Operational Research

* ~~The main origin of operational~~

* OR is the art of winning war without actually fighting it.

OR is the art of finding bad answer where worse exist.

The History Developement.

* The main origin of OR was during the II world war. At that time the military managment in England called upon a team of scientists to study the strategic and tactical problem related to air and land defence of the country.

Their mission was to formulate specific proposals and plans for aiding the military commands to arrive at the occisions on optimial utilisation of scare military resources and efforts and to implement the decisions effectively.

The OR teams were not actually engaged in military operations and in fighting the war. They were only advisors and significantly instrumental in winning the war. to the extent that the scientific and systamatic approaches involved in OR provided good intellectual support to the strategic initiatives of the military commands.

Hence OR can be associated with.

"An art of winning the war without actually fighting it"

* "As the name implies "Operational Research" was invented because the team was dealing with research on (military) operations.

* The work of OR team was given various names in United States operational Analysis, operations Evaluation, Operation Research, System Analysis, System Evaluation, System Research and Management science.

The nature and meaning of OR

Gere

1) Operation Research is the art of winning war. without actually fighting it

2) OR is scientific approach to problem solving for exective management.

3) OR is the systematic method oriented study of the basic structure charasteristics Functions and relationship of an organization to provide the executive and a sound, scientific basis for decision making.

4) OR is the art of giving bad answers to problems to which otherwise ~~were an~~ worse answers are given.

5) OR is a scientific method of providing executive departments. with quantitative basis for decision regarding the operations under their control.

Management Applications of operation Research.

Some of the areas of management decision making, where tool and techniques of OR are applied.

1. Finance : Budgeting and Investment.

   Cash flow analysis, long range capital requirement dividend policies, investment portfolios.

2. Production Management.
   1. Physical Distribution.
   2. Facilities Planning
   3. Manufacturing.
   4. Maintenance and Project scheduling.

3. Marketing
   Product Selection, Number of saleman, Advertising media.

4. Purchasing. Procurement and Exploration.
   1. Rules for buying
   2. Determination of quantities and timing of purchase
   3. Bidding policies.
   4. Strategies for exploration and exploitation of raw material
   5. Replacement policies.

⑤ Personnel Management.
Selection of personal on minimum salary, mixes of ages and skills. Recuriment policies,

⑥ Research and development.
Determination of the areas of concentration of research and development. Project selection. Determination of time, cost of development project, Reliability and alternative design.

Modelling in Operations Research

Definition : A model is actually in OR is representation of an actual object or situation It show relation and inter relation of action and reaction in terms of cause and effect.

The main objective of a model is to provide means for analysing the behaviour of the system. for the purpose of improving its performance.

* Models can be classified according to the following characteristics.

1. Classification by structure.

ⓐ Iconic Models :- Iconic models represents the system as it is by scaling it up or down. It is an image of original object.

Eg: ① a toy airplane is an iconic model of a real one.

② A model of an atom is scaled up so

as to make it visible to the naked eyes.

(b) Analogue model: The model in which one set of properties is used to represent another set of properties are called analogue models.

(c) Symbolic models: The symbolic model is one which employs a set of mathematical symbols to represent the decision variables of the system.

II Classification by Purpose.

(1) Descriptive Models: A model simply describe some aspects of a situation based on observation, survey, questionnaire results or other available data.

(2) Predictive Models: Such model can answer 'what if' type questions, they can make prediction regarding certain events.

Eg:- based on the survey results and television networks, models can explain the predict the election results.

(3) Prescriptive models: When a predictive model has been repeatedly successful, it can be used to prescribe a source of action.

Date	
Page	

III Classification of Nature of Environment.

① Deterministic models : Such models assume conditions of complete certainity and perfect knowledge.

Probabilistic Models : These types of models usually handle such situations in which the managerial actions cannot be predicted with certainity.

IV classification by behaviour.

① Static Models :- These models do not consider the impact of changes that take place during planning horizon. ie they are independent of time.

② Dynamic Model : In this model, time is considered as one of the important variables and admit the impact of changes generated by time.

V Classification by Method of Solution.

① Analytical model : These models have a specific mathematical structure and thus can be solved by known analytical or mathematical techniques.

② Simulation Models : They also have a mathematical structure but they cannot be solved by purely using the tools and techniques of mathematics.

VI classification by Use of Digital Computers.

(i) Analogue and Mathematical models combined. Sometimes analogue models are also expressed in terms of mathematical symbols.

(2) Function models : Such models are grouped on the basis of the function being performed.

(3) Quantitative model : such models are used to measure the observations.

(4) Heuristic model : These models are mainly used to explore alternative strategies that were overlooked previously, whereas mathematical model are used to represent systems possessing well defined strategies.

Model In Operation Research

- **Classification By Structure.**
  1. Iconic Model
  2. Analogue Model
  3. Symbolic (Mathematical) Model
- **Classification by purpose**
  1. Descriptive
  2. Predictive
  3. Perscriptive
- **Classification by Nature.**
  1. Deterministic
  2. Probobilistic
- **Classification By Behaviours**
  1. Dynamic
  2. Static
- **Classification by Method of Solution.**
  1. Analytical Model.
  2. Simulation Model.
- **Classification by use of digital computers**
  1. Analogue & Mathematical model
  2. Function Model
  3. Quantitative Model.
  4. Heuristic Model

Date
Page

Main Characteristics of OR:

(i) Inter-disciplinary team approach :-

In OR, the optimum solution is found by a team of scientists selected from various disciplines such as mathematics, statistics, economics, engineering, physics etc

The OR team required for a big Organization may include a statistician, an economist, a mathematicial one or more engineers, a psychologist and some supporting staff like computer programmers etc.

(ii) Wholistic approach to the system.

The most of the problem tackled by OR have the characteristic that OR tries to find the best decisions relative to largest possible portion of the total organization. The nature of organization is essentially immaterial.

(iii) Imperfectness of solutions.

By OR techniques we cannot obtain perfect answers to our problems but only the quality of the solutions is improved from worse to bad answers.

(iv) Use of scientific research.

OR uses techniques of scientific research to reach the optimum solution.

(v) To optimize the total output

OR tries to optimize total return by maximizing the profit and minimizing the cost or loss.

Main Phases of OR Study

Phase I : Formulating the problem.

Before proceeding to find the solution of a problem first of all one must be able to formulate the problem in the form of an appropriate model.

(i) To take the decision.

(ii) The objectives.

(iii) The ranges of controlled variables

(iv) The uncontrolled variables that may affect the possible solutions.

(v) The restrictions or constraints on the variables.

The wrong formulation cannot yield a right decision one must be considerably careful while executing this phase.

Phase II : Constructing a mathematical model.

The second phase of the investigations is concerned with the reformulation of the problem in an appropriate form which is convenient for analysis. The most suitable form for this purpose is to construct a mathematical model representing the system under study.

→ It requires the identification of both static and dynamic structural elements.

→ A mathematical model should include the following three important basic factors.

(i) Decision variables and parameters.

(ii) Constraints or Restrictions.

(iii) Objective function.

Phase III : Deriving the solutions from the model.

This phase is devoted to the computation of those values of decision variables that maximize (or minimize) the objective function. Such solution is called an optimal solution which is always in the best interest of the problem under consideration.

Phase IV : Testing the model and its solution (updating the model)

After completing the model, it is once again tested as a whole for the errors if any. A model may be said to be valid if it can provide a reliable prediction of the system's performance. A good practitioner of Operations Research realises that his model be applicable for a longer time and thus he updates the model time to time by taking into account the past, present and future specifications of the problem.

Phase V : Controlling the solution.

This phase establishes controls over the solution with any degree of satisfaction. The model requires immediate modification as soon as the controlled variables change significantly otherwise the model goes out of control. As the conditions are constantly changing in the world, the model and the solution may not remain valid for a long time.

Phase VI Implementing the solution

The tested results of the model are implemented to work. This phase is primarily executed with the cooperation of Operational Research experts and those who are responsible for managing and operating the systems.

Scope of operations Research.

1. In Agriculture: with the explosion of population and consequent shortage of food every country is facing the problem of.

   (i) optimum allocation of land to various crops in accordance with the climatic conditions. and
   (ii) Optimum distribution of water from various resources like canal for irrigation purpose.

2. In Finance: In modern times of economic crisis, it has become very necessary for every government to have a careful planning for the economic development of her country OR-techniques can be fruitfully applied.

   (i) To maximize the per capita income with minimum resources
   (ii) To find out the profit plan for the company
   (iii) To determine the best replacement policies etc.

3. In Marketing: with the help of OR techniques a Marketing Administrator (Manager) can decide.

   (i) Where to distribute the products for sale so that the total cost of transportation etc is minimum.
   (ii) the minimum per unit sale price.
   (iii) The size of the stock to meet the future demand.

(iv) To select the best advertizing media with respect to time, cost etc.

(v) How To purchase at the minimum possible cost?

(4) In Personnel Management.

(i) To appoint the most suitable persons on minimum salary

(ii) To determine the best age of retirement for the employees.

(ii) To find out the number of persons to be appointed on full time basis when the workload is seasonal.

Limitations of Operations Research.

OR has some limitations however, these are related to the problem of model building and the time and money factors involved in application rather than its practical utility. Some of them are as follows.

(i) Magnitude of Computations.

OR models try to find out optimal solution taking into account all the factors. These factors are enormous and expressing them in quantity and establishing relationships among these require voluminous calculations which can be handled by computers.

(ii) Non Quantifiable Factors.

OR provides solution only when all elements related to a problem can be quantified. All relevant variables do not lend themselves to quantification. Factors which cannot be quantified find no place in OR study. Models in OR do not take into account quantitative factors or emotional factors which may be quite important.

(iii) Distance between User and Analyst.

OR being specialist's job requires a mathematician or statistician who might not be aware of the business problems.
III^{ly} a manager fails to understand the complex working of OR. Thus there is a gap between the two, Management itself may offer a lot of resistance due to conventional thinking.

(iv) Time and Money Costs.

The basic data are subjected to frequent changes. incorporating them into the OR models is a costly proposition. Moreover a fairly good solution at present may be more desirable than a perfect OR solution available after sometime. The computational time increases depending upon the size of the problem and accuracy of results desired.

(v) Implementation.

It must take into account the complexities of human relations and behaviour. Sometimes, resistance is offered due to psychological factors which may not have any bearing on the problem as well as its solution.

LPP → Structure → (By Reading)

Formulate → Solution (LPP)

- Graphical Method.
- Simplex Method
- Big M Method
- ....

LPP Formulation
- → statements.
- → Draw table in solution.
- → Table is given in. Question itself.

Linear Programming Problem (LPP)
(Formulation).

↓

Structure →
- → Decision variables.
- → Objective function.
- Constraints

~~Decision Variables~~.

Definition :- LP is one of the most important optimization (maximization / minimization) techniques develope in the field of Operation Research (OR).

Linear Programming Problem (LPP)

Definition :- The General LPP calls for optimizing (maximizing / minimizing) a linear function of variables called the 'objective function' subject to a set of linear equations and / or inequalities called the constraints or Restrictions.

Two methods in LPP are
① Formulation.
② Graphical Method.

Formulation of LPP.

1. The given problem must be presented in Linear programming form. This requires defining the variables of the problem. establishing inter-relationships between them and formulating the objective function and constraints.
2. A model is developed from the given problem.
2. If some constraints happen to be non linear, they are approximated to appropriate linear functions to fit the linear programming format.

Mathematical Formulation of a Linear Programming Problem.

1. Identifying a set of variables $x_1, x_2 \ldots x_n$ which are subject to certain linear conditions known as constraints which may be written in the form of inequalities.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \leq (\geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \leq (\geq)\ b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \leq (\geq)\ b_n$$

where the coefficients a_{ij} b_i $(1 \leq i \leq m,\ 1 \leq j \leq n)$ are constants and $x_1 \geq 0,\ x_2 \geq 0,\ x_n \geq 0$

The objective function involving the variables $x_1, x_2 \ldots x_n$ along with the given constants $c_1, c_2 \ldots c_n$ will be a linear function.

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n = \sum_{i=1}^{n} c_i \cdot x_i$$

The objective function Z
(maximization or minimization)

Problem

A company produces two types of Hats. Each hat of the first type requires twice as much labour time as the second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits daily sales of the first and second type to 150 and 250 hats. Assuming that the profits per hat are Rs 8 for type A and Rs 5 for type B, formulate the problem as a linear programming model in order to determine the number of hats to be produced of each type so as to maximize the profit.

Solu: Let the Company produce x_1 hats of type A and x_2 hats of type B each day.

The Profit P after selling these two product is given by Linear function.

$$P = 8x_1 + 5x_2 \quad \text{(objective function)}$$

Since the company can produce at the most 500 hats in a day and A type of hats require twice as much time as that of type B.

$$2tx_1 + tx_2 \le 500t$$ where t is labour time per unit of second type.

ie $2x_1 + x_2 \leq 500$

There are limitations on the sale of hats,

$x_1 \leq 150 \quad x_2 \leq 250$

Since company cannot produce negative quantities

$x_1 \geq 0$ & $x_2 \geq 0$

Find x_1 and x_2 such that

Profit $P = 8x_1 + 5x_2$ is maximum.

subject to the restrictions:

$2x_1 + x_2 \leq 500$

$x_1 \leq 150$

$x_2 \leq 250$

$x_1 \geq 0 \quad x_2 \geq 0$.

Steps involved in the formulation of LP Problem:

Step I :- Identify the decision variables of interest to the decision maker & express them as $x_1, x_2 \ldots$

Step 2 :- Ascertain the objective of the decision maker whether he wants to minimize or maximize.

Step III : Ascertain the cost (in case of minimization problem) or the profit (in case of maximization problem) per unit of each of the decision variables.

Step IV : Ascertain the Constraints representing the maximum availability or minimum commitment or equality and represent them as less than or equal to ($\leq$) type inequality or greater than or equal to ($\geq$) type inequality or equal to ($=$) type equality respect.

Step V Put non negativity restriction as under:

$x_j \geq 0 \quad j = 1, 2, \ldots n$

Step VI : Now formulate the LP problem as under:

Maximize (or Minimize) $z = c_1x_1 + c_2x_2 + \ldots c_nx_n$

Subject to Constraints : $a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$

(maximum availability)

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$

(minimum commitment)

$a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n = b_3$

(equality)

$\vdots$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_m \begin{matrix}\leq \\ \geq\end{matrix} b_m$$

$$x_1, x_2 \cdots x_n \geq 0 \quad (\text{non-negativity})$$

where

x_j = Decision variables ie quantity of j^{th} variable of interest to the decision maker.

c_j = Constant representing per unit contribution (in case of maximization problem) or cost (in case of minimizatio problem of the j^{th} decision variables.

a_{ij} = Constant representing exchange coefficient of the j^{th} decision variable in the i^{th} constant

b_i = Constant representing i^{th} constraint requirement or availability.

Problem

A company produces two types of Hats. Each hat of the first type requires twice as much labour time as the second type. If all hats are of the second type only. The company can produce a total of 500 hats a day. The market limits daily sales of the first and second type to 150 and 250 hats. Assuming that the profits per hat are Rs 8 for type A and Rs 5 for type B. formulate the problem as a linear programming model in order to determine the number of hats to be produced of each type so as to maximize the profit.

Solu: Let the Company produce x_1 hats of type A and x_2 hats of type B each day.

The Profit P after selling these two product is given by Linear function.

$$P = 8x_1 + 5x_2 \quad \text{(objective function)}$$

Since the company can produce at the most 500 hats in a day and A type of hats require twice as much time as that of type B.

$2tx_1 + tx_2 \leq 500t$ where t is labour time per unit of second type.

ie $2x_1 + x_2 \leq 500$

There are limitations on the sale of hats,

$x_1 \leq 150 \quad x_2 \leq 250$

Since company cannot produce negative quantities

$x_1 \geq 0$ & $x_2 \geq 0$

Find x_1 and x_2 such that

Profit $P = 8x_1 + 5x_2$ is maximum.

subject to the restrictions:

$$2x_1 + x_2 \leq 500$$
$$x_1 \leq 150$$
$$x_2 \leq 250$$
$$x_1 \geq 0 \quad x_2 \geq 0.$$

(2) A firm manufactures two type of products A and B and sells them at a profit of Rs 2 on type A and Rs 3 on type B. Each product is processed on two machines G and H. Type A requires one minute of processing time on G and two minutes on H; type B requires one minute on G and one minute on H. The machine G is available for not more than 6 hour 40 minutes while machine H is available for 10 hours during any working day. Formulate the problem as a LPP.

Solu: Let x_1 be the number of products of type A and x_2 the number of products of type B.

Machine	Time of Products (min) Type A x_1 units	Type B x_2 units	Available Time (Minutes)
G	1	1	400
H	2	1	600
Profit per unit	Rs 2	Rs 3	

~~Profit on type A is Rs 2 per product~~

Profit P after selling these two products is given by linear function

$$P = 2x_1 + 3x_2 \quad \text{(objective function)}$$

The machine G takes 1 minute time on type A and 1 minute time on type B and machine G is available for more than 6 hour 40 min

$$= (6 \times 60 + 40 = 400 \text{ min})$$

$$x_1 + x_2 \leq 400 \quad \text{(First Constraint)}$$

III^{ly} machine H takes 2 minutes time on type A and 1 minute time on type B and H is available for 10 hours $= (10 \times 60 = 600$ minute$)$.

$$2x_1 + x_2 \leq 600 \quad \text{(second constraint)}$$

Since company cannot produce negative quantities

$$x_1 \geq 0 \; \& \; x_2 \geq 0.$$

Objective Function

Profit $P = 2x_1 + 3x_2$ maximum.

Subject to the Constraints.

$$x_1 + x_2 \leq 400.$$

$$2x_1 + x_2 \leq 600$$

$$x_1 \geq 0 \quad x_2 \geq 0.$$

(2) ~~A company produces 3 items A, B, C. Each unit of A requires 8 minutes, 4 minutes and 2~~

(3) A manufacturer has three machines M_1 M_2 M_3 with which he produces three different articles A, B, C. The different Machine times required per article. The amount of time available in any ~~day~~ week on each machine and the estimated profits per article are furnished in the following table.

Article	Machine time (in hrs)			Profit per article (in rupees)
	M_1	M_2	M_3	
A	8	4	2	20
B	2	3	0	6
C	3	0	1	8
Available machine hrs	250	100	60	

Soln :- Let x_1, x_2 and x_3 be the number of articles produced ~~per day of type A, type B and~~ by A, B, C.

Profit gained by the ~~producer~~ manufacturer is

$$P = 20x_1 + 6x_2 + 8x_3 \text{ (objective function)}$$

Machine M_1 requires 8 hrs for A, 2 hrs for B and 3 hrs for C to produce the article M_1 available time is 250 hrs

$$8x_1 + 2x_2 + 3x_3 \leq 250 \text{ (first Constraint)}$$

iii) M_2 requires 4 hrs for A, 3 hrs for B and 0 hrs for C and M_2 available time is 100 hrs.

$$4x_1 + 3x_2 + 0x_3 \leq 100 \quad \text{(second Constraint)}$$

M_3 requires 2 hrs for A, 0 hrs for B 1 hrs for C and M_3 available time is 60 hrs.

$$2x_1 + 0x_2 + 1x_3 \leq 60 \quad \text{(third constraint)}.$$

Since it is not possible to produce negative quantities.

$$x_1 \geq 0, \quad x_2 \geq 0 \text{ and } x_3 \geq 0$$

(non-negativity Restrictions)

objective function.

$$P = 20x_1 + 6x_2 + 8x_3$$

Subjected to the Constraints: $8x_1 + 2x_2 + 3x_3 \leq 250$

$$4x_1 + 3x_2 + 0x_3 \leq 100.$$

$$2x_1 + 0x_2 + 1x_3 \leq 60$$

$$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0.$$

(4) A radio factory produces two different types of transistors ordinary model and special model. For greater efficiency the assembly and finishing process are performed in two different workshop. The ordinary model requires 3 hours of work in workshop I and 4 hours of work in workshop II while special model requires 6 hours of work in workshop I and 4 hours in workshop II. Due to limited

resources in skilled labour and materials only 180 hours of work can be done in workshop I and 200 hours of work in workshop II, The factory makes a profit of Rs 30/- on each ordinary model and Rs 40/- on each special Model. Formulate LPP

Solu:- ~~Let x and y are are the two different transistors ie ordinary model and special model~~.

Let x be the ordinary transistor model and y be special transistor model.

Workshop I requires 3 hrs for ordinary model and 6 hours for special model to work. and workshop I works for only 180 hou

$$3x + 6y \leq 180. \quad (\text{first Constraint})$$

Workshop II requires 4 hrs for ordinary model and 4 hrs for special model to work and workshop II works only for 200 hrs.

$$4x + 4y \leq 200.$$

Profit of Rs 30 on ordinary model and 40 on special model.

$$P_{max} = 30x + 40y.$$

Since it is not possible to produce negative quantity.

$$x \geq 0 \quad y \geq 0.$$

Objective Function $P_{max} = 30x + 40y$

Subjected to Constraint:

$$3x + 6y \leq 180$$
$$4x + 4y \leq 200$$
$$x \geq 0 \quad y \geq 0.$$

Graphical Method.

LPP involving 2 decision variables can easily be solved by graphical method. The major steps involved in the solution are summarized as.

1. Identify the problem the decision variables, the objective and the restrictions.

2. Set up the mathematical formulation of the problem.

3. Plot a graph representing all the constraints of the problem and identify the feasible region (Solution space or Commonly shaded region). The feasible region is the intersection of all the regions represented by the constraints of the problem and is restricted to the first quadrant only.

4. The feasible region obtained in step 3 may be bounded or unbounded. Compute the Coordinates of all the corner points of the feasible region.

Note :- Inequality - Constraint corresponding to the line is '≤' then the region below the line lying in the I quadrant is shaded.

'≥' then the region above the line in the I quadrant is shaded.

Problems

1. Solve by graphical Method.

$$\text{Max } Z = 5x_1 + 3x_2$$

subject to

$$3x_1 + 5x_2 \le 15$$
$$5x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0$$

Solu :-

$3x_1 + 5x_2 = 15$

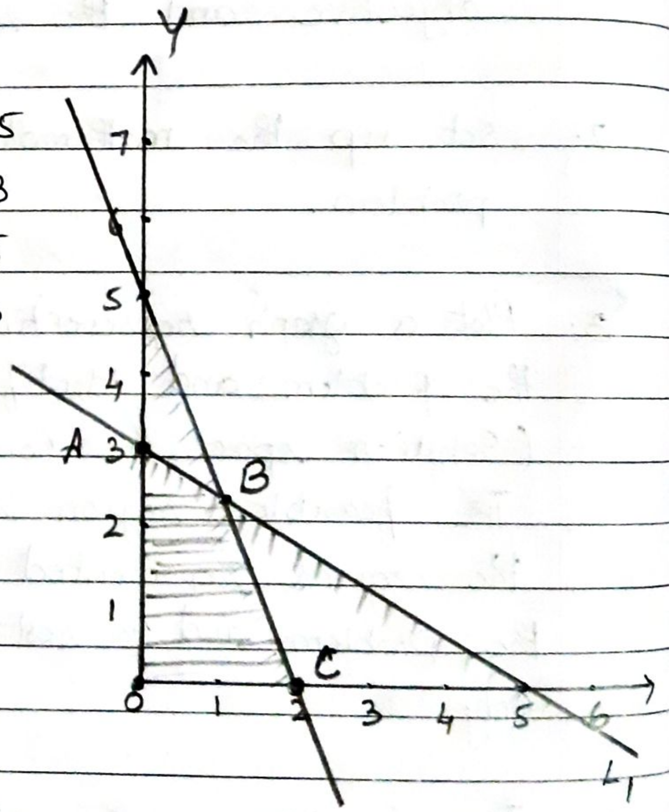
$x_1 = 0 \quad 5x_2 = 15$

$x_2 = 3$

$x_2 = 0 \quad 3x_1 = 15$

$x_1 = 5$

$L_1 : (0,3) \ (5,0)$



$5x_1 + 2x_2 \le 10$

$x_1 = 0 \quad 2x_2 = 10$

$x_2 = 5$

$x_2 = 0 \quad 5x_1 = 10$

$x_1 = 2$

$L_2 : (2,0) \ (0,5)$

Extreme points are O, A, B, C

$O(0,0) \quad A = (0,3) \quad B = \left(\frac{20}{19}, \frac{45}{19}\right) \quad C = (2,0)$

B is found out by solving two equ L_1 & L_2

$$3x_1 + 5x_2 = 15$$
$$5x_1 + 2x_2 = 10$$

$$x_1 = \frac{20}{19} \qquad x_2 = \frac{45}{19}$$

$O : (0,0) \qquad Z_{\cancel{max}} = 5(0) + 3(0) = 0$

$A : (0,3) \qquad Z_{\cancel{max}} = 5(0) + 3(3) = 9$

$B = \left(\frac{20}{19}, \frac{45}{19}\right) \qquad Z_{\cancel{max}} = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19}$

$C : (2,0) \qquad Z = 5(2) + 3(0) = 10$

$Z_B = \frac{235}{19}$ is maximum for the point $\left(\frac{20}{19}, \frac{45}{19}\right)$

optimum Solution.

(2) Max $Z = 4x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1000$

$x_1 + x_2 \leq 800$

$x_1 \leq 400$

$x_2 \leq 700.$

$x_1, x_2 \geq 0.$

Soln: $2x_1 + x_2 = 1000$

$x_1 = 0 \qquad x_2 = 1000$

$x_2 = 0 \qquad x_1 = 500$

$L_1 : (0, 1000) \; (500, 0)$

$x_1 + x_2 = 800$

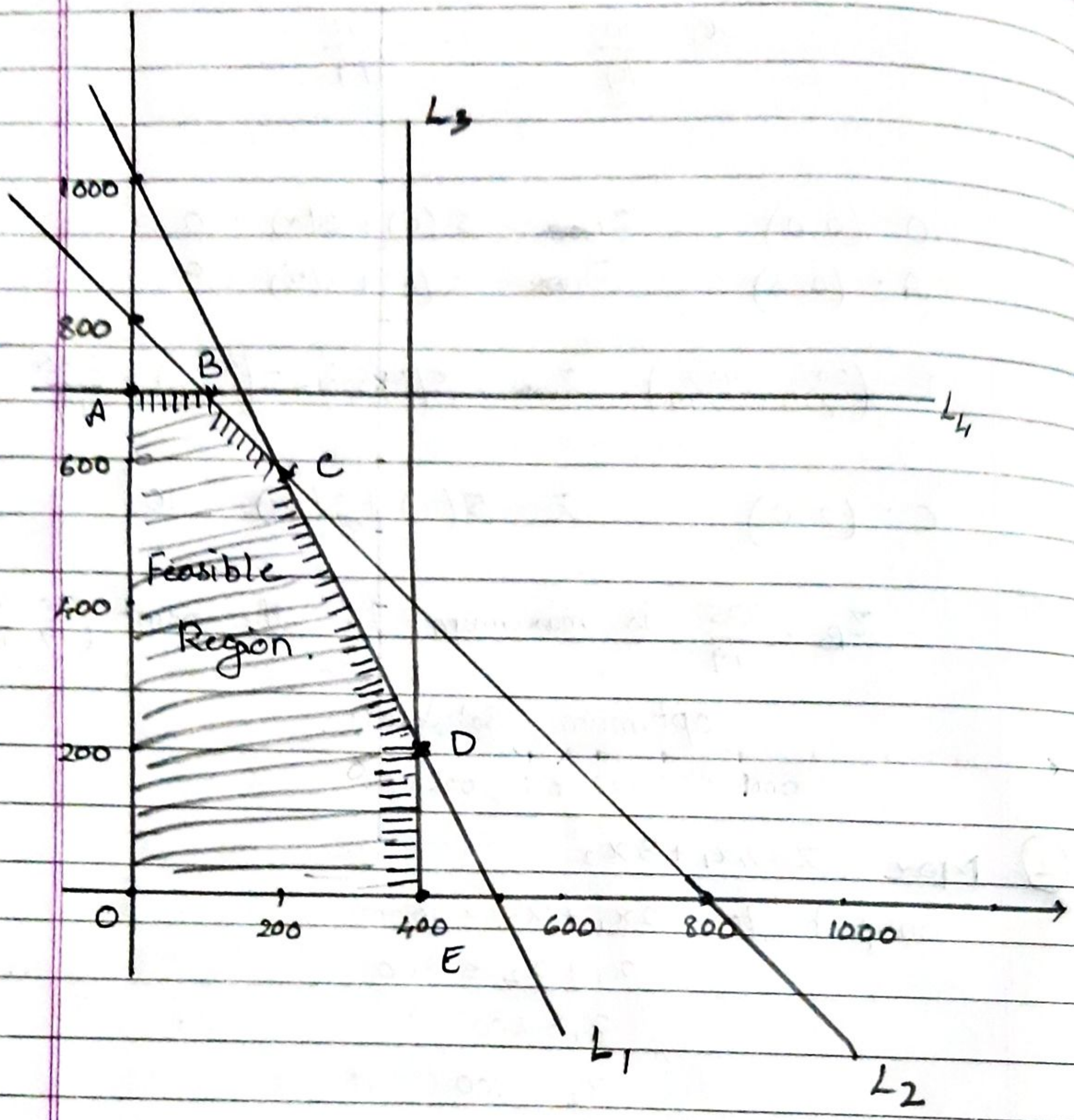
$x_1 = 0 \qquad x_2 = 800$

$x_2 = 0 \qquad x_1 = 800.$

$L_2 : (800, 0) \; (0, 800)$

$L_3 : x_1 = 400.$

$L_4 : x_2 = 700$



$O : (0,0)$

$A : (0, 700)$

B $\quad L_2 : L_4$

$$x_1 + x_2 = 800.$$

$$x_2 = 700$$

$$x_1 = 100$$

$B : (100, 700)$

$C : \quad L_1 : L_2 \qquad 2x_1 + x_2 = 1000$

$$x_1 + x_2 = 800$$

$x_1 = 200$

$x_2 = 800 - 200 = 600$

C : (200, 600)

D : $L_3 : L_1$

$x_1 = 400$

$2x_1 + x_2 = 1000$

$2(400) + x_2 = 1000$

$x_2 = 1000 - 800$

$x_2 = 200$

D : (400, 200)

E : (400, 0)

Extreme point $\quad Z = 4x_1 + 3x_2$

O : (0, 0) : $Z = 0$

A : (0, 700) $\quad Z = 4(0) + 3(700) = 2100$

B : (100, 700) $\quad Z = 4(100) + 3(700) = 2500$

C : (200, 600) $\quad Z = 4(200) + 3(600) = 2600$ *

D : (400, 200) $\quad Z = 4(400) + 3(200) = 1800$

E : (400, 0) $\quad Z = 4(400) + 3(0) = 1600$

$\therefore$ optimal solution $(x_1, x_2) = (200, 600)$

$Z_{max} = 2600$.

③ Min $Z = 20x_1 + 40x_2$

Subject to $36x_1 + 6x_2 \geq 108$

$3x_1 + 12x_2 \geq 36$

$20x_1 + 10x_2 \geq 100$

$x_1, x_2 \geq 0$

Solu: $L_1 : 36x_1 + 6x_2 = 108$

$x_1 = 0 \quad 6x_2 = 108$

$x_2 = 18$

$x_2 = 0 \quad 36x_1 = 108$

$x_1 = 3$

$L_1 : (3,0) \ (0,18)$

$L_2 : 3x_1 + 12x_2 = 36$

$x_1 = 0 \quad 12x_2 = 36$

$x_2 = 3$

$x_2 = 0 \quad 3x_1 = 36$

$x_1 = 12$

$L_2 : (12,0) \ (0,3)$

$L_3 : 20x_1 + 10x_2 = 100$

$x_1 = 0 \quad 10x_2 = 100$

$x_2 = 10$

$x_2 = 0 \quad 20x_1 = 100$

$x_1 = 5$

$L_3 : (5,0) \ (0,10)$

B: $L_1 : L_3$

$36x_1 + 6x_2 = 108$

$20x_1 + 10x_2 = 100$

$x_1 = 2 \quad x_2 = 6$

C: L_2, L_3

$3x_1 + 12x_2 = 36$

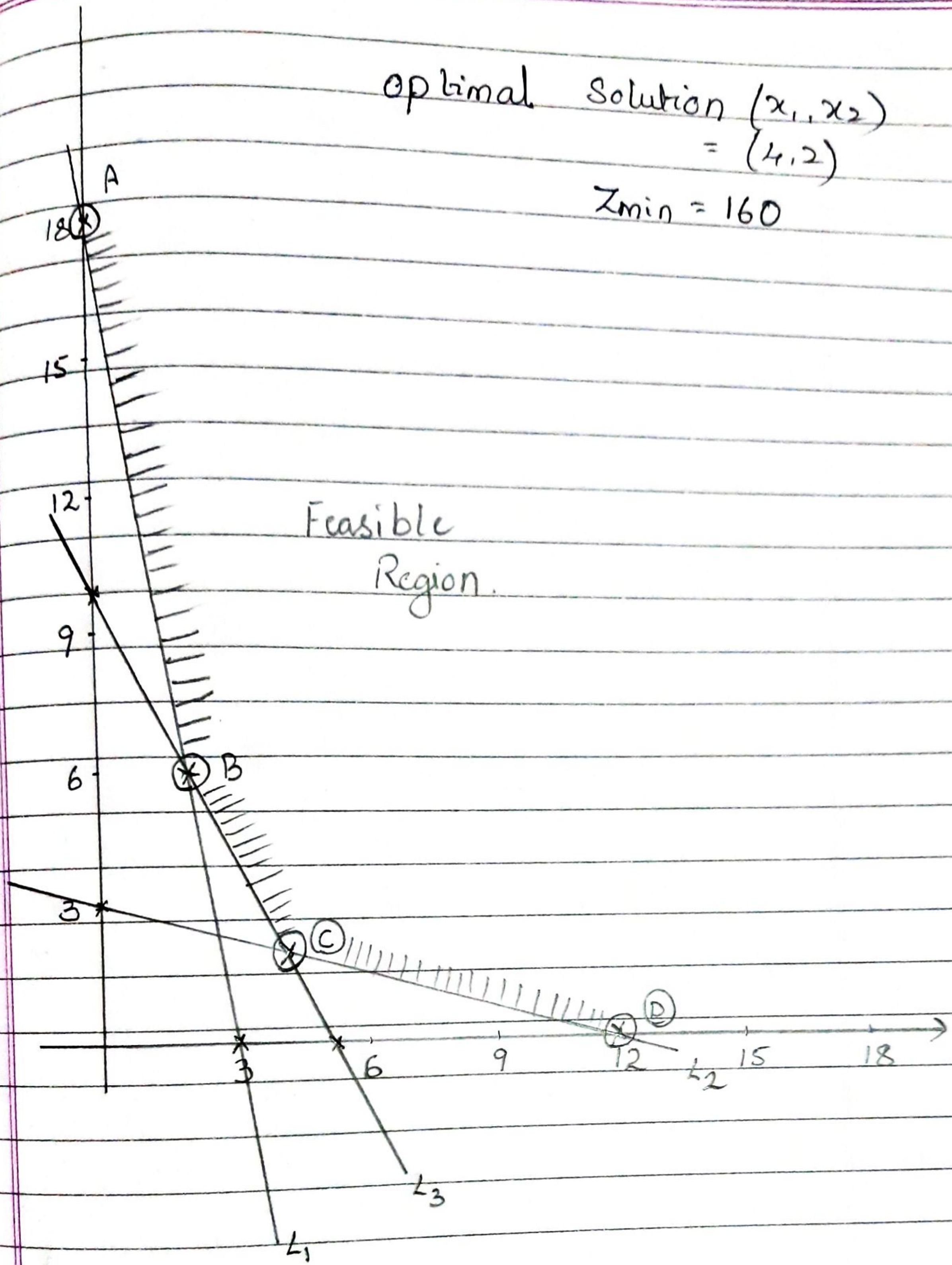
$20x_1 + 10x_2 = 100$

$x_1 = (4)$

$x_2 = 2$

Optimal Solution $(x_1, x_2) = (4, 2)$

$Z_{min} = 160$



Extreme points

$Z = 20x_1 + 40x_2$

A : (0, 18) : $20(0) + 40(18) = 720$

B : (2, 6) $20(2) + 40(6) = 280$

$\boxed{C : (4, 2) \quad 20(4) + 40(2) = 160}$ *

D : (12, 0) $20(12) + 40(0) = 240$

(3) Min $Z = 20x_1 + 40x_2$

Subject to $36x_1 + 6x_2 \geq 108$

$3x_1 + 12x_2 \geq 36$

$20x_1 + 10x_2 \geq 100$

$x_1, x_2 \geq 0$

Solu: L_1 : $36x_1 + 6x_2 = 108$

$x_1 = 0$ $\quad 6x_2 = 108$

$x_2 = 18$

$x_2 = 0$ $\quad 36x_1 = 108$

$x_1 = 3$

$L_1 : (3,0)\ (0,18)$

$L_2 : 3x_1 + 12x_2 = 36$

$x_1 = 0$ $\quad 12x_2 = 36$

$x_2 = 3$

$x_2 = 0$ $\quad 3x_1 = 36$

$x_1 = 12$

$L_2 : (12,0)\ (0,3)$

$L_3 : 20x_1 + 10x_2 = 100$

$x_1 = 0$ $\quad 10x_2 = 100$

$x_2 = 10$

$x_2 = 0$ $\quad 20x_1 = 100$

$x_1 = 5$

$L_3 : (5,0)\ (0,10)$

B: $L_1 : L_3$

$36x_1 + 6x_2 = 108$

$20x_1 + 10x_2 = 100$

$x_1 = 2$ $\quad x_2 = 6$

C: L_2, L_3

$3x_1 + 12x_2 = 36$

$20x_1 + 10x_2 = 100$

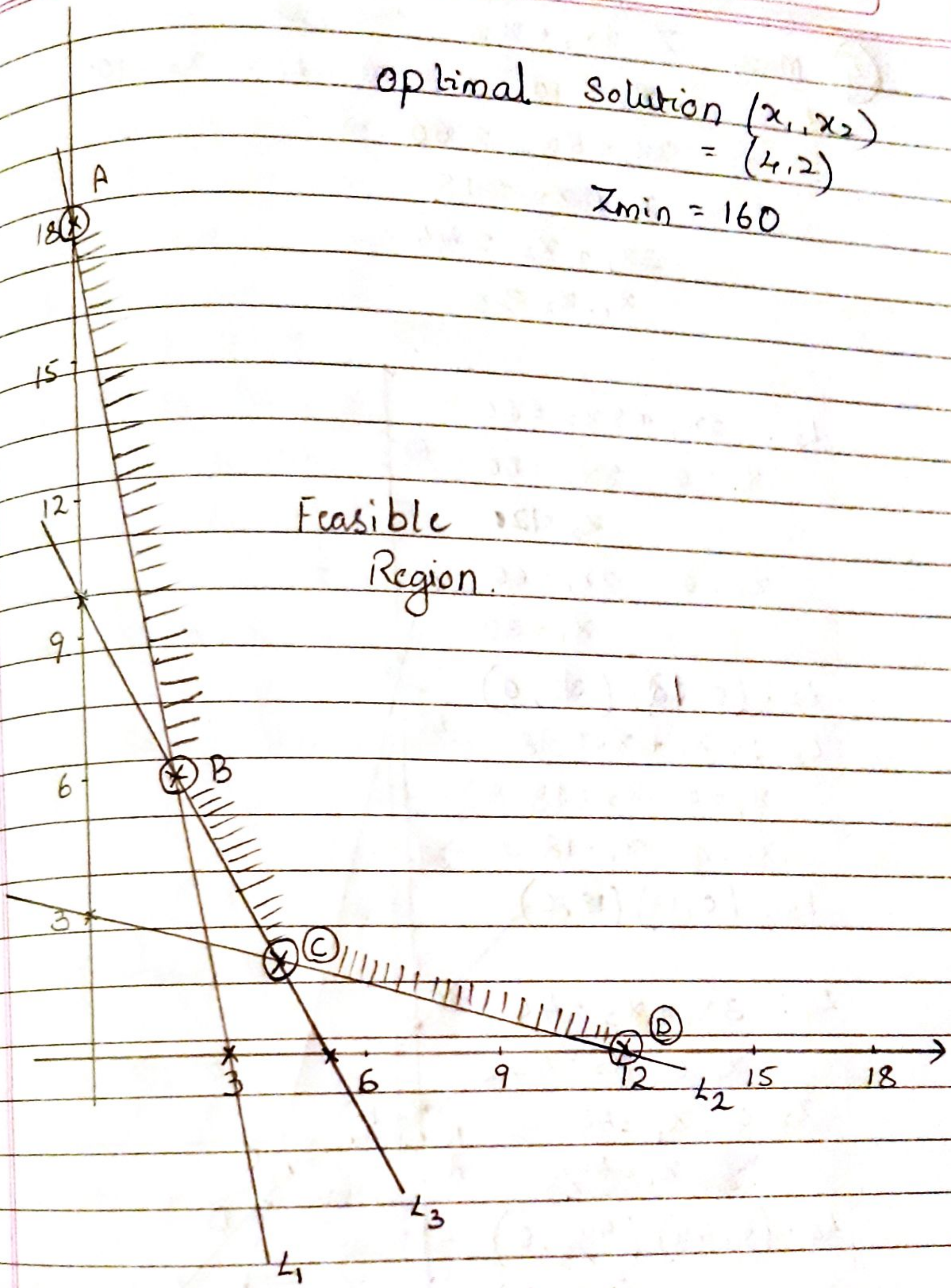
$x_1 = (4)$

$x_2 = 2$

Date
Page

optimal solution $(x_1, x_2) = (4, 2)$

$Z_{min} = 160$



Extreme points

$Z = 20x_1 + 40x_2$

A : (0. 18) : $20(0) + 40(18) = 720$

B : (2, 6) $20(2) + 40(6) = 280$

$\boxed{C : (4, 2) \quad 20(4) + 40(2) = 160}$ *

D : (12, 0) $20(12) + 40(0) = 240$

(4) Max $Z = 2x_1 + x_2$

$x_2 \le 10 \Rightarrow L_1 : x_2 = 10$

$2x_1 + 5x_2 \le 60 \Rightarrow$

$x_1 + x_2 \le 18$

$3x_1 + x_2 \le 44$

$x_1, x_2 \geq 0$

$L_2 : 2x_1 + 5x_2 = 60$

$x_1 = 0 \quad 5x_2 = 60$

$x_2 = 12$

$x_2 = 0 \quad 2x_1 = 60$

$x_1 = 30$

$L_2 : (0, 12)\ (30, 0)$

$L_3 : x_1 + x_2 = 18$

$x_1 = 0 \quad x_2 = 18$

$x_2 = 0 \quad x_1 = 18$

$L_3 : (0, 18)\ (18, 0)$

$L_4 : 3x_1 + x_2 = 44$

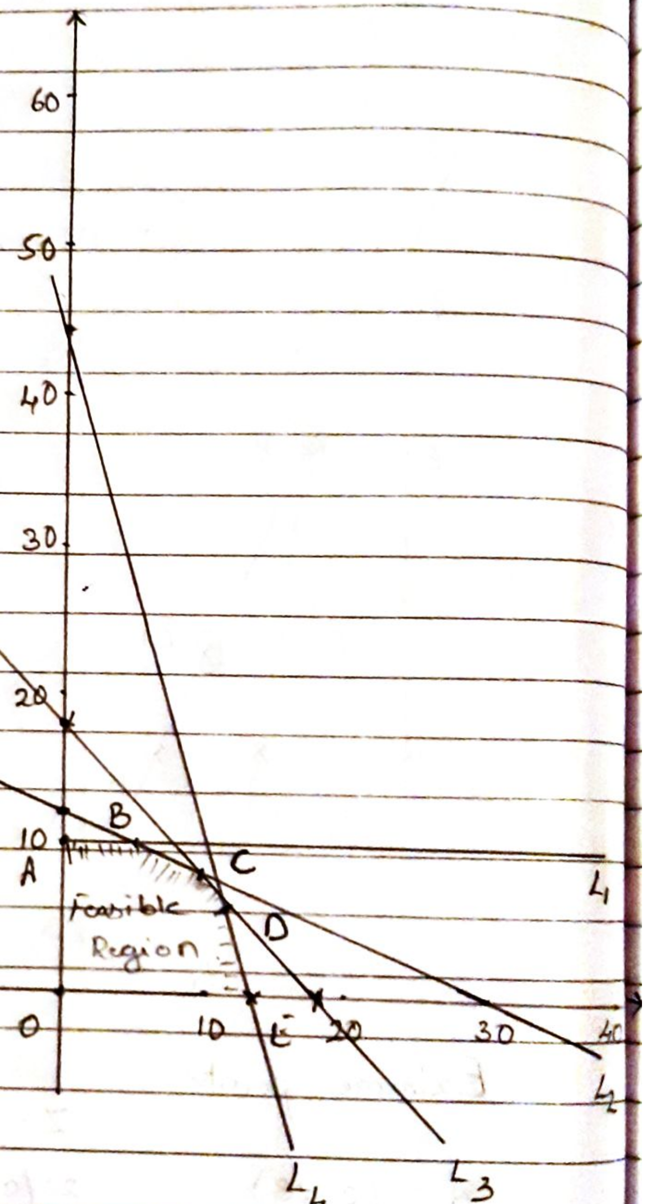
$x_1 = 0 \quad x_2 = 44$

$x_2 = 0 \quad 3x_1 = 44$

$x_1 = 44/3$

$L_4 : (0, 44)\ (44/3, 0)$

$(14.7, 0)$



The Corner points of feasible region are O, A, B, C, D, E

So the Coordinates for the corner points are

O $(0, 0)$

A $(0, 10)$

Date
Page

B: Solve the equation L_1 & L_2

$$x_2 = 10$$

$$2x_1 + 5x_2 = 60$$

$$2x_1 + 5(10) = 60$$

$$2x_1 = 60 - 50 = 10$$

$$x_1 = 5$$

B: $(5, 10)$:

C: Solve the equation. L_2 & L_3

$$2x_1 + 5x_2 = 60$$

$$x_1 + x_2 = 18$$

$$x_1 = 10 \quad x_2 = 8$$

C: $(10, 8)$

D: Solve the equation L_3 & L_4.

$$x_1 + x_2 = 18$$

$$3x_1 + x_2 = 44$$

$$x_1 = 13 \quad x_2 = 5$$

~~D(13.5)~~ D$(13, 5)$

E: $(44/3, 0)$

Extreme points $\quad Z_{max} = 2x_1 + x_2$

x_1

O$(0,0)$: $Z = 0$
A$(0,10)$: $Z = 2(0) + 10 = 10$.
B$(5,10)$: $Z = 2(5) + 10 = 20$
C$(10,8)$: $Z = 2(10) + 8 = 28$
D$(13,5)$: $Z = 2(13) + 5 = 31$
E$(44/3, 0)$: $Z = 2\left(\frac{44}{3}\right) + 0 = \frac{88}{3} = 29.3$

* optimal solution : $(x_1, x_2) = (13, 5)$

$$Z_{max} = 31.$$

Special Cases in Graphical Method.

* Unbounded Solution
* No Solution
* Multiple Solutions.

1. Max $z = 3x_1 + 2x_2$

Subjected to : $x_1 - x_2 \le 1$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

L_1 $x_1 - x_2 = 1$

$x_1 = 0$ $x_2 = -1$

$x_2 = 0$ $x_1 = 1$

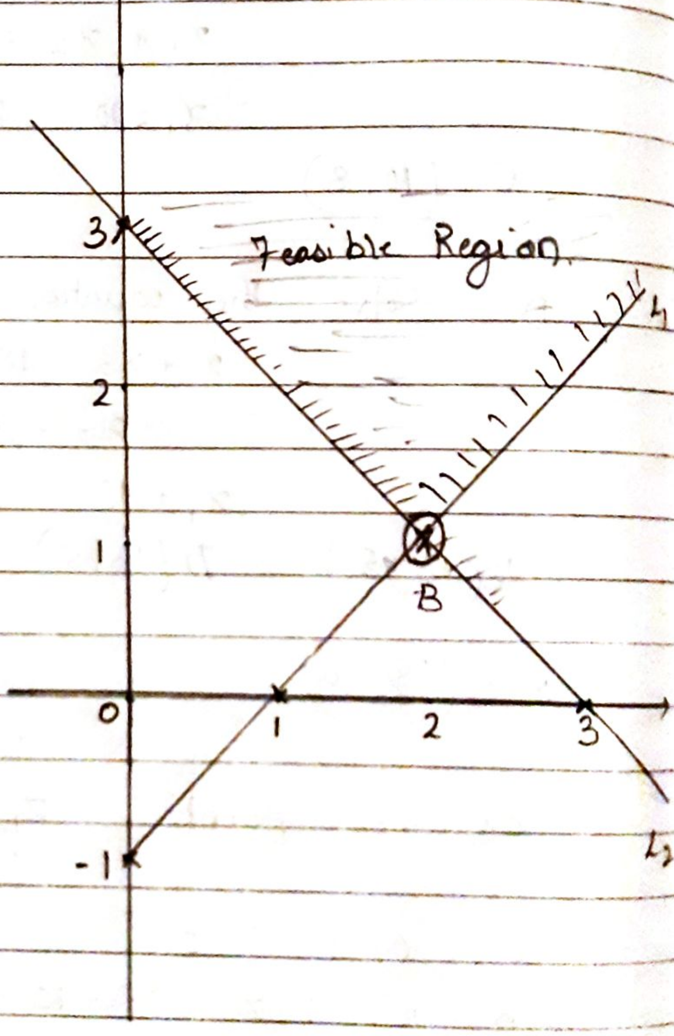
$L_1 = (0, -1)\ (1, 0)$

L_2 : $x_1 + x_2 = 3$

$x_1 = 0$ $x_2 = 3$

$x_2 = 0$ $x_1 = 3$

L_2 : $(0, 3)\ (3, 0)$



Extreme points.

$A = (0 \cdot 3)$

B : L_1 & L_2

$x_1 - x_2 = 1$

$x_1 + x_2 = 3$

$2x_1 = 4$

$x_1 = 2$

$2 - x_2 = 1$

$x_2 = 2 - 1$

$x_2 = 1$

$B = (2, 1) =$

$Z_{max} : 3x_1 + 2x_2$

$A(0,3) \quad : Z = 6$

$B(2,1) \quad : Z = 3(2) + 2(1) = 8$

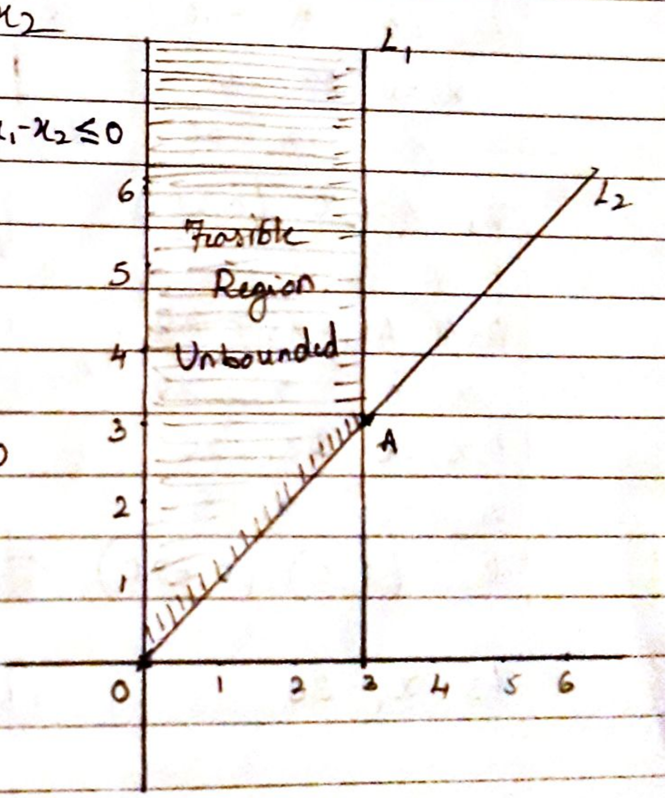
The values of objective function at corner points are 6 and 8. But their exists infinite number of points in the feasible region which is unbounded. The value of objective function will be more than the value of these two corner points, ie the maximum value of the objective function occurs at a point at ∞. Hence the given problem has unbounded solution.

(2) Max $Z = -3x_1 + 2x_2$

$x_1 \leq 3$

$x_1 - x_2 \leq 0$

$x_1, x_2 \geq 0$



$L_1 = x_1 = 3$

$L_2 = x_1 - x_2 = 0$

$x_1 = 0 \quad x_2 = 0$

$x_1 = x_2$

$O : (0.0) \quad Z_{max} = 0$

$A : L_1 \text{ \& } L_2$

$L_1 \quad x_1 = 3$

$x_1 - x_2 = 0$

$3 - x_2 = 0$

$x_2 = 3$

$A(3.3) \quad Z_{max} = -9 + 2(3) = -9 + 6 = -3$

As feasible region is unbounded & Z goes to infinity

$\therefore$ the solution is unbounded.

(3) Max $Z = 20x_1 + 30x_2$

Subject to $2x_1 + x_2 \le 40$

$4x_1 - x_2 \le 20$

$x_1 \ge 30$

$x_1, x_2 \ge 0$

$L_1 : 2x_1 + x_2 = 40$

$x_1 = 0 \quad x_2 = 40$

$x_2 = 0 \quad 2x_1 = 40$

$x_1 = 20$

$L_1 : (20, 0) \; (0, 40)$

$L_2 : 4x_1 - x_2 = 20$

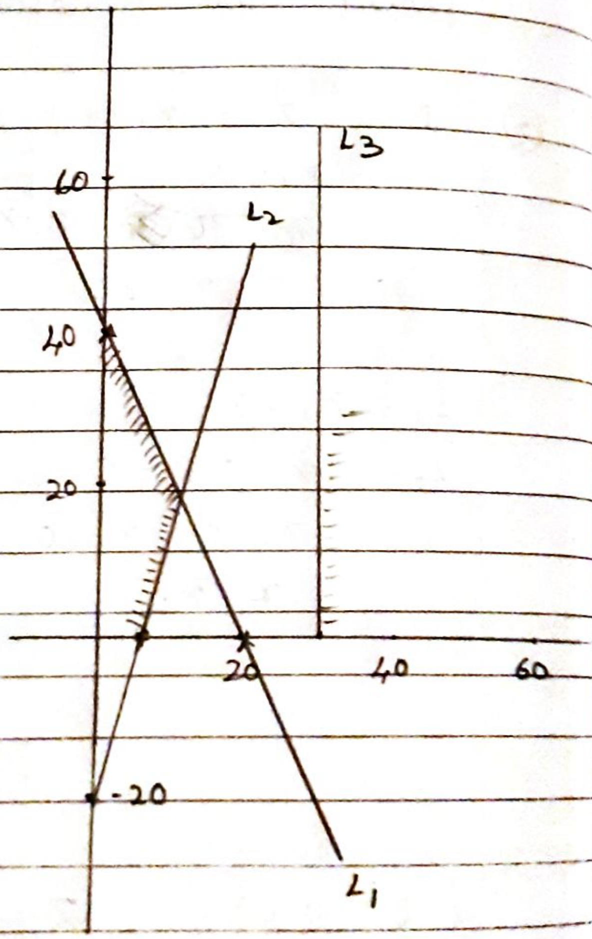
$x_1 = 0 \quad x_2 = -20$

$x_2 = 0 \quad 4x_1 = 20$

$x_1 = 5$

$L_2 : (5, 0) \; (0, -20)$

$L_3 \quad x_1 = 30$



There is no common feasible region generated (i.e. no commonly shaded region) by three constraints together.

$\therefore$ There exist no solution.

(4) Max $z = 3x_1 - 2x_2$

Subjected to : $x_1 + x_2 \leq 1$

$2x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0$

$L_1 = x_1 + x_2 = 1$

$x_1 = 0 \quad x_2 = 1$

$x_2 = 0 \quad x_1 = 1$

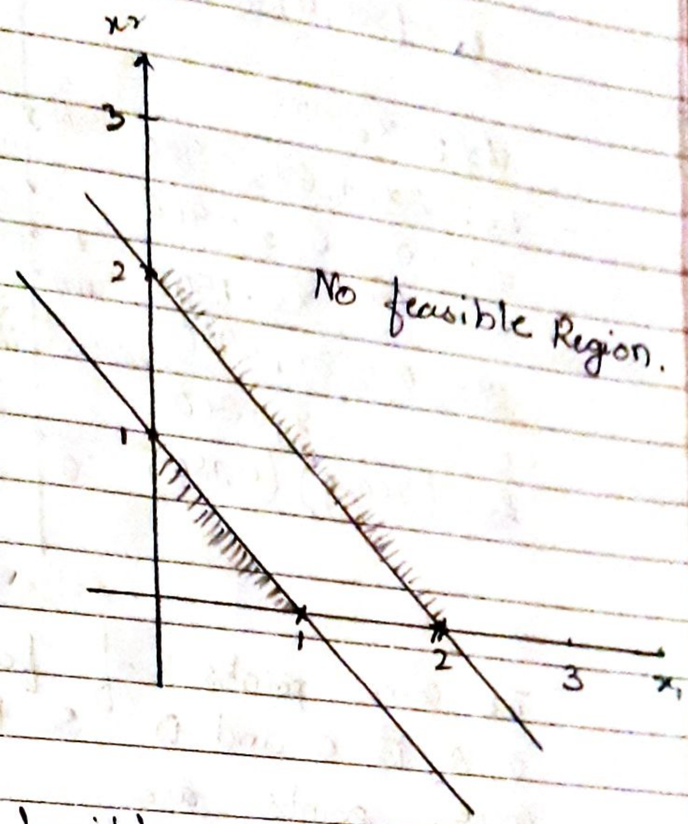
$L_1 : (1,0)\ (0,1)$

$L_2 = 2x_1 + 2x_2 = 4$

$x_1 = 0 \quad x_2 = 2$

$x_2 = 0 \quad x_1 = 2$

$L_2 : (2,0)\ (0,2)$



$\therefore$ There is no feasible Region $\therefore$ there exist no solution.

(5) Multiple optimal Solution.

Max $Z = 8x_1 + 16x_2$

Subjected to : $x_1 + x_2 \leq 200$

$x_2 \leq 125$

$3x_1 + 6x_2 \leq 900$

$x_1, x_2 \geq 0$

$L_1 = x_1 + x_2 = 200$

$x_1 = 0 \quad x_2 = 200$

$x_2 = 0 \quad x_1 = 200$

$L_1 : (200, 0)\ (0, 200)$

$L_2 : x_2 = 125$

$L_3 : 3x_1 + 6x_2 = 900$

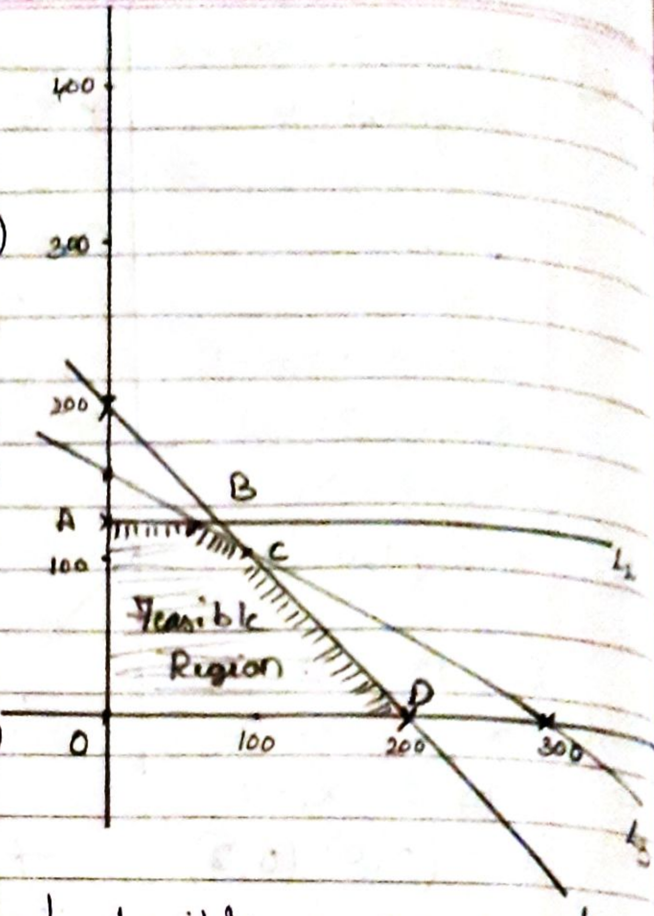
$x_1 = 0 \quad 6x_2 = 900$

$x_2 = 150$

$x_2 = 0 \quad 3x_1 = 900$

$x_1 = 300$

$L_3 : (300, 0)\ (0, 150)$



The corner points of feasible region are O, A, B, C and D so the coordinates for the corner points are

O (0, 0)

A (0, 125)

B : solve the equations of L_3 & L_2

$x_2 = 125$

$3x_1 + 6x_2 = 900$

$x_1 = 50$

B (50, 125)

C : Solve the equations of L_3 & L_1

$x_1 + x_2 = 200$

$3x_1 + 6x_2 = 900$

$x_1 = 100 \quad x_2 = 100$

D : (200, 0)

$Z_{max} = 8x_1 + 16x_2$

Ex

O : (0, 0)   Z = 0

A : (0, 125)   Z = 8(0) + 16(125) = 2000

B : (50, 125)   Z = 50(8) + 16(125) = 2400

C : (100, 100)   Z = 8(100) + 16(100) = 2400

D : (200, 0)   Z = 8(200) + 16(0) = 1600

Max Z = 2400 which is achieved at both B & C corner points. It can be achieved not only at B and C but every point between B and C. Hence the given problem has multiple optimal solutions.

(6) Max $Z = 4x_1 + 3x_2$

subject to :

$4x_1 + 3x_2 \le 24$

$x_1 \le 4.5$

$x_2 \le 6$

$x_1 \ge 0 \quad x_2 \ge 0$

Soln:

$L_1 : 4x_1 + 3x_2 = 24$

$x_1 = 0 \quad 3x_2 = 24$

$x_2 = 8$

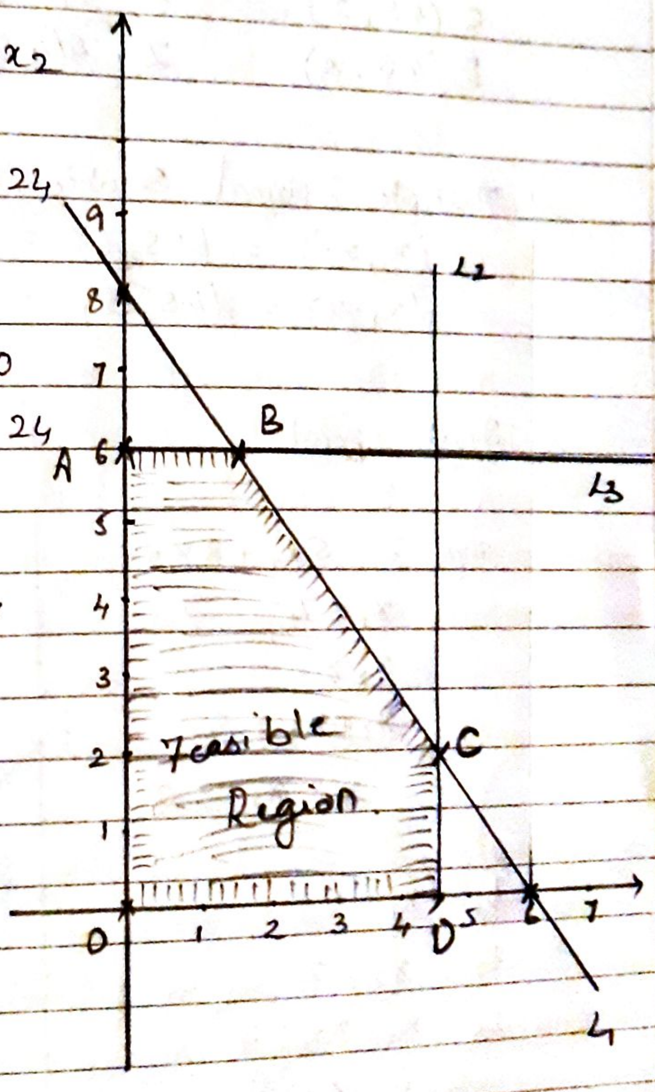
$x_2 = 0 \quad 4x_1 = 24$

$x_1 = 6$

$L_1 : (0, 8)\ (6, 0)$

$L_2 = x_1 = 4.5$

$L_3 \quad x_2 = 6$



O(0, 0)

A(0, 6)

B Solve L_1 & L_3

$4x_1 + 3x_2 = 24$

$x_2 = 6$

$x_1 = 6/4 = 1.5$

C: Solve L_1 & L_2

$4x_1 + 3x_2 = 24$

$x_1 = 4.5$ $\qquad$ $3x_2 = 6$

$x_2 = 2$

$C(4.5, 2)$

$D(4.5, 0)$

$Z_{max} = 4x_1 + 3x_2$

$O(0,0)$	$Z_{max} = 0$
$A(0, 6)$	$Z = 4(0) + 3(6) = 18$
$B(1.5, 6)$	$Z = 4(1.5) + 3(6) = 24$ *
$C(4.5, 2)$	$Z = 4(4.5) + 3(2) = 24$
$D(4.5, 0)$	$Z = 4(4.5) + 3(0) = 18$

Multiple optimal solution.

$(x_1, x_2) = (1.5, 6) = Z_{max} = 24$

$(x_1, x) = (4.5, 2) = Z_{max} = 24$

Single point

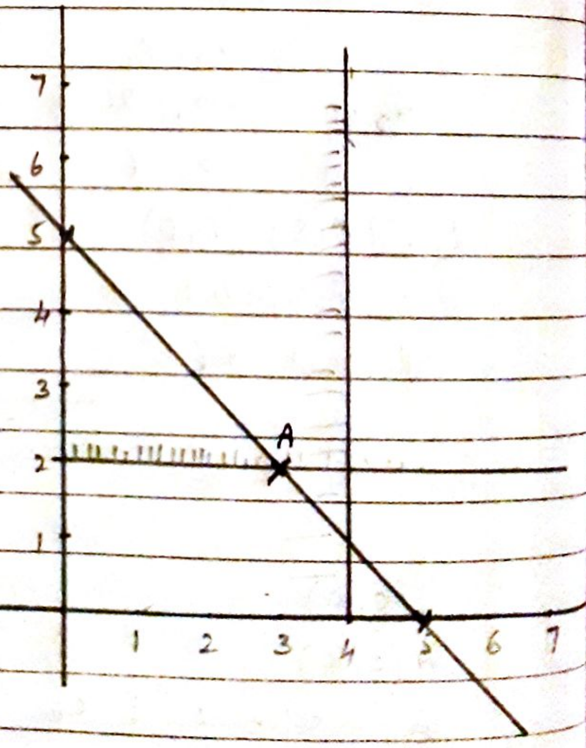
Min $Z = 5x_1 + 8x_2$

$x_1 \leq 4$

$x_2 \geq 2$

$x_1 + x_2 = 5$

$x_1, x_2 \geq 0$



$L_1 : x_1 = 4$

$L_2 : x_2 = 2$

L_3 $x_1 + x_2 = 5$

$x_1 = 0$ $\quad x_2 = 5$

$x_2 = 0$ $\quad x_1 = 5$

$L_3 : (0, 5)\ (5, 0)$

$x_1 + x_2 = 5$

$x_2 = 2$ $\qquad x_1 = 5 - 2 = 3$

$A(3,2)$ $\quad Z_{max} = 5(3) + 8(2)$
$= 15 + 16 = 31$

single point : $(3,2)$ $\quad Z = 31$ is the optimal solution

Basic Definition

Feasible Solution

The set of values of decision variables x_j $(j = 1, 2, \dots n)$ which satisfy all the constraints and non-negativity conditions of an LP problem simultaneously is said to be the feasible solution to that linear programming Problem.

Infeasible solution.

The set of values of decision variables $x_j = (j = 1, 2 \dots n)$ which do not satisfy all the constraints and non-negativity conditions of an LP problem simultaneously is said to constitute the infeasible solution to that linear programming Problem.

Basic Solution

For a set of m simultaneous equations in n variables a solution obtained by ~~using~~ setting (n-m) variables equal to zero and solving for remaining m equations in m variables is called a basic solutions.

Optimum basic feasible solutions.

A beasic feasible solution which optimizes (Maximizes or Minimizes) the objective function value of the given LP problem is called an optimum basic feasible solution.

Unbounded Solution.

A solution which increases or decreases the value of objective function of the LP problem indefinitely is called unbounded solution.

A medical scientist claims to have found a cure for common cold that consists of three drugs called K, S & H. His results indicate that the minimum daily adult dosage for effective treatment is 10mg of drug K, 6mg of drug S and 3mg of drug H. Two substances are readily available for preparing pills or drugs. Each unit of substance A contain 6mg, 1mg & 2mg of drugs K, S & H respectively and each unit of B contain 2mg, 3mg & 2mg of the same drugs. Substance A costs Rs 3 per unit & substance B costs Rs 5 per unit. Find the least cost combination of two substances that will yield a pill designed to contain the minimum daily recommended adult dosage. Solve the above problem by Graphical method.

Solu: Let x_1, x_2 be the no of units for substance A & B

LPP model

$$\text{Min } Z = 3x_1 + 5x_2$$

$$6x_1 + 2x_2 = 10 \quad (\text{drug of } K$$

$$x_1 + 3x_2 = 6 \qquad S$$

$$2x_1 + 2x_2 = 8 \qquad H$$

$$x_1, x_2 \geq 0$$

$L_1 : 6x_1 + 2x_2 = 10$

$x_1 = 0 \quad 2x_2 = 10$

$x_2 = 5$

$x_2 = 0 \quad 6x_1 = 10$

$x_1 = 10/6 = 5/3$

$L_1 : (5/3, 0)\ (0, 5)$

$L_2 : x_1 + 3x_2 = 6$

$x_1 = 0 \quad 3x_2 = 6$

$x_2 = 2$

$x_2 = 0 \quad x_1 = 6$

$L_2 = (6, 0)\ (0, 2)$

$L_3 : 2x_1 + 2x_2 = 8$

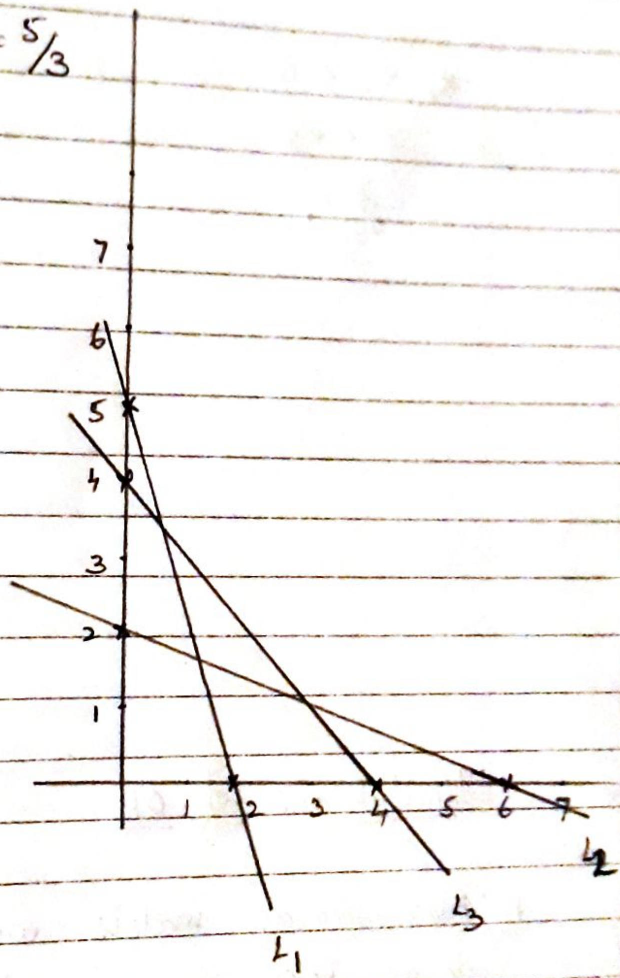
$x_1 = 0 \quad 2x_2 = 8$

$x_2 = 4$

$x_2 = 0 \quad 2x_1 = 8$

$x_1 = 4$

$L_3 : (4, 0)\ (0, 4)$

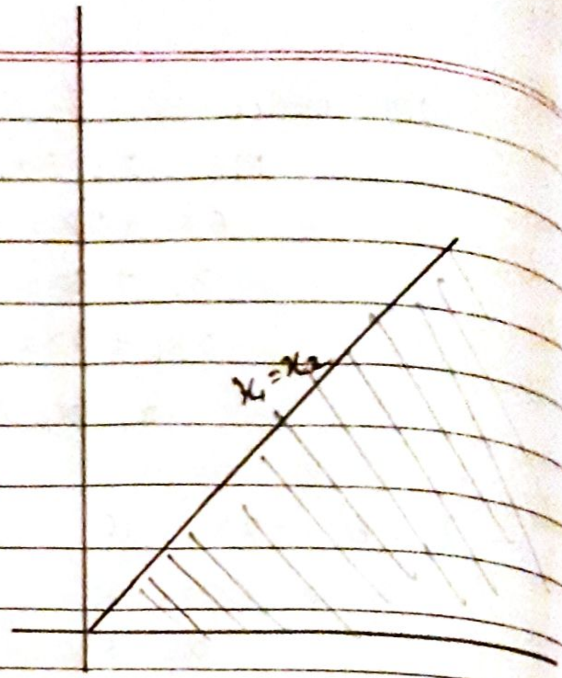


No Feasible Region $\therefore$ No solution.

$x_1 - x_2 \geq 0$

$x_1 \geq x_2$

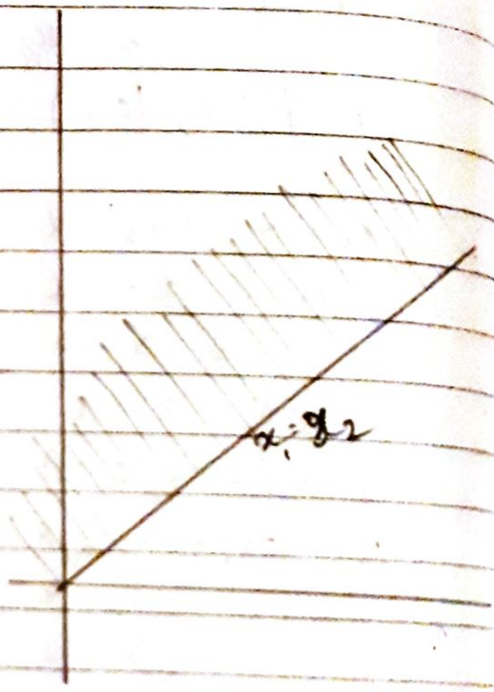
$x_2 \leq x_1$



$x_1 - x_2 \leq 0$

$x_1 \leq x_2$

$x_2 \geq x_1$



Limitations of OR

1. Mathematical models which are essence of OR, do not take into account qualitative factors or emotional factors which are quite real. All influencing factors which cannot be quantified find no place in mathematical models.

2. Mathematical model are applicable to only specific categories of problems.

3. OR tries to find optimal solution taking all the factors of the problem into account. Present day problems involve numerous such factors: expressing them in quantity and establishing relations among them requires huge calculations.

4) Being a new field, there is a resistance from the employees to the new proposals.

5) Management may offer a lot of resistance due to conventional thinking.

6) OR is meant for men and not that men are meant for it.