In [ ]:
```python
import numpy as np
import torch as torch
import torch.nn as nn
import torch.nn.functional as F
import torch.optim as optim
import seaborn as sns
import pandas as pd
import matplotlib.pyplot as plt
from datetime import datetime
import random
from scipy.stats import norm

import os
import pathlib

from Model import *
from utils import *

torch.autograd.set_detect_anomaly(True)
start_time=datetime.now().strftime('%B %d - %H:%M:%S')
```

In [ ]:
```python
#Global parameters 1, 0.75, 0.5, 0.25
GlobalParams1=Params(param_type='k1',target_type='indicator',trick='clamp',loss_type='BCELoss',delta=0.01,w=0.25,lr=0.0001)
dB1 = SampleBMIncr(GlobalParams=GlobalParams1)
init_x1 =  Sample_Init(GlobalParams=GlobalParams1)
init_c1= torch.zeros_like(init_x1)
GlobalParams2=Params(param_type='k2',target_type='indicator',trick='clamp',loss_type='BCELoss',delta=0.01,w=0.25,lr=0.0001)
dB2 = SampleBMIncr(GlobalParams=GlobalParams2)   ## TODO: same dB?????
init_x2 =  Sample_Init(GlobalParams=GlobalParams2)
init_c2= torch.zeros_like(init_x2)

NT1=GlobalParams1.NT1
NT2=GlobalParams1.NT2
dt=GlobalParams1.dt
device=GlobalParams1.device
learning_rate = GlobalParams1.lr

#Forward Loss
forward_losses = []

#How many batches
MaxBatch= 500

#How many optimization steps per batch
OptimSteps= 25

#Train on a single batch?
single_batch = True
```

```
#Set up main models for y0, yt1, and z (z will be list of models)
y0_model_main1 = Network(scaler_type='sigmoid')
yt1_model_main1 = Network(scaler_type='sigmoid')
zy_models_main1 = [Network() for i in range(NT2)]
main_models1=Main_Models(GlobalParams=GlobalParams1)
main_models1.create(y0_model=y0_model_main1,
                    yt1_model=yt1_model_main1,
                    zy_models=zy_models_main1,
                    forward_loss=forward_losses,
                    dB=dB1,
                    init_x=init_x1,
                    init_c=init_c1)

y0_model_main2 = Network(scaler_type='sigmoid')
yt1_model_main2 = Network(scaler_type='sigmoid')
zy_models_main2 = [Network() for i in range(NT2)]
main_models2=Main_Models(GlobalParams=GlobalParams2)
main_models2.create(y0_model=y0_model_main2,
                    yt1_model=yt1_model_main2,
                    zy_models=zy_models_main2,
                    forward_loss=forward_losses,
                    dB=dB2,
                    init_x=init_x2,
                    init_c=init_c2)

pop1_dict={'dB':dB1,
           'init_x':init_x1 ,
           'init_c':init_c1 ,
           'GlobalParams':GlobalParams1,
           'main_models':main_models1}

pop2_dict={'dB':dB2,
           'init_x':init_x2 ,
           'init_c':init_c2 ,
           'GlobalParams':GlobalParams2,
           'main_models':main_models2}
```

In [ ]:

```
#Define optimization parameters
params=[]
params = list(main_models1.y0_model.parameters())+\
         list(main_models1.yt1_model.parameters())+\
         list(main_models2.y0_model.parameters())+\
         list(main_models2.yt1_model.parameters())

for i in range(NT2):
    params += list(main_models1.zy_models[i].parameters())
    params += list(main_models2.zy_models[i].parameters())

#Set up optimizer and scheduler
optimizer = optim.Adamax(params, lr=learning_rate)
scheduler = torch.optim.lr_scheduler.StepLR(optimizer, step_size=100, gamma=0.95)
```

```python
for k in range(0,MaxBatch):

    print("Batch Number: ", k+1)
    sloss=0
    #optimize main network wrt the foward loss
    for l in range(0,OptimSteps):
        optimizer.zero_grad()
        loss = get_foward_loss(pop1_dict=pop1_dict, pop2_dict=pop2_dict)
        loss.backward()
        # torch.nn.utils.clip_grad_norm_(parameters=params,max_norm=0.7)
        optimizer.step()
        scheduler.step()
        nloss = loss.detach().numpy()
        sloss += nloss

    avgloss = sloss/OptimSteps
    print("Average Error Est: ", avgloss)
    forward_losses.append(avgloss)

    #Generate a new batch if using multiple batches
    if(not single_batch):

        dB1 = SampleBMIncr(GlobalParams=GlobalParams1)
        init_x1 =  Sample_Init(GlobalParams=GlobalParams1)
        init_c1= torch.zeros_like(init_x1)
        pop1_dict={'dB':dB1,
                'init_x':init_x1 ,
                'init_c':init_c1 ,
                'GlobalParams':GlobalParams1,
                'main_models':main_models1}

        dB2 = SampleBMIncr(GlobalParams=GlobalParams2)   ## TODO: same dB?????
        init_x2 =  Sample_Init(GlobalParams=GlobalParams2)
        init_c2= torch.zeros_like(init_x2)
        pop2_dict={'dB':dB2,
                'init_x':init_x2 ,
                'init_c':init_c2 ,
                'GlobalParams':GlobalParams2,
                'main_models':main_models2}
end_time=datetime.now().strftime('%B %d - %H:%M:%S')
```

In [ ]:

```python
plot=plot_results(pop1_dict=pop1_dict, pop2_dict=pop2_dict, loss=forward_losses)
plot.FwdLoss(log=True)
plot.Inventory_And_Price()
plot.Decomposition_Inventory()
plot.Key_Processes()
plot.Terminal_Convergence()
```

# Save The Models

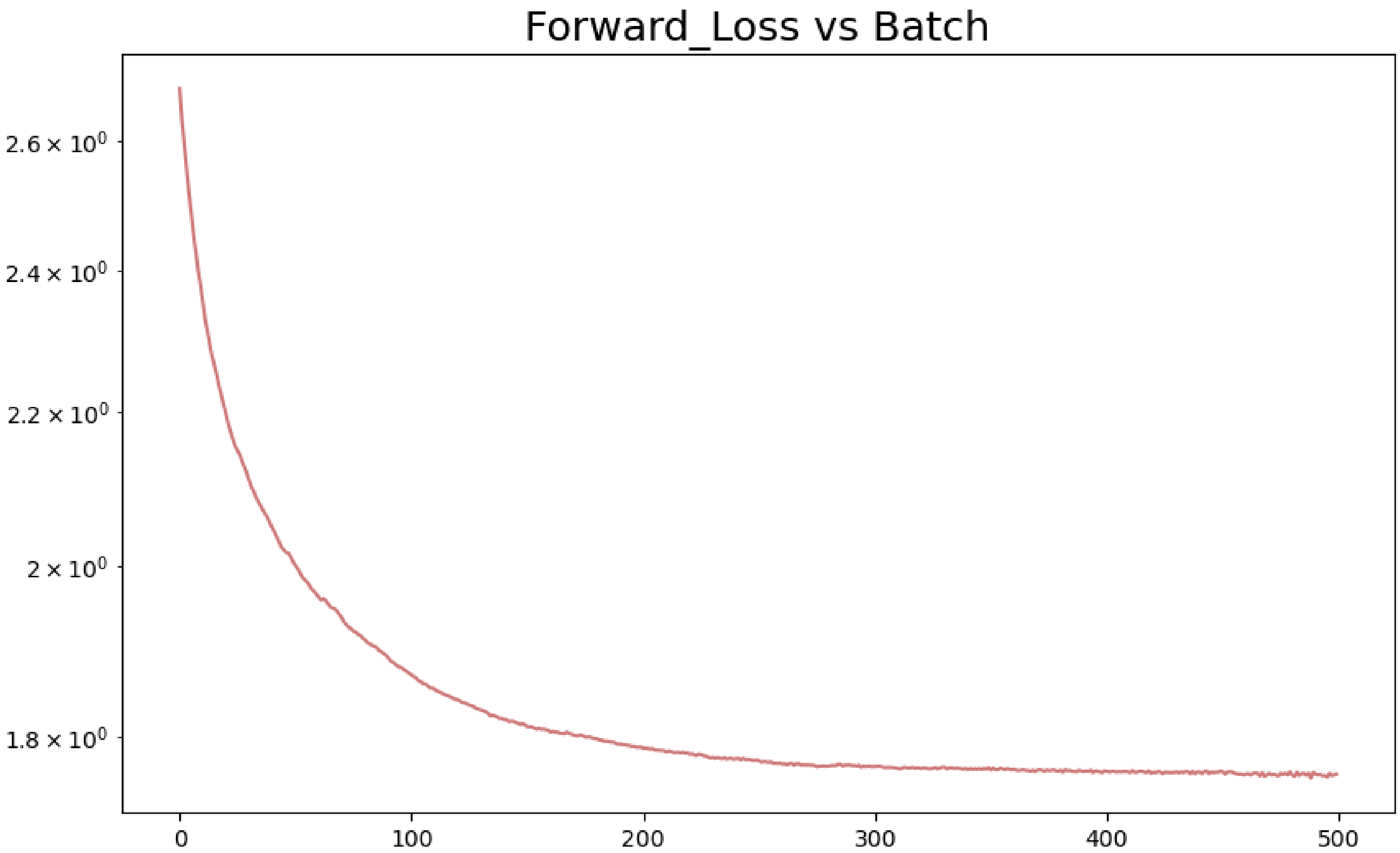
In [ ]:
```
print(f"{len(main_models1.loss)} steps\nStarted @ {start_time}\nSaved @ {end_time}")   ## to examine whether the loss attribute is updated in t
dir_path=pathlib.Path(os.getcwd(),
                      'Results',
                      'Best Models Saved',
                      'sigmoid_ind_0.0001lr_500steps_BCE_0.25w')  ## 0.25, 0.5,  0.75,  1.0
dir_path.mkdir()
path1=pathlib.Path(dir_path,'pop1.pt')
path2=pathlib.Path(dir_path,'pop2.pt')
main_models1.save_entire_models(path=path1)
main_models2.save_entire_models(path=path2)
```

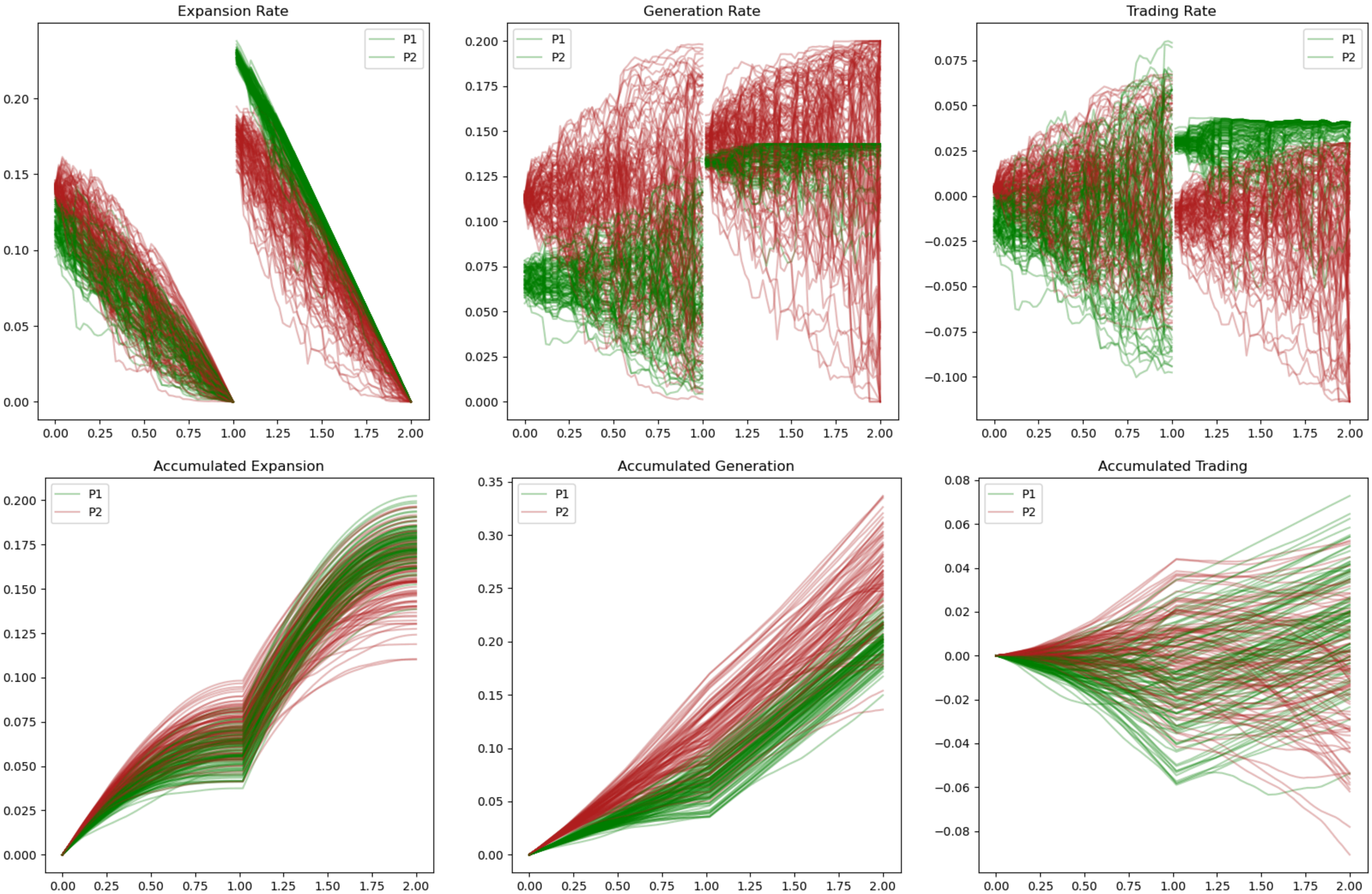
```
500 steps
Started @ September 01 - 13:04:57
Saved @ September 02 - 07:48:19
```

## Load The Models

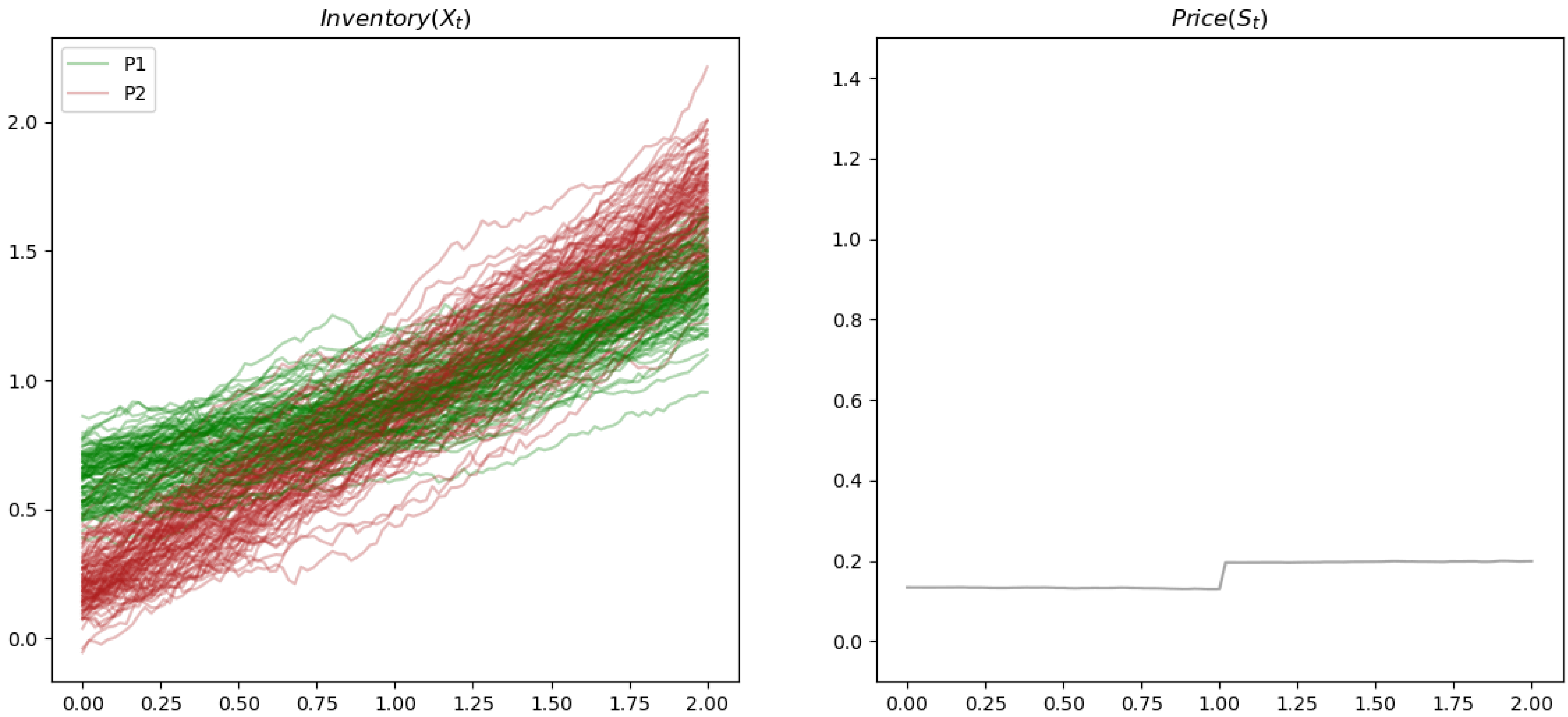
In [ ]:
```
# GlobalParams1=Params(param_type='k1',target_type='indicator',trick='clamp',loss_type='MSELoss',delta=0.03,K=0.9,lr=0.01)
# GlobalParams2=Params(param_type='k2',target_type='indicator',trick='clamp',loss_type='MSELoss',delta=0.03,K=0.9,lr=0.01)
models1=Main_Models(GlobalParams=GlobalParams1)
models2=Main_Models(GlobalParams=GlobalParams2)
path_dir=pathlib.Path(os.getcwd(),
                      "Results",
                      "Best Models Saved",
                      'sigmoid_ind_0.0001lr_500steps_BCE_0.25w')
path1=pathlib.Path(path_dir,'pop1.pt')
path2=pathlib.Path(path_dir,'pop2.pt')
model_dict1=models1.load_entire_models(path=path1,overwrite=True)
model_dict2=models2.load_entire_models(path=path2,overwrite=True)

dB1=model_dict1['dB']
init_x1=model_dict1['init_x']
init_c1=model_dict1['init_c']
pop1_dict= {'dB':dB1,
            'init_x':init_x1,
            'init_c':init_c1,
            'GlobalParams':GlobalParams1,
            'main_models':models1}

dB2=model_dict2['dB']
init_x2=model_dict2['init_x']
init_c2=model_dict2['init_c']

pop2_dict= {'dB':dB2,
            'init_x':init_x2 ,
            'init_c':init_c2 ,
            'GlobalParams':GlobalParams2,
            'main_models':models2}
dt=GlobalParams1.dt
```

```
NT1=GlobalParams1.NT1
NT2=GlobalParams1.NT2
NumTrain=GlobalParams1.NumTrain
K=GlobalParams1.K
loss=models1.loss

plot=plot_results(pop1_dict=pop1_dict, pop2_dict=pop2_dict, loss=loss)
plot.FwdLoss(log=True)
plot.Decomposition_Inventory()
plot.Inventory_And_Price()
plot.Terminal_Convergence()
plot.Key_Processes()
```

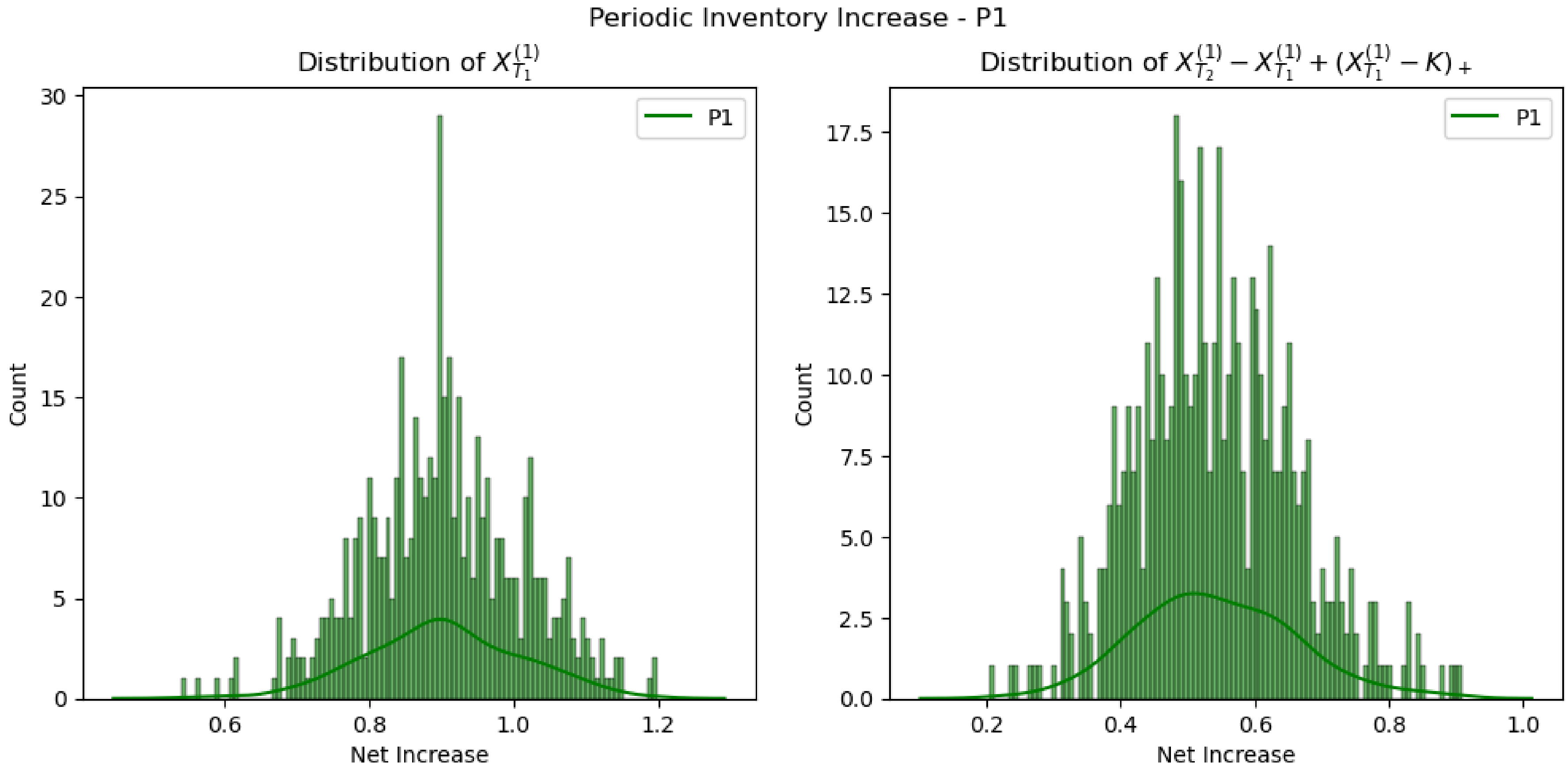




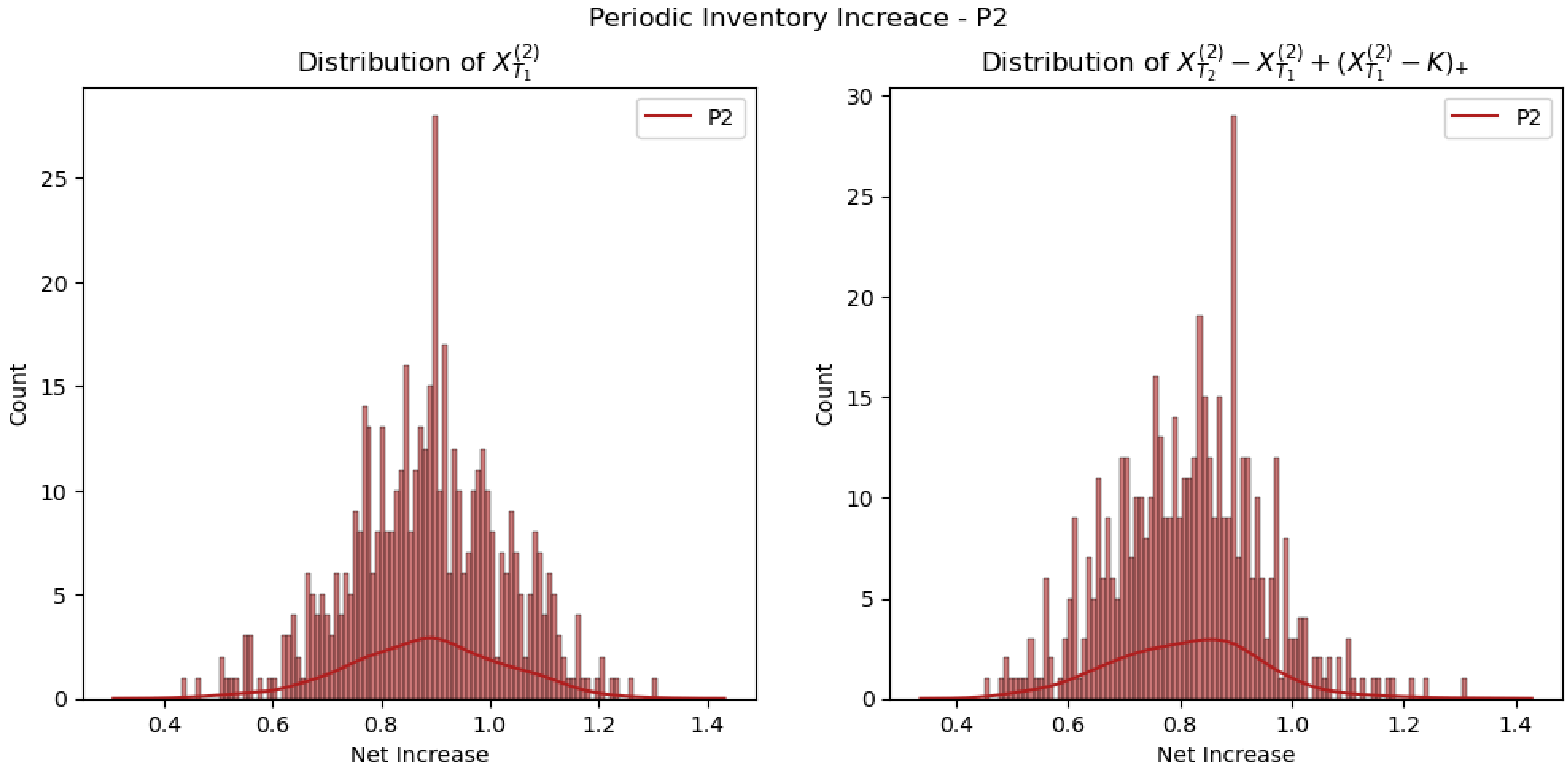
Expansion Rate
Generation Rate
Trading Rate
Accumulated Expansion
Accumulated Generation
Accumulated Trading
P1
P2

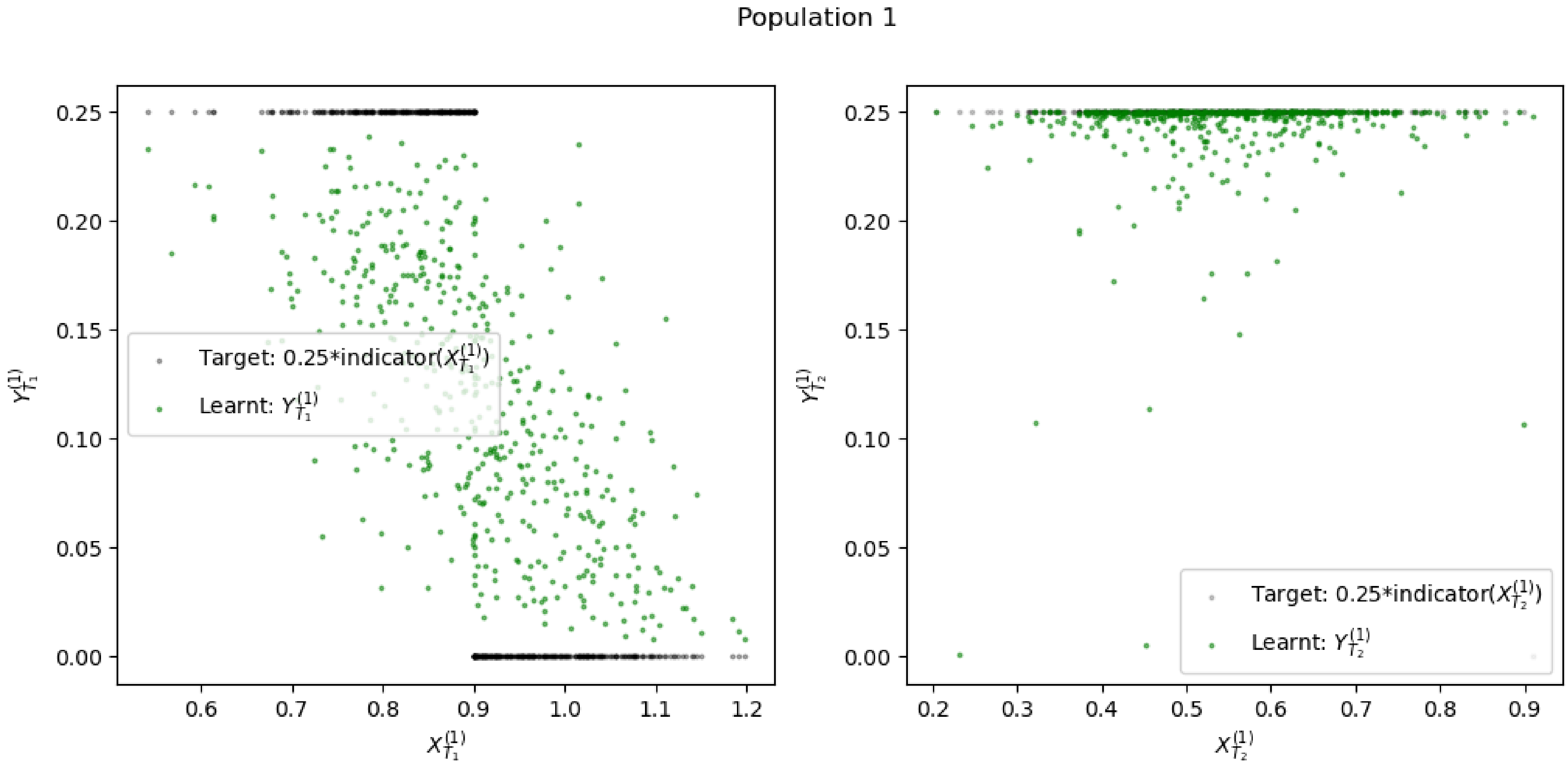


$Inventory(X_t)$
P1
P2
$Price(S_t)$

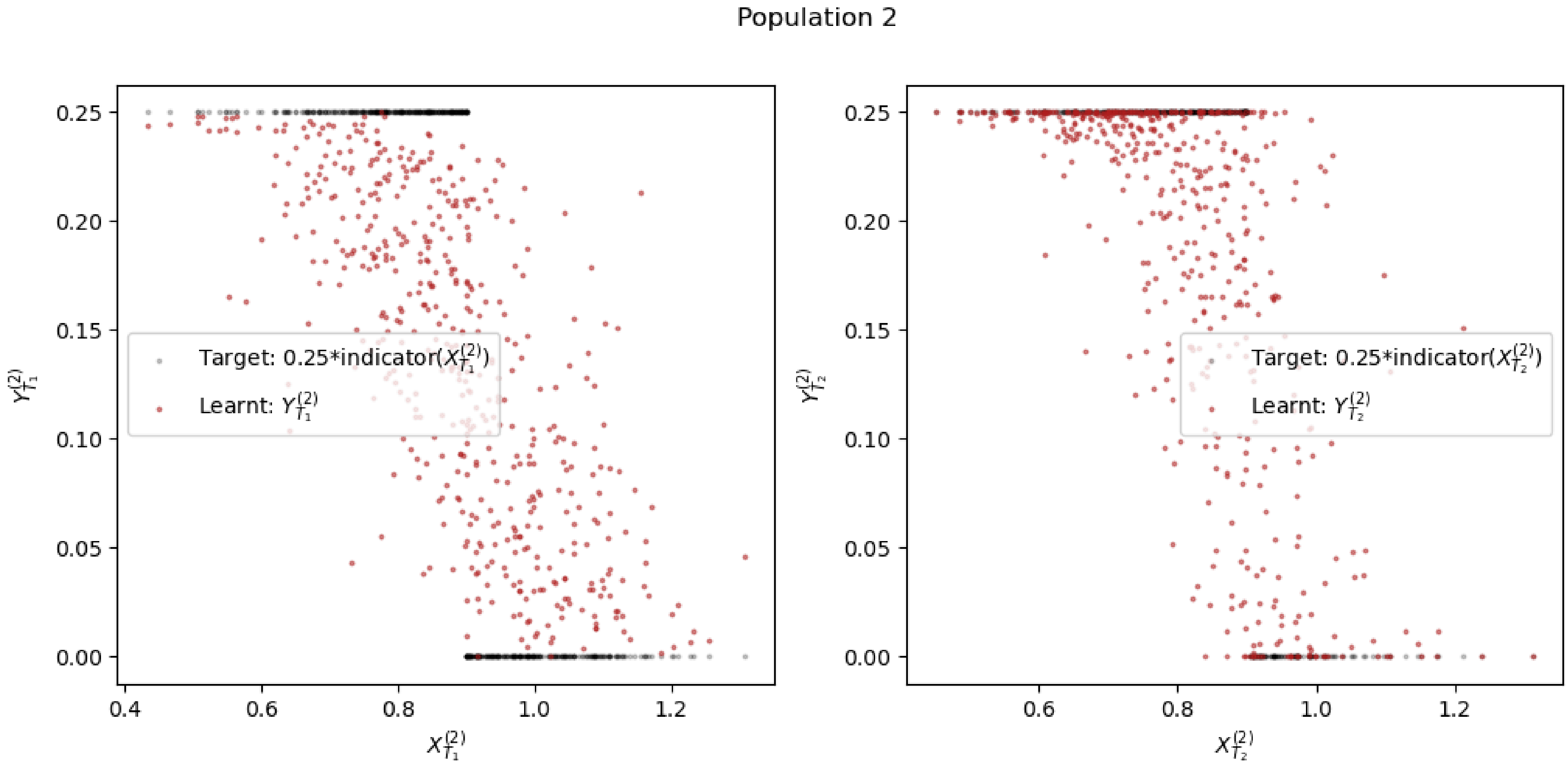


Periodic Inventory Increase - P1
Distribution of $X^{(1)}_{T_1}$
Distribution of $X^{(1)}_{T_2} - X^{(1)}_{T_1} + (X^{(1)}_{T_1} - K)_+$
Count
Net Increase
P1
0.6
0.8
1.0
1.2
0
5
10
15
20
25
30
0.2
0.4
0.6
0.8
1.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

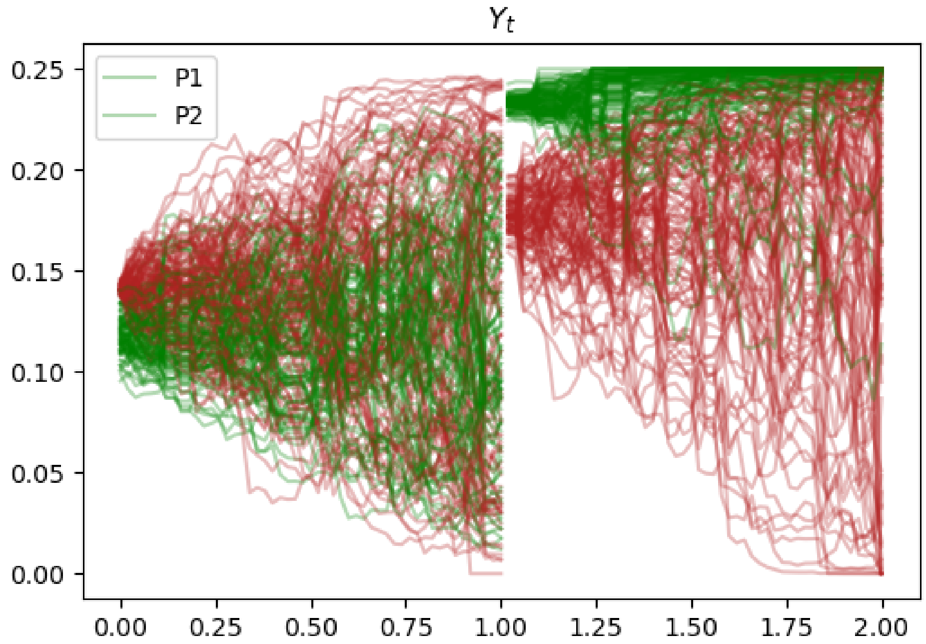

Periodic Inventory Increace - P2
Distribution of $X^{(2)}_{T_1}$
Distribution of $X^{(2)}_{T_2} - X^{(2)}_{T_1} + (X^{(2)}_{T_1} - K)_+$
P2
Count
Net Increase



Population 1
Target: 0.25*indicator($X_{T_1}^{(1)}$)
Learnt: $Y_{T_1}^{(1)}$
$Y_{T_1}^{(1)}$
$X_{T_1}^{(1)}$
0.25
0.20
0.15
0.10
0.05
0.00
0.6
0.7
0.8
0.9
1.0
1.1
1.2
Target: 0.25*indicator($X_{T_2}^{(1)}$)
Learnt: $Y_{T_2}^{(1)}$
$Y_{T_2}^{(1)}$
$X_{T_2}^{(1)}$
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9



Population 2
Target: 0.25*indicator($X^{(2)}_{T_1}$)
Learnt: $Y^{(2)}_{T_1}$
$Y^{(2)}_{T_1}$
$X^{(2)}_{T_1}$
0.25
0.20
0.15
0.10
0.05
0.00
0.4
0.6
0.8
1.0
1.2
Target: 0.25*indicator($X^{(2)}_{T_2}$)
Learnt: $Y^{(2)}_{T_2}$
$Y^{(2)}_{T_2}$
$X^{(2)}_{T_2}$


$Y_t$
P1
P2
0.25
0.20
0.15
0.10
0.05
0.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00