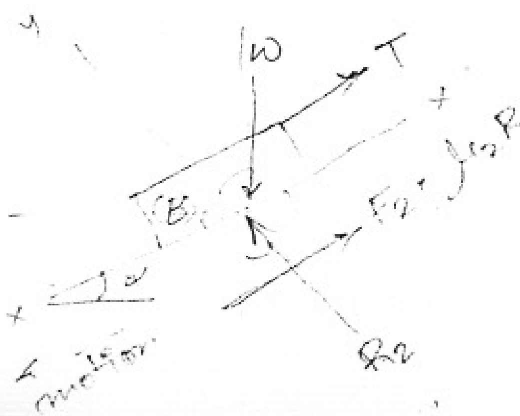
$\Sigma H = 0$

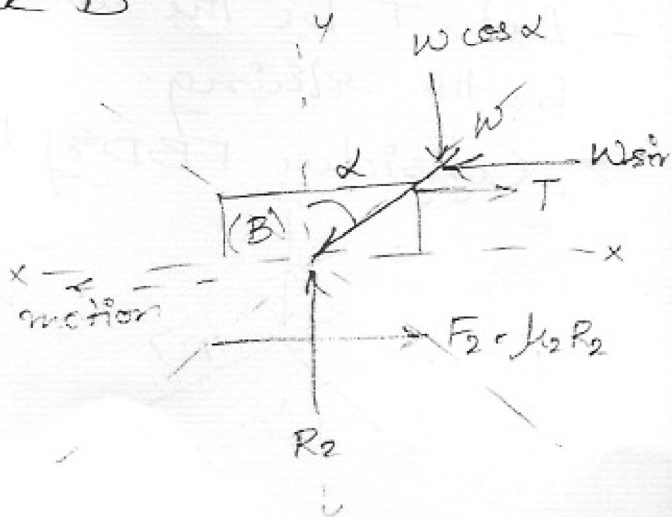
$F_1 - T = 0$

$T = \mu_1 R_1$

$T = \mu_1 W \longrightarrow (2)$

⇒ Considering FBD of block B





- Resolving the forces

$\Sigma V = 0$

$R_2 - W\cos\alpha = 0$

$R_2 = W\cos\alpha \longrightarrow (3)$

$\Sigma H = 0$

$T - W\sin\alpha + F_2 = 0$

$T - W\sin\alpha + \mu W\cos\alpha = 0 \longrightarrow (4)$

- By substituting (2) & (4)

$\mu W - W\sin\alpha + \mu W\cos\alpha = 0$

$\mu W(1+\cos\alpha) = W\sin\alpha$

$$\therefore \mu = \frac{\sin\alpha}{1+\cos\alpha}$$

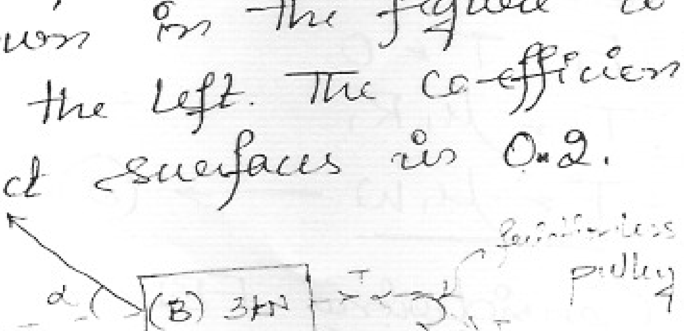
$$\mu = \frac{\cancel{2}\sin(\alpha/2)\cos(\alpha/2)}{\cancel{2}\cos^2(\alpha/2)}$$

$$\mu = \tan(\alpha/2)$$

$$\therefore (\alpha/2) = \tan^{-1}\mu$$

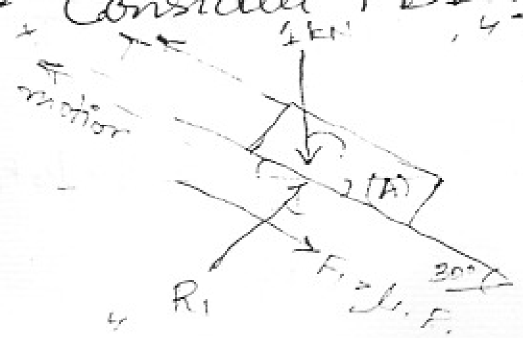
$$\underline{\underline{\alpha = 2\tan^{-1}\mu}}$$

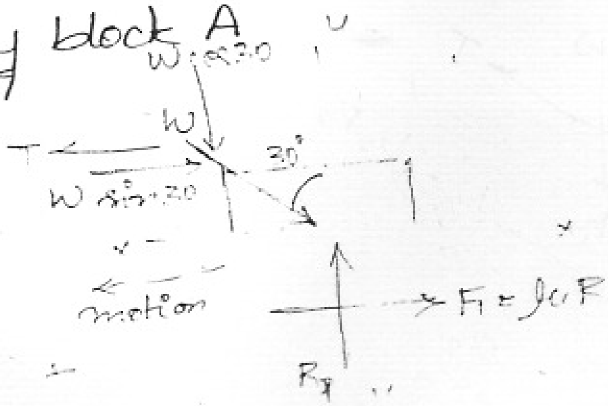
7) Find the least value of P reqd. to cause the system of block shown in the figure to have impending motion to the left. The co-efficient of friction for all contact surfaces is 0.2.



- Let T be the tension in the string.

⇒ Consider FBD of block A





- Resolving the forces

$\Sigma V = 0$

$R_1 - 1\cos 30 = 0$

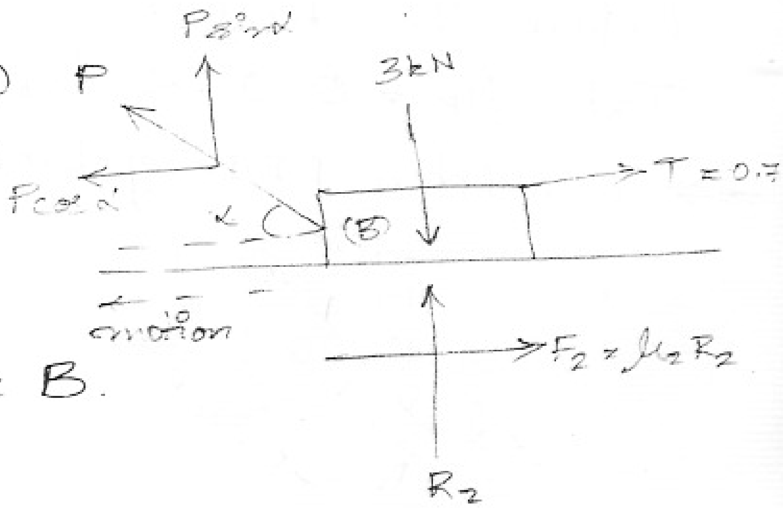
$R_1 = 0.867\,kN$

$\Sigma H = 0$

$F_1 + W\sin 30 - T = 0$

$0.2 + 1\sin 30 - T = 0$

$T = 0.7\,kN$



⇒ Consider FBD of block B.

- Resolving the forces

$\Sigma V = 0$

$R_2 - 3 + P\sin\alpha = 0$

$R_2 = 3 - P\sin\alpha \longrightarrow (1)$

$\Sigma H = 0$

$F_2 + T - P\cos\alpha = 0$

$0.2(3 - P\sin\alpha) + 0.7 - P\cos\alpha = 0$

$0.6 - 0.2P\sin\alpha + 0.7 - P\cos\alpha = 0$

$\therefore 0.2P\sin\alpha + P\cos\alpha = 1.3 \longrightarrow (2)$

- For least value of P

$$\frac{dP}{d\alpha} = 0$$

$\therefore$ Differentiating eqn (2) w.r.t α.

$0.2P\cos\alpha - P\sin\alpha = 0$

$0.2\cancel{P}\cos\alpha = \cancel{P}\sin\alpha$

$$0.2 = \frac{\sin\alpha}{\cos\alpha}$$

$\alpha = 11.3° \longrightarrow (3)$

- Substituting (3) in (2)

$0.2 P \sin 11.3 + P \cos 11.3 = 1.3$

$0.039P + 0.98P = 1.3$

$\underline{P = 1.27 kN}$

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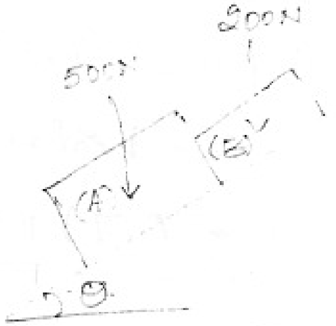
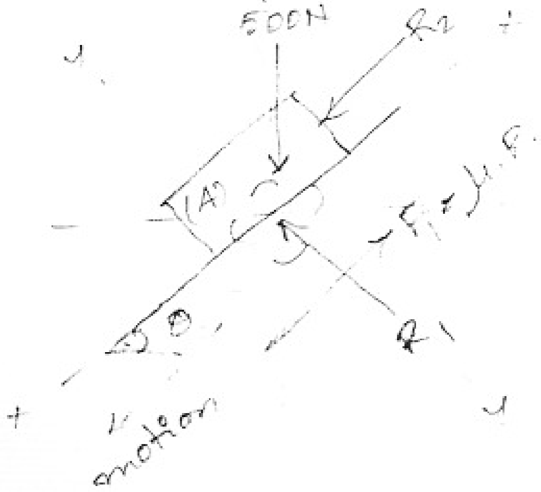
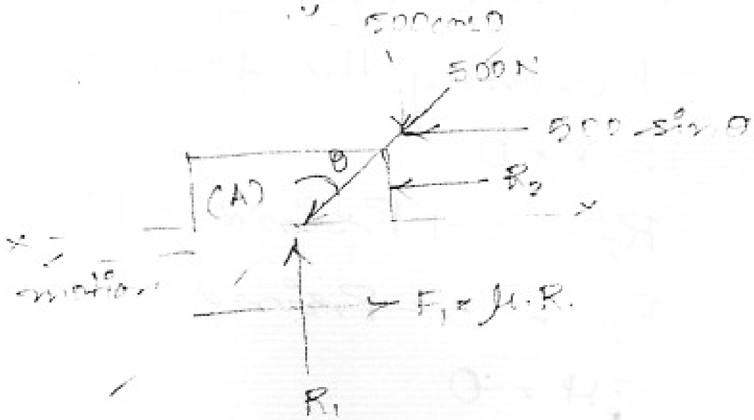
(3) What is the maximum L^{e} that can be reache before the bodies slip down the incline? Take $\mu_B = 0.2$ & $\mu_A = 0.3$



⟹ Consider FBD of block A





- Resolving the forces

$\Sigma V = 0$

$R_1 - 500 \cos\theta = 0$

$R_1 = 500 \cos\theta \longrightarrow (1)$

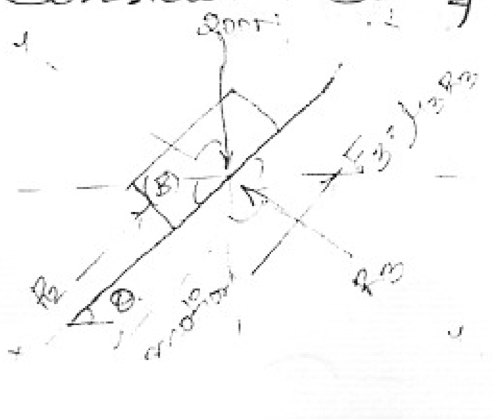
$\Sigma H = 0$

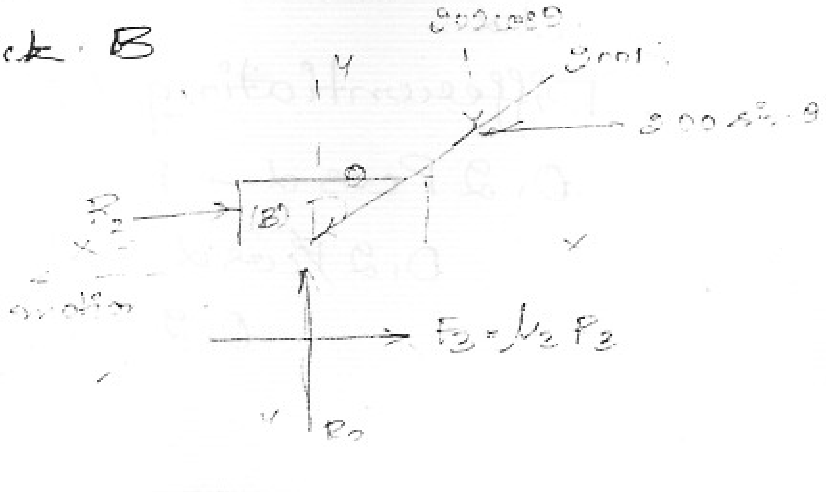
$F_1 - 500 \sin\theta - R_2 = 0$

$0.3 \times 500 \cos\theta - 500 \sin\theta - R_2 = 0$

$R_2 = 150 \cos\theta - 500 \sin\theta \longrightarrow (2)$

⟹ Consider FBD of block B





- Resolving the forces

$\Sigma V = 0$

$R_3 - 200\cos\theta = 0$

$R_3 = 200\cos\theta \longrightarrow (3)$

$\Sigma H = 0$

$F_3 + R_2 - 200\sin\theta = 0$

$0.2 \times 200\cos\theta + R_2 - 200\sin\theta = 0$

$\therefore R_2 = 200\sin\theta - 40\cos\theta \longrightarrow (4)$

- By substituting (2) & (4)

$$150\cos\theta - 500\sin\theta = 200\sin\theta - 40\cos\theta$$

$$190\cos\theta = 700\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{190}{700}$$

$$\tan\theta = 0.271$$

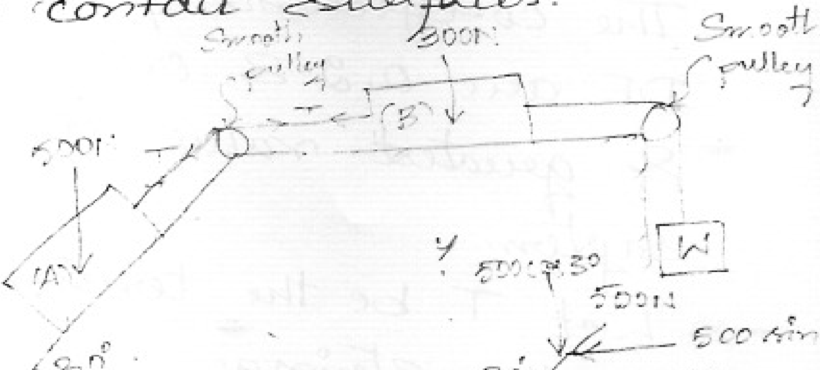
$$\theta = 15.18^\circ$$

- $\therefore R_1 = 500\cos 15.18 = 482.5N$

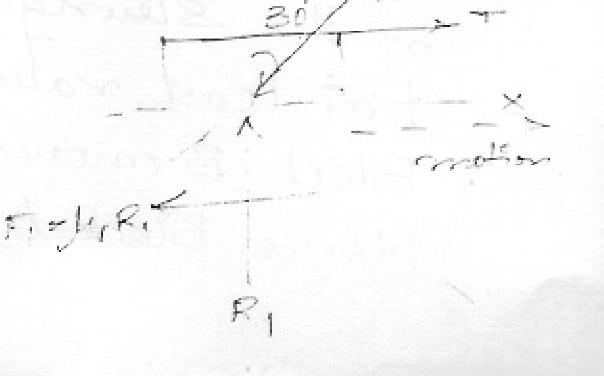
$R_2 = 150\cos 15.18 - 500\sin 15.18 = 13.84N$

$R_3 = 200\cos 15.18 = 193.1N$

9) Determine value of W to cause motion downward. Take $\mu = 0.3$ for all contact surfaces.



⇒ Consider FBD of A



- Resolving the forces

$\Sigma V = 0$

$R_1 - 500\cos 30 = 0$

$\underline{R_1 = 433N}$

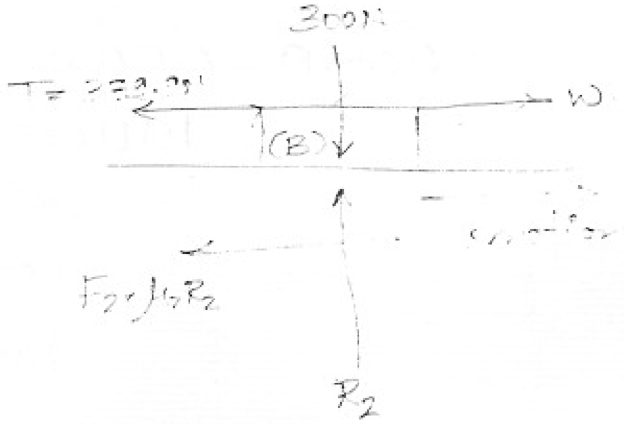
$\Sigma H = 0$

$T - 500\sin 30 - F_1 = 0$

$T - 500\sin 30 - 0.3 \times 433 = 0$

$\underline{T = \cancel{154.9N}\ 379.9N}$

⇒ Considering FBD of B



- Resolving the forces

$\Sigma V = 0$

$R_2 - 300 = 0$

$\underline{R_2 = 300N}$

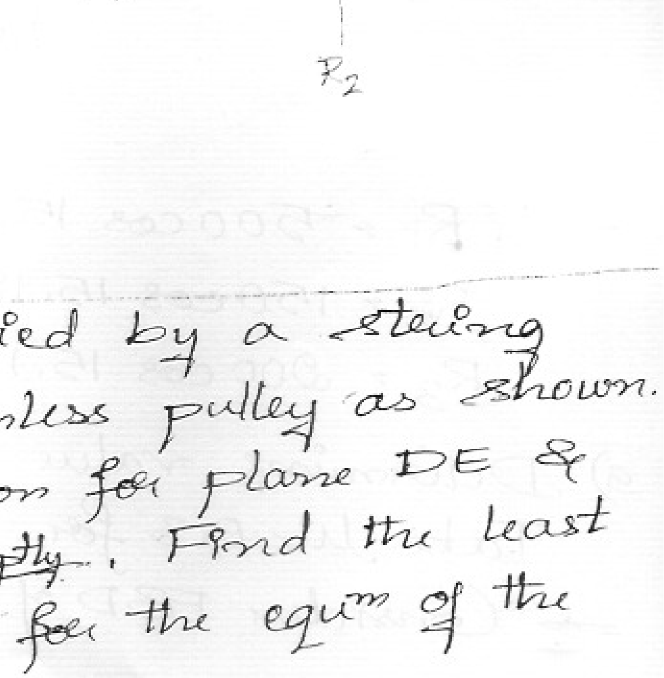
$\Sigma H = 0.$

$W - T - F_2 = 0$

$W - 379.9 - 0.3 \times 300 = 0$

$\underline{W = 469.9N}$

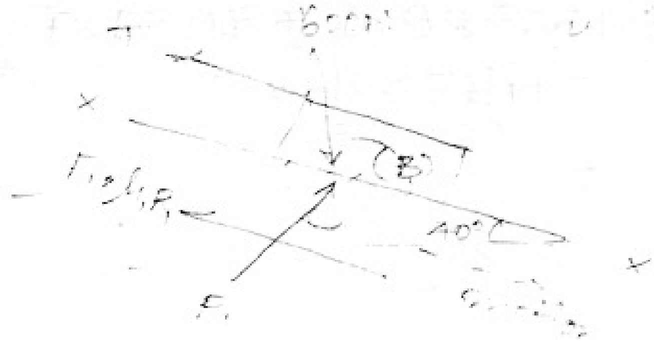
10) Two blocks A & B are tied by a string passing through a frictionless pulley as shown. The co-efficient of friction for plane DE & DF are 0.2 & 0.25 respty. Find the least & greatest value of W for the equm of the system.

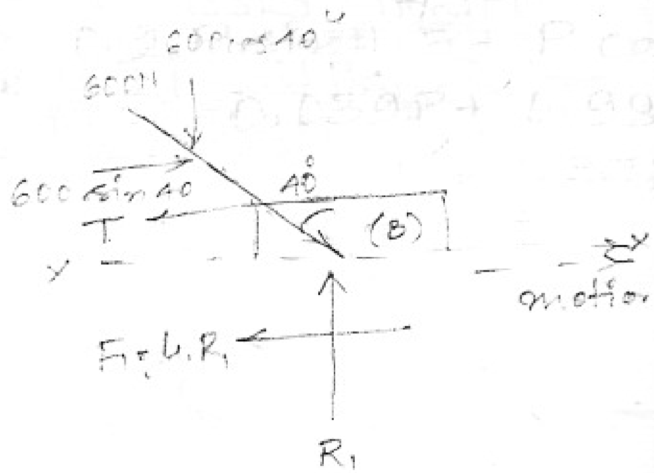


→ Let T be the tension in the string.

- For least value of W, Block B moves down. Hence Block A moves up

⇒ Consider FBD of (B)





- Resolving the forces

$\Sigma V = 0$

$R_1 - 600\cos 40 = 0$

$\underline{R_1 = 459.6N}$

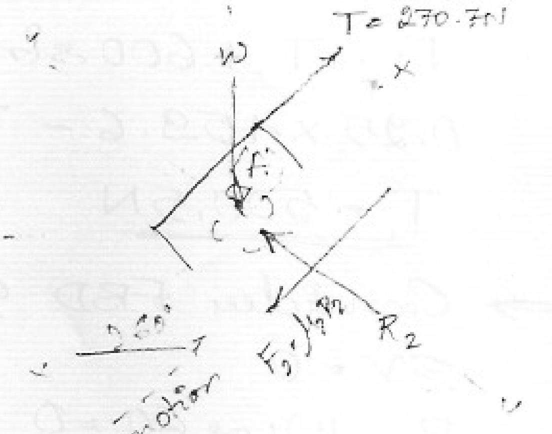
$\Sigma H = 0$

$600\sin 40 - T - F_1 = 0$

$600\sin 40 - T - 0.25 \times 459.6 = 0$

$\underline{T = 270.7N}$

⇒ Consider FBD of (A)



- Resolving the forces

$\Sigma V = 0$

$R_2 - W\cos 60 = 0$

$R_2 = W\cos 60 \longrightarrow (1)$

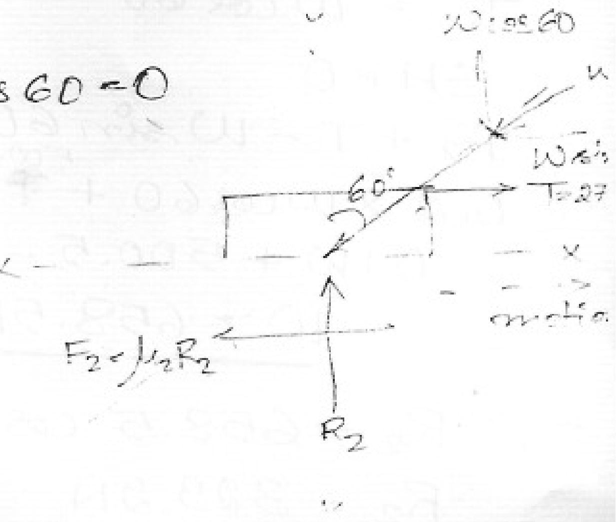
$\Sigma H = 0$

$T - W\sin 60 - F_2 = 0$

$270.7 - W\sin 60 - 0.2 \times W\cos 60 = 0$

$270.7 - 0.86W - 0.1W = 0$

$\underline{W = 281.9N}$

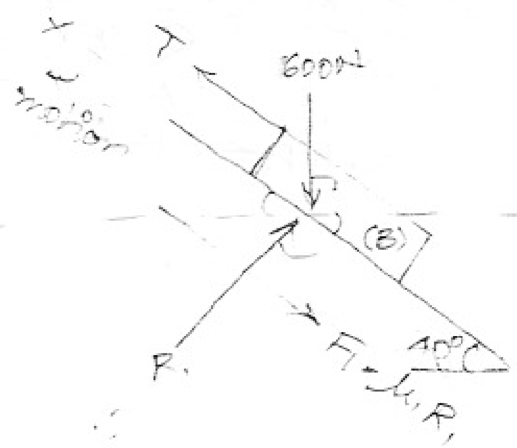


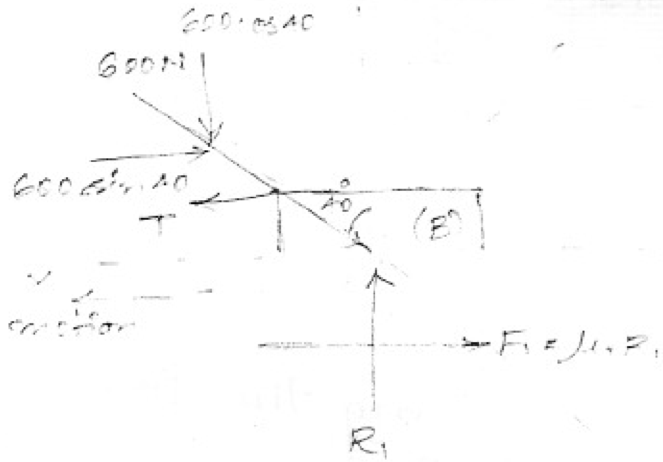
$\therefore R_2 = 281.9 \times \cos 60$

$\underline{R_2 = 140.9N}$

For highest value of W, Block B moves up. Hence Block A moves down.

→ Consider FBD of (B)





- Resolving the forces.

$\Sigma V = 0$

$R_1 - 600\cos 40 = 0$

$\underline{R_1 = 459.6N.}$

$\Sigma H = 0$

$F_1 - T + 600 \sin 40 = 0$

$0.25 \times 459.6 - T + 600 \sin 40 = 0$

$\underline{T = 500.5N}$

→ Consider FBD of (A)

$\Sigma V = 0$

$R_2 - W\cos 60 = 0$

$R_2 = W\cos 60 \longrightarrow (1)$

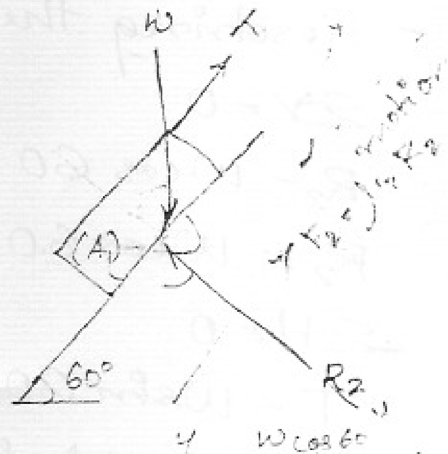
$\Sigma H = 0$

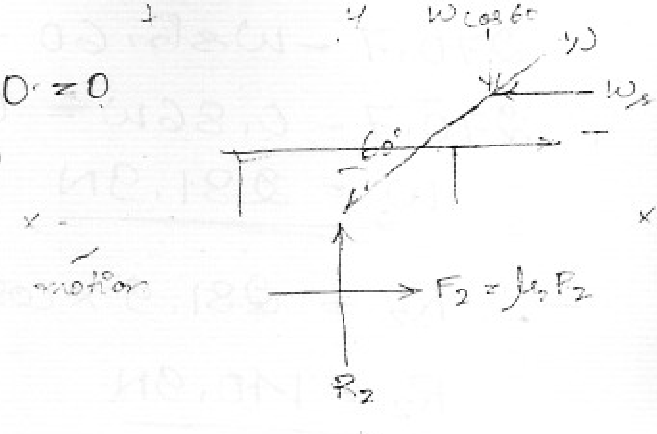
$F_2 + T - W\sin 60 = 0$

$0.2 \times W\cos 60 + \overset{500.5}{T} - W\sin 60 = 0$

$0.1W + 500.5 - 0.86W = 0$

$\underline{W = 658.5N}$





$\therefore R_2 = 658.5 \cos 60$

$\underline{R_2 = 329.2N}$

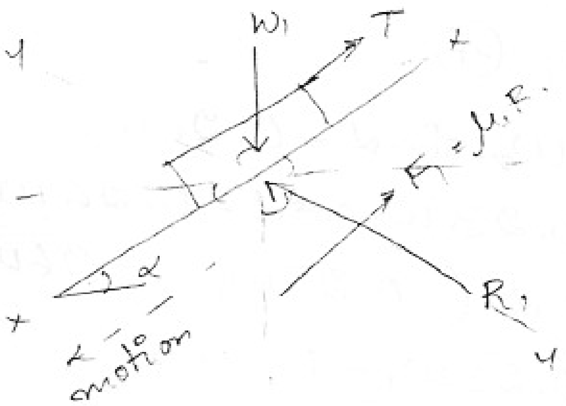
11) Two blocks W_1 & W_2 connected together by a piece of string resting on an inclined plane as shown. If $\mu_1 = 0.23$ & $\mu_2 = 0.26$, determine the angle of the included plane for sliding to impend.

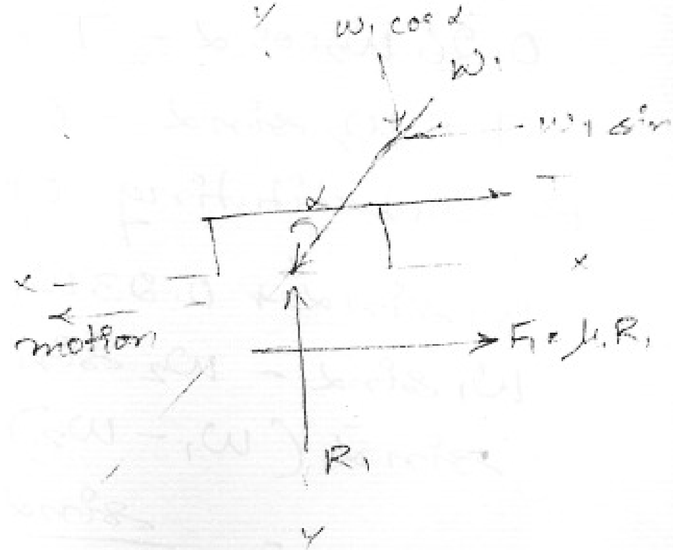
→ Here $\mu_1 < \mu_2$

∴ W_1 tends to slide down fast compared to W_2.

Hence the string is subjected to tension.

→ Consider FBD of (1)





- Resolving the forces

$\Sigma V = 0$

$R_1 - W_1 \cos\alpha = 0$

$R_1 = W_1 \cos\alpha \longrightarrow (1)$

$\Sigma H = 0$

$F_1 - W_1 \sin\alpha + T = 0$.

$0.23(W_1 \cos\alpha) - W_1 \sin\alpha + T = 0$

$T = W_1 \sin\alpha + 0.23 W_1 \cos\alpha \longrightarrow (2)$

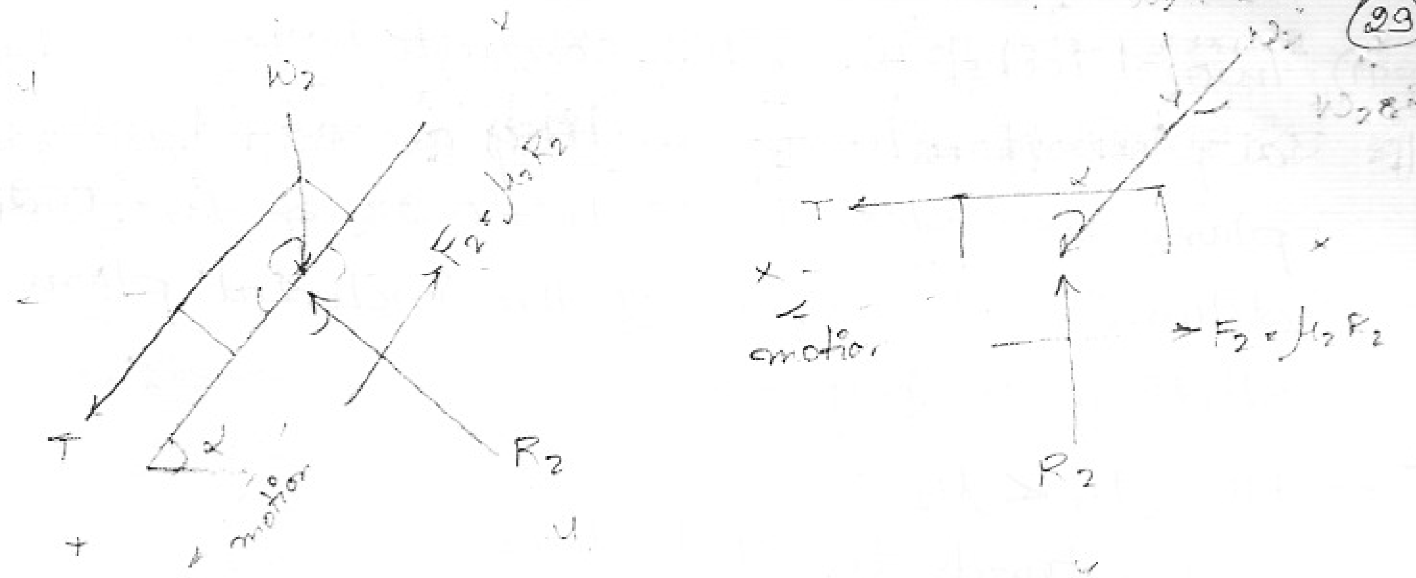
→ Consider FBD of (2)

- Resolving the forces

$\Sigma V = 0$

$R_2 - W_2 \cos\alpha = 0$

$R_2 = W_2 \cos\alpha \longrightarrow (3)$



$\Sigma H = 0$

$F_2 - T - W_2 \sin\alpha = 0$

$0.26\, W_2 \cos\alpha - T - W_2 \sin\alpha = 0$

$T = W_2 \sin\alpha - 0.26\, W_2 \cos\alpha \longrightarrow (4)$

By substituting eqn (2) & (4)

$W_1 \sin\alpha + 0.23\, W_1 \cos\alpha = W_2 \sin\alpha - 0.26\, W_2 \cos\alpha$

$W_1 \sin\alpha - W_2 \sin\alpha = -0.23\, W_1 \cos\alpha - 0.26\, W_2 \cos\alpha$

$\sin\alpha\,(W_1 - W_2) = \cos\alpha\,(-0.23 W_1 - 0.26 W_2)$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{-0.23 W_1 - 0.26 W_2}{W_1 - W_2}$$

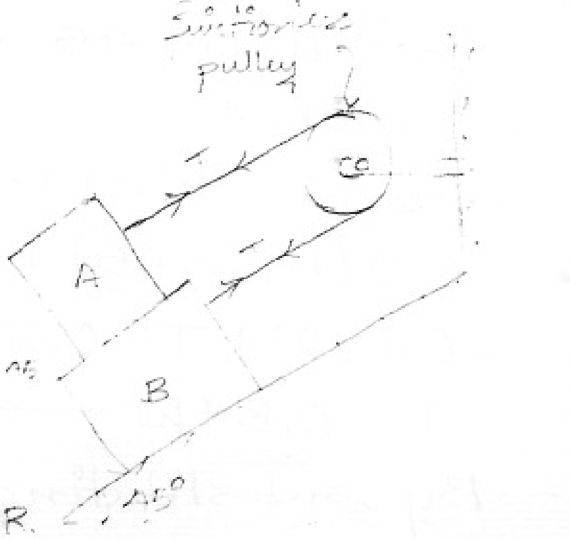
$\{\tan^{-1} x = -\tan^{-1} x\}$

$$\tan\alpha = \frac{0.23 W_1 + 0.26 W_2}{W_1 - W_2}$$

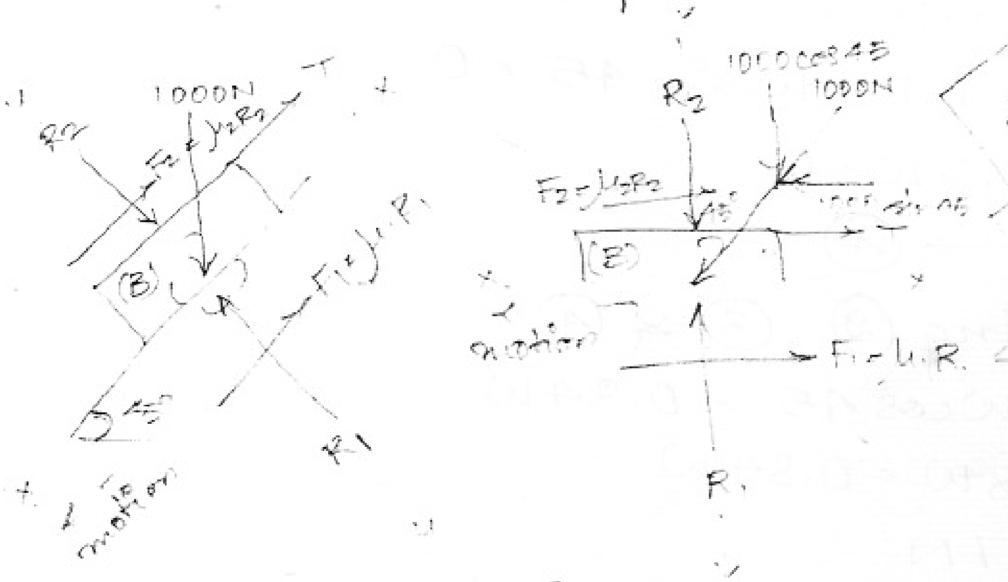
$$\alpha = \tan^{-1}\left(\frac{0.23 W_1 + 0.26 W_2}{W_2 - W_1}\right)$$

2) Two blocks A & B weighing W & 1000 N respt. rests one over the other on an inclined plane making an $\angle^{le}$ of 45° with horizontal plane making an $\angle^{le}$ ... The blocks A & B are connected by an inextensible string passing round the smooth frictionless pulley. The co-efficient of friction

of all contact surfaces are 0.2. Determine the weight W of block A & the tension in the string.



→ Consider FBD of (B)



- Resolving the forces

$\Sigma V = 0$

$R_1 - R_2 - 1000 \cos 45 = 0$

$R_1 = R_2 + 1000 \cos 45 \longrightarrow (1)$

$\Sigma H = 0$

$F_1 + T - 1000 \sin 45 + F_2 = 0$

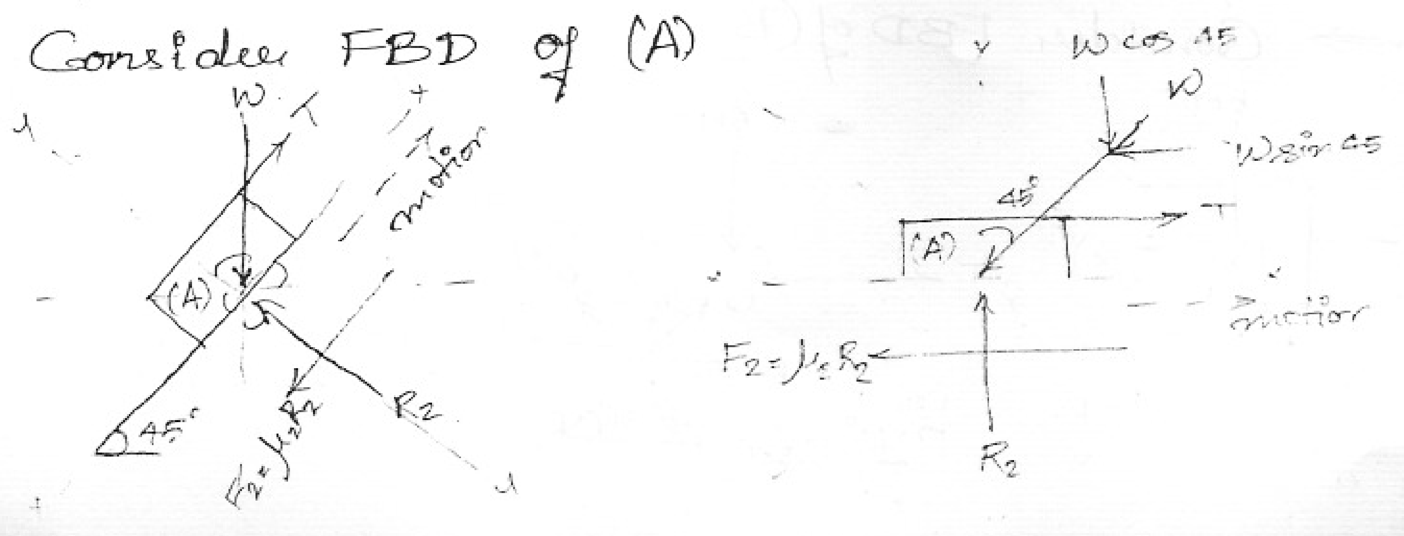
$0.2 (R_2 + 1000 \cos 45) + 0.2 R_2 + T - 1000 \sin 45 = 0$

$0.2 R_2 + 141.42 + 0.2 R_2 + T - 707.1 = 0$

$0.4 R_2 + T - 565.6 = 0$

$T = 565.6 - 0.4 R_2 \longrightarrow (2)$

→ Consider FBD of (A)



- Resolving the forces

$\Sigma V = 0$

$R_2 - W\cos 45 = 0$

$R_2 = W\cos 45 \longrightarrow (3)$

$\Sigma H = 0$

$-F_2 + T - W\sin 45 = 0$

$0.2(W\cos 45) + T - W\sin 45 = 0$

$-0.14W + T - 0.7W = 0$

$T = 0.84W \longrightarrow (4)$

- By substituting (2), (3) & (4)

$565.6 - 0.4W\cos 45 = 0.84W$

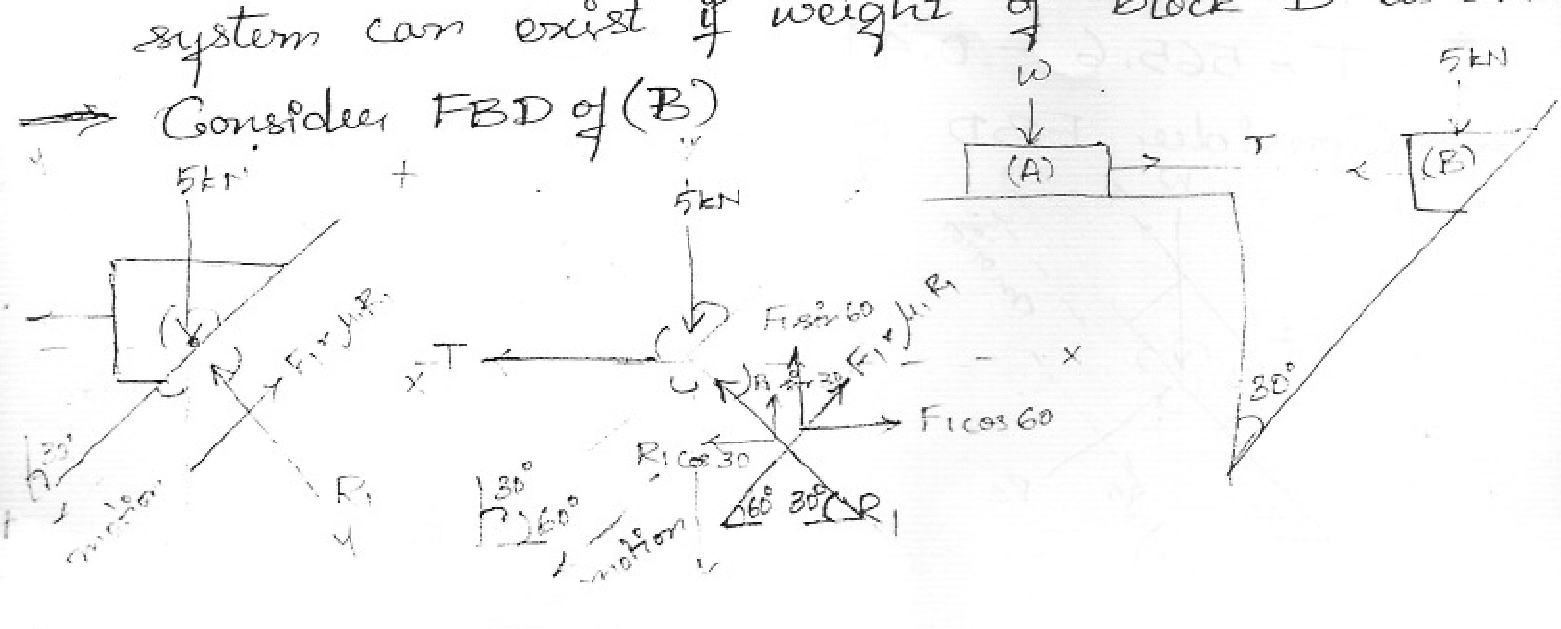
$565.6 - 0.28W = 0.84W$

$\underline{W = 503.7\,N}$

- $\therefore T = 0.84 \times 503.7 = \underline{423.12\,N}$

13) Two blocks connected by a horizontal link AB are supported on 2 rough planes as shown. The co-efficient of friction on the horizontal plane is 0.4. The limiting Lle of friction for block B on the inclined plane is 20°. What is the small weight W of the block A for which equm of the system can exist if weight of block B is 5kN

$\Rightarrow$ Consider FBD of (B)



∴ Resolving the forces

$\Sigma V = 0$

$R_1 \sin 30 + F_1 \sin 60 - 5 = 0$

$0.5R_1 + 0.311 R_1 - 5 = 0$

$0.811 R_1 - 5 = 0$

$\underline{\underline{R_1 = 6.15\ kN}}$

w.k.t $\tan \alpha = \frac{F}{R}$

∴ $\mu = \tan 20°$

$\mu = 0.36$

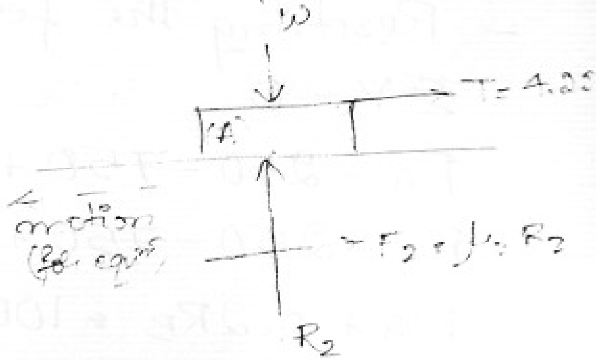
$\Sigma H = 0$

$F_1 \cos 60 - R_1 \cos 30 - T = 0$

$0.36 \times 6.15 \cos 60 - 6.15 \cos 30 - T = 0$

$1.1 - 5.32 - T = 0$

$\underline{\underline{T = 4.22\ kN}}$



→ Consider FBD of (A)

- Resolving the forces

$\Sigma V = 0$

$R_2 - W = 0$

$R_2 = W \longrightarrow (1)$

$\Sigma H = 0$

$F_2 + T = 0$

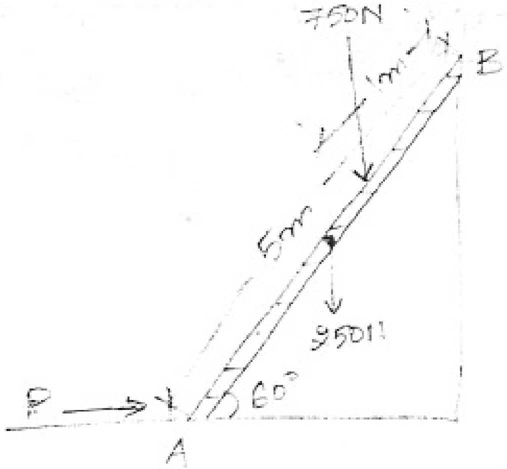
$0.4 R_2 + 4.22 = 0$

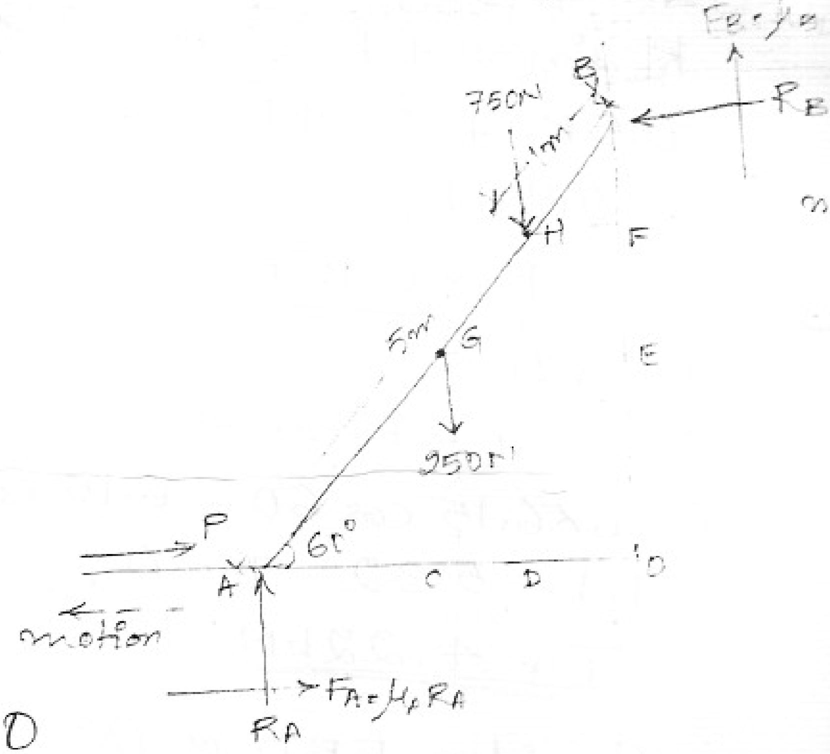
$\underline{\underline{R_2 = 10.5\ kN = W}}$

⇒ Problems on Ladder friction

1) A ladder weighing 250N & length 5m is placed against a vertical wall as shown. The co-efficient of friction b/w ladder & w is 0.2 & b/w ladder & floor is 0.3. The ladder has to support a man weighing 750N @ a distance of 1m along the ladder from wo

Calculate the minimum horizontal force to be applied @ A to prevent the slipping.





⇒ Consider FBD

- Resolving the forces

$\Sigma V = 0$

$R_A - 250 - 750 + F_B = 0$

$R_A - 250 - 750 + 0.2 R_B = 0$

$R_A + 0.2 R_B = 1000 \longrightarrow (1)$

$\Sigma H = 0$

$F_A - R_B + P = 0$

$0.3 R_A - R_B + P = 0$

$0.3 R_A - R_B = P \longrightarrow (2)$

- Taking moment of forces abt A

$250 \times AC + 750 \times AD - R_B \times BO - F_B \times AO = 0$

- $\therefore \sin 60 = \frac{O}{H} = \frac{O}{5} \Longrightarrow BO = \underline{\underline{4.33m}}$ } Δ^{le} AOB

$\cos 60 = \frac{A}{H} = \frac{A}{5} \Longrightarrow AO = \underline{\underline{2.5m}}$

$\cos 60 = \frac{A}{H} = \frac{A}{4} \Longrightarrow AD = \underline{\underline{2m}}$ } Δ^{le} ADH

$\cos 60 = \frac{A}{H} = \frac{A}{2.5} \Longrightarrow AC = \underline{\underline{\cancel{2.48}m}}$ 1.25 } Δ^{le} ACG

$\therefore 250 \times \cancel{2.48} + 750 \times 2 - R_B \times 4.33 - 0.2 R_B \times 2.5 = 0$

$\cancel{620}\ 312.5 + 1500 - 4.33 R_B - 0.5 R_B = 0$

$\underline{R_B = 375.2\,N}$

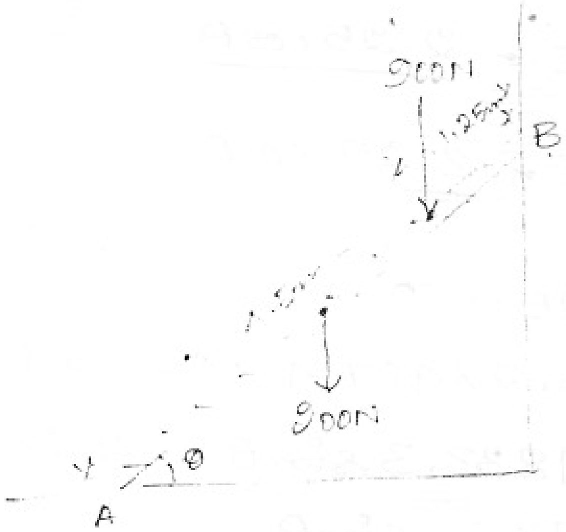
$\therefore R_A = 1000 - 0.2 \times 375.2$

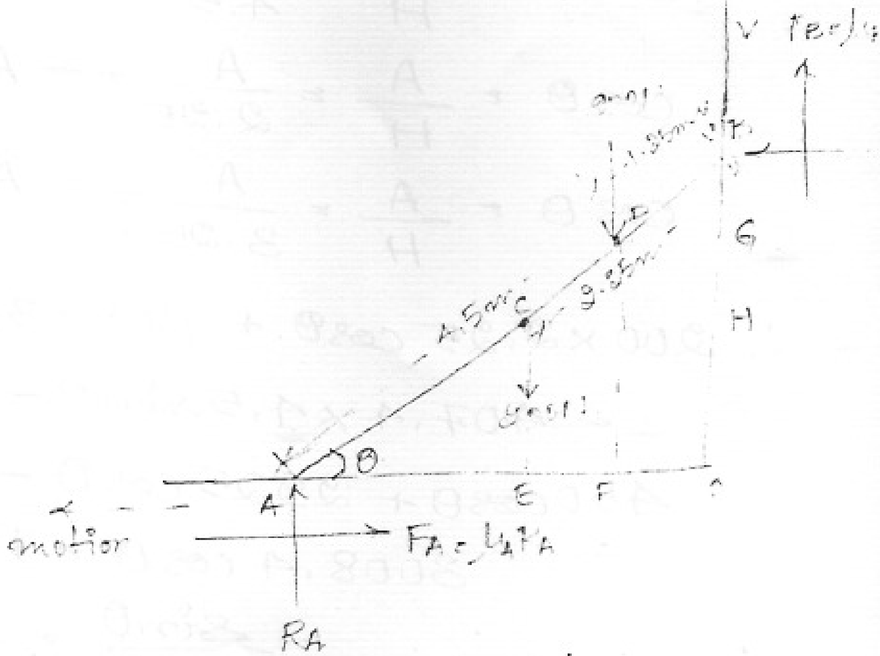
$\underline{R_A = 924.9\,N}$

$\therefore P = 0.3 \times 924.9 - 375.2$

$\underline{P = 97.7\,N}$

2) A uniform ladder of weight 200N of length 4.5m rests on a horizontal ground & leans on a rough vertical wall. The μ b/w ladder & floor is 0.4 and b/w ladder & wall is 0.2. When a weight of 900N is placed on the ladder @ a distance of 1.25m from the top of the ladder i@ the point of sliding. Find the ∠ made by the ladder with horizontal, if no horizontal load applied @ the foot of the ladder to prevent motion





⇒ Let θ be the ∠ made by the ladder

⇒ Consider FBD of the ladder.

- Resolving the forces

$\Sigma V = 0$

$R_A + F_B - 900 - 200 = 0$

$R_A + 0.2R_B - 1100 = 0$

$R_A + 0.2R_B = 1100 \longrightarrow (1)$

$\Sigma H = 0$

$F_A - R_B = 0$

$0.4R_A - R_B = 0$

$R_B = 0.4R_A \longrightarrow (2)$

- By substituting (1) & (2)

$R_A + 0.2(0.4R_A) - 1100 = 0$

$1.08R_A = 1100$

$R_A = 1018.5N$

- $\therefore R_B = 0.4 \times 1018.5 = 407.4N$

- Taking moment of forces abt A

$200 \times AE + 900 \times AF - R_B \times BO - F_B \times AO = 0$

$\therefore \sin\theta = \frac{O}{H} = \frac{O}{4.5} \Rightarrow BO = 4.5\sin\theta$

$\cos\theta = \frac{A}{H} = \frac{A}{4.5} \Rightarrow AO = 4.5\cos\theta$

$\cos\theta = \frac{A}{H} = \frac{A}{2.25} \Rightarrow AE = 2.25\cos\theta$

$\cos\theta = \frac{A}{H} = \frac{A}{3.25} \Rightarrow AF = 3.25\cos\theta$

- $\therefore 200 \times 2.25\cos\theta + 900 \times 3.25\cos\theta$

$- 407.4 \times 4.5\sin\theta - 0.2 \times 407.4 \times 4.5\cos\theta = 0$

$450\cos\theta + 2925\cos\theta - 1833.3\sin\theta - 366.6\cos\theta$

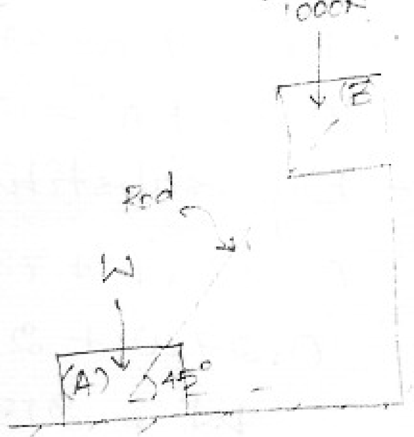
$3008.4\cos\theta = 1833.3\sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{3008.4}{1833.3}$

$\tan\theta = 1.64$

$\theta = 58.64°$

3) Two blocks A & B are held in position by a rod AB as shown. The weight of block B is 1000N. Find the weight of block A such that the block will not slide away from wall. Take L^{e} of friction for all contact surfaces as



⇒ $\tan\phi = \mu$

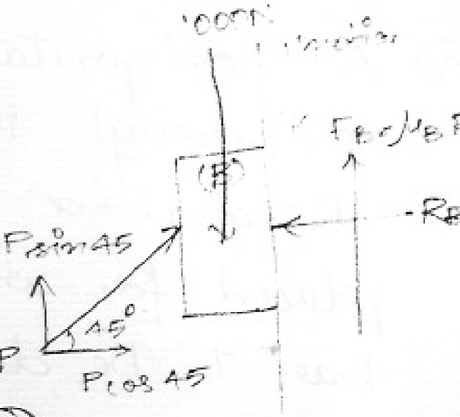
$\tan 15 = \mu$

$\therefore \underline{\mu = 0.267}$

⇒ Consider FBD of block (B)

Since the blocks are connected by rod, the rod will be subjected to compressive force.

- Resolving the forces



$\Sigma V = 0$

$F_B - 1000 + P\sin 45 = 0$

$0.27 R_B - 1000 + P\sin 45 = 0$

$0.27 R_B + P\sin 45 = 1000 \rightarrow (1)$

$\Sigma H = 0$

$P\cos 45 - R_B = 0$

$R_B = P\cos 45 \longrightarrow (2)$

- By substituting (1) & (2)

$0.27 P\cos 45 + P\sin 45 = 1000$

$0.19P + 0.707P = 1000$

$\underline{P = 1114.7 N}$

- $\therefore R_B = 1114.7 \cos 45$

$R_B = \underline{788.2N}$

⇒ Consider FBD of block (A)

- Resolving the forces

$\Sigma V = 0$

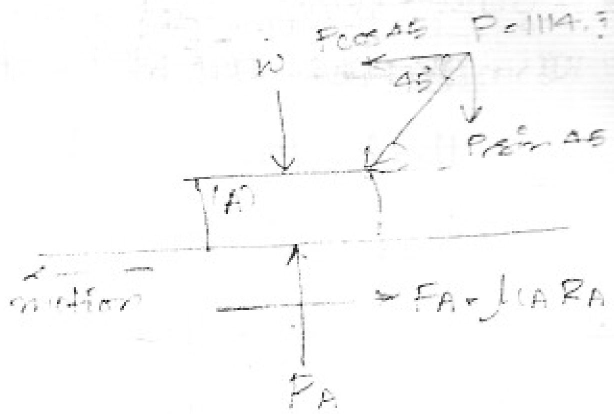
$R_A - W - P\sin 45 = 0$

$R_A = W + 788 \rightarrow (1)$

$\Sigma H = 0$

$F_A - P\cos 45 = 0$

$0.27\,R_A - 788 = 0 \rightarrow (2)$



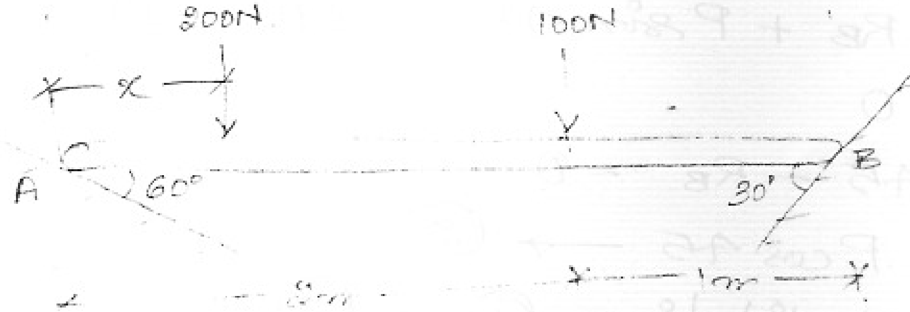
- By substituting (2) & (1)

$0.27(W+788) - 788 = 0$

$0.27W + 212.7 - 788 = 0$

$\underline{W = 2130N}$

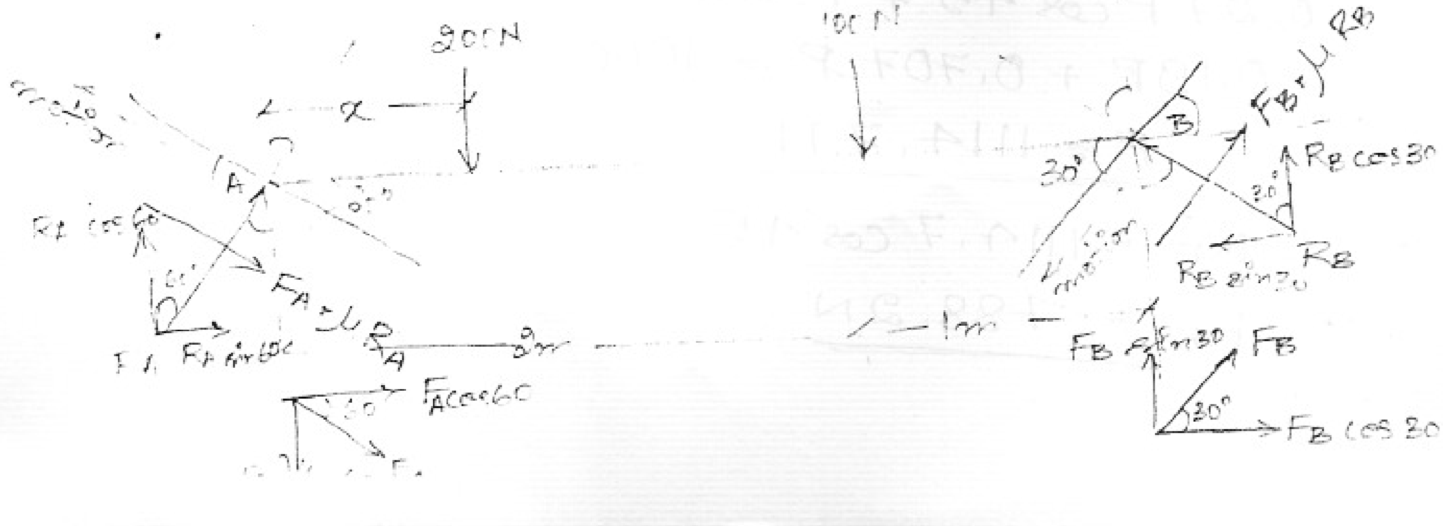
7) A horizontal bar 3m long is supported on two rough inclined planes @ A & B. Find the distance x from A of the load of 200N be placed for which the impending motion of the bar to be down the plane @ B. Take $\phi = 17°$



$\tan\phi = \mu$

$\mu = \tan 17$

$\underline{\mu = 0.3}$



⇒ Consider the FBD

- Resolving the forces

$\Sigma V = 0$

$R_A \cos 60 + R_B \cos 30 + F_B \sin 30 - F_A \sin 60 - 200 - 100$

$R_A \cos 60 + R_B \cos 30 + 0.3 R_B \sin 30 - 0.3 R_A \sin 60 - 300$

$0.5 R_A + 0.86 R_B + 0.15 R_B - 0.26 R_A - 300 = 0$

$0.24 R_A + 1.01 R_B = 300 \longrightarrow (1)\ (\div)\ 0.24$

$\Sigma H = 0$

$R_A \sin 60 - R_B \sin 30 + F_A \cos 60 + F_B \cos 30 = 0$

$R_A \sin 60 - R_B \sin 30 + 0.3 R_A \cos 60 + 0.3 R_B \cos 30 =$

$0.86 R_A - 0.5 R_B + 0.15 R_A + 0.26 R_B = 0$

$1.01 R_A - 0.24 R_B = 0 \longrightarrow (2)\ (\div)\ 1.01$

- By substituting equ^n (1) & (2)

$\cancel{0.24} R_A + \overset{4.2}{\cancel{1.01}} R_B = \overset{1250}{\cancel{300}}$

$R_A - 0.23 R_B = 0$

$4.43 R_B = 1250$

$\underline{R_B = 282.1 N}$

$\therefore R_A = 0.23 R_B = 0.23 \times 282.1 = \underline{64.8 N}$

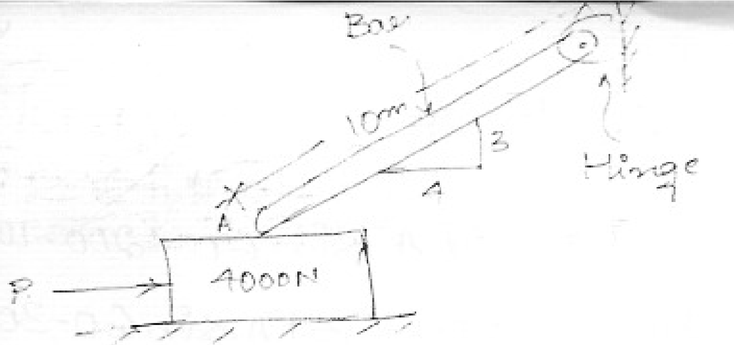
- Taking moment abt (A)

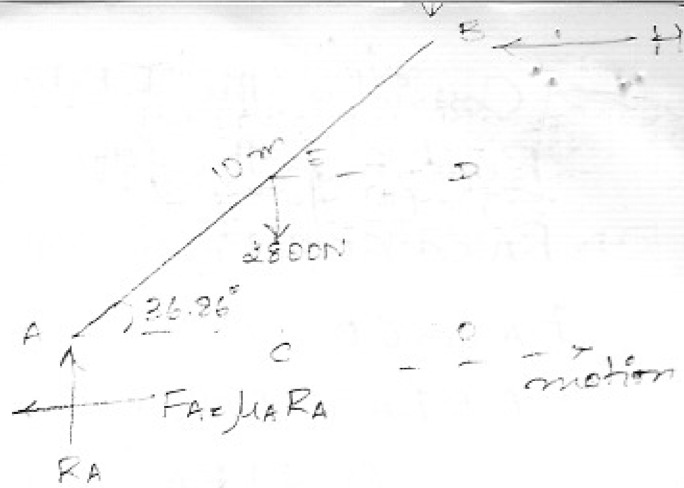
$200 \times x + 100 \times 2 - R_B \cos 30 \times 3 - F_B \sin 30 \times 3 = 0$

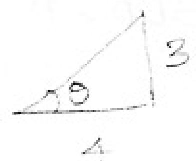
$200x + 200 - 732.9 - 126.9 = 0$

$\underline{x = 3.29 m \text{ from } A}$

5) A uniform bar AB, 10m long weighing 2800N is hinged @ A & rests upon a block weighing 4000N @ A as shown. Find the horizontal force P reqd. to start the movement of 4000N block if coefficient of friction is 0.4.







$$\tan\theta = \frac{O}{A} = \frac{3}{4}$$

$$\theta = 36.86^\circ$$

⇒ Consider FBD of bar

- Resolving the forces

$\Sigma V = 0$

$R_A - 2800 - V_B = 0 \longrightarrow (1)$

$\Sigma H = 0$

$-H_B - F_A = 0$

$-H_B - 0.4 R_A = 0 \longrightarrow (2)$

- Taking moment abt B.

$R_A \times AO + F_A \times BO - 2800 \times ED = 0$

- $\therefore \sin 36.86 = \frac{O}{H} = \frac{O}{10} \Longrightarrow BO = \underline{5.99 m}$

$\cos 36.86 = \frac{A}{H} = \frac{A}{10} \longrightarrow AO = \underline{8 m}$

- $\cos 36.86 = \frac{A}{H} = \frac{A}{5} \Longrightarrow ED = \underline{4 m}$

- $\therefore 8 R_A + 0.4 R_A \times 5.99 - 2800 \times 4 = 0$

$10.4 R_A - 11200 = 0$

$\underline{R_A = 1076.9 N}$

- $\therefore 1076.9 - 2800 - V_B = 0$

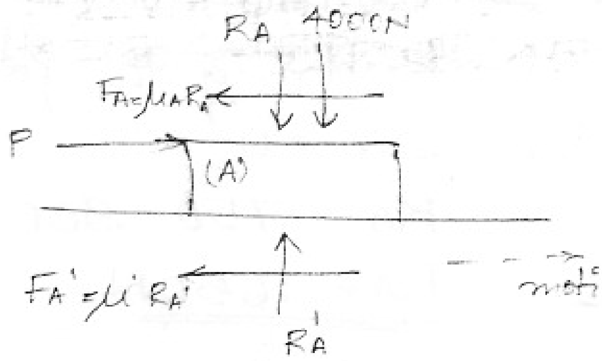
$\underline{V_B = 1723.1 N}$

- $H_B = 0.4 \times 1076.9 = \underline{430.7 N}$

→ Consider FBD of Block

- Resolving the forces

$\Sigma V = 0$



$R_A' - 4000 - R_A = 0$

$R_A' = 4000 - 1076.9$

$R_A' = \underline{2923.1N}$

$\Sigma M = 0$

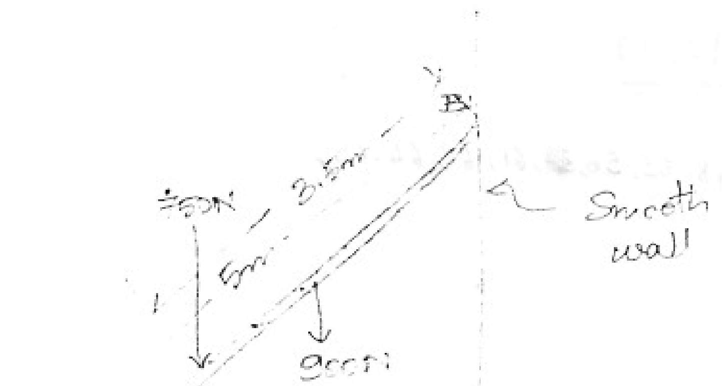
$P - F_A - F_A' = 0$

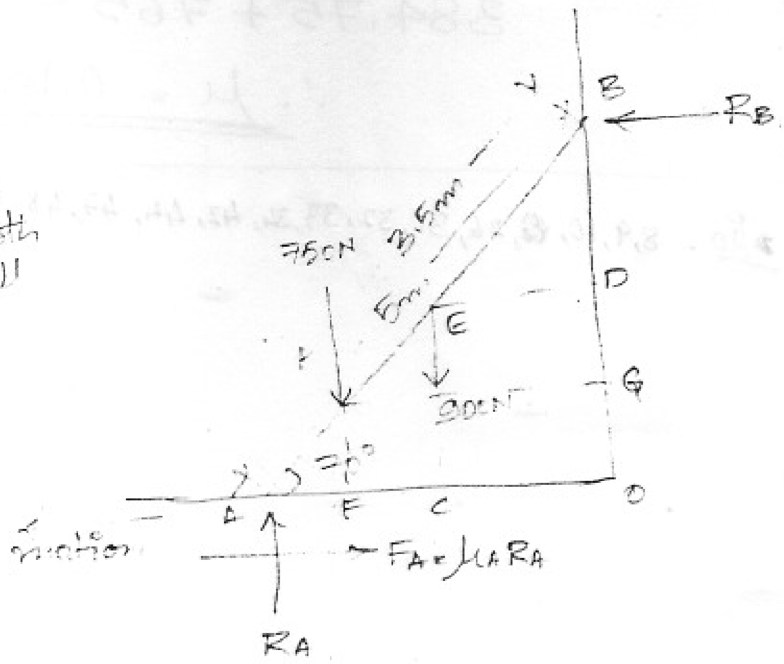
$P - 0.4 \times 1076.9 - 0.4 \times 2923.1$

$\underline{\underline{P = 1600N}}$

---

6) A ladder 5m long rests on a horizontal floor & leans against a smooth vertical wall @ an ∠le of 70° with the floor. The weight of ladder is 900N. The ladder is @ the verge of slipping when a man weighing 750N stands on it @ a distance of 3.5m measured along the ladder from the top of the ladder. Determine the co-efficient of friction b/w ladder & floor.





- Let μ_A be the co-efficient of friction b/w ladder & floor

→ Consider FBD of the ladder

- Resolving the forces

$\Sigma V = 0$

$R_A - 750 - 900 = 0$

$\underline{R_A = 1650N}$

$\Sigma H = 0$

$F_A - R_B = 0$

$\mu R_A - R_B = 0$

$\therefore R_B = \mu 1650 \longrightarrow (1)$

- Taking moment abt A ($\Sigma M_A = 0$)

$750 \times AF + 900 \times AC - R_B \times BO = 0$

- $\therefore \sin 70 = \frac{O}{H} = \frac{O}{5} \Longrightarrow \underline{BO = 4.69m}$

$\cos 70 = \frac{A}{H} = \frac{A}{1.5} \Longrightarrow \underline{AF = 0.513m}$

$\cos 70 = \frac{A}{H} = \frac{A}{2.5} \Longrightarrow \underline{AC = 0.85m}$

- $\therefore 750 \times 0.513 + 900 \times 0.85 - \mu 1650 \times 4.69 = 0$

$384.75 + 765 - 7738.5\mu = 0$

$\therefore \underline{\mu = 0.149}$

2/10 - 8, 9, 10, [illegible], 26, 31, 32, 33, 34, 42, 44, 47, 48, 55, 56, [illegible], 61, 63, 64, 65,

UNIT- 8

CENTROID OF PLANE FIGURES [redacted]

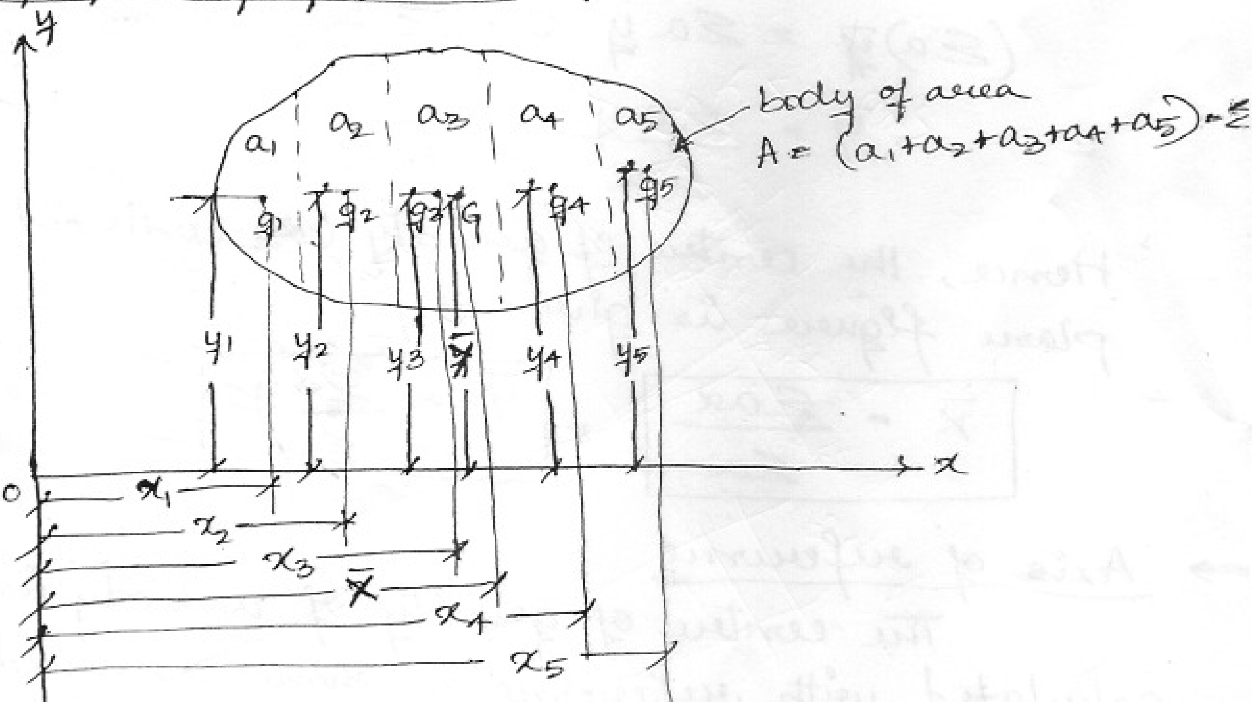
→ **Centre of gravity (C.G or G)**

Centre of gravity of a body is a point through which resultant of force of gravity (or) the whole weight of the body acts. It is represented by C.G or G. There is only one centre of gravity for any position of body. It refers to the bodies with mass & weight.

→ **Centroid**

Centroid of an area is a point @ which the whole area of the plane figure is assumed to be concentrated. The plane figures may be $\bigcirc^{le}$, Δ^{le}, $\square^{or}$, $\square^{le}$, $\square^{le}$ etc that have area but no mass. It is also represented by C.G or G.

→ **Center of gravity of plane figure**



- The figure shows a plane figure of total area A and unit thickness.
- Let G be C.G of the whole area with co-ordinates $(\bar{x}, \bar{y})$

- Divide this area A into a no of small areas a_1, a_2, a_3, a_4 & a_5 with centroid g_1, g_2, g_3, g_4 & g_5 resp.

$\therefore$ Then $A = \Sigma a = a_1 + a_2 + a_3 + a_4 + a_5 \longrightarrow (1)$

- Let (x_1, y_1), $(x_2 y_2)$, (x_3, y_3), (x_4, y_4) & (x_5, y_5) be the co-ordinates of centroids resp.

$$\left.\begin{array}{l}\therefore \text{Moment of total area} \\ \text{A about OY axis}\end{array}\right\} = A\bar{x} = (\Sigma a)\bar{x} \longrightarrow (2)$$

- Sum of moments of all areas about OY axis

$$= a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + a_5 x_5$$

$$= \Sigma ax \longrightarrow (3)$$

- Equating (2) & (3)

$$(\Sigma a)\bar{x} = \Sigma ax$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a}$$

- (11)ly, if we equate the moments of areas about OX axis to the moment of total area about OX, we get,

$$(\Sigma a)\bar{y} = \Sigma a y$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

- Hence, the centre of gravity (or) centroid of any plane figure is given by,

$$\boxed{\bar{x} = \frac{\Sigma ax}{\Sigma a}}, \quad \boxed{\bar{y} = \frac{\Sigma ay}{\Sigma a}}$$

$\Rightarrow$ Axis of reference

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference.

Note : Axis of reference to:

Leftmost point to find $\bar{x}$ is +ve

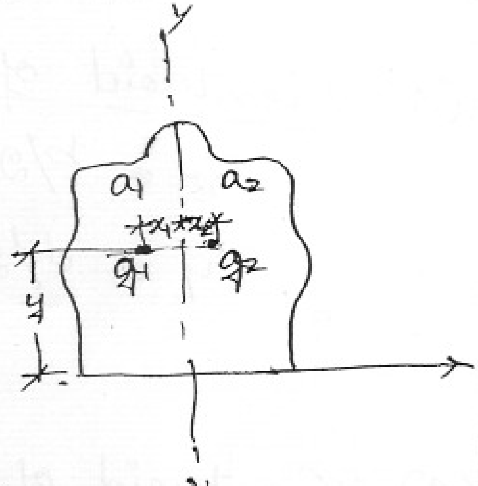
Lowermost point to find $\bar{y}$ is +ve

If the co-ordinates are on the left side & below or lower side of the reference axis, then $\bar{x}$ & $\bar{y}$ is considered as -ve for calculation.

⇒ Axis of symmetry.

Axis of symmetry is the axis along the C.G of a body that is symmetric in section.

- Consider a body as shown in figure which is symmetric w.r.t y axis.



- Here w.r.t y axis is axis of symmetry. Hence it is taken as reference axis.
- Consider 2 elemental area a_1 & a_2 that are equal in size (area) & equidistance for reference axis.
- w.r.t a_1 is equal to a_2 ($\because$ of symmetry)

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{A} \quad (\because x_1 = -x_2 \,;\, a_1 = a_2)$$

$$\underline{\bar{x} = 0}$$

- Hence the distance of centroid from symmetric axis is zero i.e, centroid lies on the axis of symmetry.

⇒ Determination of centroid of simple figures from method of integration

- For a plane figure with area A, the centroid axis is represented as $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\Sigma ax}{A} \;;\; \bar{y} = \frac{\Sigma ay}{A}$$

- Hence general expression for an elemental area dA & its distance from reference axis is

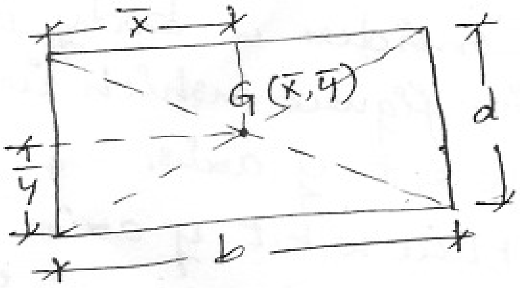
$$\bar{x} = \frac{\int dA\, x}{A} = \frac{\int x\, dA}{A}$$

$$\text{ully } \bar{y} = \frac{\int dA\, y}{A} = \frac{\int y\, dA}{A}.$$

(1) Centroid of rectangle

$\bar{x} = b/2$

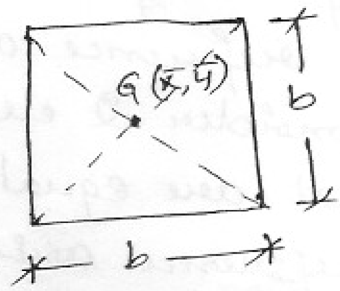
$\bar{y} = d/2$



(2) Centroid of square

$\bar{x} = b/2$

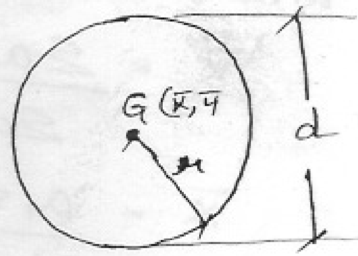
$\bar{y} = b/2$



(3) Centroid of a circle

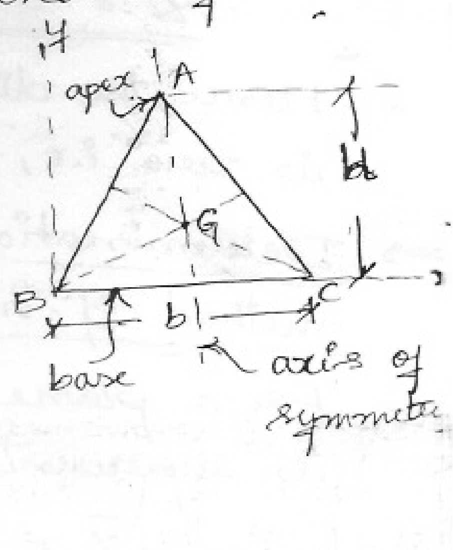
$\bar{x} = d/2 = r$

$\bar{y} = d/2 = r$

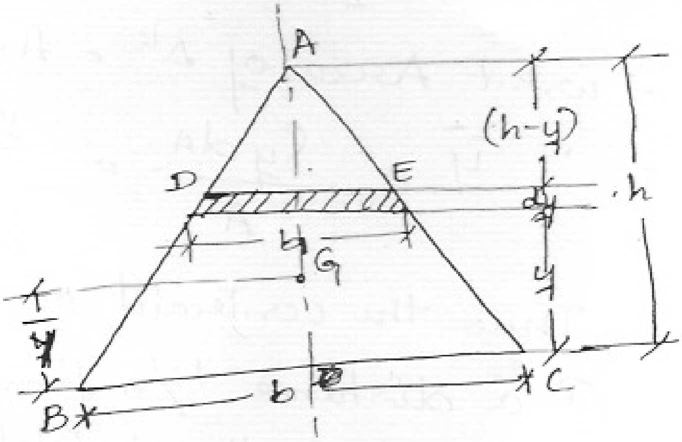


(4) Centroid of a triangle

- Consider a Δ^{le} ABC with base 'b' and height 'h' as shown in figure.
- Select a reference axis as x & y
- Due to symmetry, the centroid should lie on symmetric axis



- To locate the centroid, select the reference axis as shown in figure.



- Consider an elemental area as in figure with base b_1 & height dy

$\therefore$ Area of elemental strip $\underline{dA = b_1 dy}$

- Consider (|||)^{lar} Δ^{les} ABC & ADE

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \frac{b(h-y)}{h}$$

$$b_1 = b\left(1-\frac{y}{h}\right)$$

$$dA = b\left(1-\frac{y}{h}\right)dy$$

w.k.t $\bar{y} = \dfrac{\int y\, dA}{A}$

$$\therefore \int y\, dA = \int_0^h y\, b\left(1-\frac{y}{h}\right)dy$$

$$= b\int_0^h y\left(1-\frac{y}{h}\right)dy$$

$$= b\int_0^h y\, dy - b\int_0^h \frac{y^2}{h}\, dy$$

$$= \left.\frac{by^2}{2}\right]_0^h - \left.\frac{by^3}{3h}\right]_0^h$$

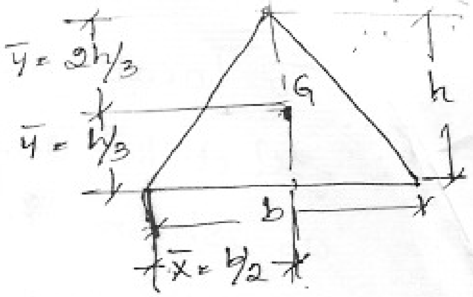
$$= b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h$$

$$= b\left[\frac{h^2}{2} - \frac{h^3}{3h}\right]$$

$$= b\left[\frac{3h^2 - 2h^2}{6}\right]$$

$$\int y\, dA = \frac{bh^2}{6}$$

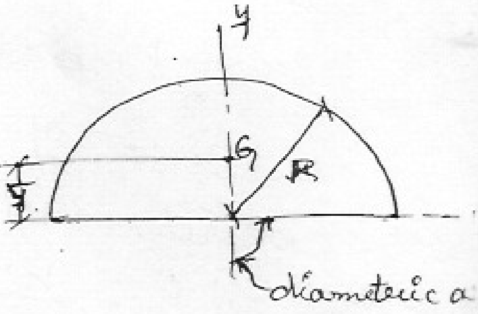
- w.k.t Area of $\Delta^{le} = A = \frac{1}{2} b h$



- $\therefore \bar{y} = \frac{\int y \, dA}{A} = \frac{\cancel{b}h^2/\cancel{6}3}{\frac{1}{2}\cancel{b}h} = \frac{h}{3}$

- Thus the centroid of a Δ^{le} is @ a distance $h/3$ from the base (or) $2h/3$ from apex, where h is the height of a Δ^{le}.

(5) <u>Centroid of semi circle</u>



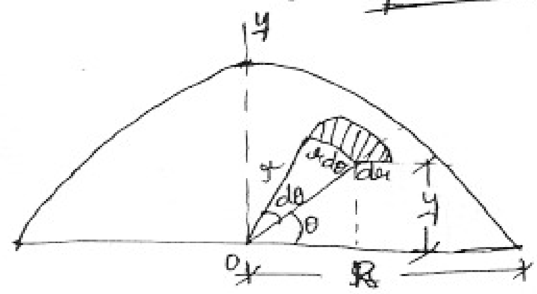
- Consider a $\bigcirc^{le}$ with radius 'R' as shown in figure

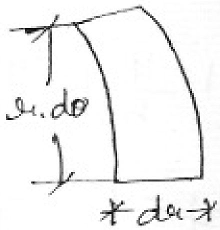
- Select a reference axis x & y

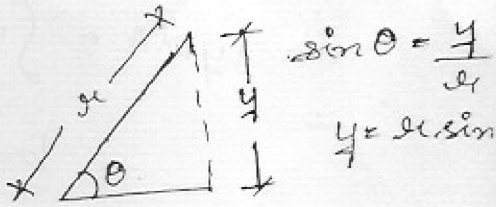
- Due to symmetry, the centroid should lie on diametric axis

- Consider an elemental area of sides $r d\theta$ & dr as shown in figure. Here elemental area is considered as $\square^{le}$ $\therefore$ $\underline{dA = r.d\theta.dr.y}$

& $\underline{y = r \sin\theta.}$







- w.k.t $\bar{y} = \frac{\int y \, dA}{A}$

$\therefore \int y \, dA = \int r \, d\theta . dr . r \sin\theta$

$= \int_0^R \int_0^\pi r^2 \, dr \sin\theta \, d\theta$

$= \frac{r^3}{3} \left(-\cos\theta\right) \Big]_0^R \Big]_0^\pi$ $\quad \left\{ \begin{array}{l} \cos\pi = -1 \\ \cos 0 = 1 \end{array} \right\}$

$= \frac{R^3}{3} - (-1) - (-1)$

$\int y \, dA = \frac{R^3}{3} (1+1) = \frac{2R^3}{3}$

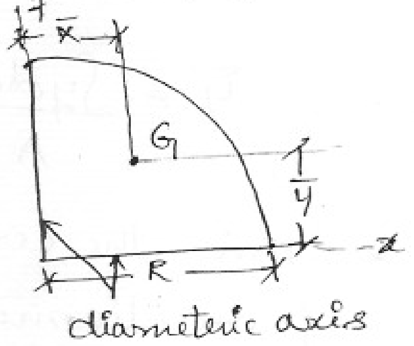
∴ w.k.t Area of $\bigcirc^{le} = \pi r^2$

Area of $D^{le} = \dfrac{\pi r^2}{2}$

$$\therefore \bar{y} = \frac{\int y\,dA}{A} = \frac{\frac{2R^3}{3}}{\frac{\pi R^2}{2}} = \frac{4R}{3\pi}$$

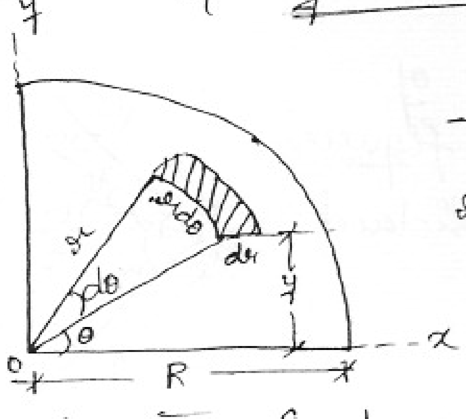
- Thus the centroid of a D^{le} lies @ a distance $4R/3\pi$ from diametric axis, where R is the radius of $\bigcirc^{le}$.

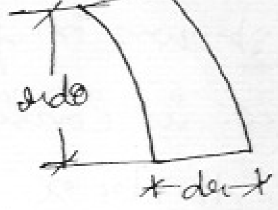
(6) <u>Centroid of quadrant of a $\bigcirc^{le}$</u>

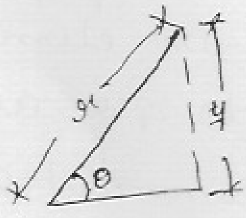


- Consider a D^{le} with radius 'R' as shown in figure
- Select a reference axis x & y
- Due to symmetry, the centroid should lie as shown in figure.
- Consider an elemental area of sides $r\,d\theta$ & dr as shown in figure. Here elemental area is considered as $\square^{le}$ ∴ <u>$dA = r.d\theta.dr.y$</u>

& <u>$y = r\sin\theta$</u>







$\sin\theta = \dfrac{y}{r}$

$y = r\sin\theta$

- w.k.t $\bar{y} = \dfrac{\int y\,dA}{A}$

$$\therefore \int y\,dA = \int r.\,d\theta.\,dr.\,r\sin\theta$$

$$= \int_0^R\int_0^{\pi/2} r^2\,dr\,\sin\theta\,d\theta$$

$$= \left[\left.\frac{r^3}{3}(-\cos\theta)\right]_0^R\right]_0^{\pi/2}$$

$$= \frac{R^3}{3}\left[-(\cos \pi/2 - \cos 0)\right]$$

$$= \frac{R^3}{3}\left[-(0-1)\right]$$

$$\int y\,dA = \frac{R^3}{3}$$

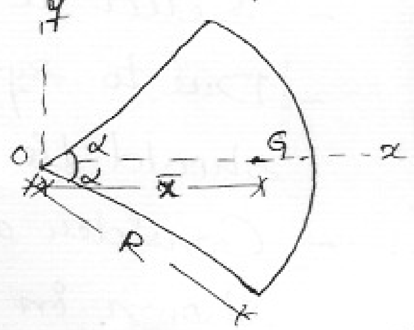
- w.k.t Area of $\bigcirc^{le} = \frac{\pi r^2}{2}$

  Area of $D^{le} = \frac{\pi . r^2}{4}$

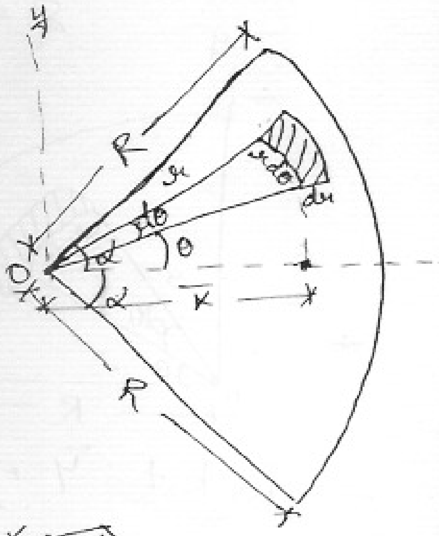
- $\therefore \bar{y} = \frac{\int y\,dA}{A} = \frac{R^3/3}{\pi R^2/4} = \frac{4R}{3\pi}$

- Thus the centroid of a D^{le} lies @ a distance $4R/3\pi$ from diametric axis, where R is the radius of O^{le}.

(7) Centroid of sector of a circle.



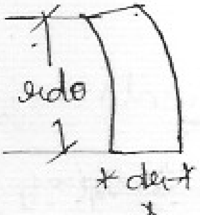
- Consider a $\triangle^{r}$ of O^{le} with radius 'R' as shown in figure with angle 2α
- Select a reference axis x & y
- Due to symmetry, the centroid should lie on x axis as shown in figure.



- Consider an elemental area of sides $rd\theta$ & r as shown in figure. Hence elemental area is considered as $\square^{le}$ $\therefore$ $dA = r\,d\theta\,dr$ x

  $x = r\cos\theta$

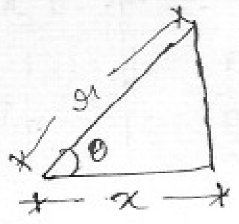
- w.k.t $\bar{x} = \frac{\int x\,dA}{A}$



$\therefore \int x\,dA = \int r . d\theta . dr . r\cos\theta$

$$= \int_0^R \int_{-\alpha}^{\alpha} r^2\,dr \cos\theta\,d\theta$$

$$= \frac{r^3}{3}\Big]_0^R \sin\theta\Big]_{-\alpha}^{\alpha}$$



$\cos\theta = \frac{x}{r}$

$x = r\cos\theta$

$$= \frac{R^3}{3}\left[\sin(+\alpha) - \sin(-\alpha)\right]$$

$$= \frac{R^3}{3}\left[\sin\alpha + \sin\alpha\right]$$

$$\int x\,dA = \frac{R^3}{3}\,2\sin\alpha$$

- w.k.t Area of Δ^{le} = $r\,d\theta\,dr$.

$$\int A\,dA = \int r\,d\theta\,dr$$

$$= \int_0^R\int_{-\alpha}^{\alpha} r\,d\theta\,dr$$

$$= \frac{R^2}{2}\,\theta\Big]_{-\alpha}^{\alpha}$$

$$= \frac{R^2}{2}\,\alpha-(-\alpha)$$

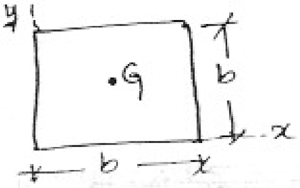
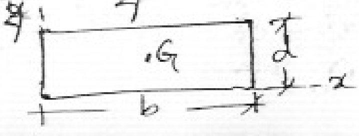
$$= \frac{R^2}{\cancel{2}}\,\cancel{2}\alpha$$

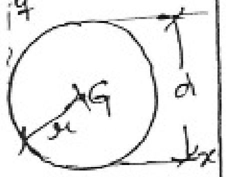
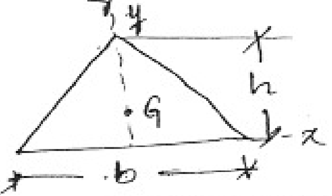
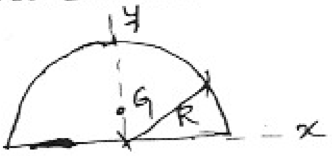
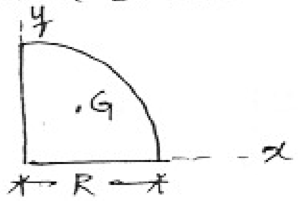
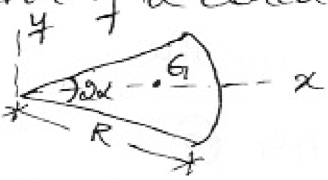
$$\int A\,dA = R^2\alpha$$

- $\therefore \bar{x} = \dfrac{\int x\,dA}{\int A\,dA} = \dfrac{2/3\,R^3\sin\alpha}{R^2\alpha} = \dfrac{2R\sin\alpha}{3\alpha}$

- Thus the centroid of Δ^{le} of a O^{le} lies @ a distance $\dfrac{2R\sin\alpha}{3\alpha}$ from x axis, where R is the radius of O^{le} & α is the half the angle of sector @ O.

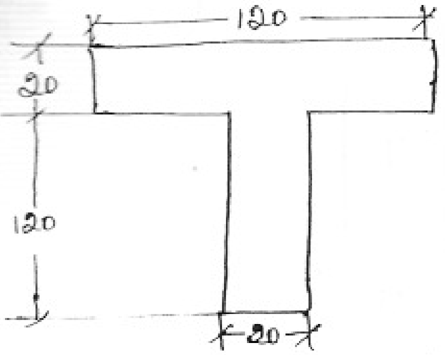
⇒ Centroid of common figures.

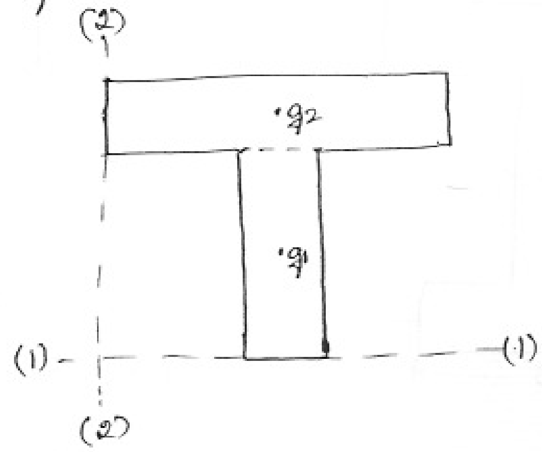
Sl no.	Shape	Area (A)	$\bar{x}$	$\bar{y}$
1)	Square 	$b \times b$	$\frac{b}{2}$	$\frac{b}{2}$
2)	Rectangle 	$b \times d$	$\frac{b}{2}$	$\frac{d}{2}$

Sl no.	Shape	Area (A)	$\bar{x}$	$\bar{y}$
3)	Circle 	$\frac{\pi d^2}{4} = \pi r^2$	$\frac{d}{2} = r$	$\frac{d}{2} = r$
4)	Triangle 	$\frac{1}{2} b h$	—	a) $h/3$ from ba b) $2h/3$ from ap
5)	Semi circle 	$\frac{\pi r^2}{2}$	0	$\frac{4R}{3\pi}$
6)	Quarter circle 	$\frac{\pi r^2}{4}$	$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$
7)	Sector of a circle 	αR^2	$\frac{2R\sin\alpha}{3\alpha}$	0

→ Problems on centre of gravity.

1) Locate the centroid of the T-section shown in figure (mm)





→ The figure is symmetrical about Y-Y axis.

→ Divide the figure into simple figure & consider the reference (1)-(1) & (2)-(2) as shown.

Component	Area [a] (mm²)	Distance of centroid from (1)-(1) [y] (mm)	Moment of area abt (1)-(1) [ay] (mm³)
g_1 (□ᴿ)	$120 \times 20 = 2400$	$\frac{120}{2} = 60$	144000
g_2 (□ᴿ)	$120 \times 20 = 2400$	$120 + \frac{20}{2} = 130$	312000
	$\Sigma a = 4800$		$\Sigma ay = 456000$

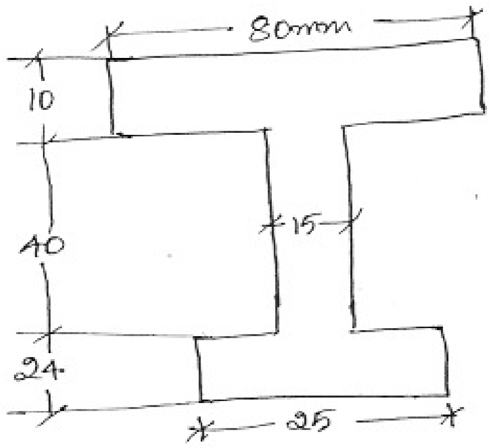
→ $\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{456000}{4800} = \underline{95mm}$.

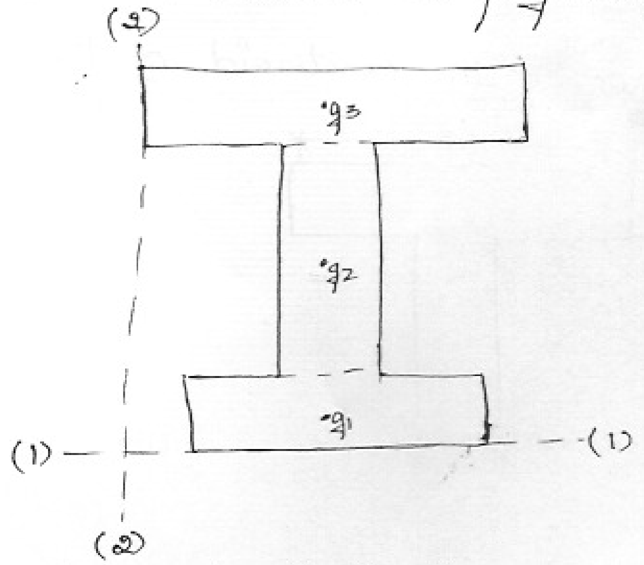
Component	Area [a] (mm²)	Distance of centroid from (2)-(2) [x] (mm)	Moment of area abt (2)-(2) [ax] (mm³)
g_1 (□ᴿ)	$120 \times 20 = 2400$	$\frac{120}{2} = 60$	144000
g_2 (□ᴿ)	$120 \times 20 = 2400$	$\frac{120}{2} = 60$	144000
	$\Sigma a = 4800$		$\Sigma ax = 288000$

→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{288000}{4800} = \underline{60mm}$

→ ∴ The co-ordinates of centroid of given T-section are $(\bar{X}, \bar{Y}) = (60mm, 90mm)$

2) Locate the centroid of I-section shown in figure (mm)





→ The figure is symmetrical about Y-Y axis

→ Divide the figure into simple figure & consider the reference (1)-(1) & (2)-(2) as shown.

Component	Area [a] (mm^2)	Dist of centroid from (1)-(1) [y] (mm)	Moment of area abt (1)-(1) [ay] (mm^3)
g_1 (▭)	$25 \times 24 = 600$	$\frac{24}{2} = 12$	7200
g_2 (▭)	$15 \times 40 = 600$	$24 + \frac{40}{2} = 44$	26400
g_3 (▭)	$80 \times 10 = 800$	$24 + 40 + \frac{10}{2} = 69$	55200
	$\Sigma a = 2000$		$\Sigma ay = 88800$

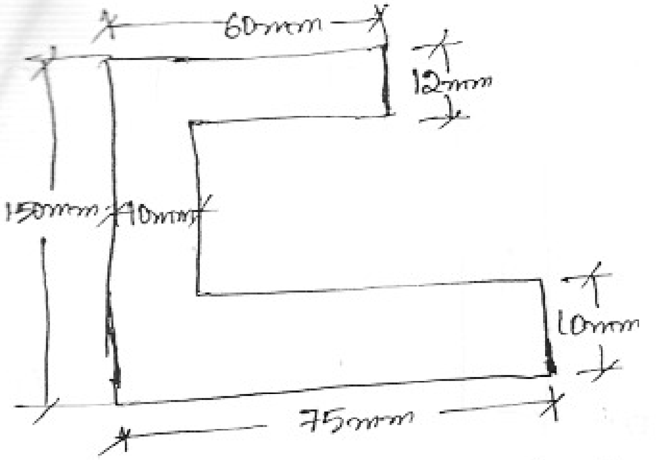
→ $\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{88800}{2000} = \underline{44.4 mm}$

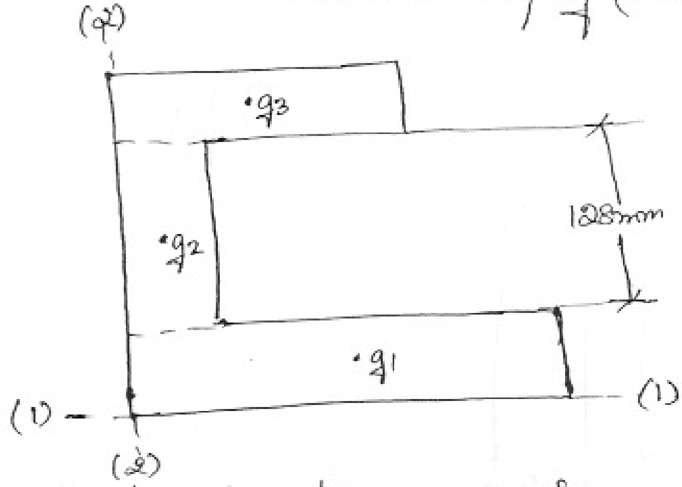
Component	Area [a] (mm^2)	Dist of centroid from (2)-(2) [x] (mm)	Moment of area abt (2)-(2) [ax] (mm^3)
g_1 (▭)	$25 \times 24 = 600$	$\frac{80}{2} = 40$	24000
g_2 (▭)	$15 \times 40 = 600$	$\frac{80}{2} = 40$	24000
g_3 (▭)	$80 \times 10 = 800$	$\frac{80}{2} = 40$	32000
	$\Sigma a = 2000$		$\Sigma ax = 8000$

→ $\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{80000}{2000} = \underline{40 mm}$

→ ∴ The co-ordinates of centroid of given I-section are $(\bar{x}, \bar{y}) = (40mm, 44.4mm)$

3) Locate the centroid of the 'C' section shown in fig (mm)





→ The figure is not symmetrical about any axis

→ Divide the figure into simple figure & consider the reference (1)-(1) & (2)-(2) as shown

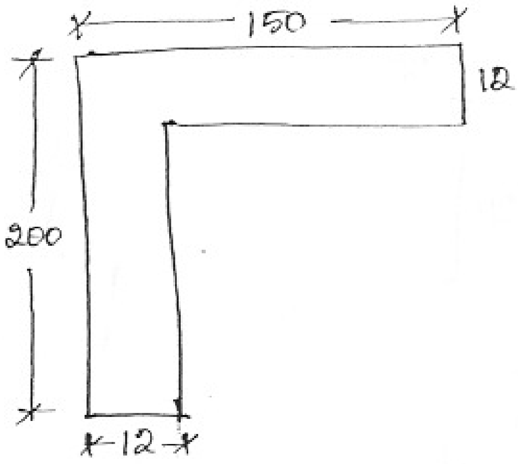
Component	Area [a] (mm²)	Centroid dist from (1)-(1) [y] (mm)	Centroid dist from (2)-(2) [x] (mm)	Moment of area about (1)-(1) [ay] (mm³)	Moment of area about (2)-(2) [ax] (mm³)
g_1 (□le)	$75 \times 10 = 750$	$\frac{10}{2} = 5$	$\frac{75}{2} = 37.5$	3750	28125
g_2 (□le)	$128 \times 10 = 1280$	$10 + \frac{128}{2} = 74$	$\frac{10}{2} = 5$	94720	6400
g_3 (□le)	$12 \times 60 = 720$	$150 - \frac{12}{2} = 144$	$\frac{60}{2} = 30$	103680	21600
	$\Sigma a = 2750$			$\Sigma ay = 202150$	$\Sigma ax = 56125$

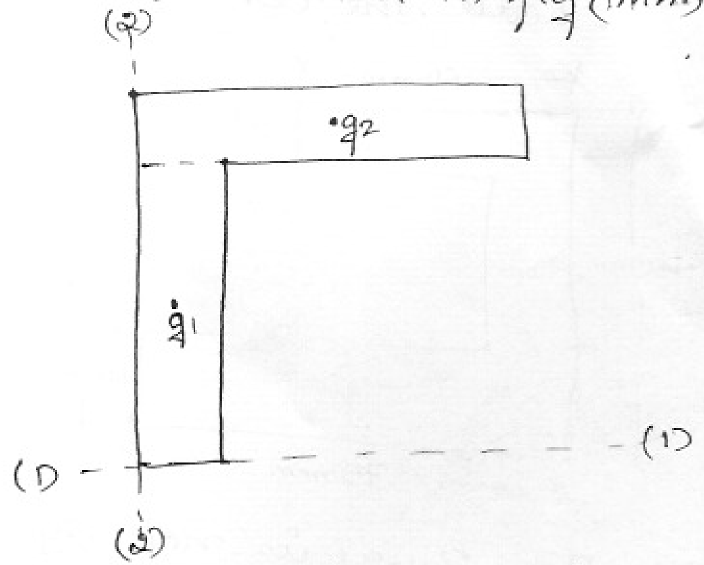
→ $\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{56125}{2750} = \underline{20.41mm}$

$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{202150}{2750} = \underline{73.51mm}$

→ ∴ The co-ordinates of centroid of given C-section are $(\bar{x}, \bar{y}) = (20.41mm, 73.51mm)$

4) Locate the centroid of 'L' section shown in fig (mm)





→ The figure is not symmetrical about any axis

→ Divide the figure into simple figure & consider the reference (1)-(1) & (2)-(2) as shown.

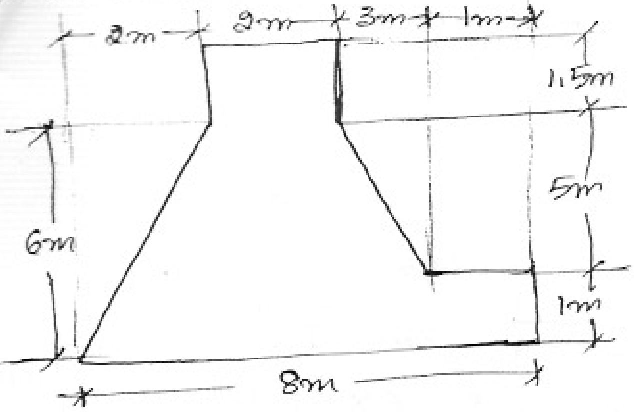
Component	Area [a] (mm^2)	Centroid dist from (1)-(1) [y] (mm)	Centroid dist from (2)-(2) [x] (mm)	Moment of area abt (1)-(1) [ay] (mm^3)	Moment of area abt (2) [ax] (mm^3)
g_1	$188 \times 12 = 2256$	$\frac{188}{2} = 94$	$\frac{12}{2} = 6$	212064	13536
g_2	$150 \times 12 = 1800$	$\frac{12}{2} + 188 = 194$	$\frac{150}{2} = 75$	349200	135000
	$\Sigma a = 4056$			$\Sigma ay = 561264$	$\Sigma ax = 148536$

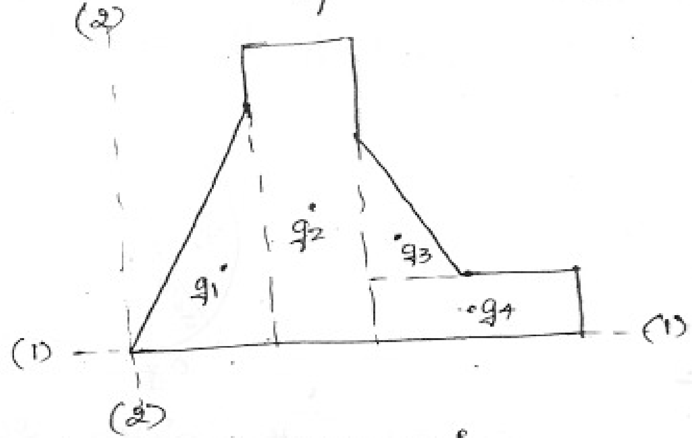
$$\rightarrow \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{148536}{4056} = 36.62 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{561264}{4056} = 138.37 \text{ mm}$$

⇒ ∴ The coordinates of centroid of given L-section are $(\bar{x}, \bar{y}) = (36.62 \text{ mm}, 138.37 \text{ mm})$

5) Determine the centroid of the section of concrete dam





→ The figure is not symmetrical about any axis

→ Divide the figure into simple figure & consider the reference (1) – (1) & (2) – (2) as shown.

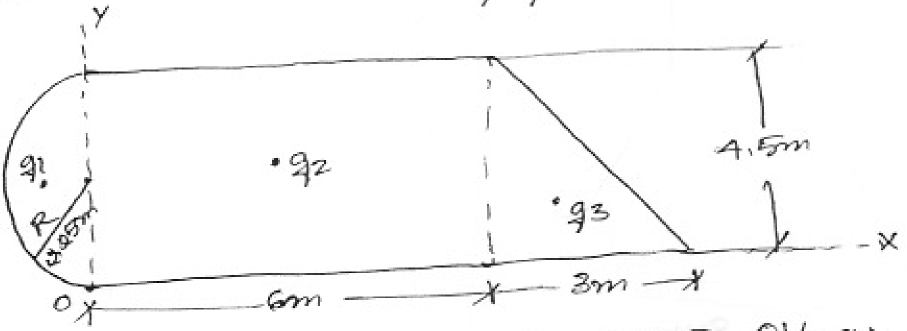
Component	Area [a] (m^2)	Centroidal dist from (1)–(1) [y] (m)	Centroidal dist from (2)–(2) [x] (m)	Moment of area abt (1)–(1) [ay] (m^3)	Moment of area abt (2)–(2) [ax] (m^3)
g_1 (Δ^{le})	$\frac{1}{2} \times 2 \times 6 = 6$	$\frac{6}{3} = 2$	$\frac{2 \times 2}{3} = 1.33$	12	7.98
g_2 (□^{le})	$2 \times 7.5 = 15$	$\frac{7.5}{2} = 3.75$	$2 + \frac{2}{2} = 3$	56.25	45
g_3 (Δ^{le})	$\frac{1}{2} \times 3 \times 5 = 7.5$	$1 + \frac{5}{3} = 2.67$	$2+2+\frac{3}{3} = 5$	20.025	37.5
g_4 (□^{le})	$1 \times 4 = 4$	$\frac{1}{2} = 0.5$	$2+2+\frac{4}{2} = 6$	2	24
	$\Sigma a = 32.5$			$\Sigma ay = 90.275$	$\Sigma ax = 114.4$

$$\rightarrow \bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{114.48}{32.5} = \underline{3.52m}$$

$$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{90.275}{32.5} = \underline{2.77m}$$

→ ∴ The co-ordinates of centroid of given dam section are $(\bar{X}, \bar{Y}) = (3.52m, 2.77m)$

6) Locate the centroid of the area shown in figure with respect to the axis shown in figure



→ Consider the given reference axis OX & OY as shown

→ The figure is not symmetrical about any axis

→ Divide the figure into simple figures.

Component	Area [a] (m^2)	Centroidal dist from (1)-(1) [y] (m)	Centroidal dist from (2)-(2) [x] (m)	Moment of area abt (1)-(1) [ay] (m^3)	Moment of area abt (2)-(2) [ax] (m^3)
g_1 (□^{le})	$\frac{\pi R^2}{2} = 7.95$	$\frac{4.5}{2} = 2.25$	$-\frac{4R}{3\pi} = -0.954$ (-ve ∵ g_1 lie on left side of axis)	17.89	-7.58
g_2 (▭^{le})	$6 \times 4.5 = 27$	$\frac{4.5}{2} = 2.25$	$\frac{6}{2} = 3$	60.75	81
g_3 (Δ^{le})	$\frac{1}{2} \times 3 \times 4.5 = 6.75$	$\frac{4.5}{3} = 1.5$	$6 + \frac{3}{3} = 7$	10.125	47.25
	$\Sigma a = 41.7$			$\Sigma ay = 88.77$	$\Sigma ax = 120.67$

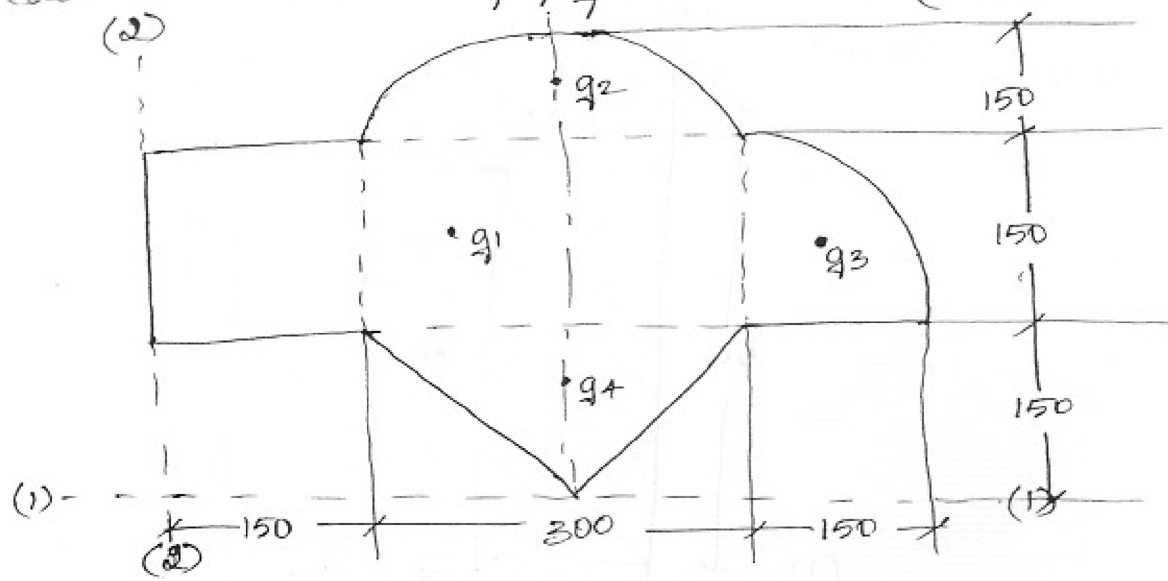
→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{120.67}{41.7} = \underline{\underline{2.89m}}$

$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{88.77}{41.7} = \underline{\underline{2.13m}}$

→ ∴ The co-ordinates of centroid of given section are

$(\bar{X}, \bar{Y}) = (2.89m \times 2.13m)$

7) Locate the centroid of figure shown. (mm)



→ The figure is not symmetrical about any axis.

→ Divide the figure into simple figure & consider the reference (1) - (1) & (2) - (2) as shown.

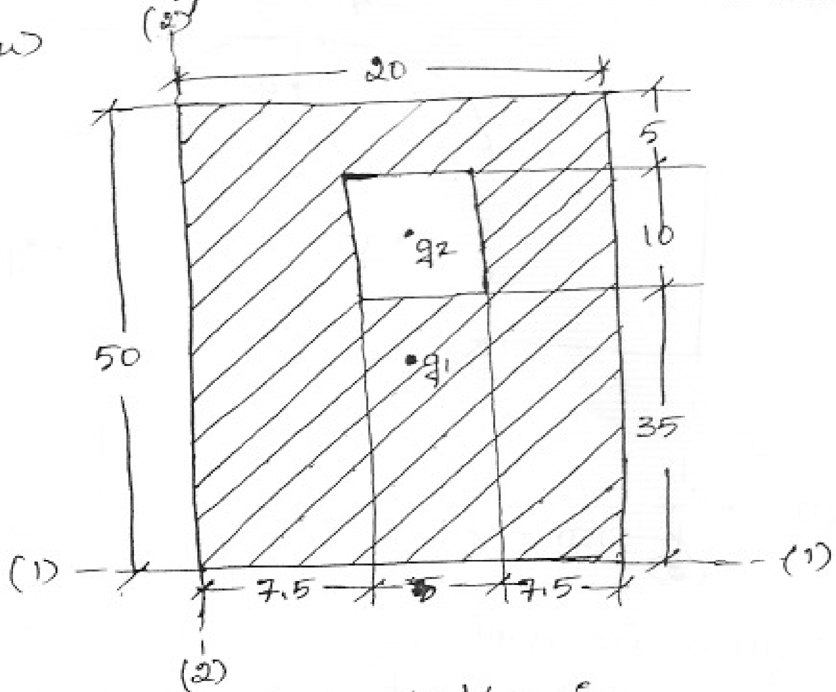
Component	Area [a] (mm^2)	Centroid dist from (1) - (1) [y] (mm)	Centroid dist from (2) - (2) [x] (mm)	Moment of area abt (1) - (1) [ay] (mm^3)	Moment of area abt (2) - (2) [ax] (mm^3)
g1 (▭ʳ)	$450 \times 150 = 67500$	$150 + \frac{150}{2} = 225$	$\frac{450}{2} = 225$	1.52×10^7	1.52×10^7
g2 (◠ʳ)	$\frac{\pi 150^2}{2} = 35343$	$150 + 150 + \frac{4 \times 150}{3\pi} = 363.6$	$150 + \frac{300}{2} = 300$	1.285×10^7	1.06×10^7
g3 (◔ʳ)	$\frac{\pi 150^2}{4} = 17671.5$	$150 + \frac{4 \times 150}{3\pi} = 213.6$	$450 + \frac{4 \times 150}{3\pi} = 513.6$	3.775×10^6	9.076×10^6
g4 (▽ʳ)	$\frac{1}{2} \times 300 \times 150 = 22500$	$\frac{2 \times 150}{3} = 100$	$150 + \frac{300}{2} = 300$	2.25×10^6	6.75×10^6
	$\Sigma a = 143014.5$			$\Sigma ay = 3.408 \times 10^7$	$\Sigma ax = 4.163 \times 10^7$

→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{4.163 \times 10^7}{143014.5} = \underline{291 \text{ mm}}$

$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{3.408 \times 10^7}{143014.5} = \underline{238.3 \text{ mm}}$

→ ∴ The co-ordinates of centroid of given section are $(\bar{X}, \bar{Y}) = (291, 238.3)$ mm

HW 8) Locate the centroid of the shaded area shown in figure (mm)



→ The figure is symmetrical about Y axis.

→ Consider the reference axis (1)-(1) & (2)-(2)

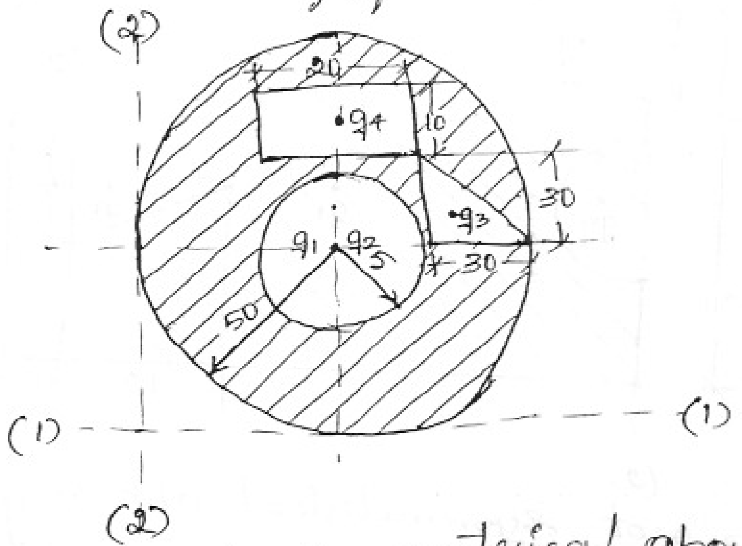
Component	Area [a] (mm^2)	Centroid dist from (1)-(1) [y] (mm)	Centroid dist from (2)-(2) [x] (mm)	Moment of area abt (1)-(1) [ay] (mm^3)	Moment of area abt (2)-(2) [ax] (mm^3)
g_1 (□^le)	$20 \times 50 = 1000$	$\frac{50}{2} = 25$	$\frac{20}{2} = 10$	25000	10000
Deduction g_2 (□^le)	$10 \times 5 = 50$	$35 + \frac{10}{2} = 40$	$7.5 + \frac{5}{2} = 10$	2000	500
	$\Sigma a = 1050$			$\Sigma ay = 23000$	$\Sigma ax = 10500$

→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{10500}{1050} = 10\ mm$

$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{23000}{1050} = 24.21\ mm$

→ ∴ The co-ordinates of the centroid of shaded area are $(\bar{X}, \bar{Y}) = (10mm, 24.21mm)$

9) Find the co-ordinates of the centroid of the shaded area as shown in figure. (mm)



→ The figure is not symmetrical about any axis

→ Consider the reference (1)-(1) & (2)-(2) as shown.

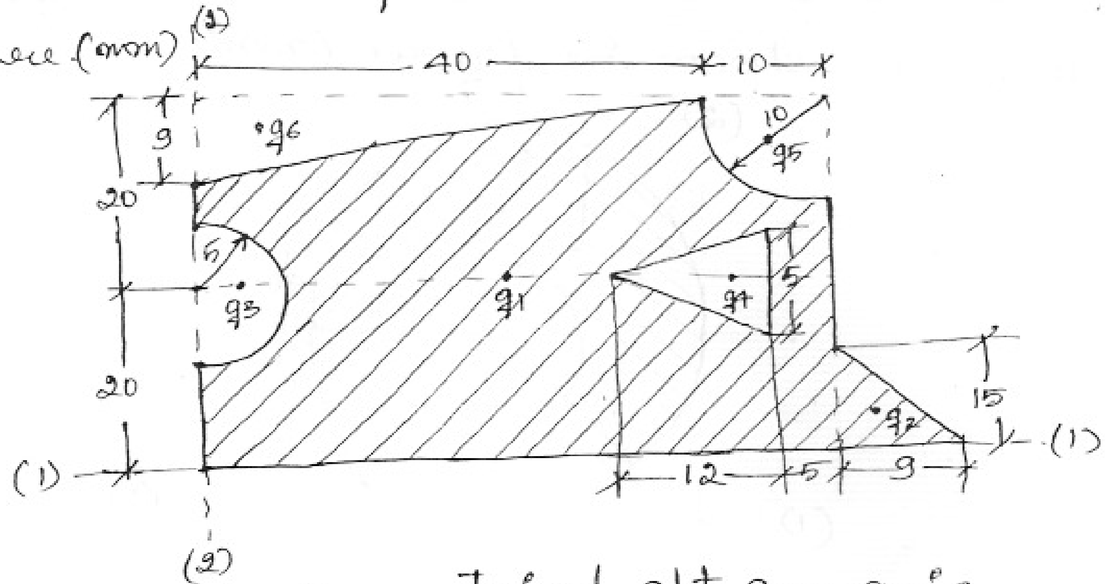
Component	Area [a] (mm^2)	Centroid dist from (1)-(1) [y] (mm)	Centroid dist from (2)-(2) [x] (mm)	Moment of area abt (1)-(1) [ay] (mm^3)	Moment of area abt (2)-(2) [ax] (mm^3)
g1 (big cle) r = 50mm	$\pi 50^2 = 7964$	$\frac{100}{2} = 50$	$\frac{100}{2} = 50$	392700	392700
Deductions g2 (small cle) r = 5mm	$\pi \times 5^2 = 78.54$	50	50	3927	3927
g3 (Δ^{le})	$\frac{1}{2} \times 30 \times 30 = 450$	$50 + \frac{30}{3} = 60$	$50+20+\frac{30}{3} = 80$	27000	36000
g4 (▭^{le})	$20 \times 10 = 200$	$50+30+10/2 = 85$	$\frac{100}{2} = 50$	17000	10000
	$\Sigma a = 7125.46$			$\Sigma ay = 344773$	$\Sigma ax = 34277$

$$\rightarrow \bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{342773}{7125.46} = \underline{48.11mm}$$

$$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{344773}{7125.46} = \underline{48.39mm}$$

→ ∴ The co-ordinates of the centroid of the shaded area are $(\bar{X}, \bar{Y}) = (48.11mm, 48.39mm)$

10) Locate the centroid of the shaded area as shown in figure (mm)



→ The figure is not symmetrical abt any axis

→ Consider the reference (1)-(1) & (2)-(2) as shown.

Component	Area [a] (mm^2)	Centroid dist from (1)-(1) [y] (mm)	Centroid dist from (2)-(2) [x] (mm)	Moment of area abt (1)-(1) [ay] (mm^3)	Moment of area abt (2)-(2) [ax] (mm^3)
g_1 (□le)	$50 \times 40 = 2000$	$\frac{40}{2} = 20$	$\frac{50}{2} = 25$	40000	50000
g_2 (Δle)	$\frac{1}{2} \times 9 \times 15 = 67.5$	$\frac{15}{3} = 5$	$50 + \frac{9}{3} = 53$	337.5	3577.5
Deductions g_3 (D le)	$\frac{\pi 5^2}{2} = 39.27$	20	$\frac{4 \times 5}{3\pi} = 2.12$	785.4	53.25
g_4 (◁le)	$\frac{1}{2} \times 5 \times 12 = 30$	20	$50 - 5 - \frac{12}{3} = 41$	600	1230
g_5 (◻le)	$\frac{\pi 10^2}{4} = 78.54$	$40 - \frac{4 \times 10}{3\pi} = 35.76$	$50 - \frac{4 \times 10}{3\pi} = 45.76$	2808.6	3594
g_6 (◺le)	$\frac{1}{2} \times 9 \times 40 = 180$	$40 - \frac{9}{3} = 37$	$\frac{40}{3} = 13.33$	6660	2400
	$\Sigma a = 1739.69$			$\Sigma ay = 24983.5$	$\Sigma ax = 46270.$..

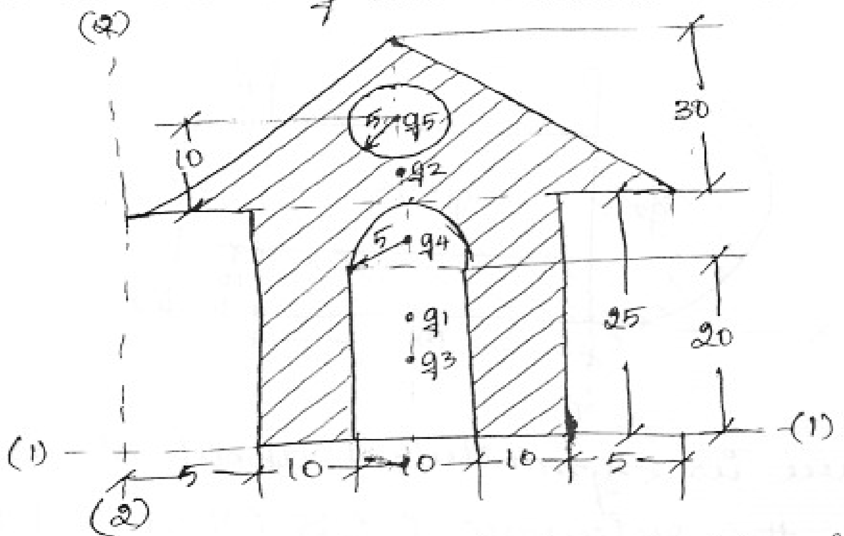
$$\rightarrow \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{46270.25}{1739.69} = \underline{26.6 mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{24983.5}{1739.69} = \underline{14.36 mm}$$

→ ∴ The co-ordinates of centroid of shaded area are $(\bar{x}, \bar{y}) = (26.6 mm, 14.36 mm)$

HW:

(1) Locate the centroid of the shaded area shown in fig.



→ The figure is symmetrical about Y axis

→ Consider the reference (1)-(1) & (2)-(2) as shown

Component	Area [a] (mm^2)	Centroid dist from (1)-(1) [y] (mm)	Centroid dist from (2)-(2) [x] (mm)	Moment of area abt (1)-(1) [ay] (mm^3)	Moment of area abt (2)-(2) [ax] (mm^3)
g1 (□le)	$30\times25 = 750$	$\frac{25}{2} = 12.5$	$5+\frac{30}{2} = 20$	9375	15000
g2 (Δle)	$\frac{1}{2}\times40\times30 = 600$	$25+\frac{30}{3} = 35$	$\frac{40}{2} = 20$	21000	12000
Deductions					
g3 (□le)	$10\times20 = 200$	$\frac{20}{2} = 10$	$15+\frac{10}{2} = 20$	2000	4000
g4 (◠le)	$\frac{\pi 5^2}{2} = 39.3$	$20+\frac{4\times5}{3\pi} = 22.12$	$15+\frac{10}{2} = 20$	869.3	786
g5 (○le)	$\pi 5^2 = 78.5$	$25+10 = 35$	$15+\frac{10}{2} = 20$	2748.9	1570
	$\Sigma a = 1032.2$			$\Sigma ay = 24756.8$	$\Sigma ax = 2064$

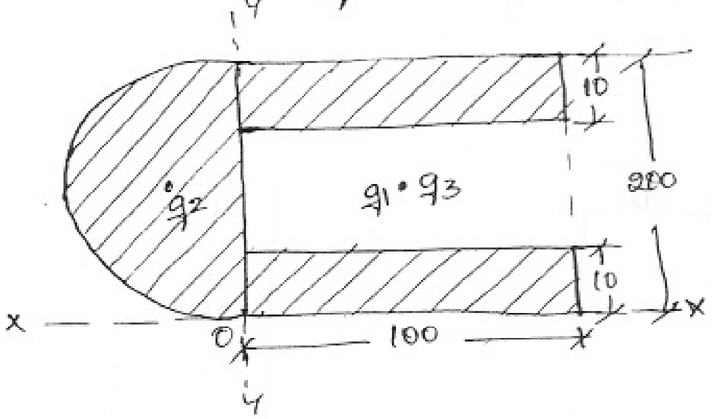
$$\rightarrow \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{20644}{1032.2} = \underline{\underline{20\,mm}}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{24756.8}{1032.2} = \underline{\underline{23.99\,mm}}$$

→ ∴ The co-ordinates of centroid of shaded area are

$(\bar{x}, \bar{y}) = (20\,mm, 23.99\,mm)$

12) Locate the centroid of shaded area w.r.t O (mm).



→ The figure is symmetrical about X-axis

→ Consider the reference OX & OY as shown in figure & divide the figure into simple figures.

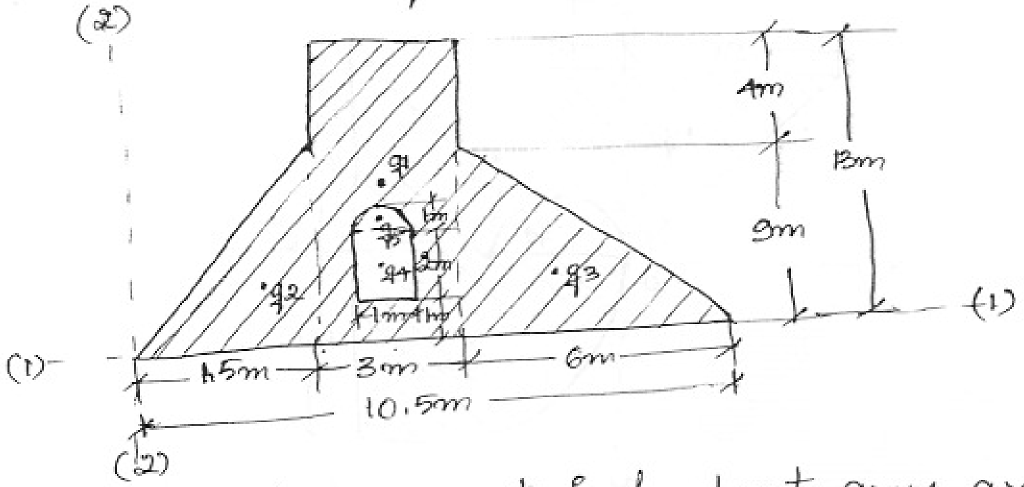
Component	Area [a] (mm^2)	Centroid dist from OX [y] (mm)	Centroid dist from OY [x] (mm)	Moment of area abt OX [ay] (mm^3)	Moment of area abt OY [ax] (mm^3)
g_1 (▭^{le})	$100 \times 200 = 20000$	$\frac{200}{2} = 100$	$\frac{100}{2} = 50$	2000000	1000000
g_2 (◖^{le})	$\frac{\pi(100)^2}{2} = 15708$	$\frac{200}{2} = 100$	$-\frac{4 \times 100}{3\pi} = -42.4$	1570800	−666019.
Deductions g_3 (▭^{le})	$100 \times 180 = 18000$	$\frac{200}{2} = 100$	$\frac{100}{2} = 50$	1800000	900000
	$\Sigma a = 17708$			$\Sigma ay = 1770800$	$\Sigma ax = -566019$

→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{-566019.2}{17708} = \underline{-31.96 mm}$

$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{1770800}{17708} = \underline{100 mm}$

→ ∴ The co-ordinates of centroid of the shaded area are $(\bar{X}, \bar{Y}) = (-31.96 mm, 100 mm)$

13) : Locate the centroid of hatched area



→ The figure is not symmetrical about any axis.

→ Consider the reference (1)-(1) & (2)-(2) as shown.

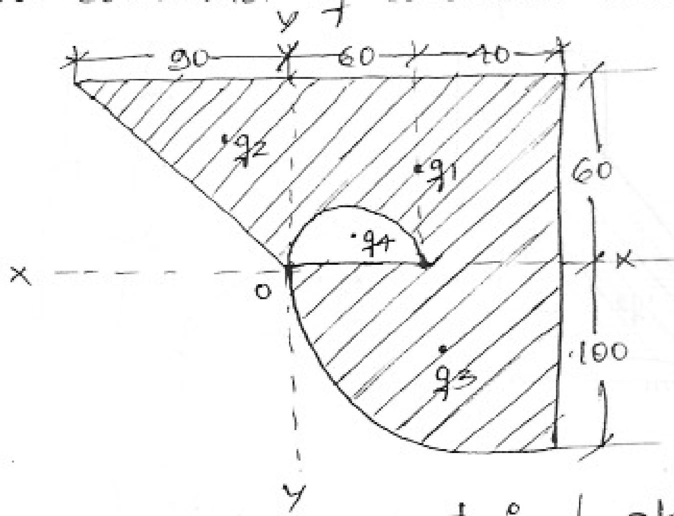
Component	Area [a] (m^2)	Centroid dist from (1)-(1) [y] (m)	Centroid dist from (2)-(2) [x] (m)	Moment of area abt (1)-(1) [ay] (m^3)	Moment of area abt (2)-(2) [ax] (m^3)
a_1 (□^{le})	$13\times3 = 39$	$\frac{13}{2} = 6.5$	$1.5+\frac{3}{2} = 3$	253.5	117
a_2 ($\triangle^{le}$)	$\frac{1}{2}\times1.5\times9 = 6.75$	$\frac{9}{3} = 3$	$\frac{1.5}{3}\times2 = 1$	20.25	6.75
a_3 ($\triangle^{le}$)	$\frac{1}{2}\times6\times9 = 27$	$\frac{9}{3} = 3$	$4.5+\frac{6}{3} = 6.5$	81	175.5
Deductions					
a_4 (□^{le})	$1\times2 = 2$	$1+\frac{2}{2} = 2$	$1.5+\frac{3}{2} = 3$	4	6
a_5 (◠^{le})	$\frac{\pi(0.5)^2}{2} = 0.39$	$3+\frac{4\times0.5}{3\pi} = 3.21$	$1.5+\frac{3}{2} = 3$	1.25	1.17
	$\Sigma a = 70.36$			$\Sigma ay = 349.5$	$\Sigma ax = 292.0$

$$\rightarrow \bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{292.08}{70.36} = \underline{4.15m}$$

$$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{349.5}{70.36} = \underline{4.97m}$$

→ ∴ The co-ordinates of centroid of hatched area are $(\bar{X}, \bar{Y}) = (4.15m, 4.97m)$

14) Locate the centroid of shaded area shown (mm)



→ The figure is not symmetrical about any axis

→ Consider the reference OY & OX as shown in figure & divide the given figure in to simple figures

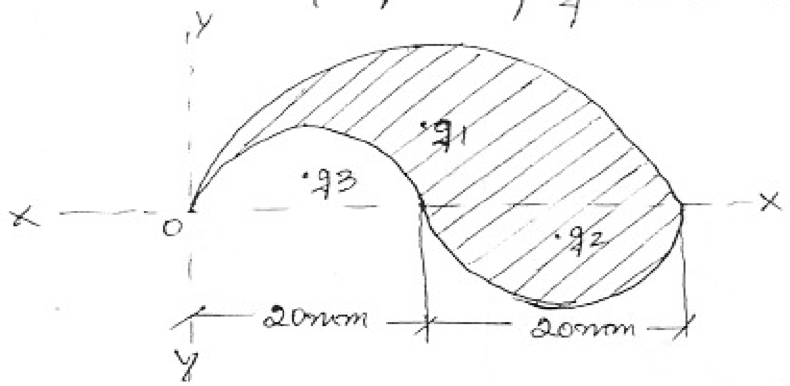
Component	Area [a] (mm^2)	Centroid dist' from OY [y] (mm)	Centroid dist' from OX [x] (mm)	Moment of area abt OY [ay] (mm^3)	Moment of area abt OX [ax] (mm^3)
g1 (▭le)	60×100 = 6000	$\frac{60}{2} = 30$	$\frac{100}{2} = 50$	180000	300000
g2 (◸le)	½×90×60 = 2700	$\frac{2\times60}{3} = 40$	$-\frac{90}{3} = -30$ (-ve ∵ x is on LHS of O)	108000	-81000
g3 (◺le)	$\frac{\pi(100)^2}{4} = 7854$	$-\frac{4\times100}{3\pi} = -42.4$ (-ve ∵ y is @ bottom of O)	$100-\frac{4\times100}{3\pi} = 57.6$	-333009.6	452390.4
Deduction g4 (◠le)	$\frac{\pi(30)^2}{2} = 1413.7$	$\frac{4\times30}{3\pi} = 12.72$	$\frac{60}{2} = 30$	17982.3	42411
	Σa = 15140.3			Σay = -62991.9	Σax = 628979.4

→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{628979.4}{15140.3} = \underline{41.54\,mm}$

$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{-62991.9}{15140.3} = \underline{-4.16\,mm}$

→ ∴ The coordinates of centroid of shaded area are $(\bar{X}, \bar{Y}) = (41.54\,mm, -4.16\,mm)$

15) Locate the C.G of the figure shown hatched



→ The figure is not symmetrical about any axis

→ Consider the reference OX & OY as shown

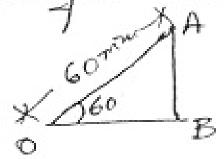
Components	Area [a] (mm^2)	Centroid dist from OX [y] (mm)	Centroid dist from OY [x] (mm)	Moment of area abt OX [ay] (mm^3)	Moment of area abt OY [ax] (mm^3)
a_1 (◠^{le})	$\frac{\pi(20)^2}{2}$ = 628.32	$\frac{4 \times 20}{3\pi}$ = 8.48	20	5382.14	12566.4
a_2 (◡^{le})	$\frac{\pi(10)^2}{2}$ = 157.08	$\frac{-4 \times 10}{3\pi}$ = −4.24 (−ve ∵ y is @ bottom of O)	$20 + \frac{20}{2} = 30$	−666.02	4712.4
Deductions a_3 (◠^{le})	$\frac{\pi(10)^2}{2}$ = 157.08	$\frac{4 \times 10}{3\pi}$ = 4.24	$\frac{20}{2} = 10$	666.02	1570.8
	$\Sigma a = 628.32$			$\Sigma ay = 4050.1$	$\Sigma ax = 1570$

→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{15708}{628.32} = 25mm$

$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{4050.1}{628.32} = 6.45mm$

→ ∴ The co-ordinates of centroid of the figure shown hatched are $(\bar{X}, \bar{Y}) = (25mm, 6.45mm)$

16) Determine the centroid of the shaded area bounded by a circular arc to a total $2\alpha = 60°$ & a straight vertical line AB as shown in figure.

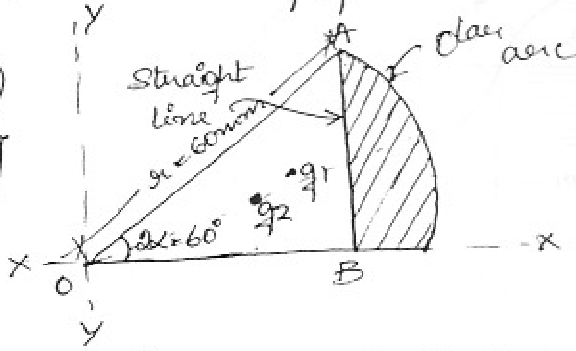


$AB = 60 \sin 60 = \underline{51.96}$

$OB = 60 \cos 60 = \underline{30mm}$

{ α is in radians
$\therefore \alpha \times \pi/180°$ }

Here $\alpha = 0.523$



→ The figure is not symmetrical about any axis

→ Consider the reference OX & OY as shown & divide the figure into simple figures.

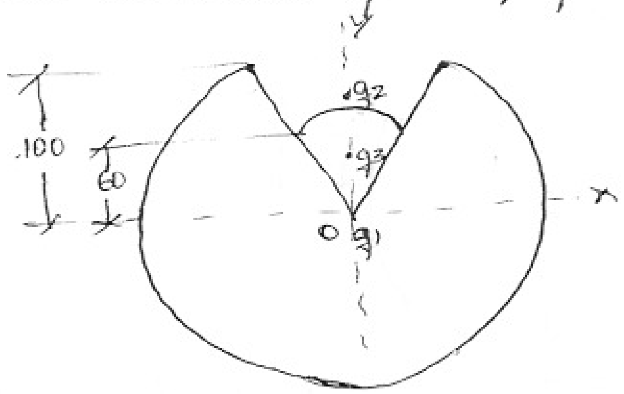
Component	Area [a] (mm^2)	Centroid dist from OX [y] (mm)	Centroid dist from OY [x] (mm)	Moment of area abt OX [ay] (mm^3)	Moment of area abt OY [ax] (mm^3)
g_1 (sector)	$\frac{30 \times \pi \times 60^2}{180} = 1884.96$	$\left[\frac{2}{3} \times \frac{\sin 30}{0.523} \times 60\right] \times \sin 30 = 19.12$	$\left[\frac{2}{3} \times \frac{\sin 30}{0.523} \times 60\right] \times \cos 30 = 33$	36002.74	62203.68
Deduction g_2 (triangle)	$\frac{1}{2} \times 30 \times 51.96 = 779.42$	$\frac{51.96}{3} = 17.32$	$\frac{2}{3} \times 30 = 20$	13499.55	15588.4
	$\Sigma a = 1108.54$			$\Sigma ay = 22503.19$	$\Sigma ax = 46615$

$$\rightarrow \bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{46615.28}{1108.54} = \underline{42.05mm}$$

$$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{22503.19}{1108.54} = \underline{20.3mm}$$

→ $\therefore$ The co-ordinates of centroid of the shaded area are $(\bar{X}, \bar{Y}) = (42.05mm, 20.3mm)$

7) Locate the centroid of the figure shown. (mm)



→ The figure is symmetrical about the given axis

→ Consider the reference OX & OY as shown & divide the figure into simple figures.

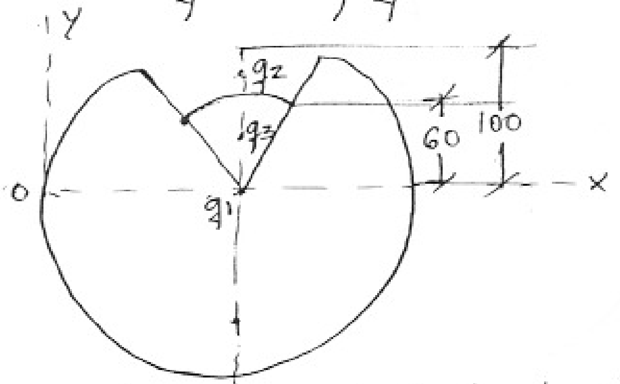
Component	Area [a] (mm²)	Centroid dist from OX [y] (mm)	Centroid dist from OY [x] (mm)	Moment of area abt OX [ay] (mm³)	Moment of area abt OY [ax] (mm³)
g_1 (Ok)	$\pi \times (100)^2 = 31415.9$	0	0	0	0
(−) g_2 (big Deg)	$\frac{30 \times \pi}{180} \times 100^2 = 5235.9$	$\frac{2}{3} \times 100 \times \frac{\sin 30}{0.523} = 63.66$	0	333317.39	0
Deduction g_3 (small Deg) (+)	$\frac{30 \times \pi}{180} \times 60^2 = 1882.8$	$\frac{2}{3} \times 60 \times \frac{\sin 30}{0.523} = 38.24$	0	71998.27	0
	$\Sigma a = 28062.8$			$\Sigma ay = 261319.12$	$\Sigma ax = 0$

→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{0}{28062.8} = \underline{0\,mm}$

→ $\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{261319.12}{28062.8} = \underline{9.31\,mm}$

→ ∴ The coordinates of centroid of the ~~shaded area~~ figure shown are $(\bar{X}, \bar{Y}) = (0\,mm, 9.31\,mm)$

HW 17) Locate the centroid of the figure shown (mm)



→ The figure is not symmetrical about the given axis

→ Consider the reference OX & OY as shown & divide the figure into simple figures.

Component	Area [a] (mm^2)	Centroid dist from OX [y] (mm)	Centroid dist from OY [x] (mm)	Moment of area abt OX [ay] (mm^3)	Moment of area abt OY [ax] (mm^3)
g_1 (ck)	$\pi \times (100)^2 = 31415.9$	0	100	0	3141590
g_2 (big Dei)	$\frac{30 \times \pi}{180} \times 100^2 = 5235.9$	$\frac{2}{3} \times 100 \times \frac{\sin 30}{0.523} = 63.66$	100	333317.39	523590
reduction g_3 (small Dei)	$\frac{30 \times \pi}{180} \times 60^2 = 1882.8$	$\frac{2}{3} \times 60 \times \frac{\sin 30}{0.523} = 38.24$	100	71998.27	188280
	$\Sigma a = 28062.8$			$\Sigma ay = 261319.12$	$\Sigma ax = 347691$

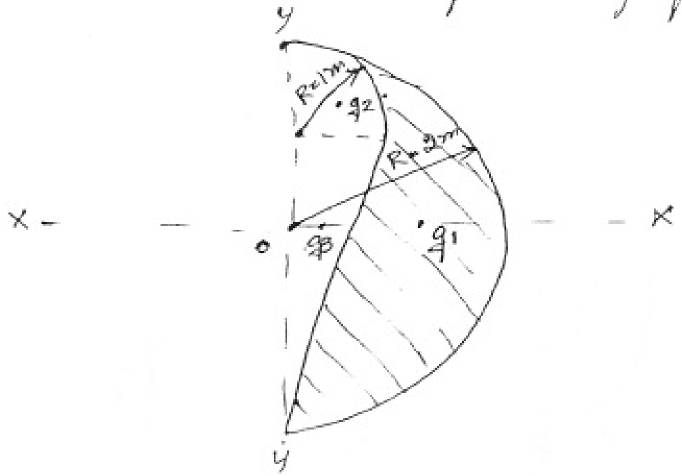
→ $\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{3476900}{28062.8} = \underline{9.31 mm}$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{261319.12}{28062.8} = \underline{123.89 mm}$.

→ ∴ The co-ordinates of centroid of the figure shown are

$(\bar{y}, \bar{x}) = (9.31mm, 123.89mm)$

18). Locate the centroid of the figure shown.



→ The figure is not symmetrical about the given axi

→ Consider the reference OX & OY as shown & divide the figure into simple figures.

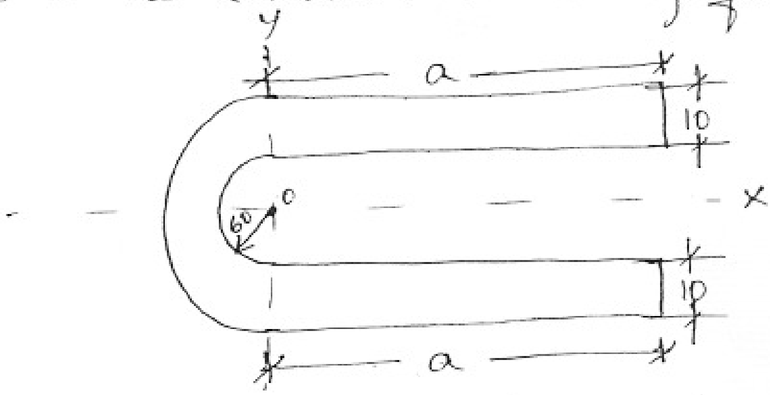
Component	Area [a] (mm²)	Centroid dist from OX [y] (mm)	Centroid dist from OY [x] (mm)	Moment of area abt OX [ay] (mm³)	Moment of a abt OY [ax] (mm³)
g_1 (D^{le})	$\frac{\pi \times (2)^2}{2}$ = 6.28	0	$\frac{4\times 2}{3\pi}$ = 0.848	0	5.325
Deduction g_2 (D^{le})	$\frac{\pi \times (1)^2}{4}$ = 0.785	$1 + \frac{4\times(1)}{3\pi}$ = 1.424	$\frac{4\times 1}{3\pi}$ = 0.424	1.1178	0.3328
g_3 (D^{le})	$\frac{1}{2}\times 3\times 1$ = 1.5	0	$\frac{1}{3}$ = 0.333	0	0.499
	Σa = 3.995			Σay = −1.1178	Σax = 4.49

$$\rightarrow \bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{4.4927}{3.995} = \underline{1.124m}$$

$$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{-1.1178}{3.995} = \underline{-0.28m}$$

→ ∴ The co-ordinates of centroid of the shaded area are $(\bar{X}, \bar{Y}) = (1.124m, -0.28m)$

19) Find a such that the centroid is about the axis O as shown in the figure.



→ The figure is symmetrical about X-axis.
→ Consider the reference OX & OY as shown & divide the figure into simple figures.

Component	Area [a] (mm^2)	Centroid dist from OX [y] (mm)	Centroid dist from OY [x] (mm)	Moment of area abt OX [ay] (mm^3)	Moment of area abt OY [ax] (mm^3)
a_1 (rectangle)	$140a$	0	$a/2 = 0.5a$	0	$70a^2$
a_2 (semicircle)	$\frac{\pi(70)^2}{2} = 7696.9$	0	$-\frac{4(70)}{3\pi} = -29.7$	0	-228597
Deduction					
a_3 (rectangle)	$120a$	0	$a/2 = 0.5a$	0	$60a^2$
a_4 (semicircle)	$\frac{\pi(60)^2}{2} = 5654.8$	0	$-\frac{4(60)}{3\pi} = -25.46$	0	$-143971.$
	$\Sigma a = 3882.1$			$\Sigma ay = 0.$	$\Sigma ax =$

→ $\bar{y} = \frac{\Sigma ay}{\Sigma a} = \underline{0mm}$

→ To find $\bar{X}$

$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = 0.$

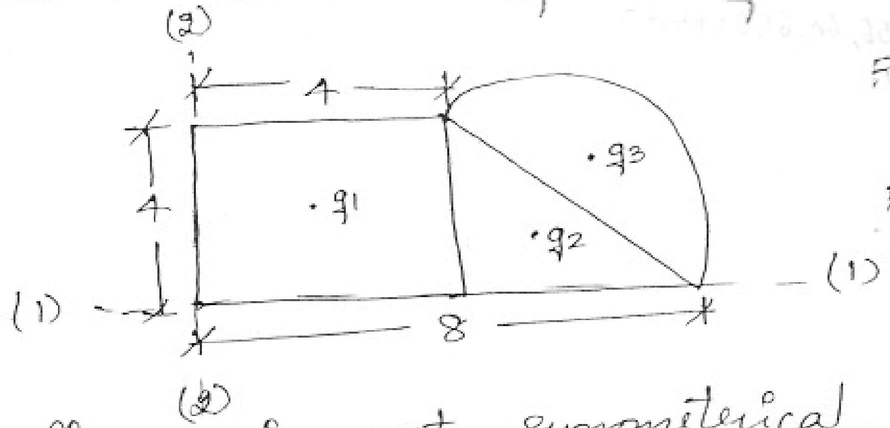
$70a^2 - 228597.93 - 60a^2 + 143971.2 = 0$

$10a^2 = 84626.73$

$\underline{a = 92mm}$

→ $\therefore \bar{X} = \frac{\Sigma ay}{\Sigma a} = \frac{13.27}{3882.1} = \underline{0.00341mm}$

20) Locate the centroid of the given figure (m)



$R = \frac{\sqrt{4^2+4^2}}{2} = \frac{5.656}{2}$

$R = 2.83\,m$

→ The figure is not symmetrical about any axis.
→ Divide the figure into simple figure & consider the reference (1)-(1) & (2)-(2) as shown.

Component	Area [a] (m^2)	Centroid dist from (1)-(1) [y] (m)	Centroid dist from (2)-(2) [x] (m)	Moment of area abt (1)-(1) [ay] (m^3)	Moment of a abt (2)-(2) [ax] (m^3)
g1 (□ler)	$4\times4 = 16$	$\frac{4}{2} = 2$	$\frac{4}{2} = 2$	32	32
g2 (◺le)	$\frac{1}{2}\times4\times4 = 8$	$\frac{4}{3} = 1.33$	$4+\frac{4}{3} = 5.33$	10.64	42.64
g3 (⌓le)	$\frac{\pi(2.82)^2}{2} = 12.49$	$4/2 + \frac{4\times2.82}{3\pi}\times\sin45 = 2.846$	$4+\frac{4}{2}+\frac{4\times2.82}{3\pi}\times\cos45 = 6.846$	35.54	85.50
	$\Sigma a = 36.49$			$\Sigma ay = 78.180$	$\Sigma ax = 160$

→ $\bar{X} = \frac{\Sigma ax}{\Sigma a} = \frac{160.14}{36.49} = \underline{4.388\,m}$

$\bar{Y} = \frac{\Sigma ay}{\Sigma a} = \frac{78.18}{36.49} = \underline{2.142\,m}$

→ ∴ The co-ordinates of centroid of given section are $(\bar{X}, \bar{Y}) = (4.388, 2.142)\,m$.

UNIT - 4

BRD

MOMENT OF INERTIA OF PLANE AREAS (MI (or) I)

→ **Inertia**

It represents the property of matter by the virtue of which it resists any change in its state of rest or of uniform motion.

→ **First moment of area**

It is defined as the product of the area (A) & its $\perp^{r}$ distance (x) to the point of consider (i.e, $A \times x$). The SI unit is m^3.

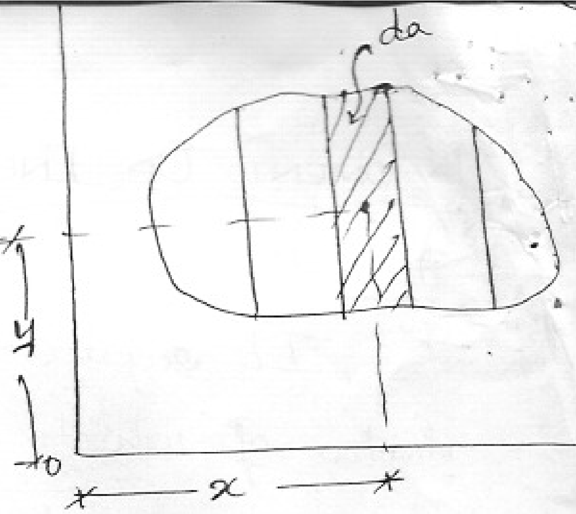
→ **Moment of Inertia (MI or I)**

It is defined as the product of First mome of area (Ax) & its $\perp^{r}$ distance (x) to the point of consideration (i.e, $Ax \times x = Ax^2$). This is also called a Moment of moment of Area (or) Second moment of area. MI plays a very important role in resisting change in rotational motion. This concept can be extended to be applicable to plane figures also. The strength of members subjected to bending depends on MI of its c/s area. The SI unit is m^4.

→ **MI by the method of integration**

- Consider a plane lamina as shown in the fig whose MI is to be found out about X-X axis & Y-Y axis.
- Divide the whole area into a no of strips

- Consider one of the strips (i.e, the shaded strip)
- Let da = Area of shaded strip



x & y = distance of C.G of strip from y-y & x-x axis resp^tly.

- MI of strip abt y-y axis
  $= da\, x^2 \longrightarrow (1)$

  MI of strip abt x-x axis
  $= da\, y^2 \longrightarrow (2)$

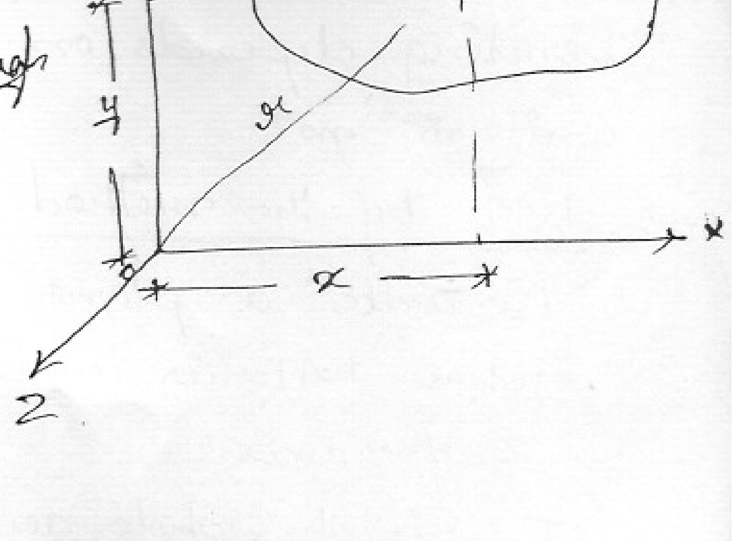
- MI of lamina is found by integrating the abov equns ∴ $I_{yy} = \int da\, x^2 = \Sigma da\, x^2$

  $I_{xx} = \int da\, y^2 = \Sigma da y^2$

⇒ **Polar moment of Inertia** (I_{zz} or I_P or J)

MI about an axis $\perp^{r}$ to the plane of an area is known as polar moment of inertia. The axis $\perp^{r}$ to the xy plane is Z-axis. The SI unit is m^4.

- Consider an area in xy plane as shown.



- MI about $\perp^{r}$ axis (i.e; Z axis) passing through origin O is given by

$I_{zz} = \Sigma da\, r^2$

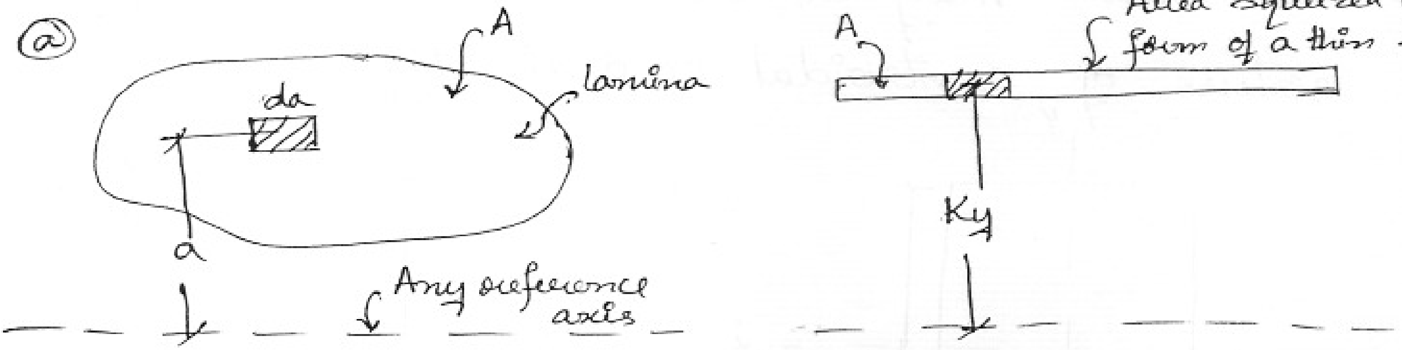
$= \Sigma da\, (x^2+y^2)$

$= \Sigma da x^2 + \Sigma da y^2$

$I_{zz} = I_{yy} + I_{xx}$

Radius of Gyration

It may be defined as the distance from reference axis where the whole area of the body assumed to be squeezed & concentrated so as to not alter the MI about the reference axis.



- Consider a lamina divided into an elemental area "da" @ a distance "a" w.r.t reference axis as sh in figure (a)
- Figure (b) shows the elemental area placed @ a distance "Ky" from reference axis, as if the whole lamina is squeezed in the form of a thin strip. This distance "Ky" is the Radius of Gyration.
- Mathematically

$$K = \sqrt{\frac{I}{A}} \quad (or) \quad I = AK^2$$

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} \; ; \quad K_{yy} = \sqrt{\frac{I_{yy}}{A}}$$

- Polar. Radius of Gyration

$$K_P = \sqrt{\frac{I_{zz} \; (or) \; I_P}{A}}$$

Moment of Inertia (?)

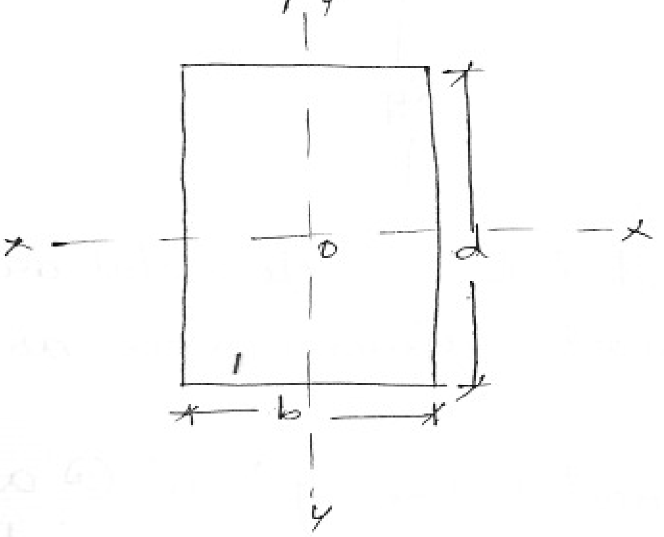
It is the product of an area of the figure & the centroidal co-ordinate of the area w.r.t reference axis.

- ∴ Mathematically

I

I

- It is useful finding in unsymmetrical sec^{ns}
- The value may be either +ve (or) -ve depending values of centroidal co-ordinates

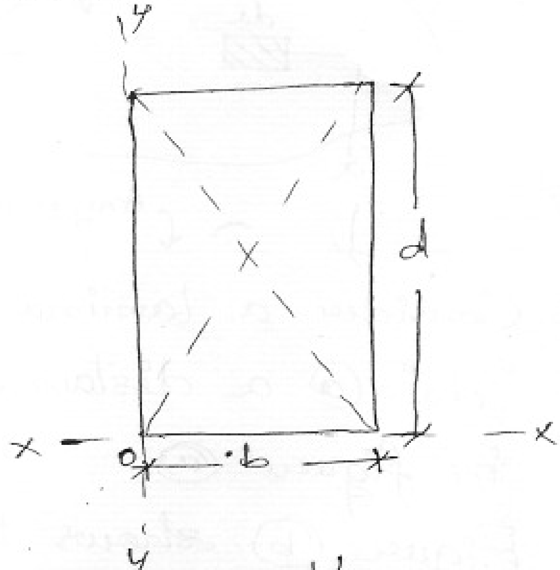


$\bar{x} = 0$

$\bar{y} = 0$

$A = bd$

$I = 0$



$\bar{x} = d/2$

$\bar{y} = b/2$

$A = bd$

$I = b^2d^2/4$

⇒ <u>Theorems of Moment of inertia</u>

1) <u>Parallel Axis theorem</u>

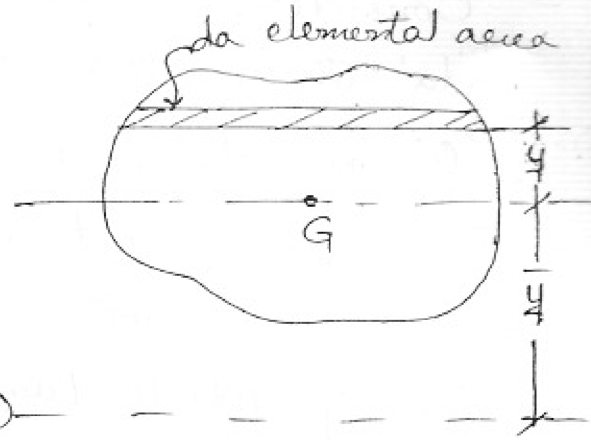
" If MI of a plane area about an axis through its CG is denoted by I_{xx}, then MI of the area about any reference axis ①-① $\parallel^{el}$ to x-axis & @ the distance of $\bar{y}$ from CG is given by $\underline{I_{①-①} = I_{xx} + A\bar{y}^2}$ "

- Consider an elemental area da $\parallel^{el}$ to X-X axis
- Let I_{xx} = MI abt XX axis

- Let I_{1-1} = MI abt ①-① axis

  A = area of the body

  $\bar{y}$ = Distance of CG of the body from ①-①



- Here Ixx & I_{1-1} & elemental area are ||el to each other. Hence it is called ||el axis theorem

- MI of element abt ①-① = $da(\bar{y}+y)^2$

  MI of whole body abt ①-① = $\Sigma da(\bar{y}+y)^2$

$$I_{①-①} = \Sigma da(\bar{y}^2+y^2+2y\bar{y})$$

$$= \Sigma da\bar{y}^2 + \Sigma day^2 + 2\Sigma day\bar{y}$$

$$I_{①-①} = \bar{y}^2\Sigma da + \Sigma day^2 + 2\bar{y}\Sigma day$$

- w.k.t $I_{xx} = \Sigma day^2$

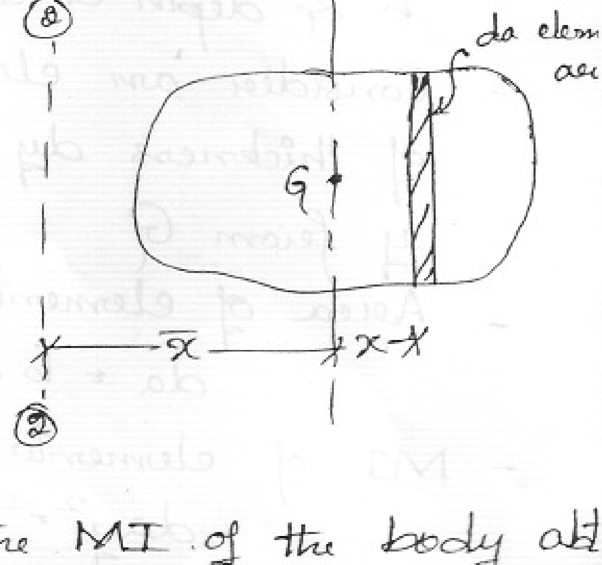
  $0 = \Sigma day$ (i.e, moment of areas abt centroid is zero)

$$\therefore I_{①-①} = A\bar{y}^2 + I_{xx} + 0$$

$$I_{①-①} = I_{xx} + A\bar{y}^2$$

- (III)^{ly} if $I_{②-②}$ axis is ||el to Y-Y axis @ a distance $\bar{x}$ from centroid, then

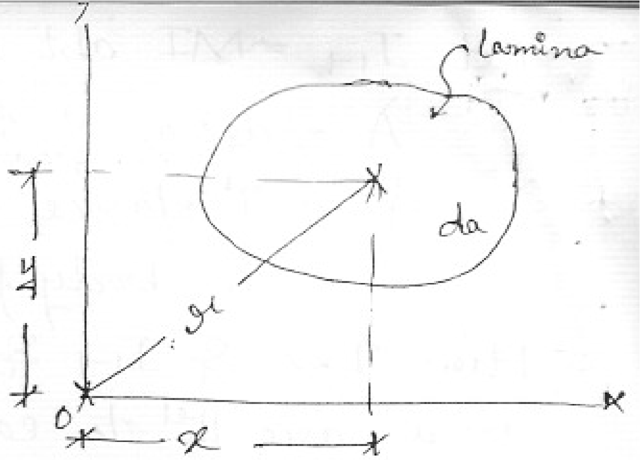
$$I_{②-②} = I_{YY} + A\bar{x}^2$$



→ Perpendicular axis theorem

"If Ixx & Iyy be the MI of the body abt XX & YY axes respectly, then MI of I_{zz} abt ZZ axis is given by $I_{zz} = I_{xx} + I_{yy}$".

- Consider an elemental area 'da' @ a distance "r" from "O" as shown in the figure.



- MI of element abt ZZ axis
  $= da\, r^2$
- MI of whole lamina = I_{zz}

$$I_{zz} = \int da\, r^2$$
$$= \int da\,(x^2+y^2)$$
$$= \int da\, x^2 + \int da\, y^2 \longrightarrow (1)$$

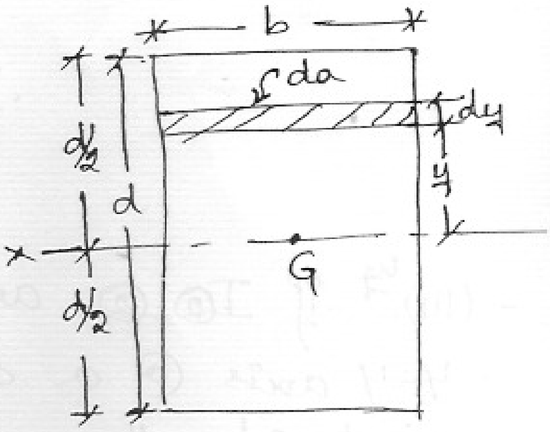
- w.k.t $\int da\, x^2 = I_{yy}$ & $\int da\, y^2 = I_{xx} \longrightarrow (2)$
- $\therefore$ From (1) & (2)

$$I_{zz} = I_{xx} + I_{yy}$$

- Here I_{zz} is also known as Polar MI (I_P)

⟹ MI of regular figures

1) MI of $\square^{le}$



- Consider a $\square^{lar}$ plane of width b & depth d as shown in fig
- Consider an elemental strip of thickness dy @ a distance y from G
- Area of elemental strip

$$da = b\,dy$$

- MI of elemental area abt XX axis

$$da\,y^2 = (b\,dy)\,y^2$$
$$da\,y^2 = b y^2\,dy$$

- MI of whole $\square^{le}$

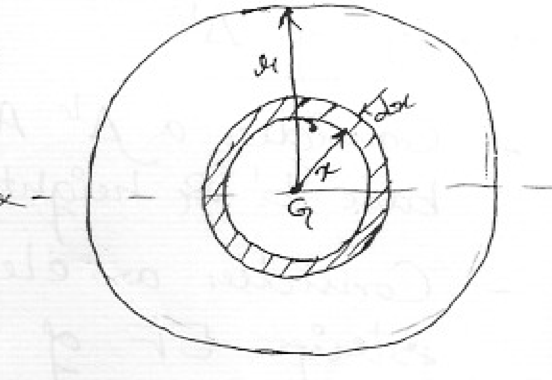
$$I_{xx} = \int_{-d/2}^{d/2} y^2\, b\, dy$$

$$= b\left(\frac{y^3}{3}\right)_{-d/2}^{d/2}$$

$$= b\left(\frac{d^3}{24} + \frac{d^3}{24}\right)$$

$$\therefore I_{xx} = \frac{bd^3}{12} \quad \& \quad I_{yy} = \frac{db^3}{12}$$

2) <u>MI of $\bigcirc^{le}$</u>



- Consider a $\bigcirc^{le}$ with 'r' radius & an elemental ring @ a distance "x" from the centre & of thickness "dx"
- Area of elemental ring

$$da = (2\pi x)\,dx$$

- MI of elemental area from centre

$$da\,x^2 = (2\pi x\,dx)\,x^2$$

$$da\,x^2 = 2\pi x^3\,dx$$

- MI of whole $\bigcirc^{le}$ from centre

$$I_{zz} = \int_0^r 2\pi x^3\,dx$$

$$= 2\pi\left(\frac{x^4}{4}\right)_0^r$$

$$= \frac{2\pi x^4}{4}$$

$$I_{zz} = \frac{\pi x^4}{2} \longrightarrow (1)$$

- w.k.t $I_{zz} = I_{xx} + I_{yy}$

$\quad \& \quad I_{xx} = I_{yy}$

$\therefore$ For plane lamina

$$I_{zz} = 2I_{xx} \longrightarrow (2)$$

- From ① & ②

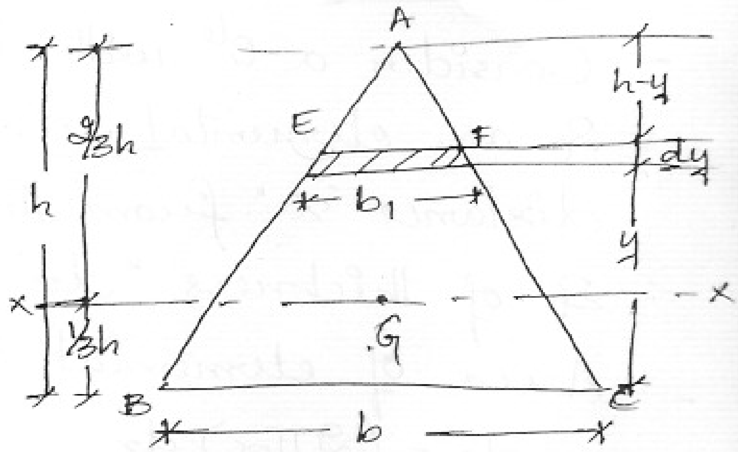
$$2I_{xx} = \frac{\pi r^4}{2}$$

$$I_{xx} = \frac{\pi r^4}{4} \quad \& \quad I_{yy} = \frac{\pi r^4}{4}$$

- w.k.t $d = 2r$

$$\therefore I_{xx} = \frac{\pi d^4}{64} \quad \& \quad I_{yy} = \frac{\pi d^4}{64}$$

3) MI of Δ^{le}



- Consider a Δ^{le} ABC of base 'b' & height 'h'
- Consider an elemental strip EF of thickness 'dy' @ a distance 'y' from base BC
- Δ^{le} ABC & Δ^{le} AEF are (|||)^{lar}

$$\therefore \frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \left(1 - \frac{y}{h}\right)b$$

- Area of element EF

$$da = b_1\, dy$$

$$= \left(1 - \frac{y}{h}\right) b \cdot dy$$

- MI of elemental strip abt base BC

$$= da\, y^2$$

$$= \left\{\left(1 - \frac{y}{h}\right) b\, dy\right\} y^2$$

$$= \left(y^2 - \frac{y^3}{h}\right) b\, dy$$

- MI of whole Δ^{le} abt base BC

$$I_{BC} = b\int_0^h \left(y^2 - \frac{y^3}{h}\right) dy$$

$$= b\left(\frac{y^3}{3} - \frac{y^4}{4h}\right)_0^h$$

$$= b\left(\frac{h^3}{3} - \frac{h^4}{4h}\right)$$

$$= b\left(\frac{h^3}{3} - \frac{h^3}{4}\right)$$

$$I_{base} = \frac{bh^3}{12}$$

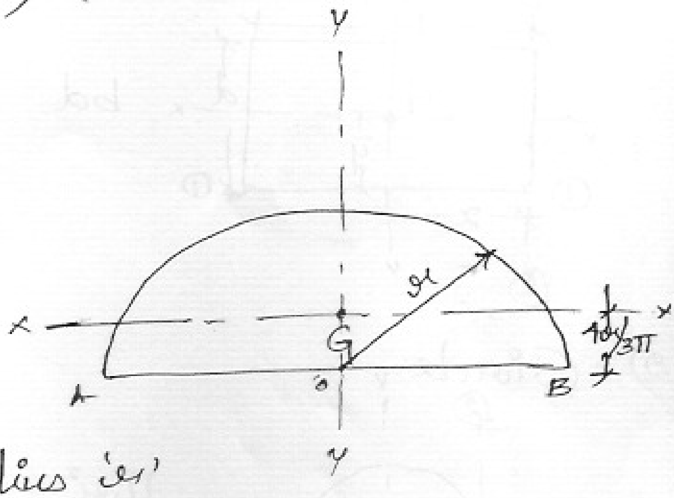
- MI abt centroid G

w.k.t $I_{AB} = I_{xx} + A\bar{y}^2$

$$\frac{bh^3}{12} = I_{xx} + \left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2$$

$$I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18}$$

$$I_{xx} = \frac{bh^3}{36}$$



4) MI of semi-O^{le}

- Consider a semi O^{le} of radius 'r'
- MI of semi O^{le} abt the ϕ AB

$$= \frac{1}{2} \text{ MI of } O^{le}$$

$$I_{AB} = \frac{1}{2} \times \frac{\pi r^4}{4}$$

$$I_{AB} = \frac{\pi r^4}{8}$$

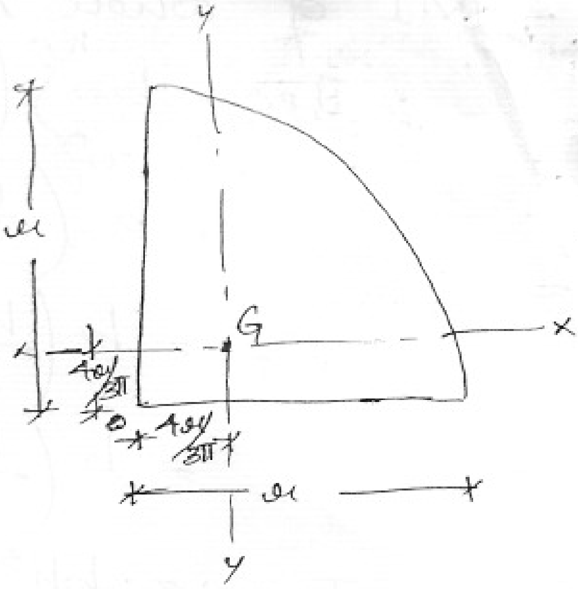
- MI abt centroidal axis

$$I_{AB} = I_{xx} + A\bar{y}^2$$

$$\frac{\pi r^4}{8} = I_{xx} + \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2$$

$$I_{xx} = \frac{\pi r^4}{8} - \frac{8r^4}{9\pi}$$

$$\underline{I_{xx} = 0.11\, r^4}$$



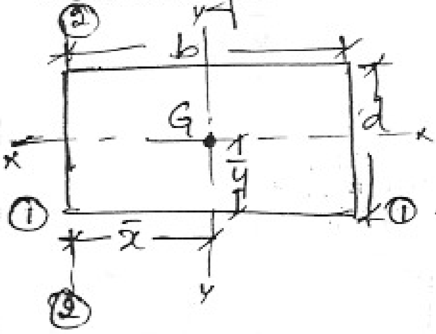
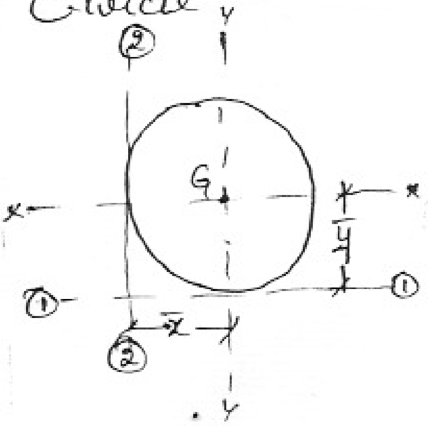
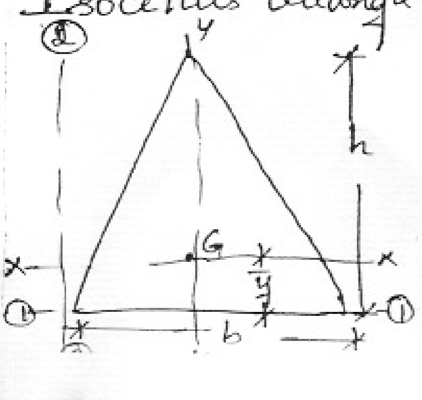
$$- \; I_{yy} = \frac{1}{2} \times \text{MI of } \bigcirc^{le}$$

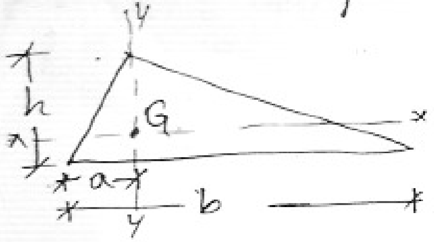
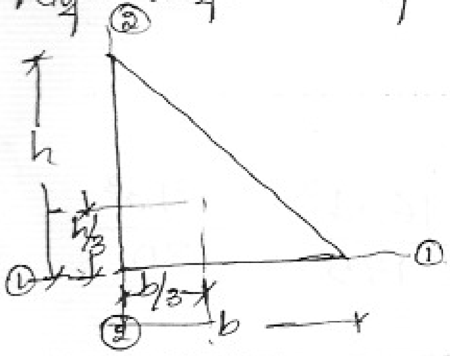
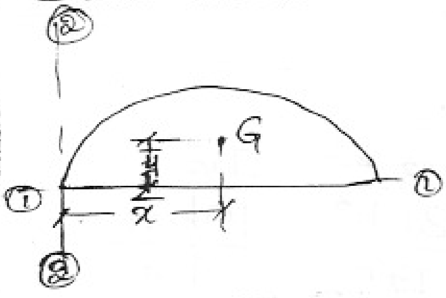
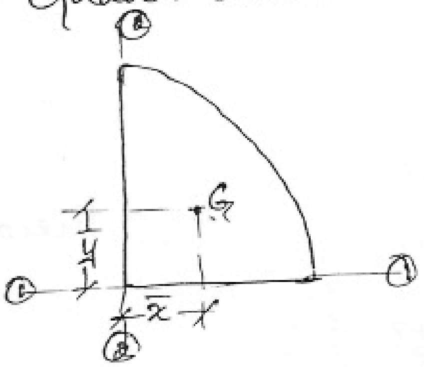
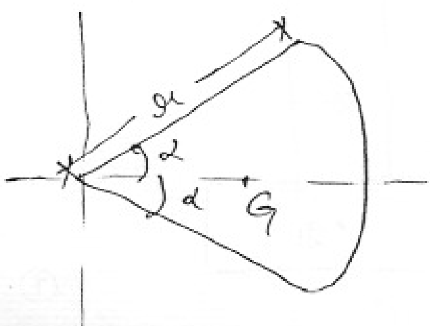
$$= \frac{1}{2} \times \frac{\pi r^4}{4}$$

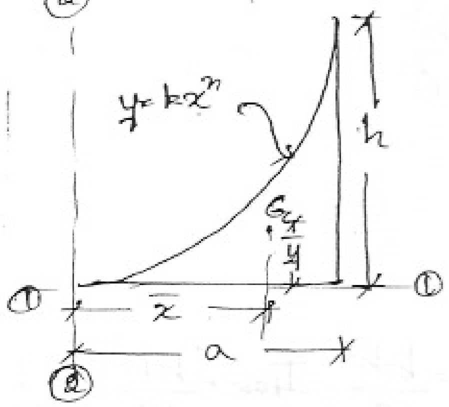
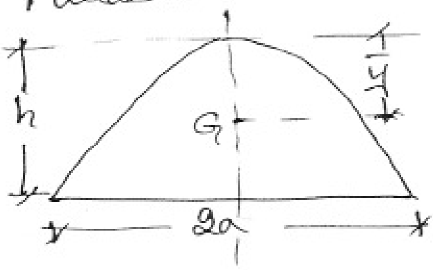
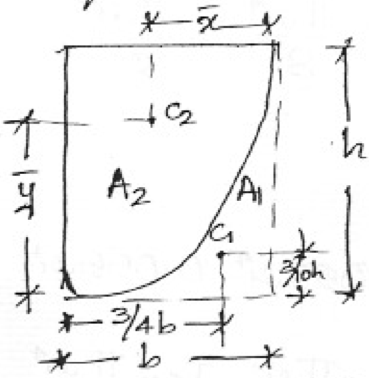
$$\underline{I_{yy} = \frac{\pi r^4}{8}}$$

5) MI of quater $\bigcirc^{le}$

$$I_{xx} = I_{yy} = \underline{0.055\, r^4}$$

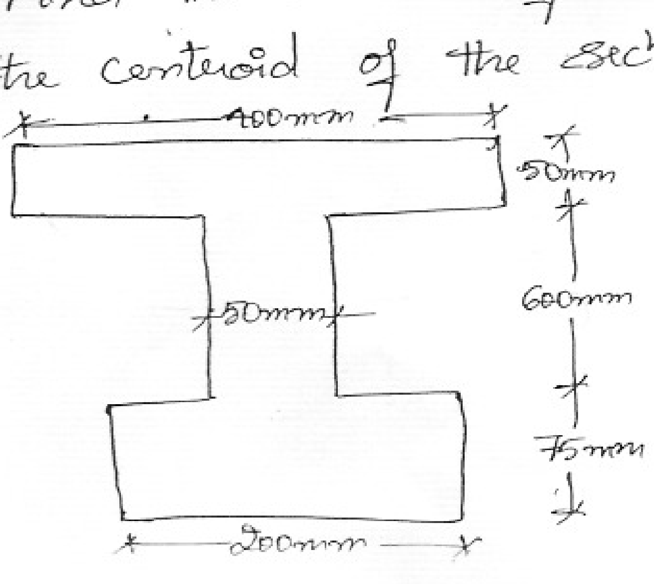
Sl No	Shape	Area	$\bar{X}$ from ②-②	$\bar{Y}$ from ①-①	I_{xx}	I_{yy}	I_{xy}
1)	Rectangle 	bd	$b/2$	$d/2$	$\frac{bd^3}{12}$	$\frac{db^3}{12}$	0
2)	Circle 	$\pi r^2 = \frac{\pi d^2}{4}$	r	r	$\frac{\pi r^4}{4} = \frac{\pi d^4}{64}$	$\frac{\pi r^4}{4} = \frac{\pi d^4}{64}$	0
3)	Isocellus triangle 	$\frac{1}{2}bh$	$b/2$	$h/3$ (from base); $2h/3$ (from apex)	$\frac{bh^3}{36}$; $I_{base} = \frac{bh^3}{12}$	$\frac{hb^3}{48}$	—

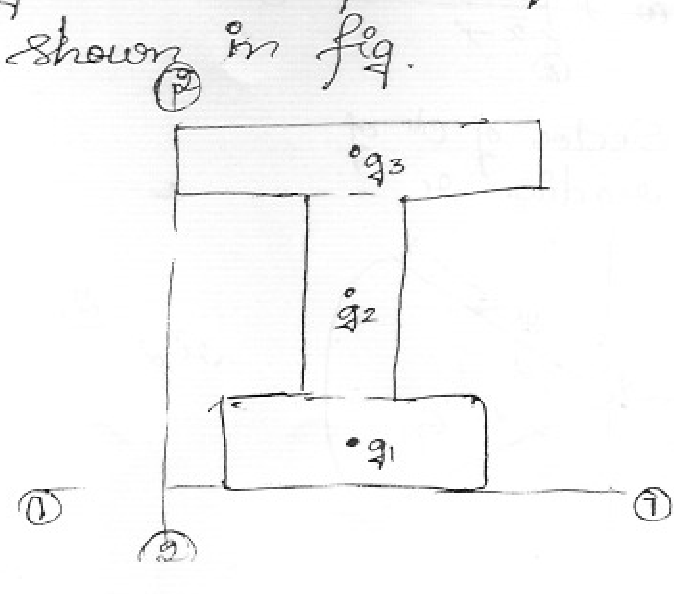
Sl. no	Shape	Area	$\bar{x}$ from ②-②	$\bar{y}$ from ①-①	I_{xx}	I_{yy}	I_{xy}
4)	Scalene triangle 	$\frac{1}{2}bh$	$\frac{a-b}{3}$	$h/3$ (from base) $2h/3$ (from apex)	$\frac{bh^3}{36}$	—	—
5)	Right angled triangle 	$\frac{1}{2}bh$	$b/3$ (from base) $2b/3$ (from apex)	$h/3$ (from base) $2h/3$ (from apex)	$I_{base} = \frac{bh^3}{12}$	$I_{base} = \frac{hb^3}{12}$	$-\frac{b^2h^2}{72}$
6)	Semi Circle 	$\frac{\pi r^2}{2}$	r	$\frac{4r}{3\pi}$ (from ϕ)	$0.11r^4$ $I\phi = \frac{\pi r^4}{8}$	$\frac{\pi r^4}{8}$	0
7)	Quater circle 	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$ (from ϕ)	$\frac{4r}{3\pi}$ (from ϕ)	$0.055r^4$ $I\phi = \frac{\pi r^4}{16}$	$0.055r^4$ $I\phi = \frac{\pi r^4}{16}$	-0.0165
8)	Sector of ck of radius r 	$r^2\alpha$	$\frac{2r\sin\alpha}{3\alpha}$	0	—	—	—

Sl no	Shape	Area	$\bar{x}$ from ② - ②	$\bar{y}$ from ① - ①	I_{xx}	I_{yy}	I_{xy}
9)	Segment of parabola (Concave upwards) 	$\frac{ab}{n+1}$	$\frac{3a}{4}$	$\frac{3h}{10}$	—	—	—
10)	Parabola 	$\frac{4ah}{3}$	0	$\frac{3h}{5}$	$\frac{16ah^3}{175}$	$\frac{8ha^3}{30}$	0
11)	Semi parabola 	$A_1 = \frac{bh}{3}$ $A_2 = \frac{2bh}{3}$	$\frac{3}{4}b$ $\frac{3}{8}b$	$\frac{3}{10}h$ $\frac{3}{5}h$	$I_{x_2} = \frac{8bh^3}{175}$	$I_{y_2} = \frac{19b^3h}{480}$	—

⇒ Problems on MI for simple figures

1) Find the MI along horizontal axis passing through the centroid of the secⁿ shown in fig.





- The figure is symmetrical abt Y-Y axis
- Divide the figure into simple figures & conside the reference ①-① & ②-② as shown.

Sl no	Compts	Area [a] (mm²)	Dist of centroid from ①-① [y] (mm)	Moment of area abt ①-① [ay] (mm³)	MI abt ①-① [ay²] (mm⁴)	Igx (mm⁴)
1)	g_1 ▭	$200\times75 = 15000$	$\frac{75}{2} = 37.5$	562500	21093750	$\frac{200\times75^3}{12} = 7031251$
2)	g_2 ▭	$50\times600 = 30000$	$\frac{600}{2}+75 = 375$	11250000	4.21×10^3	$\frac{50\times600^3}{12} = 9\times10^3$
3)	g_3 ▭	$400\times50 = 20000$	$75+600+\frac{50}{2} = 700$	14000000	9.8×10^3	$\frac{400\times50}{12} = 416666$
	$\Sigma =$	65000		25812500	1.404×10^{10}	9.112×10

- $\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{25812500}{65000} = \underline{397.11\,mm}$

- $I_{①-①} = \Sigma I_{gx} + \Sigma ay^2$

$= 9.112\times10^8 + 1.404\times10^{10}$

$= \underline{1.495\times10^{10}\,mm^4}$

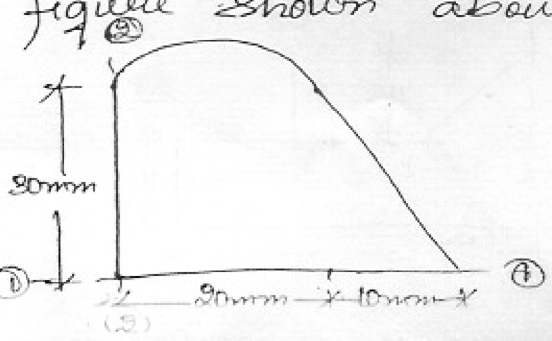
- $I_{①-①} = I_{xx} + A\bar{y}^2$

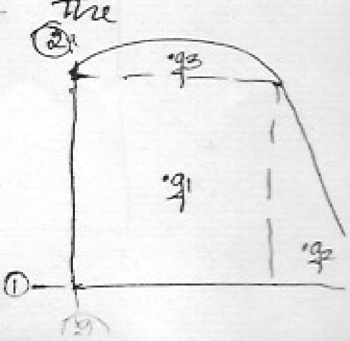
$\therefore I_{xx} = I_{①-①} - A\bar{y}^2$

$= 1.495\times10^{10} - (65000\times397.11^2)$

$I_{xx} = \underline{4.7\times10^9\,mm^4}$

2) Find the MI of the figure shown about the horizontal axis.





→ Divide the fig into simple figures & consider the reference ①-① & ②-② as shown.

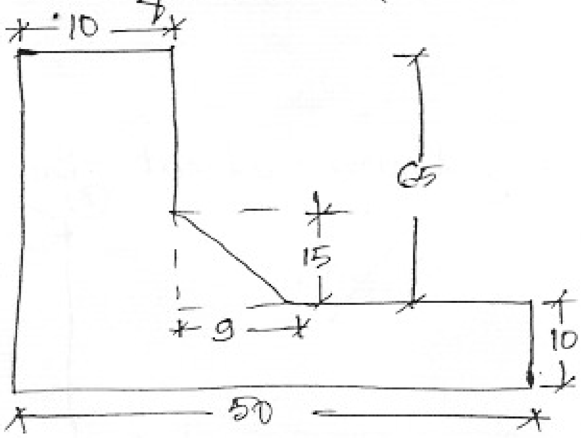
Sl no	Compts	Area $[a]$ (mm^2)	Distance of centroid from ①-① $[y]$ (mm)	Moment of area abt ①-① $[ay]$ (mm^3)	MI abt ①-① $[ay^2]$ (mm^4)	I_{gx} (mm^4)
1)	g_1 rectangle	$30\times20 = 600$	$\frac{30}{2} = 15$	9000	135000	$\frac{20\times30^3}{12} = 45000$
2)	g_2 Δ^{le}	$\frac{1}{2}\times10\times30 = 150$	$\frac{30}{3} = 10$	1500	15000	$\frac{10\times30^3}{36} = 7500$
3)	g_3 (semi cle)	$\frac{\pi\times10^2}{2} = 157$	$30\times0.424r = 30\times0.424\times10 = 34.34$	5375.7	334063.3	53600
	$\Sigma =$	907		15875.7	334063.3	53600

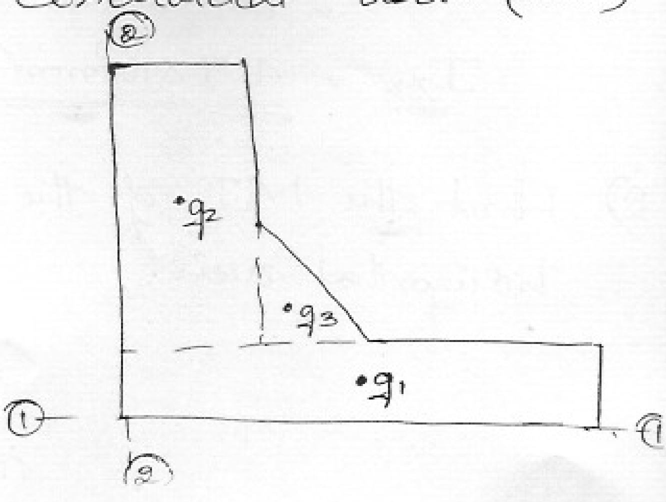
$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{15875.7}{907} = \underline{17.5\,mm}$$

$$I_{①-①} = \Sigma I_{gx} + \Sigma ay^2$$
$$= 53600 + 334063.3$$
$$= \underline{\underline{387663.3\,mm^4}}$$

$$I_{①-①} = I_{xx} + A\bar{y}^2$$
$$I_{xx} = I_{①-①} - A\bar{y}^2$$
$$= 387663.3 - 907\times17.5^2$$
$$= \underline{109894.5\,mm^4}$$

) Find the MI of the secn shown in figure for both horizontal & vertical centroidal axis (mm)





→ Divide the figure into simple figures & consider the reference ①-① & ②-② as shown.

Sl no	Compts	Area [a] (mm^2)	Distance of centroid abt ①-① [y] (mm)	Moment of area abt ①-① [ay] (mm^3)	MI abt ①-① [ay^2] (mm^4)	I_{gx} (mm^4)
1)	g_1 le	$50 \times 10 = 500$	$\frac{10}{2} = 5$	2500	12500	$\frac{50 \times 10^3}{12} = 4166.$
2)	g_2 le	$10 \times 65 = 650$	$10 + \frac{65}{2} = 42.5$	27625	1174062.5	$\frac{10 \times 65}{12} = 22885.$
3)	g_3 le	$\frac{1}{2} \times 9 \times 15 = 67.5$	$10 + \frac{1}{3} \times 15 = 15$	1012.5	15187.5	$\frac{9 \times 15^3}{36} = 843.$
	$\Sigma = 1217.5$			31137.5	1201750	233864

Sl no	Compts	Area [a] (mm^2)	Distance of centroid abt ②-② [x] (mm)	Moment of area abt ②-② [ax] (mm^3)	MI abt ②-② [ax^2] (mm^4)	I_{gy} (mm^4)
1)	g_1 le	$50 \times 10 = 500$	$\frac{50}{2} = 25$	12500	312500	$\frac{10 \times 50^3}{12} = 104166$
2)	g_2 le	$10 \times 65 = 650$	$\frac{10}{2} = 5$	3250	16250	$\frac{65 \times 1}{12} = 5416$
3)	g_3 le	$\frac{1}{2} \times 9 \times 15 = 67.5$	$10 + \frac{9}{3} = 13$	877.5	11407.5	$\frac{15 \times 9}{36} = 303.$
	$\Sigma = 1217.5$			16627.5	340157.5	109887.

$$\rightarrow \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{31137.5}{1217.5} = \underline{25.58 mm}$$

$$I_{①-①} = \Sigma I_{gx} + \Sigma ay^2$$
$$= 233864.7 + 1201750$$
$$= \underline{1435614.7 mm^4}$$

$$I_{①-①} = I_{xx} + A\bar{y}^2$$
$$I_{xx} = I_{①-①} - A\bar{y}^2$$
$$= 1435614.7 - (1217.5 - 25.58^2)$$
$$I_{xx} = \underline{638960 mm^4}$$

$$\rightarrow \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{16627.5}{1217.5} = \underline{13.66 mm}$$

$$I_{②-②} = \Sigma I_{gy} + \Sigma ax^2$$
$$= 109887.2 + 340157.5$$
$$= \underline{450044.7 mm^4}$$

$$I_{②-②} = I_{yy} + A\bar{x}^2$$
$$I_{yy} = I_{②-②} - A\bar{x}^2$$
$$= 450044.7 - (1217.5 - 13.66^2)$$
$$I_{yy} = \underline{222864.56 mm^4}$$

$$\rightarrow I_P = I_{zz} = I_{xx} + I_{yy}$$
$$= 638960 + 222864.56$$
$$= \underline{861824.56 mm^4}$$

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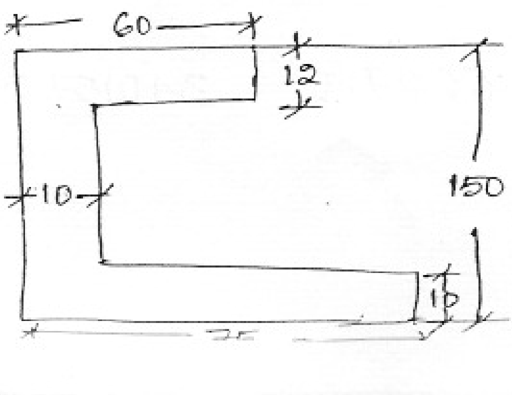
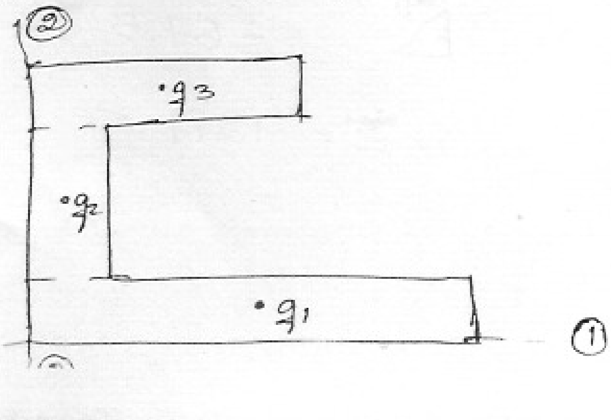
4) Find the radius of gyration of the area shown abt the horizontal centroidal axis (mm)





→ Divide the figure into simple figures & consider the reference ①-① as shown.

Sl no	Compt	Area [a] (mm^2)	Distance of centroid from ①-① [y] (mm)	Moment of area abt ①-① [ay] (mm^3)	MI abt ①-① [ay^2] (mm^4)	I_{gx} (mm^4)
1)	g_1 □ le	75×10 = 750	$\frac{10}{2} = 5$	3750	18750	$\frac{75 \times 10^3}{12}$ = 6250
2)	g_2 □ le	10×128 = 1280	$10 + \frac{128}{2}$ = 74	94720	7009280	$\frac{10 \times 128^3}{12}$ = 1747…
3)	g_3 (□ le)	60×12 = 720	$138 + \frac{12}{2}$ = 144	103680	14929920	$\frac{60 \times 12^3}{12}$ = 864…
	Σ =	2750		202150	21957950	1762516…

$$\rightarrow \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{202150}{2750} = \underline{73.5\,mm}$$

$$\rightarrow I_{①-①} = \Sigma I_{gx} + \Sigma ay^2$$

$$= 1762516.7 + 21957950$$

$$= \underline{23720467\ mm^4}$$

$$\rightarrow I_{①-①} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{①-①} - A\bar{y}^2$$

$$= 23720467 - (2750 \times 73.5^2)$$

$$I_{xx} = \underline{8864279.2\ mm^4}$$

→ Radius of Gyration

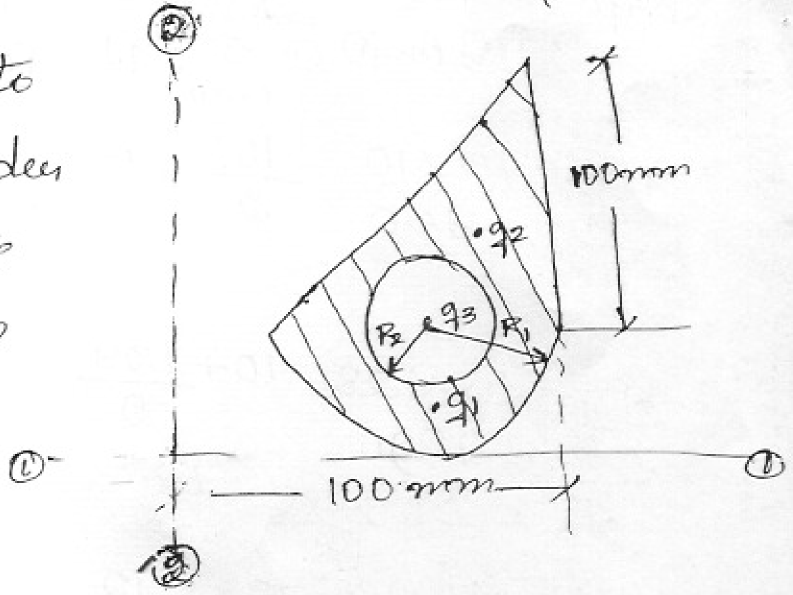
$$K_{xx} = \sqrt{\frac{I_{xx}}{A}}$$

$$= \sqrt{\frac{8864279.2}{2750}}$$

$$K_{xx} = 56.78\,mm$$

5) Determine the second moment of area (MI) about horizontal centroidal axis for shaded area shown. Also find the radius of gyration abt the same axis. Take $R_1 = 50mm$ & $R_2 = 20mm$

→ Divide the figure into simple figures & consider the reference ①-① & ②-② as shown in the figure.



Sl no	Compts	Area [a] (mm²)	Dist of centroid from ①-① [y] (mm)	Moment of area abt ①-① [ay] (mm³)	MI abt ①-① [ay²] (mm⁴)	Igx (mm⁴)
1)	g_1 (semi cle)	$\frac{\pi \times 50^2}{2} = 3927$	$50 - 0.424 \times 50 = 28.8$	113097.6	3257210.9	$0.11 \times 50^4 = 687500$
2)	g_2 (Δ^{le})	$\frac{1}{2} \times 100 \times 100$	$50 + \frac{100}{3} = 83.33$	416666.7	34720833	$\frac{100 \times 100^3}{36} = 2777777.$
3)	g_3 (cle) Deduction	$\pi \times 20^2 = 1256.6$	50	62831.9	3141592.7	$\pi \frac{20^4}{4} = 125663.$
	$\Sigma =$	7670.4		466932.4	34836451	3339614.

$$\rightarrow \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{466932.4}{7670.4} = \underline{60.88\ mm}$$

$$\rightarrow I_{①-①} = \Sigma I_{gx} + \Sigma ay^2$$
$$= 3339614.1 + 34836451$$
$$= \underline{38176065\ mm^4}$$

$$\rightarrow I_{1-1} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{1-1} - A\bar{y}^2$$

$$= 38176065 - (7670.4 - 60.88^2)$$

$$= \underline{\underline{9746690.9\ mm^4}}$$

$$\rightarrow K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{9746690.9}{7670.4}} = \underline{\underline{35.65\ mm}}$$

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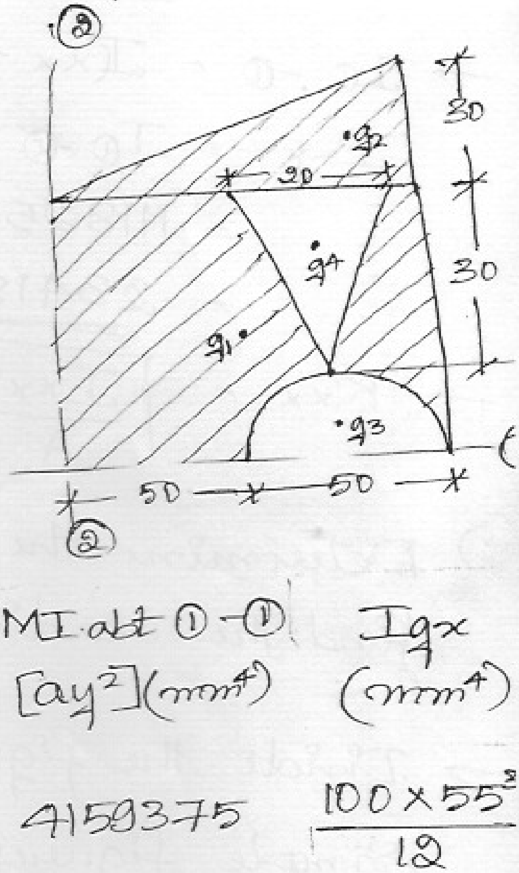
6) Find the radius of gyration abt XX axis of the shaded area shown in figure (mm)

→ Divide the figure into single figures & consider the reference ①-① as shown in the figure (mm)



SL no	Compts	Area [a] (mm^2)	Distance of centroid from ①-① [y] (mm)	Moment of area abt ①-① [ay] (mm^3)	MI abt ①-① [ay^2] (mm^4)	I_{gx} (mm^4)
1)	g_1 (▭le)	$100 \times 55 = 5500$	$\frac{55}{2} = 27.5$	151250	4159375	$\frac{100 \times 55^3}{12} = 138645$[illegible]
2)	g_2 (Δ^{le})	$\frac{1}{2} \times 100 \times 30 = 1500$	$55 + \frac{30}{3} = 65$	97500	6337500	$\frac{100 \times 30}{36} = 7500$
3)	g_3 (Semi cle) Deduction	$\frac{\pi \times 25^2}{2} = 981.8$	$0.425 \times 25 = 10.62$	104[illegible].62	110[illegible].72	0.11×2[illegible] $= 42968$
4)	g_4 (▽le) Deduction	$\frac{1}{2} \times 30 \times 20 = 300$	$25 + \frac{2}{3} \times 30 = 45$	13500	607500	$\frac{20 \times 30}{36} = 15000$

Sl no	Compts	[a] (mm²)	[y] (mm)	[ay] (mm³)	[ay²] (mm⁴)	Igx (mm⁴)
	Σ =	5718.2		224818.38	9779060	1403489.

$$\rightarrow \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{224818.38}{5718.2} = \underline{39.31\text{mm}}$$

$$\rightarrow I_{①-①} = \Sigma I_{gx} + \Sigma ay^2$$

$$= 1403489.5 + 9779060$$

$$= \underline{11182550 \text{ mm}^4}$$

$$\rightarrow I_{①-①} = I_{xx} + A\bar{y}^2$$

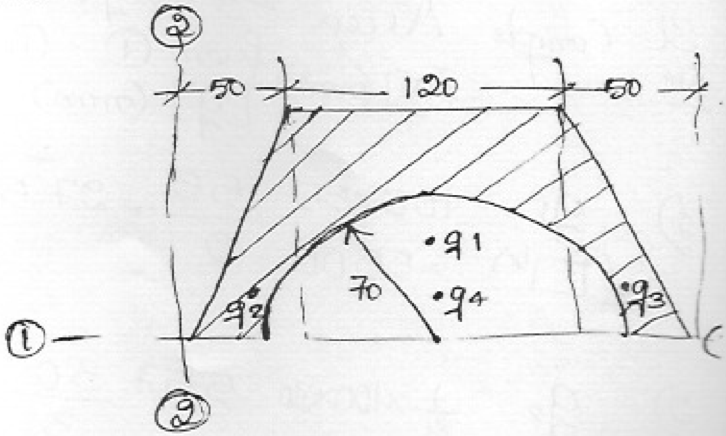
$$I_{xx} = I_{①-①} - A\bar{y}^2$$

$$= 11182550 - (5718.2 - 39.31^2)$$

$$= \underline{2341855.4 \text{ mm}^4}$$

$$\rightarrow K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{2341855.4}{5718.2}} = \underline{20.34\text{mm}}$$

8) Determine the MI abt ~~horizontal~~ axis & also find the radius of gyration (mm)



→ Divide the figure into simple figures & consider the reference ①-① as shown in the figure.

Sl no	Compts	Area $[a]$ (mm^2)	Dist of centroid from ①-① $[y]$ (mm)	Moment of area abt ①-① $[ay]$ (mm^3)	MI abt ①-① $[ay^2]$ (mm^4)	I_{gx} (mm^4)
1)	g_1 (□^{le})	$120 \times 120 = 14400$	$\frac{120}{2} = 60$	864000	51840000	$\frac{120 \times 120^3}{12} = 172800$
2)	g_2 (Δ^{le})	$\frac{1}{2} \times 50 \times 120 = 3000$	$\frac{120}{3} = 40$	120000	4800000	$\frac{50 \times 120^3}{36} = 24000$
3)	g_3 (Δ^{le})	$\frac{1}{2} \times 50 \times 120 = 3000$	$\frac{120}{3} = 40$	120000	4800000	$\frac{50 \times 120^3}{36} = 24000$
4)	g_4 (semi cle) Deduction	$\frac{\pi \times 70^2}{2} = 7697$	$0.424 \times 70 = 29.68$	228447	6780306	$0.11 \times 70^4 = 26411$
		$\Sigma = 12703$		875553	54659694	1943890

$$\Rightarrow \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{875553}{12703} = \underline{68.93 mm}$$

$$\Rightarrow I_{①-①} = \Sigma I_{gx} + \Sigma ay^2$$

$$= 19438900 + 54659694$$

$$= \underline{74098594 \, mm^4}$$

$$\Rightarrow I_{①-①} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{①-①} - A\bar{y}^2$$

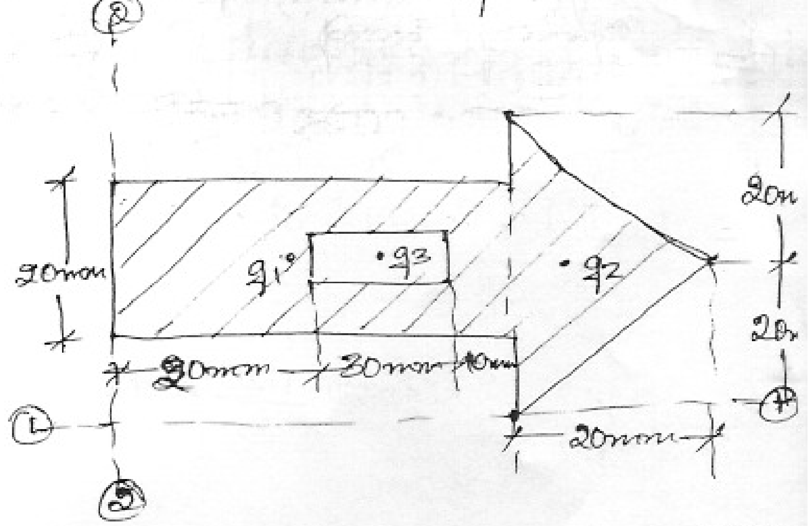
$$= 74098594 - (12703 - 68.93^2)$$

$$= \underline{13742260 \, mm^4}$$

$$\rightarrow K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{13742260}{12703}} = \underline{32.89 mm}$$

8) Find the radius of gyration of the shaded area abt an axis normal to the symmetrical a

→ Divide the figure into simple figures & consider the reference ②-② as shown in the figure.



Sl no	Compts	Area [a] (mm²)	Dist of centroid from ②-② [x] (mm)	Moment of area abt ②-② [ax] (mm³)	MI abt ②-② [ax²] (mm⁴)	I_{gy} (mm⁴)
1)	g_1 ▭	$60 \times 20 = 1200$	$\frac{60}{2} = 30$	36000	1080000	$\frac{20 \times 60^3}{12} = 36000$
2)	g_2 Δ	$\frac{1}{2} \times 40 \times 20 = 400$	$60 + \frac{20}{3} = 66.67$	26666.7	1777777.8	$\frac{40 \times 20}{36} = 8888.$
3)	g_3 ▭ Deduction	$10 \times 30 = 300$	$20 + \frac{30}{2} = 35$	10500	367500	$\frac{10 \times 30^3}{12} = 2250$
	$\Sigma =$	1300		52166.7	2490277.8	346388

$$\rightarrow \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{52166.7}{1300} = \underline{40.13 mm}$$

$$\rightarrow I_{②-②} = \Sigma I_{gy} + \Sigma ax^2$$

$$= 346388.9 + 2490277.8$$

$$I_{②-②} = \underline{2836666.7 mm^4}$$

$$\rightarrow I_{②-②} = I_{yy} + A\bar{x}^2$$

$$I_{yy} = I_{②-②} - A\bar{x}^2$$

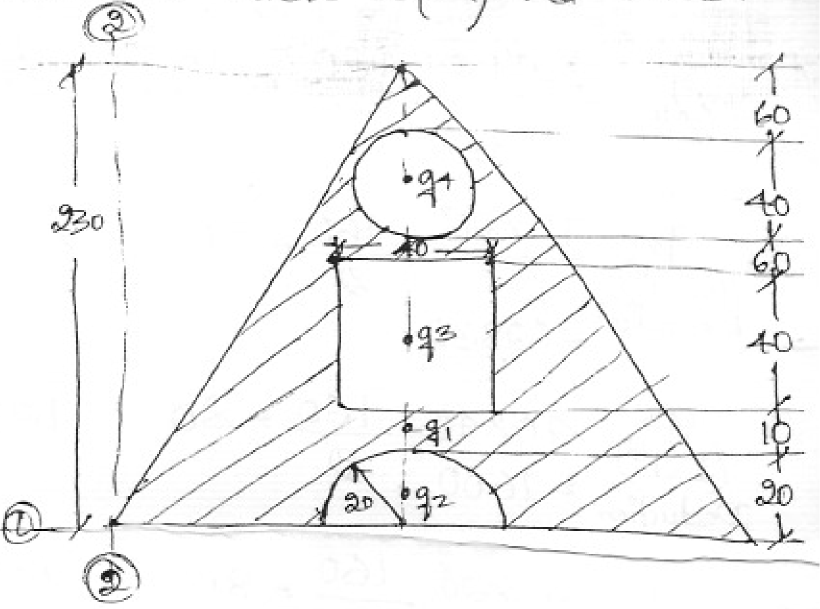
$$= 2836666.7 - (1300 - 40.13^2)$$

$$= \underline{743124.7 mm^4}$$

$$\rightarrow K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{743124.7}{1300}} = \underline{23.91 mm}$$

9) Find the MI of shaded area shown abt horizontal axis & vertical axis & find their radius of gyration.

→ Divide the figure into simple figures & consider the reference ① - ① & ② - ② as shown in the figure.



Sl no	Compts	Area [a] (mm^2)	Dist of centroid from ① - ① [y] (mm)	Moment of area abt ① - ① [ay] (mm^3)	MI abt ① - ① [ay^2] (mm^4)	Igx (mm^4)
1)	g1 (Δ^{le})	$\frac{1}{2} \times 160 \times 230$ = 18400	$\frac{230}{3}$ = 76.67	1410728	1.082×10^8	$\frac{160 \times 23}{32}$ = 54075
2)	g2 (semicircle) Deduction	$\frac{\pi \times 20^2}{2}$ = 682.32	0.424×20 = 8.48	5328.1	45182.6	0.11×2 = 1760
3)	g3 (□) Deduction	40×40 = 1600	$20 + 10 + \frac{40}{2}$ = 50	80000	4000000	$\frac{40 \times 40}{12}$ =
4)	g4 (○) Deduction	$\pi \times 20^2$ = 1256.4	$20+10+40+60+40/2$ = 150	188496	28274400	$\frac{\pi \times 20}{4}$ = 125663
		Σ = 14915.04		1136903.9	7588047	537189

$$\rightarrow \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{1136903.9}{14915.04} = \underline{76.22 mm}$$

$$\rightarrow I_{①-①} = \Sigma I_{gx} + \Sigma ay^2 = 53718959 + 7588047 = \underline{1290070}$$

$$I_{xx} = I_{①-①} - A\bar{y}^2$$

$$= 129007006 - (14915.04 \times 76.22^2)$$

$$I_{xx} = \underline{42358254.13 mm^4}$$

$$\rightarrow K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{42358254.13}{14915.04}} = \underline{53.29 mm}$$

Sl no	Compts	Area [a] (mm²)	Dist of centroid from ②-② [x] (mm)	Moment of area abt ②-② [ax] (mm³)	MI abt ②-② [ax²] (mm⁴)	I_{gy} (mm⁴)
1)	g_1 (Δ^{le})	$\frac{1}{2} \times 160 \times 230 = 18400$	$\frac{160}{2} = 80$	1472000	3.98×10^{16}	$2\left(\frac{230 \times 80}{12}\right) = 1962666$
2)	g_2 (semicircle) Deduction	$\frac{\pi \times 20^2}{2} = 628.32$	$\frac{160}{2} = 80$	50265.6	1.58×10^{12}	$\frac{\pi \times 20^4}{8} = 62831.8$
3)	g_3 (square) Deduction	$40 \times 40 = 1600$	$\frac{160}{2} = 80$	128000	2.62×10^{13}	$\frac{40 \times 40^3}{12} = 213333$
4)	g_4 (circle) Deduction	$\pi \times 20^2 = 1256.64$	$\frac{160}{2} = 80$	100531.2	1.27×10^{13}	$\frac{\pi \times 20^4}{4} = 125663$
	Σ	$= 14915.04$		1193203.2	3.97×10^{16}	19224837

$$\rightarrow \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{1193203.2}{14915.04} = \underline{30 mm}$$

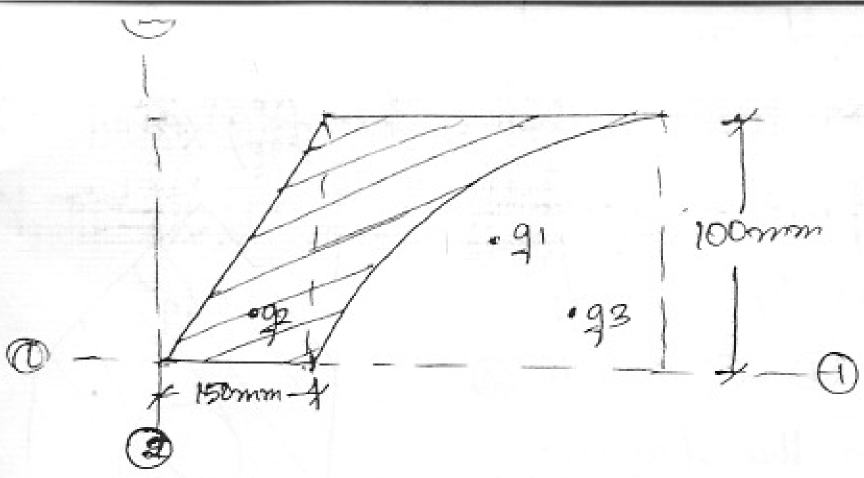
$$\rightarrow I_{②-②} = \Sigma I_{gy} + \Sigma ax^2$$
$$= 19224837.8 + 3.97 \times 10^{16}$$
$$= \underline{3.97 \times 10^{6}\ mm^4}$$

$$\rightarrow I_{②-②} = I_{yy} + A\bar{y}^2$$
$$I_{yy} = I_{②-②} - A\bar{y}^2$$
$$= 3.97 \times 10^{6} - (14915.04 \times 30^2)$$
$$= \underline{\underline{3.96 \times 10^{16}\ mm^4}}$$

$$\rightarrow K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{3.96 \times 10^6}{14915.04}} =$$

10) Find the radius of gyration abt horizontal axis.

→ Divide the figure into simple figures & consider reference ①-① & ②-② as shown in the figure.



Sl no	Compts	Area [a] (mm²)	Dist of centroid from ①-① [y] (mm)	Moment of area abt ①-① [ay] (mm³)	MI abt ①-① [ay²] (mm⁴)	Igy (mm⁴)
1)	g_1 (□le)	100×100 = 10000	$\frac{100}{2} = 50$	500000	25000000	$\frac{100\times100^3}{12}$ =
2)	g_2 (Δle)	$\frac{1}{2}$×150×100 = 7500	$\frac{100}{3} = 33.33$	249975	833666.8	$\frac{50\times100^3}{36}$ =
3)	g_3 (◔le)	$\frac{\pi\times100^2}{4}$ Deduction = 7854	0.424×100 = 42.4	333009.6	1419607	0.055×10 =
	Σ = 9646			416965.7	19212060	7000000

$$\rightarrow \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{416965.7}{9646} = \underline{43.23\,mm}$$

$$\rightarrow I_{①-①} = \Sigma I_{gx} + \Sigma ay^2$$
$$= 7000000 + 19212060$$
$$= \underline{26212060\,mm^4}$$

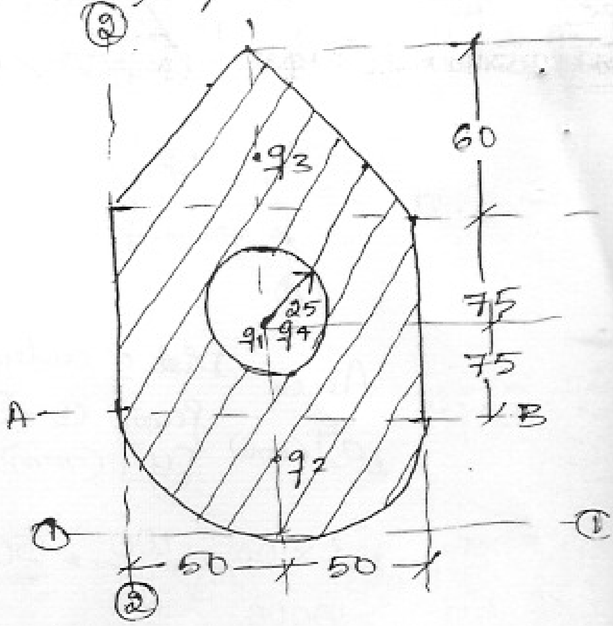
$$\rightarrow I_{①-①} = I_{xx} + A\bar{y}^2$$
$$I_{xx} = I_{①-①} - A\bar{y}^2$$
$$= 26212060 - (9646 \times 43.23^2)$$
$$= \underline{8185297.85\,mm^4}$$

$$\rightarrow K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{8185297.85}{9646}} = \underline{29.13\,mm}$$

11) Find the second moment of area of shaded area abt AB axis shown in fig (mm)

→ Divide the figure into simple figures & consider the reference ① - ① & ② - ② as shown in the figure.

→ MI is required to find abt AB axis.



Sl no	Compts	Area [a] (mm^2)	Dist of centroid from ① - ① [y] (mm)	Moment of area abt ① - ① [ay] (mm^3)	MI abt ① - ① [ay^2] (mm^4)	I_{gx} (mm^4)
1)	g_1 (□^{le})	$100 \times 150 = 15000$	$50+75 = 125$	1875000	2.34×10^8	$\frac{100 \times 150^3}{12} = 281250$
2)	g_2 (semicircle)	$\frac{\pi \times 50^2}{2} = 3927$	$50 - 0.424 \times 50 = 28.8$	113097.6	3257210.9	$0.11 \times 50^4 = 687501$
3)	g_3 (Δ^{le})	$\frac{1}{2} \times 100 \times 60 = 3000$	$50+150+\frac{60}{3} = 220$	660000	1.452×10^8	$\frac{100 \times 60^3}{36} = 60000$
4)	g_4 (O^{le}) Deduction	$\pi \times 25^2 = 1963.5$	$50+75 = 125$	245437.5	3068688	$\frac{\pi \times 24^4}{4} = 306796$
		$\Sigma = 19963.5$		2402660.1	3.522×10^8	2910570.

$$\rightarrow \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{2402660.1}{19963.5} = \underline{120.35 mm}$$

$$\rightarrow I_{①-①} = \Sigma ay^2 + \Sigma I_{gx}$$

$$= 3.522 \times 10^8 + 29105704$$

$$I_{①-①} = \underline{3.813 \times 10^8 \ mm^4}$$

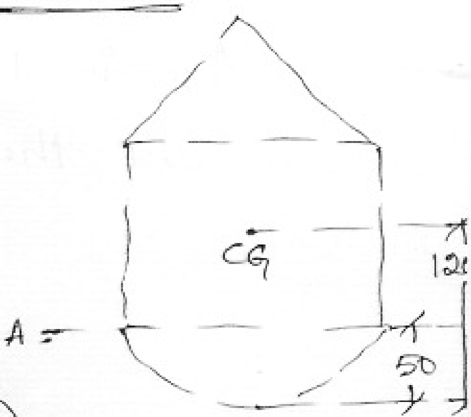
$\rightarrow I_{①-①} = I_{xx} + A\bar{y}^2$

$I_{xx} = I_{①-①} - A\bar{y}^2$

$= 3.813 \times 10^8 - (19963.5 \times 120.35^2)$

$I_{xx} = \underline{92151924 \text{ mm}^4}$

$\rightarrow K_{xx} = \sqrt{\dfrac{I_{xx}}{A}} = \sqrt{\dfrac{92151924}{19963.5}} = \underline{67.94 \text{mm}}$



$\rightarrow$ MI abt AB axis

distance b/w CG & AB axis

$\bar{y} = 120.35 - 50 = \underline{70.35 \text{mm}}$

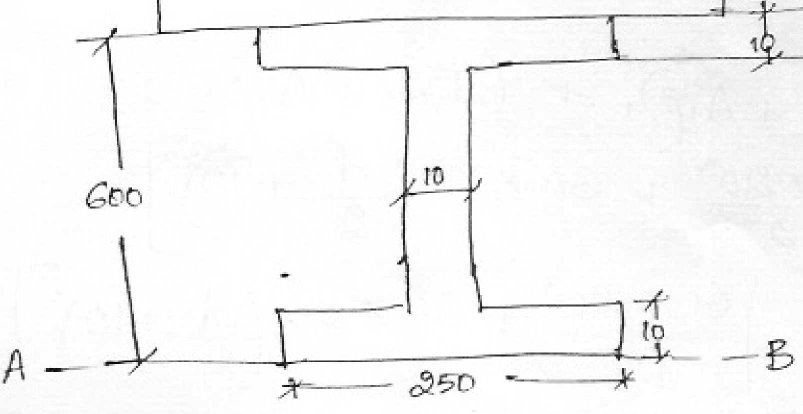
$\rightarrow I_{AB} = I_{xx} + A\bar{y}^2$

$= 92151924 + (19963.5 \times 70.35^2)$

$= \underline{1.9095 \times 10^8 \text{ mm}^4}$

$\rightarrow K_{AB} = \sqrt{\dfrac{I_{AB}}{A}} = \sqrt{\dfrac{1.9095 \times 10^8}{19963.5}} = \underline{97.8 \text{mm}}$

12) The strength of a 600mm deep & 250mm wide I-section beam of uniform thickness 10mm is increased by welding a 350mm wide & 20mm thick plate to its upper flange as shown in fig. Determine the ↑se in moment of inertia of the section about the base AB. (mm)



→ MI of whole solution abt base AB } = MI of I sec^n abt base (+) MI of plate abt base

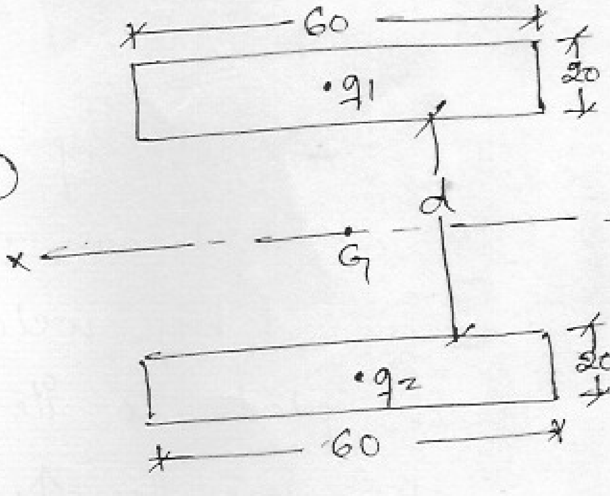
→ Here it is reqd to find the MI abt the base AB of the additional plate of size 350 X 20mm welded to top flange only.

→ ∴ Ise in MI of the sec^n abt the base AB } = MI of only extra plate abt base } = ay^2

$$= (350 \times 20)\left(600 + \frac{20}{2}\right)^2$$

$$= \underline{\underline{2.6047 \times 10^9 \text{ mm}^4}}$$

13) Determine the distance 'd' @ which the two ▭lar blocks 20X60mm be placed such that $I_{xx} = I_{yy}$ as shown



→ MI of an area abt its own axis } = $da \cdot y^2$ → ①

→ MI of an area abt any other reference axis
$= I_{xx} + A\bar{y}^2$ → ②

→ Consider two elements g_1 & g_2 from ② & apply equ^n ② in case as MI is to be found abt X - X axis

→ $I_{xx} = (I_{xx} + Ay^2)_1 + (I_{xx} + Ay^2)_2$

$$= \left[\frac{60 \times 20^3}{12} + 60 \times 20\left(\frac{d}{2} + 10\right)^2\right]$$

$$+ \left[\frac{60 + 20^3}{12} + 60 \times 20\left(\frac{d}{2} + 10\right)^2\right]$$

$$= 8\times10^4 + 2400\left(\frac{d}{2}+10\right)^2 \longrightarrow 3$$

$\rightarrow I_{yy} = 2\left[I_{yy}\right]_{1\&2}$

$$= 2\times20\times\frac{60^3}{12}$$

$$= 72\times10^4 \longrightarrow 4$$

$\rightarrow$ Equating I_{xx} & I_{yy}

$$8\times10^4 + 2400\left(\frac{d^2}{4}+100+10d\right) = 72\times10^4$$

$$\frac{d^2}{4}+10d+100 = 266.67$$

$$d^2 + 4d - 666.68 = 0$$

$$\underline{\underline{d = 23.89mm}}$$

$\rightarrow$ $\therefore$ The distance b/w the two $\sqcap^{lar}$ block is $d = 23.89mm$ ~~also~~ such that $I_{xx} = I_{yy}$