**MODULE 1. LINEAR ALGEBRA AND GRAPH THEORY**

| **Q.No** | **QUESTION BANK** |
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| 1. | **a)** Define a vector space and a subspace of a vector space. Give an example each.<br>**b)** Let $V$ be a vector space over the field $F$. Show that the intersection of any collection of subspaces of $V$ is a subspace of $V$. |
| 2. | **a)** Define linearly dependent and linearly independent set. If $W_1$ and $W_2$ are finite-dimensional subspaces of a vector space $V$, then prove that $W_1 + W_2$ is finite-dimensional and $dimW_1 + dimW_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.<br>**b)** Express the vector $b = \begin{bmatrix} 2 \\ 13 \\ 6 \end{bmatrix}$ as a linear combination of the vectors, $v_1 = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$. |
| 3. | **a)** Write the vector $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ as a linear combination of the vectors, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$.<br>**b)** Determine whether the following set of vectors is linearly independent or linearly dependent. If the set is linearly dependent, express one vector in the following set as a linear combination of the other: $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 7 \\ 11 \end{bmatrix} \right\}$ |
| 4. | **a)** Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ a \\ 5 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 4 \\ b \end{bmatrix}$ be vectors in $\mathbb{R}^3$. Determine a condition on the scalars $a, b$ so that the set of vectors $\{v_1, v_2, v_3\}$ is linearly dependent.<br>**b)** Find the value(s) of $h$ for which the following set of vectors $\left\{ v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} h \\ 1 \\ -h \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 2h \\ 3h+1 \end{bmatrix} \right\}$ is linearly independent. |
| 5. | **a)** Define the basis for a vector. Find a basis for span(s), where $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}$.<br>**b)** Let $S = \{v_1, v_2, v_3, v_4, v_5\}$ where $v_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 5 \\ -1 \\ 5 \end{bmatrix}$, $v_4 = \begin{bmatrix} 1 \\ 1 \\ 4 \\ -1 \end{bmatrix}$, $v_5 = \begin{bmatrix} 2 \\ 7 \\ 0 \\ 2 \end{bmatrix}$. Find a basis for the span(s). |
| 6. | **a)** Define the null space $\mathcal{N}(A)$ and the range $\mathcal{R}(A)$ of the matrix $A$. Find a basis of the null space of the matrix $B = \begin{bmatrix} 1 & 1 & 2 \\ -2 & -2 & -4 \end{bmatrix}$.<br>**b)** Define the rank and the nullity of a matrix. Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 3 & 2 \\ 1 & 3 & 2 \end{bmatrix}$. Find a basis for the range $\mathcal{R}(A)$ of $A$ that consists of columns of $A$ and also find the rank and nullity of the matrix $A$. |

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| 7. | **a)** Let $A = \begin{bmatrix}1 & 0 & 3 & -2\\ 0 & 3 & 1 & 1\\ 1 & 3 & 4 & -1\end{bmatrix}$. For each of the following vectors, determine whether the vectors are in the null space $\mathcal{N}(A)$. (a) $\begin{bmatrix}-3\\ 0\\ 1\\ 0\end{bmatrix}$ (b) $\begin{bmatrix}-4\\ -1\\ 2\\ 1\end{bmatrix}$ (c) $\begin{bmatrix}0\\ 0\\ 0\\ 0\end{bmatrix}$ (d) $\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$<br>Then describe the null space $\mathcal{N}(A)$ of the matrix $A$.<br>**b)** Let $A = \begin{bmatrix}1 & 2 & 1\\ 3 & 6 & 4\end{bmatrix}$ and let $a = \begin{bmatrix}-3\\ 1\\ 1\end{bmatrix}, b = \begin{bmatrix}-2\\ 1\\ 0\end{bmatrix}, c = \begin{bmatrix}1\\ 1\end{bmatrix}$. For each of the vectors $a, b, c$, determine whether the vectors are in (i) the null space $\mathcal{N}(A)$ and (ii) for the range $\mathcal{R}(A)$. |
| 8. | **a)** Define the following:<br>i. linear transformation<br>ii. kernel of a linear transformation<br>iii. range of a linear transformation<br>iv. rank of a linear transformation<br>v. nullity of a linear transformation<br>**b)** Prove that the null space of an $m \times n$ matrix $A$ is a subspace of $\mathbb{R}^n$.<br>**(OR)**<br>Prove that the set of all solutions to a system $Ax = 0$ of $m$ homogeneous linear equations in $n$ unknowns is a subspace of $\mathbb{R}^n$. |
| 9. | **a)** Let $V$ and $W$ be vector spaces over the field $F$ and let $T$ be a linear transformation from $V$ into $W$. Suppose that $V$ is finite-dimensional, then prove that $rank(T) + nullity(T) = dimV$.<br>**b)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation. Let $u = \begin{bmatrix}1\\ 2\end{bmatrix}, v = \begin{bmatrix}3\\ 5\end{bmatrix}$ be 2-dimensional vectors. Suppose that $T(u) = T\left(\begin{bmatrix}1\\ 2\end{bmatrix}\right) = \begin{bmatrix}-3\\ 5\end{bmatrix}$ and $T(v) = T\left(\begin{bmatrix}3\\ 5\end{bmatrix}\right) = \begin{bmatrix}7\\ 1\end{bmatrix}$. Let $w = \begin{bmatrix}x\\ y\end{bmatrix} \in \mathbb{R}^2$. Find the formula for $T(w)$ in terms of $x$ and $y$. |
| 10. | **a)** Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $T(e_1) = \begin{bmatrix}1\\ 0\end{bmatrix}, T(e_2) = \begin{bmatrix}0\\ 1\end{bmatrix}, T(e_3) = \begin{bmatrix}1\\ 0\end{bmatrix}$, where $e_1, e_2, e_3$ are the standard basis for $\mathbb{R}^3$. Then find the rank and nullity of $T$.<br>**b)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T\left(\begin{bmatrix}3\\ 2\end{bmatrix}\right) = \begin{bmatrix}1\\ 2\\ 3\end{bmatrix}, T\left(\begin{bmatrix}4\\ 3\end{bmatrix}\right) = \begin{bmatrix}0\\ -5\\ 1\end{bmatrix}$.<br>i) Find the matrix representation of $T$ (with respect to the standard basis for $\mathbb{R}^2$).<br>ii) Determine the rank and nullity of $T$. |

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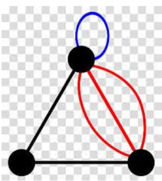
| 11 | a) | Define a graph. Draw the diagram of a graph where the degrees of vertices are 1,1,1,2,3,4,5,7. |
| | b) | Define simple and multi graph with an example. |
| 12 | a) | Show that in a graph the number of vertices of odd degree is even. |
| | b) | Define degree of a vertex. State Handshaking property. |
| 13 | a) | Show that every simple graph has at least two vertices of the same degree. |
| | b) | Indicate the degree of each vertex and verify the handshaking property.  |

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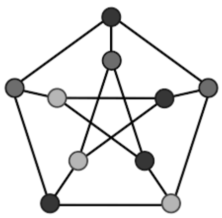
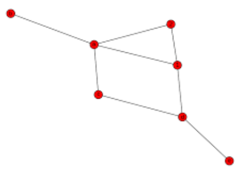
| 14 | a) | Show that every cubic graph has an even number of vertices. |
| | b) | Explain a Regular graph with an example. |
| 15 | a) | Discuss the Konigsberg bridge problem. |
| | b) | Define with an example: (a) Subgraph of a graph (b) spanning sub graph (c) Complement of a graph. |
| 16 | a) | Define complete bipartite graph. Find the complement of the complete bipartite graph $K_{3,3}$. |
| | b) | Define Bipartite graph. Show that the complement of a bipartite graph need not be a bipartite graph. |
| 17 | a) | Define Pendant and isolated vertex with an example |
| | b) | Draw the complete graphs $K_2, K_3, K_4, K_5 \text{and} K_6$. |
| 18 | a) | Define Eulerian trail and Eulerian circuit with an example. |
| | b) | Define Hamiltonian path and Hamiltonian cycle with an example. |
| 19 | a) | Explain Adjacency matrix and incidence matrix with an example. |
| | b) | Draw the graph represented by the given adjacency matrices:<br>i) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ |
| 20 | a) | Write the adjancency matrix for the following graphs<br>i)<br><br>ii)<br> |
| | b) | Write the incidence matrix for the above graphs. |