DMS Question Bank

MODULE 1:

1. Illustrate the different operations on Sets with suitable Venn diagram representation.

2. State the principal of inclusion- exclusion, principal of disjunctive counting and Principal of duality for the equality of sets with an example.

3. An integer is selected at random from 3 through 17 inclusive. If A is the event that a number 3 is divisible by chosen and B is the event that the chosen exceeds 10. Determine Pr (A), Pr (B), Pr ($A \cap B$), Pr ($A \cup B$).

4. Prove that S and T are disjoint if and only if SUT = $S \Delta T$, if S, T $\subseteq$ U

5. For any two sets A and B, prove that: (i)$A - (A - B) = A \cap B$ (ii) $A - B = A - (A \cap B)$

6. State the Laws of Set Theory.

7. Determine and prove for any three sets A, B, C that (A - B) - C = A - (BUC) = (A-C) - (B-C).
   (i)Determine the sets A, B where A-B= {1, 3, 7, 11}, B-A= {2, 6, 8} and $A \cap B$ = {4,9}.
   (ii) Determine the sets C, D where C-D= {1,2 ,4}, D-C= {7,8} and $C \cup D$= {1, 2, 4, 5, 7, 8, 9}

8. A girl rolls a fair die three times. What is the probability that (a) her second and third rolls are both larger than her first roll? (b) The result of her second roll is greater than that of the first roll and the result of her third roll is greater than that of the second?

9. A survey of 500 television viewers of a sports channel produced the following information: 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football, 50 do not watch any of the three kinds of the games.

   a) How many viewers in the survey watch all three kinds of games?
   b) How many viewers watch exactly one of the sports?

10. Using Venn diagram, For any three sets A,B,C. Prove that $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

11. Use membership table, Prove the Distributive law and De-Morgan’s law.

12. Let U= {1,2,3,4,5,6,7,8,9}, A= {1,2,4,6,8}, B= {2,4,5,9}, C= {x| x is a positive integer and x^2<=16}. Determine the following:
    a) AU(B-C) b) (A-B)-C c) (AUB)-C d) (AUB) – ($B \cap C$) e) (AUB)-C

13. Using truth table prove that the following are logically equivalent:
    a) $(P \veebar Q)$ and $(P \vee Q) \wedge (\sim P \rightarrow \sim Q)$ b) $[P \rightarrow (Q \wedge R)] \Longleftrightarrow [(P \rightarrow Q) \wedge (P \rightarrow R)]$

MODULE 2:

1. Construct the truth table for the following:

   i) $(p \vee \neg q)$ **ii)** $\neg (p \wedge q)$ **iii)** $p \rightarrow (p \vee \neg q)$ **iv)** $p \vee (q \vee r)$

2. Determine the validity of the following argument:

   If I study, then I will not fail in examination.
   If I do not watch TV in the evenings, I will study.
   I failed in the examination.

   ∴ I must have watched TV in the evenings.

3. Examine the logical equivalence between the following compound propositions.
   i. $[p \wedge (p \rightarrow q) \wedge r] \Rightarrow [(p \vee q) \rightarrow r]$
   ii. $\{[(p \vee (q \vee r)] \wedge \neg q\} \Rightarrow p \vee r$

4. Explain in detail all the Rules of Inference.

5. Determine whether the following argument is valid for which the universe is the set of all students

   No Engineering student is bad in studies
   Anil is not bad in studies

   ∴ Anil is an Engineering student

6. Test the validity of the following argument:

   I will get grade A in this course or I will not graduate.
   If I do not graduate, I will join the army
   I got grade A

   ∴ I will not join the army.

7. Prove the following logical equivalence without using truth table
   i) $[\neg p \wedge (\neg q \vee r)] \vee [(q \wedge r) \vee (p \wedge q)] \Leftrightarrow r$
   ii) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$

8. For any prepositions p, q, r proves the following:
   i) $p \uparrow (q \uparrow r) \Leftrightarrow \neg p \vee (q \wedge r)$
   ii) $p \downarrow (q \downarrow r) \Leftrightarrow \neg p \vee (q \vee r)$

9. Show that for any two proposition p and q,
   i) $(P \veebar Q) \vee (P \leftrightarrow Q)$ is a tautology
   ii) $(P \veebar Q) \wedge (P \leftrightarrow Q)$ is a contradiction
   iii) $(P \veebar Q) \wedge (P \rightarrow Q)$ is a contingency

10. Consider the following open statements with set of all real numbers as the universe.
    $p(x): x \geq 0$, $q(x): x^2 \geq 0$, $r(x): x^2 - 3x - 4$, $s(x): x^2 - 3 > 0$.
    Determine the truth value of the following statements.
    i) $\exists x, p(x) \wedge q(x)$ ii) $\forall x, p(x) \rightarrow q(x)$ iii) $\forall x, r(x) \vee s(x)$ iv) $\forall x, r(x) \rightarrow p(x)$

11. Test the validity of the following argument:

$(\sim p \vee q) \rightarrow r$

$r \rightarrow (s \vee t)$

$\sim s \wedge \sim u$

$\sim u \rightarrow \sim t$

$\therefore p$

MODULE 3:

1. Define the Pigeonhole Principle with its generalization. Also determine how many persons must be chosen in order that at least five of them will have the birth days in the same calendar month.

2. Examine with a valid proof that a function f: A → B is invertible if and only if it is one-to-one and onto.

3. With a neat diagram explain various types of functions.

4. Define Cartesian products of two sets. For non-empty sets A, B, C.
Prove that $(i) A \times (B - C) = (A \times B) - (A \times C)$ (ii) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

5. Let A={1, 2, 3} and B= {-1, 0} and R be a relation from A to B defined by R= {(1,1), (1,0), (2,-1), (3,0)}. Is R a function from A to B

6. Let f and g be functions from R to R defined by f(x) =ax+b and $g(x)=1-x+x^2$. If $(g \circ f)(x) = 9x^2-9x+3$, determine a, b.

7. Consider the function f and g defined by $f(x) = x^3$ and $g(x) = x^2 +1$, for all x belongs R. Find$(g \circ f)$, $(f \circ g)$, f^2 and g^2.

8. Prove that, If a function f: A → B is invertible then it has a unique inverse. Further if f(a) = b then $f^{-1}(b) = a$

9. Show that:
   (a) if any 5 number from 1 to 8 are chosen, then two of them will have their sum equal to 9.
   (b) If 30 dictionaries in a library contain a total of 61,327 pages, then at least one of the dictionaries must have at least 2044.

10. Let A={α, β, γ}, B={θ, η}, C={λ, μ, υ}. Find (A U B) X C, A U (B X C), (A X B)U C and A X (B U C)

11. Let f, g, h be functions from Z to Z defined by f(x)=x-1, g(x)=3x and

$$h(x)=\begin{cases} 0 \text{ if } x \text{ is even} \\ 1 \text{ if } x \text{ is odd} \end{cases}$$

Determine $((f \circ (g\circ h))(x)$ and $((f \circ g)\circ h)(x)$ and verify that $f \circ (g \circ h) = (f\circ g)\circ h$.

MODULE 4:

1. Prove that the group (Z_4, +) is cyclic. Find all its generators.
2. Let R and S be relations on a set A. Prove the following:
   a. If R and S are reflexive, so are R∩S and RUS.
   b. If R and S are symmetric, so are R∩S and RUS.
   c. If R and S are anti-symmetric, so is R∩S.
   d. If R and S are transitive, so is R∩S.
3. Consider the symmetric group S3 and the subgroup H = {p0, p5} thereof. Find all the right cosets of H in S3 and hence obtain a right coset decomposition of S3
4. Define a cyclic group. Prove that every cyclic group is abelian but converse is not true.
5. Define partial order. Let A= {1, 2, 3, 4, 6, 8, 12}. On A, define the relation R by aRb if and only if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation.
6. Define abelian group. Prove that a group is abelian if and only if for all a,b belongs to G $(a,b)^{-1}=a^{-1}b^{-1}$
7. Define Relations and its properties with an example.
8. Prove that, for any elements a, b, in a group G, we have
   (i) $(a^{-1})^{-1} = a$ (ii) $(ab)^{-1} = b^{-1}a^{-1}$
9. Define i) Group ii) Sub-group iii) coset iv) abelian group with an example.
10. Prove that a group G is abelian if and only if $(ab)^2= a^2b^2$ for all a, b ϵ G.
11. Let A= {1, 2, 3, 4, 6} and R be a relation on A defined by aRb if and only if a is a multiple of b. represent the relation R as a matrix and draw its digraph.
12. Let A= {a, b, c, d} and R be a relation on A that has the matrix

$$M_R=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Construct the digraph of R and list the in-degrees and out-degrees of all vertices.

MODULE 5:

1. If the letters of the acronym WYSIWYG are arranged in random order, find the probability that
    a. The arrangement has both consecutive W's and consecutive Y's
    b. The arrangement starts and ends with the letter W.

2. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?

3. How many arrangements are there for all letters in the word SOCIOLOGICAL? In how many of these arrangements, i) A & G are adjacent ii) all the vowels are adjacent.

4. In a class of 10 students, five are to be chosen and seated in a row for a picture. How many such linear arrangements are possible?

5. In how many ways can eight men and eight women be seated in a row if (a) any person may sit next to any other (b) men and women must occupy alternate seats (c) generalize this result for n men and n women

6. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun or byte occurs?

7. Find the number of integers between 1 and 10000 inclusive, which are divisible by none of 5, 6 or 8.

8. A machine that inserts letters into envelopes goes haywire and inserts letters randomly into envelops. What is the probability that in a group of 100 letters (a) no letters is put into the correct envelope (b) exactly 1 letter is put into the correct envelope.

9. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2,3 or 5

10. A committee of eight is to be formed from 16 men and 10 women. In how many ways can the committee be formed if (a) there are no restrictions (b) there must be 4 men and 4 women (c) there should be even number of women (d) more women then men