

# Methods to simulate the effects of sex ratios on ecosystems

## Summary

In nature, some species deviate from the uniform sex ratio, and exhibit an adaptive sex ratio variation. In order to study this phenomenon and explore the impact of the change in the sex ratio of lampreys on its population and the larger ecosystem, our team built the following mathematical models:

Firstly, we investigated the impact of the change in the sex ratio of lampreys **on its population and larger ecosystem**. For its population, we established a differential equation model based on the **Logistic equation** to predict the number of lampreys offspring. We consider the **number of offspring** from lampreys populations as a visualization factor for the effects of changes in the sex ratio of lampreys on their populations. Then we collected data on the change of the sex ratio of lampreys near the Great Lakes in the United States from 2012 to 2017 and used the counted **Least Square Method** to fit the model with the data to obtain the required growth rate parameters. For larger ecosystems, we regarded lampreys as predators in the ecosystem and used the **Lotka-Volterra Model** to judge the relationship between lampreys and their prey. The changes in the population numbers of predators and prey over time reflected the impact of changes in the sex ratio of lampreys on the ecosystem dynamics.

Secondly, we used the **Lotka-Volterra Model** and **Topsis Model** to construct a dynamic evaluation model of the lampreys population, and obtained the advantages and disadvantages of the lampreys population. Taking lampreys as **predators and prey** as two situations, we applied the predator model respectively and obtained the population dynamics models of lampreys and food under different sex ratios. We simulated lampreys that cannot perform sex transition, took them and lampreys that can perform sex transition as two evaluation objects, determined seven evaluation indicators for **Topsis analysis**, and obtained their weights by using **Analytic Hierarchy Process**. Last, the optimal and the worst solutions are determined and the intensity and similarity between the population scheme and the optimal and the worst solutions are calculated.

Finally, we selected changes in the prey lamprey population (such as plankton) within the ecosystem as an indicator of ecosystem stability. By incorporating variations in predator sex ratios using the **Lotka-Volterra Model** and examining how alterations in lamprey sex ratios impact prey dynamics, we employed the **Monod Model** to predict and estimate prey growth rates. Subsequently, **phase trajectories** were utilized to analyze the temporal patterns of predator and prey populations, assessing their stability, periodicity, or chaotic behavior within ecosystems. Following the previous Lotka-Volterra model, we developed a simplified model that considers the effect of sex ratio on the reproductive rate of lampreys (parasites) and the interactions between parasites and hosts (other large fish).

**Keywords:** Logistic   Least Square Method   Lotka-Volterra Model   Topsis Model  
Analytic Hierarchy Process   Monod Model

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# 1 Introduction

## 1.1 Problem Background

Most species have a distinction between males and females at the time of fertilization and exhibit a 1:1 sex ratio at birth, while few exhibit "adaptive sex ratio variation" in which the sex changes spontaneously based on acquired factors, such as the temperature of the nest incubating eggs of the American alligator influences the sex ratios at birth. Such changes significantly influence the resulting sex ratio.

When studying the species of lampreys, American biologists often observe the strange phenomenon that a group of adult lampreys are almost all male or all female. Through in-depth study, they find that the sex determination in sea lamprey is influenced by larval growth rate, which is affected by various environmental factors, such as food resources and environmental temperature. Given the multifaceted and pivotal roles lampreys assume within diverse ecosystems, comprehending the ramifications of their sex transformations on both their population and the larger ecosystem is of vital importance.

## 1.2 Question Restatement

- According to the requirements, we should first develop a mathematical model to predict how the larger ecosystem will change in the sex ratio of lamprey populations.
- Secondly, we need to discuss the advantages and the disadvantages to the lampreys population.
- We proceed to deepen inquiry under the discussion of question one, considering the impact on the stability of the ecosystem by the lampreys' changes of sex ratio. The "stability" factor should be focused separately.
- Finally, we are required to study whether ecosystems with changing sex ratios in lamprey populations provide an advantage to other species in the ecosystem, such as parasites.

## 1.3 Our Work

In this project, we primarily centered our efforts around the **Lotka-Volterra Model**, establishing four distinct models: **Model 1 Lampreys Growth Model**, **Model 2 Dynamic Evaluation Model**, **Model 3 Plankton's Growth Rate Predicting Model**, and **Model 4 Parasitic Ecosystem Model**.

We employed the **Least Squares Method** for numerical fitting and the **Analytic Hierarchy Process (AHP)** to determine the weights of assessment indicators, which were then analyzed and evaluated using the **Topsis** method. Furthermore, we conducted a

sensitivity analysis for each model and provided an evaluation of their strengths and weaknesses.

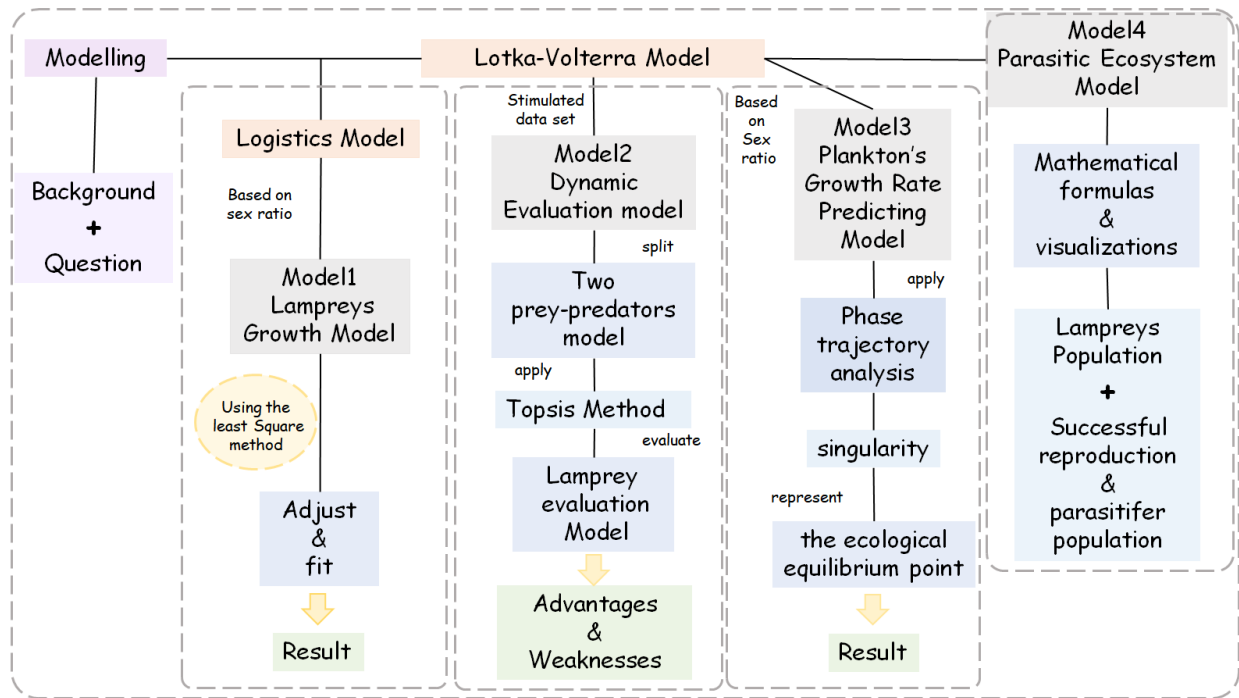


Figure 1: flow chart

## 2 Assumption and Justification

- **There is no human intervention through lampreys' growth.** The problem aims to study the impact of the lamprey sex ratio on ecosystems. As Human intervention usually idealizes the reproduction of lamprey offspring such as artificial breeding, we need to exclude human-induced effects and focus on the differences in population structure caused by sex ratio.
- **Neither sex will suddenly become extinct.** Similar to the above assumption, we do not consider a sudden environmental drastic change. Instead, we emphasize interspecific effects due to the sex ratio and learn about changes in its growth and mortality rates.
- **No differences exist between individuals of a single-sex.** In the context of analyzing the effects of sex ratio on ecosystems, the most important consideration is how offspring birth rates change under these conditions. Therefore, we do not consider factors such as lamprey size.
- **The range of activity of lampreys is fixed.** Unlike plants, fish can move to different waters, which might cause amount difference through lampreys'

migration. To Individually consider the growth rate and mortality rate as the increase and decrease of lampreys, we assume that they only live in a certain area.

- **Lamprey populations grow exponentially when there are no predators and food is plentiful.** The food resources of the prey are sufficient, and its growth is not limited by other factors.
- **The environment is closed and does not take into account the effects of migration, disease, and other biological populations.** Considering only predator-prey interactions, the effect of lampreys' population size is only affected by their predators and food. The more lamprey predators there are, the less food there is, and the smaller the lamprey population; Conversely, the more.
- **Lampreys can choose their sex subject to preconditions that change the sex ratio.** Lampreys do not choose their sex at will but adjust according to environmental factors to reduce interference from outside factors. To simplify the service model, the preconditions for sex ratio change were constrained, and the population size was not affected by sex selection resulting in large and inconsistent effects.
- **Lampreys only appear in the ecosystem as predators.** To make the model more realistic, we searched for some biological information about lampreys and found that lampreys are an invasive species with almost no natural enemies except humans. Besides, lampreys are not the only food source for humans, so we do not consider lampreys as prey.

### 3 Notations

Table 1: Notation

| Variable                          | Notation | Meaning  |
|-----------------------------------|----------|--|
| Lampreys' initial population size | $P_0$    | The initial population amount in a certain area              |
| Lampreys' population size         | $P$      | The population amount in a certain area                      |
| Growth rate                       | $r$      | Lamprey's growth rate  |
| environmental carrying capacity   | $K$      | the maximum value at which the population size can be stable |

Table 2: Notation

| Variable                             | Notation | Meaning  |
|--------------------------------------|----------|--|
| environmental carrying capacity      | $K$      | the maximum value at which the population size can be stable                               |
| Initial population size of prey      | $X_0$    | Initial population of plankton   |
| Initial population size of predator  | $Y_0$    | Initial population of lamprey  |
| Rate at which predators destroy prey | $a$      | How quickly lamprey wipe out plankton  |
| Death rate                           | $m$      | Death rate of the Lamprey in the absence of plankton                                       |
| Half saturation constant             | $K_s$    | The nutrient concentration is such that the growth rate is half of the maximum growth rate |
| Nutrient concentration               | $S$      | Measuring nutrient concentration at time points  |
| Proportion of males                  | $S_r$    | The proportion of males in a specific place at a certain time                              |
| Growth rate function of predator     | $b(S_r)$ | Lamprey's growth rate function affected by sex ratio                                       |
| Initial reproductive rate            | $b_0$    | Lamprey's growth rate function without any influence                                       |
| rate of change in reproductive rate  | $b_1$    | The rate of change in reproductive rate when the sex ratio deviates from an ideal value    |

## 4 Developing the Lamprey Growth Model

### 4.1 Model overview

To explore this question, our model will attempt to predict how lamprey offspring numbers and environmental carrying capacity will change—these factors will serve as dependent variables. Specifically, because we are interested in studying the conversion rates of these quantities, we will develop a model based on differential equations that

can naturally characterize changes over time.

However, we believe that only analyzing the system across time is not sufficient, as different environments have a strong influence on the growth of lampreys into different sexes. First, unlike other animals, the sex of lampreys at birth is unknown, and the environment limits their growth rate, causing them to grow different gonads. For example, because the water temperature at that location is unsuitable for survival or resources are scarce, the likelihood of lampreys growing into females increases, and lamprey mortality will also increase. Therefore, in add.

In the model we will include parameters that allow us to explore and capture the following conditions:

- Number of lamprey sexes in a given area
- Number of lamprey offspring in a given area
- Average temperature in a specific area
- Allocable resources for a specific region

## 4.2 Growth in the Number of Lampreys' Genders

To establish a differential equation model that accounts for the influence of external environmental factors (such as food supply) on the sex ratio of lampreys, we can consider the following formulation:

Let  $S(t)$  represent the number of male lampreys and  $F(t)$  represent the number of female eels. Let  $E(t)$  denote the environmental factors (such as food supply).

The model can be expressed by the following set of differential equations:

- For male lampreys:

$$\frac{dS}{dt} = r_s E(t) \cdot S - d_s \cdot S$$

where  $\frac{dS}{dt}$  represents the rate of change of male lampreys' population over time,  $r_s E(t)$  is the growth rate of the male population which is a function of the environmental factor  $E(t)$  and  $d_s$  is the natural death rate of the male population.

- For female lampreys:

$$\frac{dF}{dt} = r_F E(t) \cdot F - d_F \cdot F$$

where  $\frac{dF}{dt}$  represents the rate of change of the female eel population over time,  $r_s E(t)$  is the growth rate of the female population which is a function of the environmental factor  $E(t)$ , and  $d_F$  is the natural death rate of the female population.

In this model, the growth rates of both male and female eels are influenced by environmental factors, which are encapsulated in the functions  $r_s E(t)$  and  $r_F E(t)$ , respectively.

### 4.3 Logistics Model in Population Growth of Lamprey's Offspring

The logistic growth model is widely used to describe population growth. It has a strong nonlinear fitting ability and is suitable for depicting complex growth conditions, which is suitable for this question. The basic form of the model is given by the differential equation:

$$\frac{dP}{dt} = rP \left( 1 - \frac{P}{K} \right)$$

### 4.4 Initial Condition and Parameter Estimation

In the context of fitting a logistic growth model using the least squares method, our objective is to minimize the sum of the squared differences between the predicted and actual observed values. Suppose we have  $n$  observations, and for each time point  $t_i$ , we have the actual population size  $P_i$  and the model's predicted population size  $\hat{P}(t_i; P_0, r, K)$ . The least squares method aims to find the values of  $P_0$ ,  $r$ , and  $K$  that minimize the error function  $E(P_0, r, K)$ , which is defined as follows:

$$E(P_0, r, K) = \sum_{i=1}^n \left[ P_i - \hat{P}(t_i; P_0, r, K) \right]^2$$

Based on biological data from juvenile sea lampreys sampled in natural streams in Michigan and Quebec, Canada, we can get lampreys' initial population size  $P_0$ :

Table 3: Result

| Location   | $P_0$ | Location   | $P_0$ |
|------------|-------|------------|-------|
| Bear River | 63.33 | Hog Island | 44.92 |
| Carp River | 60.59 | Pine River | 60.29 |
| Ford River | 75.24 |            |       |



## 4.5 Model Analysis

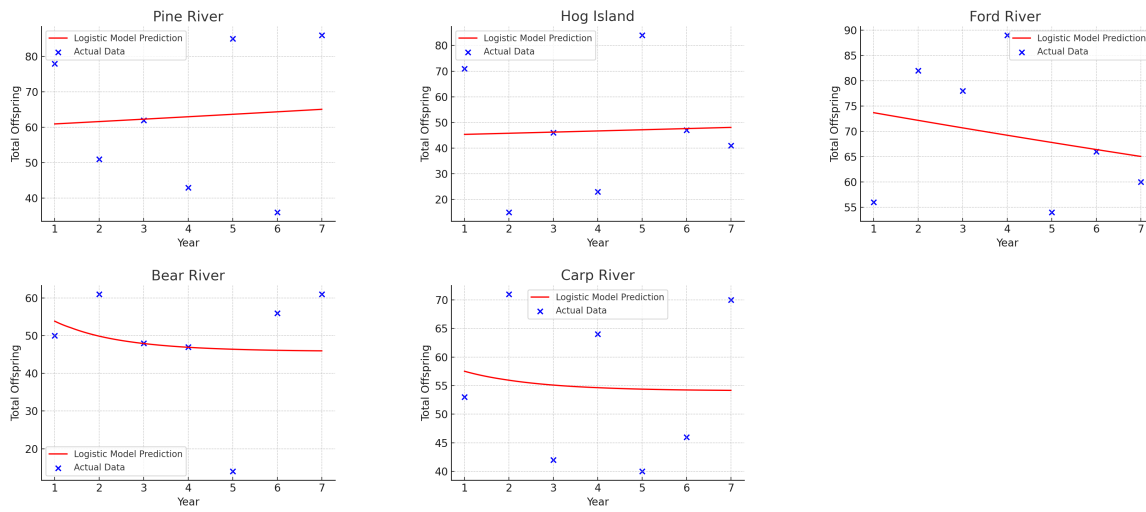


Figure 2: Relationship between actual data and predictions for each location

The graphs depict the relationship between the actual data (represented by blue dots) and the predictions made by the Logistic growth model (indicated by red lines) for each location. Each subplot corresponds to a specific location.

Upon close examination, it's evident that the model aligns well with the actual data at certain locations, indicating a strong predictive performance. In these cases, the logistic growth model seems to adequately capture the underlying dynamics of the population growth.

However, at other locations, the model appears to fit poorly. This could be attributed to the unique characteristics of the data at these specific sites or to the fact that the logistic model's assumptions may not hold true in these scenarios. For instance, the model assumes a certain form of growth that is not unlimited and is capped by a carrying capacity, and a consistent intrinsic growth rate.

## 5 Dynamic evaluation model of lampreys population

### 5.1 Model overview

Considering that lampreys can perform sex selection and are restricted by predators, food, and living space in the ecosystem, the Volterra prey-predator model was used to establish the population dynamics model of Lampreys.

It is strategically deployed to pinpoint the optimal strategic advantages by evaluating the simulated survival conditions of gender-selective lampreys, using the predator model as a foundation. With parameters set, TOPSIS analysis meticulously sifts

through to highlight the strengths and weaknesses inherent in the survival strategies of gender-selective lampreys.

### a. Lotka-Volterra model

The Volterra predator-prey model is adeptly utilized to analyze the intricate dynamics of populations within ecosystems. It elegantly distills the complex interactions between predators and their prey into robust mathematical equations, adeptly capturing the ebb and flow of population sizes over time. Grounded firmly in biological tenets, the model is enhanced by incorporating additional nuanced parameters, such as shifts in gender ratios, thus tailoring it to mirror the complexities of real-world ecosystems.

It is expressed by the following differential equation:

$$\begin{aligned}\frac{dP_0}{dt} &= aP_0 - bP_0F_0 \\ \frac{dF_0}{dt} &= -cF_0 + dbP_0F_0\end{aligned}$$

Where  $P$  is the number of predators,  $F$  is the number of prey,  $a, b, c, d$  are model parameters, representing growth rate, predation rate, etc.

### b. TOPSIS analysis and evaluation model

TOPSIS analysis, traditionally aligned with multi-criteria decision-making conundrums, steps beyond its conventional role to aid in the decision-making process informed by the model's outcomes.

- Construct decision matrix  $X$
- Normalized decision matrix  $R$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

- Construct the weighted standardized decision matrix  $V$

$$v_{ij} = w_j \cdot r_{ij}$$

- Determine the ideal positive solution  $A^*$  and the ideal negative solution  $A^-$

$$A^* = \max_i(v_{ij}), \quad j = 1, 2, \dots, n.$$

$$A^- = \min_i(v_{ij}), \quad j = 1, 2, \dots, n.$$

- Calculate the distance  $S^*$  and  $S^-$  from the positive and negative ideal solutions for each scheme

$$S_i^* = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^*)^2}$$

$$S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}$$

- Calculate relative proximity  $C_i$

$$C_i = \frac{S_i^-}{S_i^* + S_i^-}$$

Rank according to relative proximity All schemes are sorted by  $C_i$  value in descending order. The larger the  $C_i$  value, the better the scheme is.

## 5.2 Modeling

### a. Lotka-Volterra Lampreys' population dynamics model

We need to carry out the application of two Lotka-Volterra models, the first lampreys as predators, small fish as bait; Second lampreys as prey, and indigenous, predatory fish as predators. The population dynamic model of lampreys was synthesized.

- Scenario one: lampreys as predators

According to the Lotka-Volterra model, we will follow the following steps:

1. Set model parameters
2. Simulate the sex ratio of lampreys for each food availability (" low "and" high ")
3. Put the sex ratio into the model to simulate the population dynamics model of lampreys and food under different sex ratios
4. Analyze and visualize the results

Under hypothetical conditions, the Lotka-Volterra model was used to construct the predator population limit and light interaction of lampreys:

$$\begin{aligned}\frac{dP_1}{dt} &= a_1 P_1 - b_1 P_1 F_1 \\ \frac{dF_1}{dt} &= -c_1 F_1 + d_1 b_1 P_1 F_1\end{aligned}$$

$P_1$ : the number of lampreys in the population

$F_1$ : the amount of food

$a_1$ : the natural growth rate of lampreys

$b_1$ : the efficiency with which lampreys prey on food

$c_1$ : the natural mortality rate of food

$d_1$ : the growth rate of lampreys after food prey

Model parameters:  $a_1 = 0.1$ ,  $b_1 = 0.02$ ,  $c_1 = 0.1$ ,  $d_1 = 0.01$ . Then the equation becomes

$$\begin{aligned}\frac{dP_1}{dt} &= 0.1P_1 - 0.02P_1F_1 \\ \frac{dF_1}{dt} &= -0.1F_1 + 0.01 * 0.02P_1F_1\end{aligned}$$

• Scenario two: Lampreys as prey

According to the Lotka-Volterra model, we will follow the following steps:

1. Set model parameters
2. Simulate the sex ratio of lampreys for each food availability (" low "and" high ")
3. Put the sex ratio into the model to simulate the population dynamics model of lampreys and food under different sex ratios
4. Analyze and visualize the results

Under hypothetical conditions, the Lotka-Volterra model was used to construct food constraints and light interactions on lampreys populations:

$$\begin{aligned}\frac{dP_2}{dt} &= a_2P_2 - b_2P_2F_2 \\ \frac{dF_2}{dt} &= -c_2F_2 + d_2b_2P_1F_2\end{aligned}$$

$P_2$ : the number of lampreys' predator populations

$F_2$ : the amount of lampreys

$a_2$ : the natural growth rate of lampreys' predators

$b_2$ : the efficiency of lampreys predators

$c_2$ : the natural mortality rate of lampreys

$d_2$ : the growth rate of lampreys predators after lampreys are preyed upon

Model parameters:  $a_2 = 0.1, b_2 = 0.02, c_2 = 0.1, d_2 = 0.01$ . Then the equation becomes

$$\begin{aligned}\frac{dP_2}{dt} &= 0.1P_2 - 0.02P_1F_2 \\ \frac{dF_2}{dt} &= -0.1F_2 + 0.01 * 0.02P_2F_2\end{aligned}$$

## b. Lampreys evaluation model

By considering whether lampreys can perform sex selection and the influence of the maximum population size of lampreys, the advantages and disadvantages of lampreys are studied by screening indicators.

Firstly, lampreys that can perform sex selection and lampreys that cannot perform sex selection are taken as two research objects.

Secondly, reproductive success rate, genetic diversity, niche occupation, social effect, resistance to diseases and natural enemies, ecosystem impact, and reduction of random fluctuations were analyzed as seven indexes.

Finally, Topsis analysis was carried out.

### 5.3 Result

The visualization results are as follows:

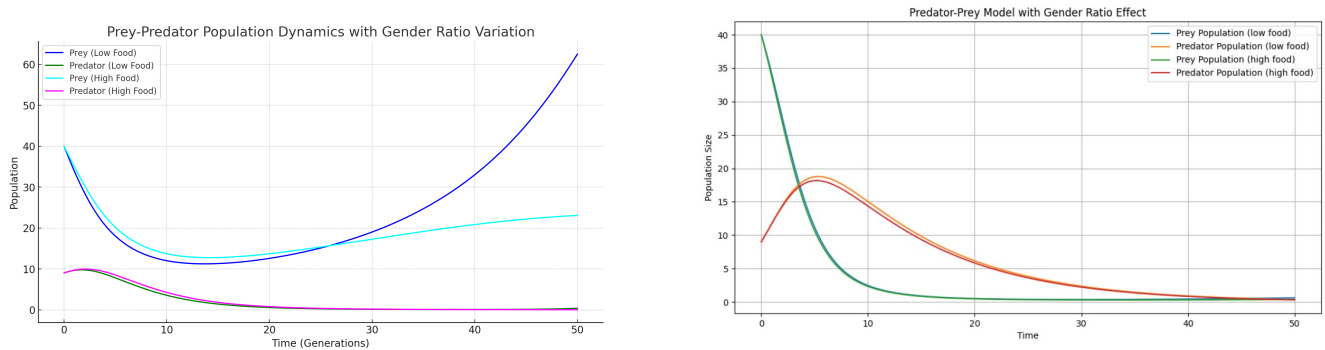


Figure 3: Scenario one & two

Scenario one: sees a lamprey as a predator

Scenario two: view lampreys as prey

According to the above analysis, sex selection in lampreys can increase the maximum population size and decrease the minimum population size, so the advantages and disadvantages brought by this change can be further studied.

According to the literature and part of the situation, we have formulated the following rating table:

| Evaluation criteria                  | Operability | Adaptability | Ecological | Community | recommendation Standards and Weights | Ecosystem Stability | Impact of Potential Risk Costs |
|--------------------------------------|-------------|--------------|------------|-----------|--------------------------------------|---------------------|--------------------------------|
| Lampreys with adjustable sex ratio   | 7           | 8            | 6          | 5         | 7                                    | 8                   | 6                              |
| Lampreys with unadjustable sex ratio | 5           | 6            | 7          | 4         | 5                                    | 6                   | 6                              |

Figure 4: Topsis parameter index rating table

We first normalized the scoring matrix, then weighted each evaluation parameter with the same weight (**it is worth noting here that different parameters may have different importance according to the actual situation, and different data sets use the analytic hierarchy process to change the weight of the evaluation parameters**), and then determine the ideal optimal solution and the worst solution.

Based on these data, we calculate the distance to the ideal solution and the worst solution for each population solution, as well as the closeness.

The analysis results are as follows:

The relative proximity of lampreys for sex selection was **0.8152**, ranking first.

Lampreys, which are not sex-selective, has the second-highest relative proximity of **0.1848**

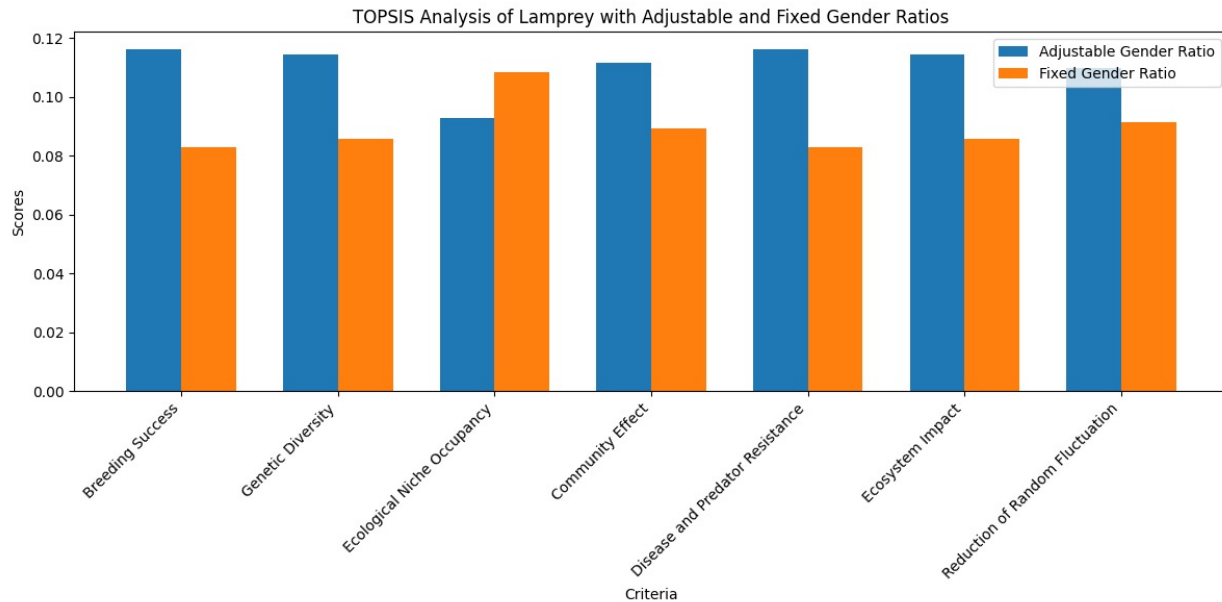


Figure 5: TOPSIS Analysis of Lamprey with Adjustable and Fixed Gender Ratios

According to TOPSIS analysis and its visualization results, lampreys that can be sexually selected perform better on all assessment parameters except niche occupation, so we can draw the following conclusions:

- **Advantages:**

- High reproductive success rate** : quantitative effect reduces individual survival pressure
- Large genetic diversity**: helps to adapt to the new environment through the natural environment when the environment changes
- Strong social effect**: population size maintains the social structure and improves the efficiency of collective defense, hunting, or other social activities
- Strong ability to resist diseases and natural enemies**: smaller loss ratio, more able to resist diseases and natural enemies
- Ecosystem impact**: Acts as a material cycle to promote the flow of energy in the ecosystem for consumers
- Reduce the possibility of random fluctuations**: random events have little impact on the overall population

In short, it has a positive impact on ecological adaptability and viability

- **Weaknesses:**

a. **Niche occupation:** The reason may be that a fixed sex ratio may make a population more specialized in a particular niche

## 6 Lotka-Volterra Model based on sex ratio

### 6.1 Model Overview

To reflect changes in predator sex ratio in the Lotka-Volterra model, we need to introduce additional mechanisms to describe how sex ratio affects the predator's reproductive rate, mortality, or predation rate. According to relevant biological material, we found that lampreys mainly feed on zooplankton. Since we cannot directly find and measure zooplankton populations, we choose to use the Monod model to predict and assume the growth rate of prey (plankton) using water quality data. We then use phase trajectories to analyze the trajectory of predator and prey populations evolving, the stability, and the cyclical or chaotic behavior of the ecosystem. This analysis can provide a deep intuitive understanding of the dynamic behavior of ecosystems.

In the model we will include parameters that allow us to explore and capture the following conditions:

- Maximum growth rate of plankton under ideal conditions
- half saturation constant
- Measuring nutrient concentration at time points
- Growth rate of prey (plankton)
- Growth rate of predator (lamprey)
- Initial population size of the predator
- Environmental carrying capacity of predators
- Predation rate coefficient
- The rate at which prey consumed is converted into offspring of the predator
- Proportion of male lamprey individuals
- natural mortality rate of predators

Note that the model as a whole is dimensionless; that is, we do not measure the total amount of natural resources, plankton content, etc. in a specific space. This was done because the purpose of this study was to determine the impact of lampreys on ecosystem stability rather than to predict the precise number of lampreys present in a specific area given a precise set of parameter values. The latter investigation most likely cannot be carried out with phenomenological, mechanistic mathematical models that would provide insight into the nature and behavior of lamprey interactions with prey, which is the ultimate goal and overarching question of this study statement. Therefore, we believe that it makes more sense to develop a dimensionless model that can be applied to any given lamprey sex ratio data, and the results provide basic information about how lampreys act in the ecosystem under different sex ratios.

## 6.2 Predator-Prey Model in Predicting Plankton's Growth Rate

To establish a predator-prey model that considers the impact of sex ratio, we can modify the Lotka-Volterra equation as follows:

Set sex ratio  $S_r$  to represent the proportion of male individuals. Assuming that sex ratio affects the reproductive rate, we can define a new reproductive rate function  $b(S_r)$  that is a function of sex ratio  $S_r$ . Therefore, the Lotka-Volterra equation considering the influence of the sex ratio can be expressed as: 1. Rate of change of plankton (prey):

$$\frac{dX}{dt} = r \cdot X - a \cdot X \cdot Y$$

2. Rate of change of predators:

$$\frac{dY}{dt} = S_r Y \cdot a \cdot X \cdot Y - m \cdot Y$$

where  $b(S_r)$  is a function of sex ratio. We assume  $b(S_r)$  is a linear function,

$$b(S_r) = b_0 + b_1 \cdot (S_r - S_{r0})$$

We can build a Monod model through the water quality data of the East AuGres River and microorganisms at a specific time, and obtain the initial population and growth rate data of plankton (prey) from the model, specifically as the following equation:

$$r = \frac{r_{\max} \cdot S}{K_s + S}$$

## 6.3 Model Analysis

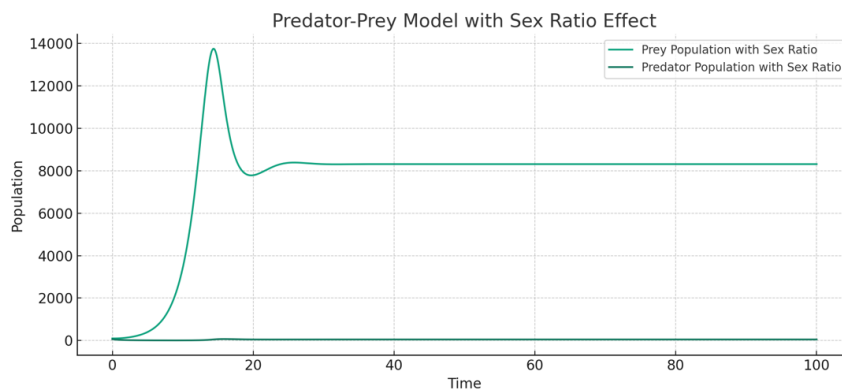


Figure 6: Predator-Prey Model with Sex Ratio Effect

The prey population experiences a rapid increase followed by a sharp decline, which suggests a classic predator-prey dynamic where the prey initially thrives due to



low predator numbers, then rapidly diminishes as predators consume them. After the sharp decline in prey, the predator population also decreases, likely due to a reduced food source. Both populations appear to stabilize after the initial fluctuations, which suggests that the system reaches a steady state. Moreover, the influence of the sex ratio is not explicitly shown in the curve shapes but is integrated into the model affecting the predator population dynamics.

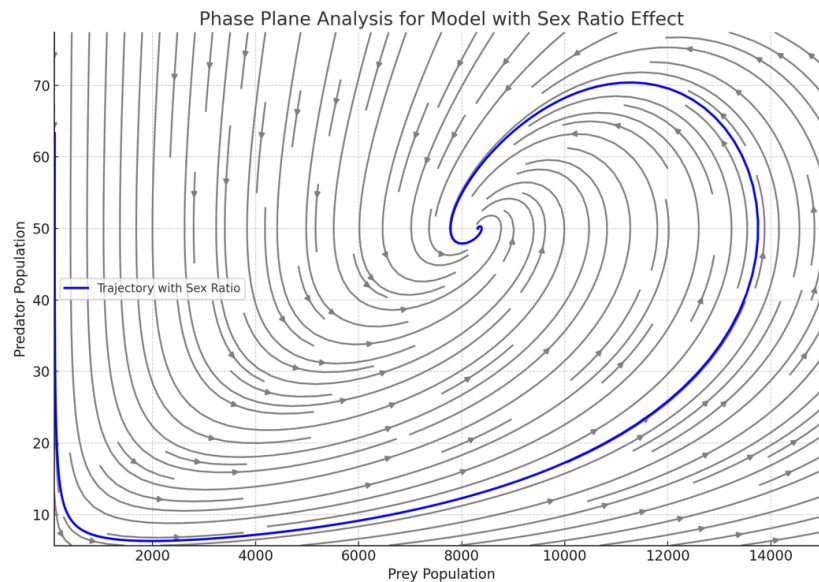


Figure 7: Phase Plane Analysis for Model with Sex Ratio Effect

The plot shows the phase plane and trajectory of the predator-prey model, with streamlines indicating the direction and speed of the system's evolution under different initial conditions. The orange line represents the trajectory of the population under a specific initial condition (marked by the red point), forming a closed orbit and reflecting the periodic behavior of the predator and prey populations.

In practice, we encountered technical problems, possibly due to errors in numerical methods, function definitions, or parameter passing.

Simplifying assumptions of the model (e.g., predators are affected by only one prey population, without considering other possible food sources or predators) may limit its application in complex real ecosystems.

## 7 Lamprey-Based Parasitic Ecosystem Model

### 7.1 Model overview

This model is a simplified model where we mainly consider the impact of sex ratio on the reproductive rate of lampreys (parasites), and the interaction between parasites

and hosts (other large fish). In reality, more factors may need to be considered, such as the age structure of hosts and parasites, seasonal changes, etc.

## 7.2 Modeling

The dynamics of the host can be expressed by the following differential equation:

$$\frac{dH}{dt} = r_H \cdot H - a \cdot H \cdot P - d_H \cdot H - b \cdot H \cdot P$$

Here,  $r_H \cdot H$  represents the natural growth of the host,  $a \cdot H \cdot P$  represents the rate at which the host is infected by the parasite,  $d_H \cdot H$  represents the natural mortality rate of the host,  $b \cdot H \cdot P$  represents the excess death of the host due to parasitic infection. The

dynamics of the parasite can be expressed by the following differential equation:

$$\frac{dP}{dt} = r_P \cdot P \cdot \left(1 - \gamma \cdot \frac{P}{H}\right) - d_P \cdot P$$

Here,  $r_P \cdot P$  represents the natural growth of parasites,  $\gamma \cdot \frac{P}{H}$  represents the effect of the sex ratio on the growth rate of the parasite (for example, if the sex ratio of the parasite is unbalanced, it may affect its ability to reproduce),  $d_P \cdot P$  represents the natural mortality rate of the parasite.

Here we take the lamprey population size and growth rate calculated from the previous logistic model as the initial population size and natural growth rate of the parasite in this parasitism model, and perform calculations. The result is as shown in the figure:

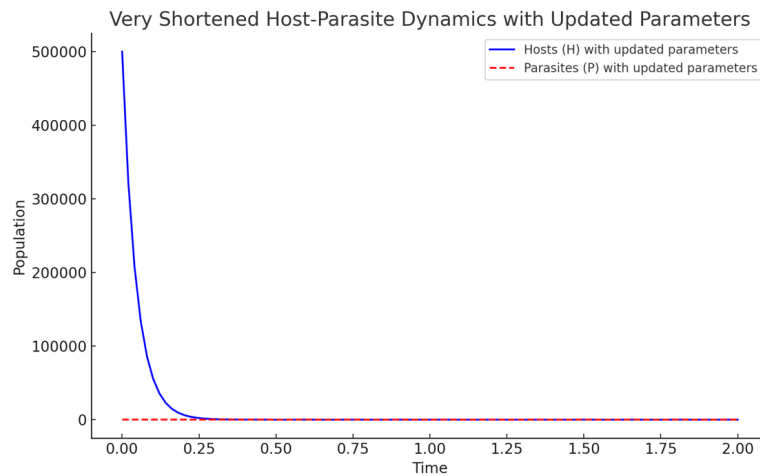


Figure 8: Very Shortened Host-Parasite Dynamics with Updated Parameters

## 8 Sensitivity Analysis

### 8.1 Predator Population to Sex Ratio

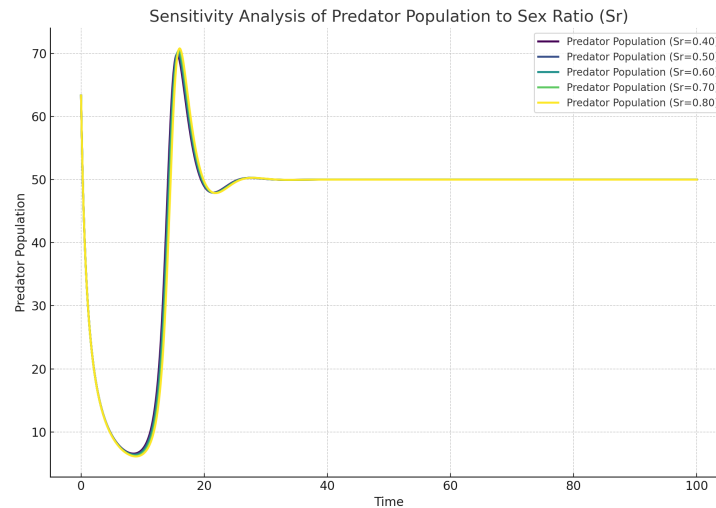


Figure 9: Sensitivity Analysis of Predator Population to Sex Ratio

The sensitivity analysis presented above elucidates the influence of sex ratio ( $S_r$ ) on the dynamics of predator populations. Altering the sex ratio unveils distinct temporal patterns in predator populations, highlighting the following findings:

- Predator Population Dynamics**

The size and fluctuation patterns of predator populations are significantly altered by varying sex ratios, underscoring the sex ratio as a pivotal determinant in population dynamics. Moreover, even though predator populations exhibit cyclical fluctuations across all scenarios, the amplitude and periodicity of these cycles—encompassing maximum and minimum population levels—shift with different sex ratios.

- Ecosystem Management and Forecasting**

Comprehending the interplay between sex ratios and predator populations is pivotal for effective ecosystem governance and prognostication of demographic shifts. Should a particular sex ratio engender an unsustainable surge or plummet in predator numbers, it may compel ecosystem managers to recalibrate the sex ratio or deploy alternative conservation tactics.

- Parameter Sensitivity**

The analysis confirms the model's susceptibility to sex ratio variations, indicating that precise assessment and vigilant monitoring of actual sex ratios within natural habitats are indispensable for a profound grasp and prognostication of ecological population trends.

In essence, this sensitivity analysis sheds light on the prospective ramifications of sex ratio modifications within ecosystems, furnishes valuable perspectives for ecological stewardship, and lays the groundwork for subsequent explorations into population dynamics.

## 8.2 Alpha Growth Rate on Lamprey and Predator population

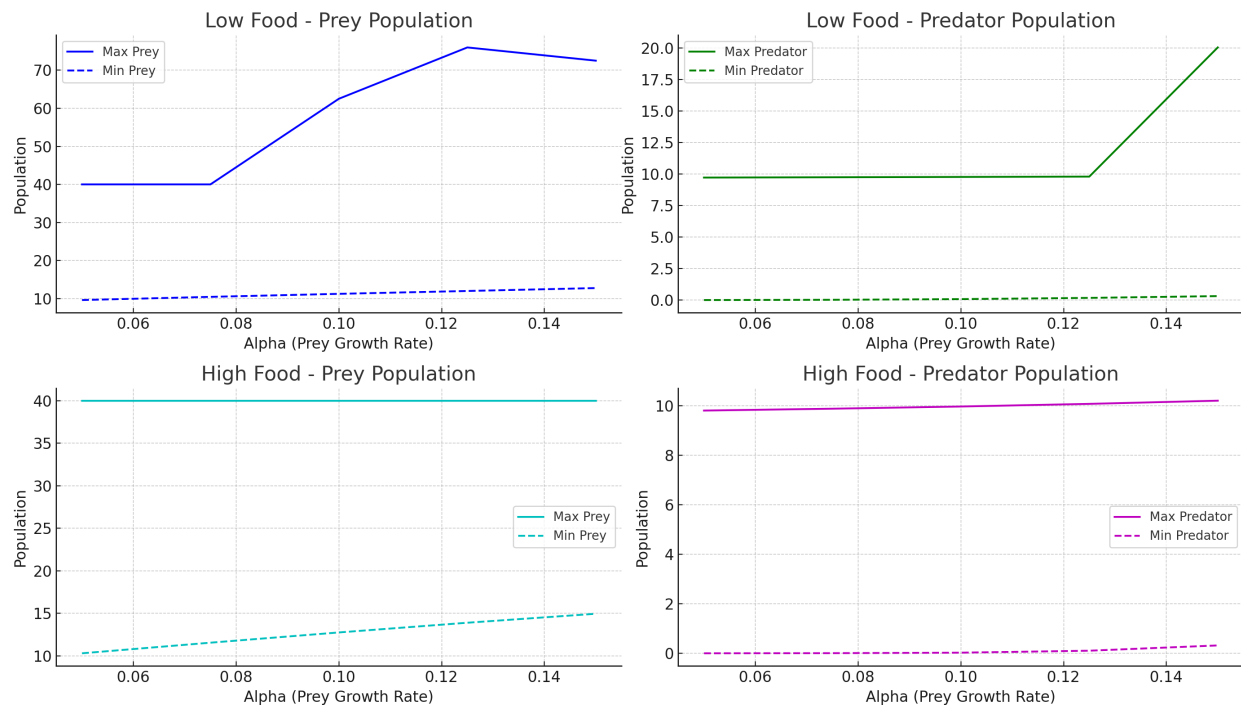


Figure 10: sensitive analysis of Alpha Growth Rate on Lamprey and Predator population

Low food availability (Figures 1 and 2) and high food availability (Figures 3 and 4)

The figure above shows how changes in prey growth rate (alpha) affect lampreys (prey) and predator populations under conditions of low and high food availability. The results of sensitivity analysis showed that an increase in prey growth rate (alpha) led to an increase in the maximum number of Jameel (prey) and predator populations, especially under conditions of high food availability. This reflects that the rate of prey growth is an important factor for population dynamics, and the availability of food has a significant effect on the population size level.

### 8.3 Gamma on Parasite Population

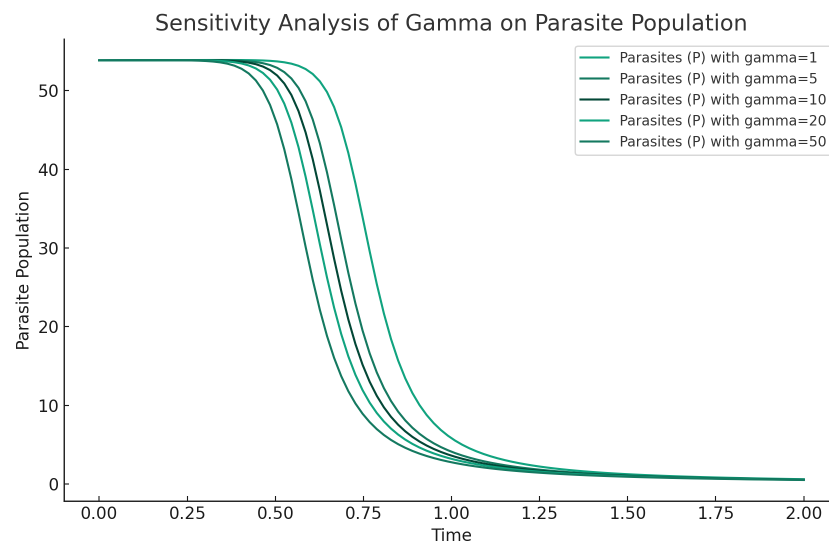


Figure 11: Sensitivity Analysis of Gamma on Parasite Population

We conducted a sensitivity analysis of the sex ratio affecting coefficient ( $\gamma$ ) of parasite growth rate. We consider different values of  $\gamma$  (1,5,10,20,50) and model how the parasite population changes over time in each case. As shown in the figure, with the increase of  $\gamma$ , the number of parasites significantly decreases, indicating that  $\gamma$  has a significant effect on the size of the parasite population, reflecting the sensitivity of the model to parameter changes.

## 9 Model Evaluation and Further Discussion

### 9.1 Strength

- **Predator model**

Predicting population dynamics Models can predict how different population sizes will change over time, helping researchers understand how interactions between species affect entire ecosystems.

- **Analytical Hierarchy Process**

Having a clear and systematic framework to analyze the problem helps us understand the weight of each evaluation indicator of the problem and their relationship with each other.

- **Topsis model**

The ideal best solution and the ideal worst solution can be considered simultaneously. The evaluation method is novel and powerful; it requires less data. Helps us build models when there is a lack of lamprey research data.

## 9.2 Weakness and Possible Improvement

- **Predator model**

**The model is relatively simplified**, which may limit its application in complex real ecosystems. For example, a predator is only affected by one prey population, and other possible food sources or predators are not considered. We can consider more biological details, such as competition within prey populations, predator starvation effects, etc. Conduct sensitivity analysis of parameters to understand which parameters have a decisive impact on model results. Use more data to calibrate and validate the model to ensure the accuracy and reliability of model predictions.

- **Analytical Hierarchy Process**

**It relies heavily on data and literature.** The weight determination criteria for each indicator of the decision-making evaluation object are set by the decision-maker, which is highly subjective. We can refer to more literature and consider many factors to ensure authority.

- **Topsis model**

**Insufficient consideration of relative importance.** When calculating the distance from the ideal solution, all attributes are considered independent of each other, which may ignore the interaction between attributes or the relative difference in importance. Adaptability in dynamic environments. To adapt to the changing decision-making environment, the dynamic TOPSIS model is developed, which can adapt to time changes or decision-making needs in different scenarios.

## 10 Reference & Report of AI

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# Report of AI

OpenAI ChatGPT (Feb 2-5, 2024 version, ChatGPT)

**Query1: < To reflect the changes in the gender ratio of the lamprey population and the relationship with other species in the ecosystem (such as parasites) using a multi-species Lotka-Volterra model, what do we need to provide?>**

Output: <To use the multi-species Lotka-Volterra model to study the changes in the gender ratio of the lamprey population and its relationship with other species in the ecosystem (such as parasites), you need to prepare the following data and information:

1. Basic Population Data: The size of the lamprey population, gender ratios (number of males and females). The size of the parasite population, and possibly other relevant species population data.
2. Biological Parameters: Lifecycle information for lampreys, including fertility rates, age of maturity, average lifespan. Lifecycle information for parasites, including how they interact with their hosts (infection rates, impact on host health, etc.). If possible, include the impact of gender ratios on the reproduction and survival rates of lampreys.
3. Interaction Coefficients: The strength of interactions between populations, such as predation, competition, parasitism, etc., typically represented by interaction coefficients in the Lotka-Volterra model.
4. Environmental Factors: Environmental factors affecting the dynamics of the lamprey and parasite populations, such as temperature, water quality, availability of food resources, etc.
5. Historical Data: Data on historical numbers of lamprey and parasite populations, which helps with model calibration and validation.

**Query2: <Range of Reproduction Rates>**

Output: <The range of reproduction rates (commonly denoted by  $r$ ) can vary significantly based on different species, ecological environments, and specific biological characteristics. In ecology and population dynamics models, the reproduction rate is a crucial parameter that affects the growth and decline of populations. Here are some general guidelines:

1. **Values around 0**: - If  $r$  is close to 0, it implies that the population is roughly stable, with birth and death rates approximately balanced. -  $r > 0$  indicates that the population is growing. -  $r < 0$  indicates that the population is decreasing.
2. **Positive values less than 1**: - For some slow-breeding species, such as large mammals, the value of  $r$  might be relatively small, like 0.01 to 0.1.
3. **Values close to or greater than 1**: - For fast-breeding species, especially some small invertebrates, insects, or microbes, the value of  $r$  can be relatively large, approaching 1, or even greater than 1 in some cases. - Such high reproduction rates are usually balanced with higher mortality rates or significant environmental constraints (such as food, space, etc.).
4. **Upper and lower bounds**: - In reality, the value of  $r$  is limited by the biology of the species and environmental conditions and usually will not be infinitely large. - In practical population models, other parameters might be introduced to limit population growth, such as the carrying capacity  $K$  (in the Logistic growth model).

In your model, an  $r_L = 0.617$  appears to be a relatively high value, especially if it represents a short-term reproduction rate (like daily or monthly). If it represents a longer-term reproduction rate (like annually), it might be more reasonable. When determining a reasonable reproduction rate, it's important to consider its relationship with the specific species' life cycle, breeding cycle, and environmental conditions.

When conducting model simulations, the correct parameter range is crucial for obtaining realistic and reliable results. For a species like the lamprey, it's advisable to further investigate professional biological and ecological literature to determine a reproduction rate range suitable for the species and specific



environmental conditions.>

**Query3:< What mathematical models can reflect the relationship between the ecosystem's variation in the gender ratio of the lamprey population and other species in the ecosystem (such as parasites)? >**

Output:<To reflect the relationship between the variation in the gender ratio of the lamprey population and other species in the ecosystem (such as parasites), the following mathematical models can be considered:

1. **\*\*Multi-Species Lotka-Volterra Model\*\***: - This is an extended Lotka-Volterra model that can not only simulate the relationship between lampreys and their prey but also include other species, such as parasites. The model can incorporate the impact of gender ratio on the reproduction and mortality rates of lampreys, as well as the impact of lampreys on the life cycle of parasites.
2. **\*\*Host-Parasite Dynamics Model\*\***: - This model is specifically used to study the interaction between hosts and parasites and can include the gender structure of the host, the transmission mode of parasites, their life cycle, and the impact on the host population. The model can consider how the variation in the gender ratio of lampreys affects the spread of parasites and the health of the host population.
3. **\*\*Structured Population Model\*\***: - These models consider the internal structure of the population, such as age, size, or gender ratio. For lampreys, the model can consider the impact of changes in gender ratio on population growth and ecological interactions by dividing the population into different gender groups.

When choosing a model, it is important to consider the specific circumstances of the problem, the type and quality of available data, and the assumptions and limitations of the model. Typically, a combination of multiple models and methods can provide more comprehensive and in-depth insights.>

**Query4:<Scope of Application of the Monod Model>**

Output:<The Monod model is a mathematical model that describes the relationship between microbial growth rate and substrate concentration. It is particularly useful in the fields of environmental engineering and bioengineering, used to depict the growth dynamics of microbial populations under specific conditions. The main applications of the Monod model include:

1. **\*\*Biological Treatment Systems\*\***: Used for the design and optimization of wastewater treatment, biodegradation, and other biological treatment systems. In these systems, understanding how microbes utilize substrates (such as organic pollutants) is crucial for process control and system design.
2. **\*\*Fermentation Industry\*\***: In the industrial fermentation process (such as the production of antibiotics, alcohol brewing, etc.), the Monod model can help optimize the supply of substrates, ensuring that the microbial growth rate is maximized, thereby increasing production.
3. **\*\*Ecological Studies\*\***: In the study of microbial ecology in natural water bodies (such as lakes, oceans), the Monod model helps understand the impact of different nutrient concentrations on the structure and function of microbial communities.
4. **\*\*Bioreactor Design and Control\*\***: The Monod model is very important in the design and operation of bioreactors, especially in determining the size of the reactor, mixing intensity, oxygen supply, and substrate feeding strategy.

Although the Monod model is very useful in many fields, it also has limitations. For example, it assumes that environmental conditions are constant and only considers the effect of a single limiting substrate on growth. In natural environments or complex industrial processes, microbial growth is affected by multiple factors, and more complex models may be needed to describe it. In addition, when the substrate concentration is very low or very high, the Monod model may no longer be applicable, and other models may be needed to describe microbial growth behavior under these extreme conditions.>