Q.1 b. Given: $V_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ $V_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}$

$V_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ & $y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$

i) Consider $A = [V_1 \ V_2 \ V_3]$

ii) Then $[A : y] = \left[\begin{array}{ccc:c} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & h \end{array}\right]$

$R_2 + R_1$, $R_3 + 2R_1$

$$\left[\begin{array}{ccc:c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 3 & -6 & h-8 \end{array}\right]$$

$\therefore$ $R_3 - 3R_2$

$$\left[\begin{array}{ccc:c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & h-5 \end{array}\right]$$

Now $Ax = y$ will have solⁿ if

rank (A) = rank $[A : y]$

Now rank $(A) = 2$

rank $[A : y] = 2$ if $h - 5 = 0$

$\therefore$ $h = 5$

$\therefore$ for $h = 5$ $Ax = y$ will have solⁿ

$\therefore$ $[V_1 \ V_2 \ V_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = y$ if $h = 5$

$\therefore$ for $h = 5$ $\exists\ x_1, x_2, x_3$

$x_1 v_1 + x_2 v_2 + x_3 v_3 = y$

$\therefore$ $y \in \text{Span}\{v_1, v_2, v_3\}$ if $\underline{h = 5}$

2a) Let $v = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$

Let $a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + c \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1 & 2 & 2 \\ 0 & -2 & 0 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

$\therefore$ $R_3 + \frac{R_2}{2}$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1/2 \end{bmatrix}$$

$\therefore$ $a + 2b + 2c = 1$

$-2b = 3$

$4c = 1/2$

$\therefore$ $c = 1/8$ $\quad b = -3/2$

$\therefore$ $a - 3 + \frac{1}{4} = 1$

$\therefore$ $a = 4 - \frac{1}{4} = \frac{15}{4}$

$$\therefore \frac{15}{4} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \frac{3}{2} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + \frac{1}{8} \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

2b) Can be solved like previous problem.

3a) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $v_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$

& $w = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

i) Clearly $w \neq v_i$, $i = 1, 2, 3$

$\therefore w \notin \{v_1, v_2, v_3\}$

There are 3 vectors in $\{v_1, v_2, v_3\}$

ii) There are infinitely many vectors in the span $\{v_1, v_2, v_3\}$ ($\because \exists$ atleast one $v_i \neq 0 \Rightarrow k v_i \in \text{span}\{v_1, v_2, v_3\}, \forall k \in \mathbb{R}$
$\Rightarrow$ uncountably many $v_i \in \text{span}\{v_1, v_2, v_3\}$)

iii) Let if possible

$a v_1 + b v_2 + c v_3 = w$

$\Rightarrow [v_1 \; v_2 \; v_3] \begin{bmatrix} a \\ b \\ c \end{bmatrix} = w$

$\Rightarrow AX = w$ where $A = [v_1, v_2, v_3]$
$X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$\therefore$ Consider $[A : w] = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{array}\right]$

$R_3 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 5 & 10 & 5 \end{array}\right]$$

$R_3 - 5R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ rank $[A]$ = rank $[A|w] = 2$

$\Rightarrow$ For $Ax = w$ solⁿ exists $\Rightarrow$ w is in the span $\{v_1, v_2, v_3\}$

$\Rightarrow$ and also

$$a + 2b + 4c = 3$$
$$b + 2c = 1$$

put $c = t$

$$b = 1 - 2t$$
$$\therefore a + 2 - 4t + 4t = 3$$
$$\therefore a = 1$$

$\therefore$ $\begin{bmatrix} 1 \\ 1-2t \\ t \end{bmatrix}$ is the solⁿ set of $Ax = w$.

$\therefore$ for $t = 1$ we have $x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$$\therefore 1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ w is in the spanned subspace of $\{v_1, v_2, v_3\}$

Q.4b

$$N(A) = \{ X \,/\, AX = 0 \}$$

Consider $AX = 0$

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 + R_1/3 \qquad R_3 + 2R_1/3$

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 0 & 0 & 5/3 & 10/3 & -10/3 \\ 0 & 0 & 13/3 & 26/3 & -26/3 \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - \frac{13}{5} R_2$

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 0 & 0 & 5/3 & 10/3 & -10/3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$

put $x_1 = t, \; x_4 = r, \; x_5 = s$

$$-3t + 6x_2 - x_3 + r - 7s = 0$$

$$\frac{5x_3}{3} + \frac{10r}{3} - \frac{10s}{3} = 0$$

$$\therefore \; x_3 = 2(s - r)$$

$$\therefore \; -3t + 6x_2 - 2(s - r) + r - 7s = 0$$

$$6x_2 = 3t + 9s - 3r$$

$$x_2 = \underline{t + 3s - r}$$

$\therefore N(A) = \left\{ \begin{bmatrix} t \\ \frac{t+3s-r}{2} \\ 2(s-r) \\ r \\ s \end{bmatrix} \Big/ t, s, r \in \mathbb{R} \right\}$

$= \left\{ t \begin{bmatrix} 1 \\ 1/2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} 0 \\ -1/2 \\ -2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 3/2 \\ 2 \\ 0 \\ 1 \end{bmatrix} \Big/ t; s, r \in \mathbb{R} \right\}$

$= \text{span} \left\{ \begin{bmatrix} 1 \\ 1/2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1/2 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3/2 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$.

5 a) Given $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & 8 & 6 \end{bmatrix}$

$u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$

i) A is a 3×4 matrix & u is 4×1 matrix

consider $Au = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & 8 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ -13 \end{bmatrix}$

$\therefore Au \neq 0 \Rightarrow u$ is not in Nul(A)

$\therefore$ Col A is subspace of $\mathbb{R}^3$ & $u \in \mathbb{R}^4$

$\therefore u \notin$ Col A

ii consider $Ax = v$

If x exist $\Rightarrow$ v is in Col A

$\therefore$ consider $[A|V]$

$$= \left[\begin{array}{cccc|c} 2 & 4 & -2 & 1 & 3 \\ -2 & -5 & 7 & 3 & -1 \\ 3 & 7 & -8 & 6 & 3 \end{array}\right]$$

$R_2 + R_1$, $R_3 - \frac{3R_1}{2}$

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 1 & 3 \\ 0 & -1 & 5 & 4 & 2 \\ 0 & 1 & -5 & 9/2 & -3/2 \end{array}\right]$$

$\therefore$ $R_3 + R_2$

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 1 & 3 \\ 0 & -1 & 5 & 4 & 2 \\ 0 & 0 & 0 & 17/2 & -1/2 \end{array}\right]$$

$\therefore$ rank $[A|V]$ = rank(A) = 3

$\therefore$ Soln x exist $\Rightarrow$ V is in Col A

But $Nul(A) \subseteq \mathbb{R}^4$ & $v \in \mathbb{R}^3$

$\therefore$ $v \notin Nul(A)$

b) Let $A = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 1 & 3 & 4 & -1 \end{bmatrix}$

a) $u_1 = \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\therefore$ $Au_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$u_1 \notin Nul(A)$

b) $u_2 = \begin{bmatrix} -4 \\ -1 \\ 2 \\ 1 \end{bmatrix}$ $\therefore Au_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\therefore u_2 \in Nul(A)$.

c) $u_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\therefore Au_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\therefore u_3 \in Nul(A)$

d) $u_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^3$ & $Nul(A) \subseteq \mathbb{R}^4$

$\therefore u_4 \notin Nul(A)$

Consider $AX = 0$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 1 & 3 & 4 & -1 \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 0 & 3 & 1 & 1 \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ Rank $(A) = 2$

$\Rightarrow$ Nul (A) has dim 2

put $x_2 = t$ $x_3 = r$

$3t + r + x_4 = 0$

$\Rightarrow x_4 = -(3t + r)$

$\therefore x_1 = -3r + (2)(-3t - r)$

$= -6t - 5r$

$$\therefore Nul(A) = \left\{ \begin{bmatrix} -6t - 5r \\ t \\ r \\ -(3t + r) \end{bmatrix} \middle/ t, r \in \mathbb{R} \right\}$$

6 b) Let $A = [V_1 ; V_2 ; V_3 , V_4]$

$$= \begin{bmatrix} 1 & 1 & -1 & -2 \\ 0 & 2 & -2 & -2 \\ -1 & 3 & 0 & 7 \\ 0 & 4 & 1 & 11 \end{bmatrix}$$

$R_3 + R_1$

$$\begin{bmatrix} 1 & 1 & -1 & -2 \\ 0 & 2 & -2 & -2 \\ 0 & 4 & -1 & 5 \\ 0 & 4 & 1 & 11 \end{bmatrix}$$

$R_3 - 2R_2 , \; R_4 - 2R_2$

$$\begin{bmatrix} 1 & 1 & -1 & -2 \\ 0 & 2 & -2 & -2 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 5 & 15 \end{bmatrix}$$

$\therefore \; R_4 - \frac{5R_3}{3}$

$$\begin{bmatrix} 1 & 1 & -1 & -2 \\ 0 & 2 & -2 & -2 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank $A = 3 < 4$.

i.e. $\{V_1, V_2, V_3, V_4\}$ are linearly dep.

7 a) $V_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} 1 \\ a \\ 5 \end{bmatrix}$ $V_3 = \begin{bmatrix} 0 \\ 4 \\ b \end{bmatrix}$

If V_1, V_2, V_3 are linearly dep then

$$\begin{vmatrix} 1 & 1 & 0 \\ 2 & a & 4 \\ 0 & 5 & b \end{vmatrix} = 0$$

$\Rightarrow (ab - 20) - (2b) = 0$

$\Rightarrow ab - 2b = 20$

$\Rightarrow b(a-2) = 20$

$\therefore a - 2 = \frac{20}{b}, \; b \neq 0$

$\therefore a = \frac{20}{b} + 2, \; b \neq 0$

is the cond^n on a, b for V_1, V_2, V_3 to be li...

8b & 9b can be solved in same manner.

9a) Given: V is a v.s

& $\mathcal{B} = \{b_1, b_2, \ldots b_n\}$ is a basis,

T.P then any set in V containing more than n vectors is lin. dep.

Let $S = \{v_1, v_2 \ldots v_m\}$ $n > m$ and all v_i are distinct

$\therefore$ $\mathcal{B}$ is a basis of V

$\exists$ scalars a_{ij} in F s.t

$$v_j = \sum_{i=1}^{n} a_{ij} b_i$$

For any m scalars $c_1, c_2 \ldots c_m$

we ~~have~~ Let

$$0 = c_1 v_1 + \ldots c_m v_m = \sum_{j=1}^{m} c_j v_j$$

$$= \sum_{j=1}^{m} c_j \sum_{i=1}^{n} a_{ij} b_i$$

$$= \sum_{i=1}^{m} \left[\sum_{j=1}^{m} (a_{ij} c_j) \right] b_i \Rightarrow \sum_{j=1}^{m} a_{ij} c_j = 0 \quad \forall i$$

But if A is $n \times m$ matrix & $n < m$, then homogeneous system of eqns $AX = 0$ has a non trivial solution

$\Rightarrow$ Not all c_j are zero

$\Rightarrow$ $v_1 \ldots v_m$ are linearly dependent

7 b) Let $v_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $v_2 = \begin{bmatrix}h\\1\\-h\end{bmatrix}$ $v_3 = \begin{bmatrix}1\\2h\\3h+1\end{bmatrix}$

are linearly indep

$$\Rightarrow \begin{vmatrix}1 & h & 1\\ 0 & 1 & 2h \\ 0 & -h & 3h+1\end{vmatrix} \neq 0$$

$\Rightarrow (3h+1) + 2h^2 \neq 0$

$\Rightarrow 2h^2 + 3h + 1 \neq 0$

$\Rightarrow 2h^2 + 2h + h + 1 \neq 0$

$\Rightarrow (2h+1)(h+1) \neq 0$

$\Rightarrow h \neq -\frac{1}{2}$ or $h \neq -1$ is the cond^n on h for v_1, v_2 & v_3 to be lin indep.

8 a) $S = \left\{ \begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}-1\\-2\\-1\end{bmatrix}, \begin{bmatrix}2\\6\\-2\end{bmatrix}, \begin{bmatrix}1\\1\\3\end{bmatrix} \right\}$

$\therefore$ Consider $A = \begin{bmatrix}1 & 2 & 1\\ -1 & -2 & -1\\ 2 & 6 & -2 \\ -1 & 1 & 3\end{bmatrix}$

$R_2 + R_1$; $R_3 - 2R_1$; $R_4 - R_1$

$$\begin{bmatrix}1 & 2 & 1\\ 0 & 0 & 0\\ 0 & 2 & -4\\ 0 & -1 & 2\end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$$\begin{bmatrix}1 & 2 & 1\\ 0 & -1 & 2\\ 0 & 2 & -4\\ 0 & 0 & 0\end{bmatrix} \xrightarrow{R_3 + 2R_2} \begin{bmatrix}1 & 2 & 1\\ 0 & -1 & 2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$\therefore$ basis for span(S) is $\left\{ \begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}0\\-1\end{bmatrix} \right\}$

& dim = 2

10) a) Given : $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a L.T.

$$T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}-3\\5\end{bmatrix} \quad T\left(\begin{bmatrix}3\\5\end{bmatrix}\right) = \begin{bmatrix}7\\1\end{bmatrix}$$

Let $\begin{bmatrix}x\\y\end{bmatrix} = a\begin{bmatrix}1\\2\end{bmatrix} + b\begin{bmatrix}3\\5\end{bmatrix}$

$$\therefore \begin{bmatrix}1 & 3\\2 & 5\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$$

$\therefore R_2 - 2R_1$

$$\begin{bmatrix}1 & 3\\0 & -1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}x\\y-2x\end{bmatrix}$$

$\therefore -b = y-2x \qquad \therefore a+3b = x$

$\therefore b = 2x-y \qquad \therefore a = x - 6x + 3y$

$\qquad\qquad\qquad\qquad = -5x+3y$

$$\therefore \begin{bmatrix}x\\y\end{bmatrix} = (-5x+3y)\begin{bmatrix}1\\2\end{bmatrix} + (2x-y)\begin{bmatrix}3\\5\end{bmatrix}$$

$$\therefore T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = [-5x+3y]\, T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) + (2x-y)\, T\left(\begin{bmatrix}3\\5\end{bmatrix}\right)$$

$$= (-5x+3y)\begin{bmatrix}-3\\5\end{bmatrix} + (2x-y)\begin{bmatrix}7\\1\end{bmatrix}$$

$$= \begin{bmatrix}29x-16y\\-23x+14y\end{bmatrix}$$

b) $T: \mathbb{R}^2 \to \mathbb{R}^3$ is a L.T

s.t $T\left(\begin{bmatrix}3\\2\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix}$ & $T\left(\begin{bmatrix}4\\3\end{bmatrix}\right) = \begin{bmatrix}0\\-5\\1\end{bmatrix}$

Let $a\begin{bmatrix}3\\2\end{bmatrix} + b\begin{bmatrix}4\\3\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

$\Rightarrow 3a + 4b = 1$

$2a + 3b = 0$

$\Rightarrow a + b = 1$

$\Rightarrow a = 1 - b$

$\therefore 3(1-b) + 4b = 1$

$\therefore 3 + b = 1$ $\quad\therefore \begin{bmatrix}1\\0\end{bmatrix} = 3\begin{bmatrix}3\\2\end{bmatrix} - 2\begin{bmatrix}4\\3\end{bmatrix}$

$\therefore b = -2$

$a = 3$

$\therefore T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = 3\,T\left(\begin{bmatrix}3\\2\end{bmatrix}\right) - 2\,T\left(\begin{bmatrix}4\\3\end{bmatrix}\right)$

$= 3\begin{bmatrix}1\\2\\3\end{bmatrix} - 2\begin{bmatrix}0\\-5\\1\end{bmatrix} = \begin{bmatrix}3\\16\\7\end{bmatrix}$

$\therefore \cancel{T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) =}$

IIIly Let $\begin{bmatrix}0\\1\end{bmatrix} = c\begin{bmatrix}3\\2\end{bmatrix} + d\begin{bmatrix}4\\3\end{bmatrix}$

$\Rightarrow 3c + 4d = 0$

$2c + 3d = 1$ $\quad\therefore \begin{bmatrix}0\\1\end{bmatrix} = -4\begin{bmatrix}3\\2\end{bmatrix} + 3\begin{bmatrix}4\\0\end{bmatrix}$

$\Rightarrow c = \frac{-4d}{3}$

$\therefore 2\left(\frac{-4d}{3}\right) + 3d = 1$ $\quad\therefore T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$

$\therefore -8d + 9d = 3$ $\quad = -4\,T\left(\begin{bmatrix}3\\2\end{bmatrix}\right) + 3\,T\left(\begin{bmatrix}4\\3\end{bmatrix}\right)$,

$\therefore d = 3$

$\therefore c = -4$

$$= -4\begin{bmatrix}1\\2\\3\end{bmatrix} + 3\begin{bmatrix}0\\-5\\1\end{bmatrix}$$

$$= \begin{bmatrix}-4\\-23\\-9\end{bmatrix}$$

$\therefore$ The matrix representatⁿ of T wrt std basis is

$$A = \begin{bmatrix}3 & -4\\16 & -23\\7 & 9\end{bmatrix}$$