Eastern Economy Edition

Engineering Economics

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Engineering Economics

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ENGINEERING ECONOMICS
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To

My Great-Grand-father

CONTENTS

PREFACE

Efficient functioning of any business organization would enable it to provide goods/services at a lower price. In the process of managing organizations, the managers at different levels should take appropriate economic decisions which will help in minimizing investment, operating and maintenance expenditures besides increasing the revenue, savings and such other gains of the organization. These can be achieved through Engineering Economics which deals with the methods that enable one to make economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

This book on Engineering Economics is the outgrowth of my several years of teaching postgraduate courses in industrial engineering and production engineering and a year of teaching water resources management (all at Anna University, Chennai). It is intended as a text for these disciplines. It can also be used as a text for the undergraduate engineering courses and as a reference for management (project management) and commerce (financial management) courses. Besides, professional engineers and project consultants undertaking economic decision analysis would find the book useful.

I have tried not only to give a comprehensive coverage of the various aspects of engineering economic analysis but provided an exhaustive appendix on Interest Tables for a wide range of interest rates (0.25 – 50%) and a period ranging from one year to 100 years. These tables, along with the topics discussed, will, I believe, help both students and teachers in carrying out economic analysis and solving problems.

The book contains about 100 well-structured worked-out examples to illustrate the concepts explained. Each chapter also has a set of questions and problems to test the student's power of comprehending the topics.

I wish to thank Prof. Dr. K.N. Balasubramanian, formerly Head of the Department of Industrial Engineering Division of Anna University, who gave me the opportunity of teaching the subject to M.E. Industrial Engineering class continuously for seven years during my stay there as a faculty. This enabled me to enrich my knowledge and expertise in the subject. Many of my colleagues and academic friends helped me by giving valuable suggestions on the structure and content of this text and these were instrumental in improving the quality and presentation of this book. I wish to express my profound gratitude and appreciation to all of them.

Any suggestions for improving the contents would be warmly appreciated.

R. Panneerselvam

1

INTRODUCTION

This chapter discusses the elements of economics and the interaction between its various components. This is followed by an analysis of the need and scope of engineering economics. Later, elements of cost and break-even analysis are presented.

1.1 ECONOMICS

Economics is the science that deals with the production and consumption of goods and services and the distribution and rendering of these for human welfare.

The following are the economic goals.

- A high level of employment
- Price stability
- Efficiency
- An equitable distribution of income
- Growth

Some of the above goals are interdependent. The economic goals are not always complementary; in many cases they are in conflict. For example, any move to have a significant reduction in unemployment will lead to an increase in inflation.

1.1.1 Flow in an Economy

The flow of goods, services, resources and money payments in a simple economy are shown in Fig. 1.1. Households and businesses are the two major entities in a simple economy. Business organizations use various economic resources like land, labour and capital which are provided by households to produce consumer goods and services which will be used by them. Business organizations make payment of money to the households for receiving various resources. The households in turn make payment of money to business organizations for receiving consumer goods and services. This cycle shows the interdependence between the two major entities in a simple economy.

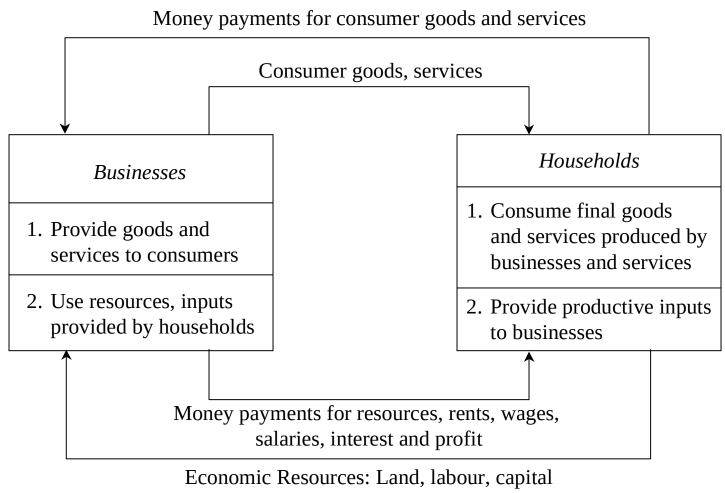


Fig. 1.1 Flow of goods, services, resources and money payments in a simple economy.

1.1.2 Law of Supply and Demand

An interesting aspect of the economy is that the demand and supply of a product are interdependent and they are sensitive with respect to the price of that product. The interrelationships between them are shown in Fig. 1.2.

From Fig. 1.2 it is clear that when there is a decrease in the price of a product, the demand for the product increases and its supply decreases. Also, the product is more in demand and hence the demand of the product increases. At the same time, lowering of the price of the product makes the producers restrain from releasing more quantities of the product in the market. Hence, the supply of the product is decreased. The point of intersection of the supply curve and the demand curve is known as the *equilibrium point*. At the price corresponding to this point, the quantity of supply is equal to the quantity of demand. Hence, this point is called the *equilibrium point*.

Factors influencing demand

The shape of the demand curve is influenced by the following factors:

- Income of the people
- Prices of related goods
- Tastes of consumers

If the income level of the people increases significantly, then their purchasing power will naturally improve. This would definitely shift the demand curve to the north-east direction of Fig. 1.2. A converse situation will shift the demand curve to the south-west direction.

If, for instance, the price of television sets is lowered drastically its demand would naturally go up. As a result, the demand for its associated product,

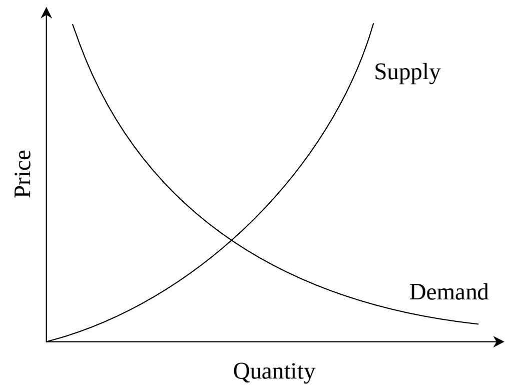


Fig. 1.2 Demand and supply curve.

namely VCDs would also increase. Hence, the prices of related goods influences the demand of a product.

Over a period of time, the preference of the people for a particular product may increase, which in turn, will affect its demand. For instance, diabetic people prefer to have sugar-free products. If the incidence of diabetes rises naturally there will be increased demand for sugar-free products.

Factors influencing supply

The shape of the supply curve is affected by the following factors:

- Cost of the inputs
- Technology
- Weather
- Prices of related goods

If the cost of inputs increases, then naturally, the cost of the product will go up. In such a situation, at the prevailing price of the product the profit margin per unit will be less. The producers will then reduce the production quantity, which in turn will affect the supply of the product. For instance, if the prices of fertilizers and cost of labour are increased significantly, in agriculture, the profit margin per bag of paddy will be reduced. So, the farmers will reduce the area of cultivation, and hence the quantity of supply of paddy will be reduced at the prevailing prices of the paddy.

If there is an advancement in technology used in the manufacture of the product in the long run, there will be a reduction in the production cost per unit. This will enable the manufacturer to have a greater profit margin per unit at the prevailing price of the product. Hence, the producer will be tempted to supply more quantity to the market.

Weather also has a direct bearing on the supply of products. For example, demand for woollen products will increase during winter. This means the prices of woollen goods will be incresed in winter. So, naturally, manufacturers will supply more volume of woollen goods during winter.

Again, take the case of television sets. If the price of TV sets is lowered significantly, then its demand would naturally go up. As a result, the demand for associated products like VCDs would also go up. Over a period of time, this will lead to an increase in the price of VCDs, which would result in more supply of VCDs.

1.2 CONCEPT OF ENGINEERING ECONOMICS

Science is a field of study where the basic principles of different physical systems are formulated and tested. Engineering is the application of science. It establishes varied application systems based on different scientific principles.

From the discussions in the previous section, it is clear that price has a major role in deciding the demand and supply of a product. Hence, from the organization's point of view, efficient and effective functioning of the organization would certainly help it to provide goods/services at a lower cost which in turn will enable it to fix a lower price for its goods or services.

The following section discusses the different types of efficiency and their impact on the operation of businesses and the definition and scope of engineering economics.

1.2.1 Types of Efficiency

Efficiency of a system is generally defined as the ratio of its output to input. The efficiency can be classified into *technical efficiency* and *economic efficiency*.

Technical efficiency

It is the ratio of the output to input of a physical system. The physical system may be a diesel engine, a machine working in a shop floor, a furnace, etc.

$$\text{Technical efficiency (\%)} = \frac{\text{Output}}{\text{Input}} \times 100$$

The technical efficiency of a diesel engine is as follows:

$$\text{Technical efficiency (\%)} = \frac{\text{Heat equivalent of mechanical energy produced}}{\text{Heat equivalent of fuel used}} \times 100$$

In practice, technical efficiency can never be more than 100%. This is mainly due to frictional loss and incomplete combustion of fuel, which are considered to be unavoidable phenomena in the working of a diesel engine.

Economic efficiency

Economic efficiency is the ratio of output to input of a business system.

$$\text{Economic efficiency (\%)} = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{\text{Worth}}{\text{Cost}} \times 100$$

‘Worth’ is the annual revenue generated by way of operating the business and ‘cost’ is the total annual expenses incurred in carrying out the business. For the survival and growth of any business, the economic efficiency should be more than 100%.

Economic efficiency is also called ‘productivity’. There are several ways of improving productivity.

- Increased output for the same input
- Decreased input for the same output
- By a proportionate increase in the output which is more than the proportionate increase in the input
- By a proportionate decrease in the input which is more than the proportionate decrease in the output
- Through simultaneous increase in the output with decrease in the input.

Increased output for the same input. In this strategy, the output is increased while keeping the input constant. Let us assume that in a steel plant, the layout of the existing facilities is not proper. By slightly altering the location of the billet-making section, and bringing it closer to the furnace which produces hot metal, the scale formation at the top of ladles will be considerably reduced. The molten metal is usually carried in ladles to the billet-making section. In the long run, this would give more yield in terms of tonnes of billet produced. In this exercise, there is no extra cost involved. The only task is the relocation of the billet-making facility which involves an insignificant cost.

Decreased input for the same output. In this strategy, the input is decreased to produce the same output. Let us assume that there exists a substitute raw material to manufacture a product and it is available at a lower price. If we can identify such a material and use it for manufacturing the product, then certainly it will reduce the input. In this exercise, the job of the purchase department is to identify an alternate substitute material. The process of identification does not involve any extra cost. So, the productivity ratio will increase because of the decreased input by way of using cheaper raw materials to produce the same output.

Less proportionate increase in output is more than that of the input. Consider the example of introducing a new product into the existing product mix of an organization. Let us assume that the existing facilities are not fully utilized and the R&D wing of the company has identified a new product which has a very good market and which can be manufactured with the surplus facilities of the organization. If the new product is taken up for production, it will lead to—

- an increase in the revenue of the organization by way of selling the new product in addition to the existing product mix and

The

- an increase in the material cost and operation and maintenance cost of machineries because of producing the new product.

If we examine these two increases closely, the proportionate increase in the revenue will be more than the proportionate increase in the input cost. Hence, there will be a net increase in the productivity ratio.

When proportionate decrease in input is more than that of the output. Let us consider the converse of the previous example, i.e. dropping an uneconomical product from the existing product mix. This will result in the following:

- A decrease in the revenue of the organization
- A decrease in the material cost, and operation and maintenance cost of machinery

If we closely examine these two decreases, we will see that the proportionate decrease in the input cost will be more than the proportionate decrease in the revenue. Hence, there will be a net increase in the productivity ratio.

Simultaneous increase in output and decrease in input. Let us assume that there are advanced automated technologies like robots and automated guided vehicle system (AGVS), available in the market which can be employed in the organization we are interested in. If we employ these modern tools, then:

- There will be a drastic reduction in the operation cost. Initially, the cost on equipment would be very high. But, in the long run, the reduction in the operation cost would break-even the high initial investment and offer more savings on the input.
- These advanced facilities would help in producing more products because they do not experience fatigue. The increased production will yield more revenue.

In this example, in the long run, there is an increase in the revenue and a decrease in the input. Hence, the productivity ratio will increase at a faster rate.

1.2.2 Definition and Scope of Engineering Economics

As stated earlier, efficient functioning of any business organization would enable it to provide goods/services at a lower price. In the process of managing organizations, the managers at different levels should take appropriate economic decisions which will help in minimizing investment, operating and maintenance expenditures besides increasing the revenue, savings and other related gains of the organization.

Definition

Engineering economics deals with the methods that enable one to take economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

Scope

The issues that are covered in this book are elementary economic analysis, interest formulae, bases for comparing alternatives, present worth method, future worth method, annual equivalent method, rate of return method, replacement analysis, depreciation, evaluation of public alternatives, inflation adjusted investment decisions, make or buy decisions, inventory control, project management, value engineering, and linear programming.

1.3 ELEMENTS OF COSTS

Cost can be broadly classified into *variable cost* and *overhead cost*. Variable cost varies with the volume of production while overhead cost is fixed, irrespective of the production volume.

Variable cost can be further classified into direct material cost, direct labour cost, and direct expenses. The overhead cost can be classified into factory overhead, administration overhead, selling overhead, and distribution overhead.

Direct material costs are those costs of materials that are used to produce the product. Direct labour cost is the amount of wages paid to the direct labour involved in the production activities. Direct expenses are those expenses that vary in relation to the production volume, other than the direct material costs and direct labour costs.

Overhead cost is the aggregate of indirect material costs, indirect labour costs and indirect expenses. Administration overhead includes all the costs that are incurred in administering the business. Selling overhead is the total expense that is incurred in the promotional activities and the expenses relating to sales force. Distribution overhead is the total cost of shipping the items from the factory site to the customer sites.

The selling price of a product is derived as shown below:

(a) Direct material costs + Direct labour costs + Direct expenses = Prime cost

(b) Prime cost + Factory overhead = Factory cost

(c) Factory cost + Office and administrative overhead = Costs of production

(d) Cost of production + Opening finished stock – Closing finished stock = Cost of goods sold

(e) Cost of goods sold + Selling and distribution overhead = Cost of sales

(f) Cost of sales + Profit = Sales

(g) Sales/Quantity sold = Selling price per unit

In the above calculations, if the opening finished stock is equal to the closing finished stock, then the cost of production is equal to the cost of goods sold.

1.4 OTHER COSTS/REVENUES

The following are the costs/revenues other than the costs which are presented in the previous section:

- Marginal cost
- Marginal revenue
- Sunk cost
- Opportunity cost

1.4.1 Marginal Cost

Marginal cost of a product is the cost of producing an additional unit of that product. Let the cost of producing 20 units of a product be Rs. 10,000, and the cost of producing 21 units of the same product be Rs. 10,045. Then the marginal cost of producing the 21st unit is Rs. 45.

1.4.2 Marginal Revenue

Marginal revenue of a product is the incremental revenue of selling an additional unit of that product. Let, the revenue of selling 20 units of a product be Rs. 15,000 and the revenue of selling 21 units of the same product be Rs. 15,085. Then, the marginal revenue of selling the 21st unit is Rs. 85.

1.4.3 Sunk Cost

This is known as the past cost of an equipment/asset. Let us assume that an equipment has been purchased for Rs. 1,00,000 about three years back. If it is considered for replacement, then its present value is not Rs. 1,00,000. Instead, its present market value should be taken as the present value of the equipment for further analysis. So, the purchase value of the equipment in the past is known as its sunk cost. The sunk cost should not be considered for any analysis done from nowonwards.

1.4.4 Opportunity Cost

In practice, if an alternative (X) is selected from a set of competing alternatives (X,Y), then the corresponding investment in the selected alternative is not available for any other purpose. If the same money is invested in some other alternative (Y), it may fetch some return. Since the money is invested in the selected alternative (X), one has to forego the return from the other alternative (Y). The amount that is foregone by not investing in the other alternative (Y) is known as the opportunity cost of the selected alternative (X). So the opportunity cost of an alternative is the return that will be foregone by not investing the same money in another alternative.

Consider that a person has invested a sum of Rs. 50,000 in shares. Let the expected annual return by this alternative be Rs. 7,500. If the same amount is invested in a fixed deposit, a bank will pay a return of 18%. Then, the corresponding total return per year for the investment in the bank is Rs. 9,000. This return is greater than the return from shares. The foregone excess return of Rs. 1,500 by way of not investing in the bank is the opportunity cost of investing in shares.

1.5 BREAK-EVEN ANALYSIS

The main objective of break-even analysis is to find the cut-off production volume from where a firm will make profit. Let

s = selling price per unit
v = variable cost per unit
FC = fixed cost per period
Q = volume of production

The total sales revenue (S) of the firm is given by the following formula:

$$S = s \times Q$$

The total cost of the firm for a given production volume is given as

$$\begin{aligned} TC &= \text{Total variable cost} + \text{Fixed cost} \\ &= v \times Q + FC \end{aligned}$$

The linear plots of the above two equations are shown in Fig. 1.3. The intersection point of the total sales revenue line and the total cost line is called

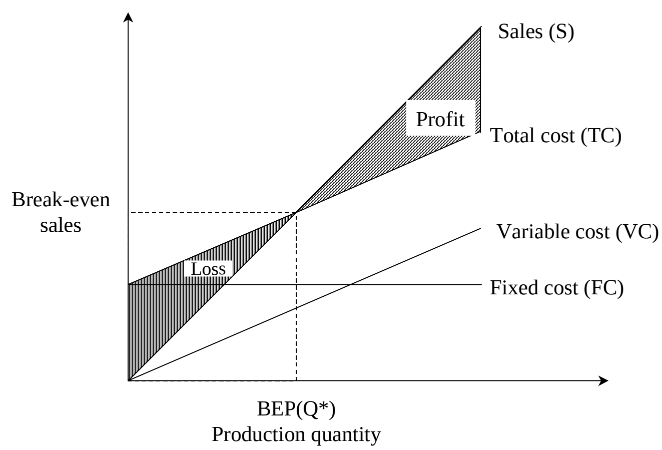


Fig. 1.3 Break-even chart.

the break-even point. The corresponding volume of production on the X-axis is known as the break-even sales quantity. At the intersection point, the total cost is equal to the total revenue. This point is also called the no-loss or no-gain situation. For any production quantity which is less than the break-even quantity, the total cost is more than the total revenue. Hence, the firm will be making loss.

For any production quantity which is more than the break-even quantity, the total revenue will be more than the total cost. Hence, the firm will be making profit.

$$\text{Profit} = \text{Sales} - (\text{Fixed cost} + \text{Variable costs})$$
$$= s \times Q - (FC + v \times Q)$$

The formulae to find the break-even quantity and break-even sales quantity

$$\text{Break-even quantity} = \frac{\text{Fixed cost}}{\text{Selling price/unit} - \text{Variable cost/unit}}$$

$$= \frac{FC}{s - v} \text{ (in units)}$$

$$\text{Break-even sales} = \frac{\text{Fixed cost}}{\text{Selling price/unit} - \text{Variable cost/unit}} \times \text{Selling price/unit}$$

$$= \frac{FC}{s - v} \times s \text{ (Rs.)}$$

The contribution is the difference between the sales and the variable costs. The margin of safety (M.S.) is the sales over and above the break-even sales. The formulae to compute these values are

$$\text{Contribution} = \text{Sales} - \text{Variable costs}$$
$$\text{Contribution/unit} = \text{Selling price/unit} - \text{Variable cost/unit}$$
$$\text{M.S.} = \text{Actual sales} - \text{Break-even sales}$$
$$= \frac{\text{Profit}}{\text{Contribution}} \times \text{sales}$$

M.S. as a per cent of sales = (M.S./Sales) × 100

EXAMPLE 1.1 Alpha Associates has the following details:

Fixed cost = Rs. 20,00,000
Variable cost per unit = Rs. 100
Selling price per unit = Rs. 200

Find
(a) The break-even sales quantity,
(b) The break-even sales
(c) If the actual production quantity is 60,000, find (i) contribution; and (ii) margin of safety by all methods.

Solution

Fixed cost (FC) = Rs. 20,00,000
Variable cost per unit (v) = Rs. 100
Selling price per unit (s) = Rs. 200

(a) Break-even quantity $= \dfrac{FC}{s - v} = \dfrac{20{,}00{,}000}{200 - 100}$

$= 20{,}00{,}000/100 = 20{,}000$ units

(b) Break-even sales $= \dfrac{FC}{s - v} \times s$ (Rs.)

$= \dfrac{20{,}00{,}000}{200 - 100} \times 200$

$= \dfrac{20{,}00{,}000}{100} \times 200 =$ Rs. 40,00,000

(c) (i) Contribution = Sales – Variable cost

$= s \times Q - v \times Q$

$= 200 \times 60{,}000 - 100 \times 60{,}000$

$= 1{,}20{,}00{,}000 - 60{,}00{,}000$

= Rs. 60,00,000

(ii) Margin of safety

METHOD I

M.S. = Sales – Break-even sales

$= 60{,}000 \times 200 - 40{,}00{,}000$

$= 1{,}20{,}00{,}000 - 40{,}00{,}000 =$ Rs. 80,00,000

METHOD II

$$\text{M.S.} = \frac{\text{Profit}}{\text{Contribution}} \times \text{Sales}$$

Profit = Sales – $(FC + v \times Q)$

$= 60{,}000 \times 200 - (20{,}00{,}000 + 100 \times 60{,}000)$

$= 1{,}20{,}00{,}000 - 80{,}00{,}000$

= Rs. 40,00,000

$$\text{M.S.} = \frac{40{,}00{,}000}{60{,}00{,}000} \times 1{,}20{,}00{,}000 = \text{Rs. } 80{,}00{,}000$$

$$\text{M.S. as a per cent of sales} = \frac{80{,}00{,}000}{1{,}20{,}00{,}000} \times 100 = 67\%$$

1.6 PROFIT/VOLUME RATIO (*P*/*V* RATIO)

P/*V* ratio is a valid ratio which is useful for further analysis. The different formulae for the *P*/*V* ratio are as follows:

$$P/V \text{ ratio} = \frac{\text{Contribution}}{\text{Sales}} = \frac{\text{Sales} - \text{Variable costs}}{\text{Sales}}$$

The relationship between BEP and *P*/*V* ratio is as follows:

$$\text{BEP} = \frac{\text{Fixed cost}}{P/V \text{ ratio}}$$

The following formula helps us find the M.S. using the *P*/*V* ratio:

$$\text{M.S.} = \frac{\text{Profit}}{P/V \text{ ratio}}$$

EXAMPLE 1.2 Consider the following data of a company for the year 1997:

Sales = Rs. 1,20,000

Fixed cost = Rs. 25,000

Variable cost = Rs. 45,000

Find the following:

(a) Contribution
(b) Profit
(c) BEP
(d) M.S.

Solution

(a) Contribution = Sales – Variable costs
= Rs. 1,20,000 – Rs. 45,000
= Rs. 75,000

(b) Profit = Contribution – Fixed cost
= Rs. 75,000 – Rs. 25,000
= Rs. 50,000

(c) BEP

$$P/V \text{ ratio} = \frac{\text{Contribution}}{\text{Sales}}$$

$$= \frac{75{,}000}{1{,}20{,}000} \times 100 = 62.50\%$$

$$\text{BEP} = \frac{\text{Fixed cost}}{P/V \text{ ratio}} = \frac{25{,}000}{62.50} \times 100 = \text{Rs. } 40{,}000$$

$$\text{M.S.} = \frac{\text{Profit}}{P/V \text{ ratio}} = \frac{50{,}000}{62.50} \times 100 = \text{Rs. } 80{,}000$$

EXAMPLE 1.3 Consider the following data of a company for the year 1998:

Sales = Rs. 80,000
Fixed cost = Rs. 15,000
Variable cost = 35,000

Find the following:

(a) Contribution
(b) Profit
(c) BEP
(d) M.S.

Solution

(a) Contribution = Sales – Variable costs
= Rs. 80,000 – Rs. 35,000
= Rs. 45,000

(b) Profit = Contribution – Fixed cost
= Rs. 45,000 – Rs. 15,000
= Rs. 30,000

(c) BEP

$$P/V \text{ ratio} = \frac{\text{Contribution}}{\text{Sales}} = \frac{45{,}000}{80{,}000} \times 100 = 56.25\%$$

$$\text{BEP} = \frac{\text{Fixed cost}}{P/V \text{ ratio}} = \frac{15{,}000}{56.25} \times 100 = \text{Rs. } 26{,}667$$

(d) $$\text{M.S.} = \frac{\text{Profit}}{P/V \text{ ratio}} = \frac{30{,}000}{56.25} \times 100 = \text{Rs. } 53{,}333.33$$

QUESTIONS

1. Define economics. Also discuss the flow of goods, services, resources and money payments in a simple economy with the help of a suitable diagram.
2. Illustrate the effect of price on demand and supply; illustrate with the help of a diagram.
3. Discuss the factors which influence demand and supply.
4. Distinguish between technical efficiency and economic efficiency by giving examples.

5. What are the ways by which the economic efficiency can be improved?
6. Give the definition and scope of engineering economics.
7. Clearly explain the method of deriving the selling price of a product.
8. Define the following costs with examples:

   (a) Sunk cost

   (b) Opportunity cost

   (c) Marginal cost

   (d) Marginal revenue.

9. Define break-even point. Draw a break-even chart and explain its components.
10. Krishna Company Ltd. has the following details:

    Fixed cost = Rs. 40,00,000
    Variable cost per unit = Rs. 300
    Selling price per unit = Rs. 500

    Find

    (a) The break-even sales quantity
    (b) The break-even sales
    (c) If the actual production quantity is 1,20,000, find the following:

       (i) Contribution
       (ii) Margin of safety by all methods

11. Consider the following data of a company for the year 1998.

    Sales = Rs. 2,40,000
    Fixed cost = Rs. 50,000
    Variable cost = Rs. 75,000

    Find the following:

    (a) Contribution
    (b) Profit
    (c) BEP
    (d) Margin of safety

2

ELEMENTARY ECONOMIC ANALYSIS

2.1 INTRODUCTION

Whether it is a business situation or a day-to-day event in somebody's personal life, there are a large number of economic decision making involved. One can manage many of these decision problems by using simple economic analysis. For example, an industry can source its raw materials from a nearby place or from a far-off place. In this problem, the following factors will affect the decision:

- Price of the raw material
- Transportation cost of the raw material
- Availability of the raw material
- Quality of the raw material

Consider the alternative of sourcing raw materials from a nearby place with the following characteristics:

- The raw material is more costly in the nearby area.
- The availability of the raw material is not sufficient enough to support the operation of the industry throughout the year.
- The raw material requires pre-processing before it is used in the production process. This would certainly add cost to the product.
- The cost of transportation is minimal under this alternative.

On the other hand, consider another alternative of sourcing the raw materials from a far-off place with the following characteristics:

- The raw material is less costly at the far off place.
- The cost of transportation is very high.
- The availability of the raw material at this site is abundant and it can support the plant throughout the year.
- The raw material from this site does not require any pre-processing before using it for production.

Under such a situation, the procurement of the raw material should be decided in such a way that the overall cost is minimized.

The above example clearly highlights the various components of cost that are involved in each of the alternatives of the decision-making process as well as a method of taking a suitable decision.

2.2 EXAMPLES FOR SIMPLE ECONOMIC ANALYSIS

In this section, the concept of simple economic analysis is illustrated using suitable examples in the following areas:

- Material selection for a product
- Design selection for a product
- Design selection for a process industry
- Building material selection for construction activities
- Process planning/Process modification

2.2.1 Material Selection for a Product/Substitution of Raw Material

The cost of a product can be reduced greatly by substitution of the raw materials. Among various elements of cost, raw material cost is most significant and it forms a major portion of the total cost of any product. So, any attempt to find a suitable raw material will bring a reduction in the total cost in any one or combinations of the following ways:

- Cheaper raw material price
- Reduced machining/process time
- Enhanced durability of the product

Therefore, the process of raw material selection/substitution will result in finding an alternate raw material which will provide the necessary functions that are provided by the raw material that is presently used. In this process, if the new raw material provides any additional benefit, then it should be treated as its welcoming feature. This concept is demonstrated with two numerical problems.

EXAMPLE 2.1 In the design of a jet engine part, the designer has a choice of specifying either an aluminium alloy casting or a steel casting. Either material will provide equal service, but the aluminium casting will weigh 1.2 kg as compared with 1.35 kg for the steel casting.

The aluminium can be cast for Rs. 80.00 per kg. and the steel one for Rs. 35.00 per kg. The cost of machining per unit is Rs. 150.00 for aluminium and Rs. 170.00 for steel. Every kilogram of excess weight is associated with a penalty of Rs. 1,300 due to increased fuel consumption. Which material should be specified and what is the economic advantage of the selection per unit?

Solution *(a) Cost of using aluminium metal for the jet engine part:*

Weight of aluminium casting/unit = 1.2 kg
Cost of making aluminium casting = Rs. 80.00 per kg
Cost of machining aluminium casting per unit = Rs. 150.00
Total cost of jet engine part made of aluminium/unit

= Cost of making aluminium casting/unit
+ Cost of machining aluminium casting/unit
= $80 \times 1.2 + 150 = 96 + 150$
= Rs. 246

(b) Cost of jet engine part made of steel/unit:
Weight of steel casting/unit = 1.35 kg
Cost of making steel casting = Rs. 35.00 per kg
Cost of machining steel casting per unit = Rs. 170.00
Penalty of excess weight of steel casting = Rs. 1,300 per kg

Total cost of jet engine part made of steel/unit
= Cost of making steel casting/unit
+ Cost of machining steel casting/unit
+ Penalty for excess weight of steel casting
= $35 \times 1.35 + 170 + 1{,}300(1.35 - 1.2)$
= Rs. 412.25

DECISION The total cost/unit of a jet engine part made of aluminium is less than that for an engine made of steel. Hence, aluminium is suggested for making the jet engine part. The economic advantage of using aluminium over steel/unit is Rs. 412.25 – Rs. 246 = Rs. 166.25

EXAMPLE 2.2 A company manufactures dining tables which mainly consist of a wooden frame and a table top. The different materials used to manufacture the tables and their costs are given in Table 2.1.

Table 2.1 Data for Example 2.2

Description of item	*Quantity*	*Cost*
Wood for frame and legs	0.1 m^3	Rs. 12,000/m^3
Table top with sunmica finish	1	Rs. 3,000
Leg bushes	4	Rs. 10/bush
Nails	100 g	Rs. 300/kg
Total labour	15 hr	Rs. 50/hr

In view of the growing awareness towards deforestation and environmental conservation, the company feels that the use of wood should be minimal. The wooden top therefore could be replaced with a granite top. This would require additional wood for the frame and legs to take the extra weight of the granite top. The materials and labour requirements along with cost details to manufacture a table with granite top are given in Table 2.2.

Table 2.2 Data for Example 2.2

Description of item	*Quantity*	*Cost*
Wood for frame and legs	0.15 m^3	Rs. 12,000/m^3
Granite table top	1.62 m^2	Rs. 800/m^2
Leg bushes	4	Rs. 25/bush
Nails	50 g	Rs. 300/kg
Total labour	8 hr	Rs. 50/hr

If the cost of the dining table with a granite top works out to be lesser than that of the table with wooden top, the company is willing to manufacture dining tables with granite tops. Compute the cost of manufacture of the table under each of the alternatives described above and suggest the best alternative. Also, find the economic advantage of the best alternative.

Solution *(a) Cost of table with wooden top*

Cost of wood for frame and legs	= 12,000 × 0.1	= Rs. 1,200
Cost of wooden top		= Rs. 3,000
Cost of bushes	= 10 × 4	= Rs. 40
Cost of nails	= 300 × (100/1,000)	= Rs. 30
Cost of labour	= 50 × 15	= Rs. 750
	Total	= Rs. 5,020

(b) Cost of table with granite top

Cost of wood for frame and legs	= 12,000 × 0.15	= Rs. 1,800
Cost of granite top	= 800 × 1.62	= Rs. 1,296
Cost of bushes	= 25 × 4	= Rs. 100
Cost of nails	= 300 × (50/1,000)	= Rs. 15
Cost of labour	= 50 × 8	= Rs. 400
	Total	= Rs. 3,611

The cost of a table with granite top works out to be less than that of a table with a wooden top. Hence, the table with granite top should be selected by the manufacturer.

(c) Economic advantage

Cost of a table with wooden top	= Rs. 5,020
Cost of a table with granite top	= Rs. 3,611
Economic advantage of table with granite top	= Rs. 1,409

2.2.2 Design Selection for a Product

The design modification of a product may result in reduced raw material requirements, increased machinability of the materials and reduced labour. Design is an important factor which decides the cost of the product for a specified level of performance of that product.

The elementary economic analysis applied to the selection of design for a product is illustrated with two example problems.

EXAMPLE 2.3 Two alternatives are under consideration for a tapered fastening pin. Either design will serve the purpose and will involve the same material and manufacturing cost except for the lathe and grinder operations.

Design A will require 16 hours of lathe time and 4.5 hours of grinder time per 1,000 units. Design B will require 7 hours of lathe time and 12 hours of grinder time per 1,000 units. The operating cost of the lathe including labour is Rs. 200 per hour. The operating cost of the grinder including labour is Rs. 150 per hour. Which design should be adopted if 1,00,000 units are required per year and what is the economic advantage of the best alternative?

Solution

Operating cost of lathe including labour = Rs. 200 per hr
Operating cost of grinder including labour = Rs. 150 per hr

(a) Cost of design A

No. of hours of lathe time per 1,000 units = 16 hr
No. of hours of grinder time per 1,000 units = 4.5 hr

Total cost of design A/1,000 units

= Cost of lathe operation per 1,000 units
+ Cost of grinder operation per 1,000 units
= $16 \times 200 + 4.5 \times 150$
= Rs. 3,875

Total cost of design A/1,00,000 units = $3,875 \times 1,00,000/1,000$
= Rs. 3,87,500

(b) Cost of design B

No. of hours of lathe time per 1,000 units = 7 hr
No. of hours of grinder time per 1,000 units = 12 hr

Total cost of design B/1,000 units

= Cost of lathe operation/1,000 units
+ Cost of grinder operation/1,000 units
= $7 \times 200 + 12 \times 150$
= Rs. 3,200

Total cost of design B/1,00,000 units = $3,200 \times 1,00,000/1,000$
= Rs. 3,20,000

DECISION The total cost/1,00,000 units of design B is less than that of design A. Hence, design B is recommended for making the tapered fastening pin.

Economic advantage of the design B over design A per 1,00,000 units
= Rs. 3,87,500 – Rs. 3,20,000
= Rs. 67,500.

EXAMPLE 2.4 **(*Design selection for a process industry*).** The chief engineer of refinery operations is not satisfied with the preliminary design for storage tanks to be used as part of a plant expansion programme. The engineer who submitted the design was called in and asked to reconsider the overall dimensions in the light of an article in the *Chemical Engineer*, entitled "How to size future process vessels?"

The original design submitted called for 4 tanks 5.2 m in diameter and 7 m in height. From a graph of the article, the engineer found that the present ratio of height to diameter of 1.35 is 111% of the minimum cost and that the minimum cost for a tank was when the ratio of height to diameter was 4 : 1. The cost for the tank design as originally submitted was estimated to be Rs. 9,00,000. What are the optimum tank dimensions if the volume remains the same as for the original design? What total savings may be expected through the redesign?

Solution *(a) Original design*

Number of tanks = 4
Diameter of the tank = 5.2 m
Radius of the tank = 2.6 m
Height of the tank = 7 m
Ratio of height to diameter = 7/5.2 = 1.35
Volume/tank = $(22/7)r^2h = (22/7)(2.6)^2 \times 7$
$= 148.72\ \text{m}^3$

(b) New design

Cost of the old design = 111% of the cost of the new design (optimal design)

Optimal ratio of the height to diameter = 4:1

$$h : d = 4 : 1$$
$$4d = h$$
$$d = h/4$$
$$r = h/8$$

Volume = $(22/7)r^2h = 148.72$ (since, the volume remains the same)
$(22/7)(h/8)^2h = 148.72$

$$h^3 = \frac{148.72}{(22/7)} \times 64 = 3{,}028.48$$

$$h = 14.47\ \text{m}$$

$$r = h/8 = 14.47/8 = 1.81\ \text{m}$$

Therefore,

Diameter of the new design $= 1.81 \times 2$

$= 3.62$ m

Cost of the new design $= 9{,}00{,}000 \times (100/111)$

= Rs. 8,10,810.81

Expected savings by the redesign = Rs. 9,00,000 – Rs. 8,10,810.81

= Rs. 89,189.19

2.2.3 Building Material Selection

As discussed in the introduction to this chapter, the sourcing of raw materials will have a significant effect on the cost of any product. Hence, it is assumed that the price of raw material is location dependent. While sourcing a raw material, the cost of transportation is to be considered in conjunction with the price of the raw material. This concept is demonstrated with a numerical example.

EXAMPLE 2.5 In the design of buildings to be constructed in Alpha State, the designer is considering the type of window frame to specify. Either steel or aluminium window frames will satisfy the design criteria. Because of the remote location of the building site and lack of building materials in Alpha State, the window frames will be purchased in Beta State and transported for a distance of 2,500 km to the site. The price of window frames of the type required is Rs. 1,000 each for steel frames and Rs. 1,500 each for aluminium frames. The weight of steel window frames is 75 kg each and that of aluminium window frame is 28 kg each. The shipping rate is Re 1 per kg per 100 km. Which design should be specified and what is the economic advantage of the selection?

Solution

Distance between Alpha State and Beta State = 2,500 km
Transportation cost = Re 1/kg/100 km

(a) Steel window frame
Price of steel window frame/unit = Rs 1,000
Weight of steel window frame/unit = 75 kg

Total cost of steel window frame/unit

= Price of steel window frame/unit
+ Transportation cost of steel window frame/unit

$= 1{,}000 + (75 \times 2{,}500 \times 1)/100$

= Rs. 2,875

(b) Aluminium window frame
Price of aluminium window frame/unit = Rs. 1,500
Weight of aluminium window frame/unit = 28 kg

Total cost of aluminium window frame/unit

= Price of aluminium window frame/unit
+ Transportation cost of aluminium window frame/unit
= $1,500 + (28 \times 2,500 \times 1)/100$
= Rs. 2,200

DECISION The total cost/unit of the aluminium window frame is less than that of steel window frame. Hence, aluminium window frame is recommended.

The economic advantage/unit of the aluminium window frame over the steel window frame = Rs. 2,875 – 2,200
= Rs. 675

2.2.4 Process Planning/Process Modification

While planning for a new component, a feasible sequence of operations with the least cost of processing is to be considered. The process sequence of a component which has been planned in the past is not static. It is always subject to modification with a view to minimize the cost of manufacturing the component. So, the objective of process planning/process modification is to identify the most economical sequence of operations to produce a component.

The steps in process planning are as follows:

1. Analyze the part drawing to get an overall picture of what is required.
2. Make recommendations to or consult with product engineers on product design changes.
3. List the basic operations required to produce the part to the drawing or specifications.
4. Determine the most practical and economical manufacturing method and the form or tooling required for each operation.
5. Devise the best way to combine the operations and put them in sequence.
6. Specify the gauging required for the process.

Steps 3–5 aim to determine the most practical and economical sequence of operations to produce a component. This concept is demonstrated with a numerical problem.

EXAMPLE 2.6 The process planning engineer of a firm listed the sequences of operations as shown in Table 2.3 to produce a component.

Table 2.3 Data for Example 2.6

Sequence	*Process sequence*
1	Turning – Milling – Shaping – Drilling
2	Turning – Milling – Drilling
3	All operations are performed with CNC machine

The details of processing times of the component for various operations and their machine hour rates are summarized in Table 2.4.

Table 2.4 Machine Hour Rates and Processing Times (minutes) for Example 2.6

Operation	*Machine hour rate* (Rs.)	*Process sequence* 1	2	3
Turning	200	5	5	–
Milling	400	8	14	–
Shaping	350	10	–	–
Drilling	300	3	3	–
CNC operations	1,000	–	–	8

Find the most economical sequence of operations to manufacture the component.

Solution *(a) Cost of component using process sequence 1*. The process sequence 1 of the component is as follows:

Turning – Milling – Shaping – Drilling

The calculations for the cost of the above process sequence are summarized in Table 2.5.

Table 2.5 Workings for Process Sequence 1

Operation No.	*Operation*	*Time* (min)	(hr)	*Machine hour rate* (Rs.)	*Cost* (Rs.)
1	Turning	5	0.083	200	16.60
2	Milling	8	0.133	400	53.20
3	Shaping	10	0.167	350	58.45
4	Drilling	3	0.050	300	15.00
					Total: 143.25

(b) Cost of component using process sequence 2. The process sequence 2 of the component is as follows:

Turning – Milling – Drilling

The calculations for the cost of the above process sequence are given in Table 2.6.

Table 2.6 Workings for Process Sequence 2

Operation No.	*Operation*	*Time* (min)	(hr)	*Machine hour rate* (Rs.)	*Cost* (Rs.)
1	Turning	5	0.083	200	16.60
2	Milling	14	0.233	400	93.20
3	Drilling	3	0.050	300	15.00
					Total: 124.80

(c) Cost of component using process sequence 3. The process sequence 3 of the component is as follows:

Only CNC operations

The calculations for the cost of the above process sequence are summarized in Table 2.7

Table 2.7 Workings for Process Sequence 3

Operation No.	*Operation*	*Time* (min)	(hr)	*Machine hour rate* (Rs.)	*Cost* (Rs.)
1	CNC operations	8	0.133	1,000	133

The process sequence 2 has the least cost. Therefore, it should be selected for manufacturing the component.

QUESTIONS

1. List and explain the different situations deserving elementary economic analysis.

2. Explain the steps in the process planning.

3. In the design of an aircraft jet engine part, the designer has a choice of specifying either an aluminium alloy casting or a steel casting. Either material will provide equal service, but the aluminium casting will weigh 5 kg as compared with 7 kg for the steel casting.

The aluminium part can be cast for Rs. 125 per kg and the steel part can be cast for Rs. 60 per kg. The cost of machining per unit is Rs. 200 for the aluminium part and Rs. 250 for the steel part. Every kilogram of excess weight is associated with a penalty of Rs. 2,500 due to increased fuel consumption. Which material should be specified and what is the economic advantage of the selection per unit?

4. Two alternatives are under consideration for a hexagonal bolt fastening pin. Either design will serve equally well and will involve the same material and manufacturing cost except for the lathe and grinder operations.

Design A will require 20 hours of lathe time and 8 hours of grinder time per 10,000 units. Design B will require 10 hours of lathe time and 22 hours of grinder time per 10,000 units. The operating cost of the lathe including labour is Rs. 400 per hour. The operating cost of the grinder including labour is Rs. 300 per hour. Which design should be adopted if 10,00,000 units are required per year and what is the economic advantage of the best alternative?

5. A building contractor can source door frames from either a nearby shop or a far-off forest area. The cost details are as summarized in the following table. The total requirement of wood for the construction work is 75 tons.

Items	*Nearby shop*	*Far-off forest area*
Distance to site	Negligible	900 km
Transportation cost per ton per km	Negligible	Rs. 100
Material cost/ton	Rs. 2,000	Rs. 1,250

Find the best alternative for buying the wooden frames. Also find the economic advantage of the best decision.

6. Consider Example 2.4. Rework this example if the ratio of the height to diameter corresponding to the minimum cost is 6:1 instead of 4:1.

7. The process planning engineer of a firm listed down the sequences of operations, as shown in the following table to produce a component:

Sequence	*Process sequence*
1	Turning – Milling – Shaping – Drilling
2	Turning – Milling – Drilling
3	All operations are performed with CNC machine

The details of process time for the components for various operations and their machine hour rates are tabulated now.

Operation	*Machine hour rate* (Rs.)	*Process sequence* 1	2	3
Turning	300	8	8	
Milling	350	10	14	
Shaping	380	12		
Drilling	350	5	5	
CNC operation	1,200			10

Find the most economical sequence of operations to manufacture the component.

3

INTEREST FORMULAS AND THEIR APPLICATIONS

3.1 INTRODUCTION

Interest rate is the rental value of money. It represents the growth of capital per unit period. The period may be a month, a quarter, semiannual or a year. An interest rate 15% compounded annually means that for every hundred rupees invested now, an amount of Rs. 15 will be added to the account at the end of the first year. So, the total amount at the end of the first year will be Rs. 115. At the end of the second year, again 15% of Rs. 115, i.e. Rs. 17.25 will be added to the account. Hence the total amount at the end of the second year will be Rs. 132.25. The process will continue thus till the specified number of years.

3.2 TIME VALUE OF MONEY

If an investor invests a sum of Rs. 100 in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of every year will be as shown in Table 3.1.

Table 3.1 Compound Amounts

(amount of deposit = Rs. 100.00)

Year end	*Interest* (Rs.)	*Compound amount* (Rs.)
0		100.00
1	15.00	115.00
2	17.25	132.25
3	19.84	152.09
4	22.81	174.90
5	26.24	201.14

The formula to find the future worth in the third column is

$$F = P \times (1 + i)^n$$

where

P = principal amount invested at time 0,

F = future amount,

i = interest rate compounded annually,
n = period of deposit.

The maturity value at the end of the fifth year is Rs. 201.14. This means that the amount Rs. 201.14 at the end of the fifth year is equivalent to Rs. 100.00 at time 0 (i.e. at present). This is diagrammatically shown in Fig. 3.1. This explanation assumes that the inflation is at zero percentage.

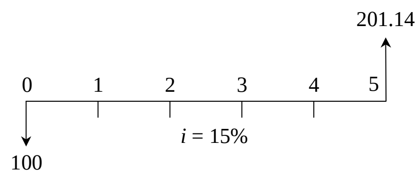


Fig. 3.1 Time value of money.

Alternatively, the above concept may be discussed as follows: If we want Rs. 100.00 at the end of the *n*th year, what is the amount that we should deposit now at a given interest rate, say 15%? A detailed working is shown in Table 3.2.

Table 3.2 Present Worth Amounts

(rate of interest = 15%)

End of year (n)	*Present worth*	*Compound amount after n year(s)*
0		100
1	86.96	100
2	75.61	100
3	65.75	100
4	57.18	100
5	49.72	100
6	43.29	100
7	37.59	100
8	32.69	100
9	28.43	100
10	24.72	100

The formula to find the present worth in the second column is

$$P = \frac{F}{(1+i)^n}$$

From Table 3.2, it is clear that if we want Rs. 100 at the end of the fifth year, we should now deposit an amount of Rs. 49.72. Similarly, if we want Rs. 100.00 at the end of the 10th year, we should now deposit an amount of Rs. 24.72.

Also, this concept can be stated as follows:

A person has received a prize from a finance company during the recent festival contest. But the prize will be given in either of the following two modes:

1. Spot payment of Rs. 24.72 or
2. Rs. 100 after 10 years from now (this is based on 15% interest rate compounded annually).

If the prize winner has no better choice that can yield more than 15% interest rate compounded annually, and if 15% compounded annually is the common interest rate paid in all the finance companies, then it makes no difference whether he receives Rs. 24.72 now or Rs. 100 after 10 years.

On the other hand, let us assume that the prize winner has his own business wherein he can get a yield of 24% interest rate (more than 15%) compounded annually, it is better for him to receive the prize money of Rs. 24.72 at present and utilize it in his business. If this option is followed, the equivalent amount for Rs. 24.72 at the end of the 10th year is Rs. 212.45. This example clearly demonstrates the time value of money.

3.3 INTEREST FORMULAS

While making investment decisions, computations will be done in many ways. To simplify all these computations, it is extremely important to know how to use interest formulas more effectively. Before discussing the effective application of the interest formulas for investment-decision making, the various interest formulas are presented first.

Interest rate can be classified into *simple interest rate* and *compound interest rate*.

In simple interest, the interest is calculated, based on the initial deposit for every interest period. In this case, calculation of interest on interest is not applicable. In compound interest, the interest for the current period is computed based on the amount (principal plus interest up to the end of the previous period) at the beginning of the current period.

The notations which are used in various interest formulae are as follows:

P = principal amount

n = No. of interest periods

i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F = future amount at the end of year n

A = equal amount deposited at the end of every interest period

G = uniform amount which will be added/subtracted period after period to/from the amount of deposit A1 at the end of period 1

3.3.1 Single-Payment Compound Amount

Here, the objective is to find the single future sum (F) of the initial payment (P) made at time 0 after n periods at an interest rate i compounded every period. The cash flow diagram of this situation is shown in Fig. 3.2.

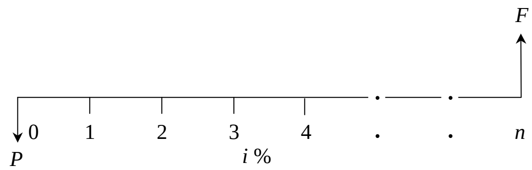


Fig. 3.2 Cash flow diagram of single-payment compound amount.

The formula to obtain the single-payment compound amount is

$$F = P(1 + i)^n = P(F/P, i, n)$$

where

$(F/P, i, n)$ is called as single-payment compound amount factor.

EXAMPLE 3.1 A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution

P = Rs. 20,000
i = 18% compounded annually
n = 10 years

$$F = P(1 + i)^n = P(F/P, i, n)$$
$$= 20{,}000\ (F/P,\ 18\%,\ 10)$$
$$= 20{,}000 \times 5.234 = \text{Rs. } 1{,}04{,}680$$

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

3.3.2 Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount (P) of a single future sum (F) which will be received after n periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.3.

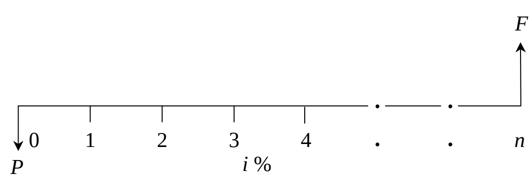


Fig. 3.3 Cash flow diagram of single-payment present worth amount.

The formula to obtain the present worth is

$$P = \frac{F}{(1 + i)^n} = F(P/F, i, n)$$

where

$(P/F, i, n)$ is termed as *single-payment present worth factor.*

EXAMPLE 3.2 A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution

F = Rs. 1,00,000
i = 15%, compounded annually
n = 10 years
$P = F/(1 + i)^n = F(P/F, i, n)$
$= 1,00,000\ (P/F, 15\%, 10)$
$= 1,00,000 \times 0.2472$
= Rs. 24,720

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

3.3.3 Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the nth interest period at an interest rate of i compounded at the end of each interest period. The corresponding cash flow diagram is shown in Fig. 3.4.

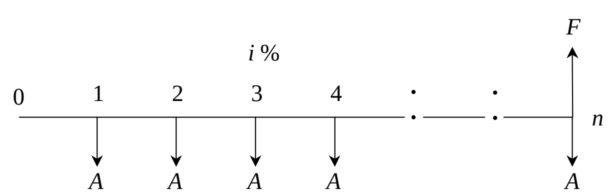


Fig. 3.4 Cash flow diagram of equal-payment series compound amount.

In Fig. 3.4,
A = equal amount deposited at the end of each interest period
n = No. of interest periods
i = rate of interest
F = single future amount

The formula to get F is

$$F = A\,\frac{(1 + i)^n - 1}{i} = A(F/A, i, n)$$

where
$(F/A, i, n)$ is termed as *equal-payment series compound amount factor*.

EXAMPLE 3.3 A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution

A = Rs. 10,000
n = 25 years
i = 20%
F = ?

The corresponding cash flow diagram is shown in Fig. 3.5.

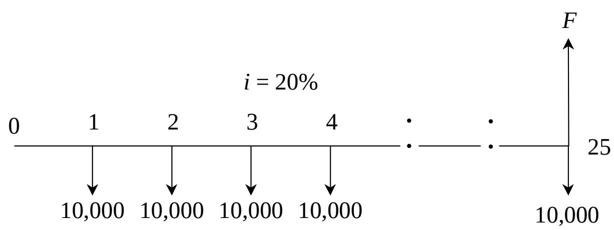


Fig. 3.5 Cash flow diagram of equal-payment series compound amount.

$$F = A\,\frac{(1+i)^n - 1}{i}$$
$$= A(F/A,\ i,\ n)$$
$$= 10{,}000(F/A,\ 20\%,\ 25)$$
$$= 10{,}000 \times 471.981$$
$$= \text{Rs. } 47{,}19{,}810$$

The future sum of the annual equal payments after 25 years is equal to Rs. 47,19,810.

3.3.4 Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i.

The corresponding cash flow diagram is shown in Fig. 3.6.

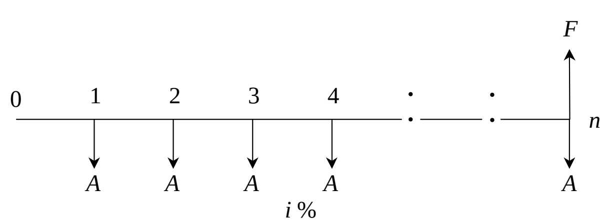


Fig. 3.6 Cash flow diagram of equal-payment series sinking fund.

In Fig. 3.6,

A = equal amount to be deposited at the end of each interest period
n = No. of interest periods
i = rate of interest
F = single future amount at the end of the nth period

The formula to get F is

$$A = F \frac{i}{(1+i)^n - 1} = F(A/F,\ i,\ n)$$

where

$(A/F, i, n)$ is called as *equal-payment series sinking fund factor.*

EXAMPLE 3.4 A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

Solution

F = Rs. 5,00,000
n = 15 years
i = 18%
A = ?

The corresponding cash flow diagram is shown in Fig. 3.7.

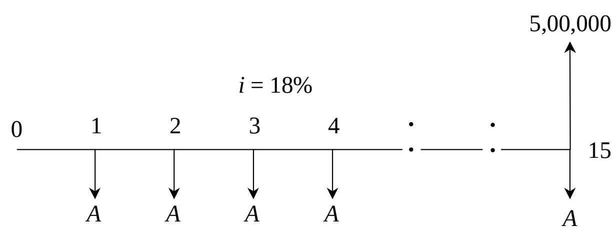


Fig. 3.7 Cash flow diagram of equal-payment series sinking fund.

$$A = F \frac{i}{(1+i)^n - 1} = F(A/F,\ i,\ n)$$
$$= 5{,}00{,}000(A/F,\ 18\%,\ 15)$$
$$= 5{,}00{,}000 \times 0.0164$$
$$= \text{Rs. } 8{,}200$$

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

3.3.5 Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal payment made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.8. Here,

P = present worth
A = annual equivalent payment
i = interest rate
n = No. of interest periods

The formula to compute P is

$$P = A\,\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A,\ i,\ n)$$

where

$(P/A, i, n)$ is called *equal-payment series present worth factor.*

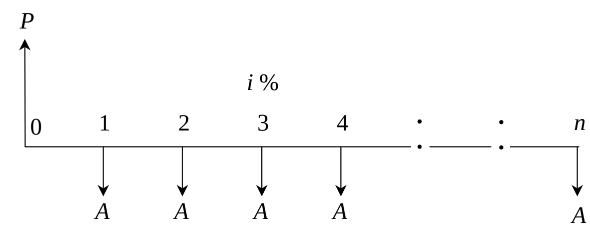


Fig. 3.8 Cash flow diagram of equal-payment series present worth amount.

EXAMPLE 3.5 A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

Solution

A = Rs. 10,00,000
i = 15%
n = 20 years
P = ?

The corresponding cash flow diagram is illustrated in Fig. 3.9.

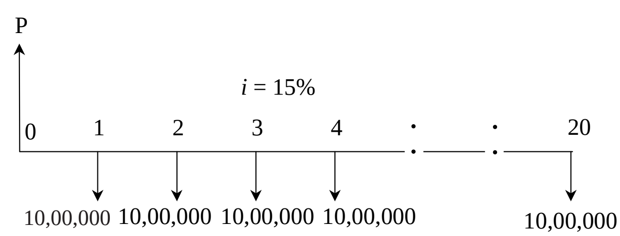


Fig. 3.9 Cash flow diagram of equal-payment series present worth amount.

$$\begin{aligned} P &= A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A,\ i,\ n) \\ &= 10{,}00{,}000 \times (P/A,\ 15\%,\ 20) \\ &= 10{,}00{,}000 \times 6.2593 \\ &= \text{Rs. } 62{,}59{,}300 \end{aligned}$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

3.3.6 Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period (see Fig. 3.10).

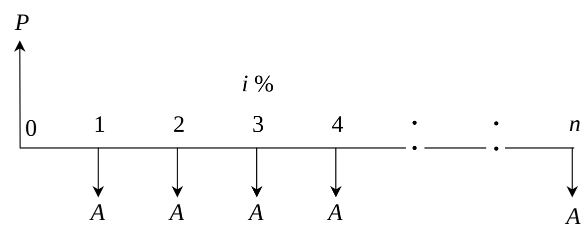


Fig. 3.10 Cash flow diagram of equal-payment series capital recovery amount.

In Fig. 3.10,

P = present worth (loan amount)
A = annual equivalent payment (recovery amount)
i = interest rate
n = No. of interest periods

The formula to compute P is as follows:

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P,\ i,\ n)$$

where,

(A/P, i, n) is called *equal-payment series capital recovery factor*.

EXAMPLE 3.6 A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

Solution

P = Rs. 10,00,000
i = 18%
n = 15 years
A = ?

The corresponding cash flow diagram is shown in Fig. 3.11.

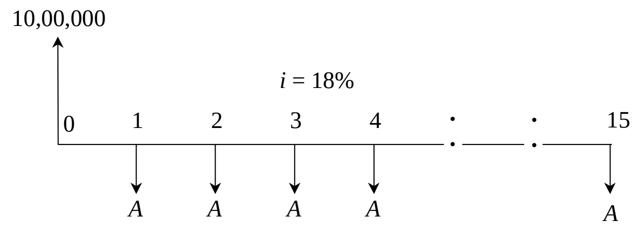


Fig. 3.11 Cash flow diagram of equal-payment series capital recovery amount.

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P,\ i,\ n)$$

$$= 10{,}00{,}000 \times (A/P,\ 18\%,\ 15)$$

$$= 10{,}00{,}000 \times (0.1964)$$

$$= \text{Rs. } 1{,}96{,}400$$

The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

3.3.7 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n - 1$ years with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.12.

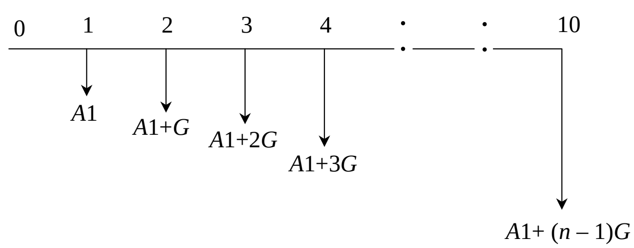


Fig. 3.12 Cash flow diagram of uniform gradient series annual equivalent amount.

The formula to compute A under this situation is

$$A = A1 + G\ \frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 + G\ (A/G,\ i,\ n)$$

where

(A/G, i, n) is called *uniform gradient series factor.*

EXAMPLE 3.7 A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 4,000, at the end of the first year, and thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

Solution Here,

$A1$ = Rs. 4,000
G = Rs. 500
i = 15%
n = 10 years
A = ? & F = ?

The cash flow diagram is shown in Fig. 3.13.

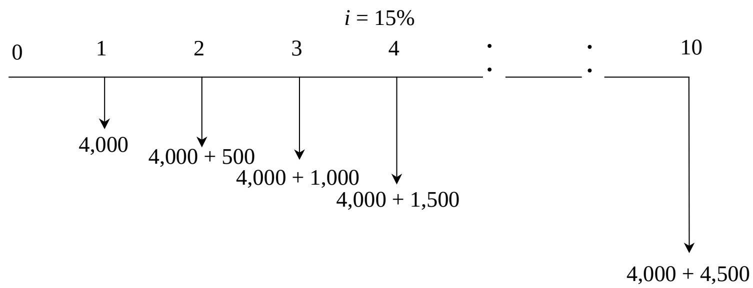


Fig. 3.13 Cash flow diagram of uniform gradient series annual equivalent amount.

$$A = A1 + G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$
$$= A1 + G(A/G, i, n)$$
$$= 4{,}000 + 500(A/G, 15\%, 10)$$
$$= 4{,}000 + 500 \times 3.3832$$
$$= \text{Rs. } 5{,}691.60$$

This is equivalent to paying an equivalent amount of Rs. 5,691.60 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$F = A(F/A, i, n)$$
$$= A(F/A, 15\%, 10)$$
$$= 5{,}691.60(20.304)$$
$$= \text{Rs. } 1{,}15{,}562.25$$

At the end of the 10th year, the compound amount of all his payments will be Rs. 1,15,562.25.

EXAMPLE 3.8 A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 8,500 at the end of the first year and thereafter he wishes to deposit the amount with an annual decrease of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

Solution Here,

$A1$ = Rs. 8,500

G = –Rs. 500

i = 15%

n = 10 years

A = ? & F = ?

The cash flow diagram is shown in Fig. 3.14.

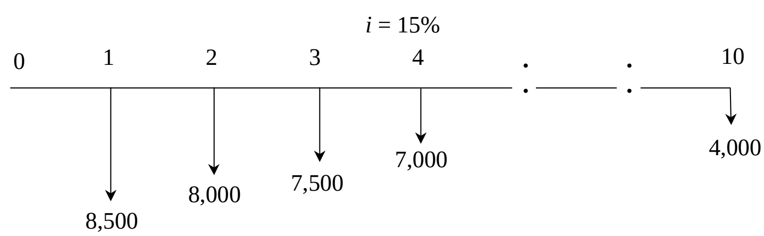


Fig. 3.14 Cash flow diagram of uniform gradient series annual equivalent amount.

$$A = A1 - G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 - G\,(A/G,\ i,\ n)$$

$$= 8{,}500 - 500(A/G,\ 15\%,\ 10)$$

$$= 8{,}500 - 500 \times 3.3832$$

$$= \text{Rs. } 6{,}808.40$$

This is equivalent to paying an equivalent amount of Rs. 6,808.40 at the end of every year for the next 10 years.

The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$F = A(F/A,\ i,\ n)$$

$$= A(F/A,\ 15\%,\ 10)$$

$$= 6{,}808.40(20.304)$$

$$= \text{Rs. } 1{,}38{,}237.75$$

At the end of the 10th year, the compound amount of all his payments is Rs. 1,38,237.75.

3.3.8 Effective Interest Rate

Let i be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year. For example, compounding may be monthly, quarterly, or semi-annually. Compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in a year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is

$$\text{Effective interest rate, } R = (1 + i/C)^C - 1$$

where,

i = the nominal interest rate

C = the number of interest periods in a year.

EXAMPLE 3.9 A person invests a sum of Rs. 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

Solution

P = Rs. 5,000

n = 10 years

i = 12% (Nominal interest rate)

F = ?

METHOD 1

No. of interest periods per year = 4

No. of interest periods in 10 years = $10 \times 4 = 40$

Revised No. of periods (No. of quarters), $N = 40$

Interest rate per quarter, r = 12%/4

= 3%, compounded quarterly.

$$F = P(1 + r)^N = 5{,}000(1 + 0.03)^{40}$$

$$= \text{Rs. } 16{,}310.19$$

METHOD 2

No. of interest periods per year, $C = 4$

Effective interest rate, $R = (1 + i/C)^C - 1$

$= (1 + 12\%/4)^4 - 1$

= 12.55%, compounded annually.

$$F = P(1 + R)^n = 5{,}000(1 + 0.1255)^{10}$$

$$= \text{Rs. } 16{,}308.91$$

3.4 BASES FOR COMPARISON OF ALTERNATIVES

In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives. Let us assume that an organization has a huge sum of money for potential investment and there are three different projects whose initial outlay and annual revenues during their lives are known. The executive has to select the best alternative among these three competing projects.

There are several bases for comparing the worthiness of the projects. These bases are:

1. Present worth method
2. Future worth method
3. Annual equivalent method
4. Rate of return method

These methods are discussed in detail in Chapters 4–7.

QUESTIONS

1. Explain the time value of money.

2. Give practical applications of various interest formulas.

3. A person deposits a sum of Rs. 1,00,000 in a bank for his son's education who will be admitted to a professional course after 6 years. The bank pays 15% interest rate, compounded annually. Find the future amount of the deposited money at the time of admitting his son in the professional course.

4. A person needs a sum of Rs. 2,00,000 for his daughter's marriage which will take place 15 years from now. Find the amount of money that he should deposit now in a bank if the bank gives 18% interest, compounded annually.

5. A person who is just 30 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 30 years starting from the end of next year. The bank gives 15% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

6. A company is planning to expand its business after 5 years from now. The expected money required for the expansion programme is Rs. 5,00,00,000. The company can invest Rs. 50,00,000 at the end of every year for the next five years. If the assured rate of return of investment is 18% for the company, check whether the accumulated sum in the account would be sufficient to meet the fund for the expansion programme. If not, find the difference in amounts for which the company should make some other arrangement after 5 years.

7. A financial institution introduces a plan to pay a sum of Rs. 15,00,000 after 10 years at the rate of 18%, compounded annually. Find the annual equivalent amount that a person should invest at the end of every year for the next 10 years to receive Rs. 15,00,000 after 10 years from the institution.

8. A company is planning to expand its business after 5 years from now. The money required for the expansion programme is Rs. 4,00,00,000. What annual equivalent amount should the company deposit at the end of every year at an interest rate of 15% compounded annually to get Rs. 4,00,00,000 after 5 years from now?

9. A company wants to set-up a reserve which will help it to have an annual equivalent amount of Rs. 15,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made as the reserve amount now.

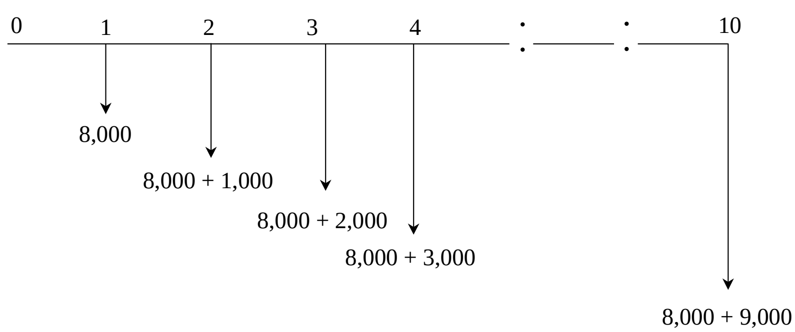
10. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually. Suggest the best alternative to the customers.

11. A company takes a loan of Rs. 20,00,000 to modernize its boiler section. The loan is to be repaid in 20 equal installments at 12% interest rate, compounded annually. Find the equal installment amount that should be paid for the next 20 years.

12. A bank gives loan to a company to purchase an equipment which is worth of Rs. 5,00,000, at an interest rate of 18% compounded annually. This amount should be repaid in 25 yearly equal installments. Find the installment amount that the company has to pay to the bank.

13. A working woman is planning for her retired life. She has 20 more years of service. She would like to deposit 10% of her salary which is Rs. 5,000 at the end of the first year and thereafter she wishes to deposit the same amount (Rs. 5,000) with an annual increase of Rs. 1,000 for the next 14 years with an interest rate of 18%. Find the total amount at the end of the 15th year of the above series.

14. Consider the following cash flow diagram. Find the total amount at the end of the 10th year at an interest rate of 12%, compounded annually.



15. A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 10,000, at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 10,000) with an annual increase of Rs. 2,000 for the next 9 years with an interest rate of 20%. Find the total amount at the end of the 10th year of the above series.

16. A person is planning for his retired life. He has 10 more years of service. He would like to deposit Rs. 30,000 at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 30,000) with an annual decrease of Rs. 2,000 for the next 9 years with an interest rate of 18%. Find the total amount at the end of the 10th year of the above series.

17. A person invests a sum of Rs. 50,000 in a bank at a nominal interest rate of 18% for 15 years. The compounding is monthly. Find the maturity amount of the deposit after 15 years.

4

PRESENT WORTH METHOD OF COMPARISON

4.1 INTRODUCTION

In this method of comparison, the cash flows of each alternative will be reduced to time zero by assuming an interest rate *i*. Then, depending on the type of decision, the best alternative will be selected by comparing the present worth amounts of the alternatives.

The sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem.

In a cost dominated cash flow diagram, the costs (outflows) will be assigned with positive sign and the profit, revenue, salvage value (all inflows), etc. will be assigned with negative sign.

In a revenue/profit-dominated cash flow diagram, the profit, revenue, salvage value (all inflows to an organization) will be assigned with positive sign. The costs (outflows) will be assigned with negative sign.

In case the decision is to select the alternative with the minimum cost, then the alternative with the least present worth amount will be selected. On the other hand, if the decision is to select the alternative with the maximum profit, then the alternative with the maximum present worth will be selected.

4.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.1.

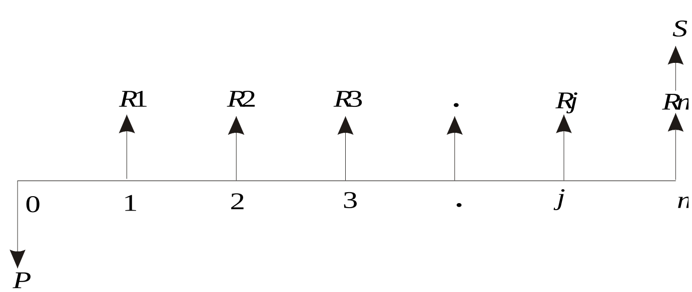


Fig. 4.1 Revenue-dominated cash flow diagram.

In Fig. 4.1, P represents an initial investment and Rj the net revenue at the end of the jth year. The interest rate is i, compounded annually. S is the salvage value at the end of the nth year.

To find the present worth of the above cash flow diagram for a given interest rate, the formula is

$$PW(i) = -P + R1[1/(1+i)^1] + R2[1/(1+i)^2] + \ldots$$
$$+ Rj[1/(1+i)^j] + Rn[1/(1+i)^n] + S[1/(1+i)^n]$$

In this formula, expenditure is assigned a negative sign and revenues are assigned a positive sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the alternative with the maximum present worth amount should be selected as the best alternative.

4.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.2.

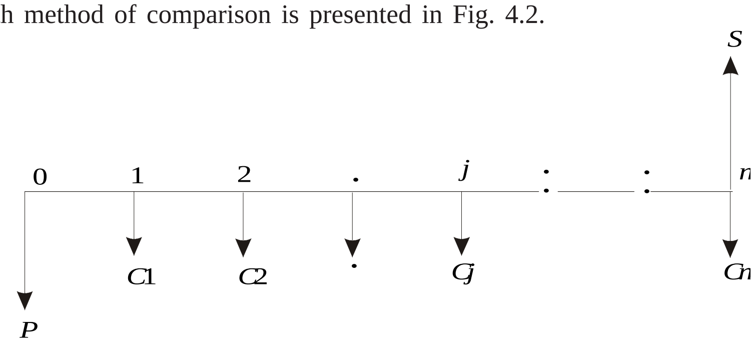


Fig. 4.2 Cost-dominated cash flow diagram.

In Fig. 4.2, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year.

To compute the present worth amount of the above cash flow diagram for a given interest rate i, we have the formula

$$PW(i) = P + C1[1/(1+i)^1] + C2[1/(1+i)^2] + \ldots + Cj[1/(1+i)^j]$$
$$+ Cn[1/(1+i)^n] - S[1/(1+i)^n]$$

In the above formula, the expenditure is assigned a positive sign and the revenue a negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the alternative with the minimum present worth amount should be selected as the best alternative.

4.4 EXAMPLES

In this section, the concept of present worth method of comparison applied to the selection of the best alternative is demonstrated with several illustrations.

EXAMPLE 4.1 Alpha Industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized in Table 4.1. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

Table 4.1

	Initial outlay (Rs.)	*Annual revenue* (Rs.)	*Life* (years)
Technology 1	12,00,000	4,00,000	10
Technology 2	20,00,000	6,00,000	10
Technology 3	18,00,000	5,00,000	10

Solution In all the technologies, the initial outlay is assigned a negative sign and the annual revenues are assigned a positive sign.

TECHNOLOGY 1

Initial outlay, P = Rs. 12,00,000
Annual revenue, A = Rs. 4,00,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is as shown in Fig. 4.3.

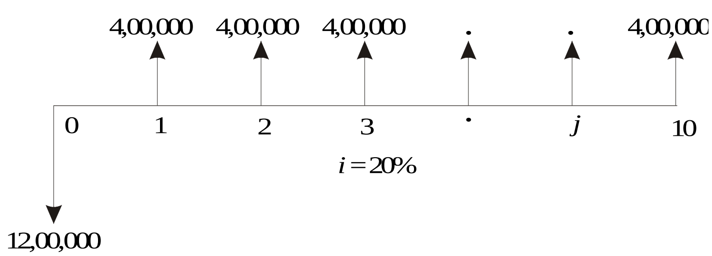


Fig. 4.3 Cash flow diagram for technology 1.

The present worth expression for this technology is

$$\begin{aligned} PW(20\%)_1 &= -12,00,000 + 4,00,000 \times (P/A, 20\%, 10) \\ &= -12,00,000 + 4,00,000 \times (4.1925) \\ &= -12,00,000 + 16,77,000 \\ &= \text{Rs. } 4,77,000 \end{aligned}$$

TECHNOLOGY 2

Initial outlay, P = Rs. 20,00,000
Annual revenue, A = Rs. 6,00,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is shown in Fig. 4.4.

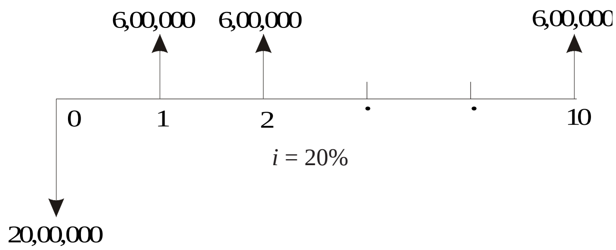


Fig. 4.4 Cash flow diagram for technology 2.

The present worth expression for this technology is

$$\begin{aligned} PW(20\%)_2 &= -20,00,000 + 6,00,000 \times (P/A, 20\%, 10) \\ &= -20,00,000 + 6,00,000 \times (4.1925) \\ &= -20,00,000 + 25,15,500 \\ &= \text{Rs. } 5,15,500 \end{aligned}$$

TECHNOLOGY 3

Initial outlay, P = Rs. 18,00,000
Annual revenue, A = Rs. 5,00,000
Interest rate, i = 20%, compounded annually
Life of this technology, n = 10 years

The cash flow diagram of this technology is shown in Fig. 4.5.

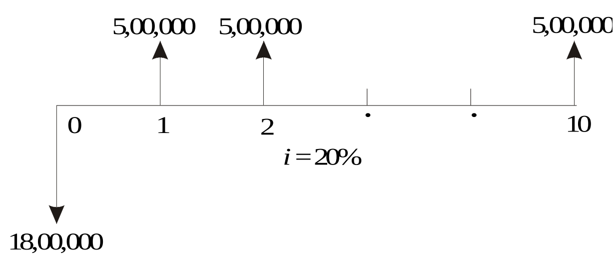


Fig. 4.5 Cash flow diagram for technology 3.

The present worth expression for this technology is

$$\begin{aligned} PW(20\%)_3 &= -18,00,000 + 5,00,000 \times (P/A, 20\%, 10) \\ &= -18,00,000 + 5,00,000 \times (4.1925) \\ &= -18,00,000 + 20,96,250 \\ &= \text{Rs. } 2,96,250 \end{aligned}$$

From the above calculations, it is clear that the present worth of technology 2 is the highest among all the technologies. Therefore, technology 2 is suggested for implementation to expand the production.

EXAMPLE 4.2 An engineer has two bids for an elevator to be installed in a new building. The details of the bids for the elevators are as follows:

Bid	*Engineer's estimates*		
	Initial cost (Rs.)	*Service life* (years)	*Annual operations & maintenance cost* (Rs.)
Alpha Elevator Inc.	4,50,000	15	27,000
Beta Elevator Inc.	5,40,000	15	28,500

Determine which bid should be accepted, based on the present worth method of comparison assuming 15% interest rate, compounded annually.

Solution

Bid 1: Alpha Elevator Inc.

Initial cost, P = Rs. 4,50,000
Annual operation and maintenance cost, A = Rs. 27,000
Life = 15 years
Interest rate, i = 15%, compounded annually.

The cash flow diagram of bid 1 is shown in Fig. 4.6.

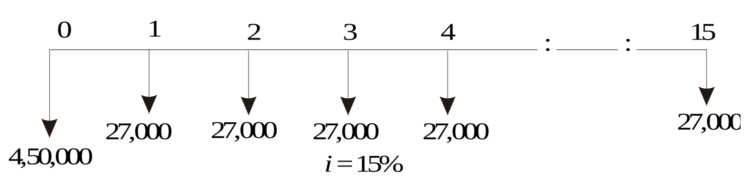


Fig. 4.6 Cash flow diagram for bid 1.

The present worth of the above cash flow diagram is computed as follows:

$$
\begin{aligned}
PW(15\%) &= 4{,}50{,}000 + 27{,}000(P/A,\ 15\%,\ 15)\\
&= 4{,}50{,}000 + 27{,}000 \times 5.8474\\
&= 4{,}50{,}000 + 1{,}57{,}879.80\\
&= \text{Rs. } 6{,}07{,}879.80
\end{aligned}
$$

Bid 2: Beta Elevator Inc.

Initial cost, P = Rs. 5,40,000
Annual operation and maintenance cost, A = Rs. 28,500
Life = 15 years
Interest rate, i = 15%, compounded annually.

The cash flow diagram of bid 2 is shown in Fig. 4.7.

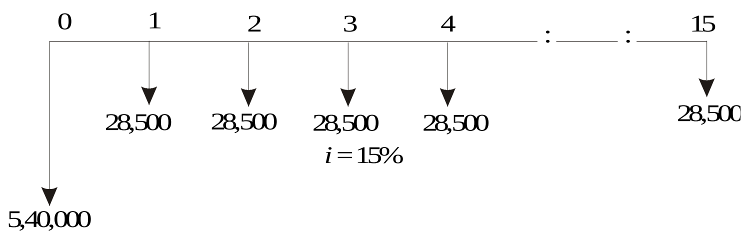


Fig. 4.7 Cash flow diagram for bid 2.

The present worth of the above cash flow diagram is computed as follows:

$$
\begin{aligned}
PW(15\%) &= 5{,}40{,}000 + 28{,}500(P/A,\ 15\%,\ 15) \\
&= 5{,}40{,}000 + 28{,}500 \times 5.8474 \\
&= 5{,}40{,}000 + 1{,}66{,}650.90 \\
&= \text{Rs. } 7{,}06{,}650.90
\end{aligned}
$$

The total present worth cost of bid 1 is less than that of bid 2. Hence, bid 1 is to be selected for implementation. That is, the elevator from Alpha Elevator Inc. is to be purchased and installed in the new building.

EXAMPLE 4.3 Investment proposals A and B have the net cash flows as follows:

Proposal	*End of years*				
	0	1	2	3	4
A (Rs.)	–10,000	3,000	3,000	7,000	6,000
B (Rs.)	–10,000	6,000	6,000	3,000	3,000

Compare the present worth of A with that of B at i = 18%. Which proposal should be selected?

Solution

Present worth of A at *i* = 18%. The cash flow diagram of proposal A is shown in Fig. 4.8.

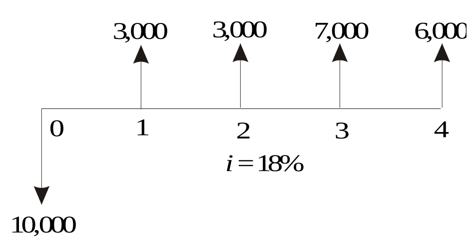


Fig. 4.8 Cash flow diagram for proposal A.

The present worth of the above cash flow diagram is computed as

$$\begin{aligned} PW_A(18\%) &= -10{,}000 + 3{,}000(P/F, 18\%, 1) + 3{,}000(P/F, 18\%, 2) \\ &\quad + 7{,}000(P/F, 18\%, 3) + 6{,}000(P/F, 18\%, 4) \\ &= -10{,}000 + 3{,}000\ (0.8475) + 3{,}000(0.7182) \\ &\quad + 7{,}000(0.6086) + 6{,}000(0.5158) \\ &= \text{Rs. } 2{,}052.10 \end{aligned}$$

Present worth of B at *i* = 18%. The cash flow diagram of the proposal B is shown in Fig. 4.9.

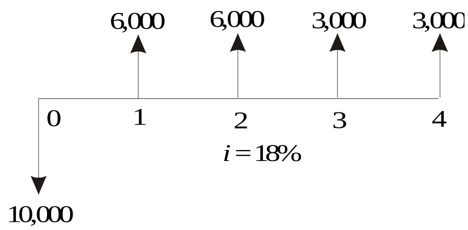


Fig. 4.9 Cash flow diagram for proposal B.

The present worth of the above cash flow diagram is calculated as

$$\begin{aligned} PW_B(18\%) &= -10{,}000 + 6{,}000(P/F, 18\%, 1) + 6{,}000(P/F, 18\%, 2) \\ &\quad + 3{,}000(P/F, 18\%, 3) + 3{,}000(P/F, 18\%, 4) \\ &= -10{,}000 + 6{,}000(0.8475) + 6{,}000(0.7182) \\ &\quad + 3{,}000(0.6086) + 3{,}000(0.5158) \\ &= \text{Rs. } 2{,}767.40 \end{aligned}$$

At i = 18%, the present worth of proposal B is higher than that of proposal A. Therefore, select proposal B.

EXAMPLE 4.4 A granite company is planning to buy a fully automated granite cutting machine. If it is purchased under down payment, the cost of the machine is Rs. 16,00,000. If it is purchased under installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of Rs. 2,00,000 each. Suggest the best alternative for the company using the present worth basis at i = 18%, compounded annually.

Solution There are two alternatives available for the company:

1. Down payment of Rs. 16,00,000
2. Down payment of Rs. 4,00,000 and 10 annual equal installments of Rs. 2,00,000 each

Present worth calculation of the second alternative. The cash flow diagram of the second alternative is shown in Fig. 4.10.

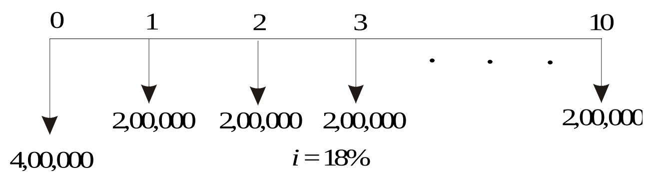


Fig. 4.10 Cash flow diagram for the second alternative.

The present worth of the above cash flow diagram is computed as

$$\begin{aligned} PW(18\%) &= 4{,}00{,}000 + 2{,}00{,}000(P/A,\ 18\%,\ 10) \\ &= 4{,}00{,}000 + 2{,}00{,}000 \times 4.4941 \\ &= \text{Rs. } 12{,}98{,}820 \end{aligned}$$

The present worth of this option is Rs. 12,98,820, which is less than the first option of complete down payment of Rs. 16,00,000. Hence, the company should select the second alternative to buy the fully automated granite cutting machine.

EXAMPLE 4.5 A finance company advertises two investment plans. In plan 1, the company pays Rs. 12,000 after 15 years for every Rs. 1,000 invested now. In plan 2, for every Rs. 1,000 invested, the company pays Rs. 4,000 at the end of the 10th year and Rs. 4,000 at the end of 15th year. Select the best investment plan from the investor's point of view at i = 12%, compounded annually.

Solution *Plan 1.* The cash flow diagram for plan 1 is illustrated in Fig. 4.11.

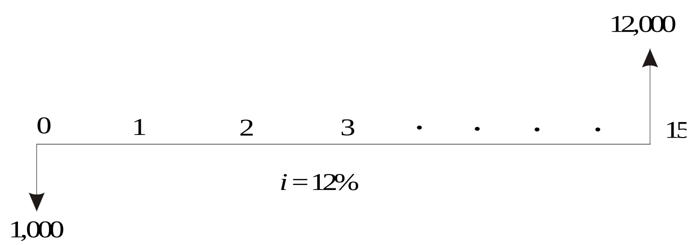


Fig. 4.11 Cash flow diagram for plan 1.

The present worth of the above cash flow diagram is calculated as

$$\begin{aligned} PW(12\%) &= -1{,}000 + 12{,}000(P/F,\ 12\%,\ 15) \\ &= -1{,}000 + 12{,}000(0.1827) \\ &= \text{Rs. } 1{,}192.40 \end{aligned}$$

Plan 2. The cash flow diagram for plan 2 is shown in Fig. 4.12.

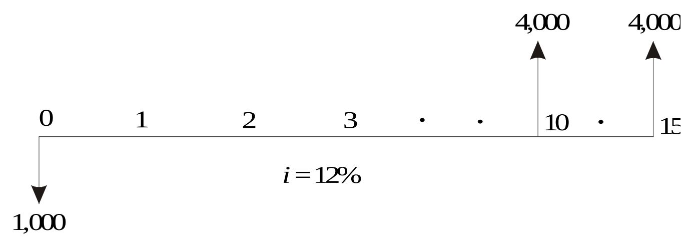


Fig. 4.12 Cash flow diagram for plan 2.

The present worth of the above cash flow diagram is computed as

$$PW(12\%) = -1{,}000 + 4{,}000(P/F, 12\%, 10) + 4{,}000(P/F, 12\%, 15)$$
$$= -1{,}000 + 4{,}000(0.3220) + 4{,}000(0.1827)$$
$$= \text{Rs. } 1{,}018.80$$

The present worth of plan 1 is more than that of plan 2. Therefore, plan 1 is the best plan from the investor's point of view.

.

EXAMPLE 4.6 Novel Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 8,00,000 at the end of the 20th year. Innovative Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 15,00,000 at the end of the 25th year. Which is the best investment alternative? Use present worth base with i = 12%.

Solution *Novel Investment Ltd's plan.* The cash flow diagram of Novel Investment Ltd's plan is shown in Fig. 4.13.

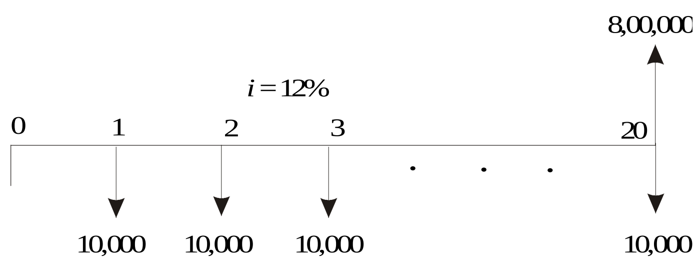


Fig. 4.13 Cash flow diagram for Novel Investment Ltd.

The present worth of the above cash flow diagram is computed as

$$PW(12\%) = -10{,}000(P/A, 12\%, 20) + 8{,}00{,}000(P/F, 12\%, 20)$$
$$= -10{,}000(7.4694) + 8{,}00{,}000(0.1037)$$
$$= \text{Rs. } 8{,}266$$

Innovative Investment Ltd's plan. The cash flow diagram of the Innovative Investment Ltd's plan is illustrated in Fig. 4.14.

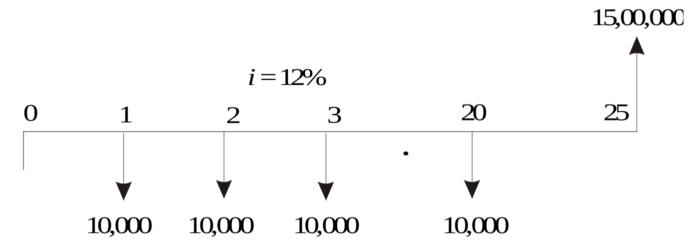


Fig. 4.14 Cash flow diagram for Innovative Investment Ltd.

The present worth of the above cash flow diagram is calculated as

$$PW(12\%) = -10{,}000(P/A,\ 12\%,\ 20) + 15{,}00{,}000(P/F,\ 12\%,\ 25)$$
$$= -10{,}000(7.4694) + 15{,}00{,}000(0.0588)$$
$$= \text{Rs. } 13{,}506$$

The present worth of Innovative Investment Ltd's plan is more than that of Novel Investment Ltd's plan. Therefore, Innovative Investment Ltd's plan is the best from investor's point of view.

EXAMPLE 4.7 A small business with an initial outlay of Rs. 12,000 yields Rs. 10,000 during the first year of its operation and the yield increases by Rs. 1,000 from its second year of operation up to its 10th year of operation. At the end of the life of the business, the salvage value is zero. Find the present worth of the business by assuming an interest rate of 18%, compounded annually.

Solution

Initial investment, P = Rs. 12,000

Income during the first year, A = Rs. 10,000

Annual increase in income, G = Rs. 1,000

n = 10 years

i = 18%, compounded annually

The cash flow diagram for the small business is depicted in Fig. 4.15.

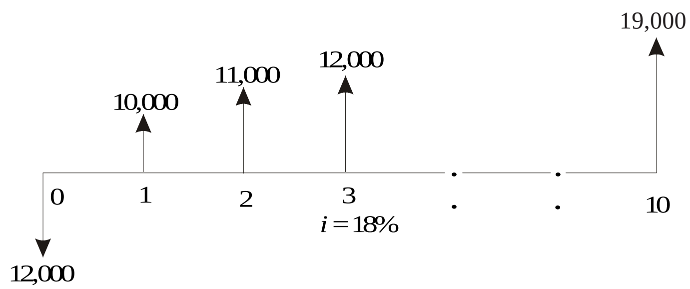


Fig. 4.15 Cash flow diagram for the small business.

The equation for the present worth is

$$\begin{aligned} PW(18\%) &= -12{,}000 + (10{,}000 + 1{,}000 \times (A/G, 18\%, 10)) \times (P/A, 18\%, 10) \\ &= -12{,}000 + (10{,}000 + 1{,}000 \times 3.1936) \times 4.4941 \\ &= -12{,}000 + 59{,}293.36 \\ &= \text{Rs. } 47{,}293.36 \end{aligned}$$

The present worth of the small business is Rs. 47,293.36.

QUESTIONS

1. A project involves an initial outlay of Rs. 30,00,000 and with the following transactions for the next five years. The salvage value at the end of the life of the project after five years is Rs. 2,00,000. Draw a cash flow diagram of the project and find its present worth by assuming i = 15%, compounded annually.

End of year	*Maintenance and operating expense* (Rs.)	*Revenue* (Rs.)
1	2,00,000	9,00,000
2	2,50,000	10,00,000
3	3,00,000	12,00,000
4	3,00,000	13,00,000
5	4,00,000	12,00,000

2. Find the present worth of the following cash flow series. Assume i = 15%, compounded annually.

End of year	0	1	2	3	4	5
Cash flow (Rs.)	–10,000	30,000	30,000	30,000	30,000	30,000

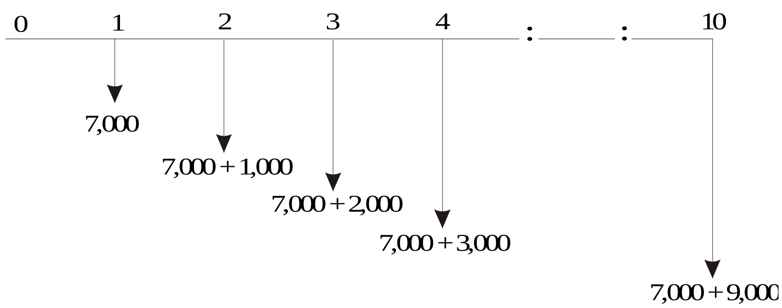
3. Consider the following cash flow series over a 20-year period. Assuming the interest rate as 18% compounded annually, compute the present worth of the series; give your comments.

End of year	*Cash flow* (Rs.)
0	–50,00,000
1	6,00,000
2	6,00,000
.	.
.	.
.	.
20	6,00,000

4. The cost of erecting an oil well is Rs. 1,50,00,000. The annual equivalent yield from the oil well is Rs. 30,00,000. The salvage value after its useful life of 10 years is Rs. 2,00,000. Assuming an interest rate of 18%, compounded annually, find out whether the erection of the oil well is financially feasible, based on the present worth method.

5. The details of the feasibility report of a project are as shown below. Check the feasibility of the project based on present worth method, using i = 20%.
Initial outlay = Rs. 50,00,000
Life of the project = 20 years.
Annual equivalent revenue = Rs. 15,00,000
Modernizing cost at the end of the 10th year = Rs. 20,00,000
Salvage value at the end of project life = Rs. 5,00,000.

6. Consider the following cash flow diagram. Find the present worth using an interest rate of 15%, compounded annually.



7. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually.
You are asked to advise the best alternative for the customers based on the present worth method of comparison.

8. The cash flows of two project proposals are as given below. Each of the project has an expected life of 10 years. Select the best project based on present worth method of comparison using an interest rate of 18%, compounded annually.

	Initial outlay (Rs.)	*Annual equivalent revenue* (Rs.)	*Salvage value after 10 years* (Rs.)
Project 1	–7,50,000	2,00,000	50,000
Project 2	–9,50,000	2,25,000	1,00,000

9. A company has two alternatives for satisfying its daily travel requirements of its employees for the next five years:

Alternative 1: Renting a vehicle at a cost of Rs. 10,00,000 per year.

Alternative 2: Buying a vehicle for Rs. 5,00,000 with an operating and maintenance cost of Rs. 3,50,000 per year. The salvage value of the vehicle after five years is Rs. 1,00,000.

Select the best alternative based on the present worth method of comparison using the interest rate of 20%, compounded annually.

10. A working woman is planning for her retired life. She has 20 more years of service. She would like to have an annual equivalent amount of Rs. 3,00,000, starting from the end of the first year of her retirement. Find the single amount that should be deposited now so that she receives the above mentioned annual equivalent amount at the end of every year for 20 years after her retirement. Assume i = 15%, compounded annually.

5

FUTURE WORTH METHOD

5.1 INTRODUCTION

In the future worth method of comparison of alternatives, the future worth of various alternatives will be computed. Then, the alternative with the maximum future worth of net revenue or with the minimum future worth of net cost will be selected as the best alternative for implementation.

5.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the future worth method of comparison is presented in Fig. 5.1.

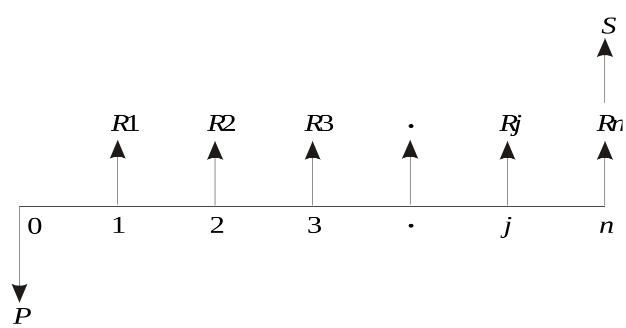


Fig. 5.1 Revenue-dominated cash flow diagram.

In Fig. 5.1, P represents an initial investment, Rj the net-revenue at the end of the jth year, and S the salvage value at the end of the nth year.

The formula for the future worth of the above cash flow diagram for a given interest rate, i is

$$FW(i) = -P(1 + i)^n + R1(1 + i)^{n-1} + R2(1 + \mathrm{i})^{n-2} + \ldots$$
$$+ Rj(1 + i)^{n-j} + \ldots + Rn + S$$

In the above formula, the expenditure is assigned with negative sign and the revenues are assigned with positive sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the maximum future worth amount should be selected as the best alternative.

5.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the future worth method of comparison is given in Fig. 5.2.

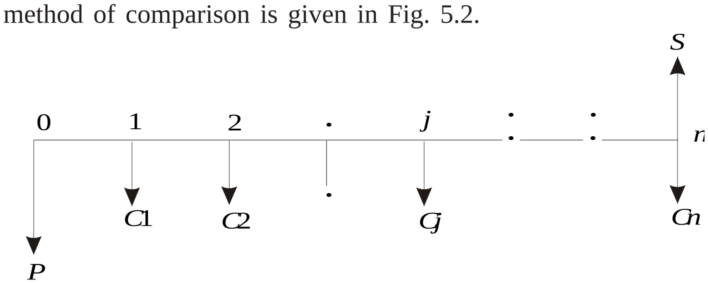


Fig. 5.2 Cost-dominated cash flow diagram.

In Fig. 5.2, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year.

The formula for the future worth of the above cash flow diagram for a given interest rate, i is

$$FW(i) = P(1 + i)^n + C1(1 + i)^{n-1} + C2(1 + i)^{n-2} + \ldots + Cj(1 + i)^{n-j} + \ldots + Cn - S$$

In this formula, the expenditures are assigned with positive sign and revenues with negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the minimum future worth amount should be selected as the best alternative.

5.4 EXAMPLES

In this section, several examples highlighting the applications of the future worth method of comparison are presented.

EXAMPLE 5.1 Consider the following two mutually exclusive alternatives:

	End of year				
Alternative	0	1	2	3	4
A (Rs.)	–50,00,000	20,00,000	20,00,000	20,00,000	20,00,000
B (Rs.)	–45,00,000	18,00,000	18,00,000	18,00,000	18,00,000

At i = 18%, select the best alternative based on future worth method of comparison.

Solution ***Alternative A***

Initial investment, P = Rs. 50,00,000
Annual equivalent revenue, A = Rs. 20,00,000
Interest rate, i = 18%, compounded annually
Life of alternative A = 4 years

The cash flow diagram of alternative A is shown in Fig. 5.3.

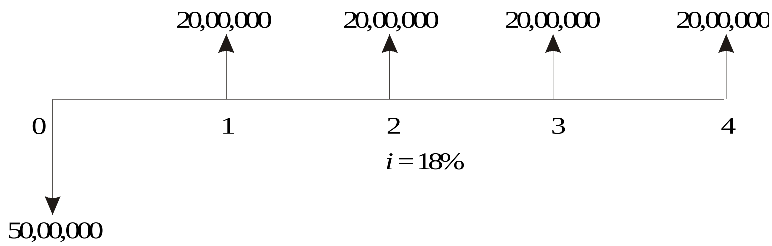


Fig. 5.3 Cash flow diagram for alternative A.

The future worth amount of alternative B is computed as

$$FW_A(18\%) = -50{,}00{,}000(F/P, 18\%, 4) + 20{,}00{,}000(F/A, 18\%, 4)$$
$$= -50{,}00{,}000(1.939) + 20{,}00{,}000(5.215)$$
$$= \text{Rs. } 7{,}35{,}000$$

Alternative B
Initial investment, P = Rs. 45,00,000
Annual equivalent revenue, A = Rs. 18,00,000
Interest rate, i = 18%, compounded annually
Life of alternative B = 4 years

The cash flow diagram of alternative B is illustrated in Fig. 5.4.

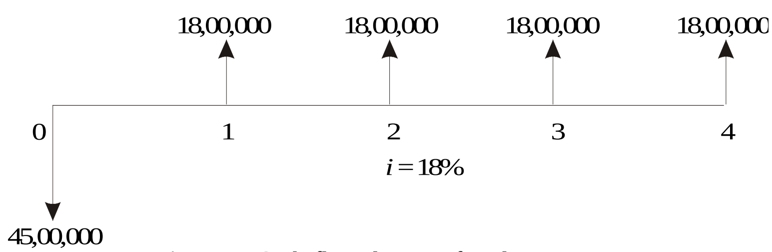


Fig. 5.4 Cash flow diagram for alternative B.

The future worth amount of alternative B is computed as

$$FW_B(18\%) = -45{,}00{,}000(F/P, 18\%, 4) + 18{,}00{,}000\ (F/A, 18\%, 4)$$
$$= -45{,}00{,}000(1.939) + 18{,}00{,}000(5.215)$$
$$= \text{Rs. } 6{,}61{,}500$$

The future worth of alternative A is greater than that of alternative B. Thus, alternative A should be selected.

EXAMPLE 5.2 A man owns a corner plot. He must decide which of the several alternatives to select in trying to obtain a desirable return on his investment. After much study and calculation, he decides that the two best alternatives are as given in the following table:

	Build gas station	*Build soft ice-cream stand*
First cost (Rs.)	20,00,000	36,00,000
Annual property taxes (Rs.)	80,000	1,50,000
Annual income (Rs.)	8,00,000	9,80,000
Life of building (years)	20	20
Salvage value (Rs.)	0	0

Evaluate the alternatives based on the future worth method at i = 12%.

***Alternative 1*—Build gas station**

First cost = Rs. 20,00,000

Net annual income = Annual income – Annual property tax

= Rs. 8,00,000 – Rs. 80,000

= Rs. 7,20,000

Life = 20 years

Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is depicted in Fig. 5.5.

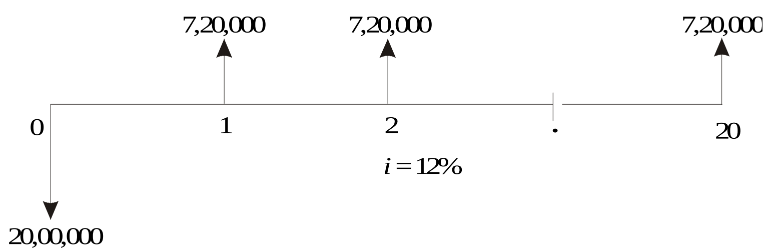


Fig. 5.5 Cash flow diagram for alternative 1.

The future worth of alternative 1 is computed as

$$FW_1(12\%) = -20,00,000\,(F/P,\ 12\%,\ 20) + 7,20,000\,(F/A,\ 12\%,\ 20)$$

$$= -20,00,000\,(9.646) + 7,20,000\,(72.052)$$

$$= \text{Rs. } 3,25,85,440$$

***Alternative 2*—Build soft ice-cream stand**

First cost = Rs. 36,00,000

Net annual income = Annual income – Annual property tax

= Rs. 9,80,000 – Rs. 1,50,000

= Rs. 8,30,000

Life = 20 years

Interest rate = 12%, compounded annually

The cash flow diagram for this alternative is shown in Fig. 5.6.

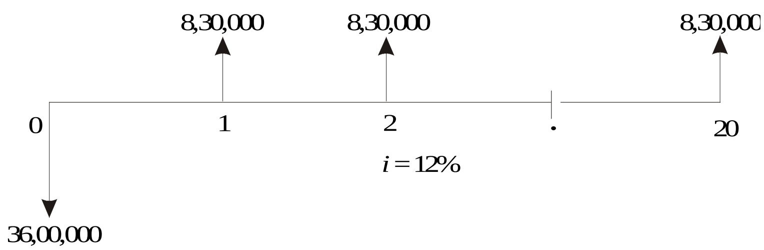


Fig. 5.6 Cash flow diagram for alternative 2.

The future worth of alternative 2 is calculated as

$$FW_2(12\%) = -36{,}00{,}000\,(F/P,\ 12\%,\ 20) + 8{,}30{,}000\,(F/A,\ 12\%,\ 20)$$
$$= -36{,}00{,}000\,(9.646) + 8{,}30{,}000\,(72.052)$$
$$= \text{Rs. } 2{,}50{,}77{,}560$$

The future worth of alternative 1 is greater than that of alternative 2. Thus, building the gas station is the best alternative.

EXAMPLE 5.3 The cash flow diagram of two mutually exclusive alternatives are given in Figs. 5.7 and 5.8.

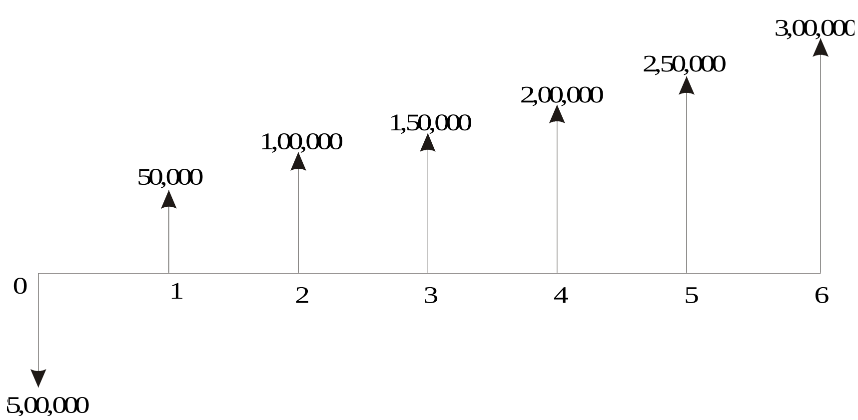


Fig. 5.7 Cash flow diagram for alternative 1.

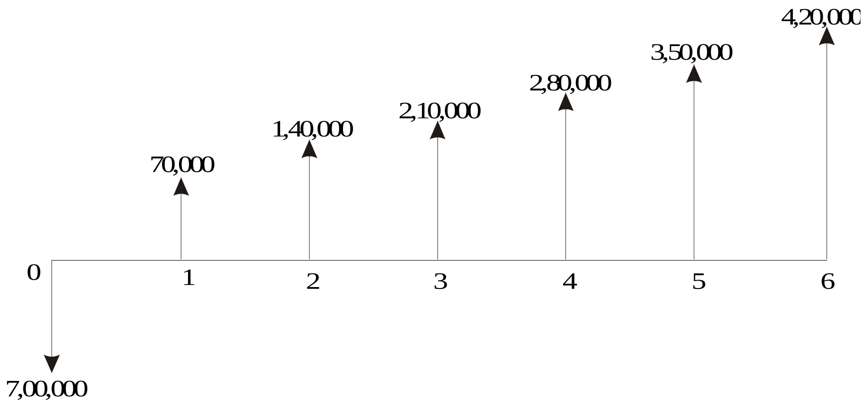


Fig. 5.8 Cash flow diagram for alternative 2.

(a) Select the best alternative based on future worth method at i = 8%.
(b) Rework part (a) with i = 9% and 20%

(*a*) *Evaluation at* *i = 8%*

***Alternative 1*—This comes under equal payment gradient series.**

P = Rs. 5,00,000

$A1$ = Rs. 50,000

G = Rs. 50,000

i = 8%

n = 6 years

The formula for the future worth of alternative 1 is

$$\begin{aligned} FW_1(8\%) &= -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] \times (F/A, 8\%, 6) \\ &= -5{,}00{,}000(1.587) + [50{,}000 + 50{,}000(2.2764)] \times 7.336 \\ &= -79{,}35{,}000 + 1{,}63{,}820 \times 7.336 \\ &= -79{,}35{,}000 + 12{,}01{,}784 \\ &= \text{Rs. } 4{,}08{,}283.52 \end{aligned}$$

***Alternative 2*—This comes under equal payment gradient series.**

P = Rs. 7,00,000

$A1$ = Rs. 70,000

G = Rs. 70,000

i = 8%

n = 6 years

The formula for the future worth of alternative 2 is

$$FW_2(8\%) = -P(F/P,\ 8\%,\ 6) + [A1 + G(A/G,\ 8\%,\ 6)] \times (F/A,\ 8\%,\ 6)$$
$$FW_2(8\%) = -7{,}00{,}000 \times 1.587 + [70{,}000 + 70{,}000 \times 2.2764] \times 7.336$$
$$= -11{,}10{,}900 + 16{,}82{,}497$$
$$= \text{Rs. } 5{,}71{,}596.93$$

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(*b*) *(i) Evaluation at* $i = 9\%$: ***Alternative 1***

P = Rs. 5,00,000

$A1$ = Rs. 50,000

G = Rs. 50,000

n = 6 years

The formula for the future worth of alternative 1 is as follows:

$$FW_1(9\%) = -P(F/P,\ 9\%,\ 6) + [A1 + G(A/G,\ 9\%,\ 6)] \times (F/A,\ 9\%,\ 6)$$
$$= -5{,}00{,}000\ (1.677) + [50{,}000 + 50{,}000\,(2.2498)] \times 7.523$$
$$= -8{,}38{,}500 + 12{,}22{,}412.27$$
$$= \text{Rs. } 3{,}83{,}912.27$$

Alternative 2

P = Rs. 7,00,000

$A1$ = Rs. 70,000

G = Rs. 70,000

n = 6 years

The formula for the future worth of the alternative 2 is

$$FW_2(9\%) = -P(F/P,\ 9\%,\ 6) + [A1 + G(A/G,\ 9\%,\ 6)] \times (F/A,\ 9\%,\ 6)$$
$$= -7{,}00{,}000 \times 1.677 + [70{,}000 + 70{,}000 \times 2.2498] \times 7.523$$
$$= -11{,}73{,}900 + 17{,}11{,}377.18$$
$$= \text{Rs. } 5{,}37{,}477.18$$

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(ii) Evaluation at $i = 20\%$: ***Alternative 1***

P = Rs. 5,00,000

$A1$ = Rs. 50,000

G = Rs. 50,000

n = 6 years

The formula for the future worth of alternative 1 is

$$\begin{aligned} FW_1(20\%) &= -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \times (F/A, 20\%, 6) \\ &= -5{,}00{,}000(2.986) + [50{,}000 + 50{,}000(1.9788)] \times 9.93 \\ &= -14{,}93{,}000 + 14{,}78{,}974.20 \\ &= \text{Rs. } -14{,}025.80 \end{aligned}$$

The negative sign of the future worth amount indicates that alternative 1 incurs loss.

Alternative 2

P = Rs. 7,00,000

$A1$ = Rs. 70,000

G = Rs. 70,000

n = 6 years

The formula for the future worth of alternative 2 is

$$\begin{aligned} FW_2(20\%) &= -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] \times (F/A, 20\%, 6) \\ &= -7{,}00{,}000 \times 2.986 + [70{,}000 + 70{,}000 \times 1.9788] \times 9.93 \\ &= -20{,}90{,}200 + 20{,}70{,}563.88 \\ &= \text{Rs. } -19{,}636.12 \end{aligned}$$

The negative sign of the above future worth amount indicates that alternative 2 incurs loss. Thus, none of the two alternatives should be selected.

EXAMPLE 5.4 M/S Krishna Castings Ltd. is planning to replace its annealing furnace. It has received tenders from three different original manufacturers of annealing furnace. The details are as follows.

	Manufacturer		
	1	2	3
Initial cost (Rs.)	80,00,000	70,00,000	90,00,000
Life (years)	12	12	12
Annual operation and maintenance cost (Rs.)	8,00,000	9,00,000	8,50,000
Salvage value after 12 years	5,00,000	4,00,000	7,00,000

Which is the best alternative based on future worth method at i = 20%?

Solution ***Alternative 1—Manufacturer 1***

First cost, P = Rs. 80,00,000

Life, n = 12 years

Annual operating and maintenance cost, A = Rs. 8,00,000
Salvage value at the end of furnace life = Rs. 5,00,000

The cash flow diagram for this alternative is shown in Fig. 5.9.

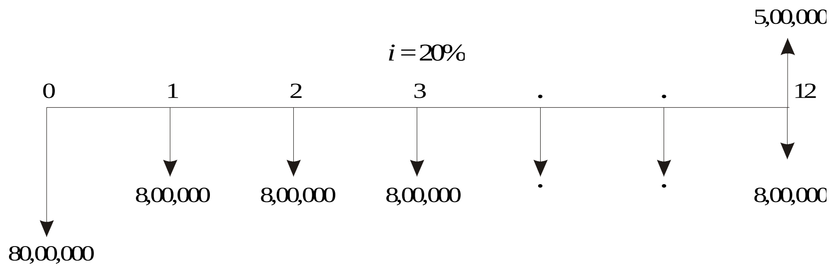


Fig. 5.9 Cash flow diagram for manufacturer 1.

The future worth amount of alternative 1 is computed as

$$\begin{aligned} FW_1(20\%) &= 80{,}00{,}000(F/P, 20\%, 12) + 8{,}00{,}000(F/A, 20\%, 12) - 5{,}00{,}000 \\ &= 80{,}00{,}000(8.916) + 8{,}00{,}000(39.581) - 5{,}00{,}000 \\ &= \text{Rs. } 10{,}24{,}92{,}800 \end{aligned}$$

Alternative 2— Manufacturer 2

First cost, P = Rs. 70,00,000
Life, n = 12 years
Annual operating and maintenance cost, A = Rs. 9,00,000
Salvage value at the end of furnace life = Rs. 4,00,000

The cash flow diagram for this alternative is given in Fig. 5.10.

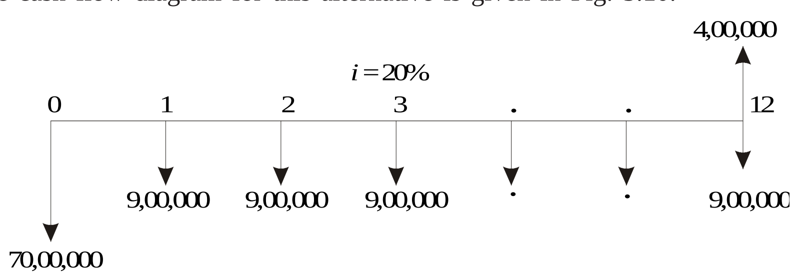


Fig. 5.10 Cash flow diagram for manufacturer 2.

The future worth amount of alternative 2 is computed as

$$\begin{aligned} FW_2(20\%) &= 70{,}00{,}000(F/P, 20\%, 12) + 9{,}00{,}000(F/A, 20\%, 12) - 4{,}00{,}000 \\ &= 70{,}00{,}000(8.916) + 9{,}00{,}000\ (39.581) - 4{,}00{,}000 \\ &= \text{Rs. } 9{,}76{,}34{,}900 \end{aligned}$$

Alternative 3—Manufacturer 3

First cost, P = Rs. 90,00,000
Life, n = 12 years

Annual operating and maintenance cost, A = Rs. 8,50,000
Salvage value at the end of furnace life = Rs. 7,00,000

The cash flow diagram for this alternative is illustrated in Fig. 5.11.

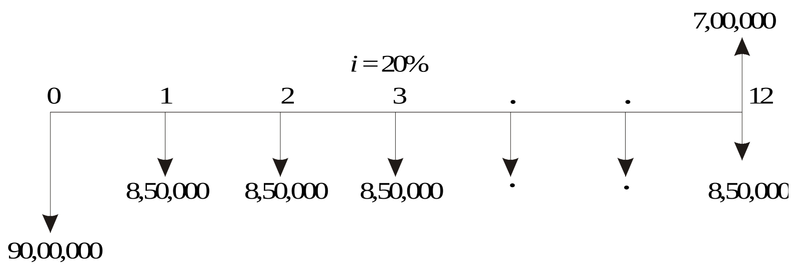


Fig. 5.11 Cash flow diagram for manufacturer 3.

The future worth amount of alternative 3 is calculated as

$$\begin{aligned} FW_3(20\%) &= 90{,}00{,}000(F/P,\ 20\%,\ 12) + 8{,}50{,}000(F/A,\ 20\%,\ 12) - 7{,}00{,}000 \\ &= 90{,}00{,}000(8.916) + 8{,}50{,}000(39.581) - 7{,}00{,}000 \\ &= \text{Rs. } 11{,}31{,}87{,}850 \end{aligned}$$

The future worth cost of alternative 2 is less than that of the other two alternatives. Therefore, M/s. Krishna castings should buy the annealing furnace from manufacturer 2.

EXAMPLE 5.5 A company must decide whether to buy machine A or machine B:

	Machine A	*Machine B*
Initial cost	Rs. 4,00,000	Rs. 8,00,000
Useful life, in years	4	4
Salvage value at the end of machine life	Rs. 2,00,000	Rs. 5,50,000
Annual maintenance cost	Rs. 40,000	0

At 12% interest rate, which machine should be selected? (Use future worth method of comparison).

Solution Machine A

Initial cost of the machine, P = Rs. 4,00,000

Life, n = 4 years

Salvage value at the end of machine life, S = Rs. 2,00,000

Annual maintenance cost, A = Rs. 40,000

Interest rate, i = 12%, compounded annually.

The cash flow diagram of machine A is given in Fig. 5.12.

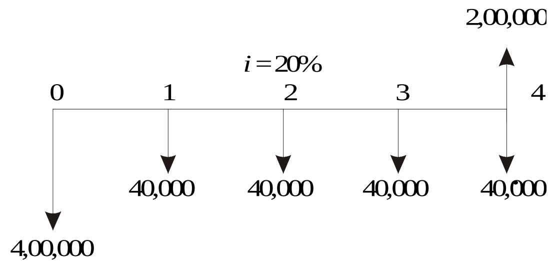


Fig. 5.12 Cash flow diagram for machine *A*.

The future worth function of Fig. 5.12 is

$$FW_A(12\%) = 4{,}00{,}000 \times (F/P,\ 12\%,\ 4) + 40{,}000 \times (F/A,\ 12\%,\ 4) - 2{,}00{,}000$$
$$= 4{,}00{,}000 \times (1.574) + 40{,}000 \times (4.779) - 2{,}00{,}000$$
$$= \text{Rs. } 6{,}20{,}760$$

Machine B

Initial cost of the machine, P = Rs. 8,00,000

Life, n = 4 years

Salvage value at the end of machine life, S = Rs. 5,50,000

Annual maintenance cost, A = zero.

Interest rate, i = 12%, compounded annually.

The cash flow diagram of the machine *B* is illustrated in Fig. 5.13.

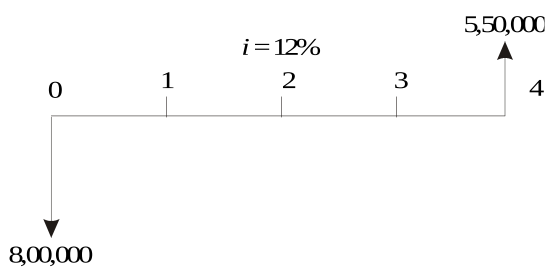


Fig. 5.13 Cash flow diagram for machine *B*.

The future worth function of Fig 5.13 is

$$FW_B(12\%) = 8{,}00{,}000 \times (F/P,\ 12\%,\ 4) - 5{,}50{,}000$$
$$= 8{,}00{,}000 \times (1.574) - 5{,}50{,}000$$
$$= \text{Rs. } 7{,}09{,}200$$

The future worth cost of machine *A* is less than that of machine *B*. Therefore, machine *A* should be selected.

QUESTIONS

1. A suburban taxi company is considering buying taxis with diesel engines instead of petrol engines. The cars average 50,000 km a year, with a useful life of three years for the taxi with the petrol engine and four years for the diesel taxi. Other comparative information are as follows:

	Diesel	*Petrol*
Vehicle cost	Rs. 5,00,000	Rs. 4,00,000
Fuel cost per litre	Rs. 9.00	Rs. 24.00
Mileage, in km/litre	30	20
Annual insurance premium	Rs. 500	Rs. 500
Salvage value at the end of vehicle life	Rs. 70,000	Rs. 1,00,000

Determine the more economical choice based on the future worth method of comparison if the interest rate is 15%, compounded annually.

2. A motorcycle is sold for Rs. 50,000. The motorcycle dealer is willing to sell it on the following terms:

(a) Make no down payment but pay Rs. 1,500 at the end of each of the first four months and Rs. 3,000 at the end of each month after that for 18 continuous months.

(b) Make no down payment but pay a total amount of Rs. 90,000 at the end of the 22nd month; till that time the buyer should mortgage property worth of Rs. 50,000, at present.

Based on these terms and a 12% annual interest rate compounded monthly, find the best alternative for the buyer based on the future worth method of comparison.

3. Consider the following two mutually exclusive alternatives.

	A	*B*
Cost	Rs. 4,000	Rs. 6,000
Uniform annual benefit	Rs. 640	Rs. 960
Useful life (years)	20	20

Using a 15% interest rate, determine which alternative should be selected based on the future worth method of comparison.

4. A company must decide whether to buy machine *A* or machine *B*:

	Machine A	*Machine B*
Initial cost	Rs. 4,00,000	Rs. 8,00,000
Useful life, (years)	5	5
Salvage value at the end of machine life	Rs. 2,00,000	Rs. 5,50,000
Annual maintenance cost	Rs. 40,000	0

At 15% interest rate, which machine should be selected? (Use the future worth method of comparison.)

5. Due to increasing awareness of customers, two different television manufacturing companies started a marketing war. The details of advertisements of the companies are as follows:

	Brand X	*Brand Y*
Selling price of a TV set	Rs. 15,000	Rs. 10,000
Amount returned to buyer after 5 years	Rs. 8,000	–

Select the most economical brand from the customer's point of view using the future worth method of comparison, assuming an interest rate of 15%, compounded annually.

6. Alpha Finance Company is coming with an option of accepting Rs. 10,000 now and paying a sum of Rs. 1,60,000 after 20 years. Beta Finance Company is coming with a similar option of accepting Rs. 10,000 now and paying a sum of Rs. 3,00,000 after 25 years. Compare and select the best alternative based on the future worth method of comparison with 15% interest rate, compounded annually.

7. An insurance company gives an endowment policy for a person aged 30 years. The yearly premium for an insured sum of Rs. 1,00,000 is Rs. 4,000. The policy will mature after 25 years. Also, the person is entitled for a bonus of Rs. 75 per thousand per year at the end of the policy. If a person survives till the end of the 25th year:

(a) What will be the total sum that he will get from the insurance company at that time?

(b) Instead of paying the premiums for the insurance policy, if the person invests an equal sum of Rs. 4,000 at the end of each year for the next 25 years in some other scheme which is having similar tax benefit, find the future worth of the investment at 15% interest rate, compounded annually.

(c) Rate the above alternatives assuming that the person is sure of living for the next 25 years.

6

ANNUAL EQUIVALENT METHOD

6.1 INTRODUCTION

In the annual equivalent method of comparison, first the annual equivalent cost or the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent revenue in the case of revenue-based comparison or with the minimum annual equivalent cost in the case of cost-based comparison will be selected as the best alternative.

6.2 REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is presented in Fig. 6.1.

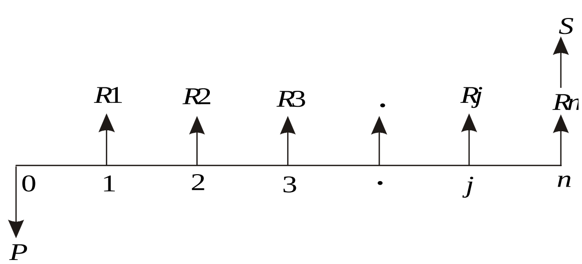


Fig. 6.1 Revenue-dominated cash flow diagram.

In Fig. 6.1, P represents an initial investment, Rj the net revenue at the end of the jth year, and S the salvage value at the end of the nth year.

The first step is to find the net present worth of the cash flow diagram using the following expression for a given interest rate, i:

$$PW(i) = -P + R1/(1+i)^1 + R2/(1+i)^2 + \ldots$$
$$+ Rj/(1+i)^j + \ldots + Rn/(1+i)^n + S/(1+i)^n$$

In the above formula, the expenditure is assigned with a negative sign and the revenues are assigned with a positive sign.

In the second step, the annual equivalent revenue is computed using the following formula:

$$A = PW(i)\,\frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= PW(i)\ (A/P,\ i,\ n)$$

where $(A/P, i, n)$ is called *equal payment series capital recovery factor*.

If we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent revenues are to be computed and compared. Finally, the alternative with the maximum annual equivalent revenue should be selected as the best alternative.

6.3 COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the annual equivalent method of comparison is illustrated in Fig. 6.2.

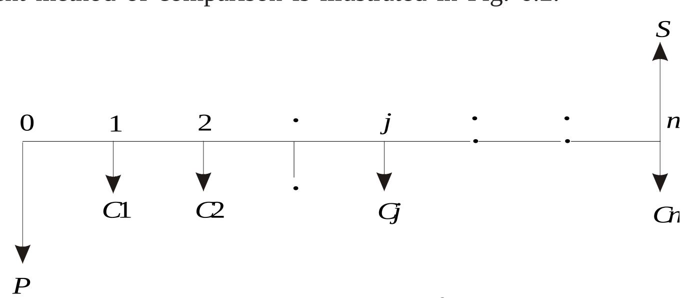


Fig. 6.2 Cost-dominated cash flow diagram.

In Fig. 6.2, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year.

The first step is to find the net present worth of the cash flow diagram using the following relation for a given interest rate, i.

$$PW(i) = P + C1/(1+i)^1 + C2/(1+i)^2 + \ldots$$
$$+ Cj/(1+i)^j + \ldots + Cn/(1+i)^n - S/(1+i)^n$$

In the above formula, each expenditure is assigned with positive sign and the salvage value with negative sign. Then, in the second step, the annual equivalent cost is computed using the following equation:

$$A = PW(i)\,\frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= PW(i)\ (A/P,\ i,\ n)$$

where $(A/P, i, n)$ is called as equal-payment series capital recovery factor.

As in the previous case, if we have some more alternatives which are to be compared with this alternative, then the corresponding annual equivalent costs are to be computed and compared. Finally, the alternative with the minimum annual equivalent cost should be selected as the best alternative.

If we have some non-standard cash flow diagram, then we will have to follow the general procedure for converting each and every transaction to time zero and then convert the net present worth into an annual equivalent cost/revenue depending on the type of the cash flow diagram. Such procedure is to be applied to all the alternatives and finally, the best alternative is to be selected.

6.4 ALTERNATE APPROACH

Instead of first finding the present worth and then figuring out the annual equivalent cost/revenue, an alternate method which is as explained below can be used. In each of the cases presented in Sections 6.2 and 6.3, in the first step, one can find the future worth of the cash flow diagram of each of the alternatives. Then, in the second step, the annual equivalent cost/revenue can be obtained by using the equation:

$$A = F\frac{i}{(1+i)^n - 1}$$

$$= F(A/F, i, n)$$

where $(A/F, i, n)$ is called *equal-payment series sinking fund factor.*

6.5 EXAMPLES

In this section, the application of the annual equivalent method is demonstrated with several numerical examples.

EXAMPLE 6.1 A company provides a car to its chief executive. The owner of the company is concerned about the increasing cost of petrol. The cost per litre of petrol for the first year of operation is Rs. 21. He feels that the cost of petrol will be increasing by Re.1 every year. His experience with his company car indicates that it averages 9 km per litre of petrol. The executive expects to drive an average of 20,000 km each year for the next four years. What is the annual equivalent cost of fuel over this period of time?. If he is offered similar service with the same quality on rental basis at Rs. 60,000 per year, should the owner continue to provide company car for his executive or alternatively provide a rental car to his executive? Assume i = 18%. If the rental car is preferred, then the company car will find some other use within the company.

Solution

Average number of km run/year = 20,000 km
Number of km/litre of petrol = 9 km

Therefore,

Petrol consumption/year = 20,000/9 = 2222.2 litre
Cost/litre of petrol for the 1st year = Rs. 21
Cost/litre of petrol for the 2nd year = Rs. 21.00 + Re. 1.00
= Rs. 22.00
Cost/litre of petrol for the 3rd year = Rs. 22.00 + Re. 1.00
= Rs. 23.00
Cost/litre of petrol for the 4th year = Rs. 23.00 + Re. 1.00
= Rs. 24.00
Fuel expenditure for 1st year = 2222.2 × 21 = Rs. 46,666.20
Fuel expenditure for 2nd year = 2222.2 × 22 = Rs. 48,888.40
Fuel expenditure for 3rd year = 2222.2 × 23 = Rs. 51,110.60
Fuel expenditure for 4th year = 2222.2 × 24 = Rs. 53,332.80

The annual equal increment of the above expenditures is Rs. 2,222.20 (G).

The cash flow diagram for this situation is depicted in Fig. 6.3.

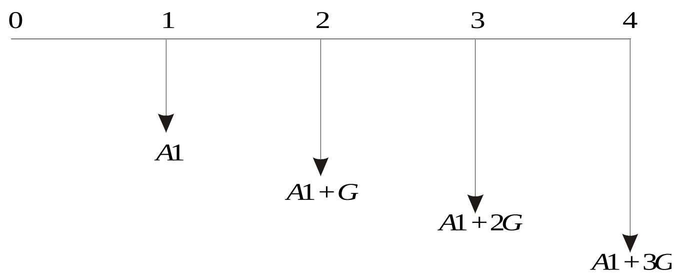


Fig. 6.3 Uniform gradient series cash flow diagram.

In Fig. 6.3, $A1$ = Rs. 46,666.20 and G = Rs. 2,222.20

$$\begin{aligned} A &= A1 + G(A/G,\ 18\%,\ 4) \\ &= 46{,}666.20 + 2222.2(1.2947) \\ &= \text{Rs. } 49{,}543.28 \end{aligned}$$

The proposal of using the company car by spending for petrol by the company will cost an annual equivalent amount of Rs. 49,543.28 for four years. This amount is less than the annual rental value of Rs. 60,000. Therefore, the company should continue to provide its own car to its executive.

EXAMPLE 6.2 A company is planning to purchase an advanced machine centre. Three original manufacturers have responded to its tender whose particulars are tabulated as follows:

Manufacturer	*Down payment* (Rs.)	*Yearly equal installment* (Rs.)	*No. of installments*
1	5,00,000	2,00,000	15
2	4,00,000	3,00,000	15
3	6,00,000	1,50,000	15

Determine the best alternative based on the annual equivalent method by assuming i = 20%, compounded annually.

Solution ***Alternative 1***

Down payment, P = Rs. 5,00,000
Yearly equal installment, A = Rs. 2,00,000
n = 15 years
i = 20%, compounded annually
The cash flow diagram for manufacturer 1 is shown in Fig. 6.4.

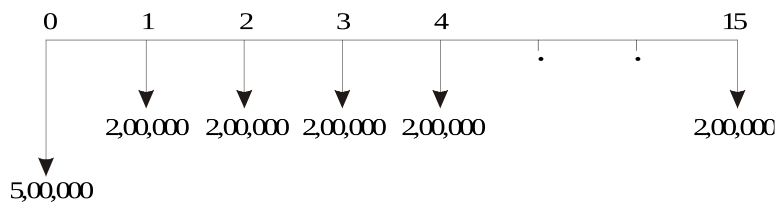


Fig. 6.4 Cash flow diagram for manufacturer 1.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_1(20\%) &= 5{,}00{,}000(A/P,\ 20\%,\ 15) + 2{,}00{,}000 \\ &= 5{,}00{,}000(0.2139) + 2{,}00{,}000 \\ &= 3{,}06{,}950 \end{aligned}$$

Alternative 2

Down payment, P = Rs. 4,00,000
Yearly equal installment, A = Rs. 3,00,000
n = 15 years
i = 20%, compounded annually

The cash flow diagram for the manufacturer 2 is shown in Fig. 6.5.

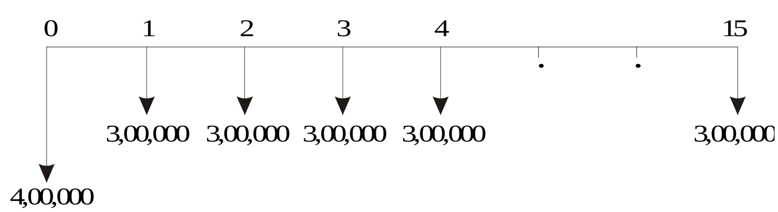


Fig. 6.5 Cash flow diagram for manufacturer 2.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_2(20\%) &= 4{,}00{,}000(A/P,\ 20\%,\ 15) + 3{,}00{,}000 \\ &= 4{,}00{,}000(0.2139) + 3{,}00{,}000 \\ &= \text{Rs. } 3{,}85{,}560. \end{aligned}$$

Alternative 3

Down payment, P = Rs. 6,00,000
Yearly equal installment, A = Rs. 1,50,000
n = 15 years
i = 20%, compounded annually

The cash flow diagram for manufacturer 3 is shown in Fig. 6.6.

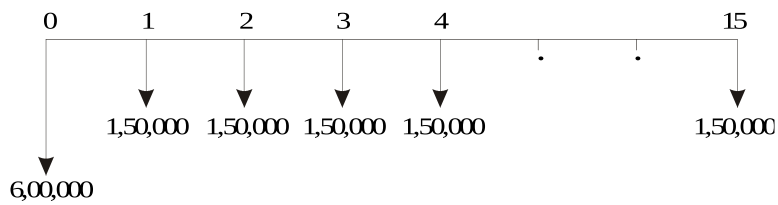


Fig. 6.6 Cash flow diagram for manufacturer 3.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_3(20\%) &= 6{,}00{,}000(A/P,\ 20\%,\ 15) + 1{,}50{,}000 \\ &= 6{,}00{,}000(0.2139) + 1{,}50{,}000 \\ &= \text{Rs. } 2{,}78{,}340. \end{aligned}$$

The annual equivalent cost of manufacturer 3 is less than that of manufacturer 1 and manufacturer 2. Therefore, the company should buy the advanced machine centre from manufacturer 3.

EXAMPLE 6.3 A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values.

	Alternative	
	A	B
Investment (Rs.)	– 1,50,000	– 1,75,000
Annual equal return (Rs.)	+ 60,000	+ 70,000
Salvage value (Rs.)	+ 15,000	+ 35,000

Determine the best alternative based on the annual equivalent method by assuming i = 25%.

Solution Alternative A

Initial investment, P = Rs. 1,50,000
Annual equal return, A = Rs. 60,000
Salvage value at the end of machine life, S = Rs. 15,000
Life = 5 years
Interest rate, i = 25%, compounded annually

The cash flow diagram for alternative A is shown in Fig. 6.7.

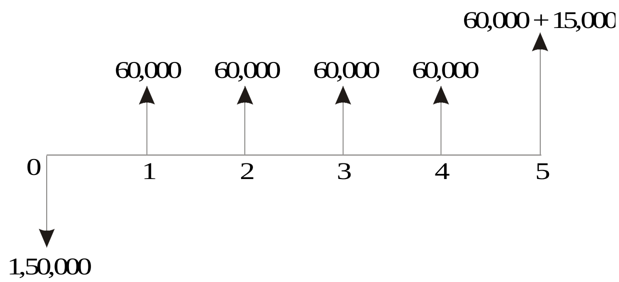


Fig. 6.7 Cash flow diagram for alternative A.

The annual equivalent revenue expression of the above cash flow diagram is as follows:

$$AE_A(25\%) = -1,50,000(A/P, 25\%, 5) + 60,000 + 15,000(A/F, 25\%, 5)$$
$$= -1,50,000(0.3718) + 60,000 + 15,000(0.1218)$$
$$= \text{Rs. } 6,057$$

Alternative B

Initial investment, P = Rs. 1,75,000
Annual equal return, A = Rs. 70,000
Salvage value at the end of machine life, S = Rs. 35,000
Life = 5 years
Interest rate, i = 25%, compounded annually

The cash flow diagram for alternative B is shown in Fig. 6.8.

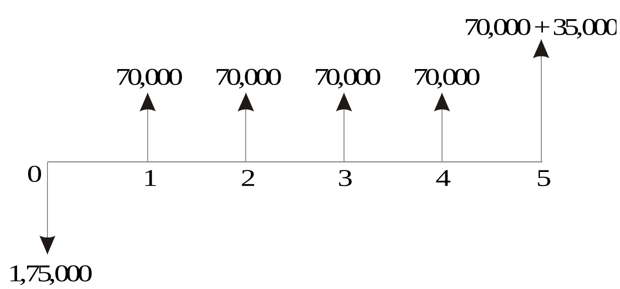


Fig. 6.8 Cash flow diagram for alternative B.

The annual equivalent revenue expression of the above cash flow diagram is

$$AE_B(25\%) = -1,75,000(A/P, 25\%, 5) + 70,000 + 35,000(A/F, 25\%, 5)$$
$$= -1,75,000(0.3718) + 70,000 + 35,000(0.1218)$$
$$= \text{Rs. } 9,198$$

The annual equivalent net return of alternative B is more than that of alternative A. Thus, the company should select alternative B.

EXAMPLE 6.4 A certain individual firm desires an economic analysis to determine which of the two machines is attractive in a given interval of time. The minimum attractive rate of return for the firm is 15%. The following data are to be used in the analysis:

	Machine X	*Machine Y*
First cost	Rs. 1,50,000	Rs. 2,40,000
Estimated life	12 years	12 years
Salvage value	Rs. 0	Rs. 6,000
Annual maintenance cost	Rs. 0	Rs. 4,500

Which machine would you choose? Base your answer on annual equivalent cost.

Solution Machine X

First cost, P = Rs. 1,50,000
Life, n = 12 years
Estimated salvage value at the end of machine life, S = Rs. 0.
Annual maintenance cost, A = Rs. 0.
Interest rate, i = 15%, compounded annually.

The cash flow diagram of machine X is illustrated in Fig. 6.9.

Fig. 6.9 Cash flow diagram for machine X.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_X(15\%) &= 1{,}50{,}000(A/P, 15\%, 12) \\ &= 1{,}50{,}000(0.1845) \\ &= \text{Rs. } 27{,}675 \end{aligned}$$

Machine Y

First cost, P = Rs. 2,40,000

Life, n = 12 years

Estimated salvage value at the end of machine life, S = Rs. 60,000

Annual maintenance cost, A = Rs. 4,500

Interest rate, i = 15%, compounded annually.

The cash flow diagram of machine Y is depicted in Fig. 6.10.

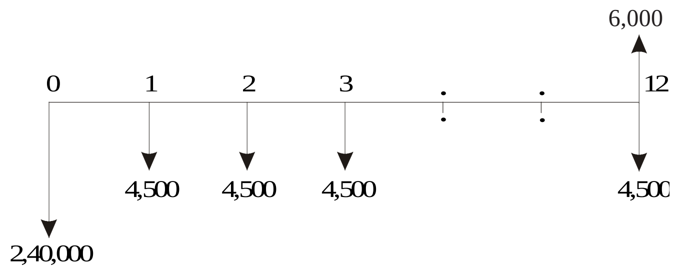


Fig. 6.10 Cash flow diagram for machine Y.

The annual equivalent cost expression of the above cash flow diagram is

$$AE_Y(15\%) = 2{,}40{,}000(A/P,\ 15\%,\ 12) + 4{,}500 - 6{,}000(A/F,\ 15\%,\ 12)$$
$$= 2{,}40{,}000(0.1845) + 4{,}500 - 6{,}000(0.0345)$$
$$= \text{Rs. } 48{,}573$$

The annual equivalent cost of machine X is less than that of machine Y. So, machine X is the more cost effective machine.

EXAMPLE 6.5 Two possible routes for laying a power line are under study. Data on the routes are as follows:

		Around the lake	*Under the lake*
Length		15 km	5 km
First cost	(Rs.)	1,50,000/km	7,50,000/km
Useful life	(years)	15	15
Maintenance cost	(Rs.)	6,000/km/yr	12,000/km/yr
Salvage value	(Rs.)	90,000/km	1,50,000/km
Yearly power loss	(Rs.)	15,000/km	15,000/km

If 15% interest is used, should the power line be routed around the lake or under the lake?

Solution Alternative 1—Around the lake

First cost = 1,50,000 × 15 = Rs. 22,50,000

Maintenance cost/yr = 6,000 × 15 = Rs. 90,000

Power loss/yr = 15,000 × 15 = Rs. 2,25,000

Maintenance cost and power loss/yr = Rs. 90,000 + Rs. 2,25,000
= Rs. 3,15,000

Salvage value = 90,000 × 15 = Rs. 13,50,000

The cash flow diagram for this alternative is shown in Fig. 6.11.

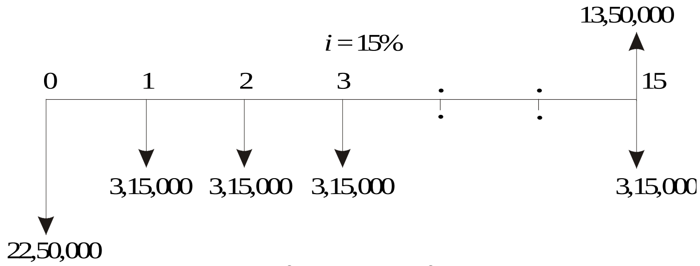


Fig. 6.11 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_1(15\%) &= 22{,}50{,}000(A/P,\ 15\%,\ 15) + 3{,}15{,}000 - 13{,}50{,}000(A/F,\ 15\%,\ 15) \\ &= 22{,}50{,}000(0.1710) + 3{,}15{,}000 - 13{,}50{,}000(0.0210) \\ &= \text{Rs. } 6{,}71{,}400 \end{aligned}$$

Alternative 2—Under the lake

First cost = 7,50,000 × 5 = Rs. 37,50,000
Maintenance cost/yr = 12,000 × 5 = Rs. 60,000
Power loss/yr = 15,000 × 5 = Rs. 75,000
Maintenance cost and power loss/yr = Rs. 60,000 + Rs. 75,000
= Rs. 1,35,000
Salvage value = 1,50,000 × 5 = Rs. 7,50,000

The cash flow diagram for this alternative is shown in Fig. 6.12.

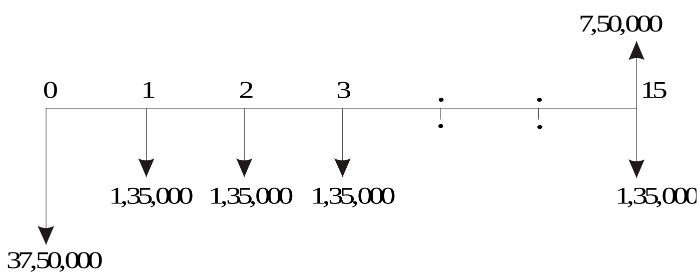


Fig. 6.12 Cash flow diagram for alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_2(15\%) &= 37{,}50{,}000(A/P,\ 15\%,\ 15) + 1{,}35{,}000 - 7{,}50{,}000(A/F,\ 15\%,\ 15) \\ &= 37{,}50{,}000(0.1710) + 1{,}35{,}000 - 7{,}50{,}000(0.0210) \\ &= \text{Rs. } 7{,}60{,}500 \end{aligned}$$

The annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, select the route around the lake for laying the power line.

EXAMPLE 6.6 A suburban taxi company is analyzing the proposal of buying cars with diesel engines instead of petrol engines. The cars average 60,000 km a year with a useful life of three years for the petrol taxi and four years for the diesel taxi. Other comparative details are as follows:

	Diesel	*Petrol*
Vehicle cost (Rs.)	3,90,000	3,60,000
Fuel cost per litre (Rs.)	8	20
Mileage in km/litre	30	20
Annual repairs (Rs.)	9,000	6,000
Annual insurance premium (Rs.)	15,000	15,000
Resale value at the end of vehicle life (Rs.)	60,000	90,000

Determine the more economical choice if interest rate is 20%, compounded annually.

Solution Alternative 1— Purchase of diesel taxi

Vehicle cost = Rs. 3,90,000
Life = 4 years
Number of litres/year 60,000/30 = 2,000 litres
Fuel cost/yr = 2,000 × 8 = Rs. 16,000
Fuel cost, annual repairs and insurance premium/yr
= Rs. 16,000 + Rs. 9,000 + Rs. 15,000 = Rs. 40,000
Salvage value at the end of vehicle life = Rs. 60,000

The cash flow diagram for alternative 1 is shown in Fig. 6.13.

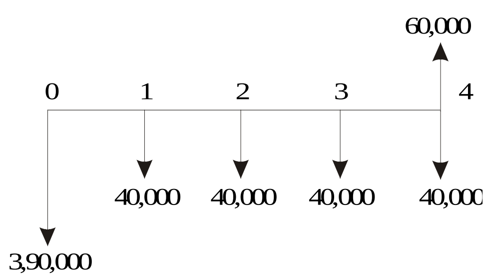


Fig. 6.13 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(20\%) = 3{,}90{,}000(A/P, 20\%, 4) + 40{,}000 - 60{,}000(A/F, 20\%, 4)$$
$$= 3{,}90{,}000(0.3863) + 40{,}000 - 60{,}000(0.1863)$$
$$= \text{Rs. } 1{,}79{,}479$$

Alternative 2—Purchase of petrol taxi

Vehicle cost = Rs. 3,60,000
Life = 3 years
Number of litres/year 60,000/20 = 3,000 litres
Fuel cost/yr = 3,000 × 20 = Rs. 60,000

Fuel cost, annual repairs and insurance premium/yr
= Rs. 60,000 + Rs. 6,000 + Rs. 15,000 = Rs. 81,000
Salvage value at the end of vehicle life = Rs. 90,000

The cash flow diagram for alternative 2 is shown in Fig. 6.14.

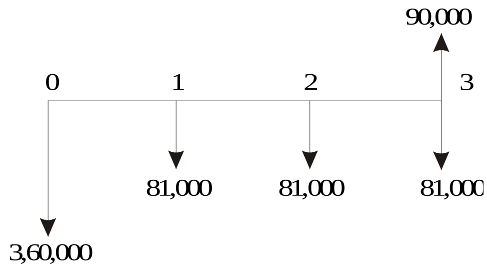


Fig. 6.14 Cash flow diagram for alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(20\%) &= 3{,}60{,}000(A/P,\ 20\%,\ 3) + 81{,}000 - 90{,}000(A/F,\ 20\%,\ 3) \\ &= 3{,}60{,}000(0.4747) + 81{,}000 - 90{,}000(0.2747) \\ &= \text{Rs. } 2{,}27{,}169 \end{aligned}$$

The annual equivalent cost of purchase and operation of the cars with diesel engine is less than that of the cars with petrol engine. Therefore, the taxi company should buy cars with diesel engine. (*Note:* Comparison is done on common multiple lives of 12 years.)

EXAMPLE 6.7 Ramu, a salesman, needs a new car for use in his business. He expects that he will be promoted to a supervisory job at the end of third year and so his concern now is to have a car for the three years he expects to be "on the road". The company will reimburse their salesman each month the fuel cost and maintenance cost. Ramu has decided to drive a low-priced automobile. He finds, however, that there are two different ways of obtaining the automobile. In either case, the fuel cost and maintenance cost are borne by the company.

(a) Purchase for cash at Rs. 3,90,000.

(b) Lease a car. The monthly charge is Rs. 10,500 on a 36-month lease payable at the end of each month. At the end of the three-year period, the car is returned to the leasing company.

Ramu believes that he should use a 12% interest rate compounded monthly in determining which alternative to select. If the car could be sold for Rs. 1,20,000 at the end of the third year, which option should he use to obtain it?

Alternative 1—Purchase car for cash
Purchase price of the car = Rs. 3,90,000
Life = 3 years = 36 months
Salvage value after 3 years = Rs. 1,20,000

Interest rate = 12% (nominal rate, compounded annually)
= 1% compounded monthly

The cash flow diagram for alternative 1 is shown in Fig. 6.15.

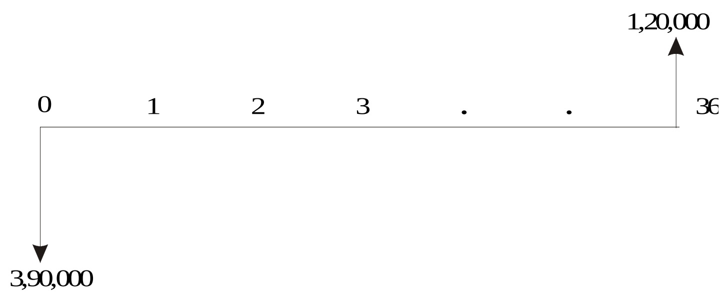


Fig. 6.15 Cash flow diagram for alternative 1.

The monthly equivalent cost expression [*ME*(1%)] of the above cash flow diagram is

$$ME(1\%) = 3{,}90{,}000(A/P,\ 1\%,\ 36) - 1{,}20{,}000(A/F,\ 1\%,\ 36)$$
$$= 3{,}90{,}000(0.0332) - 1{,}20{,}000(0.0232)$$
$$= \text{Rs. } 10{,}164$$

Alternative 2—Use of car under lease
Monthly lease amount for 36 months = Rs. 10,500

The cash flow diagram for alternative 2 is illustrated in Fig. 6.16.

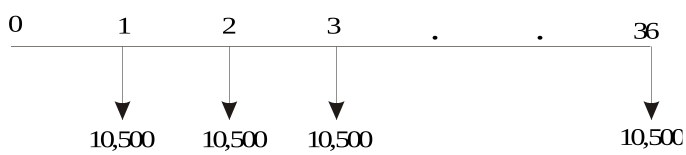


Fig. 6.16 Cash flow diagram for alternative 2.

Monthly equivalent cost = Rs.10,500.

The monthly equivalent cost of alternative 1 is less than that of alternative 2. Hence, the salesman should purchase the car for cash.

EXAMPLE 6.8 A company must decide whether to buy machine A or machine B.

	Machine A	*Machine B*
Initial cost (Rs.)	3,00,000	6,00,000
Useful life (years)	4	4
Salvage value at the end of machine life (Rs.)	2,00,000	3,00,000
Annual maintenance (Rs.)	30,000	0

At 15% interest rate, which machine should be purchased?

Solution Machine A

Initial cost = Rs. 3,00,000
Useful life (years) = 4
Salvage value at the end of machine life = Rs. 2,00,000
Annual maintenance = Rs. 30,000
Interest rate = 15%, compounded annually

The cash flow diagram of machine A is depicted in Fig. 6.17.

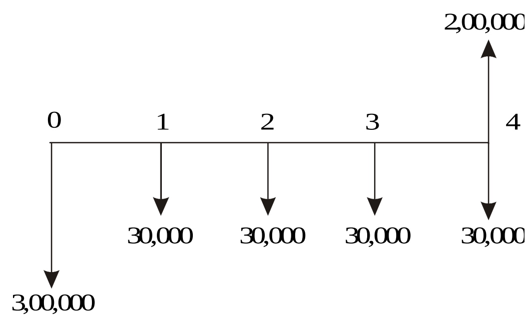


Fig. 6.17 Cash flow diagram for machine A.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(15\%) = 3{,}00{,}000(A/P,\ 15\%,\ 4) + 30{,}000 - 2{,}00{,}000(A/F,\ 15\%,\ 4)$$
$$= 3{,}00{,}000(0.3503) + 30{,}000 - 2{,}00{,}000(0.2003)$$
$$= \text{Rs. } 95{,}030$$

Machine B

Initial cost = Rs. 6,00,000
Useful life (years) = 4
Salvage value at the end of machine life = Rs. 3,00,000
Annual maintenance = Rs. 0.
Interest rate = 15%, compounded annually

The cash flow diagram of machine B is illustrated in Fig. 6.18.

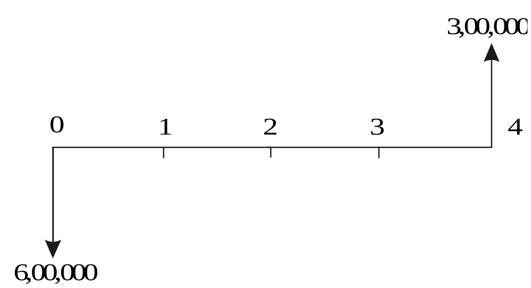


Fig. 6.18 Cash flow diagram for machine B.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(15\%) &= 6{,}00{,}000(A/P,\ 15\%,\ 4) - 3{,}00{,}000(A/F,\ 15\%,\ 4) \\ &= 6{,}00{,}000(0.3503) - 3{,}00{,}000(0.2003) \\ &= \text{Rs. } 1{,}50{,}090 \end{aligned}$$

Since the annual equivalent cost of machine A is less than that of machine B, it is advisable to buy machine A.

EXAMPLE 6.9 Jothi Lakshimi has arranged to buy some home recording equipment. She estimates that it will have a five year useful life and no salvage value at the end of equipment life. The dealer, who is a friend has offered Jothi Lakshimi two alternative ways to pay for the equipment.

(a) Pay Rs. 60,000 immediately and Rs. 15,000 at the end of one year.

(b) Pay nothing until the end of fourth year when a single payment of Rs. 90,000 must be made.

If Jothi Lakshimi believes 12% is a suitable interest rate, which alternative is the best for her?

Solution Alternative 1

Down payment = Rs. 60,000
Payment after one year = Rs. 15,000

The cash flow diagram for alternative 1 is shown in Fig. 6.19.

Fig. 6.19 Cash flow diagram for alternative 1.

The present worth equation of the above cash flow diagram is

$$\begin{aligned} PW(12\%) &= 60{,}000 + 15{,}000(P/F,\ 12\%,\ 1) \\ &= 60{,}000 + 15{,}000(0.8929) \\ &= 73{,}393.50 \end{aligned}$$

The above present worth is represented in Fig. 6.20.

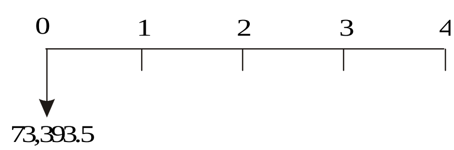


Fig. 6.20 Resultant cash flow diagram.

The annual equivalent expression of the above cash flow diagram is

$$AE(12\%) = 73{,}393.5(A/P,\ 12\%,\ 4)$$
$$= 73{,}393.5(0.3292)$$
$$= \text{Rs. } 24{,}161.14$$

Alternative 2

Payment after four years = Rs. 90,000

The cash flow diagram for alternative 2 is shown in Fig. 6.21.

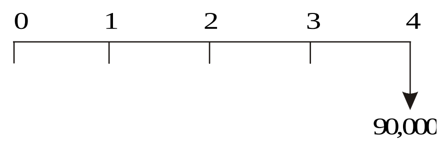


Fig. 6.21 Cash flow diagram of alternative 2.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(12\%) = 90{,}000(A/F,\ 12\%,\ 4)$$
$$= 90{,}000(0.2092)$$
$$= \text{Rs. } 18{,}828$$

The annual equivalent cost of alternative 2 is less than that of alternative 1. Hence, Jothi Lakshimi should select alternative 2 for purchasing the home equipment.

EXAMPLE 6.10 A transport company has been looking for a new tyre for its truck and has located the following alternatives:

Brand	*Tyre warranty* (months)	*Price per tyre* (Rs.)
A	12	1,200
B	24	1,800
C	36	2,100
D	48	2,700

If the company feels that the warranty period is a good estimate of the tyre life and that a nominal interest rate (compounded annually) of 12% is appropriate, which tyre should it buy?

Solution In all the cases, the interest rate is 12%. This is equivalent to 1% per month.

Brand A

Tyre warranty = 12 months

Price/tyre = Rs. 1,200

The cash flow diagram for brand A is shown in Fig. 6.22.

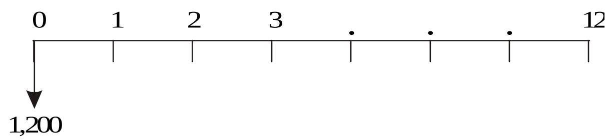


Fig. 6.22 Cash flow diagram of brand A.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(1\%) = 1{,}200(A/P, 1\%, 12)$$
$$= 1{,}200(0.0888)$$
$$= \text{Rs. } 106.56$$

Brand B

Tyre warranty = 24 months

Price/tyre = Rs. 1,800

The cash flow diagram for brand B is shown in Fig. 6.23.

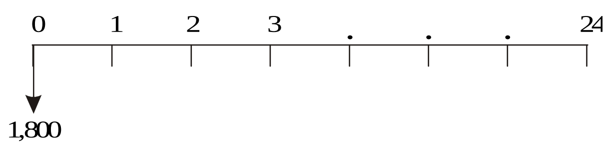


Fig. 6.23 Cash flow diagram of brand B.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(1\%) = 1{,}800(A/P, 1\%, 24)$$
$$= 1{,}800(0.0471)$$
$$= \text{Rs. } 84.78$$

Brand C

Tyre warranty = 36 months

Price/tyre = Rs. 2,100

The cash flow diagram for brand C is shown in Fig. 6.24.

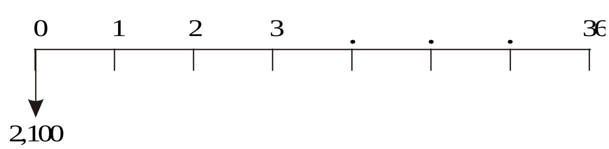


Fig. 6.24 Cash flow diagram of brand C.

The annual equivalent expression of the above cash flow diagram is

$$AE(1\%) = 2{,}100(A/P, 1\%, 36)$$
$$= 2{,}100(0.0332)$$
$$= \text{Rs. } 69.72$$

Brand D

Tyre warranty = 48 months

Price/tyre = Rs. 2,700

The cash flow diagram for brand D is shown in Fig. 6.25.

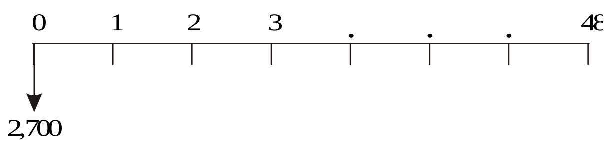


Fig. 6.25 Cash flow diagram of brand D.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(1\%) &= 2{,}700\,(A/P,\ 1\%,\ 48) \\ &= 2{,}700\,(0.0263) \\ &= \text{Rs. } 71.01 \end{aligned}$$

Here, minimum common multiple lives of tyres is considered. This is 144 months. Therefore, the comparison is made on 144 month's basis.

The annual equivalent cost of brand C is less than that of other brands. Hence, it should be used in the vehicles of the trucking company. It should be replaced four times during the 144-month period.

QUESTIONS

1. A company has three proposals for expanding its business operations. The details are as follows:

Alternative	*Initial cost* (Rs.)	*Annual revenue* (Rs.)	*Life* (years)
*A*1	25,00,000	8,00,000	10
*A*2	20,00,000	6,00,000	10
*A*3	30,00,000	10,00,000	10

Each alternative has insignificant salvage value at the end of its life. Assuming an interest rate of 15%, compounded annually, find the best alternative for expanding the business operations of the company using the annual equivalent method.

2. An automobile dealer has recently advertised for its new car. There are three alternatives of purchasing the car which are explained below.

Alternative 1 The customer can take delivery of a car after making a down payment of Rs. 25,000. The remaining money should be paid in 36 equal monthly installments of Rs. 10,000 each.

Alternative 2 The customer can take delivery of the car after making a down payment of Rs. 1,00,000. The remaining money should be paid in 36 equal monthly installments of Rs. 7,000 each.

Alternative 3 The customer can take delivery of the car by making full payment of Rs. 3,00,000.

Suggest the best alternative of buying the cars for the customers by assuming an interest rate of 20% compounded annually. Use the annual equivalent method.

3. A small-scale industry is in the process of buying a milling machine. The purchase value of the milling machine is Rs. 60,000. It has identified two banks for loan to purchase the milling machine. The banks can give only 80% of the purchase value of the milling machine as loan. In Urban Bank, the loan is to be repaid in 60 equal monthly installments of Rs. 2,500 each. In State Bank, the loan is to be repaid in 40 equal monthly installments of Rs. 4,500 each. Suggest the most economical loan scheme for the company, based on the annual equivalent method of comparison. Assume a nominal rate of 24%, compounded monthly.

4. There are two alternatives of replacing a machine. The details of the alternatives are as follows:

Alternative 1

Purchase value of the new machine	: Rs. 2,00,000
Life of the machine	: 10 years
Salvage value of the new machine at the end of its life	: Rs. 20,000
Annual operation and maintenance cost	: Rs. 40,000
Buyback price of the existing machine	: Rs. 25,000

Alternative 2

Purchase value of the new machine	: Rs. 3,00,000
Life of the machine	: 10 years
Salvage value of the new machine at the end of its life	: Rs. 15,000
Annual operation and maintenance cost	: Rs. 35,000
Buyback price of the existing machine	: Rs. 5,000

Suggest the best replacement option for the company using the annual equivalent cost method of comparison by assuming 20% interest rate, compounded annually.

5. A company receives two options for purchasing a copier machine for its office.

Option 1 Make a down payment of Rs. 30,000 and take delivery of the copier machine. The remaining money is to be paid in 24 equal monthly installments of Rs. 4,500 each.

Option 2 Make a full payment of Rs. 1,00,000 and take delivery of the copier machine.

Suggest the best option for the company to buy the copier machine based on the annual equivalent method of comparison by assuming 15% interest rate, compounded annually.

6. Find the best alternative using the annual equivalent method of comparison. Assume an interest rate of 15% compounded annually.

Alternative	*A*	*B*	*C*
Initial cost (Rs.)	5,00,000	8,00,000	6,00,000
Annual receipt (Rs.)	2,00,000	1,50,000	1,20,000
Life (years)	10	10	10
Salvage value (Rs.)	1,00,000	50,000	30,000

7

RATE OF RETURN METHOD

7.1 INTRODUCTION

The rate of return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero. In this method of comparison, the rate of return for each alternative is computed. Then the alternative which has the highest rate of return is selected as the best alternative.

In this type of analysis, the expenditures are always assigned with a negative sign and the revenues/inflows are assigned with a positive sign.

A generalized cash flow diagram to demonstrate the rate of return method of comparison is presented in Fig. 7.1.

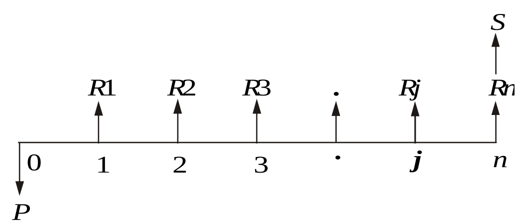


Fig. 7.1 Generalized cash flow diagram.

In the above cash flow diagram, *P* represents an initial investment, *Rj* the net revenue at the end of the *j*th year, and *S* the salvage value at the end of the *n*th year.

The first step is to find the net present worth of the cash flow diagram using the following expression at a given interest rate, *i*.

$$PW(i) = -P + R1/(1 + i)^1 + R2/(1 + i)^2 + \dots + Rj/(1 + i)^j + \dots + Rn/(1 + i)^n + S/(1 + i)^n$$

Now, the above function is to be evaluated for different values of *i* until the present worth function reduces to zero, as shown in Fig. 7.2.

In the figure, the present worth goes on decreasing when the interest rate is increased. The value of *i* at which the present worth curve cuts the *X*-axis is the rate of return of the given proposal/project. It will be very difficult to find the exact value of *i* at which the present worth function reduces to zero.

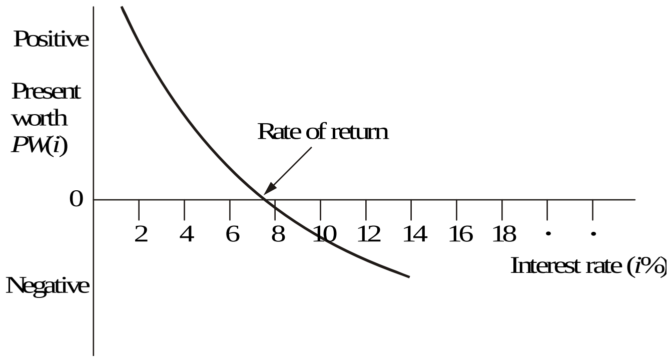


Fig. 7.2 Present worth function graph.

So, one has to start with an intuitive value of i and check whether the present worth function is positive. If so, increase the value of i until $PW(i)$ becomes negative. Then, the rate of return is determined by interpolation method in the range of values of i for which the sign of the present worth function changes from positive to negative.

7.2 EXAMPLES

In this section, the concept of rate of return calculation is demonstrated with suitable examples.

EXAMPLE 7.1 A person is planning a new business. The initial outlay and cash flow pattern for the new business are as listed below. The expected life of the business is five years. Find the rate of return for the new business.

Period	0	1	2	3	4	5
Cash flow (Rs.)	–1,00,000	30,000	30,000	30,000	30,000	30,000

Solution

Initial investment = Rs. 1,00,000
Annual equal revenue = Rs. 30,000
Life = 5 years
The cash flow diagram for this situation is illustrated in Fig. 7.3.

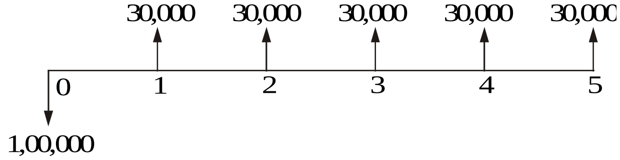


Fig. 7.3 Cash flow diagram.

The present worth function for the business is

$PW(i) = -1{,}00{,}000 + 30{,}000(P/A, i, 5)$

When $i = 10\%$,

$$\begin{aligned} PW(10\%) &= -1{,}00{,}000 + 30{,}000(P/A, 10\%, 5) \\ &= -1{,}00{,}000 + 30{,}000(3.7908) \\ &= \text{Rs. } 13{,}724. \end{aligned}$$

When $i = 15\%$,

$$\begin{aligned} PW(15\%) &= -1{,}00{,}000 + 30{,}000(P/A, 15\%, 5) \\ &= -1{,}00{,}000 + 30{,}000(3.3522) \\ &= \text{Rs. } 566. \end{aligned}$$

When $i = 18\%$,

$$\begin{aligned} PW(18\%) &= -1{,}00{,}000 + 30{,}000(P/A, 18\%, 5) \\ &= -1{,}00{,}000 + 30{,}000(3.1272) \\ &= \text{Rs. } -6{,}184 \end{aligned}$$

$$\begin{aligned} i &= 15\% + \frac{566 - 0}{566 - (-6184)} \times (3\%) \\ &= 15\% + 0.252\% \\ &= 15.252\% \end{aligned}$$

Therefore, the rate of return for the new business is 15.252%.

EXAMPLE 7.2 A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is Rs. 20,00,000. The annual net profit is Rs. 3,50,000. Find the rate of return for the new business.

Solution

Life of the product line (n) = 10 years
Initial outlay = Rs. 20,00,000
Annual net profit = Rs. 3,50,000
Scrap value after 10 years = 0

The cash flow diagram for this situation is shown in Fig. 7.4.

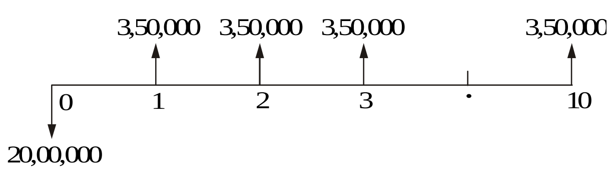


Fig. 7.4 Cash flow diagram.

The formula for the net present worth function of the situation is

$$PW(i) = -20{,}00{,}000 + 3{,}50{,}000(P/A, i, 10)$$

When $i = 10\%$,

$$\begin{aligned} PW(10\%) &= -20{,}00{,}000 + 3{,}50{,}000(P/A, 10\%, 10) \\ &= -20{,}00{,}000 + 3{,}50{,}000(6.1446) \\ &= \text{Rs. } 1{,}50{,}610. \end{aligned}$$

When $i = 12\%$,

$$\begin{aligned} PW(12\%) &= -20{,}00{,}000 + 3{,}50{,}000(P/A, 12\%, 10) \\ &= -20{,}00{,}000 + 3{,}50{,}000(5.6502) \\ &= \text{Rs. } -22{,}430. \end{aligned}$$

$$i = 10\% + \frac{1{,}50{,}610 - 0}{1{,}50{,}610 - (-22{,}430)} \times (2\%)$$

$$= 11.74\ \%$$

Therefore, the rate of return of the new product line is 11.74%

EXAMPLE 7.3 A firm has identified three mutually exclusive investment proposals whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%.

	Alternative		
	*A*1	*A*2	*A*3
Investment	Rs. 1,50,000	Rs. 2,10,000	Rs. 2,55,000
Annual net income	Rs. 45,570	Rs. 58,260	Rs. 69,000

Find the best alternative based on the rate of return method of comparison.

Solution ***Calculation of rate of return for alternative*** *A*1

Initial outlay = Rs. 1,50,000

Annual profit = Rs. 45,570

Life = 5 years

The cash flow diagram for alternative *A*1 is shown in Fig. 7.5.

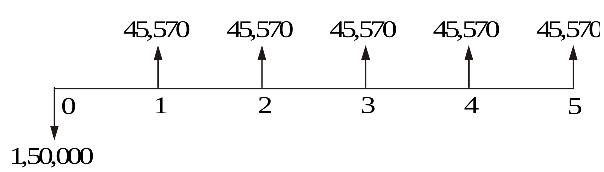


Fig. 7.5 Cash flow diagram for alternative *A*1.

The formula for the net present worth of alternative $A1$ is given as

$$PW(i) = -1{,}50{,}000 + 45{,}570(P/A, i, 5)$$

When i = 10%,

$$\begin{aligned} PW(10\%) &= -1{,}50{,}000 + 45{,}570(P/A, 10\%, 5) \\ &= -1{,}50{,}000 + 45{,}570(3.7908) \\ &= \text{Rs. } 22{,}746.76 \end{aligned}$$

When i = 12%,

$$\begin{aligned} PW(12\%) &= -1{,}50{,}000 + 45{,}570(P/A, 12\%, 5) \\ &= -1{,}50{,}000 + 45{,}570(3.6048) \\ &= \text{Rs. } 14{,}270.74 \end{aligned}$$

When i = 15%,

$$\begin{aligned} PW(15\%) &= -1{,}50{,}000 + 45{,}570(P/A, 15\%, 5) \\ &= -1{,}50{,}000 + 45{,}570(3.3522) \\ &= \text{Rs. } 2{,}759.75 \end{aligned}$$

When i = 18%,

$$\begin{aligned} PW(18\%) &= -1{,}50{,}000 + 45{,}570(P/A, 18\%, 5) \\ &= -1{,}50{,}000 + 45{,}570(3.1272) \\ &= \text{Rs. } -7{,}493.50 \end{aligned}$$

Therefore, the rate of return of the alternative $A1$ is

$$\begin{aligned} i &= 15\% + \frac{2{,}759.75 - 0}{2{,}759.75 - (-7{,}493.50)} \times (3\%) \\ &= 15\% + 0.81\% \\ &= 15.81\% \end{aligned}$$

Calculation of rate of return for alternative $A2$

Initial outlay = Rs. 2,10,000
Annual profit = Rs. 58,260
Life of alternative $A2$ = 5 years

The cash flow diagram for alternative $A2$ is shown in Fig. 7.6.

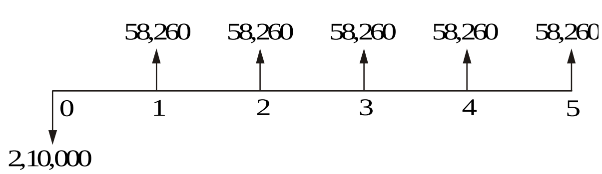


Fig. 7.6 Cash flow diagram for alternative $A2$.

The formula for the net present worth of this alternative is

$PW(i) = -2{,}10{,}000 + 58{,}260(P/A, i, 5)$

When $i = 12\%$,

$$\begin{aligned} PW(12\%) &= -2{,}10{,}000 + 58{,}260(P/A, 12\%, 5) \\ &= -2{,}10{,}000 + 58{,}260(3.6048) \\ &= \text{Rs. } 15.65 \end{aligned}$$

When $i = 13\%$,

$$\begin{aligned} PW(13\%) &= -2{,}10{,}000 + 58{,}260(P/A, 13\%, 5) \\ &= -2{,}10{,}000 + 58{,}260(3.5172) \\ &= \text{Rs. } -5{,}087.93 \end{aligned}$$

Therefore, the rate of return of alternative $A2$ is

$$\begin{aligned} i &= 12\% + \frac{15.65 - 0}{15.65 - (-5{,}087.93)} \times (1\%) \\ &= 12\% + 0\% \\ &= 12\% \end{aligned}$$

Calculation of rate of return for alternative $A3$

Initial outlay = Rs. 2,55,000
Annual profit = Rs. 69,000
Life of alternative $A3$ = 5 years

The cash flow diagram for alternative $A3$ is depicted in Fig. 7.7.

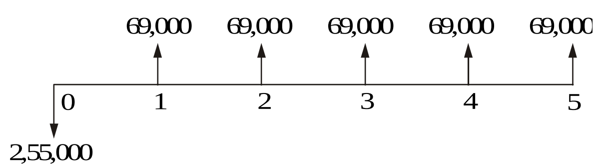


Fig. 7.7 Cash flow diagram for alternative $A3$.

The formula for the net present worth of this alternative $A3$ is

$PW(i) = -2{,}55{,}000 + 69{,}000(P/A, i, 5)$

When $i = 11\%$,

$$\begin{aligned} PW(11\%) &= -2{,}55{,}000 + 69{,}000(P/A, 11\%, 5) \\ &= -2{,}55{,}000 + 69{,}000(3.6959) \\ &= \text{Rs. } 17.1 \end{aligned}$$

When $i = 12\%$,

$$\begin{aligned} PW(12\%) &= -2{,}55{,}000 + 69{,}000(P/A, 12\%, 5) \\ &= -2{,}55{,}000 + 69{,}000(3.6048) \\ &= \text{Rs. } -6{,}268.80 \end{aligned}$$

Therefore, the rate of return for alternative *A*3 is

$$i = 11\% + \frac{17.1 - 0}{17.1 - (-6{,}268.80)} \times 1\%$$

$$= 11\%$$

The rates of return for the three alternatives are now tabulated.

Alternative	*A*1	*A*2	*A*3
Rate of return	15.81%	12%	11%

From the above data, it is clear that the rate of return for alternative *A*3 is less than the minimum attractive rate of return of 12%. So, it should not be considered for comparison. The remaining two alternatives are qualified for consideration. Among the alternatives *A*1 and *A*2, the rate of return of alternative *A*1 is greater than that of alternative *A*2. Hence, alternative *A*1 should be selected.

EXAMPLE 7.4 For the cash flow diagram shown in Fig. 7.8, compute the rate of return. The amounts are in rupees.

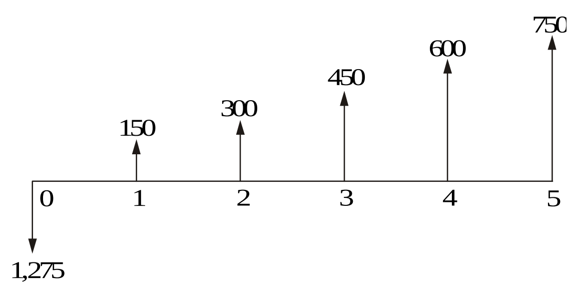


Fig. 7.8 Cash flow diagram.

Solution For the positive cash flows of the problem,

$$A1 = \text{Rs. } 150, \qquad G = \text{Rs. } 150$$

The annual equivalent of the positive cash flows of the uniform gradient series is given by

$$A = A1 + G(A/G, i, n)$$
$$= 150 + 150(A/G, i, 5)$$

The formula for the present worth of the whole diagram

$$= -1{,}275 + [150 + 150(A/G, i, 5)] \times (P/A, i, 5)$$

$$PW(10\%) = -1{,}275 + [150 + 150(A/G, 10\%, 5)] \times (P/A, 10\%, 5)$$
$$= -1{,}275 + [150 + 150(1.8101)] \times (3.7908)$$
$$= \text{Rs. } 322.88$$

$$\begin{aligned} PW(12\%) &= -1{,}275 + [150 + 150(A/G, 12\%, 5)] \times (P/A, 12\%, 5) \\ &= -1{,}275 + [150 + 150(1.7746)] \times (3.6048) \\ &= \text{Rs. } 225.28 \end{aligned}$$

$$\begin{aligned} PW(15\%) &= -1{,}275 + [150 + 150(A/G, 15\%, 5)] \times (P/A, 15\%, 5) \\ &= -1{,}275 + [150 + 150(1.7228)] \times (3.3522) \\ &= \text{Rs. } 94.11 \end{aligned}$$

$$\begin{aligned} PW(18\%) &= -1{,}275 + [150 + 150(A/G, 18\%, 5)] \times (P/A, 18\%, 5) \\ &= -1{,}275 + [150 + 150(1.6728)] \times (3.1272) \\ &= \text{Rs. } -21.24 \end{aligned}$$

Therefore, the rate of return for the cash flow diagram is

$$\begin{aligned} i &= 15\% + \frac{94.11 - 0}{94.11 - (-21.24)} \times 3\% \\ &= 15\% + 2.45\% = 17.45\% \end{aligned}$$

EXAMPLE 7.5 A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are tabulated below. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 12%. Suggest the best alternative to the company.

	Initial investment (Rs.)	*Yearly revenue* (Rs.)
Alternative 1	5,00,000	1,70,000
Alternative 2	8,00,000	2,70,000

Solution Alternative 1

Initial outlay = Rs. 5,00,000
Annual revenue = Rs. 1,70,000
Life of alternative 1 = 5 years

The cash flow diagram for alternative 1 is illustrated in Fig. 7.9.

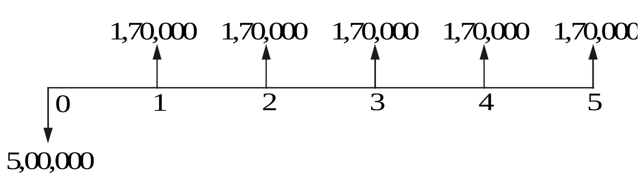


Fig. 7.9 Cash flow diagram for alternative 1.

The formulae for the net present worth of alternative 1 are as follows:

$$PW_1(i) = -5,00,000 + 1,70,000(P/A, i, 5)$$

$$PW_1(15\%) = -5,00,000 + 1,70,000(P/A, 15\%, 5)$$
$$= -5,00,000 + 1,70,000(3.3522)$$
$$= \text{Rs. } 69,874$$

$$PW_1(17\%) = -5,00,000 + 1,70,000(P/A, 17\%, 5)$$
$$= -5,00,000 + 1,70,000(3.1993)$$
$$= \text{Rs. } 43,881$$

$$PW_1(20\%) = -5,00,000 + 1,70,000(P/A, 20\%, 5)$$
$$= -5,00,000 + 1,70,000(2.9906)$$
$$= \text{Rs. } 8,402$$

$$PW_1(22\%) = -5,00,000 + 1,70,000(P/A, 22\%, 5)$$
$$= -5,00,000 + 1,70,000(2.8636)$$
$$= \text{Rs. } -13,188$$

Therefore, the rate of return of alternative 1 is

$$i = 20\% + \frac{8,402 - 0}{8,402 - (-13,188)} \times 2\%$$
$$= 20.78\%$$

Alternative 2

Initial outlay = Rs. 8,00,000
Annual revenue = Rs. 2,70,000
Life = 5 years

The cash flow diagram for alternative 2 is depicted in Fig. 7.10.

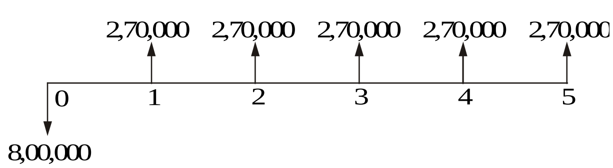


Fig. 7.10 Cash flow diagram for alternative 2.

The formula for the net present worth of alternative 2 is:

$$PW_2(i) = -8,00,000 + 2,70,000(P/A, i, 5)$$

$$PW_2(20\%) = -8,00,000 + 2,70,000(P/A, 20\%, 5)$$
$$= -8,00,000 + 2,70,000(2.9906)$$
$$= \text{Rs. } 7,462$$

$$\begin{aligned} PW_2(22\%) &= -8,00,000 + 2,70,000(P/A, 22\%, 5) \\ &= -8,00,000 + 2,70,000(2.8636) \\ &= \text{Rs. } -26,828 \end{aligned}$$

Thus, the rate of return of alternative 2 is

$$\begin{aligned} i &= 20\% + \frac{7,462 - 0}{7,462 - (-26,828)} \times 2\% \\ &= 20.435\% \end{aligned}$$

Since the rate of return of alternative 1 is greater than that of the alternative 2, select alternative 1.

QUESTIONS

1. Consider the following cash flow of a project:

Year	0	1	2	3	4	5
Cash flow	–10,000	4,000	4,500	5,000	5,500	6,000

Find the rate of return of the project.

2. A person invests a sum of Rs. 2,00,000 in a business and receives equal net revenue of Rs. 50,000 for the next 10 years. At the end of the 10th year, the salvage value of the business is Rs. 25,000. Find the rate of return of the business.

3. A company is in the process of selecting the best alternative among the following three mutually exclusive alternatives:

Alternative	*Initial investment*	*Annual revenue* (Rs.)	*Life* (years)
*A*1	Rs. 5,00,000	1,00,000	10
*A*2	Rs. 8,00,000	1,40,000	10
*A*3	Rs. 3,00,000	70,000	10

Find the best alternative based on the rate of return method of comparison.

4. A shipping firm is considering the purchase of a materials handling system for unloading ships at a dock. The firm has reduced their choice to three different systems, all of which are expected to provide the same unloading speed. The initial costs and the operating costs estimated for each system are now tabulated.

System	*Initial cost*	*Annual operating expenses*
*S*1	Rs. 6,50,000	Rs. 91,810
*S*2	Rs. 7,80,000	Rs. 52,600
*S*3	Rs. 7,50,000	Rs. 68,417

The life of each system is estimated to be five years and the firm's minimum attractive rate of return is 15%. If the firm must select one of the materials handling systems, which one is the most desirable?.

5. A firm has identified three mutually exclusive alternatives. The life of all three alternatives is estimated to be five years. The minimum attractive rate of return is 12%. Find the best alternative based on the rate of return method.

Alternative	*A*1	*A*2	*A*3
Initial investment (Rs.)	2,00,000	2,80,000	3,60,000
Annual income (Rs.)	52,000	72,000	1,00,000

6. An automobile company is planning to buy a robot for its forging unit. It has identified two different companies for the supply of the robot. The details of cost and incremental revenue of using robots are summarized in the following table:

	Brand	
	Speedex	*Giant*
Initial cost (Rs.)	5,00,000	9,00,000
Annual incremental revenue (Rs.)	80,000	2,50,000
Life (years)	8	8
Life-end salvage value (Rs.)	40,000	60,000

The minimum attractive return for the company is 12%. Suggest the best brand of robot to the company based on the rate of return method.

7. A bank introduces two different investment schemes whose details are as follows: Find the best investment alternative from the investor's point of view.

	Alpha Bank	*Beta Bank*
Deposit amount (Rs.)	1,00,000	2,00,000
Period of deposit (years)	5 years	3 years
Maturity amount (Rs.)	3,00,000	4,50,000

8. A company is planning for its expansion programme which will take place after five years. The expansion requires an equal sum of Rs. 5,00,000 for consecutive three years. Gamma Bank has recently introduced a scheme in this line. If the company invests Rs. 7,00,000 now with this bank, it will make equal repayments of Rs. 5,00,000 for three consecutive years starting from the end of the fifth year from now. The minimum attractive rate of return for the company is 12%. Suggest whether the company should invest with the Gamma Bank for its expansion programme.

9. Consider the following table which summarizes data of two alternatives.

	First cost	*Annual return*	*Life*
Alternative 1	Rs. 5,00,000	Rs. 1,50,000	10 yrs
Alternative 2	Rs. 8,00,000	Rs. 2,50,000	10 yrs

Find the best alternative based on the rate of return method of comparison.

10. A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are given in the following table. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 15%. Suggest the best alternative to the company.

	Initial investment (Rs.)	*Yearly revenue* (Rs.)
Alternative 1	4,50,000	1,50,000
Alternative 2	7,50,000	2,50,000

8

REPLACEMENT AND MAINTENANCE ANALYSIS

8.1 INTRODUCTION

Organizations providing goods/services use several facilities like equipment and machinery which are directly required in their operations. In addition to these facilities, there are several other items which are necessary to facilitate the functioning of organizations.

All such facilities should be continuously monitored for their efficient functioning; otherwise, the quality of service will be poor. Besides the quality of service of the facilities, the cost of their operation and maintenance would increase with the passage of time. Hence, it is an absolute necessity to maintain the equipment in good operating conditions with economical cost. Thus, we need an integrated approach to minimize the cost of maintenance. In certain cases, the equipment will be obsolete over a period of time.

If a firm wants to be in the same business competitively, it has to take decision on whether to replace the old equipment or to retain it by taking the cost of maintenance and operation into account.

There are two basic reasons for considering the replacement of an equipment—physical impairment of the various parts or obsolescence of the equipment.

Physical impairment refers only to changes in the physical condition of the machine itself. This would lead to a decline in the value of the service rendered, increased operating cost, increased maintenance cost or a combination of these.

Obsolescence is due to improvement of the tools of production, mainly improvement in technology.

So, it would be uneconomical to continue production with the same machine under any of the above situations. Hence, the machines are to be periodically replaced.

Sometimes, the capacity of existing facilities may be inadequate to meet the current demand. Under such situation, the following alternatives will be considered.

- Replacement of the existing equipment with a new one.
- Augmenting the existing one with an additional equipment.

8.2 TYPES OF MAINTENANCE

Maintenance activity can be classified into two types: preventive maintenance and breakdown maintenance. *Preventive maintenance* (PM) is the periodical inspection and service activities which are aimed to detect potential failures and perform minor adjustments or repairs which will prevent major operating problems in future. *Breakdown maintenance* is the repair which is generally done after the equipment has attained down state. It is often of an emergency nature which will have associated penalty in terms of expediting cost of maintenance and down time cost of equipment. Preventive maintenance will reduce such cost up to a point. Beyond that point, the cost of preventive maintenance will be more when compared to the breakdown maintenance cost. The total cost, which is the sum of the preventive maintenance cost and the breakdown maintenance cost, will go on decreasing with an increase in the level of maintenance up to a point. Beyond that point, the total cost will start increasing. The level of maintenance corresponding to the minimum total cost is the optimal level of maintenance. The concepts are demonstrated in Fig. 8.1.

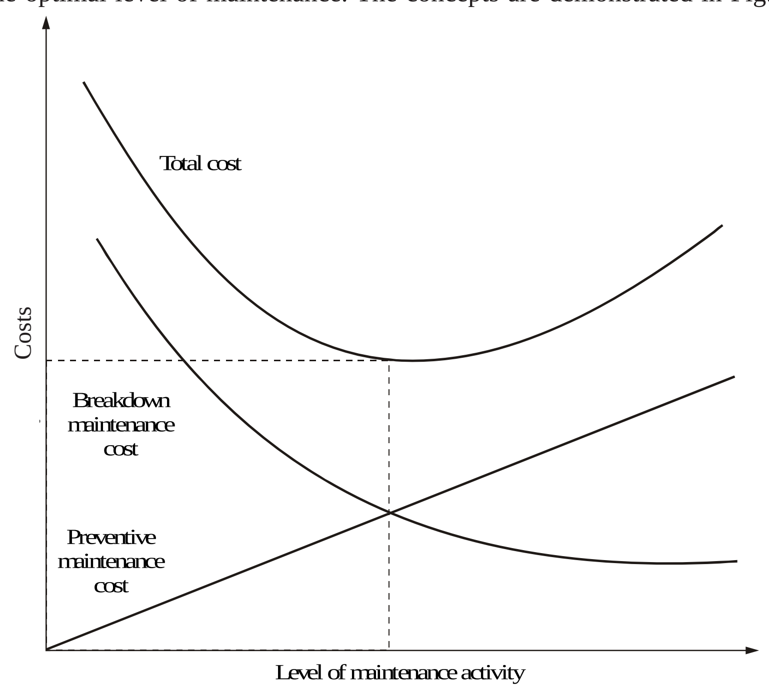


Fig. 8.1 Maintenance costs.

8.3 TYPES OF REPLACEMENT PROBLEM

Replacement study can be classified into two categories:

(a) Replacement of assets that deteriorate with time (Replacement due to gradual failure, or wear and tear of the components of the machines).

This can be further classified into the following types:

(i) Determination of economic life of an asset.

(ii) Replacement of an existing asset with a new asset.

(b) Simple probabilistic model for assets which fail completely (replacement due to sudden failure).

8.4 DETERMINATION OF ECONOMIC LIFE OF AN ASSET

Any asset will have the following cost components:

- Capital recovery cost (average first cost), computed from the first cost (purchase price) of the machine.
- Average operating and maintenance cost (O & M cost)
- Total cost which is the sum of capital recovery cost (average first cost) and average maintenance cost.

A typical shape of each of the above costs with respect to life of the machine is shown in Fig. 8.2.

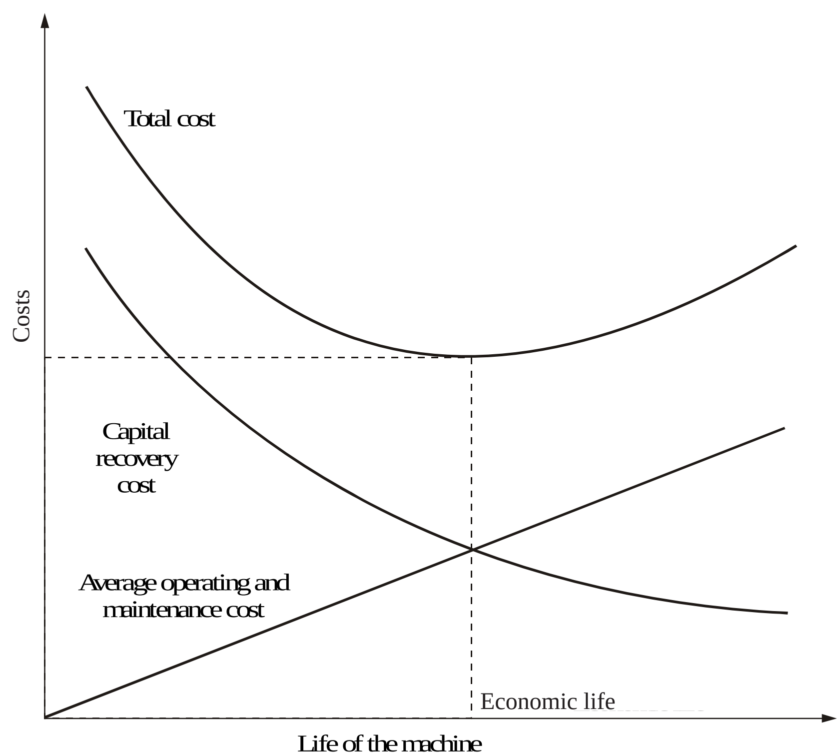


Fig. 8.2 Chart showing economic life.

From Fig. 8.2, it is clear that the capital recovery cost (average first cost) goes on decreasing with the life of the machine and the average operating and maintenance cost goes on increasing with the life of the machine. From the beginning, the total cost continues to decrease up to a particular life and then it starts increasing. The point where the total cost is minimum is called the *economic life* of the machine.

If the interest rate is more than zero per cent, then we use interest formulas to determine the economic life.

The replacement alternatives can be evaluated based on the present worth criterion and annual equivalent criterion. The basics of these criteria are already presented in Chapter 3.

EXAMPLE 8.1 A firm is considering replacement of an equipment, whose first cost is Rs. 4,000 and the scrap value is negligible at the end of any year. Based on experience, it was found that the maintenance cost is zero during the first year and it increases by Rs. 200 every year thereafter.

(a) When should the equipment be replaced if i = 0%?

(b) When should the equipment be replaced if i = 12%?

(a) When i = 0%. In this problem,

(i) First cost = Rs. 4,000

(ii) Maintenance cost is Rs. 0 during the first year and it increases by Rs. 200 every year thereafter.

This is summarized in column B of Table 8.1.

Table 8.1 Calculations to Determine Economic Life (First cost = Rs. 4,000, Interest = 0%)

End of year (*n*)	*Maintenance cost at end of year*	*Summation of maintenance costs*	*Average cost of maintenance through year given*	*Average first cost if replaced at year end given*	*Average total cost through year given*
		ΣB	C/A	4,000/A	D + E
A	B (Rs.)	C (Rs.)	D (Rs.)	E (Rs.)	F (Rs.)
1	0	0	0	4,000.00	4,000.00
2	200	200	100	2,000.00	2,100.00
3	400	600	200	1,333.33	1,533.33
4	600	1,200	300	1,000.00	1,300.00
5	800	2,000	400	800.00	1,200.00
6	1,000	3,000	500	666.67	1,166.67*
7	1,200	4,200	600	571.43	1,171.43

*Economic life of the machine = 6 years

Column C summarizes the summation of maintenance costs for each replacement period. The value corresponding to any end of year in this column represents the total maintenance cost of using the equipment till the end of that particular year.

$$\text{Average total cost} = \frac{\text{First cost (FC) + Summation of maintenance cost}}{\text{Replacement period}}$$

$$= \frac{\text{FC}}{n} + \frac{\text{Column C}}{n}$$

$$= \begin{array}{c}\text{Average first cost for}\\ \text{the given period}\end{array} + \begin{array}{c}\text{Average maintenance cost}\\ \text{for the given period}\end{array}$$

$$\text{Column F} = \text{Column E} + \text{Column D}$$

The value corresponding to any end of year (n) in Column F represents the average total cost of using the equipment till the end of that particular year.

For this problem, the average total cost decreases till the end of year 6 and then it increases. Therefore, the optimal replacement period is six years, i.e. economic life of the equipment is six years.

(b) When interest rate, i = 12%. When the interest rate is more than 0%, the steps to be taken for getting the economic life are summarized with reference to Table 8.2.

Table 8.2 Calculations to Determine Economic Life (First cost = Rs. 4,000, Interest = 12%)

End of year (*n*)	*Maintenance cost at end of year*	*P/F*, 12%, *n*	*Present worth as of beginning of year 1 of maintenance costs*	*Summation of present worth of maintenance costs through year given*	*Present worth of cumulative maintenance cost & first cost*	*A/P*, 12%, *n*	*Annual equivalent total cost through year given*
			(B × C)	Σ D	E + Rs. 4,000		F × G
A	B (Rs)	C	D (Rs.)	E (Rs.)	F (Rs.)	G	H (Rs.)
1	0	0.8929	0.00	0.00	4,000.00	1.1200	4,480.00
2	200	0.7972	159.44	159.44	4,159.44	0.5917	2,461.14
3	400	0.7118	284.72	444.16	4,444.16	0.4163	1,850.10
4	600	0.6355	381.30	825.46	4,825.46	0.3292	1,588.54
5	800	0.5674	453.92	1,279.38	5,279.38	0.2774	1,464.50
6	1,000	0.5066	506.60	1,785.98	5,785.98	0.2432	1,407.15
7	1,200	0.4524	542.88	2,328.86	6,328.86	0.2191	1,386.65*
8	1,400	0.4039	565.46	2,894.32	6,894.32	0.2013	1,387.83
9	1,600	0.3606	576.96	3,471.28	7,471.28	0.1877	1,402.36
10	1,800	0.3220	579.60	4,050.88	8,050.88	0.1770	1,425.00

*Economic life of the machine = 7 years

The steps are summarized now:

1. Discount the maintenance costs to the beginning of year 1.

$$\text{Column D} = \text{Column B} \times \frac{1}{(1+i)^n}$$

$$= \text{Column B} \times (P/F,\ i,\ n) = \text{Column B} \times \text{Column C.}$$

2. Find the summation of present worth of maintenance costs through the year given (Column E = Σ Column D).

3. Find Column F by adding the first cost of Rs. 4,000 to Column E.
4. Find the annual equivalent total cost through the years given.

$$\text{Column H} = \text{Column F} \times \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= \text{Column F} \times (A/P,\ 12\%,\ n) = \text{Column F} \times \text{Column G}$$

5. Identify the end of year for which the annual equivalent total cost is minimum.

For this problem, the annual equivalent total cost is minimum at the end of year 7. Therefore, the economic life of the equipment is seven years.

EXAMPLE 8.2 The following table gives the operation cost, maintenance cost and salvage value at the end of every year of a machine whose purchase value is Rs. 20,000.

Find the economic life of the machine assuming interest rate, i = 15%.

End of year (n)	*Operation cost at the end of year (Rs.)*	*Maintenance cost at the end of year (Rs.)*	*Salvage value at the end of year (Rs.)*
1	3,000	300	9,000
2	4,000	400	8,000
3	5,000	500	7,000
4	6,000	600	6,000
5	7,000	700	5,000
6	8,000	800	4,000
7	9,000	900	3,000
8	10,000	1,000	2,000
9	11,000	1,100	1,000
10	12,000	1,200	0

Solution

First cost = Rs. 20,000

Interest rate = 15%

The other details are summarized in Table 8.3 along with regular calculations for determining the economic life.

Table 8.3 Calculations to Determine Economic Life

(First Cost = Rs. 20,000, Interest Rate = 15%)

End of year (*n*)	*Operation cost at the end of year*	*Maintenance cost at the end of year*	*Sum of operation and maintenance costs at the end of year*	*P/F*, 15%, *n*	*Present worth as of beginning of year 1 of sum of operation & maintenance cost*	*Cumulative sum of column F through year designated*	*Salvage value at the end of year*	*Present worth as of beginning of year 1 of salvage value*	*Total present worth*	*A/P*, 15%, *n*	*Annual equlant. total cost through year given*
			B + C		D × E	Σ F		H × E	G + 20,000 – I		J × K
A	B (Rs.)	C (Rs.)	D (Rs.)	E	F (Rs.)	G (Rs.)	H (Rs.)	I (Rs.)	J (Rs.)	K	L (Rs.)
1	3,000	300	3,300	0.8696	2,869.68	2,869.68	9,000	7,826.40	15,043.28	1.1500	17,299.77
2	4,000	400	4,400	0.7562	3,326.84	6,196.52	8,000	6,048.80	20,147.72	0.6151	12,392.86
3	5,000	500	5,500	0.6575	3,616.25	9,812.77	7,000	4,602.50	25,210.27	0.4380	11,042.01
4	6,000	600	6,600	0.5718	3,773.88	13,586.65	6,000	3,430.80	30,155.85	0.3503	10,563.59
5	7,000	700	7,700	0.4972	3,828.44	17,415.09	5,000	2,486.00	34,929.09	0.2983	10,419.35 *
6	8,000	800	8,800	0.4323	3,804.24	21,219.33	4,000	1,729.20	39,490.13	0.2642	10,433.29
7	9,000	900	9,900	0.3759	3,721.41	24,940.74	3,000	1,127.70	43,813.04	0.2404	10,532.66

*Economic Life = 5 years

Total annual equivalent cost

$$= \left[\begin{array}{c}\text{Cumulative sum of}\\ \text{present worth as of}\\ \text{beginning of year 1}\\ \text{of operation and}\\ \text{maintenance costs}\end{array} + \begin{array}{c}\text{First}\\ \text{cost}\end{array} - \begin{array}{c}\text{Present worth}\\ \text{as of beginning}\\ \text{of year 1 of}\\ \text{salvage value}\end{array}\right] \times (A/P, 15\%, n)$$

i.e. Column L = (Column G + 20,000 – Column I) × Column K
= Column J × Column K

In Column L, the annual equivalent total cost is minimum for $n = 5$. Therefore, the economic life of the machine is five years.

EXAMPLE 8.3 A company has already identified machine A and determined the economic life as four years by assuming 15% interest rate. The annual equivalent total cost corresponding to the economic life is Rs. 2,780.

Now, the manufacturer of machine B has approached the company. Machine B, which has the same capacity as that of machine A, is priced at Rs. 6,000. The maintenance cost of machine B is estimated at Rs. 1,500 for the first year and an equal yearly increment of Rs. 300 thereafter.

If the money is worth 15% per year, which machine should be purchased? (Assume that the scrap value of each of the machines is negligible at any year.)

Determination of economic life and corresponding annual equivalent total cost of machine B. The details of machine B are summarized in Table 8.4 along with the usual calculations to determine the economic life.

Table 8.4 Calculations to Determine Economic Life (First Cost = Rs. 6,000, Interest = 15%)

End of year (*n*)	*Maintenance cost for end of year*	*P/F, 15%, n*	*Present worth as of beginning of year 1 of maintenance costs*	*Summation of present worth of maintenance costs through year given*	*Column E + Rs. 6000*	*A/P, 15%, n*	*Annual equivalent total cost through year given*
			B × C	Σ D			F × G
A	B (Rs)	C	D (Rs.)	E (Rs.)	F (Rs.)	G	H (Rs.)
1	1,500	0.8696	1,304.40	1,304.40	7,304.40	1.1500	8,400.06
2	1,800	0.7561	1,360.98	2,665.38	8,665.38	0.6151	5,330.08
3	2,100	0.6575	1,380.75	4,046.13	10,046.13	0.4380	4,400.21
4	2,400	0.5718	1,372.32	5,418.45	11,418.45	0.3503	3,999.88
5	2,700	0.4972	1,342.44	6,760.89	12,760.89	0.2983	3,806.57
6	3,000	0.4323	1,296.90	8,057.79	14,057.79	0.2642	3,714.07
7	3,300	0.3759	1,240.47	9,298.26	15,298.26	0.2404	3,677.70
8	3,600	0.3269	1,176.84	10,475.10	16,475.10	0.2229	3,672.30 *
9	3,900	0.2843	1,108.77	11,583.87	17,583.87	0.2096	3,685.58
10	4,200	0.2472	1,038.24	12,622.11	18,622.11	0.1993	3,711.39

*Economic life of the machine = 8 years

Column B of Table 8.4 summarizes the yearly maintenance costs of machine B. The first cost of machine B is equal to Rs. 6,000.

Annual equivalent total cost

$$= \left[\begin{array}{l}\text{Summation of present worth} \\ \text{of maintenance cost through} \\ \text{year}\end{array} + \begin{array}{c}\text{First} \\ \text{cost}\end{array}\right] \times (A/P,\ 15\%,\ n)$$

$$= (\text{Column E} + \text{Rs. } 6{,}000) \times \text{Column G}$$

$$= \text{Column F} \times \text{Column G}$$

In Column H, the minimum annual equivalent total cost occurs when n is equal to 8. Hence the economic life of machine B is 8 years and the corresponding annual equivalent total cost is Rs. 3,672.30.

RESULT

Minimum annual equivalent total cost for machine A = Rs. 2,780
Minimum annual equivalent total cost for machine B = Rs. 3,672.30

Since the minimum annual equivalent total cost of machine A is less than that of machine B, machine A is selected as the best machine which has the economic life of four years. (*Note:* Selection of the best machine is based on the minimum annual equivalent total cost. The comparison is made over the minimum common multiple of the lives of machine A and machine B, i.e. $4 \times 2 = 8$ years).

8.5 REPLACEMENT OF EXISTING ASSET WITH A NEW ASSET

In this section, the concept of comparison of replacement of an existing asset with a new asset is presented. In this analysis, the annual equivalent cost of each alternative should be computed first. Then the alternative which has the least cost should be selected as the best alternative. Before discussing details, some preliminary concepts which are essential for this type of replacement analysis are presented.

8.5.1 Capital Recovery with Return

Consider the following data of a machine. Let

P = purchase price of the machine,

F = salvage value of the machine at the end of machine life,

n = life of the machine in years, and

i = interest rate, compounded annually

The corresponding cash flow diagram is shown in Fig. 8.3.

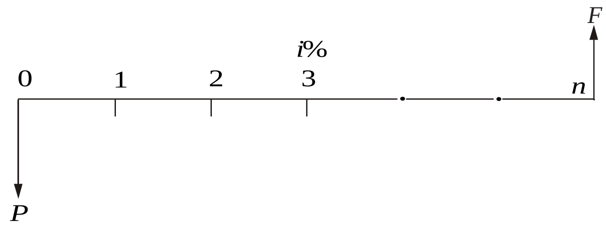


Fig. 8.3 Cash flow diagram of machine.

The equation for the annual equivalent amount for the above cash flow diagram is

$$AE(i) = (P - F) \times (A/P, i, n) + F \times i$$

This equation represents the *capital recovery with return*.

8.5.2 Concept of Challenger and Defender

If an existing equipment is considered for replacement with a new equipment, then the existing equipment is known as the *defender* and the new equipment is known as *challenger*.

Assume that an equipment has been purchased about three years back for Rs. 5,00,000 and it is considered for replacement with a new equipment. The supplier of the new equipment will take the old one for some money, say, Rs. 3,00,000. This should be treated as the present value of the existing equipment and it should be considered for all further economic analysis. The purchase value of the existing equipment before three years is now known as *sunk cost*, and it should not be considered for further analysis.

EXAMPLE 8.4 Two years ago, a machine was purchased at a cost of Rs. 2,00,000 to be useful for eight years. Its salvage value at the end of its life is Rs. 25,000. The annual maintenance cost is Rs. 25,000. The market value of the present machine is Rs. 1,20,000. Now, a new machine to cater to the need of the present machine is available at Rs. 1,50,000 to be useful for six years. Its annual maintenance cost is Rs. 14,000. The salvage value of the new machine is Rs. 20,000. Using an interest rate of 12%, find whether it is worth replacing the present machine with the new machine.

Solution ***Alternative 1—Present machine***

Purchase price = Rs. 2,00,000

Present value (P) = Rs. 1,20,000

Salvage value (F) = Rs. 25,000

Annual maintenance cost (A) = Rs. 25,000

Remaining life = 6 years

Interest rate = 12%

The cash flow diagram of the present machine is illustrated in Fig. 8.4. The

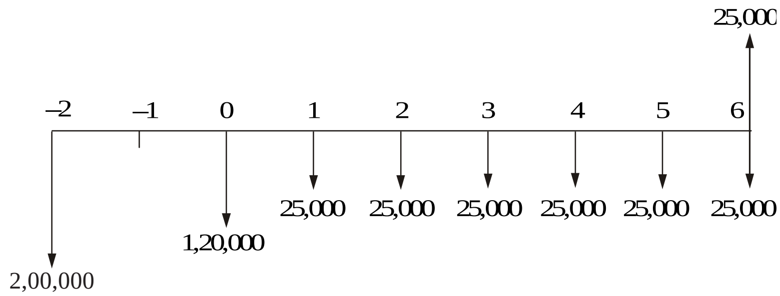


Fig. 8.4 Cash flow diagram for alternative 1.

annual maintenance cost for the preceding periods are not shown in this figure. The annual equivalent cost is computed as

$$AE(12\%) = (P - F)(A/P, 12\%, 6) + F \times i + A$$
$$= (1,20,000 - 25,000)(0.2432) + 25,000 \times 0.12 + 25,000$$
$$= \text{Rs. } 51,104$$

Alternative 2—New machine

Purchase price (P) = Rs. 1,50,000
Salvage value (F) = Rs. 20,000
Annual maintenance cost (A) = Rs. 14,000
Life = 6 years
Interest rate = 12%

The cash flow diagram of the new machine is depicted in Fig. 8.5.

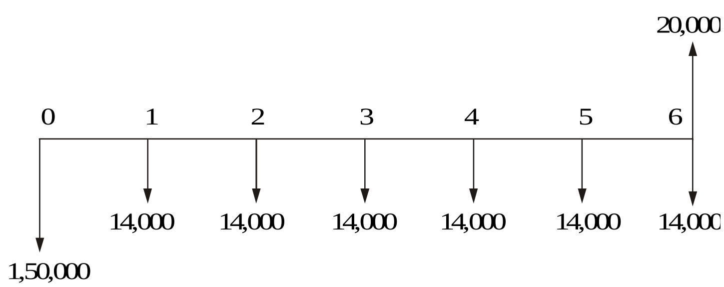


Fig. 8.5 Cash flow diagram for alternative 2.

The formula for the annual equivalent cost is

$$AE(12\%) = (P - F)(A/P, 12\%, 6) + F \times i + A$$
$$= (1,50,000 - 20,000)(0.2432) + 20,000 \times 0.12 + 14,000$$
$$= \text{Rs. } 48,016$$

Since the annual equivalent cost of the new machine is less than that of the present machine, it is suggested that the present machine be replaced with the new machine.

EXAMPLE 8.5 A diesel engine was installed 10 years ago at a cost of Rs. 50,000. It has a present realizable market value of Rs. 15,000. If kept, it can be expected to last five years more, with operating and maintenance cost of Rs. 14,000 per year and to have a salvage value of Rs. 8,000 at the end of the fifth year. This engine can be replaced with an improved version costing Rs. 65,000 which has an expected life of 20 years. This improved version will have an estimated annual operating and maintenance cost of Rs. 9,000 and ultimate salvage value of Rs. 13,000. Using an interest rate of 15%, make an annual equivalent cost analysis to determine whether to keep or replace the old engine.

Solution ***Alternative 1—Old diesel engine***

Purchase price = Rs. 50,000
Present value (P) = Rs. 15,000
Salvage value (F) = Rs. 8,000
Annual operating and maintenance cost (A) = Rs. 14,000
Remaining life (n) = 5 years
Interest rate = 15%

The cash flow diagram of the old diesel engine is shown in Fig. 8.6.

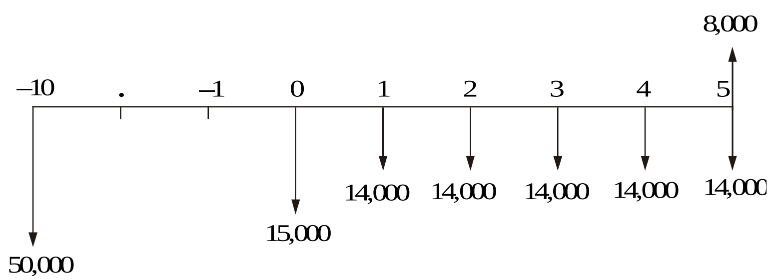


Fig. 8.6 Cash flow diagram for alternative 1.

The formula for the annual equivalent cost is

$$\begin{aligned} AE(15\%) &= (P - F)(A/P,\ 15\%,\ 5) + F \times i + A \\ &= (15{,}000 - 8{,}000)(0.2983) + 8{,}000 \times 0.15 + 14{,}000 \\ &= \text{Rs. } 17{,}288.10 \end{aligned}$$

Alternative 2—New diesel engine

Present value (P) = Rs. 65,000
Salvage value (F) = Rs. 13,000
Annual operating and maintenance cost (A) = Rs. 9,000
Life (n) = 20 years
Interest rate = 15%

The cash flow diagram of the new diesel engine is shown in Fig. 8.7.

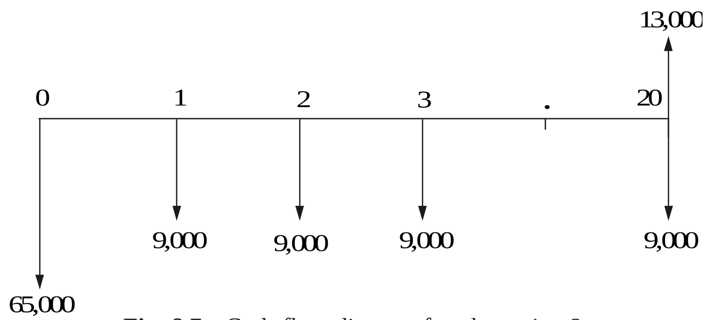


Fig. 8.7 Cash flow diagram for alternative 2.

The formula for the annual equivalent cost is

$$\begin{aligned} AE(15\%) &= (P - F)(A/P,\ 15\%,\ 20) + F \times i + A \\ &= (65{,}000 - 13{,}000)(0.1598) + 13{,}000 \times 0.15 + 9{,}000 \\ &= \text{Rs. } 19{,}259.60 \end{aligned}$$

For comparing the engines based on equal lives (20 years), the annual equivalent figures are given in Fig. 8.8. Equal lives are nothing but the least common multiple of the lives of the alternatives.

Old engine		New engine	
	0		0
17,288.10	1	19,259.60	1
17,288.10	2	19,259.60	2
17,288.10	3	19,259.60	3
17,288.10	4	19,259.60	4
17,288.10	5	19,259.60	5
17,288.10	6	19,259.60	6
17,288.10	7	19,259.60	7
17,288.10	8	19,259.60	8
17,288.10	9	19,259.60	9
17,288.10	10	19,259.60	10
17,288.10	11	19,259.60	11
17,288.10	12	19,259.60	12
17,288.10	13	19,259.60	13
17,288.10	14	19,259.60	14
17,288.10	15	19,259.60	15
17,288.10	16	19,259.60	16
17,288.10	17	19,259.60	17
17,288.10	18	19,259.60	18
17,288.10	19	19,259.60	19
17,288.10	20	19,259.60	20

Fig. 8.8 Cash flow diagram of alternatives based on common lives.

Since the annual equivalent cost of the old diesel engine is less than that of the new diesel engine, it is suggested to keep the old diesel engine. Here, an important assumption is that the old engine will be replaced four times during the 20 years period of comparison.

EXAMPLE 8.6 A steel highway bridge must either be reinforced or replaced. Reinforcement would cost Rs. 6,60,000 and would make the bridge fit for an additional five years of service. If it is reinforced, it is estimated that its net salvage value would be Rs. 4,00,000 at the time it is retired from service. The new prestressed concrete bridge would cost Rs. 15,00,000 and would meet the foreseeable requirements of the next 40 years. Such a bridge would have no salvage value. It is estimated that the annual maintenance cost of the reinforced bridge would exceed that of the concrete bridge by Rs. 96,000. If the bridge is replaced by a new prestressed concrete bridge, the scrap value of the steel would exceed the demolition cost by Rs. 4,20,000. Assume that the money costs the state 10%. What would you recommend?

Solution There are two alternatives:

1. Reinforce the existing bridge.
2. Replace the existing bridge by a new prestressed concrete bridge.

Alternative 1—Reinforce the existing bridge

Cost of reinforcement (P) = Rs. 6,60,000
Salvage value after 5 years (F) = Rs. 4,00,000
The excess annual maintenance cost over prestressed concrete bridge (A)
= Rs. 96,000
Life (n) = 5 years
Interest rate (i) = 10%

The cash flow diagram of alternative 1 is illustrated in Fig. 8.9.

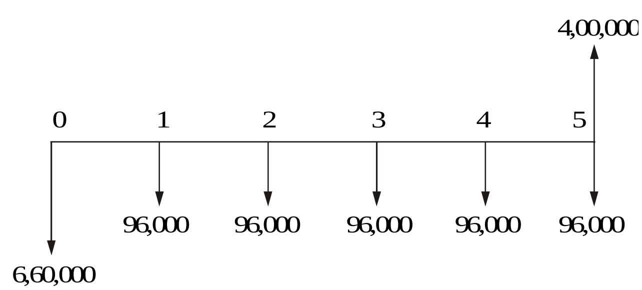


Fig. 8.9 Cash flow diagram for alternative 1.

The annual equivalent cost of the alternative 1 is computed as

$$\begin{aligned} AE(10\%) &= (P - F)(A/P, 10\%, 5) + F \times i + A \\ &= (6,60,000 - 4,00,000)(0.2638) + 4,00,000 \times 0.10 + 96,000 \\ &= \text{Rs. } 2,04,588 \end{aligned}$$

Alternative 2—Replace the existing bridge by a new prestressed concrete bridge

Cost of prestressed concrete bridge (P) = Rs. 15,00,000
Excess scrap value of steel over the demolition cost of the current bridge (X) = Rs. 4,20,000

Life (n) = 40 years
Interest rate (i) = 10%

Note that the excess maintenance cost of the reinforced bridge over the prestressed concrete bridge is included in alternative 1.

The cash flow diagram for alternative 2 is shown in Fig. 8.10.

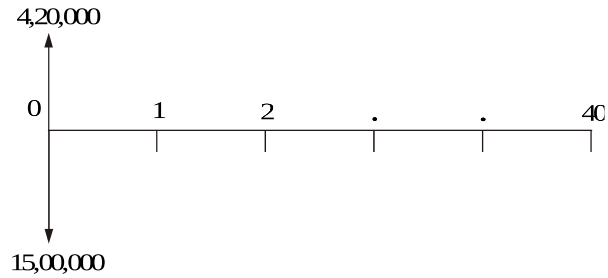


Fig. 8.10 Cash flow diagram for alternative 2.

The annual equivalent cost of alternative 2 is calculated as

$$
\begin{aligned}
AE(10\%) &= (P - X)\,(A/P,\ 10\%,\ 40)\\
&= (15{,}00{,}000 - 4{,}20{,}000) \times 0.1023\\
&= \text{Rs. } 1{,}10{,}484
\end{aligned}
$$

The annual equivalent cost of alternative 2 is less than that of alternative 1. Based on equal lives comparison over 40 years, alternative 2 is selected as the best alternative.

Thus, it is suggested to go in for prestressed concrete bridge.

EXAMPLE 8.7 Three years back, a municipality purchased a 10 hp motor for pumping drinking water. Its useful life was estimated to be 10 years. Due to the fast development of that locality, the municipality is unable to meet the current demand for water with the existing motor. The municipality can cope with the situation either by augmenting an additional 5 hp motor or replacing the existing 10 hp motor with a new 15 hp motor. The details of these motors are now tabulated.

	Old 10 hp motor	*New 5 hp motor*	*New 15 hp motor*
Purchase cost (P) Rs.	25,000	10,000	35,000
Life in years (n)	10	7	7
Salvage value at the end of machine life (Rs.)	1,500	800	4,000
Annual operating & maintenance cost (Rs.)	1,600	1,000	500

The current market value of the 10 hp motor is Rs. 10,000. Using an interest rate of 15%, find the best alternative.

Solution There are two alternatives to cope with the situation:

1. Augmenting the present 10 hp motor with an additional 5 hp motor.
2. Replacing the present 10 hp motor with a new 15 hp motor.

Alternative 1—Augmenting the present 10 hp motor with an additional 5 hp motor

Total annual equivalent cost = Annual equivalent cost of 10 hp motor + Annual equivalent cost of 5 hp motor

Calculation of annual equivalent cost of 10 hp Motor

Present market value of the 10 hp motor (P) = Rs. 10,000
Remaining life (n) = 7 years
Salvage value at the end of motor life (F) = Rs. 1,500
Annual operation and maintenance cost (A) = Rs. 1,600
Interest rate, i = 15%

The cash flow diagram of this alternative is shown in Fig. 8.11.

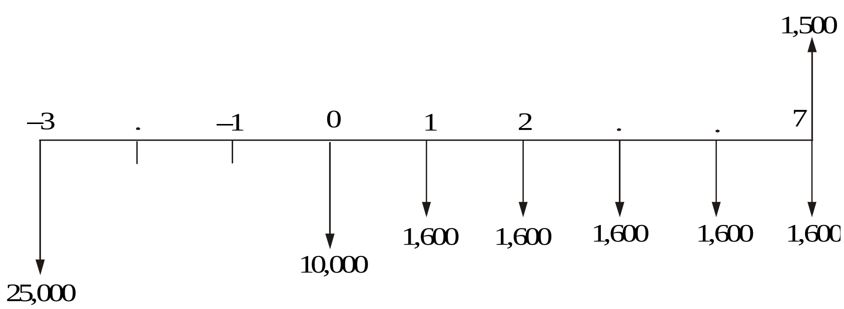


Fig. 8.11 Cash flow diagram for 10 hp motor.

The annual equivalent cost of the 10 hp motor is calculated as

$$\begin{aligned} AE(15\%) &= (P - F)(A/P, 15\%, 7) + F \times i + A \\ &= (10,000 - 1,500)(0.2404) + 1,500 \times 0.15 + 1,600 \\ &= \text{Rs. } 3,868.40 \end{aligned}$$

Calculation of annual equivalent cost of 5 hp motor

Purchase value of the 5 hp motor (P) = Rs. 10,000
Life (n) = 7 years
Salvage value at the end of motor life (F) = Rs. 800
Annual operation and maintenance cost (A) = Rs. 1,000
Interest rate, i = 15%

The cash flow diagram of the 5 hp motor is illustrated in Fig. 8.12.

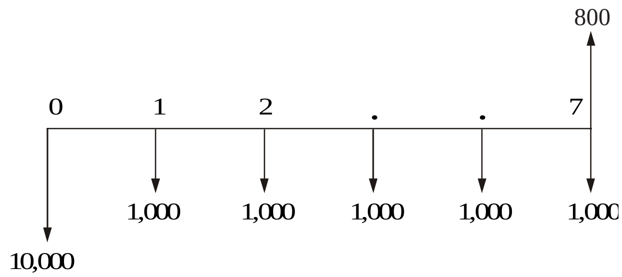


Fig. 8.12 Cash flow diagram for 5 hp motor.

The annual equivalent cost of the 5 hp motor is computed as

$$AE(15\%) = (P - F)(A/P, 15\%, 7) + F \times i + A$$
$$= (10{,}000 - 800)(0.2404) + 800 \times 0.15 + 1{,}000$$
$$= \text{Rs. } 3{,}331.68$$

Total annual equivalent cost of the alternative 1 = Rs. 3,868.40
+ Rs. 3,331.68
= Rs. 7,200.08

Alternative 2—Replacing the present 10 hp motor with a new 15 hp motor

Purchase value of the 15 hp motor (P) = Rs. 35,000
Life (n) = 7 years
Salvage value at the end of motor life (F) = Rs. 4,000
Annual operation and maintenance cost (A) = Rs. 500
Interest rate, i = 15%

The cash flow diagram of this alternative is shown in Fig. 8.13.

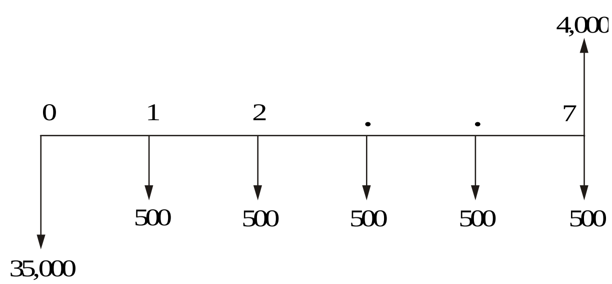


Fig. 8.13 Cash flow diagram for alternative 2.

The annual equivalent cost of alternative 2 is

$$AE(15\%) = (P - F)\ (A/P, 15\%, 7) + F \times i + A$$
$$= (35{,}000 - 4{,}000)(0.2404) + 4{,}000 \times 0.15 + 500$$
$$= \text{Rs. } 8{,}552.40$$

The total annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, it is suggested that the present 10 hp motor be augmented with a new 5 hp motor.

EXAMPLE 8.8 A machine was purchased two years ago for Rs. 10,000. Its annual maintenance cost is Rs. 750. Its life is six years and its salvage value at the end of its life is Rs. 1,000. Now, a company is offering a new machine at a cost of Rs. 10,000. Its life is four years and its salvage value at the end of its life is Rs. 4,000. The annual maintenance cost of the new machine is Rs. 500. The company which is supplying the new machine is willing to take the old machine for Rs. 8,000 if it is replaced by the new machine. Assume an interest rate of 12%, compounded annually.

(a) Find the comparative use value of the old machine.
(b) Is it advisable to replace the old machine?

Solution Old machine Let the comparative use value of the old machine be X.

Remaining life (n) = 4 years.
Salvage value of the old machine (F) = Rs. 1,000
Annual maintenance cost (A) = Rs. 750
Interest rate, i = 12%

The cash flow diagram of the old machine is depicted in Fig. 8.14.

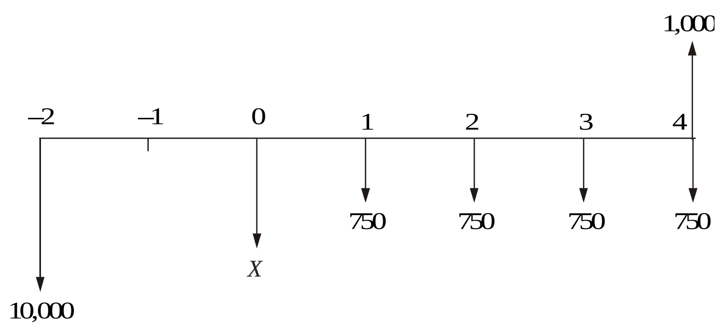


Fig. 8.14 Cash flow diagram for old machine.

The annual equivalent cost of the old machine is computed as

$$AE(12\%) = (X - F)(A/P,\ 12\%,\ 4) + F \times i + A$$
$$= (X - 1{,}000)(0.3292) + 1{,}000 \times 0.12 + 750$$

New machine

Cost of the new Machine (P) = Rs. 10,000
Life (n) = 4 years.
Salvage value of the new machine (F) = Rs. 4,000
Annual Maintenance cost (A) = Rs. 500
Interest rate, i = 12%

The cash flow diagram of the new machine is illustrated in Fig. 8.15.

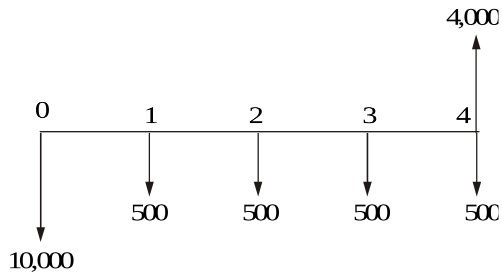


Fig. 8.15 Cash flow diagram for new machine.

The annual equivalent cost of the new machine is illustrated as

$$
\begin{aligned}
AE(12\%) &= (P - F)\,(A/P,\ 12\%,\ 4) + F \times i + A \\
&= (10{,}000 - 4{,}000)(0.3292) + 4{,}000 \times 0.12 + 500 \\
&= \text{Rs. } 2{,}955.20
\end{aligned}
$$

Now, equate the annual equivalent costs of the two alternatives and solve for X.

$$(X - 1{,}000)(0.3292) + 1{,}000 \times 0.12 + 750 = 2{,}955.20$$

$$X = \text{Rs. } 7{,}334.14$$

The comparative use value of the old machine is Rs. 7,334.14, which is less than the price (Rs. 8,000) offered by the company which is supplying the new machine in the event of replacing the old machine by the new machine.

Therefore, it is advisable to replace the old machine with the new one.

8.6 SIMPLE PROBABILISTIC MODEL FOR ITEMS WHICH FAIL COMPLETELY

Electronic items like transistors, resistors, tubelights, bulbs, etc. could fail all of a sudden, instead of gradual deterioration. The failure of the item may result in complete breakdown of the system. The system may contain a collection of such items or just one item, say a tubelight. Therefore, we use some replacement policy for such items which would avoid the possibility of a complete breakdown.

The following are the replacement policies which are applicable for this situation.

(i) Individual replacement policy. Under this policy, an item is replaced immediately after its failure.

(ii) Group replacement policy. Under this policy, the following decision is made:

At what equal intervals are all the items to be replaced simultaneously with a provision to replace the items individually which fail during a fixed group replacement period?

There is a trade-off between the individual replacement policy and the group replacement policy. Hence, for a given problem, each of the replacement policies is evaluated and the most economical policy is selected for implementation. This is explained with two numerical problems.

EXAMPLE 8.9 The failure rates of transistors in a computer are summarized in Table 8.5.

Table 8.5 Failure Rates of Transistors in Computers

End of week	1	2	3	4	5	6	7
Probability of failure to date	0.07	0.18	0.30	0.48	0.69	0.89	1.00

The cost of replacing an individual failed transistor is Rs. 9. If all the transistors are replaced simultaneously, it would cost Rs. 3.00 per transistor. Any one of the following two options can be followed to replace the transistors:

(a) Replace the transistors individually when they fail (individual replacement policy).

(b) Replace all the transistors simultaneously at fixed intervals and replace the individual transistors as they fail in service during the fixed interval (group replacement policy).

Find out the optimal replacement policy, i.e. individual replacement policy or group replacement policy. If group replacement policy is optimal, then find at what equal intervals should all the transistors be replaced.

Solution Assume that there are 100 transistors in use.

Let, p_i be the probability that a transistor which was new when placed in position for use, fails during the ith week of its life. Hence,

$$p_1 = 0.07, \quad p_2 = 0.11, \quad p_3 = 0.12, \quad p_4 = 0.18,$$
$$p_5 = 0.21, \quad p_6 = 0.20, \quad p_7 = 0.11$$

Since the sum of p_is is equal to 1 at the end of the 7th week, the transistors are sure to fail during the seventh week.

Assume that

(a) transistors that fail during a week are replaced just before the end of the week, and

(b) the actual percentage of failures during a week for a sub-group of transistors with the same age is same as the expected percentage of failures during the week for that sub-group of transistors.

Let

Ni = the number of transistors replaced at the end of the ith week

$N0$ = number of transistors replaced at the end of the week 0 (or at the beginning of the first week).

= 100

$N1$ = number of transistors replaced at the end of the 1st week
$= N0 \times p1 = 100 \times 0.07 = 7$
$N2$ = number of transistors replaced at the end of the 2nd week
$= N0 \times p2 + N1 \times p1$
$= 100 \times 0.11 + 7 \times 0.07 = 12$
$N3 = N0 \times p3 + N1 \times p2 + N2 \times p1$
$= 100 \times 0.12 + 7 \times 0.11 + 12 \times 0.07$
$= 14$
$N4 = N0 \times p4 + N1 \times p3 + N2 \times p2 + N3 \times p1$
$= 100 \times 0.18 + 7 \times 0.12 + 12 \times 0.11 + 14 \times 0.07$
$= 21$
$N5 = N0 \times p5 + N1 \times p4 + N2 \times p3 + N3 \times p2 + N4 \times p1$
$= 100 \times 0.21 + 7 \times 0.18 + 12 \times 0.12 + 14 \times 0.11 + 21 \times 0.07$
$= 27$
$N6 = N0 \times p6 + N1 \times p5 + N2 \times p4 + N3 \times p3 + N4 \times p2 + N5 \times p1$
$= 100 \times 0.2 + 7 \times 0.21 + 12 \times 0.18 + 14 \times 0.12 + 21 \times 0.11 + 27 \times 0.07$
$= 30$
$N7 = N0 \times p7 + N1 \times p6 + N2 \times p5 + N3 \times p4 + N4 \times p3 + N5 \times p2 + N6 \times p1$
$= 100 \times 0.11 + 7 \times 0.2 + 12 \times 0.21 + 14 \times 0.18 + 21 \times 0.12 + 27 \times 0.11 + 30 \times 0.07$
$= 25$

Calculation of individual replacement cost

$$\text{Expected life of each transistor} = \sum_{i=1}^{7} i \times p_i$$

$$= 1 \times 0.07 + 2 \times 0.11 + 3 \times 0.12 + 4 \times 0.18 + 5 \times 0.21 + 6 \times 0.2 + 7 \times 0.11$$

$$= 4.39 \text{ weeks}$$

Average No. of failures/week = 100/4.39 = 23 (approx.)

Therefore,

Cost of individual replacement

= (No. of failures/week × Individual replacement cost/transistor)

= 23 × 9 = Rs. 207.

Determination of group replacement cost

Cost of transistor when replaced simultaneously = Rs. 3
Cost of transistor when replaced individually = Rs. 9

The costs of group replacement policy for several replacement periods are summarized in Table 8.6.

Table 8.6 Calculations of Cost for Preventive Maintenance

End of week A	*Cost of replacing 100 transistors at a time* B (Rs.)	*Cost of replacing transistors individually during given replacement period* C (Rs.)	*Total cost* (B + C) D (Rs.)	*Average cost/week* (D/A) E (Rs.)
1	300	$7 \times 9 = 63$	363	363.00
2	300	$(7 + 12) \times 9 = 171$	471	235.50
3	300	$(7 + 12 + 14) \times 9 = 297$	597	199.00
4	300	$(7 + 12 + 14 + 21) \times 9 = 486$	786	196.50*
5	300	$(7 + 12 + 14 + 21 + 27) \times 9 = 729$	1,029	205.80
6	300	$(7 + 12 + 14 + 21 + 27 + 30) \times 9 = 999$	1,299	216.50
7	300	$(7 + 12 + 14 + 21 + 27 + 30 + 25) \times 9 = 1{,}224$	1,524	217.71

*Indicates the minimum average cost/week.

From Table 8.6, it is clear that the average cost/week is minimum for the fourth week. Hence, the group replacement period is four weeks.

Individual replacement cost/week = Rs. 207
Minimum group replacement cost/week = Rs. 196.50

Since the minimum group replacement cost/week is less than the individual replacement cost/week, the group replacement policy is the best, and hence all the transistors should be replaced once in four weeks and the transistors which fail during this four-week period are to be replaced individually.

EXAMPLE 8.10 An electronic equipment contains 1,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs. 10. If all the resistors are replaced at the same time, the cost per resistor is Rs. 4. The per cent surviving, $S(i)$ at the end of month i is tabulated as follows:

i	0	1	2	3	4	5	6
$S(i)$	100	96	89	68	37	13	0

What is the optimum replacement plan?

Solution Let pi be the probability of failure during the month i. Then,

$$p1 = (100 - 96)/100 = 0.04 \qquad p4 = (68 - 37)/100 = 0.31$$
$$p2 = (96 - 89)/100 = 0.07 \qquad p5 = (37 - 13)/100 = 0.24$$
$$p3 = (89 - 68)/100 = 0.21 \qquad p6 = (13 - 0)/100 = 0.13$$

It is clear that no resistor can survive beyond six months. Hence, a resistor which has survived for five months would certainly fail during the sixth month. We assume that the resistors failing during a month are accounted at the end of the month.

Let

Ni = No. of resistors replaced at the end of the ith month.

$$\begin{aligned}
N0 &= 1{,}000\\
N1 &= N0p1 = 1{,}000 \times 0.04 = 40\\
N2 &= N0p2 + N1p1 = 1{,}000 \times 0.07 + 40 \times 0.04 = 72\\
N3 &= N0p3 + N1p2 + N2p1 = 1{,}000 \times 0.21 + 40 \times 0.07\\
&\quad + 72 \times 0.04 = 216\\
N4 &= N0p4 + N1p3 + N2p2 + N3p1\\
&= 1{,}000 \times 0.31 + 40 \times 0.21 + 72 \times 0.07 + 216 \times 0.04 = 332\\
N5 &= N0p5 + N1p4 + N2p3 + N3p2 + N4p1\\
&= 1{,}000 \times 0.24 + 40 \times 0.31 + 72 \times 0.21 + 216 \times 0.07\\
&\quad + 332 \times 0.04 = 296\\
N6 &= N0p6 + N1p5 + N2p4 + N3p3 + N4p2 + N5p1\\
&= 1{,}000 \times 0.13 + 40 \times 0.24 + 72 \times 0.31 + 216 \times 0.21 + 332 \times 0.07\\
&\quad + 296 \times 0.04\\
&= 242
\end{aligned}$$

Determination of individual replacement cost

$$\begin{aligned}
\text{Expected life of each resistor} &= \sum_{i=1}^{6} i \times p_i\\
&= 1 \times 0.04 + 2 \times 0.07 + 3 \times 0.21 + 4 \times 0.31\\
&\quad + 5 \times 0.24 + 6 \times 0.13\\
&= 4.03 \text{ months.}
\end{aligned}$$

Average number of failures/month = 1,000/4.03 = 248 (approx.)

Therefore,

Cost of individual replacement
= (No. of failures/month × individual replacement cost/resistor)
= 248 × 10 = Rs. 2,480.

Determination of group replacement cost

Cost/resistor when replaced simultaneously = Rs. 4.00
Cost/resistor when replaced individually = Rs. 10.00

The costs of group replacement policy for several replacement periods are summarized in Table 8.7.

Table 8.7 Calculations of Costs for Preventive Maintenance

End of month	*Cost of replacing 1,000 resistors at a time*	*Cost of replacing resistors individually during given replacement period*	*Total cost (B + C)*	*Average cost/ month (D/A)*
A	B (Rs.)	C (Rs.)	D (Rs.)	E (Rs.)
1	4,000	40 × 10 = 400	4,400	4,400.00
2	4,000	(40 + 72)10 = 1,120	5,120	2,560.00
3	4,000	(40 + 72 + 216)10 = 3,280	7,280	2,426.67*
4	4,000	(40 + 72 + 216 + 332)10 = 6,600	10,600	2,650.00

*Indicates the minimum average cost/month.

From Table 8.7, it is clear that the average cost/month is minimum for the third month. Thus, the group replacement period is three months.

Summary

Individual replacement cost/month = Rs. 2,480.00
Minimum group replacement cost/month = Rs. 2,426.67

Since the minimum group replacement cost/month is less than the individual replacement cost/month, the group replacement policy is the best and hence all the resistors are to be replaced once in three months and the resistors which fail during this three months period are to be replaced individually.

QUESTIONS

1. List and explain the different types of maintenance.

2. Discuss the reasons for replacement.

3. Define 'economic life' of an equipment.

4. Distinguish between breakdown maintenance and preventive maintenance.

5. A firm is considering replacement of an equipment, whose first cost is Rs. 1,750 and the scrap value is negligible at any year. Based on experience, it was found that the maintenance cost is zero during the first year and it increases by Rs. 100 every year thereafter.

(a) When should the equipment be replaced if i = 0%?

(b) When should the equipment be replaced if i = 12%?

6. The following table gives the operation cost, maintenance cost and salvage value at the end of every year of a machine whose purchase value is Rs. 20,000.

(a) Find the economic life of the machine assuming interest rate (i) of 0%

(b) Find the economic life of the machine assuming interest rate of 15%.

End of year (n)	*Operation cost*	*Maintenance cost*	*Salvage value*
1	2,000	200	10,000
2	3,000	300	9,000
3	4,000	400	8,000
4	5,000	500	7,000
5	6,000	600	6,000
6	7,000	700	5,000
7	8,000	800	4,000
8	9,000	900	3,000
9	10,000	1,000	2,000
10	11,000	1,100	1,000

7. A manufacturer is offered two machines A and B. A is priced at Rs. 8,000 and maintenance costs are estimated at Rs. 500 for the first year and an equal increment of Rs. 100 from year 2 to year 5, and Rs. 1,500 for the sixth year and an equal increment of Rs. 500 from year 7 onwards.

Machine B which has the same capacity is priced at Rs. 6,000. The maintenance costs of the machine B are estimated at Rs. 1,000 for the first year and an equal yearly increment of Rs. 200 thereafter.

If the money is worth 15% per year, which machine should be purchased? (Assume that the scrap value of each of the machines is negligible at any year.)

8. Three years back, a machine was purchased at a cost of Rs. 3,00,000 to be useful for 10 years. Its salvage value at the end of its estimated life is Rs. 50,000. Its annual maintenance cost is Rs. 40,000. The market value of the present machine is Rs. 2,00,000. A new machine to cater to the need of the present machine is available at Rs. 2,50,000 to be useful for 7 years. Its annual maintenance cost is Rs. 14,000. The salvage value of the new machine is Rs. 20,000. Using an interest rate of 15%, find whether it is worth replacing the present machine with the new one.

9. A steel highway bridge must either be reinforced or replaced. Reinforcement would cost Rs. 8,60,000 and would make the bridge adequate for an additional seven years of service. If it is reinforced, it is estimated that its net salvage value would be Rs. 5,00,000 at the time it is retired from service. The new prestressed concrete bridge would cost Rs. 18,00,000 and would meet the foreseeable requirements of the next 35 years. Such a bridge would have no salvage value. It is estimated that the annual maintenance cost of the reinforced bridge would exceed that of the concrete bridge by Rs. 1,00,000. If the bridge is replaced by a new prestressed concrete bridge, the scrap value of the steel would exceed the demolition cost by Rs. 5,20,000. Assume that the money costs the state 12%. What would you recommend?

10. Three years back, a municipality purchased a 10 hp motor for pumping drinking water. Its useful life was estimated to be 10 years. Its annual operation and maintenance cost is Rs. 1,500. Due to rapid development of that locality, the municipality is unable to meet the current demand for water with the existing motor. The municipality can cope with the situation either by augmenting an additional 5 hp motor or replacing the existing 10 hp motor with a new 15 hp motor. The details of these motors are given in the following table.

	Old 10 hp motor	*New 5 hp motor*	*New 15 hp motor*
Purchase cost (P) Rs.	20,000	8,000	30,000
Life in years (n)	10	7	7
Salvage value at the end of machine life (Rs.)	1,200	800	3,500
Annual operating & maintenance cost (Rs.)	1,500	900	450

The current market value of the 10 hp motor is Rs. 10,000. Using an interest rate of 18% find the best alternative.

11. The failure rates of transistors in a computer are summarized in the following table.

End of week	1	2	3	4	5	6	7
Probability of failure to date	0.09	0.17	0.27	0.50	0.65	0.90	1.00

The cost of replacing an individual failed transistor is Rs. 8. If all the transistors are replaced simultaneously, it would cost Rs. 4 per transistor. Any one of the following two options can be followed to replace the transistors:

(a) Replace the transistors individually when they fail (individual replacement policy).

(b) Replace all the transistors simultaneously at fixed intervals and replace the individual transistors as they fail in service during the fixed interval (Group replacement policy).

Find out which is the optimal replacement policy, i.e. the individual replacement policy or the group replacement policy. If the group replacement policy is optimal, then find at what equal intervals should all the transistors be replaced.

12. An electronic equipment contains 1,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs. 7. If all the resistors are replaced at the same time, the cost per resistor is Rs. 4. The per cent surviving, $S(i)$ at the end of month i is tabulated now.

i	0	1	2	3	4	5	6
$S(i)$	100	90	85	65	35	15	0

What is the optimum replacement plan?

9

DEPRECIATION

9.1 INTRODUCTION

Any equipment which is purchased today will not work for ever. This may be due to wear and tear of the equipment or obsolescence of technology. Hence, it is to be replaced at the proper time for continuance of any business. The replacement of the equipment at the end of its life involves money. This must be internally generated from the earnings of the equipment. The recovery of money from the earnings of an equipment for its replacement purpose is called *depreciation fund* since we make an assumption that the value of the equipment decreases with the passage of time. Thus, the word "depreciation" means *decrease* in value of any physical asset with the passage of time.

9.2 METHODS OF DEPRECIATION

There are several methods of accounting depreciation fund. These are as follows:

1. Straight line method of depreciation
2. Declining balance method of depreciation
3. Sum of the years—digits method of depreciation
4. Sinking-fund method of depreciation
5. Service output method of depreciation

These are now discussed in detail.

9.2.1 Straight Line Method of Depreciation

In this method of depreciation, a fixed sum is charged as the depreciation amount throughout the lifetime of an asset such that the accumulated sum at the end of the life of the asset is exactly equal to the purchase value of the asset. Here, we make an important assumption that inflation is absent.

Let

P = first cost of the asset,

F = salvage value of the asset,

n = life of the asset,
B_t = book value of the asset at the end of the period t,
D_t = depreciation amount for the period t.

The formulae for depreciation and book value are as follows:

$$D_t = (P - F)/n$$
$$B_t = B_{t-1} - D_t = P - t \times [(P - F)/n]$$

EXAMPLE 9.1 A company has purchased an equipment whose first cost is Rs. 1,00,000 with an estimated life of eight years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Determine the depreciation charge and book value at the end of various years using the straight line method of depreciation.

Solution

$$\begin{aligned} P &= \text{Rs. } 1{,}00{,}000 \\ F &= \text{Rs. } 20{,}000 \\ n &= 8 \text{ years} \\ D_t &= (P - F)/n \\ &= (1{,}00{,}000 - 20{,}000)/8 \\ &= \text{Rs. } 10{,}000 \end{aligned}$$

In this method of depreciation, the value of D_t is the same for all the years. The calculations pertaining to B_t for different values of t are summarized in Table 9.1.

Table 9.1 D_t and B_t Values under Straight line Method of Depreciation

End of year (t)	*Depreciation* (D_t)	*Book value* ($B_t = B_{t-1} - D_t$)
0		1,00,000
1	10,000	90,000
2	10,000	80,000
3	10,000	70,000
4	10,000	60,000
5	10,000	50,000
6	10,000	40,000
7	10,000	30,000
8	10,000	20,000

If we are interested in computing D_t and B_t for a specific period (t), the formulae can be used. In this approach, it should be noted that the depreciation is the same for all the periods.

EXAMPLE 9.2 Consider Example 9.1 and compute the depreciation and the book value for period 5.

$$
\begin{aligned}
P &= \text{Rs. } 1{,}00{,}000 \\
F &= \text{Rs. } 20{,}000 \\
n &= 8 \text{ years} \\
D_5 &= (P - F)/n \\
&= (1{,}00{,}000 - 20{,}000)/8 \\
&= \text{Rs. } 10{,}000 \text{ (This is independent of the time period.)} \\
B_t &= P - t \times (P - F)/n \\
B_5 &= 1{,}00{,}000 - 5 \times (1{,}00{,}000 - 20{,}000)/8 \\
&= \text{Rs. } 50{,}000
\end{aligned}
$$

9.2.2 Declining Balance Method of Depreciation

In this method of depreciation, a constant percentage of the book value of the previous period of the asset will be charged as the depreciation amount for the current period. This approach is a more realistic approach, since the depreciation charge decreases with the life of the asset which matches with the earning potential of the asset. The book value at the end of the life of the asset may not be exactly equal to the salvage value of the asset. This is a major limitation of this approach.

Let

P = first cost of the asset,

F = salvage value of the asset,

n = life of the asset,

B_t = book value of the asset at the end of the period t,

K = a fixed percentage, and

D_t = depreciation amount at the end of the period t.

The formulae for depreciation and book value are as follows:

$$
\begin{aligned}
D_t &= K \times B_{t-1} \\
B_t &= B_{t-1} - D_t = B_{t-1} - K \times B_{t-1} \\
&= (1 - K) \times B_{t-1}
\end{aligned}
$$

The formulae for depreciation and book value in terms of P are as follows:

$$
\begin{aligned}
D_t &= K(1 - K)^{t-1} \times P \\
B_t &= (1 - K)^t \times P
\end{aligned}
$$

While availing income-tax exception for the depreciation amount paid in each year, the rate K is limited to at the most $2/n$. If this rate is used, then the corresponding approach is called the *double declining balance method of depreciation.*

EXAMPLE 9.3 Consider Example 9.1 and demonstrate the calculations of the declining balance method of depreciation by assuming 0.2 for K.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

K = 0.2

The calculations pertaining to D_t and B_t for different values of t are summarized in Table 9.2 using the following formulae:

$$D_t = K \times B_{t-1}$$

$$B_t = B_{t-1} - D_t$$

Table 9.2 D_t and B_t according to Declining Balance Method of Depreciation

End of year (n)	*Depreciation* (D_t)	*Book value* (B_t)
0		1,00,000.00
1	20,000.00	80,000.00
2	16,000.00	64,000.00
3	12,800.00	51,200.00
4	10,240.00	40,960.00
5	8,192.00	32,768.00
6	6,553.60	26,214.40
7	5,242.88	20,971.52
8	4,194.30	16,777.22

If we are interested in computing D_t and B_t for a specific period t, the respective formulae can be used.

EXAMPLE 9.4 Consider Example 9.1 and calculate the depreciation and the book value for period 5 using the declining balance method of depreciation by assuming 0.2 for K.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

K = 0.2

$$D_t = K(1 - K)^{t-1} \times P$$

$$D_5 = 0.2(1 - 0.2)^4 \times 1,00,000$$

$$= \text{Rs. } 8,192$$

$$B_t = (1 - K)^t \times P$$
$$B_5 = (1 - 0.2)^5 \times 1{,}00{,}000$$
$$= \text{Rs. } 32{,}768$$

9.2.3 Sum-of-the-Years-Digits Method of Depreciation

In this method of depreciation also, it is assumed that the book value of the asset decreases at a decreasing rate. If the asset has a life of eight years, first the sum of the years is computed as

$$\text{Sum of the years} = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$
$$= 36 = n(n + 1)/2$$

The rate of depreciation charge for the first year is assumed as the highest and then it decreases. The rates of depreciation for the years 1–8, respectively are as follows: 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36, and 1/36.

For any year, the depreciation is calculated by multiplying the corresponding rate of depreciation with $(P - F)$.

$$D_t = \text{Rate} \times (P - F)$$
$$B_t = B_{t-1} - D_t$$

The formulae for D_t and B_t for a specific year t are as follows:

$$D_t = \frac{n - t + 1}{n(n + 1)/2}\,(P - F)$$

$$B_t = (P - F)\,\frac{(n - t)}{n}\,\frac{(n - t + 1)}{(n + 1)} + F$$

EXAMPLE 9.5 Consider Example 9.1 and demonstrate the calculations of the sum-of-the-years-digits method of depreciation.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

$$\text{Sum} = n(n + 1)/2 = 8 \times 9/2 = 36$$

The rates for years 1–8, are respectively 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36 and 1/36.

The calculations of D_t and B_t for different values of t are summarized in Table 9.3 using the following formulae:

$$D_t = \text{Rate} \times (P - F)$$
$$B_t = B_{t-1} - D_t$$

Table 9.3 D_t and B_t under Sum-of-the-years-digits Method of Depreciation

End of year (n)	*Depreciation* (D_t)	*Book value* (B_t)
0		1,00,000.00
1	17,777.77	82,222.23
2	15,555.55	66,666.68
3	13,333.33	53,333.35
4	11,111.11	42,222.24
5	8,888.88	33,333.36
6	6,666.66	26,666.70
7	4,444.44	22,222.26
8	2,222.22	20,000.04

If we are interested in calculating D_t and B_t for a specific t, then the usage of the formulae would be better.

EXAMPLE 9.6 Consider Example 9.1 and find the depreciation and book value for the 5th year using the sum-of-the-years-digits method of depreciation.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

$$D_t = \frac{n - t + 1}{n(n + 1)/2}\,(P - F)$$

$$D_5 = \frac{8 - 5 + 1}{8(8 + 1)/2}\,(1,00,000 - 20,000)$$

$$= \text{Rs. } 8,888.88$$

$$B_t = (P - F)\,\frac{n - t}{n}\;\frac{n - t + 1}{n + 1} + F$$

$$B_5 = (1,00,000 - 20,000)\,\frac{8 - 5}{8}\;\frac{8 - 5 + 1}{8 + 1} + 20,000$$

$$= 80,000 \times (3/8) \times (4/9) + 20,000$$

$$= \text{Rs. } 33,333.33$$

9.2.4 Sinking Fund Method of Depreciation

In this method of depreciation, the book value decreases at increasing rates with respect to the life of the asset. Let

P = first cost of the asset,

F = salvage value of the asset,

n = life of the asset,

i = rate of return compounded annually,

A = the annual equivalent amount,

B_t = the book value of the asset at the end of the period t, and

D_t = the depreciation amount at the end of the period t.

The loss in value of the asset ($P - F$) is made available an the form of cumulative depreciation amount at the end of the life of the asset by setting up an equal depreciation amount (A) at the end of each period during the lifetime of the asset.

$$A = (P - F) \times [A/F, i, n]$$

The fixed sum depreciated at the end of every time period earns an interest at the rate of i% compounded annually, and hence the actual depreciation amount will be in the increasing manner with respect to the time period. A generalized formula for D_t is

$$D_t = (P - F) \times (A/F, i, n) \times (F/P, i, t - 1)$$

The formula to calculate the book value at the end of period t is

$$B_t = P - (P - F) (A/F, i, n) (F/A, i, t)$$

The above two formulae are very useful if we have to calculate D_t and B_t for any specific period. If we calculate D_t and B_t for all the periods, then the tabular approach would be better.

EXAMPLE 9.7 Consider Example 9.1 and give the calculations regarding the sinking fund method of depreciation with an interest rate of 12%, compounded annually.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

i = 12%

$$\begin{aligned} A &= (P - F) \times [A/F, 12\%, 8] \\ &= (1,00,000 - 20,000) \times 0.0813 \\ &= \text{Rs. } 6,504 \end{aligned}$$

In this method of depreciation, a fixed amount of Rs. 6,504 will be depreciated at the end of every year from the earning of the asset. The depreciated amount will earn interest for the remaining period of life of the asset at an interest rate of 12%, compounded annually. For example, the calculations of net depreciation for some periods are as follows:

Depreciation at the end of year 1 (D_1) = Rs. 6,504.

Depreciation at the end of year 2 (D_2) = 6,504 + 6,504 × 0.12

= Rs. 7,284.48

Depreciation at the end of the year 3 (D_3)

$$= 6{,}504 + (6{,}504 + 7{,}284.48) \times 0.12$$
$$= \text{Rs. } 8{,}158.62$$

Depreciation at the end of year 4 (D_4)

$$= 6{,}504 + (6{,}504 + 7{,}284.48 + 8{,}158.62) \times 0.12$$
$$= \text{Rs. } 9{,}137.65$$

These calculations along with book values are summarized in Table 9.4.

Table 9.4 D_t and B_t according to Sinking Fund Method of Depreciation

End of year t	*Fixed depreciation* (Rs.)	*Net depreciation* D_t (Rs.)	*Book value* B_t (Rs.)
0	6,504	–	1,00,000.00
1	6,504	6,504.00	93,496.00
2	6,504	7,284.48	86,211.52
3	6,504	8,158.62	78,052.90
4	6,504	9,137.65	68,915.25
5	6,504	10,234.17	58,681.08
6	6,504	11,462.27	47,218.81
7	6,504	12,837.74	34,381.07
8	6,504	14,378.27	20,002.80

$$B_t = B_{t-1} - D_t$$

EXAMPLE 9.8 Consider Example 9.1 and compute D_5 and B_7 using the sinking fund method of depreciation with an interest rate of 12%, compounded annually.

Solution

P = Rs. 1,00,000

F = Rs. 20,000

n = 8 years

i = 12%

$$D_t = (P - F)\,(A/F, i, n)\,(F/P, i, t - 1)$$
$$D_5 = (P - F)\,(A/F, 12\%, 8)\,(F/P, 12\%, 4)$$
$$= (1{,}00{,}000 - 20{,}000) \times 0.0813 \times 1.574$$
$$= \text{Rs. } 10{,}237.30$$

This is almost the same as the corresponding value given in the table. The minor difference is due to truncation error.

$$B_t = P - (P - F)\,(A/F, i, n)\,(F/A, i, t)$$
$$B_7 = P - (P - F)\,(A/F, 12\%, 8)\,(F/A, 12\%, 7)$$
$$= 1{,}00{,}000 - (1{,}00{,}000 - 20{,}000) \times 0.0813 \times 10.089$$
$$= 34{,}381.10$$

9.2.5 Service Output Method of Depreciation

In some situations, it may not be realistic to compute depreciation based on time period. In such cases, the depreciation is computed based on service rendered by an asset. Let

P = first cost of the asset

F = salvage value of the asset

X = maximum capacity of service of the asset during its lifetime

x = quantity of service rendered in a period.

Then, the depreciation is defined per unit of service rendered:

$$\text{Depreciation/unit of service} = (P - F)/X$$

$$\text{Depreciation for } x \text{ units of service in a period} = \frac{P - F}{X}(x)$$

EXAMPLE 9.9 The first coat of a road laying machine is Rs. 80,00,000. Its salvage value after five years is Rs. 50,000. The length of road that can be laid by the machine during its lifetime is 75,000 km. In its third year of operation, the length of road laid is 2,000 km. Find the depreciation of the equipment for that year.

Solution

P = Rs. 80,00,000

F = Rs. 50,000

X = 75,000 km

x = 2,000 km

$$\text{Depreciation for } x \text{ units of service in a period} = \frac{P - F}{X}x$$

$$\text{Depreciation for year 3} = \frac{(80{,}00{,}000 - 50{,}000)}{75{,}000} \times 2{,}000$$

$$= \text{Rs. } 2{,}12{,}000$$

QUESTIONS

1. Define the following:
(a) Depreciation
(b) Book value

2. Distinguish between declining balance method of depreciation and double declining balance method of depreciation.

3. The Alpha Drug Company has just purchased a capsulating machine for Rs. 20,00,000. The plant engineer estimates that the machine has a useful life of five years and a salvage value of Rs. 25,000 at the end of its useful life. Compute the depreciation schedule for the machine by each of the following depreciation methods:

(a) Straight line method of depreciation

(b) Sum-of-the-years digits method of depreciation

(c) Double declining balance method of depreciation

4. A company has recently purchased an overhead travelling crane for Rs. 25,00,000. Its expected life is seven years and the salvage value at the end of the life of the overhead travelling crane is Rs. 1,00,000. Using the straight line method of depreciation, find the depreciation and the book value at the end of third and fourth year after the crane is purchased.

5. An automobile company has purchased a wheel alignment device for Rs. 10,00,000. The device can be used for 15 years. The salvage value at the end of the life of the device is 10% of the purchase value. Find the following using the double declining balance method of depreciation:

(a) Depreciation at the end of the seventh year

(b) Depreciation at the end of the twelfth year

(c) Book value at the end of the eighth year

6. A company has purchased a bus for its officers for Rs. 10,00,000. The expected life of the bus is eight years. The salvage value of the bus at the end of its life is Rs. 1,50,000. Find the following using the sinking fund method of depreciation:

(a) Depreciation at the end of the third and fifth year

(b) Book value at the end of the second year and sixth year

7. Consider Problem 4 and find the following using the sum-of-the-years-digits method of depreciation:

(a) Depreciation at the end of the fourth year

(b) Depreciation at the end of the seventh year

(c) Book value at the end of the fifth year

(d) Book value at the end of the eighth year

8. A company has purchased a Xerox machine for Rs. 2,00,000. The salvage value of the machine at the end of its useful life would be insignificant. The maximum number of copies that can be taken during its lifetime is 1,00,00,000. During the fourth year of its operation, the number of copies taken is 9,00,000. Find the depreciation for the fourth year of operation of the Xerox machine using the service output method of depreciation.

9. A heavy construction firm has been awarded a contract to build a large concrete dam. It is expected that a total of eight years will be required to complete the work. The firm will buy Rs.1,80,00,000 worth of special equipment for the job. During the preparation of the job cost estimate, the following utilization schedule was computed for the special equipment:

Year	1	2	3	4	5	6	7	8
Hours/yr	6,000	4,000	4,000	1,600	800	800	2,200	2,200

At the end of the job, it is estimated that the equipment can be sold at auction for Rs. 18,00,000.

Prepare the depreciation schedule for all the years of operation of the equipment using the service output method of depreciation.

10

EVALUATION OF PUBLIC ALTERNATIVES

10.1 INTRODUCTION

In evaluating alternatives of private organizations, the criterion is to select the alternative with the maximum profit. The profit maximization is the main goal of private organizations while providing goods/services as per specifications to their customers. But the same criterion cannot be used while evaluating public alternatives. Examples of some public alternatives are constructing bridges, roads, dams, establishing public utilities, etc.

The main objective of any public alternative is to provide goods/services to the public at the minimum cost. In this process, one should see whether the benefits of the public activity are at least equal to its costs. If yes, then the public activity can be undertaken for implementation. Otherwise, it can be cancelled. This is nothing but taking a decision based on Benefit-Cost ratio (BC) given by

$$\text{BC ratio} = \frac{\text{Equivalent benefits}}{\text{Equivalent costs}}$$

The benefits may occur at different time periods of the public activity. For the purpose of comparison, these are to be converted into a common time base (present worth or future worth or annual equivalent). Similarly, the costs consist of initial investment and yearly operation and maintenance cost. These are to be converted to a common time base as done in the equivalent benefits. Now the ratio between the equivalent benefits and equivalent costs is known as the "Benefit-Cost ratio". If this ratio is at least one, the public activity is justified; otherwise, it is not justified. Let

B_P = present worth of the total benefits

B_F = future worth of the total benefits

B_A = annual equivalent of the total benefits

P = initial investment

P_F = future worth of the initial investment

P_A = annual equivalent of the initial investment

C = yearly cost of operation and maintenance

C_P = present worth of yearly cost of operation and maintenance

C_F = future worth of yearly cost of operation and maintenance

$$\text{BC ratio} = \frac{B_P}{P + C_P} = \frac{B_F}{P_F + C_F} = \frac{B_A}{P_A + C}$$

10.2 EXAMPLES

EXAMPLE 10.1 In a particular locality of a state, the vehicle users take a roundabout route to reach certain places because of the presence of a river. This results in excessive travel time and increased fuel cost. So, the state government is planning to construct a bridge across the river. The estimated initial investment for constructing the bridge is Rs. 40,00,000. The estimated life of the bridge is 15 years. The annual operation and maintenance cost is Rs. 1,50,000. The value of fuel savings due to the construction of the bridge is Rs. 6,00,000 in the first year and it increases by Rs. 50,000 every year thereafter till the end of the life of the bridge. Check whether the project is justified based on BC ratio by assuming an interest rate of 12%, compounded annually.

Solution

Initial investment = Rs. 40,00,000
Annual operation and maintenance = Rs. 1,50,000
Annual fuel savings during the first year = Rs. 6,00,000
Equal increment in fuel savings in the following years = Rs. 50,000
Life of the project = 15 years
Interest rate = 12%

The cash flow diagram of the project is shown in Fig. 10.1.

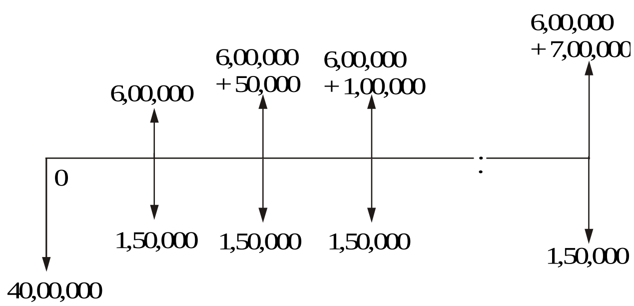


Fig. 10.1 Cash flow diagram for constructing bridge.

$$\begin{aligned}\text{Total present worth of costs} &= \text{Initial investment } (P) \\ &\quad + \text{Present worth of annual operating} \\ &\qquad \text{and maintenance cost } (C_P) = P + C_P \\ &= \text{Rs. } 40{,}00{,}000 + 1{,}50{,}000 \times (P/A, 12\%, 15) \\ &= \text{Rs. } 40{,}00{,}000 + 1{,}50{,}000 \times 6.8109 \\ &= \text{Rs. } 50{,}21{,}635\end{aligned}$$

Total present worth of fuel savings (B_P):

$A1$ = Rs. 6,00,000
G = Rs. 50,000
n = 15 years
i = 12%

$$\text{Annual equivalent fuel savings } (A) = A1 + G\,(A/G, 12\%, 15)$$
$$= 6{,}00{,}000 + 50{,}000\ (4.9803)$$
$$= \text{Rs. } 8{,}49{,}015$$

$$\text{Present worth of the fuel savings } (B_P) = A(P/A, 12\%, 15)$$
$$= 8{,}49{,}015\ (6.8109)$$
$$= \text{Rs. } 57{,}82{,}556$$

$$\text{BC ratio} = \frac{B_P}{P + C_P} = \frac{57{,}82{,}556}{50{,}21{,}635} = 1.1515$$

Since the BC ratio is more than 1, the construction of the bridge across the river is justified.

EXAMPLE 10.2 A state government is planning a hydroelectric project for a river basin. In addition to the production of electric power, this project will provide flood control, irrigation and recreation benefits. The estimated benefits and costs that are expected to be derived from this project are as follows:

Initial cost = Rs. 8,00,00,000
Annual power sales = Rs. 60,00,000
Annual flood control savings = Rs. 30,00,000
Annual irrigation benefits = Rs. 50,00,000
Annual recreation benefits = Rs. 20,00,000
Annual operating and maintenance costs = Rs. 30,00,000
Life of the project = 50 years

Check whether the state government should implement the project (Assume i = 12%)

Solution

Initial cost = Rs. 8,00,00,000
Annual power sales = Rs. 60,00,000
Annual flood control savings = Rs. 30,00,000
Annual irrigation benefits = Rs. 50,00,000
Annual recreation benefits = Rs. 20,00,000
Annual operating and maintenance costs = Rs. 30,00,000
Life of the project = 50 years, i = 12%

Total annual benefits = Flood control savings + Irrigation benefits + Recreation benefits
= Rs. 30,00,000 + Rs. 50,00,000 + Rs. 20,00,000
= Rs. 1,00,00,000

Present worth of the benefits = Total annual benefits × (P/A, 12%, 50)

= 1,00,00,000 × (8.3045)

= Rs. 8,30,45,000

Present worth of costs = Initial cost + Present worth of annual operating and maintenance cost – Present worth of power sales

= Rs. 8,00,00,000 + 30,00,000 × (P/A, 12%, 50) – 60,00,000 (P/A, 12%, 50)

= Rs. 8,00,00,000 + 30,00,000 × 8.3045 – 60,00,000 × 8.3045

= Rs. 5,50,86,500

$$\text{BC ratio} = \frac{\text{Present worth of benefits}}{\text{Present worth of costs}}$$

$$= \frac{8,30,45,000}{5,50,86,500} = 1.508$$

Since, the BC ratio is more than 1, the state government can implement the hydroelectric project.

EXAMPLE 10.3 Two mutually exclusive projects are being considered for investment. Project *A*1 requires an initial outlay of Rs. 30,00,000 with net receipts estimated as Rs. 9,00,000 per year for the next 5 years. The initial outlay for the project *A*2 is Rs. 60,00,000, and net receipts have been estimated at Rs. 15,00,000 per year for the next seven years. There is no salvage value associated with either of the projects. Using the benefit cost ratio, which project would you select? Assume an interest rate of 10%.

Solution Alternative A1

Initial cost (P) = Rs. 30,00,000

Net benefits/year (B) = Rs. 9,00,000

Life (n) = 5 years

Annual equivalent of initial cost = P × (A/P, 10%, 5)

= 30,00,000 × 0.2638

= Rs. 7,91,400

$$\text{Benefit-cost ratio} = \frac{\text{Annual equivalent benefit}}{\text{Annual equivalent cost}}$$

= 9,00,000/7,91,400

= 1.137

Alternative A2

Initial cost (P) = Rs. 60,00,000

Net benefits/year (B) = Rs. 15,00,000

Life (n) = 7 years

Annual equivalent of initial cost = $P \times (A/P, 10\%, 7)$

= 60,00,000 × 0.2054

= Rs. 12,32,400

$$\text{BC ratio} = \frac{\text{Annual equivalent benefit}}{\text{Annual equivalent cost}}$$

$$= 15{,}00{,}000/12{,}32{,}400 = 1.217$$

The benefit-cost ratio of alternative $A2$ is more than that of alternative $A1$. Hence, alternative $A2$ is to be selected. The comparison is made on a 35-year period which is the minimum common multiple of the lives of alternatives 1 and 2.

EXAMPLE 10.4 An inland state is presently connected to a seaport by means of a railroad system. The annual goods transported is 1,00,00,000 ton km. The average transport charge is Rs. 30/ton/km. Within the next 20 years, the transport is likely to increase by 10,00,000 ton km per year.

It is proposed to broaden a river flowing from the state to the seaport at a cost of Rs. 2,50,00,00,000. This will make the river navigable to barges and will reduce the transport cost to Rs. 10.00/ton/km. The project will be financed by 10% bond at par. There would be some side effects of the change-over as follows.

1. The railroad would be bankrupt and be sold for no salvage value. The right of way, worth about Rs. 3,00,00,000, will revert to the state.
2. 300 employees will be out of employment. The state will have to pay to each of them a welfare cheque of Rs. 48,000/year.
3. The reduction in the income from the taxes on the railroad will be compensated by the taxes on the barges.

What is the benefit-cost ratio based on the next 20 years of operation? Also, check whether broadening the river is justified.

Solution ***Cost to the state***

Life of the project = 20 years

Total cost of the project, P = Rs. 2,50,00,00,000

Annual goods transported = 1,00,00,000 ton km

Current average transport cost = Rs. 30/ton/km

Annual increase in goods transported = 10,00,000 ton km

Compensation for employees = Rs. 48,000/year/employee for 300 employees.

Annual compensation ($C1$) = 48,000 × 300 = Rs. 1,44,00,000

Annual equivalent initial cost ($C2$) $= P \times (A/P, 10\%, 20)$

$= 2{,}50{,}00{,}00{,}000 \times 0.1175$

= Rs. 29,37,50,000

Total annual equivalent cost of the project, C_A

= Annual equivalent initial cost

\+ Annual compensation

= Rs. 29,37,50,000 + Rs. 1,44,00,000

= Rs. 30,81,50,000

Benefit to the state

Worth of the right of the way of the railroad to the state

= Rs. 3,00,00,000

Annual equivalent of the above amount ($A1$)

$= 3{,}00{,}00{,}000 \times (A/P, 10\%, 20)$

$= 3{,}00{,}00{,}000 \times 0.1175$

= Rs. 35,25,000

Average goods transported/year = 1,00,00,000 ton/year

Average transport cost savings for year 1, ($A2$)

$= 1{,}00{,}00{,}000(30 - 10)$

= Rs. 20,00,00,000

Average annual increase in goods transport = 10,00,000 tons km.

Average increase in transportation cost savings/year (G)

= Rs. 2,00,00,000

Equivalent annual average transport cost savings ($A3$)

$= A2 + G(A/G, 10\%, 20)$

$= 20{,}00{,}00{,}000 + 2{,}00{,}00{,}000 \times 6.5081$

= Rs. 33,01,62,000

Total annual equivalent benefits to the state, B_A

$= A1 + A3$

= Rs. 35,25,000 + Rs. 33,01,62,000

= Rs. 33,36,87,000

$$\text{BC ratio} = \frac{\text{Total annual equivalent benefit}}{\text{Total annual equivalent cost}}$$

$$= \frac{B_A}{C_A}$$

$$= \text{Rs. } 33{,}36{,}87{,}000/\text{Rs. } 30{,}81{,}50{,}000$$

$$= 1.08287 = 1.1 \text{ (approx.)}$$

Since the BC ratio is more than 1, the project is justified.

EXAMPLE 10.5 A government is planning a hydroelectric project for a river basin. In addition to the production of electric power, this project will provide flood control, irrigation and recreation benefits. The estimated benefits and costs that are expected from the three alternatives under consideration are given in the following table.

	A (Rs.)	*B* (Rs.)	*C* (Rs.)
Initial cost (*P*)	15,00,00,000	25,00,00,000	40,00,00,000
Annual equivalent benefits & cost			
(a) Operating & maintenance cost	20,00,000	25,00,000	35,00,000
(b) Power sales/year	1,00,00,000	1,20,00,000	1,80,00,000
(c) Flood control savings	25,00,000	35,00,000	50,00,000
(d) Irrigation benefits	35,00,000	45,00,000	60,00,000
(e) Recreation benefits	10,00,000	20,00,000	35,00,000

If the interest rate is 9% and the life of projects is estimated to be 50 years, by comparing the BC ratios, determine which project should be selected.

Solution

Benefits/year = Flood control savings + irrigation benefits
+ Recreation benefits.

Costs/year = Annual equivalent cost of the initial cost
+ Operating and maintenance cost/year
– Power sales/year

Based on these guidelines, the computation of benefits, costs and BC ratio for each of the projects are summarized in Table 10.1.

Table 10.1 Illustration of Example 10.5 (Interest rate = 9%, n = 50 years)

	A (Rs.)	*B* (Rs.)	*C* (Rs.)
1. Initial cost (P)	15,00,00,000	25,00,00,000	40,00,00,000
2. Annual equivalent of the initial cost [$P \times (A/P, 9\%, 50)$] i.e. ($P \times 0.0912$)	1,36,80,000	2,28,00,000	3,64,80,000
3. Operating & maintenance cost	20,00,000	25,00,000	35,00,000
4. Power sales/year	1,00,00,000	1,20,00,000	1,80,00,000
5. Cost/year (2 + 3 – 4)	56,80,000	1,33,00,000	2,19,80,000
6. Flood control savings	25,00,000	35,00,000	50,00,000
7. Irrigation benefits	35,00,000	45,00,000	60,00,000
8. Recreation benefits	10,00,000	20,00,000	35,00,000
9. Benefit/year (6 + 7 + 8)	70,00,000	1,00,00,000	1,45,00,000
10. Benefit-cost ratio	1.2324	0.7519	0.6597

From the last row of Table 10.1, it is clear that alternative *A* is the only eligible alternative because the BC ratio of each of the other two alternatives is less than one. Since *A* is the only eligible alternative, it is selected as the best alternative for implementation.

QUESTIONS

1. Discuss the difference in evaluating alternatives of private and public organizations.

2. Consider the evaluation of the alternative of constructing a bridge across a river. List the different benefits and costs related to this alternative.

3. In a particular locality of a state, presently, the vehicle users take a roundabout route to reach certain places because of the presence of a river. This results in excessive time of travel and increased fuel cost. So, the state government is planning to construct a bridge across the river. The estimated initial investment for constructing the bridge is Rs. 40,00,000. The estimated life of the bridge is 15 years. The annual operation and maintenance cost is Rs. 2,50,000. The value of fuel savings due to the construction of the bridge is Rs. 6,00,000 in the first year and it increases by Rs. 50,000 every year thereafter till the end of the life of the bridge. Check whether the project is justified based on BC ratio by assuming an interest rate of 20%, compounded annually.

4. A state government is planning a hydroelectric project for a river basin. In addition to the production of electric power, this project will provide flood control, irrigation and recreation benefits. The estimated benefits and costs that are expected to be derived from this project are listed below.

Initial cost = Rs. 18,00,00,000
Annual power sales = Rs. 1,20,00,000
Annual flood control savings = Rs. 50,00,000
Annual irrigation benefits = Rs. 80,00,000
Annual recreation benefits = Rs. 40,00,000
Annual operating and maintenance costs = Rs. 50,00,000
Life of the project = 40 years

Check whether the state government should implement the project (assume i = 15%).

5. Two mutually exclusive projects are being considered for investment. Project $A1$ requires an initial outlay of Rs. 50,00,000 with net receipts estimated to be Rs. 11,00,000 per year for the next eight years. The initial outlay for the project $A2$ is Rs. 80,00,000, and net receipts have been estimated at Rs. 20,00,000 per year for the next eight years. There is no salvage value associated with either of the projects. Using the BC ratio, which project would you select? Assume an interest rate of 15%.

6. An inland state is presently connected to a seaport by means of a railroad system. The annual goods transported amount to 1,50,00,000 ton kilometre. The average transport charge is Rs. 40/ton/km. Within the next 25 years, the transport is likely to increase by 15,00,000 ton kilometre per year.

It is proposed to improve a river flowing from the state to the seaport at a cost of Rs. 3,00,00,00,000. This will make the river navigable to barges and will reduce the transport cost to Rs. 12.00/ton/km. The project will be financed by 12% bond at par. There would be some side effects of the change-over as follows.

(a) The railroad would be bankrupt and would be sold for no salvage value. The right of way, worth about Rs. 4,00,00,000, will revert to the state.

(b) 400 employees will be out of employment. The state will have to pay to each of them a welfare cheque of Rs. 60,000/year.

(c) The reduction in the income from the taxes on the railroad will be compensated by the taxes on the barges.

What is the BC ratio based on the next 25 years of operation? Also, check whether broadening the river is justified.

7. A government is planning a hydroelectric project for a river basin. Besides the production of electric power, this project will provide flood control, irrigation and recreation benefits. The estimated benefits and costs expected from the three alternatives under consideration are listed in the following table:

	A (Rs.)	*B* (Rs.)	*C* (Rs.)
Initial cost (*P*)	25,00,00,000	35,00,00,000	50,00,00,000
Annual equivalent benefits & cost			
(a) Operating & maintenance cost	30,00,000	35,00,000	45,00,000
(b) Power sales/year	2,00,00,000	2,20,00,000	2,80,00,000
(c) Flood control savings	35,00,000	45,00,000	60,00,000
(d) Irrigation benefits	45,00,000	55,00,000	70,00,000
(e) Recreation benefits	20,00,000	30,00,000	45,00,000

If the interest rate is 10% and the life of the projects is estimated to be 40 years, by comparing the BC ratios, determine which project should be selected.

11

INFLATION ADJUSTED DECISIONS

11.1 INTRODUCTION

A general inflationary trend in the cost of goods is common everywhere due to various interacting factors. If the rate of inflation is very high, it will produce extremely serious consequences for both individuals and institutions.

Inflation is the rate of increase in the prices of goods per period. So, it has a compounding effect. Thus, prices that are inflated at a rate of 7% per year will increase 7% in the first year, and for the next year the expected increase will be 7% of these new prices. The same is true for succeeding years and hence the rate of inflation is compounded in the same manner that an interest rate is compounded. If the average inflation over six years period is 7%, then the prices at the beginning of the seventh year would be 150% that of the first year by assuming 100% for the prices at the beginning of the first year of the six-year period.

If economic decisions are taken without considering the effect of inflation into account, most of them would become meaningless and as a result the organizations would end up with unpredictable return.

But there is always difficulty in determining the rate of inflation. The world-wide trend/wish is to curtail inflation. But due to various reasons, it is very difficult to have zero inflation. For practical decision making, an average estimate may be assumed depending on the period of the proposals under consideration. Hence, we need a procedure which will combine the effects of inflation rate and interest rate to take realistic economic decision.

11.2 PROCEDURE TO ADJUST INFLATION

A procedure to deal with this situation is summarized now.

1. Estimate all the costs/returns associated with an investment proposal in terms of today's rupees.
2. Modify the costs/returns estimated in step 1 using an assumed inflation rate so that at each future date they represent the costs/returns at that date in terms of the rupees that must be expended/received at that time, respectively.
3. As per our requirement, calculate either the annual equivalent amount or future amount or present amount of the cash flow resulting from step 2 by considering the time value of money.

EXAMPLE 11.1 Suppose a 40-year old man is planning for his retirement. He plans to retire at the age of 60 and estimates that he can live comfortably on Rs. 24,000 per year in terms of today's rupee value. He can invest his savings at 15% compounded annually. Assume an average inflation rate of 9% for the next 30 years.

What equal amount should he save each year until he retires so that he can make withdrawals at the end of each year commencing from the end of the 21st year from now that will allow him to live as comfortably as he desires for 10 years beyond his retirement?

Solution

Step 1. The estimated future requirement per year in terms of today's rupees from his age 61 through 70 is Rs. 24,000.

Step 2. Modification of the costs estimated in step 1 is summarized in Table 11.1. The formula which is given below is used to get future equivalent of Rs. 24,000 with the inflation of 9% per year (IR-inflation rate).

$$F = P\,(1 + IR)^n$$

Table 11.1 Inflated Future Requirements

End of year	*Age* (years)	*Inflated value of Rs. 24,000 at each year end*
21	61	$24{,}000 \times (1 + 0.09)^{21}$ = Rs. 1,46,611
22	62	$24{,}000 \times (1 + 0.09)^{22}$ = Rs. 1,59,806
23	63	$24{,}000 \times (1 + 0.09)^{23}$ = Rs. 1,74,189
24	64	$24{,}000 \times (1 + 0.09)^{24}$ = Rs. 1,89,866
25	65	$24{,}000 \times (1 + 0.09)^{25}$ = Rs. 2,06,954
26	66	$24{,}000 \times (1 + 0.09)^{26}$ = Rs. 2,25,580
27	67	$24{,}000 \times (1 + 0.09)^{27}$ = Rs. 2,45,882
28	68	$24{,}000 \times (1 + 0.09)^{28}$ = Rs. 2,68,011
29	69	$24{,}000 \times (1 + 0.09)^{29}$ = Rs. 2,92,132
30	70	$24{,}000 \times (1 + 0.09)^{30}$ = Rs. 3,18,424

Step 3. Now, the calculation of the equivalent amount of cash flow as per the requirement is presented.

The overall cash flow diagram for the savings and withdrawal in terms of future rupees is shown in Fig. 11.1.

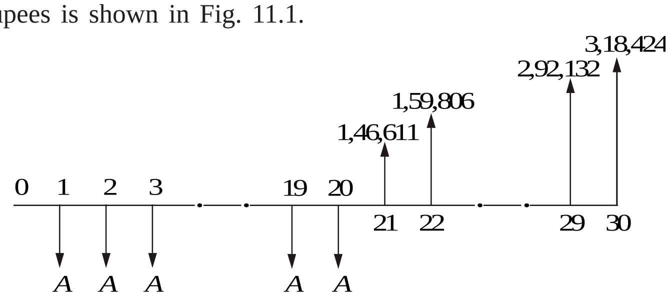


Fig. 11.1 Overall cash flow diagram.

The sum of the present equivalents of the year end withdrawals from the year 21 to 30 is computed by assuming the end of the year 20 as the base (time zero) and it is shown at the end of the year 20 in Fig. 11.2. The method of computing the present equivalent of the withdrawals is as follows:

$$\begin{aligned} PW(i = 15\%) = \; & 1,46,611/(1 + 0.15)^1 + 1,59,806/(1 + 0.15)^2 \\ & + 1,74,189/(1 + 0.15)^3 + 1,89,866/(1 + 0.15)^4 \\ & + 2,06,954/(1 + 0.15)^5 + 2,25,580/(1 + 0.15)^6 \\ & + 2,45,882/(1 + 0.15)^7 + 2,68,011/(1 + 0.15)^8 \\ & + 2,92,132/(1 + 0.15)^9 + 3,18,424/(1 + 0.15)^{10} \\ = \; & \text{Rs. } 10,13,631. \end{aligned}$$

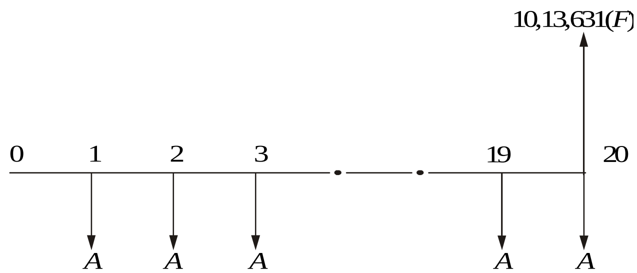


Fig. 11.2 Equivalent cash flow diagram.

The annual equivalent amount (*A*), which should be invested from the end of year 1 (age 41) to year 20 (age 60), is computed using the following formula.

$$\begin{aligned} A &= F(A/F, 15\%, 20) \\ &= 10,13,631 \times (0.0098) \\ &= \text{Rs. } 9,934 \end{aligned}$$

Recommendation: The person has to invest an amount of Rs. 9,934 at the end of every year starting from his age 41 (year 1) through 60 (year 20) which will enable him to make withdrawals at the end of every year starting from his age 61 (year 21) through 70 (year 30) as shown in the Table 11.1 (also in Fig. 11.1).

11.3 INFLATION ADJUSTED ECONOMIC LIFE OF MACHINE

(Panneerselvam, 1998)

In any industrial/service organization, equipment/machinery forms an important element. The productivity of any organization is a function of many factors. It is largely affected by efficient and effective use of machinery and equipment. So, operations and maintenance of these equipment are very important to the organization.

A machine which is purchased today cannot be used forever. It has a definite economic lifetime. After the economic life, the machine should be replaced with a substitute machine with similar operational capabilities. This kind of analysis is called *replacement analysis*.

The elements of costs involved in the replacement analysis are as follows:

1. Purchase cost (initial cost)
2. Annual operation and maintenance cost
3. Salvage value at the end of every year, if it is significant

The trade-off between different cost elements is shown in Fig. 11.3.

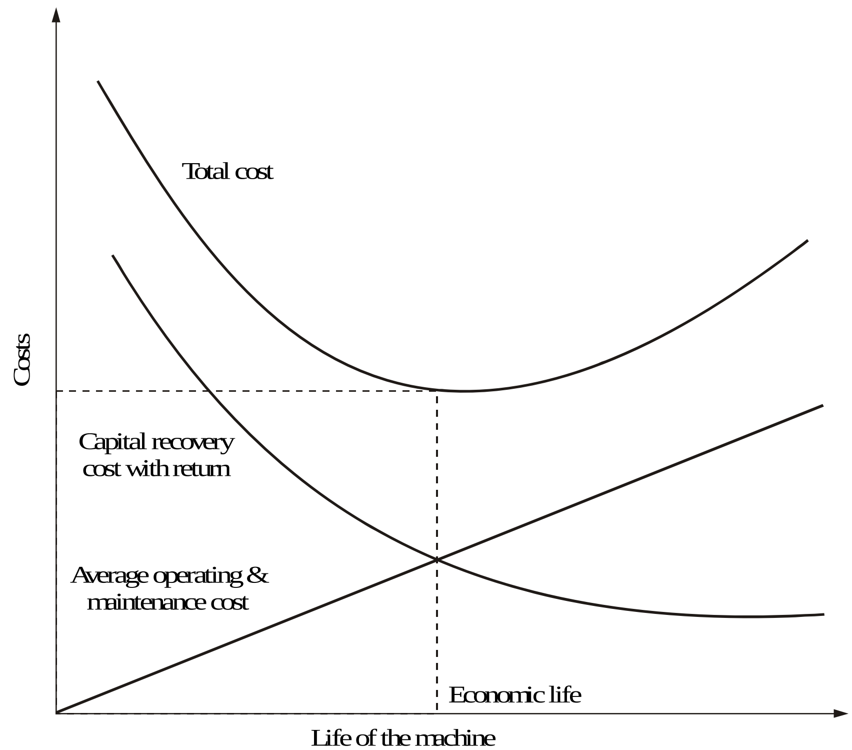


Fig. 11.3 Chart showing economic life.

From Fig. 11.3, it is clear that the sum of operation and maintenance cost increases with the life of the machine. But the capital recovery with return decreases with the life of the machine. The total cost of the machine goes on decreasing initially but it starts increasing after some years. The year with the minimum total cost is called as the economic life of the machine.

11.3.1 Limitation of Existing Model

In the case where the machine is replaced due to wear and tear, the following costs are considered (refer Chapter 8):

1. Initial cost
2. Operation and maintenance cost
3. Salvage value

In the existing model to deal with this type of replacement analysis, the different cost elements are estimated without taking the effect of inflation into account.

The annual cost of operation and maintenance of the machine will increase with the age of the machine due to decline in efficiency of the machine. In the existing model, this increase in the operation and maintenance cost is taken into account. But the increase in the operation and maintenance cost due to inflation is not considered. Similarly, in the existing model, the salvage value is estimated without taking into account the effect of inflation.

To highlight this particular fact on salvage value, an example is now given.

The internal combustion engines (R.A. Lister) which were made in England during pre-independence of India are still functioning well. Their resale value is going up year after year. This may be partly due to inflation and partly due to good quality of the engine parts. So, consideration of the effect of the inflation on the economic life of the machine is a realistic approach.

In replacement analysis, a discount rate is usually assumed to reflect the time value of money. First the concept of replacement analysis is demonstrated without taking the inflation into account. Then, the same is demonstrated by taking the effect of inflation into account. At the end, a comparison between the two models is presented.

11.3.2 Economic Life Determination without Inflationary Effect

The determination of economic life of a machine without considering the effect of inflation is demonstrated using the following example.

EXAMPLE 11.2 A machine costs Rs. 5,00,000. Its annual operation cost during the first year is Rs. 40,000 and it increases by Rs. 5,000 every year thereafter. The maintenance cost during the first year is Rs. 60,000 and it increases by Rs. 6,000 every year thereafter. The resale value of the machine is Rs. 4,00,000 at the end of the first year and it decreases by Rs. 50,000 every year thereafter. Assume an interest rate (discounting factor) of 20%.

The method of finding the economic life of the machine with a discounting factor of 20% at zero inflation rate is summarized in Table 11.2. From the table it is clear that the total annual equivalent cost is minimum if the machine is used for 14 years. Hence, the economic life of the machine is 14 years.

11.3.3 Economic Life Determination with Inflationary Effect

The illustration in Section 11.3.2 is reconsidered for analyzing the effect of inflation on the economic life of the machine. An average annual inflation rate of 6% is assumed for discussion. The corresponding steps are explained in Table 11.3.

From the Table 11.3, it is clear that the total annual equivalent cost is minimum if the machine is used for three years. Thus, the economic life of the machine is three years.

Table 11.2 Determination of Economic Life of the Machine without Inflation

End of year (n) (1)	*Operation cost* (2) (Rs.)	*Maintenance cost* (3) (Rs.)	*Operation & maint. cost* (2) + (3) (4) (Rs.)	*P/F, i, n* (i = 20%) (5)	*Present worth of column 4* (4) × (5) (6) (Rs.)	*Cumulative of column (6)* (7) (Rs.)	*Salvage value (S)* (8) (Rs.)	*Present worth of salvage value* (8) × (5) (9) (Rs.)	*Total present worth* 5,00,000 + *Column* 7 – 9 (10) (Rs.)	*A/P, i, n* (i = 20%) (11)	*Annual equivalent amount* (10) × (11) (12) (Rs.)
1	40,000	60,000	1,00,000	0.8333	83,330.00	83,330.00	4,00,000	3,33,320.00	2,50,010.00	1.2000	3,00,012.00
2	45,000	66,000	1,11,000	0.6945	77,089.50	1,60,419.50	3,50,000	2,43,075.00	4,17,344.50	0.6546	2,73,193.70
3	50,000	72,000	1,22,000	0.5787	70,601.40	2,31,020.90	3,00,000	1,73,610.00	5,57,410.90	0.4747	2,64,602.90
4	55,000	78,000	1,33,000	0.4823	64,145.90	2,95,166.80	2,50,000	1,20,575.00	6,74,591.80	0.3863	2,60,594.80
5	60,000	84,000	1,44,000	0.4019	57,873.60	3,53,040.40	2,00,000	80,380.00	7,72,660.40	0.3344	2,58,377.60
6	65,000	90,000	1,55,000	0.3349	51,909.50	4,04,949.90	1,50,000	50,235.00	8,54,714.90	0.3007	2,57,012.70
7	70,000	96,000	1,66,000	0.2791	46,330.60	4,51,280.50	1,00,000	27,910.00	9,23,370.50	0.2774	2,56,142.90
8	75,000	1,02,000	1,77,000	0.2326	41,170.20	4,92,450.70	50,000	11,630.00	9,80,820.70	0.2606	2,55,601.80
9	80,000	1,08,000	1,88,000	0.1938	36,434.40	5,28,885.10	0	0.00	10,28,885.00	0.2481	2,55,266.30
10	85,000	1,14,000	1,99,000	0.1615	32,138.50	5,61,023.60	0	0.00	10,61,023.00	0.2385	2,53,054.10
11	90,000	1,20,000	2,10,000	0.1346	28,266.00	5,89,289.60	0	0.00	10,89,289.00	0.2311	2,51,734.80
12	95,000	1,26,000	2,21,000	0.1122	24,796.20	6,14,085.80	0	0.00	11,14,085.00	0.2253	2,51,003.50
13	1,00,000	1,32,000	2,32,000	0.0935	21,692.00	6,35,777.80	0	0.00	11,35,777.00	0.2206	2,50,552.50
14	1,05,000	1,38,000	2,43,000	0.0779	18,929.70	6,54,707.50	0	0.00	11,54,707.00	0.2169	2,50,456.00***
15	1,10,000	1,44,000	2,54,000	0.0649	16,484.60	6,71,192.10	0	0.00	11,71,192.00	0.2139	2,50,517.90

***Total annual equivalent cost is minimum if the machine is used for 14 years.

Table 11.3 Determination of Economic Life of the Machine with Inflationary Effect

End of year (*n*) (1)	*Opera-tion cost* (2) (Rs.)	*Mainte-nance cost* (3) (Rs.)	*Sum of operation & mainte-nance cost* (2) + (3) (4) (Rs.)	*F/P, i, n* (*i* = 6%) (5)	*Inflated operation & maint. cost* (4) × (5) (6) (Rs.)	*P/F, i, n* (*i* = 20%) (7)	*Present worth of column* 6 (6) × (7) (8) (Rs.)	*Cumulative of column* (8) (9) (Rs.)	*Salvage value* (*S*) (10) (Rs.)	*Inflated salvage value* (10) × (5) (11) (Rs.)	*Present worth of column* 11 (11) × (7) (12) (Rs.)	*Total present worth* 5,00,000 + *Column* 9 – 12 (13) (Rs.)	*A/P, i, n* (*i* = 20%) (14)	*Annual equivalent amount of total present worth* (13) × (14) (15) (Rs.)
1	40,000	60,000	1,00,000	1.060	1,06,000	0.8333	88,329.80	88,329.80	4,00,000	4,24,000	3,53,319.20	2,35,010.6	1.2000	2,82,012.70
2	45,000	66,000	1,11,000	1.124	1,24,764	0.6945	86,648.59	1,74,978.30	3,50,000	3,93,400	2,73,216.30	4,01,762.0	0.6546	2,62,993.40
3	50,000	72,000	1,22,000	1.191	1,45,302	0.5787	84,086.26	2,59,064.60	3,00,000	3,57,300	2,06,769.50	5,52,295.1	0.4747	2,62,174.50 ***
4	55,000	78,000	1,33,000	1.262	1,67,846	0.4823	80,952.12	3,40,016.70	2,50,000	3,15,500	1,52,165.60	6,87,851.1	0.3863	2,65,716.80
5	60,000	84,000	1,44,000	1.338	1,92,672	0.4019	77,434.87	4,17,451.60	2,00,000	2,67,600	1,07,548.40	8,09,903.2	0.3344	2,70,831.60
6	65,000	90,000	1,55,000	1.419	2,19,945	0.3349	73,659.58	4,91,111.20	1,50,000	2,12,850	71,283.46	9,19,827.7	0.3007	2,76,592.20
7	70,000	96,000	1,66,000	1.504	2,49,664	0.2791	69,681.22	5,60,792.40	1,00,000	1,50,400	41,976.64	10,18,815.0	0.2774	2,82,619.50
8	75,000	1,02,000	1,77,000	1.594	2,82,138	0.2326	65,625.29	6,26,417.70	50,000	79,700	18,538.22	11,07,879.0	0.2606	2,88,713.40
9	80,000	1,08,000	1,88,000	1.689	3,17,532	0.1938	61,537.70	6,87,955.40	0	0	0.00	11,87,955.0	0.2481	2,94,731.70
10	85,000	1,14,000	1,99,000	1.791	3,56,409	0.1615	57,560.05	7,45,515.50	0	0	0.00	12,45,515.0	0.2385	2,97,055.40

***Total annual equivalent cost is minimum if machine is used for three years.

Comparison of results

The results of the two approaches are summarized in Table 11.4 . From the table, it is clear that the inflation has an effect on the economic life of the machine. Since it is meaningful and realistic to analyze this type of problem by considering the effect of inflation, the second approach should be used for such analysis.

Table 11.4 Results of the Two Approaches

Approach	*Minimum annual equivalent cost* (Rs.)	*Corresponding life* (years)
Replacement analysis without inflation effect	2,50,456.00	14
Replacement analysis with inflation effect	2,62,174.50	3

EXAMPLE 11.3 A company has received quotes for its recent advertisement for the purchase of a sophisticated milling machine. The data are as per the estimate in today's rupee value.

	Machine X	*Machine Y*
Purchase price (Rs.)	15,00,000	20,00,000
Machine life (years)	7	7
Salvage value at the end of machine life (Rs.)	2,00,000	3,00,000
Annual operating & maintenance cost (Rs.)	3,00,000	2,50,000

Assuming an average annual inflation of 5% for the next five years, determine the best machine based on the present worth method. Interest rate is 15%, compounded annually.

Solution

Average annual inflation rate = 5%
Interest rate = 15% compounded annually

Machine X

Purchase price = Rs.15,00,000
Machine life = 7 years
Salvage value at the end of machine life = Rs. 2,00,000
Annual operating & maintenance cost = Rs. 3,00,000

The computation of the present worth of the annual operating and maintenance costs of the machine X is summarized in Table 11.5.

Table 11.5 Computation of the Present Worth of the Annual Operating and Maintenance Costs of Machine X

End of year (n)	*Annual operating & maintenance cost* (Rs.)	*Inflation factor* (F/P, 5%, n)	*Inflated annual operating & maintenance cost* (Rs.)	P/F, 15%, n	*Present worth of inflated annual operating & maintenance cost*
A	B	C	D	E	F (Rs.)
			B × C		D × E
1	3,00,000	1.050	3,15,000	0.8696	2,73,924
2	3,00,000	1.102	3,30,600	0.7561	2,49,967
3	3,00,000	1.158	3,47,400	0.6575	2,28,416
4	3,00,000	1.216	3,64,800	0.5718	2,08,593
5	3,00,000	1.276	3,82,800	0.4972	1,90,328
6	3,00,000	1.340	4,02,000	0.4323	1,73,785
7	3,00,000	1.407	4,22,100	0.3759	1,58,667
					Rs. 14,83,680

The equation for the present worth of the machine X is

$$\begin{aligned}
PW_X(15\%) &= \text{Purchase price} \\
&\quad + \text{Present worth of inflated annual and operating cost} \\
&\quad - \text{Present worth of the salvage value} \\
&= 15{,}00{,}000 + 14{,}83{,}680 - 2{,}00{,}000 \times (\text{inflation factor}) \\
&\quad \times (P/F, 15\%, 7) \\
&= 15{,}00{,}000 + 14{,}83{,}680 - 2{,}00{,}000(F/P, 5\%, 7)\ (P/F, 15\%, 7) \\
&= 15{,}00{,}000 + 14{,}83{,}680 - 2{,}00{,}000 \times 1.407 \times 0.3759 \\
&= \text{Rs. } 28{,}77{,}901.74
\end{aligned}$$

Machine Y

Purchase price = Rs. 20,00,000
Machine life = 7 years
Salvage value at the end of machine life = Rs. 3,00,000
Annual operating & maintenance cost = Rs. 2,50,000

The computation of the present worth of the annual operating and maintenance costs of the machine Y is summarized in Table 11.6.

Table 11.6 Computation of the Present Worth of the Annual Operating and Maintenance Costs of Machine Y

End of year (n)	*Annual operating & maintenance cost* (Rs.)	*Inflation factor* (F/P, 5%, n)	*Inflated annual operating & maintenance cost* (Rs.)	P/F, 15%, n	*Present worth of inflated annual operating & maintenance cost*
A	B	C	D	E	F (Rs.)
			B × C		D × E
1	2,50,000	1.050	2,62,500	0.8696	2,28,270
2	2,50,000	1.102	2,75,500	0.7561	2,08,306
3	2,50,000	1.158	2,89,500	0.6575	1,90,346
4	2,50,000	1.216	3,04,000	0.5718	1,73,827
5	2,50,000	1.276	3,19,000	0.4972	1,58,607
6	2,50,000	1.340	3,35,000	0.4323	1,44,821
7	2,50,000	1.407	3,51,750	0.3759	1,32,223
					Rs. 12,36,400

The expression for the present worth of machine Y is

$$
\begin{aligned}
PW_Y(15\%) &= \text{Purchase price} \\
&\quad + \text{present worth of inflated annual and operating cost} \\
&\quad - \text{present worth of the salvage value} \\
&= 20{,}00{,}000 + 12{,}36{,}400 - 3{,}00{,}000 \times \\
&\quad (\text{inflation factor}) \times (P/F, 15\%, 7) \\
&= 20{,}00{,}000 + 12{,}36{,}400 - 3{,}00{,}000\ (F/P, 5\%, 7)\ (P/F, 15\%, 7) \\
&= 20{,}00{,}000 + 12{,}36{,}400 - 3{,}00{,}000 \times 1.407 \times 0.3759 \\
&= \text{Rs. } 30{,}77{,}732.61
\end{aligned}
$$

Remark. Since the present worth cost of machine X is less than that of machine Y, select machine X.

EXAMPLE 11.4 A company is planning to start an employee welfare fund. It needs Rs. 50,00,000 during the first year and it increases by Rs. 5,00,000 every year thereafter up to the end of the 5th year. The above figures are in terms of todays rupee value. The annual average rate of inflation is 6% for the next five years. The interest rate is 18%, compounded annually. Find the single deposit which will provide the required series of fund towards employees welfare scheme after taking the inflation rate into account.

Solution

Fund requirement during the first year = Rs. 50,00,000
Annual increase in the fund requirement = Rs. 5,00,000
Annual inflation rate = 6%
Interest rate = 18%, compounded annually

The computation of the present worth of the annual fund requirements is summarized in Table 11.7.

Table 11.7 Computation of the Present Worth of the Annual Requirements

End of year (n)	*Annual fund requirements* (Rs.)	*Inflation factor* (F/P, 6%, n)	*Inflated annual fund requirements* (Rs.)	*P/F*, 18%, n	*Present worth of inflated annual fund requirements*
A	B	C	D	E	F (Rs.)
			B × C		D × E
1	50,00,000	1.060	53,00,000	0.8475	44,91,750
2	55,00,000	1.124	61,82,000	0.7182	44,39,912
3	60,00,000	1.191	71,46,000	0.6086	43,49,056
4	65,00,000	1.262	82,03,000	0.5158	42,31,107
5	70,00,000	1.338	93,66,000	0.4371	40,93,879
					Rs. 2,16,05,704

The value of the single deposit to be made now to receive the specified series for the next five years is Rs. 2,16,05,704.

QUESTIONS

1. Define inflation.

2. Discuss the impact of inflation on investment decision.

3. Suppose a 50-year old man is planning for his retirement. He plans to retire at the age of 60 and estimates that he can live comfortably on Rs. 40,000 per year in terms of today's rupee value. Let us assume the average inflation rate for the next 20 years is 7% per year. This is only an assumption. He can invest his savings at 20%, compounded annually.

What equal amount should he save each year until he retires so that he can make withdrawals that will allow him to live as comfortably as he desires for 10 years beyond his retirement?

4. A machine costs Rs. 7,00,000. Its annual operation cost during the first year is Rs. 60,000 and it increases by Rs. 7,000 every year thereafter. The maintenance cost during the first year is Rs. 80,000 and it increases by Rs. 10,000 every year thereafter. The resale value of the machine is Rs. 3,00,000 at the end of the first year and it decreases by Rs. 75,000 every year thereafter. Assume an interest rate (discounting factor) of 20% and inflation of 5%, compounded yearly. Find the inflation adjusted economic life of the machine.

12

INVENTORY CONTROL

12.1 INTRODUCTION

Inventory is essential to provide flexibility in operating a system. Inventory can be classified into raw materials inventory, in-process inventory, and finished goods inventory. The raw material inventories obviate the need for dependency between suppliers and plants. The work-in-process inventories eliminate dependency between machines of a product line. The finished goods inventory remove dependency between plant and its customers/market.

The main functions of inventory are:

1. Smoothing out irregularities in supply
2. Minimizing the production cost
3. Allowing organizations to cope with perishable materials

Inventory decisions

Two basic inventory decisions are generally taken by managers:

1. When should the inventory of an item be replaced?
2. How much of an item is to be ordered when the inventory of that item has to be replenished?

Costs trade-off

If we place frequent orders, the cost of order will be more, but the inventory carrying cost will be less. On the other hand, if we place less frequent orders, the ordering cost will be less, but the carrying cost will be more. These are shown in Fig. 12.1. In the figure, for an increase in Q (order size), the carrying cost increases and the ordering cost decreases.

In Fig. 12.1, the total cost curve represents the sum of the ordering cost and carrying cost for each order size. The order size at which the total cost is minimum is called the *Economic Order Quantity (EOQ)* or Q^* (optimal order size).

There are different models of inventory. The inventory models can be classified into deterministic models and probabilistic models. The various deterministic models are as follows:

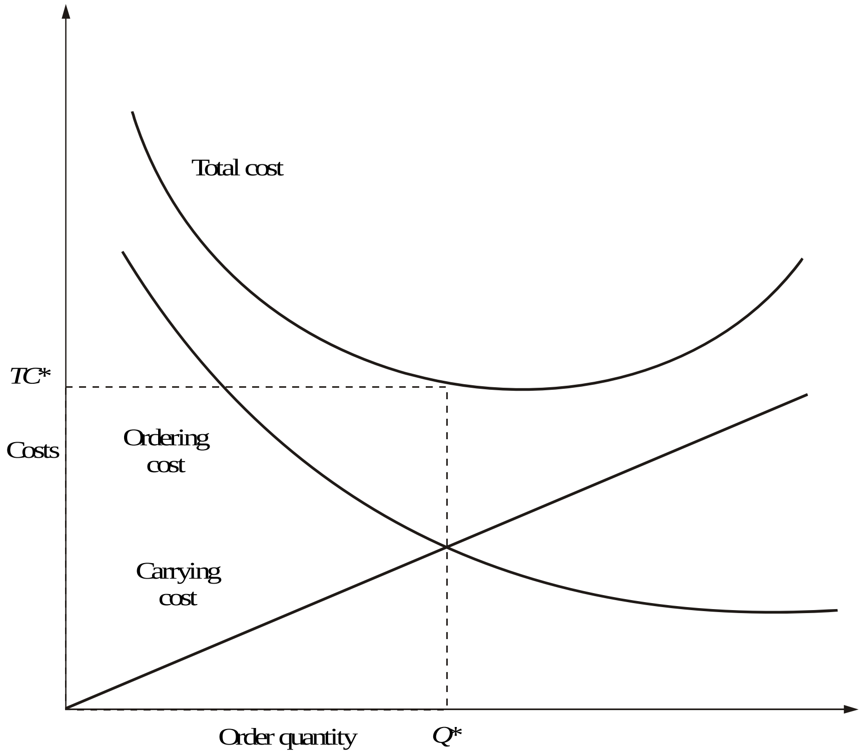


Fig. 12.1 Trade-off between costs.

1. Purchase model with instantaneous replenishment and without shortages
2. Manufacturing model without shortages
3. Purchase model with instantaneous replenishment and with shortages
4. Manufacturing model with shortages

These models are explained in the following sections.

12.2 PURCHASE MODEL WITH INSTANTANEOUS REPLENISHMENT AND WITHOUT SHORTAGES

In this model of inventory, orders of equal size are placed at periodical intervals. The items against an order are replenished instantaneously and they are consumed at a constant rate. The purchase price per unit is same irrespective of order size. Let

D = annual demand in units
C_o = ordering cost/order
C_c = carrying cost/unit/year
P = purchase price/unit
Q = order size

Then, the corresponding model is shown in Fig. 12.2.

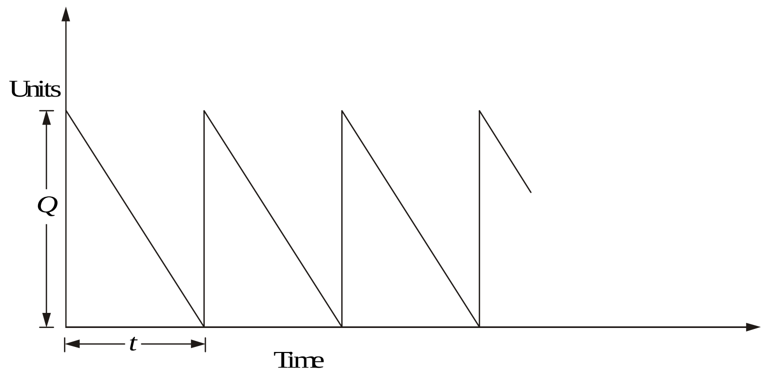


Fig. 12.2 Purchase model without shortages.

For this model, the formula for the EOQ is as follows:

$$Q^* = \sqrt{\frac{2C_oD}{C_c}}$$

$$\text{No. of orders} = \frac{D}{Q^*}$$

$$\text{Time between orders} = \frac{Q^*}{D}$$

EXAMPLE 12.1 Krishna Industry needs 24,000 units/year of a bought-out component which will be used in its main product. The ordering cost is Rs.150 per order and the carrying cost per unit per year is 18% of the purchase price per unit. The purchase price per unit is Rs. 75. Find

(a) Economic order quantity
(b) No. of orders per year
(c) Time between successive orders

Solution

(a) D = 24,000 units/year

C_o = Rs. 150/order

Purchase price/unit = Rs. 75.

C_c = Rs. 75 × 0.18 = Rs. 13.5/unit/year

Therefore,

$$\text{EOQ} = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2 \times 150 \times 24{,}000}{13.50}}$$

$$= 730 \text{ units (approx.)}$$

$$\text{No. of orders/year} = \frac{D}{Q^*} = \frac{24{,}000}{730} = 32.88$$

$$\text{Time between successive orders} = \frac{Q^*}{D} = 730/24{,}000$$
$$= 0.0304 \text{ year}$$
$$= 0.37 \text{ month}$$
$$= 11 \text{ days}$$

12.3 MANUFACTURING MODEL WITHOUT SHORTAGES

If a company manufactures its component which is required for its main product, then the corresponding model of inventory is called the "manufacturing model". This model will be with or without shortages. The rate of consumption of items is uniform throughout the year. The cost of production per unit is same irrespective of production lot size. Let

r = annual demand of an item
k = production rate of the item (No. of units produced per year)
C_o = cost per set-up.
C_c = carrying cost per unit per year.
p = cost of production per unit
$t1$ = period of production as well as consumption satisfying requirement for the period
$t2$ = period of consumption only.
Q = production size

The operation of the manufacturing model without shortages is shown in Fig. 12.3.

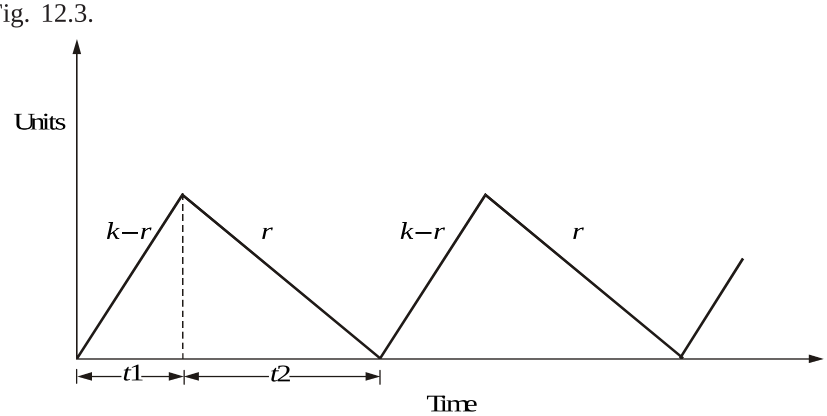


Fig. 12.3 Manufacturing model without shortages.

During the period $t1$, the item is produced at the rate of k units per period and simultaneously it is consumed at the rate of r units per period. So, during this period, the inventory is built at the rate of $k - r$ units per period. During the period $t2$, the production of the item is discontinued but the consumption of that item is continued. Hence, the inventory is decreased at the rate of r units per period during this period.

The various formulae for this situation are as follows:

$$Q^* = \text{EOQ} = \sqrt{\frac{2C_o r}{C_c(1 - r/k)}}$$

$$t1^* = Q^*/k$$

$$t2^* = \frac{Q^*[1 - r/k]}{r} = \frac{(k - r)t1}{r}$$

$$\text{Cycle time} = t1^* + t2^*$$

$$\text{No. of set-ups/year} = r/Q^*$$

EXAMPLE 12.2 If a product is to be manufactured within the company, the particulars are:

r = 12,000 units/year
k = 24,000 units/year
C_o = Rs. 175/set-up
C_c = Rs. 15/unit/year

Find the EOQ and cycle time.

Solution

$$Q^* = \text{EOQ} = \sqrt{\frac{2C_o r}{C_c(1 - r/k)}}$$

$$= \sqrt{\frac{2 \times 175 \times 12{,}000}{15(1 - 12{,}000/24{,}000)}}$$

$$= 749 \text{ units (approx.)}$$

$$t1^* = \frac{Q^*}{k} = 749/24{,}000$$

$$= 0.031 \text{ year} = 0.372 \text{ month} = 11 \text{ days (approx.)}$$

$$t2^* = \frac{Q^*}{r}\left(1 - \frac{r}{k}\right) = \frac{749}{12{,}000}\left(1 - \frac{12{,}000}{24{,}000}\right)$$

$$= 0.031 \text{ year} = 0.372 \text{ month} = 11 \text{ days}$$

Therefore,

$$\text{Cycle time} = t1^* + t2^*$$

$$= 11 + 11 = 22 \text{ days}$$

$$\text{No. of set-ups/year} = r/Q^* = 12{,}000/749 = 16.02$$

12.4 PURCHASE MODEL WITH SHORTAGES (Instantaneous Supply)

In this model, the items on order will be received instantaneously and they are consumed at a constant rate. The purchase price per unit is same irrespective of order size. If there is no stock at the time of receiving a request for the items, it is assumed that it will be satisfied at a later date with a penalty. This is called *backordering*. The operation of this model is illustrated in Fig. 12.4.

The variables which are used in this model are

D = demand/period
C_c = carrying cost/unit/period
C_o = ordering cost/order
C_s = shortage cost/unit/period

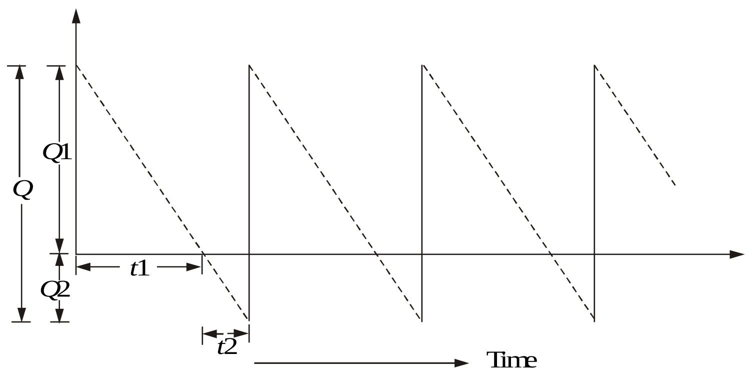


Fig. 12.4 Purchase model of inventory with shortages.

In the above model,

Q = order size,
$Q1$ = maximum inventory,
$Q2$ = maximum stock-out,
$t1$ = period of positive stock,
$t2$ = period of shortage.

$$Q^* = \text{EOQ} = \sqrt{\frac{2C_oD}{C_c}\frac{(C_s + C_c)}{C_s}}$$

$$Q1^* = \sqrt{\frac{2C_oD}{C_c}\frac{C_s}{C_s + C_c}}$$

$$Q2^* = Q^* - Q1^*$$

$$t^* = Q^*/D$$

$$t1^* = Q1^*/D$$

$$t2^* = Q2^*/D$$

$$\text{No. of orders/year} = D/Q^*$$

EXAMPLE 12.3 The annual demand for a component is 30,000 units. The carrying cost is Rs. 2.00/unit/year, the ordering cost is Rs. 100.00/order, and the shortage cost is Rs. 12.00/unit/year. Find the optimal values of the following:

(a) Ordering quantity
(b) Maximum inventory
(c) Maximum shortage quantity
(d) Cycle time
(e) Inventory period ($t1$)
(f) Shortage period ($t2$)

Solution

D = 30,000 units/year
C_c = Rs. 2.00/unit/year
C_o = Rs. 100/order
C_s = Rs. 12/unit/year

$$Q^* = \text{EOQ} = \sqrt{\frac{2C_oD}{C_c}\,\frac{(C_s + C_c)}{C_s}}$$

$$= \sqrt{\frac{2 \times 100 \times 30{,}000}{2}\,\frac{(12 + 2)}{12}}$$

$$= 1{,}871 \text{ units}$$

$$Q1^* = \sqrt{\frac{2C_oD}{C_c}\,\frac{C_s}{C_s + C_c}}$$

$$= \sqrt{\frac{2 \times 100 \times 30{,}000}{2}\,\frac{12}{(12 + 2)}}$$

$$= 1{,}604 \text{ units}$$

$Q2^* = 1{,}871 - 1{,}604 = 267$ units
$t^* = Q^*/D = (1{,}871/30{,}000) \times 365 = 23$ days (approx.)
$t1^* = Q1^*/D = (1{,}604/30{,}000) \times 365 = 20$ days
$t2^* = t^* - t1^* = 23 - 20 = 3$ days
No. of orders/year = D/Q^* = 30,000/1,871 = 16.03

12.5 MANUFACTURING MODEL WITH SHORTAGES

In this model, the items are produced and consumed simultaneously for a portion of the cycle time. During the remaining cycle time, the items are only consumed. The cost of production per unit is same irrespective of production lot size. In this model, stock-out is permitted. It is assumed that the stock-out units will be satisfied from the units which will be produced at a later date with a penalty. This is called backordering. The operation of this model is shown in Fig. 12.5. The variables which are used in this model are given now.

Let

r = annual demand of an item per period

k = production rate of the item (No. of units produced per period)

C_o = cost per set-up.

C_c = carrying cost per unit per period

C_s = shortage cost per unit per period

p = cost of production per unit

$t1$ = period of production as well as consumption satisfying period's requirement

$t2$ = period of consumption only

$t3$ = period of shortage

$t4$ = period of production as well as consumption satisfying back order

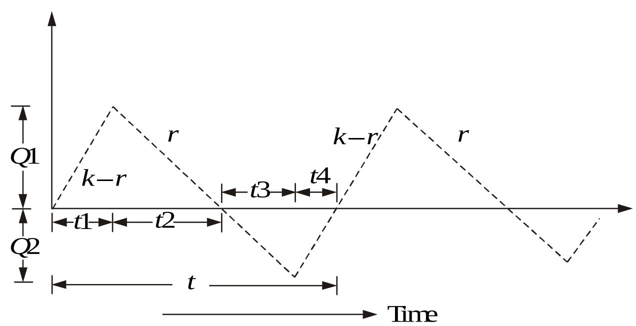


Fig. 12.5 Manufacturing model of inventory with shortages.

In the above model,

Q = production size
$Q1$ = maximum inventory
$Q2$ = maximum stock-out

The formulae for the optimal values of the above variables are presented now.

$$Q^* = \text{EOQ} = \sqrt{\frac{2C_o}{C_c}\,\frac{kr}{k-r}\,\frac{C_c+C_s}{C_s}}$$

$$Q1^* = \sqrt{\frac{2C_o}{C_c}\,\frac{r(k-r)}{k}\,\frac{C_s}{C_c+C_s}}$$

$$Q2^* = \sqrt{\frac{2C_o\, C_c}{C_s(C_c + C_s)}\,\frac{r(k-r)}{k}}$$

$$Q1^* = \frac{k-r}{k}\, Q^* - Q2^*$$

$$t^* = Q^*/r$$

$$t1^* = Q1^*/(k-r)$$

$$t2^* = Q1^*/r$$

$$t3^* = Q2^*/r$$

$$t4^* = Q2^*/(k-r)$$

EXAMPLE 12.4 The demand for an item is 12,000/year. Its production rate is 2,000/month. The carrying cost is Re. 0.20/unit/month and the set-up cost is Rs. 400.00/set-up. The shortage cost is Rs. 15.00/unit/year. Find the various parameters of the inventory system.

Solution

r = 12,000 units/year

k = 2,000 × 12 = 24,000 units/year

C_o = Rs. 400/set-up.

C_c = Rs. (0.20 × 12) = Rs. 2.40/year

C_s = Rs. 15.00/unit/year

$$Q^* = \text{EOQ} = \sqrt{\frac{2 \times 400}{2.40}\,\frac{24{,}000 \times 12{,}000}{(24{,}000 - 12{,}000)}\,\frac{(2.40 + 15)}{15}}$$

= 3,046 units (approx.)

$$Q2^* = \sqrt{\frac{2 \times 400 \times 2.40}{15(2.40 + 15)}\,\frac{12{,}000(24{,}000 - 12{,}000)}{24{,}000}}$$

= 210

$$Q1^* = \frac{24{,}000 - 12{,}000}{24{,}000}\, 3{,}046 - 210$$

= 1,313

t^* = $[Q^*/r]$365 days = [3,046/12,000]365 days = 93 days

$t1^*$ = $[Q1^*/(k-r)]$365 days = [1,313/(24,000 – 12,000)]365 days
= 40 days

$t2^*$ = $[Q1^*/r]$365 days = [1,313/12,000]365 days = 40 days

$t3^*$ = $[Q2^*/r]$365 days = [210/12,000]365 days = 7 days

$t4^*$ = $[Q2^*/(k-r)]$365 days = [210/(24,000 – 12,000)]365 days
= 7 days

QUESTIONS

1. What are the reasons for stocking items in inventory?

2. List and explain different types of costs in the inventory system.

3. What are the types of model of inventory system? Explain them in brief.

4. Explain the cost trade-off of the purchase model of inventory system without shortage.

5. Alpha Industry estimates that it will sell 12,000 units of its product for the forthcoming year. The ordering cost is Rs. 100 per order and the carrying cost per unit per year is 20% of the purchase price per unit. The purchase price per unit is Rs. 50. Find:

(a) Economic order size

(b) No. of orders per year

(c) Time between successive orders

6. A product is to be manufactured within the company whose details are as follows:

r = 24,000 units/year

k = 48,000 units/year

C_o = Rs. 200/set-up

C_c = Rs. 20/unit/year

Find the EOQ and cycle time.

7. The annual demand for an automobile component is 24,000 units. The carrying cost is Rs. 0.40/unit/year, the ordering cost is Rs. 20.00/order, and the shortage cost is Rs. 10.00/unit/year. Find the optimal values of the following:

(a) Economic order quantity

(b) Maximum inventory

(c) Maximum shortage quantity

(d) Cycle time

(e) Inventory period ($t1$)

(f) Shortage period ($t2$).

8. The demand for an item is 18,000 per year. Its production rate is 3,000/month. The carrying cost is Re. 0.15/unit/month, and the set-up cost is Rs. 500.00/set-up. The shortage cost is Rs. 20.00/unit/year. Find various parameters of the inventory system.

13

MAKE OR BUY DECISION

13.1 INTRODUCTION

In the process of carrying out business activities of an organization, a component/product can be made within the organization or bought from a subcontractor. Each decision involves its own costs. So, in a given situation, the organization should evaluate each of the above make or buy alternatives and then select the alternative which results in the lowest cost. This is an important decision since it affects the productivity of the organization. In the long run, the make or buy decision is not static. The make option of a component/product may be economical today; but after some time, it may turn out to be uneconomical to make the same.

Thus, the make or buy decision should be reviewed periodically, say, every 1 to 3 years. This is mainly to cope with the changes in the level of competition and various other environmental factors.

13.2 CRITERIA FOR MAKE OR BUY

In this section the criteria for make or buy are discussed.

Criteria for make

The following are the criteria for make:

1. The finished product can be made cheaper by the firm than by outside suppliers.
2. The finished product is being manufactured only by a limited number of outside firms which are unable to meet the demand.
3. The part has an importance for the firm and requires extremely close quality control.
4. The part can be manufactured with the firm's existing facilities and similar to other items in which the company has manufacturing experience.

Criteria for buy

The following are the criteria for buy:

1. Requires high investments on facilities which are already available at suppliers plant.
2. The company does not have facilities to make it and there are more profitable opportunities for investing company's capital.
3. Existing facilities of the company can be used more economically to make other parts.
4. The skill of personnel employed by the company is not readily adaptable to make the part.
5. Patent or other legal barriers prevent the company for making the part.
6. Demand for the part is either temporary or seasonal.

13.3 APPROACHES FOR MAKE OR BUY DECISION

Types of analysis followed in make or buy decision are as follows:

1. Simple cost analysis
2. Economic analysis
3. Break-even analysis

13.3.1 Simple Cost Analysis

The concept is illustrated using an example problem.

EXAMPLE 13.1 A company has extra capacity that can be used to produce a sophisticated fixture which it has been buying for Rs. 900 each. If the company makes the fixtures, it will incur materials cost of Rs. 300 per unit, labour costs of Rs. 250 per unit, and variable overhead costs of Rs. 100 per unit. The annual fixed cost associated with the unused capacity is Rs. 10,00,000. Demand over the next year is estimated at 5,000 units. Would it be profitable for the company to make the fixtures?

Solution We assume that the unused capacity has alternative use.

Cost to make

Variable cost/unit = Material + labour + overheads

= Rs. 300 + Rs. 250 + Rs. 100

= Rs. 650

Total variable cost = (5,000 units) (Rs. 650/unit)

= Rs. 32,50,000

Add fixed cost associated
with unused capacity + Rs. 10,00,000

Total cost = Rs. 42,50,000

Cost to buy

Purchase cost = (5,000 units) (Rs. 900/unit)

= Rs. 45,00,000

Add fixed cost associated
with unused capacity + Rs. 10,00,000

Total cost = Rs. 55,00,000

The cost of making fixtures is less than the cost of buying fixtures from outside. Therefore, the organization should make the fixtures.

13.3.2 Economic Analysis

The following inventory models are considered to illustrate this concept:

- Purchase model
- Manufacturing model

The formulae for EOQ and total cost (TC) for each model are given in the following table:

Purchase model	*Manufacturing model*
$Q1 = \sqrt{\dfrac{2C_oD}{C_c}}$	$Q2 = \sqrt{\dfrac{2C_oD}{C_c\,(1 - r/k)}}$
$TC = D \times P + \dfrac{DC_o}{Q1} + \dfrac{Q1 \times C_c}{2}$	$TC = D \times P + \dfrac{DC_o}{Q2} + C_c\,(k - r)\,\dfrac{Q2}{2 * k}$

where

D = demand/year

P = purchase price/unit

C_c = carrying cost/unit/year

C_o = ordering cost/order or set-up cost/set-up

k = production rate (No. of units/year)

r = demand/year

$Q1$ = economic order size

$Q2$ = economic production size

TC = total cost per year

EXAMPLE 13.2 An item has a yearly demand of 2,000 units. The different costs in respect of make and buy are as follows. Determine the best option.

	Buy	*Make*
Item cost/unit	Rs. 8.00	Rs. 5.00
Procurement cost/order	Rs. 120.00	
Set-up cost/set-up		Rs. 60.00
Annual carrying cost/ item/year	Rs. 1.60	Rs. 1.00
Production rate/year		8,000 units

Solution

Buy option

D = 2,000 units/year

C_o = Rs. 120/order

C_c = Rs. 1.60/unit/year

$$Q1 = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2 \times 2{,}000 \times 120}{1.60}}$$

$$= 548 \text{ units (approx.)}$$

$$TC = DP + \frac{DC_o}{Q1} + \frac{Q1C_c}{2}$$

$$= 2{,}000 \times 8 + \frac{2{,}000 \times 120}{548} + \frac{548 \times 1.60}{2}$$

$$= \text{Rs. } 16{,}876.36$$

Make option

C_o = Rs. 60/set-up

r = 2,000 units/year

C_c = Re 1/unit/year

k = 8,000 units/year

$$Q2 = \sqrt{\frac{2C_or}{C_c[1 - (r/k)]}}$$

$$= \sqrt{\frac{2 \times 60 \times 2{,}000}{1.0\,(1 - 2{,}000/8{,}000)}} = 566 \text{ units (approx.)}$$

$$TC = DP + \frac{D \times C_o}{Q2} + C_c\,(k - r)\,\frac{Q2}{2 \times k}$$

$$= 2{,}000 \times 5.00 + \frac{2{,}000 \times 60}{566} + 1.0\,(8{,}000 - 2{,}000)\,\frac{566}{2 \times 8{,}000}$$

$$= \text{Rs. } 10{,}424.26$$

Result: The cost of making is less than the cost of buying. Therefore, the firm should go in for the making option.

13.3.3 Break-even Analysis

The break-even analysis chart is shown in Fig. 13.1. In the figure

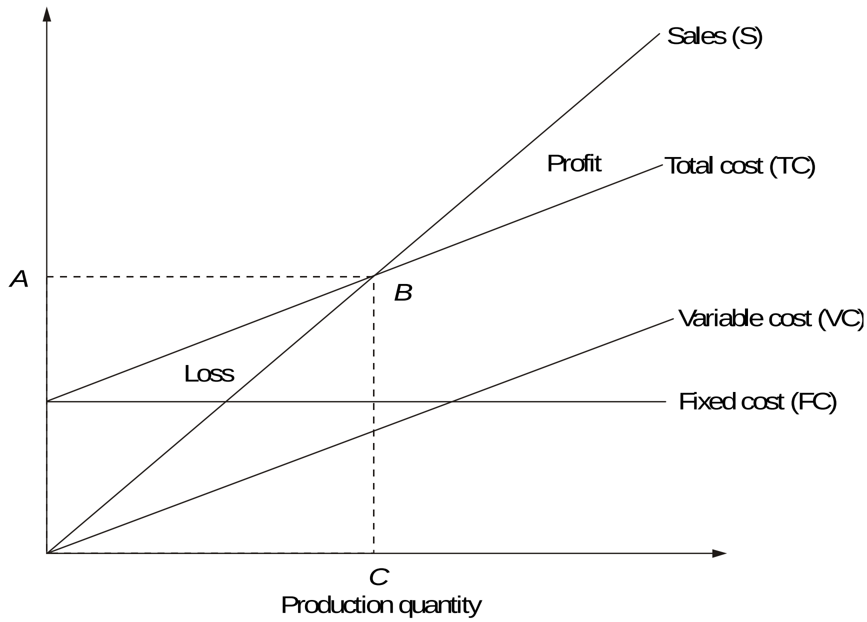


Fig. 13.1 Break-even chart.

TC = total cost

FC = fixed cost

TC = FC + variable cost

B = the intersection of TC and sales (no loss or no gain situation)

A = break-even sales

C = break-even quantity/break-even point (BEP)

The formula for the break-even point (BEP) is

$$\text{BEP} = \frac{FC}{\text{Selling price/unit} - \text{Variable cost/unit}}$$

EXAMPLE 13.3 A manufacturer of TV buys TV cabinet at Rs. 500 each. In case the company makes it within the factory, the fixed and variable costs would be Rs. 4,00,000 and Rs. 300 per cabinet respectively. Should the manufacturer make or buy the cabinet if the demand is 1,500 TV cabinets?.

Solution

Selling price/unit (SP) = Rs. 500
Variable cost/unit (VC) = Rs. 300
Fixed cost (FC) = Rs. 4,00,000

$$\text{BEP} = \frac{4{,}00{,}000}{500 - 300} = 2{,}000 \text{ units}$$

Since the demand (1,500 units) is less than the break-even quantity, the company should buy the cabinets for its TV production.

EXAMPLE 13.4 There are three alternatives available to meet the demand of a particular product. They are as follows:

(a) Manufacturing the product by using process A

(b) Manufacturing the product by using process B

(c) Buying the product

The details are as given in the following table:

Cost elements	*Manufacturing the product by process A*	*Manufacturing the product by process B*	*Buy*
Fixed cost/year (Rs.)	5,00,000	6,00,000	
Variable/unit (Rs.)	175	150	
Purchase price/unit (Rs.)			125

The annual demand of the product is 8,000 units. Should the company make the product using process A or process B or buy it?

Solution

$$\begin{aligned}
\text{Annual cost of process A} &= \text{FC} + \text{VC} \times \text{Volume} \\
&= 5{,}00{,}000 + 175 \times 8{,}000 \\
&= \text{Rs. } 19{,}00{,}000
\end{aligned}$$

$$\begin{aligned}
\text{Annual cost of process B} &= \text{FC} + \text{VC} \times \text{Volume} \\
&= 6{,}00{,}000 + 150 \times 8{,}000 \\
&= \text{Rs. } 18{,}00{,}000
\end{aligned}$$

$$\begin{aligned}
\text{Annual cost of buy} &= \text{Purchase price/unit} \times \text{Volume} \\
&= 125 \times 8{,}000 \\
&= \text{Rs. } 10{,}00{,}000
\end{aligned}$$

Since the annual cost of buy option is the minimum among all the alternatives, the company should buy the product.

QUESTIONS

1. Briefly explain the various criteria for make or buy decisions.

2. What are the approaches available for make or buy decisions? Explain any one of them with a suitable example.

3. An automobile company has extra capacity that can be used to produce gears that the company has been buying for Rs. 300 each. If the company makes gears, it will incur materials cost of Rs. 90 per unit, labour costs of Rs. 120 per unit, and variable overhead costs of Rs. 30 per unit. The annual fixed cost associated with the unused capacity is Rs. 2,40,000. Demand for next year is estimated at 4,000 units.

(a) Would it be profitable for the company to make the gears?

(b) Suppose the capacity could be used by another department for the production of some agricultural equipment that would cover its fixed and variable cost and contribute Rs. 90,000 to profit which would be more advantageous, gear production or agricultural equipment production?

4. An item has an yearly demand of 1,000 units. The different costs with regard to make and buy are as follows. Determine the best option.

	Buy	*Make*
Item cost/unit	Rs. 6.00	Rs. 5.90
Procurement cost/order	Rs. 10.00	
Set-up cost/set-up		Rs. 50.00
Annual carrying cost/item/year	Rs. 1.32	Rs. 1.30
Production rate/year		6,000 units

5. A manufacturer of motor cycles buys side boxes at Rs. 240 each. In case he makes it himself, the fixed and variable costs would be Rs. 30,00,000 and Rs. 90 per side box respectively. Should the manufacturer make or buy the side boxes if the demand is 2,500 side boxes?

6. There are three alternatives available to meet the demand of a particular product. They are as follows:

(a) Manufacturing the product by using process A

(b) Manufacturing the product by using process B

(c) Buying the product

The details are as follows:

Cost elements	*Manufacturing the product by using process A* (Rs.)	*Manufacturing the product by using process B* (Rs.)	*Buy* (Rs.)
Fixed cost/year	1,00,000	3,00,000	
Variable cost/unit	75	70	
Purchase price/unit			80

The annual demand of the product is 10,000 units.

(a) Should the company make the product using process A or process B or buy it?

(b) At what annual volume should the company switch from buying to manufacturing the product by using process A?

(c) At what annual volume, should the company switch from process A to B?

14

PROJECT MANAGEMENT

14.1 INTRODUCTION

A project consists of interrelated activities which are to be executed in a certain order before the entire task is completed. The activities are interrelated in a logical sequence, which is known as *precedence relationship*. An activity of a project cannot be started until all its immediate preceding activities are completed. Some of the typical projects are as follows:

- Construction of a house
- Commissioning of a factory
- Construction of a ship
- Fabrication of a steam boiler
- Construction of a bridge
- Construction of a dam
- Commissioning of a power plant
- Shutdown maintenance of major equipment/plants
- State level professional course admission process
- New product launching
- Launching a new weapon system
- Conducting national elections
- Research to develop a new technology
- Construction of railway coaches

Project management is generally applied for constructing public utilities, large industrial projects, and organizing mega events. Project management is considered to be an important field in production scheduling mainly because many of the industrial activities can also be viewed as project management problems. For example, fabrication of boilers, construction of railway coaches, launching satellites, product launching, organizing R&D activities, etc. can be viewed as project management problems.

From the examples, one can recognize the fact that many of the projects are repeated either by the same organization or by different organizations. Though they are repeatable by nature, each project is unique in itself. In the case of the new product launching of an organization, the first launching will be done at some key city. Subsequent launchings will be done at other cities as per priorities fixed by the marketing department of the organization. The project schedule which is prepared for the first city cannot be applied to other cities without any modification because the time estimates of different activities of the project (product launching) will be different for each city due to environmental conditions. In some cases, addition or deletion of some activities will take place. So, if a project is repeated at a different place/different time, then a detailed planning effort is required.

Project is represented in the form of a network for the purpose of analytical treatment to get solutions for scheduling and controlling its activities. A network consists of a set of arcs which are connected meaningfully through a set of nodes. The precedence relationship among various activities of a project can be conveniently represented using a network. So, the collection of precedence relationships among various activities of a project is known as project network.

There are two methods of representing any project in the network form. There are: (i) Activities on Arrows Diagram (AOA diagram), and (ii) Activities on Nodes Diagram (AON Diagram).

The AOA diagram is commonly used in project management. This concept is demonstrated through two examples.

EXAMPLE 14.1 A construction company has listed various activities that are involved in constructing a community hall. These are summarized along with predecessor(s) details in Table 14.1.

Table 14.1 Details of Activities and Predecessor(s) for Constructing Community Hall

Activity	*Description*	*Immediate predecessor(s)*
A	Plan approval	
B	Site preparation	
C	Arranging foundation materials	*A*
D	Excavation for foundation	*B*
E	Carpentry work for door and window main supporting frames	*A*
F	Laying foundation	*C,D*
G	Raising wall from foundation to window base	*F*
H	Raising wall from window base to lintel level	*E,G*
I	Roofing	*H*
J	Electrical wiring & fitting	*I*
K	Plastering	*J,L*
L	Making doors and windows & fitting them	*A*
M	White washing	*K*
N	Clearing the site before handing over	*M*

Draw a project network for the above project.

Solution The project network summarizing the precedence relationships of various activities of constructing the community hall is shown in Fig. 14.1. In this figure, activities *A* and *B* are concurrent activities. Activities *C*, *E* and *L* can be started only after completing activity *A*. Activity *D* follows activity *B*.

Activity *F* can be started only when activities *C* and *D* are completed. One can use similar logic to infer the remaining precedence relationships in the Fig. 14.1.

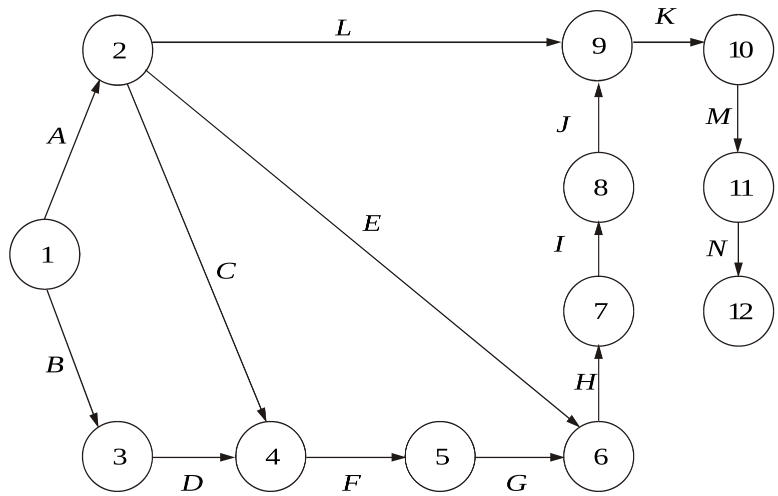


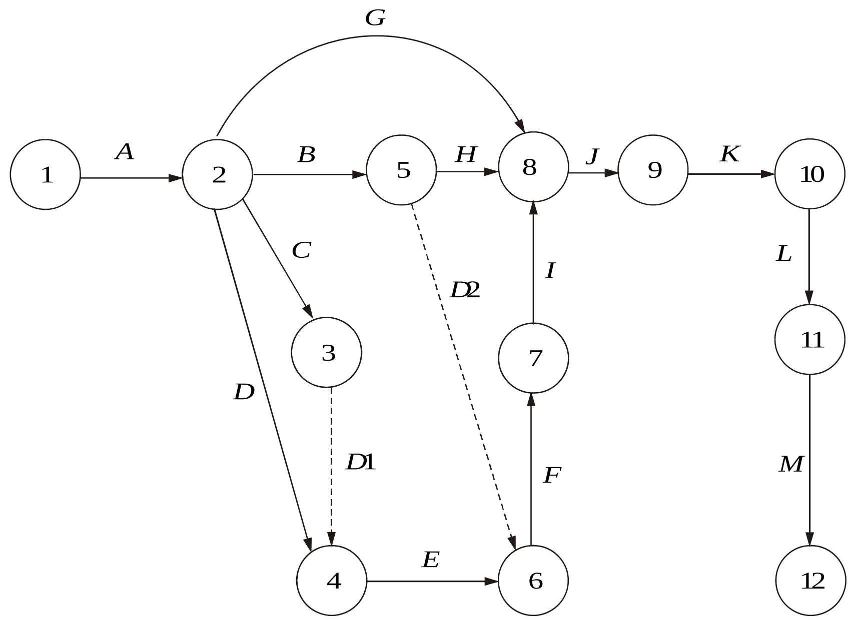
Fig. 14.1 Project network for construction of a community hall.

EXAMPLE 14.2 Nowadays, in all professional institutes, students selection is done through a written test, group discussion and interview. Prior to the written test, there are so many activities, which are to be performed right from deciding the date of conducting all other events. Though the system looks simple, the precedence relationships among the activities necessitate coordination of these activities in completing the whole task of admitting students to any professional course. A delay at any stage would lead to ineffective operation of the system. Hence, this problem may be treated as a project consisting of activities with precedence relationships as shown in Table 14.2 (Rajkumar and Panneerselvam 1992). Construct a CPM network for this problem.

Table 14.2 Precedence Relationships for Professional Course Admission Process

Activity	*Description*	*Immediate predecessors*
A	Decide the date, time of the entrance test, group discussion and interview	
B	Decide the examination venue(s)	*A*
C	Call for application through advertisement	*A*
D	Print application form	*A*
E	Despatch applications	*D*, *C*
F	Receive and process applications and send hall tickets	*E*, *B*
G	Set question paper	*A*
H	Arrange invigilators, and examiner	*B*
I	Make seating arrangements	*F*
J	Conduct examination	*G*, *H*, *I*
K	Valuation and announcement of entrance test result	*J*
L	Conduct group discussion and interview	*K*
M	Publish admission result	*L*

Solution The CPM network for the above problem is shown in Fig. 14.2.



*$D1$ and $D2$ are dummy activities

Fig. 14.2 Network for professional course admission process.

Figure 14.2 shows how the precedence relations between various activities of the admission project are defined. In this figure, activity *A* is the starting activity. Activities *B*, *C* and *D* can be started only after activity *A* is completed. Activity *E* can be started after completing activities *D* and *C*. The precedence relationship between all other activities are defined in this manner. Activity *M* is the final activity representing the end of the selection process.

There are two main basic techniques in project management, namely, Critical Path Method (CPM), and Project Evaluation and Review Technique (PERT). CPM was developed by E.I. Du Pont de Nemours & Company as an application to construction projects and was later extended by Mauchly Associates. PERT was developed by a consulting firm for scheduling the research and development activities for the polaris missile program of US Navy. In CPM, the activities timings are deterministic in nature. But in PERT, each activity will have three time estimates: optimistic time, most likely time, and pessimistic time. There are some more advanced topics in project management which are as follows:

- Crashing of a project network
- Resource levelling
- Resource allocation

These advanced topics are beyond the scope of this book.

14.2 PHASES OF PROJECT MANAGEMENT

Project management has three phases: planning, scheduling and controlling.

The planning phase involves the following steps:

(a) Dividing the project into distinct activities.

(b) Estimating time requirement for each activity.

(c) Establishing precedence relationships among the activities.

(d) Construction of the arrow diagram (network).

The scheduling phase determines the start and end times of each and every activity. These can be summarized in the form of a time chart/Gantt chart. For each non-critical activity, the amount of slack time must be shown on the same time chart. This will be useful at the time of adjusting non-critical activities for resource levelling/resource allocation.

The control phase uses the arrow diagram and time chart for continuous monitoring and progress reporting. In this phase, the network will be updated if there is any variation in the proposed schedule.

14.3 GUIDELINES FOR NETWORK CONSTRUCTION

The terminologies which are used in network construction are explained as follows:

Node: Generally, a node represents the starting or ending of an activity.

Branch/Arc: A branch represents the actual activity which consumes some kind of resource.

Precedence relations of activities: For any activity, the precedence relations provide the following information:

1. The activities that precede it
2. The activities that follow it
3. The activities that may be concurrent with it.

Network construction requires a detailed list of individual activities of a project, estimates of activity durations, and specifications of precedence relationships among various activities of the project.

Rules for network construction

The following are the primary rules for constructing an AOA diagram:

1. The starting event and ending event of an activity are called *tail event* and *head event*, respectively.
2. The network should have a unique starting node.

3. The network should have a unique completion node.
4. No activity has to be represented by more than one arc in the network.
5. No two activities need to have the same starting node and the same ending node.
6. Dummy activity is an imaginary activity indicating precedence relationship only. Duration of a dummy activity is zero.

14.4 CRITICAL PATH METHOD (CPM)

As stated earlier, CPM deals with project management involving deterministic time estimates. In this section, the concept of CPM is demonstrated through an example.

EXAMPLE 14.3 Consider Table 14.3 summarizing the details of a project involving 16 activities.

Table 14.3 Project Details

Activity	*Immediate predecessor*	*Duration (months)*
A		1
B		4
C		2
D	*B*	2
E	*B*	3
F	*A, D*	3
G	*A, D*	2
H	*B*	1
I	*C, E*	3
J	*C, E*	2
K	*F*	3
L	*G, H, I*	6
M	*G, H, I*	5
N	*J*	2
P	*K, L*	4
Q	*M, N*	7

(a) Construct the CPM network.

(b) Determine the critical path.

(c) Compute total floats and free floats for non-critical activities.

Solution **(*a*)** The CPM network is shown in Fig. 14.3.

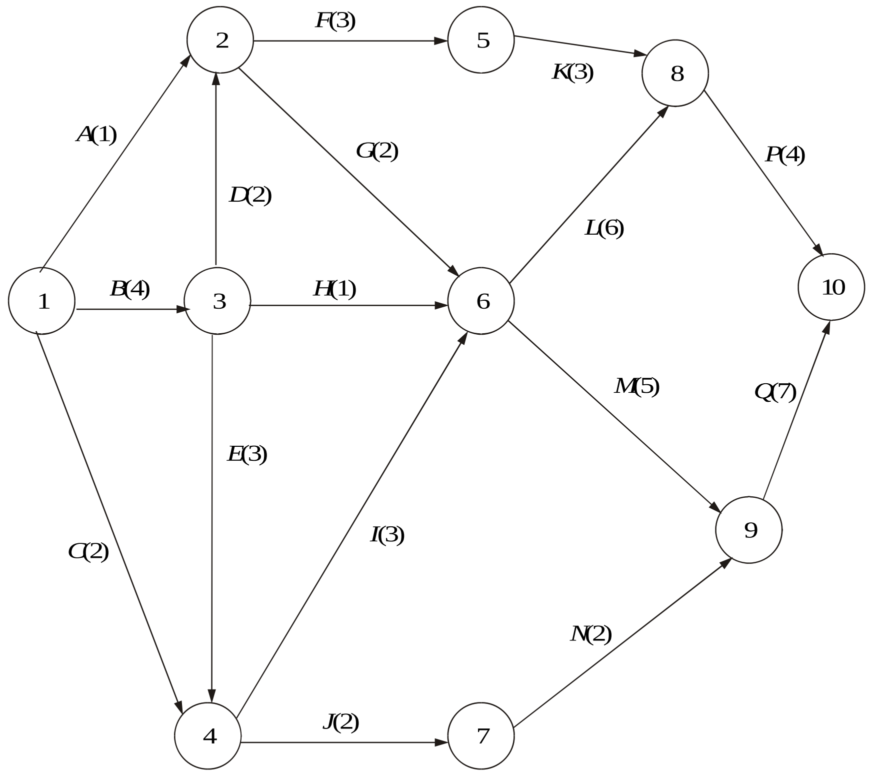


Fig. 14.3 CPM network for Example 14.3.

(b) Determination of critical path. The critical path of a project network is the longest path in the network. This can be identified by simply listing out all the possible paths from the start node of the project (node 1) to the end node of the project (node 10), and then selecting the path with the maximum sum of activity times on that path.

This method has several drawbacks. In a large network, one may commit mistake in listing all the paths. Moreover, this method will not provide necessary details like total floats and free floats for further analysis.

A different approach is therefore to be used to identify the critical path. This consists of two phases: *Phase 1* determines the earliest start times (ES) of all the nodes. This is called the *forward pass*; *phase 2* determines the latest completion times (LC) of various nodes. This is called the *backward pass*. Let

D_{ij} = duration of the activity (i, j),

ES_j = earliest start time of all the activities which emanate from node j,

LC_j = latest completion time of all the activities that end at node j.

(i) Determination of earliest start times (ES_j): During forward pass, use the following formula to compute the earliest start time for all nodes:

$$ES_j = \max_i \; [ES_i + D_{ij}]$$

The calculations of ES_j are as follows:

NODE 1: For node 1, $ES_1 = 0$

NODE 3: $ES_3 = \max\limits_{i=1} [ES_i + D_{i3}]$

$= \max [ES_1 + D_{1,3}] = 0 + 4 = 4$

NODE 2: $ES_2 = \max\limits_{i=1,3} [ES_i + D_{i2}]$

$= \max [(ES_1 + D_{1,2}), (ES_3 + D_{3,2})]$

$= \max (0 + 1, 4 + 2) = 6$

NODE 4: $ES_4 = \max\limits_{i=1,3} [ES_i + D_{i4}]$

$= \max [ES_1 + D_{1,4}, ES_3 + D_{3,4}]$

$= \max (0 + 2, 4 + 3) = 7$

Similarly, the ES_j values for all other nodes are computed and summarized in Fig. 14.4.

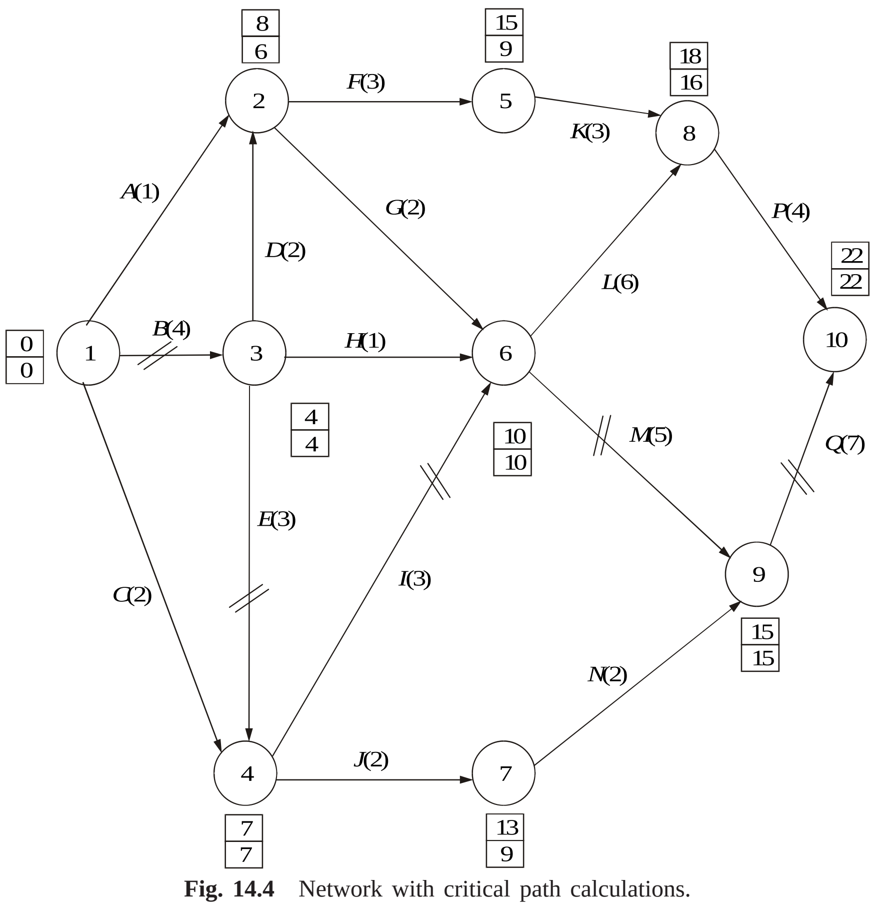


Fig. 14.4 Network with critical path calculations.

(ii) Determination of latest completion times (LC_i). During backward pass, the following formula is used to compute the latest completion times (LC_i):

$$LC_i = \min_j [LC_j - D_{ij}]$$

NODE 10: For the last node 10, $LC_{10} = ES_{10} = 22$.

NODE 9: $LC_9 = \min_{j=10} [LC_j - D_{9,j}]$

$= \min [LC_{10} - D_{9,10}] = 22 - 7 = 15.$

NODE 8: $LC_8 = \min_{j=10} [LC_j - D_{8,j}]$

$= \min [LC_{10} - D_{8,10}] = 22 - 4 = 18.$

NODE 7: $LC_7 = \min_{j=9} [LC_j - D_{7,j}]$

$= \min [LC_9 - D_{7,9}] = 15 - 2 = 13.$

NODE 6: $LC_6 = \min_{j=8,9} [LC_j - D_{6,j}]$

$= \min [LC_8 - D_{6,8}, LC_9 - D_{6,9}]$

$= \min [18 - 6, 15 - 5] = 10$

Similarly, the LC_j values for all other nodes are summarized in Fig. 14.4.

An activity (i, j) is said to be critical if all the following conditions are satisfied:

(i) $ES_i = LC_i$

(ii) $ES_j = LC_j$

(iii) $ES_j - ES_i = LC_j - LC_i = D_{ij}$

By applying the above conditions to the activities in Fig. 14.4, the critical activities are identified and are shown in the same figure with cross lines on them. The corresponding critical path is 1–3–4–6–9–10 (*B–E–I–M–Q*).

(*c*) Determination of Total Floats and Free Floats

TOTAL FLOATS. It is the amount of time that the completion time of an activity can be delayed without affecting the project completion time.

$$TF_{ij} = LC_j - ES_i - D_{ij}$$
$$= LC_j - (ES_i + D_{ij}) = LC_j - EC_{ij}$$

where EC_{ij} is the earliest completion of the activity (i, j).
Also, $TF_{ij} = LS_{ij} - ES_i$, where LS_{ij} is the latest start of the activity (i, j).

$$LS_{ij} = LC_j - D_{ij}$$

FREE FLOATS. It is the amount of time that the activity completion time can be delayed without affecting the earliest start time of immediate successor activities in the network.

$$FF_{ij} = ES_j - ES_i - D_{ij}$$
$$= ES_j - (ES_i + D_{ij}) = ES_j - EC_{ij}$$

A summary of calculations of total floats and free floats of the activities is given in Table 14.4.

Table 14.4 Total Floats and Free Floats

Activity (i, j)	*Duration* (D_{ij})	*Total float* (TF_{ij})	*Free float* (FF_{ij})
1–2	1	7	5
1–3	4	0	0
1–4	2	5	5
2–5	3	6	0
2–6	2	2	2
3–2	2	2	0
3–4	3	0	0
3–6	1	5	5
4–6	3	0	0
4–7	2	4	0
5–8	3	6	4
6–8	6	2	0
6–9	5	0	0
7–9	2	4	4
8–10	4	2	2
9–10	7	0	0

Any critical activity will have zero total float (TF_{ij}) and zero free float (FF_{ij}). Based on this property, one can determine the critical activities. From Table 14.4, one can check that the total floats and free floats for the activities (1,3), (3,4), (4,6), (6,9) and (9,10) are zero. Therefore, they are critical activities.

The corresponding critical path is 1–3–4–6–9–10 (*B*–*E*–*I*–*M*–*Q*).

14.5 GANTT CHART/TIME CHART

The next stage after completing network calculations is to draw a Gantt chart/ time chart. The start time and completion time of each and every activity will be represented on this chart. This chart gives a clear calendar schedule for the whole project.

This chart is also used for resource levelling purpose. When there are limitations on the available resource(s) (manpower, equipment, money, etc.), using this chart, one can adjust the schedule of non-critical activities, depending upon their total floats to minimize the peak requirement of resource(s). This helps to level the resource requirements smoothly throughout the project execution. Even if there is no restriction on any resource, it is the usual practice to smooth out the resource requirements by resource levelling technique.

The time chart for the example problem is given in Fig. 14.5.

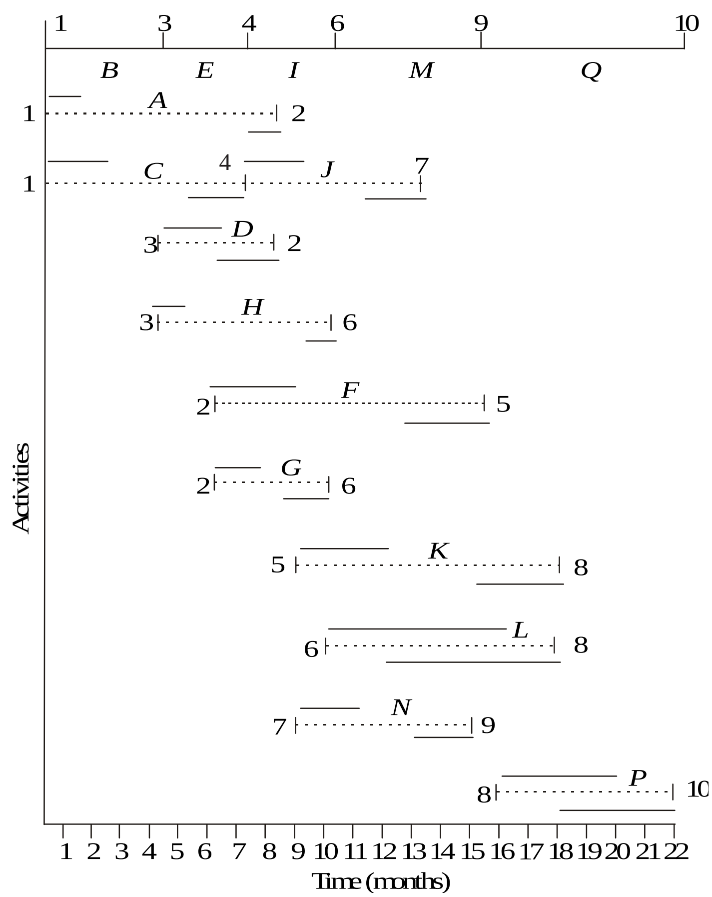


Fig. 14.5 Time chart/Gantt chart.

In Fig. 14.5, the thick line represents the schedule for various critical activities [(1,3), (3,4), (4,6), (6,9) and (9,10)]. The horizontal dashed lines represent the total time span over which a non-critical activity can be performed. As stated earlier, a non-critical activity will have float in excess of its time duration. So, it is possible to adjust the start and completion time of any non-critical activity over its entire range shown by the dashed lines without delaying the project completion time. Infinite number of schedules are possible for the non-critical activities. But to give a finite number of schedules, we consider only two types of schedules: 1) earliest start schedule, and 2) latest start schedule. These are shown by continuous horizontal lines over and below the dashed line showing the time range for execution of a given non-critical activity, respectively.

14.6 PERT (PROJECT EVALUATION AND REVIEW TECHNIQUE)

Earlier, we assumed deterministic estimates for time durations of various activities in a project. But, in reality, activity durations may be probabilistic. Hence, probabilistic considerations are incorporated while obtaining time durations of the activities in a project.

The following three estimates are used:

a = optimistic time

b = pessimistic time

m = most likely time

The *optimistic time* is a time estimate if the execution is very good. The *pessimistic time* is a time estimate if the execution is very badly done. The *most likely time* is a time estimate if execution is normal. The probabilistic data for project activities generally follow beta distribution. The formula for mean (μ) and variance (σ^2) of the beta distribution are as follows:

$$\mu = (a + 4m + b)/6$$
$$\sigma^2 = [(b - a)/6]^2$$

The range for the time estimates is from a to b. The most likely time will be anywhere in between the range from a to b.

The expected project completion time = $\Sigma\ \mu_i$
where μ_i is the expected duration of the ith critical activity.

The variance of the project completion time = $\Sigma_i\ \sigma_i^2$
where σ_i^2 is the variance of the ith critical activity in the critical path.

As a part of statistical analysis, we may be interested in knowing the probability of completing the project on or before a given due date (C), or we may be interested in knowing the expected project completion time if the probability of completing the project is given.

For the purpose of this analysis, the Beta distribution is approximated to standard normal distribution whose statistic is given by

$$z = (x - \mu)/\sigma$$

where x is the actual project completion time.

μ = expected project completion time (sum of the expected durations of the critical activities).

σ = standard deviation of the expected project completion time (square root of the sum of the variances of all the critical activities).

Therefore, $P(x \leq C)$ represents the probability that the project will be completed on or before the C time units. This can be converted into the standard normal statistic z as follows:

$$P[(x - \mu)/\sigma\ ] \leq (C - \mu)/\sigma = P[z \leq (C - \mu)/\sigma]$$

EXAMPLE 14.4 Consider the following (Table 14.5) summarizing the details of a project involving nine activities.

Table 14.5 Details of Project with Nine Activities

Activity	*Predecessor(s)*	*Duration* (weeks)		
		a	*m*	*b*
A		2	2	8
B		1	2	3
C		1	5	9
D	*A*	1	2	9
E	*B*	1	2	3
F	*B*	1	2	3
G	*C*	1	4	7
H	*D, E, F, G*	6	7	8
I	*F, G*	1	2	9

The project comprises the following steps:

(a) Construct the project network.

(b) Find the expected duration and variance of each activity.

(c) Find the critical path and the expected project completion time.

(d) What is the probability of completing the project on or before 19 weeks?

(e) If the probability of completing the project is 0.85, find the expected project completion time.

Solution (a) The project network is shown in Fig. 14.6.

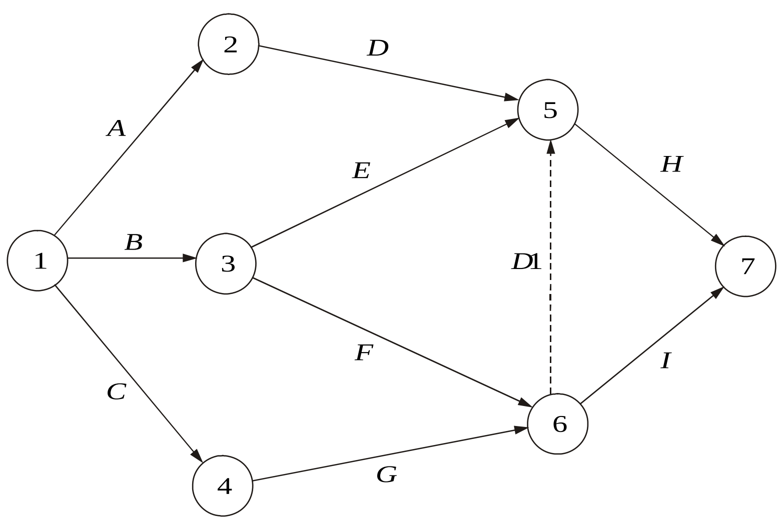


Fig. 14.6 Network for Example 14.4.

(b) The expected duration and variance of each activity are as shown in Table 14.6.

Table 14.6 Computations of Expected Duration and Variance

Activity	*Duration* (weeks)			*Expected duration*	*Variance*
	a	*m*	*b*		
A	2	2	8	3	1.00
B	1	2	3	2	0.11
C	1	5	9	5	1.78
D	1	2	9	3	1.78
E	1	2	3	2	0.11
F	1	2	3	2	0.11
G	1	4	7	4	1.00
H	6	7	8	7	0.11
I	1	2	9	3	1.78

(c) The calculations of critical path based on expected durations are summarized in Fig. 14.7.

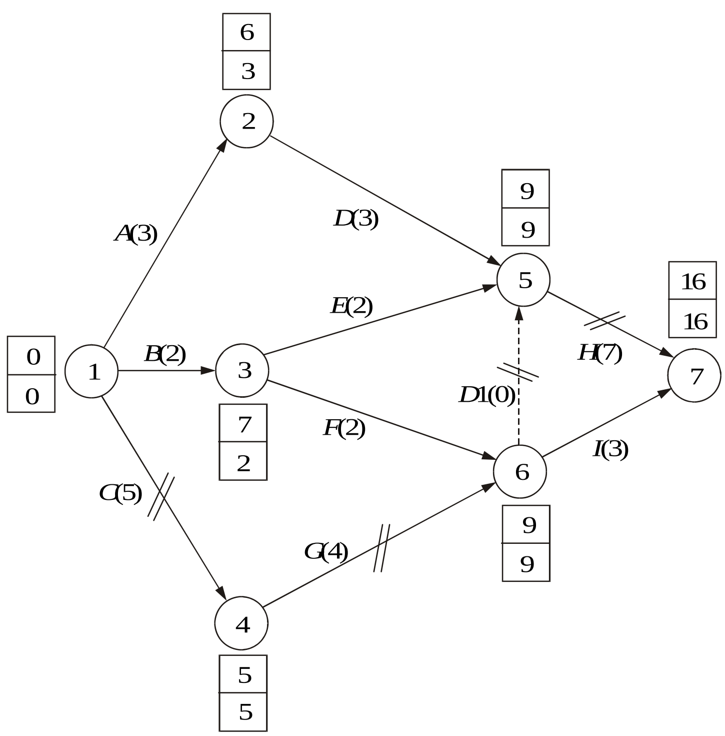


Fig. 14.7 Network for Example 14.4 with PERT calculations.

The critical path is $C–G–D1–H$, and the corresponding project completion time is 16 weeks.

(d) The sum of the variances of all the activities on the critical path is 2.89 weeks (1.78 + 1.00 + 0.00 + 0.11). Therefore,

$$\sigma = \sqrt{2.89} = 1.7 \text{ weeks.}$$

$$P(x \le 19) = P[(x - \mu)/\sigma \le (19 - 16)/1.7]$$

$$= P(z \le 1.77)$$

$$= 0.9616 \text{ (value obtained from standard normal table).}$$

Thus, the probability of completing the project on or before 19 weeks is 0.9616.

(e) $P(x \le C) = 0.85$

$P[(x - \mu)/\sigma \le (C - \mu)/\sigma] = 0.85$

$P[z \le (C - 16)/1.7] = 0.85$

From the standard normal table, the value of z is 1.04 when the cumulative probability is 0.85. Therefore,

$$(C - 16)/1.7 = 1.04$$

$$C = 1.04 \times 1.7 + 16 = 17.768 \text{ weeks}$$

$$= 18 \text{ weeks (approx.)}$$

The project will be completed in 17.768 weeks (approx. 18 weeks) if the probability of completing the project is 0.85.

QUESTIONS

1. Explain different phases of project management.

2. Distinguish between CPM and PERT.

3. Discuss the guidelines for network construction.

4. The activities involved in Alpha Garment Manufacturing Company are given in the following table, with their time estimates. Draw the network for the given activities and carry out the critical path calculations.

Activity	*Description*	*Immediate predecessor(s)*	*Duration* (days)
A	Forecast sales volume		10
B	Study competitive market		7
C	Design item and facilities	*A*	5
D	Prepare production plan	*C*	3
E	Estimate cost of production	*D*	2
F	Set sales price	*B, E*	1
G	Prepare budget	*E, F*	14

5. Consider the following problem involving activities from A to J.

Activity	*Immediate predecessor*	*Duration* (months)
A		1
B	A	4
C	A	2
D	A	2
E	D	3
F	D	3
G	E	2
H	F, G	1
I	C, H	3
J	B	2

(a) Construct the CPM network.

(b) Determine the critical path.

(c) Compute the total floats and free floats for non-critical activities.

6. Consider the following data of a project:

Activity	*Predecessor(s)*	*Duration (weeks)*		
		a	m	b
A		1	2	3
B		2	2	8
C	A	6	7	8
D	B	1	2	3
E	A	1	4	7
F	C, D	1	5	9
G	C, D, E	1	2	3
H	F	1	2	9

(a) Construct the project network.

(b) Find the expected duration and variance of each activity.

(c) Find the critical path and the expected project completion time.

(d) What is the probability of completing the project on or before 20 weeks?

(e) If the probability of completing the project is 0.8, find the expected project completion time.

15

VALUE ANALYSIS/ VALUE ENGINEERING

15.1 INTRODUCTION

Value analysis is one of the major techniques of cost reduction and cost prevention. It is a disciplined approach that ensures necessary functions for minimum cost without sacrificing quality, reliability, performance, and appearance. According to the Society of American Value Engineers (SAVE),

> Value Analysis is the systematic application of recognized techniques which identify the function of a product or service, establish a monetary value for the function and provide the necessary function reliably at the lowest overall cost.

It is an organized approach to identify unnecessary costs associated with any product, material part, component, system or service by analysing the function and eliminating such costs without impairing the quality, functional reliability, or the capacity of the product to give service.

15.2 WHEN TO APPLY VALUE ANALYSIS

One can definitely expect very good results by initiating a VA programme if one or more of the following symptoms are present:

1. Company's products show decline in sales.
2. Company's prices are higher than those of its competitors.
3. Raw materials cost has grown disproportionate to the volume of production.
4. New designs are being introduced.
5. The cost of manufacture is rising disproportionate to the volume of production.
6. Rate of return on investment has a falling trend.
7. Inability of the firm to meet its delivery commitments.

15.2.1 Value Analysis vs. Value Engineering

Often the terms *value analysis* and *value engineering* are used synonymously.

Though the philosophy underlying the two is same, i.e. identification of unnecessary cost, yet they are different. The difference lies in the time and the stage at which the techniques are applied.

Value analysis is the application of a set of techniques to an existing product with a view to improve its value. It is thus a remedial process. *Value engineering* is the application of exactly the same set of techniques to a new product at the design stage, project concept or preliminary design when no hardware exists to ensure that bad features are not added. *Value engineering,* therefore, is a preventive process.

Value

The term 'value' is used in different ways and, consequently, has different meanings. The designer equates the value with reliability; a purchase person with price paid for the item; a production person with what it costs to manufacture, and a sales person with what the customer is willing to pay. Value, in value investigation, refers to "economic value", which itself can be divided into four types: cost value, exchange value, use value, and esteem value. These are now briefly described.

Cost value. It is the summation of the labour, material, overhead and all other elements of cost required to produce an item or provide a service compared to a base.

Exchange value. It is the measure of all the properties, qualities and features of the product, which make the product possible of being traded for another product or for money. In a conventional sense, *exchange value* refers to the price that a purchaser will offer for the product, the price being dependent upon satisfaction (value) which he derives from the product. Value derived from the product consists of two parts "use value" and "esteem value", which are now described.

Use value. It is known as the function value. The *use value* is equal to the value of the functions performed. Therefore, it is the price paid by the buyer (buyer's view), or the cost incurred by the manufacturer (manufacturer's view) in order to ensure that the product performs its intended functions efficiently. The use value is the fundamental form of economic value. An item without "use value" can have neither "exchange value" nor "esteem value".

Esteem value. It involves the qualities and appearance of a product (like a TV set), which attract persons and create in them a desire to possess the product. Therefore, *esteem value* is the price paid by the buyer or the cost incurred by the manufacturer beyond the use value.

Performance

The performance of a product is the measure of functional features and properties that make it suitable for a specific purpose. Appropriate performance requires that (a) the product reliably accomplish the intended use of work or service requirement (functional requirements), (b) the product provide protection against accident, harmful effects on body and danger to human life (safety requirements), (c) the product give trouble-free service cover during its specified life span (reliability requirements), (d) service and maintenance work can be carried out on the product with ease and with simple tools (maintainability requirements), and (e) appearance of the product creates an impression on the buyer and induces in him or her the desire to own the product (appearance requirements).

Performance and cost must be interwoven. Desired performance at the least cost should be achieved by selecting appropriate materials and manufacturing operations, which is the measure of value. Therefore, the value of the product is the ratio of performance (utility) to cost. Thus,

$$\text{Value} = \frac{\text{Performance (utility)}}{\text{Cost}}$$

Value can be increased by increasing the utility for the same cost or by decreasing the cost for the same utility. Satisfactory performance at lesser cost through identification and development of low cost alternatives is the philosophy of Value analysis.

15.3 FUNCTION

Function is the purpose for which the product is made. Identification of the basic functions and determination of the cost currently being spent on them are the two major considerations of value analysis.

Function identifies the characteristics which make the product/component/part/item/device to work or sell. “Work functions” lend performance value while “sell functions” provide esteem value. Verbs like “support”, “hold”, “transmit”, “prevent”, “protect”, “exhibits”, “control”, etc., are used to describe work functions, while “attract”, enhance”, “improve”, “create”, etc., are used to describe “sell” functions. For example, in a “bus driver cabin”, the functional analysis of some of the parts are given in Table 15.1.

Table 15.1 Functional Analysis of Some Parts of a Bus Driver Cabin

Component of study	*Functional analysis*	
	Verb	*Noun*
Steering wheel	Control	Direction
Gear box	Change	Speed
Brake system	Stop	Vehicle
Wiper	Clear	Water
Horn	Make	Sound
Side mirror	Show	Side traffic

Classification of the functions

Rarely do all functions assume equal importance. Usually, some functions are more important than others. Functions can be classified into the following three categories:

1. Primary function
2. Secondary function
3. Tertiary function

Primary functions are the basic functions for which the product is specially designed to achieve. Primary functions, therefore, are the most essential functions whose non-performance would make the product worthless, e.g. a photo frame exhibits photographs, a chair supports weight, a fluorescent tube gives light.

Secondary functions are those which, if not in-built, would not prevent the device from performing its primary functions, e.g., arms of a chair provide support for hands. Secondary functions are usually related to convenience. The product can still work and fulfill its intended objective even if these functions are not in-built and yet they may be necessary to sell the product.

Tertiary functions are usually related to esteem appearance. For example, Sunmica top of a table gives esteem appearance for the table.

Let us consider a single example of painting a company bus to explain all the above three functions. Here, the primary function of painting is to avoid corrosion. The secondary function is to identify the company to which the bus belongs by the colour of the paint (e.g. blue colour for Ashok Leyland Ltd.). The tertiary function is to impart a very good appearance to the bus by using brilliant colours.

15.4 AIMS

The aims of value engineering are as follows:

1. Simplify the product.
2. Use (new) cheaper and better materials.
3. Modify and improve product design.
4. Use efficient processes.
5. Reduce the product cost.
6. Increase the utility of the product by economical means.
7. Save money or increase the profits.

The value content of each piece of a product is assessed using the following questions:

1. Does its use contribute to value?
2. Is its cost proportionate to its usefulness?
3. Does it need all its features?

These three questions pertain to the function of the part which may decide the elimination of parts.

- Is there anything better for the intended use?
- Can company or vendor standard be used?
- Can a usable part be made by a lower-cost method?
- Is it made with the proper tooling, considering volume?
- Does the part yield suitable profit?
- Can another vendor furnish the same at a lower cost?

15.5 VALUE ENGINEERING PROCEDURE

The basic steps of value engineering are as follows:

(a) Blast (i) Identify the product.
(ii) Collect relevant information.
(iii) Define different functions.

(b) Create (iv) Different alternatives.
(v) Critically evaluate the alternatives.

(c) Refine (vi) Develop the best alternative.
(vii) Implement the alternative.

Step 1: Identify the product. First, identify the component for study. In future, any design change should add value and it should not make the product as obsolete one. Value engineering can be applied to a product as a whole or to sub-units.

Step 2: Collect relevant information. Information relevant to the following must be collected:

- Technical specifications with drawings
- Production processes, machine layout and instruction sheet
- Time study details and manufacturing capacity
- Complete cost data and marketing details
- Latest development in related products

Step 3: Define different functions. Identify and define the primary, secondary and tertiary functions of the product or parts of interest. Also, specify the value content of each function and identify the high cost areas.

Step 4: Different alternatives. Knowing the functions of each component part and its manufacturing details, generate the ideas and create different alternatives so as to increase the value of the product. Value engineering should be done after a **brain storming** session. All feasible or non-feasible suggestions are recorded without any criticism; rather, persons are encouraged to express their views freely.

Basic principles of brain storming

Some of the important principles of brain storming which are useful in value analysis are now listed.

(i) *A quality idea comes from quantity of ideas.* If the number of ideas generated is more, the more good solutions do turn up.

(ii) *Creative ideas emerge from unconventional thinking.* This is possible when members of the group "talk off the top of their heads" and voice weird ideas as they flash through their minds, regardless of how stupid or impractical they may appear. Often, non-technical personnel can prove to be the greatest innovators in technical areas since their viewpoints are objective and they do not know that some of their ideas are technically not feasible at all. So it is preferable to include one or two non-technical persons in the study team. Members are to be told by the team leader in the beginning of the session itself, not to breathe a word of criticism of even the most weirdest idea.

(iii) *Spontaneous evaluation of ideas curbs imaginative thinking and retards the flow of creative ideas.* The group should not evaluate the alternatives suggested by its member immediately since immediate evaluation may curb imaginative thinking and slow down the flow of creative ideas.

(iv) *Hitch-hiking on the ideas often lead to better ideas.* Participants have to improve upon ideas of other members either directly or by combining more ideas in addition to contributing ideas of their own. A brilliant idea may not be a practical one initially, or it may look to be silly or useless but discussions can convert it into a valuable one.

(v) *Creativity is a regenerative process and the recording of ideas as they emerge helps serve as a catalyst to generate more ideas.* Memory may not retain all ideas or recall them when they are needed. So, a stenographer may be asked to record ideas simultaneously. A tape recorder can also be used for this purpose or even ideas can be written on a blackboard. These recorded ideas can be reviewed at some later date.

(vi) *When ideas cease to flow, short diversions enable the mind to rebound with new ideas after recuperation.* Members of the syndicate may reach a stage where new ideas do not come. At such a stage, short diversions—rest, favourite sport, hobby, lunch or tea break, etc.—may be taken during which members are advised to sleep over the ideas and report fresh after the break. Such short diversions enable mind to recoup and rebound with new ideas.

Step 5: Critically evaluate the alternatives. Different ideas recorded under step 4 are compared, evaluated and critically assessed for their virtues, validity and feasibility as regards their financial and technical requirements. The ideas technically found and involving lower costs are further developed.

Step 6: Develop the best alternative. Detailed development plans are made for those ideas which emerged during step 5 and appear most suitable and promising. Development plans comprise drawing the sketches, building of models, conducting discussions with the purchase section, finance section, marketing division, etc.

Step 7: Implement the alternative. The best alternative is converted into a proto-type manufacturing model which ultimately goes into operation and its results are recorded.

15.6 ADVANTAGES AND APPLICATION AREAS

Advantages

The advantages of value engineering are as follows:

1. It is a much faster cost reduction technique.
2. It is a less expensive technique.
3. It reduces production costs and adds value to sales income of the product.

Applications

The various application areas of value engineering are machine tool industries, industries making accessories for machine tools, auto industries, import substitutes, etc.

QUESTIONS

1. Define value analysis(VA)/ value engineering (VE).

2. Discuss the symptoms favouring the applications of VA/VE.

3. Define value. What are the types of value?

4. What are the types of functions? Explain them with examples.

5. What are the aims of value engineering?

6. Briefly explain the steps of value engineering.

7. Discuss the advantages and application areas of value engineering.

16

LINEAR PROGRAMMING

16.1 INTRODUCTION

A generalized format of the linear programming (LP) problem is

Maximize $Z = C1X1 + C2X2 + C3X3 + ... + CnXn$

subject to

$$
\begin{array}{l}
a11\ X1 + a12\ X2 + ... + a1n\ Xn \le b1 \\
a21\ X1 + a22\ X2 + ... + a2n\ Xn \le b2 \\
\vdots \qquad\quad \vdots \qquad\qquad \vdots \qquad \vdots \\
ai1\ X1 + ai2\ X2 + ... + ain\ Xn \le bi \\
\vdots \qquad\quad \vdots \qquad\qquad \vdots \qquad \vdots \\
am1\ X1 + am2\ X2 + ... + amn\ Xn \le bm
\end{array}
$$

where, $X1, X2, X3, ..., Xn \ge 0$.

If the number of variables in the problem is two, then one can use the graphical method to solve the problem. If the number of variables in the problem is more than two, one should use the simplex method to solve the problem.

16.2 DEVELOPMENT OF LP MODELS

Modelling is an art. One can develop this expertise only by seeing more and more models. In this section, the concept of model building is demonstrated using two example problems.

EXAMPLE 16.1 A company manufactures two types of products, *P*1 and *P*2. Each product uses a lathe and milling machine. The processing time for *P*1 on the lathe is five hours and on the milling machine it is four hours. The processing time for *P*2 on the lathe is 10 hours and on the milling machine it is four hours. The maximum number of hours per week available on the lathe and the milling machine are 60 hours and 40 hours, respectively. The profit per unit of selling *P*1 and *P*2 are Rs. 6.00 and Rs. 8.00, respectively. Formulate a linear programming model to determine the production volume of each product such that the total profit is maximized.

Solution The data of the problem are summarized in Table 16.1.

Table 16.1 Details of Products

Machine	*Product*		*Limit on machine hours*
	*P*1	*P*2	
Lathe	5	10	60
Milling machine	4	4	40
Profit/unit (Rs.)	6	8	

Let $X1$ be the production volume of the product, $P1$, and
$X2$ be the production volume of the product, $P2$.

The corresponding linear programming model to determine the production volume of each product such that the total profit is maximized is as shown below:

$$\text{Maximize } Z = 6X1 + 8X2$$

subject to

$$5X1 + 10X2 \leq 60$$
$$4X1 + 4X2 \leq 40$$
$$X1, X2 \geq 0$$

EXAMPLE 16.2 A nutrition scheme for babies is proposed by a committee of doctors. Babies can be given two types of food (Type I and Type II), which are available in standard packets weighing 50 grams. The cost per packet of these foods is Rs. 2.00 and Rs. 3.00 respectively. The vitamin availability in each type of food per packet and the minimum vitamin requirement in each type of vitamin is summarized in Table 16.2. Develop a linear programming model to determine the optimal combination of food types with the minimum cost such that the minimum required vitamin in each type is satisfied.

Table 16.2 Details of Food Types

	Vitamin availability per packet		*Minimum daily required vitamin*
Vitamin	Food type 1	Food type 2	
1	1	1	6
2	7	1	14
Cost/packet (Rs.)	2	3	

Solution Let, $X1$ be the No. of packets of food type 1 suggested for babies, and $X2$ be the No. of packets of food type 2 suggested for babies.

A linear programming model to determine the number of packets of each food type to be suggested for babies with the minimum cost such that the minimum daily required vitamin in each food type is satisfied is as shown below.

Minimize $Z = 2X1 + 3X2$
subject to

$$X1 + X2 \geq 6$$
$$7X1 + X2 \geq 14$$
$$X1, X2 \geq 0$$

16.3 GRAPHICAL METHOD

As already stated, if the number of variables in any linear programming problem is two, one can use the graphical methods to solve it. In this section, the same is demonstrated with two example problems.

EXAMPLE 16.3 Solve the following LP problem using the graphical method.

Maximize $Z = 6X1 + 8X2$
subject to

$$5X1 + 10X2 \leq 60$$
$$4X1 + 4X2 \leq 40$$
$$X1, X2 \geq 0$$

Solution Given the following LP model:

Maximize $Z = 6X1 + 8X2$
subject to

$$5X1 + 10X2 \leq 60$$
$$4X1 + 4X2 \leq 40$$
$$X1, X2 \geq 0$$

The introduction of the non-negative constraints $X1 \geq 0$ and $X2 \geq 0$ will eliminate the second, third and fourth quadrants of the *XY* plane as shown in Fig. 16.1.

Compute the coordinates to plot equations relating to the constraints on the *XY* plane as shown below.

Consider the first constraint in the form

$$5X1 + 10X2 = 60$$

When $X1 = 0$,

$$10X2 = 60, \qquad X2 = 6$$

When $X2 = 0$,

$$5X1 = 60, \qquad X1 = 12$$

Now, plot constraint 1 as shown in the Fig. 16.1.

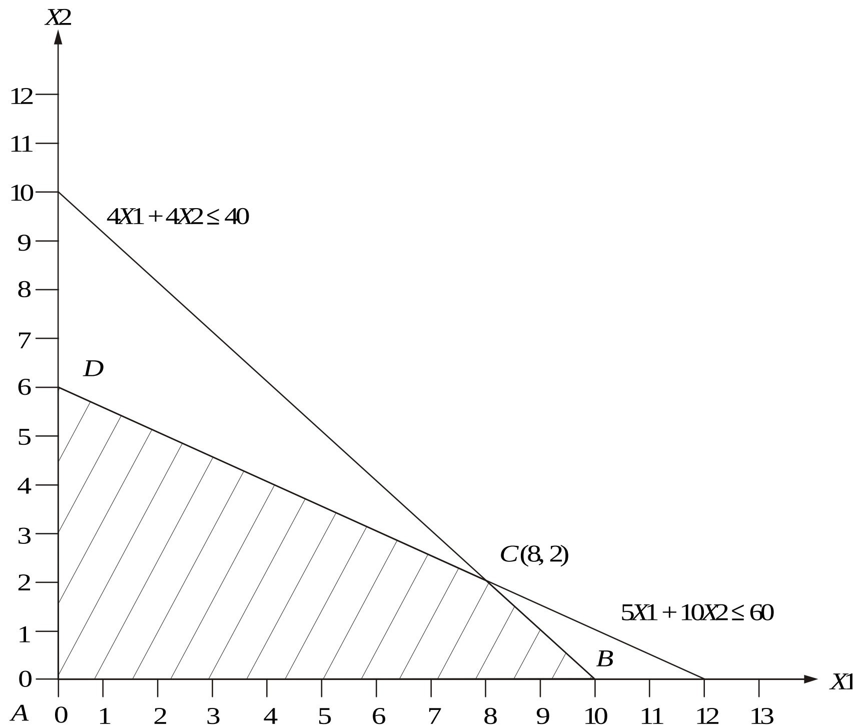


Fig. 16.1 Graphical plot of Example 16.3.

Consider the second constraint in the form

$4X1 + 4X2 = 40$

When $X1 = 0$,

$4X2 = 40, \quad X2 = 10$

When $X2 = 0$,

$4X1 = 40, \quad X1 = 10$

Now, plot constraint 2 as shown in Fig. 16.1.

The closed polygon *A–B–C–D* is the feasible region.

The objective function value at each of the corner points of the closed polygon is computed as follows by substituting its coordinates in the objective function:

$Z(A) = 6 \times 0 + 8 \times 0 = 0$

$Z(B) = 6 \times 10 + 8 \times 0 = 60$

$Z(C) = 6 \times 8 + 8 \times 2 = 48 + 16 = 64$

$Z(D) = 6 \times 0 + 8 \times 6 = 48$

Since the type of the objective function is maximization, the solution corresponding to the maximum Z value should be selected as the optimum solution. The Z value is maximum for the corner point C. Hence, the corresponding solution is

$$X1^* = 8, \quad X2^* = 2, \quad Z\,(\text{optimum}) = 64$$

EXAMPLE 16.4 Solve the following LP problem using the graphical method:

Minimize $Z = 2X1 + 3X2$
subject to

$$X1 + X2 \geq 6$$
$$7X1 + X2 \geq 14$$
$$X1, X2 \geq 0$$

Solution Consider the following LP problem:

Minimize $Z = 2X1 + 3X2$
subject to

$$X1 + X2 \geq 6$$
$$7X1 + X2 \geq 14$$
$$X1, X2 \geq 0$$

The introduction of the non-negative constraints $X1 \geq 0$ and $X2 \geq 0$ will eliminate the second, third and fourth quadrants of the *XY*-plane as shown in Fig. 16.2.

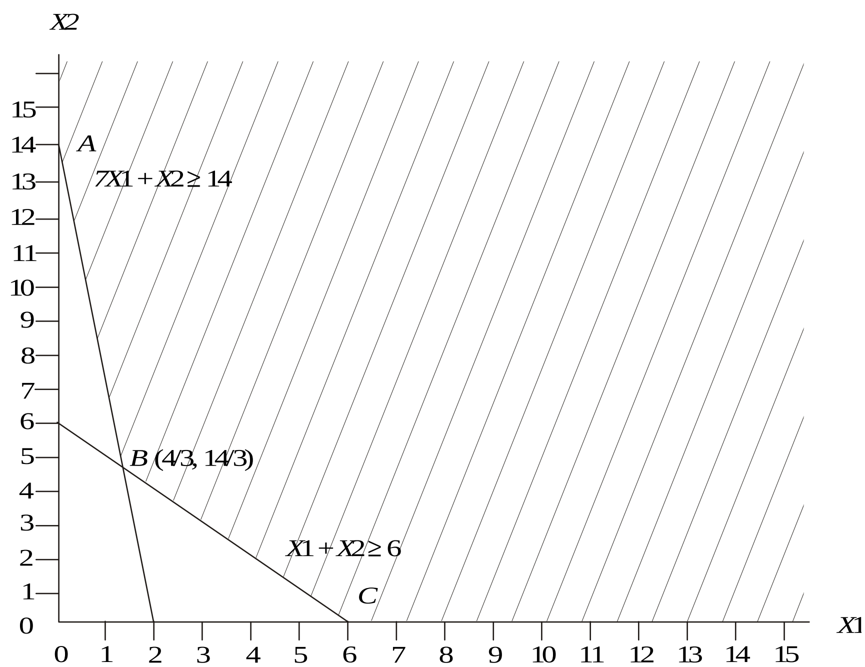


Fig. 16.2 Graphical plot of Example 16.4.

Compute the coordinates to plot the equations relating to the constraints on the *XY*-plane as shown below.

Consider the first constraint in the form

$X1 + X2 = 6$
When $X1 = 0$, $X2 = 6$
When $X2 = 0$, $X1 = 6$

Now, plot constraint 1 as shown in Fig. 16.2.

Consider the second constraint in the form

$7X1 + X2 = 14$

When $X1 = 0, \quad X2 = 14$

When $X2 = 0,$

$7X1 = 14, \quad X1 = 2$

Now, plot constraint 2 as shown in Fig. 16.2.

The optimum solution will be in any one of the corners *A*, *B* and *C*.

The objective function value at each of these corner points of the feasible solution space is computed as follows by substituting its coordinates in the objective function:

$Z(A) = 2 \times 0 + 3 \times 14 = 42$

$Z(B) = 2(4/3) + 3(14/3) = 50/3 = 16.67$

$Z(C) = 2 \times 6 + 3 \times 0 = 12$

Since the type of the objective function is minimization, the solution corresponding to the minimum *Z*-value is to be selected as the optimum solution. The *Z*-value is minimum for the corner point *C*. Accordingly, the corresponding solution is presented as

$$X1^* = 6, \quad X2^* = 0, \; Z \text{ (optimum)} = 12.$$

16.4 SIMPLEX METHOD

This method is demonstrated using a numerical problem which is given below.

EXAMPLE 16.5 Consider the linear programming model of Example 16.1 as shown below and solve it by the simplex method.

Maximize $Z = 6X1 + 8X2$

subject to

$5X1 + 10X2 \le 60$

$4X1 + 4X2 \le 40$

$X1, X2 \ge 0$

Solution The general form of the linear programming problem is

Maximize $Z = 6X1 + 8X2$

subject to

$5X1 + 10X2 \le 60$

$4X1 + 4X2 \le 40$

$X1, X2 \ge 0$

The canonical form of the above LP problem is

Maximize $Z = 6X1 + 8X2 + 0S1 + 0S2$
subject to

$5X1 + 10X2 + S1 = 60$

$4X1 + 4X2 + S2 = 40$

$X1, X2, S1, S2 \geq 0$
($S1$, $S2$ are slack variables).

The initial Simplex table of the above problem is shown in Table 16.3.

Table 16.3 Initial Simplex Table

CB_i	C_j	6	8	0	0		
	Basic variable	$X1$	$X2$	$S1$	$S2$	Solution	Ratio
0	$S1$	5	10	1	0	60	60/10 = 6**
0	$S2$	4	4	0	1	40	40/4 = 10
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	6	8*	0	0		

* Key column, ** Key row.

The value at the intersection of the key row and the key column is called the *key element*.

$$Z_j = \sum_{i=1}^{2} CB_i \; * \; T_{ij}$$

where T_{ij} is the technological coefficient for the ith row and the jth column of the table.

$C_j - Z_j$ is the relative contribution. Here, C_j is the objective coefficient for the jth variable. The objective function value of the problem at this iteration is zero, i.e. the value of Z_j against the solution column is the value of the objective function.

Optimality condition

For maximization problem, if all $C_j - Z_j \leq 0$, then optimality is reached. Otherwise, select the entering variable with the maximum $C_j - Z_j$ value.

[For minimization problem, if all $C_j - Z_j$ are greater than or equal to zero, the optimality is reached; otherwise, select the entering variable with the most negative value.]

In Table 16.3, all the values for $C_j - Z_j$ are not less than or equal to zero.

Hence, the solution can be improved further. $C_j - Z_j$ is maximum for the variable $X2$. Therefore, $X2$ enters the basis. This is known as the *entering variable*, and the corresponding column is called the *key column*.

Feasibility condition

To maintain the feasibility of the solution in each iteration, do the following:

1. Find the ratio between the solution value and the respective value in the key column value in each row.
2. Then, select the variable from the present set of basic variables w.r.t. the minimum ratio (break tie randomly). This is the leaving variable and the corresponding row is called the *key row*. The value at the intersection of the key row and key column is called the *key element* or *pivot element*.

In Table 16.3, the leaving variable is $S1$ and row 1 is the key row. The key element is 10.

The next iteration is shown in Table 16.4. In this table, the basic variable $S1$ of Table 16.3 is replaced by $X2$.

$$\text{New value} = \text{Old value} - \frac{\text{Key column value} \times \text{key row value}}{\text{Key value}}$$

Sample calculation. New value for row 2 and column $X1$

$$\text{New value} = 4 - \frac{4 \times 5}{10} = 4 - \frac{20}{10} = 4 - 2 = 2$$

Table 16.4 Iteration 1

CB_i	C_j	6	8	0	0	Solution	Ratio
	Basic variable	$X1$	$X2$	$S1$	$S2$		
8	$X2$	1/2	1	1/10	0	6	6/(1/2) = 12
0	$S2$	2	0	–2/5	1	16	16/2 = 8**
	Z_j	4	8	4/5	0	48	
	$C_j - Z_j$	2*	0	–4/5	0		

* Key column, ** Key row.

The solution in Table 16.4 is not optimal. Hence, the variable $X1$ is selected as the entering variable and after computing the ratios, $S2$ is selected as the leaving variable. The next iteration is shown in Table 16.5.

Table 16.5 Iteration 2

CB_i	C_j	6	8	0	0	
	Basic variable	$X1$	$X2$	$S1$	$S2$	Solution
8	$X2$	0	1	1/5	–1/4	2
6	$X1$	1	0	–1/5	1/2	8
	Z_j	6	8	2/5	1	64
	$C_j - Z_j$	0	0	–2/5	–1	

In Table 16.5, all the values for $C_j - Z_j$ are 0 and negative. Therefore, the optimality is reached.

The corresponding optimal solution is

$X1$ (production volume of $P1$) = 8 units

$X2$ (production volume of $P2$) = 2 units

Optimal objective value Z (total profit) = Rs. 64.00.

EXAMPLE 16.6 Consider the linear programming model (shown below) of Example 16.2, and solve it by using the simplex method.

Minimize $Z = 2X1 + 3X2$

subject to

$X1 + X2 \geq 6$

$7X1 + X2 \geq 14$

$X1, X2 \geq 0$

Solution The general form of the linear programming problem is as follows:

Minimize $Z = 2X1 + 3X2$

subject to

$X1 + X2 \geq 6$

$7X1 + X2 \geq 14$

$X1, X2 \geq 0$

Standard form. The standard form of the above model is as follows:

Minimize $Z = 2X1 + 3X2$

subject to

$X1 + X2 - S1 = 6$

$7X1 + X2 - S2 = 14$

$X1, X2, S1$ and $S2 \geq 0$

$S1$ and $S2$ are called *surplus variables* which are introduced to balance the constraints.

Canonical form. The canonical form of the above model is

Minimize $Z = 2X1 + 3X2 + M1 \times R1 + M2 \times R2$

subject to

$X1 + X2 - S1 + R1 = 6$

$7X1 + X2 - S2 + R2 = 14$

$X1, X2, S1, S2; R1$ and $R2 \geq 0$

Here, $R1$ and $R2$ are called *artificial variables* which are introduced to have basic variables in each of the constraints. These artificial variables are included in the model for its solution. So, in the final solution, these variables should not be available. This is achieved by including these variables in the objective function with a very high positive coefficient, M, since the objective function is minimization type (a very high negative coefficient, M for maximization objective type). Even, a small value of $R1$ or $R2$ will increase the value of the objective function infinitely which is against the objective of minimization. So, the solution procedure should necessarily assign zero value to each of the artificial variables in the final solution.

The initial table of the model is shown in Table 16.6.

Table 16.6 Initial Table

	C_j	2	3	0	0	M	M		
CB_i	Basic variable	$X1$	$X2$	$S1$	$S2$	$R1$	$R2$	Sol.	Ratio
M	$R1$	1	1	–1	0	1	0	6	6
M	$R2$	7	1	0	–1	0	1	14	2**
	Z_j	$8M$	$2M$	$-M$	$-M$	M	M	$20M$	
	$C_j - Z_j$	$2-8M$*	$3-2M$	M	M	0	0		

* Key column, **Key row.

Optimality condition. For the minimization problem, if all $C_j - Z_j$ are greater than or equal to zero, optimality is reached; otherwise, select the entering variable with the most negative value.

In Table 16.6, all the values for $C_j - Z_j$ are not greater than or equal to zero. Hence, the solution can be improved further. $C_j - Z_j$ has maximum negative value for the variable $X1$. So, $X1$ enters the basis. This is known as the *entering variable*, and the corresponding column is called the *key column*. If $C_j - Z_j$ is a function of M, then ignore the constant numeric terms in it while making comparison with another $C_j - Z_j$. The feasibility condition is same as that for the maximization problem. The ratio is minimum for $R2$. Therefore, it is called the *key row* and it leaves the basis. The modified table after applying the pivot operation is shown in Table 16.7.

Table 16.7 Iteration 1

CB_i	Cj	2	3	0	0	M	M	Solution	Ratio
	Basic variable	$X1$	$X2$	$S1$	$S2$	$R1$	$R2$		
M	$R1$	0	6/7	–1	1/7	1	–1/7	4	14/3**
2	$X1$	1	1/7	0	–1/7	0	1/7	2	14
	Z_j	2	$\frac{6}{7}M + \frac{2}{7}$	$-M$	$\frac{M}{7} - \frac{2}{7}$	M	$-\frac{M}{7} + \frac{2}{7}$	$4M + 4$	
	$C_j - Z_j$	0	$-\frac{6}{7}M^* + \frac{19}{7}$	M	$-\frac{M}{7} + \frac{2}{7}$	0	$\frac{8}{7}M - \frac{2}{7}$		

*Key column, **Key row.

In Table 16.7, the column $X2$ becomes the key column and the row $R1$ becomes the key row. So, $R1$ is replaced by $X2$ in the following (Table 16.8) with corresponding pivot operations.

Table 16.8 Iteration 2

CB_i	C_j	2	3	0	0	M	M	Solution	Ratio
	Basic variable	$X1$	$X2$	$S1$	$S2$	$R1$	$R2$		
3	$X2$	0	1	–7/6	1/6	7/6	–1/6	14/3	28**
2	$X1$	1	0	1/6	–1/6	–1/6	1/6	4/3	
	Z_j	2	3	–19/6	1/6	19/6	–1/6	50/3	
	$C_j - Z_j$	0	0	19/6	–1/6*	$M - \frac{19}{6}$	$M + 1/6$		

*Key column, **Key row.

In Table 16.8, column $S2$ becomes the key column and the row $X2$ becomes the key row. So, $X2$ is replaced by $S2$ as shown in Table 16.9 with the corresponding pivot operations.

Table 16.9 Iteration 3

CB_i	C_j	2	3	0	0	M	M	Solution
	Basic variable	$X1$	$X2$	$S1$	$S2$	$R1$	$R2$	
0	$S2$	0	6	–7	1	7	–1	28
2	$X1$	1	1	–1	0	1	0	6
	Z_j	2	2	–2	0	2	0	12
	$C_j - Z_j$	0	1	2	0	$M - 2$	M	

Since all the $C_j - Z_j$ are equal to 0 and above, optimality is reached. The corresponding optimal solutions are

$X1 = 6, \quad X2 = 0, \quad Z \text{ (opt.)} = 12$

Interpretation of the results:

$X1$ (No. of packets of type 1 food) = 6

$X2$ (No. of packets of type 2 food) = 0

Corresponding daily total cost of food = Rs. 12.00

Thus, the babies are to be given six packets of type 1 food daily.

QUESTIONS

1. A manufacturer of leather belts makes three types of belts A,B and C which are processed on three machines M1, M2 and M3. Belt A requires two hours on machine M1 and three hours on machine M2. Belt B requires three hours on machine M1, two hours on machine M2 and two hours on machine M3, and belt C requires five hours on machine M2, and four hours on machine M3. There are 80 hours of time per week available on machine M1, 100 hours of time available on machine M2, and 150 hours of time on machine M3. The profit gained from belt A is Rs. 5.00 per unit, from belt B is Rs. 7.00 per unit, and from belt C is Rs. 6.00 per unit. What should be the weekly production of each type of belts so that the profit is maximum?

2. Solve the following linear programming problem:

Maximize $Z = X1 + 2X2 + 3X3 + X4$

subject to

$X1 + 2X2 + 3X3 \leq 15$

$2X1 + X2 + 5X3 \leq 20$

$X1 + 2X2 + X3 + X4 \leq 10$

$X1, X2, X3, X4 \geq 0.$

3. Consider the following linear programming problem:

(a) Solve it graphically.

(b) Solve it using the simplex method.

Maximize $Z = 8X1 + 18X2$

subject to

$10X1 + 24X2 \leq 120$

$15X1 + 25X2 \leq 140$

$20X1 + 80X2 \leq 200$

$X1, X2 \geq 0.$

4. Consider the following linear programming problem:

(a) Solve it graphically.

(b) Solve it using the simplex method.

Minimize $Z = 20X1 + 8X2$

subject to

$15X1 + 25X2 \geq 180$

$10X1 + 15X2 \geq 160$

$12X1 + 16X2 \geq 120$

$X1, X2 \geq 0.$

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APPENDIX—INTEREST TABLES

Interest Table for Annual Compounding with $i = 0.25\%$

n	$F/p,i,n$	$P/F,i,n$	$F/A,i,n$	$A/F,i,n$	$P/A,i,n$	$A/P,i,n$	$A/G,i,n$
1	1.003	0.9975	1.000	1.0000	0.9975	1.0025	0.0000
2	1.005	0.9950	2.003	0.4994	1.9926	0.5019	0.5048
3	1.008	0.9925	3.008	0.3325	2.9851	0.3350	1.0020
4	1.010	0.9901	4.015	0.2491	3.9752	0.2516	1.5012
5	1.013	0.9876	5.025	0.1990	4.9628	0.2015	1.9965
6	1.015	0.9851	6.038	0.1656	5.9479	0.1681	2.4988
7	1.018	0.9827	7.053	0.1418	6.9306	0.1443	2.9938
8	1.020	0.9802	8.070	0.1239	7.9108	0.1264	3.4907
9	1.023	0.9778	9.091	0.1100	8.8886	0.1125	3.9865
10	1.025	0.9753	10.113	0.0989	9.8639	0.1014	4.4830
11	1.028	0.9729	11.139	0.0898	10.8369	0.0923	4.9779
12	1.030	0.9705	12.167	0.0822	11.8074	0.0847	5.4744
13	1.033	0.9681	13.197	0.0758	12.7754	0.0783	5.9676
14	1.036	0.9656	14.230	0.0703	13.7411	0.0728	6.4633
15	1.038	0.9632	15.266	0.0655	14.7043	0.0680	6.9569
16	1.041	0.9608	16.304	0.0613	15.6652	0.0638	7.4498
17	1.043	0.9584	17.344	0.0577	16.6237	0.0602	7.9441
18	1.046	0.9561	18.388	0.0544	17.5797	0.0569	8.4373
19	1.049	0.9537	19.434	0.0515	18.5334	0.0540	8.9293
20	1.051	0.9513	20.482	0.0488	19.4847	0.0513	9.4211
21	1.054	0.9489	21.534	0.0464	20.4336	0.0489	9.9116
22	1.056	0.9465	22.587	0.0443	21.3802	0.0468	10.4033
23	1.059	0.9442	23.644	0.0423	22.3244	0.0448	10.8944
24	1.062	0.9418	24.703	0.0405	23.2662	0.0430	11.3840
25	1.064	0.9395	25.765	0.0388	24.2057	0.0413	11.8736
26	1.067	0.9371	26.829	0.0373	25.1428	0.0398	12.3629
27	1.070	0.9348	27.896	0.0358	26.0776	0.0383	12.8520
28	1.072	0.9325	28.966	0.0345	27.0102	0.0370	13.3407
29	1.075	0.9301	30.038	0.0333	27.9403	0.0358	13.8284
30	1.078	0.9278	31.114	0.0321	28.8681	0.0346	14.3166
31	1.080	0.9255	32.191	0.0311	29.7937	0.0336	14.8043
32	1.083	0.9232	33.272	0.0301	30.7169	0.0326	15.2906
33	1.086	0.9209	34.355	0.0291	31.6378	0.0316	15.7773
34	1.089	0.9186	35.441	0.0282	32.5564	0.0307	16.2632
35	1.091	0.9163	36.530	0.0274	33.4727	0.0299	16.7491
36	1.094	0.9140	37.621	0.0266	34.3868	0.0291	17.2342
37	1.097	0.9118	38.715	0.0258	35.2985	0.0283	17.7192
38	1.100	0.9095	39.812	0.0251	36.2080	0.0276	18.2036
39	1.102	0.9072	40.911	0.0244	37.1153	0.0269	18.6878
40	1.105	0.9049	42.014	0.0238	38.0202	0.0263	19.1708
41	1.108	0.9027	43.119	0.0232	38.9229	0.0257	19.6544
42	1.111	0.9004	44.226	0.0226	39.8233	0.0251	20.1368
43	1.113	0.8982	45.337	0.0221	40.7215	0.0246	20.6187

Interest Table for Annual Compounding with i = 0.25% (Cont.)

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
44	1.116	0.8960	46.450	0.0215	41.6175	0.0240	21.1010
45	1.119	0.8937	47.567	0.0210	42.5112	0.0235	21.5826
46	1.122	0.8915	48.686	0.0205	43.4028	0.0230	22.0641
47	1.125	0.8893	49.807	0.0201	44.2920	0.0226	22.5441
48	1.127	0.8871	50.932	0.0196	45.1791	0.0221	23.0245
49	1.130	0.8848	52.059	0.0192	46.0640	0.0217	23.5048
50	1.133	0.8826	53.189	0.0188	46.9466	0.0213	23.9840
51	1.136	0.8804	54.322	0.0184	47.8270	0.0209	24.4631
52	1.139	0.8782	55.458	0.0180	48.7052	0.0205	24.9412
53	1.141	0.8760	56.597	0.0177	49.5813	0.0202	25.4191
54	1.144	0.8739	57.738	0.0173	50.4552	0.0198	25.8977
55	1.147	0.8717	58.883	0.0170	51.3269	0.0195	26.3752
56	1.150	0.8695	60.030	0.0167	52.1964	0.0192	26.8514
57	1.153	0.8673	61.180	0.0163	53.0637	0.0188	27.3278
58	1.156	0.8652	62.333	0.0160	53.9289	0.0185	27.8042
59	1.159	0.8630	63.489	0.0158	54.7919	0.0183	28.2795
60	1.162	0.8609	64.647	0.0155	55.6529	0.0180	28.7554
61	1.165	0.8587	65.809	0.0152	56.5115	0.0177	29.2295
62	1.167	0.8566	66.974	0.0149	57.3682	0.0174	29.7046
63	1.170	0.8544	68.141	0.0147	58.2226	0.0172	30.1784
64	1.173	0.8523	69.311	0.0144	59.0749	0.0169	30.6521
65	1.176	0.8502	70.485	0.0142	59.9251	0.0167	31.1252
66	1.179	0.8481	71.661	0.0140	60.7732	0.0165	31.5981
67	1.182	0.8460	72.840	0.0137	61.6192	0.0162	32.0704
68	1.185	0.8438	74.022	0.0135	62.4630	0.0160	32.5423
69	1.188	0.8417	75.207	0.0133	63.3048	0.0158	33.0138
70	1.191	0.8396	76.395	0.0131	64.1444	0.0156	33.4850
71	1.194	0.8375	77.586	0.0129	64.9820	0.0154	33.9556
72	1.197	0.8355	78.780	0.0127	65.8174	0.0152	34.4259
73	1.200	0.8334	79.977	0.0125	66.6508	0.0150	34.8956
74	1.203	0.8313	81.177	0.0123	67.4821	0.0148	35.3650
75	1.206	0.8292	82.380	0.0121	68.3113	0.0146	35.8342
76	1.209	0.8272	83.586	0.0120	69.1385	0.0145	36.3027
77	1.212	0.8251	84.795	0.0118	69.9636	0.0143	36.7708
78	1.215	0.8230	86.007	0.0116	70.7867	0.0141	37.2391
79	1.218	0.8210	87.222	0.0115	71.6076	0.0140	37.7061
80	1.221	0.8189	88.440	0.0113	72.4266	0.0138	38.1733
81	1.224	0.8169	89.661	0.0112	73.2435	0.0137	38.6398
82	1.227	0.8149	90.885	0.0110	74.0584	0.0135	39.1059
83	1.230	0.8128	92.113	0.0109	74.8712	0.0134	39.5717
84	1.233	0.8108	93.343	0.0107	75.6819	0.0132	40.0367
85	1.236	0.8088	94.576	0.0106	76.4907	0.0131	40.5016
86	1.240	0.8068	95.813	0.0104	77.2975	0.0129	40.9663
87	1.243	0.8047	97.052	0.0103	78.1023	0.0128	41.4305
88	1.246	0.8027	98.295	0.0102	78.9050	0.0127	41.8936
89	1.249	0.8007	99.541	0.0100	79.7057	0.0125	42.3571
90	1.252	0.7987	100.790	0.0099	80.5045	0.0124	42.8199
91	1.255	0.7967	102.041	0.0098	81.3012	0.0123	43.2824
92	1.258	0.7948	103.297	0.0097	82.0960	0.0122	43.7443
93	1.261	0.7928	104.555	0.0096	82.8888	0.0121	44.2058
94	1.265	0.7908	105.816	0.0095	83.6796	0.0120	44.6671
95	1.268	0.7888	107.081	0.0093	84.4685	0.0118	45.1280
96	1.271	0.7869	108.349	0.0092	85.2553	0.0117	45.5881
97	1.274	0.7849	109.619	0.0091	86.0402	0.0116	46.0481
98	1.277	0.7829	110.893	0.0090	86.8232	0.0115	46.5075
99	1.280	0.7810	112.171	0.0089	87.6042	0.0114	46.9667
100	1.284	0.7790	113.451	0.0088	88.3832	0.0113	47.4253

Interest Table for Annual Compounding with $i = 0.5\%$

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
1	1.005	0.9950	1.000	1.0000	0.9950	1.0050	0.0000
2	1.010	0.9901	2.005	0.4988	1.9851	0.5038	0.4994
3	1.015	0.9851	3.015	0.3317	2.9702	0.3367	0.9964
4	1.020	0.9802	4.030	0.2481	3.9505	0.2531	1.4944
5	1.025	0.9754	5.050	0.1980	4.9259	0.2030	1.9903
6	1.030	0.9705	6.076	0.1646	5.8964	0.1696	2.4864
7	1.036	0.9657	7.106	0.1407	6.8621	0.1457	2.9794
8	1.041	0.9609	8.141	0.1228	7.8230	0.1278	3.4743
9	1.046	0.9561	9.182	0.1089	8.7791	0.1139	3.9665
10	1.051	0.9513	10.228	0.0978	9.7304	0.1028	4.4593
11	1.056	0.9466	11.279	0.0887	10.6770	0.0937	4.9505
12	1.062	0.9419	12.336	0.0811	11.6190	0.0861	5.4411
13	1.067	0.9372	13.397	0.0746	12.5562	0.0796	5.9304
14	1.072	0.9326	14.464	0.0691	13.4887	0.0741	6.4193
15	1.078	0.9279	15.537	0.0644	14.4166	0.0694	6.9068
16	1.083	0.9233	16.614	0.0602	15.3399	0.0652	7.3941
17	1.088	0.9187	17.697	0.0565	16.2586	0.0615	7.8803
18	1.094	0.9141	18.786	0.0532	17.1728	0.0582	8.3659
19	1.099	0.9096	19.880	0.0503	18.0824	0.0553	8.8504
20	1.105	0.9051	20.979	0.0477	18.9874	0.0527	9.3344
21	1.110	0.9006	22.084	0.0453	19.8880	0.0503	9.8172
22	1.116	0.8961	23.194	0.0431	20.7841	0.0481	10.2997
23	1.122	0.8916	24.310	0.0411	21.6757	0.0461	10.7807
24	1.127	0.8872	25.432	0.0393	22.5629	0.0443	11.2613
25	1.133	0.8828	26.559	0.0377	23.4457	0.0427	11.7409
26	1.138	0.8784	27.692	0.0361	24.3240	0.0411	12.2197
27	1.144	0.8740	28.830	0.0347	25.1980	0.0397	12.6976
28	1.150	0.8697	29.975	0.0334	26.0677	0.0384	13.1749
29	1.156	0.8653	31.124	0.0321	26.9331	0.0371	13.6513
30	1.161	0.8610	32.280	0.0310	27.7941	0.0360	14.1268
31	1.167	0.8567	33.441	0.0299	28.6508	0.0349	14.6012
32	1.173	0.8525	34.609	0.0289	29.5033	0.0339	15.0752
33	1.179	0.8482	35.782	0.0279	30.3515	0.0329	15.5482
34	1.185	0.8440	36.961	0.0271	31.1956	0.0321	16.0204
35	1.191	0.8398	38.145	0.0262	32.0354	0.0312	16.4917
36	1.197	0.8356	39.336	0.0254	32.8710	0.0304	16.9622
37	1.203	0.8315	40.533	0.0247	33.7025	0.0297	17.4320
38	1.209	0.8274	41.736	0.0240	34.5299	0.0290	17.9008
39	1.215	0.8232	42.944	0.0233	35.3531	0.0283	18.3688
40	1.221	0.8191	44.159	0.0226	36.1723	0.0276	18.8360
41	1.227	0.8151	45.380	0.0220	36.9873	0.0270	19.3023
42	1.233	0.8110	46.607	0.0215	37.7983	0.0265	19.7679
43	1.239	0.8070	47.840	0.0209	38.6053	0.0259	20.2326
44	1.245	0.8030	49.079	0.0204	39.4083	0.0254	20.6966
45	1.252	0.7990	50.324	0.0199	40.2072	0.0249	21.1597
46	1.258	0.7950	51.576	0.0194	41.0022	0.0244	21.6219
47	1.264	0.7910	52.834	0.0189	41.7932	0.0239	22.0832
48	1.270	0.7871	54.098	0.0185	42.5804	0.0235	22.5439
49	1.277	0.7832	55.368	0.0181	43.3635	0.0231	23.0036
50	1.283	0.7793	56.645	0.0177	44.1428	0.0227	23.4626

Interest Table for Annual Compounding with i = 0.5% (Cont.)

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
51	1.290	0.7754	57.928	0.0173	44.9182	0.0223	23.9206
52	1.296	0.7716	59.218	0.0169	45.6898	0.0219	24.3780
53	1.303	0.7677	60.514	0.0165	46.4575	0.0215	24.8344
54	1.309	0.7639	61.817	0.0162	47.2214	0.0212	25.2902
55	1.316	0.7601	63.126	0.0158	47.9815	0.0208	25.7449
56	1.322	0.7563	64.441	0.0155	48.7378	0.0205	26.1989
57	1.329	0.7525	65.764	0.0152	49.4903	0.0202	26.6520
58	1.335	0.7488	67.092	0.0149	50.2391	0.0199	27.1044
59	1.342	0.7451	68.428	0.0146	50.9842	0.0196	27.5559
60	1.349	0.7414	69.770	0.0143	51.7256	0.0193	28.0066
61	1.356	0.7377	71.119	0.0141	52.4633	0.0191	28.4564
62	1.362	0.7340	72.475	0.0138	53.1973	0.0188	28.9054
63	1.369	0.7304	73.837	0.0135	53.9277	0.0185	29.3536
64	1.376	0.7267	75.206	0.0133	54.6544	0.0183	29.8011
65	1.383	0.7231	76.582	0.0131	55.3775	0.0181	30.2476
66	1.390	0.7195	77.965	0.0128	56.0970	0.0178	30.6933
67	1.397	0.7159	79.355	0.0126	56.8129	0.0176	31.1382
68	1.404	0.7124	80.752	0.0124	57.5253	0.0174	31.5823
69	1.411	0.7088	82.155	0.0122	58.2342	0.0172	32.0256
70	1.418	0.7053	83.566	0.0120	58.9395	0.0170	32.4681
71	1.425	0.7018	84.984	0.0118	59.6412	0.0168	32.9097
72	1.432	0.6983	86.409	0.0116	60.3395	0.0166	33.3505
73	1.439	0.6948	87.841	0.0114	61.0344	0.0164	33.7905
74	1.446	0.6914	89.280	0.0112	61.7258	0.0162	34.2297
75	1.454	0.6879	90.727	0.0110	62.4137	0.0160	34.6681
76	1.461	0.6845	92.180	0.0108	63.0982	0.0158	35.1056
77	1.468	0.6811	93.641	0.0107	63.7793	0.0157	35.5424
78	1.476	0.6777	95.109	0.0105	64.4570	0.0155	35.9783
79	1.483	0.6743	96.585	0.0104	65.1314	0.0154	36.4133
80	1.490	0.6710	98.068	0.0102	65.8024	0.0152	36.8476
81	1.498	0.6677	99.558	0.0100	66.4700	0.0150	37.2810
82	1.505	0.6643	101.056	0.0099	67.1343	0.0149	37.7136
83	1.513	0.6610	102.561	0.0098	67.7953	0.0148	38.1454
84	1.520	0.6577	104.074	0.0096	68.4531	0.0146	38.5765
85	1.528	0.6545	105.594	0.0095	69.1075	0.0145	39.0066
86	1.536	0.6512	107.122	0.0093	69.7588	0.0143	39.4360
87	1.543	0.6480	108.658	0.0092	70.4067	0.0142	39.8645
88	1.551	0.6447	110.201	0.0091	71.0515	0.0141	40.2923
89	1.559	0.6415	111.752	0.0089	71.6930	0.0139	40.7191
90	1.567	0.6383	113.311	0.0088	72.3313	0.0138	41.1452
91	1.574	0.6352	114.878	0.0087	72.9665	0.0137	41.5705
92	1.582	0.6320	116.452	0.0086	73.5985	0.0136	41.9949
93	1.590	0.6289	118.034	0.0085	74.2274	0.0135	42.4187
94	1.598	0.6257	119.624	0.0084	74.8531	0.0134	42.8415
95	1.606	0.6226	121.223	0.0082	75.4757	0.0132	43.2634
96	1.614	0.6195	122.829	0.0081	76.0953	0.0131	43.6847
97	1.622	0.6164	124.443	0.0080	76.7117	0.0130	44.1051
98	1.630	0.6134	126.065	0.0079	77.3251	0.0129	44.5246
99	1.638	0.6103	127.695	0.0078	77.9354	0.0128	44.9434
100	1.647	0.6073	129.334	0.0077	78.5427	0.0127	45.3614

Interest Table for Annual Compounding with i = 0.75%

n	$F/p,i,n$	$P/F,i,n$	$F/A,i,n$	$A/F,i,n$	$P/A,i,n$	$A/P,i,n$	$A/G,i,n$
1	1.008	0.9926	1.000	1.0000	0.9926	1.0075	0.0000
2	1.015	0.9852	2.008	0.4981	1.9777	0.5056	0.4993
3	1.023	0.9778	3.023	0.3308	2.9556	0.3383	0.9956
4	1.030	0.9706	4.045	0.2472	3.9261	0.2547	1.4920
5	1.038	0.9633	5.076	0.1970	4.8895	0.2045	1.9862
6	1.046	0.9562	6.114	0.1636	5.8456	0.1711	2.4792
7	1.054	0.9490	7.160	0.1397	6.7947	0.1472	2.9710
8	1.062	0.9420	8.213	0.1218	7.7367	0.1293	3.4623
9	1.070	0.9350	9.275	0.1078	8.6717	0.1153	3.9514
10	1.078	0.9280	10.344	0.0967	9.5997	0.1042	4.4396
11	1.086	0.9211	11.422	0.0876	10.5208	0.0951	4.9265
12	1.094	0.9142	12.508	0.0800	11.4350	0.0875	5.4122
13	1.102	0.9074	13.602	0.0735	12.3425	0.0810	5.8967
14	1.110	0.9007	14.704	0.0680	13.2431	0.0755	6.3799
15	1.119	0.8940	15.814	0.0632	14.1371	0.0707	6.8618
16	1.127	0.8873	16.932	0.0591	15.0244	0.0666	7.3425
17	1.135	0.8807	18.059	0.0554	15.9052	0.0629	7.8221
18	1.144	0.8742	19.195	0.0521	16.7793	0.0596	8.3003
19	1.153	0.8676	20.339	0.0492	17.6470	0.0567	8.7771
20	1.161	0.8612	21.491	0.0465	18.5082	0.0540	9.2529
21	1.170	0.8548	22.653	0.0441	19.3630	0.0516	9.7274
22	1.179	0.8484	23.823	0.0420	20.2114	0.0495	10.2007
23	1.188	0.8421	25.001	0.0400	21.0535	0.0475	10.6726
24	1.196	0.8358	26.189	0.0382	21.8893	0.0457	11.1435
25	1.205	0.8296	27.385	0.0365	22.7190	0.0440	11.6130
26	1.214	0.8234	28.591	0.0350	23.5424	0.0425	12.0812
27	1.224	0.8173	29.805	0.0336	24.3597	0.0411	12.5483
28	1.233	0.8112	31.029	0.0322	25.1709	0.0397	13.0140
29	1.242	0.8052	32.261	0.0310	25.9761	0.0385	13.4786
30	1.251	0.7992	33.503	0.0298	26.7753	0.0373	13.9420
31	1.261	0.7932	34.755	0.0288	27.5685	0.0363	14.4040
32	1.270	0.7873	36.015	0.0278	28.3559	0.0353	14.8649
33	1.280	0.7815	37.285	0.0268	29.1374	0.0343	15.3245
34	1.289	0.7757	38.565	0.0259	29.9130	0.0334	15.7829
35	1.299	0.7699	39.854	0.0251	30.6829	0.0326	16.2400
36	1.309	0.7641	41.153	0.0243	31.4471	0.0318	16.6959
37	1.318	0.7585	42.462	0.0236	32.2055	0.0311	17.1506
38	1.328	0.7528	43.780	0.0228	32.9584	0.0303	17.6040
39	1.338	0.7472	45.109	0.0222	33.7056	0.0297	18.0561
40	1.348	0.7416	46.447	0.0215	34.4472	0.0290	18.5071
41	1.358	0.7361	47.795	0.0209	35.1834	0.0284	18.9568
42	1.369	0.7306	49.154	0.0203	35.9140	0.0278	19.4054
43	1.379	0.7252	50.523	0.0198	36.6392	0.0273	19.8526
44	1.389	0.7198	51.901	0.0193	37.3590	0.0268	20.2986
45	1.400	0.7144	53.291	0.0188	38.0735	0.0263	20.7434
46	1.410	0.7091	54.690	0.0183	38.7826	0.0258	21.1869
47	1.421	0.7039	56.101	0.0178	39.4865	0.0253	21.6292
48	1.431	0.6986	57.521	0.0174	40.1851	0.0249	22.0703
49	1.442	0.6934	58.953	0.0170	40.8785	0.0245	22.5102
50	1.453	0.6882	60.395	0.0166	41.5668	0.0241	22.9489

Interest Table for Annual Compounding with i = 0.75% (Cont.)

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
51	1.464	0.6831	61.848	0.0162	42.2499	0.0237	23.3862
52	1.475	0.6780	63.312	0.0158	42.9280	0.0233	23.8224
53	1.486	0.6730	64.787	0.0154	43.6010	0.0229	24.2574
54	1.497	0.6680	66.273	0.0151	44.2690	0.0226	24.6911
55	1.508	0.6630	67.770	0.0148	44.9320	0.0223	25.1236
56	1.520	0.6581	69.278	0.0144	45.5901	0.0219	25.5549
57	1.531	0.6532	70.798	0.0141	46.2432	0.0216	25.9849
58	1.542	0.6483	72.329	0.0138	46.8915	0.0213	26.4138
59	1.554	0.6435	73.871	0.0135	47.5350	0.0210	26.8414
60	1.566	0.6387	75.425	0.0133	48.1737	0.0208	27.2678
61	1.577	0.6339	76.991	0.0130	48.8077	0.0205	27.6929
62	1.589	0.6292	78.568	0.0127	49.4369	0.0202	28.1169
63	1.601	0.6245	80.157	0.0125	50.0615	0.0200	28.5396
64	1.613	0.6199	81.759	0.0122	50.6814	0.0197	28.9611
65	1.625	0.6153	83.372	0.0120	51.2966	0.0195	29.3814
66	1.637	0.6107	84.997	0.0118	51.9073	0.0193	29.8005
67	1.650	0.6061	86.635	0.0115	52.5135	0.0190	30.2183
68	1.662	0.6016	88.284	0.0113	53.1151	0.0188	30.6350
69	1.675	0.5972	89.947	0.0111	53.7123	0.0186	31.0504
70	1.687	0.5927	91.621	0.0109	54.3050	0.0184	31.4646
71	1.700	0.5883	93.308	0.0107	54.8933	0.0182	31.8776
72	1.713	0.5839	95.008	0.0105	55.4773	0.0180	32.2895
73	1.725	0.5796	96.721	0.0103	56.0568	0.0178	32.7000
74	1.738	0.5753	98.446	0.0102	56.6321	0.0177	33.1094
75	1.751	0.5710	100.185	0.0100	57.2031	0.0175	33.5176
76	1.765	0.5667	101.936	0.0098	57.7698	0.0173	33.9246
77	1.778	0.5625	103.701	0.0096	58.3323	0.0171	34.3303
78	1.791	0.5583	105.478	0.0095	58.8907	0.0170	34.7349
79	1.805	0.5542	107.269	0.0093	59.4448	0.0168	35.1382
80	1.818	0.5500	109.074	0.0092	59.9949	0.0167	35.5403
81	1.832	0.5459	110.892	0.0090	60.5408	0.0165	35.9413
82	1.845	0.5419	112.724	0.0089	61.0827	0.0164	36.3410
83	1.859	0.5378	114.569	0.0087	61.6206	0.0162	36.7396
84	1.873	0.5338	116.428	0.0086	62.1544	0.0161	37.1369
85	1.887	0.5299	118.302	0.0085	62.6843	0.0160	37.5331
86	1.901	0.5259	120.189	0.0083	63.2102	0.0158	37.9280
87	1.916	0.5220	122.090	0.0082	63.7322	0.0157	38.3218
88	1.930	0.5181	124.006	0.0081	64.2503	0.0156	38.7143
89	1.945	0.5143	125.936	0.0079	64.7646	0.0154	39.1057
90	1.959	0.5104	127.881	0.0078	65.2751	0.0153	39.4959
91	1.974	0.5066	129.840	0.0077	65.7817	0.0152	39.8849
92	1.989	0.5029	131.814	0.0076	66.2846	0.0151	40.2727
93	2.004	0.4991	133.802	0.0075	66.7837	0.0150	40.6593
94	2.019	0.4954	135.806	0.0074	67.2791	0.0149	41.0447
95	2.034	0.4917	137.824	0.0073	67.7708	0.0148	41.4289
96	2.049	0.4881	139.858	0.0072	68.2589	0.0147	41.8120
97	2.064	0.4844	141.907	0.0070	68.7433	0.0145	42.1939
98	2.080	0.4808	143.971	0.0069	69.2241	0.0144	42.5745
99	2.095	0.4772	146.051	0.0068	69.7014	0.0143	42.9541
100	2.111	0.4737	148.147	0.0068	70.1751	0.0143	43.3324

Interest Table for Annual Compounding with i = 1%

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
1	1.010	0.9901	1.000	1.0000	0.9901	1.0100	0.0000
2	1.020	0.9803	2.010	0.4975	1.9704	0.5075	0.4976
3	1.030	0.9706	3.030	0.3300	2.9410	0.3400	0.9934
4	1.041	0.9610	4.060	0.2463	3.9020	0.2563	1.4873
5	1.051	0.9515	5.101	0.1960	4.8534	0.2060	1.9801
6	1.062	0.9420	6.152	0.1625	5.7955	0.1725	2.4710
7	1.072	0.9327	7.214	0.1386	6.7282	0.1486	2.9601
8	1.083	0.9235	8.286	0.1207	7.6517	0.1307	3.4477
9	1.094	0.9143	9.369	0.1067	8.5660	0.1167	3.9335
10	1.105	0.9053	10.462	0.0956	9.4713	0.1056	4.4179
11	1.116	0.8963	11.567	0.0865	10.3676	0.0965	4.9005
12	1.127	0.8874	12.682	0.0788	11.2551	0.0888	5.3814
13	1.138	0.8787	13.809	0.0724	12.1337	0.0824	5.8607
14	1.149	0.8700	14.947	0.0669	13.0037	0.0769	6.3383
15	1.161	0.8613	16.097	0.0621	13.8650	0.0721	6.8143
16	1.173	0.8528	17.258	0.0579	14.7179	0.0679	7.2886
17	1.184	0.8444	18.430	0.0543	15.5622	0.0643	7.7613
18	1.196	0.8360	19.615	0.0510	16.3983	0.0610	8.2323
19	1.208	0.8277	20.811	0.0481	17.2260	0.0581	8.7016
20	1.220	0.8195	22.019	0.0454	18.0455	0.0554	9.1694
21	1.232	0.8114	23.239	0.0430	18.8570	0.0530	9.6354
22	1.245	0.8034	24.472	0.0409	19.6604	0.0509	10.0997
23	1.257	0.7954	25.716	0.0389	20.4558	0.0489	10.5625
24	1.270	0.7876	26.973	0.0371	21.2434	0.0471	11.0236
25	1.282	0.7798	28.243	0.0354	22.0231	0.0454	11.4830
26	1.295	0.7720	29.526	0.0339	22.7952	0.0439	11.9409
27	1.308	0.7644	30.821	0.0324	23.5596	0.0424	12.3970
28	1.321	0.7568	32.129	0.0311	24.3164	0.0411	12.8516
29	1.335	0.7493	33.450	0.0299	25.0658	0.0399	13.3044
30	1.348	0.7419	34.785	0.0287	25.8077	0.0387	13.7556
31	1.361	0.7346	36.133	0.0277	26.5423	0.0377	14.2052
32	1.375	0.7273	37.494	0.0267	27.2696	0.0367	14.6531
33	1.389	0.7201	38.869	0.0257	27.9897	0.0357	15.0994
34	1.403	0.7130	40.258	0.0248	28.7027	0.0348	15.5441
35	1.417	0.7059	41.660	0.0240	29.4086	0.0340	15.9871
36	1.431	0.6989	43.077	0.0232	30.1075	0.0332	16.4285
37	1.445	0.6920	44.508	0.0225	30.7995	0.0325	16.8682
38	1.460	0.6852	45.953	0.0218	31.4847	0.0318	17.3063
39	1.474	0.6784	47.412	0.0211	32.1630	0.0311	17.7427
40	1.489	0.6717	48.886	0.0205	32.8347	0.0305	18.1776
41	1.504	0.6650	50.375	0.0199	33.4997	0.0299	18.6108
42	1.519	0.6584	51.879	0.0193	34.1581	0.0293	19.0424
43	1.534	0.6519	53.398	0.0187	34.8100	0.0287	19.4723
44	1.549	0.6454	54.932	0.0182	35.4554	0.0282	19.9006
45	1.565	0.6391	56.481	0.0177	36.0945	0.0277	20.3273
46	1.580	0.6327	58.046	0.0172	36.7272	0.0272	20.7523
47	1.596	0.6265	59.626	0.0168	37.3537	0.0268	21.1757
48	1.612	0.6203	61.223	0.0163	37.9740	0.0263	21.5975
49	1.628	0.6141	62.835	0.0159	38.5881	0.0259	22.0177
50	1.645	0.6080	64.463	0.0155	39.1961	0.0255	22.4363

Interest Table for Annual Compounding with i = 1% (Cont.)

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
51	1.661	0.6020	66.108	0.0151	39.7981	0.0251	22.8533
52	1.678	0.5961	67.769	0.0148	40.3942	0.0248	23.2686
53	1.694	0.5902	69.447	0.0144	40.9843	0.0244	23.6823
54	1.711	0.5843	71.141	0.0141	41.5687	0.0241	24.0944
55	1.729	0.5785	72.852	0.0137	42.1472	0.0237	24.5049
56	1.746	0.5728	74.581	0.0134	42.7200	0.0234	24.9138
57	1.763	0.5671	76.327	0.0131	43.2871	0.0231	25.3211
58	1.781	0.5615	78.090	0.0128	43.8486	0.0228	25.7268
59	1.799	0.5560	79.871	0.0125	44.4046	0.0225	26.1308
60	1.817	0.5504	81.670	0.0122	44.9550	0.0222	26.5333
61	1.835	0.5450	83.486	0.0120	45.5000	0.0220	26.9341
62	1.853	0.5396	85.321	0.0117	46.0396	0.0217	27.3334
63	1.872	0.5343	87.174	0.0115	46.5739	0.0215	27.7311
64	1.890	0.5290	89.046	0.0112	47.1029	0.0212	28.1272
65	1.909	0.5237	90.937	0.0110	47.6266	0.0210	28.5216
66	1.928	0.5185	92.846	0.0108	48.1451	0.0208	28.9145
67	1.948	0.5134	94.774	0.0106	48.6586	0.0206	29.3058
68	1.967	0.5083	96.722	0.0103	49.1669	0.0203	29.6955
69	1.987	0.5033	98.689	0.0101	49.6702	0.0201	30.0837
70	2.007	0.4983	100.676	0.0099	50.1685	0.0199	30.4702
71	2.027	0.4934	102.683	0.0097	50.6619	0.0197	30.8552
72	2.047	0.4885	104.710	0.0096	51.1504	0.0196	31.2386
73	2.068	0.4837	106.757	0.0094	51.6340	0.0194	31.6204
74	2.088	0.4789	108.825	0.0092	52.1129	0.0192	32.0006
75	2.109	0.4741	110.913	0.0090	52.5870	0.0190	32.3793
76	2.130	0.4694	113.022	0.0088	53.0565	0.0188	32.7564
77	2.152	0.4648	115.152	0.0087	53.5213	0.0187	33.1320
78	2.173	0.4602	117.304	0.0085	53.9815	0.0185	33.5059
79	2.195	0.4556	119.477	0.0084	54.4371	0.0184	33.8783
80	2.217	0.4511	121.671	0.0082	54.8882	0.0182	34.2492
81	2.239	0.4467	123.888	0.0081	55.3348	0.0181	34.6185
82	2.261	0.4422	126.127	0.0079	55.7771	0.0179	34.9862
83	2.284	0.4379	128.388	0.0078	56.2149	0.0178	35.3524
84	2.307	0.4335	130.672	0.0077	56.6484	0.0177	35.7170
85	2.330	0.4292	132.979	0.0075	57.0777	0.0175	36.0801
86	2.353	0.4250	135.309	0.0074	57.5026	0.0174	36.4416
87	2.377	0.4208	137.662	0.0073	57.9234	0.0173	36.8016
88	2.400	0.4166	140.038	0.0071	58.3400	0.0171	37.1601
89	2.424	0.4125	142.439	0.0070	58.7525	0.0170	37.5170
90	2.449	0.4084	144.863	0.0069	59.1609	0.0169	37.8724
91	2.473	0.4043	147.312	0.0068	59.5652	0.0168	38.2263
92	2.498	0.4003	149.785	0.0067	59.9656	0.0167	38.5786
93	2.523	0.3964	152.283	0.0066	60.3619	0.0166	38.9294
94	2.548	0.3925	154.806	0.0065	60.7544	0.0165	39.2787
95	2.574	0.3886	157.354	0.0064	61.1430	0.0164	39.6265
96	2.599	0.3847	159.927	0.0063	61.5277	0.0163	39.9727
97	2.625	0.3809	162.526	0.0062	61.9086	0.0162	40.3174
98	2.652	0.3771	165.152	0.0061	62.2858	0.0161	40.6606
99	2.678	0.3734	167.803	0.0060	62.6592	0.0160	41.0023
100	2.705	0.3697	170.481	0.0059	63.0289	0.0159	41.3425

Interest Table for Annual Compounding with i = 1.25%

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
1	1.013	0.9877	1.000	1.0000	0.9877	1.0125	0.0000
2	1.025	0.9755	2.013	0.4969	1.9631	0.5094	0.4974
3	1.038	0.9634	3.038	0.3292	2.9265	0.3417	0.9921
4	1.051	0.9515	4.076	0.2454	3.8781	0.2579	1.4848
5	1.064	0.9398	5.127	0.1951	4.8179	0.2076	1.9756
6	1.077	0.9282	6.191	0.1615	5.7460	0.1740	2.4642
7	1.091	0.9167	7.268	0.1376	6.6628	0.1501	2.9507
8	1.104	0.9054	8.359	0.1196	7.5682	0.1321	3.4352
9	1.118	0.8942	9.463	0.1057	8.4624	0.1182	3.9176
10	1.132	0.8832	10.582	0.0945	9.3456	0.1070	4.3979
11	1.146	0.8723	11.714	0.0854	10.2179	0.0979	4.8762
12	1.161	0.8615	12.860	0.0778	11.0794	0.0903	5.3525
13	1.175	0.8509	14.021	0.0713	11.9302	0.0838	5.8266
14	1.190	0.8404	15.196	0.0658	12.7706	0.0783	6.2987
15	1.205	0.8300	16.386	0.0610	13.6006	0.0735	6.7687
16	1.220	0.8197	17.591	0.0568	14.4204	0.0693	7.2366
17	1.235	0.8096	18.811	0.0532	15.2300	0.0657	7.7025
18	1.251	0.7996	20.046	0.0499	16.0296	0.0624	8.1663
19	1.266	0.7898	21.297	0.0470	16.8194	0.0595	8.6281
20	1.282	0.7800	22.563	0.0443	17.5994	0.0568	9.0878
21	1.298	0.7704	23.845	0.0419	18.3698	0.0544	9.5454
22	1.314	0.7609	25.143	0.0398	19.1307	0.0523	10.0010
23	1.331	0.7515	26.458	0.0378	19.8821	0.0503	10.4546
24	1.347	0.7422	27.788	0.0360	20.6243	0.0485	10.9061
25	1.364	0.7330	29.136	0.0343	21.3574	0.0468	11.3555
26	1.381	0.7240	30.500	0.0328	22.0813	0.0453	11.8029
27	1.399	0.7150	31.881	0.0314	22.7964	0.0439	12.2482
28	1.416	0.7062	33.280	0.0300	23.5026	0.0425	12.6915
29	1.434	0.6975	34.696	0.0288	24.2001	0.0413	13.1327
30	1.452	0.6889	36.129	0.0277	24.8890	0.0402	13.5719
31	1.470	0.6804	37.581	0.0266	25.5694	0.0391	14.0091
32	1.488	0.6720	39.051	0.0256	26.2414	0.0381	14.4442
33	1.507	0.6637	40.539	0.0247	26.9051	0.0372	14.8773
34	1.526	0.6555	42.046	0.0238	27.5606	0.0363	15.3083
35	1.545	0.6474	43.571	0.0230	28.2080	0.0355	15.7373
36	1.564	0.6394	45.116	0.0222	28.8474	0.0347	16.1643
37	1.583	0.6315	46.680	0.0214	29.4789	0.0339	16.5892
38	1.603	0.6237	48.263	0.0207	30.1026	0.0332	17.0121
39	1.623	0.6160	49.867	0.0201	30.7186	0.0326	17.4330
40	1.644	0.6084	51.490	0.0194	31.3271	0.0319	17.8519
41	1.664	0.6009	53.134	0.0188	31.9280	0.0313	18.2688
42	1.685	0.5935	54.798	0.0182	32.5214	0.0307	18.6836
43	1.706	0.5862	56.483	0.0177	33.1076	0.0302	19.0964
44	1.727	0.5789	58.189	0.0172	33.6865	0.0297	19.5072
45	1.749	0.5718	59.916	0.0167	34.2583	0.0292	19.9160
46	1.771	0.5647	61.665	0.0162	34.8230	0.0287	20.3228
47	1.793	0.5577	63.436	0.0158	35.3808	0.0283	20.7276
48	1.815	0.5509	65.229	0.0153	35.9316	0.0278	21.1303
49	1.838	0.5441	67.044	0.0149	36.4757	0.0274	21.5311
50	1.861	0.5373	68.882	0.0145	37.0130	0.0270	21.9299

Interest Table for Annual Compounding with i = 1.25% (Cont.)

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
51	1.884	0.5307	70.743	0.0141	37.5437	0.0266	22.3267
52	1.908	0.5242	72.628	0.0138	38.0679	0.0263	22.7215
53	1.932	0.5177	74.535	0.0134	38.5856	0.0259	23.1143
54	1.956	0.5113	76.467	0.0131	39.0969	0.0256	23.5052
55	1.980	0.5050	78.423	0.0128	39.6018	0.0253	23.8940
56	2.005	0.4987	80.403	0.0124	40.1006	0.0249	24.2809
57	2.030	0.4926	82.408	0.0121	40.5932	0.0246	24.6658
58	2.055	0.4865	84.438	0.0118	41.0797	0.0243	25.0488
59	2.081	0.4805	86.494	0.0116	41.5602	0.0241	25.4297
60	2.107	0.4746	88.575	0.0113	42.0347	0.0238	25.8088
61	2.134	0.4687	90.682	0.0110	42.5035	0.0235	26.1858
62	2.160	0.4629	92.816	0.0108	42.9664	0.0233	26.5609
63	2.187	0.4572	94.976	0.0105	43.4236	0.0230	26.9340
64	2.215	0.4516	97.163	0.0103	43.8752	0.0228	27.3052
65	2.242	0.4460	99.378	0.0101	44.3211	0.0226	27.6745
66	2.270	0.4405	101.620	0.0098	44.7616	0.0223	28.0418
67	2.299	0.4350	103.890	0.0096	45.1967	0.0221	28.4072
68	2.327	0.4297	106.189	0.0094	45.6263	0.0219	28.7706
69	2.356	0.4244	108.516	0.0092	46.0507	0.0217	29.1321
70	2.386	0.4191	110.873	0.0090	46.4698	0.0215	29.4917
71	2.416	0.4140	113.259	0.0088	46.8838	0.0213	29.8494
72	2.446	0.4088	115.675	0.0086	47.2926	0.0211	30.2051
73	2.477	0.4038	118.121	0.0085	47.6964	0.0210	30.5590
74	2.507	0.3988	120.597	0.0083	48.0952	0.0208	30.9109
75	2.539	0.3939	123.104	0.0081	48.4891	0.0206	31.2609
76	2.571	0.3890	125.643	0.0080	48.8782	0.0205	31.6091
77	2.603	0.3842	128.214	0.0078	49.2624	0.0203	31.9553
78	2.635	0.3795	130.817	0.0076	49.6419	0.0201	32.2996
79	2.668	0.3748	133.452	0.0075	50.0166	0.0200	32.6421
80	2.701	0.3702	136.120	0.0073	50.3868	0.0198	32.9826
81	2.735	0.3656	138.821	0.0072	50.7524	0.0197	33.3213
82	2.769	0.3611	141.557	0.0071	51.1135	0.0196	33.6582
83	2.804	0.3566	144.326	0.0069	51.4701	0.0194	33.9931
84	2.839	0.3522	147.130	0.0068	51.8223	0.0193	34.3262
85	2.875	0.3479	149.969	0.0067	52.1702	0.0192	34.6574
86	2.911	0.3436	152.844	0.0065	52.5138	0.0190	34.9868
87	2.947	0.3393	155.755	0.0064	52.8531	0.0189	35.3143
88	2.984	0.3351	158.702	0.0063	53.1883	0.0188	35.6400
89	3.021	0.3310	161.685	0.0062	53.5193	0.0187	35.9639
90	3.059	0.3269	164.706	0.0061	53.8462	0.0186	36.2859
91	3.097	0.3229	167.765	0.0060	54.1691	0.0185	36.6060
92	3.136	0.3189	170.862	0.0059	54.4880	0.0184	36.9244
93	3.175	0.3150	173.998	0.0057	54.8030	0.0182	37.2409
94	3.215	0.3111	177.173	0.0056	55.1141	0.0181	37.5557
95	3.255	0.3072	180.388	0.0055	55.4213	0.0180	37.8686
96	3.296	0.3034	183.643	0.0054	55.7247	0.0179	38.1797
97	3.337	0.2997	186.938	0.0053	56.0244	0.0178	38.4890
98	3.378	0.2960	190.275	0.0053	56.3204	0.0178	38.7965
99	3.421	0.2923	193.654	0.0052	56.6128	0.0177	39.1022
100	3.463	0.2887	197.074	0.0051	56.9015	0.0176	39.4061

Interest Table for Annual Compounding with i = 1.5%

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
1	1.015	0.9852	1.000	1.0000	0.9852	1.0150	0.0000
2	1.030	0.9707	2.015	0.4963	1.9559	0.5113	0.4962
3	1.046	0.9563	3.045	0.3284	2.9122	0.3434	0.9900
4	1.061	0.9422	4.091	0.2444	3.8544	0.2594	1.4811
5	1.077	0.9283	5.152	0.1941	4.7826	0.2091	1.9700
6	1.093	0.9145	6.230	0.1605	5.6972	0.1755	2.4563
7	1.110	0.9010	7.323	0.1366	6.5982	0.1516	2.9402
8	1.126	0.8877	8.433	0.1186	7.4859	0.1336	3.4216
9	1.143	0.8746	9.559	0.1046	8.3605	0.1196	3.9006
10	1.161	0.8617	10.703	0.0934	9.2222	0.1084	4.3770
11	1.178	0.8489	11.863	0.0843	10.0711	0.0993	4.8510
12	1.196	0.8364	13.041	0.0767	10.9075	0.0917	5.3225
13	1.214	0.8240	14.237	0.0702	11.7315	0.0852	5.7914
14	1.232	0.8118	15.450	0.0647	12.5433	0.0797	6.2580
15	1.250	0.7999	16.682	0.0599	13.3432	0.0749	6.7221
16	1.269	0.7880	17.932	0.0558	14.1312	0.0708	7.1837
17	1.288	0.7764	19.201	0.0521	14.9076	0.0671	7.6428
18	1.307	0.7649	20.489	0.0488	15.6725	0.0638	8.0995
19	1.327	0.7536	21.797	0.0459	16.4261	0.0609	8.5537
20	1.347	0.7425	23.124	0.0432	17.1686	0.0582	9.0055
21	1.367	0.7315	24.470	0.0409	17.9001	0.0559	9.4547
22	1.388	0.7207	25.837	0.0387	18.6208	0.0537	9.9016
23	1.408	0.7100	27.225	0.0367	19.3308	0.0517	10.3460
24	1.430	0.6995	28.633	0.0349	20.0304	0.0499	10.7879
25	1.451	0.6892	30.063	0.0333	20.7196	0.0483	11.2274
26	1.473	0.6790	31.514	0.0317	21.3986	0.0467	11.6644
27	1.495	0.6690	32.987	0.0303	22.0676	0.0453	12.0990
28	1.517	0.6591	34.481	0.0290	22.7267	0.0440	12.5311
29	1.540	0.6494	35.999	0.0278	23.3760	0.0428	12.9608
30	1.563	0.6398	37.539	0.0266	24.0158	0.0416	13.3881
31	1.587	0.6303	39.102	0.0256	24.6461	0.0406	13.8129
32	1.610	0.6210	40.688	0.0246	25.2671	0.0396	14.2353
33	1.634	0.6118	42.298	0.0236	25.8789	0.0386	14.6553
34	1.659	0.6028	43.933	0.0228	26.4817	0.0378	15.0728
35	1.684	0.5939	45.592	0.0219	27.0755	0.0369	15.4880
36	1.709	0.5851	47.276	0.0212	27.6606	0.0362	15.9007
37	1.735	0.5764	48.985	0.0204	28.2371	0.0354	16.3110
38	1.761	0.5679	50.720	0.0197	28.8050	0.0347	16.7189
39	1.787	0.5595	52.480	0.0191	29.3645	0.0341	17.1244
40	1.814	0.5513	54.268	0.0184	29.9158	0.0334	17.5275
41	1.841	0.5431	56.082	0.0178	30.4589	0.0328	17.9282
42	1.869	0.5351	57.923	0.0173	30.9940	0.0323	18.3265
43	1.897	0.5272	59.792	0.0167	31.5212	0.0317	18.7225
44	1.925	0.5194	61.689	0.0162	32.0405	0.0312	19.1160
45	1.954	0.5117	63.614	0.0157	32.5523	0.0307	19.5072
46	1.984	0.5042	65.568	0.0153	33.0564	0.0303	19.8960
47	2.013	0.4967	67.552	0.0148	33.5531	0.0298	20.2824
48	2.043	0.4894	69.565	0.0144	34.0425	0.0294	20.6665
49	2.074	0.4821	71.608	0.0140	34.5246	0.0290	21.0482
50	2.105	0.4750	73.682	0.0136	34.9996	0.0286	21.4275

Interest Table for Annual Compounding with i = 1.5% (Cont.)

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
51	2.137	0.4680	75.788	0.0132	35.4676	0.0282	21.8045
52	2.169	0.4611	77.925	0.0128	35.9287	0.0278	22.1792
53	2.201	0.4543	80.093	0.0125	36.3829	0.0275	22.5515
54	2.234	0.4475	82.295	0.0122	36.8305	0.0272	22.9215
55	2.268	0.4409	84.529	0.0118	37.2714	0.0268	23.2891
56	2.302	0.4344	86.797	0.0115	37.7058	0.0265	23.6545
57	2.336	0.4280	89.099	0.0112	38.1338	0.0262	24.0175
58	2.372	0.4217	91.436	0.0109	38.5555	0.0259	24.3782
59	2.407	0.4154	93.807	0.0107	38.9709	0.0257	24.7366
60	2.443	0.4093	96.214	0.0104	39.3802	0.0254	25.0927
61	2.480	0.4032	98.657	0.0101	39.7834	0.0251	25.4466
62	2.517	0.3973	101.137	0.0099	40.1807	0.0249	25.7981
63	2.555	0.3914	103.654	0.0096	40.5721	0.0246	26.1474
64	2.593	0.3856	106.209	0.0094	40.9578	0.0244	26.4943
65	2.632	0.3799	108.802	0.0092	41.3377	0.0242	26.8390
66	2.672	0.3743	111.434	0.0090	41.7120	0.0240	27.1815
67	2.712	0.3688	114.106	0.0088	42.0808	0.0238	27.5217
68	2.752	0.3633	116.817	0.0086	42.4441	0.0236	27.8596
69	2.794	0.3580	119.570	0.0084	42.8021	0.0234	28.1953
70	2.835	0.3527	122.363	0.0082	43.1548	0.0232	28.5288
71	2.878	0.3475	125.199	0.0080	43.5023	0.0230	28.8600
72	2.921	0.3423	128.076	0.0078	43.8446	0.0228	29.1891
73	2.965	0.3373	130.998	0.0076	44.1819	0.0226	29.5159
74	3.009	0.3323	133.963	0.0075	44.5141	0.0225	29.8405
75	3.055	0.3274	136.972	0.0073	44.8415	0.0223	30.1629
76	3.100	0.3225	140.027	0.0071	45.1641	0.0221	30.4831
77	3.147	0.3178	143.127	0.0070	45.4818	0.0220	30.8011
78	3.194	0.3131	146.274	0.0068	45.7949	0.0218	31.1169
79	3.242	0.3084	149.468	0.0067	46.1034	0.0217	31.4306
80	3.291	0.3039	152.710	0.0065	46.4072	0.0215	31.7421
81	3.340	0.2994	156.001	0.0064	46.7066	0.0214	32.0514
82	3.390	0.2950	159.341	0.0063	47.0016	0.0213	32.3586
83	3.441	0.2906	162.731	0.0061	47.2922	0.0211	32.6637
84	3.493	0.2863	166.172	0.0060	47.5786	0.0210	32.9666
85	3.545	0.2821	169.664	0.0059	47.8606	0.0209	33.2674
86	3.598	0.2779	173.209	0.0058	48.1386	0.0208	33.5660
87	3.652	0.2738	176.807	0.0057	48.4124	0.0207	33.8626
88	3.707	0.2698	180.459	0.0055	48.6822	0.0205	34.1571
89	3.762	0.2658	184.166	0.0054	48.9479	0.0204	34.4494
90	3.819	0.2619	187.929	0.0053	49.2098	0.0203	34.7397
91	3.876	0.2580	191.748	0.0052	49.4678	0.0202	35.0279
92	3.934	0.2542	195.624	0.0051	49.7219	0.0201	35.3140
93	3.993	0.2504	199.558	0.0050	49.9723	0.0200	35.5980
94	4.053	0.2467	203.552	0.0049	50.2191	0.0199	35.8800
95	4.114	0.2431	207.605	0.0048	50.4621	0.0198	36.1600
96	4.176	0.2395	211.719	0.0047	50.7016	0.0197	36.4379
97	4.238	0.2359	215.895	0.0046	50.9375	0.0196	36.7138
98	4.302	0.2325	220.133	0.0045	51.1700	0.0195	36.9877
99	4.367	0.2290	224.435	0.0045	51.3990	0.0195	37.2595
100	4.432	0.2256	228.802	0.0044	51.6246	0.0194	37.5293

Interest Table for Annual Compounding with i = 1.75%

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
1	1.018	0.9828	1.000	1.0000	0.9828	1.0175	0.0000
2	1.035	0.9659	2.018	0.4957	1.9487	0.5132	0.4959
3	1.053	0.9493	3.053	0.3276	2.8980	0.3451	0.9886
4	1.072	0.9330	4.106	0.2435	3.8310	0.2610	1.4784
5	1.091	0.9169	5.178	0.1931	4.7479	0.2106	1.9655
6	1.110	0.9011	6.269	0.1595	5.6490	0.1770	2.4495
7	1.129	0.8856	7.378	0.1355	6.5347	0.1530	2.9308
8	1.149	0.8704	8.508	0.1175	7.4051	0.1350	3.4091
9	1.169	0.8554	9.656	0.1036	8.2605	0.1211	3.8845
10	1.189	0.8407	10.825	0.0924	9.1012	0.1099	4.3571
11	1.210	0.8263	12.015	0.0832	9.9275	0.1007	4.8268
12	1.231	0.8121	13.225	0.0756	10.7396	0.0931	5.2936
13	1.253	0.7981	14.457	0.0692	11.5377	0.0867	5.7575
14	1.275	0.7844	15.710	0.0637	12.3220	0.0812	6.2185
15	1.297	0.7709	16.985	0.0589	13.0929	0.0764	6.6767
16	1.320	0.7576	18.282	0.0547	13.8505	0.0722	7.1320
17	1.343	0.7446	19.602	0.0510	14.5951	0.0685	7.5844
18	1.367	0.7318	20.945	0.0477	15.3269	0.0652	8.0339
19	1.390	0.7192	22.311	0.0448	16.0461	0.0623	8.4806
20	1.415	0.7068	23.702	0.0422	16.7529	0.0597	8.9245
21	1.440	0.6947	25.116	0.0398	17.4476	0.0573	9.3655
22	1.465	0.6827	26.556	0.0377	18.1303	0.0552	9.8036
23	1.490	0.6710	28.021	0.0357	18.8013	0.0532	10.2388
24	1.516	0.6594	29.511	0.0339	19.4607	0.0514	10.6713
25	1.543	0.6481	31.028	0.0322	20.1088	0.0497	11.1009
26	1.570	0.6369	32.571	0.0307	20.7458	0.0482	11.5276
27	1.597	0.6260	34.141	0.0293	21.3718	0.0468	11.9515
28	1.625	0.6152	35.738	0.0280	21.9870	0.0455	12.3726
29	1.654	0.6046	37.363	0.0268	22.5917	0.0443	12.7909
30	1.683	0.5942	39.017	0.0256	23.1859	0.0431	13.2063
31	1.712	0.5840	40.700	0.0246	23.7699	0.0421	13.6189
32	1.742	0.5740	42.412	0.0236	24.3439	0.0411	14.0287
33	1.773	0.5641	44.155	0.0226	24.9080	0.0401	14.4358
34	1.804	0.5544	45.927	0.0218	25.4624	0.0393	14.8400
35	1.835	0.5449	47.731	0.0210	26.0073	0.0385	15.2414
36	1.867	0.5355	49.566	0.0202	26.5428	0.0377	15.6400
37	1.900	0.5263	51.434	0.0194	27.0691	0.0369	16.0359
38	1.933	0.5172	53.334	0.0187	27.5863	0.0362	16.4290
39	1.967	0.5083	55.267	0.0181	28.0947	0.0356	16.8193
40	2.002	0.4996	57.234	0.0175	28.5943	0.0350	17.2068
41	2.037	0.4910	59.236	0.0169	29.0853	0.0344	17.5916
42	2.072	0.4826	61.273	0.0163	29.5679	0.0338	17.9736
43	2.109	0.4743	63.345	0.0158	30.0421	0.0333	18.3529
44	2.145	0.4661	65.453	0.0153	30.5082	0.0328	18.7295
45	2.183	0.4581	67.599	0.0148	30.9663	0.0323	19.1033
46	2.221	0.4502	69.782	0.0143	31.4165	0.0318	19.4744
47	2.260	0.4425	72.003	0.0139	31.8590	0.0314	19.8428
48	2.300	0.4349	74.263	0.0135	32.2939	0.0310	20.2085
49	2.340	0.4274	76.563	0.0131	32.7212	0.0306	20.5715
50	2.381	0.4200	78.903	0.0127	33.1413	0.0302	20.9318

Interest Table for Annual Compounding with i = 1.75% (Cont.)

n	$F/p, i, n$	$P/F, i, n$	$F/A, i, n$	$A/F, i, n$	$P/A, i, n$	$A/P, i, n$	$A/G, i, n$
51	2.422	0.4128	81.283	0.0123	33.5541	0.0298	21.2895
52	2.465	0.4057	83.706	0.0119	33.9598	0.0294	21.6444
53	2.508	0.3987	86.171	0.0116	34.3585	0.0291	21.9967
54	2.552	0.3919	88.679	0.0113	34.7504	0.0288	22.3463
55	2.597	0.3851	91.231	0.0110	35.1355	0.0285	22.6933
56	2.642	0.3785	93.827	0.0107	35.5140	0.0282	23.0376
57	2.688	0.3720	96.469	0.0104	35.8860	0.0279	23.3793
58	2.735	0.3656	99.157	0.0101	36.2516	0.0276	23.7183
59	2.783	0.3593	101.893	0.0098	36.6109	0.0273	24.0548
60	2.832	0.3531	104.676	0.0096	36.9640	0.0271	24.3886
61	2.881	0.3471	107.508	0.0093	37.3111	0.0268	24.7199
62	2.932	0.3411	110.389	0.0091	37.6522	0.0266	25.0485
63	2.983	0.3352	113.321	0.0088	37.9874	0.0263	25.3746
64	3.035	0.3295	116.304	0.0086	38.3169	0.0261	25.6981
65	3.088	0.3238	119.339	0.0084	38.6407	0.0259	26.0191
66	3.142	0.3182	122.428	0.0082	38.9589	0.0257	26.3375
67	3.197	0.3127	125.570	0.0080	39.2716	0.0255	26.6534
68	3.253	0.3074	128.768	0.0078	39.5790	0.0253	26.9667
69	3.310	0.3021	132.021	0.0076	39.8811	0.0251	27.2775
70	3.368	0.2969	135.331	0.0074	40.1780	0.0249	27.5858
71	3.427	0.2918	138.700	0.0072	40.4697	0.0247	27.8916
72	3.487	0.2868	142.127	0.0070	40.7565	0.0245	28.1949
73	3.548	0.2818	145.614	0.0069	41.0383	0.0244	28.4957
74	3.610	0.2770	149.162	0.0067	41.3153	0.0242	28.7941
75	3.674	0.2722	152.773	0.0065	41.5875	0.0240	29.0900
76	3.738	0.2675	156.446	0.0064	41.8551	0.0239	29.3835
77	3.803	0.2629	160.184	0.0062	42.1180	0.0237	29.6745
78	3.870	0.2584	163.987	0.0061	42.3764	0.0236	29.9631
79	3.938	0.2540	167.857	0.0060	42.6304	0.0235	30.2493
80	4.006	0.2496	171.795	0.0058	42.8800	0.0233	30.5330
81	4.077	0.2453	175.801	0.0057	43.1253	0.0232	30.8144
82	4.148	0.2411	179.878	0.0056	43.3664	0.0231	31.0934
83	4.220	0.2369	184.026	0.0054	43.6033	0.0229	31.3700
84	4.294	0.2329	188.246	0.0053	43.8362	0.0228	31.6443
85	4.369	0.2289	192.540	0.0052	44.0651	0.0227	31.9162
86	4.446	0.2249	196.910	0.0051	44.2900	0.0226	32.1858
87	4.524	0.2211	201.356	0.0050	44.5110	0.0225	32.4531
88	4.603	0.2173	205.880	0.0049	44.7283	0.0224	32.7180
89	4.683	0.2135	210.482	0.0048	44.9418	0.0223	32.9807
90	4.765	0.2098	215.166	0.0046	45.1517	0.0221	33.2410
91	4.849	0.2062	219.931	0.0045	45.3579	0.0220	33.4991
92	4.934	0.2027	224.780	0.0044	45.5606	0.0219	33.7549
93	5.020	0.1992	229.714	0.0044	45.7598	0.0219	34.0085
94	5.108	0.1958	234.734	0.0043	45.9556	0.0218	34.2598
95	5.197	0.1924	239.842	0.0042	46.1480	0.0217	34.5089
96	5.288	0.1891	245.039	0.0041	46.3371	0.0216	34.7557
97	5.381	0.1858	250.327	0.0040	46.5229	0.0215	35.0004
98	5.475	0.1827	255.708	0.0039	46.7056	0.0214	35.2429
99	5.571	0.1795	261.183	0.0038	46.8851	0.0213	35.4831
100	5.668	0.1764	266.753	0.0037	47.0615	0.0212	35.7213

INDEX

Engineering Economics

R. Panneerselvam

Designed as a text book for undergraduate students in various engineering disciplines — mechanical, civil and industrial engineering — and for postgraduate students in industrial engineering and water resource management, this comprehensive and well-organized book shows how complex economic decisions can be made from a number of given alternatives. It provides the managers not only a sound basis but also a clear-cut approach to decision making. These decisions will ultimately result in minimizing costs and/or maximizing benefits to their organizations. What is more, the book adequately illustrates these approaches with numerical problems and *Indian cases.*

After giving an overview of the subject, the text discusses, in a simple and easy-to-read style, such topics as interest formulas and their applications, methods like present worth method of comparison, future worth method, annual equivalent method, rate of return method, and evaluation of public alternatives. Besides, it deals with depreciation, inflation adjusted decisions, and inventory control. Finally, the book analyzes other important areas, for instance, make or buy decision, project management, value analysis/value engineering, and linear programming.

A distinguishing feature of the book is that it has an Appendix on interest tables for a wide range of interest rates (0.25% – 50%) and for a period ranging from one year to 100 years. This book, which is profusely illustrated with worked-out examples and diagrams, should prove extremely useful not only as a text book but also as a reference for those offering courses in such management areas as project management, production management and financial management.

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