

In [ ]:

lr=0.005

target: sigma=0.03

norm\_clip(max\_norm=1)

optimizer: AdamW(decay=5e-4)

scheduler: StepLR(step=50, gamma=0.995)

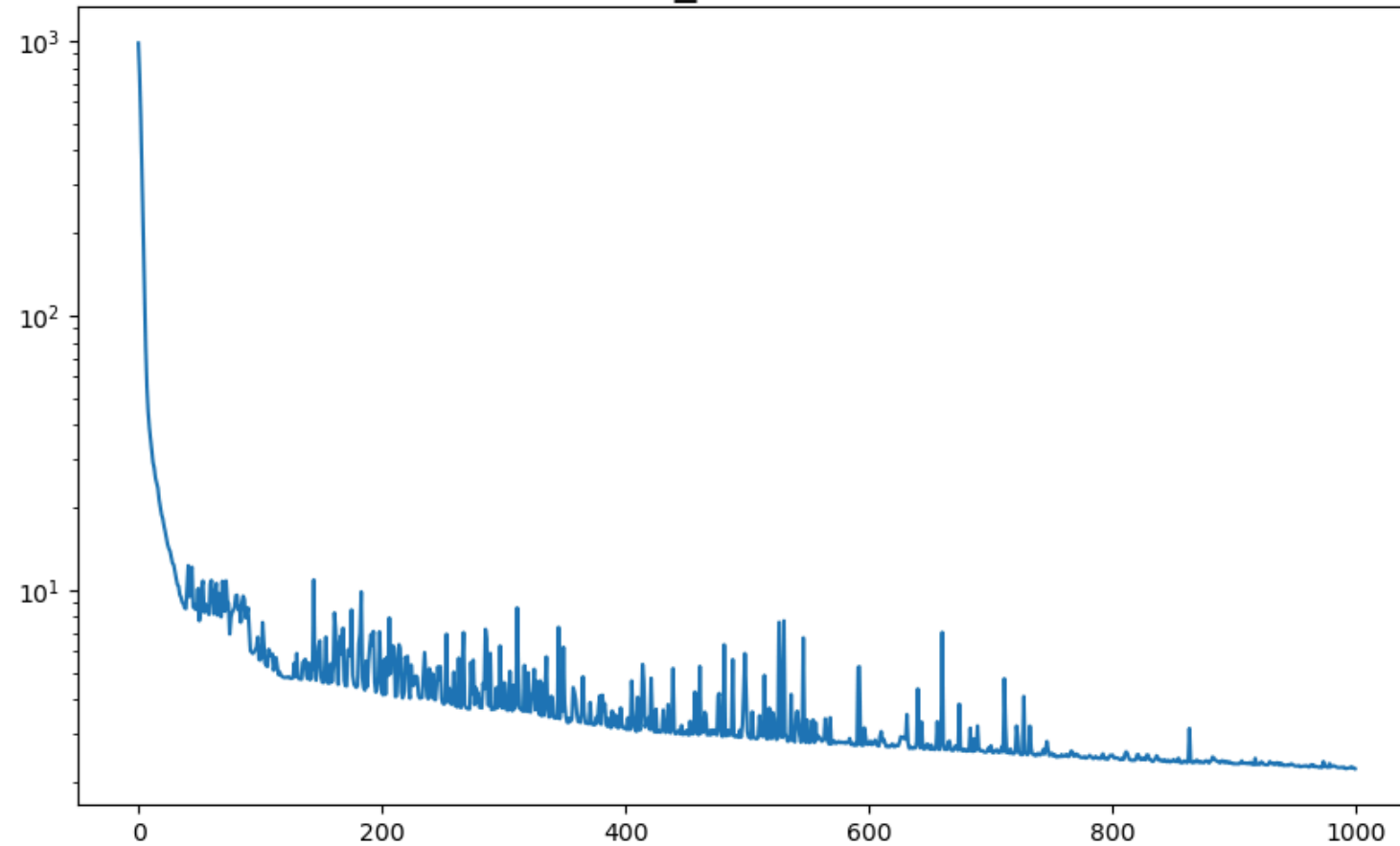
MaxBatch=1000

OptimStep=20

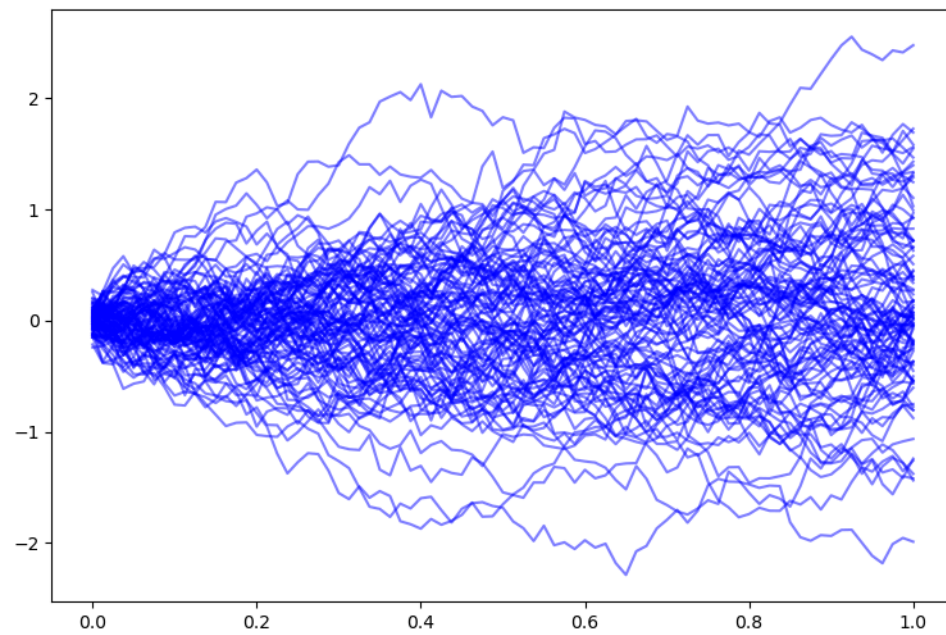
Average Error Est: 2.257347595691681  
Batch Number: 996  
Average Error Est: 2.266151475906372  
Batch Number: 997  
Average Error Est: 2.2806038856506348  
Batch Number: 998  
Average Error Est: 2.2574188828468325  
Batch Number: 999  
Average Error Est: 2.2589728593826295  
Batch Number: 1000  
Average Error Est: 2.243794345855713

/var/folders/xb/rvrt2lnd1bs3y0fbz2c5wbwm0000gn/T/ipykernel\_3331/3736784659.py:143: RuntimeWarning: overflow encountered in exp  
x\_sigmoid=1/(1+np.exp(self.x\_path[:,-1]/sigma))

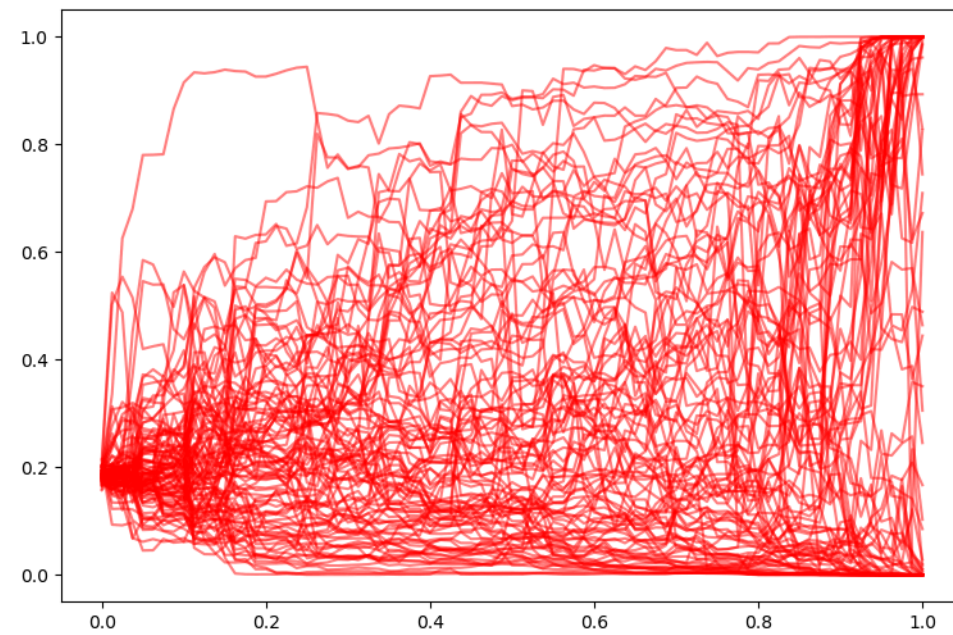
Forward\_Loss vs Batch



X



Y Values



```
In [ ]: import numpy as np
import torch as torch
import torch.nn as nn
import torch.nn.functional as F
import torch.optim as optim
import seaborn as sns
import pandas as pd
import matplotlib.pyplot as plt
import time
import random
from scipy.stats import norm
```

```
In [ ]: #Model and Params
#Numbers
NumTrain=500
NT=80
dt=1/NT
sigma=0.03

#Forward Loss
forward_losses = []
#Network Class for FBSDE
class Network(nn.Module):
    def __init__(self, lr, input_dims, fc1_dims, fc2_dims, n_outputs):
        """
        lr: learning rate
        """
        super(Network, self).__init__()

        #Pass input parameters
        self.input_dims = input_dims
        self.fc1_dims = fc1_dims
        self.fc2_dims = fc2_dims
        self.n_out = n_outputs

        #Construct network
        self.fc1 = nn.Linear(*self.input_dims, self.fc1_dims)
        nn.init.xavier_uniform_(self.fc1.weight)
        self.fc2 = nn.Linear(self.fc1_dims, self.fc2_dims)
        nn.init.xavier_uniform_(self.fc2.weight)
        self.fc3 = nn.Linear(self.fc2_dims, self.n_out)
        nn.init.xavier_uniform_(self.fc3.weight)

        self.optimizer = optim.Adam(self.parameters(), lr=lr)
        self.device = torch.device('cuda:0' if torch.cuda.is_available() else 'cpu')
        self.to(self.device)

    def forward(self, input):
        x = F.relu(self.fc1(input))
```

```

        x= F.relu(self.fc2(x))
        output = self.fc3(x)
        return output

```

*## Functions*

```

def Sample_Init(N,mean=0,sd=0.1):
    '''

```

```

        Generate N samples of x0
    '''

```

```

    xi = np.random.normal(mean,sd,size=N)

```

```

    return torch.FloatTensor(xi).view(-1,1)

```

```

def SampleBMIncr(T, Npaths, Nsteps):

```

*# Returns Matrix of Dimension Npaths x Nsteps With Sample Increments of of BM*

*# Here an increment is of the form dB*

```

    dt = T / Nsteps

```

```

    dB = np.sqrt(dt) * np.random.randn(Npaths, Nsteps)

```

```

    return torch.FloatTensor(dB)

```

```

def target(x,sigma=sigma):

```

```

    x=x.detach().numpy()

```

```

    return torch.FloatTensor(-x/sigma)

```

*# Forward Loss*

```

def get_foward_loss_coupled(dB, init_x,NT, target,y0_model, z_models):

```

```

    x = init_x

```

```

    # y = torch.rand_like(x)

```

```

    y_tilde=y0_model(x)

```

```

    y=torch.sigmoid(y_tilde)

```

```

    for j in range(1, NT+1):

```

```

        z = z_models[j-1](x)

```

```

        x =x+ y*dt+ dB[:,j].view(-1,1)

```

```

        y_tilde = (y_tilde +(z**2)*(1-2/(1+torch.exp(y_tilde)))/2*dt + z * dB[:,j].view(-1,1))#.clamp(min=-1,max=1)

```

```

        y=torch.sigmoid(y_tilde)

```

```

    loss=torch.mean((y_tilde-target(x))**2)

```

```

    return loss

```

```

def get_target_path_coupled(dB, init_x,NumBM, NT,y0_model, z_models):

```

```

    x_path = torch.ones(NumBM,NT+1)

```

```

    y_path = torch.ones(NumBM,NT+1)

```

```

    x = init_x

```

```

    # y = torch.rand_like(x)

```

```

    y_tilde=y0_model(x)

```

```

    y=torch.sigmoid(y_tilde)

```

```

    x_path[:,0] = x.squeeze()

```

```

    y_path[:,0] = y.squeeze()

```

```

    for j in range(1, NT+1):

```

```

z = z_models[j-1](x)
x += y*dt+ dB[:,j].view(-1,1)
y_tilde = (y_tilde +(z**2)*(1-2/(1+torch.exp(y_tilde)))/2 *dt + z * dB[:,j].view(-1,1)).clamp(min=-1,max=1)
y=torch.sigmoid(y_tilde)
x_path[:,j] = x.squeeze()
y_path[:,j] = y.squeeze()
return x_path.detach(), y_path.detach()

```

```

class plot_results():
    def __init__(self, loss=forward_losses, sigma=sigma, Npaths=100, NumTrain=NumTrain, NT=NT):
        self.loss=loss
        self.x_path, self.y_path=get_target_path_coupled(dB, init_x, y0_model=y0_model_main, z_models=z_models_main, NumBM=NumTrain, NT=NT)
        self.number_of_paths=np.minimum(Npaths, NumTrain)
        self.sigma=sigma

    def FwdLoss(self, log=True):
        plt.figure(figsize=(10,6))
        plt.title("Forward Loss vs Batch", fontsize=18)
        plt.plot(self.loss)

        if log==True:
            plt.yscale('log')

    def results(self, seed=0):
        random.seed(seed)
        idx_list = np.random.choice(NumTrain, self.number_of_paths, replace = False)
        x_plot = self.x_path.detach().numpy()[idx_list]
        y_plot = self.y_path.detach().numpy()[idx_list]
        t = np.array([i for i in range(NT+1)]) * 1/(NT)
        plt.figure(figsize=(20,6))
        plt.subplot(121)
        for i in range(self.number_of_paths):
            plt.plot(t, x_plot[i], color="blue", alpha=0.5)
        plt.title("X")

        plt.subplot(122)
        for i in range(self.number_of_paths):
            plt.plot(t, y_plot[i], color="red", alpha=0.5)
        plt.title("Y Values")

    def Integrated Plots
        random.seed(seed)
        idx=random.randint(0, self.number_of_paths)
        plt.figure(figsize=(10,8))
        plt.subplot()
        plt.plot(t, x_plot[idx], color="blue", alpha=0.5, label='X')
        plt.plot(t, y_plot[idx], color="black", linestyle='--', alpha=0.5, label="Y Values")
        plt.hlines(y=[0,1], xmin=0, xmax=1, colors='firebrick', linestyle='-.')
        plt.title("Comparison of A Particular Path")
        plt.legend()

```

```

def qq_plot(self, sigma=sigma):
    plt.figure()
    plt.title("QQ-Plot")
    x_sigmoid=1/(1+np.exp(self.x_path[:,-1]/sigma))
    plt.scatter(x_sigmoid, self.y_path[:,-1], s=3)
    plt.plot(np.linspace(0,1,5), np.linspace(0,1,5), linestyle='--', linewidth=1, color='r')

```

```

In [ ]: ## Train
torch.autograd.set_detect_anomaly(True)

dB = SampleBMIncr(1, Npaths=NumTrain, Nsteps=NT+1)
init_x = Sample_Init(N=NumTrain)

#Forward Loss
forward_losses = []
#How many batches?
MaxBatch= 1000

#How many optimization steps per batch
OptimSteps= 20

#Set Learning rate
learning_rate = 0.005

#Train on a single batch?
single_batch = True

#Set up main models for y0 and z (z will be list of models)
layer_dim = 10
y0_model_main = Network(lr=learning_rate, input_dims=[1], fc1_dims=layer_dim, fc2_dims=layer_dim,
                        n_outputs=1)
z_models_main = [Network(lr=learning_rate, input_dims=[1], fc1_dims=layer_dim, fc2_dims=layer_dim,
                        n_outputs=1) for i in range(NT)]

#Define optimization parameters
# params = list(y0_model_main.parameters())
params=[]
for i in range(NT):
    params += list(z_models_main[i].parameters())

#Set up optimizer and scheduler
optimizer = optim.AdamW(params, lr=learning_rate, weight_decay=5e-4)
scheduler = torch.optim.lr_scheduler.StepLR(optimizer, step_size=50, gamma=0.995)

for k in range(0, MaxBatch):

    print("Batch Number: ", k+1)
    sloss=0

```

*#optimize main network wrt the foward loss*

```
for l in range(0,OptimSteps):
```

```
    optimizer.zero_grad()
```

```
    loss = get_foward_loss_coupled(dB, init_x,NT=NT,target=target, y0_model=y0_model_main, z_models=z_models_main)
```

```
    # print(loss)
```

```
    loss.backward()
```

```
    # print(params)
```

```
    torch.nn.utils.clip_grad_norm_(parameters=params,max_norm=1)
```

```
    optimizer.step()
```

```
    scheduler.step()
```

```
    nloss = loss.detach().numpy()
```

```
    sloss += nloss
```

```
    # print('OptimStep: ' + str(l+1))
```

```
    # print('forward_loss: ' + str(nloss))
```

```
avgloss = sloss/OptimSteps
```

```
print("Average Error Est: ", avgloss)
```

```
forward_losses.append(avgloss)
```

*#Generate a new batch if using multiple batches*

```
if(not single_batch):
```

```
    dB = SampleBMIncr(1, Npaths=NumTrain, Nsteps=NT+1)
```

```
    init_x = Sample_Init(N=NumTrain)
```

```
plot=plot_results(loss=forward_losses)
```

```
plot.FwdLoss()
```

```
plot.results()
```

```
plot.qq_plot()
```



Batch Number: 1  
Average Error Est: 983.2191223144531  
Batch Number: 2  
Average Error Est: 755.0713348388672  
Batch Number: 3  
Average Error Est: 522.91279296875  
Batch Number: 4  
Average Error Est: 319.2924629211426  
Batch Number: 5  
Average Error Est: 185.80145416259765  
Batch Number: 6  
Average Error Est: 113.15606498718262  
Batch Number: 7  
Average Error Est: 76.98015537261963  
Batch Number: 8  
Average Error Est: 55.06469612121582  
Batch Number: 9  
Average Error Est: 44.8608585357666  
Batch Number: 10  
Average Error Est: 39.19727382659912  
Batch Number: 11  
Average Error Est: 35.60523166656494  
Batch Number: 12  
Average Error Est: 32.81537599563599  
Batch Number: 13  
Average Error Est: 29.285586452484132  
Batch Number: 14  
Average Error Est: 27.92152318954468  
Batch Number: 15  
Average Error Est: 25.673546504974365  
Batch Number: 16  
Average Error Est: 24.553220558166505  
Batch Number: 17  
Average Error Est: 23.59891300201416  
Batch Number: 18  
Average Error Est: 21.4233341217041  
Batch Number: 19  
Average Error Est: 20.129490661621094  
Batch Number: 20  
Average Error Est: 18.930263233184814  
Batch Number: 21  
Average Error Est: 18.27023849487305  
Batch Number: 22  
Average Error Est: 17.203854084014893  
Batch Number: 23  
Average Error Est: 16.397400236129762  
Batch Number: 24  
Average Error Est: 15.459060716629029  
Batch Number: 25  
Average Error Est: 14.701130390167236  
Batch Number: 26

Comparison of A Particular Path

