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☆ Obtain a PDA to accept a string of balanced Parentheses. The parentheses to be considered are $(,),[,]$

Step 1: Let q_0 be the start state and z_0 be the initial symbol on the stack. The state q_0 itself is the final state accepting ε (empty string).

Step 2: In state q_0, if the first scanned Parenthesis is '(' or '[', push the scanned symbol on to the stack and change the state to q_1. The transition defined for this can be of the form.

$$\delta(q_0, (, z_0) = (q_1, (z_0))$$

$$\delta(q_0, [, z_0) = (q_1, [z_0))$$

Step 3: if at least one parentheses either '(' or '[' is present on the stack and if the scanned symbol is left parentheses, then push the left parentheses on to the stack. The transitions defined for this can be of the form.

$$\delta(q_1, (, () = (q_1, (())$$

$$\delta(q_1, (, [) = (q_1, ([)$$

$$\delta(q_1, [, () = (q_1, [()$$

$$\delta(q_1, [, [) = (q_1, [[)$$

Step 4: if the scanned symbol is ')' and if the top of the stack is '(' pop an element from the stack similarly. if the scanned symbol ']' and if the top of the top of the stack is '[' pop an element from the stack. The transitions are.

$$\delta(q_1,), ( ) = (q_1, \varepsilon)$$
$$\delta(q_1,], [ ) = q_1, \varepsilon) .$$

Step 5: when top of the stack is Z_0, it indicates that so far all the parentheses have been matched at this point, on ε-transitions the PDA enters into state q_0 and all the steps from step 1 are repeated. The transition for this can be of the form.

$$\delta(q_1, \varepsilon, Z_0) = (q_0, Z_0)$$

So, the PDA to accept the language consisting of balanced parentheses is given by.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

where,

$$Q = \{q_0, q_1\}$$
$$\Sigma = \{(,), [,]\}$$
$$\Gamma = \{ (, [, Z_0 \}$$

$$\delta (q_0, C, z_0) = (q_1, C z_0)$$

$$\delta (q_0, [, z_0) = (q_1, [z_0)$$

$$\delta (q_1, C, C) = (q_1, CC)$$

$$\delta (q_1, C, [) = (q_1, C[)$$

$$\delta (q_1, [, C) = (q_1, [C)$$

$$\delta (q_1, [, [) = (q_1, [[)$$

$$\delta (q_1,), C) = (q_1, \varepsilon)$$

$$\delta (q_1,], [) = (q_1, \varepsilon)$$

$$\delta (q_1, \varepsilon, z_0) = (q_0 z_0)$$

Acceptance:

$$(q_0, [()], z_0) \vdash (q_1, ()], [z_0)$$

$$\vdash (q_1,)] , [z_0)$$

$$\vdash (q_1,] , [q_0)$$

$$\vdash (q_1, \varepsilon, z_0)$$

$$\vdash (q_0, \varepsilon, z_0)$$

final configuration.