Module – 3 (Vector Differential Calculus)

Sl. No	Question	year	Marks
1	a) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find the components of its velocity and acceleration in the direction of $i - 2j + 2k$ at t=1. b) Find the directional derivative of $\emptyset = 4xz^3 - 3x^2y^2z$ at (2,-1,2) along 2i-3j+6k.	Mar 2001	5 5
2	a)If the directional derivatives of $\varphi = axy^2 + byz + cz^2x^3 at\ (-1, 1, 2)$ has a maximum magnitude of 32 units in the direction parallel to y-axis, find a, b, c. b) In which direction the directional derivative of x^2yz^3 is maximum at (2,1,-1) and find the magnitude of this maximum.	Mar 2001 Jan 2009	5 5
3	a)Find the directional derivative of the following $\varphi = x^2yz + 4xz^2$ at (1,-2,-1) along 2i-j-2k. b) Find the unit normal to the surface $\phi = 2xz - y^2$ at (1,3,2)	July 2010	5 5
4	a) Determine the unit normal vector to the surface $x^2y - 2xz + 2y^2z^4 = 10$ at (2,1,-1) b) Find the angle between the normal to the surface $xy = z^2$ at the points (4,1,2) and (3,3,-3).	Jan 2015	5 5
5	a).Find the angle between the surfaces $x^2 + y^2 + z^2 = 9, x^2 + y^2 - z = 3$ at the point (2,-1, 2). a)If $f = \nabla(x^3y + y^3z + z^3x - x^2y^2z^2)$ find div$f$ and curl$f$ at the point (1,2,3).	Jan 2015	5 5
6	a) If $\vec{F} = 2xy^3z^4i + 3x^2y^2z^4j + 4x^2y^3z^3k$, find (i) ($\nabla.\overrightarrow{F}$),(ii) $\nabla \times \overrightarrow{F}$. b) If $\vec{F} = (x + y + 1)i + j - (x + y)k$, then prove that $\vec{F}.curl\vec{F} = 0$.	Dec 2011	5 5
7	a) If $\varphi = xy + yz + zx$ and $\vec{F} = x^2yi + y^2zj + z^2xk$ find $\vec{F}.grad\varphi$ and $\vec{F} \times grad\varphi$ at the point (3, -1, 2) b) Show that $F = yzi + zxj + xyk$ is irrotational. Find ϕ so that $\overrightarrow{F} = \nabla\phi$	Dec 2009	5 5
8	a) For what value of 'a' vector point function $\vec{F} = (2x + 3y)i - (3x + 4y)j + (y - az)k$ is solenoidal b.) Is $\vec{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$, is irrotational.	Dec 2010	5 5
9	a) Show that the vector field $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational. And find its scalar potential such that $\vec{F} = \nabla\varphi$. b) If $\vec{r} = xi + yj + zk$ and$\lvert \vec{r} \rvert = r$. Find grad div$\left(\frac{\vec{r}}{r}\right)$.	Dec 2010	5 5
10	a) Find the constants ' a ' and ' b ' such that $\overrightarrow{F} = (axy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (bxz^2 - y)k$ is irrotational and also find a scalar potential function φ such that $\vec{F} = \nabla\varphi$. b) If $\phi = x^3 + y^3 + z^3 - 3xyz$, find $\nabla\phi$, $\lvert\nabla\phi\rvert$ at (2,1,-2)	Jun 2012	5 5

11	a) Find a, b, c such that $\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational and also find scalar potential. b) P.T $\vec{V}$ is solenoidal and $\vec{F}$ is irrotational. If $\vec{V} = 3xy^2z^2i + y^3z^2j - 2y^2z^3k$ and $\overrightarrow{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$	July 2011	5 5
12	a)If $F = 3xyi - y^2j$,evaluate$\int F.dR$, where C is the curve in the xy-plane$y = 2x^2$ from (0,0)to(1,2). b) A vector field is given by$F = siny\ i + x(1 + cosy)j$.evaluate the line integral over a circular path given by$x^2 + y^2 = a^2, z = 0$.	June 2012	5 5
13	a) If $\vec{F} = xyi + yzj + zxk$,evaluate$\int_C \vec{F}.\overrightarrow{dr}$, where Cis the curve represented by x= t, y= t^2,z= t^3, $-1 \leq t \leq 1$. b) Evaluate $\int_C \vec{F}.\overrightarrow{dr}$, where $\vec{F} = xyi + (x^2 + x^2)j$ along i. The path of the straight line from (0,0) to (1,0) and the to (1,1).		5 5
14	a) Find the work done in moving a particle in the force field $F = 3x^2i + (2xz - y)j + zk$ along a)the straight line from (0,0,0)to (2,1,3) b) curve defined by $x^2 = 4y, 3x^3 = 8z$ from x=0 tox=2.	JNTU(2002)	5 5
15	a)Evaluate $\int_S F.Nds$ where $F = 2x^2yi - y^2j + 4xz^2k$ and S is the closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes x=0,x=2,y=0and z=0. b) Evaluate using Gauss Divergence Theorem for the vector $\vec{F} = 4xzi - y^2j + yzk$ over the unit cube	Jun 2010 July 2006	5 5
16	a) Using Green's Theorem find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$. b) Evaluate using Gauss Divergence Theorem for the vector $\vec{F} = (x^2 - z^2)i + 2xyj + (y^2 + z^2)k$,S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	Jan 2010 July 2010	5 5
17	a) Evaluate using Stoke's Theorem for the vector $\vec{F} = (x^2 - y^2)i + 2xyj$ taken around the rectangle bounded by $= \pm a, y = 0, y = b$. b) Evaluate using Stoke's Theorem for the vector $\vec{F} = (2x - y)i - yz^2j - y^2zk$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, C is its boundary.	Jan 2011 July 2011	5 5
18	a) Evaluate using Gauss Divergence Theorem $\int \vec{A}.\hat{n}ds$, where $\vec{A} = x^3i + y^3j + z^3k$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. b) Evaluate using Gauss Divergence Theorem for the vector $\vec{F} = 2xyi + yz^2j + xzk$,S is the rectangular parallelepiped bounded $0 \leq x \leq 2, 0 \leq y \leq 1, 0 \leq z \leq 3$.	Jun 2010 July 2006	5 5
19	a) Verify Green's Theorem in a plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. b) Using Green's Theorem evaluate $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$	Jan 2010 July 2010	5 5
20	a) Using Stoke's Theorem evaluate $\vec{F} = 2xyi + ((x^2 - y^2)j$ over the circle $x^2 + y^2 = 1, z = 0$. b) Evaluate using Stoke's Theorem for the vector $\vec{F} = (x^2 + y^2)i - 2xyj$ taken around the rectangle bounded by $= 0, x = a, y = 0, y = b$.	Jun 2012	5 5