URBAN EDGE 16 / 8 / 2019

MODULE 1: INTRODUCTION

SET: Set is a collection of well defined objects. The objects in the set (or which make up a set) are called its members or elements.

→ We denote set by capital letters such as A, B, C..... and lowercase letters are used to represent element

→ For a set we write $x \in A$ if x is an element of A and $y \notin A$ if y is not an element of A.

→ A set can be designated by listing its elements within flower brackets

(ex) $A = \{1, 3, 5, 7, 9\}$. Here $1 \in A$ but $6 \notin A$.

FINITE SET: If no. of elements in a set is finite then we say that the set is a finite set.

(ex) $A = \{1, 4, 9, \ldots\ldots, 64, 81\}$ or $A = \{x^2 \mid x^2 < 100\}$

INFINITE SET: Set having infinitely many elements are called infinite set.

(ex) $B = \{1, 3, 5, 7, 9 \ldots\ldots\}$ or $B = \{2k+1 \mid k \in U\}$

SINGLETON SET: A set containing only one element

(ex) $C = \{1\}$

METHODS TO REPRESENT SETS:

1) Tabulation method
2) Rule method

TABULATION METHOD:

All the elements of the set are written down within flower brackets. If there are too many elements, the first few elements are written down in such a way as to indicate clearly what the others are.

(ex) $A = \{1, 4, 9, \ldots\ldots, 64, 81\}$

RULE METHOD:

We specify the set by stacking a characteristic property which all the elements of the set possess and which no other object possesses.

(ex) $B = \{1, 3, 5, 7 \ldots\}$

$B = \{x \mid x \text{ is a positive odd integer}\}$

NULL SET: Set containing no elements at all. is called null set or empty set. It is denoted by $\{\}$ or ϕ or $\emptyset$

(ex) The set of all positive integers less than 10 which are divisible 11 which is null set

EQUAL SETS: The sets A and B are said to be equal if they have precisely the same elements, then we write ~~A and B~~ $A = B$

$A = \{1, 2, 3, 4\}$ then $A = B$

$B = \{x \mid 0 < x < 5, x \in N\}$

SUBSET: Given two sets A and B we say that A is a subset of B or A is contained in B if every element of A is an element of B as well.

→ The statement A is a subset of B is symbollically written as "$A \subseteq B$."

→ Here the symbol "$\subseteq$" stands for "is a subset of" or "is contained in".

→ For A is not a subset of $B \Rightarrow A \nsubseteq B$.

Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$ $C = \{2, 3, 5, 6\}$

Here $A \subseteq B$ but $A \nsubseteq C$.

In such situation we say that A is PROPERLY CONTAINED in or is a proper subset of B (B also contains elements not in A).

Thus set A is a proper subset of B if
(i) $A \subseteq B$
(ii) B possess atleast one element not in A.
Symbollically we write $A \subset B$ i.e., "A is a proper subset of B".

POWER SET:

If A is a set from the universe (U) the power set of A is denoted by P(A) is the collection of all possible subsets of A.

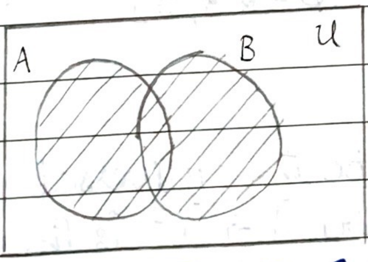
(ex) $A = \{1, 2, 3, 4\}$

$P = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{2,3,4\}, \{1,2,4\}, \{1,3,4\}, A\}$

In general for any finite set A with $|A| = n \geq 0$ we find that A has 2^n subsets and that $|P(A)| = 2^n$.

SET OPERATIONS:

1. UNION: Let A and B be sets, the union of the sets A and B denoted by $A \cup B$, is the set that contains those elements that are in A or B or in both.



$A \cup B$ is shaded region
$A \cup B = \{x / x \in A \vee x \in B\}$

(ex) $A = \{1, 2, 3\}$ $B = \{\cancel{1,2}, 3, 4, 5\}$ $A \cup B = \{1, 2, 3, 4, 5\}$

2. INTERSECTION: Let A and B be sets. The intersection of sets A and B denoted by $A \cap B$, is the set containing those elements in both A and B i.e.,
$A \cap B = \{x / x \in A \wedge x \in B\}$

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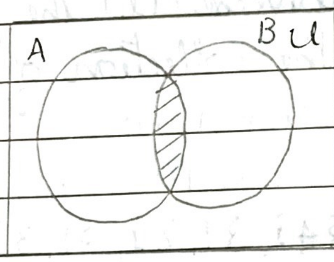
(ex) Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$

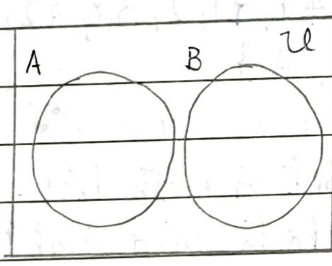
$A \cap B = \{3\}$

DISJOINT SETS: Two sets are said to be disjoint if their intersection is empty and have no common element.

(ex) Let $A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$



$A \cap B$ is the shaded region.



disjoint

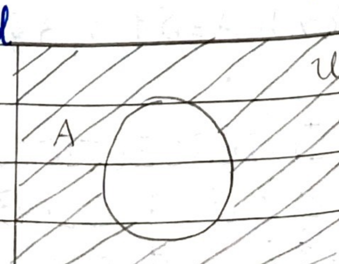
SET DIFFERENCE: Let A and B be two sets. The difference of A and B denoted by $A - B$ is the set containing elements in A but not in B.

i.e., $A - B = \{x \mid x \in A \wedge x \notin B\}$

(ex) Let $A = \{1, 3, 5, 6\}$ $B = \{3, 5\}$

$A - B = \{1, 6\}$

COMPLEMENT: Let U be the universal set. The complement of A denoted by $\bar{A}$ or A^c is the complement of A with respect to U denoted by $U - A$.



$\bar{A} = \{x \mid x \notin A\}$

(ex) Let U be the universal set

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3\}$

$\bar{A} = U - A = \{4, 5, 6, 7, 8, 9\}$

(ex) Let $A = \{a, e, i, o, u\}$ where universal set is the set of letters of english alphabet.
$\bar{A} = U - \bar{A} = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$.

SYMMETRIC DIFFERENCE: For $A, B \subseteq U$, we define the symmetric difference

$$A \Delta B = \{x \mid (x \in A \vee x \in B) \wedge x \notin A \cap B\}$$
$$= \{x \mid x \in A \cup B \wedge x \notin A \cap B\}$$

(ex) Let $U = \{1, 2, 3, \ldots, 9, 10\}$, $A = \{1, 2, 3, 4, 5\}$
$B = \{3, 4, 5, 6, 7\}$ and $C = \{7, 8, 9\}$ we have
$= \{1, 2, 3, 4, 5, 6, 7\} \wedge \{1, 2, 6, 7\}$
$A \Delta B = \{1, 2, 6, 7\}$
$A \Delta C = \{1, 2, 3, 4, 5, 7, 8, 9\}$

SET IDENTITIES:

IDENTITY	NAME
$A \cup \phi = A$	Identity laws.
$A \cap U = A$	
$A \cup U = U$	Domination laws.
$A \cap \phi = \phi$	
$A \cup A = A$	Idempotent laws.
$A \cap A = A$	
$(\bar{\bar{A}}) = A$	complementation
$A \cup (B \cup C) = (A \cup B) \cup C$	Associative law
$A \cap (B \cap C) = (A \cap B) \cap C$	
$A \cup B = B \cup A$	commutative laws.
$A \cap B = B \cap A$	
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	distributive laws.
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	

Identity	Name
$\overline{A \cup B} = \bar{A} \cap \bar{B}$	de Morgan's laws
$\overline{A \cap B} = \bar{A} \cup \bar{B}$	
$A \cup (A \cap B) = A$	Absorption laws.
$A \cap (A \cup B) = A$	
$A \cup \bar{A} = U$	complement.
$A \cap \bar{A} = \phi$	

PROBLEMS:

imp 1. Prove that $\overline{A \cap B} = \bar{A} \cup \bar{B}$ (using laws of sets)

ans To show that $\overline{A \cap B} = \bar{A} \cup \bar{B}$, first we show that $\overline{A \cap B} \subseteq \bar{A} \cup \bar{B}$ and $\bar{A} \cup \bar{B} \subseteq \overline{A \cap B}$

(i) Suppose, first we show that $\overline{A \cap B} = \bar{A} \cup \bar{B}$.

Suppose that $x \in \overline{A \cap B} \Rightarrow x \notin A \cap B$ (By definition of complement).

$\Rightarrow \sim[(x \in A) \wedge (x \in B)]$ (By definition of intersection]

$\sim(x \in A)$ or $\sim(x \in B)$ (By applying de Morgan's law of logic).

$\Rightarrow x \notin A$ or $x \notin B$ [By definition of negation]

$\Rightarrow x \in \bar{A}$ or $x \in \bar{B}$ [By definition of complement]

$\Rightarrow x \in \bar{A} \cup \bar{B}$.

$\overline{A \cap B} = \bar{A} \cup \bar{B} \longrightarrow (1)$

(ii) we show that $\bar{A} \cup \bar{B} \subseteq \overline{A \cap B}$. Suppose that $x \in \bar{A} \cup \bar{B}$

$\Rightarrow x \in \bar{A}$ or $x \in \bar{B}$ [By definition of union]

$x \notin A$ or $x \notin B$ [By definition of complement]

$\sim(x \in A)$ or $\sim(x \in B)$ [By definition of de Morgans law]

$\sim[x \in A \wedge x \in B]$ is true.

$\sim[x \in A \cap B]$ holds

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \in \overline{A \cap B}$

$\bar{A} \cup \bar{B} \subseteq \overline{A \cap B} \rightarrow (2)$

From 1 and 2 we have $\overline{A \cap B} = \bar{A} \cup \bar{B}$

2. For all sets A and B, prove the De-Morgan's laws $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (using Rule method).

ans We have, $\bar{A} \cap \bar{B} = \{x \mid x \in (\bar{A} \cap \bar{B})\}$

$= \{x \mid x \in \bar{A} \text{ and } x \in \bar{B}\}$

$= \{x \mid x \notin A \text{ and } x \notin B\}$

$= \{x \mid x \notin (A \cup B)\}$.

$= \{x \mid x \in (\overline{A \cup B})\}$

3. $\overline{A \cap B} = \bar{A} \cup \bar{B}$

We have $\bar{A} \cup \bar{B} = \{x \mid x \in (\bar{A} \cup \bar{B})\}$

$= \{x \mid x \in \bar{A} \text{ or } x \in \bar{B}\} \Rightarrow \{x \mid x \notin A \text{ or } x \notin B\}$

$\Rightarrow \{x \mid x \notin (A \cap B)\} = \{x \mid x \in (\overline{A \cup B})\}$.

4. For all sets A, B, C prove the associative law:

a) $A \cup (B \cup C) = (A \cup B) \cup C$.

ans Let $D = B \cup C$ and E as $A \cup B$ (Let us assume)

Take any $x \in A \cup (B \cup C)$.

$\Rightarrow x \in A \cup D$

$\Rightarrow x \in A$ or $x \in D$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow (x \in E)$ or $(x \in C)$

$\Rightarrow x \in E \cup C$

$\Rightarrow x \in (A \cup B) \cup C$.

Thus $A \cup (B \cup C) \subseteq (A \cup B) \cup C \longrightarrow (1)$

To prove $(A \cup B) \cup C \subseteq A \cup (B \cup C)$

Take any $x \in (A \cup B) \cup C$.

$(x \in A$ or $x \in B)$ or $x \in C$

$x \in A$ or $(x \in B$ or $x \in C)$

$x \in A$ or $(x \in B \cup C)$

$x \in A \cup (B \cup C)$.

$(A \cup B) \cup C \subseteq A \cup (B \cup C) \longrightarrow (2)$

from (1) and (2)
$A \cup (B \cup C) = (A \cup B) \cup C$

5. $A \cap (B \cap C) = (A \cap B) \cap C$

ans Let $B \cap C = D$ and $(A \cap B) = E$

Take any $x \in A \cap (B \cap C)$

$x \in (A \cap D)$

$x \in A$ and $x \in D$ (substituting for D)

$\Rightarrow x \in A$ and $x \in (B \cap C)$

$\Rightarrow x \in A$ and $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ and $x \in C$

$\Rightarrow x \in (A \cap B) \cap C$

$\Rightarrow (A \cap B) \cap C \subseteq A \cap (B \cap C) \longrightarrow (1)$

Take any $x \in (A \cap B) \cap C$

$x \in E \cap C$

$x \in E$ and $x \in C$

$(x \in A$ and $x \in B)$ and $x \in C$

$x \in A$ and $(x \in B$ and $x \in C)$

$x \in A \cap (B \cap C) \longrightarrow (2)$ $A \cap (B \cap C) \subseteq (A \cap B) \cap C$

from (1) and 2 $A \cap (B \cap C) = (A \cap B) \cap C$

6. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ans First we show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Take any $x \in A \cap (B \cup C)$.

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$(x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$x \in (A \cap B)$ or $(x \in A \cap C)$

$x \in (A \cap B) \cup (A \cap C)$

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

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To show $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

$\Rightarrow$ Consider any $x \in (A \cap B) \cup (A \cap C)$

$(x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow x \in A \cap (B \cup C)$

$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \rightarrow (ii)$

From (i) and (ii)

$A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$

7. $\overline{(A \cap B) \cup C} = (\bar{A} \cup \bar{B}) \cap \bar{C}$

ans Using deMorgan's law:

$(\overline{A \cap B}) \cap \bar{C} \Rightarrow (\bar{A} \cup \bar{B}) \cap \bar{C}$ (Again using deMorgan's law).

$\Rightarrow \overline{(A \cap B) \cup C} = (\bar{A} \cup \bar{B}) \cap \bar{C}$

8. $A - B = A \cap \bar{B}$

ans Let $A - B = \{x \mid x \in A$ and $x \notin B\}$

$= \{x \mid x \in (A \cap \bar{B})\}$

$= A \cap \bar{B}$

$A - B = A \cap \bar{B}$

~~9. $A - (A \cap B) = A \cap (\overline{A \cap B})$~~.

9. $A \cap (B - C) = (A \cap B) - C$

Let us consider an element x such that

$A \cap (B - C) = \{x \mid x \in A$ and $(x \in B$ and $x \notin C)\}$

$\Rightarrow \{x \mid (x \in A$ and $x \in B)$ and $x \notin C\}$

$\Rightarrow (A \cap B) - C$

Now Let us consider $x \in$

$\Rightarrow (A \cap B) - C \Rightarrow \{x \mid (x \in A$ and $x \in B)$ and $x \notin C\}$

$\Rightarrow \{x \mid x \in A$ and $(x \in B$ and $x \notin C)\}$

$\Rightarrow x \in A \cap (B - C)$

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10) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ans First we show that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Consider $x \in A \cup (B \cap C)$

$x \in A$ or ($x \in B$ and $x \in C$)

($x \in A$ or $x \in B$) and ($x \in A$ ~~and~~ or $x \in C$)

$x \in (A \cup B) \cap (A \cup C) \rightarrow (1) = A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Now we need to show that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

$x \in (A \cup B) \cap (A \cup C)$

($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

$\Rightarrow$ ~~$x \in A$ or $x \in B$~~ ($x \in A$ and $x \in A$) or ($x \in B$ and $x \in C$)

$x \in A \cup (B \cap C) \rightarrow (2) \Rightarrow (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

From (1) and (2)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

11) $A - (A \cap B) = A - B$

ans $A - B = A \cap \bar{B}$ (proved already)

considering LHS in this form

$A - (A \cap B) = A \cap (\overline{A \cap B})$ ∵ complement law

$A - (A \cap B) = A \cap (\bar{A} \cup \bar{B})$ ∵ distributive law

$= (A \cap \bar{A}) \cup (A \cap \bar{B})$

$= \phi \cup (A - B)$

$A - (A \cap B) = A - B$

12) $A - (A - B) = A \cap B$

ans consider $A - (A - B)$ take $A - D$

$A - D = A \cap \bar{D}$

$= A \cap (\overline{A - B}) = A \cap (\overline{A \cap \bar{B}})$

$= A \cap (\bar{A} \cup B) = (A \cap \bar{A}) \cup (A \cap B)$ De Morgan's law

$= \phi \cup (A \cap B)$ [By complement law]

$= A \cap B$

$A - (A - B) = A \cap B$

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13) $\overline{A-B} = \bar{A} \cup (A \cap B)$

ans $A-B = A \cap \bar{B}$ (proved)

$\overline{A \cap \bar{B}} = \bar{A} \cup B \rightarrow$ (LHS) (DeMorgan's law)

RHS $(\bar{A} \cup A) \cap (\bar{A} \cup B) = \mathcal{U} \cap (\bar{A} \cup B) = \bar{A} \cup B$

(distributive law) (complement law).

From LHS and RHS : $\overline{A-B} = \bar{A} \cup (A \cap B)$

~~property~~ $\cancel{\overline{A-B} = \bar{A} \cup B}$, $\cancel{A-(A \cap B) = A-B}$

~~using LHS~~

$\cancel{\overline{A-B} = \overline{A-(A \cap B)}}$

$A-(A \cap B) = A-B$

$\overline{A-B} = \bar{A} \cup B$

$\overline{A-B} = \overline{A-(A \cap B)} = \bar{A} \cup (A \cap B)$

14) $A \cap (B-C) = (A \cap B) - C$

ans $A \cap (B-C) = A \cap (B \cap \bar{C}) = (A \cap B) \cap \bar{C}$

$\Rightarrow (A \cap B) - C$

15) $(A-B) \cap (A-C) = A-(B \cup C)$ deMorgan's law

ans Taking $A-(B \cup C) = A \cap (\overline{B \cup C}) = A \cap (\bar{B} \cap \bar{C})$

$(A \cap \bar{B}) \cap (A \cap \bar{C}) = (A-B) \cap (A-C)$

16. Prove the following

i) $A \Delta B = (B \cap \bar{A}) \cup (A \cap \bar{B}) = (B-A) \cup (A-B)$

(ii) $\overline{A \Delta B} = \bar{A} \Delta B = A \Delta \bar{B}$.

ans We have $A \Delta B = (A \cup B) - (A \cap B)$.

$= (A \cup B) \cap (\overline{A \cap B})$

$= (A \cup B) \cap (\bar{A} \cup \bar{B})$

$[(A \cap \bar{A}) \cup B \cap \bar{A}] \cup (A \cap \bar{B}) \cup (B \cap \bar{B})$

$= \phi \cup (B \cap \bar{A}) \cup (A \cap \bar{B}) \cup \phi$

$\Rightarrow (B \cap \bar{A}) \cup (A \cap \bar{B})$

$B \cap \bar{A} = B - A$

$A \cap \bar{B} = A - B$

$\Rightarrow (B \cap \bar{A}) \cup (A \cap \bar{B})$

$\Rightarrow \underline{(B - A) \cup (A - B)}$

ii) ans: $A \Delta B = (A \cup B) - (A \cap B)$

$\Rightarrow \overline{A \Delta B} = \overline{(A \cup B) - (A \cap B)}$

$\overline{A - B} = \bar{A} \cup B$

$\overline{(A \cup B) - (A \cap B)} = (\overline{A \cup B}) \cup (A \cap B)$

$= (\bar{A} \cap \bar{B}) \cup (A \cap B)$

$(\bar{A} \cup A) \cap (\bar{A} \cup B) \cap (\bar{B} \cup A) \cap (\bar{B} \cup B)$

$U \cap (\bar{A} \cup B) \cap (\bar{B} \cup A) \cap (U)$

$(\bar{A} \cup B) \cap (\bar{B} \cup A)$

$= \cancel{\overline{A - B} \cap \overline{B - A}}$

$\cancel{(\bar{A} \cap \bar{B}) \cup (\bar{A} \cap A) \cup (B \cap \bar{B}) \cup (B \cap A)}$

$\cancel{(\bar{A} \cap \bar{B}) \cup \phi \cup \phi \cup (B \cap A)}$

$\cancel{(\bar{A} \cap \bar{B}) \cup (B \cap A)}$

$= \cancel{(\overline{A \cup B}) \cap (\bar{A} \cap A)}$

$\underline{A \Delta B = (B \cap \bar{A}) \cup (A \cap \bar{B})}$

$\overline{A \Delta B} = \overline{(B \cap \bar{A}) \cup (A \cap \bar{B})} = \overline{(B \cap \bar{A})} \cap \overline{(A \cap \bar{B})}$

$(\bar{B} \cup A) \cap (\bar{A} \cup B)$

$(\bar{B} \cap \bar{A}) \cup ($

Using the laws of set Theory, simplify the following:-

(i) $A\cap(B-A)$ (ii) $(A-B)\cup(A\cap B)$

(iii) $\bar{A}\cup\bar{B}\cup(A\cap B\cap\bar{C})$

ans $A\cap(B-A) = A\cap(B\cap\bar{A})$ $\because A-B=\bar{A}\cap B$

$=(A\cap\bar{A})\cap B = \phi\cap B = \underline{\underline{\phi}}$

(ii) $(A-B)\cup(A\cap B)$ $(A\cap\bar{B})\cup(A\cap B)$

$\cancel{\Rightarrow (A\cup A)\cap(A\cup B)\cap(\bar{B}\cup A)\cap(\bar{B}\cap B)}$

$= \cancel{A\cap(A\cup B)\cap(\bar{B}\cup A)\cap\phi}$

$\cancel{(A\cup B)\cap\phi}$

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Prove the following (for any sets A, B, C):

i) If $A \cap C = B \cap C$ and $A \cup C = B \cup C$ then $A = B$

ii) If $A \Delta C = B \Delta C$ then $A = B$

ans i) Take any $x \in A \cup C$ $x \in A$ or $x \in C$

$x \in B \cup C$ (as $A \cup C = B \cup C$) $\Rightarrow x \in B$ or $x \in C$

If $x \in A$ and $x \in B \Rightarrow A$ is subset of B

If $x \in C$, then $x \in A \cap C = B \cap C$ so that $x \in B$

Thus in this case $A \subseteq B$.

Next Take any $y \in B$

$\Rightarrow y \in B \cup C = A \cup C$, $y \in A$ or $y \in C$

If $y \in A$, then it follows that B is subset of A.

If $y \in C$, then $y \in B \cap C = A \cap C$, so $y \in A$

Thus in this case $B \subseteq A$

So, we have proved that $A \subseteq B$ and $B \subseteq A$ which means that $\underline{\underline{A = B}}$

ii ans) ~~Take any $x \in A$, then $x \in A \cup C$~~

We have $A \Delta C = (C \cap \bar{A}) \cup (A \cup \bar{C})$ [$\because$ using

$A \Delta B = (B \cap \bar{A}) \cup (A \cap \bar{B})$

and $B \Delta C = (C \cap \bar{B}) \cup (B \cap \bar{C})$

Take any $x \in A$, suppose $x \in C$ then

$x \notin (C \cap \bar{A})$ and $x \in (A \cap \bar{C})$ and as such

$x \notin [(C \cap \bar{A}) \cup (A \cap \bar{C})]$

$x \notin A \Delta B \Rightarrow x \notin B \Delta C$ (as $A \Delta B = B \Delta C$)

Therefore, $x \in B$, because if $x \notin B$, then $x \in \bar{B}$ so that $x \in (C \cap \bar{B})$ which in turn yields $x \in B \Delta C$.

Thus if $x \in C$ we have $A \subseteq B$.

Next suppose $x \notin C$. Then $x \in \bar{C}$ so that $x \in (A \cap \bar{C})$

This yields $x \in A \Delta C$

Since $A \Delta C = B \Delta C$, it follows $x \in B \Delta C$

Therefore $x \in C \cap \bar{B}$ or $x \in B \cap \bar{C}$

URBAN EDGE

Since $x \notin C$, this yield $x \in B$
Thus if $x \notin C$, we have $A \subseteq B$.
We have proved that $A \subseteq B$. In similar way it follows that $B \subseteq A$.
Thus if $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$.

Addition Principle:

$|A|$ or $O(A)$ = no. of elements in the set $= n(A)$ or cardinality of A or order of A.

If A and B are 2 sets (non-empty finite sets)

1. $|A \cup B| = |A| + |B| - |A \cap B|$
2. $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

For disjoint sets A and B:
$|A \cup B| = |A| + |B| - \phi$
$\Rightarrow |A \cup B| = |A| + |B|$

→ Suppose we consider the union of 2 finite sets A and B & we want to determine cardinality of $A \cup B$ which is obviously a finite set.
Counting the no. of elements in $A \cup B$, we include all elements in $A \cup B$, we include all elements in A and B, but exclude all elements common to both A and B.
This principle of inclusion and exclusion is called Addition principle or rule.

Note :- The addition can be extended to three or more finite sets.
(ex) For three non-empty finite sets:
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

PROBLEMS:

1. A computer company requires 30 programmers to handle systems programming job, 40 programmers for applications. If the company appoints 55 programmers to carryout these jobs.

i) How many handling only system programming jobs.

ii) How many handling only application programming.

ans Let A denote the set of programmers who handle system programming and B denote set of programmers who handle applications programming.

Then $A \cup B$ is the set of programmers appointed to carry out these jobs

Let $|A| = 30$ $|B| = 40$, $|A \cup B| = 55$

By addition rule $= |A \cup B| = |A| + |B| - (A \cap B|$

$|A \cap B| = |A| + |B| - |A \cup B|$

$= 30 + 40 - 55$

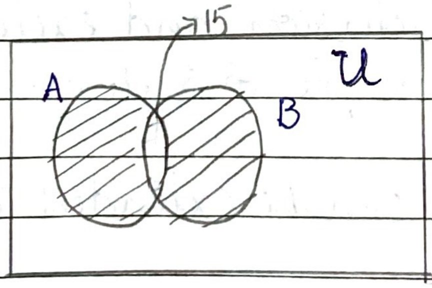
15 programmers handling both jobs

No. of programmers handling only system programming jobs.

$|A - B| = |A| - |A \cap B| = 30 - 15 = 15$

No. of programmers handling only application

$|B - A| = |B| - |B \cap A| = 40 - 15 = 25$



URBAN EDGE

2.) A survey of 500 television viewers of a sports channel produced the following information 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football 70 watch cricket & Hockey, 50 watch Hockey and football & 50 do not watch any of the three kinds of games. Find the no. of viewers who watch.

i) all the three kinds of games.

ii) exactly one of the sports.

(iii) Watch only Hockey & football but not cricket.

(iv) only football.

$n(C) = 285 \quad n(H) = 195 \quad n(F) = 115$

$n(C \cap F) = 45 \quad \cancel{n(C \cap H) = 70} \quad \cancel{n(}$

$n(C \cap H) = 70 \quad n(H \cap F) = 50$

$U = 500 =$ Total no. of TV viewers.

$n(\overline{C \cap H \cap F}) = 50 \quad n(C \cup H \cup F) = 500 - 50 = 450$

gen. formula

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$\quad - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

$\cancel{n(C \cup H \cup F)} = \cancel{285 + 195} + \cancel{115 - 45 - 70 - 50} +$
$\cancel{450}$

$n(C \cup H \cup F) = |C| + |H| + |F| - |C \cap H| - |C \cap F| + |C \cap H \cap F|$

$450 = 285 + 195 + 115 - 45 - 50 - 70 + |C \cap H \cap F|$

$= 20 = |C \cap H \cap F|$

The no. of viewers watching all 3 games is 20.

ii) Exactly one of the sports:

only cricket viewers $= |C1| = |C - H - F| =$

$= |C| - |C \cap H| - |C \cap F| + |C \cap H \cap F|$

$|C1| = 285 - 70 - 45 + 20$

$= 190.$

$|H1| = n(H) - n(H \cap F) - n(C \cap H) + |C \cap H \cap F|$

$= 195 - 50 - 70 + 20$

$= \underline{95}$

(iv) only Football

$|F1| = |F - C - H| =$

$n(F) - n(H \cap F) - n(C \cap F) + |C \cap H \cap F|$

$= 115 - 50 - 45 + 20$

$= \underline{\underline{40}}$

(iii) Hockey and Football but not cricket

$n(H \cap F) - n(H \cap C \cap F)$

$= 50 - 20$

$= \underline{\underline{30}}$

PROBABILITY:

→ Probability is the branch of mathematics that deals with the study of possible outcome of given events with the outcomes relative likelihoods & distributions.

→ Consider the set of all possible outcomes of a random trial; like tossing of a coin, Throwing a die etc where each outcome has the same likelihood of occurance. Such a set is called the sample space of the ~~the~~ trial.

→ Sample space is the set of all possible outcomes. ~~of~~ So it is also known as event space / possibility space.

→ Event: Subset of sample space, consisting of favorable outcomes is called an event.

→ Suppose S is a non-empty finite sample space & A is an event. Then $P(A) = \frac{|A|}{|S|} \longrightarrow (1)$ is called the probability that A occurs or probability of A.

In general,

$$p(\text{event}) = P(A) = \frac{\text{No. of favorable outcomes}}{\text{Total no. of outcomes}} = \frac{n(A)}{n(S)}$$

Note: 1) Since $|A| \geq 0$ and $|S| > 0$, we have $P(A) \geq 0$

2) If $|A| = 0$ then $P(A) = 0$.

3) Since A is subset of S we have $|A| \leq |S|$ so that $P(A) \leq 1$ with the inequality holding only if $A = S$. Thus $P(A)$ satisfy the inequality $0 \leq P(A) \leq 1$

Probability of non-occurance of an Event

Since A is the set of all favorable outcomes, its complement $(\bar{A})$ in S is the set of all non-favorable outcomes.

Then the probability of $\bar{A}$ is given by the formula

$$P(\bar{A}) = \frac{|\bar{A}|}{|S|} = \frac{|S|-|A|}{|S|} = \underline{1-P(A)} \longrightarrow (2)$$

Note: $P(\bar{A}) = 0$ iff $P(A) = 1$ &
$P(\bar{A}) = 1$ iff $P(A) = 0$

Addition Theorm:-

1m

Suppose A and B are two events in a sample space, then $A \cup B$ is an event S consisting of outcomes that are in A or B or both and $A \cap B$ is an event in S consisting of events common to A and B.

Accordingly by the principle of addition, we have,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$P[A \cup B] = \frac{\cancel{P}|(A)| + \cancel{P}(B)| - \cancel{P}(A \cap B)|}{|S|} = \frac{\cancel{P}|(A \cup B)|}{|S|}$$

$$= \frac{|A|}{|S|} + \frac{|B|}{|S|} - \frac{|A \cap B|}{|S|}$$

$$P(A) + P(B) - P(A \cap B)$$

This result is called the addition theorm for probability.

Mutually exclusive sets:-

Two events A and B in a sample space are said to be mutually exclusive if $A \cap B = \phi$

Then $P(A \cap B) = 0$. and the addition theorm reduces to $P(A \cup B) = P(A) + P(B)$

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Conditional Probability

If E is an event in a finite sample space S with $P(E) > 0$, then the probability that an event A in S occurs when E has already occured is called the probability of A relative to E or the conditional probability of A given E. This is probability, denoted by $P\left(\frac{A}{E}\right)$, which is defined by

$$P\left(\frac{A}{E}\right) = \frac{|A \cap E|}{|E|} \Rightarrow |A \cap E| = |E| \cdot P\left[\frac{A}{E}\right]$$

so that $$P(A \cap E) = \frac{|A \cap E|}{|S|} = \frac{|E| \cdot P(A/E)}{|S|} = \frac{|E|}{|S|} \cdot P(A/E)$$
$$= \underline{P(E) \cdot P(A/E)}$$

AXIOMS OF PROBABILITY:

Let S denote a sample space with probability measure P defined over it, such that probability of an event $A \subseteq S$ is given by $P(A)$. Then the probability obeys the following axioms.

(1) $P(A) \geq 0$

(2) $P(S) = 1$

(3) If $\{A_1, A_2, \ldots A_j\}$ is a sequence of mutually exclusive events such that $A_i \cap A_j = \phi$ $\forall i, j$ then
$P(A_1 \cup A_2 \cup \cdots \cup A_j \cup \cdots) = P(A_1) + P(A_2) + \cdots + P(A_j) + \cdots$

(4) The conditional probability of A given B is defined by
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

5) The events A, B are said to be statistically independent if $P(A \cap B) = P(A) \cdot P(B)$.

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PROBLEMS

1. A card is drawn from a well sheuffled pack of cards(52). Find the probability that the card of drawn will be

(i) Red (ii) Black queen (iii) King of diamond

ans i) No. of. red cards = 26

Total no. of. cards = 52

$$P(R) = \frac{\text{No. of. red cards}}{\text{Total no. of. cards}} = \frac{26}{52} = \frac{1}{2}$$

ii) No. of. black queens = 2

Total no. of. cards = 52

$$P(BQ) = \frac{2}{52} = \frac{1}{26}$$

iii) King of diamond = 1

Total no. of. cards = 52

$$P(KD) = \frac{\text{no. of. king of diamond}}{\text{Total no. of. cards}} = \frac{1}{52}$$

Face cards = J, Q, K = 3 cards.

Mutually Independent events:

The events A and E in a sample space S are said to be mutually independent if probability of occurance of A is independent of probability of occurance of E. So that $P(A) = P\left(\frac{A}{E}\right)$. For such events $P(A \cap E) = P(A) \cdot P(E)$.

[$\because P(A \cap E) = P(E) \cdot P(A/E) \rightarrow$ cond. probability

This result is called the product rule/Multiplication ~~Rule~~ theorm for the mutually independent events.

URBAN EDGE

PROBLEMS:

1. A message communicated through a channel consists of zero's & one's that is transmitted. Because of noise in the channel, a 1 that is transmitted could be received as 0 & vice versa. The probability that a transmitted 1 is received as zero is 0.05 & the probability that a transmitted 0 is received as 1 is 0.1.

Find the probability that

i) The message 101 is received as 110

ii) 011 received as 001 and

iii) 110 is received correctly.

ans From the given, the probability that transmitted 1 is received as 0 is $P_1 = 0.05$. The probability that 0 is received as 1 is $P_2 = 0.1$.

Therefore probability that transmitted 1 is received as 1 is $P_3 = 1 - P_1 = 1 - 0.05 = 0.95$

Probability that transmitted 0 is received as 0 is $P_4 = 1 - P_2 = 1 - 0.1 = 0.9$.

Further note that the events of receiving the signals 0 and 1 are mutually independent.

i) The probability of receiving the message 101 as 110 is $P_3 \times P_2 \times P_1 = 0.95 \times 0.1 \times 0.05 = 0.00475$

ii) The prob of receiving message 011 as 001 is $P_4 \times P_1 \times P_3 = 0.9 \times 0.95 \times 0.05 = 0.04275$

iii) 110 received correctly $= P_3 \times P_3 \times P_4 = 0.81225$

2. Find the prob of the occurance of exactly 2 heads in 3 tosses of an honest coin

ans When an honest coin is tossed, head (H) or tail (T) occurs with equal likelihood. Thus for each toss, there are 2 possible outcomes.

$\therefore$ No. of outcomes in these tosses $= 2 \times 2 \times 2 = 2^3 = 8$.

URBAN EDGE

$S = \{HHH, HTH, HTT, THH, THT, TTT, HHT, TTH\}$

The no of favorable outcomes which contains exactly 2 heads are = HHT, THH, HTH

prob of occurance = $\frac{3}{8}$

3. The prob the IC will have defective etching is 0.12, the prob that they will have a crack defect is 0.29 and the " " " both defects is 0.07. What is the prob that a newly manufactured chip will have;
 i) an etching / crack defect.
 ii) neither defect.

ans Let A is the event that chip has etching defect & B is the event that it has crack defect.
Then we have $P(A) = 0.12$ $P(B) = 0.29$ $P(A \cap B) = 0.07$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.12 + 0.29 - 0.07 = 0.34$

(i) $P(\overline{A \cup B}) = 1 - P(A \cup B)$
$= 1 - 0.34 = \underline{\underline{0.66}}$

4. If A and B be events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ & $P(A \cup B) = \frac{1}{2}$. Find (a) $P(A/B)$ (b) $P(B/A)$ (c) $P(A \cap B^{c})$ (d) $P(A/B^{c})$.

ans. (a) $P(A/B) = \frac{P(A \cap B)}{P(B)}$

using Addition Theorm

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{1}{3} + \frac{1}{4} - \frac{1}{2} = \frac{4+3-6}{12} = \underline{\underline{\frac{1}{12}}}$

$$P\left(\frac{A}{B}\right) = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{4}{12} = \frac{1}{3}$$

b) $P\left(\frac{B}{A}\right) = \frac{P(B\cap A)}{P(A)} = \frac{\frac{1}{12}}{\frac{1}{3}} = \frac{1}{4}$

c) $P(A\cap B^c) = P(A) - P(A\cap B)$

$= \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$ //

d) $P(A/B^c) = \frac{P(A\cap B^c)}{P(B^c)} = \frac{\frac{1}{4}}{1-P(B)} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$ //

5. If A and B are events with $P(A\cup B) = 7/8$, $P(A\cap B) = \frac{1}{4}$ and $P(\bar{A}) = 5/8$, find $P(A)$, $P(B)$ and $P(A\cap\bar{B})$

6. Two dice are thrown. (a) the prob of getting odd number on one and a multiple of 3 on another.

b) one of the dice showed 3 and the sum of 2 dice is 9

c) Sum on the 2 dice is 9

d) Sum on 2 dice is 13

6 ans List of all possible outcomes $(6\times6=36)$

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
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(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

(a) The prob of getting an odd no & multiple of 3 on another $= \frac{11}{36}$

(b) one die should 3, Sum on other is 9 $= \frac{2}{36} = \frac{1}{18}$

(iii) $\frac{4}{36}$

(iv) 0

6) Biconditional/double implication ($p \longleftrightarrow q$).

1) Negation:
A proposition obtained by inserting the word 'not' at an appropriate place in a given proposition is called negation of the given proposition.

(ex)- p: 7 is a prime number True/1
$\sim p$: 7 is not a prime no. False/0.

Truth Table: or

p	$\sim p$
T	F
F	T

p	$\sim p$
1	0
0	1

2) CONJUNCTION:
A compound proposition obtained by ~~defining~~ combining two given propositions, by inserting the word 'and' in between them is called conjunction of given propositions

(ex) p: $\sqrt{2}$ is an irrational no
q: 9 is a prime number.

And is denoted by '$\wedge$'

$p \wedge q$: $\sqrt{2}$ is an irrational number and 9 is a prime number.

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \wedge q$
1	1	1
0	1	0
1	0	0
0	0	0

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3) DISJUNCTION:

A compound proposition is obtained by combining the given 2 propositions by inserting the word 'or' in between them is called disjunction of the given propositions, symbolically denoted by 'V'.

(ex) p: $\sqrt{2}$ is an irrational number

q: 9 is a prime number.

p v q is read as 'p ~~and~~ or q'

p v q: $\sqrt{2}$ is an irrational number or 9 is a prime number.

Truth Table:

p	q	p v q
0	0	0
0	1	1
1	0	1
1	1	1

4) EXCLUSIVE DISJUNCTION:

In the conjunction p v q of 2 propositions p & q the symbol V is used in the exclusive sense

That is p $\underline{\vee}$ q is taken to be true when both p and q or p or q is true

Truth Table:

p	q	p $\underline{\vee}$ q
0	0	0
0	1	1
1	0	1
1	1	1

5) CONDITIONAL PROPOSITION: ($p \rightarrow q$)

A compound proposition obtained by combining 2 proposition by inserting the word if, then at appropriate places are called a conditional proposition

(ex) p: $\sqrt{2}$ is an irrational no.

q: 9 is a prime number

$p \rightarrow q$ is read as 'p implies q'

if $\sqrt{2}$ is an irrational no, then 9 is a prime number.

$p \rightarrow q \neq q \rightarrow p$

Truth Table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

6) BICONDITIONAL ($p \longleftrightarrow q$)

URBAN EDGE

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