Module 4. (IT)
Sampling Distribution

Q.No		Question	Marks
1	a)	Explain the following i) Null hypothesis ii) Alternative hypothesis iii) Type I and type II error iv) Level of significance v) Standard error	10
	b)	A population has mean 75 and standard deviation 12. a) Random samples of size 121 are taken. Find the mean and standard deviation of the sample. b) How would the answers to part a) change if the size of the samples were 400 instead of 121?	5
2	a)	A population has mean 5.75 and standard deviation 1.02. a) Random samples of size 81 are taken. Find the mean and standard deviation of the sample. b) How would the answers to part a) change if the size of the samples were 25 instead of 81?	5
	b)	The weights of 1500 ball bearings are normally distributed with a mean of 635 gms and S.D of 1.36gms. If 300 random samples of size are drawn from this population, determine the expected mean and S.D of the sampling distribution of means if sampling is done a) with replacement b) without replacement.	5
3	a)	A population consists of 4 numbers 3, 7, 11, 15. a) Find the mean and S.D. of the sampling distribution of means by considering samplings of size 2 with replacement. b) If n, n denotes respectively the population size and sample size, σ and $\sigma_{\bar{x}}$ respectively denotes population S.D. and S.D. of the sampling distribution of means without replacement. i. $\sigma_{\bar{x}}{}^2 = \frac{\sigma^2}{n}\left[\frac{N-n}{N-1}\right]$ ii. $\mu_{\bar{x}} = \mu$ where $\mu_{\bar{x}}$ is the mean of this distribution and μ is the population mean.	10
	b)	Certain tubes manufactured by a company have mean life time of 800 hours and S.D of 60hours. Find the probability that a random sample of 16 tubes from the group will have a mean life time a) between 790 hours and 810 hours b) less than 785 hours c) more than 820 hours d) between 770 hours and 830 hours.	10
4	a)	A prototype automotive tire has a design life of 38500 miles with S.D. of 2500 miles. Five such tires are manufactured and tested. On the assumption that the actual population S.D. is 2500 miles, find the probability that the sample mean will be less than 36000 miles. Assume that the distribution of lifetimes of such tires is normal.	5
	b)	An automobile battery manufacturer claims that its midgrade battery has a mean life of 50 months with a S.D. of 6 months. Suppose the distribution of battery lives of this particular brand is approximately normal. a) On the assumption that the manufacturer claims are true, find the probability that a randomly selected battery of this type will last less than 48 months. b) On the same assumption, find the probability that mean of a random sample of 36 such batteries will be less than 48 months.	5
5	a)	The weights of 1500 ball bearings are normally distributed with a mean of 635 gms and	10

		S.D. of 1.36 gms. If 300 random samples of size 36 are drawn from this population. In the case of random sampling with replacement, find how many random samples would have their mean a)between 634.76gms and 635.24 gms, b) greater than 635.6 gms, c) less than 634.2 gms, d)less than 634.5 gms or more than 635.24 gms	
	b)	500 ball bearings have a mean weight of 142.30 gms and S.D. of 8.5 gms. Find the probability that a random sample of 100 ball bearings chosen from this group will have a combined weight a) between 14061 and 14175 gms b) more than 14460 gms	5
6	a)	The mean and S.D of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find a) 95% b) 99% confidence limits for mean of the maximum loads of all cables by the company.	10
	b)	A sample of 900 men is found to have a mean height of 64inch. If this sample has been drawn from a normal population with standard deviation 20 inch, find the 99% confidence limits for the mean height of the men in the population.	5
7	a)	A sample of 5000 students in a college was taken and their average height was found to be 62.5Kg with a standard deviation of 22kg. Find the 95% confidential limits of the average weight of the students in the entire University.	5
	b)	Systolic blood pressure of 566 males was taken. Mean BP was found to be 128.8mm and SD 13.05mm. Find 95% confidence limits of BP within which the populations mean would lie.	5
8	a)	Standard deviation of blood sugar level in a population is 6 mg%. If population mean is not known, within what limits is it likely to lie if a random sample of 100 has a mean of 80mg%?	5
	b)	To know the mean weights of all 10 year old boys in Delhi a sample of 225 was taken. The mean weight of the sample was found to be 67 pounds with s.d. of 12 pounds. What can we infer about the mean weight of the population?	5
9	a)	The mean and S.D of the diameters of a sample of 250 rivet heads manufactured by a company are 7.2642 mm and 0.0058mm respectively. Find (a) 99% (b) 95% confidence limits for the mean diameter of all the rivet heads manufactured by the company.	10
	b)	Spring break can be a very expensive holiday. A sample of 80 students is surveyed, and the average amount spent by students on travel and beverages is \$593.84. The sample standard deviation is approximately \$369.34.Construct a 95% confidence interval for the population mean amount of money spent by spring breakers.	5
10	a)	400 items are sampled from a normally distributed population with a sample mean $\bar{x}$ of 22.1 and a population standard deviation(σ) of 12.8. Construct a 95% confidence interval for the true population mean.	5
	b)	The mean and S.D. marks of a sample of 100 students are 67.45 and 2.92 respectively. Find (a) 95% (b) 99% confidence intervals for estimating the marks of the population.	10
11	a)	A machine is expected to produce nails of length 3 inches. A random sample of 25 nails gave an average length of 3.1 inch with standard deviation 0.3. Can it be said that the machine is producing nails as per specification?($t_{0.05}$ for 24 d.f. is 2.064)	
	b)	Ten individuals are chosen at random from a population and their heights in inches are found to be 63,63,66,67,68,69,70,70,71,71. Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.05}$=2.262 for 9 d.f.)	10
12	a)	A certain stimulus administered to each of 12 patients resulted in the following increases of blood pressure: 5,2,8,-1,3,0,-2,1,5,0,4,6. Can it be concluded that the stimulus will in general be accompanied by an increase in blood pressure. ($t_{0.05}$ for 11 d.f. is 2.2)	10
	b)	A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a standard deviation of 0.04 inch. On the basis of this sample, would you say that the work is inferior? ($t_{0.05}$=2.262 for 9 d.f.)	5

<table>
<tr><td>13</td><td>a)</td><td>Show that 95% confidence limits for the mean μ of the population are $\bar{x} \pm \frac{\sigma_s}{\sqrt{n}} t_{0.05}$.
Deduce that for a random sample of 16 values with mean 41.5 inches and the sum of the squares of the deviation from the mean 135 sq inches and drawn from a normal population, 95% confidence limits for the mean of the population are 39.9 and 43.1 inches.</td><td>5</td></tr>
<tr><td></td><td>b)</td><td>A random sample of 10 measurements of the diameter of a sphere gave a mean of 12 cm and standard deviation 0.15 cm. Find 95% confidence limits for the actual diameter. ($t_{0.05}$=2.262 for 9 d.f.)</td><td>5</td></tr>
<tr><td>14</td><td>a)</td><td>A random sample of 10 boys had the following I.Q.: 70, 120, 110, 101, 88, 83,95, 98, 107,100. Do these data support the assumption of a population mean I.Q. of 100 at 5% level of significance</td><td>10</td></tr>
<tr><td></td><td>b)</td><td>A random sample of size 25 from a normal population has the mean 47.5 and s.d 8.4. Does this information refute the claim that the mean of the population is 42.1.</td><td>5</td></tr>
<tr><td>15</td><td>a)</td><td>A process for making certain bearings is under control if the diameter of the bearings have the mean 0.5 cm. What can we say about this process if a sample of 10 of these bearings has a mean diameter of 0.506 cm. and S.D. of 0.004cm?($t_{0.05}$=2.262 for 9 d.f.)</td><td>5</td></tr>
<tr><td></td><td>b)</td><td>A machine is supposed to produce washers of mean thickness 0.12cm. A sample of 10 washers was found to have a mean thickness of 0.128cm and standard deviation 0.008. Test whether the machine is working in proper order at 5% level of significance.</td><td>5</td></tr>
<tr><td>16</td><td>a)</td><td>The number of automobile accidents per week in a certain community are as follows: 12,8,20,2,14,10,15,6,9,4. Are these frequencies in agreement with the belief that the accident conditions were the same during this 10 week period ?</td><td>10</td></tr>
<tr><td></td><td>b)</td><td>A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4:3:2:1 for the respective categories. ($\chi^2_{0.05}$=7.81 for 3 d.f.)</td><td>10</td></tr>
<tr><td>17</td><td>a)</td><td>The following figures show the distribution of digits in numbers chosen at random from a telephone directory. Test whether the digits may be taken to occur equally frequently in the directory.
<table>
<tr><td>Digits</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>1026</td><td>1107</td><td>997</td><td>966</td><td>1075</td><td>933</td><td>1107</td><td>972</td><td>964</td><td>853</td></tr>
</table></td><td>10</td></tr>
<tr><td></td><td>b)</td><td>Fit a Poisson distribution for the following data and test the goodness of fit given that ($\chi^2_{0.05}$=7.81 for 3 d.f.)
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>f</td><td>122</td><td>60</td><td>15</td><td>2</td><td>1</td></tr>
</table></td><td>10</td></tr>
<tr><td>18</td><td>a)</td><td>The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given below. Test the goodness of fit in respect of Poisson distribution of fit to the given data ($\chi^2_{0.05}$=9.49 for 4 d.f)
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>f</td><td>173</td><td>168</td><td>37</td><td>18</td><td>3</td><td>1</td></tr>
</table></td><td>10</td></tr>
<tr><td></td><td>b)</td><td>In experiments on pea breeding, the following frequencies of seeds were obtained:
<table>
<tr><td>Round & yellow</td><td>Wrinkled & yellow</td><td>Round & green</td><td>Wrinkled & green</td><td>Total</td></tr>
<tr><td>315</td><td>101</td><td>108</td><td>32</td><td>556</td></tr>
</table>
Theory predicts that the frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment.</td><td>10</td></tr>
</table>

19 a) 200 digits were chosen at random from a set of tables. The frequencies of the digits are shown below. Use the chi square test to assess the correctness of the hypothesis that the digits were distributed in equal number in the tables from which these were chosen. **10**

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	18	19	23	21	16	25	22	20	21	15

b) Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0.05. **10**

x	0	1	2	3	4
f	419	352	154	56	19

20 a) A pair of dice are shown 360 times and the frequency of each sum is indicated below. Would you say that the dice are fair on the basis of the chi square test at 0.05 level of significance? **10**

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	8	24	35	37	44	65	51	42	26	14	14

b) In 1000 extensive sets of trials for an event of small probability, the frequencies f_0 of the number x of successes proved to be: **10**

x	0	1	2	3	4	5	6	7
f_0	305	366	210	80	28	9	2	1

Fit a Poisson distribution to the data and test the goodness of fit.