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Complex Variables and Distributions [18MA4GCCVD].

abelian group - commutative, associative, inverse.

$a \times \underline{1} = a$; 1 is identity in multiplication

$a + \underline{0} = a$; 0 is identity in addition.

$\therefore\ a \times \underline{\left(\frac{1}{a}\right)} = 1$; $\frac{1}{a}$ is inverse.

$a + \underline{(-a)} = 0$; $-a$ is inverse.

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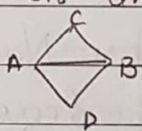
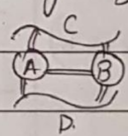
Module 1 :- Linear Algebra and Graph Theory

Graph Theory :

→ Introduction :- Graph is very convenient and natural way of representing the relation between the objects. We represent object by vertices and relationship between them by lines. Example :- Chemical molecules, map colouring etc. The graph has the mathematical modelling. The origin of graph theory was traced back to Euler's work on the Konigberg's bridge problem in 17^{th} century.



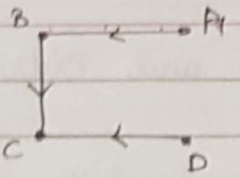


Definations

1) Graph :- A Graph is a pair of V, E. The elements of V are called vertices and all elements of E are called undirected edges or edges, and it is denoted by $G = (V, E)$

2) Directed Graph or digraph :- The directed graph is a pair of V, E. The elements of V are called vertices and the elements of E are called directed edge. The set V is called vertex set. The set E is called directed edge set. Denoted by D(V, E).

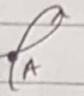
Example :-



$V = \{A, B, C, D\}$
$E = \{AB, BC, DC\}$.

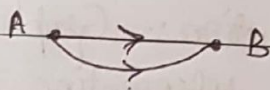
AB is the directed edge where A is initial value and B is the terminal vertex.

3) Loop :- A directed edge beginning and ending at the same vertex A and it is denoted by edge AA is called directed loop.

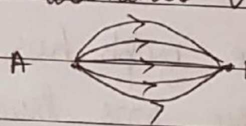


$V = A$ $\quad E = \{AA\}$ $\quad$ {Degree of Loop is 2}

4) Parallel directed edges :- Two directed edges having the same initial vertex and same terminal vertex are called parallel directed edges.

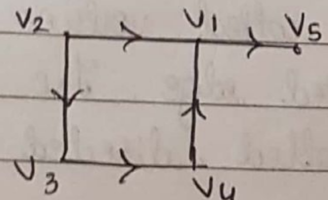


5) Multiple directed edges :- Two or more directed edges having same initial ~~valu~~ vertex and same terminal vertex are called multiple directed edges.



6) Degree :- Number of edges a vertex has is called degree
→ Out - degree and In - degree
If V is the vertex of a digraph d, the number of edges for which V is the initial vertex is called the outgoing degree or out-degree of V
The number of edges for which V is the terminal vertex is called the incoming degree or in-degree.
The out-degree of V is denoted by d^+ and indegree of V is denoted by d^-.

Example :-



$d^+(V_1) = 1$	$d^+(V_2) = 2$
$d^-(V_1) = 2$	$d^-(V_2) = 0$
$d^+(V_3) = 1$	$d^+(V_4) = 1$
$d^-(V_3) = 1$	$d^-(V_4) = 1$
$d^+(V_5) = 0$	
$d^-(V_5) = 1$	

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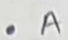
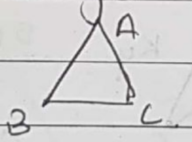
7) Null graph:- A graph containing no edges and vertices is called an null graph.

8) Trivial Graph:- A null graph ~~whi~~ with only one vertex is called a trivial graph.
Example:- 

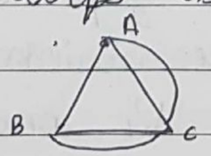
9) Finite Graph:- A graph with only a finite number of vertices as well as a finite number of edges is called a finite graph. Otherwise, it is called infinite graph.

10) Order and Size:-
The number of vertices in a graph is called order of the graph and the number of edges in ~~a~~ it is called its size. It is denoted by (n,m) where n is the order / vertices number;
m is the size / edges. (no. of);

⇒ Simple graph
A graph which does not contain loops and multiple edges is called a simple graph.
Example:-



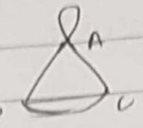
→ Multi graph
A graph which contains multiple edges ~~but~~ but no loops is called a multi graph.
Example:-



⇒ General graph
A graph which contains multiple edges or

loops or both are called a general graph.

Example



11) <u>Incidence</u>:-

When a vertex V of the a graph G is an end ~~edg~~ vertex of a small edge of the graph G. then we say edge E is incident on the vertex V.

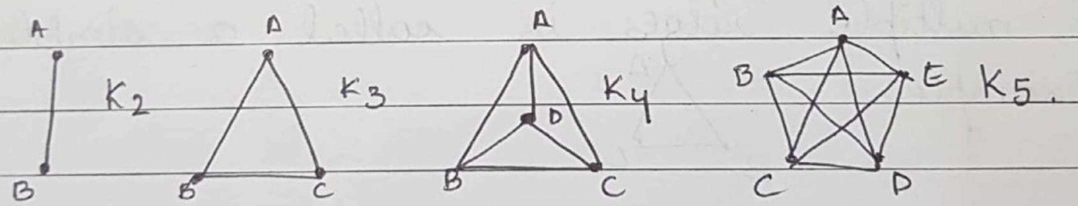
Two non ~~population~~ i.e. parallel edges are said to be adjacent edges if they are incident on a common vertex.

Two vertices are said to be adjacent vertices if there is an edge joining them.

12) <u>Complete Graph</u>:-

A simple graph of order greater that or equal 2 in which there is an edge between ~~eq~~ every pair of vertices is called a Complete graph. It is denoted by ~~g~~ Kn.

where n is ~~vertices~~ number of vertices

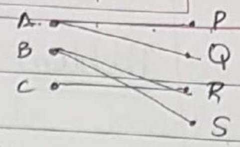


K5 is Kuratowski's first Graph.

13) Bipartite Graph :- A simple graph G is such that, its vertex set V is the union of two mutual disjoint non-empty set V_1 and V_2, which are such that each edges in G, joines a vertex in V_1 and a vertex in V_2. Then G is called a bipartite graph, and it is denoted by

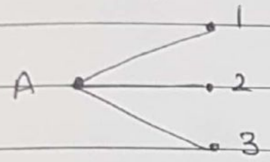
$G = (V_1, V_2 : E)$.

Example :- $V_1\{A, B, C\}$ $V_2\{P, Q, R, S\}$

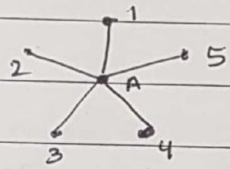


14) The Complete Bipartite Graph :- A bipartite graph G, i.e. $G = (V_1, V_2 : E)$ is called a complete bipartite graph if there is an edge between every vertex in V_1 and every vertex of V_2. It is represented by $K_{n,m}$
where n is V_1 vertices (no. of).
m is number of V_2 vertices.

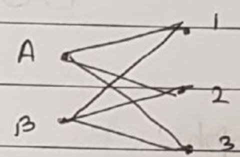
Example :-



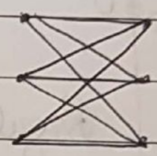
$K_{1,3}$



$K_{1,5}$



$K_{2,3}$

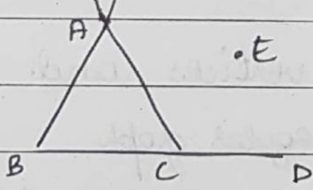


$K_{3,3}$.

$K_{3,3}$ is Kuratowski's second graph.

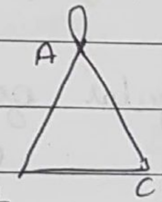
15) Vertex degree :-
The number of edges of G, that are incident on with the loops counted twice, is called the degree of vertex V.

Example :-



$d(A) = 4$
$d(B) = 2$
$d(C) = 3$
$d(D) = 1$ – pendant
$d(E) = 0$ — isolated vertex

CD is pendent edge



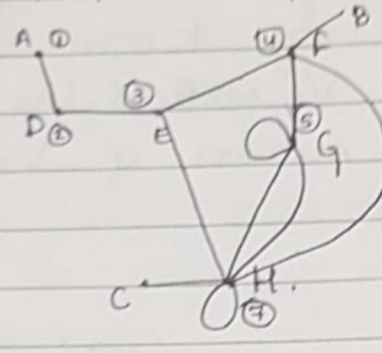
$d(A) = 4$
$d(B) = 2$
$d(C) = 2$.

16) Isolated and Pendant vertex
A vertex is an isolated vertex if and only if i[t] vertex is zero. A vertex of degree one is called pendant vertex. An edge incident on the pendant vertex is called the pendant edge

Problem

1) Draw the diagram of a graph where the degree of the vertices are (1, 1, 1, 2, 3, 4, 5, 7)

Solution Let (A, B, C, D, E, F, G, H). (1, 1, 1, 2, 3, 4, 5, 7).



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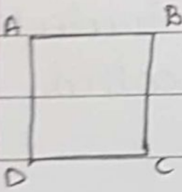
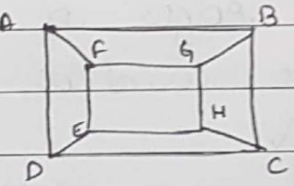
Regular Graph:-

A Graph in which all the vertices has same degree is called a regular graph. A Regular graph in which all the vertices of degree K is called K regular graph.

3 regular graph is called a cubic graph / ~~h~~ hyper cube



- 2 regular Graph.



3 - regular graph Hypercube.

If a regular graph has 10 vertices and 15 edges - Petersen graph.

Example:-  3 - regular graph

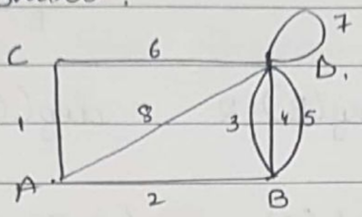
Handshaking Property:-

STATEMENT:- The sum of the degree of all vertices of a graph is ~~to~~ even and ~~is~~ ~~eq~~ this no. is equal to twice the number of edges in the graph.

$$\sum_{i=1}^{n} d(v_i) = 2m$$

This is because of the ~~gre~~ fact that, while counting the degree of the vertices, each edge is counted twice which is same as two hands are involved in each handshake.

Example:-



m

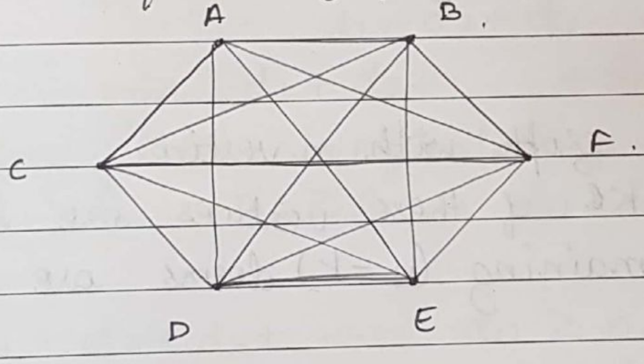
$2($[illegible]$) = 14$

$2(7) = 14.$

$$\deg(A) + \deg(B) + \deg(C) + \deg(D)$$
$$3 + 4 + 2 + 7 = 14.$$

Problem

1) Draw the complete graph of K_6.



K_6 complete graph

NOTE :- A Complete Graph is a simple graph in which there is an edge between every pair of vertices.

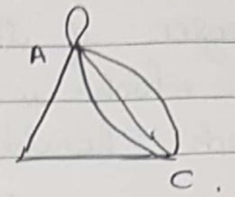
i.e., $m = {}^nC_2$

$$= \frac{n(n-1)}{2}.$$

$$\left\{ \begin{array}{l} \dfrac{n!}{(n-r)!\,2!} \\ \dfrac{n!}{(n-2)!\,2!} = \dfrac{n(n-1)}{2}. \end{array} \right.$$

where n – no. of vertices.
m – no. of edges

3) Indicate the degree of each vertex and verify the handshaking property.



Solution :- $deg(A) = 6$ $\quad deg(B) = 2$ $\quad deg(C) = 4$.

$\therefore d(A) + d(B) + d(C) = 6 + 2 + 4 = 12.$

$\therefore m = 12$

$$\sum_{i=1}^{n} d(v_i) = 2m$$

$$12 = 2(6)$$

4) Show that in a graph the number of odd vertices is even.

Solution: Consider a graph with n vertices.
Suppose K of these vertices are of odd degree so that remaining $(n-k)$ terms are of even degree.

Odd degree vertices are $v_1, v_2, v_3 \ldots v_k$ and even degree vertices are $v_{k+1} \ldots v_n$.

By using the handshaking property, the sum of the degrees of the vertices is equal to the twice the number of edges.

i.e., $\sum_{i=1}^{n} d(v_i) = 2m = \text{even}$

$$\{d(v_1) + \ldots + d(v_k)\} + \{d(v_{k+1}) + \ldots + d_n\} = \text{even}$$

$$\sum_{i=1}^{k} d(v_i) + \sum_{i=k+1}^{n} d(v_i) = \text{even}$$

$$\sum_{i=1}^{k} d(v_i) + \text{even} = \text{even}.$$

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$\Rightarrow \sum_{i=1}^{k} d(v_i) = \text{even}.$

The sum of the ~~vertices~~ degrees of the ~~odd~~ vertices of odd degree is even.

This proves the theorem.

5) Property - 2.

If G is a graph with n vertices and m edges. Let δ is the minimum and Δ is the maximum of the degrees of vertices. Show that $\delta \leq \frac{2m}{n} \leq \Delta$.

Solution:- $d_1, d_2, d_3 \ldots d_n$ be the degrees of the vertices. Then by the handshaking property.

$$\sum_{i=1}^{n} d(v_i) = 2m$$

$$\therefore d_1 + d_2 + d_3 \ldots + d_n = 2m \quad \text{—— (I)}.$$

Since, $\delta = \min(d_1 + d_2 + \ldots + d_n)$.

adding these inequalities, we get

$d_1 \geq \delta \quad d_2 \geq \delta \quad d_n \geq \delta.$

$$d_1 + d_2 + d_3 + \ldots d_n \geq n\delta.$$

$$2m \geq n\delta \quad \text{—— (II)}.$$

Similarly, $\Delta = \max(d_1 + d_2 + \ldots + d_n)$.

$d_1 \leq \Delta \quad d_2 \leq \Delta \ldots \ldots d_n \leq \Delta.$

adding these inequalities, we get.

$$d_1 + d_2 + d_3 + \ldots + d_n \leq n\Delta.$$

$$2m \leq n\Delta \quad \text{—— (III)}$$

by (II) and (III), we get $2m \leq n\Delta \quad 2m \geq n\delta$.

$$n\delta \leq 2m \leq n\Delta.$$

$$\delta \leq \frac{2m}{n} \leq \Delta$$

Hence proved.

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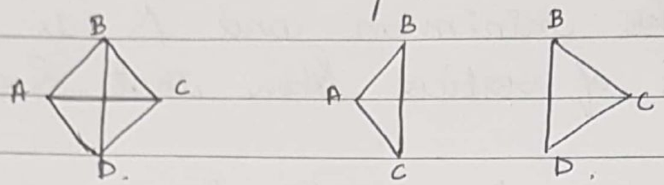
Sub-Graphs

Two graphs G and G_1, G_1 is sub ~~set~~ graph of G if the following condition ~~holds~~. holds

i) All the vertices and all the edges of G_1 are in G.

ii) Each edge of G_1 has the same end vertices in G.



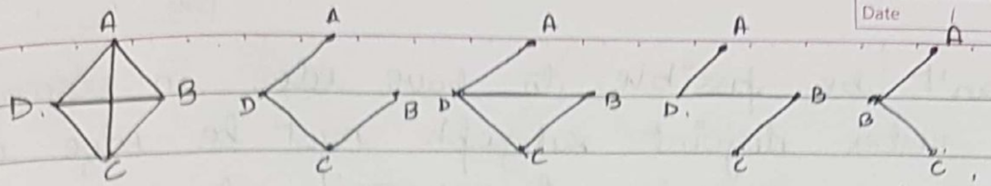
"G" G_1 G_2

The consequences of a sub-graph.

1) Every graph is a subgraph of itself.
2) Every simple graph of n-vertices is a sub-graph of a complete graph K_n.
3) If G_1 is the sub-graph of G_2 and G_2 is a subgraph of graph G, [then. it is transitive law]
4) A single vertex in a graph G_1 is a ~~of~~ subgraph of G.
5) A single edge in a graph G together with its end vertices is a subgraph of G.

Spanning sub-graph

A sub-graph G_1 of a graph G is a spanning subgraph of G whenever the vertex at G_1 contains all the vertices of G. Thus, A graph and all its spanning sub-graphs has the same vertex set.



"G" 'G1' 'G2' 'G3' 'G4'

G_1, G_2, G_3 are spanning subgraph and G_4 is not a spanning subgraph.

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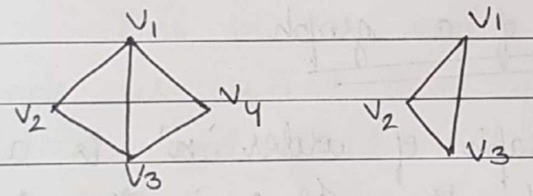
Induced subgraph

Given a subgraph there is graph G_1, every edge of G_1 on every edge ~~of~~ $\{A, B\}$ G where (A, B) belongs ~~to V_1~~ to V_1 is an edge of G_1 also then G_1 is called a subgraph of G induced by V_1.

Induced subgraph is denoted by $\langle V_1 \rangle$

$V_1 = \{v_1, v_2, v_3\}$.

Example:-



$V = \{V_1, V_2, V_3\}$.

Edge disjoint and vertex disjoint subgraph.

Let G be a graph G_1 and G_2 are the two subgraph of G, then G_1 and G_2 are said to be edge disjoint if they donot have any common edge.

1) G_1 and G_2 are said to be vertex disjoint if they do not have any common edge and any common vertex.

NOTE:- Edge disjoint subgraph may have common vertices ~~Subgra~~ Subgraphs that have no vertices in common can't be possible to have edge in common.

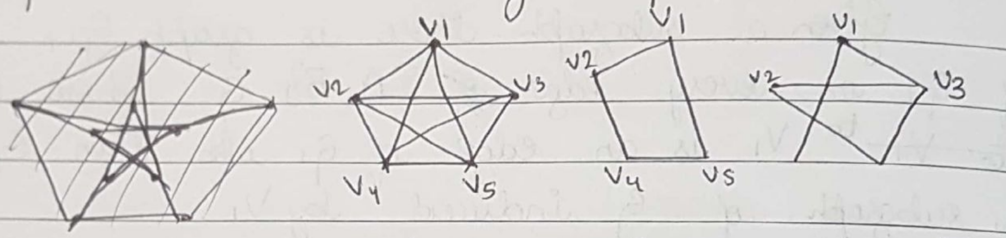
2) Two vertex disjoint subgraph must be edge disjoint but the converse is not neccesarily true.

The complement of a subgraph

A graph G and a subgraph ~~G_1 of~~ G_1 of G

The subgraph of G obtained from G all the edges that belongs to G_1, is called the complement of G_1 in G and is denoted by $G - G_1$ or $\overline{G_1}$

Example:-



G $\quad$ G_1 $\quad$ $G - G_1 = \overline{G_1}$

The complement of a graph

Every simple graph of order 'n' is a subgraph of the complete graph K_n. If G is the simple graph of order n, the complement of G is K_n and is called the complement of G and is denote by $\overline{G}$.

The complement $\overline{G}$ of a simple graph G of n vertices is the graph which is obtained by deleting those edges in K_n which belongs to G.

Therefore, $\overline{G} = K_n - G$.

NOTE: (i) The $\overline{\overline{G}} = G$.

(ii) The complement of K_n is a null graph of order n and vice versa.

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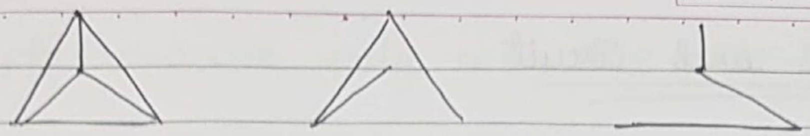
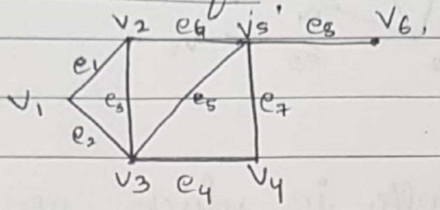
Example:-



K_4 $\qquad$ G $\qquad$ $\bar{G} = K_4 - G$

Walk and their classification



G is considered a finite alternating sequence of vertices and edges of the form $v_1, e_1, v_2\, e_6\, v_5\, e_8\, v_6$ which begins and ends with the vertices which is such that each edge in the sequence is incident on the vertices preceding.

The number of edges present in the walk is called its length. The vertex with which walk begins is called initial vertex and the vertex with which the walk ends is called the final vertex.

[The initial and final vertices of a walk is together called as terminal vertices].

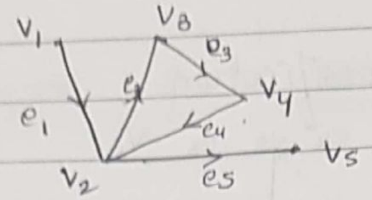
1) A Walk that begins and ends at the same vertex is called closed walk
2) An open walk is a walk that begins and end at two different vertices.

Example:- $v_1\, e_1\, v_2\, e_6\, v_5\, e_8\, v_6$ → open walk
$v_1\, e_1\, v_2\, e_6\, v_5\, e_7\, v_4\, e_4\, v_3\, e_2\, v_1$ → close walk

~~Trial and circuit~~

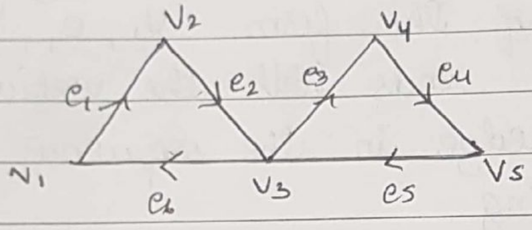
Trail and Circuit

If in an open walk no edges appears more than once, then the walk is called a trail.



$V_1\ e_1\ V_2\ e_2\ V_3\ e_3\ V_4\ e_4\ V_2\ e_5\ V_5$

A close walk in which no edge appears more than once, is called a circuit.



$V_1\ e_1\ V_2\ e_2\ V_3\ e_3\ V_4\ e_4\ V_5\ e_5\ V_3\ e_6\ V_1$

⇒ The following facts of a walk are as follows:-

1) A walk can be open or closed. In a walk a vertex and an edge can appear more than once.
2) A trail :- A trail is an open walk in which a vertex can appear more than once but an edge can not appear more than once.
3) A circuit is a closed ~~path~~ walk in which a vertex can appear more than once but an edge can't appear more than once.
4) A path is an open walk in which neither vertex nor an edge can appear more than once.
5) Every path is a trail but a trail need not be a path.
6) A Cycle is a close walk in which neither vertex nor an edge can appear more than once.

Every cycle is a circuit but circuit need not be a cycle.

7) If a cycle contains only one edge it has to be a loop.

8) Two parallel edges form a cycle.
In a simple graph, a cycle must have atleast 3 edges. A cycle formed by three edges forms a triangle.

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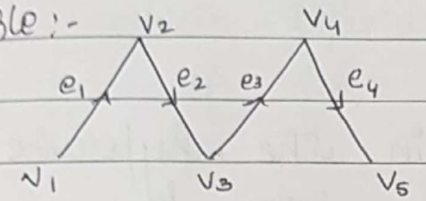
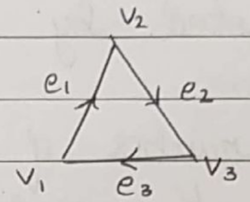
Example:-



path



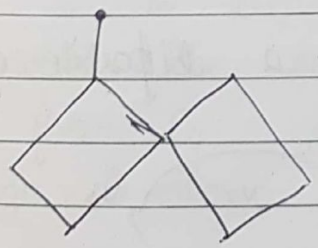
$V_1 e_1 V_2 e_2 V_3 e_3 V_1$

Cycle.

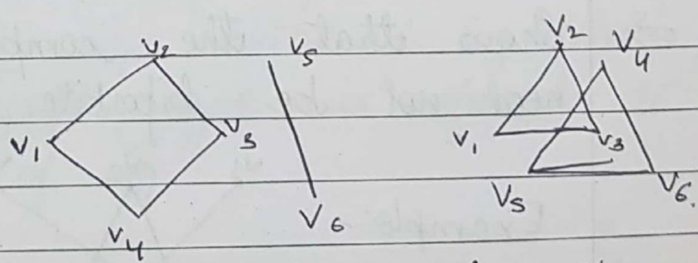
Connected and dis-connected graph

The two vertices in 'G' are said to be connected if there is atleast one path, from one vertex to the other. 'G' is connected if every pair of distinct vertices in 'G' are connected. Otherwise 'G' is called a disconnected graph.

1) G is said to be connected if there is atleast one path between every two distinct vertices in 'G'.
2) G is disconnected if 'G' has atleast one pair of distinct vertices between which there is no path.



Connected graph



Disconnected graph.

Component of a graph

A Graph 'G' consists of one or more connected graph. Each such connected graph is a subgraph of G and is called component of a graph G.

A connected has only one component and a disconnected has two or more components.

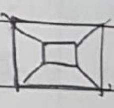
The number of components of a graph 'G' is denoted by $K(G)$

NOTE:- The number of edges in the hypercube ~~is given by~~ of K component is given by:-

$$Q_K = K2^{K-1}$$

⇒. Show that every cubic graph has an even number of vertices

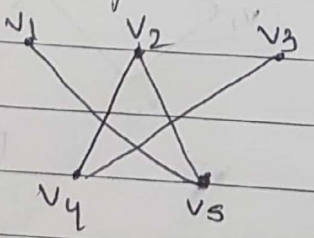
'G' be a cubic graph of n vertices i.e., ~~every vertex of odd order of degree 3,~~ every vertex of odd degree.

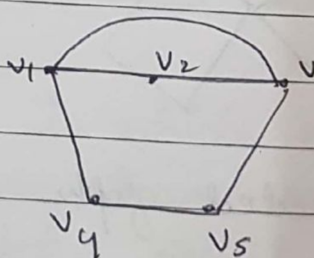


In a graph 'G'. The number of odd degree vertices are even by using this property, The cubic graph has even number of vertices.

⇒ Show that the complement of a bipartite graph need not be bipartite.

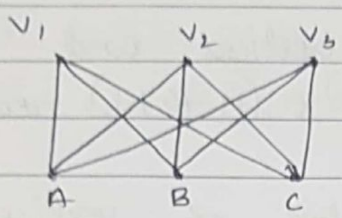
Example:-



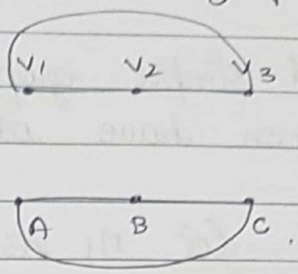


Problems:-

(i) Find the compliment of complete Bipartite graph of $K_{3,3}$



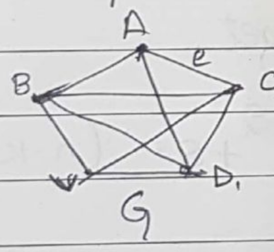
Complete Bipartite graph of $K_{3,3}$.

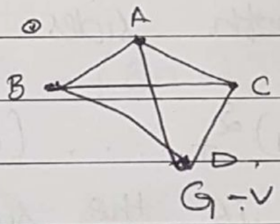


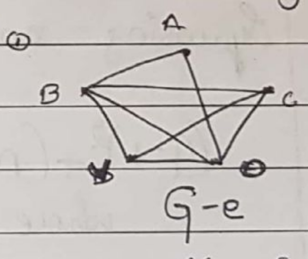
Compliment of Bipartite graph $K_{3,3}$.

Deletion

If V is a vertex in a graph 'G', then 'G − V' denoted the subgraph of 'G' obtained by deleting V and all edges inciden on V from G. This subgraph G−V is called vertex deleted subgraph of 'G'.







NOTE:- Deletion of a vertex always result in deletion of all edges incident on that vertex

If e is an edge in a graph 'G', then 'G−e' denotes the subgraph of 'G' obtained by deleting e from 'G', this subgraph 'G−e' is referred to a edge-deleted subgraph of 'G'.

NOTE:- The deletion of an edge does not alter, the number of vertices. ~~Sh~~

An edge deleted subgraph of a graph is a spanning subgraph of a graph.

⇒ 7) A simple graph with n vertices and K components can have atmost $\frac{1}{2}(n-k) \times (n-k+1)$ edges

Let n_1 be the number of vertices in 1 component and n_2 be the number of vertices in 2 component and so on . . , n_k be the number of vertices in k component.

then, $n_1 + 2 + n_3 + \dots + n_K = n$. ——— (I).

Let (I) be the number of vertices in k components

$(n_1 - 1) + (n_2 - 1) + (n_3 - 1) + \dots (n_K - 1) = n(-1 + 1 + 1 \dots + 1)$

$(n_1 - 1) + (n_2 - 1) + \dots + (n_K - 1) = n - k)$.

Squaring on both sides, we get.

$(n_1 - 1)^2 + (n_2 - 1)^2 + \dots (n_K - 1)^2 + S = (n - k^2)$ ——— (II).

where S is the sum,

$i = 1, 2, 3 \dots k$

$j = 1, 2, 3 \dots K \quad i \neq j$

$S = 2(n_i - 1)(n_j - 1)$.

~~Since~~ $2(n_p - 1)(n_j - 1)$ ⇒ Since $2(n_i - 1)(n_j - 1)$

Since $n_1 \dots k$ is ≥ 1, We have S. greater than ≥ 0.

i.e., $S \geq 0$.

expanding (II), we get.

$(n_1 - 1)^2 + (n_2 - 1)^3 + \dots + (n_K - 1)^2 \leq (n - k^2)$

$n_1^2 + n_2^2 + \dots + n_K^2 - 2(n_1 + n_2 + \dots n_k)$

$\leq n^2 - 2nk + k^2$

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$$\sum_{i=1}^{k} n_i^2 - 2n + k \leq n^2 - 2nk + k^2.$$

$$n_1^2 - 2n_1 + 1, \sum_{i=1}^{k} n_i^2 \leq n^2 - 2nk + k^2 + 2n - k,$$

$$\leq n^2 - 2n(k-1) + k(k-1).$$

$$\leq n^2 - (2n-k)(k-1) \text{ ——— (iii)}$$

If 'G' is a simple graph of each of the components of graph 'G' is a simple graph. Therefore, the max. edges ~~of~~ with a high component can be

$$\frac{1}{2} n_i(n_i - 1).$$

∴ The maximum number of edges when 'G' have

$$N = \sum_{i=1}^{k} \frac{1}{2} n_i(n_i - 1).$$

$$= \frac{1}{2}\left[\sum_{i=1}^{k} n_i^2 - \sum_{i=1}^{k} n_i\right]$$

$$= \frac{1}{2}\left[n^2 - (2n-k)(k-1) - n\right]$$

$$= \frac{1}{2}\left[n^2 - 2nk + k^2 + 2n - k - n\right]$$

$$= \frac{1}{2}\left[(n-k)^2 + (n-k)\right]$$

$$= \frac{1}{2}\left[(n-k)\left[n-k+1\right]\right].$$

∴ The number of edges in G cannot exceed

$$\frac{1}{2}(n-k)(n-k+1).$$

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3) Show that every simple graph of order greater than or equal to 2 must have 2 vertices of same degree.

Solution: Let 'G' be a simple graph with n vertices suppose all the vertices have different degree, then every vertex should have a degree. Since all such degrees must be d between 0 and n-1, i.e., the degree must be 0, 1, 2, 3, ... n-1. Let A be the vertex whose degree is zero and B be the vertex whose degree is n-1.

Then n-1 edges are incident on B, this means that B is joined to all other vertices by an edge and in particular to A also.

Hence, the degree of A is not zero, This is contradiction. Hence all vertices of G cannot have different degrees, atleast 2 of them must have the same degree.

Euler's circuit and Euler trails

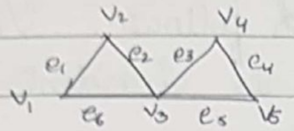
A connected graph 'G', if there is a circuit in G, that connets all the edges of 'G' Then that circuit is called an ~~an~~ Euler's circuit.

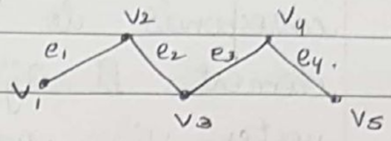
If there is a ~~t~~ trail if 'G' that contains all ~~the~~ the edges of G, then that trail is called eulerian trail of G.

Eulerian circuits and trails ~~incl~~ includes all the edges. They automacally should include all the vertices as well.

A connected graph that contains, an eulerian ~~trail~~ circuit is called euler graph.
A connected graph that contains an euleian an trail is called semieuler graph.

Example:- • Eulerian circuit • Eulerian trail





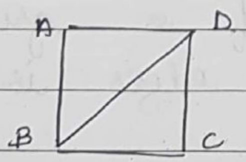
$V_1 e_1 V_2 e_2 V_3 e_3 V_4 e_4 V_5 e_5 V_3 e_6 V_1$ $V_1 e_1 V_2 e_2 V_3 e_3 V_4 e_4 V_5$

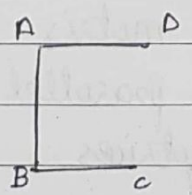
Hamilton path ~~of~~ and Hamilton Cycle.

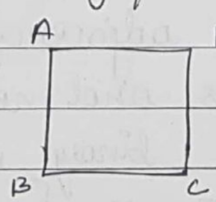
~~Hamiltonic G~~

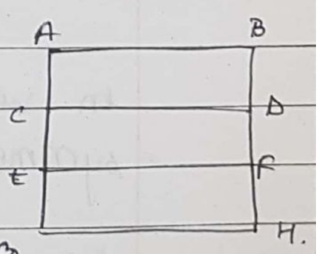
Let G be the connected graph, If there is a cycle in G that contains all the vertices of G, then the cycle is called Hamiltonian cycle in G.
Hamiltonian cycle in a graph of n-vertices, consists of exactly n edges because, a cycle will n-vertices has n-edges.
A Hamiltonian cycle in a graph 'G' must include all the vertices in G. This doesnt mean that it should include all the edges of 'G'. A path in a connected graph which include every vertex of a graph is called Hamiltonian path.
HP in graph 'G' meets every vertex of G, the length of HP in a connected graph of n-vertices is n-1.

Example:-









CBAD. 4 vertices 3 edges

4 vertices 4 edges.

ABDC EFHG (Hamilton path)

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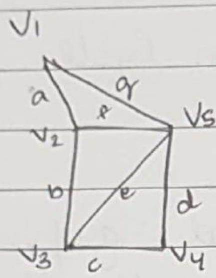
Incident Matrix

'G' be a graph with n vertices m edges and no self loops. ~~define~~ Define n by m matrix, $A = [a_{ij}]$ where n rows corresponds to ~~m~~ vertices and m column corresponds to the edges as follows. The matrix element $a_{ij} = 1$, if j^{th} edge is incident on i^{th} vertex v_i. and $= 0$ otherwise.

Example:-



$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$A = [a_{ij}]$

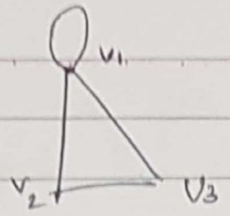
• Matrix A is called the vertex edge incident matrix or simply incident matrix.

The incident matrix contains only two elements 0 and 1 and such a matrix is called binary matrix and write with ~~witho~~ no loops.

Adjacency Matrix

The adjacency matrix of a graph G with m vertices and no parallel edges is an n/m symmetric binary matrices

Let $X = [x_{ij}]$, $x_{ij} = 1$ if there is an edge bridge i^{th} and j^{th} vertex. and equal to zero if there is no edge between them.

Example:- 

$$\begin{array}{c|ccc} & V_1 & V_2 & V_3 \\ V_1 & 1 & 1 & 1 \\ V_2 & 1 & 0 & 1 \\ V_3 & 1 & 1 & 0 \end{array}$$

A simple graph and its adjacency matrix.

1) The entries along the principle diagonal are all zeros, if and only the graph has no self loop.
2) The self loop along the i^{th} vertex. ~~The self loop~~ is i.e, $[x_{ij}] = 1$

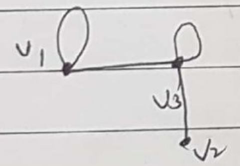
The definations for adjany matrices makes no provision for parallel ~~cut~~ ~~for~~ edges &

Ex:- graph without parallel ~~junct~~ edges.

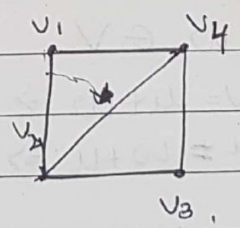
9) Draw the graph corresponds to the given ~~adj~~ adjacenty matrix.

$$V = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{matrix} v_1 \\ v_2 \\ V_3 \end{matrix}$$

Solution



10)
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix}$$



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Linear Algebra

Defination of a vector space,

A non-empty set V is said to be a vector space over the field F, if any two vectors, $\alpha, \beta \in$ V this implies $\alpha+\beta$ belongs to V (Vector addition) and any vector $\alpha \in$ V and the scalar $c \in$ F. ~~Scalar me~~ $\Rightarrow c\alpha \in V$ (scalar multiplication) with other ~~proties~~ properties as follows.

i) V^+ is an abelian Group,

ii) $c(\alpha+\beta) = c\alpha + c\beta$ where $\alpha, \beta \in V$, $c \in F$ ~~then~~

iii) $(c_1+c_2)\alpha = c_1\alpha + c_2\alpha$, where $c_1, c_2 \in F$, $\alpha \in V$.

iv) $(c_1 \cdot c_2)\alpha = c_1(c_2\alpha)$, $c_1, c_2 \in F$, $\alpha \in V$

v) $1 \cdot \alpha = \alpha$, where $\alpha \in V$, and 1 is unit element function

V^+ is abelian means:-

1) Commutative property $\Rightarrow u+v = v+u$

2) $(u+v)+w = u+(v+w)$

3) $u+0 = u$, $u \in V$ $0 \in F$.

4) $u+(-u)=0$, $u \in V$ & $-u \in V$.

~~zero vector~~

The properties of vector space:-

- Let V be the ~~st~~ vector space. ~~uvw~~ $u, v, w \in V$

(i) $u+v = u+w \Rightarrow v = w$ Left cancellation law

(ii) $v+u = w+u \Rightarrow v = w$ Right cancellation law.

Zero vector: zero is unique, for each u belongs to V, The adding inverse $-u$ is unique.

~~zero~~ $0 \cdot v = 0$ for all v, $v \in V$ where 0 belongs to ~~expat~~ R is zero vector;

$a \cdot 0 = 0$, $\forall v$, $v \in V$ & ~~to~~ where a is scalar.
$a \cdot v = 0$ then $a = 0$ or $v = 0$,
$(-1) v = -v$. where $-v \in V$ (vector space)

31/1/2020

The Geometrical Representation of a vector.

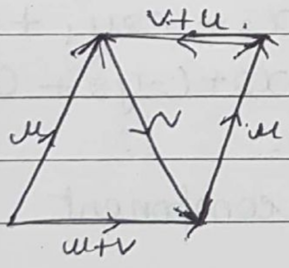
Let V be the set of all arrows in a 3-D space with 2 arrows regarded as equal ~~to~~ if they have same length ~~&~~ and point in the same direction. ~~If~~ ~~I~~ Defined by llegram rule and for each v in V define cv to be the arrow whose length is c times the length of v pointing in the same direction as v. If $c \geq 0$, otherwise pointing in the opposite direction.

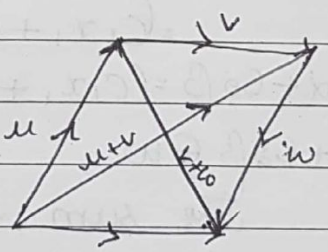
NOTE:- An ~~arrow~~ arrow of '0' length is ~~a~~ the single point and represents the zero vector

(-)ve of v is, $(-1)v$

Example:-

$(V, +)$





$v + u = u + v$ $\qquad (u+v)+w = u+(v+w)$

Sub Space

~~A~~ non-empty subset W of a vector space V is said to be the subspace of V, if and only if

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w itself is the vector space, with same vector addition and scalar multiplication of V.

(i) '0' vector of V is in w
(ii) w is closed under vector addition, i.e, for every $(\alpha, \beta) \in w$, also $(\alpha+\beta) \in w$.
(iii) w is closed under scalar multiplication, i.e, where $\alpha \in w$, c is scalar, then $c\alpha \in w$

Example:- $w = \{(x_1, x_2, x_3)\}$ such that $x_1+x_2+x_3=0\}$ at a vector space $V_3(R)$ and is a subspace of $V_3(R)$.

Solu:- Let $\alpha=(x_1, x_2, x_3)$ & β be (y_1, y_2, y_3) be any two elements of w,

$x_1+x_2+x_3=0$ and $y_1+y_2+y_3=0$ —— (1)

$\alpha+\beta = (x_1, x_2, x_3)+(y_1, y_2, y_3)$
$= x_1+x_2+x_3+y_1+y_2+y_3 = 0+0=0$

$c_1\alpha+c_2\beta = c_1(x_1, x_2, x_3) + c_2(y_1, y_2, y_3)$
$= (c_1x_1, c_1x_2, c_1x_3) + (c_2y_1, c_2y_2, c_2y_3)$
$= (c_1x_1+c_1x_2+c_1x_3+c_2y_1+c_2y_2+c_2y_3)$
$c_1\alpha+c_2\beta = (c_1x_1+c_2y_1+c_1x_2+c_2y_2+c_1x_3+c_2y_3)$
$c_1\alpha+c_2\beta \in w$,
The sum of the component $c_1\alpha+c_2\beta$ is 0

This is the subspace of w.

Let V be the vector space over the field f, show that the intersection of any collection of subspaces of V is a subspace of V.

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Let w_1, and w_2 be the two subspace of V, $0 \in W_1$ and $0 \in W_2$ implies $0 \in W_1 \cap W_2$ $\therefore$ $W_1 \cap W_2 \neq \phi$

i.e., w is non-empty.

For every c_1, c_2 belongs to field f, $\alpha, \beta \in W_1 \cap W_2$, then $\alpha, \beta \in W_1$ and $\alpha, \beta \in W_2$,

This implies $c_1\alpha + c_2\beta \in W_1$ and $c_1\alpha + c_2\beta \in W_2$

$\Rightarrow c_1\alpha + c_2\beta \in W_1 \cap W_2$

Hence, $W_1 \cap W_2$ is the subspace of the vector field

NOTE:- The union of 2 subspaces of vector field need not be the subspace of the vector field.

Linear Combinations of Vectors.

The expression $c_1\alpha_1 + c_2\alpha_2$ till $c_n\alpha_n$ is called a linear combination of the vectors $\alpha_1 + \alpha_2 + \dots \alpha_n \in \mathbb{R}^n$ (vector).

where $c_1, c_2, \dots c_n$ are the scalars, then the linear combination is given by

$$\beta = c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3 + \dots + c_n\alpha_n.$$

$$\beta = \sum_{i=1}^{n} c_j \alpha_i$$

Linearly independent and ~~independent~~ linearly dependent vectors

The set of vectors $v_1, v_2 \dots v_k$ is said to be linearly indepent, if there are only scalars $c_1, c_2 \dots c_k$ satisfies $c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3 \dots + c_n\alpha_n = 0$,

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We also say that the vector v_1, v_2, till v_k, are linearly independent.

$c_1 v_1 + c_2 v_2 + c_3 v_3 + \dots + c_k v_k \neq 0$.

If the vectors are not linearly independent then they ~~to~~ are called linearly dependent.

4/08/2020

A subspace spanned by a set

A span of a set of vector :-

A span $\{v_1, v_2, \dots v_p\}$ v_1 to v_p denotes the set of all vectors that can be written as linear combination of ~~v_1 to v_p~~ $\{v_1, \dots v_p\}$.

Problems :-

10) Express the vectors $b = \begin{bmatrix} 2 \\ 13 \\ 6 \end{bmatrix}$ as a linear combination of vectors $v_1 = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$

Solution :- $b = x_1 v_1 + x_2 v_2 + x_3 v_3$

$$\begin{bmatrix} 2 \\ 13 \\ 6 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 + x_2 + x_3 \\ 5x_1 + 2x_2 + 4x_3 \\ -x_1 + x_2 + 3x_3 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 13 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 2 & 4 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad Ax = 0$$

$[A:B]$

$$\begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 5 & 2 & 4 & : & 13 \\ -1 & 1 & 3 & : & 6 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 5R_1 \\ R_3 \to R_3 + R_1 . \end{array} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 0 & -3 & -1 & : & 3 \\ 0 & 2 & 4 & : & 8 \end{bmatrix} \quad \begin{array}{l} R_3 \to 1/2 R_3 \\ R_3 \to -R_2 . \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 0 & 3 & 1 & : & -3 \\ 0 & 1 & 2 & : & 4 \end{bmatrix} \quad R_2 \leftrightarrow R_3 \Rightarrow \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 0 & 1 & 2 & : & 4 \\ 0 & 3 & 1 & : & -3 \end{bmatrix} \quad R_3 \to R_3 - 3R_2 \Rightarrow \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & -5 & : & 15 \end{bmatrix}$$

$x_1 + x_2 + x_3 = 2$ $\Rightarrow$ $x_3 = 3$

$x_2 + 2x_3 = 4$ $\qquad x_2 = -2$

$-5x_3 = 3.$ $\qquad x_1 = 1$

$\therefore b = 1V_1 - 2V_2 + 3V_3.$

Null space :- The null space of $m \times n$ matrix A is written as Null A is the set of all solution of the homogenous equation, $AX = 0$.

The set notation ~~Null A is equal to~~

Null $A = \{ x : x \text{ in all } R^n \ni AX = 0 \}$.

Ex:- Let $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$ and $u \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$. Determine u that belongs to null space of A.

$$\begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 5-9+4 \\ -25+27-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$\therefore$ v is a null of A.

Column space of a matrix:-

The column space of an $n \times m$ matrix A written as col. A is the set of all linear combination of columns of A.

If $A = [a_1, a_2, \ldots a_n]$.
then $\text{col } A = \text{span}\{a_1, a_2, \ldots a_n\}$

The Column space of an $n \times m$ matrix A is a subspace of R^n

Example:- $$W = \left\{ \begin{bmatrix} 6a-b \\ a+b \\ -7a \end{bmatrix} : a, b \in R \right\}$$

$$W = \left\{ a \begin{bmatrix} 6 \\ 1 \\ -7 \end{bmatrix} + b \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\text{Span} = \left\{ \begin{bmatrix} 6 \\ 1 \\ -7 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$A = \begin{bmatrix} 6 & -1 \\ 1 & 1 \\ -7 & 0 \end{bmatrix}$$

NOTE:- The Column space of a $m \times n$ matrix X. ~~It~~ Is all in R^n, if and only if the equation is of the form $AX = B$, has a solution for each B in R^m.

1) Write the vector ~~$\{1, 3, -1\}$~~ $= \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ as a linear combination of the vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$

Solution:- $b = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ $V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $V_2 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $V_3 = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$

$$b = x_1 V_1 + x_2 V_2 + x_3 V_3.$$

$$\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & -2 & 0 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$b = AX$ $\quad [A : b]$.

$$\left[\begin{array}{ccc:c} 1 & 2 & 2 & 1 \\ 0 & -2 & 0 & 3 \\ 0 & 1 & 4 & -1 \end{array}\right] R_2 \leftrightarrow R_3 \Rightarrow \left[\begin{array}{ccc:c} 1 & 2 & 2 & 1 \\ 0 & 1 & 4 & 3 \\ 0 & -2 & 0 & -1 \end{array}\right] R_3 \to R_3 + 2R_2 \Rightarrow \left[\begin{array}{ccc:c} 1 & 2 & 2 & 1 \\ 0 & 1 & 4 & -1 \\ 0 & 0 & 8 & 1 \end{array}\right].$$

$x_1 + 2x_2 + 2x_3 = 1 \qquad \longrightarrow \qquad x_3 = 1/8.$

$x_2 + 4x_3 = -1 \qquad\qquad x_2 = -3/2$

$8x_3 = 1 \qquad\qquad x_1 = 15/4$

$$b = \tfrac{15}{4} v_1 - \tfrac{3}{2} v_2 + \tfrac{1}{8} v_3.$$

Verification :- $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \begin{bmatrix} x_1 + 2x_2 + 2x_3. \\ -2x_2 \\ x_2 + 4x_3 \end{bmatrix} = \begin{bmatrix} 15/4 + 2(-3/2) + 2(1/8) \\ -2(-3/2) \\ -3/2 + 4(1/8) \end{bmatrix} \to \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}.$

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12) For what value of H, y will be the span of $\{v_1, v_2, v_3\}$ where $v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ $v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}$ $v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$

Solution:- $y = x_1 v_1 + x_2 v_2 + x_3 v_3$.

$R_2 \to R_2 + R_1$
$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix} = \begin{bmatrix} 1 & 5 & -3 \\ -1 & -4 & 1 \\ -2 & -7 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc:c} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & h \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 3 & -6 & h-8 \end{array}\right] R_3 \to R_3 - 3R_2 \Rightarrow \left[\begin{array}{ccc:c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & h-5 \end{array}\right] \qquad \because r = n.$$

when $h = 5$, the matrix becomes consisten

13) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $v_2 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $v_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$ $w = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

(i) Is w in $\{v_1, v_2, v_3\}$? How many vectors are in $\{v_1, v_2, v_3\}$?
(ii) How many vectors are in span $\{v_1, v_2, v_3\}$?
(iii) Is w in the subspace spanned $\{v_1, v_2, v_3\}$? Why?

Solution: (i) w is not in $\{v_1, v_2, v_3\}$. There are 3 vectors in $\{v_1, v_2, v_3\}$.

ii) $w = x_1 V_1 + x_2 V_2 + x_3 V_3$.

$$\begin{bmatrix}3\\1\\2\end{bmatrix} = x_1\begin{bmatrix}1\\0\\-1\end{bmatrix} + x_2\begin{bmatrix}2\\1\\3\end{bmatrix} + x_3\begin{bmatrix}4\\2\\6\end{bmatrix}$$

$$\begin{bmatrix}3\\1\\2\end{bmatrix} = \begin{bmatrix}1&2&4\\0&1&2\\-1&3&6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \Rightarrow \left[\begin{array}{ccc:c}1&2&4&3\\0&1&2&1\\-1&3&6&2\end{array}\right] \quad R_3 \to R_3 + R_1$$

$$\left[\begin{array}{ccc:c}1&2&4&3\\0&1&2&1\\0&5&10&5\end{array}\right] \; R_3 \to R_3 - 5R_2 \Rightarrow \left[\begin{array}{ccc:c}1&2&4&3\\0&1&2&1\\0&0&0&0\end{array}\right]$$

$r = 2 \qquad n = 3 \qquad n$:- no. of unknowns.

$r < n$. ~~System is~~

It has infinitely many solution.

iii) $x_1 + 2x_2 + 4x_3 = 3$.

$x_2 + 2x_3 = 1$.

x_3 is free variable and $x_3 = 1$

$x_2 + 2(1) = 1 \Rightarrow x_2 = -2 + 1 \Rightarrow x_2 = -1$

$x_1 - 2 + 4 = 3$.

$x_1 = 1$

w is a subspace spanned by $\{V_1, V_2, V_3\}$

$$w = 1\begin{bmatrix}1\\0\\-1\end{bmatrix} - 1\begin{bmatrix}2\\1\\3\end{bmatrix} + 1\begin{bmatrix}4\\2\\6\end{bmatrix}$$

$$\begin{bmatrix}3\\1\\2\end{bmatrix} = \begin{bmatrix}1-2+4\\0-1+2\\-1-3+6\end{bmatrix}$$

iv) Show that w is the subspace of $\mathbb{R}^4$ spanned by $\{v_1, v_2, v_3\}$ where $w = \begin{bmatrix}-9\\7\\4\\8\end{bmatrix}$ $v_1 = \begin{bmatrix}7\\-4\\-2\\9\end{bmatrix}$ $v_2 = \begin{bmatrix}-4\\5\\-1\\-7\end{bmatrix}$ $v_3 = \begin{bmatrix}-9\\4\\4\\7\end{bmatrix}$

Solution:- $\left[\begin{array}{ccc:c}7&-4&-9&-9\\-4&5&4&7\\-2&-1&4&4\\9&-7&7&8\end{array}\right]$ $R_2 \to R_2 + 4/7 R_1$, $R_3 \to R_3 + 2/7 R_1$, $R_4 \to R_4 - 9/7 R_1$ $\Rightarrow \left[\begin{array}{ccc:c}7&-4&-9&-9\\0&19/7&-8/7&13/7\\0&-15/7&10/7&10/7\\0&-13/7&130/7&130/7\end{array}\right]$ $R_2 \to 7R_2$, $R_3 \to 7R_3$, $R_4 \to 7R_4$

$$\left[\begin{array}{ccc:c}7&-4&-9&-9\\0&19&-8&13\\0&-15&10&10\\0&-13&130&130\end{array}\right] \; R_3 \to -\frac{1}{5}R_3,\; R_2 \leftrightarrow R_3 \Rightarrow \left[\begin{array}{ccc:c}7&-4&-9&-9\\0&3&-2&-2\\0&19&-8&13\\0&-13&\cancel{130}\,132&137\end{array}\right] \; R_3 \to R_3 - \frac{19}{3}R_2,\; R_4 \to R_4 + \frac{13}{3}R_2$$

$$\left[\begin{array}{ccc:c}7&-4&-9&-9\\0&3&-2&-2\\0&0&14/3&77/3\\0&0&\cancel{364}\,70/3&385/3\end{array}\right] \; R_3 \to 3R_3,\; R_4 \to 3R_4 \Rightarrow \left[\begin{array}{ccc:c}7&-4&-9&-9\\0&3&-2&-2\\0&0&14&77\\0&0&\cancel{364}\,70&385\end{array}\right] \; R_4 \to R_4 - \frac{70}{14}R_3$$

$\frac{70}{14}$

$$\left[\begin{array}{ccc:c}7&-4&-9&-9\\0&3&-2&-2\\0&0&14&77\\0&0&0&\cancel{1037}\;0\end{array}\right]$$

$\rho(A) = 3 \qquad \rho(A:w) = 3$

$\rho(A) = \rho(A:w)$

consistent. $x_3 = 5.5 \quad x_2 = 3$

$x_1 = 7.5$.

$\Rightarrow w = 7.5V_1 + 3V_2 + 5.5V_3$

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15) Let $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$ $u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$ & $v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$.

(i) Determine u is in null A ?

~~Let A =~~ ~~(ii)~~ Could u be in column A ?

(iii) Determine v is in column A. Could v be in null A ?

Solution :- (i) Null A = $AX = 0$

$Au = 0$

$$\begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \therefore$$

u is not a solution for $AX = 0$, so u is not a null A.

→ u cannot be a column of A because 4 entries in u. Since Column A is a subspace of R_3.

(ii) $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$: $u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$

$[A : v]$ $\left[\begin{array}{cccc|c} 2 & 4 & -2 & 1 & 3 \\ -2 & -5 & 7 & 3 & -1 \\ 3 & 7 & -8 & 6 & 3 \end{array}\right]$ $R_2 \to R_2 + R_1$
$R_3 \to R_3 - \frac{3}{2} R_1$.

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 1 & 3 \\ 0 & -1 & 5 & 4 & 2 \\ 0 & -5 & -5 & 9/2 & -3/2 \end{array}\right]$$

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It is that $AX = V$ is consistent. So V is a column of A with only ~~It is consistent~~ 3 entries.

~~We~~ V could not be possible to be the null of A Since null A is the subspace of R_4.

16) Find a spanning set for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

Solution $AX = 0$.

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[A:0] \begin{bmatrix} -3 & 6 & -1 & 1 & -7 & : & 0 \\ 1 & -2 & 2 & 3 & -1 & : & 0 \\ 2 & -4 & 5 & 8 & -4 & : & 0 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & -2 & 2 & 3 & -1 & : & 0 \\ -3 & 6 & -1 & 1 & -7 & : & 0 \\ 2 & -4 & 5 & 8 & -4 & : & 0 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 + 3R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\begin{bmatrix} 1 & -2 & 2 & 3 & -1 & : & 0 \\ 0 & 0 & 5 & 10 & -10 & : & 0 \\ 0 & 0 & 1 & 2 & -2 & : & 0 \end{bmatrix} \quad R_2 \to \tfrac{1}{5}R_2 \rightarrow \begin{bmatrix} 1 & -2 & 2 & 3 & -1 & : & 0 \\ 0 & 0 & 1 & 2 & -2 & : & 0 \\ 0 & 0 & 1 & 2 & -2 & : & 0 \end{bmatrix} \quad \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\begin{bmatrix} 1 & -2 & 0 & -1 & 3 & : & 0 \\ 0 & 0 & 1 & 2 & -2 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$n - r$

$5 - 2 = 3$.

x_2, x_4, x_5 are free variables.

$$x_1 - 2x_2 - x_4 + 3x_5 = 0.$$

$$x_3 + 2x_4 - 2x_5 = 0.$$

$$x_1 = 2x_2 + x_4 - 3x_5 \qquad x_3 = -2x_4 + 2x_5.$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2x_2 + x_4 - 3x_5 \\ x_2 \\ -2x_4 + 2x_5 \\ x_4 \\ x_5 \end{bmatrix} \Rightarrow x_2 \underbrace{\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{u} + x_4 \underbrace{\begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}}_{v} + x_5 \underbrace{\begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}}_{w}$$

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$\begin{bmatrix} 1 & -2 & 2 & 3 & -1 \end{bmatrix}$

Every linear combination of u,v,w is an element of all gata, thus ~~ENEw~~ $\{u, v, w\}$ is a space of null A.
Let.

17) Let $A = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 1 & 3 & 4 & -1 \end{bmatrix}$ For each of the following vectors. Determine the vectors are ~~null~~ N(A). Then describe the N(A) of the matrix A.

(a) $\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} -4 \\ -1 \\ 2 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. (d) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

(a) $Au = 0$. $\begin{bmatrix} -3 + 0 + 3 + 0. \\ 0 + 0 + 1 + 0. \\ -3 + 0 + 4 + 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. not a N(A).

(b). $Av = 0$. $\begin{bmatrix} -4 + 0 + 6 - 2 \\ 0 - 3 + 2 + 1 \\ -4 - 3 + 8 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ V is null A N(A).

(c) $Aw = 0$. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ w is null A.

(d) AX $\Rightarrow$ A order 3×4, X order 3×1. } matrix multiplication is not posible.

The size of matrix A is $\overset{n \times m}{3 \times 4}$ and size of matrix X is $\overset{m \times p}{3 \times 1}$. since $m \neq p$, matrix multiplication is not possible.
AX is not defined
so X is not a null of A

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18) Determine whether the following set of vectors is linearly independent or linearly dependent?
If the set is linearly dependent, express one vector in the following set as the linear combination of the other.

Solution: $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 7 \\ 11 \end{bmatrix} \right\}$.

$V_1 \quad V_2 \quad V_3 \quad V_4$.

$$x_1 V_1 + x_2 V_2 + x_3 V_3 + x_4 V_4 = 0.$$

$$[A:B] = \left[\begin{array}{cccc:c} 1 & 1 & -1 & -2 & 0 \\ 0 & 2 & -2 & -2 & 0 \\ -1 & 3 & 0 & 7 & 0 \\ 0 & 4 & 1 & 11 & 0 \end{array}\right] \quad R_3 \to R_3 + R_1$$

$$\left[\begin{array}{cccc:c} 1 & 1 & -1 & -2 & 0 \\ 0 & 2 & -2 & -2 & 0 \\ 0 & 4 & -1 & 5 & 0 \\ 0 & 4 & 1 & 11 & 0 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 2R_2. \end{array}$$

$$\left[\begin{array}{cccc:c} 1 & 1 & -1 & -2 & 0 \\ 0 & 2 & -2 & -2 & 0 \\ 0 & 0 & 3 & 9 & 0 \\ 0 & 0 & 5 & 15 & 0 \end{array}\right] \quad R_2 \to 1/2 R_2 \quad R_3 \to 1/3 R_3 \quad R_4 \to 1/5 R_4$$

$$\left[\begin{array}{cccc:c} 1 & 1 & -1 & -2 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right] \quad R_4 \to R_4 - R_3$$

$$\left[\begin{array}{cccc:c} 1 & 1 & -1 & -2 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$r = n$.
matrix is consistent.

$r = 3 \quad n = 4 \quad n - r = 1$ $\quad x_4$ is free variable
$x_4 = 1$

$x_1 + x_2 - x_3 - 2x_4 = 0.$
$x_2 - x_4 - x_4 = 0$ $\quad x_2 = -2$
$x_3 + 3x_4 = 0$ $\quad x_1 = 1$
$x_3 = -3x_4 = -3$

$0 = 1V_1 - 2V_2 - 3V_3 + 1V_4$

$$\begin{bmatrix}1\\0\\-1\\0\end{bmatrix} - 2\begin{bmatrix}1\\2\\3\\4\end{bmatrix} - 3\begin{bmatrix}-1\\-2\\0\\1\end{bmatrix} + \begin{bmatrix}-2\\-2\\7\\4\end{bmatrix} = 0.$$

$$\begin{bmatrix}-2\\-2\\7\\11\end{bmatrix} = -1\begin{bmatrix}1\\0\\-1\\0\end{bmatrix} + 2\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + 3\begin{bmatrix}-1\\-2\\0\\1\end{bmatrix} \Rightarrow \begin{bmatrix}-1+2-3\\0+4-6\\1+6+0\\0+8+3\end{bmatrix} = \begin{bmatrix}-2\\-2\\7\\11\end{bmatrix},$$

So V_4 is ~~the~~ one vector obtained by linear combination of other 3 vectors.

This set is linearly dependent.

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Basis

Let H be the subspace of the vector space V. An index set of vectors i.e., $B = \{b_1, b_2, b_3, \ldots b_p\}$ in B is a traces for H. If B is linearly independent set and the subspace spanned by B coincides with H. i.e., $H = \text{span}\{b_1, b_2, \ldots b_p\}$

NOTE :- (i) H = V because every vector space is a subspace of itself.

(ii) A basis V is linearly independent set than spans V.

(iii) $H \neq V$ includes the requirement that each of the vectors $b_1, b_2, \ldots b_p$ belongs to H.

The dimension of a vector space :

If V is spanned by finite set then V is said to be finite dimensional and dimension of V is written as dim V.

Is the number of vectors in a basis for H. The dimension of the '0' vector space is written by $\{0\}$ and is defined to be zero.

If V is not spanned by a finite set then V is said to be infinite dimensional.

Ex: $H = \left\{ \begin{bmatrix} a-3b+6c \\ 5a+4d \\ b-2c-d \\ 5a \end{bmatrix} ; a, b, c, d \in R \right\}$

$$H = a\begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 4 \\ -1 \\ 5 \end{bmatrix}$$

v_1 v_2 v_3 v_4.

$H = \{v_1, v_2, v_4\}$ = dim 3.

$v_1 \neq 0$ v_2 is not a multiple of v_1,
v_3 is multiple of v_2 by the spanning set, we discard v_3, then v_4 is not a linear combination of v_1 and v_2, so the set $\{v_1, v_2, v_4\}$ is linearly independent and hence is a basis of H.
Thus dimensional of H is 3.

Standard ~~Beat~~ Basis
The set of n vectors, $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$ ... $e_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$ is the Basis of R^n, this basis is called standard basis for R^n.

16) Find the values of S of H for which the following set of vectors: $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} h \\ 1 \\ h \end{bmatrix}$ $v_3 = \begin{bmatrix} 1 \\ 2h \\ 3h+1 \end{bmatrix}$ is linearly independent.

Solution $x_1v_1 + x_2v_2 + x_3v_3 = 0$.

$$\begin{bmatrix} 1 & h & 1 \\ 0 & 1 & 2h \\ 0 & -h & 3h+1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & h & 1 & : 0 \\ 0 & 1 & 2h & : 0 \\ 0 & -h & 3h+1 & : 0 \end{bmatrix} \quad R_3 \to R_3 + hR_2$$

$$\begin{bmatrix} 1 & h & 1 & : 0 \\ 0 & 1 & 2h & : 0 \\ 0 & 0 & 3h+1+2h^2 & : 0 \end{bmatrix}$$

$2h^2 + 3h + 1 \neq 0$
$2h^2 + 2h + h + 1 \neq 0$
$2h(h+1) + 1(h+1) \neq 0$
$(h+1)(2h+1) \neq 0$
$h = -1, -1/2$

If this is an homogeneous system of equation has only zero solution, $x_1 = x_2 = x_3 = 0$

Then the vectors v_1, v_2, v_3 are linearly independent.

We see that the homogenerous system has zero-solution if and only if, $2h^2+3h+1 \neq 0$, to find out the h value.

Therefore, the vector v_1, v_2, v_3 are linearly independent for all H except -1 and $-1/2$.

17) Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 \\ a \\ 5 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 \\ 4 \\ b \end{bmatrix}$ be the vectors in R^3.

Determine a condition on the scalars. a, b so that the set of vectors $\{v_1, v_2, v_3\}$ is linearly dependent.

18) ~~Solution~~. Find a basis of span S where. ~~S is equal to~~.

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}$$

Solution $x_1 V_1 + x_2 V_2 + x_3 V_3 + x_4 V_4 = 0$.

$$\left[\begin{array}{cccc:c} 1 & -1 & 2 & 1 & 0 \\ 2 & -2 & 6 & 1 & 0 \\ 1 & -1 & -2 & 3 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{array} \Rightarrow \left[\begin{array}{cccc:c} 1 & -1 & 2 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & -4 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc:c} 1 & -1 & 0 & 2 & 0 \\ 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] R_2 \to \tfrac{1}{2} R_2 \Rightarrow \left[\begin{array}{cccc:c} 1 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -\tfrac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix} \right\}$$

Since the above matrix has leading 1^{st} and 3^{rd} column. We conclude that the 1^{st} and 3^{rd} ~~column~~ vector of S forms a basis of span 'S' $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix} \right\}$

dim of vector is 2

19) Let $S=\{v_1, v_2, v_3, v_4, v_5\}$, $v_1=\begin{bmatrix}1\\2\\2\\-1\end{bmatrix}$ $v_2=\begin{bmatrix}1\\3\\1\\1\end{bmatrix}$ $v_3=\begin{bmatrix}1\\5\\-1\\5\end{bmatrix}$ $v_4=\begin{bmatrix}1\\1\\4\\-1\end{bmatrix}$

$v_5=\begin{bmatrix}2\\7\\0\\2\end{bmatrix}$. Find the basis for the span S...

Solution: $x_1V_1 + x_2V_2 + x_3V_3 + x_4V_4 + x_5V_5 = 0$

$$\begin{bmatrix}1&1&1&1&2\\2&3&5&1&7\\2&1&-1&4&0\\-1&1&5&-1&2\end{bmatrix} \quad \begin{matrix}R_2\to R_2-2R_1\\R_3\to R_3-2R_1\\R_4\to R_4+R_1\end{matrix}$$

$$\begin{bmatrix}1&1&1&1&2\\0&1&3&-1&3\\0&-1&-3&2&-4\\0&2&6&0&4\end{bmatrix} \quad \begin{matrix}R_1\to R_1-R_2\\R_3\to R_3+R_2\\R_4\to R_4 2R_2\end{matrix}$$

$$\begin{bmatrix}1&0&-2&2&-1\\0&1&3&-1&3\\0&0&0&1&-1\\0&0&0&2&-2\end{bmatrix}$$

$\Longrightarrow$ V_1, V_2, V_4 will be the basis

$$A=\begin{bmatrix}1&1&1&1&2\\2&3&5&1&7\\2&1&-1&4&0\\-1&1&5&-1&2\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}1&2&2&-1\\1&3&1&1\\1&5&-1&5\\1&1&4&-1\\2&7&0&2\end{bmatrix} \quad \begin{matrix}R_2\to R_2-R_1\\R_3\to R_3-R_1\\R_4\to R_4-R_1\\R_5\to R_5-2R_1\end{matrix}$$

$$A^T=\begin{bmatrix}1&2&2&-1\\0&1&-1&2\\0&3&-3&6\\0&-1&2&0\\0&3&-4&4\end{bmatrix} \quad \begin{matrix}R_3\to R_3-3R_2\\R_4\to R_4+R_2\\R_5\to R_5-3R_2\end{matrix} \Rightarrow \begin{bmatrix}1&2&2&-1\\0&1&-1&2\\0&0&0&0\\0&0&1&2\\0&0&-1&-2\end{bmatrix} \quad R_5\to R_5+R_4$$

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$$A=\begin{bmatrix}1&2&2&1\\0&1&-1&2\\0&0&0&0\\0&0&1&2\\0&0&0&0\end{bmatrix}.$$

$\Rightarrow$ corresponding vectors to the non-zero rows will be basis of the ~~vector~~ S:

$$\left\{\underset{v_1}{\begin{bmatrix}1\\2\\2\\-1\end{bmatrix}}, \underset{v_2}{\begin{bmatrix}1\\3\\1\\1\end{bmatrix}}, \underset{V3}{\begin{bmatrix}1\\1\\4\\1\end{bmatrix}}\right\}$$

$\therefore V_1, V_2, V_4$ will be basis.

$\dim S = 3$.

$w = au+bv$

$T: R^2 \to R^3$

$T: R^3 \to R^4$

$T = R^2 \to R^4$

The linear Transformation

T forms a vector space V into a vector space w is a rule that assigns to each vector v in V, an unique vector T(x) in w, such that,

$$T(u+v) = T(u) + T(V) \rightarrow \text{for every } u = V.$$
$$T(c, u) = cT(u) \rightarrow \text{for every } u \text{ in } V$$
and all scalar c.

NOTE :- Let R^n and R^m be the linear transformation then exist an ~~e~~ unique matrix A. Such that T(~~x~~) ~~and~~ for ~~T(x)~~ every value of x.

Infact, A is ~~abaltering~~ matrix the. n×m. When j^{th} column is the vector. ~~to~~ where e_j is the j^{th} column of the identity matrix in R^n.

For $A [T(e_1) .. T(e_2) .... T(e_n)]$
when $Ax = (T(e_1) .... T(e_n)]$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \quad e_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \quad e_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \qquad L : R^n \rightarrow R^m$$

Q0) Let T from. R^1 to R_2 be a linear transformation.
~~Let u = 35 be the two dimentational vector space.~~
~~suppe~~ ~~S~~ $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ be the 2D vector space

Suppose that $T(u) = T\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$. and $T(v) = T\begin{bmatrix} 3 \\ 5 \end{bmatrix} T = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

$w = \begin{bmatrix} x \\ y \end{bmatrix}$. T(w) in terms of x and y.

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Solution: $w = au + bv$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = a\begin{bmatrix} 1 \\ 2 \end{bmatrix} + b\begin{bmatrix} 3 \\ 5 \end{bmatrix} \qquad x = a + b3, \quad y = 2a + 5b$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \left[\begin{array}{cc:c} 1 & 3 & x \\ 2 & 5 & y \end{array}\right] \quad R_2 \to R_2 - 2R_1$$

$$\left[\begin{array}{cc:c} 1 & 3 & x \\ 0 & -1 & y-2x \end{array}\right]$$

$a + 3b = x \qquad -b = y - 2x \qquad b = 2x - y.$

$a + 6x - 3y - x \Rightarrow \quad a = 3y - 5x. \qquad w = au + bv$

$T(w) = T(au + bv).$

$$T(w) = aT(u) + bT(v) \Rightarrow T\begin{bmatrix} x \\ y \end{bmatrix} = 3y - 5x\, T\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2x - y\, T\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$T\begin{bmatrix} x \\ y \end{bmatrix} = 3y - 5x\begin{bmatrix} -3 \\ 5 \end{bmatrix} + 2x - y\begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9y + 15x + 14 - 7y \\ 15y - 25x + 2x - y \end{bmatrix} \Rightarrow \begin{bmatrix} 29x - 16y \\ -23x + 14y \end{bmatrix}$$

21) Let T from R^2 to R^3 be the linear transformation

$$T\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad T\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$$

Find the matrix representation of T. (with respect to standard basis).

Solution: $e_1 = au + bv$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = a\begin{bmatrix} 3 \\ 2 \end{bmatrix} + b\begin{bmatrix} 4 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{cc:c} 3 & 4 & 1 \\ 2 & 3 & 0 \end{array}\right] \quad R_2 \to R_2 - \tfrac{2}{3}R_1$$

$$\left[\begin{array}{cc:c} 3 & 4 & 1 \\ 0 & 1/3 & -2/3 \end{array}\right] \Rightarrow 3a + 4b = 1$$

$\frac{1}{3}b = -2/3$

$b = -2$

$3a + 4(-2) = 1$

$3a = 1 + 8 = 9$

$a = 9/3 = 3$

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$e_1 = 3u - 2v \qquad e_2 = au + bv \Rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix} = a\begin{bmatrix} 3 \\ 2 \end{bmatrix} + b\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$$\left.\begin{aligned} 3a + 4b &= 0. \\ 2x + 3b &= 1. \end{aligned}\right\} b = 3 \quad a = -4.$$

$e_2 = -4u + 3v.$

$T(e_1) = 3T(u) - 2T(v)$

$T(e_2) = -4T(u) + 3T(v).$

$T(e_2) = 4T\begin{bmatrix} 3 \\ 2 \end{bmatrix} - 2T\begin{bmatrix} 4 \\ 3 \end{bmatrix}.$

$T(e_1) = 3\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix} \qquad Te_1 = \begin{bmatrix} 3 & 0 \\ 6 & 10 \\ 9 & 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 16 \\ 7 \end{bmatrix}.$

$T(e_2) = -4T(u) + 3T(v).$

$T(e_2) = -4T\begin{bmatrix} 3 \\ 2 \end{bmatrix} + 3T\begin{bmatrix} 4 \\ 3 \end{bmatrix} = -4\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 3\begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$

$T(e_2) = \begin{bmatrix} -4 & 10 \\ -8 & -15 \\ -12 & 3 \end{bmatrix} = \begin{bmatrix} -4 \\ -23 \\ -9 \end{bmatrix}$

$A = (T(e_1), T(e_3)) \qquad A = \begin{bmatrix} 3 & -4 \\ 16 & -23 \\ 7 & -9 \end{bmatrix}$

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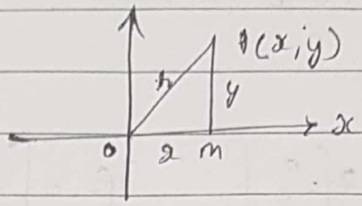
Module 2 :- Complex ~~tr~~ variables.

Def:- If $z = x + iy$ is said to be the complex nos where x and y are the ~~rall~~ real number and $i = \sqrt{-1}$, then $\bar{z} = x - iy$ is the conjugate ~~ro~~ of the complex number

Geometrical Representation:-



$\cos\theta = x/r$
$\sin\theta = y/r$

~~OP = R $\angle NOP = \theta$ ... equall no,~~
$OP = r$ $\angle MOP = \theta$. The point, $p = p(x, y)$ so $\cos\theta = x/r$ and $\sin\theta = y/r$) so $x = r\cos\theta$, $y = r\sin\theta$.

$\therefore z = r(\cos\theta + i\sin\theta)$, i.e, called the polar form of a complex no. The modulus of z is given as

$|z| = r = \sqrt{x^2 + y^2}$ and the argument of $z = \theta = \tan^{-1}(y/x)$ $x \neq 0$.

The properties of z

$z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

$z_1 \pm z_2 = (x_1 \pm x_2) + (y_1 + iy_2)$

$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \left[\frac{x_1 + iy_1}{x_2 + iy_2}\right] \times \frac{x_2 - iy_2}{x_2 - iy_2}$$

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$$= \frac{(x_1 x_2 + y_1 y_2) + i(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2}.$$

$\because \frac{a+ib}{c} = \frac{a}{c} + i\frac{b}{c}$

$\overline{\alpha z_1} = \alpha \underline{x_1} + \underline{i \alpha y_1}$

$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ +

$\overline{z_1 + z_2} = \overline{z_1} \cdot \overline{z_2}$

$|z_1 + z_2| \underline{\leq} |z_1| + |z_2|$

$|z_1 - z_2| \geq |z_1| - |z_2|$