MODULE 3: SINGLE PHASE AC CKTS

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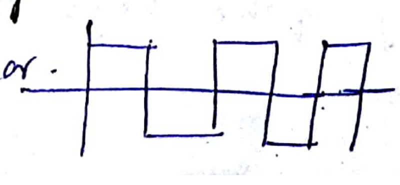
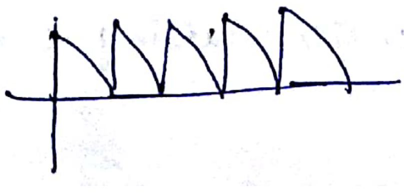
3(a) Single phase AC ckts:

Introduction to AC fundamentals:

- An alternating current ckt written in short form as a.c ckts, consists of alternating vtg source or source due to which, alternating current flows thro' the various elements of the ckt viz. the resistance, inductance & capacitance, which may be connected in the ckt independently or in all possible combinations.
- An alternating vtg or current is a quantity, whose magnitude continuously changes with time but which can have only 2 directions, either +ve or -ve. (which changes periodically both in magnitude & direction)
- Periodic means recurring & hence during equal periods of time known as time period, the nature of the w/f of an alternating quantity is the same.
- An alternating quantity may have any shape of w/f such as rectangular, sinusoidal, sawtooth etc.
- But usually, the alternating vtgs generated & the alternating currents flowing thro' the electric ckts are having periodic sinusoidal w/fs.

rectangular



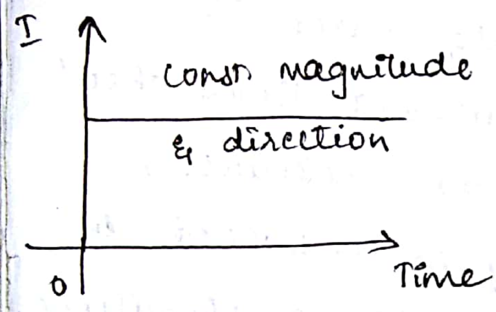


Sawtooth

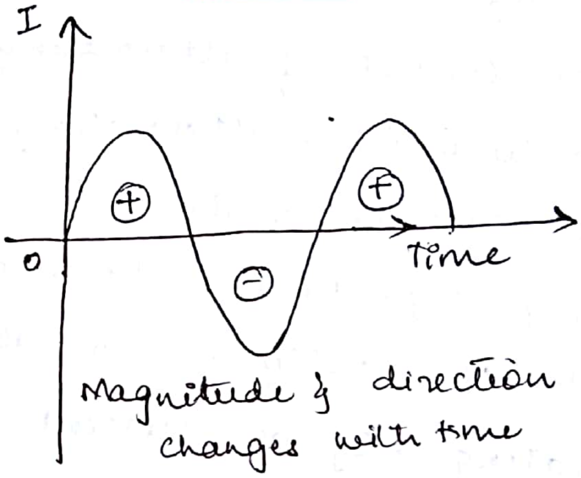
* Electric supply used for commercial & domestic purposes is alternating.

- DC supply has const. magnitude w.r.to time.

(a) Direct current

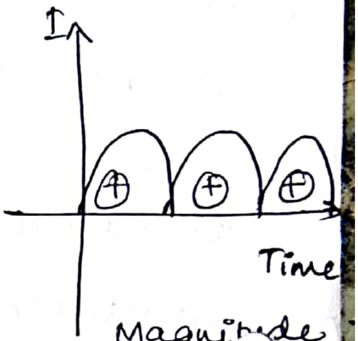


(b) Alternating current



Magnitude & direction changes with time

(c) pulsating DC



Magnitude changing but direction same.

Advantages of Purely Sinusoidal w/f :

- The voltages in ac s/s can be raised or lowered with the help of a device called transformer.

* A transformer is a static device, which transfers electrical energy from one ckt to another ckt (usually from one AC vtg level to another) without changing its frequency.

- Basic use of this transformer is to increase or decrease the AC vtg level.
- If its is used to increase the vtg, then it is stepup transfo.
- ——— " ——— decrease ——— " ——— step down t/f
- If there is no change in the vtg, then it is one to one t/f.

* In dc s/s, raising & lowering of the vtgs is not easy.

Advantages (Continued...) pure sinusoidal w/f.

- In practice, a quantity which undergoes variations in magnitude as well as in its instantaneous values, in alternating direction w.r.t some reference is called an alternating quantity. A sinusoidal wave, even after repeated differentiation or integration, remains sinusoidal of same frequency

- There can be rectangular ac, triangular ac & so on. Out of all these types of alternating w/f, purely sinusoidal w/f is preferred for ac s/s.

The advantages of using sinusoidal w/f are -

(i) Mathematically, it is very easy to write the eq^ns for purely sinusoidal w/f.

(ii) Any other type of w/f can be resolved into a series of sine or cosine waves of fundamental & higher frequencies, sum of all these waves gives the original w/f. Hence, it is always better to have sinusoidal w/f as the std. w/f.

(iii) Sine & cosine waves are the only waves which can pass thro linear ckts containing resistance, inductance, capacitance without distortion. In case of other w/f's, there is a possibility of distortion when it passes thro linear ckt.

(iv) The integration & derivative of a sinusoidal functn is again a sinusoidal function.

- This makes the analysis of linear electrical n/tws with sinusoidal i/ps very easy. Sum & difference of a no. of sinusoids of same freq. but of diff amplitudes & phase angle is a sinusoid of same frequency.

Generation of AC Voltage:

- The basic principle of an ac generation is the principle of electromagnetic inductance.
- The sine wave is generated acc. to Faraday's law of Electromagnetic Induction

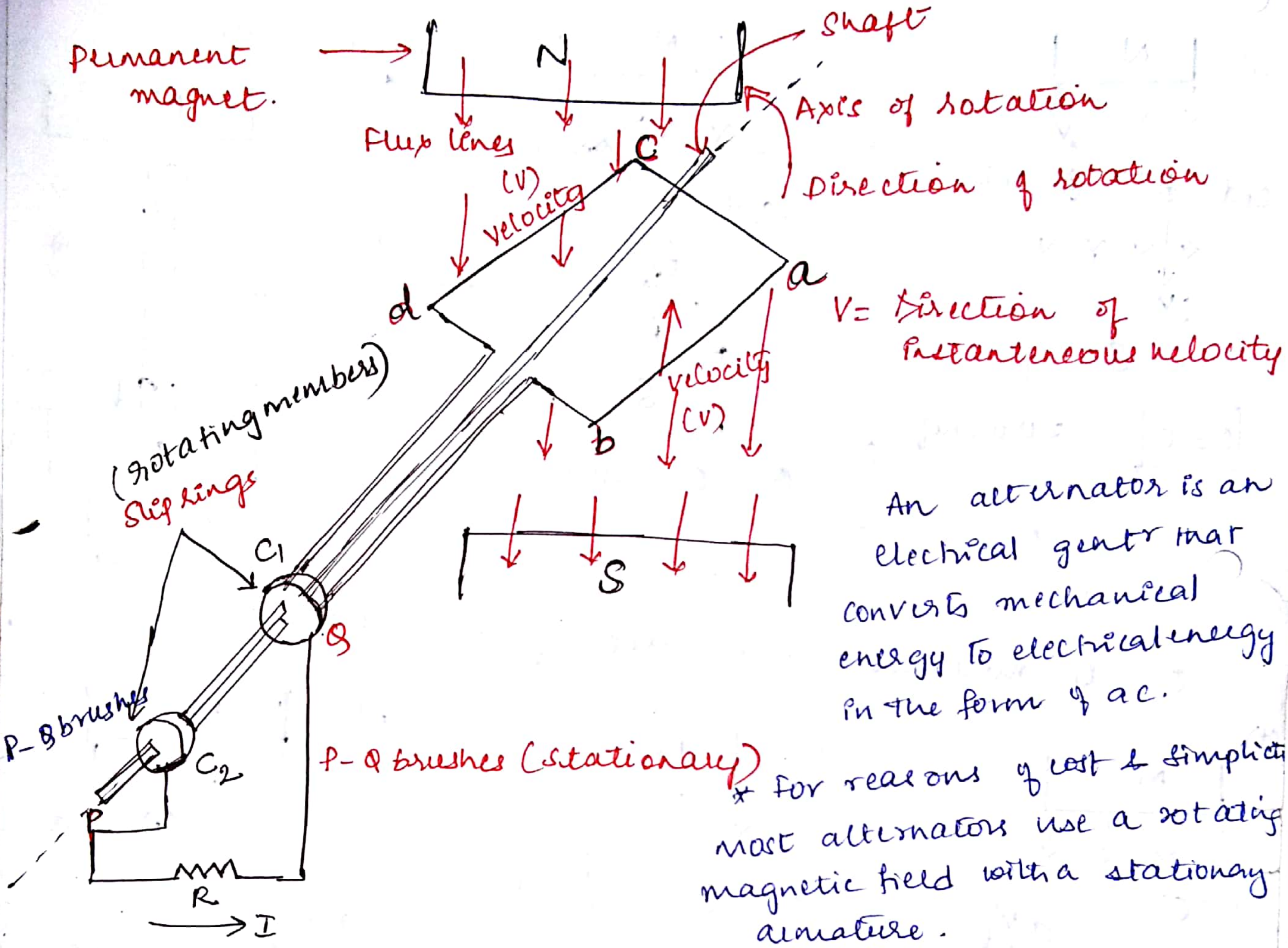
Alternator → electrical generator converts mechanical energy to electrical energy

* Let us study how an alternator produces a sine wave, with the help of simplest form of an alternator called single turn or single loop alternator.

- It consists of permanent magnet having two poles. A single turn rectangular coil is kept in the vicinity of permanent magnet.
- The coil is made up of 2 conductors namely a-b & c-d. Such two conductors are connected at one end to form a coil.
- The coil is so placed so that it can be rotated about its own axis.
- The remaining 2 ends C_1 & C_2 of the coil are connected to the rings mounted on the shaft called slip rings.

* Slip rings are also rotating members of the alternator.
* The 2 brushes P & Q are resting on the slip rings. The brushes are stationary & just making contact with slip rings.

SINGLE TURN ALTERNATOR:



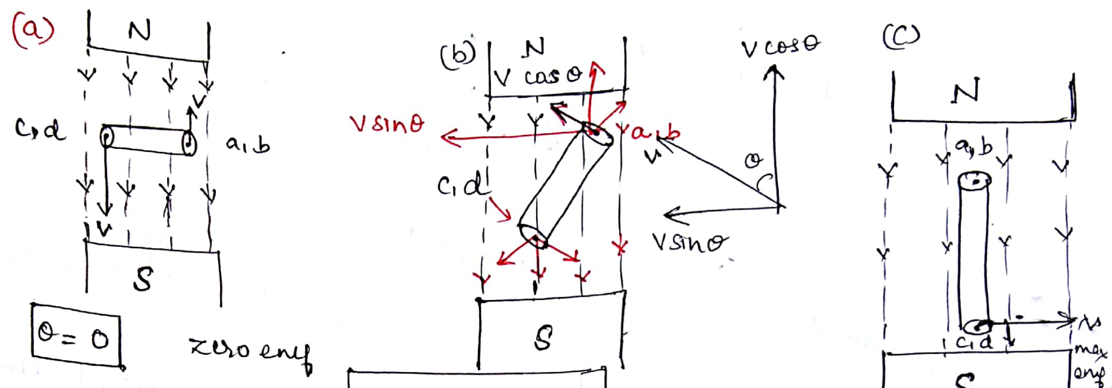
An alternator is an electrical generator that converts mechanical energy to electrical energy in the form of a.c.

* For reasons of cost & simplicity most alternators use a rotating magnetic field with a stationary armature.

WORKING: The coil is rotated in anticlockwise direction.

- While rotating, the conductors ab & cd cut the lines of flux of the permanent magnet.
- Due to Faraday's law of EMI, an emf gets induced in the conductors.
- This emf drives a current thro' resistance 'R' connected across the brushes P & Q.
- Magnitude of the induced emf depends on the position of the coil in magnetic field.

Consider diff instants & diff positions of the coil

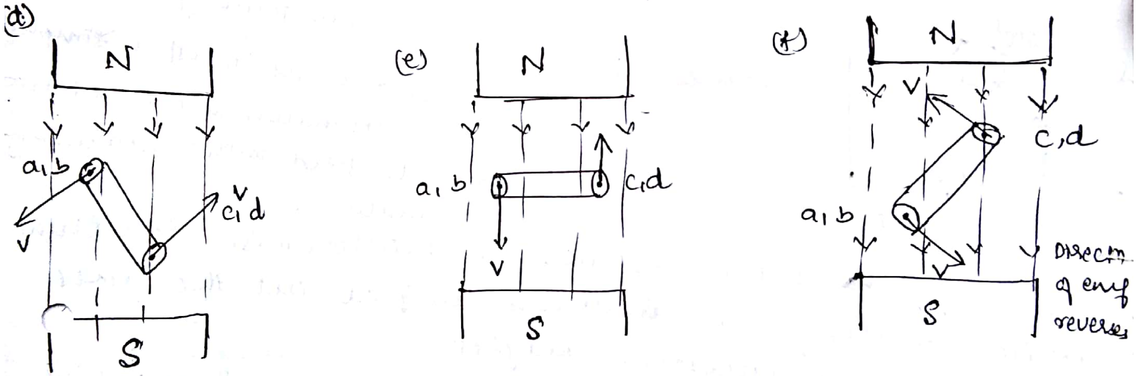


$\boxed{\theta = 0}$ zero emf

$\boxed{(b)\ 0^\circ < \theta < 90^\circ}$

$\boxed{(c)\ \theta = 90^\circ}$

θ is measured w.r.to axis of flux (vertical)

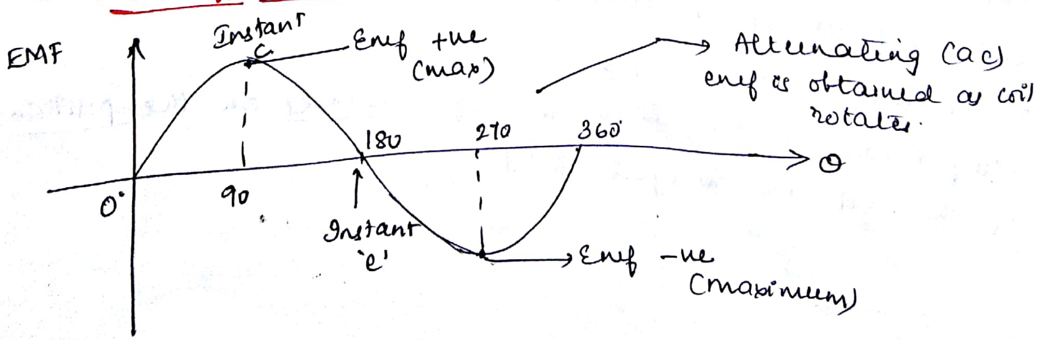


(d) $90^\circ < \theta < 180$

(e) $\theta = 180^\circ$

(f) $180^\circ < \theta < 270^\circ$

The different instants of induced emf



Consider diff. instants & diff. positions of the coil.

* Instant 1: Let the initial position of coil be as shown in fig.

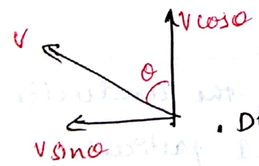
- Plane of the coil is ⊥lar to the direction of the magnetic field.
- Instantaneous component of velocity of conductors ab & cd is ||lel to the magnetic field as shown. & there cannot be any cutting of the flux lines by the conductors.
- Hence no emf will be generated in the conductors ab & cd.
- The angle 'θ' is measured from the plane of the magnetic flux.

Instant 2:

- When the coil is rotated in anticlockwise direction thro' some ∠le 'θ', then the velocity will have 2 components

$V \sin\theta \rightarrow$ ⊥lar to flux lines

$V \cos\theta \rightarrow$ ||lel to flux lines



- Due to $V \sin\theta$ component, there will be cutting of flux & proportionally there will be induced emf in conductors ab & cd.

= 360°. the emf in a

Instant 3: As angle 'θ' ↑'s, the component of velocity acting ⊥lar to flux lines ↑'s, hence induced emf ↑'s.

- At θ = 90°, the plane of the coil is ||lel to the plane of the magnetic field, while the component of velocity cutting the lines of flux is at its max.

* So induced emf in this position is at its max.

- So as θ ↑'s from 0–90°, emf induced in the conductors ↑'s gradually from 0 – max. value.

Instant 4: As the coil continues to rotate from θ = 90 to 180°, the component of velocity, ⊥lar to the magnetic field starts decreasing, hence gradually decreasing the magnitude of the induced emf.

Instant 5: In this position, velocity component is fully ||lel to the lines of flux ||lar to Instant 1.

- No cutting of flux
- No induced emf in both conductors

Instant 6: As coil rotates beyond θ = 180°, the conductor ab until now, cutting flux lines in 1 particular direction reverses the direction of cutting flux lines. Similar is the behavior of conductor cd.

- This change in directn of induced emf occurs because the directn of rotation of conductors ab & cd reverses w.r.to field as 'θ' varies from 180° to 360°. This process continues as coil rotates further

- So, as θ varies from 0 to 360°, the emf in a conductor ab or cd varies in an alternating manner i.e zero, increasing to achieve max. in one direction, decreasing to zero, increasing to achieve max. in other direcn & again decreasing to zero.

- This set of variation repeats for every revolution as the conductor rotates in a circular motion with a certain speed.

- The variation of emf in a conductor can be graphically represented as shown.

- From the w/f it is clear that the w/f generated by the instantaneous values of the induced emf in any conductor (ab or cd) is purely sinusoidal in nature

- The angle 'θ' in radians & the angular velocity 'ω' in radians/sec. of the coil are related to each other thro' time 't' by the equation,

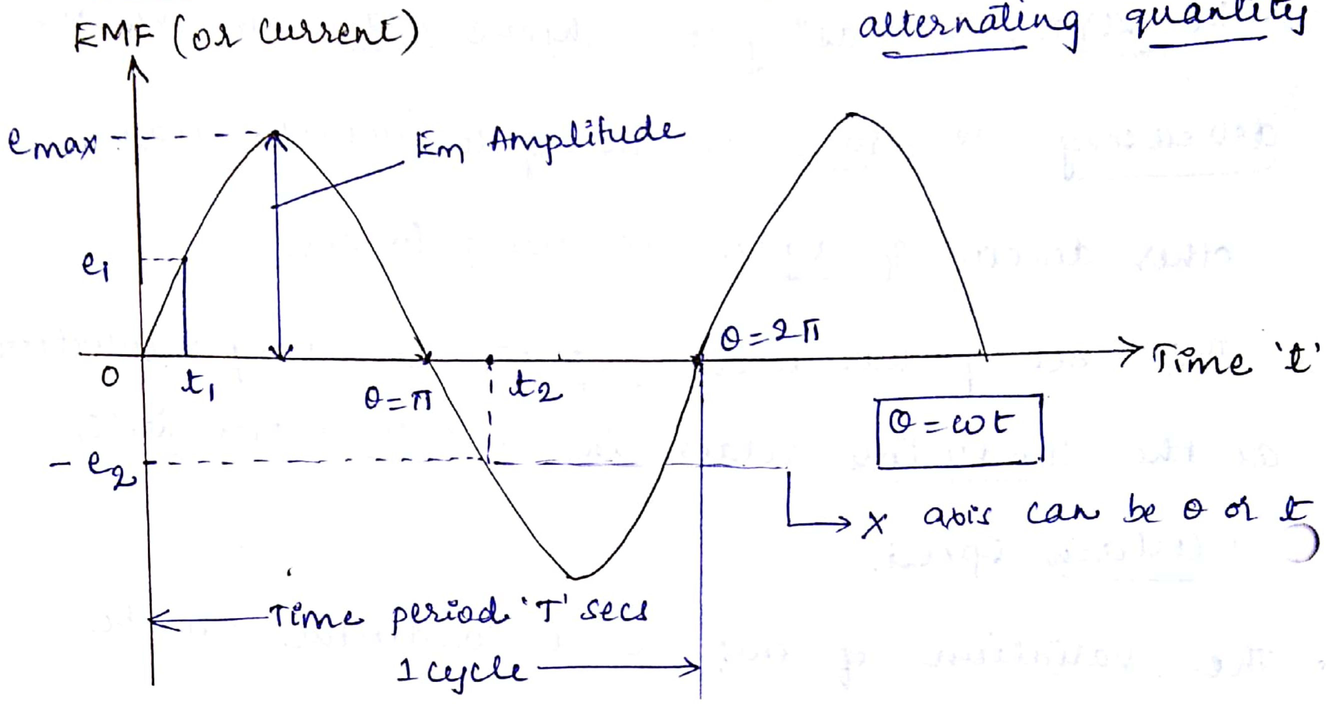
$$\boxed{\theta = \omega t \text{ radians}}$$

- Hence the w/f of alternating quantity can be shown w.r.to time 't' rather than 'θ' as 'ω' is const.

- There are 2 poles & for 1 complete rotation of conductor one electrical cycle of an induced emf is generated.

Standard Definitions Related to Alternating Quantity.

Representation of an alternating quantity



(i) Instantaneous value: The value of an alternating quantity at a particular instant is known as its instantaneous value.

Eg: e_1 & $-e_2$ are the instantaneous values of an alternating emf at the instants t_1 & t_2 respectively.

(ii) Waveform: The graph of instantaneous values of an alternating quantity plotted against time is called its waveform.

(iii) CYCLE: Each repetition of a set of the +ve & -ve instantaneous values of the alternating quantity is called a cycle.

- Such repetition occurs at regular intervals of time.
- Such a w/f which exhibits variations that reoccur after a regular time interval is called periodic w/f.

- A cycle can also be defined as that interval of time during which a complete set of non-repeating events or w/f variations occur (containing the +ve as well as -ve loops).

- One such cycle of the alternating quantity is as shown.

** One cycle corresponds to 2π radians or $360°$

(iv) Time period (T): The time taken by an alternating quantity to complete its one cycle is known as its time period denoted by 'T' seconds.

- After every 'T' seconds, the cycle of an alternating quantity repeats.

(v) Frequency (f): The no. of cycles completed by an alternating quantity/sec is known as its frequency. It is denoted by 'f' & it is measured in cycles/sec which is known as Hertz denoted as 'Hz'.

- As time period 'T' is time for 1 cycle i.e. secs/cycle & frequency is cycles/sec, we can say that frequency is reciprocal of the time period.

$$\boxed{f = \frac{1}{T}\ Hz}$$

- As time period ↑'s, frequency ↓'s while as time period ↓'s frequency ↑'s.

- In our nation, std. frequency of an alternating vtg & current is 50Hz.

vi) Amplitude : The max. value attained by an alternating quantity during +ve or -ve half cycle is called its amplitude.

- It is denoted as "Em or Im".
- Thus Em → peak value of Vtg
  Im → peak value of current.
- Amplitude is also called peak value or max value of an alternating quantity.

(vii) Angular frequency (ω): It is the frequency expressed in electrical radians/sec.

- As one cycle of an alternating quantity corresponds to 2π radians, the angular frequency can be expressed as (2π x cycles/sec).
- It is denoted by 'ω'. & its unit is radians/sec.
- The relation b/w frequency 'f' & angular frequency 'ω' is

$$\boxed{\omega = 2\pi f \ \text{rad/sec} \quad \text{OR} \quad \omega = \frac{2\pi}{T} \ \text{rad/sec}}$$

- The angle θ, the angular frequency 'ω' are related to each other thro' time as,

$$\boxed{\theta = \omega t \ \text{radians} \quad \text{or} \quad \theta = 2\pi f t \ \text{radians}}$$

- Thus the o/p of an alternating quantity can be shown wrto time 't' or angle 'θ' as 'ω' is const.

(viii) Peak to Peak value: The value of an alternating quantity from its +ve peak to -ve peak is called its peak-peak value.

- It is denoted as I_{p-p} or V_{p-p}.

$$\boxed{\text{Amplitude} = \frac{\text{Peak to Peak Value}}{2}}$$

Equation of an Alternating Quantity:

- As the std. w/f of an alternating quantity is purely sinusoidal, the eqn of an alternating vtg can be expressed as,

$$\boxed{e = E_m \sin\theta \text{ volts}}$$

where, E_m = Amplitude / Max / peak value of vtg.
e = Instantaneous value of an alternating vtg.

IIIcarly. equation of an alternating current can be expressed as,

$$\boxed{i = I_m \sin\theta}$$

where, I_m = Amplitude / Max / peak value of ~~vtg~~ current.
i = Instantaneous value of an alternating current.

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The equation can be eppressed in various forms as

Now, $\theta = \omega t$ radians

$\therefore \quad e = E_m \sin(\omega t)$

But, $\omega = 2\pi f$ rad/sec.

$\therefore \quad e = E_m \sin(2\pi f t)$

But, $f = \frac{1}{T}$ seconds

$$\boxed{e = E_m \sin\left(\frac{2\pi}{T} t\right)}$$

** In all the above eqns, the angle 'θ' is eppressed in radians.

- Hence, while calculating the instanteneous value of the emf, it is necessary to calculate the sine of the angle eppressed in radians.

Numericals:

(2) ① An alternating current of frequency 60 Hz has a max. value of 12A.

(i) write down the equation for instanteneous values.

(ii) Find the value of current after 1/360 secs.

Sol (iii) Time taken to reach 9.6A for the I time.

In the above cases assume that time is reckoned as zero when current wave is passing thro zero & increasing in the +ve direction.

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Soln: $f = 60Hz$, $I_m = 12A$, $\omega = 2\pi f = 2\pi \times 60 = 377$ rad/sec

(i) Eqn of instanteneous value is,

$$i = I_m \sin\omega t = 12 \sin 377t$$

(ii) $t = \frac{1}{360}$ sec i.e $i = 12 \sin 377 \frac{1}{360}$

$$i = 12 \underbrace{\sin 1.0472}_{\text{radian mode.}}$$

$$\boxed{i = 10.3924A}$$

(iii) $i = 9.6A$, i.e $i = I_m \sin\omega t$

$9.6 = 12 \sin 377t \Rightarrow \sin 377t = 0.8$

$377t = 0.9272$

$t = 2.459 \times 10^{-3}$ sec --- $\sin^{-1}$ in radian mode.

(2) A sinusoidally varying alternating current has max. value of $2\sqrt{2}$A. & time period of 20msec. If the (e) w/f enters into its the half cycle at $t=0$. Find the instanteneous value of the current of $t = 6$msec.

Soln: $I_m = 2\sqrt{2}$ A, $T = 20$msec, $f = \frac{1}{T} = 50Hz$

∴ Eqn of current is, $i(t) = I_m \sin(2\pi f t)$

At $t = 6$msec. $i(t) = 2.8284 \underbrace{\sin(2\pi \times 50 \times 6 \times 10^{-3})}_{\text{radian mode.}}$

$$i(t) = 2.6899A$$

Effective value or RMS value

- An alternating current varies from instant to instant, while the direct current is const. w.r.t. time.

- For the comparision of the two, a common effect to both the type of currents can be considered. Such an effect is heat produced by the 2 currents flowing thro' the resistance.

- The heating effect can be used to compare the alternating & direct ~~current~~ current.

* From this, rms value of an alternating current can be defined as,

** [The effective or rms value of an alternating current is given by that steady DC current (DC) which, when flowing thro' a given ckt for a given time, produces the same amount of heat as produced by the alternating current, which when flowing thro' the same ckt for the same time.]

Analytical Method of Obtaining RMS Value.

Steps to find rms value of an ac quantity

(1) Write the equation of an ac quantity. observe its behaviour during various time intervals.

(2) Find square of the ac quantity form its equation

(3) Find average value of square of an alternating quantity as,

$$\text{Average} = \frac{\text{Area of curve over one cycle of squared w/f}}{\text{Length of the cycle.}}$$

(4) Find square root of avg. value which gives rms value of an alternating quantity.

* Consider sinusoidally varying alternating current & square of this current as shown

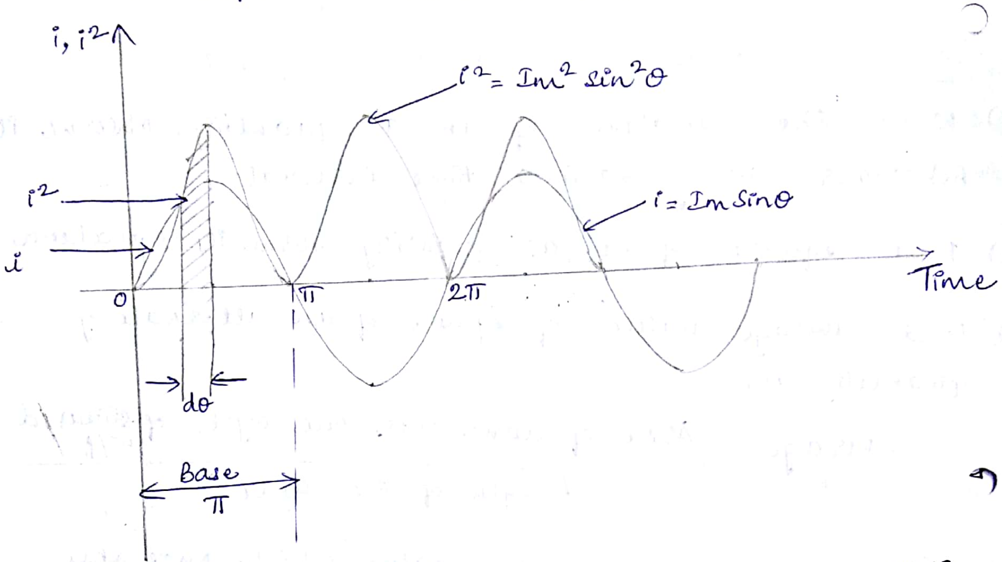
Step I: The current $i = I_m \sin\theta$.

Step II: Square of current $i^2 = I_m^2 \sin^2\theta$

- The area of curve over half a cycle can be calculated by considering an interval '$d\theta$' as shown.

$$\left.\begin{array}{l}\text{Area of square curve}\\ \text{over half cycle}\end{array}\right\} = \int_0^{\pi} i^2 \, d\theta$$ & length of the base is π.

W/f of current & square of the current



Step III: Avg value of square of the current over half cycle.

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^{\pi} i^2 d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i^2 d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta = \frac{I_m^2}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = \frac{I_m^2}{2\pi}[\pi] = \frac{I_m^2}{2}$$

Step IV: Root mean square value i.e rms value can be calculated as,

$I_{rms} = \sqrt{\text{mean or avg. value of square of current}}$.

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{\begin{aligned} \therefore\ & I_{rms} = 0.707 I_m \\ \text{|||ly}\ & V_{rms} = 0.707 V_m \end{aligned}}$$

RMS value of the sinusoidal alternating current is 0.707 times the max or peak value or amplitude of that ac.

Instantaneous values → i, e etc

RMS values → I, E etc.

Importance of RMS value:

- In case of alternating quantities, rms values are used for specifying magnitudes of alternating quantities
- The given values such as 230V, 110V are rms values of alternating quantities unless & otherwise specified to be other than rms.

* In practice, everywhere, rms values are used to analyse alternating quantities

* The ammeters & voltmeter records the rms values of current & vtg respectively.

- Heat produced due to ac is proportional to the square of the rms value of the current.

AVERAGE VALUE:

- Average value of an alternating quantity is defined as that value which is obtained by averaging all instantaneous values over a period of half cycle.
- For a symmetrical ac, the avg value over a complete cycle is zero as both the & -ve half cycles are exactly identical.
- Hence, the avg value is defined for half cycle only.
- Avg value can also be expressed by that steady current, which transfers across any ckt, the same amount of charge as is transferred by that alternating current during the same time.

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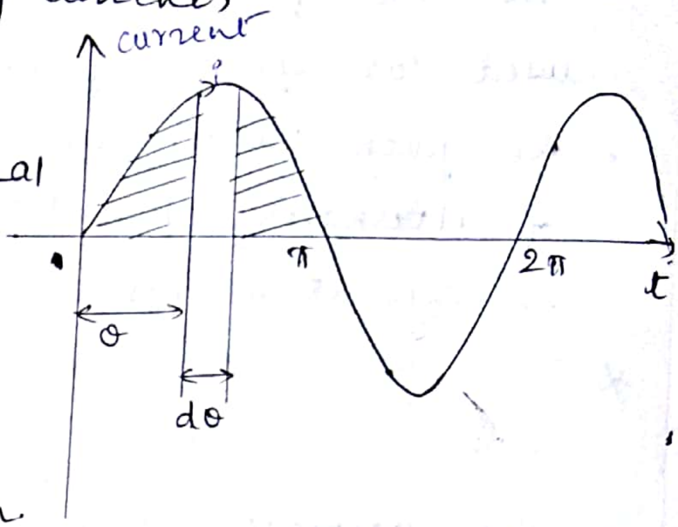
ANALYTICAL METHOD OF OBTAINING AVERAGE VALUE:

- Consider sinusoidally varying current,

  i.e $I = I_m \sin\theta$.



- consider the elementary interval of instant 'dθ' as shown in fig.
- The avg (value) instanteneous value of current in this interval is say 'i' as shown.

- Avg. value can be obtained by taking ratio of area under curve over half cycle to length of the base for half cycle

$$I_{avg} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle.}}$$

$$I_{avg} = \frac{\int_0^{\pi} i\,d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i\,d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$I_{avg} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[-\cos\pi + \cos 0\right] = \frac{I_m}{\pi}\left[2\right] = \frac{2I_m}{\pi}$$

$$\boxed{I_{avg} = 0.637 I_m}$$

- For a purely sinusoidal w/f, the avg value is expressed in terms of its max value as,

$$\boxed{I_{avg} = 0.637 I_m \quad \& \quad V_{avg} = 0.637 V_m}$$

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Importance of Avg Value.

1) The avg value is used for applications like battery charging.

2) The charge transfered in capacitor ckts is measured ent. using avg. values.

3) The avg values of vlgs & currents play an imp. role in analysis of the rectifier ckts.

4) The avg value is indicated by dc ammeters & voltmeters.

5) The avg value of purely sinusoidal w/f is always zero.

Form Factor (K_f)

- Form factor of an alternating quantity is defined as the ratio of rms value to the avg value.

$$\text{Form factor, } K_f = \frac{\text{rms value}}{\text{avg value.}}$$

- Form factor for sinusoidal alternating currents or vlgs can be obtained as,

$$K_f = \frac{0.707 I_m}{0.637 I_m} = 1.11 \text{ for sinusoidally varying quantity}$$

Crest or Peak Factor (K_p)

- Peak factor of an alternating quantity is defined as ratio of max. value to the rms vlg.

$$\text{Peak factor, } K_p = \frac{\text{max. value}}{\text{rms value}}$$

- peak factor for sinusoidally varying alternating currents & vlgs can be obtained as,

$$K_p = \frac{I_m}{0.707 I_m} = 1.414 \text{ for sinusoidal w/f.}$$

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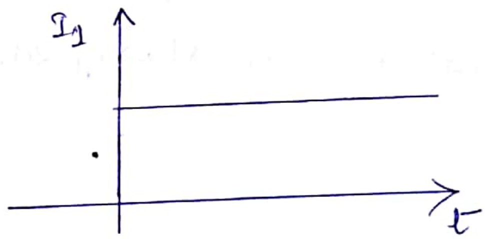
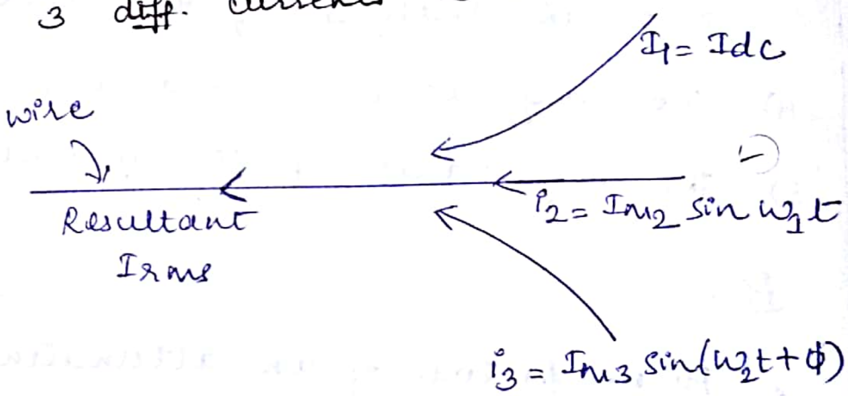
RMS value of combined w/f:

- Consider a wire carrying simultaneously more than one alternating current of diff. magnitudes & frequencies along with certain d.c. current.
- It is required to calculate resultant rms value i.e effective value of the current.

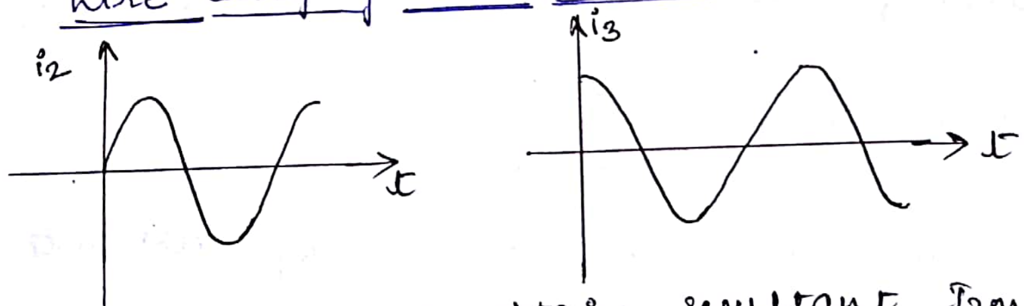
* Let the wire carries 3 diff. currents as shown



$I_1 = I_{dc}$



wire carrying 3 diff. currents simultaneously



- It is required to obtain resultant I_{rms} thro' the wire.
- Method is based on heating effect of various currents.
- Let R = Resistance of wire.
- I_{rms} = Resultant rms value of current.

  t = Time for which current is flowing.

  H = Heat produced by resultant $= I_{rms}^2 \times R \times t$.

- This heat produced is sum of heats produced by the individual current components flowing for same time 't'.

Problem based on RMS value of combined form:

(1) An a.c. current is given by $i = 10\sin\omega t + 3\sin 3\omega t + 2\sin 5\omega t$. Find the rms value of current.

Soln When a coil carries d.c. current & more than one alternating signals, then the total heat produced is the sum of the heats produced by dc component & all the alternating components.

Let R = resistance of wire
t = time for which signals are flowing.

$$H_{total} = H_{dc} + H_1 + H_2 + \ldots$$

$$H_{total} = I^2_{rms} \times R \times t \quad (\text{where } I_{rms} = \text{Total rms value})$$

$$H_{dc} = I^2_{dc} \times R \times t$$

$$H_1 = I^2_{rms1} \times R \times t, \quad H_2 = I^2_{rms2} \times R \times t \ldots$$

$$\therefore I^2_{rms} \times R \times t = I^2_{dc} Rt + I^2_{rms1} Rt + I^2_{rms2} Rt + \ldots$$

$$I_{rms} = \sqrt{I^2_{dc} + I^2_{rms1} + I^2_{rms2} + \ldots}$$

For instance, $I_{dc} = 0$

$$I_{rms1} = \frac{I_{m1}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07106\text{ A}$$

$$I_{rms2} = \frac{3}{\sqrt{2}} = 2.1213\text{ A}$$

$$I_{rms3} = \frac{2}{\sqrt{2}} = 1.4142\text{ A}$$

$$I_{rms} = \sqrt{0 + (7.07106)^2 + (2.1213)^2 + (1.4142)^2}$$

$$\boxed{I_{rms} = 7.5166\text{ A}}$$

• H_1 = Heat produced by dc component.

$= I_{dc}^2 \times R \times t$

* H_2 = Heat produced by I ac current component.

$$= I_{rms\,2}^2 \times R \times t = \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 \times R \times t$$

* H_3 = Heat produced by II ac component.

$$= I_{rms\,3}^2 \times R \times t = \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 \times R \times t.$$

Since equating total heat produced to sum of the individual heats produced.

i.e $H = H_1 + H_2 + H_3$

i.e $$I_{rms}^2 R \times t = I_{dc}^2 R t + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 R t + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 R t$$

$$\boxed{I_{rms} = \sqrt{I_{dc}^2 + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2}}$$

• The result can be extended to 'n' no. of current components flowing thro the wire.

Problems:

(1) The equation of an alternating current is given by $i = 42.42 \sin 628t$. Calculate its

(i) Max. value (ii) frequency (iii) RMS value (iv) Avg value

(v) Form factor

Soln Compare given eq^n with, $i = I_m \sin(\omega t)$

(i) $I_m = \underline{42.42A}$

(ii) $f = ?$, $\omega = 2\pi f$, $\therefore f = \frac{\omega}{2\pi} = \frac{628}{2\pi} = \underline{100 Hz}$

(iii) RMS value, $I_{rms} = ?$

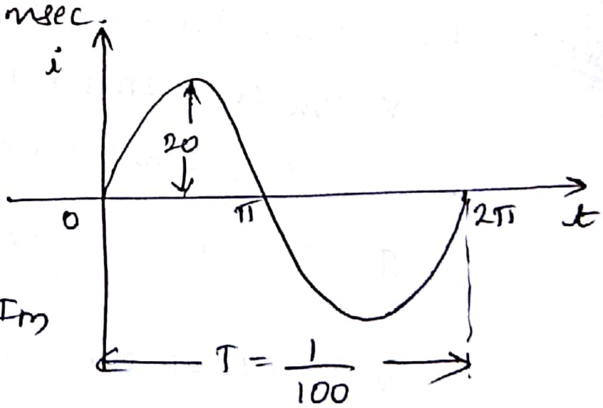
$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{42.42}{\sqrt{2}} = \underline{30A}$$

(iv) $I_{avg} = ?$ $0.637 I_m = \underline{27.0215 A}$

(v) $K_f = \frac{RMS}{Avg} = \frac{30}{27.0215} = \underline{1.11}$

---

(2) For the current wave shown, find (i) Peak current (ii) Avg value (iii) frequency (iv) Periodic time. (v) Instanteneous value at $t = 3msec$.



Soln Amplitude or peak value of current w/f $= \underline{20A}$ i.e $I_m = 20A$

(ii) Avg value, $I_{(av)} = \frac{2I_m}{\pi} = 0.637 I_m$

$$= \frac{2 \times 20}{\pi} = \underline{12.732A}$$

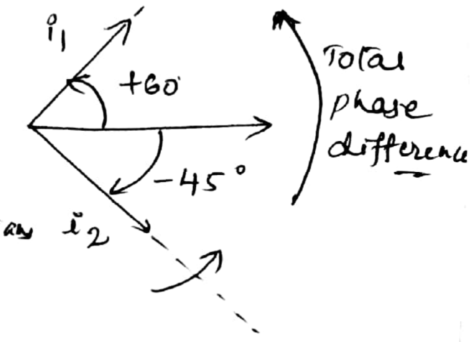
(iii) Frequency, $f = \frac{1}{\text{Time period}} = \frac{1}{1/100} = \underline{100 Hz}$

(iv) $T = 1/100 = 0.01 \underline{sec}$ (periodic time).

(v) Instanteneous value at t, $3 msec. = 3 \times 10^{-3} sec.$

$i = I_m \sin \omega t$ $\therefore$ $\boxed{i = 19.0211 A}$

(3) Two sinusoidal currents are given by $i_1 = 10\sin(\omega t + \pi/3)$ & $i_2 = 15\sin(\omega t - \pi/4)$. Calculate the phase diff. b/w them in degrees.



Soln Phase of current i_1 is $\frac{\pi}{3}$ radians.

i.e $\underline{60°}$

while phase of the current i_2 is $-\pi/4$ radian

i.e $-45°$

phase diff. b/w the two is,

$\phi = \theta_1 - \theta_2 = 60 - (-45) = \underline{105°}$

i.e $\underline{i_2 \text{ lags } i_1}$.

(4) In a ckt supplied from 50Hz the vtg & current have max. values of 500V & 10A respectively. At $t=0$, their respective values are 400V & 4A. both increasing they.

respectively values are 400V & 4A. both increasing. their instantaneous values.

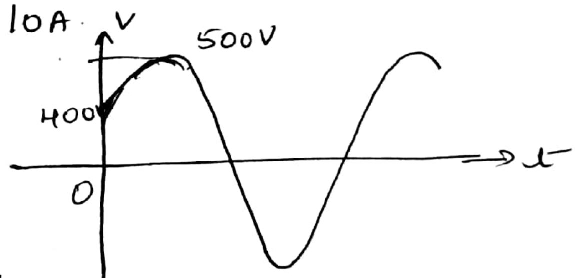
(i) write expressions for their instantaneous values.

(ii) Find the angle b/w V & I.

(iii) I at at = 0.015 sec.

As vtg is +ve at $t=0$

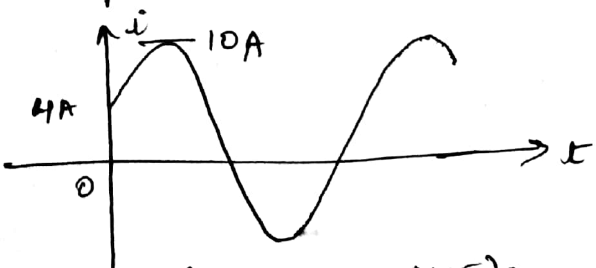
Soln $f = 50Hz$, $V_m = 500V$, $I_m = 10A$.



$V = V_m \sin(2\pi f t + \phi_1)$

$400 = 500\sin(0 + \phi_1)$

$\phi_1 = 53.13° = 0.\underline{9272}$ rad.



$V = 500\sin(100\pi t + 0.9272)$ V

$i = I_m \sin(2\pi f t + \phi_2)$

$4 = 10\sin(0 + \phi_2)$, $\underline{t=0}$

$\phi_2 = 23.57° = 0.4115$ rad. i.e $10\sin(100\pi t + 0.4115)$A

(ii) $\phi_1 = 53.13°$ for vtg, $\phi_2 = 23.57°$ for current.

ϕ = angle b/w V & I = $53.13° - 23.57° = 29.52°$

(iii) At $t = 0.015$ sec $i = -9.1652A$

(3) A vtg is defined as $-E_m \cos\omega t$. Express it in polar form.

Sol^n To express vtg in polar form, express it in the form.

$e = E_m \sin\omega t$

Now $e = -E_m \cos\omega t = -E_m \sin\left(\omega t + \frac{\pi}{2}\right)$

as $\sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t$.

$\sin(\pi + \theta) = -\sin\theta$

$= E_m \sin\left(\omega t + \frac{3\pi}{2}\right)$

Express it in polar form, $e = E_m \angle + \frac{3\pi}{2}$ radi

$= E_m \angle + 270^\circ$ v

$+270^\circ$ phase is nothing but -90°.

$$\boxed{e = E_m \angle -90^\circ V}$$

NOTE: (i) Addition & subtraction → Rectangular form

(ii) Multiplication & division → polar form.

*

(4) If $A = 4 + j7$; $B = 8 + j9$ & $C = 5 - j6$, then calculate

(i) $\frac{A+B}{C}$ (ii) $\frac{A \times B}{C}$ (iii) $\frac{A+B}{B+C}$ (iv) $\frac{B-C}{A}$.

Sol^n (i) $\frac{A+B}{C} = \frac{4+j7 + 8+j9}{7.810\angle -50.194} = \frac{12+j16}{7.810\angle -50.194} = \frac{20\angle 53.13}{7.810\angle -50.194}$

$= -0.5901 + 2.491j = \underline{\underline{2.560\angle 103.3^\circ}}$

(ii) $\frac{A \times B}{C} = \frac{8.062 \angle 60.255 \times 12.041 \angle 48.366}{7.8102 \angle -50.194}$

$= \frac{97.0745 \angle 108.621}{7.8102 \angle -50.194} = -11.589 + 4.49i$

or

$12.429 \angle 158°$

(iii) $\frac{A+B}{B+C} = \frac{4+j7+8+j9}{8+j9+5-j6} = \frac{12+16j}{13+j3}$

$= \frac{20 \angle 53.13}{13.34 \angle 12.99} = 1.499 \angle 40.14°$

$= 1.149 + j0.966$

(iv) $\frac{B-C}{A} = \frac{(8+j9)-(5-j6)}{(4+j7)} = \frac{3+j15}{4+j7} = \frac{15.297 \angle 78.69}{8.062 \angle 60.255}$

$= 1.799 + 0.599j$ or $1.897 \angle 18.435°$

⑤ The current drawn by a pure capacitor of 20 μF is 1.382A, from 220V ac supply. What is supply frequency.

Soln

$C = 20\mu F, \quad I = 1.382A, \quad V = 220V$

$X_C = \frac{V}{I} = \frac{220}{1.382} = 159.18\Omega$

$X_C = \frac{1}{2\pi fC}$ i.e $f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi \times 159.18 \times 20 \times 10^{-6}} = 50Hz$

⑥ A 50 Hz, alternating vtg of 150 V (rms) is applied independently to (i) Resistance of 10Ω

(ii) Inductance of 0.2H

(iii) Capacitance of $50\mu F$.

Find the expression for the instantaneous current in each case. Draw the phasor diagram in each case.

Soln. Case(i): $R = 10\Omega$

$V = V_m \sin \omega t$

$V = \sqrt{2}\, V_{rms} = \sqrt{2} \times 150 = \underline{212.3 \text{ volts}}$

$I_m = \frac{V_m}{R} = \frac{212.3}{10} = \underline{21.213} \text{ Amps.}$

In pure resistive ckt, current is in phase with vtg, $\therefore \phi =$ phase difference $= 0°$

$\therefore\ i = I_m \sin \omega t$

$= I_m \sin (2\pi f t)$

$i = 21.213 \sin(2\pi \times 50 \times t)$

$i = \underline{21.213 \sin(100\pi\, t)}$ Amps

phasor diagram,

Case (ii): $L = 0.2\Omega$

Inductive reactance, $X_L = \omega L = 2\pi f L$

$X_L = 2\pi \times 50 \times 0.2 = \underline{62.83\Omega}$

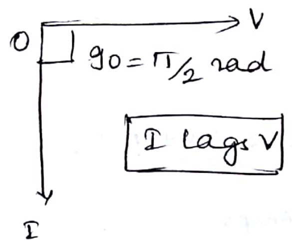
$I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83} = \underline{3.37 \text{ Amps}}$

In pure inductive ckt, current lags vtg by 90°

$\therefore \phi =$ phase diff. $= -90° = \pi/2$ radian

$i = I_m \sin(\omega t - \phi)$

$\therefore i = 3.37 \sin(100\pi t - \pi/2)$ Amps



Case (iii) :- $C = 50\ \mu F$:

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$= \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$

$$\boxed{X_C = 63.66\Omega}$$

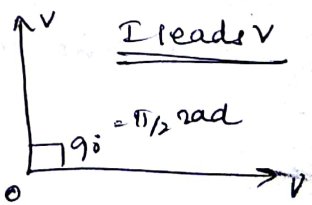
$I_m = \frac{V_m}{X_C} = \frac{212.13}{63.66} = 3.33A$

In pure capacitive ckt, current leads vtg by 90°.

$\therefore \phi =$ phase diff. $= 90° = \pi/2$ radian

$\therefore i = I_m \sin(\omega t + \phi)$

$i = 3.33 \sin(100\pi t + \pi/2)$ Amps.

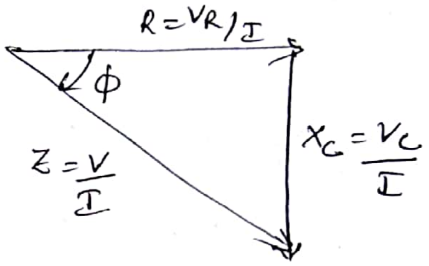


** All phasor diagrams are represented in rms values of vtg & current.

- From vtg triangle, we can write,

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

- If all the sides of the vtg Δ^{le} are divided by the current, we get a Δ^{le} called impedance Δ^{le}.



- Two sides of the Δ^{le} are 'R' & X_C & third side is impedance 'Z'.

x component of $Z = R = Z\cos\phi$

y component of $Z = X_C = Z\sin\phi$

- But, as direction of the X_C is -ve y direction, the rectangular form of impedance is denoted as,

$$\boxed{Z = R - jX_C \ \Omega}$$

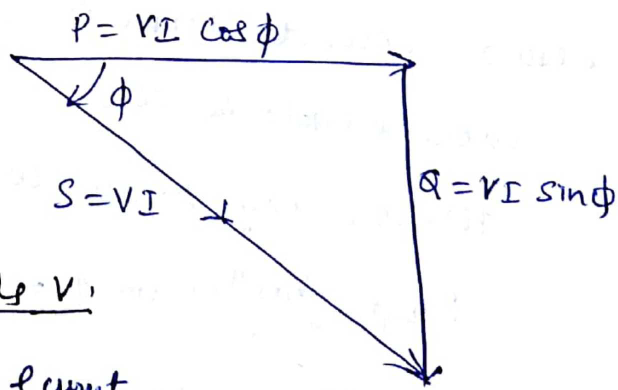
while in polar form,

$$Z = R - jX_C = |Z| \angle -\phi$$

where $|Z| = \sqrt{R^2 + X_C^2}$, $\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$

- * 'ϕ' is -ve for capacitive impedance *.

Power & Power Triangle:



- The currents leads vtg by angle ϕ, hence its expression is,

$i = I_m \sin(\omega t + \phi) \rightarrow$ as I leads V.

Apparent power:- Product of rms values of vtg & current is called Apparent power measured in VA or kVA

Power Δ^{le} - $\boxed{S = VI}$

Real power - Product of applied vtg & active component of current is real Power expressed in watts or (kW) - $P = VI\cos\phi$

Reactive power - Product of applied vtg & reactive component of current. Expressed in volt-ampere reactive (VAR) or KVAR $Q = VI\sin\phi$

... of the two alternating

- Power is the product of instanteneous values of vtg & current.

$P = V \times i = V_m \sin \omega t \times I_m \sin(\omega t + \phi)$ → as current leads vtg.

Repeated

* Power is the product of instanteneous value of vtg & current.

$\therefore P = V \times i$

$= V_m \sin \omega t * I_m \sin(\omega t + \phi)$

$= V_m I_m [\sin \omega t \cdot \sin(\omega t + \phi)]$

$= V_m I_m \left[\frac{\cos(\cancel{\omega t} - \cancel{\omega t} - \phi) - \cos(\omega t + \omega t + \phi)}{2}\right]$

$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$

$\sin A \cdot \sin B$

$\frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$P = \frac{V_m \cdot I_m \cos\phi}{2} - \frac{V_m I_m}{2} \cos(2\omega t + \phi)$

as $\cos(-\phi) = \cos\phi$

. Now, second term is cosine term, whose avg. value over a cycle is zero.

Hence, avg power consumed by ckt is,

$P_{avg} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$

$\boxed{P = VI\cos\phi \text{ watts}}$ wherer V & I are rms values

Phasor representation of an alternating Quantity:

- The sinusoidally varying alternating quantity can be represented graphically by a st. line with an arrow in the phasor representation method.
- The length of the line represents the magnitude of the quantity & arrow indicates direction.
- This is similar to a vector representation. Such a line is called phasor.

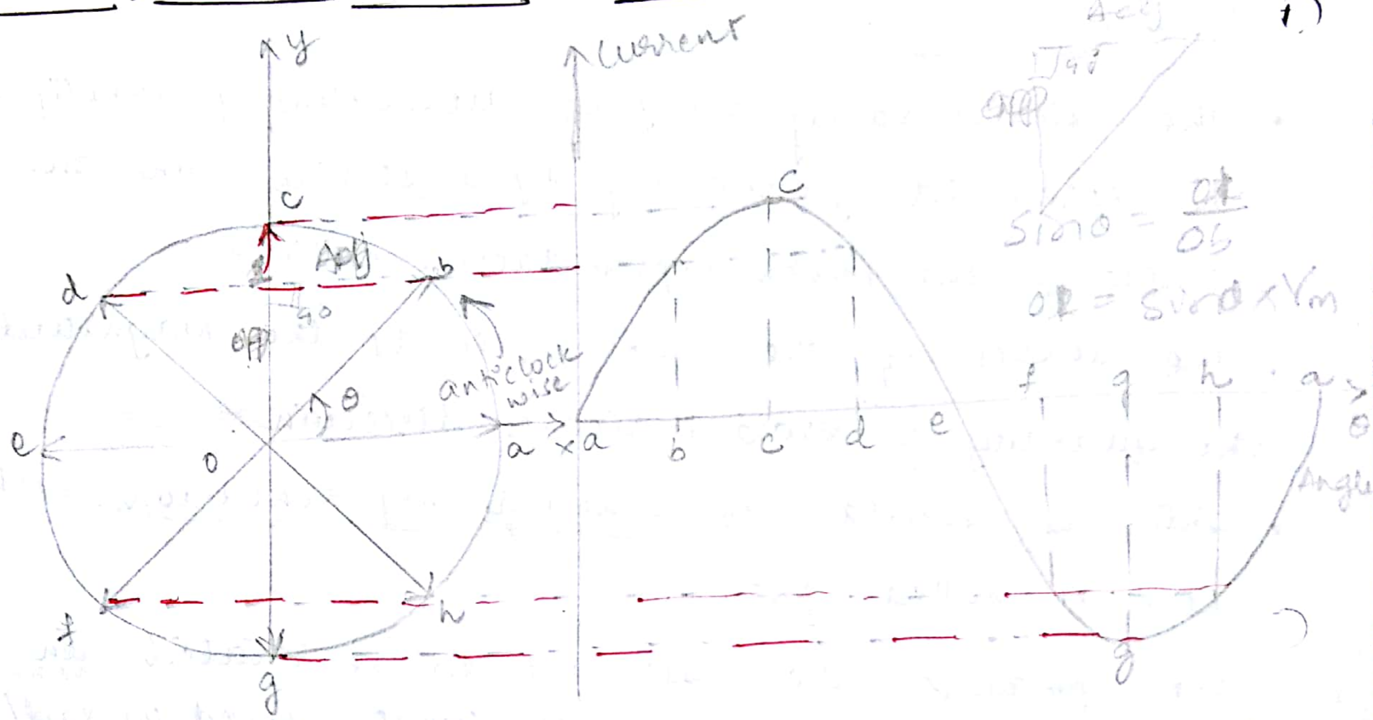
** The phasors are assumed to be rotated in anticlockwise direction with const. speed ω rad/sec

- One complete cycle of sine wave is represented by one complete rotation of phasor.

The anticlockwise direction of rotation is purely conventional direction which has been universally accepted.

- Consider a phasor, rotating in anticlockwise direction with uniform angular velocity, with its starting position 'a' as shown in fig.
- If the projections of this phasor on y axis are plotted against the angle turned through 'θ' (or time as $\theta = \omega t$) we get sine w/f.

Consider various positions shown:



Phasor representation of an alternating quantity.

(i) At pt. 'a', Y-axis projection is zero. Instantaneous value of current is also zero.

(ii) At pt. 'b', Y-axis projection is [l × (Ob) sin θ]. The length of the phasor is equal to maximum value of an alternating quantity. So, instantaneous value of the current at this position is $i = I_m \sin\theta$.

(iii) At pt. 'C', the Y axis projection 'OC' represents the entire length of the phasor i.e instantaneous value equal to the max. value of current 'I_m'.

(iv) ||ly at pt. 'd', Y axis projection becomes $I_m \sin\theta$, which is instantaneous value of the current at that instant.

||ly at points f, g, h Y axis projections give us instantaneous values of current at the respective instants & when plotted give us -ve ½ cycle of alternating quantity

(v) At point 'e', the Y axis projection is zero of instantaneous value of current at this inst. is zero.

NOTE: (Phasor representation of an alternating quantity.

- In practice, the alternating quantities are represented by their rms values.
- Hence, the length of the phasor represents rms value of the alternating quantity.
- In such case, projection on Y-axis does not give directly the instantaneous value, but as $\boxed{I_m = \sqrt{2} I_{rms}}$, the projection on Y-axis must be multiplied by $\sqrt{2}$ to get an instantaneous value of that alternating quantity.
- Phasors are always assumed to be rotated in anti-clockwise direction.
- Two alternating quantities of same frequencies can be represented on same phasor diagram.

PHASOR DIAGRAM:

- The diagram in which diff. alternating quantities of same frequency, sinusoidal in nature are represented by individual phasors, indicating exact phase inter-relationships is known as phasor diagram.

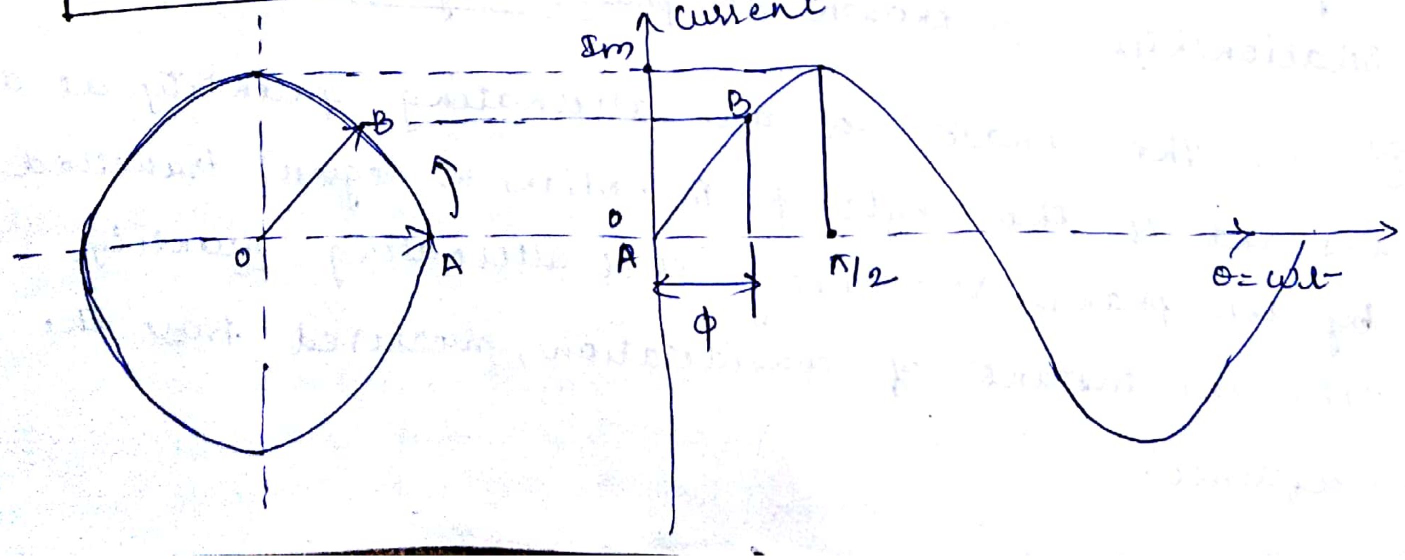
PHASE: The phase of an alternating quantity at any instant is the angle ϕ (in radians or degrees) travelled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

Concept of phase of an Alternating Quantity.

- In the analysis of alternating quantities, it is necessary to know the position of the phasor representing that alternating quantity at a particular instant.
- It is represented in terms of angle θ in radians or degrees, measured from certain reference.
- Let X-axis be the reference axis, so the phase of the alternating current is shown in fig., at the instant A is $\phi = 0^\circ$.
- While the phase of the current at the instant 'B' is the angle ϕ thro' which the phasor has travelled, measured from the reference axis. i.e X axis.
- In general, the phase ϕ of an alternating quantity varies from $\phi = 0$ to 2π radians or $\phi = 0^\circ$ to 360°.
- * In terms of phase, the equation of an alternating quantity can be modified as,

$$\boxed{e = E_m \sin(\omega t \pm \phi)}$$

where ϕ = phase of the alternating quantity.



NOTE:

(i) The phase is measured w.r.t reference direction i.e the x-axis direction.

(ii) The phase measured in anticlockwise direction is +ve while the phase measured in clockwise direction is -ve.

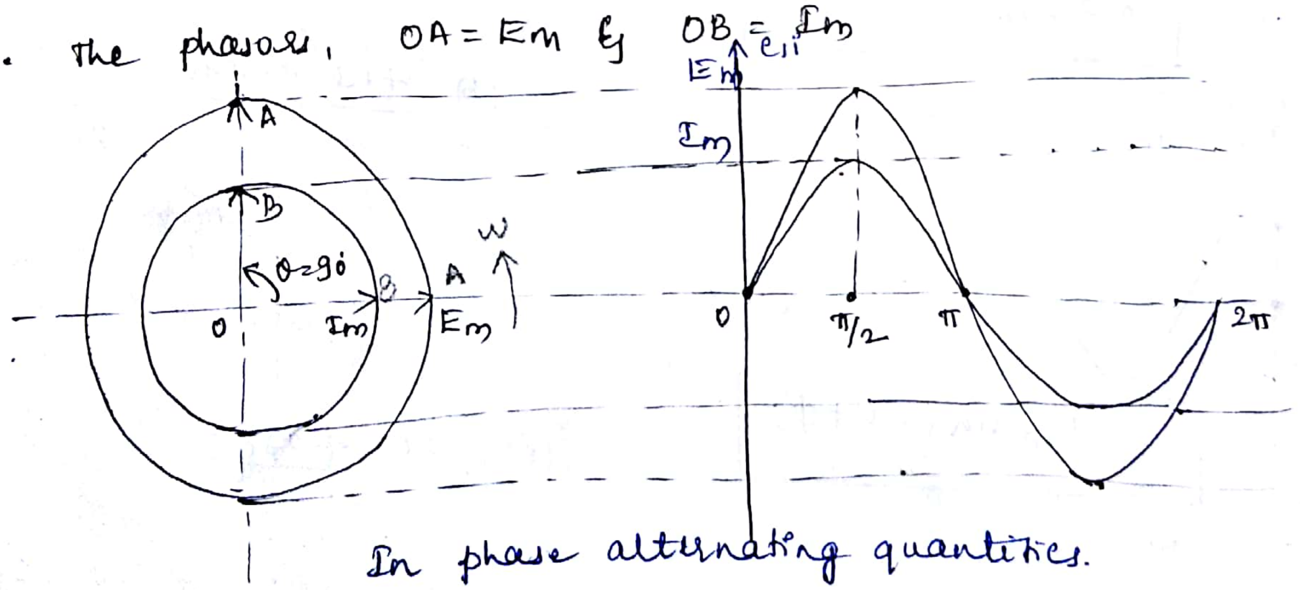
PHASE DIFFERENCE:

(i) Zero Phase Difference: Consider the two alternating quantities having same frequency 'f' Hz having diff. max. values.

$$\boxed{e = E_m \sin(\omega t) \quad \& \quad i = I_m \sin(\omega t)}$$

where $E_m > I_m$

- The phasor representation & w/fs of both quantities are as shown.
- The phasors, $OA = E_m$ & $OB = I_m$



In phase alternating quantities.

- After $\theta = \pi/2$ radians, the OA phasor achieves its max. E_m while at the same instant, the OB phasor achieves its max. I_m.
- So at any instant, we can say that the phase of vtg 'e' will be same as phase of 'i'. The diff. b/w the phases of 2 quantities is zero at any instant.

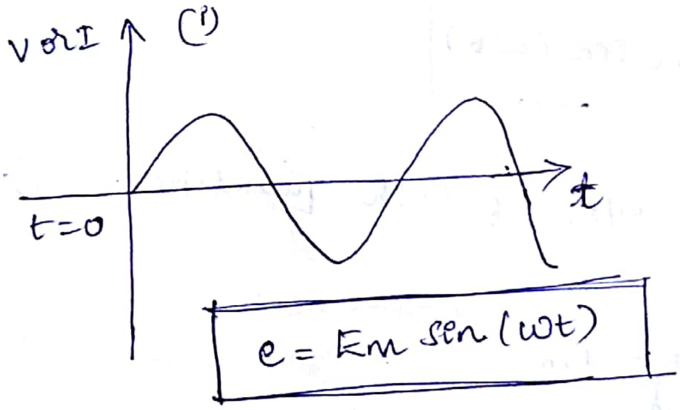
Let us consider 3 cases:

Case (i) : $\phi = 0°$

when phase of an alternating quantity is zero, it is std. pure sinusoidal quantity having instanteneous value zero at $t=0$

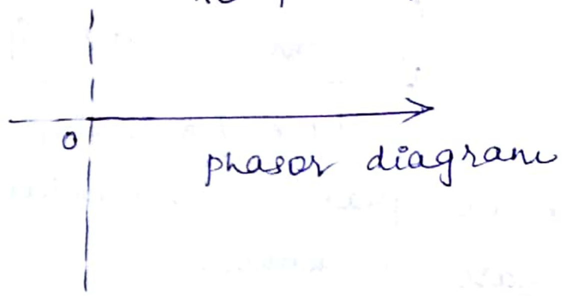
Case (ii): positive phase 'ϕ':

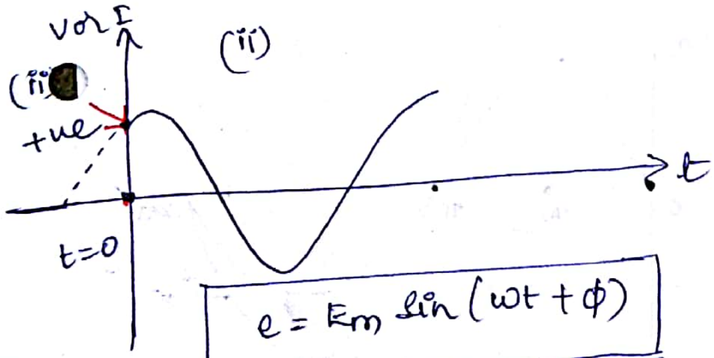
when phase of an alternating quantity is +ve it means that quantity has some +ve instanteneous value at $t=0$.



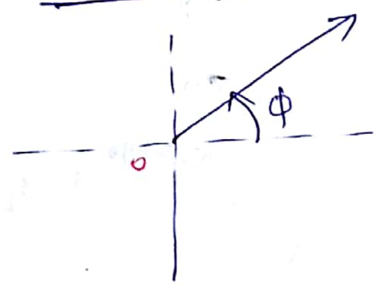
(a) Zero phase:

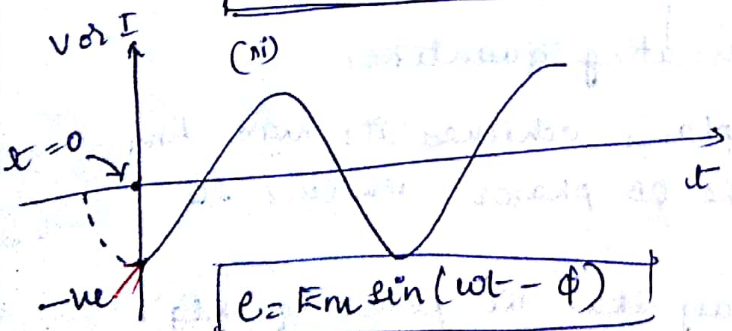
i.e phase $\phi = 0°$.



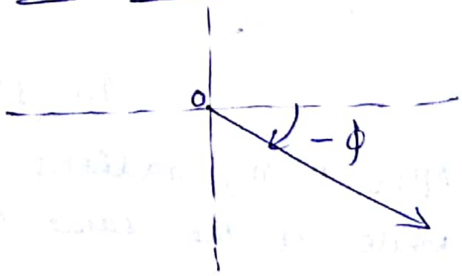


(b) +ve phase





(c) -ve phase:



Case (iii): Negative phase ϕ: when phase of an alternating quantity is -ve, it means the quantity has some -ve instanteneous value at $t=0$.

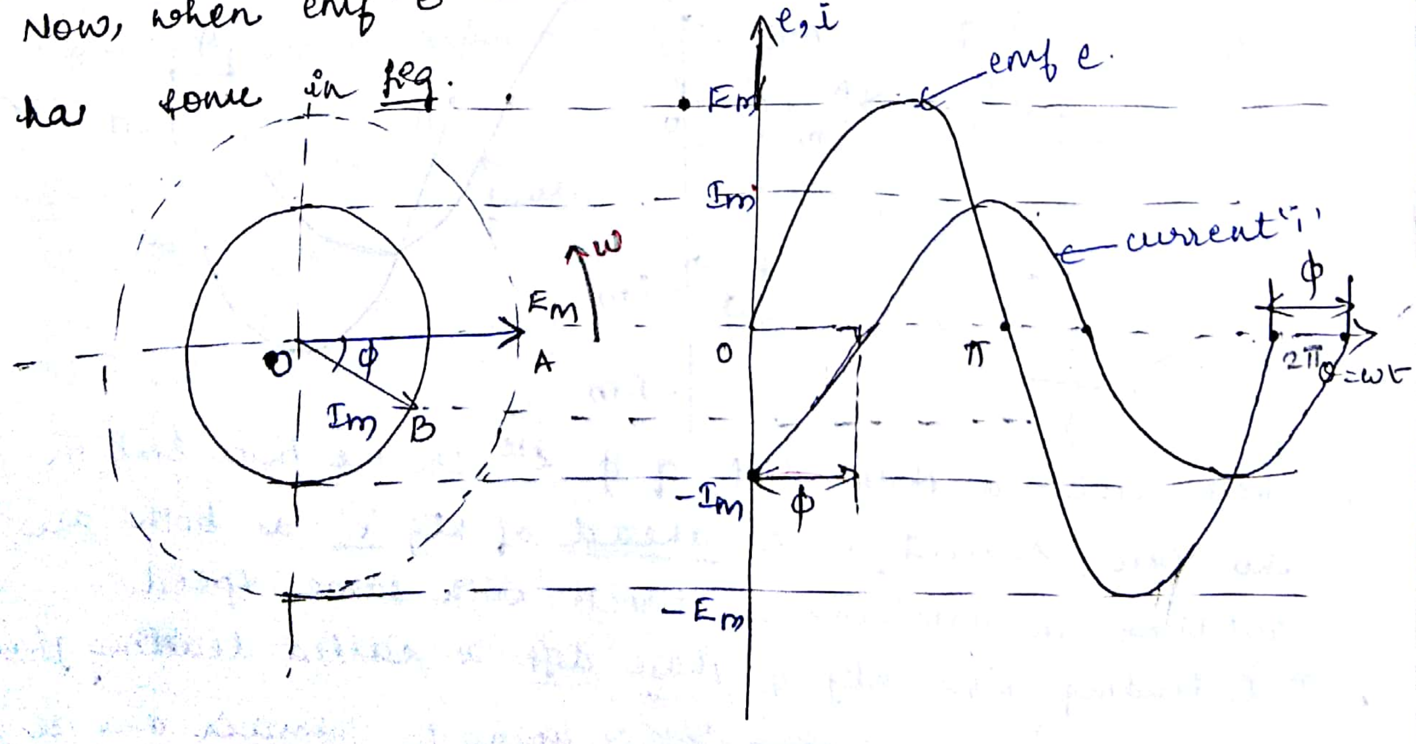
- The difference b/w the phases of the two alternating quantities is called phase difference which is nothing but the angle difference b/w the 2 phasors representing the 2 alternating quantities

** When such phase diff. b/w the 2 alternating quantities is zero, the 2 quantities are said to be in phase.

- In a.c analysis, it is not necessary that all the alternating quantities must be always in phase.
- It is possible that if one is achieving its zero value, at the same instant, the other is having some -ve value or +ve value. Such two quantities are said to have phase diff. b/w them.

(ii) Lagging Phase Difference: Consider an emf having max. value E_m & current having max. value I_m.

- Now, when emf 'e' is at its zero value, the current 'i' has some in fig.



- Thus, there exists a phase difference ϕ b/w the two phasors.
- Now, as the two are rotating in anticlockwise direct^n we can say that <u>current is falling</u> back w.r.t voltage, at all the <u>instants by angle ϕ</u>.
- This is called <u>lagging p. difference</u>.
- The current 'i' is said to lag the vtg 'e' by an <u>angle ϕ</u>
- current 'i' acheive its max. & zero <u>values</u>, ϕ angle later than the corresponding max. & zero values of vtg.
- Eq^{ns} of 2 quantities.

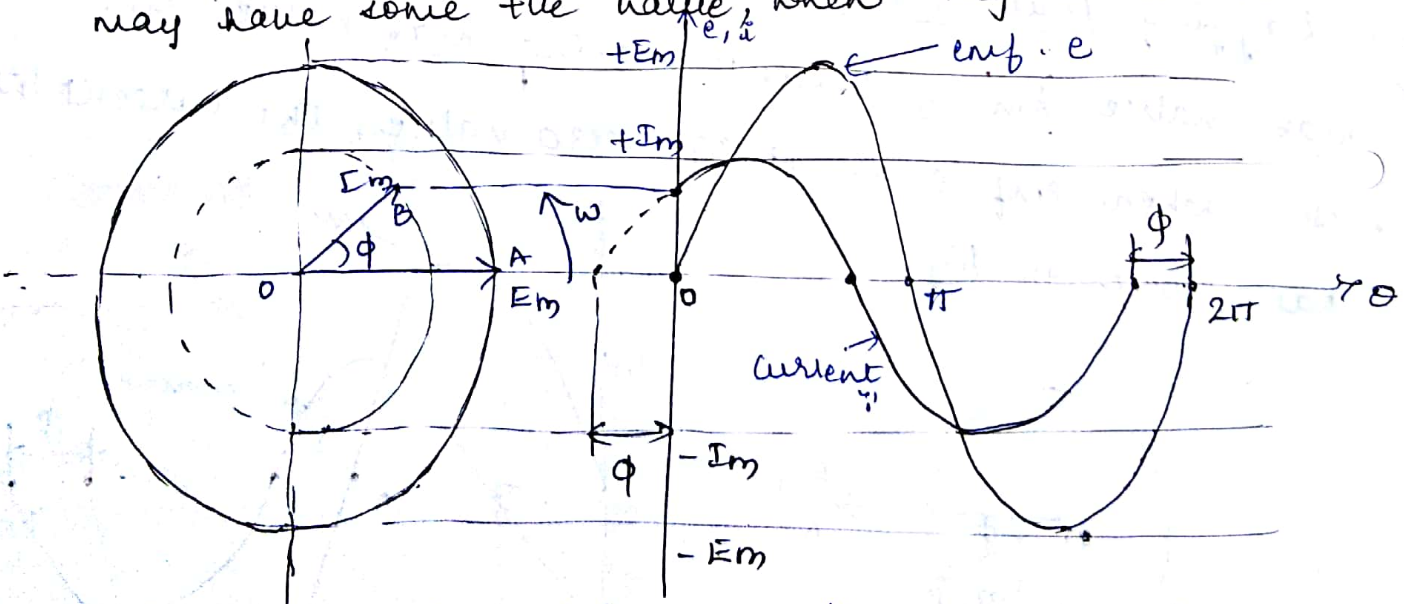
$$\boxed{\text{i.e}\quad e = E_m \sin \omega t \quad \& \quad i = I_m \sin(\omega t - \phi)}$$

* 'i' is said to lag 'e' by angle ϕ. *

(iii) <u>Leading phase difference</u> (<u>$e = E_m \sin \omega t$</u> & $i = I_m \sin(\omega t + \phi)$)

i leads 'e' by angle ϕ

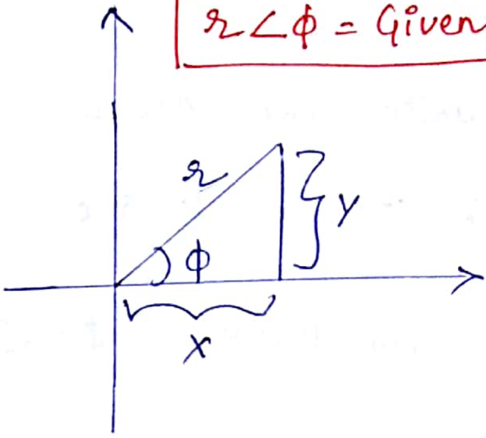
- It is possible in practice that the current 'i' may have some +ve value, when vtg 'e' is zero.



- There exists a phase diff. of ϕ $\angle^{le}$ b/w the two. But in this case, current <u>'i'</u> is <u>ahead</u> of vtg <u>'e'</u> as both are rotating in anticlockwise direct^n with same speed.
- I is leading w.r.t vtg & phase <u>diff.</u> is called leading phase diff.
- At all instants, current 'i' is going to remain ahead of vtg 'e' by angle 'ϕ'.

Polar to Rectangular Conversion:

$r\angle\phi$ = Given



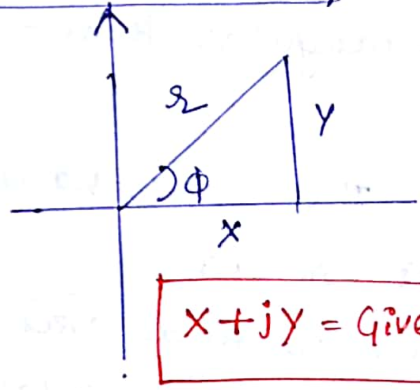
- Let a phasor is represented in polar as shown
- It is necessary to find x & y components in terms of r & φ

X component = $r\cos\phi$

Y component = $r\sin\phi$

$$\boxed{\text{Rectangular representation} = r\cos\phi + jr\sin\phi}$$

Rectangular to Polar Conversion:



$X + jY$ = Given

- Let a phasor is represented in rectangular form $X + jY$.
- It is necessary to find r & φ in terms of X & Y.
- From the fig, $\boxed{r = \sqrt{x^2 + y^2}}$ & $\boxed{\phi = \tan^{-1}\frac{y}{x}}$

$$\boxed{\text{polar representation} = r\angle\phi = \sqrt{x^2+y^2}\ \angle \tan^{-1}\frac{y}{x}}$$

- Polar form always gives rms value of an alternating quantity.

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Problems:

(1) Write the polar form of the vtg given by,

$V = 100 \sin(100\pi t + \pi/6)$ V. obtain its rectangular form.

Soln $V_m = 100$ V , $\phi = +\frac{\pi}{6}$ rad $= +30°$

$V_{rms} = \frac{V_m}{\sqrt{2}} = 70.7106$V

$\therefore$ In polar form $= 70.7106 \angle +30°$V

Rectangular form $= 61.2371 + j35.35$ V

NOTE: The rms value of an alternating quantity exists in its polar form and not in rectangular form.

- Thus to find rms value of an alternating quantity express it in polar form.

(2) Find the rms value & phase of the current.

$I = 25 + j40$A.

Soln RMS value is not 25 or 40 as it exists in rectangular form, converting it to polar form.

$I = 47.16 \angle 57.99 = I_{rms} \angle \phi$ A.

$\therefore$ rms value of current $= 47.16$ A

Phase angle $= 57.99°$

Points regarding phasor diagram:

(1) Phasor diagram can be drawn at any instant. 'X & Y' axis are not included in it.

(2) Generally, the reference phasor chosen is shown along the +ve 'X' axis direction, & at that instant other phasors are shown. (Just convenience).

** The individual phase of an alternating quantity is always referred w.r.to the +ve x axis

(3) There may be more than 2 quantities represented in phasor diagram. (V & I), or flux.. Frequency of all of them must be the same.

(4) Length of the phasor drawn equal to rms value of ac quantity rather than max value.

(5) phasors which are ahead in anticlockwise direction w.r.to reference phasor are leading. phasors behind are said to be lagging.

(6) Arrow heads represent direction.

Addition & subtraction of AC quantity:

(i) Addition & Sub. of phasor → vector addition
vector subtraction

It is possible by analytical method (or)
graphical method

i) $P = x_1 + jy_1$ $\quad Q = x_2 + jy_2$

$R = P + Q$, $= (x_1 + x_2) + j(y_1 + y_2)$

$R = P - Q$, $= (x_1 - x_2) + j(y_1 - y_2)$

$\boxed{\text{Note}}$ while performing addition & subtraction use rectangular form represn of phasors.

Multiplication & Division of phasors:
is performed using polar form represn such that

$+$ & $-$	→ Rectang
$\times$ & $\div$	→ polar form

P & Q be 2 phasors,

$P = x_1 + jy_1$ $\quad | \quad P = r_1 \angle \phi_1$

$Q = x_2 + jy_2$. $\quad | \quad Q = r_2 \angle \phi_2$

$P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2]$

$= [r_1 \times r_2] \angle \phi_1 + \phi_2$

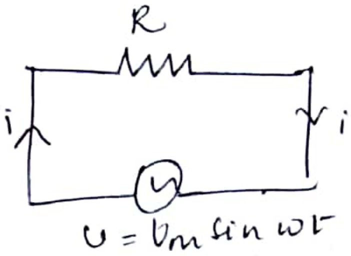
(**) $\times^{tn}$ of complex nos. in polar form, the magnitudes get multiplied while their angles get added.

$$\frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2$$

Division of complex nos in polar form the magnitudes get divided while angles get subtracted.

AC thro pure Resistance:

Consider a simple ckt consisting of pure resist 'R' shown connected across.



$v = V_m \sin \omega t$.

Acc to ohms law, $i = \frac{V}{R} = \frac{V_m \sin \omega t}{R}$

i.e. $\boxed{i = \left(\frac{V_m}{R}\right) \sin \omega t}$ instantaneous value of current.

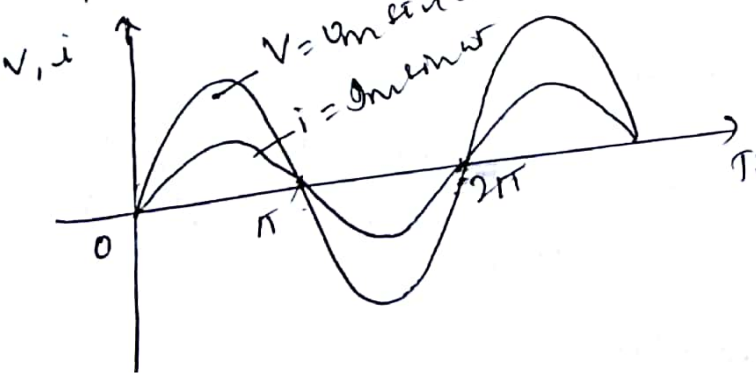
Comparing this with eqn. $i = I_m \sin(\omega t + \phi)$

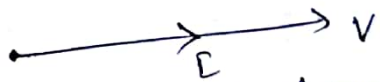
$I_m = \frac{V_m}{R}$ & $\phi = 0$

while $\phi = 0$, it indicates that it is in phase with vlg applied.

* In purely resistive ckt, 'I & V' are in phase with each other.

phasor → rms values of alternating quantities





Both in phase.

Power: The instantaneous power in ac ckts can be obtained by taking the product of inst values of current & vtg.

$$P = v \times i = V_m \sin(\omega t)\, I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t.$$

$$P = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t.$$

$\frac{V_m I_m}{2}$ → Const. power component

$\frac{V_m I_m}{2}\cos 2\omega t$ → Fluctuating power component.
→ having freq. double the frequency of applied vtg.

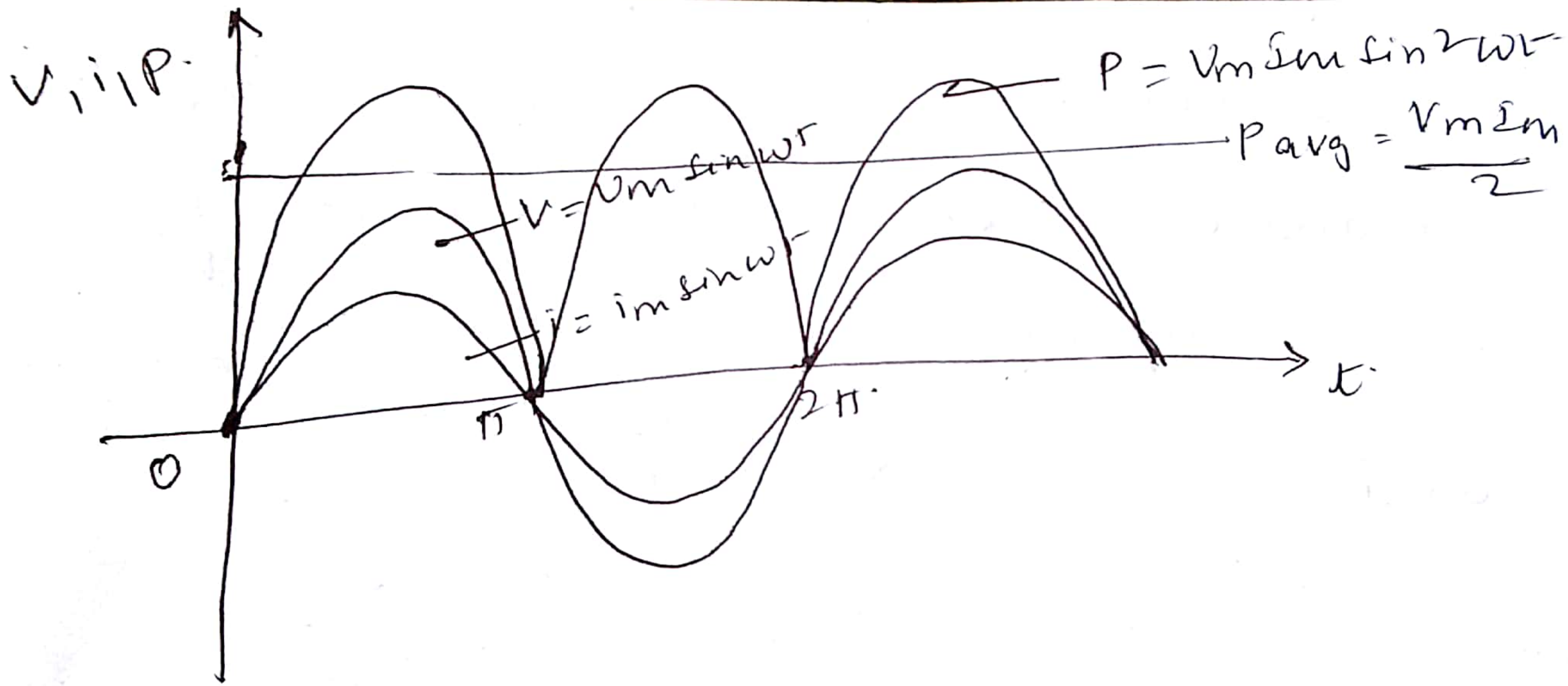
** Avg. value of fluctuating cosine component of double frequency is zero over one complete cycle.

∴ Avg power consumption over one cycle } = Const power component $\frac{V_m I_m}{2}$ = half of peak power $V_m I_m$

∴ $P_{avg} = \frac{V_m I_m}{2} \Rightarrow \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$ i.e $\boxed{P_{av} = V \times I}$ watts

$P = I^2 R$ watts.

RMS value → Capital letters.



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