DAYANANDA SAGAR COLLEGE OF ENGINEERING

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Department of Mathematics

Question Bank for IT(EC, TC, EE, ML, EI, IS)

Unit 2: Complex Variable

Q.No	Question
1.	a) Find the modulus and amplitude of $\frac{(1+i)^2}{(3+i)}$. b) Prove that $cosx = \frac{e^{ix}+e^{-ix}}{2}$, $sinx = \frac{e^{ix}-e^{-ix}}{2i}$
2.	a) Determine the region in the z-plane represented by i) $1<\|z+2i\| \leq 3$ ii) R(z) >3 iii) $\frac{\pi}{6} \leq amp\ (z) \leq \frac{\pi}{3}$ b) Define the following : i) Limit of a complex function ii) Continuity of a complex function iii) Differentiability of a complex function
3.	a) If $f(z) = u + iv$ is analytic, then prove that Cauchy- Riemann equations $u_x = v_y, v_x = -u_y$ are true. b) If $f(z) = \mathrm{u}(\mathrm{r},\theta) + \mathrm{i}\ \mathrm{v}(\mathrm{r},\theta)$ is analytic, then prove that Cauchy- Riemann equations in polar form $\frac{\partial \mathrm{u}}{\partial \mathrm{r}} = \frac{1}{\mathrm{r}}\frac{\partial \mathrm{v}}{\partial \theta}$ and $\frac{\partial \mathrm{v}}{\partial \mathrm{r}} = -\frac{1}{\mathrm{r}}\frac{\partial \mathrm{u}}{\partial \theta}$ are true.
4.	a) If $u = x^2 - y^2$, $v = x^3 - 3xy^2$ Show that $u\ and\ v$ are harmonic functions but $f(z) = u + iv$ is not analytic. b) If $u = \frac{x^2}{y}$, $y \neq 0$ and $v = x^2 + 2y^2$, then show that the curve $u = constant$ and $v = constant$ are orthogonal but $f(z) = u + iv$ is not an analytic function.
5.	a) Show that $f(z) = z^n$ is analytic. Hence find its derivative. b) Show that the function $f(z) = logz$ is analytic and hence find its derivative.
6.	a) Show that the function $f(z) = coshz$ is analytic and hence find its derivative. b) Show that $w = z + e^z$ is analytic and hence find $\frac{\mathrm{dw}}{\mathrm{dz}}$.
7.	a) Show that the function $f(z) = sin2z$ is analytic and hence find its derivative. b) Find the analytic function $f(z) = u + iv$, where $\mathrm{u} = x^2 - y^2 + \frac{x}{x^2+y^2}$
8.	a) Find the analytic function whose real part is $log\sqrt{x^2+y^2}$. b) Find the analytic function $f(z) = u + iv$, where $v = \frac{y}{x^2+y^2}$.
9.	a) Find the analytic function $f(z) = u + iv$, where $v = e^x(xsiny + ycosy)$ using Milne Thompson method. b) Find the analytic function $f(z) = u + iv$, where $u = e^{-x}\{(x^2 - y^2)cosy + 2xysiny\}$ using Milne Thompson method.
10.	a) Find the analytic function $f(z) = u(r,\theta) + iv(r,\theta)$, where $v(r,\theta) = r^2cos2\theta - rcos\theta + 2$ using Milne Thompson method. b) Find the analytic function $f(z) = u(r,\theta) + iv(r,\theta)$, where $u(r,\theta) = \frac{cos2\theta}{r^2}$ using Milne Thompson method.
11.	a) Find the analytic function $f(z) = u + iv$, where $v = -\frac{sin\theta}{r}$ using Milne Thompson method. b) Find the analytic function $f(z) = u + iv$, where $\mathrm{u} = r^2cos2\theta$

12.	a) Show that $u = e^{2x}(xcos2y - ysin2y)$ is harmonic and find its harmonic conjugate. b) Show that $u = e^{x}cosy + xy$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function.
13.	a) Show that $v = cosx\ sinhy$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function. b) Show that $v = x^3 - 3xy^2 - 3x^2 + 3y^2 + 1$ is harmonic and find its harmonic conjugate.
14.	a) Show that $u = \left(r + \frac{1}{r}\right) cos\theta$ is harmonic and find its harmonic conjugate. b) Show that $v = \left(r - \frac{1}{r}\right) sin\theta$ is harmonic and find its harmonic conjugate.
15.	a) Show that $v = rsin\theta + \frac{cos\theta}{r}$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function. b) Show that $u = \frac{cos\theta}{r}$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function.
16.	a) Show that $u = x^2 + 4x - y^2 + 2y$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function. b) Show that $v = 2xy - 2x + 4y$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function.
17.	a) Find the analytic function $f(z) = u + iv$ if $u + v = \frac{1}{r^2}(cos2\theta - sin2\theta)$ $(r \neq 0)$. b) Find the analytic function $f(z) = u + iv$ if $u + v = r(cos\theta + sin\theta) + \frac{1}{r}(cos\theta - sin\theta)$
18.	a) Find the analytic function $f(z) = u + iv$ if $u + v = (x + y) + e^{x}(cosy + siny)$. b) Find the analytic function $f(z)$ as a function of z gives the sum of its real and imaginary part is $x^3 - y^3 + 3xy(x - y)$.
19.	a) Find the analytic function $f(z) = u + iv$ if $u - v = (x - y)(x^2 + 4xy + y^2)$ b) Find the analytic function $f(z) = u + iv$ if $u - v = e^{x}(cosy - siny)$
20.	a) An electrostatic field in the xy-plane is given by the potential function $\emptyset = 3x^2y - y^3$,find the stream function. b) Two concentric circular cylinders of radii r_1, r_2 $(r_1 < r_2)$ are kept at potentials $\emptyset_1 and\ \emptyset_2$ respectively.Using complex function $\omega = alogz + c$, prove that the capacitance per unit length of the capacitor formed by them is $\frac{2\pi\lambda}{\log(r_2 \mid r_1)}$ where λ is the dielectric constant of the medium.