UNIT-5

EQUILIBRIUM OF FORCES

⇒ **Equilibrium of forces**

Any system of forces acting on a body are said to be in equ^m when the resultant of all forces is zero & algebraic sum of moments of all the forces is zero.

⇒ **Conditions of Equilibrium**

A system of forces is said to be in equ^m when $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 0$ and $\Sigma M = 0$.

i.e, For concurrent force system

$\Sigma H = 0$; $\Sigma V = 0$

& For non-concurrent force system

$\Sigma H = 0$; $\Sigma V = 0$; $\Sigma M = 0$

where; ΣH & ΣV are algebraic sum of horizonta & vertical component of forces.

ΣM = Algebraic sum of moments of forces about any point.

⇒ **Principle (or) Conditions of equ^m for diff force system**

~~2) Three force system~~

~~If a body is acted upon by 3 forces then for equ^m the resultant of any 2 forces must be equal opposite & collinear with third force~~

1) **Two force system**

If a body is acted upon by two forces, then for equ^m they must be equal in magnitude, opposite & collinear (same line)

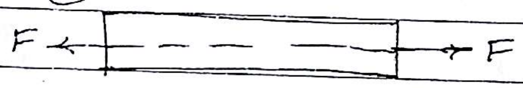
2) <u>Three force system</u>

If a body is acted upon by three forces then for equ^n the resultant of any two forces must be equal, opp & collinear with third force.

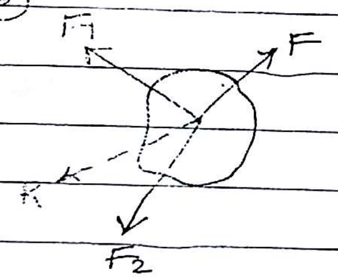
3) <u>Four force system</u>

If a body is acted upon by four forces then for equ^n the resultant of any two forces must be equal, opp & collinear with the resultant of remaining two forces.

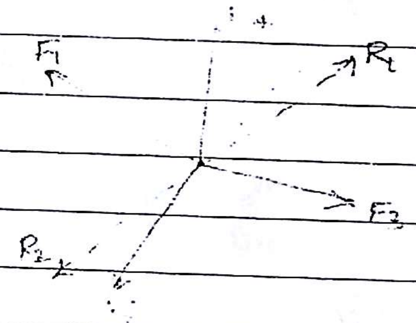
(a)



(b)



(c)



Resultant of F_1 & F_2 = R_1

Resultant of F_3 & F_4 = R_2

Resultant of F_1 & F_2

= F_3 = R

⇒ <u>Equilibrant</u>

An equ^t is a force equal in magnitude, opp in direc^n & collinear with the resultant. It is defined as a force or a moment used to keep an object in equ^n.

If an equ^t is added to a concurrent system of forces then the system will be in equ^n.

⇒ Types of forces acting on a body

a) Applied forces

They are the forces applied externally to a body. Ex: If a person stands on a ladder, his weight is an applied force to the ladder.

b) Non-applied forces

The foll. are the two types of nonapplied force.

i) Self weight

Everybody is subjected to gravitational attraction & hence has got self wt, given by the expression $(W) = mg$

where; m = mass of the body

g = gravitational attraction = $9.81\ m/sec^2$

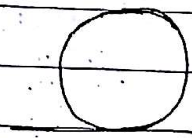
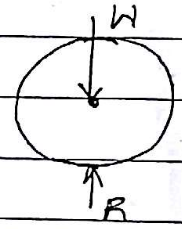
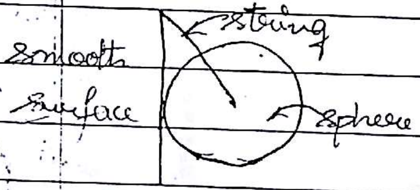
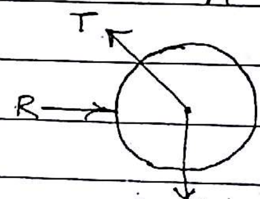
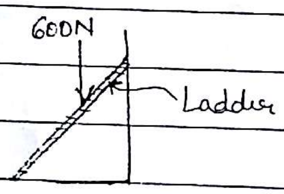
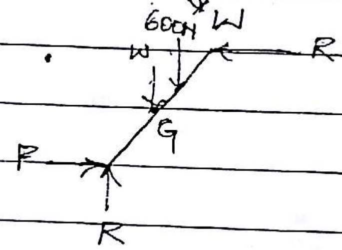
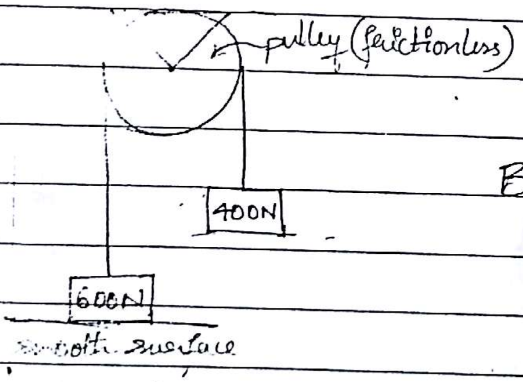
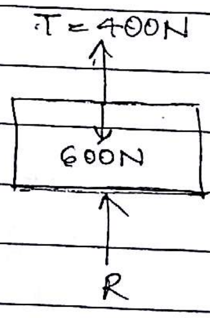
Self wt always acts in vertically downward direc^n through the center of gravity of the body. If self wt is very small compared to other forces, it may be neglected.

ii) Reactions

Reactions are the self adjusting forces developed by other bodies which are in contact with the body under consideration. Reactions are equal & opp. If surface of contact is smooth, direction of reaction is $\perp^r$ to the surface of contact.

⇒ Free body diagram

For the analysis of equ^m cond^ns, it is necessary to isolate the body under consideration from other bodies in contact & draw all forces acting on the body. This type of diagram of a body in which the body under consideration is freed from all contact surfaces & shown with all forces on it (self wt, reactions & applied forces) is called Free body diagram.

Reacting bodies	FBD reqd for	FBD
	Ball	
	Sphere tied by a string ~~Ball~~	
	Ladder	
	Block of 600N	

→ Lami's theorem

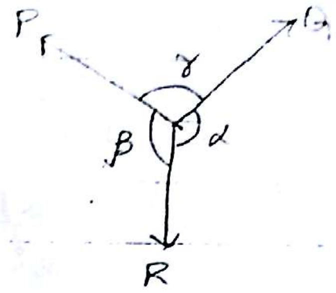
If 3 coplanar forces acting simultaneously @ a pt be in equ^{m}, then each force is proportional to sine $\angle$ b/w other two forces.

Let P, Q & R be 3 coplanar forces acting @ O be in equ^{m} & α, β, γ be corresponding opp $\angle$es, then

(2)

Chandramani
9901108108

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

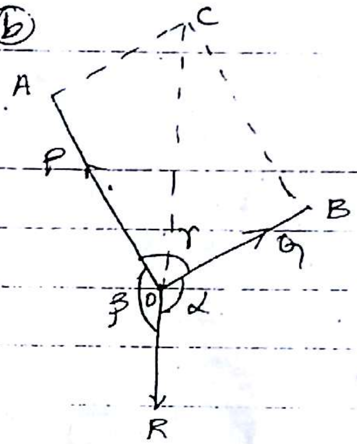


proof → With P & Q as two forces along OA & OB, complete the ▱ᵐ OACB

→ Then diagonal OC represents the resultant of P & Q according to ▱ᵐ law of forces

→ But since system is in equⁿ, the resultant of the forces P & Q must be in line with OC but opp to R.

(b)



From fig (b), In Δ^{le} AOC,

$\angle AOC = 180-\beta$

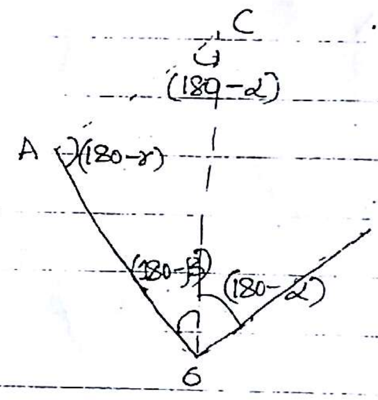
$\angle ACO = \angle COB = 180-\alpha$

$\angle CAO = 180 - \angle AOC - \angle ACO$

$\angle CAO = 180 - (180-\beta) - (180-\alpha)$

$\angle CAO = 180 - 180 + \beta - 180 + \alpha$

$\angle CAO = \alpha + \beta - 180 \longrightarrow (1)$



⇒ w.k.t $\alpha+\beta+\gamma = 360°$

Subtracting both sides by 180°

$\alpha+\beta+\gamma-180 = 360-180$

$\alpha+\beta+\gamma-180 = 180$

$\alpha+\beta-180 = 180-\gamma \longrightarrow (2)$

→ From (1) & (2)

$\angle CAO = 180-\gamma \longrightarrow (3)$

⇒ Applying sine rule

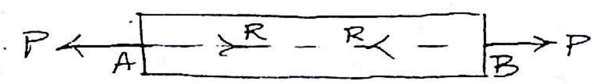
$$\frac{OA}{\sin\angle CAO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{\sin\angle OAC}$$

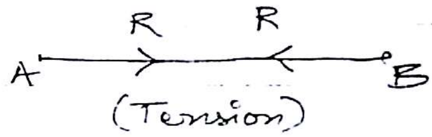
$$\frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\gamma)}$$

(or) $$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$ $\{\because \sin 180-\alpha$

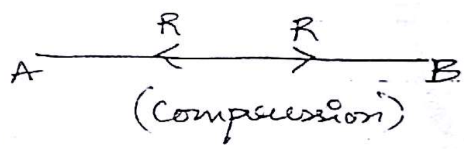
⇒ Important pts to remember for solving probl

1) If a member AB is subjected to pull (ten then resistance against external force is Aw from end A & B.





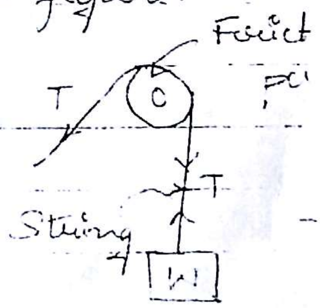
2) If a member AB is subjected to push (compr then resistance against external force is TOWA the end A & B

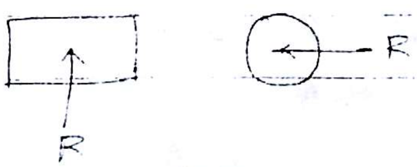


3) String, cables, wires, ropes, chains are always subjected to tension only.

4) A string carrying freely hanging load on one will be in tension. If such string passes roun frictionless pulley, then tension on either side pulley remains same as shown in figure.

5) Reactions are directed towards the centroid of the body.



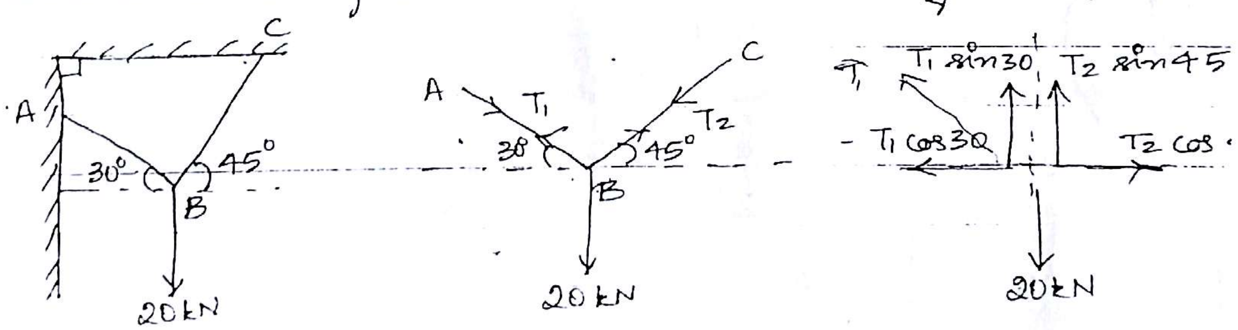


⟹ General procedure for solving problems

1) Draw FBD of a pt in a system. Consider all forces acting on a system including applied & non-applied forces (i.e, reactions & self wt)
2) If concurrent force system, then $\Sigma V = 0$; $\Sigma H = 0$
3) If non-concurrent force system, then $\Sigma V = 0$; $\Sigma H = 0$; $\Sigma M =$
4) For 3 concurrent forces in equ^n, use Lami's theorem
5) For object hanging freely from cable, Tension in cable = weight of object
6) For equ^n of system of more than one object, draw FBD of all objects. Start solving both FBD of minimum no of unknowns.

⟹ Problems

1) 2 cables are connected @ A & C as shown in figure & a force of 20kN is applied @ B. Determine the forces in the cable along BA & CB



→ Let T_1 & T_2 be tension in strings AB & CB as shown.

→ The system is in equ^n (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^n @ ⊕ of joint B

→ Resolving horizontal forces

$$T_1 \cos 30 + T_2 \cos 45 = 0$$

$$T_2 \cos 45 = T_1 \cos 30$$

$$T_2 = \frac{T_1 \cos 30}{\cos 45}$$

$$T_2 = 1.22 T_1 \longrightarrow (1)$$

→ Resolving vertical forces

$$T_1 \sin 30 + T_2 \sin 45 - 20 = 0$$

$$T_1 \sin 30 + T_2 \sin 45 = 20 \longrightarrow (2)$$

→ By substituting (1) in (2)

$$T_1 \sin 30 + 1.22 T_1 \sin 45 = 20$$

$$0.5 T_1 + 0.86 T_1 = 20$$

$$T_1 = 14.67 \text{ kN}$$

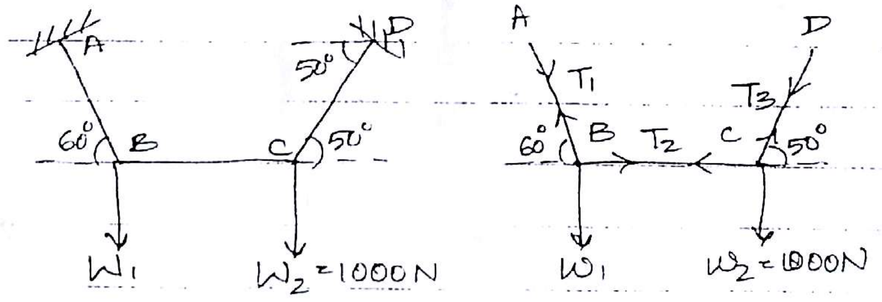
→ By substituting T_1 value in equ^n (1)

$$T_2 = 1.22 \times 14.67$$

$$T_2 = 17.9 \text{ kN}$$

---

3) Find the forces in the wires (AB, BC & CD) & the load W_1 to keep the system in equ^m with BC horizontal. Take $W_2 = 1000$N.



→ Let T_1, T_2, T_3 be tension in strings AB, BC & CD resp.

→ The system is in equ^m (i.e., $\Sigma H = \Sigma V = 0$)

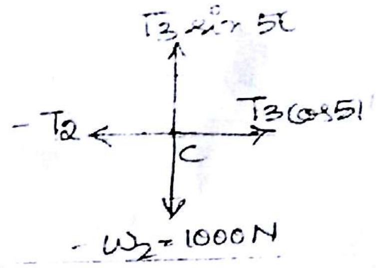
→ Considering equ^m of jt C

- Resolving horizontal forces

$$-T_2 + T_3 \cos 50 = 0$$

$$T_3 \cos 50 = T_2$$

$$0.642 T_3 = T_2$$



- Resolving vertical forces

$$T_3 \sin 50 - 1000 = 0$$

$$T_3 \sin 50 = 1000$$

$$T_3 = \underline{1305.4N}$$

$$\therefore T_2 = 1305.4 \times 0.642$$

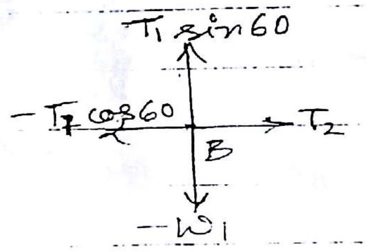
$$T_2 = \underline{838.07N}$$

→ Considering equ^{m} of jt B

- Resolving horizontal forces

$$-T_1 \cos 60 + T_2 = 0$$

$$T_1 = \frac{\cancel{838.60}}{\cos 60} = \underline{1676.14N}$$



- Resolving vertical forces

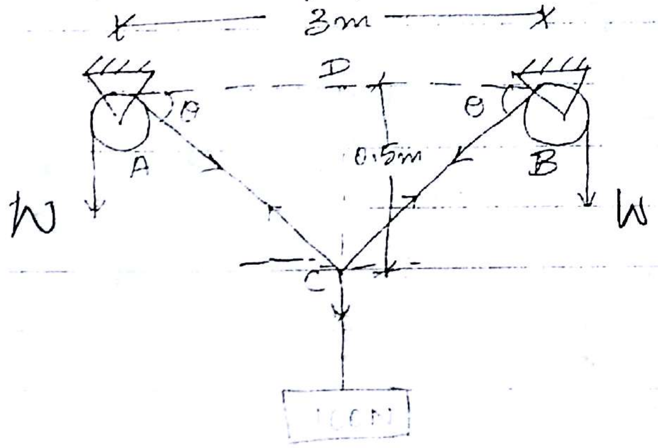
$$T_1 \sin 60 - W_1 = 0$$

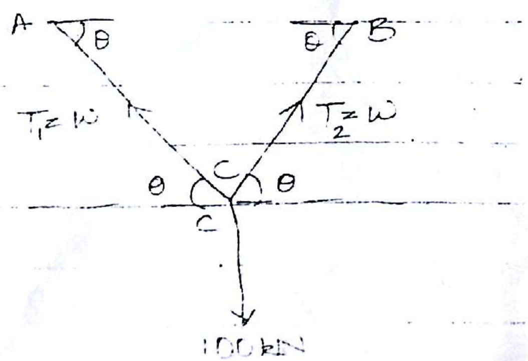
$$W_1 = 1676.14 \, \cancel{\cos}\, \sin 60$$

$$W_1 = \underline{1451.6N}$$

---

3) Find the value of W which is reqd to maintain equ^{m} configuration as shown in fig.



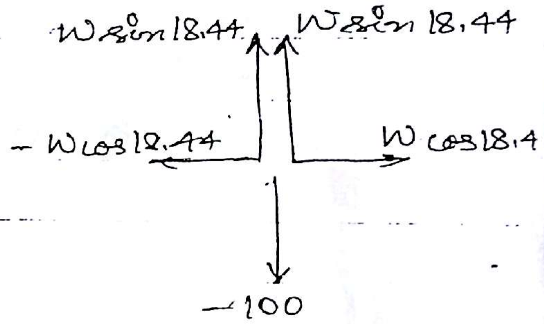


Scanned by CamScanner

→ Let T_1 & T_2 be tension in strings AC & CB as shown. As the string A & B passes over frictionless pulley, $T_1 = T_2 = W$

→ The system is in equ^{m} (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering eqn^{m} of joint C



– To find θ, From Δ^{le} ADC

$$\theta = \tan^{-1}\left(\frac{0.5}{1.5}\right) = \underline{18.44^\circ}$$

– Resolving horizontal forces

$$W\cos 18.44 - W\cos 18.44 = 0$$

– Resolving vertical forces

$$W\sin 18.44 + W\sin 18.44 - 100 = 0$$

$$2W\sin 18.44 = 100$$

$$W = \frac{100}{2\sin 18.44} = \underline{158.1N}$$

(OR)

→ By applying Lami's theorem

$$\frac{W}{\sin(90+18.44)} = \frac{W}{\sin(90+18.44)} = \frac{100}{\sin(180-18.44-18.4)}$$

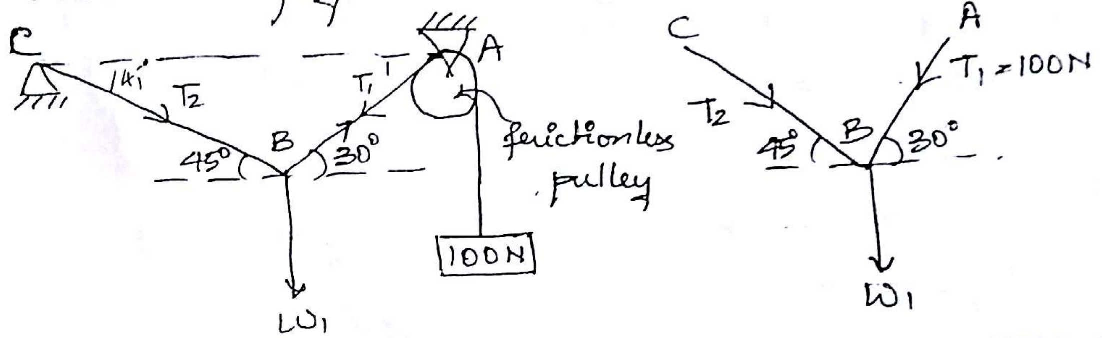
$$\frac{W}{0.948} = \frac{W}{0.948} = \frac{100}{\cancel{143.12}\ 0.6}$$

$$\therefore \frac{W}{0.948} = \frac{100}{0.6}$$

$$W = \frac{100 \times 0.948}{0.6} = \underline{158N}$$

Scanned by CamScanner

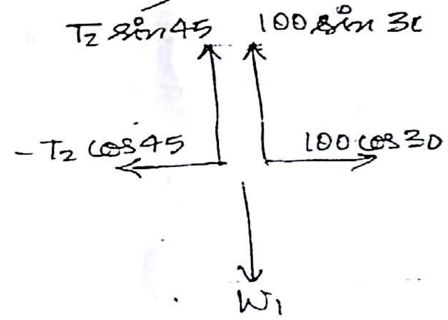
4) Find the value of W_1 for the equ^n of the system shown in figure.



→ Let T_1 & T_2 be tension in string AB & CB as shown. As the string AB passes over frictionless pulley, $T_1 = 100N$

→ The system is in equ^m (i.e., $\Sigma H = \Sigma V = 0$)

→ Considering equ^m of pt B.



- Resolving horizontal forces

$$100 \cos 30 - T_2 \cos 45 = 0$$

$$T_2 = \frac{100 \cos 30}{\cos 45}$$

$$\underline{T_2 = 122.47N}$$

- Resolving vertical forces

$$T_2 \sin 45 + 100 \sin 30 - W_1 = 0$$

$$\underline{W_1 = 136.6N}$$

(OR)

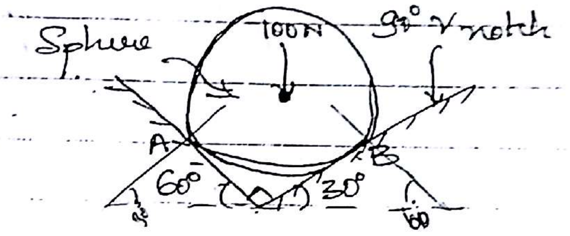
→ By applying Lami's theorem

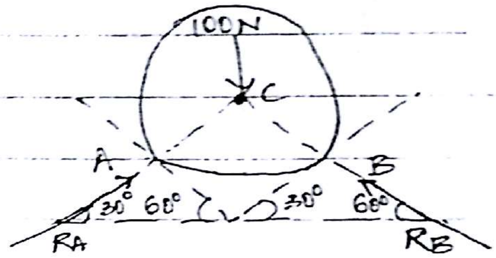
$$\frac{W_1}{\sin(180-45-30)} = \frac{T_2}{\sin(90+30)} = \frac{100}{\sin(90+45)}$$

$$\therefore T_2 = \frac{100 \sin 120}{\sin 135} = \underline{122.47N}$$

$$W_1 = \frac{100 \sin 105}{\sin 135} = \underline{136.6N}$$

5) A sphere weighing 100N is fitted in a set [illegible] notch as shown. If all the contact surfaces are smooth, then determine the reactions @ A & B.

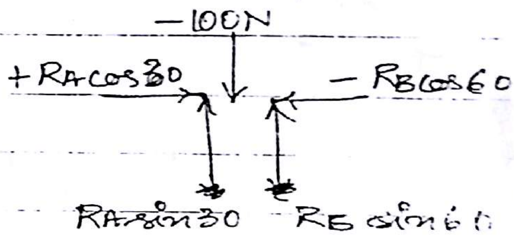




→ Let RA & RB be the reactions @ contact pts A & B

→ The system is in equi^m (i.e, ΣH = ΣV = 0)

→ Considering equi^m @ jt C



– Resolving horizontal forces

$R_A \cos 30 - R_B \cos 60 = 0$

$$R_A = \frac{R_B \cos 60}{\cos 30}$$

$R_A = 0.577 R_B \longrightarrow (1)$

– Resolving vertical forces

$-100 + R_A \sin 30 + R_B \sin 60 = 0$

$R_B \sin 60 + 0.577 R_B \sin 30 = 100$

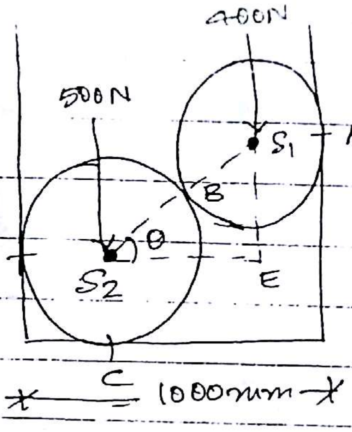
$0.86 R_B + 0.28 R_B = 100$

$1.148 R_B = 100$

$R_B = \underline{87N}$

$\therefore R_A = 0.577 \times 87 = \underline{50.23N}$

6) A horizontal channel with an inner clearance of 1000mm carries two spheres of radius 350mm & 250mm whose wts are 500N & 400N resp^ly. Find the reactions @ all pts of contacts.



→ Let R_A, R_B, R_C & R_D be the reactions @ contact pts A, B, C & D

→ The system is in equi^m (i.e., $\Sigma H = \Sigma V = 0$)

→ Consider Δ^{le} $S_1 S_2 E$

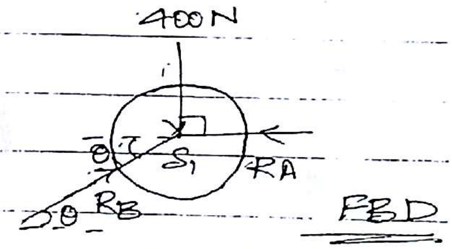
$S_1 S_2 = S_2 B + B S_1 = 350 + 250 = \underline{600mm}$

$S_2 E = 1000 - 350 - 250 = \underline{400mm}$

$S_1 E = \sqrt{(S_2 S_1)^2 - (S_2 E)^2} = \sqrt{(600)^2 - (400)^2} = \underline{447.21mm}$

$\cos\theta = \dfrac{S_2 E}{S_1 S_2} \Rightarrow \theta = \cos^{-1}\left(\dfrac{400}{600}\right) = \underline{48.18^\circ}$

→ Consider FBD of sphere @ S_1



- Resolving horizontal forces

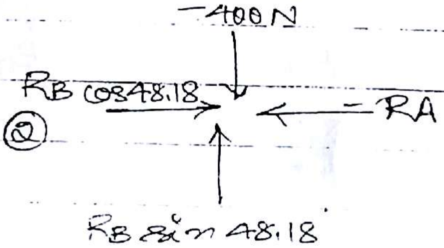
$R_B \cos 48.18 - R_A = 0$

$R_A = 0.67 R_B \rightarrow (1)$

- Resolving vertical forces

$R_B \sin 48.18 - 400 = 0$

$R_B = \dfrac{400}{\sin 48.18} = \underline{536.67N} \rightarrow (2)$



→ Considering FBD of Sphere @ S_2

- Resolving horizontal forces

$R_D - 536.67 \cos 48.18 = 0$

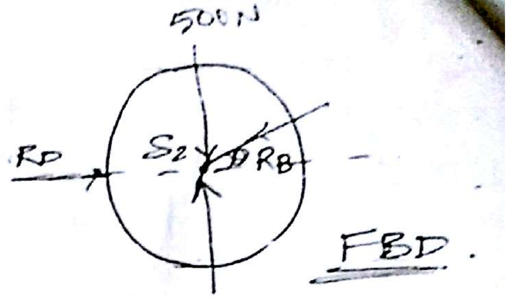
$R_D = \underline{357.8N}$

✓

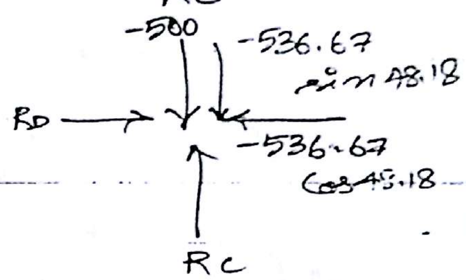
- Resolving vertical forces

$R_C - 536.67 \sin 48.18 - 0$

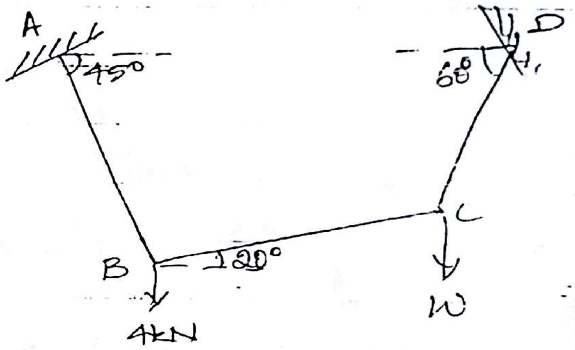
$R_C = \underline{900 N}$



FBD.



HW 7) A string is subjected to the forces 4 kN & W as shown in fig. Determine the magnitude of W & tension induced in various portions of the string.



⇒ Let T_1, T_2 & T_3 be the reactions ~~@~~ in strings AB, BC & CD resp.

→ The system is in equⁿ (i.e, $\Sigma H = \Sigma V = 0$)

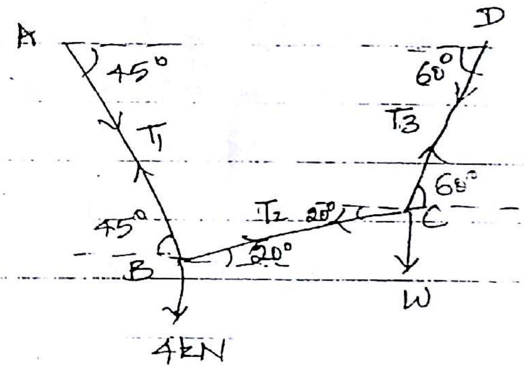
→ Considering equⁿ @ jt. B

- Resolving horizontal forces

$T_2 \cos 20 - T_1 \cos 45 = 0$

$$T_2 = \frac{T_1 \cos 45}{\cos 20}$$

$T_2 = 0.75 T_1 \longrightarrow (1)$



- Resolving vertical forces

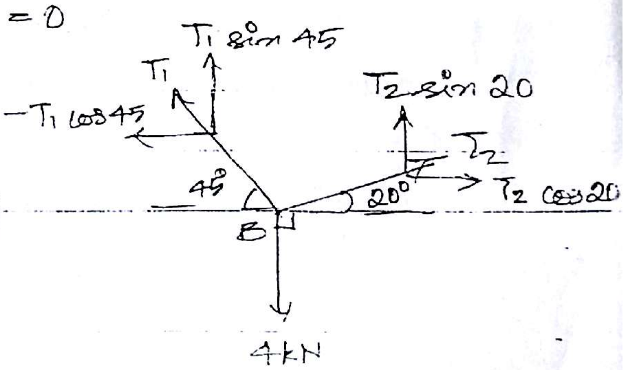
$T_1 \sin 45 + T_2 \sin 20 - 4 = 0$

$T_1 \sin 45 + 0.75 T_1 \sin 20 - 4 = 0$

$0.96 T_1 = 4$

$T_1 = 4.15 N$

$T_2 = 0.75 \times 4.15 = \underline{3.11 N}$



Scanned by CamScanner

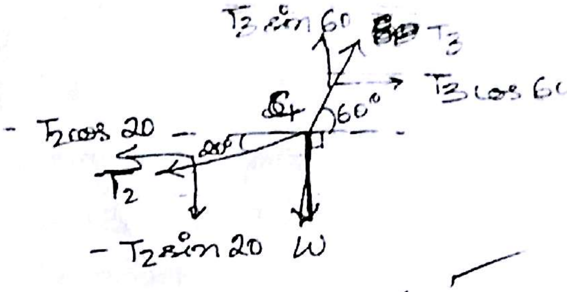
→ Considering equ^n @ jt C

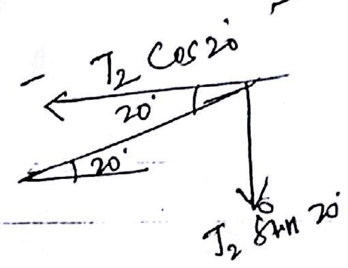
- Resolving horizontal forces

$T_3 \cos 60 - T_2 \cos 20 = 0$

$$T_3 = \frac{3.11 \cos 20}{\cos 60}$$

$\boxed{T_3 = 5.84 N}$





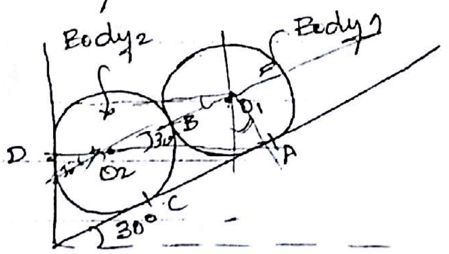
- Resolving vertical forces

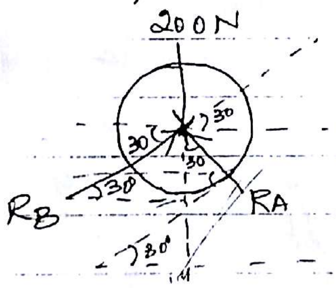
$T_3 \sin 60 - T_2 \sin 20 - W = 0$

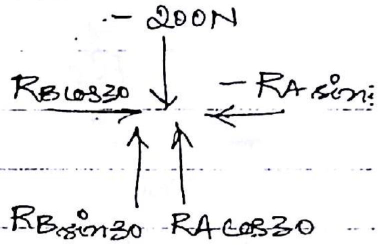
$5.84 \sin 60 - 3.11 \sin 20 = W$

$\boxed{W = 4 kN}$

8) Two identical rollers each weighing 200N are placed in a trough as shown in fig. Assuming all contact surfaces as smooth, find the reactions developed @ contact surface A, B, C, D.







→ Let R_A, R_B, R_C, R_D be the reactions @ contact pts A, B, C & D

→ The system is in equ^m (i.e, $\Sigma H = \Sigma V = 0$)

→ Consider FBD @ O_1

- Resolving horizontal forces

$R_B \cos 30 - R_A \sin 30 = 0$

$$R_B = \frac{R_A \sin 30}{\cos 30}$$

$R_B = 0.57 R_A \longrightarrow (1)$

Scanned by CamScanner

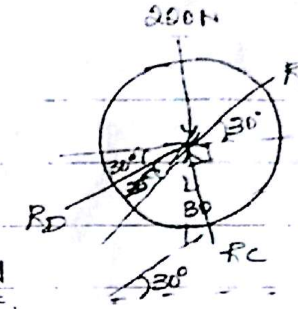
- Resolving vertical forces

$R_B \sin 30 + R_A \cos 30 - 200 = 0$

$0.57 R_A \sin 30 + R_A \cos 30 - 200 = 0$

$1.15 R_A = 200$

$R_A = \underline{173.75N}$; $R_B = \underline{99N}$



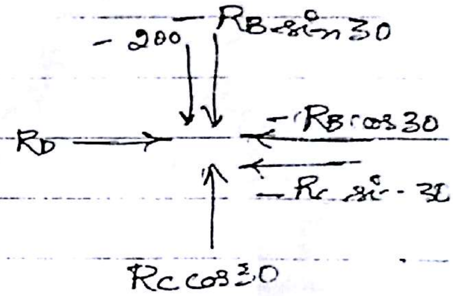
→ Consider FBD @ O_2

- Resolving horizontal forces

$R_D - R_B \cos 30 - R_C \sin 30 = 0$

$R_D - 85.73 - R_C \sin 30 = 0$

$R_D = 85.73 + R_C \sin 30$



- Resolving vertical forces

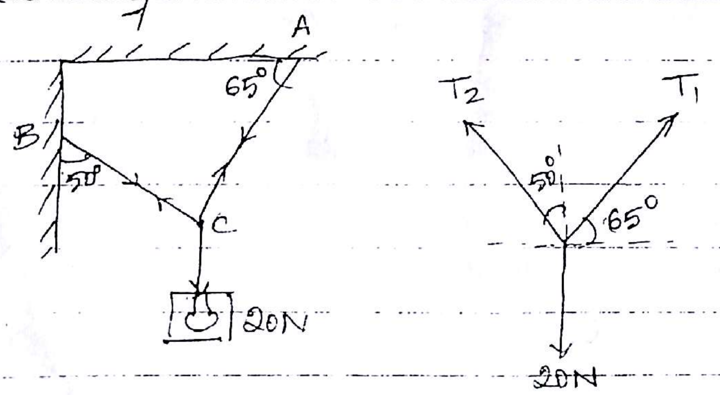
$R_C \cos 30 - R_B \sin 30 - 200 = 0$

$R_C \cos 30 - 49.5 - 200 = 0$

$R_C = \frac{49.5 + 200}{\cos 30} = \underline{288N}$

$\therefore R_D = 85.73 + \cancel{57.15}\ 288 \sin 30 = \underline{229.7N}$

Q9) An electric bulb hangs @ a pt C as shown by 2 strings AC & BC. Determine the forces in the string.



⇒ Let T_1 & T_2 be tension in strings AC & BC as shown in fig

→ The system is in equ^n (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^n @ C & applying Lami's theorem

$$\frac{T_2}{\sin(90+65)} = \frac{20}{\sin(50+(90-65))} = \frac{T_1}{\sin(180-50)}$$

$$\therefore \frac{T_2}{\sin 155} = \frac{20}{\sin 75} = \frac{T_1}{\sin 130}$$

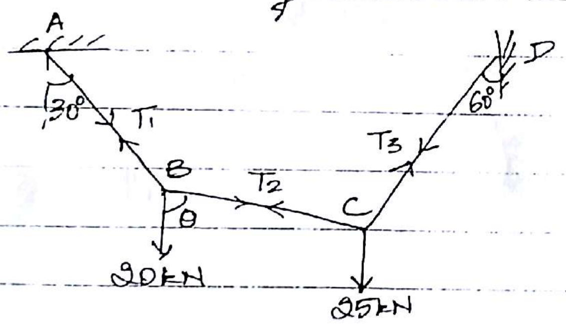
$$\therefore T_2 = \frac{20 \sin 155}{\sin 75} = \underline{8.75N}$$

$$T_1 = \frac{20 \sin 130}{\sin 75} = \underline{15.86N}$$

---

10) Determine tension in AB, BC & CD of rope & inclination of BC to vertical.

HW



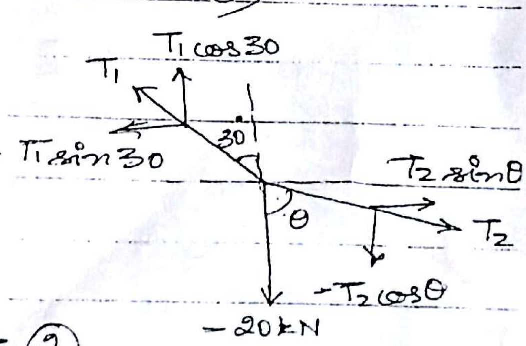
→ Let T_1, T_2 & T_3 be tension in strings AB, BC & CD

→ The system is in equ^n (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^n @ jt B.

– Resolving horizontal forces

$$T_2 \sin\theta = T_1 \sin 30 \rightarrow (1)$$



– Resolving vertical forces

$$T_1 \cos 30 - T_2 \cos\theta - 20 = 0$$

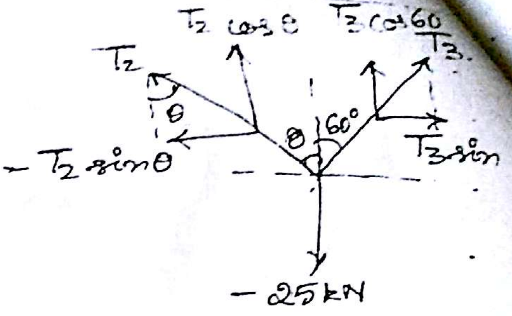
$$T_2 \cos\theta = T_1 \cos 30 - 20 \rightarrow (2)$$

→ Considering equ^n @ jt C

- Resolving horizontal forces

$T_3 \sin 60 - T_2 \sin\theta = 0$

$T_2 \sin\theta = T_3 \sin 60 \longrightarrow (3)$



- Resolving vertical forces

$T_2 \cos\theta + T_3 \cos 60 - 25 = 0$

$T_2 \cos\theta = 25 - T_3 \cos 60 \longrightarrow (4)$

→ From equ^n (1) & (3)

$T_1 \sin 30 = T_3 \sin 60$

$$T_1 = \frac{T_3 \sin 60}{\sin 30}$$

$T_1 = \underline{1.73\, T_3}$

→ From equ^n (2) & (4)

$T_1 \cos 30 - 20 = 25 - T_3 \cos 60$

$1.73 T_3 \cos 30 - 20 - 25 + T_3 \cos 60 = 0$

$1.49 T_3 - 20 - 25 + 0.5 T_3 = 0$

$1.99 T_3 - 45 = 0$

$T_3 = \underline{22.61 N}$

$\therefore T_1 = 1.73 \times 22.6 = \underline{39.12 N}$

→ From equ^n (3) & (4)

$$\frac{\cancel{T_2} \sin\theta}{\cancel{T_2} \cos\theta} = \frac{T_3 \sin 60}{25 - T_3 \cos 60}$$

$\tan\theta = 1.42$

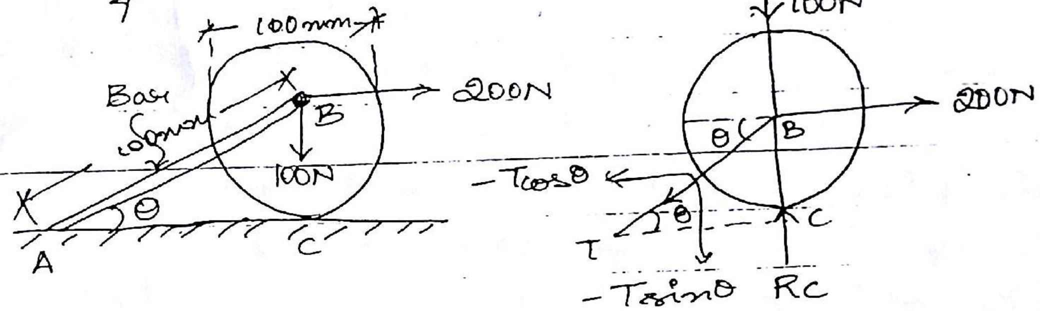
$\theta = \underline{55.03°}$

$\therefore T_2 \sin 55.03 = T_3 \sin 60$

$$T_2 = \frac{T_3 \sin 60}{\sin 55.03} = \underline{23.89 N}$$

1 (*)

11) A Oler roller of radius 50mm & wt 100N rests on horizontal surface & is held in position by an inclined bar AB of length 100mm as shown. Find the reaction @ C & tension in bar if a horizontal force of 200N acts.



→ Let T & R_C be tension in strings AB & contact pt @ C as shown.

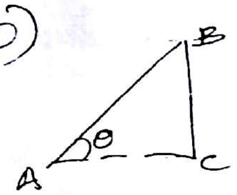
→ The system is in equ^n (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^n @ jt B

- Resolving horizontal forces

$200 - T\cos 30 = 0$

$T = \dfrac{200}{\cos 30} = \underline{\underline{230.94N}}$ ✓



$\sin\theta = \dfrac{BC}{AB}$

$\underline{\underline{\theta = 30^\circ}}$

- Resolving vertical forces

$R_C - 100 - T\sin\theta = 0$

$R_C = 100 + 230.94 \sin 30$

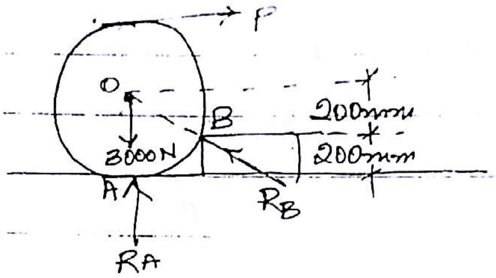
$R_C = \underline{\underline{215.47N}}$ ✓

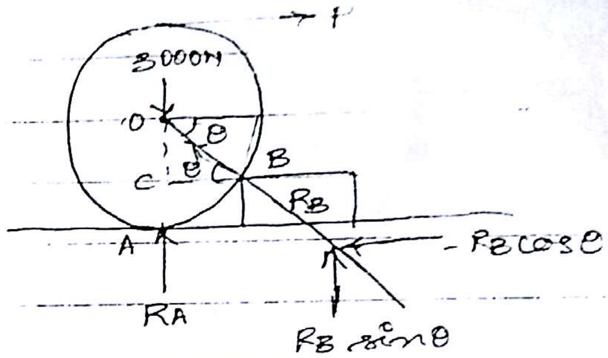
---

12) A roller of radius 400mm, weighing 3000N is to be pulled over a block of ht 200mm as shown by a horizontal force P. Determine the magnitude of P, ~~Ra~~ & R_B.

(*)

H.W.





→ The system is in equ^{m}
(i.e, $\Sigma V = \Sigma H = 0$)

→ Considering equ^{m} @ jt O

– Resolving horizontal forces

$P - R_B\cos 30 = 0$

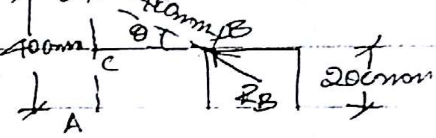
$P = \underline{0.866\,R_B}$

– Resolving vertical forces

$R_B\sin 30^\circ - 300 = 0$

$R_B = \frac{300}{\sin 30} = \underline{\underline{600\,N}}$

$\therefore\ P = 0.866 \times 600 = \underline{519.6\,N}$



$\sin\theta = \frac{OC}{OB}$

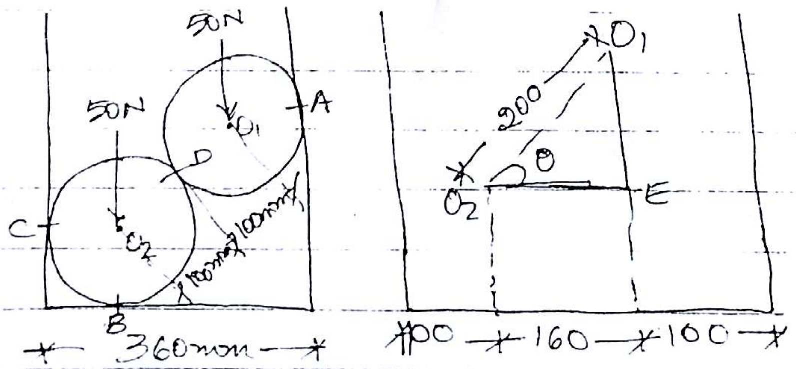
$\theta = \sin^{-1}\left(\frac{200}{400}\right)$

$\underline{\underline{\theta = 30^\circ}}$

---

13) Find the reactions @ A, B & C for the fig shown.

H.W.



$\cos\theta = \frac{O_2E}{O_1O_2}$

$\theta = \cos^{-1}\left(\frac{160}{200}\right)$

$\theta = \underline{\underline{36.86^\circ}}$

→ Let R_A, R_B, R_C be tension @ contact pts A, B, C

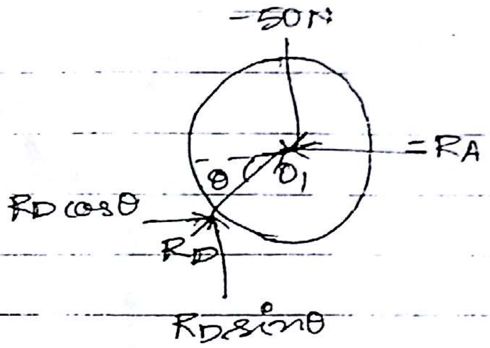
→ The system is in equ^{m} (i.e. $\Sigma V = \Sigma H = 0$)

→ Considering equ^n @ pt O_1

- Resolving horizontal forces

$R_D \cos 36.86 - R_A = 0$

$R_A = 0.8 R_D \rightarrow (1)$



- Resolving vertical forces

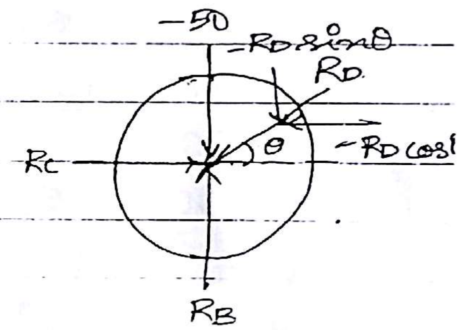
$R_D \sin 36.86 - 50 = 0$

$R_D \sin 36.86 - 50 = 0$

$R_D = \dfrac{50}{0.599} = \cancel{66.68}\ 83.35\,N$

$\therefore R_A = 0.8 \times \cancel{66.68}\ 83.35 = \cancel{53.34}\ 66.68\,N$

→ Considering equ^n @ pt O_2



- Resolving horizontal forces

$R_C - R_D \cos\theta = 0$

$R_C = 83.35 \cos 36.86$

$R_C = 66.68\,N$

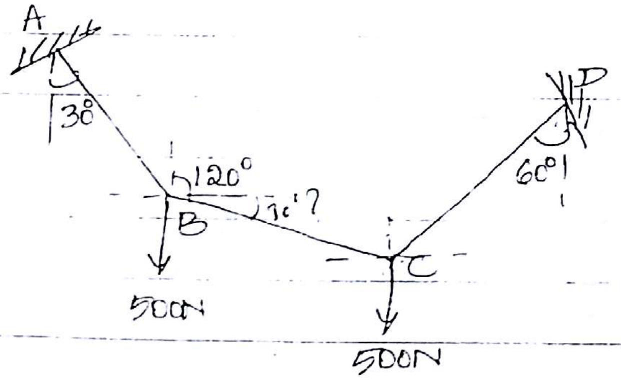
- Resolving vertical forces

$R_B - 50 - R_D \sin\theta = 0$

$R_B - 50 - 83.35\ \cancel{\cos}\ \sin 36.86 = 0$

$R_B = 100\,N$

Q14) Find the tensions in the string shown in fig.

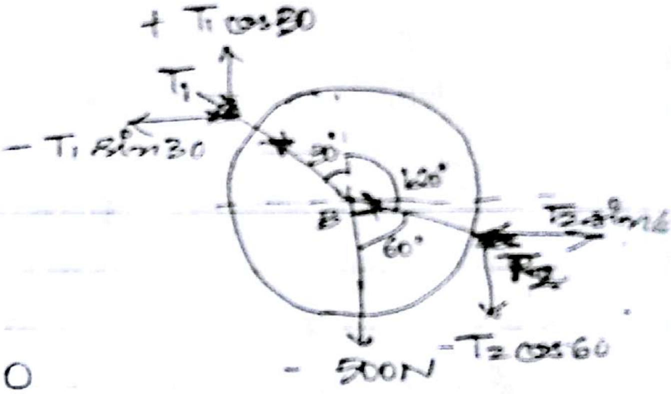


→ Let T_1, T_2, T_3 be tension in string AB, BC & CD respectively

→ The system is in equ^m (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^m @ jt B

– Resolving horizontal forces



$T_2 \sin 60 - T_1 \sin 30 = 0$

$$T_2 = \frac{T_1 \sin 30}{\sin 60}$$

$T_2 = 0.577\,T_1 \longrightarrow (1)$

– Resolving vertical forces

$-500 - T_2 \cos 60 + T_1 \cos 30 = 0$

$-500 - 0.577\,T_1 \cos 60 + T_1 \cos 30 = 0$

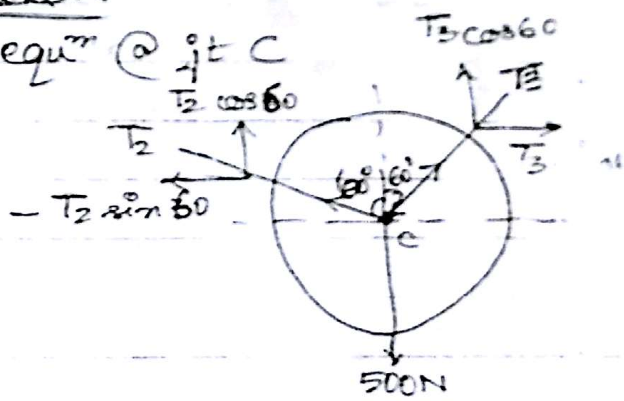
$-500 - 0.28\,T_1 + 0.86\,T_1 = 0$

$$T_1 = \frac{500}{0.58} = \underline{862.06\,N}$$

$\therefore T_2 = 0.577 \times 862.06 = \underline{500\,N}$

→ Considering ~~vertical forces~~ equ^m @ jt C

– Resolving horizontal forces



$T_3 \sin 60 - T_2 \sin 60 = 0$

$$T_3 = \frac{500 \sin 60}{\sin 60}$$

$T_3 = \underline{500\,N}$

~~Resolving vertical forces~~

→ Check (ΣV = 0)

$T_2 \cos 60 + T_3 \cos 60 - 500 = 0$

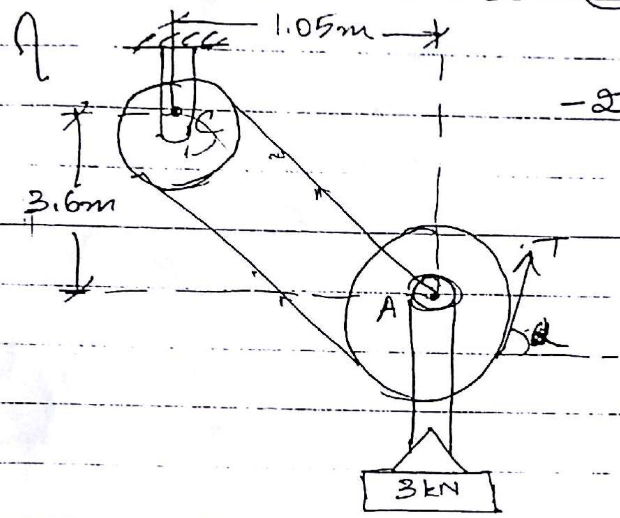
$500 \cos 60 + 500 \cos 60 - 500 = 0$

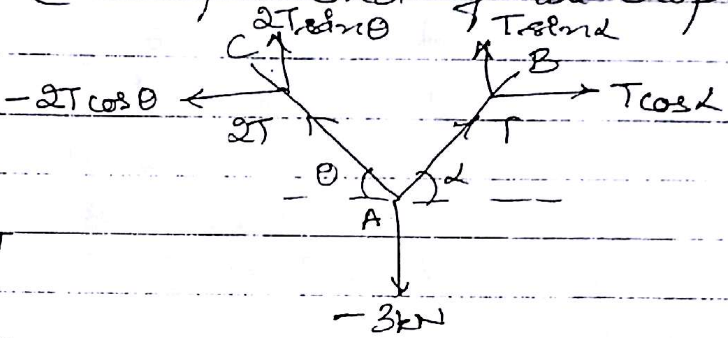
$250 + 250 - 500 = 0$

$\underline{0 = 0}$

Scanned by CamScanner

15) A 3kN crate is to be supported by the rope & pulley arrangement shown in fig. Determine the magnitude & direction of the force T, which should be exerted @ the free end of the rope.





$$\tan\theta = \frac{3.6}{1.05}$$

$$\theta = 73.73^\circ$$

→ Let T be the tension in rope AB. From the pulley system tension in rope AC = 2T

→ The system is in equ^n (i.e ΣH = ΣV = 0)

→ Considering equ^n @ jt A

– Resolving horizontal forces

$$T\cos\alpha - 2T\cos 73.73 = 0$$

$$\cancel{T}\cos\alpha = 2\cancel{T}\cos 73.73$$

$$\cos\alpha = 0.56$$

$$\alpha = 55.95^\circ$$

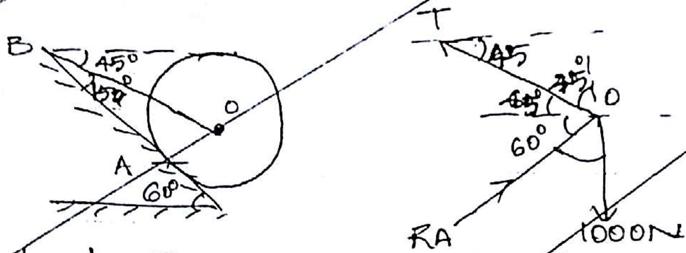
– Resolving vertical forces

$$T\sin 55.95 + 2T\sin 73.73 - 3 = 0$$

$$2.74T = 3$$

$$T = 1.09\,kN$$

16) Determine the tension in string & reaction @ contact surface for cylinder of wt 1000N placed as shown.



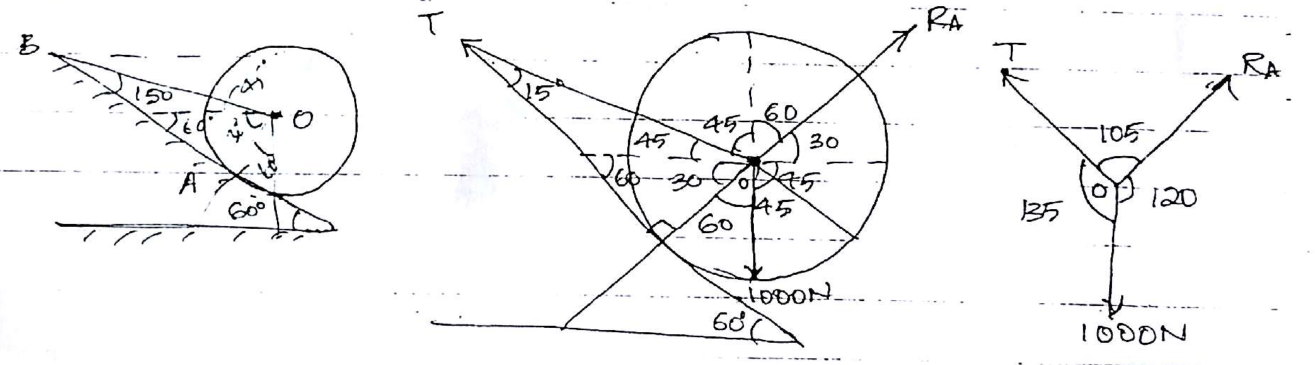
→ Let RA & T be the tension @ contact pt & in OB wire.

→ The system is in equ^n (i.e, $\Sigma V = \Sigma H = 0$)

→ Considering equ^n @ jt O & by applying Lamis theo

$$\frac{R_A}{\sin(180+45)} = \frac{1000}{\sin(60+45)} = \frac{T}{\sin}$$

16) Determine the tension in string & reaction @ contact surface for cylinder of wt 1000N placed as shown.



→ Let RA & T be the tension @ contact pt & in OB wire respectively

→ The system is in equ^n (i.e., $\Sigma V = \Sigma H = 0$)

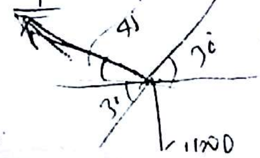
→ Considering equ^n @ jt O & by applying Lamis theo

$$\frac{R_A}{\sin 135} = \frac{T}{\sin 120} = \frac{1000}{\sin 105}$$

$T\cos 45 = R_A \cos 30$ →(i) ⇒ $R_A = T\cos 45/\cos 30$

$T\sin 45 + R_A \sin 30 = 1000$ 0.81T.

$0.707T + (0.81)T(0.5) = 1000$ $T = 899.70$ N

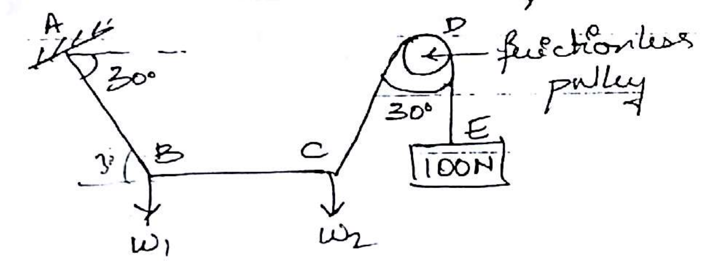


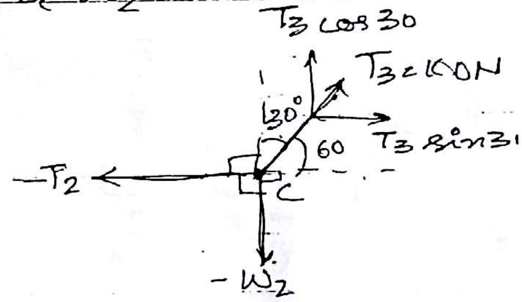
$R_A = 728.4$

$$\therefore R_A = \frac{1000 \sin 135}{\sin 105} = \underline{732.1N}$$

$$T = \frac{1000 \sin 120}{\sin 105} = \underline{896.57N}$$

HW (17) A light string ABCDE is fixed to wts W_1 & W_2 @ B & C & a wt 100N @ free end E. Find the tensions in the string & wt W_1 & W_2





→ Let T_1, T_2 & T_3 be tensions in AB, BC & CD

→ Here $T_3 = 100N$ ∵ the pulley is frictionless

→ The system is in equ^m (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^m @ jt C

- Resolving horizontal forces

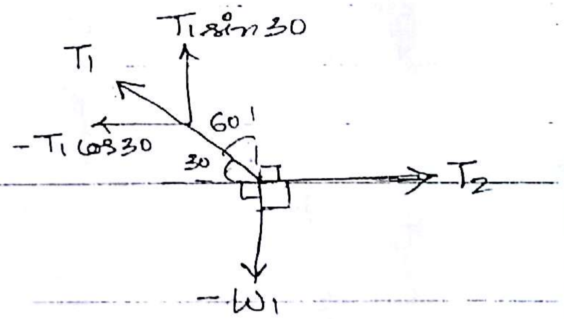
$$T_3 \sin 30 - T_2 = 0$$

$$T_2 = 100 \sin 30 = \underline{50N}$$

- Resolving vertical forces

$$T_3 \cos 30 - W_2 = 0$$

$$W_2 = 100 \cos 30 = \underline{86.6N}$$



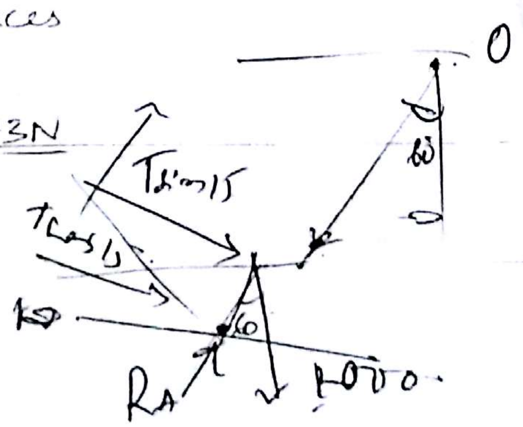
→ Considering equ^m @ jt B

- Resolving horizontal forces

$$T_2 - T_1 \cos 30 = 0$$

$$T_1 = \frac{50}{\cos 30} = \underline{57.73N}$$

R_A

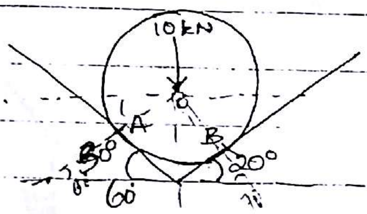


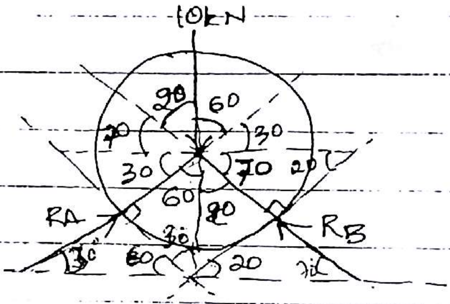
- Resolving vertical forces

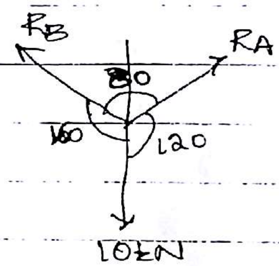
$T_1 \sin 30 - W_1 = 0$

$W_1 = 57.73 \sin 30 = \underline{\underline{28.86N}}$

HW 18) A smooth O^{lar} cylinder of radius 15m is placed in a notch as shown. Find the reactions @ contact faces.







→ Let RA & RB be tension @ contact pts A & B

→ The system is in equ^m (i.e, ΣH = ΣV = 0)

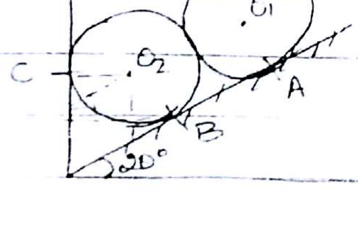
→ Considering equ^m @ jt O & applying Lami's theorem

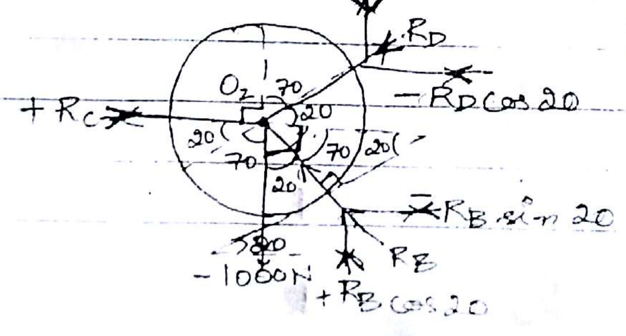
$$\frac{10}{\sin 80} = \frac{R_B}{\sin 120} = \frac{R_A}{\sin 160}$$

$$\therefore R_B = \frac{10 \sin 120}{\sin 80} = \underline{\underline{8.79N}}$$

$$R_A = \frac{10 \sin 160}{\sin 80} = \underline{\underline{3.47N}}$$

18) Two identical rollers each of wt 1000N are supported by an inclined plane & vertical wall. Find the actions @ A, B & C





Solve this by resolving & 2 forces

→ Let R_A, R_B, R_C, R_D be tension @ contact pts A, B, C, D

→ ~~...~~ The system is in equ^m (i.e, $\Sigma H = \Sigma V = 0$)

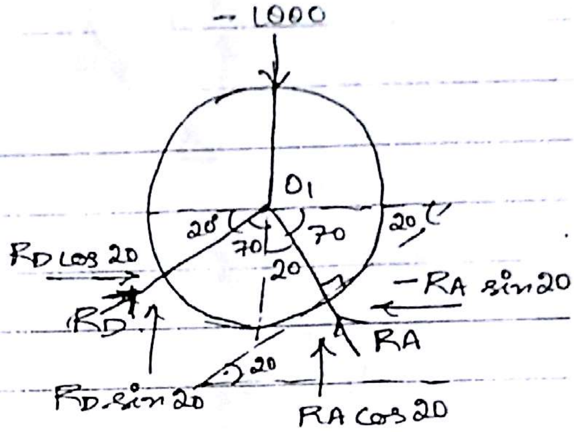
→ Considering equ^m @ jt O_1

- Resolving horizontal forces

$R_D \cos 20 - R_A \sin 20 = 0$

$$R_D = \frac{R_A \sin 20}{\cos 20}$$

$R_D = \underline{0.36 R_A}$



- Resolving vertical forces

$R_D \sin 20 + R_A \cos 20 - 1000 = 0$

$0.36 R_A \sin 20 + R_A \cos 20 - 1000 = 0$

$0.123 R_A + 0.93 R_A - 1000 = 0$

$$R_A = \frac{1000}{1.053} = \underline{941 N}$$

$\therefore R_D = 0.36 \times 941 = \underline{338.76 N}$

→ Considering equ^m @ jt O_2

- Resolving horizontal forces

$-R_D \cos 20 - R_B \sin 20 + R_C = 0$

$-338.76 \cos 20 - R_B \sin 20 + R_C = 0$

$R_C = 318.33 + R_B \sin 20$

- Resolving vertical forces

$-R_D \sin 20 + R_B \cos 20 - 1000 = 0$

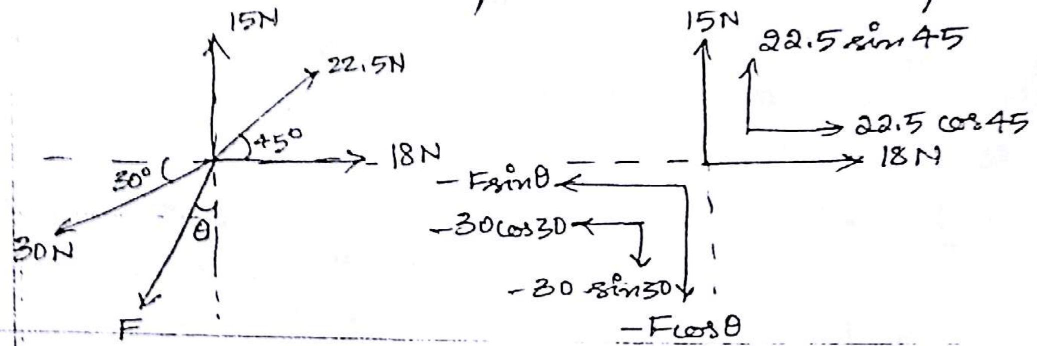
$-338.76 \sin 20 + R_B \cos 20 - 1000 = 0$

$$R_B = \frac{1115.86}{\cos 20} = \underline{1187.47 N}$$

$\therefore R_C = 318.33 + 1187.4 \sin 20 = \underline{724.47 N}$

Scanned by CamScanner

19). A system of forces acting (o a pt are in equ^n. Determine the magnitude & direc^n of force F & θ



→ The system is in equ^m (i.e, $\Sigma V = \Sigma H = 0$)

→ Resolving horizontal forces

$$22.5 \cos 45 + 18 - F \sin\theta - 30 \cos 30 = 0$$

$$F \sin\theta = 7.98 \longrightarrow (1)$$

→ Resolving vertical forces

$$15 + 22.5 \sin 45 - 30 \sin 30 - F \cos\theta = 0$$

$$F \cos\theta = 15.91 \longrightarrow (2)$$

→ By dividing equ^n (1) & (2)

$$\frac{\cancel{F} \sin\theta}{\cancel{F} \cos\theta} = \frac{7.98}{15.91}$$

$$\tan\theta = 0.5$$

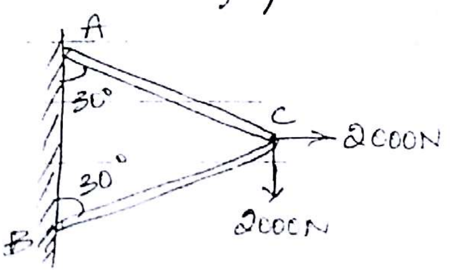
$$\theta = \underline{26.63^\circ}$$

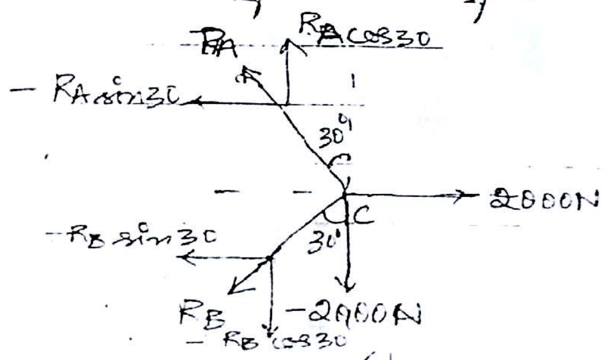
$$\therefore F \sin 26.63 = 7.98$$

$$F = \frac{7.98}{\sin 26.63} = \underline{\underline{17.8N}}$$

---

20) Determine the forces in the weightless rigid bar shown in figure.





→ Let R_A & R_B be tension in rods AC & BC resp^{ly}

→ The system is in equ^m (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^m @ jt C

- Resolving horizontal forces

$$2000 - R_B \sin 30 - R_A \sin 30 = 0$$

$$(-R_A - R_B) \sin 30 = -2000$$

$$R_A + R_B = \frac{2000}{\sin 30} = \underline{4000} \rightarrow (1)$$

- Resolving vertical forces

$$R_A \cos 30 - R_B \cos 30 - 2000 = 0$$

$$(R_A - R_B) \cos 30 - 2000 = 0$$

$$R_A - R_B = \frac{2000}{\cos 30} = \underline{2309.4} \rightarrow (2)$$

- Solving equ^{ns} (1) & (2)

$$R_A + R_B = 4000$$

$$R_A - R_B = 2309.4$$

$$2R_A = 6309.4$$

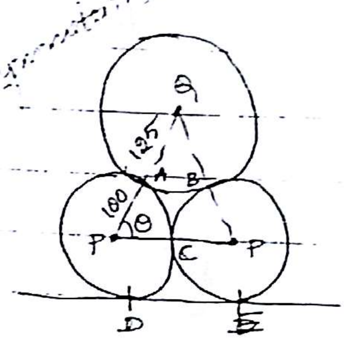
$$R_A = \underline{3154.7 N}$$

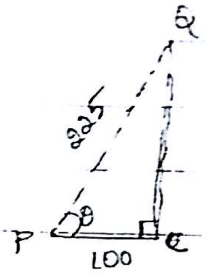
- ∴ $3154.7 + R_B = 4000$

$$R_B = \underline{845.3 N}$$

---

21) Two solid round identical cylinders each of radius 100mm & wt 50N, tied together by a thread of length 200mm, rest on a horizontal surface. They hold a third homogeneous cylinder of radius 125mm & wt 75N, placed as shown in fig. Calculate the tension in the thread & the reaction @ all contact surfaces.





$$\cos\theta = \frac{PC}{PQ}$$

$$\theta = \cos^{-1}\left(\frac{100}{225}\right)$$

$$\theta = \underline{\underline{63.61^\circ}}$$

→ Let R_A, R_B, R_C, R_D & R_E be tension contact pts A, B, C, D, E.

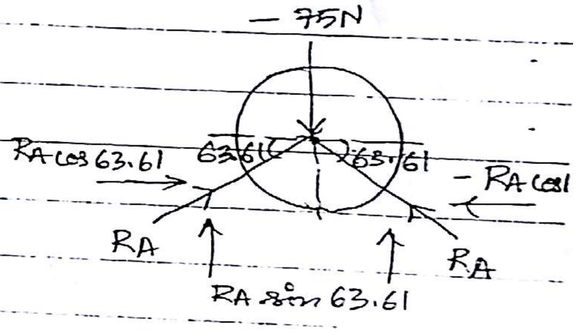
→ The system is in equ^n (i.e, $\Sigma V = \Sigma H = 0$)

→ Considering equ^n @ jt Q

- Resolving horizontal forces

$R_A \cos 63.61 - R_A \cos 63.61 = 0$

$0 = 0$



- Resolving vertical forces

$2 R_A \sin 63.61 - 75 = 0$

$R_A = \dfrac{75}{2 \sin 63.61} = \underline{\underline{41.86N}}$

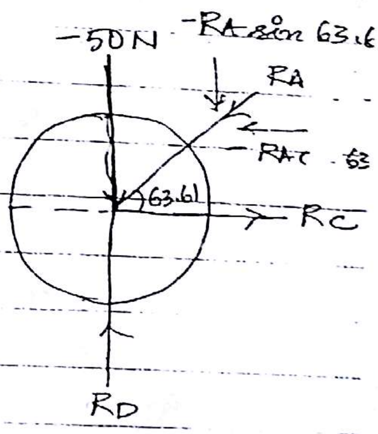
→ Considering equ^n @ jt P

- Resolving horizontal forces

$R_C - R_A \cos 63.61 = 0$

$R_C = 41.86 \cos 63.61$

$R_C = \underline{\underline{18.61N}}$



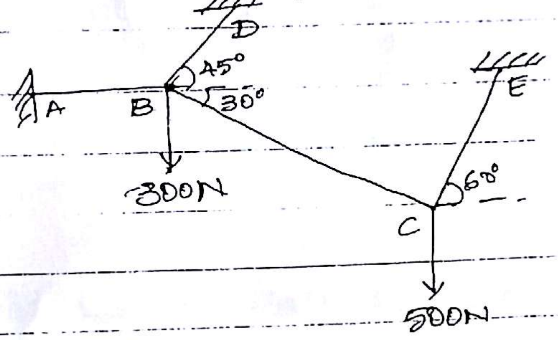
- Resolving vertical forces

$R_D - 50 - R_A \sin 63.61 = 0$

$R_D = 50 + 41.86 \sin 63.61$

$R_D = \underline{\underline{87.5N}}$

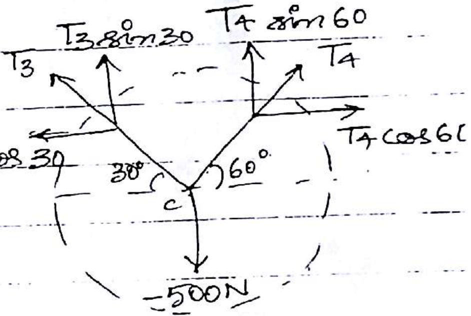
22) Figure shows a system of cables in equⁿ under 2 vertical loads of 300N & 500N. Determine the forces developed in different segments.



→ Let T_1, T_2, T_3 & T_4 be tension in AB, BD, BC & C[illegible]

→ The system is in equⁿ (i.e. $\Sigma H = \Sigma V = 0$)

→ Considering equⁿ @ jt. C



- Resolving horizontal forces

$T_4 \cos 60 - T_3 \cos 30 = 0$

$T_4 = \dfrac{T_3 \cos 30}{\cos 60}$

$T_4 = \underline{1.73 T_3}$

- Resolving vertical forces

$T_3 \sin 30 + T_4 \sin 60 - 500 = 0$

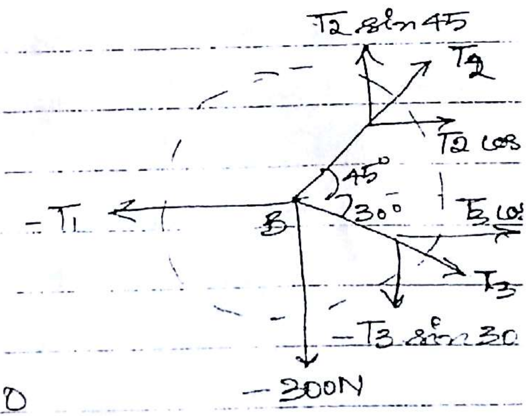
$T_3 \sin 30 + 1.73 T_3 \sin 60 - 500 = 0$

$1.99 T_3 = 500$

$T_3 = \underline{250N}$

$\therefore T_4 = 1.73 \times 250 = \underline{433N}$

→ Considering equⁿ @ jt B



- Resolving horizontal forces

$T_3 \cos 30 + T_2 \cos 45 - T_1 = 0$

$250 \cos 30 + T_2 \cos 45 - T_1 = 0$

$T_2 \cos 45 = T_1 - 216.5$

Resolving vertical forces

$T_2 \sin 45 - T_3 \sin 30 - 300 = 0$

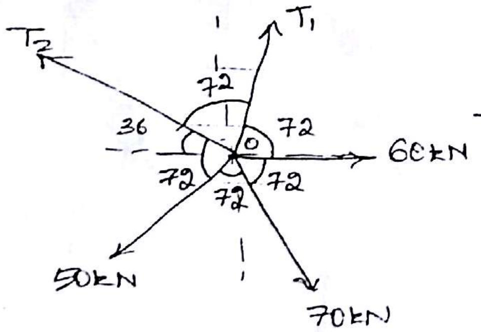
$T_2 \sin 45 - 250 \sin 30 - 300 = 0$

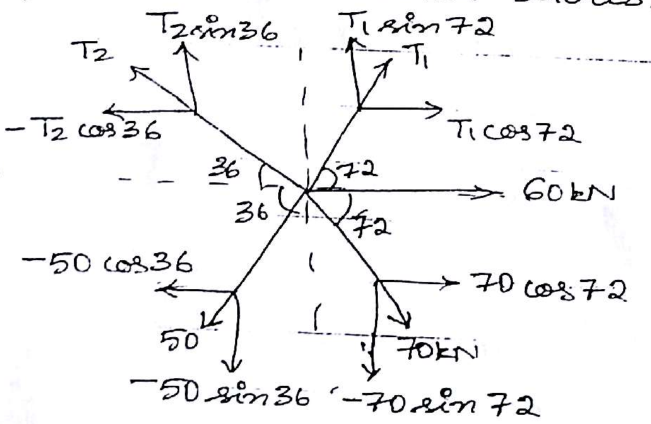
$T_2 = \underline{601.04\,N}$

$\therefore T_1 = 601.04 \cos 45 + 216.5$

$T_1 = \underline{641.5\,N}$

HW 23) Four guy wires tied @ a pt are pulled in radial directions, equally spaced from one another. If the magnitude of pulls in 3 consequtive wires is 50kN, 70kN & 60kN. Determi the magnitude of pulls on two other wires.





→ The system is in equ^n (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^n @ O

Resolving horizontal forces

$60 + T_1 \cos 72 + 70 \cos 72 - 50 \cos 36 - T_2 \cos 36 = 0$

$60 + 0.3 T_1 + 21.63 - 40.45 - 0.8 T_2 = 0$

$0.3 T_1 = 0.8 T_2 - 41.18$

$T_1 = 2.67 T_2 - 137.26 \longrightarrow (1)$

Resolving vertical forces

$T_2 \sin 36 + T_1 \sin 72 - 50 \sin 36 - 70 \sin 72 = 0$

$0.58\,T_2 + 0.95\,T_1 - 29.38 - 66.57 = 0$

$0.95\,T_1 = 95.95 - 0.58\,T_2$

$T_1 = 101 - 0.61\,T_2 \longrightarrow (2)$

→ By solving equ^n (1) & (2)

$$T_1 = -0.61\,T_2 + 101$$
$$T_1 = 2.67\,T_2 - 137.26$$
$$3.28\,T_2 = 238.26$$
$$T_2 = \underline{72.64\,N}$$

$\therefore T_1 = 101 - 0.61 \times 72.64 = \underline{56.68N}$

→ **Note** : Forces on rods

If external forces on the rod are acting only @ end pts & the rod is in equ^m, the forces inside the rod is co-axial. They are called two force members & the equ^m cond^ns are

$\Sigma H = 0$ & $\Sigma V = 0$

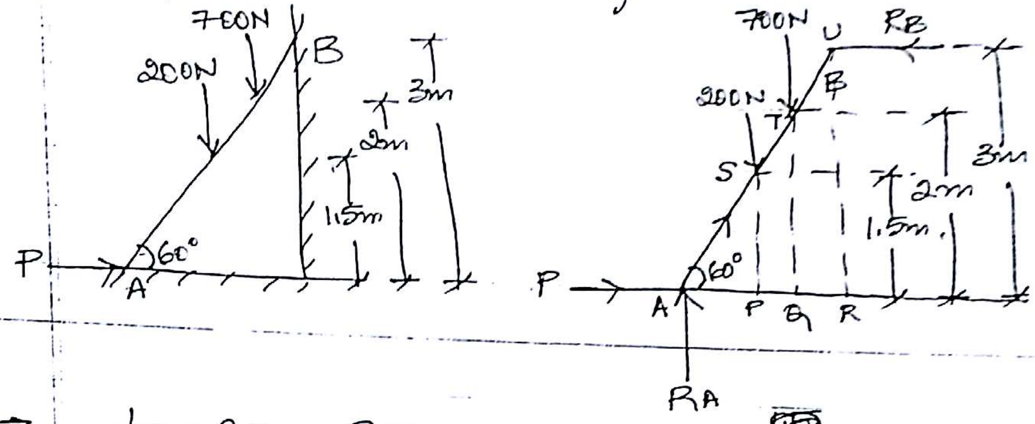
If external forces are not acting @ the end of the rod & atleast one of them is not along the axis, the force in the rod is not co-axial. Minimum no. of forces reqd for equ^m of rod is 3. Such rods are multi-force members & the equ^m cond^ns are

$\Sigma H = 0$, $\Sigma V = 0$ & $\Sigma M = 0$

---

24) A ladder weighing 200N is to be kept in position as shown resting on a smooth floor & leaning against a smooth wall. Determine the horizontal force reqd to prevent it from

slipping when a man weighing 700N is @ a ht of 2m above the floor level.



→ $\tan 60 = \frac{SP}{AP} \Rightarrow AP = \frac{1.5}{\tan 60} = \underline{0.866}$

$\tan 60 = \frac{TQ}{AQ} \Rightarrow AQ = \frac{2}{\tan 60} = \underline{1.15m}$

$\tan 60 = \frac{UR}{AR} \Rightarrow AR = \frac{3}{\tan 60} = \underline{1.73}$

→ The wall & floor has a smooth surface. Hence there will be only surface reaction acting $\perp^{r}$ to surface.

→ The system is in equ^m (i.e., $\Sigma H = \Sigma V = \Sigma M = 0$)

→ Resolving horizontal forces

$P - R_B = 0$

$\underline{P = R_B}$

→ Resolving vertical forces

$R_A - 200 - 700 = 0$

$R_A = \underline{900N}$

→ Taking moment abt pt A

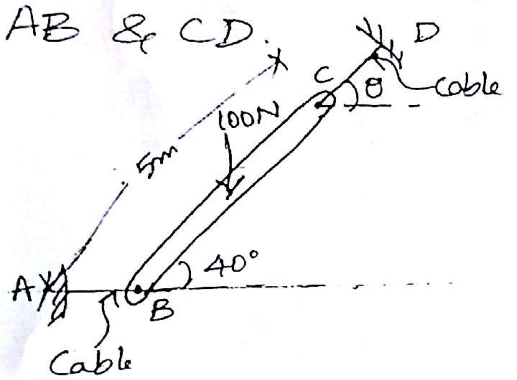
$-R_B \times 3 + 700 \times \overset{1.15}{\cancel{0.866}} + 200 \times \overset{0.866}{\cancel{\ldots}} = 0$

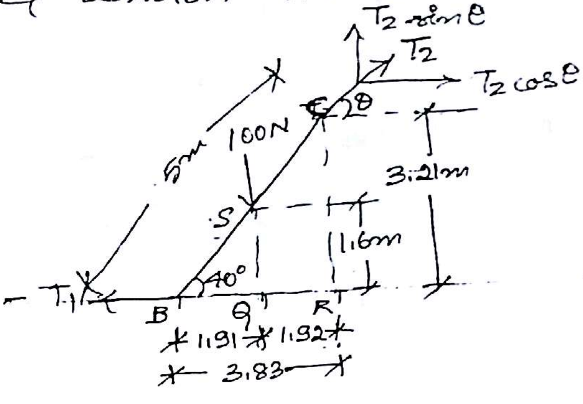
$3R_B = 978.2$

$R_B = \underline{326.1N} = P$

Scanned by CamScanner

25) A slender rod BC of length 5m & wt 100N is held in equ^m position as shown in fig. Determine the $\angle^{le}$ θ & tension in each cable AB & CD.





→ $\sin 40 = \frac{SQ}{BS} \Longrightarrow SQ = 2.5 \sin 40 = \underline{1.6m}$

$\sin 40 = \frac{CR}{BC} \longrightarrow CR = 5 \sin 40 = \underline{3.21m}$

→ $\cos 40 = \frac{PQ}{BS} \Longrightarrow BQ = 2.5 \cos 40 = \underline{1.91m}$

$\cos 40 = \frac{PR}{BC} \longrightarrow BR = 5 \cos 40 = \underline{3.83m}$

→ The system is in equ^m (i.e, $\Sigma H = \Sigma V = \Sigma M$

→ Resolving horizontal forces

$T_2 \cos\theta - T_1 = 0$

$T_1 = T_2 \cos\theta \longrightarrow (1)$

→ Resolving vertical forces

$T_2 \sin\theta - 100 = 0$

$T_2 = \frac{100}{\sin\theta} \longrightarrow (2)$

→ Taking moment abt pt C

~~100 × 1.92 + $T_2\cos\theta$ × 3.21 − $T_2\sin\theta$ × 3.83 = 0~~

$T_1 \times 3.21 - 100 \times 1.92 = 0$

$T_1 = \underline{59.81N} \longrightarrow (3)$

→ By substituting ③ in ①

$$59.81 = T_2 \cos\theta$$

$$T_2 = \frac{59.81}{\cos\theta} \longrightarrow ④$$

→ By substituting equ^ns ④ & ②

$$\frac{100}{\sin\theta} = \frac{59.81}{\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{100}{59.81}$$

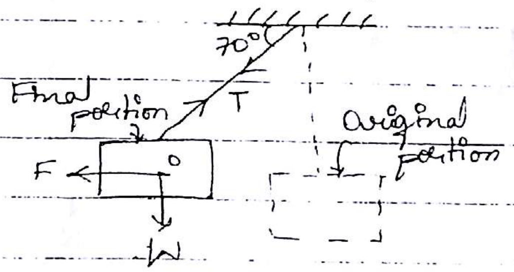
$$\tan\theta = 1.67$$

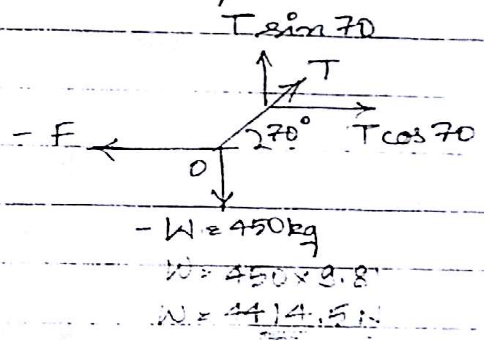
$$\theta = 59.11^\circ \longrightarrow ⑤$$

→ Substituting ⑤ in ④

$$T_2 = \frac{59.81}{\cos 59.11} = 116.49N$$

HW 26) Determine the horizontal force & the tension in the rope when a mass of 450 kg is suspended by a rope from the centering & pulled by a horizontal force untill the rope makes an ∠ of with ceiling





→ Let T be the tension in rope

→ The system is in equ^m (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^m @ jt O

Resolving horizontal forces

$T \cos 70 - F = 0$

$T = \dfrac{F}{\cos 70} \longrightarrow (1)$

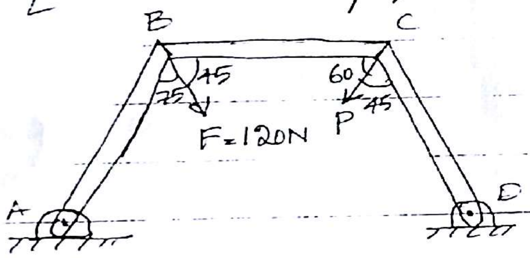
- Resolving vertical forces

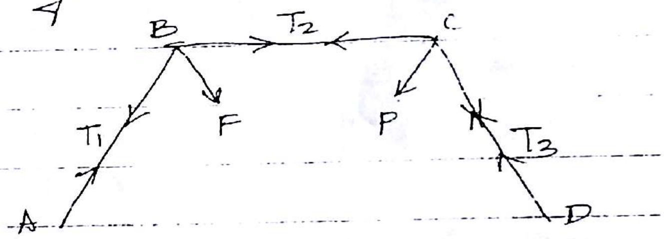
$T \sin 70 - 450 = 0$

$T = \dfrac{450 \times 9.81}{\sin 70} = 4697.81N$

$\therefore F = 4697.81 \cos 70 = 1606.78N$

27) A 4 bar mechanism consits of light pin connected members. A force F = 120N is applied @ B. Determine the magnitude of force P so as the equ^n exists by preventing movement.





→ Let T_1, T_2 & T_3 be tension in rods AB, BC & CD

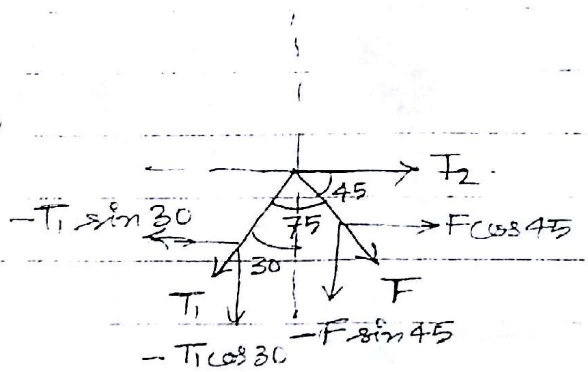
→ The system is in equ^n (i.e, $\Sigma V = \Sigma H = 0$)

→ Considering equ^n @ jt B

- Resolving horizontal forces

$F \cos 45 - T_1 \sin 30 + T_2 = 0$

$120 \cos 45 - T_1 \sin 30 + T_2 = 0$



$\cancel{T_1 = \dfrac{84.85}{\sin 30}}$

$\cancel{T_1 = 169.7N}$

$T_2 = T_1 \sin 30 + 84.85 \longrightarrow (1)$

- Resolving vertical forces

$-T_1 \cos 30 - F \sin 45 = 0$

$T_1 = \frac{120 \sin 45}{\cos 30} = \underline{97.97N}$

$\therefore T_2 = 97.97 \sin 30 + 84.85 = \underline{133.83N}$

→ Considering equ^n @ jt C

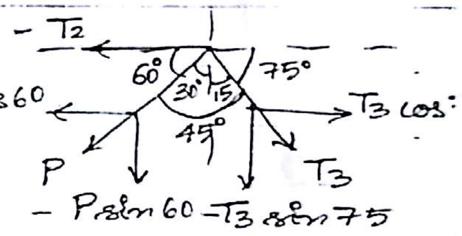
- Resolving horizontal forces

$T_3 \cos 75 - P \cos 60 - T_2 = 0$

$T_3 \cos 75 - P \cos 60 - 133.83 = 0$

$T_3 = \frac{P \cos 60 + 133.83}{\cos 75}$

$T_3 = 1.93P + 517.1 \longrightarrow (1)$



- Resolving vertical forces

$-P \sin 60 - T_3 \sin 75 = 0$

$T_3 = \frac{-P \sin 60}{\sin 75}$

$T_3 = -0.89P \longrightarrow (2)$

→ By solving equ^n (1) & (2)

$T_3 = 1.93P + 517.1$

$T_3 = -0.89P + 0$

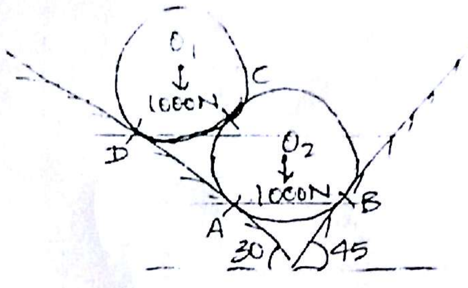
$P =$

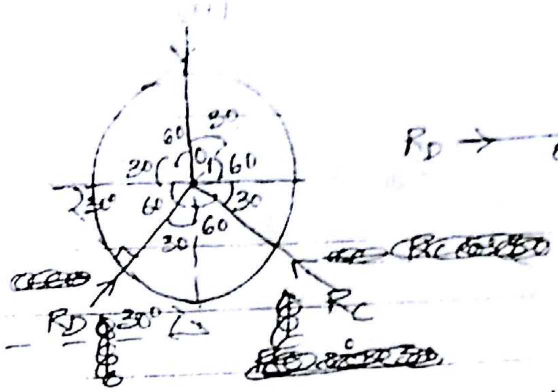
$P =$

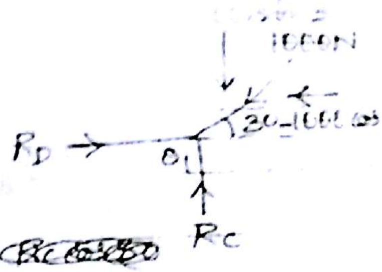
$\therefore T_3 = 0.89 \times$

---

28) A sphere of wt 1000N rests in a V groove whose sides are inclined @ $\angle$s 30° & 45° to the horizontal. Another identical sphere of same wt 1000N rests on the first sphere & in contact with the side inclined @ 30°. Find the reaction RA & RB on the lower sphere @ pts A & B resply.



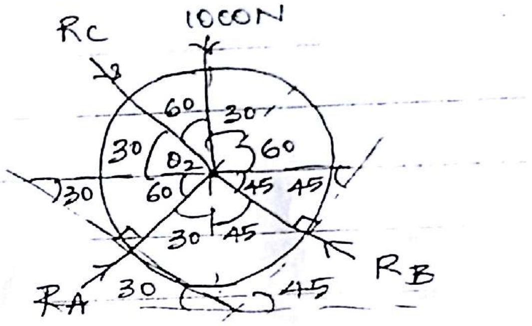




→ Let R_A, R_B, R_C & R_D be tension @ contact pts A, B, C & D

→ The system is in equ^m (i.e, $\Sigma V = \Sigma H = 0$)

→ Considering equ^m @ pt O_1



- Resolving horizontal forces

$$R_D - 1000 \cos 30 = 0$$

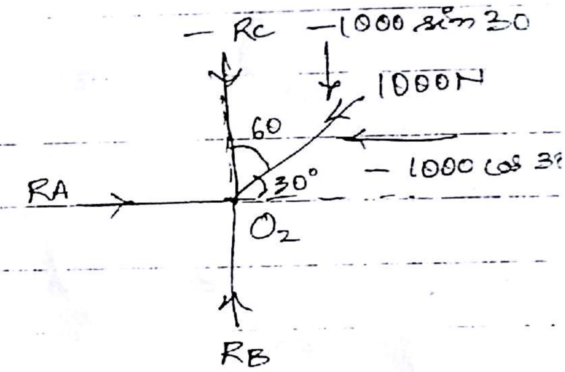
$$R_D = \underline{866.02N}$$

- Resolving vertical forces

$$R_C - 1000 \sin 30 = 0$$

$$R_C = \underline{500N}$$

→ Considering equ^m @ pt O_2



- Resolving horizontal forces

$$R_A - 1000 \cos 30 = 0$$

$$R_A = \underline{500N}$$

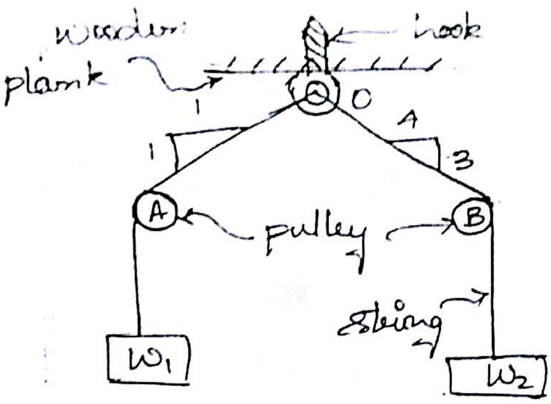
Resolving vertical forces

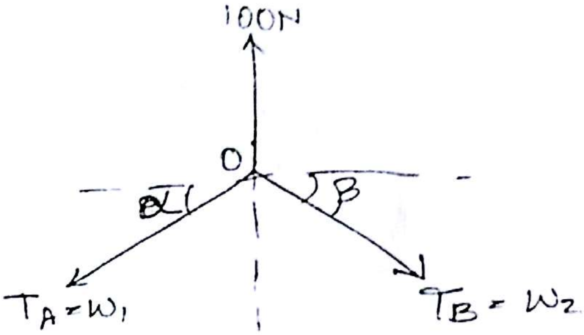
$$R_B - R_C - 1000 \sin 30 = 0$$

$$R_B = 500 + 500 = \underline{1000N}$$

AW 29) A rigid hook in the wooden plank O has to offer a resistance of 100N against pulling. Determine the wts W_1 & W_2 which can be supported from inextensible string as shown in figure. Neglect the pulley friction.

Scanned by CamScanner





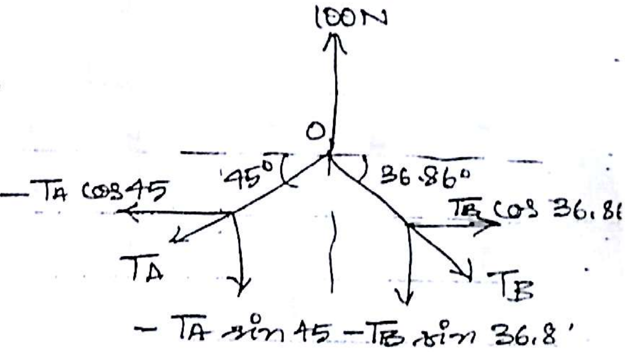
→ $\tan\alpha = \frac{1}{1} \implies \alpha = \tan^{-1} 1 = \underline{45^\circ}$

$\tan\beta = \frac{3}{4} \implies \beta = \tan^{-1} 0.75 = \underline{36.86^\circ}$

→ Let T_A & T_B be tension in strings OA & OB

→ The system is in equ^n (i.e, $\Sigma V = \Sigma H = 0$)

→ Considering equ^m @ O



– Resolving horizontal forces

$T_B \cos 36.86 - T_A \cos 45 = 0$

$T_B = \frac{T_A \cos 45}{\cos 36.86}$

$T_B = 0.88\, T_A$

– Resolving vertical forces

$100 - T_A \sin 45 - T_B \sin 36.86 = 0$

$100 - T_A \sin 45 - 0.88\, T_A \sin 36.86 = 0$

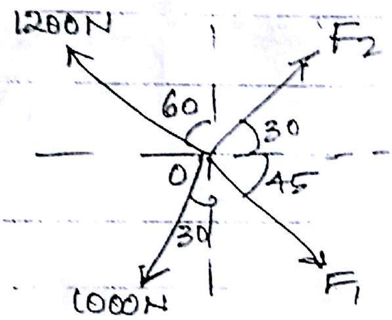
$100 - 0.7\, T_A - 0.52\, T_A = 0$

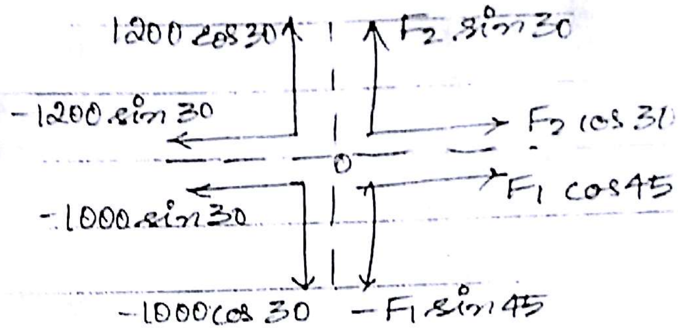
$1.22\, T_A = 100$

$T_A = \underline{81.96N}$

$\therefore T_B = 0.88 \times 81.96 = \underline{72.13N}$

30. Determine the magnitude of forces F_1 & F_2 if the system of forces shown in fig is in equ^m





→ The system is in equ^m (i.e, $\Sigma H = \Sigma V = 0$)

→ Considering equ^m @ pt O

– Resolving horizontal forces

$$F_2 \cos 30 + F_1 \cos 45 - 1200 \sin 30 - 1000 \sin 30 = 0$$

$$0.866 F_2 + 0.707 F_1 = 1039.23 \longrightarrow (1)$$

– Resolving vertical forces

$$F_2 \sin 30 + 1200 \cos 30 - F_1 \sin 45 - 1000 \cos 30 = 0$$

$$0.5 F_2 - 0.707 F_1 = 266.03 \longrightarrow (2)$$

→ By solving equ^{ns} (1) & (2)

$$0.866 F_2 + \cancel{0.707 F_1} = 1039.23$$

$$0.5 F_2 - \cancel{0.707 F_1} = 266.03$$

$$1.366 F_2 = 1305.26$$

$$F_2 = \underline{941.75 N} \longrightarrow (3)$$

→ By substituting (3) in (1)

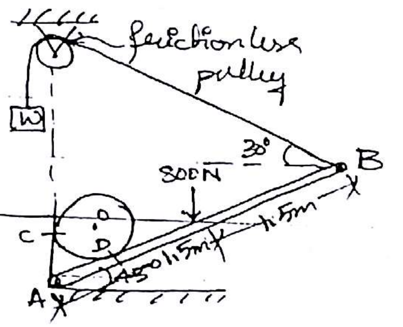
$$0.866 \times 941.75 + 0.707 F_1 = 1039.23$$

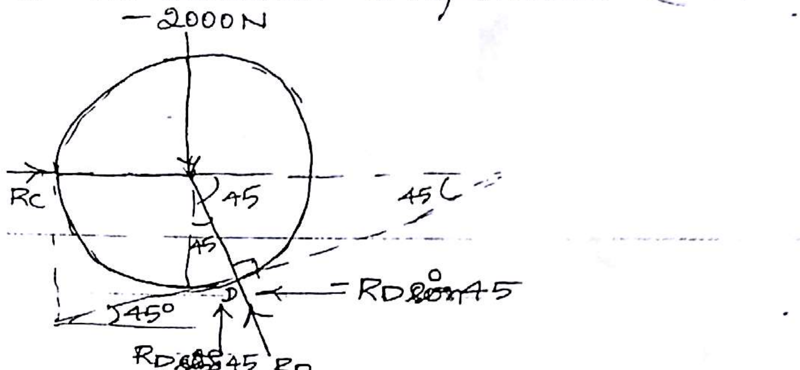
$$0.707 F_1 = 1039.23 - 815.55$$

$$F_1 = \underline{316.37 N}$$

Scanned by CamScanner

31). A cylinder 2000N is supported on a 800N member AB. Find the weight W for equ^m to exist. The Ø of cylinder is 750mm & length of member AB is 3m. Determine the reaction components @ A.





→ Let C & D be the contact pts that create R_C & R_D tension

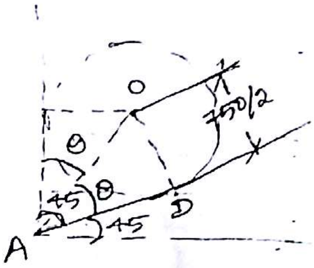
→ The system is in equ^m (i.e, $\Sigma M = \Sigma H = \Sigma V = 0$)

→ Considering equ^m @ pt O

- Resolving horizontal forces

$$R_C - R_D \sin 45 = 0$$

$$R_C = 0.707 R_D$$



- Resolving vertical forces

$$R_D \sin 45 - 2000 = 0$$

$$R_D = \frac{2000}{\sin 45} = \underline{2828.42N}$$

∴ $R_C = 0.707 \times 2828.42 = \underline{2000N}$

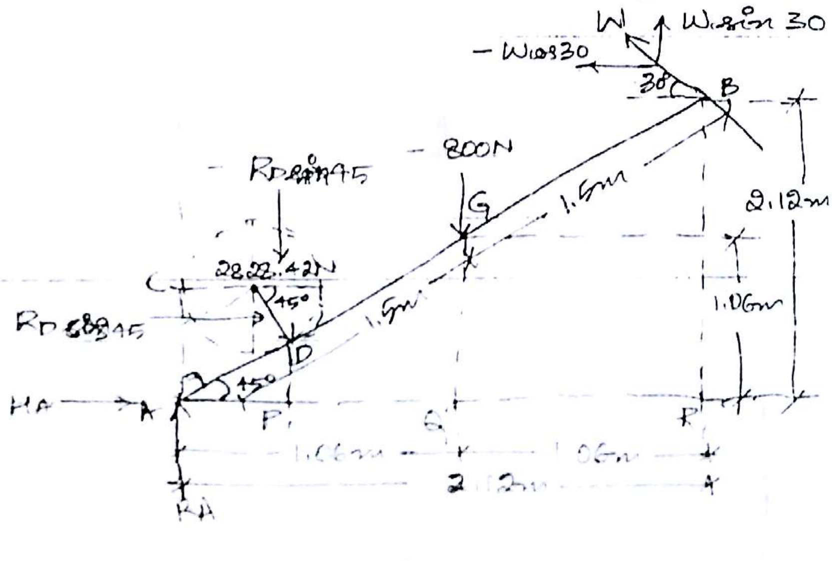
→ $\theta = 45/2$

$\theta = \underline{22.5°}$

→ $\tan\theta = \frac{OD}{AD}$

$AD = \frac{750/2}{\tan 22.5}$

$AD = \underline{0.905m}$



$\rightarrow \sin 45 = \frac{GQ}{AG} \Rightarrow GQ = 1.5 \sin 45 = \underline{1.06m}$

$\sin 45 = \frac{BR}{AB} \Rightarrow BR = 3 \sin 45 = \underline{2.12m}$

$\cos 45 = \frac{AQ}{AG} \Rightarrow AQ = 1.5 \cos 45 = \underline{1.06m}$

$\cos 45 = \frac{AR}{AB} \Rightarrow AR = 3 \cos 45 = \underline{2.12m}$

→ Considering equ^m on the member

– Resolving horizontal forces

$R_D \cos 45 - W \cos 30 + H_A = 0$

$\therefore H_A = -0.707 R_D + W\, 0.867 \rightarrow (1)$

– Resolving vertical forces

$R_A - R_D \sin 45 - 800 + W \sin 30 = 0$

$R_A - 0.707 R_D + 0.5W - 800 = 0 \rightarrow (2)$

– Taking moments abt pt @ A

$R_D \times AD + 800 \times 1.06 - W \cos 30 \times 2.12 - W \sin 30 \times 2.12 = 0$

$2828.42 \times 0.905 + 848 - 1.83 W - 1.061 W = 0$

$3407.72 - 2.89 W = 0$

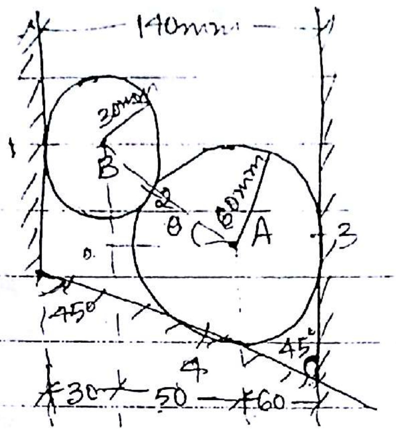
$W = \underline{1179.14 N}$

$\therefore H_A = -0.707 \times 2828.42 + 1179.14 \times 0.867 = -\underline{977.37 N} (\leftarrow$

$\therefore R_A = 0.707 \times 2828.42 \mp 0.5 \times 1179.14 + 800 = \underline{2710.12 N}$

32) Two cylinders A & B are placed in a trough as shown in the figure Find the reactions @ the contact surfaces if the surface are smooth.

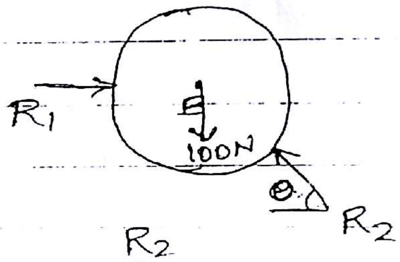
$\rightarrow \cos\theta = \frac{A}{H} = \frac{50}{90} = 0.55 \qquad \therefore \theta = \underline{56.25°}$



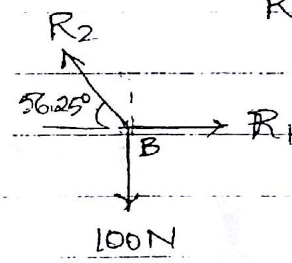
$\phi A = 120mm$
$\phi B = 60mm$
$WA = 250N$
$WB = 100N$

→ Let R_1, R_2, R_3 & R_4 be reactions @ contact points 1, 2, 3 & 4 respectively.
→ The system is in equ^n (i.e, $\Sigma M = \Sigma H = \Sigma V = 0$).
→ Considering equ^m @ B

$$\frac{100}{\sin(180-56.25)} = \frac{R_1}{\sin(90+56.25)} = \frac{R_2}{\sin 90}$$



$$\frac{100}{\sin 123.74} = \frac{R_1}{\sin 146.25} = \frac{R_2}{\sin 90}$$

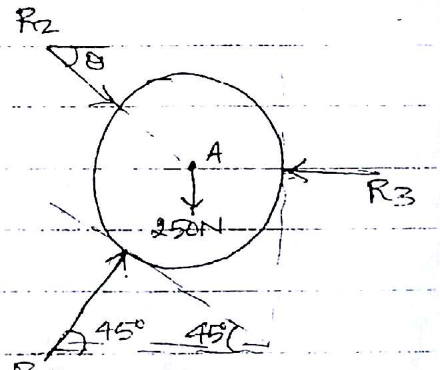


$$R_1 = \frac{100 \times \sin 146.25}{\sin 123.74}$$

$$R_1 = 66.81N$$

$$R_2 = \frac{100 \times \sin 90}{\sin 123.74}$$

$$R_2 = 120.25N$$



→ Considering equ^m @ A
- Resolving horizontal forces

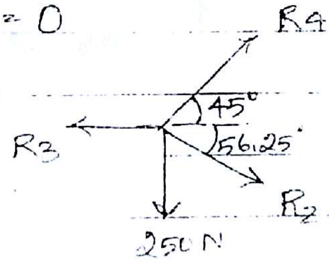
$$R_4 \cos 45 + R_2 \cos 56.25 - R_3 = 0$$

$$R_3 - R_4 \cos 45 = 66.8 \rightarrow (1)$$

- Resolving vertical forces

$$R_4 \sin 45 - 250 - R_2 \sin 56.25 = 0$$

$$R_4 = \frac{349.9}{\sin 45} = 494.5N \rightarrow (2)$$



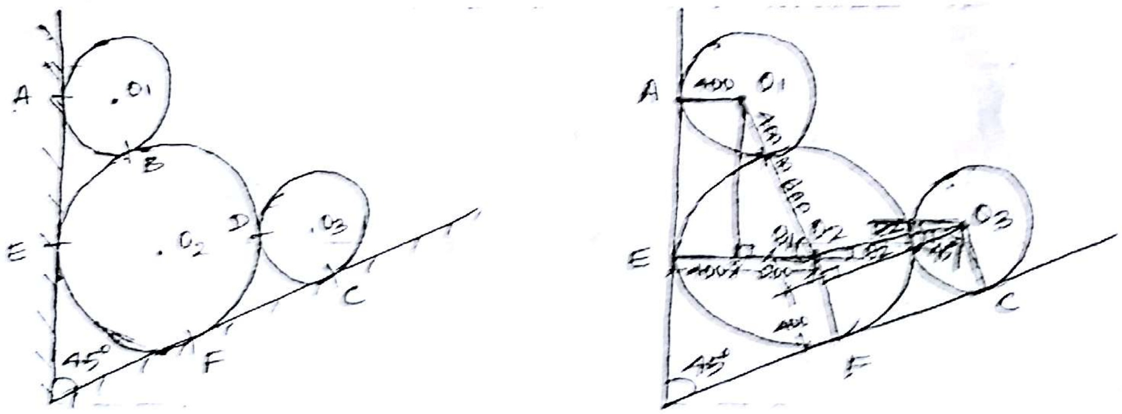
Substituting ① & ②

$R_3 - 494.5 \cos 45 = 66.8$

$\underline{R_3 = 416.8\,N}$

---

32) Three spheres O_1, O_2 & O_3 weighing 200kN, 400kN & 200kN respectively are having radii 400mm, 600mm & 400mm respectively are placed in a V shaped notch as shown in the figure. Determine the reactions @ contact if the surfaces are smooth.



→ $\cos\theta_1 = \frac{A}{H} = \frac{200}{1000}$ ∴ $\underline{\theta_1 = 72.46°}$

→ $\sin\alpha = \frac{O}{H} = \frac{200}{1000}$ ∴ $\underline{\alpha = 11.53°}$

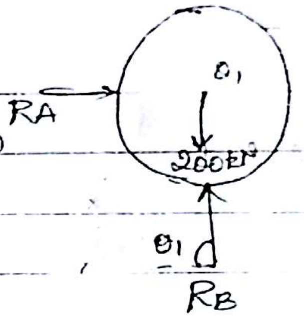
→ $\theta_2 = 45 - \alpha = 45 - 11.53 = \underline{33.46°}$

→ Let R_A, R_B, R_E, R_F, R_D, R_C be support reac @ all contact points respectively.

→ The system is in equ^m i.e, $\Sigma M = \Sigma H = \Sigma V = 0$

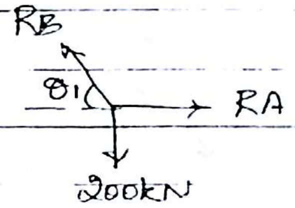
→ Considering equ^m @ O_1

$$\frac{200}{\sin(180-78.46)} = \frac{RA}{\sin(90+78.46)} = \frac{RB}{\sin 90}$$



$$\frac{200}{\sin 101.54} = \frac{RA}{\sin 168.46} = \frac{RB}{\sin 90}$$

$$\therefore RA = \frac{200 \times \sin 168.46}{\sin 101.54}$$



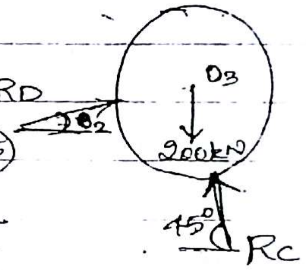
$$RA = \underline{40.83\,N}$$

$$\therefore RB = \frac{200 \times \sin 90}{\sin 101.54}$$

$$RB = \underline{204.12\,N}$$

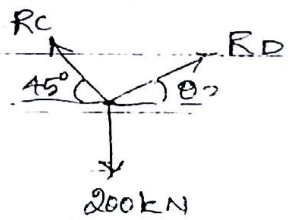
→ Considering equ^m @ O_3

$$\frac{200}{\sin(180-45-33.46)} = \frac{R_C}{\sin(90+33.46)} = \frac{R_D}{\sin(90+45)}$$



$$\frac{200}{\sin 101.54} = \frac{R_C}{\sin 123.46} = \frac{R_D}{\sin 135}$$

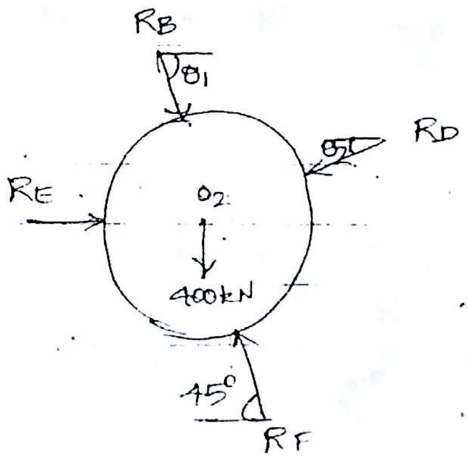
$$\therefore R_C = \frac{200 \times \sin 123.46}{\sin 101.54}$$



$$R_C = \underline{170.3\,kN}$$

$$R_D = \frac{200 \times \sin 135}{\sin 101.54}$$

$$R_D = \underline{144.3\,kN}$$



→ Considering equ^m @ O_2

Resolving horizontal forces

$$R_E + 204.12 \cos 78.46 - R_F \cos 45 - 144.3 \cos 33.46 = 0$$

$$R_E - 0.7\,R_F = 79.55 \longrightarrow (1)$$

Resolving vertical forces

$$R_F \sin 45 - 144.3 \sin 33.46 - 400 - 204.12 \sin 78.46 = 0$$