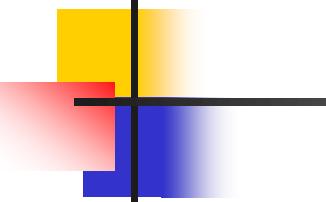


UNIT - 3

ERROR DETECTION AND CORRECTION



Note

**Data can be corrupted
during transmission.**

**Some applications require that
errors be detected and corrected.**

TYPES OF ERRORS

1. Single bit error: Only 1 bit in the data unit is changed.
2. Burst errors: 2 or more bits in the data unit are changed.

Figure 10.1 Single-bit error

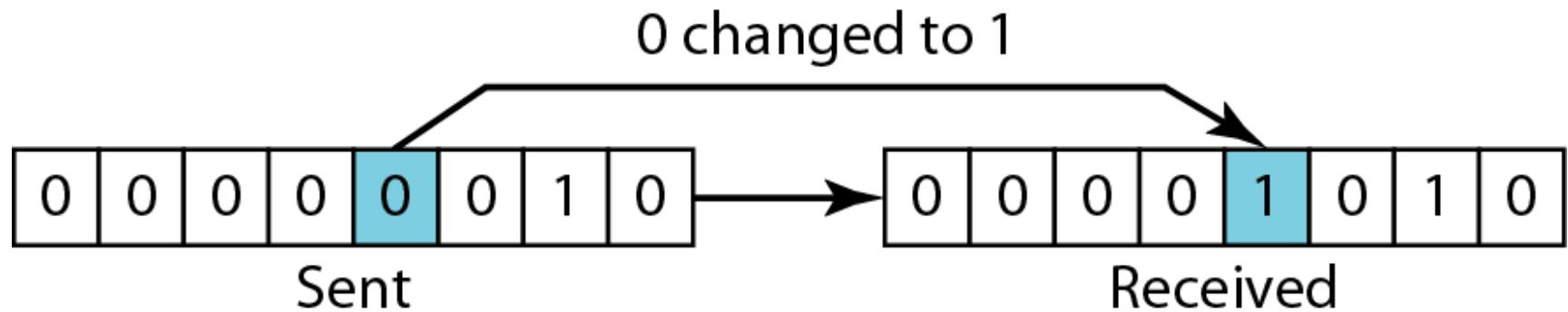
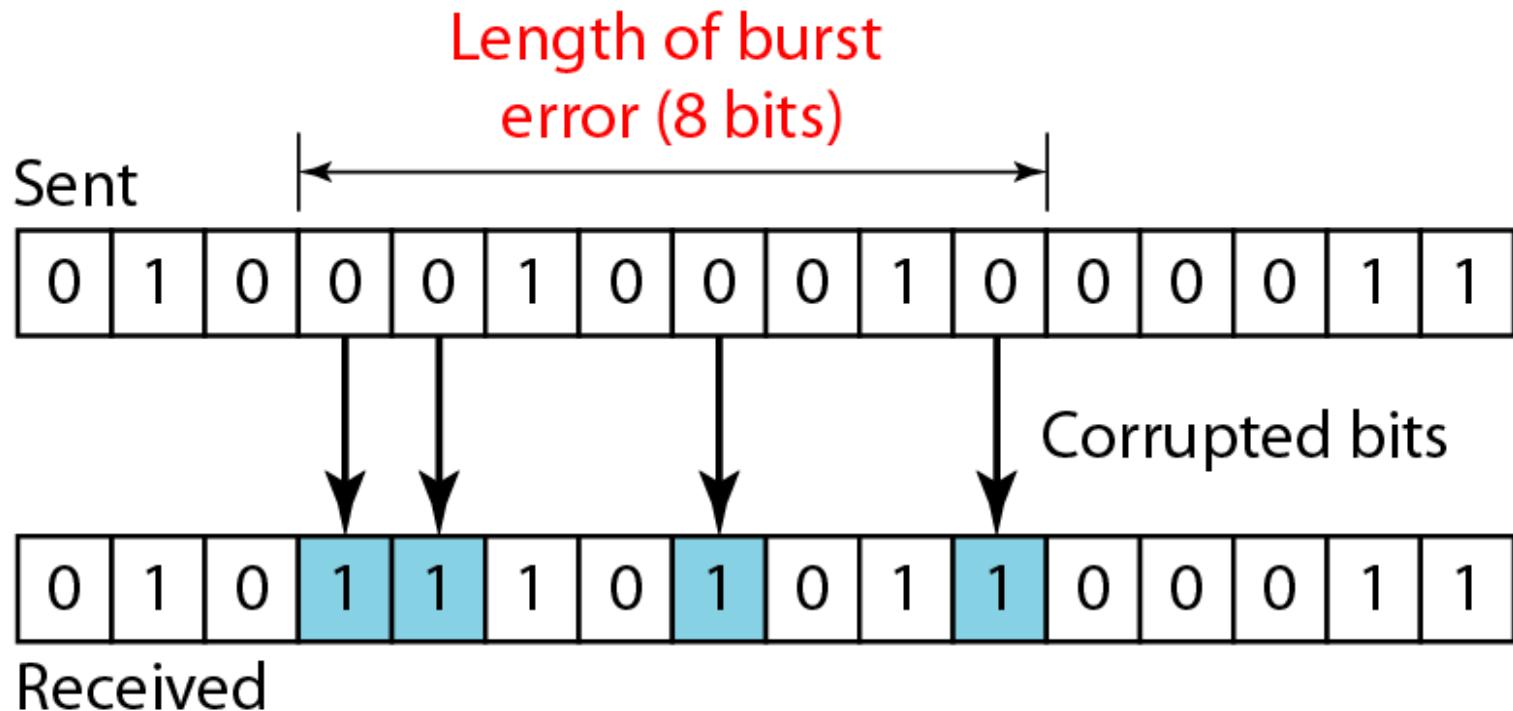


Figure 10.2 *Burst error of length 8*



BURST ERROR

- The term **burst error** means that two or more bits in the data unit have changed from 1 to 0 or from 0 to 1.
- **Burst errors does not necessarily mean that the errors occur in consecutive bits**, the length of the burst is measured from the first corrupted bit to the last corrupted bit. Some bits in between may not have been corrupted.

Error detection and correction

- In error detection we only see if error has occurred or not . Answer is simple yes or no.
- In error correction, we need to know the exact bits that are corrupted and their location in the message.

Modular arithmetic

- We define an upper limit called modulus N .
- We then use only the integers between 0 to $N-1$.

Modular arithmetic

Modulo-2 Arithmetic

Adding $0+0=0$ $0+1=1$ $1+0=1$ $1+1=0$

Subtracting $0-0=0$ $0-1=1$ $1-0=1$ $1-1=0$

Figure 10.4 XORing of two single bits or two words

$$0 \oplus 0 = 0$$

$$1 \oplus 1 = 0$$

a. Two bits are the same, the result is 0.

$$0 \oplus 1 = 1$$

$$1 \oplus 0 = 1$$

b. Two bits are different, the result is 1.

$$\begin{array}{r} 1 & 0 & 1 & 1 & 0 \\ \oplus & 1 & 1 & 1 & 0 \\ \hline 0 & 1 & 0 & 1 & 0 \end{array}$$

c. Result of XORing two patterns

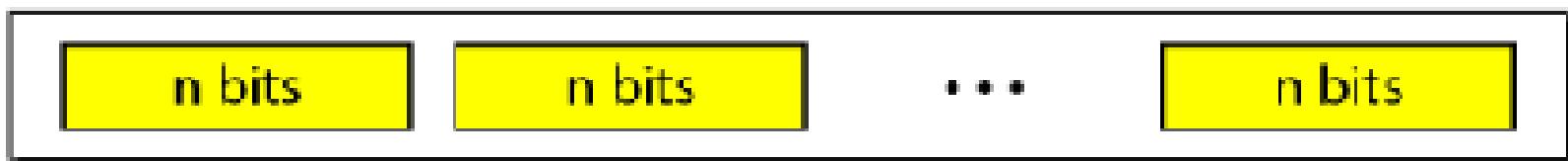


10-2 BLOCK CODING

*In block coding, we divide our message into blocks, each of k bits, called **datawords**. We add r redundant bits (adding extra bits for detecting errors at the destination) to each block to make the length $n = k + r$. The resulting n -bit blocks are called **codewords**.*



2^k Datawords, each of k bits



2ⁿ Codewords, each of n bits (only 2^k of them are valid)

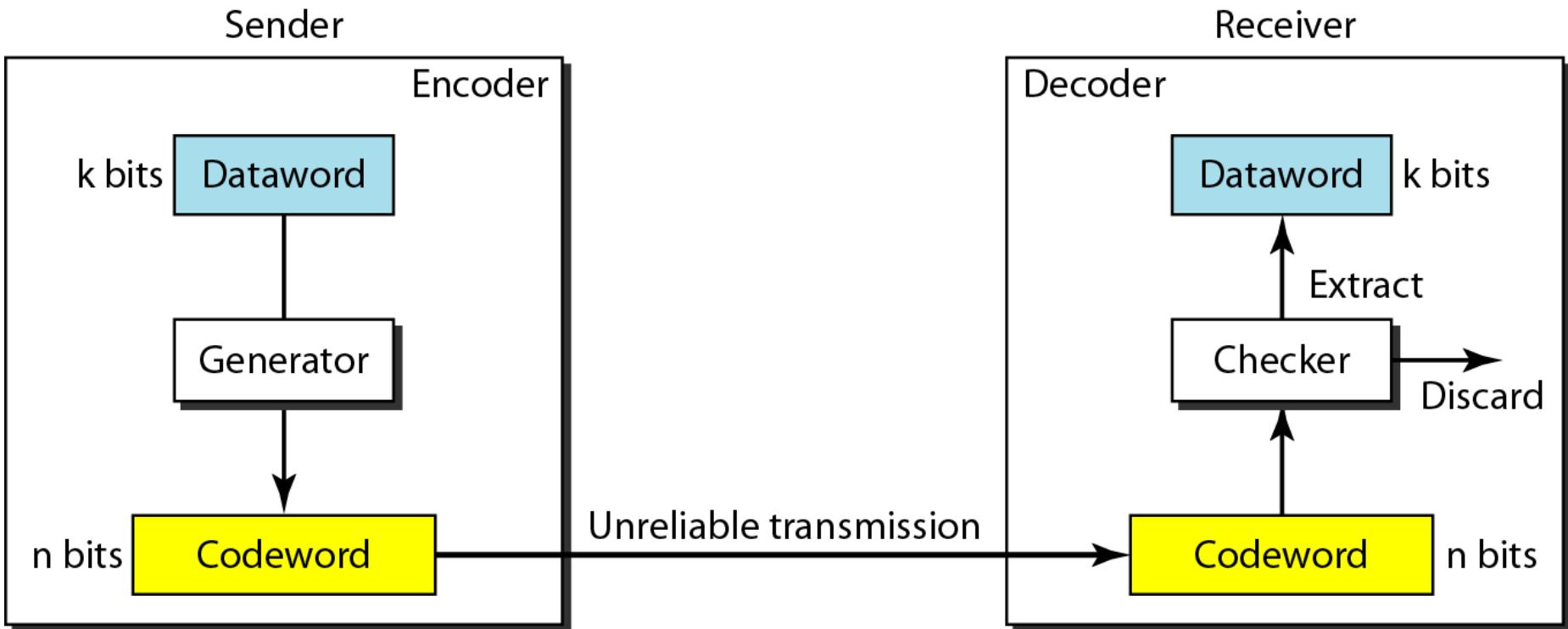
Error Detection

- If the following 2 conditions are met, the receiver can detect the change in codeword.

The receiver has (or can find) a list of valid codewords.

The original codeword has changed to an invalid one.

Figure 10.3 *The structure of encoder and decoder*



To detect or correct errors, we need to send redundant bits

Table 10.1 A code for error detection (Example 10.2)

<i>Datawords</i>	<i>Codewords</i>
00	000
01	011
10	101
11	110

What if we want to send 01? We code it as 011. If 011 is received, no problem.

What if 001 is received? Error detected.

What if 000 is received? Error occurred, but not detected.

Let's add more redundant bits to see if we can correct error.

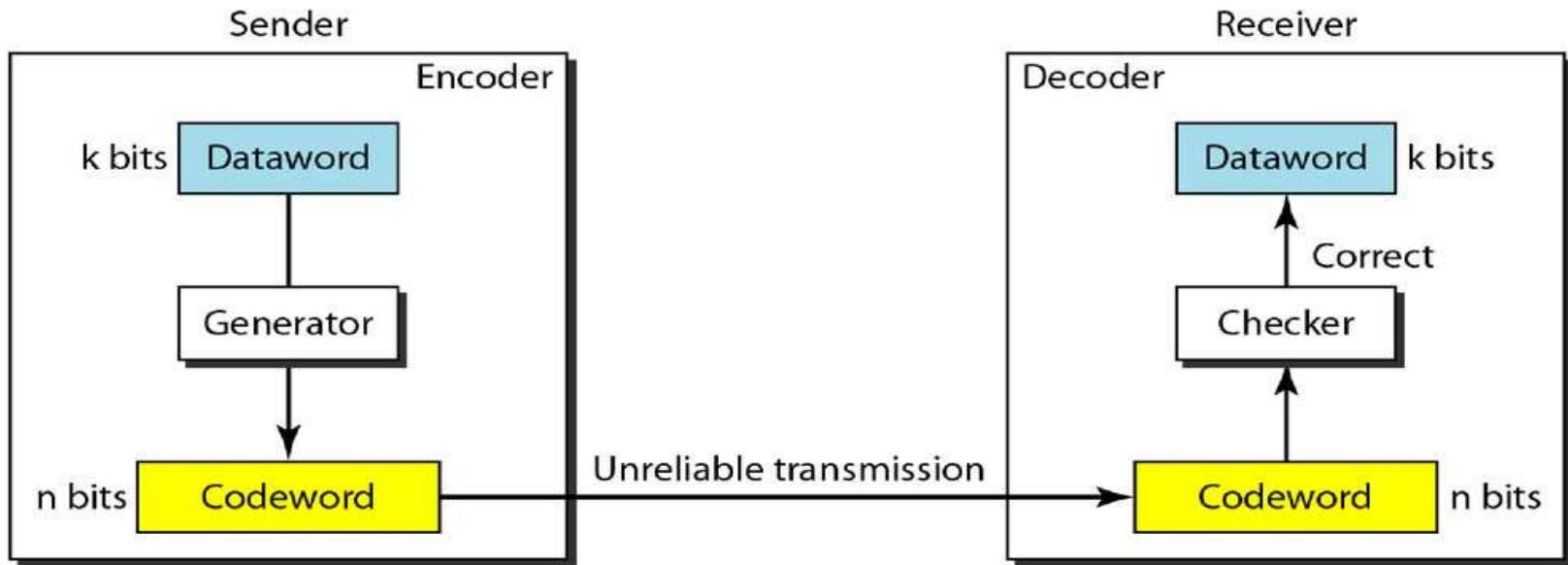
Table 10.2 *A code for error correction (Example 10.3)*

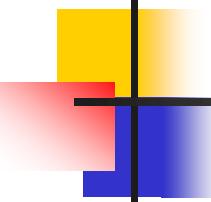
<i>Dataword</i>	<i>Codeword</i>
00	00000
01	01011
10	10101
11	11110

**Let's say we want to send 01. We then transmit 01011.
What if an error occurs and we receive 01001. If we
assume one bit was in error, we can correct.**

Error Correction

Figure 10.7 Structure of encoder and decoder in error correction





Note

The Hamming distance between two words is the number of differences between corresponding bits and can be calculated by applying XOR operation on the two words and count the number of 1's in it.

Example 10.4

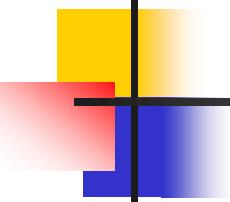
Let us find the Hamming distance between two pairs of words.

1. *The Hamming distance $d(000, 011)$ is 2 because*

$$000 \oplus 011 \text{ is } 011 \text{ (two 1s)}$$

2. *The Hamming distance $d(10101, 11110)$ is 3 because*

$$10101 \oplus 11110 \text{ is } 01011 \text{ (three 1s)}$$



Note

The minimum Hamming distance is the smallest Hamming distance between all possible pairs in a set of words.

Example 10.5

Find the minimum Hamming distance of the coding scheme in Table 10.1.

Solution

We first find all Hamming distances.

$$\begin{array}{llll} d(000, 011) = 2 & d(000, 101) = 2 & d(000, 110) = 2 & d(011, 101) = 2 \\ d(011, 110) = 2 & d(101, 110) = 2 & & \end{array}$$

The d_{min} in this case is 2.

Example 10.6

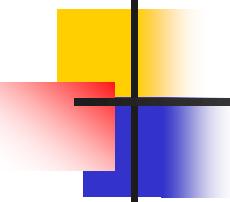
Find the minimum Hamming distance of the coding scheme in Table 10.2.

Solution

We first find all the Hamming distances.

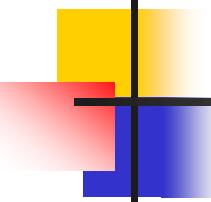
$d(00000, 01011) = 3$	$d(00000, 10101) = 3$	$d(00000, 11110) = 4$
$d(01011, 10101) = 4$	$d(01011, 11110) = 3$	$d(10101, 11110) = 3$

The d_{min} in this case is 3.



Note

To guarantee the *detection* of up to s errors in all cases, the minimum Hamming distance in a block code must be $d_{\min} = s + 1$.



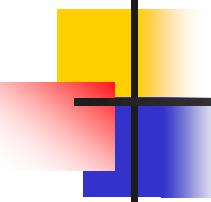
Example 10.7

The minimum Hamming distance for our first code scheme (Table 10.1) is 2. This code guarantees detection of only a single error. For example, if the third codeword (101) is sent and one error occurs, the received codeword does not match any valid codeword. If two errors occur, however, the received codeword may match a valid codeword and the errors are not detected.

Example 10.8

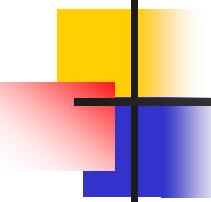
Our second block code scheme (Table 10.2) has $d_{min} = 3$. This code can detect up to two errors. Again, we see that when any of the valid codewords is sent, two errors create a codeword which is not in the table of valid codewords. The receiver cannot be fooled.

However, some combinations of three errors change a valid codeword to another valid codeword. The receiver accepts the received codeword and the errors are undetected.



Note

**To guarantee correction of up to t errors
in all cases, the minimum Hamming
distance in a block code
must be $d_{\min} = 2t + 1$.**



Example 10.9

A code scheme has a Hamming distance $d_{min} = 4$. What is the error detection and correction capability of this scheme?

Solution

*This code guarantees the detection of up to **three** errors ($s = 3$), but it can correct up to **one** error. In other words, if this code is used for error correction, part of its capability is wasted. Error correction codes need to have an odd minimum distance (3, 5, 7, . . .).*

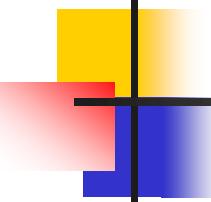
10-3 LINEAR BLOCK CODES

*Almost all block codes used today belong to a subset called **linear block codes**. A linear block code is a code in which the exclusive OR (addition modulo-2) of two valid codewords creates another valid codeword.*

Linear Block Codes

1. Simple parity check code

The **simple parity-check code** is the most familiar error-detecting **code**. In this **code**, a **k-bit** dataword is changed to an **n-bit** **codeword** where $n = k + 1$. The extra **bit**, called the **parity bit**, is selected to make the total number of 1s in the **codeword** even.



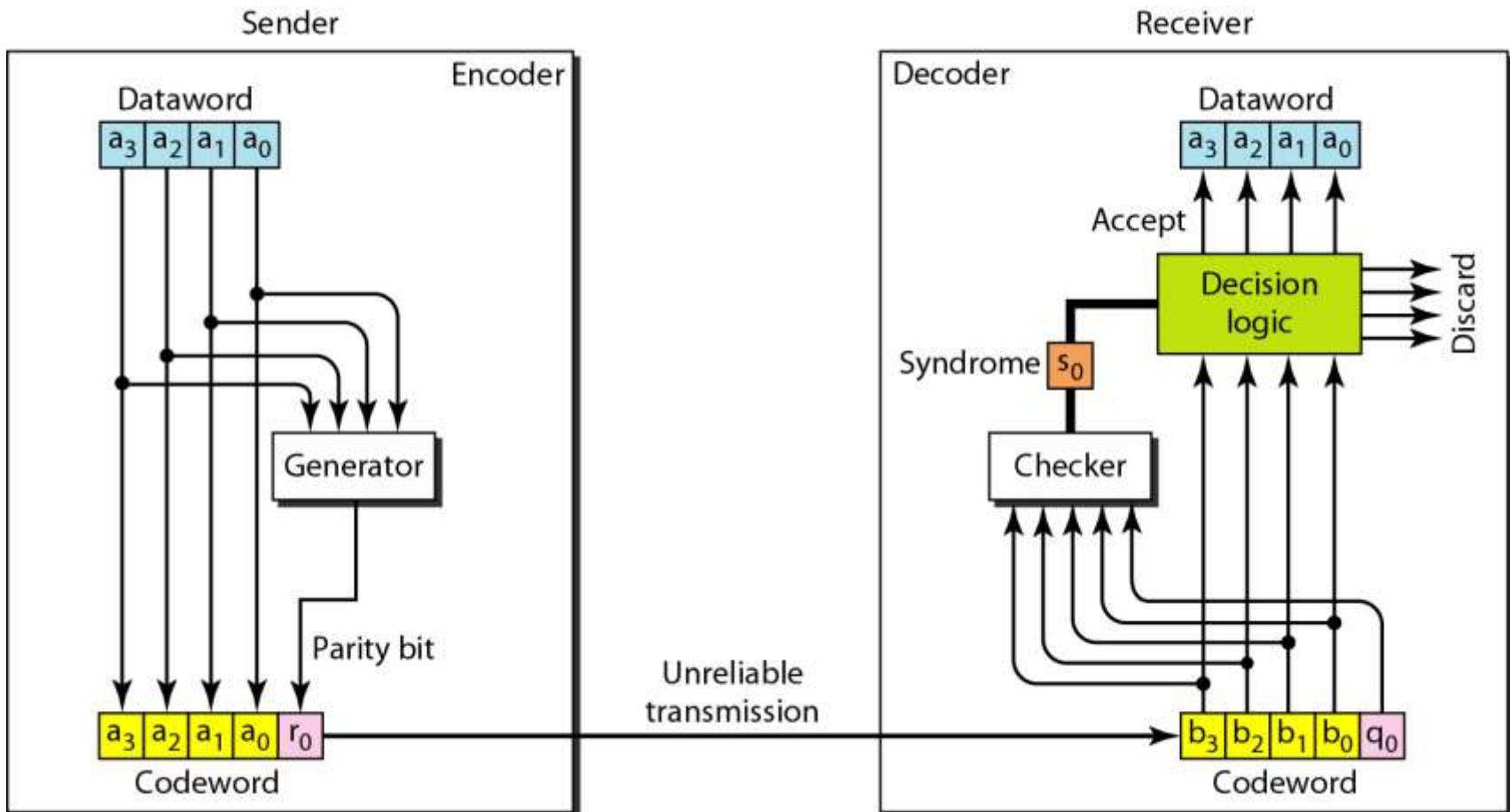
Note

A simple parity-check code is a single-bit error-detecting code in which $n = k + 1$ with $d_{\min} = 2$.

Table 10.3 *Simple parity-check code C(5, 4)*

<i>Datawords</i>	<i>Codewords</i>	<i>Datawords</i>	<i>Codewords</i>
0000	00000	1000	10001
0001	00011	1001	10010
0010	00101	1010	10100
0011	00110	1011	10111
0100	01001	1100	11000
0101	01010	1101	11011
0110	01100	1110	11101
0111	01111	1111	11110

Figure 10.10 Encoder and decoder for simple parity-check code



Encoder and Decoder

- The encoder uses a generator that takes a copy of a 4-bit data word (a_0, a_1, a_2 and a_3) and generates a parity bit r_0 . The data word bits and the parity bit create the 5-bit code word. The parity bit that is added makes the number of 1s in the code word even. This is normally done by adding the 4 bits of the data word (modulo-2).

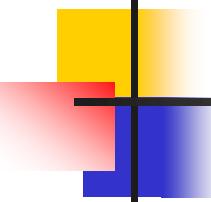
$$r_0 = a_3 + a_2 + a_1 + a_0 \text{ (modulo-2)}$$

- The sender sends the code word which may be corrupted during transmission. The receiver receives a 5-bit word. The checker at the receiver does the same thing as the generator in the sender with one exception: The addition is done over all 5 bits.

$$s_0 = b_3 + b_2 + b_1 + b_0 + q_0 \text{ (modulo-2)}$$

Encoder and Decoder

- The result, which is called the syndrome, is just 1 bit. The syndrome is passed to the decision logic analyzer. If the syndrome is 0, there is no error in the received code word, the data portion of the received code word is accepted as the data word, if the syndrome is 1, the data portion of the received code word is discarded. The data word is not created.



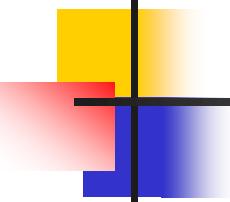
Example 10.12

Let us look at some transmission scenarios. Assume the sender sends the dataword 1011. The codeword created from this dataword is 10111, which is sent to the receiver. We examine five cases:

- 1. No error occurs; the received codeword is 10111. The syndrome is 0. The dataword 1011 is created.*
- 2. One single-bit error changes a_1 . The received codeword is 10011. The syndrome is 1. No dataword is created.*
- 3. One single-bit error changes r_0 . The received codeword is 10110. The syndrome is 1. No dataword is created.*

Example 10.12 (continued)

- 4.** An error changes r_0 and a second error changes a_3 .
The received codeword is 00110. The syndrome is 0.
The dataword 0011 is created at the receiver. Note that
here the dataword is wrongly created due to the
syndrome value.
- 5.** Three bits— a_3 , a_2 , and a_1 —are changed by errors.
The received codeword is 01011. The syndrome is 1.
The dataword is not created. This shows that the simple
parity check, guaranteed to detect one single error, can
also find any odd number of errors.



Note

**A simple parity-check code can detect
an odd number of errors.**

Figure 10.11 Two-dimensional parity-check code

1	1	0	0	1	1	1	1	1
1	0	1	1	1	0	1	1	1
0	1	1	1	0	0	1	0	0
0	1	0	1	0	0	1	1	1
0	1	0	1	0	1	0	1	1

a. Design of row and column parities

Figure 10.11 Two-dimensional parity-check code

1	1	0	0	1	1	1	1
1	0	1	1	1	0	1	1
0	1	1	1	0	0	1	0
0	1	0	1	0	0	1	1
<hr/>							
0	1	0	1	0	1	0	1

b. One error affects two parities

1	1	0	0	1	1	1	1
1	0	1	1	1	1	0	1
0	1	1	1	0	0	1	0
0	1	0	1	0	0	1	1
<hr/>							
0	1	0	1	0	1	0	1

c. Two errors affect two parities

Figure 10.11 Two-dimensional parity-check code

1	1	0	0	1	1	1	1
1	0	1	1	1	0	1	1
0	1	1	1	0	0	1	0
0	1	0	1	0	0	1	1
<hr/>							
0	1	0	1	0	1	0	1

d. Three errors affect four parities

1	1	0	0	1	1	1	1
1	0	1	1	1	1	0	1
0	1	1	1	1	0	0	0
0	1	0	1	0	0	1	1
<hr/>							
0	1	0	1	0	1	0	1

e. Four errors cannot be detected

10-4 CYCLIC CODES

Cyclic codes are special linear block codes with one extra property. In a cyclic code, if a codeword is cyclically shifted (rotated), the result is another codeword.

Topics discussed in this section:

Cyclic Redundancy Check

Hardware Implementation

Polynomials

Cyclic Code Analysis

Advantages of Cyclic Codes

Other Cyclic Codes

Cyclic Redundancy Checksum

The CRC error detection method treats the packet of data to be transmitted as a large polynomial.

The transmitter takes the message polynomial and using polynomial arithmetic, divides it by a given generating polynomial.

The quotient is discarded but the remainder is “attached” to the end of the message.

Cyclic Redundancy Checksum

The message (with the remainder) is transmitted to the receiver.

The receiver divides the message and remainder by the same generating polynomial.

If a remainder not equal to zero results, there was an error during transmission.

If a remainder of zero results, there was no error during transmission.

More Formally

- $M(x)$ - original message treated as a polynomial
- To prepare for transmission:
- Add r 0s to end of the message (where r = degree of generating polynomial)
- Divide $M(x)x^r$ by generating polynomial $P(x)$ yielding a quotient and a remainder $Q(x)+R(x)/P(x)$.

More Formally

- Add (XOR) remainder $R(x)$ to $M(x)x^r$ giving $M(x)x^r+R(x)$ and transmit.
- Receiver receives message $(M(x)x^r+R(x))$ and divides by *same* $P(x)$.
- If remainder is 0, then there were no errors during transmission.
- (Any expression which has exactly $P(x)$ as a term is evenly divisible by $P(x)$.)

Table 6-4 Error detection performance of cyclic redundancy checksum

Type of Error	Error Detection Performance
Single bit errors	100 percent
Double bit errors	100 percent, as long as the generating polynomial has at least three 1s (they all do)
Odd number of bits in error	100 percent, as long as the generating polynomial contains a factor $x + 1$ (they all do)
An error burst of length $< r+1$	100 percent
An error burst of length $= r+1$	probability = $1 - (\frac{1}{2})^{(r-1)}$
An error burst of length $> r+1$	probability = $1 - (\frac{1}{2})^r$

Common CRC Polynomials

- CRC-12: $x^{12} + x^{11} + x^3 + x^2 + x + 1$
- CRC-16: $x^{16} + x^{15} + x^2 + 1$
- CRC-CCITT: $x^{16} + x^{15} + x^5 + 1$
- CRC-32: $x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$
- ATM CRC: $x^8 + x^2 + x + 1$

CRC Example

- Given a pretend $P(x) = x^5 + x^4 + x^2 + 1$ and a message $M(x) = 1010011010$, calculate the remainder using long hand division and a shift register.

10-5 CHECKSUM

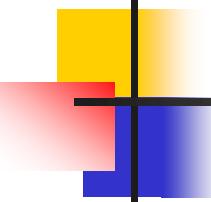
The last error detection method we discuss here is called the checksum, or arithmetic checksum. The checksum is used in the Internet by several protocols although not at the data link layer. However, we briefly discuss it here to complete our discussion on error checking

Topics discussed in this section:

Idea

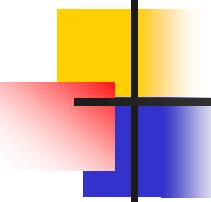
One's Complement

Internet Checksum



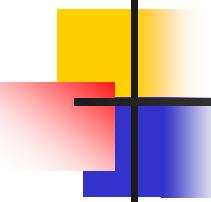
Example 10.18

Suppose our data is a list of five 4-bit numbers that we want to send to a destination. In addition to sending these numbers, we send the sum of the numbers. For example, if the set of numbers is (7, 11, 12, 0, 6), we send (7, 11, 12, 0, 6, 36), where 36 is the sum of the original numbers. The receiver adds the five numbers and compares the result with the sum. If the two are the same, the receiver assumes no error, accepts the five numbers, and discards the sum. Otherwise, there is an error somewhere and the data are not accepted.



Example 10.19

*We can make the job of the receiver easier if we send the negative (complement) of the sum, called the **checksum**. In this case, we send (7, 11, 12, 0, 6, **-36**). The receiver can add all the numbers received (including the checksum). If the result is 0, it assumes no error; otherwise, there is an error.*

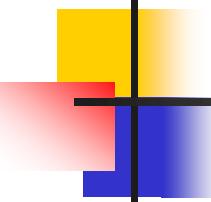


Example 10.20

How can we represent the number 21 in one's complement arithmetic using only four bits?

Solution

The number 21 in binary is 10101 (it needs five bits). We can wrap the leftmost bit and add it to the four rightmost bits. We have $(0101 + 1) = 0110$ or 6.



Example 10.21

How can we represent the number -6 in one's complement arithmetic using only four bits?

Solution

In one's complement arithmetic, the negative or complement of a number is found by inverting all bits. Positive 6 is 0110; negative 6 is 1001. If we consider only unsigned numbers, this is 9. In other words, the complement of 6 is 9. Another way to find the complement of a number in one's complement arithmetic is to subtract the number from $2^n - 1$ ($16 - 1$ in this case).

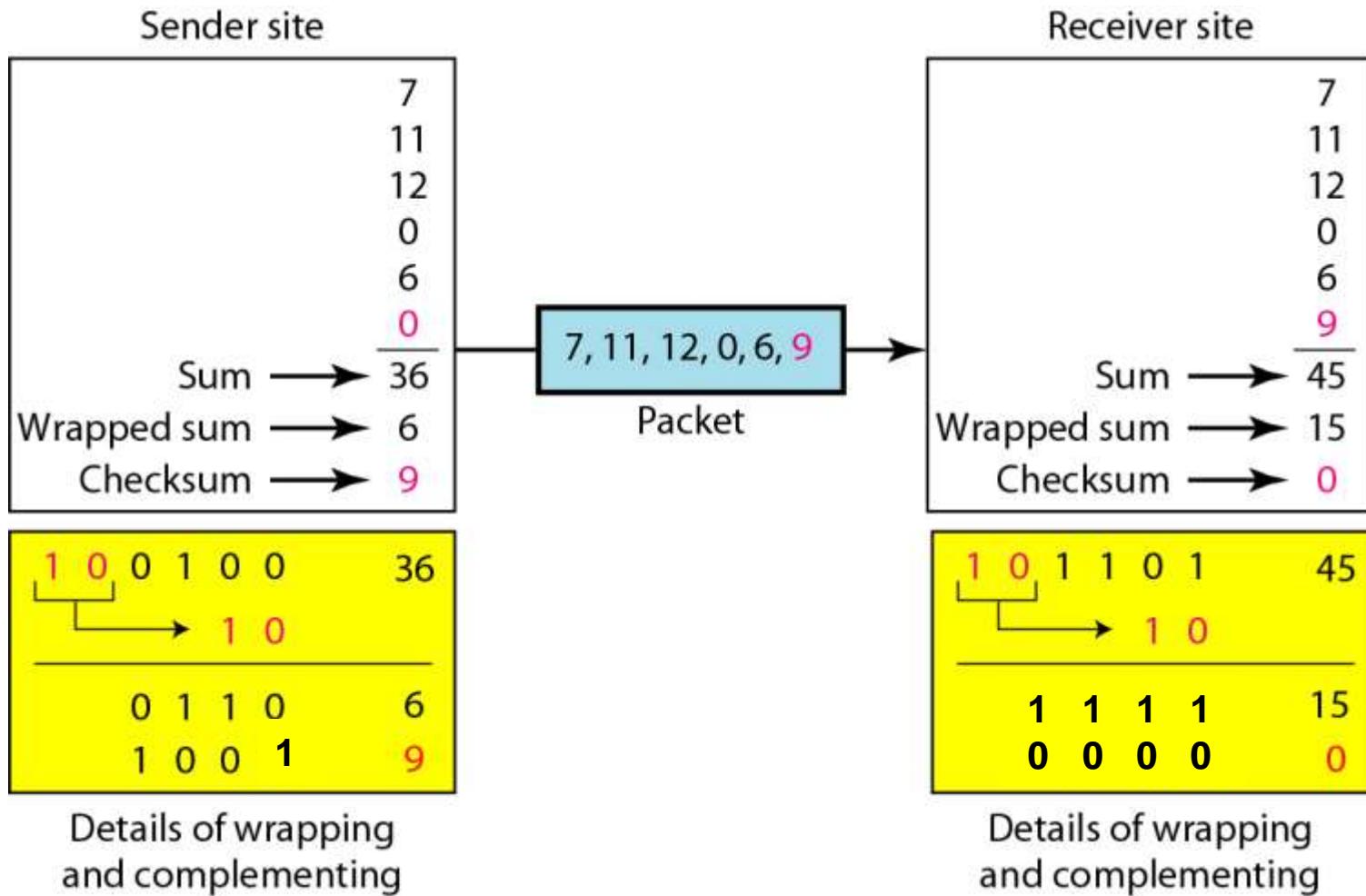
Example 10.22

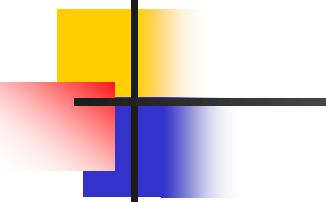
Let us redo Exercise 10.19 using one's complement arithmetic. Figure 10.24 shows the process at the sender and at the receiver. The sender initializes the checksum to 0 and adds all data items and the checksum (the checksum is considered as one data item and is shown in color). The result is 36. However, 36 cannot be expressed in 4 bits. The extra two bits are wrapped and added with the sum to create the wrapped sum value 6. In the figure, we have shown the details in binary. The sum is then complemented, resulting in the checksum value 9 ($15 - 6 = 9$). The sender now sends six data items to the receiver including the checksum 9.

Example 10.22 (continued)

The receiver follows the same procedure as the sender. It adds all data items (including the checksum); the result is 45. The sum is wrapped and becomes 15. The wrapped sum is complemented and becomes 0. Since the value of the checksum is 0, this means that the data is not corrupted. The receiver drops the checksum and keeps the other data items. If the checksum is not zero, the entire packet is dropped.

Figure 10.24 Example 10.22

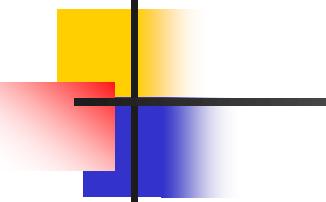




Note

Sender site:

- 1. The message is divided into 16-bit words.**
- 2. The value of the checksum word is set to 0.**
- 3. All words including the checksum are added using one's complement addition.**
- 4. The sum is complemented and becomes the checksum.**
- 5. The checksum is sent with the data.**



Note

Receiver site:

- 1. The message (including checksum) is divided into 16-bit words.**
- 2. All words are added using one's complement addition.**
- 3. The sum is complemented and becomes the new checksum.**
- 4. If the value of checksum is 0, the message is accepted; otherwise, it is rejected.**

Example 10.23

Let us calculate the checksum for a text of 8 characters (“Forouzan”). The text needs to be divided into 2-byte (16-bit) words. We use ASCII (see Appendix A) to change each byte to a 2-digit hexadecimal number. For example, F is represented as 0x46 and o is represented as 0x6F. Figure 10.25 shows how the checksum is calculated at the sender and receiver sites. In part a of the figure, the value of partial sum for the first column is 0x36. We keep the rightmost digit (6) and insert the leftmost digit (3) as the carry in the second column. The process is repeated for each column. Note that if there is any corruption, the checksum recalculated by the receiver is not all 0s. We leave this an exercise.

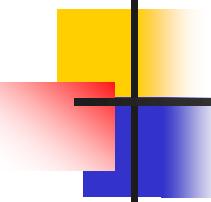
Figure 10.25 Example 10.23

1	0	1	3	Carries
4	6	6	F	(Fo)
7	2	6	F	(ro)
7	5	7	A	(uz)
6	1	6	E	(an)
0	0	0	0	Checksum (initial)
<hr/>				
8	F	C	6	Sum (partial)
<hr/>				
8	F	C	7	Sum
7	0	3	8	Checksum (to send)

a. Checksum at the sender site

1	0	1	3	Carries
4	6	6	F	(Fo)
7	2	6	F	(ro)
7	5	7	A	(uz)
6	1	6	E	(an)
7	0	3	8	Checksum (received)
<hr/>				
F	F	F	E	Sum (partial)
<hr/>				
F	F	F	F	Sum
0	0	0	0	Checksum (new)

a. Checksum at the receiver site



Error Correcting Codes

or

Forward Error Correction (FEC)

FEC is used in transmission of radio signals, such as those used in transmission of digital television (Reed-Solomon and Trellis encoding) and 4D-PAM5 (Viterbi and Trellis encoding)

Some FEC is based on Hamming Codes

Let's examine a Hamming Code

11	10	9	8	7	6	5	4	3	2	1
d	d	d	r ₈	d	d	d	r ₄	d	r ₂	r ₁

From previous edition of Forouzan

Redundancy bits calculation

r_1 will take care of these bits.

11	9	7	5	3	1				
d	d	d	r_8	d	d	r_4	d	r_2	r_1

r_2 will take care of these bits.

11	10	7	6	3	2				
d	d	d	r_8	d	d	r_4	d	r_2	r_1

r_4 will take care of these bits.

7	6	5	4							
d	d	d	r_8	d	d	d	r_4	d	r_2	r_1

r_8 will take care of these bits.

11	10	9	8							
d	d	d	r_8	d	d	d	r_4	d	r_2	r_1

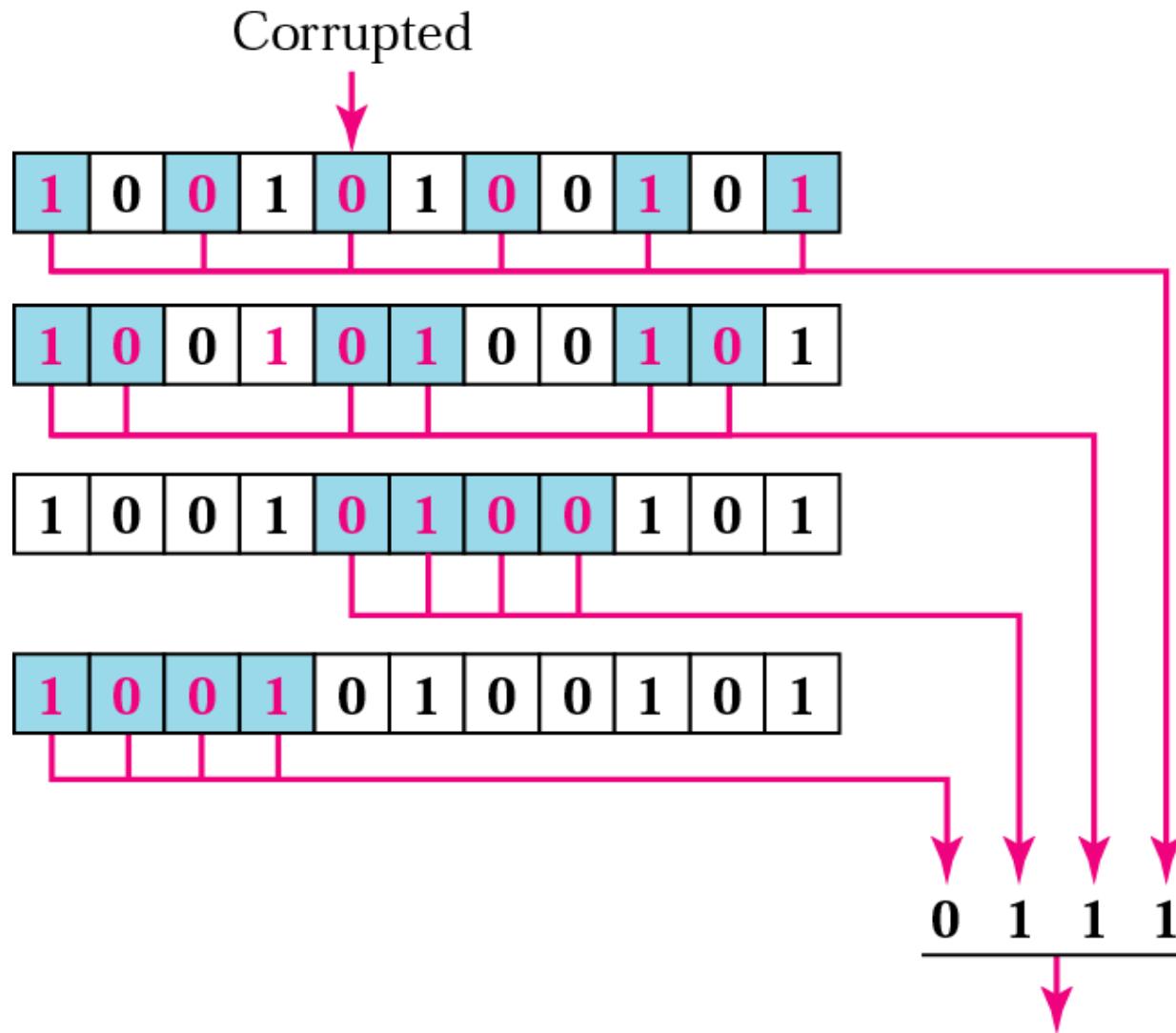
Example of redundancy bit calculation

	1	0	0		1	1	0		1		
	11	10	9	8	7	6	5	4	3	2	1
<hr/>											
Adding r_1	1	0	0		1	1	0		1		1
	11	10	9	8	7	6	5	4	3	2	1
<hr/>											
Adding r_2	1	0	0		1	1	0		1	0	1
	11	10	9	8	7	6	5	4	3	2	1
<hr/>											
Adding r_4	1	0	0		1	1	0	0	1	0	1
	11	10	9	8	7	6	5	4	3	2	1
<hr/>											
Adding r_8	1	0	0	1	1	1	0	0	1	0	1
	11	10	9	8	7	6	5	4	3	2	1

Data:
1 0 0 1 1 0 1

Code:
1 0 0 1 1 1 0 0 1 0 1

Error detection using Hamming code



The bit in position 7 is in error. 7

Review Questions

- What is the efficiency of simple parity?
- What is the efficiency of CRC?
- Be able to calculate a CRC using either shift register method or long-hand division method
- Be able to encode a character using a Hamming code, then decode, detecting and correcting an error



Data Communications
and Networking

Fourth Edition

Forouzan

Chapter 11

Data Link Control

11-1 FRAMING

*The data link layer needs to pack bits into **frames**, so that each frame is distinguishable from another. Our postal system practices a type of framing. The simple act of inserting a letter into an envelope separates one piece of information from another; the envelope serves as the delimiter.*

Topics discussed in this section:

Fixed-Size Framing

Variable-Size Framing

Figure 11.1 A frame in a character-oriented protocol

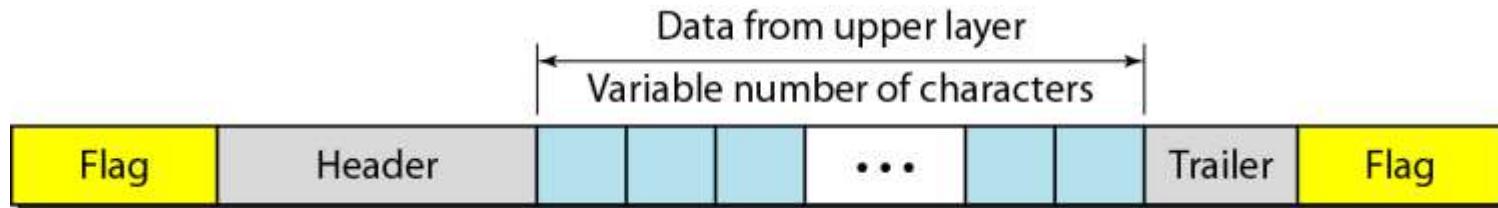
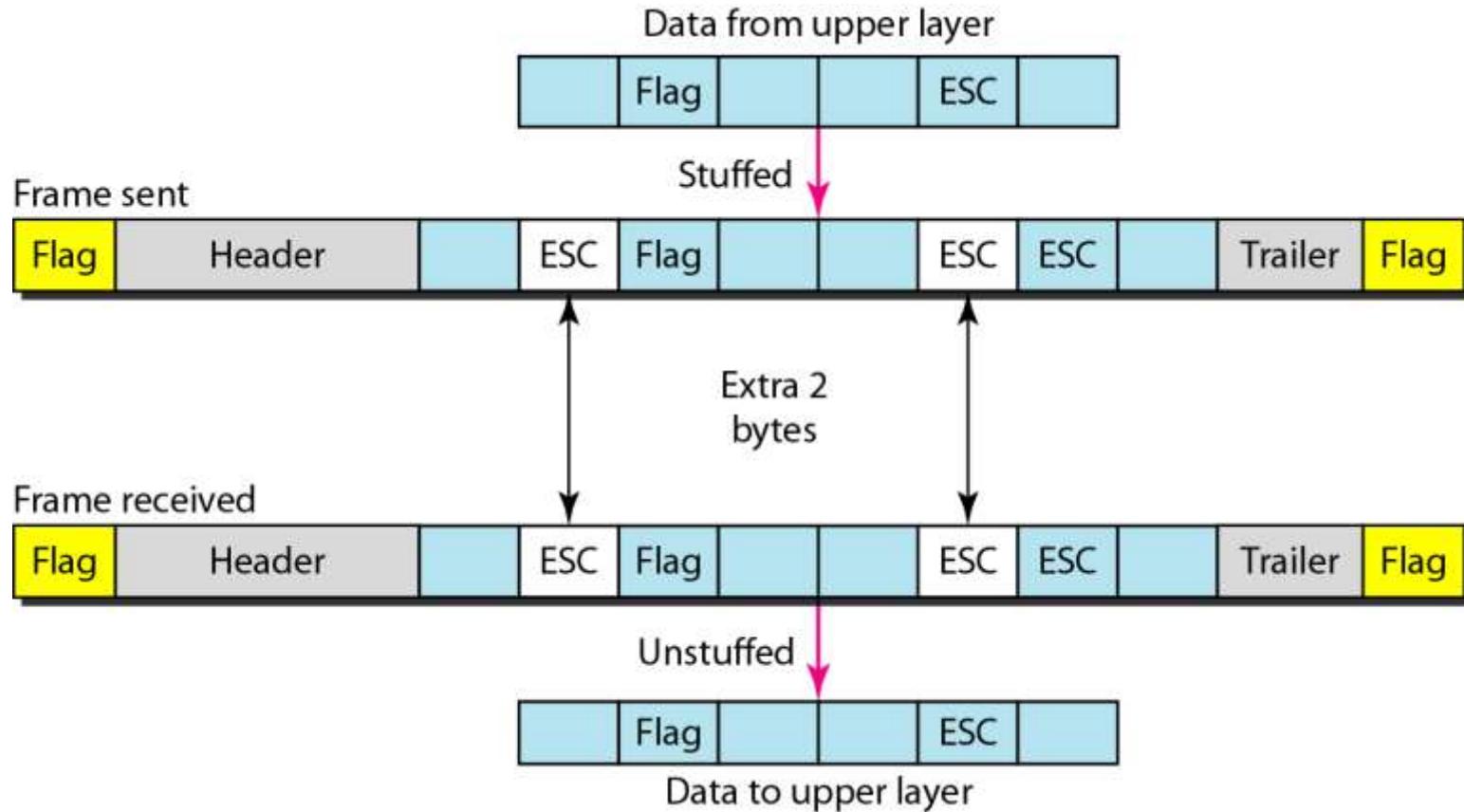
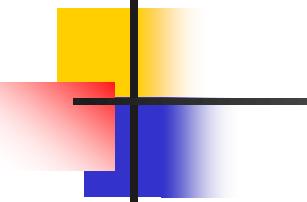


Figure 11.2 Byte stuffing and unstuffing

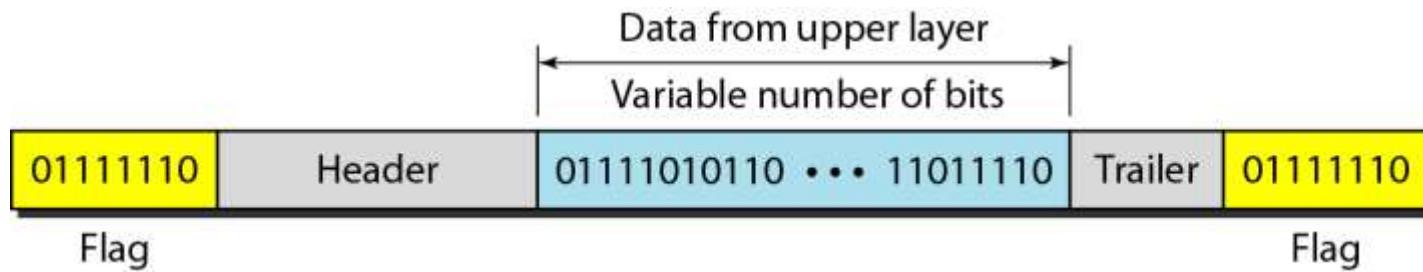


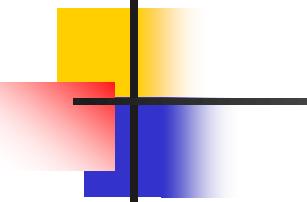


Note

Byte stuffing is the process of adding 1 extra byte whenever there is a flag or escape character in the text.

Figure 11.3 A frame in a bit-oriented protocol

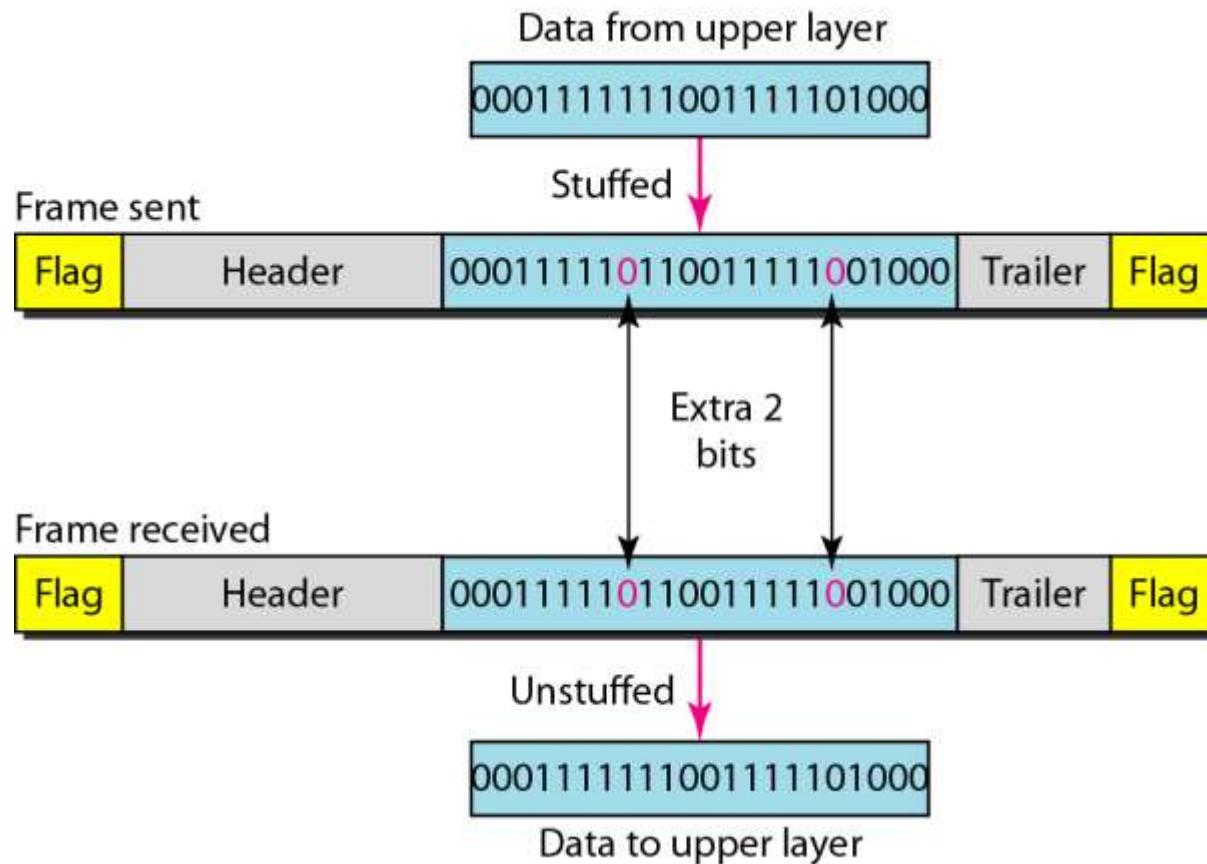




Note

Bit stuffing is the process of adding one extra 0 whenever five consecutive 1s follow a 0 in the data, so that the receiver does not mistake the pattern 0111110 for a flag.

Figure 11.4 Bit stuffing and unstuffing



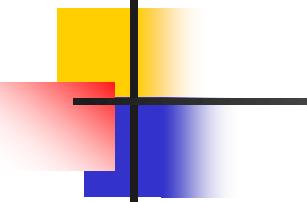
11-2 FLOW AND ERROR CONTROL

*The most important responsibilities of the data link layer are **flow control** and **error control**. Collectively, these functions are known as **data link control**.*

Topics discussed in this section:

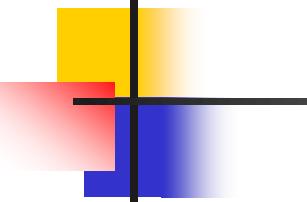
Flow Control

Error Control



Note

Flow control refers to a set of procedures used to restrict the amount of data that the sender can send before waiting for acknowledgment.



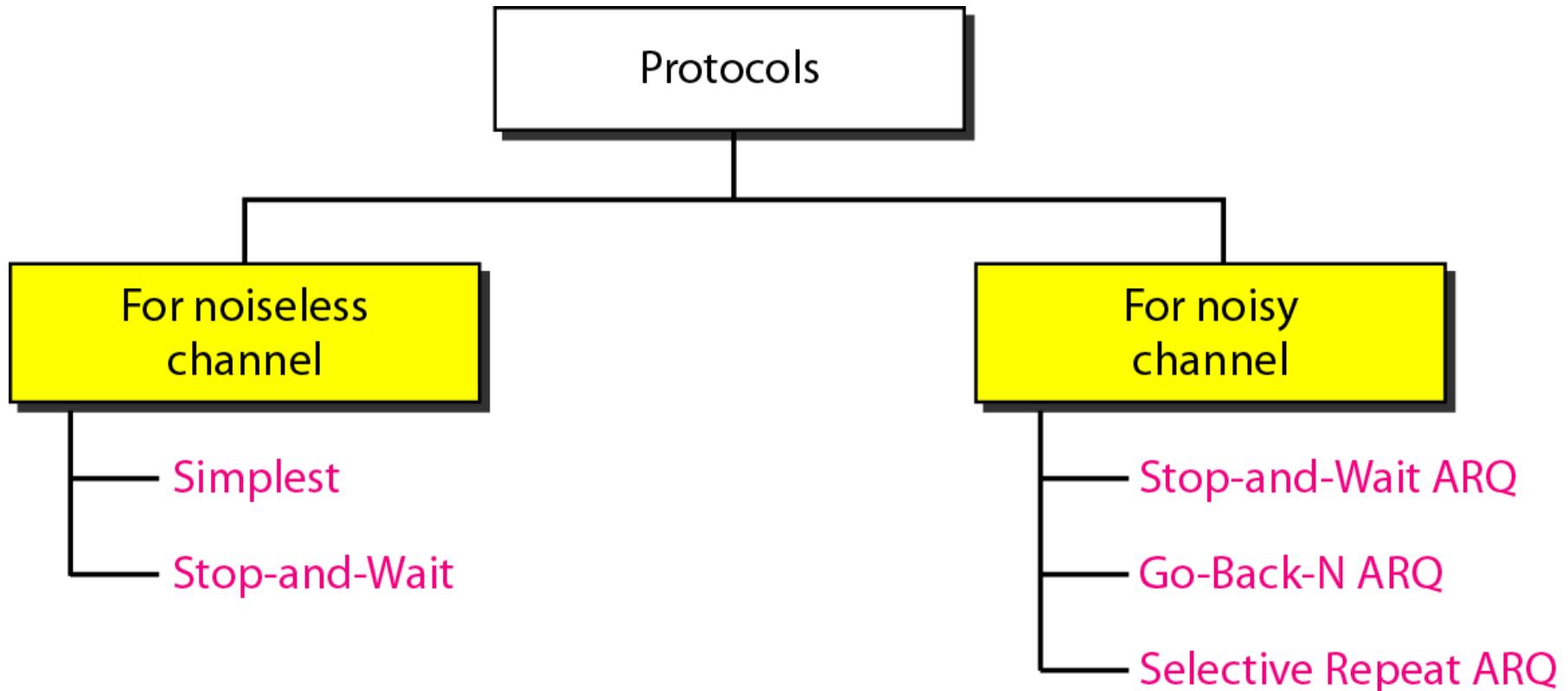
Note

Error control in the data link layer is based on automatic repeat request, which is the retransmission of data.

11-3 PROTOCOLS

Now let us see how the data link layer can combine framing, flow control, and error control to achieve the delivery of data from one node to another. The protocols are normally implemented in software by using one of the common programming languages. To make our discussions language-free, we have written in pseudocode a version of each protocol that concentrates mostly on the procedure instead of delving into the details of language rules.

Figure 11.5 *Taxonomy of protocols discussed in this chapter*



11-4 NOISELESS CHANNELS

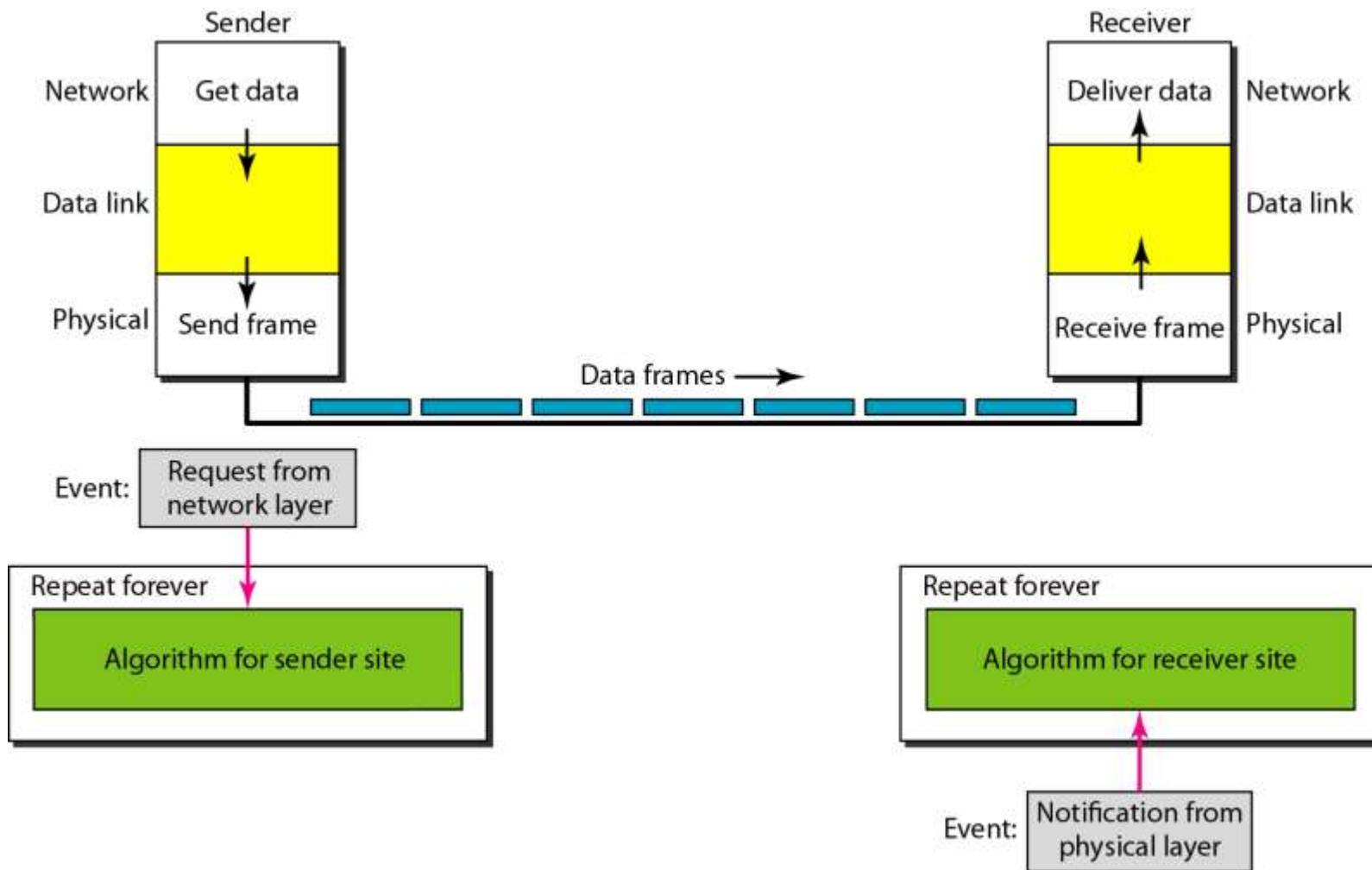
Let us first assume we have an ideal channel in which no frames are lost, duplicated, or corrupted. We introduce two protocols for this type of channel.

Topics discussed in this section:

Simplest Protocol

Stop-and-Wait Protocol

Figure 11.6 *The design of the simplest protocol with no flow or error control*



Algorithm 11.1 *Sender-site algorithm for the simplest protocol*

```
1 while(true)          // Repeat forever
2 {
3     WaitForEvent();    // Sleep until an event occurs
4     if(Event(RequestToSend)) //There is a packet to send
5     {
6         GetData();
7         MakeFrame();
8         SendFrame();      //Send the frame
9     }
10 }
```

Algorithm 11.2 *Receiver-site algorithm for the simplest protocol*

```
1 while(true)                                // Repeat forever
2 {
3     WaitForEvent();                         // Sleep until an event occurs
4     if(Event(ArrivalNotification)) //Data frame arrived
5     {
6         ReceiveFrame();
7         ExtractData();
8         DeliverData();                    //Deliver data to network layer
9     }
10 }
```

Example 11.1

Figure 11.7 shows an example of communication using this protocol. It is very simple. The sender sends a sequence of frames without even thinking about the receiver. To send three frames, three events occur at the sender site and three events at the receiver site. Note that the data frames are shown by tilted boxes; the height of the box defines the transmission time difference between the first bit and the last bit in the frame.

Figure 11.7 Flow diagram for Example 11.1

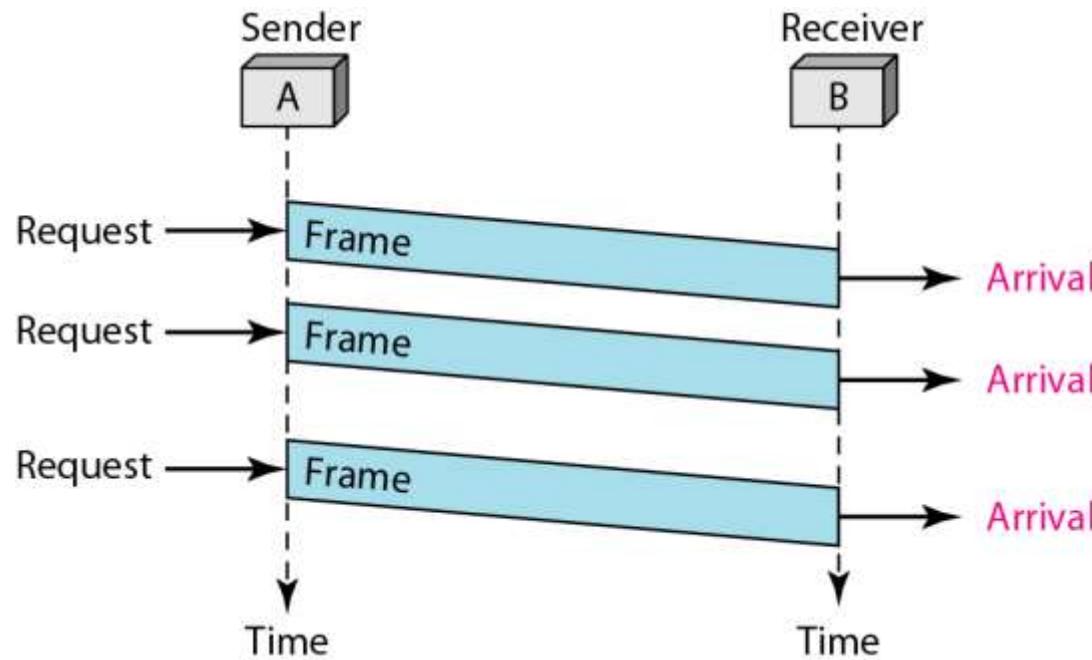
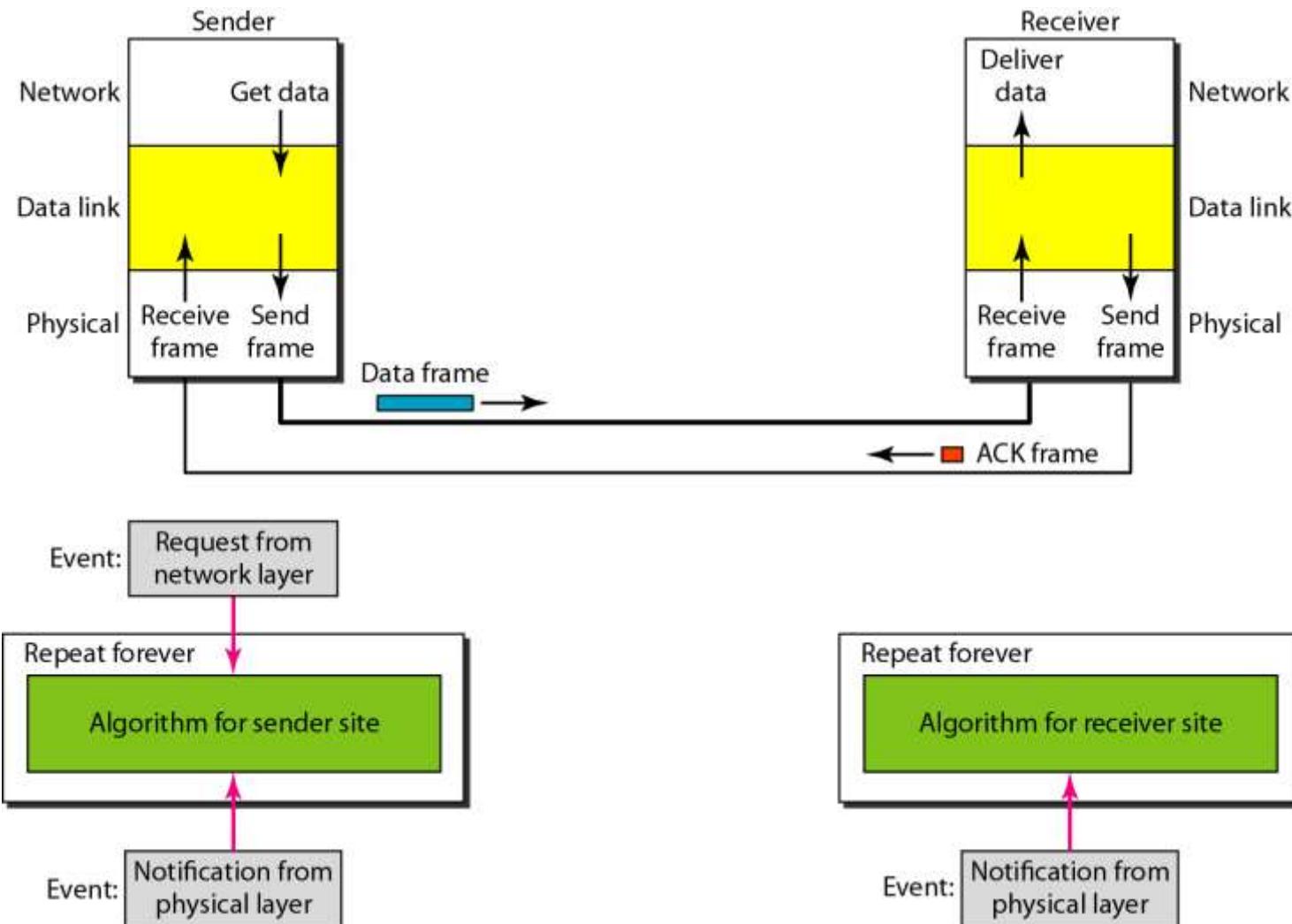


Figure 11.8 Design of Stop-and-Wait Protocol

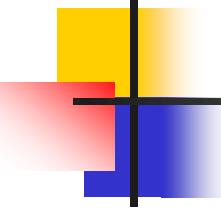


Algorithm 11.3 Sender-site algorithm for Stop-and-Wait Protocol

```
1 while(true)                                //Repeat forever
2 canSend = true                            //Allow the first frame to go
3 {
4     WaitForEvent();                      // Sleep until an event occurs
5     if(Event(RequestToSend) AND canSend)
6     {
7         GetData();
8         MakeFrame();
9         SendFrame();                     //Send the data frame
10        canSend = false;                //Cannot send until ACK arrives
11    }
12    WaitForEvent();                      // Sleep until an event occurs
13    if(Event(ArrivalNotification) // An ACK has arrived
14    {
15        ReceiveFrame();                //Receive the ACK frame
16        canSend = true;
17    }
18 }
```

Algorithm 11.4 *Receiver-site algorithm for Stop-and-Wait Protocol*

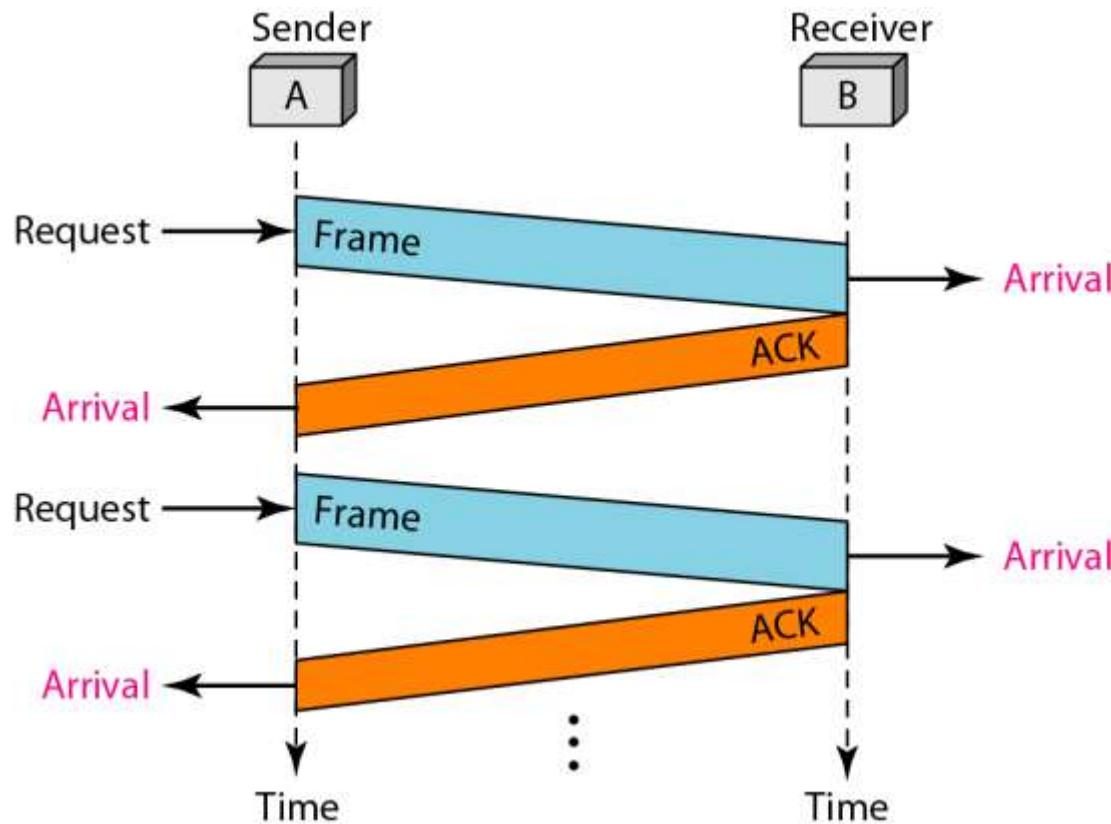
```
1 while(true)                                //Repeat forever
2 {
3     WaitForEvent();                         // Sleep until an event occurs
4     if(Event(ArrivalNotification)) //Data frame arrives
5     {
6         ReceiveFrame();
7         ExtractData();
8         Deliver(data);                  //Deliver data to network layer
9         SendFrame();                  //Send an ACK frame
10    }
11 }
```



Example 11.2

Figure 11.9 shows an example of communication using this protocol. It is still very simple. The sender sends one frame and waits for feedback from the receiver. When the ACK arrives, the sender sends the next frame. Note that sending two frames in the protocol involves the sender in four events and the receiver in two events.

Figure 11.9 Flow diagram for Example 11.2



11-5 NOISY CHANNELS

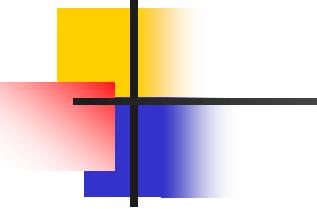
Although the Stop-and-Wait Protocol gives us an idea of how to add flow control to its predecessor, noiseless channels are nonexistent. We discuss three protocols in this section that use error control.

Topics discussed in this section:

Stop-and-Wait Automatic Repeat Request

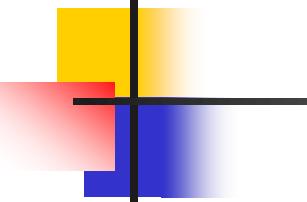
Go-Back-N Automatic Repeat Request

Selective Repeat Automatic Repeat Request



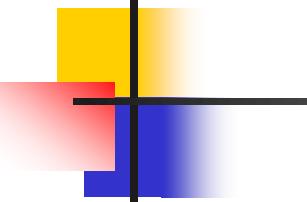
Note

Error correction in Stop-and-Wait ARQ is done by keeping a copy of the sent frame and retransmitting of the frame when the timer expires.



Note

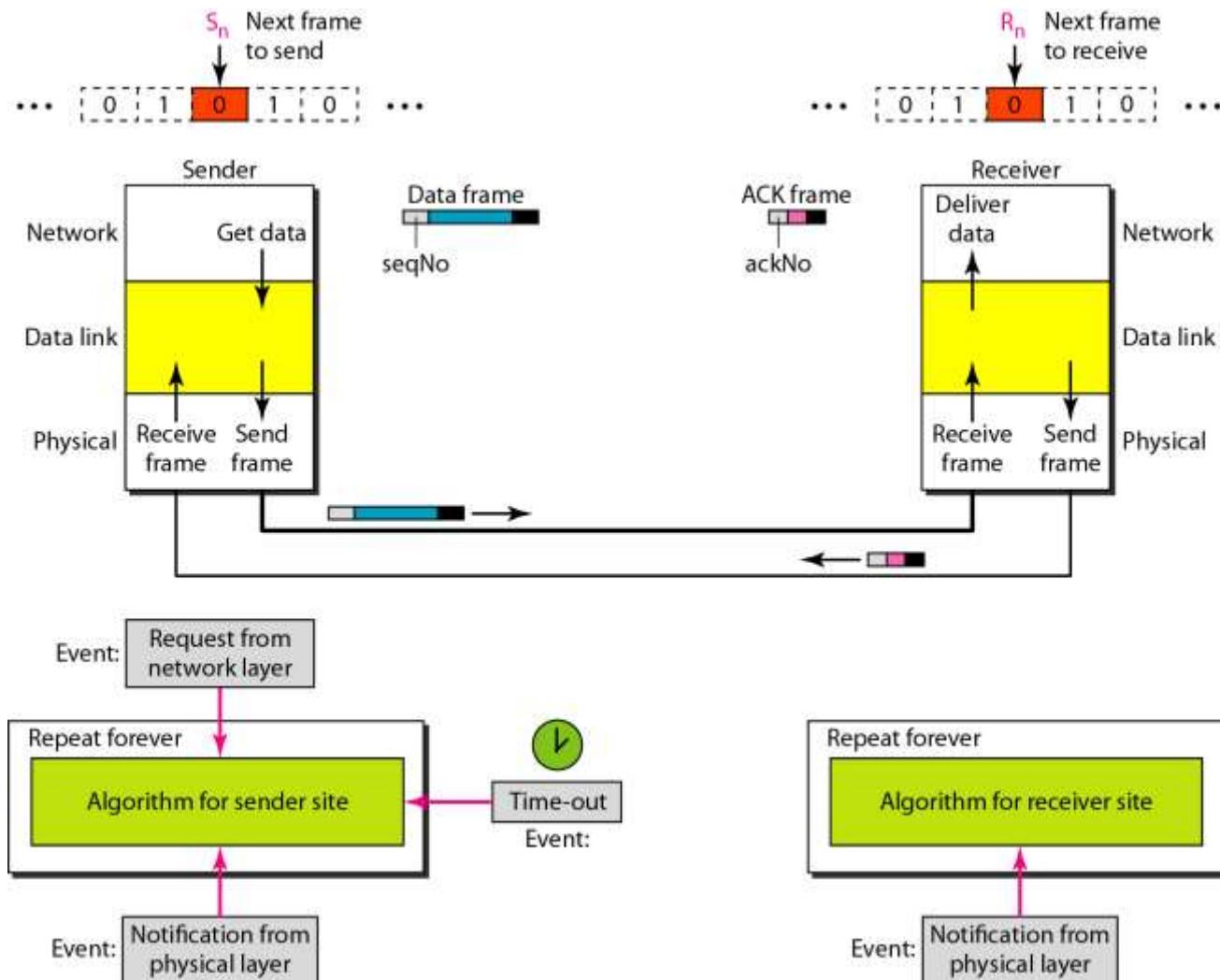
**In Stop-and-Wait ARQ, we use sequence numbers to number the frames.
The sequence numbers are based on modulo-2 arithmetic.**



Note

In Stop-and-Wait ARQ, the acknowledgment number always announces in modulo-2 arithmetic the sequence number of the next frame expected.

Figure 11.10 Design of the Stop-and-Wait ARQ Protocol



Algorithm 11.5 Sender-site algorithm for Stop-and-Wait ARQ

```
1 Sn = 0;                                // Frame 0 should be sent first
2 canSend = true;                           // Allow the first request to go
3 while(true)                               // Repeat forever
4 {
5   WaitForEvent();                         // Sleep until an event occurs
6   if(Event(RequestToSend) AND canSend)
7   {
8     GetData();
9     MakeFrame(Sn);                   //The seqNo is Sn
10    StoreFrame(Sn);                  //Keep copy
11    SendFrame(Sn);
12    StartTimer();
13    Sn = Sn + 1;
14    canSend = false;
15  }
16  WaitForEvent();                         // Sleep
```

(continued)

Algorithm 11.5 Sender-site algorithm for Stop-and-Wait ARQ (continued)

```
17  if(Event(ArrivalNotification))          // An ACK has arrived
18  {
19      ReceiveFrame(ackNo);           //Receive the ACK frame
20      if(not corrupted AND ackNo == Sn) //Valid ACK
21      {
22          Stoptimer();
23          PurgeFrame(Sn-1);        //Copy is not needed
24          canSend = true;
25      }
26  }
27
28  if(Event(TimeOut))                  // The timer expired
29  {
30      StartTimer();
31      ResendFrame(Sn-1);        //Resend a copy check
32  }
33 }
```

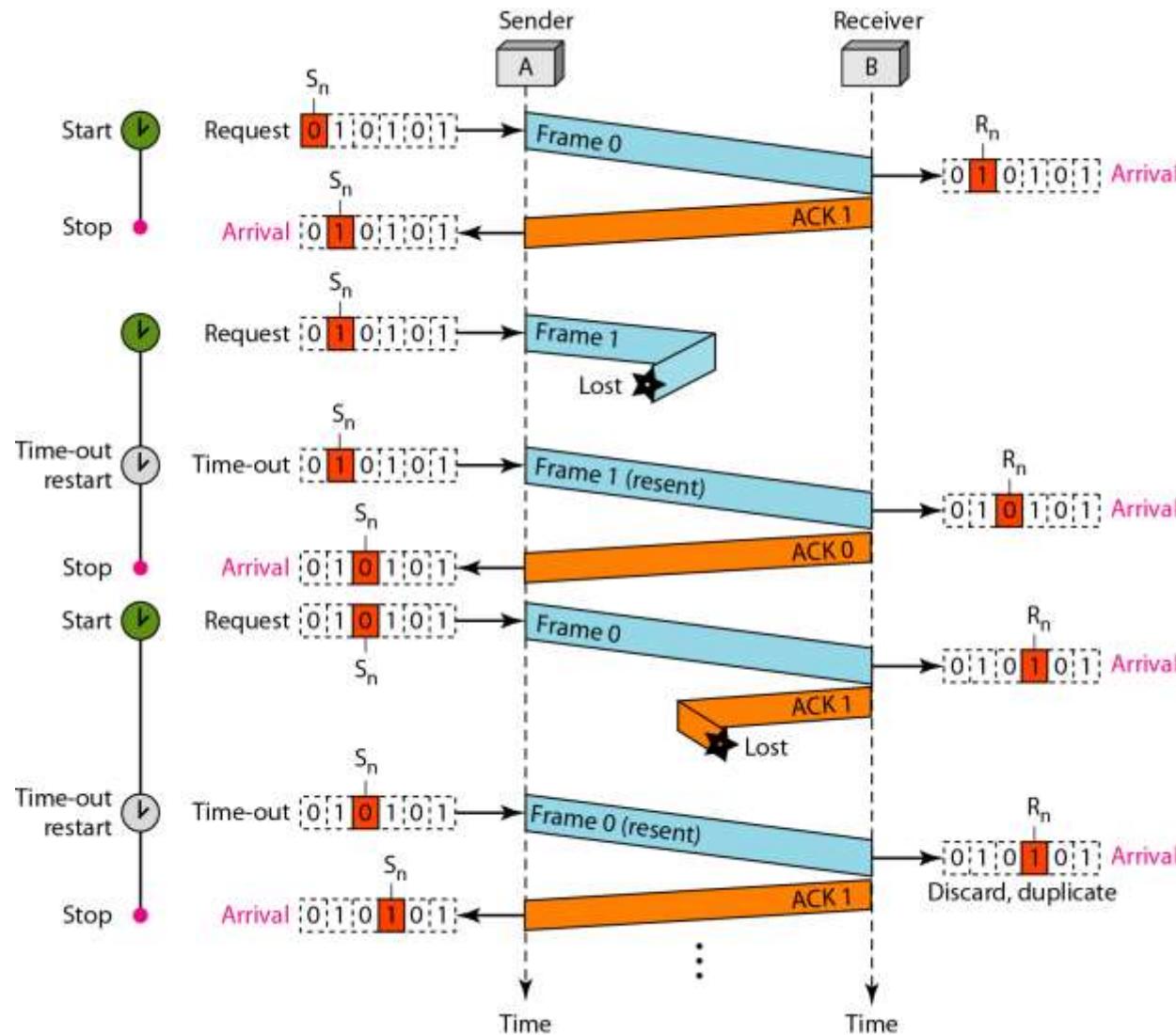
Algorithm 11.6 Receiver-site algorithm for Stop-and-Wait ARQ Protocol

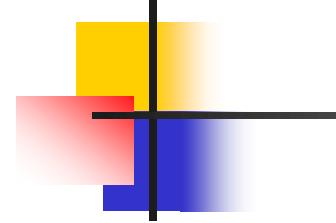
```
1 Rn = 0;                                // Frame 0 expected to arrive first
2 while(true)
3 {
4     WaitForEvent();                      // Sleep until an event occurs
5     if(Event(ArrivalNotification))      //Data frame arrives
6     {
7         ReceiveFrame();
8         if(corrupted(frame));
9             sleep();
10        if(seqNo == Rn)              //Valid data frame
11        {
12            ExtractData();
13            DeliverData();           //Deliver data
14            Rn = Rn + 1;
15        }
16        SendFrame(Rn);           //Send an ACK
17    }
18 }
```

Example 11.3

Figure 11.11 shows an example of Stop-and-Wait ARQ. Frame 0 is sent and acknowledged. Frame 1 is lost and resent after the time-out. The resent frame 1 is acknowledged and the timer stops. Frame 0 is sent and acknowledged, but the acknowledgment is lost. The sender has no idea if the frame or the acknowledgment is lost, so after the time-out, it resends frame 0, which is acknowledged.

Figure 11.11 Flow diagram for Example 11.3

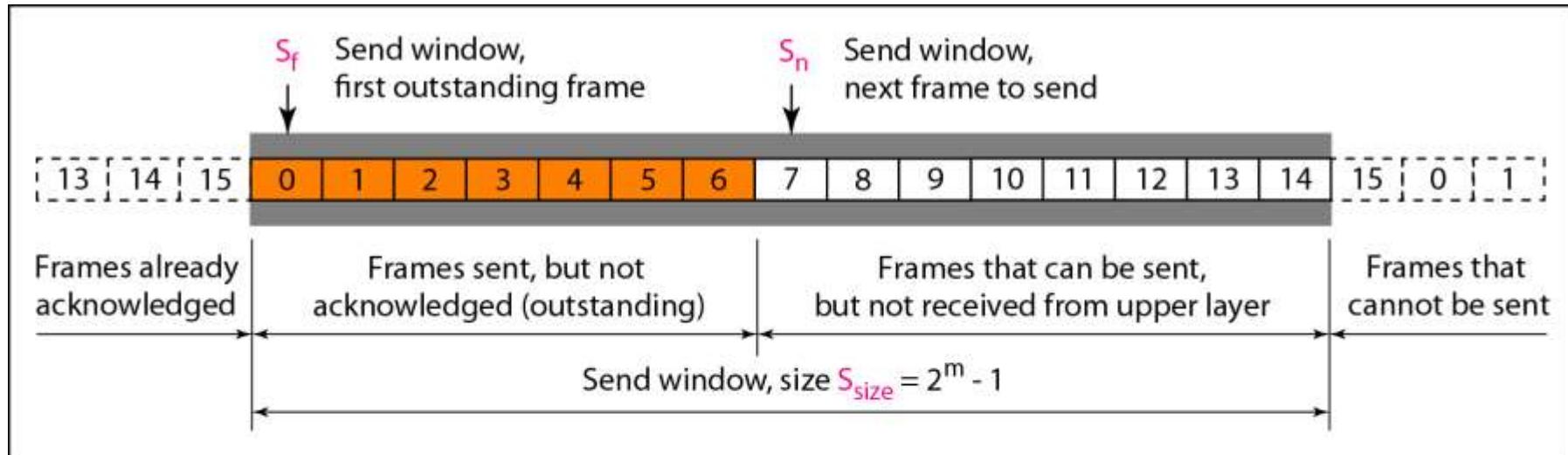




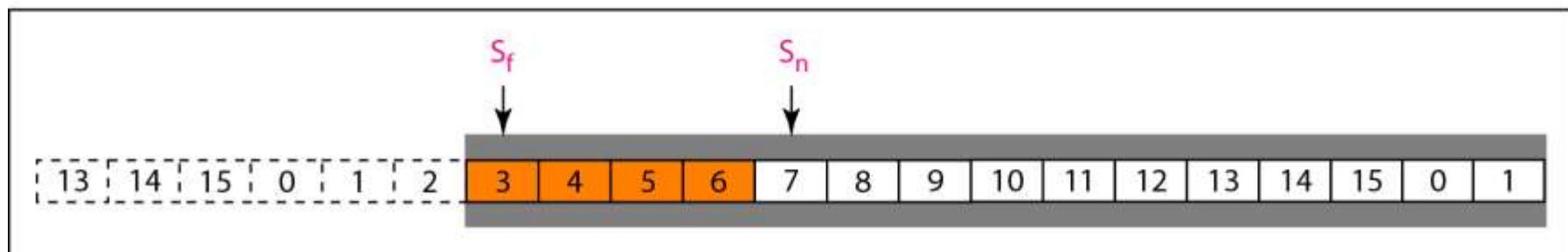
Note

In the Go-Back-N Protocol, the sequence numbers are modulo 2^m , where m is the size of the sequence number field in bits.

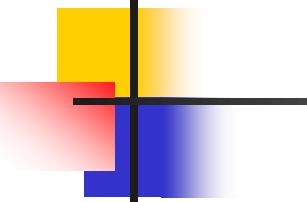
Figure 11.12 Send window for Go-Back-N ARQ



a. Send window before sliding

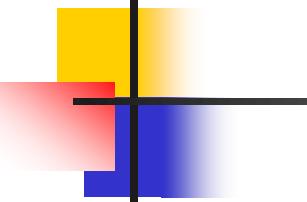


b. Send window after sliding



Note

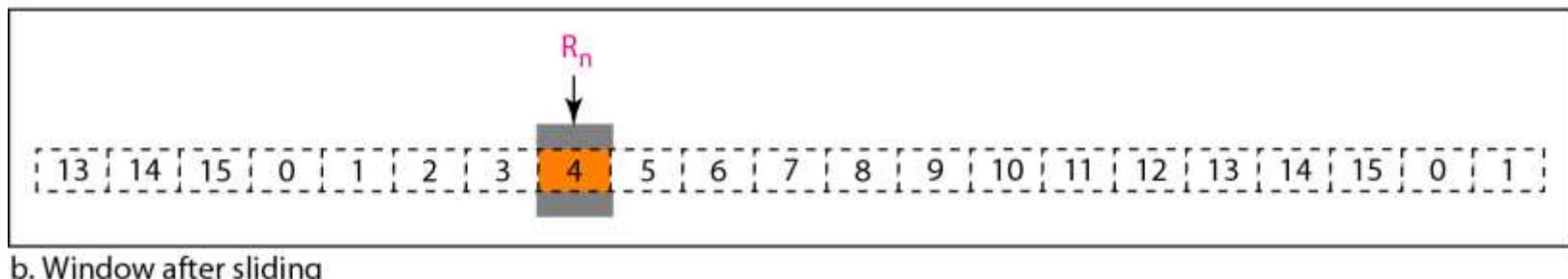
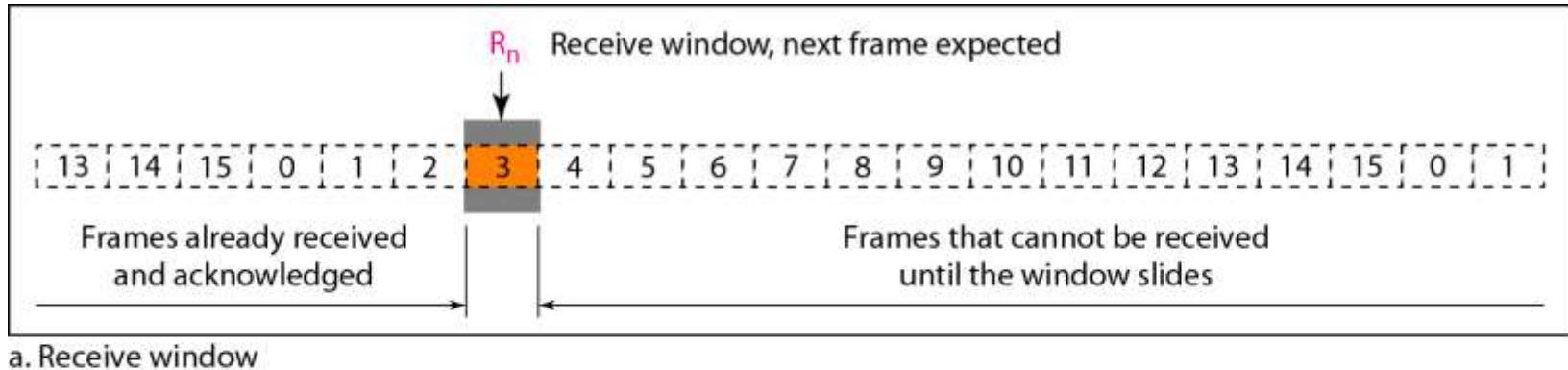
The send window is an abstract concept defining an imaginary box of size $2^m - 1$ with three variables: S_f , S_n , and S_{size} .

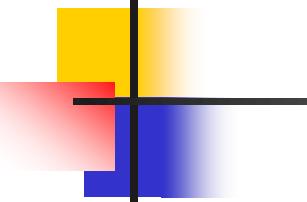


Note

The send window can slide one or more slots when a valid acknowledgment arrives.

Figure 11.13 Receive window for Go-Back-N ARQ





Note

The receive window is an abstract concept defining an imaginary box of size 1 with one single variable R_n .

The window slides when a correct frame has arrived; sliding occurs one slot at a time.

Figure 11.14 Design of Go-Back-N ARQ

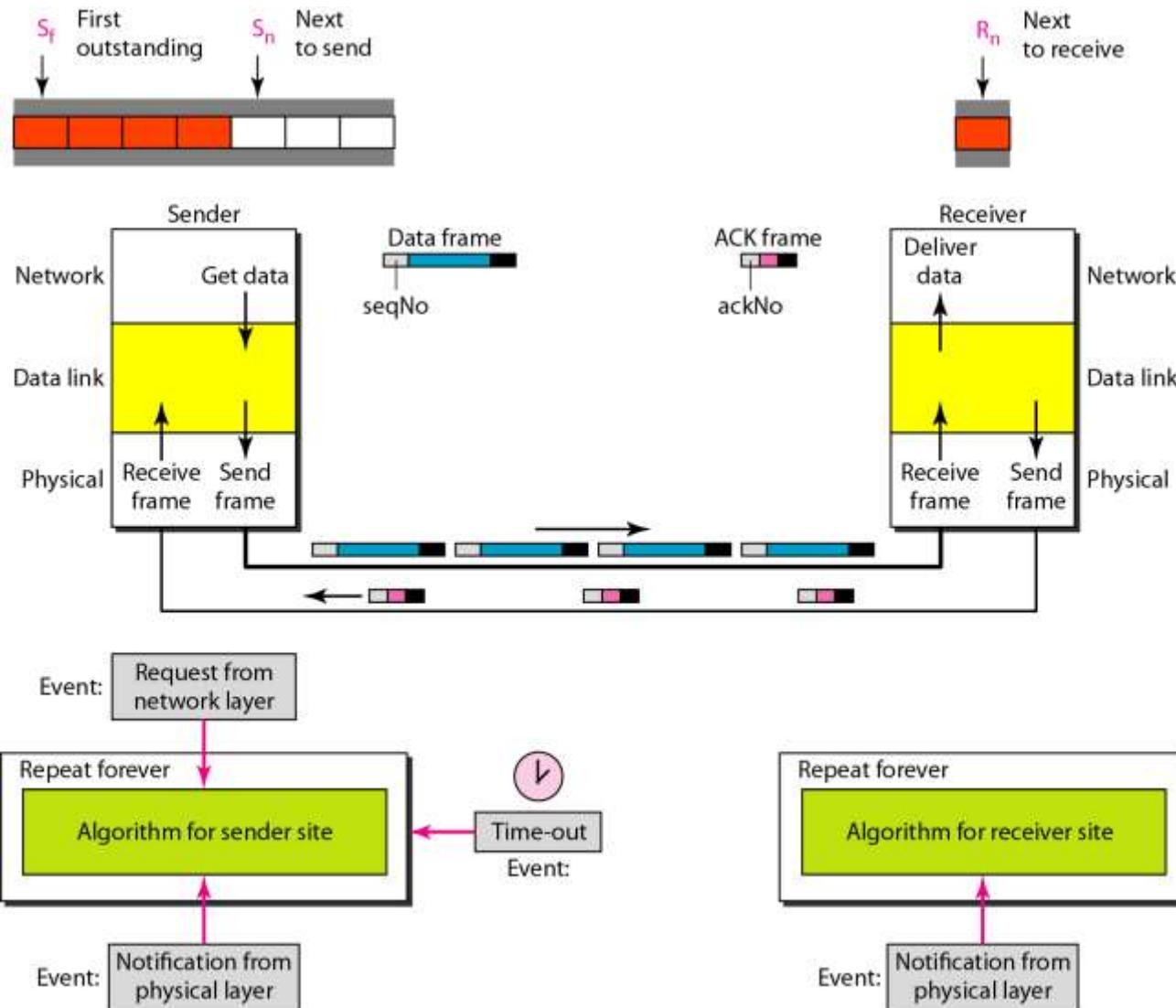
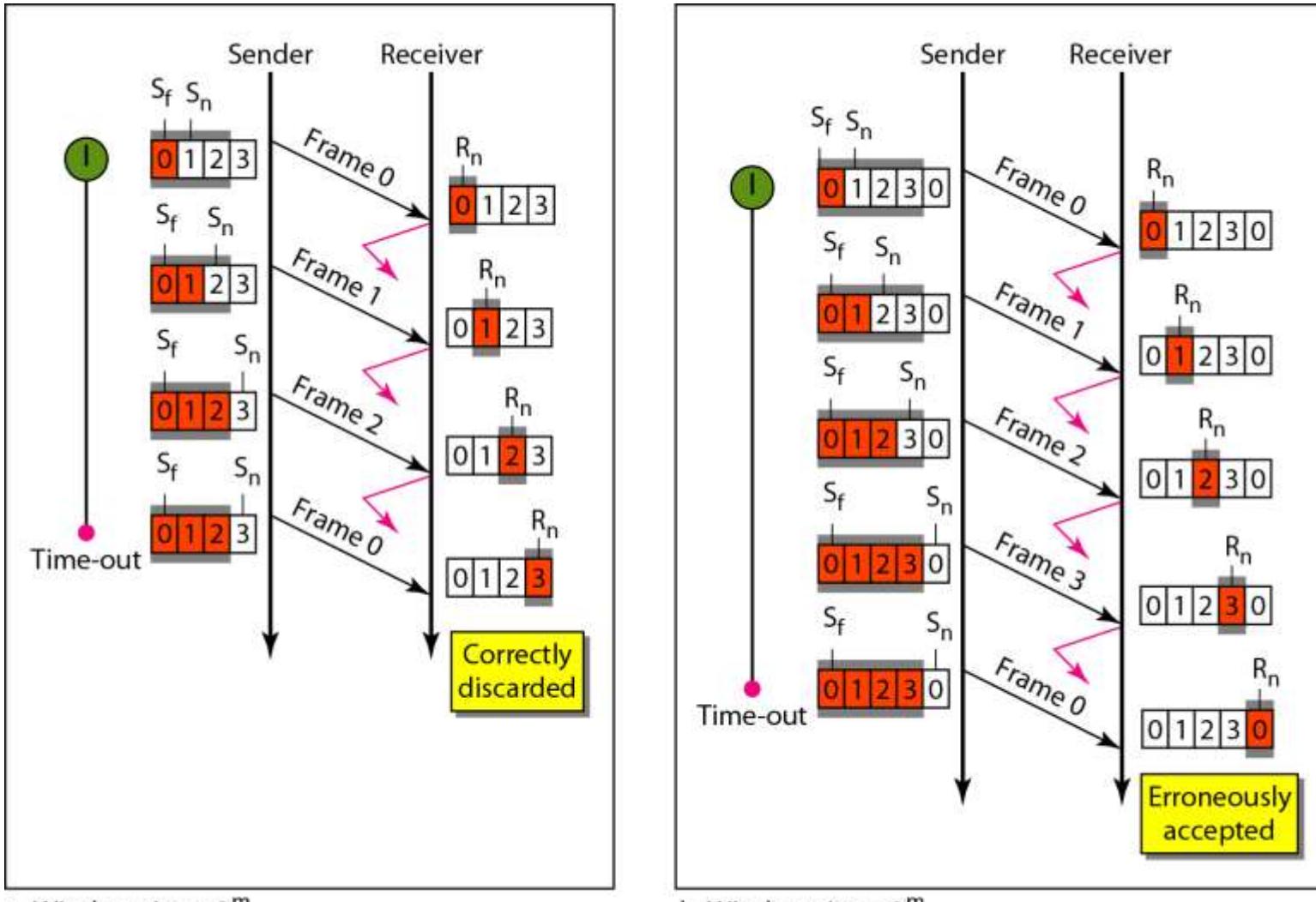
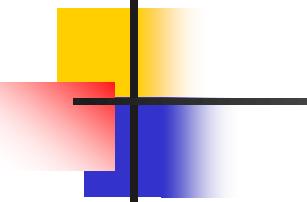


Figure 11.15 Window size for Go-Back-N ARQ



a. Window size $< 2^m$

b. Window size $= 2^m$



Note

In Go-Back-N ARQ, the size of the send window must be less than 2^m ; the size of the receiver window is always 1.

Algorithm 11.7 Go-Back-N sender algorithm

```
1 Sw = 2m - 1;  
2 Sf = 0;  
3 Sn = 0;  
4  
5 while (true) //Repeat forever  
6 {  
7   WaitForEvent();  
8   if(Event(RequestToSend)) //A packet to send  
9   {  
10     if(Sn-Sf >= Sw) //If window is full  
11       Sleep();  
12     GetData();  
13     MakeFrame(Sn);  
14     StoreFrame(Sn);  
15     SendFrame(Sn);  
16     Sn = Sn + 1;  
17     if(timer not running)  
18       StartTimer();  
19   }  
20 }
```

(continued)

Algorithm 11.7 Go-Back-N sender algorithm

(continued)

```
21  if(Event(ArrivalNotification)) //ACK arrives
22  {
23      Receive(ACK);
24      if(corrupted(ACK))
25          Sleep();
26      if((ackNo>Sf)&&(ackNo<=Sn)) //If a valid ACK
27      While(Sf <= ackNo)
28      {
29          PurgeFrame(Sf);
30          Sf = Sf + 1;
31      }
32      StopTimer();
33  }

34

35  if(Event(TimeOut)) //The timer expires
36  {
37      StartTimer();
38      Temp = Sf;
39      while(Temp < Sn);
40      {
41          SendFrame(Sf);
42          Sf = Sf + 1;
43      }
44  }
45 }
```

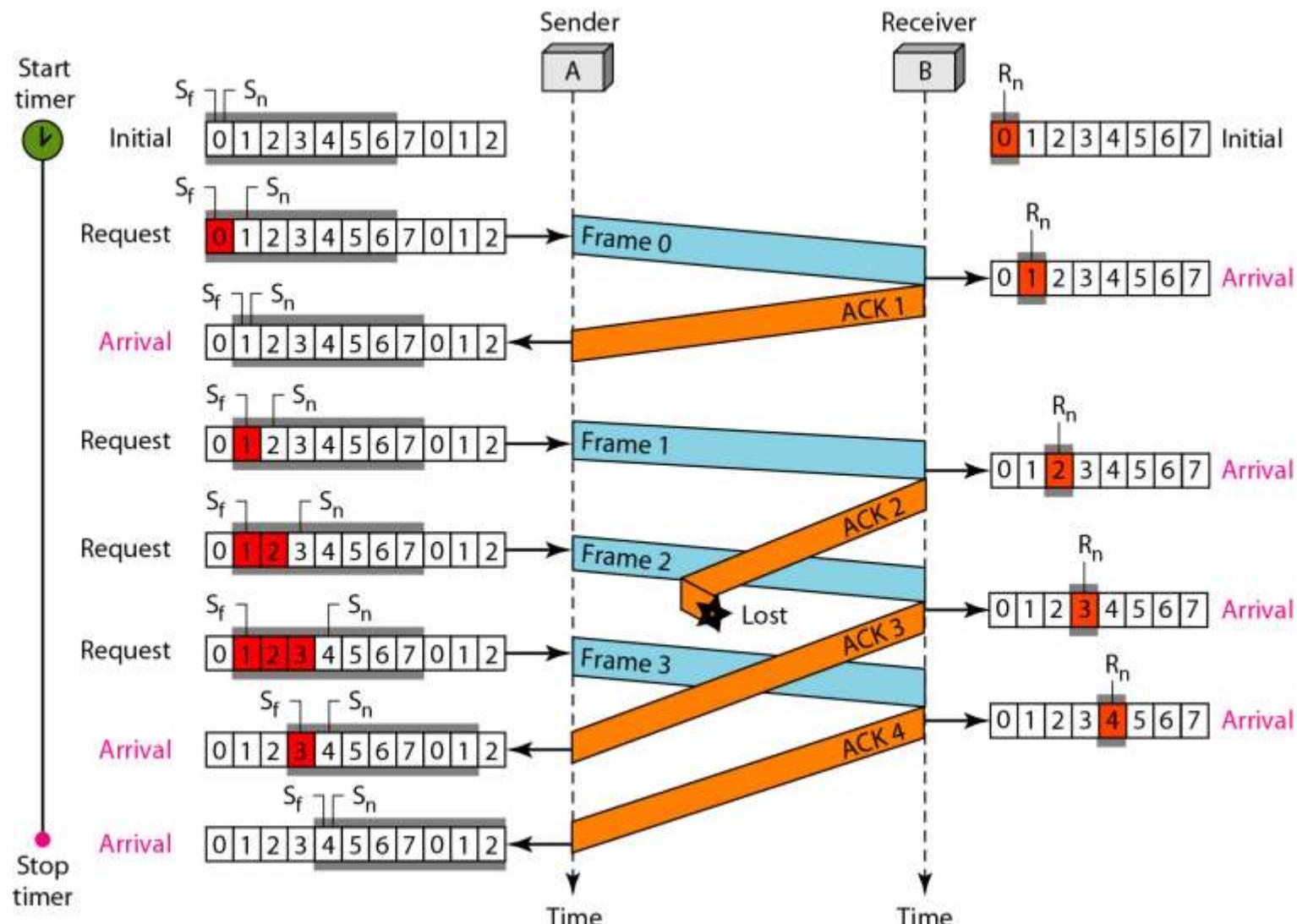
Algorithm 11.8 Go-Back-N receiver algorithm

```
1 Rn = 0;  
2  
3 while (true) //Repeat forever  
4 {  
5     WaitForEvent();  
6  
7     if(Event(ArrivalNotification)) /Data frame arrives  
8     {  
9         Receive(Frame);  
10        if(corrupted(Frame))  
11            Sleep();  
12        if(seqNo == Rn) //If expected frame  
13        {  
14            DeliverData(); //Deliver data  
15            Rn = Rn + 1; //Slide window  
16            SendACK(Rn);  
17        }  
18    }  
19 }
```

Example 11.6

Figure 11.16 shows an example of Go-Back-N. This is an example of a case where the forward channel is reliable, but the reverse is not. No data frames are lost, but some ACKs are delayed and one is lost. The example also shows how cumulative acknowledgments can help if acknowledgments are delayed or lost. After initialization, there are seven sender events. Request events are triggered by data from the network layer; arrival events are triggered by acknowledgments from the physical layer. There is no time-out event here because all outstanding frames are acknowledged before the timer expires. Note that although ACK 2 is lost, ACK 3 serves as both ACK 2 and ACK 3.

Figure 11.16 Flow diagram for Example 11.6



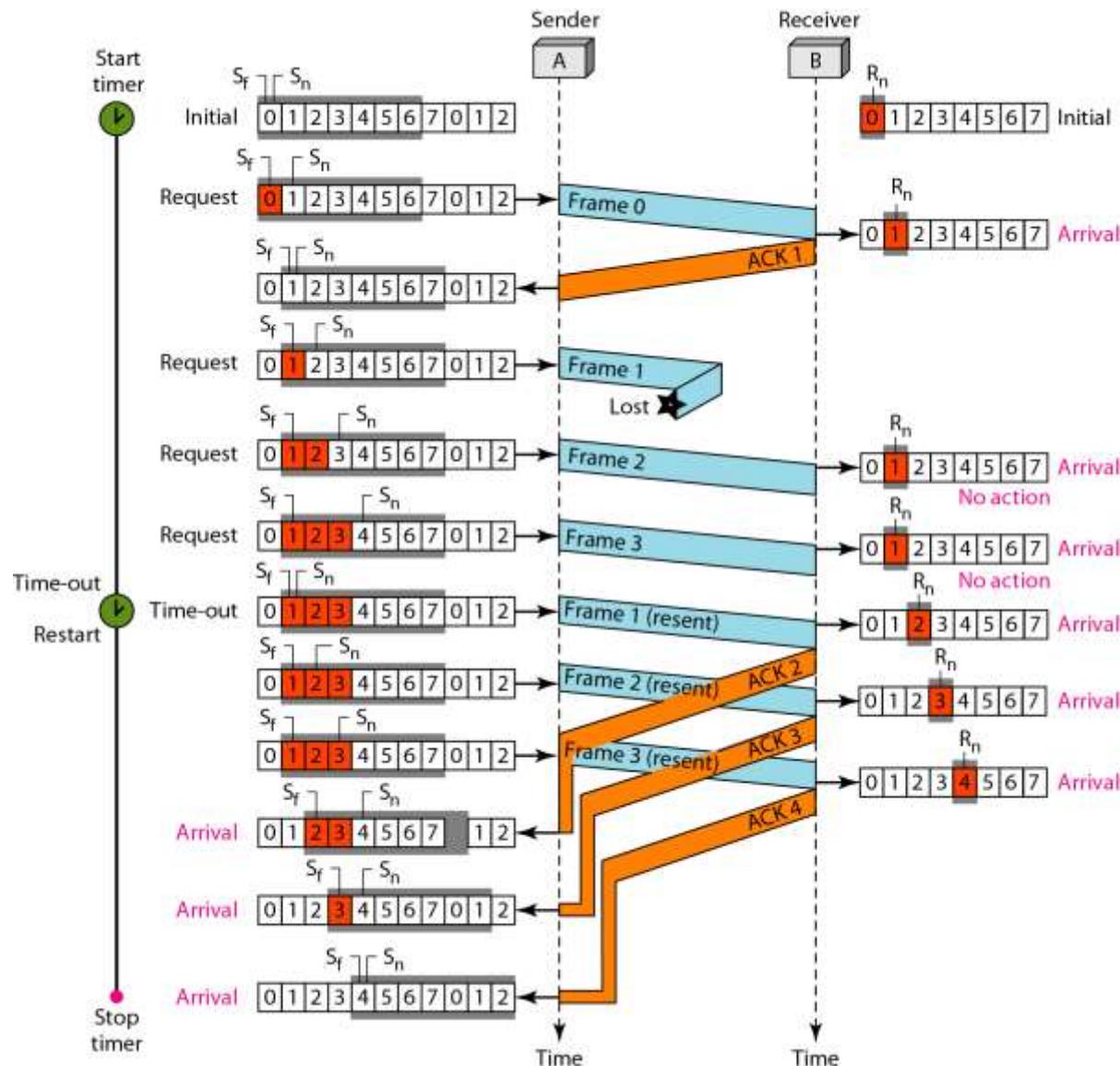
Example 11.7

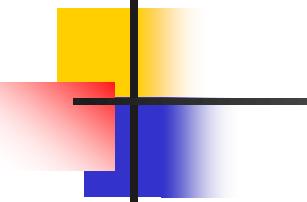
Figure 11.17 shows what happens when a frame is lost. Frames 0, 1, 2, and 3 are sent. However, frame 1 is lost. The receiver receives frames 2 and 3, but they are discarded because they are received out of order. The sender receives no acknowledgment about frames 1, 2, or 3. Its timer finally expires. The sender sends all outstanding frames (1, 2, and 3) because it does not know what is wrong. Note that the resending of frames 1, 2, and 3 is the response to one single event. When the sender is responding to this event, it cannot accept the triggering of other events. This means that when ACK 2 arrives, the sender is still busy with sending frame 3.

Example 11.7 (continued)

The physical layer must wait until this event is completed and the data link layer goes back to its sleeping state. We have shown a vertical line to indicate the delay. It is the same story with ACK 3; but when ACK 3 arrives, the sender is busy responding to ACK 2. It happens again when ACK 4 arrives. Note that before the second timer expires, all outstanding frames have been sent and the timer is stopped.

Figure 11.17 Flow diagram for Example 11.7





Note

Stop-and-Wait ARQ is a special case of Go-Back-N ARQ in which the size of the send window is 1.

Figure 11.18 Send window for Selective Repeat ARQ

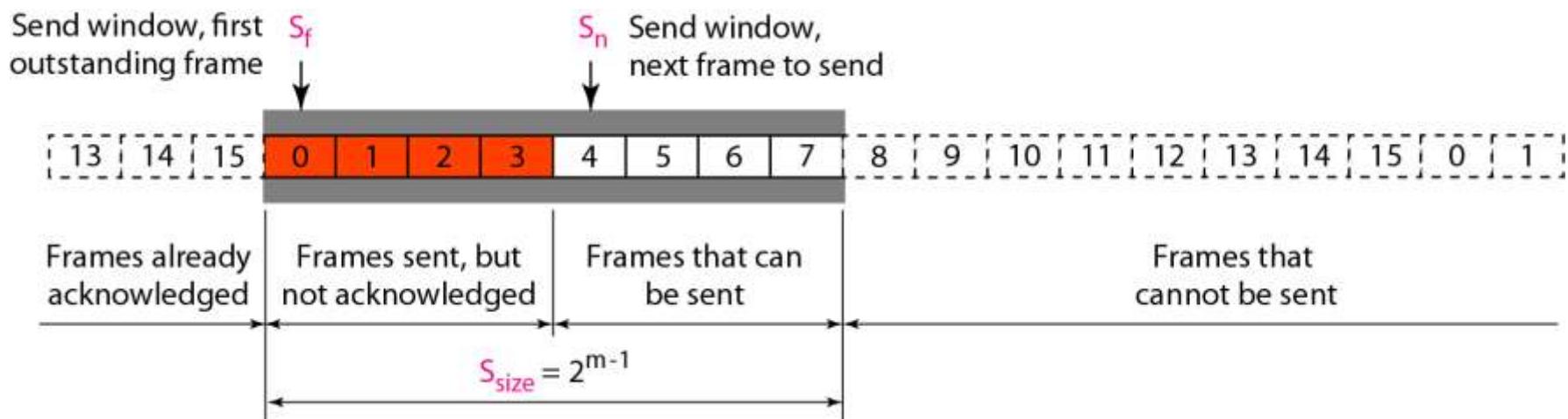


Figure 11.19 Receive window for Selective Repeat ARQ

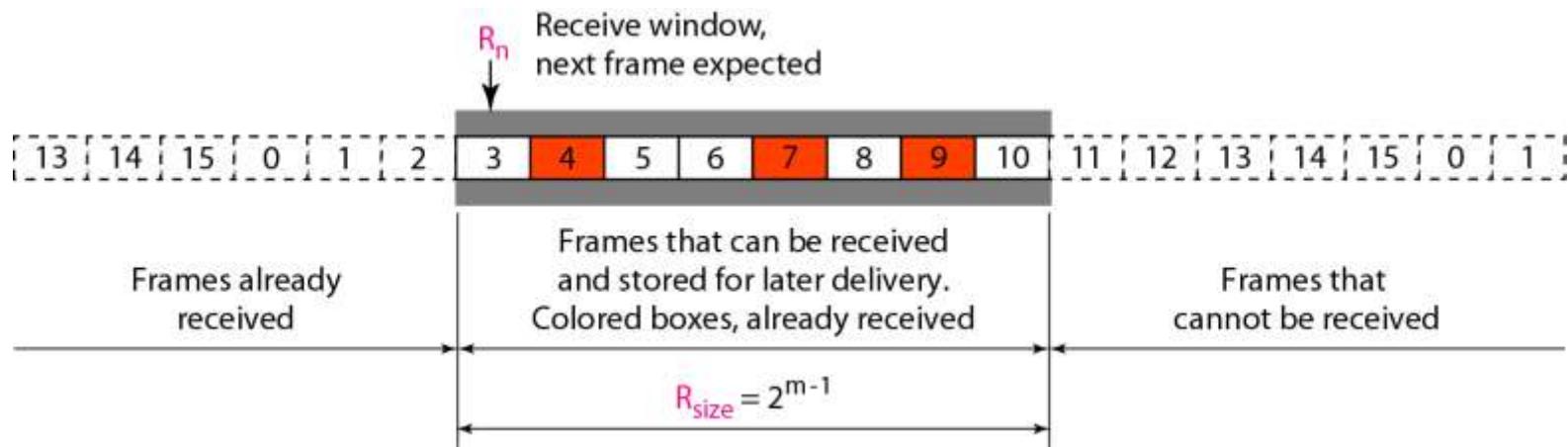


Figure 11.20 Design of Selective Repeat ARQ

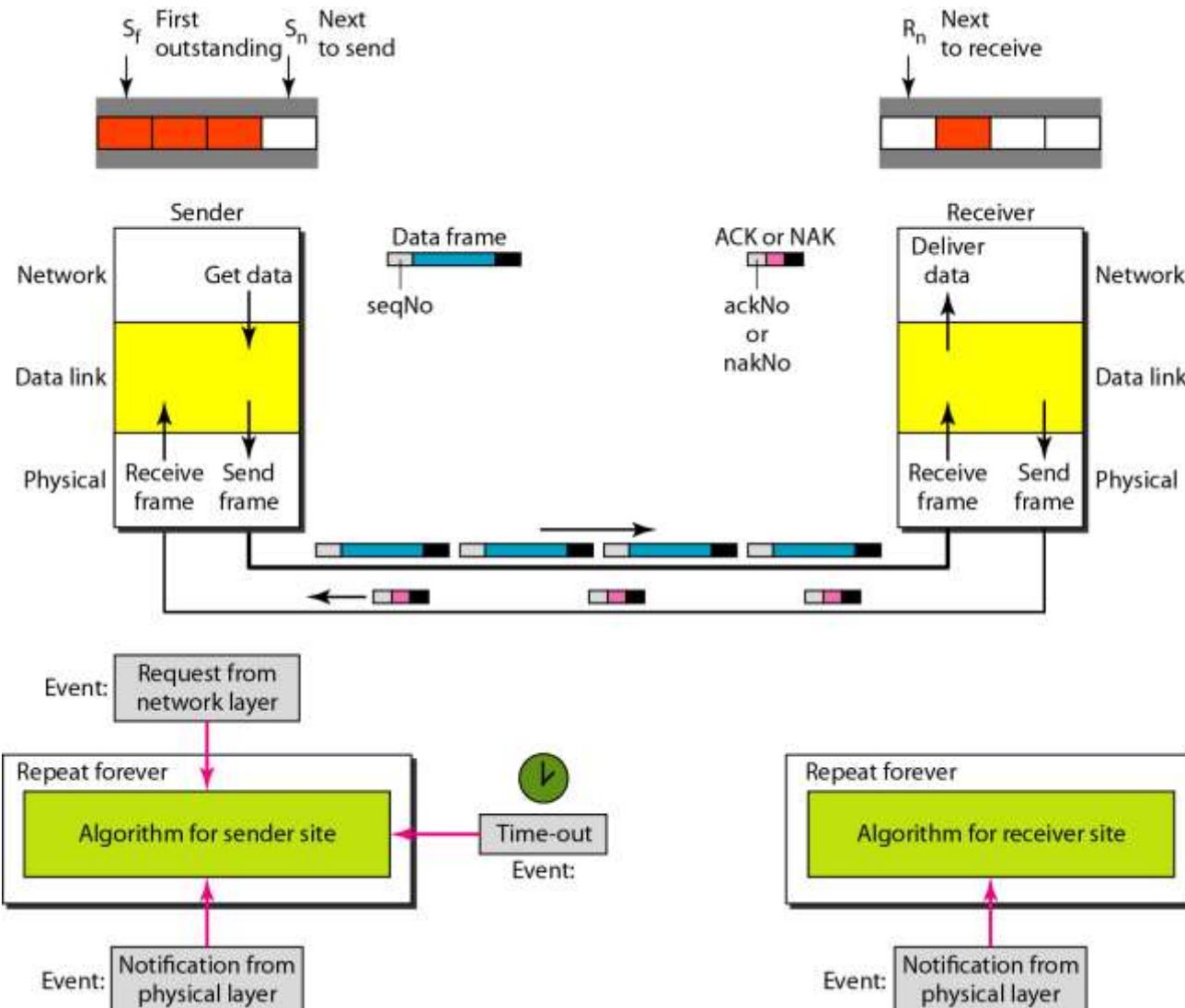
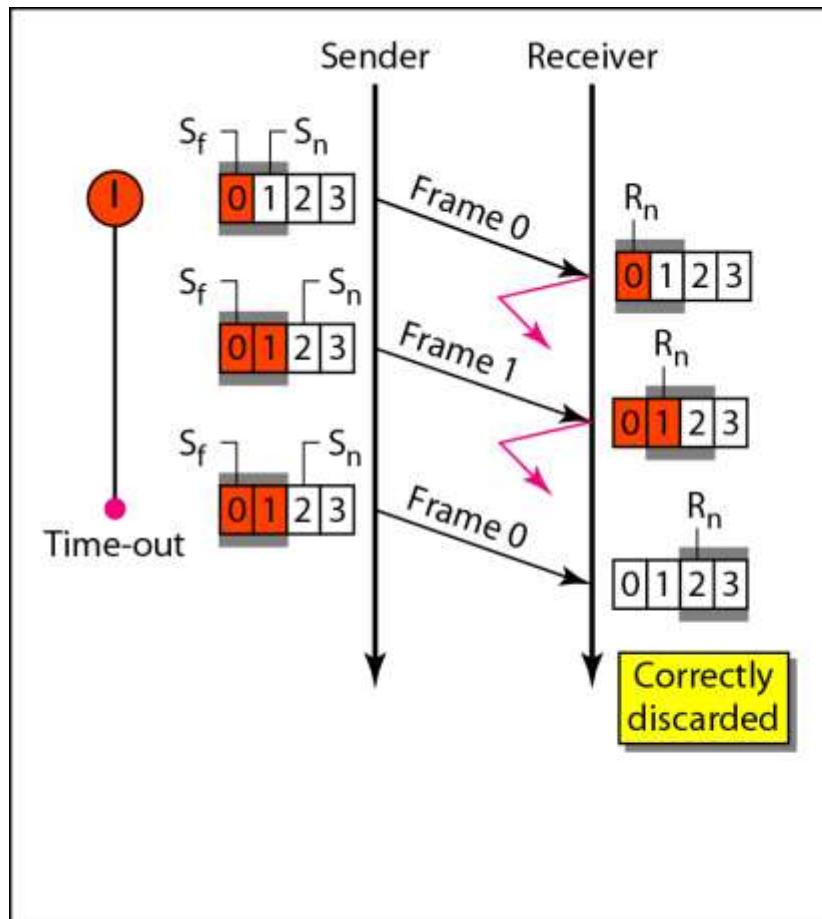
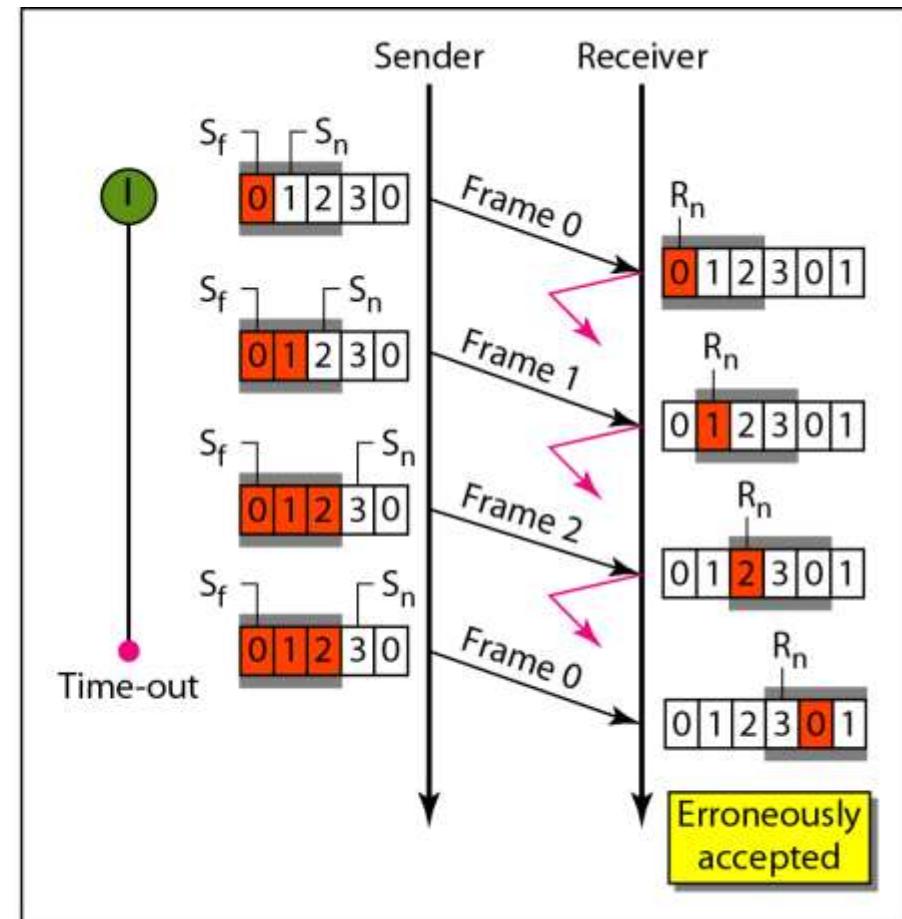


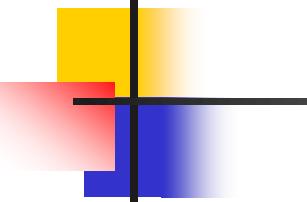
Figure 11.21 Selective Repeat ARQ, window size



a. Window size = 2^{m-1}



b. Window size > 2^{m-1}



Note

In Selective Repeat ARQ, the size of the sender and receiver window must be at most one-half of 2^m .

Algorithm 11.9 Sender-site Selective Repeat algorithm

```
1 Sw = 2m-1 ;
2 Sf = 0 ;
3 Sn = 0 ;
4
5 while (true)                                //Repeat forever
6 {
7     WaitForEvent() ;
8     if(Event(RequestToSend))                  //There is a packet to send
9     {
10         if(Sn-Sf >= Sw)                //If window is full
11             Sleep() ;
12         GetData() ;
13         MakeFrame(Sn) ;
14         StoreFrame(Sn) ;
15         SendFrame(Sn) ;
16         Sn = Sn + 1 ;
17         StartTimer(Sn) ;
18     }
19 }
```

(continued)

Algorithm 11.9 Sender-site Selective Repeat algorithm

(continued)

```
20  if(Event(ArrivalNotification)) //ACK arrives
21  {
22      Receive(frame);           //Receive ACK or NAK
23      if(corrupted(frame))
24          Sleep();
25      if (FrameType == NAK)
26          if (nakNo between Sf and Sn)
27          {
28              resend(nakNo);
29              StartTimer(nakNo);
30          }
31      if (FrameType == ACK)
32          if (ackNo between Sf and Sn)
33          {
34              while(sf < ackNo)
35              {
36                  Purge(sf);
37                  StopTimer(sf);
38                  Sf = Sf + 1;
39              }
40          }
41 }
```

(continued)

Algorithm 11.9 *Sender-site Selective Repeat algorithm*

(continued)

```
42  
43     if(Event(TimeOut(t)))          //The timer expires  
44     {  
45         StartTimer(t);  
46         SendFrame(t);  
47     }  
48 }
```

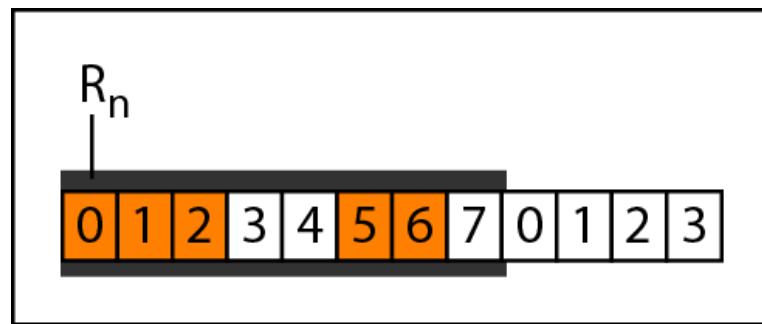
Algorithm 11.10 *Receiver-site Selective Repeat algorithm*

```
1 Rn = 0;
2 NakSent = false;
3 AckNeeded = false;
4 Repeat(for all slots)
5     Marked(slot) = false;
6
7 while (true)                                //Repeat forever
8 {
9     WaitForEvent();
10
11    if(Event(ArrivalNotification))           /Data frame arrives
12    {
13        Receive(Frame);
14        if(corrupted(Frame))&& (NOT NakSent)
15        {
16            SendNAK(Rn);
17            NakSent = true;
18            Sleep();
19        }
20        if(seqNo <> Rn)&& (NOT NakSent)
21        {
22            SendNAK(Rn);
```

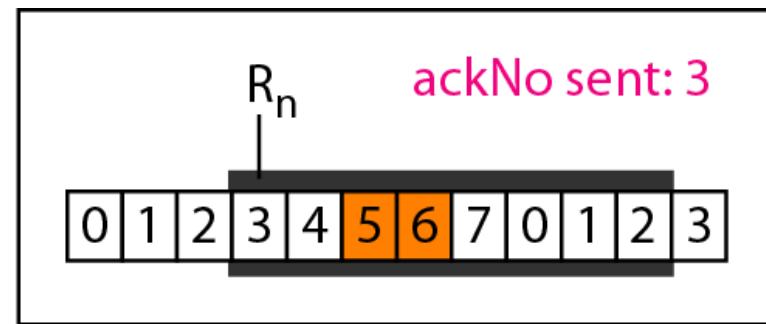
Algorithm 11.10 *Receiver-site Selective Repeat algorithm*

```
23     NakSent = true;
24     if ((seqNo in window)&&(!Marked(seqNo)))
25     {
26         StoreFrame(seqNo)
27         Marked(seqNo)= true;
28         while(Marked(Rn))
29         {
30             DeliverData(Rn);
31             Purge(Rn);
32             Rn = Rn + 1;
33             AckNeeded = true;
34         }
35         if(AckNeeded);
36         {
37             SendAck(Rn);
38             AckNeeded = false;
39             NakSent = false;
40         }
41     }
42 }
43 }
44 }
```

Figure 11.22 *Delivery of data in Selective Repeat ARQ*



a. Before delivery



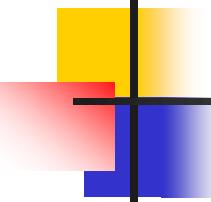
b. After delivery

Example 11.8

This example is similar to Example 11.3 in which frame 1 is lost. We show how Selective Repeat behaves in this case. Figure 11.23 shows the situation. One main difference is the number of timers. Here, each frame sent or resent needs a timer, which means that the timers need to be numbered (0, 1, 2, and 3). The timer for frame 0 starts at the first request, but stops when the ACK for this frame arrives. The timer for frame 1 starts at the second request, restarts when a NAK arrives, and finally stops when the last ACK arrives. The other two timers start when the corresponding frames are sent and stop at the last arrival event.

Example 11.8 (continued)

At the receiver site we need to distinguish between the acceptance of a frame and its delivery to the network layer. At the second arrival, frame 2 arrives and is stored and marked, but it cannot be delivered because frame 1 is missing. At the next arrival, frame 3 arrives and is marked and stored, but still none of the frames can be delivered. Only at the last arrival, when finally a copy of frame 1 arrives, can frames 1, 2, and 3 be delivered to the network layer. There are two conditions for the delivery of frames to the network layer: First, a set of consecutive frames must have arrived. Second, the set starts from the beginning of the window.



Example 11.8 (continued)

Another important point is that a NAK is sent after the second arrival, but not after the third, although both situations look the same. The reason is that the protocol does not want to crowd the network with unnecessary NAKs and unnecessary resent frames. The second NAK would still be NAK1 to inform the sender to resend frame 1 again; this has already been done. The first NAK sent is remembered (using the nakSent variable) and is not sent again until the frame slides. A NAK is sent once for each window position and defines the first slot in the window.

Example 11.8 (continued)

The next point is about the ACKs. Notice that only two ACKs are sent here. The first one acknowledges only the first frame; the second one acknowledges three frames. In Selective Repeat, ACKs are sent when data are delivered to the network layer. If the data belonging to n frames are delivered in one shot, only one ACK is sent for all of them.

Figure 11.23 Flow diagram for Example 11.8

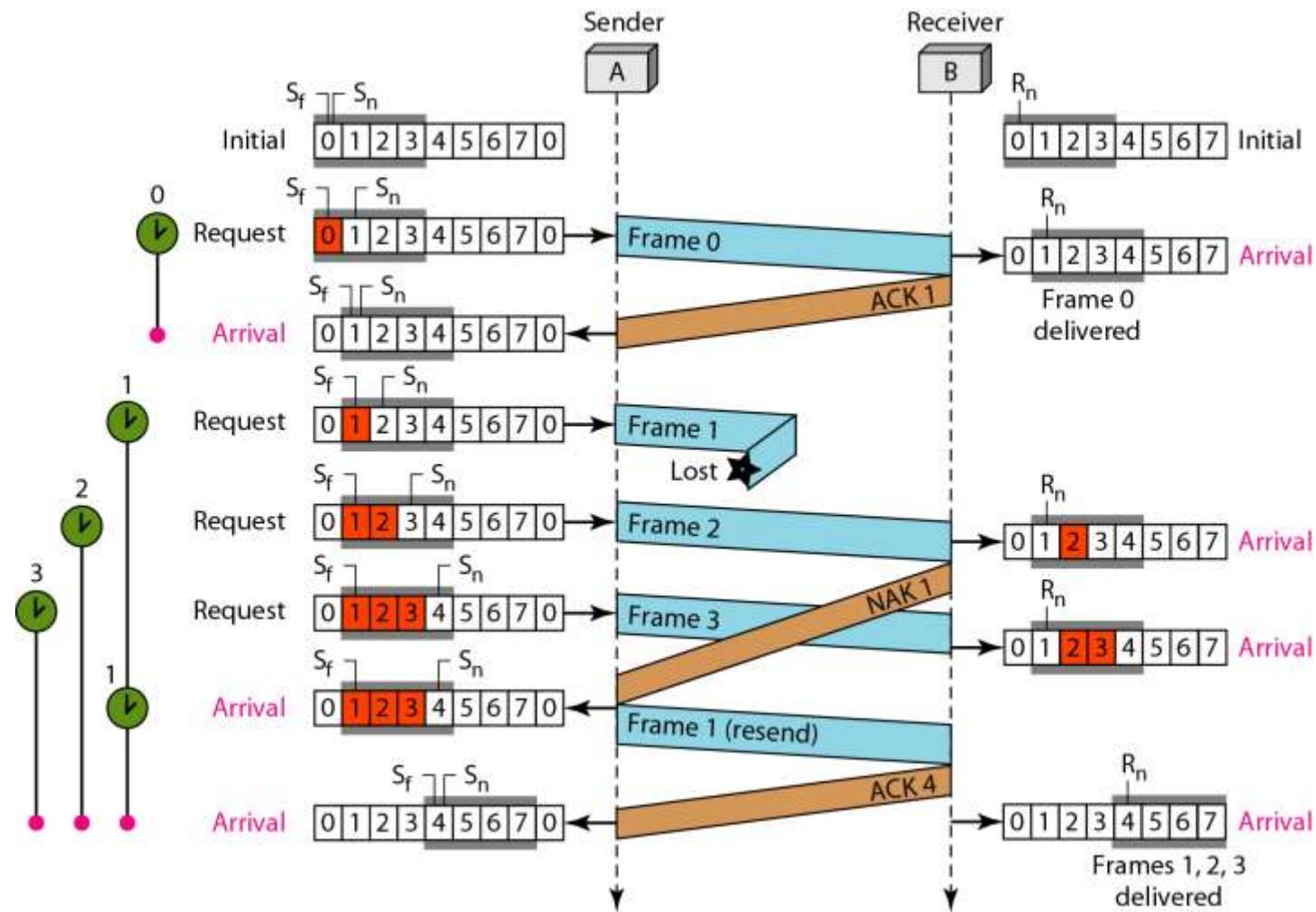
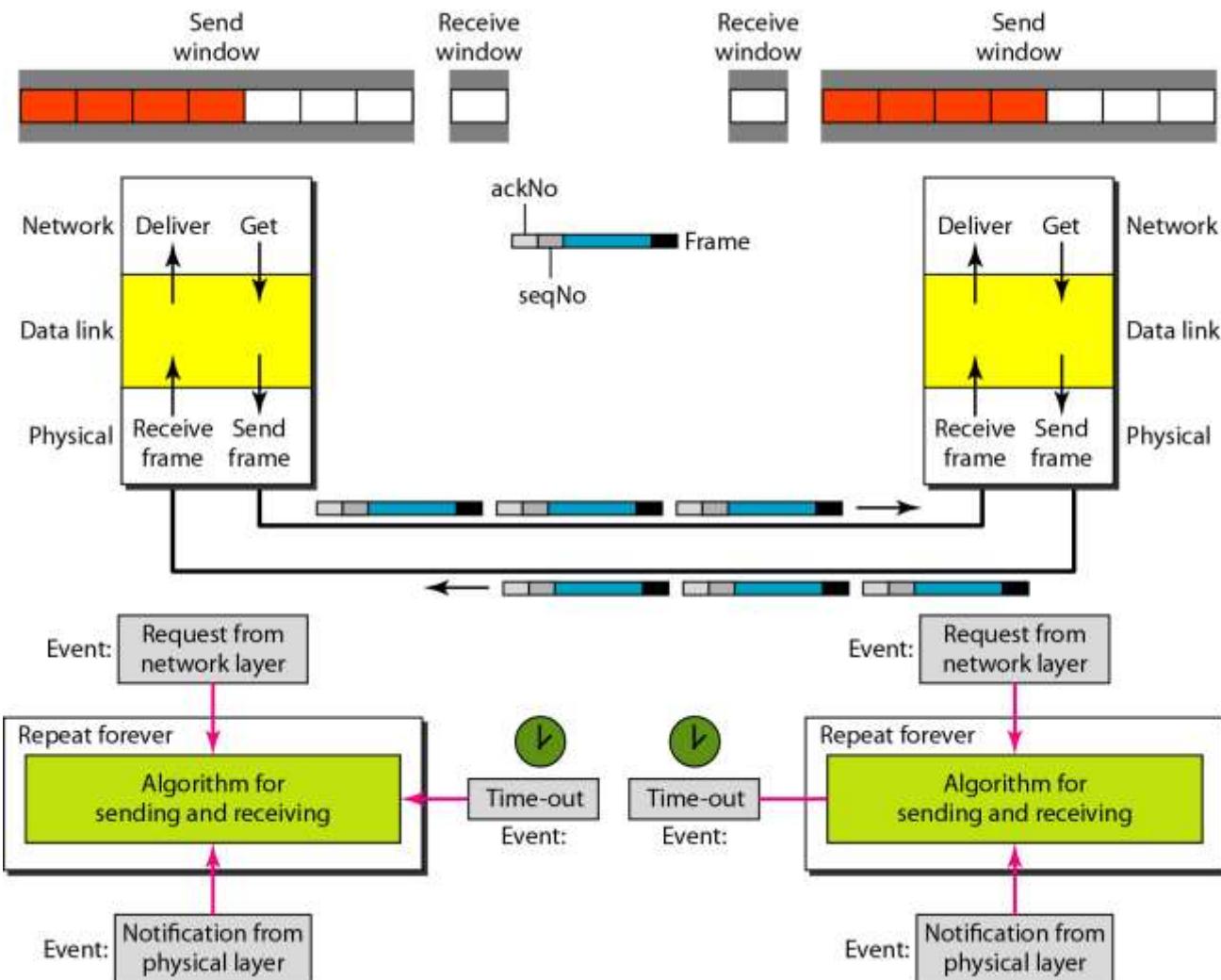


Figure 11.24 Design of piggybacking in Go-Back-N ARQ



11-6 HDLC

High-level Data Link Control (HDLC) is a bit-oriented protocol for communication over point-to-point and multipoint links. It implements the ARQ mechanisms we discussed in this chapter.

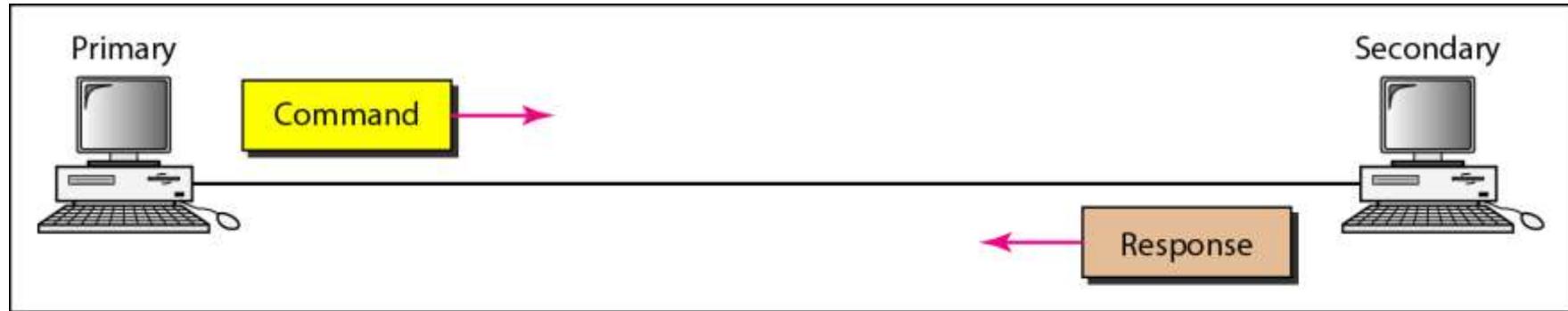
Topics discussed in this section:

Configurations and Transfer Modes

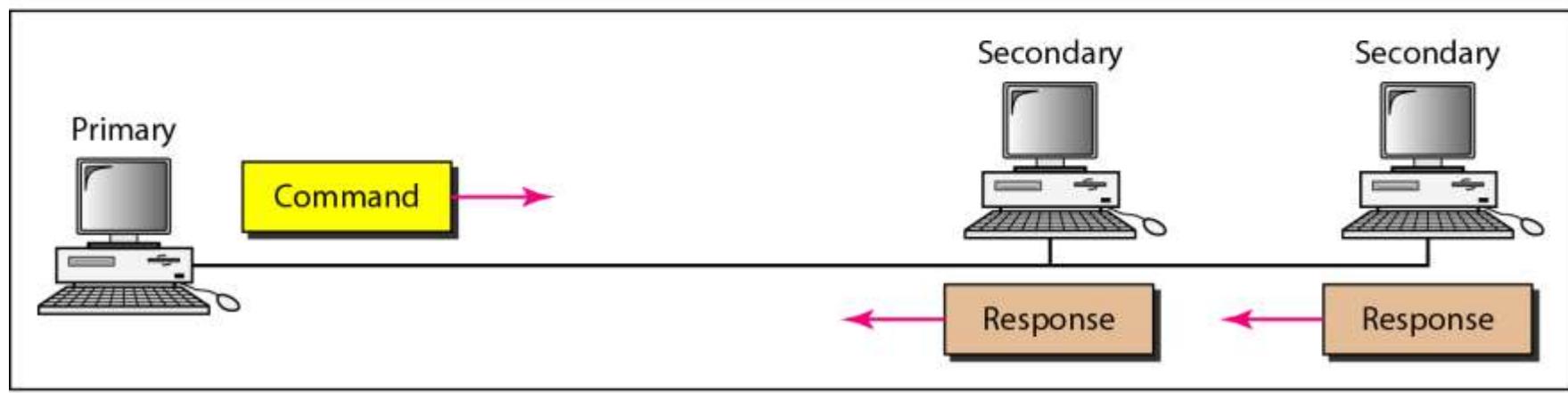
Frames

Control Field

Figure 11.25 *Normal response mode*



a. Point-to-point



b. Multipoint

Figure 11.26 *Asynchronous balanced mode*



Figure 11.27 HDLC frames

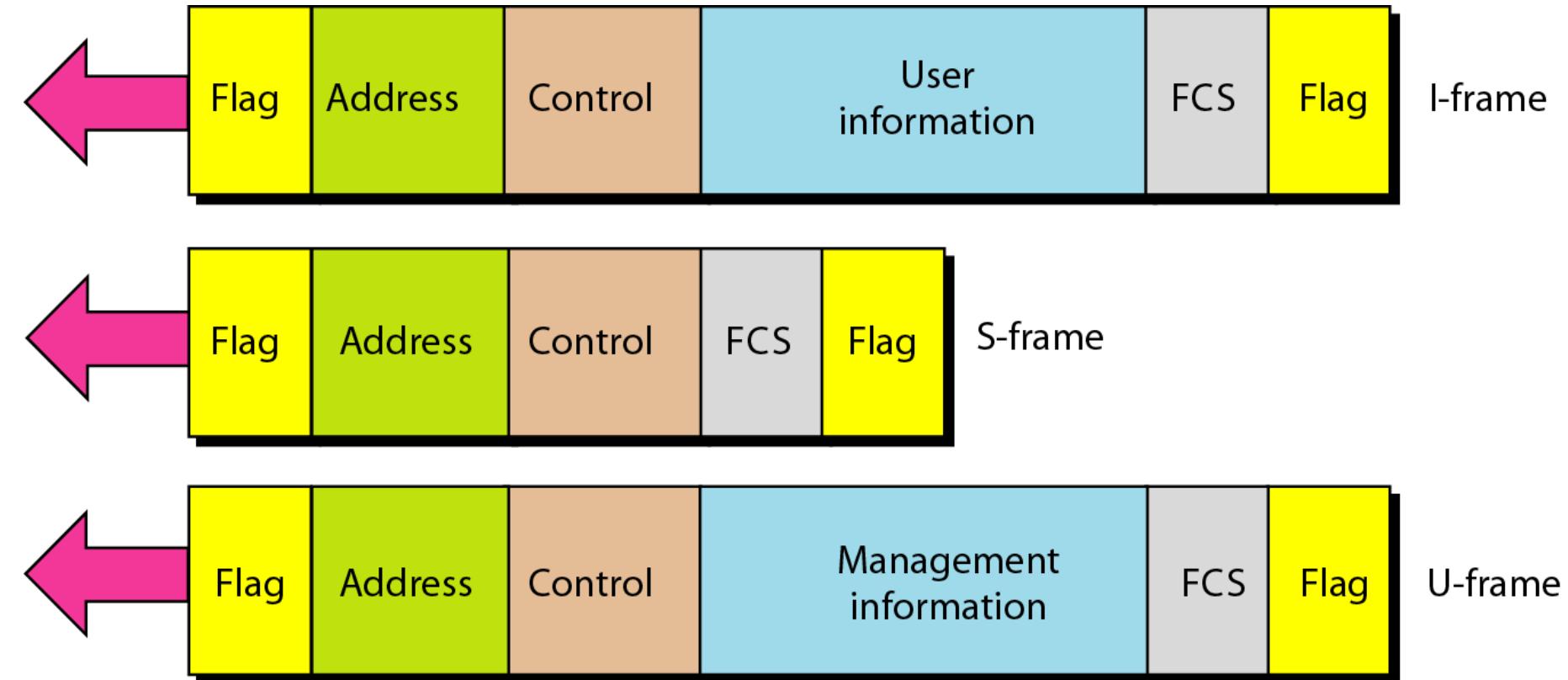


Figure 11.28 Control field format for the different frame types

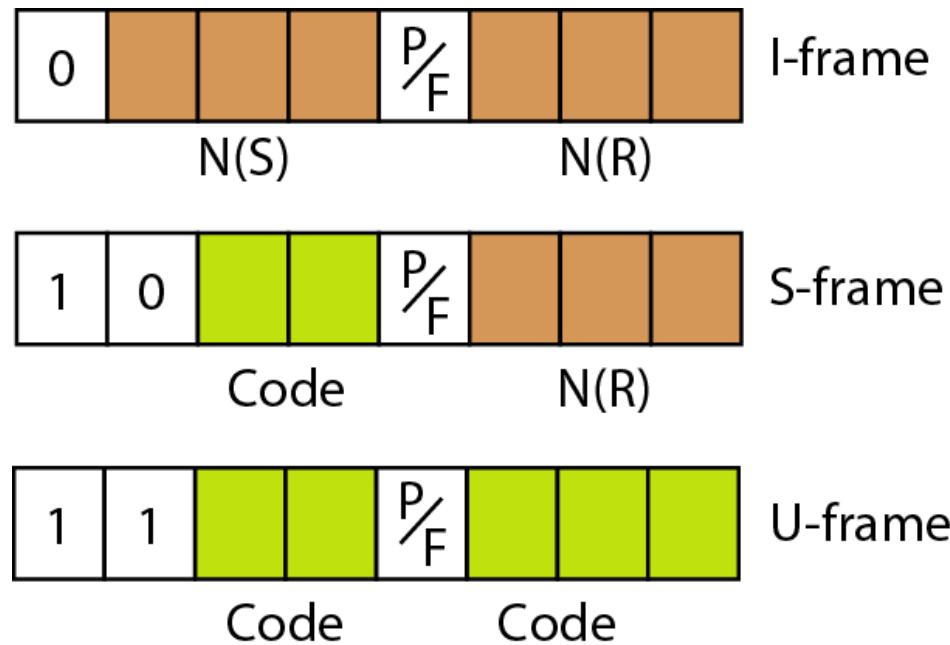


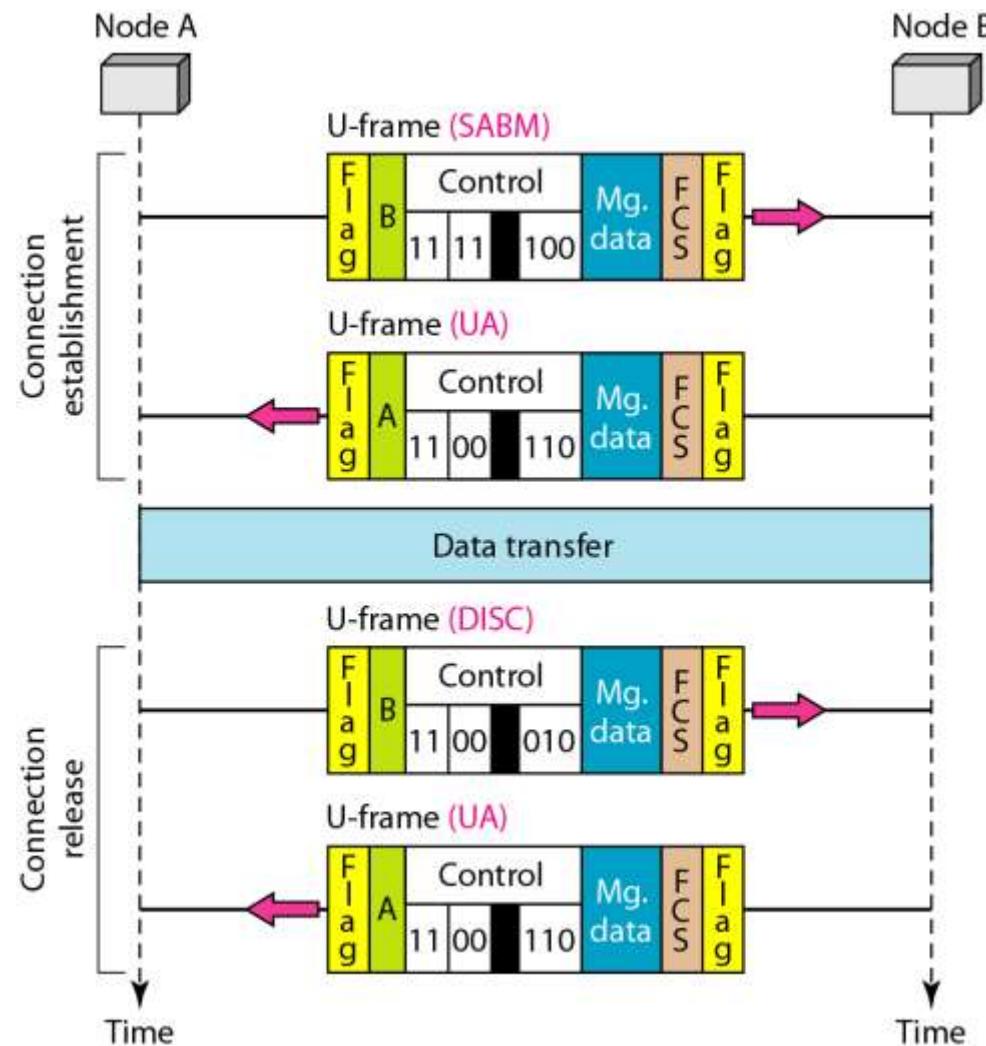
Table 11.1 *U-frame control command and response*

<i>Code</i>	<i>Command</i>	<i>Response</i>	<i>Meaning</i>
00 001	SNRM		Set normal response mode
11 011	SNRME		Set normal response mode, extended
11 100	SABM	DM	Set asynchronous balanced mode or disconnect mode
11 110	SABME		Set asynchronous balanced mode, extended
00 000	UI	UI	Unnumbered information
00 110		UA	Unnumbered acknowledgment
00 010	DISC	RD	Disconnect or request disconnect
10 000	SIM	RIM	Set initialization mode or request information mode
00 100	UP		Unnumbered poll
11 001	RSET		Reset
11 101	XID	XID	Exchange ID
10 001	FRMR	FRMR	Frame reject

Example 11.9

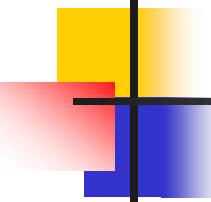
*Figure 11.29 shows how **U-frames** can be used for connection establishment and connection release. Node A asks for a connection with a set asynchronous balanced mode (SABM) frame; node B gives a positive response with an unnumbered acknowledgment (UA) frame. After these two exchanges, data can be transferred between the two nodes (not shown in the figure). After data transfer, node A sends a DISC (disconnect) frame to release the connection; it is confirmed by node B responding with a UA (unnumbered acknowledgment).*

Figure 11.29 Example of connection and disconnection



Example 11.10

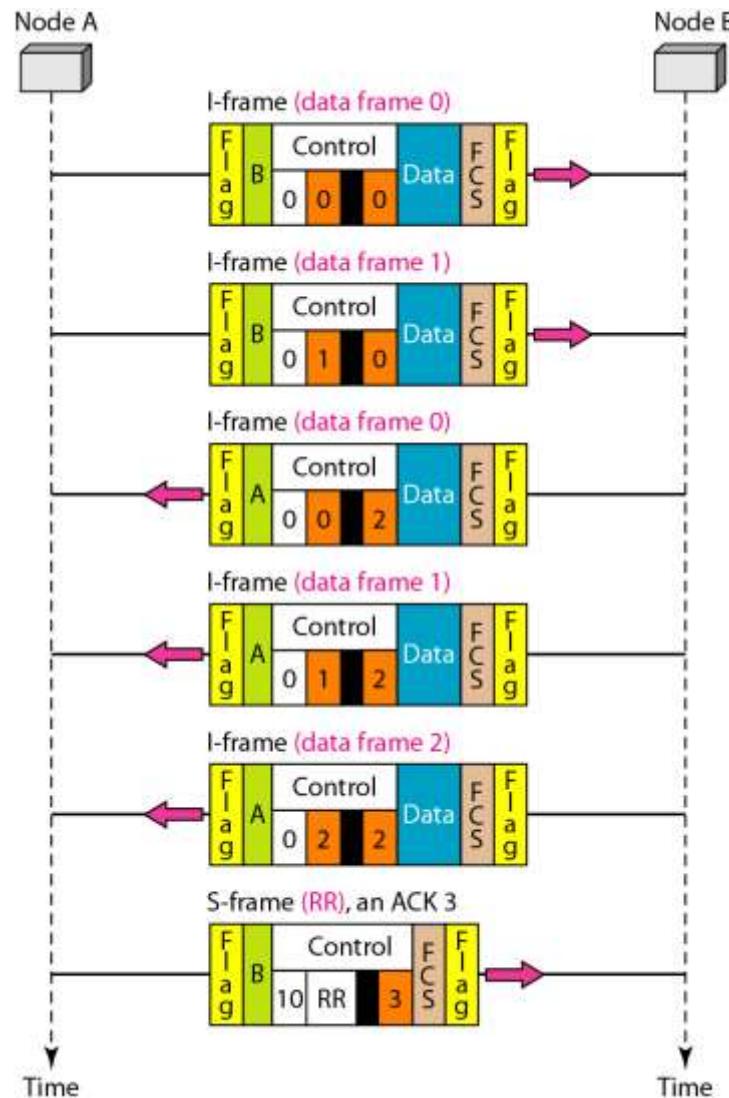
Figure 11.30 shows an exchange using piggybacking. Node A begins the exchange of information with an I-frame numbered 0 followed by another I-frame numbered 1. Node B piggybacks its acknowledgment of both frames onto an I-frame of its own. Node B's first I-frame is also numbered 0 [N(S) field] and contains a 2 in its N(R) field, acknowledging the receipt of A's frames 1 and 0 and indicating that it expects frame 2 to arrive next. Node B transmits its second and third I-frames (numbered 1 and 2) before accepting further frames from node A.



Example 11.10 (continued)

Its $N(R)$ information, therefore, has not changed: B frames 1 and 2 indicate that node B is still expecting A's frame 2 to arrive next. Node A has sent all its data. Therefore, it cannot piggyback an acknowledgment onto an I-frame and sends an S-frame instead. The RR code indicates that A is still ready to receive. The number 3 in the $N(R)$ field tells B that frames 0, 1, and 2 have all been accepted and that A is now expecting frame number 3.

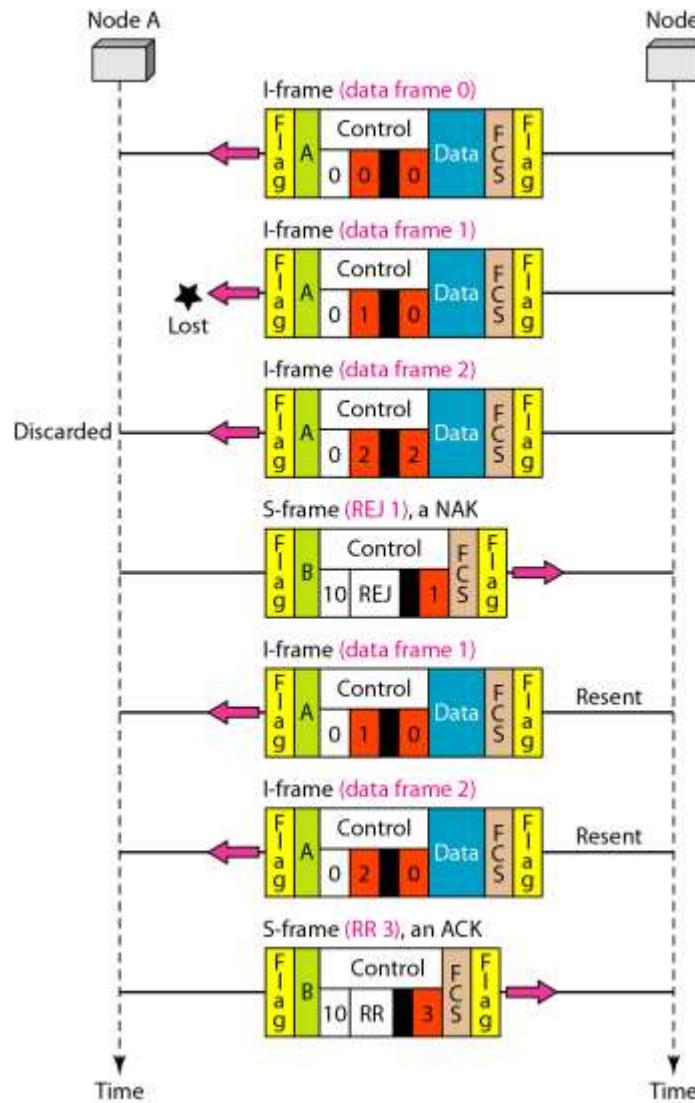
Figure 11.30 Example of piggybacking without error



Example 11.11

Figure 11.31 shows an exchange in which a frame is lost. Node B sends three data frames (0, 1, and 2), but frame 1 is lost. When node A receives frame 2, it discards it and sends a REJ frame for frame 1. Note that the protocol being used is Go-Back-N with the special use of an REJ frame as a NAK frame. The NAK frame does two things here: It confirms the receipt of frame 0 and declares that frame 1 and any following frames must be resent. Node B, after receiving the REJ frame, resends frames 1 and 2. Node A acknowledges the receipt by sending an RR frame (ACK) with acknowledgment number 3.

Figure 11.31 Example of piggybacking with error



11-7 POINT-TO-POINT PROTOCOL

*Although HDLC is a general protocol that can be used for both point-to-point and multipoint configurations, one of the most common protocols for point-to-point access is the **Point-to-Point Protocol (PPP)**. PPP is a byte-oriented protocol.*

Topics discussed in this section:

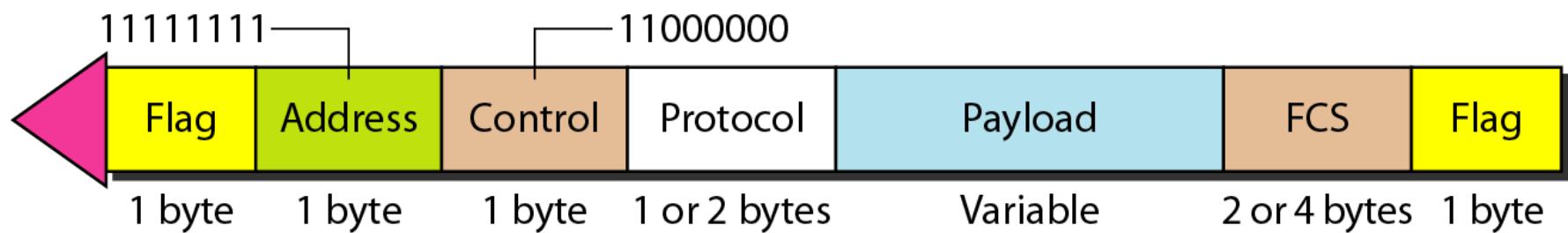
Framing

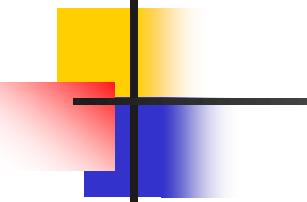
Transition Phases

Multiplexing

Multilink PPP

Figure 11.32 PPP frame format





Note

PPP is a byte-oriented protocol using byte stuffing with the escape byte 01111101.

Figure 11.33 Transition phases

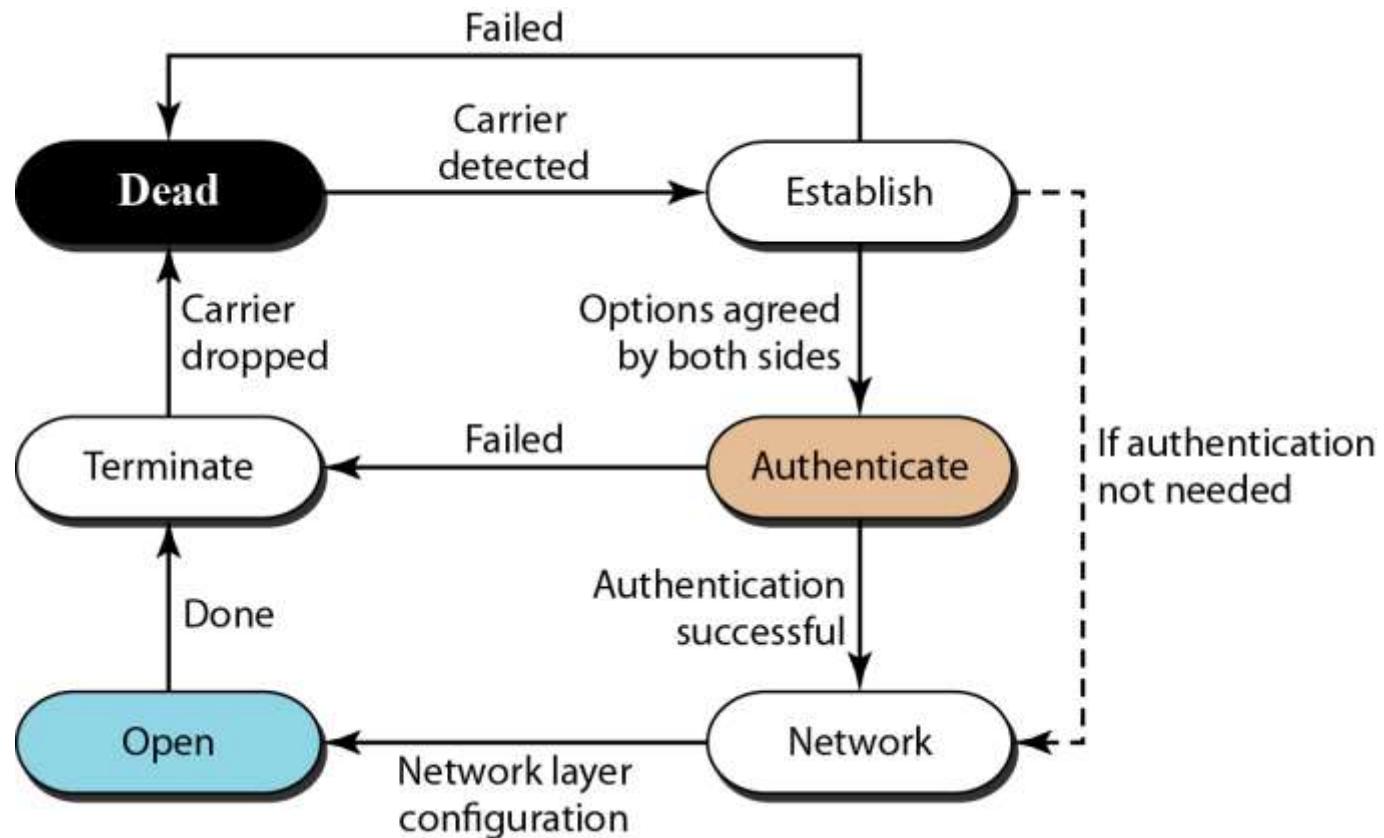


Figure 11.34 Multiplexing in PPP

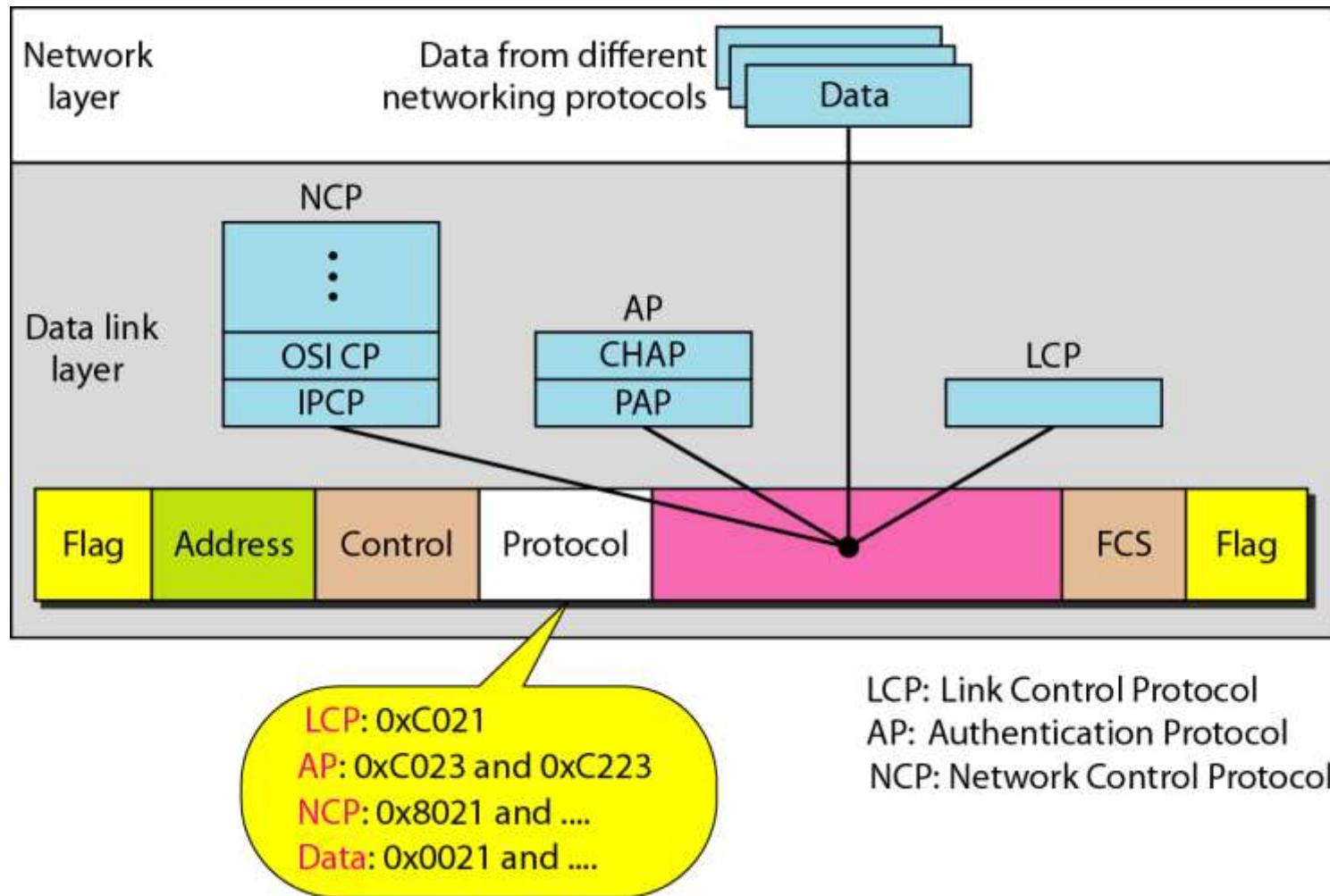


Figure 11.35 LCP packet encapsulated in a frame

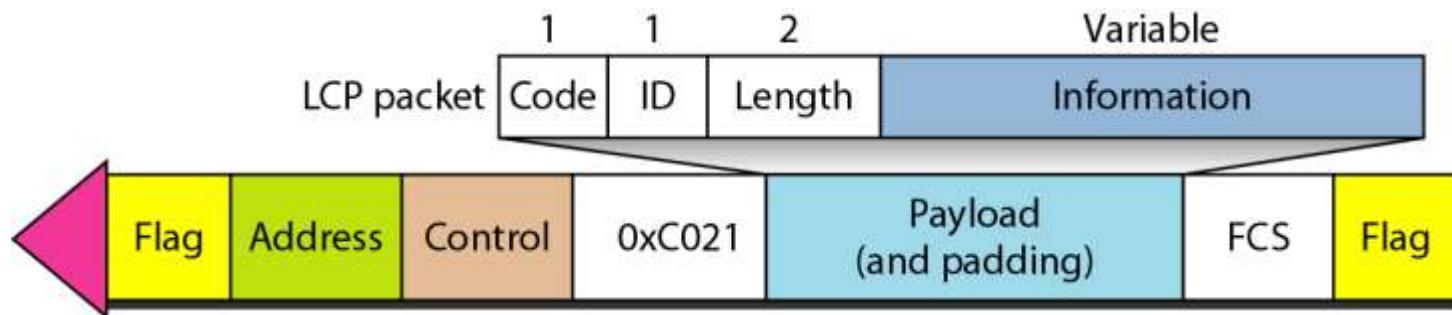


Table 11.2 *LCP packets*

<i>Code</i>	<i>Packet Type</i>	<i>Description</i>
0x01	Configure-request	Contains the list of proposed options and their values
0x02	Configure-ack	Accepts all options proposed
0x03	Configure-nak	Announces that some options are not acceptable
0x04	Configure-reject	Announces that some options are not recognized
0x05	Terminate-request	Request to shut down the line
0x06	Terminate-ack	Accept the shutdown request
0x07	Code-reject	Announces an unknown code
0x08	Protocol-reject	Announces an unknown protocol
0x09	Echo-request	A type of hello message to check if the other end is alive
0x0A	Echo-reply	The response to the echo-request message
0x0B	Discard-request	A request to discard the packet

Table 11.3 *Common options*

<i>Option</i>	<i>Default</i>
Maximum receive unit (payload field size)	1500
Authentication protocol	None
Protocol field compression	Off
Address and control field compression	Off

Figure 11.36 PAP packets encapsulated in a PPP frame

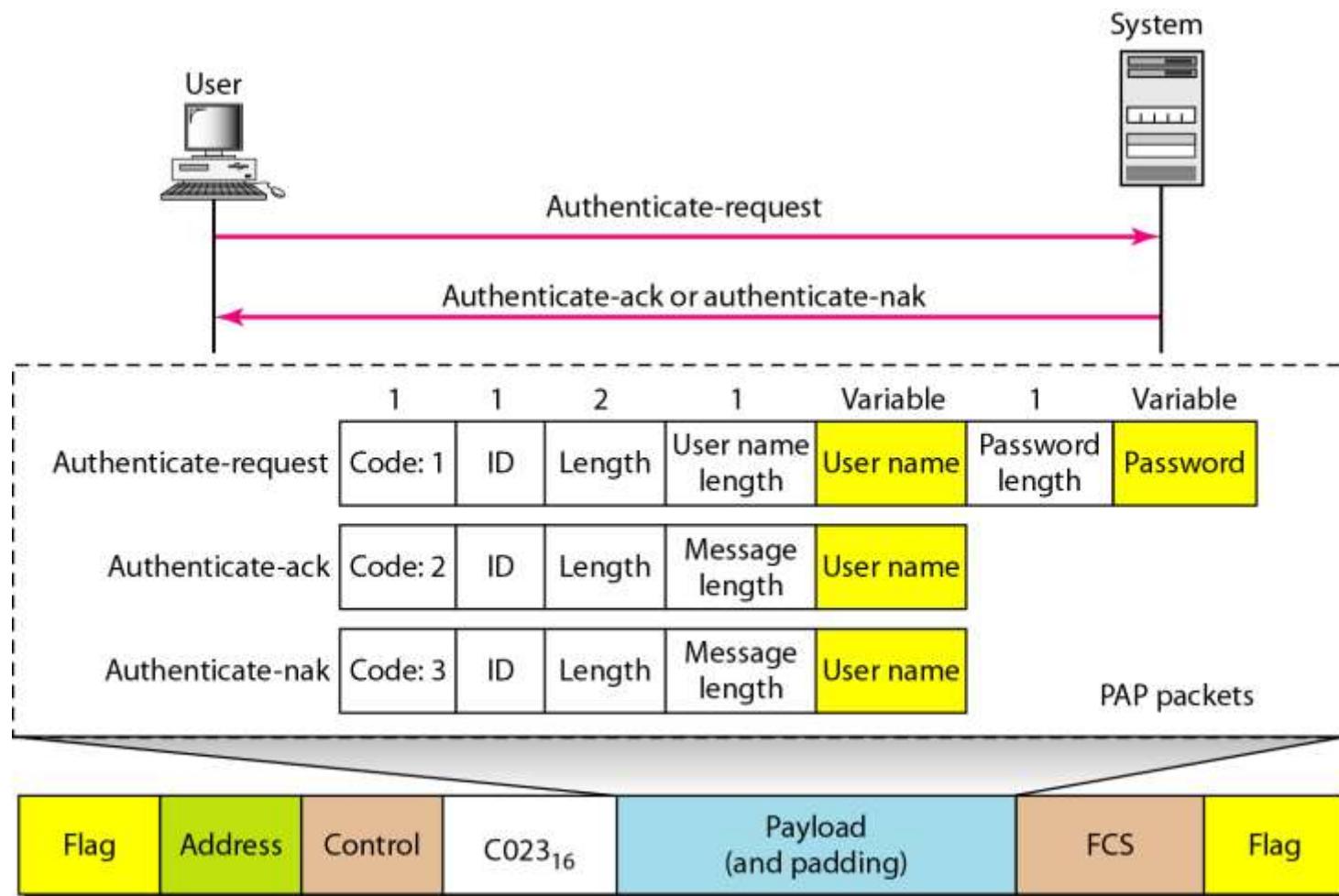


Figure 11.37 CHAP packets encapsulated in a PPP frame

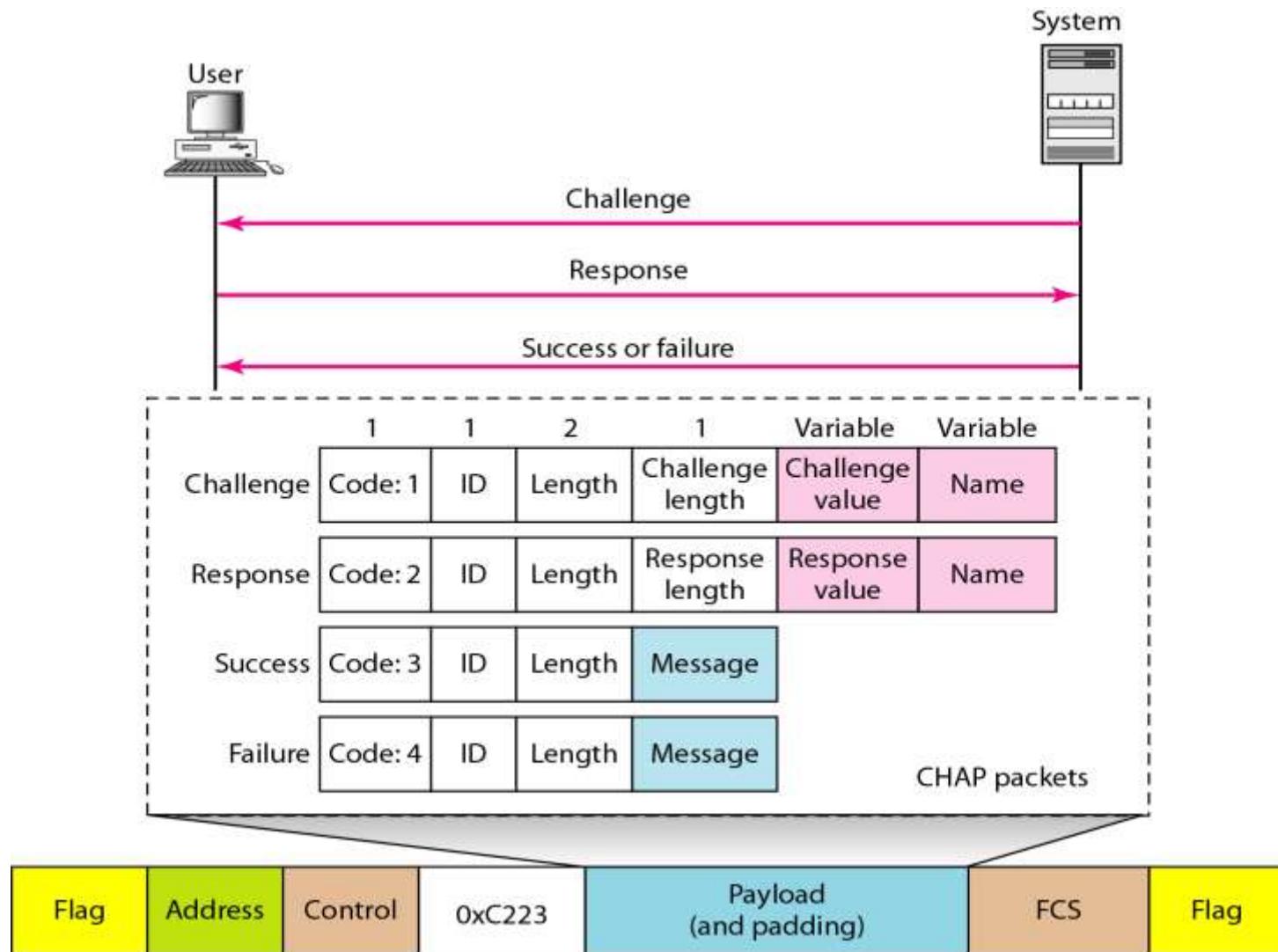


Figure 11.38 *IPCP packet encapsulated in PPP frame*

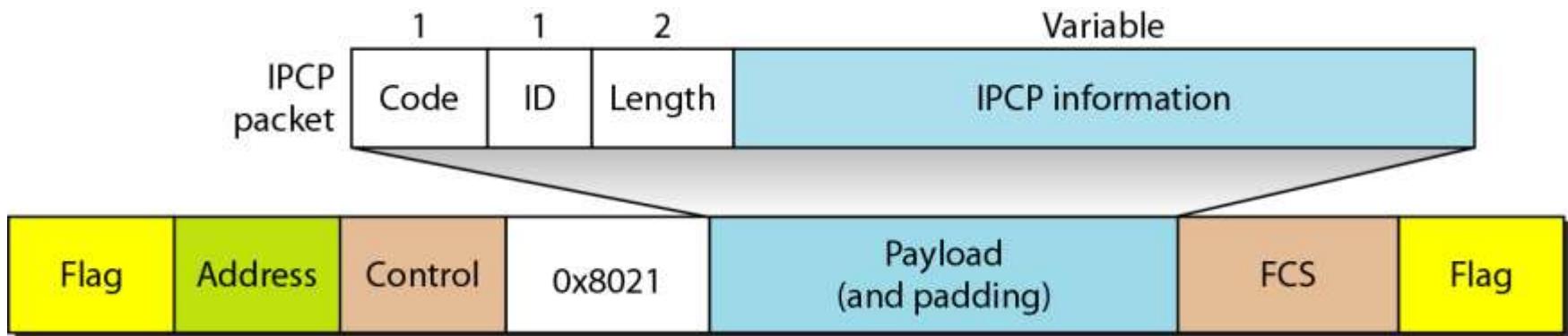


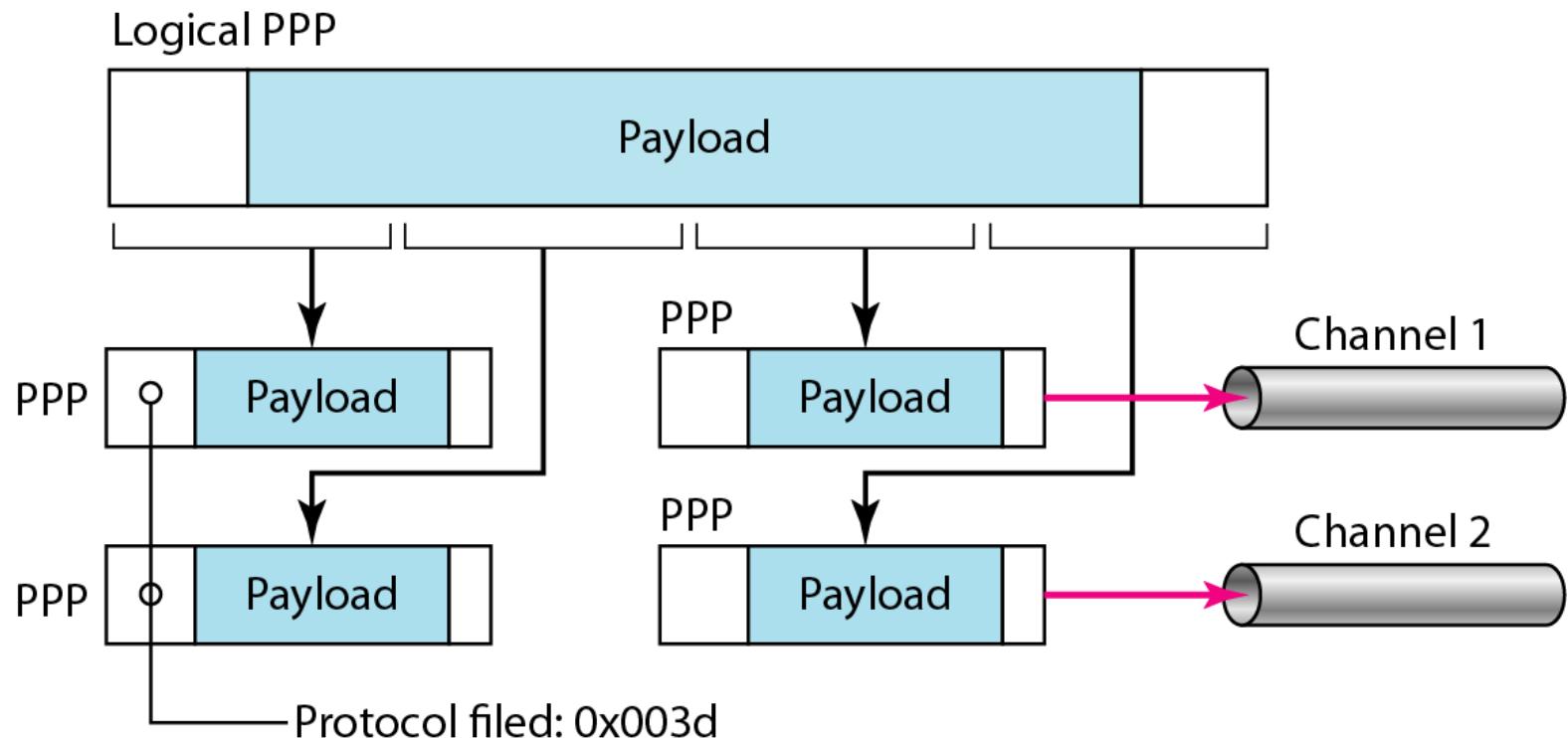
Table 11.4 *Code value for IPCP packets*

<i>Code</i>	<i>IPCP Packet</i>
0x01	Configure-request
0x02	Configure-ack
0x03	Configure-nak
0x04	Configure-reject
0x05	Terminate-request
0x06	Terminate-ack
0x07	Code-reject

Figure 11.39 *IP datagram encapsulated in a PPP frame*



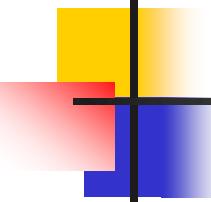
Figure 11.40 Multilink PPP



Example 11.12

Let us go through the phases followed by a network layer packet as it is transmitted through a PPP connection. Figure 11.41 shows the steps. For simplicity, we assume unidirectional movement of data from the user site to the system site (such as sending an e-mail through an ISP).

The first two frames show link establishment. We have chosen two options (not shown in the figure): using PAP for authentication and suppressing the address control fields. Frames 3 and 4 are for authentication. Frames 5 and 6 establish the network layer connection using IPCP.



Example 11.12 (continued)

The next several frames show that some IP packets are encapsulated in the PPP frame. The system (receiver) may have been running several network layer protocols, but it knows that the incoming data must be delivered to the IP protocol because the NCP protocol used before the data transfer was IPCP.

After data transfer, the user then terminates the data link connection, which is acknowledged by the system. Of course the user or the system could have chosen to terminate the network layer IPCP and keep the data link layer running if it wanted to run another NCP protocol.

Figure 11.41 An example

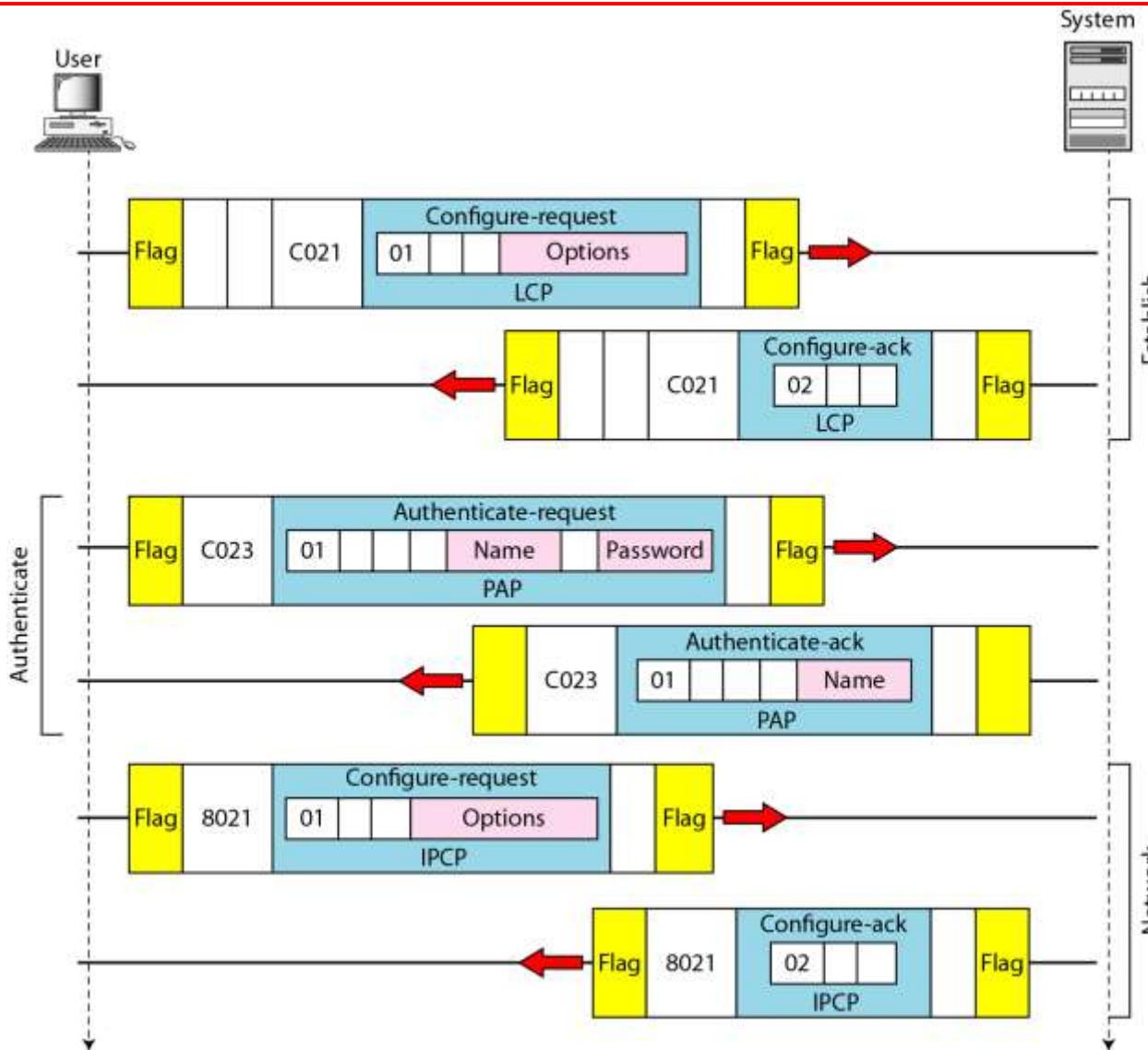
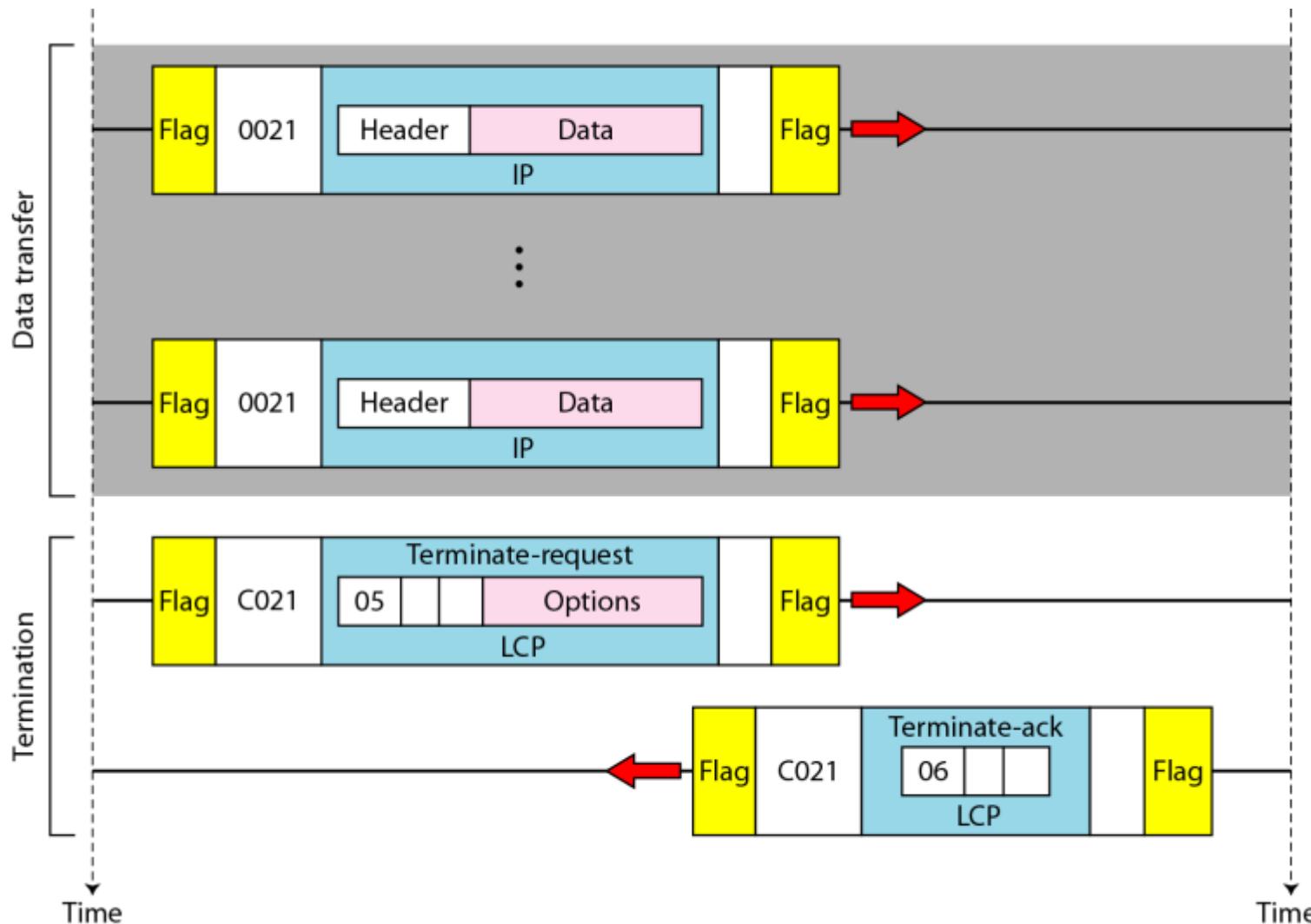


Figure 11.41 An example (continued)



Problem-02:

A bit stream 10011101 is transmitted using the standard CRC method. The generator polynomial is x^3+1 .

1. What is the actual bit string transmitted?
2. Suppose the third bit from the left is inverted during transmission. How will receiver detect this error?

Solution-

Part-01:

- The generator polynomial $G(x) = x^3 + 1$ is encoded as 1001.
- Clearly, the generator polynomial consists of 4 bits.
- So, a string of 3 zeroes is appended to the bit stream to be transmitted.
- The resulting bit stream is 10011101000.

Now, the binary division is performed as-

	10001100
1001	10011101000 <hr/> 1001 <hr/> 00001 <hr/> 0000 <hr/> 00011 <hr/> 0000 <hr/> 00110 <hr/> 0000 <hr/> 01101 <hr/> 1001 <hr/> 01000 <hr/> 1001 <hr/> 00010 <hr/> 0000 <hr/> 00100 <hr/> 0000 <hr/> 0100

← CRC

From here, CRC = 100.

Now,

- The code word to be transmitted is obtained by replacing the last 3 zeroes of 10011101000 with the CRC.

- Thus, the code word transmitted to the receiver = 10011101100.

Part-02:

According to the question,

- Third bit from the left gets inverted during transmission.
- So, the bit stream received by the receiver = 10111101100.

Now,

- Receiver receives the bit stream = 10111101100.
- Receiver performs the binary division with the same generator polynomial as-

$$\begin{array}{r}
 10101000 \\
 \hline
 1001 \quad \boxed{10111101100} \\
 1001 \\
 \hline
 00101 \\
 0000 \\
 \hline
 01011 \\
 1001 \\
 \hline
 00100 \\
 0000 \\
 \hline
 01001 \\
 1001 \\
 \hline
 00001 \\
 0000 \\
 \hline
 00010 \\
 0000 \\
 \hline
 00100 \\
 0000 \\
 \hline
 \underline{\underline{0'100}} \leftarrow \text{Remainder}
 \end{array}$$

From here,

- The remainder obtained on division is a non-zero value.
- This indicates to the receiver that an error occurred in the data during the transmission.
- Therefore, receiver rejects the data and asks the sender for retransmission.

Data word to be sent - 100100

Key - 1101 [Or generator polynomial $x^3 + x^2 + 1$]

Sender Side:

$$\begin{array}{r} 111101 \\ 1101 \quad \boxed{100100000} \quad \boxed{} \\ 1101 \\ \hline 1000 \\ 1101 \\ \hline 1010 \\ 1101 \\ \hline 1110 \\ 1101 \\ \hline 0110 \\ 0000 \\ \hline 1100 \\ 1101 \\ \hline 001 \end{array}$$

Therefore, the remainder is 001 and hence the encoded data sent is 100100001.

Receiver Side:

Code word received at the receiver side 100100001

$$\begin{array}{r} 111101 \\ 1101 \quad \boxed{100100001} \quad \boxed{} \\ 1101 \\ \hline 1000 \\ 1101 \\ \hline 1010 \\ 1101 \\ \hline 1110 \\ 1101 \\ \hline 0110 \\ 0000 \\ \hline 1101 \\ 1101 \\ \hline 0000 \end{array}$$

Therefore, the remainder is all zeros. Hence, the data received has no error.

Example 2: (Error in transmission)

Data word to be sent - 100100

Key - 1101

Sender Side:

$$\begin{array}{r} 111101 \\ 1101 \quad \boxed{100100000} \\ 1101 \\ \hline 1000 \\ 1101 \\ \hline 1010 \\ 1101 \\ \hline 1110 \\ 1101 \\ \hline 0110 \\ 0000 \\ \hline 1100 \\ 1101 \\ \hline 001 \end{array}$$

Therefore, the remainder is 001 and hence the code word sent is 100100001.

Receiver Side

Let there be error in transmission media

Code word received at the receiver side - 100000001

$$\begin{array}{r} 111010 \\ 1101 \quad \boxed{100000001} \\ 1101 \\ \hline 1010 \\ 1101 \\ \hline 1110 \\ 1101 \\ \hline 0110 \\ 0000 \\ \hline 1100 \\ 1101 \\ \hline 0011 \\ 0000 \\ \hline 011 \end{array}$$

Since the remainder is not all zeroes, the error is detected at the receiver side.

Checksum Example-

Consider the data unit to be transmitted is-

1001100111000100010010010000100

Consider 8 bit checksum is used.

Step-01:

At sender side,

The given data unit is divided into segments of 8 bits as-

10011001	11100010	00100100	10000100
----------	----------	----------	----------

Now, all the segments are added and the result is obtained as-

- $10011001 + 11100010 + 00100100 + 10000100 = 1000100011$
- Since the result consists of 10 bits, so extra 2 bits are wrapped around.
- $00100011 + 10 = 00100101$ (8 bits)
- Now, 1's complement is taken which is 11011010.
- Thus, checksum value = 11011010

Step-02:

- The data along with the checksum value is transmitted to the receiver.

Step-03:

At receiver side,

- The received data unit is divided into segments of 8 bits.
- All the segments along with the checksum value are added.
- Sum of all segments + Checksum value = $00100101 + 11011010 = 11111111$
- Complemented value = 00000000
- Since the result is 0, receiver assumes no error occurred in the data and therefore accepts it.