1 b) i Given $\mu = 75$ $\sigma = 12$ $n = 121$

$\mu_{\bar{x}} = \mu = 75$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{121}} = \frac{12}{11} = \underline{1.09}$$

ii) $n = 400$

$\therefore \mu_{\bar{x}} = \mu = 75$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{20} = \underline{0.6}$$

So if the size of the samples is changed from 121 to 400, the $\sigma_{\bar{x}}$ decreases from $\underline{1.09}$ to $\underline{0.6}$ but the mean remains the same

2a) Given: $\mu = 5.75$, $\sigma = 1.02$ and $n = 81$

$\therefore \mu_{\bar{x}} = \mu = 5.75$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\cancel{1}.02}{9} = 0.1133$$

ii $n = 25$

$\therefore \mu_{\bar{x}} = \mu = 5.75$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.02}{5} = 0.204$$

$\therefore$ $\mu_{\bar{x}}$ remains same but the $\sigma_{\bar{x}}$ changes from 0.1133 to 0.204

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2b) Here $N = 1500$

$\mu = 635$ $\quad \sigma = 1.36$ $\quad n = 36$

a) Expected mean $\mu_{\bar{x}} = \mu = 635$

$$\sigma_{\bar{x}} = \sqrt{\frac{\sigma^2}{n}} = \frac{1.36}{6} = \underline{\underline{0.227}}$$

b) $\mu_{\bar{x}} = \mu = 635$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$$

$$= \sqrt{\frac{(1.36)^2}{36} \times \frac{1500-36}{1500-1}}$$

$$= \sqrt{0.05}$$

$$= \underline{\underline{0.224}}$$

3) Population mean $\mu = \frac{1}{4}(3+7+11+15)$

$= \underline{\underline{9}}$

Population variance $\sigma^2 = \frac{1}{4}\{(3-9)^2 + (7-9)^2 + (11-9)^2 + (15-9)^2\}$

$= \underline{\underline{20}}$

a) Let us consider the samples of size 2 with replacement. They are as follows

(3, 3), (3, 7), (3, 11), (3, 15),

(7, 3) (7, 7), (7, 11), (7, 15),

(11, 3), (11, 7), (11, 11), (11, 15),

(15, 3), (15, 7) (15, 11), (15, 15),

Sampling means are as follows

3, 5, 7, 9, 5, 7, 9, 11, 7, 9, 11, 13, 9, 11, 13, 15

The frequency distribution of the sampling means is as follows

x :	3	5	7	9	11	13	15
f :	1	2	3	4	3	2	1

$$\mu_{\bar{x}} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{144}{16} = 9$$

$$\sigma_{\bar{x}}^2 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\mu_{\bar{x}})^2$$

$$= \frac{1456}{16} - 9^2$$

$$= 10$$

Thus $\mu_{\bar{x}} = 9$

$\sigma_{\bar{x}} = \sqrt{10}$

b) Let us consider samples without replacement. They are as follows

(3, 7), (3, 11), (3, 15), (7, 11), (7, 15)

(11, 15).

The sampling means are 5, 7, 9, 9, 11, 13

$$\therefore \mu_{\bar{x}} = \frac{1}{6}(5+7+9+9+11+13)$$

$$= \underline{\underline{9}}$$

Thus $\underline{\underline{\mu_{\bar{x}} = \mu}}$

$$\sigma_{\bar{x}}^2 = \frac{1}{6}\{(5-9)^2 + (7-9)^2 + (9-9)^2 + (9-9)^2 + (11-9)^2 + (13-9)^2\}$$

$$= \frac{40}{6} = \frac{20}{3}$$

Consider $\frac{\sigma^2}{n} \times \left[\frac{N-n}{N-1}\right]$

$$= \frac{20}{2} \times \left[\frac{4-2}{4-1}\right]$$

$$= 10 \times \frac{2}{3} = \frac{20}{3} = \sigma_{\bar{x}}^2$$

Thus $\underline{\underline{\frac{\sigma^2}{n} \times \left[\frac{N-n}{N-1}\right] = \sigma_{\bar{x}}^2}}$

3 b) By data $\mu = 800$ $\sigma = 60$

$n = 16$

$\therefore \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{60}{4} = \underline{\underline{15}}$

We have $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{\bar{x} - 800}{15} \sim N(0,1)$

a) $P(790 < \bar{x} < 810)$

$= P\left(\frac{790-800}{15} < z < \frac{810-800}{15}\right)$

$= P(-0.67 < z < 0.67)$

$= 2 \times P(0 < z < 0.67)$

$= \underline{\underline{0.4972}}$.

b) $P(\bar{x} < 785)$

$= P\left(z < \frac{785-800}{15}\right)$

$= P(z < -1)$

$= P(z > 1)$

$= \underline{\underline{0.1587}}$.

c) $P(\bar{x} > 820)$

$= P\left(z > \frac{820-800}{15}\right)$

$= P(z > 1.33)$

$= \underline{\underline{0.0918}}$

d) $P(770 < \bar{x} < 830)$

$= P\left(\frac{770-800}{15} < z < \frac{830-800}{15}\right)$

$= P(-2 < z < 2)$

$= 2\,P(0 < z < 2)$

$= \underline{0.9544}$.

7 a) For simplicity we use units of thousands of miles.

Then $\mu_{\bar{x}}$ = Mean of sample mean $\bar{x}$

$= \mu = \underline{\underline{38.5}}$. thousands of miles

$\therefore \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{5}} = \underline{\underline{1.11803}}$ thousands of miles

$\therefore$ the pop^n is normally distributed

$\Rightarrow$ $\bar{x}$ is also follows Normal dist^n : $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \sim N(0,1)$

$\therefore$ $P[\bar{X} < 86] = P[Z < \frac{86 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}]$

$= P[Z < -2.24]$

$= \underline{\underline{0.0125}}$

That is, if the tire perform as designed, there is only about a 1.25% chance that the average of a sample of this size would be so low.

4 b) <u>Given</u>: μ = mean life of a battery = 50 months

σ = standard deviation = 6 months

Let X: life of a battery

$\therefore$ $X \sim N(\mu, \sigma^2) \Rightarrow \frac{X - \mu}{\sigma} = Z \sim N(0,1)$

$\therefore$ $P[X < 48] = P[Z < \frac{48 - \mu}{\sigma}]$

$= P[Z < -0.33]$

$= \underline{\underline{0.3707}}$

b) n = sample size = 36

$\therefore$ sample mean $= \mu_{\bar{x}} = \mu = 50$

& $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{6}{\sqrt{36}} = 1$

$\therefore \dfrac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = Z \sim N(0, 1)$

$\therefore P[\bar{x} < 48] = P\left[Z < \dfrac{48 - 50}{1}\right]$

$= P[Z < -2]$

$= \underline{\underline{0.0228}}$

5a) X : wt of ball bearing

~~n = sample size~~

N = pop^n size = 1500

$\mu = 635$ gm

$\sigma = 1.36$ gm

No of random samples = 300

n = sample size = 36

$\therefore X \sim N(\mu, \sigma^2)$ Let $\bar{x}$: sample mean

$\therefore \bar{x} \sim N(\mu_{\bar{x}}, \sigma_{\bar{x}}^2)$

where $\mu_{\bar{x}} = \mu = \underline{\underline{635}}$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{1.36}{6} = \underline{\underline{0.2267}}$

$\therefore Z = \dfrac{\bar{x} - 635}{0.2267} \sim N(0, 1)$

i) ∴ No of random samples that have their mean between 634.76 g & 635.24 g

$= \text{No of samples} \times P[634.76 < \bar{X} < 635.24]$

$= 300 \times P[-1.06 < Z < 1.06]$

$= 300 \times 0.7108$

$= 213.24$

$\simeq \underline{\underline{213}}$

ii) No of samples that have their mean greater than 635.6 g

$= 300 \times P[\bar{X} > 635.6]$

$= 300 \times P[Z > \cancel{2.20}\ 2.467]$

$= 300 \times \cancel{0.0136}\ [0.5 - 0.4960] = 300 \times 0.004$

$= \cancel{4.08} \simeq \cancel{4}\ \ 1.2 \simeq \underline{\underline{1}}$

iii) No of samples that have their mean less than 634.2

$= 300 \times P[\bar{X} < 634.2]$

$= 300 \times P[Z < -3.53]$

$= 300 \times 0$

$= \underline{0}$

d) No of samples with mean less than 634.5 g or more than 685.24 g

$= 300 \times [P[\bar{x} < 634.5] + P[\bar{x} > 635.24]]$

$= 300 \times [P[\bar{z} < -2.21] + P[z > 1.06]]$

$= 300 \times [0.0136 + \cancel{0.3554}\ 0.1446]$

$= 300 \times [\cancel{0.3690}\ 0.1582]$

$= \cancel{443} \simeq \cancel{5}\ 47.46\ \cancel{47}$

5b) Population size $= N = 500$

$\mu = 142.3$ gms

$\sigma = 8.5$ gm

sample size $= n = 100$

x : wt of ball bearing

$\therefore$ n is large by central limit thm

$$\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \sim N(0,1)$$

where $\mu_{\bar{x}} = \mu = 142.3$ gm

$$\sigma_{\bar{x}} = \frac{8.5}{\sqrt{n}} = 0.85 \text{ gm}$$

$\therefore Z = \frac{\bar{x} - 142.3}{0.85} \sim N(0,1)$

~~iv) No of samples with their mean between 63.~~

i) P [The combined wt of the group lie between 14061gm & 14175gm]

$= P\left[\frac{14061}{n} < \bar{x} < \frac{14175}{n}\right]$

$= P[140.61 < \bar{x} < 141.75]$

$= P[-1.988 < Z < -0.647]$

$= \underline{\underline{0.2222}}.$

ii) P [The combined wt of the group is more than 14460gm]

$= P[\bar{x} > 144.6]$

$= P[Z > 3.06]$

$= \underline{\underline{0.0013}}.$

16a

6a) By data:

x : max load supported by a cable

then $\bar{x} = 11.09$, $\sigma = 0.73$ $n = 60$

a) 95 % confidence limits for the mean of maximum loads are given by

$\bar{x} \pm 1.96(\sigma/\sqrt{n}) = 11.09 \pm 1.96(0.73/\sqrt{60})$

$= 11.09 \pm 0.18$

i.e limits are 10.91 tonnes & 11.27 tonnes

b) 99 % confidence limits for the mean of maximum loads are given by

$\bar{x} \pm 2.58(\sigma/\sqrt{n}) = 11.09 \pm 2.58(0.73/\sqrt{60})$

$= 11.09 \pm 0.24$

i.e limits are 10.85 tonnes & 11.33 tonnes

16b

6b) Given $n = 900$

Let x : ht $\therefore$ $\bar{x} = 64$ inch $\sigma = 20$ inch

$\therefore$ 99% confidential limits for the mean is given by

$= \bar{x} \pm 2.58(\sigma/\sqrt{n})$

$= 64 \pm 2.58(20/30)$

$= 64 \pm 2.58(0.6667)$

$= 64 \pm 1.720086$

$\therefore$ The limits are 62.279914 & 65.720086

17a/7a) $n = 5000$ $\bar{x}$ = average wt = 62.5 kg

σ = 22 kg

a) 95% confidence limits for the mean of maximum loads are given by

$\bar{x} \pm 1.96(\sigma/\sqrt{n})$

$= 62.5 \pm 1.96(22/\sqrt{5000})$

$= 62.5 \pm 0.6098$

i.e limits are 61.8902 & 63.1098

17b/7b) Let x: systolic blood pressure

$n = 566$ $\bar{x} = 128.8$ mm $\sigma = 13.05$ mm

a) 95% limits are $\bar{x} \pm 1.96(\sigma/\sqrt{n})$

$= 128.8 \pm 1.96(13.05/\sqrt{566})$

$= 128.8 \pm 1.07506$

∴ Limits are 127.72494 & 129.87506

18a/8a) x: blood sugar level

$n = 100$, $\bar{x}$ = 80 mg% σ = 6 mg%

95% confidential limits for the mean is given by.

$\bar{x} \pm 1.96(\sigma/\sqrt{n})$

$= 80 \pm 1.176$

∴ The limits are 78.824 & 81.176

18b/8b) x : ~~wt~~ of a 10 year old boy in Delhi

$n = 225$ $\quad \bar{x} = 67$ pounds $\quad \sigma = 12$ pounds

$\therefore$ 95% confidence limit of x

is $\bar{x} \pm 1.96(\sigma/\sqrt{n})$

$= 67 \pm 1.96(0.8)$

$= 67 \pm 1.568$

i.e the limits are 65.432 pounds & 68.568 pounds

$\therefore$ 99% confidence limit of x

is $\bar{x} \pm 2.58(\sigma/\sqrt{n})$

$= 67 \pm 2.58(0.8)$

$= 67 \pm 2.064$

i.e the limits are 64.936 & 69.064 pound

19a/9a) x : diameter of rivet head

$n = 250$ $\quad \bar{x} = 7.2642$ mm $\quad \sigma = 0.0058$ mm

95% confidence limits of x $= 7.2642 \pm 1.96(0.0058/\sqrt{250})$

$= 7.2642 \pm 0.00072$

i.e the limits are ~~7.26335~~ 7.26348 &

~~7.26515 mm~~ 7.26492 mm

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99% confidence limits of x is

$= \bar{x} \pm 2.58(\sigma/\sqrt{n})$

$= 7.2642 \pm 2.58(0.0037/\sqrt{100})$

$= 7.2642 \pm 0.00095$

$\therefore$ The limits are 7.26325 & 7.26515 mm

19b
9b) x : Money spent by a student in a spring break.

$n = 80 \quad \bar{x} = 593.84\$ \quad \sigma = 369.34\$$

95% confidence limits of x is

$= \bar{x} \pm 1.96(\sigma/\sqrt{n})$

$= 593.84 \pm 1.96(369.34/\sqrt{80})$

$= 593.84 \pm 80.9352$

$\therefore$ The limits are $512.9048\$$ & 674.7752

19
20a
10 a) $n = 400 \quad \bar{x} = 22.1 \quad \sigma = 12.8$

95% confidence limit of population mean μ

$= \bar{x} \pm 1.96(\sigma/\sqrt{n})$

$= 22.1 \pm 1.96(12.8/20)$

$= 22.1 \pm 1.2544$

$= 20.8456$ & 23.3544

$\therefore$ The limits are 20.8456 & 23.3544

2cb

Tob) $\bar{x} = 67.45$ $\sigma = 2.92$ $n = 100$

$\therefore$ 95% limits are $\bar{x} \pm 1.96\,\sigma/\sqrt{n}$

$= 67.45 \pm 1.96(2.92)/10$

$= 67.45 \pm 0.57232$

$\therefore$ The limits are 66.87768 & 68.02232

$\therefore$ 99% limits are $\bar{x} \pm 2.58\,\sigma/\sqrt{n}$

$= 67.45 \pm 2.58(2.92)/10$

$= 67.45 \pm 0.75336$

$\therefore$ The limits are 66.69664 & 69.20336

1) a)

11a) Let x : length of nail

μ : pop^n mean of x = 3 inch

$n = 25$

s = sample std deviation = 0.3

$\bar{x}$ = 3.1 inch

$$\therefore t = \frac{\bar{x} - \mu}{s} \cdot \sqrt{n}$$

$$= \frac{0.1}{0.3}\sqrt{25} = 1.67 < 2.064 = t_{0.05, 24 \, df}$$

Thus the hypo that the machine is producing nails are per specifications is accepted at 5% level of significance.

11b) x : height of an individual in inches

μ = 66 inch.

$n = 10$

$$\bar{x} = \frac{\sum x_i}{n} = \underline{\underline{67.8}}$$

$$s^2 = \frac{1}{(n-1)} \sum (x_i - \bar{x})^2$$

$$= 9.067$$

$$\therefore s = \underline{\underline{3.011}}$$

$$\therefore t = \frac{\bar{x} - \mu}{s} \cdot \sqrt{n}$$

$$= \frac{67.8 - 66}{3.011} \cdot \sqrt{10} = 1.89 < 2.262$$

$$= t_{0.05, 9 \, df}.$$

Thus the hypothesis that the average height is 68 inch is accepted at 5% los.

12a) Given $n = 12$

x : ~~stimulus~~ increase in blood pressure.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{[5+2+8-1+0-2+1+5+0+4+6]}{12}$$

$$= \frac{31}{12} = 2.5833$$

$$s^2 = \frac{1}{(n-1)} \left\{ \sum x_i^2 - \left[\frac{\sum (x_i)}{n}\right]^2 \right\}$$

$$S^2 = \frac{1}{(n-1)} \sum (x_i - \bar{x})^2$$

$$= \frac{1}{(n-1)} \left[\sum x_i^2 - \frac{1}{n} (\sum x_i)^2 \right]$$

$$= \frac{1}{11} \left\{ 185 - \frac{1}{12} (31)^2 \right\}$$

$$= \underline{\underline{9.538}}$$

$$S = 3.088$$

$$\mu = 0$$

$$\therefore t = \frac{\bar{x} - \mu}{s} \cdot \sqrt{n} = 2.8979 > 2.201 = [t_{0.05}, 11 \text{ df}$$

$\therefore$ The hypo is rejected i.e with 95% confidence we can say that the stimulus in general is accompanied with increase in b.p.

12b) x : axle diameter

$n = 10$

$\bar{x} = 0.742$

$s = 0.04$

$\mu = 0.7$

$\therefore \quad t = \frac{\bar{x}-\mu}{s} \cdot \sqrt{n} = \frac{0.042}{0.04} \sqrt{10}$

$= 3.3204 > 2.262 = t_{0.05}, 9\, df$

$\therefore$ The hypo is rejected i.e on the basis of sample we can say that the cook is inferior.

13 a) If $t_{.05}$ is the ~~b~~ tabulated value of t for $n-1$ degrees of freedom at 5% l.o.s.

$\Rightarrow \quad P[\,|t| > t_{.05}] = 0.05$

$\Rightarrow \quad P[\,|t| \leq t_{.05}] = 1-0.05 = 0.95$

i.e with 95% confidence

$|t| \leq t_{.05}$

$\Rightarrow \left| \frac{\bar{x}-\mu}{\sigma_s} \cdot \sqrt{n} \right| \leq t_{.05} \qquad \because t = \frac{\bar{x}-\mu}{\sigma_s}$

$\Rightarrow -t_{.05} \leq \frac{\bar{x}-\mu}{\sigma_s} \cdot \sqrt{n} \leq t_{.05}$

$\Rightarrow$ ~~$\bar{x} - \sigma_s t$~~

$$\Rightarrow \quad -t_{0.05} \le \frac{\mu-\bar{x}}{\sigma_s}\cdot\sqrt{n} \le t_{0.05}$$

$$\Rightarrow \quad -\frac{\sigma_s t_{0.05}}{\sqrt{n}} \le \mu-\bar{x} \le \frac{\sigma_s t_{0.05}}{\sqrt{n}}$$

$$\Rightarrow \quad \bar{x} - \sigma_s\frac{t_{0.05}}{\sqrt{n}} \le \mu \le \bar{x} + \frac{\sigma_s}{\sqrt{n}} t_{0.05}$$

$\therefore$ 95% confidence limits of μ

is $\bar{x} \pm \frac{\sigma_s t_{0.05}}{\sqrt{n}}$.

13 b) x : diameter of a sphere

$n = 10$, $\bar{x} = 12\,cm$ $\quad s = 0.15\,cm$

$\therefore$ 95% confidence limits are

$$\bar{x} \pm \frac{s}{\sqrt{n}}\, t_{0.05}$$

i.e $12 \pm \frac{0.15}{\sqrt{10}}(2.262)$

i.e 12 ± 1.073

i.e $(11.893,\ 13.073)$

14 a) x : I.Q. of a boy.

$\bar{x} = (70 + 120 + 110 + 101 + 88 + 83 + 95 + 98$
$+ 107 + 100)/10$

$= \frac{972}{10} = 97.2$

$s^2 = (739.84 + 519.84 + 551.24 + 84.64$
$+ 84.64 + 201.64 + 4.84 + 0.64 + 96.04 + 7.84)/9$

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$= \frac{2071.24}{9} = 230.1378$

$S = 15.1703$

$\therefore |t| = \frac{2.8}{15.1703} \sqrt{10} = 0.5837 < 2.262$

$\therefore$ Hypothesis that the population mean i.e of 100 at 5% level of significance is accepted.

14 b) $n = 25, \quad \bar{x} = 47.5 \quad s = 8.4$

$\mu = 42.1$

$\therefore t = \frac{\bar{x} - \mu}{s} \cdot \sqrt{n}$

$= \frac{47.5 - 42}{8.4} \times \sqrt{25}$

$= \frac{45}{14} = 3.2143 > 2.064 = t_{.05}, \ d.f = 24$

$\therefore$ Hypothesis is rejected at 5% l o s.

15 a) x: diameter of a bearing.

$\mu = 0.5, \quad n = 10, \quad \bar{x} = 0.506, \quad s = 0.004$

$\therefore t = \frac{\bar{x} - \mu}{s} \cdot \sqrt{n} = \frac{0.006}{0.004} \times \sqrt{10}$

$= 4.743 > 2.262$

$\therefore$ Hypothesis is rejected at 5% los

i.e the process is not under control.

15b) x: thickness of a washer.

$\mu = 0.12$, $n = 10$, $\bar{x} = 0.128$, $\sigma = 0.008$

$\therefore\ t = \frac{\bar{x}-\mu}{s} \cdot \sqrt{n} = \frac{0.008\ \sqrt{10}}{0.008}$

$= 3.1623 > 2.262 = t_{.05},\ 9\,df$

$\therefore$ The hypothesis is rejected i.e the machine is not working in proper order at 5% l.o.s.

16a) Total no of accidents in 10 week period $= 100$

$\therefore$ no of accidents expected per week $= \frac{100}{10} = 10$

$\therefore\ O_i$	12	8	20	2	14	10	15	6	9	4
E_i	10	10	10	10	10	10	10	10	10	10

$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{1}{10}\left[4+4+100+64+16+0+25+16+1+36\right]$

$= \frac{266}{10} = \underline{\underline{26.6}} > 16.92 = \chi^2_{0.05},\ 9\,df$

$\therefore$ Hypothesis that no of accidents conditions were same during this 10 week period is rejected at 5% l.o.s.

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16b Let us take the hypo that these figures support to the general result in the ratio 4:3:2:1

The expected frequencies are

$\frac{4}{10} \times 500$, $\frac{3}{10} \times 500$, $\frac{2}{10} \times 500$, $\frac{1}{10} \times 500$

i.e 200, 150, 100, 50

$\therefore$ we have

Observed freq O_i:	220	170	90	20
Expected freq E_i:	200	150	100	50

$\therefore \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 23.67 > \chi^2_{0.05, 3df}$
$= 7.81$

$\therefore$ The hypo is rejected that is the figures don't support to the general result in the ratio 4:3:2:1 with 5% l o s.

17a We have total = 10000
no of digits = 10

$\therefore$ no of each digit if equally distributed = 1000
$= E_i$

Digits.	E_i	O_i	$(E_i - O_i)^2$
0	1000	1026	676
1	1000	1107	11449
2	1000	997	9
3	1000	966	1156
4	1000	1075	5625
5	1000	933	4489
6	1000	1107	11449
7	1000	972	784
8	1000	964	1296
9	1000	853	21609

$$\therefore \chi^2 = \Sigma \frac{(E_i - O_i)^2}{E_i}$$

$$= \frac{58542}{1000}$$

$$= 58.542 > \chi^2_{0.05}, 9\, df = 16.92$$

$\therefore$ The hypothesis is rejected at 5% l.o.s.

17b

$$\mu = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{0 + 60 + 30 + 6 + 4}{200}$$

$$= 0.5$$

$\therefore$ we have $\mu = m = 0.5$ for Poisson distribution.

$\therefore \; P(x) = \dfrac{m^x \cdot e^{-m}}{x!}$

$\therefore \; f(x) = 200 \times P(x)$

$= \dfrac{200 \times (0.5)^x \cdot e^{-0.5}}{x!}$

$= \dfrac{121.3 \, (0.5)^x}{x!}$

$\therefore \; f(0) = 121, \quad f(1) = 61, \quad f(2) = 15$

$f(3) = 3, \quad f(4) = 0.$

$\therefore$ the last of the expected freq is 0 so we shall club it with the previous one.

$\therefore$ d.f of $\chi^2 = \underline{\underline{3}}$

$\therefore$ $O_i = 122 \quad 60 \quad 15 \quad 2+1=3$

$E_i = 121 \quad 61 \quad 15 \quad 3$

$\chi^2 = \dfrac{1}{121} + \dfrac{1}{61} + 0 + 0 = 0.025 < 7.815 = \chi^2_{0.05}$

$\therefore$ The Poisson distribution gives a good fit.

18 b) ~~we have total~~

18 a) We have $m = \dfrac{\Sum f_i x_i}{\Sum f_i}$

$= 0.7825$

$\therefore P(x) = \dfrac{m^x e^{-m}}{x!} = \dfrac{(0.7825)^x \cdot e^{-0.7825}}{x!}$

$\therefore f(x) = 400 \times P(x)$

$= (182.9) \dfrac{(0.7825)^x}{x!}$

$\therefore f(0) = 183 \quad f(1) = 143, \quad f(2) = 56$

$f(3) = 15 \quad f(4) = 3, \quad f(5) = 0$

$\therefore$ the last of the expected freq is 0 we shall club it with the previous one

$\therefore$ O_i: 173 168 37 18 (3+1=4)

E_i: 183 143 56 15 (3 to = 3)

$\therefore \chi^2 = \Sum \dfrac{(O_i - E_i)^2}{E_i} = 12.297$

$\approx 12.3 > 9.49 = \chi^2_{0.05}$ 4.df

$\therefore$ With 5% l.o.s we say that Poisson fit is not good.

18 b) Total seeds = 556

We have expected frequencies of seeds of type

Round yellow $= \frac{9}{16} \times 556 = 312.75$

≈ 313

wrinkled yellow $= \frac{3}{16} \times 556 = 104.25$

≈ 104

Round & green $= \frac{3}{16} \times 556 = 104.25$

≈ 104

wrinkled & green $= \frac{1}{16} \times 556 = 34.75$

≈ 35

$\therefore$ O_i : 315 101 108 32

E_i : 313 104 104 35

$$\therefore \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(2)^2}{313} + \frac{(3)^2}{104} + \frac{(4)^2}{104} + \frac{(3)^2}{35}$$

$$= 0.0128 + 0.0865 + 0.1538 + 0.2571$$

$$= 0.5103 < 7.815 = \chi^2_{0.05},\ 3\,df$$

$\therefore$ The correspondence between theory & experiment is good.

19a) We have total digits = 200

no of digits = 10

$\therefore$ no of each digit if equally distributed $\doteq$ 20

Digits	0	1	2	3	4	5	6	7	8	9
O_i	18	19	23	21	16	25	22	20	21	15
E_i	20	20	20	20	20	20	20	20	20	20
$(O_i-E_i)^2$	4	1	9	1	16	25	4	0	1	25

$$X^2 = \sum_{i=1}^{10} \frac{(O_i-E_i)^2}{E_i} = \frac{86}{20} = 4.2 < 16.92$$
$$= \chi^2_{0.05,\,9df}$$

$\therefore$ Hypothesis is accepted i.e digits were distributed in equal number in the tables from which these were chosen.

19b)

x_i :	0	1	2	~~3~~ 3	4
f_i :	419	352	154	56	19
$x_i f_i$:	0	352	308	168	76

$$\therefore m = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{904}{1000} = 0.904$$

$\therefore e^{-m} = 0.4049$ $\quad\therefore P(x_i) = \dfrac{e^{-m} m^{x_i}}{x_i!}$

$\therefore E_i = 1000 \times P(x_i)$

Note the total is not 1000 ΣE_i

$\therefore E_i$;	404.9	366	165.4	49.8	11.3	$= \boxed{997.4}$
	$\boxed{406.2}$	366	165.4	49.8	$\boxed{12.6}$	

$\therefore$ we adjust in I^{st} & last value equally

$\therefore$ d.f of χ^2 is reduced again by 1

$$\therefore \chi^2 = \sum_{i} \frac{(E_i - O_i)^2}{E_i}$$

$$= 0.403 + 0.536 + 0.786 + 3.251$$

$$= 5.748 < \chi^2_{0.05} = 7.82$$

$\therefore$ Poisson fit is a good fit.

20 a) When a pair of fair is thrown each sample point will have same prob $= \frac{1}{36}$

$P[sum = 2] = P(1,1) = \frac{1}{36}$

$P[sum = 3] = P(1,2) + P(2,1) = \frac{2}{36}$

$P[sum = 4] = P[1,3] + P(3,1) + P(2,2) = \frac{3}{36}$

IIIly $P[sum = 5] = \frac{4}{36}$, $P[sum = 6] = \frac{5}{36}$

$P[sum = 7] = \frac{6}{36}$. $P[sum = 8] = \frac{5}{36}$ $P[sum = 9] = \frac{4}{36}$

$P[sum = 10] = \frac{3}{36}$ $P[sum = 11] = \frac{2}{36}$ $P[sum = 12] = \frac{1}{36}$

E_i = Expected freq that sum = i = $P[sum = i] \times 360$

$\therefore$ we have

Sum	2	3	4	5	6	7	8	9	10	11	12
O_i	8	24	35	37	44	65	51	42	26	14	14
E_i	10	20	30	40	50	60	50	40	30	20	10

$\therefore \chi^2 = \frac{4}{10} + \frac{16}{20} + \frac{25}{30} + \frac{9}{40} + \frac{36}{50} + \frac{25}{60} + \frac{1}{50}$

$+ \frac{4}{40} + \frac{16}{30} + \frac{36}{20} + \frac{16}{10}$

$= 7.4483 < \chi^2_{0.05}$ 10 d.f $= 18.31$

$\therefore$ The fit is good with 5% LOS.

20b Total no trials = 1000

x	f_i		xf	E_i		$(O_i - E_i)^2/E_i$
0	305		0	$300.9 = 301$		0.0532
1	365		365	361		0.0443
2	210		420	217		0.2258
3	80		240	87		0.5632
4	28		112	26		0.1538
5	9	12	45	6.26	7.7	2.401
6	2		12	1.25		
7	1		7	0.215		
			1201			3.4413

$\because$ the freq is <10 we add

$\therefore m = \frac{\Sigma xf}{\Sigma f} = \frac{1201}{1000} = 1.201$

$\therefore$ total pts are 6

$\therefore$ d.f $= 6-1 = 5$

$\therefore e^{-m} = 0.3009$

$\therefore E_i = \frac{e^{-m} m^i}{i!} \times 1000, i = 0, 1 \dots 7$

$\therefore \chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 3.4413 < 11.07 = \chi^2_{0.05, 5}$

$\therefore$ Poisson fit is good.