URBAN EDGE ___/___/___

Module 2:

Fundamentals of Logic:

Introduction:

Logic is the science that deals with methods of reasoning. Logic expressed in symbolic language is called as 'Mathematical logic'.

PROPOSITIONS:

A proposition is a statement or declaration which in a given context can be said to be either true/false, but not both.

(ex) $p: 2+5=7 \Rightarrow$ is true/1

p: All rectangles are square $\Rightarrow$ False/0.

TRUTH TABLE:

The truth or falcity of a proposition is called its truth value.

If a proposition is true we indicate its truth value by the symbol T or 1.

If the proposition is false we indicate its truth value by the symbol F or 0.

LOGICAL CONNECTIVES:

New propositions are obtained by starting with given propositions with the help of words or phrases like 'OR', 'AND', "if then", 'if & only if'.

Such words are called as Logical connectives.

(1) negation ($\sim p$)
(2) conjunction ($\wedge$)
(3) disjunction ($\vee$)
(4) exclusive OR ($\veebar$)
(5) conditional/implication $p \rightarrow q$

6) Biconditional/double implication ($p \longleftrightarrow q$).

1) Negation:
A proposition obtained by inserting the word 'not' at an appropriate place in a given proposition is called negation of the given proposition.

(ex) - p: 7 is a prime number True/1
$\sim p$: 7 is not a prime no. False/0.

Truth Table: or

p	$\sim p$
T	F
F	T

p	$\sim p$
1	0
0	1

2) CONJUNCTION:
A compound proposition obtained by ~~defini~~ combining two given propositions, by inserting the word 'and' in between them is called conjunction of given propositions
(ex) p: $\sqrt{2}$ is an irrational no
q: 9 is a prime number.
And is denoted by '$\wedge$'
$p \wedge q$: $\sqrt{2}$ is an irrational number and 9 is a prime number.

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \wedge q$
1	1	1
0	1	0
1	0	0
0	0	0

3) DISJUNCTION:

A compound proposition is obtained by combining the given 2 propositions by inserting the word 'or' in between them is called disjunction of the given propositions, symbolically denoted by '$\vee$'.

(ex) p: $\sqrt{2}$ is an irrational number

q: 9 is a prime number.

$p \vee q$ is read as 'p ~~and~~ or q'

$p \vee q$: $\sqrt{2}$ is an irrational number or 9 is a prime number.

Truth Table:

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

4) EXCLUSIVE DISJUNCTION:

In the conjunction $p \vee q$ of 2 propositions p & q the symbol $\vee$ is used in the exclusive sense.

That is $p \vee q$ is taken to be true when both p and q or p or q is true

Truth Table:

p	q	$p \underline{\vee} q$
0	0	0
0	1	1
1	0	1
1	1	1

5) CONDITIONAL PROPOSITION: ($p \rightarrow q$)

A compound proposition obtained by combining 2 proposition by inserting the word if, then at appropriate places are called a conditional proposition

(ex) p: $\sqrt{2}$ is an irrational no.
q: 9 is a prime number

$p \rightarrow q$ is read as 'p implies q'
if $\sqrt{2}$ is an irrational no, then 9 is a prime number.

$p \rightarrow q \neq q \rightarrow p$

Truth Table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

6) BICONDITIONAL ($p \longleftrightarrow q$)

If p and q be two proposition. Then the conjunction of the conditional $p \rightarrow q$ and $q \rightarrow p$ is called bi-conditional of p and q, it is denoted by $p \leftrightarrow q$ "p if and only if q".

Truth Table:

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

1. Construct the Truth Table for the following compound propositions

i) $(p \vee q) \wedge r$ (ii) $q \wedge (\sim r \rightarrow p)$ (iii) $(p \vee q) \wedge (\sim p)$

iv) $q \leftrightarrow (\sim p \vee \sim q)$ (v) $(p \rightarrow q) \rightarrow r$

(iii) ans

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge (\sim p)$
0	0	1	0	0
0	1	1	1	1
1	0	0	1	0
1	1	0	1	0

(iv)

p	q	$\sim p$	$\sim q$	$(\sim p \vee \sim q)$	$q \leftrightarrow (\sim p \vee \sim q)$
0	0	1	1	1	0
0	1	1	0	1	1
1	0	0	1	1	0
1	1	0	0	0	0

~~(v) $(p \rightarrow q) \rightarrow r$~~

~~p q r $p \rightarrow q$ $(p \rightarrow q) \rightarrow r$~~

~~0 0 0~~

(i) $(p \vee q) \wedge r$

p	q	r	$(p \vee q)$	$(p \vee q) \wedge r$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

ii)

p	q	r	$\sim r$	$\sim r \to p$	$q \wedge (\sim r \to p)$
0	0	0	1	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	1
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	0	1	1

(v) $(p \to q) \to r$

p	q	r	$(p \to q)$	$(p \to q) \to r$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

Some special Logical propositions

1) Tautology → Always True (1)
2) Contradiction → Always False (0)
3) Contingency.

TAUTOLOGY: A proposition which ~~is~~ always true.

CONTRADICTION: A proposition which is always false.

CONTINGENCY: A proposition that is neither a tautology nor a contradiction

1. For any proposition p, q, r, prove that the given propositions.

$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow [p \rightarrow r]$ is a tautology.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	1	0	1	0	1
1	1	0	1	0	0	0
1	1	1	1	1	1	1

$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow [p \rightarrow r]$
1
1
1
1
1
1
1
1

Since we got all values as True $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow [p \rightarrow r]$ is a Tautology

2. Show that $(p \rightarrow q) \wedge (q \rightarrow \sim r) \rightarrow (\sim r \rightarrow p)$ is neither Tautology nor contradiction

p	q	r	$\sim r$	$p \rightarrow q$	$q \rightarrow \sim r$	$p \rightarrow q \wedge q \rightarrow \sim r$	$\sim r \rightarrow p$
0	0	0	1	1	1	1	0
0	0	1	0	1	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	0	0	1	0	1
1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1

$(p \to q \land q \to \sim r) \to (\sim r \to p) = *$

0
1
0
1
1
1
1
1

Hence * is neither contradiction nor Tautology.

3. Show that $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a Tautology
4. PT for any proposition p, q, r the compound proposition $(p \to (q \to r)) \to [(p \to q) \to (p \to r)$ is a tautology

3ans

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$	$(p \to q) \leftrightarrow (\sim q \to \sim p)$
0	0	1	1	1	1	1
0	1	1	0	1	1	1
1	0	0	1	0	0	1
1	1	0	0	1	1	1

Hence the given proposition is a tautology

4ans

p	q	r	$q \to r$	$p \to (q \to r)$	$p \to q$	$p \to r$	$(p \to q) \to (p \to r)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	1	0	0	1
1	0	1	1	1	0	1	1
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

$(p \to (q \to r)) \to [(p \to q) \to (p \to r)]$

1

1

1

1

1

1

1

1

Hence the given expression is a Tautology

5. $p \wedge q \Leftrightarrow \sim(p \to \sim q)$ by truth Table ($\Leftrightarrow$) = $\equiv$ equivalent

ans

p	q	$\sim q$	$p \wedge q$	$p \to \sim q$	$\sim(p \to \sim q)$
0	0	1	0	1	0
0	1	0	0	1	0
1	0	1	0	1	0
1	1	0	1	0	1

Hence proved

6. $q \to p \Leftrightarrow (\sim p \to \sim q)$

p	q	$\sim p$	$\sim q$	$q \to p$	$\sim p \to \sim q$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	1	1
1	1	0	0	1	1

7. $[p \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$

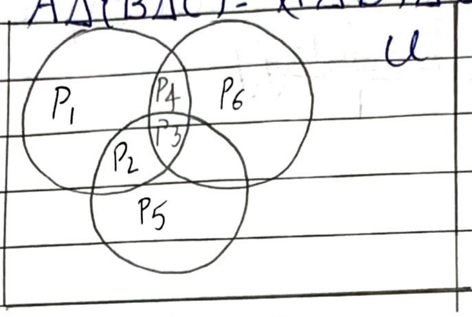
p	q	r	$q \wedge r$	$p \to q \wedge r$	$p \to q$	$p \to r$	$p \to q \wedge p \to r$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

$\frac{5}{9} \times \frac{4}{9} \times$

Set Theory:

Using venn diagram prove the following property of the symmetric difference.

$A \Delta (B \Delta C) = (A \Delta B) \Delta C$



ans $A = P_1 \cup P_2 \cup P_3 \cup P_4$

$B = P_2 \cup P_3 \cup P_5 \cup P_6$

$C = P_3 \cup P_4 \cup P_6 \cup P_7$

$A \Delta B = (A \cup B) - (A \cap B) =$

$LHS = A \Delta (B \Delta C)$

$B \Delta C = (B \cup C) - (B \cap C)$

$\Rightarrow [P_2 \cup P_3 \cup P_5 \cup P_6 \cup P_4 \cup P_7] - [P_3 \cup P_6]$

$= [P_2 \cup P_5 \cup P_4 \cup P_7]$

$A \Delta (B \Delta C) = A \cup (B \Delta C) - A \cap (B \Delta C)$

$[P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_7] - [P_2 \cup P_4]$

$= P_1 \cup P_3 \cup P_5 \cup P_7$

$A \Delta B = [P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_6] - [P_2 \cup P_3]$

$= P_1 \cup P_4 \cup P_5 \cup P_6$

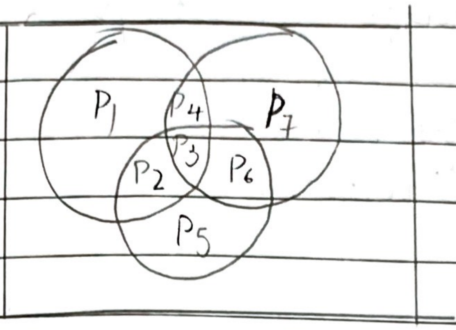
$(A \Delta B) \Delta C = ((A \Delta B) \cup C) - ((A \Delta B) \cap C)$

$= [P_1 \cup P_3 \cup P_4 \cup P_5 \cup P_6 \cup P_7] -$

$[P_4 \cup P_6]$

$= P_1 \cup P_3 \cup P_5 \cup P_7$

(ii) $\overline{(A\cup B)\cup C} = (\bar{A}\cap\bar{B})\cup\bar{C}$



Doubt of probability

ans 4 even numbers = {2, 4, 6, 8}

5 odd numbers = {1, 3, 5, 7, 9}

Let 'e' denote even and 'o' denote odd num

$P(oeo \text{ or } eoe) = P(oeo) + P(eoe)$

Since oeo and eoe are mutually exclusive.

Now

$P(\text{odd \& even \& odd}) = P(\text{odd}) . P\left(\frac{\text{even}}{\text{given odd}}\right) . P\left(\frac{\text{odd}}{\text{given odd \& even}}\right)$

$P(oeo) = \frac{5}{9} \cdot \frac{4}{8} \cdot \frac{4}{7} = \frac{10}{63}$

$P(eoe) = P(e)\,P\left(\frac{o}{e}\right)P\left(\frac{e}{eo}\right) = \frac{4}{9} \cdot \frac{5}{8} \cdot \frac{3}{7} = \frac{5}{42}$

$P(oeo \text{ or } eoe) = \frac{10}{63} + \frac{5}{42} = \frac{5}{18}$

LAWS OF LOGIC

1. LAW OF DOUBLE NEGATION:

For any proposition p $\sim(\sim p) \Leftrightarrow p$

2. IDEMPOTENT LAW

For any proposition p

(a) $(p \vee p) = p$

(b) $(p \wedge p) = p$

3. IDENTITY LAW:

For any proposition p

(a) $(P \vee F_0) \Leftrightarrow P$

(b) $(P \vee T_0) \Leftrightarrow P$

where F_0 denotes contradiction and T_0 denotes tautology.

4. INVERSE LAWS:

For any proposition p

(a) $(P \vee \sim p) \Leftrightarrow T_0$ (b) $(p \wedge \sim p) \Leftrightarrow F_0$

5. DOMINATION LAWS

For any proposition p

(a) $(p \vee T_0) \Leftrightarrow T_0$

(b) $(p \wedge F_0) \Leftrightarrow F_0$

6. COMMUTATIVE LAW

For any 2 proposition p and q

(a) $p \vee q \Leftrightarrow q \vee p$

(b) $(p \wedge q) \Leftrightarrow (q \wedge p)$

7. ABSORPTION LAWS

For any 2 propositions p and q

(a) $(p \vee (p \wedge q)) \Leftrightarrow p$

(b) $[p \wedge (p \vee q)] \Leftrightarrow p$

8. DE MORGAN'S LAW:

(a) $\sim(p \vee q) = \sim p \wedge \sim q$

(b) $\sim(p \wedge q) = \sim p \vee \sim q$

9. ASSOCIATIVE LAW:

For any 3 propositions p, q, r

(a) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$

(b) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

10. DISTRIBUTIVE LAWS:

For any 3 propositions p, q, r,

(a) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

(b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Some Standard Results:

1. $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$
2. $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$
3. $\sim(p \to q) \equiv (p \wedge \sim q)$
4. $p \to q \equiv \sim\sim(p \to q) \equiv \sim(p \wedge \sim q) \equiv \sim p \vee q$

Examples:

1. Simplify the following compound propositions using the laws of logic

i) $(p \vee q) \wedge \sim[(\sim p) \vee q]$

ans $\equiv (p \vee q) \wedge [p \wedge \sim q]$ by deMorgan's laws

$\equiv [(p \vee q) \wedge p] \wedge \sim q$ By associative law

$\Rightarrow$ $\underline{p \wedge \sim q}$ By absorption law

2) $\sim[\sim\{(p \vee q) \wedge r\} \vee \sim q]$

ans $\sim[\{\sim(p \vee q) \vee \sim r\} \vee \sim q]$ DeMorgan's law

$\sim[\{(\sim p \wedge \sim q) \vee \sim r\} \vee \sim q]$ DeMorgan's law

$\cancel{\sim[(p \vee q) \wedge r]}$

$(\sim(\sim p \wedge \sim q) \vee \sim r) \wedge q$ DeMorgan's law, Absorption

$(p \vee q) \wedge r \wedge q \Rightarrow \underline{q \wedge r}$ Negation law

3. Prove the following logical equivalence without using truth table.

$[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$

ans $[(p \vee q \vee q) \wedge (p \vee \sim q \vee q)]$ By distributive law

$[(p \vee q) \wedge (p \vee T_0)]$

$(p \vee q) \wedge T_0$

$p \vee q$

or

$p \vee (q \wedge \sim q) \vee q$

$(p \vee F_0) \vee q$

$\underline{p \vee q}$

4. $(p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim(q \vee p)$.

$\cancel{(p \rightarrow q) \wedge [(\sim q \wedge r) \vee (\sim q \wedge \sim q)]}$

$\cancel{p \rightarrow q \wedge [(\sim q \wedge r) \vee (\sim q)]}$

$\cancel{p \rightarrow q \wedge [\sim q \wedge r]}$

$\cancel{(\sim p \vee q) \wedge (\sim q \wedge r)}$

$\cancel{(\sim p \wedge \sim q) \vee (\sim p \wedge r) \vee (q \wedge \sim q) \vee (q \wedge r)}$

$\cancel{\sim(p \vee q) \vee (\sim p \wedge r) \vee (q \wedge r) \vee (F_0)}$

$\cancel{\sim(p \vee q) \vee r \wedge (\sim p \vee q) \vee F_0}$

$\cancel{\sim(p \vee q) \vee r \wedge (\sim p \vee q)}$

ans $(\sim p \vee q) \wedge [\sim q]$ [absorption law)

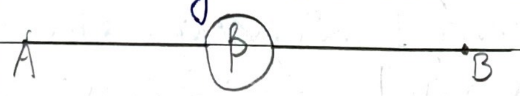
$\sim(\sim p \wedge q) \vee (q \wedge \sim q)$

$\sim(p \vee q) \vee F_0$ [Inverse laws]

$= \sim(p \vee q)$

Application

1. Switching network.

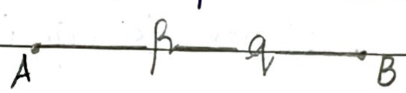


A switching network is made up of wires & switches connecting two terminals say A and B

In such a network each switch is open (so that no current flows through it) or closed (so that current flows through it).
If a switch p is open, we assign the symbol 0 to it and if a switch p is closed, we assign the symbol 1 to it.

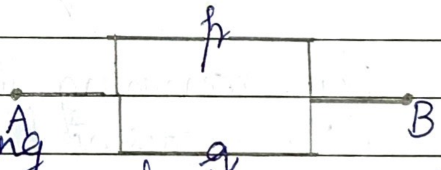
APPLICATION TO SWITCHING NETWORK

$p \wedge q$: ~~A — p B~~



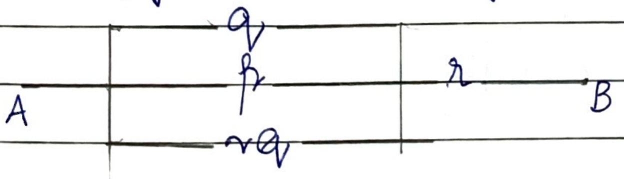
This figure shows a parallel network consisting of only one switch p. Current flows from the terminal A to terminal B, if the switch is closed, i.e., if p is assigned the symbol 1. ~~This network is~~ If switch q is opened, i.e., current doesn't flow from A to B i.e., if p is assigned the symbol 0.

$p \vee q$: Parallel Network.
The figure shows a parallel network consisting of 2 switches p and q in which current flows from A to B, if p or q or both are closed.



Problems:

1. Simplify the switching network shown below



ans Here the switches p, q and ~q are parallel and all these together are in series network with the switch 'r'
Therefore the given network is represented as

$(q \vee p \vee \sim q) \wedge r$

$\Rightarrow ([q \vee \sim q] \vee p) \wedge r$ [Inverse law]

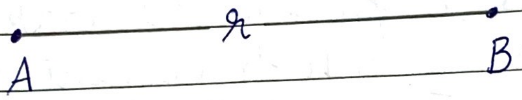
$(T_0 \vee p) \wedge r$

$(p \vee T_0) \wedge r$ [commutative]

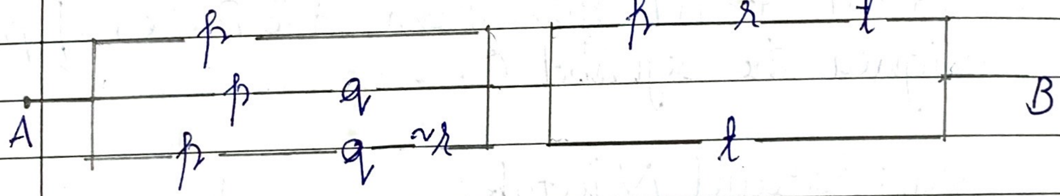
$\Rightarrow T_0 \wedge r$ [Domination law]

$= r$ By Identity law.

This shows that the given network that has four switches is equivalent to only one switch r.



2. Simplify the network shown below



By examining the given network, we find that it is represented by $u \equiv v \wedge w$

$v \Leftrightarrow \{p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)\}$

$w \Leftrightarrow (p \wedge r \wedge t) \vee t$

$v = p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)$

$= p \vee (p \wedge q \wedge \sim r)$ (Absorption law)

$p \vee [(t \vee \sim q) \wedge r]$

Special conditional statements

1. Implication ($p \to q$)
2. Converse ($q \to p$)
3. Contrapositive ($\sim q \to \sim p$)
4. Inverse ($\sim p \to \sim q$)

Examples

1. State the inverse, converse and contrapositive of "If a polygon is a square, then it is a rectangle"

p: A polygon is a square
q: It is a rectangle

Inverse $\Rightarrow$ $\sim p \to \sim q$
If a polygon is not a square then it is not a rectangle

Converse: $q \to p$
If a polygon is a rectangle then it is a square

Contrapositive:
If a polygon is not a rectangle, then it is not a square.

2. If I study well, then I shall pass.
p: I study well
q: I shall pass.

Inverse: $\sim p \Rightarrow \sim q$
If I don't study well, then I shall not pass.
Converse: $q \to p$
If I ~~shall~~ pass then I study well
Contrapositive: $\sim q \to \sim p$
If I ~~shall~~ did not pass, then I don't study well.

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Rules of Inference:-

P_1
P_2
P_3
$\vdots$
P_n } arguements/ premises

$\therefore q \rightarrow$ conclusion

Logically this is written as
$P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_n) \rightarrow q$ which is valid if this logic is tautology.

Rules of Inferences:

1. Rule of conjunctive simplification
This rule states that, for any two proposition p and q, if $p \wedge q$ is true, then p is true i.e., $(p \wedge q) \Rightarrow p$

Tabular Form: $\dfrac{p \wedge q}{\therefore p}$

2. Rule of disjunctive (A)mplification
This rule states that, for any two propositions, p and q, if p is true then $p \vee q$ is true
$p \Rightarrow (p \vee q)$

3. Rule of Syllogism:
This rule states that for any three propositions p, q, r. If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is true.
Logically, $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow p \rightarrow r$

4. Rule of detatchment/Modus Ponems:
This states that if p is true and $p \rightarrow q$ is true then q is true.
$\{p \wedge (p \rightarrow q)\} \Rightarrow q$

5) Modus Tollen's rule:
This rule state that if $p \to q$ is true and q is false, then p is also false.
$\{(p \to q) \wedge \sim q\} \Rightarrow \sim p$.

Tabular Form:

$p \to q$
$\sim q$
$\therefore \sim p$

6) Rule of disjunctive Syllogism:
This rule states that if $p \vee q$ is true and p is false, then q is true
logically $(p \vee q) \wedge (\sim p) \Rightarrow q$

Tabular form:

$p \vee q$
$\sim p$
$\therefore q$

1. Test the validity of the following arguements
(a) If I study, I will not fail in the exam.
(b) If I don't watch TV in the evening, I will study
(c) I failed in the exam
$\therefore$ I must have watched TV in the evenings

p: I study.
q: I fail in the exam
r: I watch TV in the evening

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1) Test the validity of the following arguments:
If Ravi goes out with friends he will not study. If Ravi does not study, his father become angry.
His father is not angry.
∴ Ravi has not gone out with friends

Tabular form

$p \rightarrow \sim q$

$\sim q \rightarrow r$

$\sim r$

$\therefore \sim p$

Acc to law of syllogism

$p \rightarrow \sim q \quad \sim q \rightarrow r \Rightarrow p \rightarrow r$

$p \rightarrow r \qquad \Rightarrow (p \rightarrow r) \wedge (\sim r)$

$\sim r \qquad$ This is equivalent to modus tollen's

$\sim p \qquad$ rule which implies $\sim p$ is true

∴ This is a valid arguement.

2. If I study, I will not fail in the exam.
If I do not watch TV in the evening, I will study.
I fail in the exam.
∴ I must have watched T V in the evening

Tabular form

$p \rightarrow \sim q$

$\sim r \rightarrow p$

q

$\therefore r$

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$(p \to \sim q) \equiv (\sim\sim q \to \sim p)$

$q \to \sim p$ (using contrapositive).

$\sim r \to p$ $\quad\quad \sim r \to p = \sim p \to r$.

q

$\therefore r$

$q \to \sim p$

$\sim p \to r$ $\quad = \quad$ $q \to r$ (Law of syllogism)

q $\quad\quad$ q

using Modus Ponens rule r is true

$\therefore$ The assignment is valid.

3. Test the validity of the following arguments

$p \wedge q$

$p \to (q \to r)$

$\therefore r$

This argument is logically equivalent to

$(p \wedge q) \wedge [p \to (q \to r)] \Rightarrow r$

Since $p \wedge q$ is true, both p and q are true (using conjunction).

Since p is true & $p \to (q \to r)$ is true

$q \to r$ is true

Since q is true and $q \to r$ is true, r has to be true. The argument is valid

ii) p

$p \to \sim q$

$\sim q \to \sim r$

$\therefore \sim r$

p

$p \to \sim r$ (By law of syllogism)

$\therefore \sim r$

$p \wedge (p \to \sim r) \Rightarrow \sim r$ By Modus Ponens law

The argument is valid

iii) $p \to q$

$r \to s$

$\sim q \vee \sim s$

$\sim(p \wedge r)$

(iv) $(\sim p \vee q) \to r$

ans ~~$(p \to q) \wedge (r \to s) \wedge (\sim q \vee \sim s)$~~

~~$\Rightarrow (p \to q) \wedge (r \to s) \wedge \sim(q \wedge s)$~~

~~$(\sim p \vee q) \wedge (\sim r \vee s)$~~

$(p \to q) \wedge (r \to s) \wedge (\sim q \vee \sim s)$

$\Leftrightarrow (p \to q) \wedge (r \to s) \wedge (q \to \sim s)$ using $p \to q \approx \sim p \vee q$

$(p \to \sim s) \wedge (r \to s)$ (law of syllogism)

$(p \to \sim s) \wedge (\sim s \to \sim r)$

$(p \to \sim r)$ $[p \to q = \sim p \wedge q]$

$\sim(p \wedge r)$

Quantifiers:

1. $x + 3 = 6$
2. $x^2 < 10$
3. x divides 4
4. $x = \sqrt{2}$

} declarative sentences (open sentenses) / open statements

$x \in R$ is a set of real numbers.

universe of discourse.

$p(x): x + 3 = 6$ for $x = 3$

$q(x) = x^2 < 10$ for $x = 0, \pm 1, \pm 2, \pm 3$

(ex) All squares are rectangles.

For every integer x, x^2 is a non negative integer.

Some determienants are equal to zero.

There exists a real number whose square is equal to itself.

Types of Quantifier:
1) Universal Quantifier
2) Existential

Universal Quantifier:
The words/phrases like "for all", "all", "for every", 'for each', 'for any' are called universal Quantifier.
Symbollically universal Quantifier is denoted by '$\forall$'.
(ex) - All squares are rectangle.
For all, $x \in S$, x is a rectangle
where S is the set of squares & $p(x)$: x is a rectangle. Symbollically, $\forall x \in S$, $p(x)$
2) For every integer x, x^2 is a non-negative integer.
For every $x \in Z$, x^2 is a non-negative integer
Symbollically, $\forall x \in Z$, $q(x)$

Existential Quantifiers:
The word/phrases like 'some', 'for some', 'there exists', 'for atleast one' etc are called existential quantifiers
Symbollically it is written as '$\exists$'
(ex) 3 Some determinants are equal to 0.
For some $x \in D$, x is equal to zero.
symbollical $\exists x \in D$, $r(x)$
where D is the set of determinants & $r(x)$: x is equal to zero
~~Symbollically~~

(ex) Write down the following propositions in symbolic form.
1) "All integers ~~e~~ are rational no and some rational numbers are not integers.

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2) If all triangles are right- ~~handed~~ angled then no triangle is equiangular

Sol Let $p(x) = x$ is a rational number
$q(x) = x$ is an integers
and
$Z =$ set of all integers
$q =$ set of all rational numbers
Symbollically
$\forall x \in Z$, x is a rational no.
$\wedge$ ($\exists x \in Q$, x is not an integer)
$\Rightarrow [\forall x \in Z, p(x)] \wedge [\exists x \in Q, \sim q(x)]$.

ii) Let $p(x)$: x is a right angled Δ^{le}.
$q(x)$: x is equiangular.

T: set of Δ^{les}.
symbollically: $[\forall x \in T, p(x)] \rightarrow [\forall x \in T, \sim q(x)]$

Negation of (i)
$\sim [\forall x \in Z, p(x)] \wedge [\exists x \in Q, \sim q(x)]$
$\equiv [\exists x \in Z, \sim p(x)] \vee [\forall x \in Q, q(x)]$ //

Negation of ii)
$\sim [\forall x \in T, p(x)] \longrightarrow [\forall x \in T, \sim q(x)]$
$\sim (p \rightarrow q) \equiv p \wedge \sim q$
$\equiv [\exists x \in T, p(x)] \wedge [\exists x \in T, q(x)]$

2. Write down the negations of the following
i) $\{\forall x, p(x)\} \vee \{\forall x, \sim q(x)\}$
ii) $\{\exists x, \sim p(x)\} \wedge \{\forall x, q(x)\}$
iii) $\{\exists x, p(x)\} \rightarrow \{\exists x, \sim q(x)\}$
ans $\cancel{\sim} [\exists x, \sim p(x)] \vee \{\exists x, q(x)\}$
$\equiv$

Truth value of Quantified statement:
The following rules are employed for determ-ining the truth value of quantified state-ment.

1) Rule 1: The statement, $\{\forall x \in S, p(x)\}$ is true only when $p(x)$ is true for each $x \in S$.

2) Rule 2: The statement, $\{\forall x \in S, p(x)\}$ is false only when $p(x)$ is false for every $x \in S$.

3) Rule 3: The open statement $p(x)$ is known to be true for all x in a universe S and if $a \in S$, then $p(a)$ is true. (Rule of universal specificat-ion.)

4) Rule 4: If an open statement $p(x)$ is proved to be true for any arbitrary x chosen from a set S, then the quantified statement $\{\forall x \in S, p(x)\}$ is true (Rule of universal generalisation).

Logical equivalences:

i) Two quantified statements are said to be logically equivalent if they have truth values in all possible situation

i) $\forall x, [p(x) \wedge q(x)] \Rightarrow (\forall x, p(x)) \wedge (\forall x, q(x))$.

ii) $\exists x, [p(x) \vee q(x)] \Rightarrow (\exists x, p(x)) \vee (\exists x, q(x)]$

iii) $\exists x, [p(x) \rightarrow q(x)] \Rightarrow \exists x [\sim p(x) \vee q(x)]$

$p \rightarrow q \Rightarrow \sim p \vee q$

Rule 5: Negation of a Quantified statement

$\sim [\forall x, p(x)] = [\exists x, \sim p(x)]$

$\sim [\exists x, p(x)] = [\forall x, \sim p(x)]$

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Write down the negations of the following
i) $\{\forall x, p(x)\} \vee \{\forall x, \sim q(x)\}$
ii) $\{\exists x, \sim p(x)\} \wedge \{\forall x, q(x)\}$
iii) $\{\exists x, p(x)\} \rightarrow \{\exists x, \sim q(x)\}$

ans Negation of (1)
$\sim[\forall x, p(x) \vee \forall x, \sim q(x)]$
$\{\exists x, \sim p(x) \wedge \exists x, q(x)\}$

Negation of (2)
$\sim[\exists x, \sim p(x) \wedge \forall x, q(x)]$
$\equiv \{\forall x, p(x) \vee \exists x, \sim q(x)\}$

Negation of (3)
$\sim[\exists x, p(x)] \longrightarrow \exists x, \sim q(x)]$
$\{(\exists x, p(x)) \wedge (\forall x, q(x))\}$

2. Let $p(x): x^2 - 7x + 10 = 0$, $q(x) = x^2 - 2x - 3 = 0$
$r(x): x < 0$. Determine the truth or falsity of the following statements.
When the universe U contains only the integers 2 and 5. If a statement is false, provide a counter example or explanation
i) $\forall x, p(x) \rightarrow \sim r(x)$
ii) $\forall x, q(x) \rightarrow r(x)$
iii) $\exists x, q(x) \longrightarrow r(x)$
iv) $\exists x, p(x) \rightarrow r(x)$.

ans. $p(x) = x^2 - 7x + 10 = 0 \quad x = 2, 5$
$p(x)$ is true as those elements are there in the universe. $\forall x \in U$.
Let $q(x) = x^2 - 2x - 3 = 0$
$x = 3, x = -1$
Since $x = 3$ and $x = -1$ are not the elements of U, $Q(x)$ is false for all $x \in U$

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$r(x): x<0$

Obviously, $r(x)$ is false as U contains 2, 5

$r(x)$ is false $\forall x \in U$.

i) Since $p(x)$ is true, $\sim r(x)$ is true for all $x \in U$

$\therefore$ The statement $\forall x, p(x) \to \sim r(x)$ is true.

ii) Since $q(x)$ is false and $r(x)$ is false. The statement $\forall x, q(x) \to r(x)$ is true

iii) Since $q(x)$ and $r(x)$ are false for all $x \in U$, The statement $\{\exists x, q(x) \to r(x)\}$ is true

iv) Since $p(x)$ is true for all $x \in U$ & $r(x)$ is false for all $x \in U$, the statement $p(x) \to r(x)$ is false for every $x \in U$.

$\therefore \exists x, p(x) \to r(x)$ is false.

Logical Implements involving Quantifier

A quantified statement p is said to be logically imply a quantified statement Q, if Q is true whenever P is true.

Then we write $P \Rightarrow Q$.

$p_1(x), p_2(x) \cdots\cdots p_n(x)$ be the premises and Q be the conclusion

That is we write, $P_1 \land P_2 \land \cdots \land P_n \to Q$

$\left.\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \end{array}\right\}$ premises

$\therefore Q \to$ conclusion.

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Prove the following arguements is valid:

All men are mortal
Sachin is a man
∴ Sachin is mortal

ans Let U be the set of men.

$p(x)$: x is mortal
$q(a)$: Sachin is a man
$p(a)$: Sachin is mortal

$\forall x \in S, p(x)$
$q(a)$
$\therefore p(a)$

logically
$\{\forall x \in S, p(x)\} \wedge q(a) \Rightarrow p(a)$.
$\Rightarrow \forall x, p(x)$ is true and $q(a)$ is true
∴ By rule of universal specification $p(a)$ is true. The arguement is true.

2. Find whether the arguement is valid
If a Δ^{le} has 2 equal sides it is isoceles.
If a Δ^{le} is isoceles, then it has two equal angles.
The Δ^{le} ABC does not have 2 equal angles.
∴ ABC does not have 2 equal sides

ans Let T denote set of Δ^{les}.

$p(x)$: x has 2 equal sides
$q(x)$: x is isoceles.
$r(x)$: x has 2 angles.

$\sim p(a)$: a does'nt have 2 equal sides
Symbollically

$\forall x \in T, p(x) \rightarrow q(x)$
$\forall x \in T\ q(x) \rightarrow r(x)$
$\sim r(a)$

$\therefore \sim p(a)$

$\{\forall x \in T, p(x) \rightarrow q(x) \wedge \{\forall x \in T, q(x) \rightarrow r(x)\}$
$\wedge \sim r(a)$
$\equiv \forall x \in T, \{p(x) \rightarrow r(x)\} \wedge \sim r(a)$ using law of syllogism.
$\equiv \{p(a) \rightarrow r(a)\} \wedge \sim r(a)$

$(p \rightarrow q) \wedge \sim q \rightarrow$
$\sim p(a)$ Modus Tollens.

Therefore the given arguement is valid.

3. Prove that the following arguement is not valid
All squares have four sides
The quadrilateral ABCD has four sides

Methods of proof and methods of disproof:
A proof is a valid arguement that establishes the truth of a ~~method~~ mathematical statement

Axioms: are statements which are assumed to be true.
(ex) There passes only one and only one straight line through two distinct points
Theorms: are statenments which are required to be proved. Proof of the theorm is valid arguments used to arrive at conclusion stated in the theorm.
i) By direct proof
ii) By indirect proof.

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i) Direct proof: In direct proof of the theorm which is in the form of a conditional statement, the hypothesis is assumed to be true

ii) Indirect proof: based on logical equivalent statements of the theorm i.e., if we want to prove $p \to q$ is true, then we prove that its contrapositive is true because

$$\boxed{p \to q \equiv \sim q \to \sim p}$$

Example:

1. give direct proof of:
"If 'n' is an even integer, then n^2 is an even integer

ans Hypothesis: i) n is an even integer
ii) If n is an even integer then 'n' is of the form $n = 2k$ where $k \in Z$
iii) If $n = 2k$, $k \in Z$, then $n^2 = 4k^2$
(iv) If $n^2 = 4k^2$ for $k \in Z$, then n^2 is an even integer
Conclusion: n^2 is even.

~~Indirect proof:-~~
~~In indirect proof of theorm which is in the form of a conditional statement~~

Note In indirect proof method, we can also prove the given statement by method of contradiction. That is, if we want to prove $p \to q$ is true, then it is sufficient if we prove that $\sim(p \to q)$ is false i.e., $p \wedge \sim q$

(ex) give indirect proof of:
"Set of all primes is infinite"

Let p: S is set of all primes (Assume $p \to q$ is
q: S is infinite false)
Then we require to prove $p \to q$ is true.
It is sufficient to prove that $\sim(p \to q)$ is false
i.e., $p \wedge \sim q$ is false.

$p \wedge \sim q \Rightarrow$ S is set of all primes and S is finite.
which is a contradiction of our assumption.
"$p \to q$ is false" is not a correct statement
Hence $p \to q$ is true.

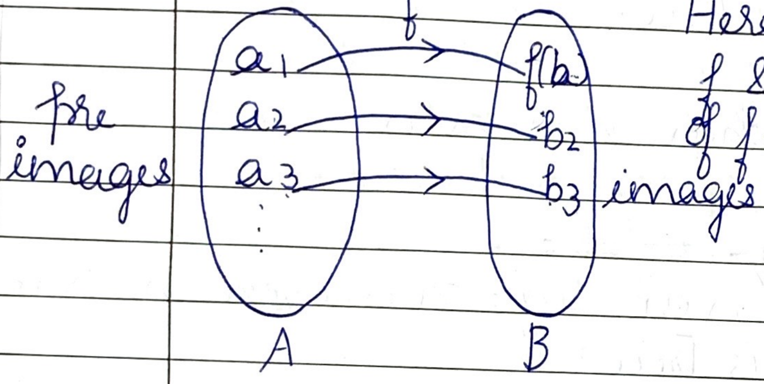
① If n^2 is even then n is even
$p \to q$
~~Assume $p \to q$ is false~~
proof: Let p: n^2 is even q: n is even. We have to prove $p \to q$ is true.
But let us assume $p \to q$ is false
$\sim(p \to q) = p \wedge \sim q$ is false
n^2 is even and n is odd $\to$ This statement is false. Then the assumption that $p \to q$ is false is false.
$p \to q$ is True

Module 3

Functions

aRa → reflexive relation
aRb = bRa → symmetric
aRb & bRc then aRc → transitive

Let A and B be non empty set, then a function / mapping f from A to B is a relation from A to B such that for each a in A there is a unique 'b' in B such that $(a, b) \in f$
we write it as $b = f(a)$
$f : A \to B$



Here A is the domain of f & B is called codomain of f.

Domain & Codomain: For the func $A \to B$, A is called domain of f, B is called codomain of f.

Range of f: The subset of B consisting of the images of all elements of A under f is called range of f. It is denoted by $f(A) \subseteq B$

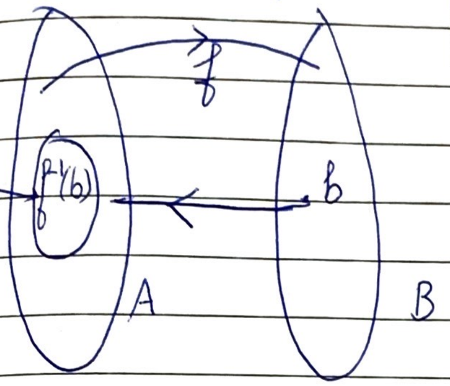
Observations:

1. Unique image in B.
2. need not have preimage in A
3. preimage need not be unique.
4. $a, b \in f$, $a f b$ $b = f(a)$.
5. $f = g$ when $f(a) = g(a)$

6. Range of $f: A \to B$ is given by
$f(A) = \{ f(x) \mid x \in A \}$

Inverse Image
For $f: A \to B$ if $b \in B$ and $f^{-1}(b)$ is defined by $f^{-1}(b) \subseteq A$. Here $f^{-1}(b)$ is called the preimage set of b under f.

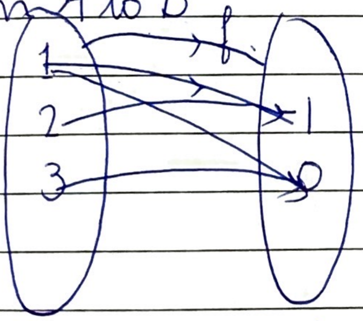
$$\boxed{f^{-1}(b) = \{ x \in A, f(x) = b \}}$$



$f^{-1}(B) = \{ x \in A \mid f(x) \in B \}$

(ex) Let $A = \{1, 2, 3\}$ and $B = \{-1, 0\}$ and R be a relation from A to B defined by
$R = \{(1, -1), (1, 0), (2, -1), (3, 0)\}$. Is R a func from A to B

ans



1 has 2 images (-1 and 0)
$\therefore$ This is not a function
We observe that under R the elements 1 is related to 2 diff elements -1 and 0 in B

$\therefore$ R is not a function