DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

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Department of Mathematics

Fourth Semester B.E. (Autonomous)

Question Bank

Module 5

<table>
<tr><th>Question Number</th><th>Question</th></tr>
<tr><td>1.</td><td>The joint probability distribution is given by

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find the (i) Marginal distribution of X and Y (ii) COV(X,Y)</td></tr>
<tr><td>2</td><td>The joint probability distribution of two random variables X and Y are given as:

X \ Y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

Find the (i) Marginal distribution of X and Y (ii) COV(X,Y)</td></tr>
<tr><td>3.</td><td>The joint probability distribution table for two random variables X and Y is as follows.

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginal distribution of X and Y. Also compute (a) Expectations of X ,Y (b) S.Ds of X,Y (c) Covariance of X and Y (d) Correlations of X and Y.</td></tr>
<tr><td>4</td><td>The joint probability distribution of two random variables X and Y are given as:

X \ Y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute the following (i) E(X) and E(Y) (ii) E(XY) (iii) COV(X,Y) (iv) $\rho(X, Y)$ (v) $\sigma_x \text{ and } \sigma_y$.</td></tr>
<tr><td>5</td><td>Suppose X and Y are independent random variables with the following respective distribution. Find the joint distribution of X and Y. Also verify that COV(X,Y)=0

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	-2	5	8
$g(y_j)$	0.3	0.5	0.2

</td></tr>
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6 If X and Y are independent random variables, find the joint probability distribution of X and Y with the following marginal distribution of X and Y.

x_i	1	2
$f(x_i)$	0.6	0.4

y_j	5	10	15
$g(y_j)$	0.2	0.5	0.3

7 Two cards are selected at a random from a box which contains five cards numbered 1,1,2,2 and 3. Find the joint distribution of x and Y where X denotes the sum and Y, the maximum of the two numbers drawn. Also determine COV(X,Y) and ρ(X,Y).

8 X and Y are independent random variables. X take values 2,5,7 with probabilities ½, ¼, ¼ respectively take values 3,4,5 with probabilities 1/3,1/3,1/3. (i) find the joint distribution of X and Y. (ii) Show that COV(X,Y)=0 (iii) Find the probability distribution of Z=X+Y

9 The joint probability distribution of two discrete random variables X and Y is given by $f(x,y) = k(2x+y)$, where x and y are integers such that $0 \le x \le 2, 0 \le y \le 3$.
(i) Find the value of the constant k (ii) Find the marginal distribution of X and Y.(iii) Show that the random variables X and Y are dependent.

10 A coin is tossed three times. Let X denotes 0 and 1according as a tail or a head occurs on the first toss. Let Y denote the total number of tails which occur. Determine (i) the marginal distribution of X and Y and (ii) The joint probability distribution of X and Y. Also, find the expected values of X+Y and XY.

11 If X and Y are independent random variables, prove the following results.
(a) E (XY) =E(X).E(Y) (b) COV(X, Y) =0 and (c) ${\sigma^2}_{X+Y} = {\sigma^2}_X + {\sigma^2}_Y$.

12 Two marbles are drawn from a box containing 3 blue, 2 red and 3 green marbles. If x is the number of blue marbles and Y is the number of red marbles. Form
(i) The joint probability distribution of X and Y (ii) Find E(x) and E(Y)

13 X and Y are random variables having joint density function $f(x,y) = \begin{cases} 4xy, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & otherwise \end{cases}$
Verify that (i) E(X+Y)=E(X) + E(Y) (ii) E(XY) = E(X).E(Y)

14 The joint density function of two continuous random variables X and Y is given by
$$f(x,y) = \begin{cases} kxy, & 0 \le x \le 4 , 1 < y < 5 \\ 0, & otherwise \end{cases}$$
Find (a) the value of k (b) E(X) (c) E(Y) (d) E(XY) (e) E(2X+3Y)

15 If X and Y are continuous random variables having joint density function
$$f(x,y) = \begin{cases} c(x^2+y^2), & 0 \le x \le 1 , 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$
Determine (i) c (ii) $P(x < \frac{1}{2}, y > \frac{1}{2})$ (iii) $P(\frac{1}{4} < x < \frac{3}{4})$ (iv) $P(y < \frac{1}{2})$

16 The joint density function of 2 continuous random variables $f(x,y) = \begin{cases} \frac{xy}{96}, & 0 < x < 4 , 1 < y < 5 \\ 0, & otherwise \end{cases}$
Find $P(x+y<3)$

17 If the joint Probability function of 2 continuous random variables X and Y is given by
$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), & 0 < x < 2, 2 < y < 4 \\ 0, & otherwise \end{cases}$$
Find (i) $P(x+y<3)$ (ii) $P(x<1, y<3)$

18 Verify that $f(x,y) = \begin{cases} e^{-(x+y)}, & x \ge 0 , y \ge 0 \\ 0, & otherwise \end{cases}$ is a density function of joint probability

	distribution. Also evaluate $(i) P(x<1)$ (ii) $P(x>y)$(ii) $P(x+y\leq 1)$
19	The joint density function of two continuous random variables X and Y is given by $f(x,y)=\begin{cases}2, & 0<x<y<1\\ 0, & otherwise\end{cases}$ Find the covariance between x and y
20	Find the constant k so that $f(x,y)=\begin{cases}k(x+1)e^{-y}, & 0<x<1, y>0\\ 0, & otherwise\end{cases}$ is a joint probability density function. Are x and y independent?
21	Define the following (i) Stochastic process (ii) Probability vector (iii) Stochastic matrix (iv) Regular Stochastic matrix
22	Explain (i) Absorbing state of Markov chain (ii) Transient state (iii) Recurrent state
23	Find the unique probability vector for the regular stochastic matrix $\begin{bmatrix}0.7 & 0.3\\ 0.8 & 0.2\end{bmatrix}$
24	Find the unique fixed probability vector for the regular stochastic matrix $\begin{bmatrix}0 & 0.75 & 0.25\\ 0.5 & 0.5 & 0\\ 0 & 1 & 0\end{bmatrix}$
25	Verify that the matrix A= $\begin{bmatrix}0 & 0 & 1\\ 0.5 & 0.25 & 0.25\\ 0 & 1 & 0\end{bmatrix}$ is a regular stochastic matrix
26	Show that P= $\begin{bmatrix}0.5 & 0.25 & 0.25\\ 0.5 & 0 & 0.5\\ 0 & 1 & 0\end{bmatrix}$ is a regular stochastic matrix. Also find the associated unique fixed probability vector.
27	Show that $v=(\,b\;a\,)$ is fixed point of the stochastic matrix $P=\begin{bmatrix}1-a & a\\ b & 1-b\end{bmatrix}$
28	Show that P= $\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ \frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}$ is a regular stochastic matrix. Also find the associated unique fixed probability vector.
29	Prove that the Markov chain whose transition probability matrix is P= $\begin{bmatrix}0 & \frac{2}{3} & \frac{1}{3}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}$ is irreducible. Also find the corresponding stationary probability vector
30	The transition probability matrix of a Markov Chain is given by P= $\begin{bmatrix}\frac{1}{2} & 0 & \frac{1}{2}\\ 1 & 0 & 0\\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4}\end{bmatrix}$ and the initial probability distribution is $p^{(0)}=(\frac{1}{2},\frac{1}{2},0)$ find $p_{13}^{(2)}$, $p_{23}^{(2)}$ $p^{(2)}$ and$p_1^{(2)}$
31	Prove that the Markov chain whose transition probability matrix is P=$\frac{1}{10}$ $\begin{bmatrix}6 & 2 & 2\\ 1 & 8 & 1\\ 6 & 0 & 4\end{bmatrix}$ is irreducible. Also find the corresponding stationary probability vector
32	The transition probability matrix of a Markov Chain is given by P= $\begin{bmatrix}1 & 0\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ and the initial probability distribution is $p^{(0)}=(\frac{1}{3},\frac{2}{3})$ find $p_{21}^{(3)}$, $p^{(3)}$ and$p_2^{(3)}$.
33	A habitual gambler is a member of two clubs A and B. He visits either of the clubs every day

	for playing cards. He never visits club A on two consecutive days. But , if he visits club B on a particular day, then the next day he is as likely to visits club B or club A. Find the transition matrix of this Markov Chain also. (a) Show that the matrix is a regular stochastic matrix and find the unique fixed probability vector. (b) If the person had visited club B on Monday, find the probability that he visits club A on Thursday.
34	A students study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long ran how often does he study?
35	A man's smoking habits are as follows. If he smokes filter cigarettes one week, he switches to non-filter cigarettes the next week with probability 0.2. on the other hand , if he smokes non filter cigarettes one week there is a probability of 0.7 that he will smoke non filter cigarettes the next week as well. In the long run how often does he smoke filter cigarettes.
36	Three boys A, B, C are throwing ball to each other. A always throws ball to B and B always throws the ball to C. C is just as likely to throw the ball to b as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball.
37	A gambler's luck follows a pattern. If he wins a game, the probability of winning the next game is 0.6. However if he loses a game, the probability of losing the next game is 0.7. There is an even chance of gambler winning the first game. If so (i) What is the probability of winning the second game?(ii)What is the probability of winning the third game? (iii) In the long run, how often he will win?
38	Each year a man trades his car for a new car in 3 brands of the popular company Maruthi Udyog limited. If he has a 'Standard' he trades it for 'Zen'. If he has a 'Zen' he trades it for a 'Esteem'. If he has a 'Esteem' he is just as likely to trade it for new 'Esteem' or for a 'Zen' or a 'Standard' one. In 1996 he bought his first car which was Esteem.(i) Find probability that he has (a) 1998 Esteem (b) 1998 Standard (c) 1999 Zen (d) 1999 Esteem. (ii) In the long run how often he will have a Esteem.
39	Two boys B_1 ,B_2 and two girls G_1,G_2 are throwing ball from one to the other. Each boy throws the ball to the other boy with probability $\frac{1}{2}$ and to each girl with probability $\frac{1}{4}$. On the other hand each girl throws the ball to each boy with probability $\frac{1}{2}$ and never to the other girl. In the long run how often does each receive the ball?
40	A Player luck follows a pattern. If he wins a game the probability of winning next game is 0.6 .However if he loses the game the probability of losing the next game is 0.7. There is an en chance of winning the first game. If so (i) what is the probability of winning second gam what is the probability of winning third game?