LAB MANUAL

Discrete and Integral Transforms

(IT: EC, TC, EE, ML, EI, IS)

COURSE CODE: 18MA3GCDIT

2019-20

DEPARTMENT OF MATHEMATICS

DAYANANDA SAGAR COLLEGE OF ENGINEERING

DAYANANDA SAGAR COLLEGE OF ENGINEERING

BANGALORE -560078

VISION

To impart quality technical education with a focus on Research and Innovation emphasizing on Development of Sustainable and Inclusive Technology for the benefit of society.

MISSION

- To provide an environment that enhances creativity and innovation in pursuit of excellence.
- To nurture framework in order to transform individuals as responsible leaders and entrepreneurs.
- To train the students to the changing technical scenarios and make them to understand the importance of sustainable and inclusive technology.

DEPARTMENT OF MATHEMATICS

VISION

To create lifelong competent and socially innovative Engineers capable of working in multicultural environment in the world.

MISSION

- Enhance Mathematical knowledge which will enable them to analyse, interpret and optimize any Engineering related problem.
- Imbibe interest in the convergence of Science and Technology through continuous research activities.
- Provide a platform to develop extraordinary professionals with high ethical values to meet the future age world.

CONTENTS

INTRODUCTION

In the age of computers, doing maths just theoretically is not enough. So to get a feel of Computational Mathematics, the department of Mathematics has introduced lab classes in Microsoft Excel for III semester (IT branches).

A spreadsheet can be considered as a set of columns and rows in a table. Excel is developed by Microsoft for Windows, macOS, Android and iOS to create spreadsheets. Calculation, graphing tools, pivot tables, and a macro programming language called Visual Basic for Applications are various features of Excel. It is used to record the data and analyse. Columns are named using alphabets and numbers are assigned to rows. A cell is the space corresponding to any one particular column and row. A cell is referred by the letter representing the corresponding column and the number representing the corresponding row. Microsoft Excel can be used instead of a calculator, to do the math!

You can enter simple formulas to add, divide, multiply, and subtract two or more numeric values. Or use the AutoSum feature to quickly total a series of values without entering them manually in a formula. Once a formula is created, it can be copied into adjacent.

The main objective of introducing this lab is to understand data by looking into the graph and also try out various options in very less time which otherwise is not feasible to do manually.

In this semester, mean, Regression analysis, Harmonic analysis of Fourier series and Linear programming problems will be covered during lab sessions. HAPPY COMPUTING!!!!

LAB 1: Linear Curve Fitting

THEORY: Given some data, it is very difficult to infer some logical and meaningful result by looking into it as it is. So after data collection, the next step is to express it graphically. Plot of X vs Y is called as a scatter plot.

If the points lie almost in a straight line then we say that X is related to Y linearly. Then we would like to find the equation of the line $y = ax + b$ that describes the data perfectly. The coefficients of the line are computed using the principle of least squares. So, first form normal equations for each constant and then solve the equations to find the coefficients.

Ex. 1 Consider the following data

X	Y
1	7.5
2	44.31
3	60.8
4	148.97
5	225.5
6	262.64
7	289.06
8	451.53
9	439.62
10	698.88

Draw scatter plot and observe whether the X and Y are linearly dependent or not. Find the equation of the straight line that fits best to the given data. Also compute the mean square error. Plot the best fit line.

PROCEDURE 1:

- Go to files, open a new EXCEL workbook
- Enter the data in the workbook as shown in the 1.1.1

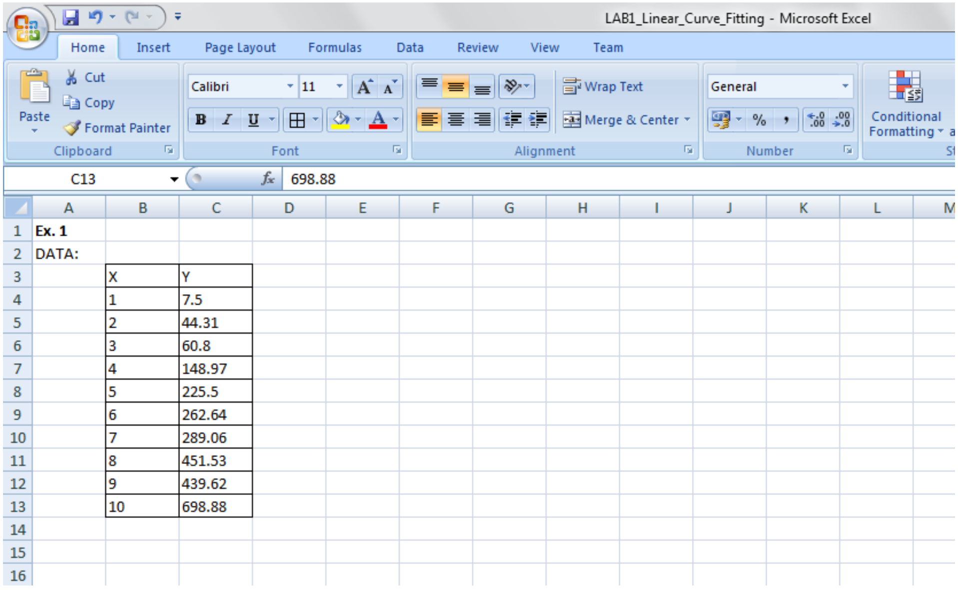


Fig 1.1.1

- Go to insert tab. Then select the data and choose scatter plot which is shown in Fig.1.1.2. One can edit the title

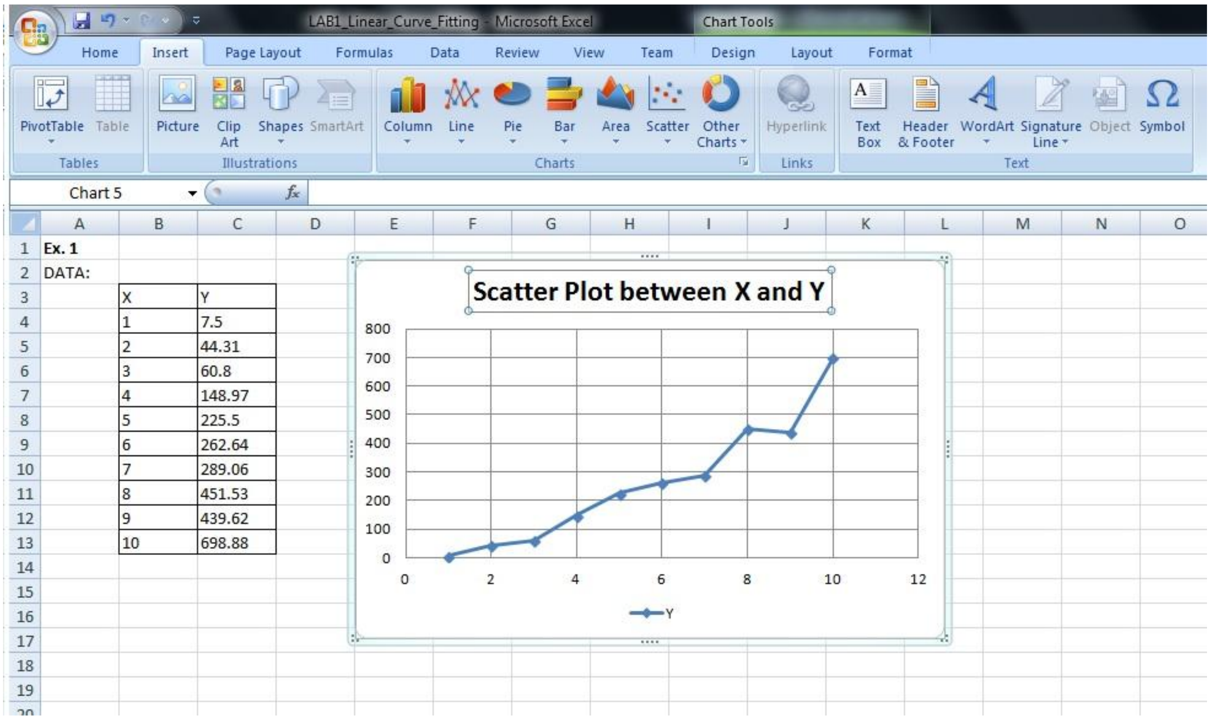


Fig 1.1.2

Observation: The plot of X versus Y almost looks linear

- Compute $\sum_x$ = sum of X_is by typing =SUM(B4:B13) in the cell or by typing =SUM(selecting the cells containing data points of X), press ENTER.
  Note that SUM() is the function used to evaluate the sum of data points
- Compute $\sum_y$ = sum of Y_is by typing =SUM(C4:C13) in the cell, press ENTER.
- Compute $\sum_{xy}$ = sum of product of Y_iX_i s by typing =SUMPRODUCT(B4:B13,C4:C13), press ENTER. Here the SUMPRODUCT() is function used to compute element wise product of arrays or vectors of same dimension and then calculates the sum.
- Compute $\sum_{xx}$ = sum of X_i^2 s by typing =SUMPRODUCT(B4:B13,B4:B13), press ENTER

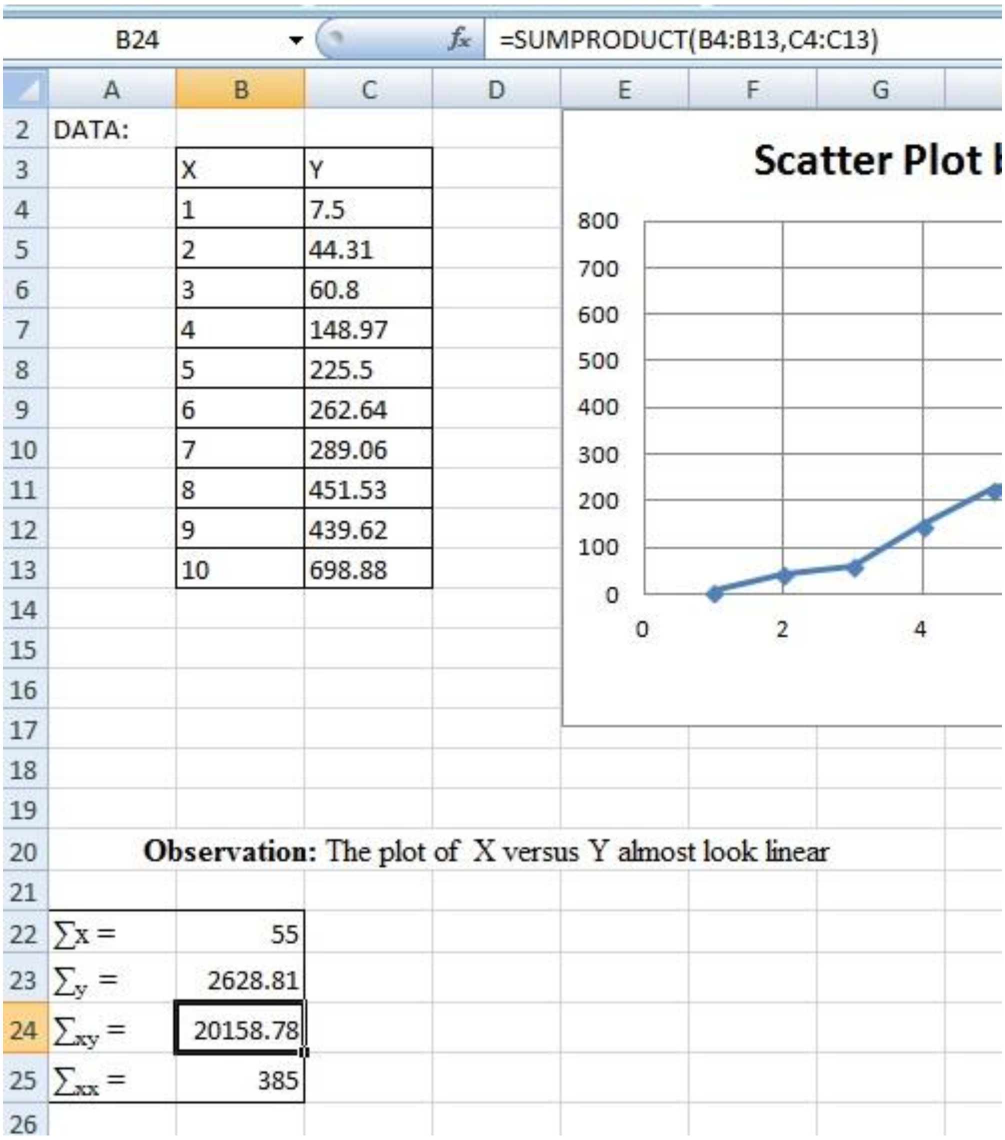


Fig 1.1.3

- The two normal equations are as follows:
  . $\sum_y = a \sum_x + n\, b$
  $\sum_{xy} = a \sum_{xx} + b \sum_x$

This can be written in the matrix form as

$$\begin{bmatrix} \Sigma_x & n \\ \Sigma_{xx} & \Sigma_x \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \Sigma_y \\ \Sigma_{xy} \end{bmatrix}$$

- Type normal equation in matrix form in the excel

Coeff	**Matrix**	**Variable**	**Value**
55	**10**	**A**	**2628.81**
385	**55**	**b**	**20158.78**

- Value of a and b are obtained by multiplying inv(coeff) to value matrix.
  So, the inverse of the matrix is calculated by the given formula

- First select the cells where the inverse has to be entered.

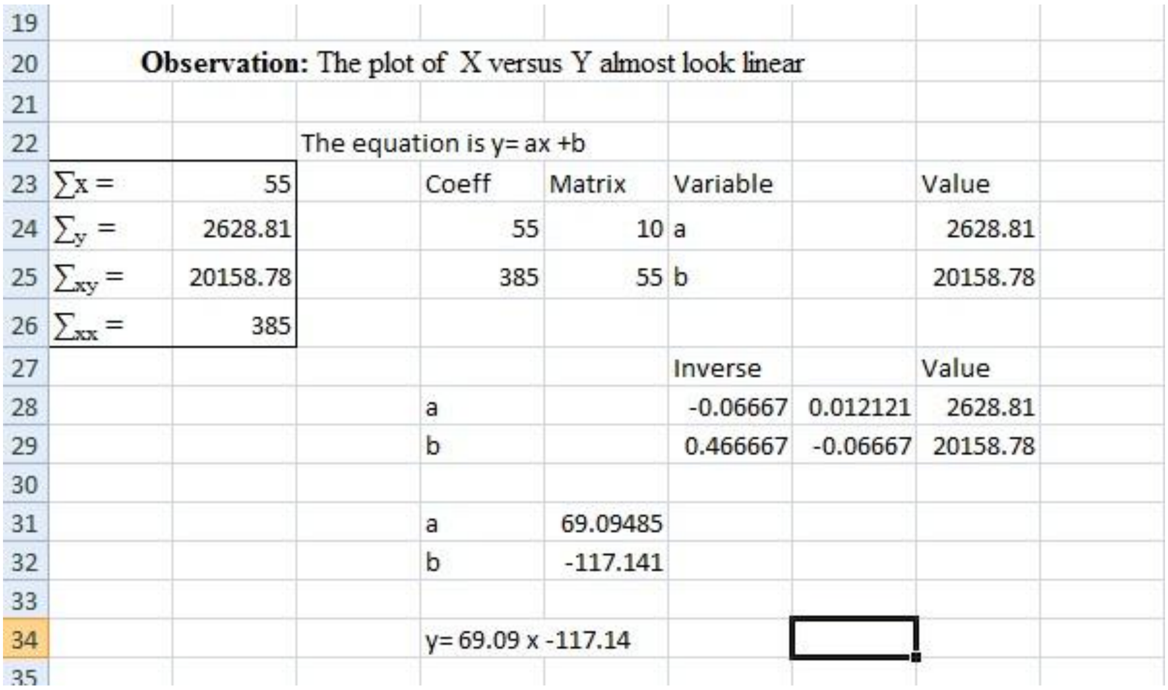
- Then type in formula bar =MINVERSE(select the matrix whose inverse has to be computed) Then press CTRL+ SHIFT+ENTER. Clearly, MINVERSE() is the function used to compute inverse of a matrix.

- Therefore by multiplying inverse matrix by value vector, we get a, b

- First select the cells where the value of a and b has to be evaluated

- Then type in formula bar =MMULT(select the inverse, select the value matrix) Then press CTRL+SHIFT+ENTER. For matrix multiplication MMULT() is used.

- Thus we get ***y=69.09x – 117.14***



19								
20	**Observation:** The plot of X versus Y almost look linear							
21								
22			The equation is y= ax +b					
23	$\sum$x =	55		Coeff	Matrix	Variable		Value
24	$\sum_y$ =	2628.81		55	10	a		2628.81
25	$\sum_{xy}$ =	20158.78		385	55	b		20158.78
26	$\sum_{xx}$ =	385						
27						Inverse		Value
28				a		-0.06667	0.012121	2628.81
29				b		0.466667	-0.06667	20158.78
30								
31				a	69.09485			
32				b	-117.141			
33								
34				y= 69.09 x -117.14				
35								

Fig1.1.4

- To evaluate the error first we will copy the data as it is once again. Then in next column we will compute y' using the formula y= 69.09x -117.14. Then in next column we can find y-y' and then $(y\text{-}y')^2$.

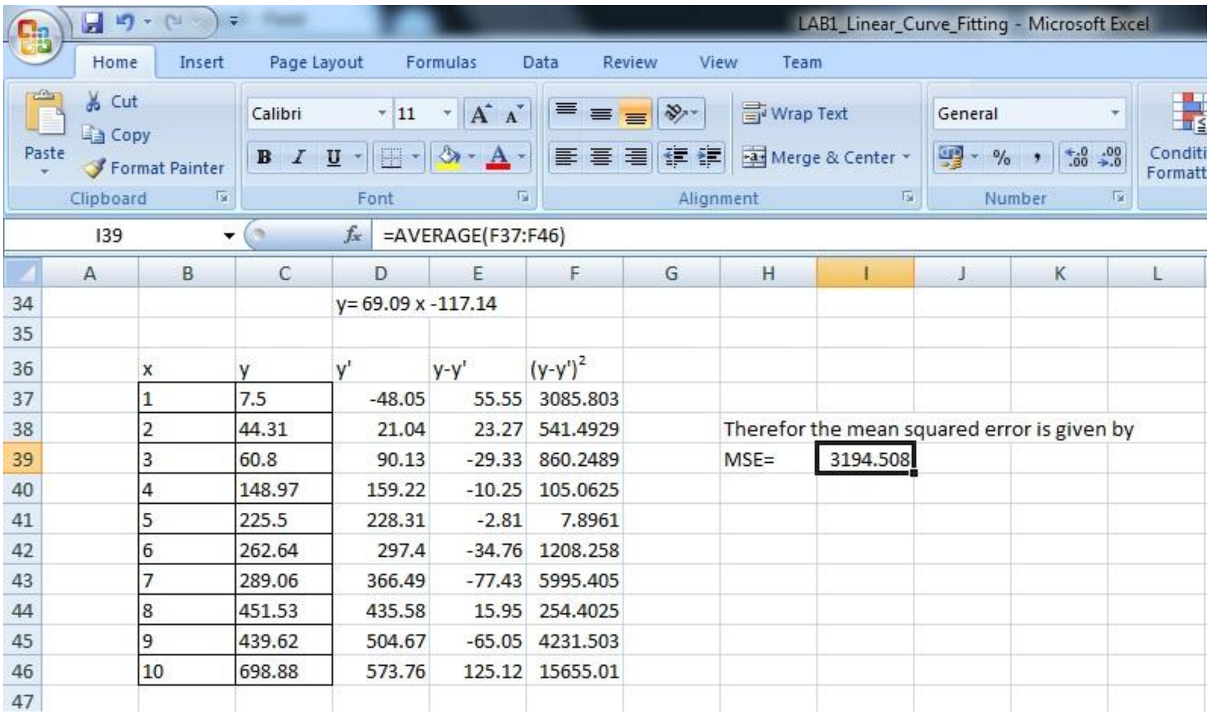
- In the column of y', select the cells and in formula bar type =69.09*(select the cells under x)-117.14, press CTRL+SHIFT+ENTER.

- In the column of y-y', select the destination cells and in formula bar type =(select the cells under y)-(select the cells under y'), press CTRL+SHIFT+ENTER.

- In the column of $(y\text{-}y')^2$, select the destination cells and in formula bar type =(select the cells under $(y\text{-}y')^2$)^2, press CTRL+SHIFT+ENTER.

- Then the mean squared error is given finding the average of the last column.

- MSE = type in the formula bar =AVERAGE(select the cells under $(y\text{-}y')^2$), press ENTER. As it can be observed that function AVERAGE() is used to find average of data points.



x	y	y'	y-y'	(y-y')²
1	7.5	-48.05	55.55	3085.803
2	44.31	21.04	23.27	541.4929
3	60.8	90.13	-29.33	860.2489
4	148.97	159.22	-10.25	105.0625
5	225.5	228.31	-2.81	7.8961
6	262.64	297.4	-34.76	1208.258
7	289.06	366.49	-77.43	5995.405
8	451.53	435.58	15.95	254.4025
9	439.62	504.67	-65.05	4231.503
10	698.88	573.76	125.12	15655.01

Fig 1.1.5

- Select cells under x and y. Then in insert tab choose scatter plot and in scatter plot option, choose layout with best fit line as shown in the Fig 1.1.6. Later we can edit the X axis and Y axis.

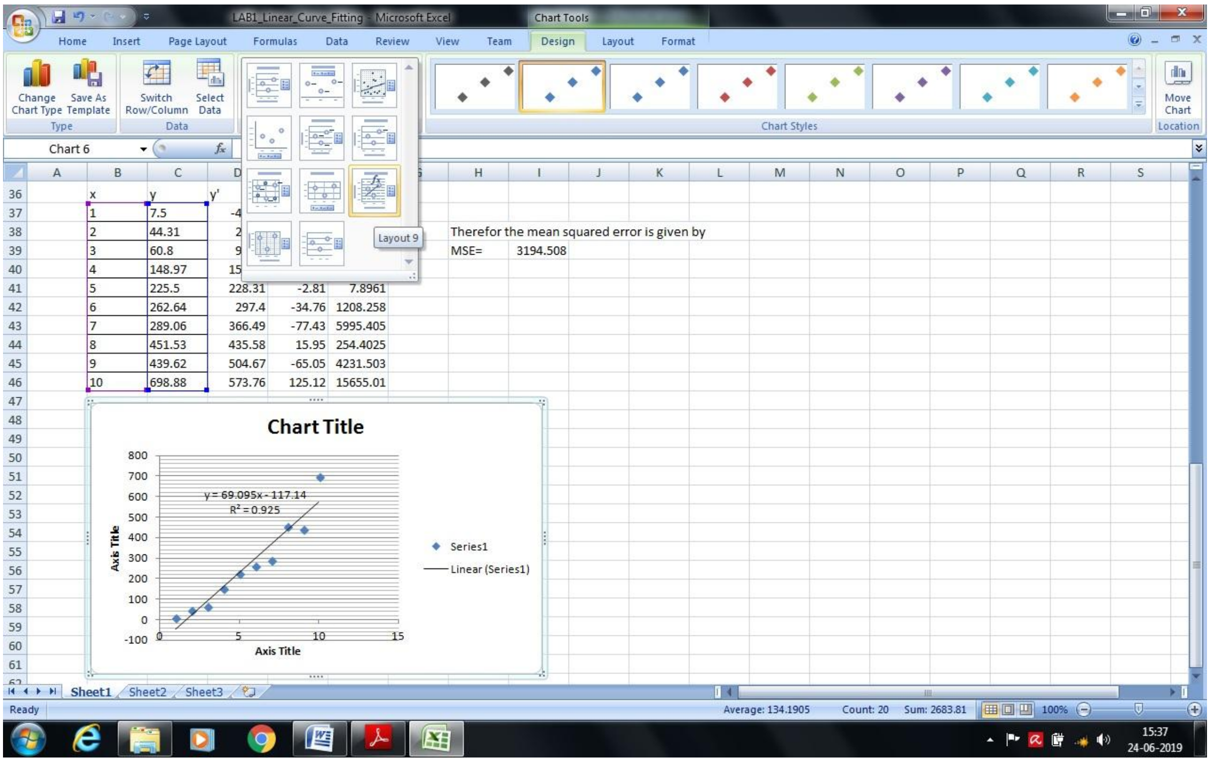


Fig 1.1.6

The final chart will look like as in Fig 1.1.7

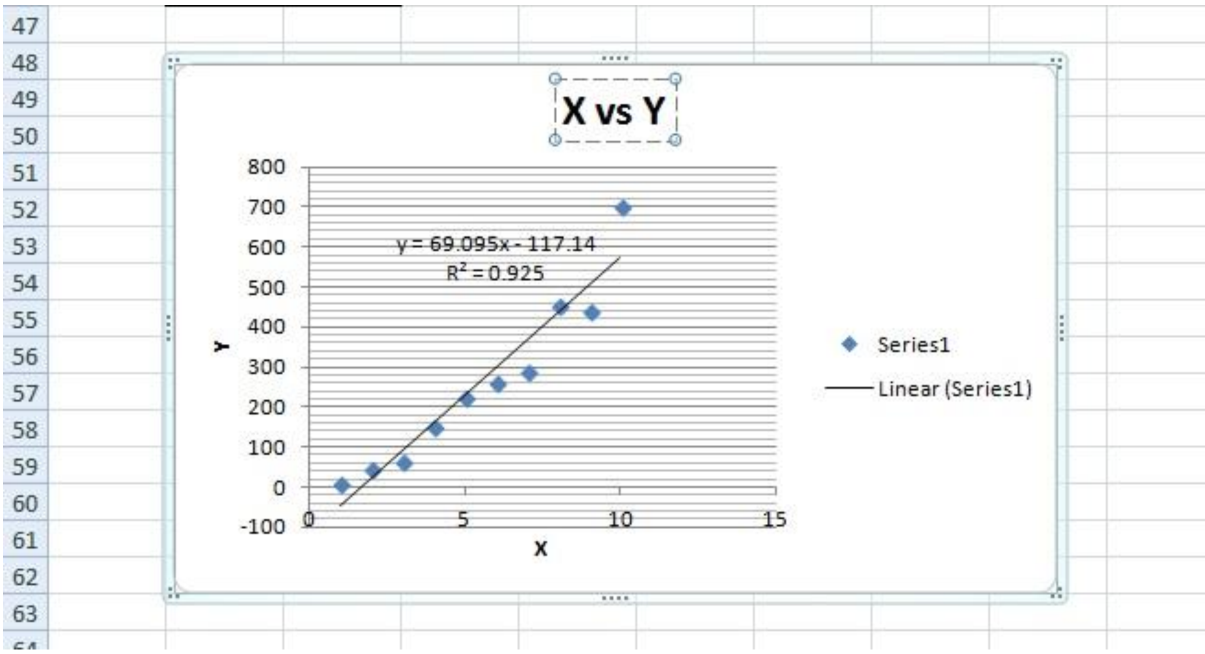


Fig 1.1.7

This completes the problem.

Ex. 2: The weights of a calf taken at weekly intervals are given below

Age (in weeks)	Weight (in Kg)
1	52.5
2	58.7
3	65.0
4	70.2
5	75.4
6	81.1
7	87.2
8	95.5
9	102.2
10	108.4

Plot the data and see whether the data is linear. Find a straight line of best fit and the mean squared error. Estimate the weight of the calf in the 11^{th} week.

Ex. 3: The latent heat of vaporisation of steam r, is given in the following table at different temperatures t:

t	R
40	1069.1
50	1063.6
60	1058.2
70	1052.7
80	-
90	1041.8
100	1036.3
110	1030.8

Plot the data and see whether the data is linear. Find the equation of the line of best fit and the mean squared error. Also estimate the missing data.

LAB 2: Non Linear Curve Fitting

THEORY: As we know that given the values of two parameters at different time points, finding relation between them is an important task. The relation need not be always linear. So here we look into some types of non linear curves and the normal equations using the principle of least squares.

Fitting a parabola: Let $y = ax^2 + bx + c$ be a parabola. To fit a parabola, the normal equations are

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$

Fitting a exponential curve: Let $y = ae^{bx}$ be a exponential curve. To fit it, the normal equations are as follows:

Let $Y = \log_{10} y$, $A = \log_{10} a$, $B = b \log_{10} e$. Then

$$\Sigma Y = nA + B\Sigma x$$

$$\Sigma xY = A\Sigma x + B\Sigma x^2$$

Fitting a logarithmic curve: Let $y = ax^b$ be a logarithmic curve. To fit it, the normal equations are as follows:

Let $Y = \log_{10} y$, $A = \log_{10} a$, $X = \log_{10} x$. Then

$$\Sigma Y = nA + b\Sigma x$$

$$\Sigma XY = A\Sigma X + b\Sigma X^2$$

By solving the normal equations, the coefficients are obtained.

One can compute R square value. It is a measure of goodness of fit. If R square value is closer to 1, it implies fit is very good whereas if the value is closer to 0, it implies the fit is not good.

Ex. 1: Fit a parabola $y = ax^2+bx+c$ in least square sense to the data. Does a better relation exists?

x	10	12	15	23	20
y	14	17	23	25	21

PROCEDURE:

- Go to files, open a new EXCEL workbook

- Enter the data in the workbook

- Compute $\sum x$ = sum of X_is by typing =SUM(select all the cells containing x values), press ENTER.
- Compute $\sum x^2$ = sum of X_i^2 s by typing =SUMPRODUCT(select all the cells containing x values, select all the cells containing x values), press ENTER
- Compute $\sum x^3$ = sum of X_i^3 s by typing =SUMPRODUCT(select all the cells containing x values, select all the cells containing x values, select all the cells containing x values), press ENTER
- Compute $\sum x^4$ = sum of X_i^4 s by typing =SUMPRODUCT(select all the cells containing x values, select all the cells containing x values, select all the cells containing x values, select all the cells containing x values), press ENTER
- Compute $\sum y$ = sum of Y_is by typing =SUM(select all the cells containing y values), press ENTER.
- Similarly compute $\sum xy$, $\sum x^2y$

- The two normal equations are as follows:

.

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$

This can be written in the matrix form as

$$\begin{bmatrix} \Sigma x^4 & \Sigma x^3 & \Sigma x^2 \\ \Sigma x^3 & \Sigma x^2 & \Sigma x \\ \Sigma x^2 & \Sigma x & n \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \Sigma x^2 y \\ \Sigma xy \\ \Sigma y \end{bmatrix}$$

- Type normal equation in matrix form in the excel

- Value of a, b, c are obtained by multiplying inv(coeff) to value matrix.
  So, the inverse of the matrix is calculated by the given formula

- First select the cells where the inverse has to be entered.

- Then type in formula bar =MINVERSE(select the matrix whose inverse has to be computed) Then press CTRL+ SHIFT+ENTER

- Therefore by multiplying inverse matrix by value vector, we get a, b, c.

- First select the cells where the value of a, b and c has to be evaluated

- Then type in formula bar =MMULT(select the inverse, select the value matrix) Then press CTRL+SHIFT+ENTER

- Therefore the required equation is y= -0.0695 x^2+3.0099 x-8.7279 (See Fig 2.1.1)

	A	B	C	D	E	F	G	H	I	J
1	**Ex. 1**									
2	DATA:	X	Y							
3		10	14							
4		12	17							
5		15	23							
6		23	25							
7		20	21							
8										
9		∑x=	80		n=	5				
10		$\sum x^2$=	1398		∑y=	100				
11		$\sum x^3$=	26270		∑xy=	1684				
12		$\sum x^4$=	521202		$\sum x^2y$=	30648				
13										
14		COEF	FICIENT	MATRIX	VARIABLE		VALUE			
15		1398	80	5	a		100			
16		26270	1398	80	b		1684			
17		521202	26270	1398	c		30648			
18										
19		INV	COEFF	MATRIX	VALUE		VARIABLE			
20		0.1933	-0.0256	0.0008	100		a=	-0.0695		
21		-6.5276	0.8557	-0.0256	1684		b=	3.0099		
22		50.5950	-6.5276	0.1933	30648		c=	-8.7279		
23										
24		Therefore the required equation is y= -0.0695 x^2+3.0099 x-8.7279								

Fig2.1.1

- Select cells under x and y. Then in insert tab choose scatter plot.
- Select any data point in the scatter plot and right click then in the pop up window choose Add Trendline as shown in the Fig 2.1.2

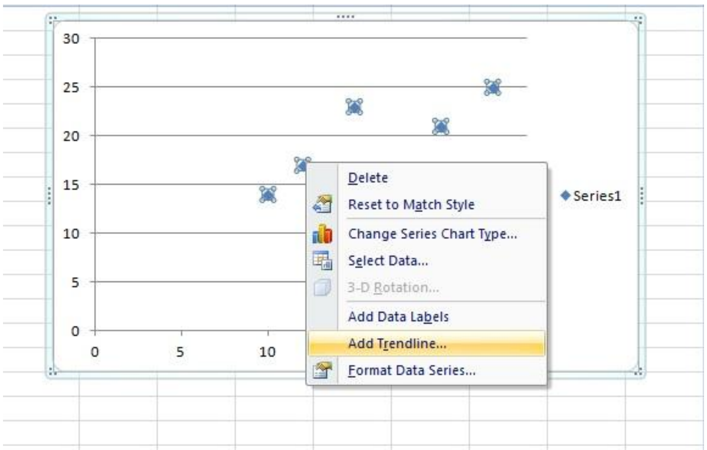


Fig 2.1.2

- A new pop up window comes up. In that first choose exponential. Also tick in 'Display equation on chart' and 'Display R- squared value on chart'. Then select line color.

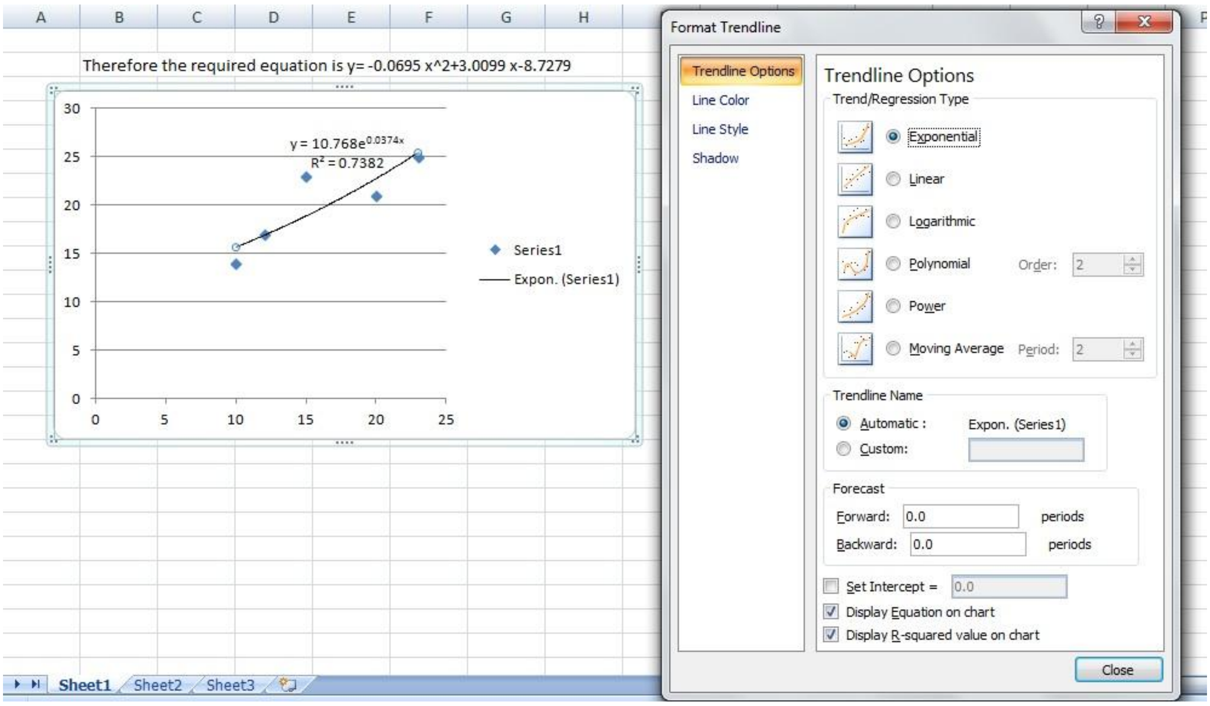


Fig 2.1.3

- In Line Color, choose solid line. In drop down menu for color, choose blue color for the trendline.

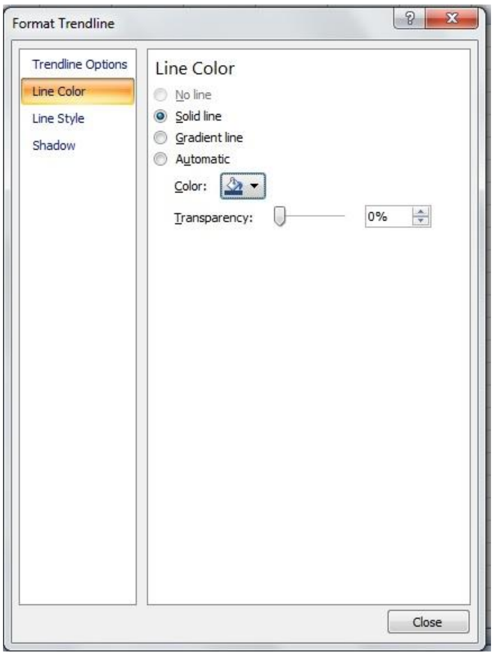


Fig 2.1.4

- Drag the equation to the side.

- Then again click any data point and right click. Choose ‘add trendline'. In Format trendline, select linear trendline. Again tick in ‘Display equation on chart’ and

'Display R- squared value on chart'. Then select line color. Choose black color and close. Drag the equation to the side.

- Repeat the same and plot logarithmic trendline. Choose green color and drag the equation to the side.
- Similarly, select polynomial trendline, choose degree 2 and plot. Then select color red for it and adjust the equation placement. This is nothing but the parabolic fit. So, compare the equation in the graph and the one obtained using normal equations. It should be the same.
- Repeat the same and plot power law trendline. Choose purple color and drag the equation to the side.
- Compare the R square value of each fit to test goodness of fit. In this case, both parabolic and power law explains the data more than other fits as R square is more.

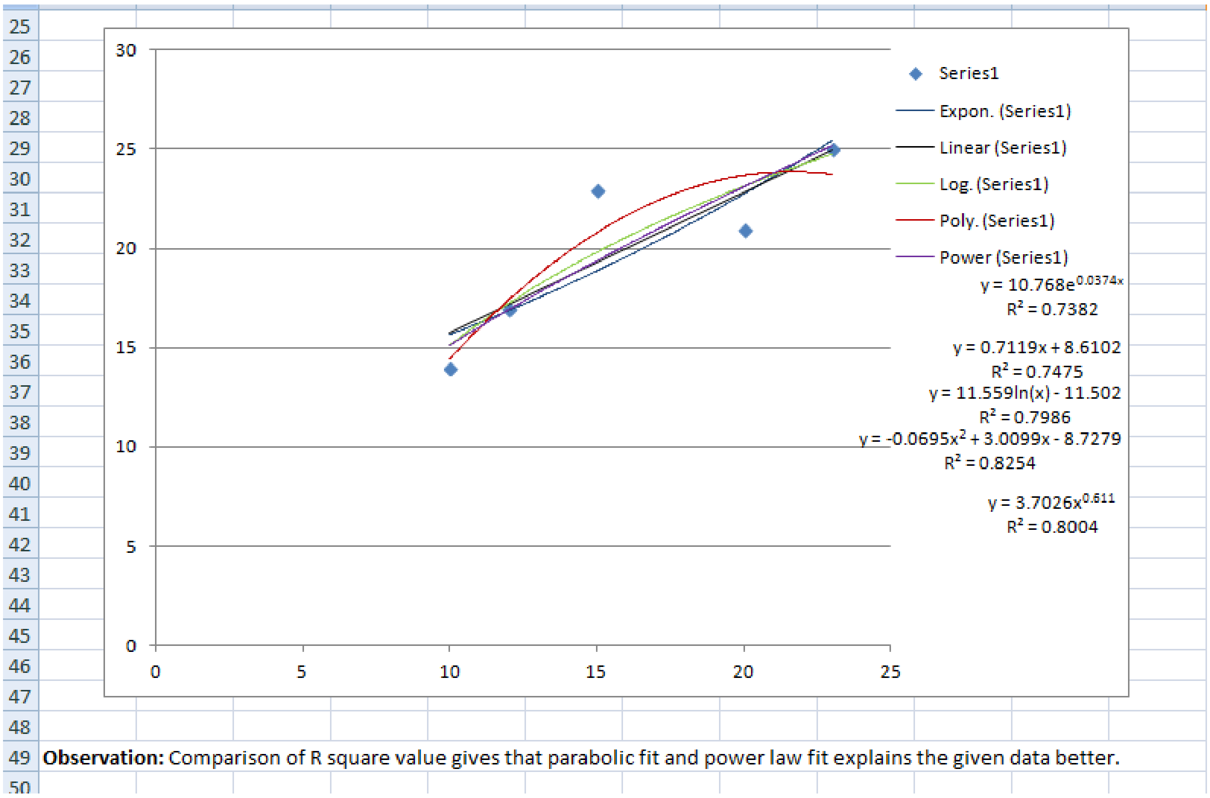


Fig 2.1.5

Ex. 2: The following table gives the result of the measurements of train resistances. V is the velocity in miles per hour. R is the resistance in pounds per ton.

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

Find a, b, c if R is related to V by the relation $R= aV^2+bV+c$. Does a better relation exists ?

Ex. 3: The voltage V across the capacitor at time t seconds is given by the following table. Use the principle of least squares to fit a curve of the form $V= ae^{bt}$.

t	0	2	4	6	8
V	150	63	28	12	5.6

Check if it is the best form of curve**.**

Ex. 4: The pressure and volume of a gas are related by the equation $pv^g = k$, g and k being constants. Fit this equation to the following set of observations:

p (kg/cm^2)	0.5	1.0	1.5	2.0	2.5	3.0
v (litres)	1.62	1.00	0.75	0.62	0.52	0.46

Check if it is the best form of curve.

LAB 3: Linear Regression

THEORY: If the points in the scatter plot lie almost in a straight line then we say that X is related to Y linearly. This linear interdependence between the two variables or data sets is quantified by the measure called as correlation coefficient. The value lies between -1 and 1. If the absolute value of correlation coefficient is closer to 1 implies that X and Y are linearly dependent, whereas correlation close to zero implies existence of hardly any linear dependence of X and Y. In such cases, the points will not be in a straight line in the scatter plot. The sign signifies the direction of linear dependence.

So, once if it is established that there is some kind of linear dependence between X and Y, then we would like to find the equation of the line that describes the data perfectly. This best fit line is called the line of regression. The line that gives the best possible value of Y in terms of X is called as the line of regression of Y on X. Similarly, the line that gives the best possible value of X in terms of Y is called as the line of regression of X on Y. The coefficients of the lines of regression are computed using the principle of least squares.

Consider the line of regression of *y* on *x*. Let the equation satisfied by the data points be of the form

$$y=a + bx.$$

By the principle of least squares, we get the normal equations as follows

$$\sum_y = a\,n + b\sum_x$$
$$\sum_{xy} = a\sum_x + b\sum_{xx}$$

This can be written in the matrix form as

$$\begin{bmatrix} n & \Sigma_x \\ \Sigma_x & \Sigma_{xx} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \Sigma_y \\ \Sigma_{xy} \end{bmatrix}.$$

Clearly, we can find *a* and *b* by solving the system of equations. The *b* is called the regression coeffieicnt of *y* on *x*. The correlation coefficient *r* is given by

$$r = \frac{\sum \mathrm{xy}}{\sigma_x \sigma_y}$$

where σ_x, σ_y are standard deviation of *x* and *y* respectively.

Note that the regression coefficient is nothing but the slope of the line and *a* is the intercept. Once the regression line of *y* on *x* is obtained, we can predict values of *y* by substituting values of x in the equation. The sum of square of difference of actual value and the predicted value is called the RSS and the TSS is nothing but the sum of square of deviation y from its mean value. RSS gives a measure of dispersion of data points around the regression line. The difference between TSS and RSS gives the sum of explained variation.

Ex. 1: The data set contains marketing budget and sales of certain company for past 17 years. Build a simple regression model to understand how the marketing budget impacts on sales. Plot the scatter plot and regression equation of y on x, correlation coefficient, TSS, RSS without using data analysis tools as well using data analysis tools.

Marketing Budget (X) (In lakhs)	Actual Sales(Y) (In crores)
127.4	10.1
364.4	21.4
150	10
128.7	9.6
285.9	17.4
200	12.5
303.3	20
315.7	21
169.8	14.7
104.9	10.1
297.7	21.5
256.4	16.6
249.1	17.1
323.1	20.7
223	15.5
235	13.5
200	12.5

Solution:

- Go to files, open a new EXCEL workbook
- Enter the data in the workbook
- Select data under x and y, select tab insert, choose scatter plot.
- Let the regression equation be given by $y = a + bx$
- So, a = type =INTERCEPT(select data in y column, select data in x column), press ENTER. As the word suggest, INTERCEPT() computes the y intercept of the line corresponding to the data
- b= type =SLOPE(select data in y column, select data in x column), press ENTER. Therefore the equation of regression line is given by
  $$y=0.053x+3.229$$
  Similarly, SLOPE() evaluates the slope.
- Correlation coefficient r= type =correl(select data in y column, select data in x column), press ENTER.
- Avg(y)=type =AVERAGE(select data in y column), press ENTER

- Compute predicted value of y= y' using the regression equation
- Compute the square of residual (y-y')^2. This can be done by typing =(select 1st cell under y-select 1st cell under y')^2,press ENTER. Then select that cell and drag till the last cell of that column, press ENTER.

- Compute the square of deviation from the mean. This can be done by typing =((select cells under y)-select cell containing avg(y))^2,press ENTER.

- Compute RSS by typing=SUM(cells under residual squares), press ENTER.

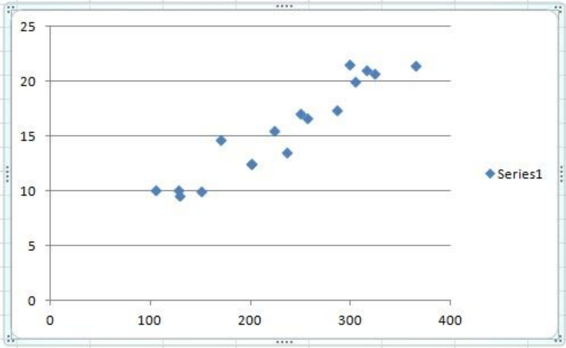
- Compute TSS by typing=SUM(cells under square of deviation from the mean), press ENTER

Ex. 1:

Data: X: Marketing Budget in Lakhs

Y: Actual Sales in crores

X	Y	y'	Residual^2	MD^2
127.4	10.1	10.00666	0.00871172	29.6064
364.4	21.4	22.61469	1.47546812	34.32581
150	10	11.20895	1.46155441	30.70464
128.7	9.6	10.07582	0.22640595	35.29758
285.9	17.4	18.43861	1.07871627	3.455225
200	12.5	13.86887	1.87380111	9.248754
303.3	20	19.36427	0.40415881	19.88111
315.7	21	20.02393	0.95272138	29.79875
169.8	14.7	12.26228	5.94249672	0.707578
104.9	10.1	8.809699	1.6648767	29.6064
297.7	21.5	19.06635	5.92263285	35.50758
256.4	16.6	16.86926	0.07250059	1.121107
249.1	17.1	16.48091	0.38327134	2.429931
323.1	20.7	20.41759	0.07975325	26.61346
223	15.5	15.09243	0.16611154	0.001696
235	13.5	15.73081	4.97652746	4.166401
200	12.5	13.86887	1.87380111	9.248754



Let the equation of regression line be y= a + b x

a=	3.229185
b=	0.053198

Therefore the equation of regression line is y = 3.2292 + 0.0532 x

r=	0.951489
Avg(y)	15.54118
RSS=	28.56351
TSS=	301.7212

Fig 3.1.1

We can solve this problem using method of least squares.

- Enter the data.
- Compute Avg(x) by typing average(select all cells under x), press ENTER.
- Similarly, compute Avg(y).
- Compute deviation of x from its mean X-Xbar by typing=(select the 1st cell under x)-type alphabet of the column containing average of x followed by the $ and the row no, press ENTER. Then drag the entire column.

- Compute deviation of y from its mean Y-Ybar.

- Compute (X-Xbar)^2 by typing =(select a cell under X-Xbar)^2, press ENTER. Then drag. In same way compute (Y-Ybar)^2

- Compute (X-Xbar)(Y-Ybar) by typing = select 1st cell under X-Xbar followed by a * and then select the 1st cell under Y-Ybar, press ENTER. Then drag.

- Compute sum of (X-Xbar)^2, (Y-Ybar)^2, (X-Xbar)(Y-Ybar)

- Compute b by typing = (sum of (X-Xbar)(Y-Ybar))/(sum of (X-Xbar)^2)

- Compute a by typing = -b*(select cell containing average(x))+(select cell containing average(y)), press ENTER.

- Therefore the equation of regression line is y = 3.2292 + 0.0532x

- Compute correlation coefficient r by typing =(select cell containing (sum of (X-Xbar)(Y-Ybar))/sqrt((select cell containing (sum of(X-Xbar)^2)* (select cell containing (sum of(Y-Ybar)^2)), press ENTER.

	A	B	C	D	E	F	G	H	I	J	K
27											
28		X	Y	X-Xbar	Y-Ybar	(X-Xbar)^2	(Y-Ybar)^2	(X-Xbar)(Y-Ybar)		∑(X-Xbar)^2=	96519.7
29		127.4	10.1	-104.035	-5.4411765	10823.342	29.6064	566.0743945		∑(Y-Ybar)^2=	301.7212
30		364.4	21.4	132.9647	5.85882353	17679.613	34.32581	779.0167474		∑(X-Xbar)(Y-Ybar)=	5134.695
31		150	10	-81.4353	-5.5411765	6631.7071	30.70464	451.2473356			
32		128.7	9.6	-102.735	-5.9411765	10554.541	35.29758	610.3685121		b=	0.053198
33		285.9	17.4	54.46471	1.85882353	2966.4042	3.455225	101.2402768		a=	3.229185
34		200	12.5	-31.4353	-3.0411765	988.17772	9.248754	95.60027682			
35		303.3	20	71.86471	4.45882353	5164.536	19.88111	320.4320415			
36		315.7	21	84.26471	5.45882353	7100.5407	29.79875	459.9861592		Therefore the equation of regression line is y = 3.2292 + 0.0532x	
37		169.8	14.7	-61.6353	-0.8411765	3798.9095	0.707578	51.84615917			
38		104.9	10.1	-126.535	-5.4411765	16011.181	29.6064	688.5008651		r=	0.951489
39		297.7	21.5	66.26471	5.95882353	4391.0112	35.50758	394.8596886			
40		256.4	16.6	24.96471	1.05882353	623.23654	1.121107	26.43321799			
41		249.1	17.1	17.66471	1.55882353	312.04183	2.429931	27.53615917			
42		323.1	20.7	91.66471	5.15882353	8402.4183	26.61346	472.8820415			
43		223	15.5	-8.43529	-0.0411765	71.154187	0.001696	0.34733564			
44		235	13.5	3.564706	-2.0411765	12.707128	4.166401	-7.276193772			
45		200	12.5	-31.4353	-3.0411765	988.17772	9.248754	95.60027682			
46											
47		Avg(x)=	231.4353								
48		Avg(y)=	15.54118								
49											

Fig 3.1.2

This completes our problem.

Ex. 2: An article in the Journal of Environmental Engineering (1989, Vol. 115(3), pp. 608–619) reported the results of a study on the occurrence of sodium and chloride in surface streams in central Rhode Island. The following data are chloride concentration y (in milligrams per liter) and roadway area in the watershed x (in percentage).

(a) Fit a simple linear regression model with y = green liquor Na2 S concentration and x = production. Find an estimate of σ2 . Draw a scatter diagram of the data and the resulting least squares fitted model.

(b) Find the fitted value of y corresponding to x = 910 and the associated residual.

(c) Find the mean green liquor Na2 S concentration when the production rate is 950 tons per day.

x	y
0.19	4.4
0.15	9.6
0.57	9.7
0.7	10.6
0.67	10.8
0.63	10.9
0.47	11.8
0.7	12.1
0.6	14.3

x	y
0.78	14.7
0.81	15
0.78	14.3
0.69	19.2
1.3	23.1
1.05	27.4
1.06	27.7
1.74	31.8
1.62	39.5

Ex. 3: The coefficient of thermal expansion of steel α is given at discrete values of temperature.Develop a simple linear regression model for the given data between temperature and α

Temperature (T) ^{0}F	Coefficient of Thermal expansion in (α) in/in ^{0}F
80	6.46E-06
60	6.36E-06
40	6.24E-06
20	6.12E-06
0	6.00E-06
-20	5.86E-06
-40	5.72E-06
-60	5.58E-06
-80	5.43E-06
-100	5.28E-06
-120	5.09E-06
-140	4.91E-06
-160	4.72E-06
-180	4.52E-06
-200	4.30E-06
-220	4.08E-06
-240	3.83E-06
-260	3.58E-06
-280	3.33E-06
-300	3.07E-06
-320	2.76E-06
-340	2.45E-06
-340	2.45E-06

LAB 4: Multiple Regression (Part I)

THEORY: Multiple regression is a generalization of linear regression. In this case there will be more than one independent variable and one dependent variable. It is of the form

$$y = a_0 + a_1x_1 + \ a_2x_2 + \cdots a_kx_k + c$$

Where a_i s are called the regression coefficients. In general, the model with k independent variable can be expressed as

$$Y = XA + c$$

Where Y= ($y_{1,}$ y_2......y_n)' is a n x 1 vector of n observations on dependent variable and

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nk} \end{bmatrix}$$

is n x k matrix of n observations of k independent variables, A=[a_1......a_k]' is a k x 1 vector of regression coefficients and c = [$c_{1,}$ c_2, ...c_n] is n x 1 vector of random error. If intercept term is present, take first column of X to be (1,1,…,1)'. To find the regression coefficients, the normal equation is given by

$$X'XA = X'y$$

If it is assumed that rank(X) =k, then $X'X$ is positive definite and inverse exists. Then

$$A = (X'X)^{-1}X'y$$

There are various measures to check the goodness of fit. Multiple R is the correlation coefficient between the actual and predicted values of y in case of multiple regression. It represents the strength of the linear relationship. If it is closer to 1, it implies the relationship is highly linear. It is the square root of r squared. R squared gives the measure of explained variation by the linear relationship so obtained. Adjusted R squared adjusts for the number of terms in the model. Hence it can be considered as a better measure as compared to R squared. Standard error of regression gives an estimate of standard deviation of the error.

ANOVA is a type of statistical testing widely used to statistically analyse data. Here df stands for degree of freedom. SS stands for sum of squares. MS stands for mean square. Now df corresponding to regression is number of independent variable and df corresponding to residual is nothing difference between number of observations and number of independent variable. SS corresponding to regression represents explained variation and is given by sum of squares of deviation of predicted value from the mean value. SS corresponding to the residual represents unexplained variation and is given by sum of squares of difference between the actual and predicted value. The MS is obtained by dividing SS by corresponding df. The F statistic is calculated as the ratio of MS corresponding to regression and MS corresponding to residuals. In the second table, the intercept and regression coefficients are given. Alongwith it, the standard error in calculation of regression coefficients, corresponding t- statistic and confidence limits are given.

Ex. 1: The following contains data on three variables that were collected in an observational study in a semiconductor manufacturing plant. In this plant, the finished semiconductor is wire-bonded to a frame. The variables reported are pull strength (a measure of the amount of force required to break the bond), the wire length, and the height of the die. We would like to find a model relating pull strength to wire length and die height. Fit a multiple regression model.

Observation Number	Pull strength y	Wire Length x1	Die Length x2
1	9.95	2	50
2	24.45	8	110
3	31.75	11	120
4	35	10	550
5	25.02	8	295
6	16.86	4	200
7	14.38	2	375
8	9.6	2	52
9	24.35	9	100
10	27.5	8	300
11	17.08	4	412
12	37	11	400
13	41.95	12	500
14	11.66	2	360
15	21.65	4	205
16	17.89	4	400
17	69	20	600
18	10.3	1	585
19	34.93	10	540
20	46.59	15	250
21	44.88	15	290
22	54.12	16	510
23	56.63	17	590
24	22.13	6	100
25	21.15	5	400

PROCEDURE:

- Go to files, open a new EXCEL workbook
- Enter the data in the workbook.
- To fit a multiple regression model using principle of least squares, define X as with first column as ones, second column as x1 and third column as x2.

- Compute X’ by typing = TRANSPOSE(select all cells of matrix X), press CTRL+SHIFT+ ENTER. Note that TRANSPOSE() is used to get transpose.

- Compute $(X'X)^{-1}$ by typing = MINVERSE((select cells under X’, select cells under X)), press CTRL+SHIFT+ENTER.

- Compute $((X'X)^{-1}X')$ by typing =MMULT(select cells under $(X'X)^{-1}$, select cells under X’),press CTRL+SHIFT+ENTER.

- Now $A = (X'X)^{-1}X'y$. So, compute A by typing = MMULT(select cells under $((X'X)^{-1}X')$, select cells under y), press CTRL+SHIFT+ENTER.

- Therefore y’ = XA. So compute y’ by typing =MMULT(select cells under X, select cells under A), press CTRL+SHIFT+ENTER.

- Compute residual by typing = (select 1st cell of y)-(select 1st cell of y’), press ENTER. Then select, drag and press ENTER. Dragging helps in copying the same formula.

- Compute average of Y by typing =AVERAGE(select cells under Y), press ENTER.

- Compute Multiple R by typing =CORREL(select cells under Y, select cells under y’), press ENTER. Clearly, CORREL() computes the correlation.

- In order to use Data tools, select tab data and click on Data Analysis. A pop up window will appear as shown in the Fig 4.1.1

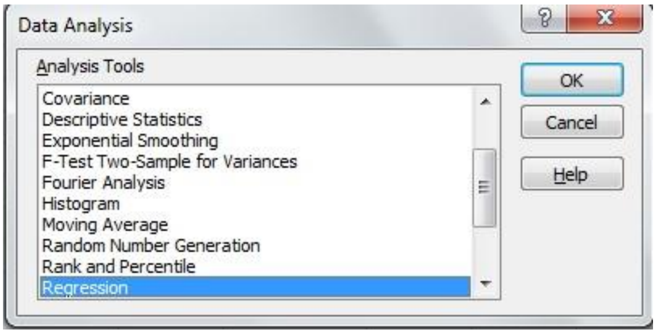


Fig. 4.1.1

- Choose Regression and press OK. A new pop up window will appear as shown in the Fig 4.1.2

- In Input Y range, select Y values including the label.

- In Input X range, select x1 and x2 values including the label.

- Since Labels are included, select label.

- In Output range, select some empty space in the worksheet.

- In Residuals, select residuals.

- Press OK.

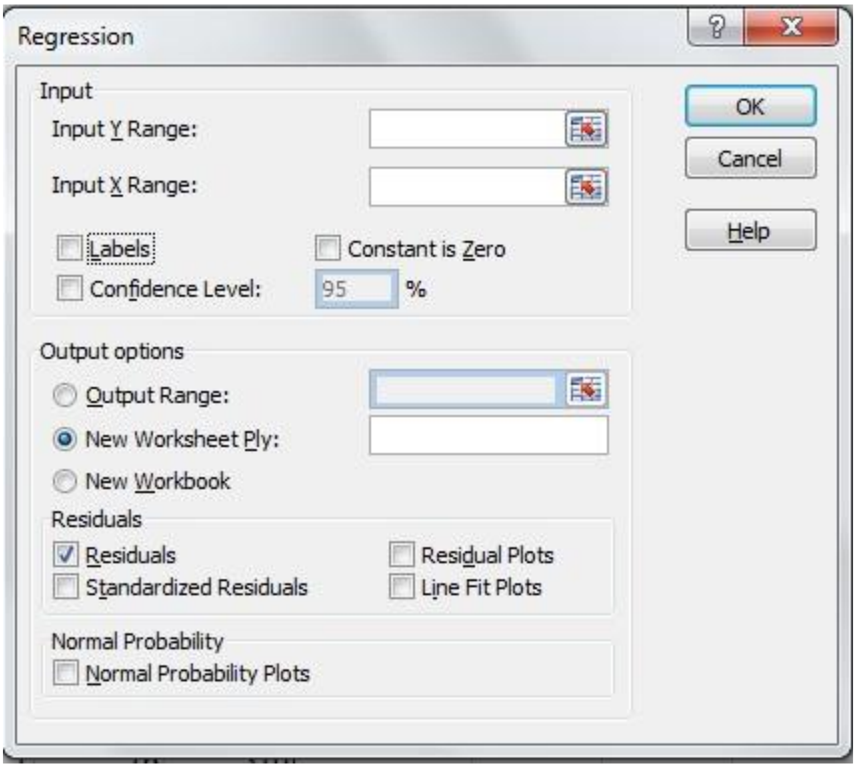


Fig. 4.1.2

Therefore, the final conclusion is shown in Fig. 4.1.3

98	**Conclusion: The regression model is Y=XA+C where A is given by**			
99	A=	2.263791434		
100		2.744269643		
101		0.012527811		
102				
103	**Clearly multiple R = 0.99 implies that data is highly linear.**			
104	**Also, out of the total variation, explained variation is significant.**			

Fig. 4.1.3

LAB 5: Multiple Linear Regression (Part II)

Ex. 1: The electric power consumed each month by a chemical plant is thought to be related to the average ambient temperature (x1), the number of days in the month (x2), the average product purity (x3), and the tons of product produced (x4). The past year's historical data are available and are presented in the following table. Fit a multiple linear regression model to these data.

x1	x2	x3	x4	Y
25	24	91	100	240
31	21	90	95	236
45	24	88	110	270
60	25	87	88	274
65	25	91	94	301
72	26	94	99	316
80	25	87	97	300
84	25	86	96	296
75	24	88	110	267
60	25	91	105	276
50	25	90	100	288
38	23	89	98	261

Ex 2: An article in IEEE Transactions on Instrumentation and Measurement (2001, Vol. 50, pp. 2033–2040) reported on a study that had analyzed powdered mixtures of coal and limestone for permittivity. The error in the density measurement was the response. The data are reported in following table:

a) Fit a multiple linear regression model to these data with the density as the response.
b) Use the model to predict the density when the dielectric constant is 2.5 and the loss factor is 0.03.

Density	Dielectric Constant	Loss Factor
0.749	2.05	0.016
0.798	2.15	0.02
0.849	2.25	0.022
0.877	2.3	0.023
0.929	2.4	0.026
0.963	2.47	0.028
0.997	2.54	0.031
1.046	2.64	0.034
1.133	2.85	0.039
1.17 2	0.94	0.042
1.215	3.05	0.045

LAB 6: Practical Harmonic Analysis

THEORY: In the absence of analytical expression for the function *f(x)*, the process of finding Fourier coefficients from the table of function values corresponding to some equidistant points numerically is called the practical harmonic analysis.

Let $y = f(x)$ in $(0, 2\pi)$, then Fourier series will be of the form

$$y = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where the Fourier coefficients are given by

$$a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)dx\,, a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cos nx\,dx\,, b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\sin nx\,dx.$$

The term $\frac{a_0}{2}$ is called the constant term and the groups of terms ($a_1 \cos x + b_1 \sin x$),

($a_2 \cos 2x + b_2 \sin 2x$) etc are called the first harmonic, second harmonic, etc respectively. Let x_0=0, x_1..................$x_m = 2\pi$ be m equidistant points. Then using mean value theorem and trapezoidal rule for integration, numerically approximation for the Fourier coefficients is given by

$$a_0 = \frac{2}{m}\sum_{r=0}^{m-1} y_r \;\;, a_n = \frac{2}{m}\sum_{r=0}^{m-1} y_r \cos nx_r\,, b_n = \frac{2}{m}\sum_{r=0}^{m-1} y_r \sin nx_r$$

The number of ordinates used should not be less than the twice the number of highest harmonic to be found.

If the length of interval i.e. is the period is T, i.e. $l = T/2$, then

$$a_n = \frac{2}{m}\sum_{r=0}^{m-1} y_r \cos n\pi\frac{x_r}{l}, b_n = \frac{2}{m}\sum_{r=0}^{m-1} y_r \sin n\pi\frac{x_r}{l}$$

Where x_r, $r = 0......m$, are equidistant points in the interval $[0, l]$ with x_0=0 and x_m=m. So, the working procedure is as follows:

1. Write down the period $y= f(x)$.
2. If the period is 2π, prepare the relavant table alongwith the summations of $y \cos x$,...etc. Compute the harmonics.
3. If the period is not 2π, equate it to $2l$ and find l and compute summations of $y \cos \theta$, $y \cos 2\theta$,.........etc where $\theta = \pi x/l$.

Ex. 1: Analyse harmonically the data given below and express y in Fourier series upto the third harmonic:

x	0	π/4	2 π /3	Π	4 π /3	5 π /3	2 π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Solution:

- Since the last value of y is a repetition of the first, only the first six values will be used. Note that for entering π in excel type =pi(), press ENTER.

Ex. 1: We have

x:	0	1.0472	2.0944	3.14159	4.18879	5.23599	6.28319
y:	1	1.4	1.9	1.7	1.5	1.2	1

We need to express y in Fourier series upto the third harmonic, so we have
y= (a0/2) + (a1 cos x + b1 sin x) + (a2 cos 2x + b2 sin 2x) + (a3 cos 3x + b3 sin 3x).

Fig 6.1.1

- Clearly the length of the interval is 2π. So, we have

$$a_0 = \frac{2}{m}\sum_{r=0}^{m-1} y_r \ , a_n = \frac{2}{m}\sum_{r=0}^{m-1} y_r \cos nx_r \, , b_n = \frac{2}{m}\sum_{r=0}^{m-1} y_r \sin nx_r$$

  Hence, we need to find the values of cos x, cos 2x, cos 3x, sin x, sin 2x and sin 3x.

- First column, under the heading x, type first 5 values of x

- Next type first 5 values of y under the heading y.

- In column 3, give heading as cos x. To compute cos, COS() and for sin, SIN() are the functions. Below it type =COS(value of x), press ENTER. Then drag till next 4 rows. Then the cos values will be evaluated for the entire column. Similarly, make columns of sin x, cos 2x, sin 2x, cos 3x, sin 3x and repeat the procedure. Select the table and right click. Choose format cells, in numbers, fix decimal points upto 4 digits. The table so obtained is shown in the Fig 6.1.2

7
8 Clearly the length of the interval is 2π. So we will be using only 5 values as the last one is repeated.
9

10	x	y	cos x	sin x	cos 2x	sin 2x	cos 3x	sin 3x
11	0.0000	1.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
12	1.0472	1.4000	0.5000	0.8660	-0.5000	0.8660	-1.0000	0.0000
13	2.0944	1.9000	-0.5000	0.8660	-0.5000	-0.8660	1.0000	0.0000
14	3.1416	1.7000	-1.0000	0.0000	1.0000	0.0000	-1.0000	0.0000
15	4.1888	1.5000	-0.5000	-0.8660	-0.5000	0.8660	1.0000	0.0000
16	5.2360	1.2000	0.5000	-0.8660	-0.5000	-0.8660	-1.0000	0.0000
17								

18

Fig 6.1.2

- Compute a_0 by typing =(2/6)*SUM(select all values of y), press ENTER.
- Compute a_1 by typing =(2/6)*MMULT(TRANSPOSE(select all values of y),select all values of cos x), press CTRL+SHIFT+ENTER.
- Compute a_2 by typing =(2/6)*MMULT(TRANSPOSE(select all values of y),select all values of cos 2x), press CTRL+SHIFT+ENTER.
- Compute *a3* by typing =(2/6)*MMULT(TRANSPOSE(select all values of y),select all values of cos 3x), press CTRL+SHIFT+ENTER.
- Compute $a_0/2$ by typing =(select value of a_0)/2, press ENTER.
- Compute b_1 by typing =(2/6)*MMULT(TRANSPOSE(select all values of y),select all values of sin x), press CTRL+SHIFT+ENTER.
- Compute b_2 by typing =(2/6)*MMULT(TRANSPOSE(select all values of y),select all values of sin 2x), press CTRL+SHIFT+ENTER.
- Compute b_3 by typing =(2/6)*MMULT(TRANSPOSE(select all values of y),select all values of sin 3x), press CTRL+SHIFT+ENTER.
- Select all the values computed and right click. Then select format cells. In that numbers, fix the decimal upto 4 places.
- Substitute the values in Fourier series expansion and get the answer.

We need to express y in Fourier series upto the third harmonic, so we have

y= (a0/2) + (a1 cos x + b1 sin x) + (a2 cos 2x + b2 sin 2x) + (a3 cos 3x + b3 sin 3x).

Clearly the length of the interval is 2π. So we will be using only 5 values as the last one is repeated.

x	y	cos x	sin x	cos 2x	sin 2x	cos 3x	sin 3x
0.0000	1.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
1.0472	1.4000	0.5000	0.8660	-0.5000	0.8660	-1.0000	0.0000
2.0944	1.9000	-0.5000	0.8660	-0.5000	-0.8660	1.0000	0.0000
3.1416	1.7000	-1.0000	0.0000	1.0000	0.0000	-1.0000	0.0000
4.1888	1.5000	-0.5000	-0.8660	-0.5000	0.8660	1.0000	0.0000
5.2360	1.2000	0.5000	-0.8660	-0.5000	-0.8660	-1.0000	0.0000

n=6;

a0=	2.9000	a0/2=	1.45
a1=	-0.3667	b1=	0.1732
a2=	-0.1000	b2=	-0.0577
a3=	0.0333	b3=	0.0000

y= 1.45 + (-0.3667 cos x + 0.1732 sin x) - (0.1 cos 2x +0.0577 sin 2x) + 0.0333 cos 3x .

Fig 6.1.3

Ex. 2: The displacement y of a part of mechanism is tabulated with corresponding angular movement x^o of the crank. Express y as a Fourier series neglecting the harmonics above the third.

x^o	0	30	60	90	120	150	180	210	280	270	300	320
y	1.80	1.10	0.30	0.16	0.50	1.30	2.16	1.25	1.30	1.52	1.76	2.00

Ex. 3: The following table gives the variations of periodic current over a period:

t(secs)	0	T/6	T/3	T/2	2T/3	5T/6	T
A(amp.)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show, by numerical analysis, that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic.
(Note: $(a_0/2)$ represents the direct current part.
$\sqrt{(a_1^2+b_1^2)}$ gives the amplitude.)

LAB 7: Linear Programming Problems

THEORY: Linear programming deals with complex optimization problem. The problem is modelled as linear function which has to be optimised subject to various constraints expressed as inequalities. The function which has to be optimised is called the objective function. Such problems arise in industry where resources are limited and one has to use it in a best possible way.

Ex. 1: A paint manufacturer produces two types of paint, one type of standard quality (S) and the other of top quality (T). To make these paints, he needs two ingredients, the pigment and the resin. Standard quality paint requires 2 units of pigment and 3 units of resin for each unit made, and is sold at a profit of Rs1 per unit. Top quality paint requires 4 units of pigment and 2 units of resin for each unit made, and is sold at a profit of Rs1.50 per unit. He has stocks of 12 units of pigment, and 10 units of resin. Formulate the above problem as a linear programming problem to maximize his profit and solve it using graphical method and *Solver*.

Solution:

- Initially covert the problem into mathematically.

	A	B	C	D	E	F	G	H	I
1	**Ex. 1:**								
2		Let x: no of units of S type paint produced and							
3		y: no of units of T type paint produced.							
4		So, $x \geq 0$, $y \geq 0$							
5									
6		For 1 unit of S type, 2 units of pigment and 3 units of resin are required.							
7		For 1 unit of T type, 4 units of pigment and 2 units of resin are required.							
8									
9		No of units of pigments used: $2x + 4y$, therefore $2x + 4y \leq 12$							
10		No of units of resin used: $3x + 2y$, therefore $3x + 2y \leq 10$							
11									
12		The profit per unit S is Rs. 1							
13		The profit per unit T is Rs. 1.5							
14									
15		So, total profit $Z= x + 1.5y$ and we have to maximize the profit.							
16									
17		Therefore the mathematical formulation of LPP is as follows:							
18									
19		Max $Z= x + 1.5\ y$, subject to constraints							
20									
21		$2x + 4y \leq 12$							
22		$3x + 2y \leq 10$							
23		$x \geq 0$, $y \geq 0$							
24									

Fig. 7.1.1

- Tabulate the objective function and the constraints

25		x	y		
26	Z	1	1.5		
27	C1	2	4	≤	12
28	C2	3	2	≤	10
29					
30					

Fig 7.1.2

- Consider C1. By putting $y = 0$, we get $x = 6$ and by putting $x = 0$, we get $y = 3$. Similarly, consider C2. By putting $y = 0$, we get $x = 10/3$ and by putting $x = 0$, we get $y = 5$. This can be tabulated in excel as shown in the Fig 7.1.3 Note that before dividing in excel select the cell, right click and then select format. In format, select data type as numbers and also you can fix the numbers after decimal.

29		
30	C1	
31	x	y
32	6	0
33	0	3
34		
35	C2	
36	x	y
37	3.333333	0
38	0	5
39		

Fig 7.1.3

- Select the data in C1 and then select insert tab, choose straight line in scatter plot as shown in the Fig 7.1.4

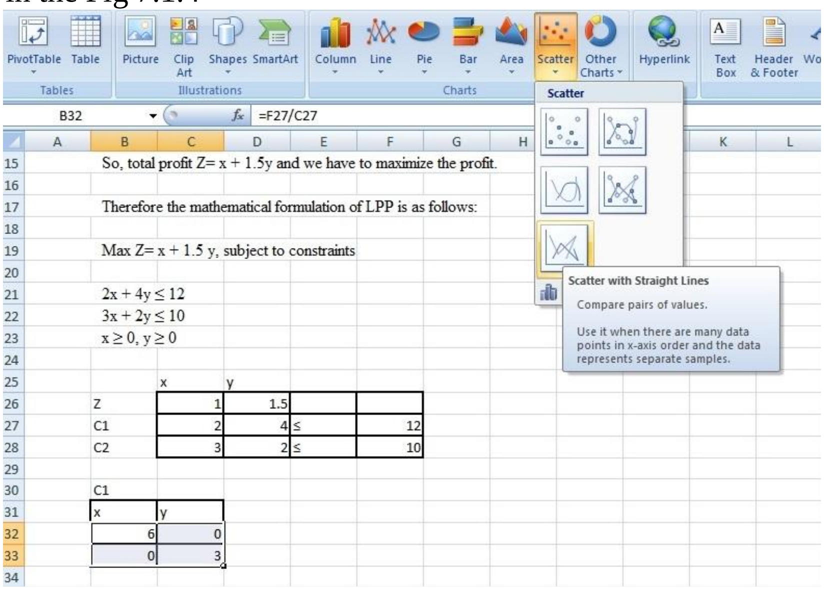


Fig 7.1.4

- After plotting the graph, right click anywhere on graph and choose select data. Click on series 1 and choose edit. For series name, type C1. In series X, delete the entries and select the data under x in C1 and similarly for y. Then click Ok. Again click add. For series name, type C2. For series X, choose data under x in C2. For series Y, choose data under y in C2. Then in the graph insert TEXT BOX and name the lines as C1 and C2.

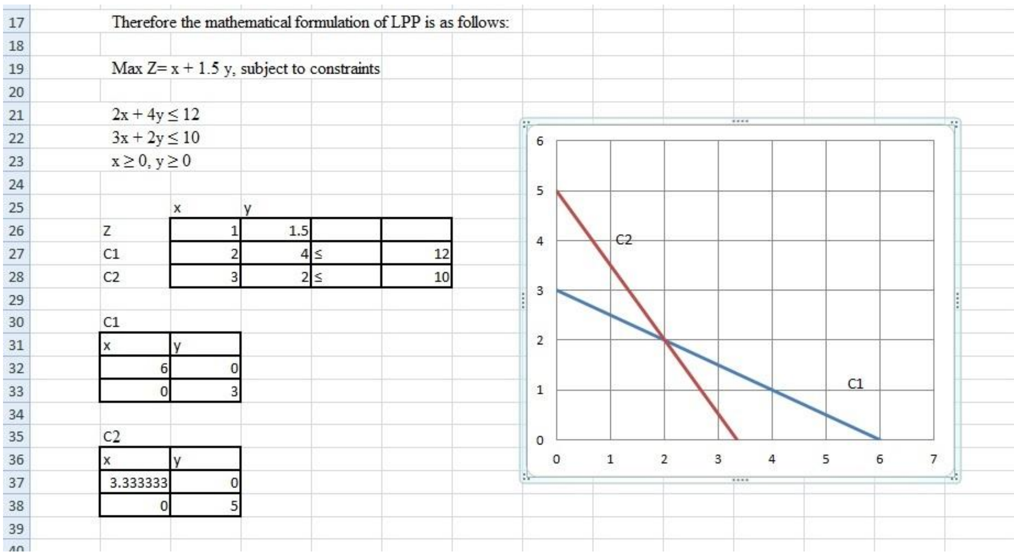


Fig 7.1.5

- To find the feasible region note that x≥0, y≥0. This implies the feasible region is in the first quadrant. Similarly, by putting x=0 and y=0, the inequality in C1 is satisfied so the region is below C1 line. By putting x=0 and y=0, the inequality in C2 is satisfied so the region is below C2 line. This can be shown in the graph by inserting arrows as shown in the Fig 7.1.6

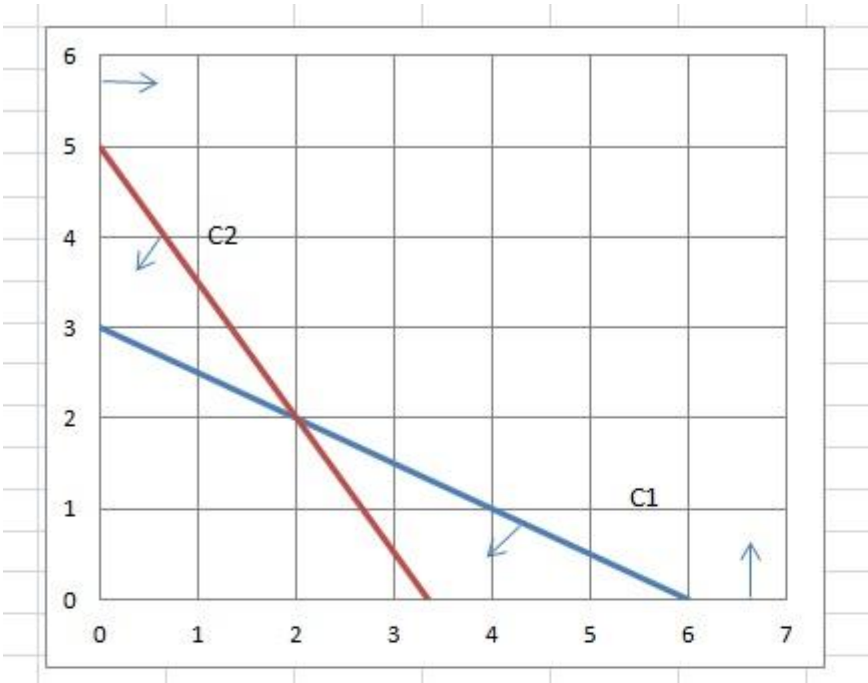


Fig 7.1.6

- The feasible region is nothing but the common region as shown in the Fig 7.1.7. To shade the region, insert shapes and from shapes select freeform and move along the edges of the region. Insert a text box and label the region as feasible region. Label the edges.

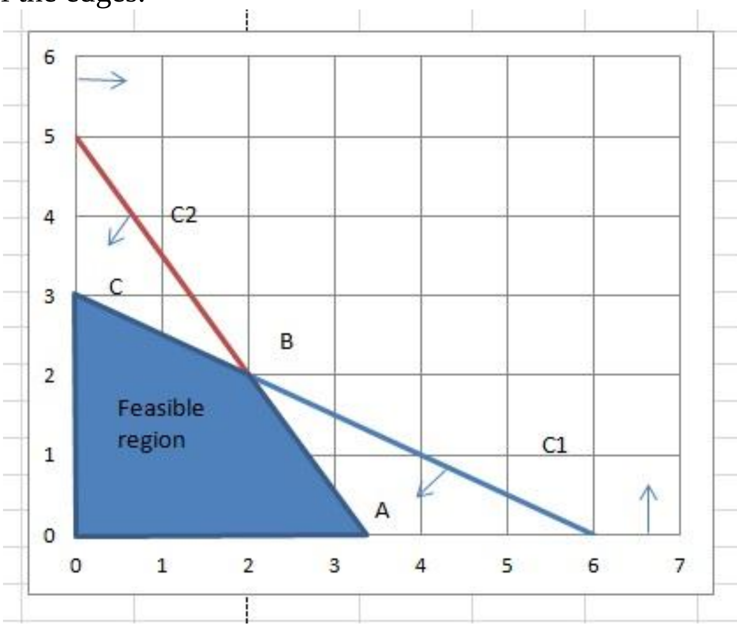


Fig 7.1.7

- To find the co-ordinates of A, make a table with coefficients of x and y in the equations of lines which intersect at A.

point A

X	Y		
1	0	=	0
3	2	=	10

- Then values of x and y are obtained using functions MMULT and MINV and press CTRL+SHIFT+ ENTER,. Then find value of Z at A by putting values of x and y at A.

point A

x	y		
0	1	=	0
3	2	=	10

x	=MMULT(MINVERSE(O18:P19),R18:R19)
y	MMULT(array1, array2)
z	

Fig 7.1.8

point A

x	y		
0	1	=	0
3	2	=	10

x	3.333333
y	0
z	3.333333

Fig 7.1.9

- Similarly, we can find co-ordinates B and C.

point B

x	y		
2	4	=	12
3	2	=	10

x	2
y	2
z	5

point C

x	y		
2	4	=	12
1	0	=	0

x	0
y	3
z	4.5

Fig 7.1.10

- Clearly, Z is maximum at B. Therefore (x, y) =(2, 2) gives the maximum, i.e. by making 2 units of S type and 2 units of T type gives maximum profit of Rs. 5/-

$2x + 4y \leq 12$
$3x + 2y \leq 10$
$x \geq 0, y \geq 0$

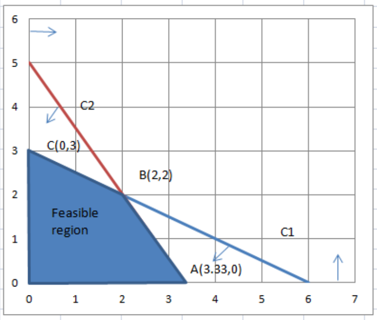
	x	y		
Z	1	1.5		
C1	2	4	≤	12
C2	3	2	≤	10

C1

x	y
6	0
0	3

C2

x	y
3.333333	0
0	5



x	3.333333
y	0
z	3.333333

point B

x	y		
2	4	=	12
3	2	=	10

x	2
y	2
z	5

point C

x	y		
2	4	=	12
1	0	=	0

x	0
y	3
z	4.5

- Clearly, Z is maximum at B. Therefore (x, y) =(2, 2) gives the maximum, i.e. by making 2 units of S type and 2 units of T type gives maximum profit of Rs. 5/-

Fig 7.1.11

PROCEDURE FOR SOLVER:

- Identify the decision variable. Here it is the number of units of S type paint and the number of units of T type paints.
- Make a table of given information with respect to the decision variables. In this case S type paint and T type paint with number of units of pigment resin required as shown in Fig 7.1.12.

45	METHOD: USING SOLVER			
46		Product	S Type	T Type
47		Pigment	2	4
48		Resin	3	2

Fig 7.1.12

- Make a table with the decision variable and their contribution

	S type	T type	Total
Decision var			
Contribution	1	1.5	

Leave the cell corresponding to number of S type and T type paint empty. In row corresponding to contribution, profit per unit has to be entered.

- To compute total contribution, type=(select cell corresponding to contribution of S type)*(select cell corresponding to number of units of S type)+ (select cell corresponding to contribution of T type)*(select cell corresponding to number of units of T type), press ENTER.

- Enter the constraints. That can be done by entering information regarding no of units of pigments and resins required below the contribution.

- In the column of Total, corresponding to pigments write the formula for total number of pigments needed in terms of decision variable and the required quantity which is =(select cell corresponding to pigments required for per unit of S type)*(select cell corresponding to number of units of S type)+ (select cell corresponding to pigments required for per unit of T type)*(select cell corresponding to number of units of T type), press ENTER.

- Similarly for resin, type=(select cell corresponding to resin required for per unit of S type)*(select cell corresponding to number of units of S type)+ (select cell corresponding to resin required for per unit of T type)*(select cell corresponding to number of units of T type), press ENTER

- Now total number of pigments and resins are to be entered in a column next to Total with title Max. (See Fig 7.1.13).

- Now we go to *Solver*. In the top panel of Excel sheet select tab Data. In that right hand corner *Solver* option is there. Select it. (See Fig 7.1.14).

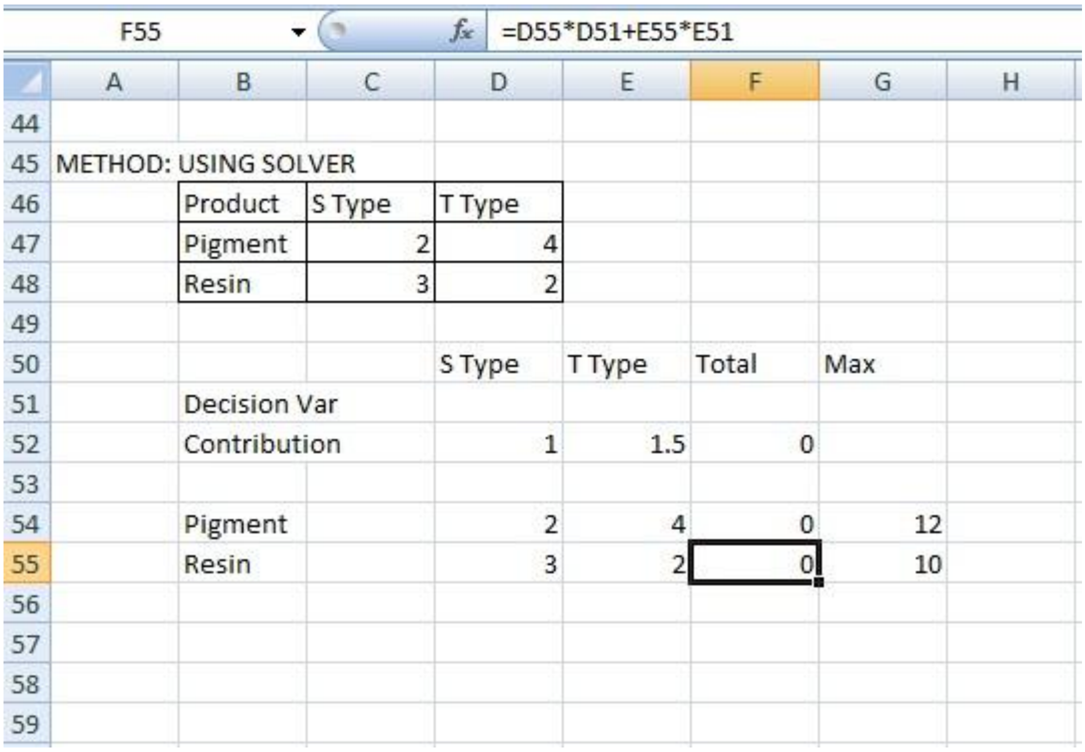


Fig 7.1.13

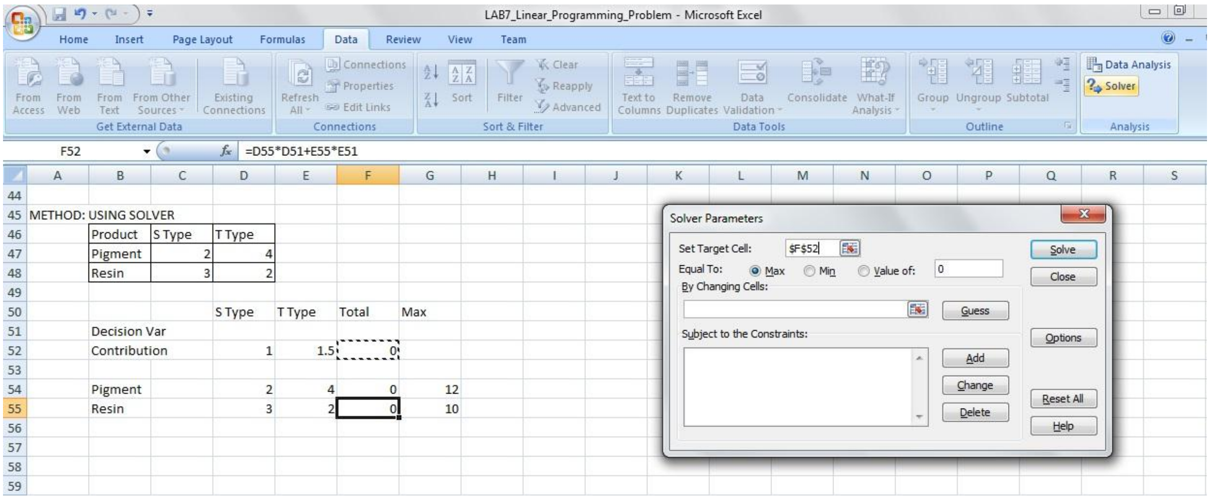


Fig 7.1.14

- In option Set Target Cells, select cell corresponding Total Contribution.
- Since we need to maximize, in option Equal to, select Max.
- Changing cells are nothing but the decision variables, so select the cells in the row of decision variable under S Type and T Type.

- To add constraints, select add and we get another pop up table as shown in the Fig 7.1.15

Fig 7.1.15

- In cell reference, select the cell under total corresponding to the first constraint (pigment).

- In constraint, select the cell under Max corresponding to the first constraint (pigment) and then press Add to add second constraint.

- In cell reference, select the cell under total corresponding to the second constraint (resin).

- Again in constraint, select the cell under Max corresponding to the second constraint (resin) and then press OK as there are no more constraints. Again the previous window pops up with added constraints.

- Press OPTIONS. Then a window pops up as shown in the Fig 7.1.16

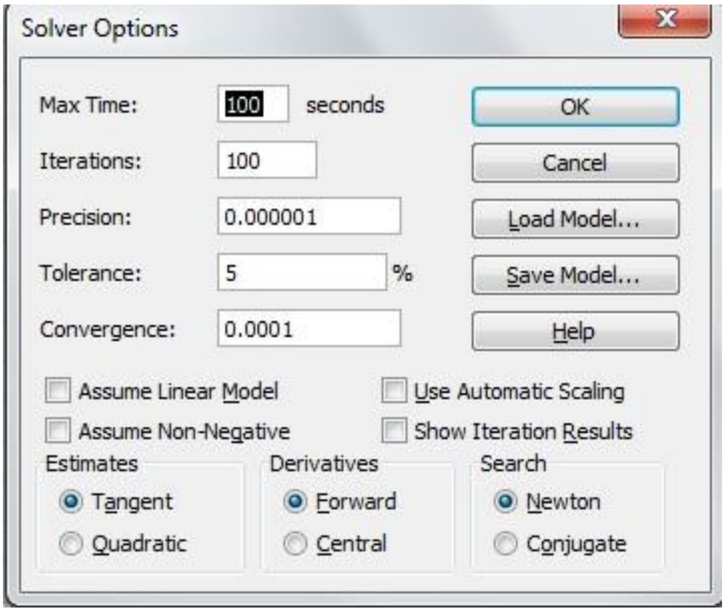


Fig 7.1.16

- Select Linear Model. Also since the units cannot be negative, select Assume Non-Negative. Then press OK. Then the previous pop up table comes.

- Select Solve. Then in next pop up table press OK. The solution is obtained in the columns of decision variable. We can see that in the Total contribution we also get the maximum value.

- Clearly, Z is maximum at B. Therefore (x, y) =(2, 2) gives the maximum, i.e. by making 2 units of S type and 2 units of T type gives maximum profit of Rs. 5/-

METHOD: USING SOLVER

Product	S Type	T Type
Pigment	2	4
Resin	3	2

	S Type	T Type	Total	Max
Decision Var	2	2		
Contribution	1	1.5	5	
Pigment	2	4	12	12
Resin	3	2	10	10

Fig 7.1.17

Ex. 2: Old hens can be bought at Rs. 20 each and young ones at Rs. 50 each. The old hens lay 3 eggs per week and the young ones lay 5 eggs per week, each egg worth Rs. 3 each. A hen (young and old) cost Rs. 10 per week to feed. Ira have only Rs. 800 to spend for hens, how many of each kind should Ira buy to give a profit of more than Rs.60 per week, assuming that Ira cannot house more than 20 hens. Solve graphically.

Ex. 3: The standard weight of a special purpose brick is 5 kg and it contains three basic ingredients B1, B2 and B3. B1 costs Rs.5 per kg and B2 costs Rs. 8 kg per kg and B3 costs Rs 4 per kg. Strength considerations state that the brick contains not more than 3 kg of B1 and not more than 1 kg of B3 and minimum of 2 kg of B2. Since the demand for the product is likely to be related to the price of the brick. Find out minimum cost of the brick satisfying the above conditions. Use *Solver.*

Reference

B. S. Grewal, J. S. Grewal, J. K. Dhanoa, Higher Engineering Mathematics, Khanna Publishers, 44^{th} Edition, 2017.

N. P. Bali, N. Ch. N. Iyengar, A Textbook of Engineering Mathematics, Laxmi Publications, 6^{th} Edition, 2004.

S. D. Sharma, H. Sharma, Operation Research, Theory, Methods and Applications, Kedar Nath Ram Nath Publishers, 15^{th} Edition, 2005.

D. C. Montgomery, G. C. Runger, Applied Statistics and Probability for Engineers, Wiley Publications, 6^{th} Edition, 2014.