MODULE-4-Probability Theory

Q.No	Questions

1.

a) The Random Variable X has the following probability mass function, find (i) k (ii) $P(X<3)$ (iii) $P(3<X\leq 5)$ (iv) variance

X	0	1	2	3	4	5
P(X)	K	3K	5K	7K	9K	11K

b) A random variable (X= x) has the following probability distributions. Find (i) k (ii) $p(x<6)$ (iii) $p(x>6)$ (iv) Mean and also find the probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

2.

a) Find 'k' such that the following distribution represents a finite probability distribution. Hence find (i)mean (ii) $p(x\leq 1)$ (iii) $p(x>1)$ (iv) $p(-1<x\leq 2)$

X	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

b) A random variable (X=x) has the following probability function for various values of x. Find (i)k (ii) $p(x<1)$ (iii) $p(x>-1)$

X	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	k

3.

a) Find the mean and variance of geometric distributions.

b) 3% of the product produced by a machine is found to be defective. Find the probability that first defective occurs in the (i) 5^{th} item inspected (ii) first five inspected (iii) mean (iv) variance.

4.

a) What is the probability that the marketing representative must select (i) more than 6 people (ii) six people, before he finds one who attended the last home? c.d.f of a Geometric R V with 1-p=0.8.

b) If the probability that a target is destroyed on any one shot is 0.5. What is the probability that it would be destroyed on (i) 6^{th} attempt (ii) more than 6 attempt (iii) Mean (iv) variance?

5.

a) Derive mean and variance for the Poisson Distribution

b) 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses which contains (i) No defective fuse (ii) 3 or more defective fuses (iii) atleast one defective fuse

6.

a) The probability that an individual's suffers a bad reaction from an injection is 0.001. Find the probability that out of 2,000 individual (i) almost 2 (ii) exactly 2 (iii) more than 2 will get bad reaction

b) In a certain factory turning out razor blade, there is a small probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades in a consignment of 10,000 packets

7.

a) A certain screw making machines produces on an average two defective out of 100 and pack them in boxes of 500. Find the probability that the box contains (i)three defectives (ii) At least one defective.

	b) The number of industrial injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0.5. Find the probability that (a) in a particular week there will be: (i) less than 2 accidents, (ii) more than 2 accidents; (b) in a three week period there will be no accidents.
8.	a) The number of misprints on a page of the *Daily Mercury* has a Poisson distribution with mean 1.2. Find the probability that the number of errors (i)on page four is 2 (ii) on page three is less than 3. b) A shop sells a particular make of video recorder. Assuming that the weekly demand for the video recorder is a Poisson variable with mean 3, find the probability that the shop sells (i) at least 3 in a week, (ii) at most 7 in a week, (iii) more than 20 in a month (4 weeks).
9.	a) The number of runs scored by Ali in an innings of a cricket match is distributed according to a Poisson distribution with mean 4.5. Find the probability that he will score: (i) exactly 4 in his next innings; (ii) at least three in his next innings; (iii) at least six in total in his next two innings. b) The number of parasites on fish hatched in the same season and living in the same pond follows a Poisson distribution with mean 3.6. Find, giving your answers to 3 decimal places, the probability that a fish selected at random will have (a) 4 or less parasites, (b) exactly 2 parasites.
10.	a) The number of bacteria in one millilitre of a liquid is known to follow a Poisson distribution with mean 3. Find the probability that a 1 ml sample will contain no bacteria. If 100 samples are taken, find the probability that at most ten will contain no bacteria. b) A van hire firm has twelve vehicles available and has found that demand follows a Poisson distribution with mean 9.5. In a month of 25 working days, on how many days would you expect: (a) demand to exceed supply; (b) all vehicles to be idle; (c) it to be possible to service 3 of the vans?
11.	a) Wireless sets are manufactured with 25 soldered joints each. On the average 1 joint in 500 is defective. How many sets can be expected to be free from defective joints in a consignment of 10,000 sets. b) A continuous random variable has the following density function P(x)= $\begin{cases} kx^2, -3 \leq x \leq 3 \\ 0 \quad elsewhere \end{cases}$ Evaluate (i) k, (ii) P(1$\leq x \leq 2$), (iii) P($x \leq 2$), (iv) P(x>1).
12.	a) A continuous random variable has the density function $f(x) = \frac{k}{1+x^2}$, $-\infty < x < \infty$. Determine (i) k, (ii) $P(x > 0)$, $(iii) P(0 < x < 1)$. b) The probability density function of continuous random variable is given by $(x) = ke^{-\lvert x \rvert}$, $-\infty < x < \infty$. Prove that $k = 1/2$ and also find mean and variance.

13.	a) The probability density function f(x) of continuous random variable is given by $P(x) = \begin{cases} kx(1-x), 0 \le x \le 1 \\ 0 \quad otherwise \end{cases}$ Determine k, $P(0 < x < 1/3)$. b) Derive mean and S.D for the Exponential Distribution.
14.	a) In a certain town the duration of a shower is exponentially distributed with mean 5min. what is the probability that the shower will last for (i) 10min or more (ii) less than 10min (iii) between 10 to 12min. b) The life of a compressor manufactured by a company is known to be 200 months on an average following an exponential distribution. Find the probability that the life of a compressor of that company is (i) < 200 months, (ii) between 100 months and 25 years.
15.	a) Studies of a single-machine-tool system showed that the time the machine operates before breaking down is exponentially distributed with a mean 10 hours. 1. Determine the failure rate and the reliability. 2. Find the probability that the machine operates for at least 12 hours before breaking down. 3. If the machine has already been operating 8 hours, what is the probability that it will last another 4 hours? b) The sale per day in a shop is exponentially distributed with an average sale amounting to Rs.100 and net profit is 8%. Find the probability that the net profit exceeds Rs. 30 on a day.
16.	a) Derive mean and variance for normal distribution. b) In examination 7% of students score less than 35% marks and 89% of students score less than 60% marks, Find the mean and standard deviation, if the marks are normally distributed. It is given that if p (z) =$\frac{1}{\sqrt{2\pi}}\int_0^z e^{-z^2/2}\,dz$ then p (1.2263) = 0.39 p(1.4757) = 0.43.
17.	a) In a normal distribution, 7% of the items are under 35 and 89% are under 63. Determine the mean and variance of the distribution. b) Suppose the weights of 800 male students are normally distributed with mean 140 pounds and S.D 10 pounds. Find the number of students whose weight are (i) between 138 and 148 pounds (ii) more than 152 pounds.
18.	a) A sales tax officer has reported that the average sales of the 500 business that he has to deal with during a year is Rs.36,000 with a standard deviation of 10,000. Assuming that the sales in these business are normally distributed, find (i) the number of business as the sales of which are Rs. 40,000. (ii) the percentage of business the sales of which are likely to range between Rs.30,000 and Rs. 40,000. b) A manufacturer knows from experience that the resistance of resistors he produces is normal with mean 100 ohms and SD 2 ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms?
19.	a) A sample of 100 dry battery cells tested to find the length of life produced by a company and following results are recorded: mean life is 12 hrs, SD is 3 hrs. Assuming data to be normally distributed, find the expected life of a dry cell. (i) have more than 15 hrs (ii) between 10 and 14 hrs.[P(0.667)=0.2486,P(1)=0.3413]. b) The mean weight of 1,000 students during medical examination was found to be 70kg and S.D weight 6kg. Assume that the weight are normally distributed, find the number of students having weight (i) less than 65kg (ii) more than 75kg (iii) between 65kg to 75kg.[P(0.83)=0.2967]
20.	a) Given that the mean height of students in a class is 158cms with SD of 20cms. Find how many students heights lie between 150cms and 170cms, if there are 100 students in the class. b) Suppose 2% of the people on the average are left handed. Find (i) the probability of finding 3 or more left handed (ii) the probability of finding ≤ 1 left handed.

MCQ

Question Number	Question
1.	The mean of the Geometric distribution is (a) Pq (b) np (c) $\sqrt{np}$ (d) $\frac{q}{p}$
2.	If the mean of the poisson distribution is m, then S.D of this distribution is $(a) m^2$ (b) $\sqrt{m}$ (c) m (d) none of these
3.	The S D of the Geometric distribution is (a) $\sqrt{npq}$ (b) $\sqrt{np}$ (c) $\frac{\sqrt{q}}{p}$ (d) pq
4.	The mean and variance of a poisson distributions are (a) same (b) 0 (c)different (d) none of these
5.	The mean of the poisson distribution is (a) m (b) $\sqrt{m}$ (c) np (d) none of these
6.	The mean of the exponential distribution is (a) $\frac{1}{\alpha}$ (b) $\frac{1}{\propto^2}$ (c) $\frac{1}{\sqrt{\propto}}$ (d) $\propto$
7.	The S.D of the exponential distribution is (a) $\frac{1}{\alpha}$ (b) $\frac{1}{\propto^2}$ (c) $\frac{1}{\sqrt{\propto}}$ (d) $\propto$
8.	The p.d .f of a continuous random variable is $f(x) = {}^{k}/_{x^3}$,$5 \le x \le 10$; 0 elsewhere, then the value of k is (a)1 (b) 50 (c) 200/3 (d) 200
9.	The marks obtained by the students were found normally distributed with mean 75 and variance 100, the percentage of students who scored more than 75 marks is (a) 70% (b) 50% (c) 25% (d) 65%
10.	The variance of poisson distribution with parameter $\lambda = 2$ is (a)4 (b) 2 (c) 0 (d) 1
11.	The area under the whole normal curve is (a) 1 (b) 0.5 (c) -0.5 (d) 0